Teaching geometry in the elementary classroom

Virginia Lee Copper Rogers

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TEACHING GEOMETRY IN THE
ELEMENTARY CLASSROOM

A Project
Presented to the
Faculty of
California State University,
San Bernardino

In Partial Fulfillment
of the Requirements for the Degree
Masters of Arts
in
Education: Elementary Education

by
Virginia Lee Copper Rogers
June 1995
TEACHING GEOMETRY IN THE ELEMENTARY CLASSROOM

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ABSTRACT

Geometry has always been an important part of mathematics. The geometry content appropriate to the elementary classroom is informal in organization, and should be presented in a way that allows students to construct meaning. Classroom activities should encourage students to observe, comment on, compare observations, make hypotheses, and then prove or disprove their hypotheses.

However, much of the geometry presented in elementary classrooms today, if presented at all, focuses mainly on identification of basic shapes. Geometric problem solving, connections to other areas of mathematics, and exploration of geometric concepts and principles, receive scant attention.

Mathematics educators are working for major reforms. The 1989 *Curriculum and Evaluation Standards for School Mathematics* state that "...the mathematics curriculum should include two- and three-dimensional geometry so that students can:

1) describe, model, draw, and classify shapes;

2) investigate and predict the results of
combining, subdividing, and changing shapes;
3) develop spatial sense;
4) relate geometric ideas to number and measurement ideas;
5) recognize and appreciate geometry in their world" (p.48).

Evidence also suggests that the development of geometric thinking progresses through a hierarchy of levels. Curriculum development and instruction must take this into account as more complex concepts and strategies require a firm foundation of basic skills.

In order to enable students to meet the Standards' goals, teachers must change their methods of instruction. A constructivist approach based on how children think and learn, will help structure classroom activities to accomplish these goals. Students need to explore geometric concepts, share their thinking, and develop more complex, abstract, and powerful mathematical structures for meaningful problem solving.

This project includes theoretical background, resources, and activities to enable teachers to meet these challenges in the elementary classroom. Also
included are additional lessons which help integrate geometry into other subject areas, so that students will develop a holistic view of mathematics.
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SECTION 1: INTRODUCTION

Traditionally, the elementary mathematics curriculum has been "...preoccupied with computation and other traditional skills" (National Council of Teachers of Mathematics [NCTM], 1989, p. 15). The study of geometry and development of spatial sense often receives superficial, if any, attention (Burger, 1990). Identification of basic shapes is repeated at every grade level, however, based on tests, many students appear to have an incomplete knowledge of geometric shapes (Rowan, 1990). The geometry knowledge of elementary students has been minimal.

For example, on the 1982 National Assessment of Educational Progress, fewer than 10% of 13-year-olds could find the measure of the third angle of a triangle given the measures of the other two angles. Only 20% of 13-year-olds were able to obtain the length of the hypotenuse given the lengths of the other two sides (Usiskin, 1989). These test results indicate that the Pythagorean theorem is taught to more students in this age group than the triangle-sum theorem, which illustrates the curriculum-performance connection. "In
grades K-8, there is no consistent agreement regarding the content, sequence, or timing of geometry to be taught" (Usiskin, 1989, p. 18). A sequential geometry curriculum, with an emphasis on concept development, and connections with the broader spectrum of mathematics, appears to be lacking (Burger, 1988).

Why study geometry? "Geometry organizes and clarifies our visual experiences and provides visual models of mathematical concepts" (Burger, 1988, p. 2). An understanding of geometry gives people more appreciation for the world they live in. The structure of the solar system, geological formations, plants, animals, art, architecture, machines, and virtually everything humans create, have some element of geometric form. Many professionals such as architects, engineers, and land developers use geometry daily. Geometry is also connected to the study of other mathematics, such as fractions, ratio and proportion, and measurement.

Spatial reasoning, an integral part of geometry, is an important form of problem solving. "A primary goal for the study of mathematics is to give children
experiences that promote the ability to solve problems" (NCTM, 1993, p. v). The Curriculum and Evaluation Standards for School Mathematics states:

"...[Children's] spatial capabilities frequently exceed their numerical skills, and tapping these strengths can foster an interest in mathematics and improve number understandings and skills" (NCTM, 1989, p. 48).

Finally, and perhaps most importantly, the study of geometry is enjoyable and motivating for the students (Van De Walle, 1990).

Recent reform efforts resulted in a set of standards for teaching mathematics by the National Council of Teachers of Mathematics (NCTM, 1989). The Curriculum and Evaluation Standards for School Mathematics has five goals. Students will:

1) learn to value mathematics
2) be confident in their ability to do mathematics
3) become mathematical problem solvers
4) communicate mathematically
5) reason mathematically (NCTM, 1989)

Changes in teaching practices are called for if we are to help students accomplish these goals in all strands of mathematics, including geometry (NCTM, 1989).

Teachers are urged to structure classroom
activities which encourage students' development of spatial relationships and explorations of geometric concepts and language (NCTM, 1993). In the early grades, geometry should be informal and experiential (Mathematics Framework, 1992). The goal is to "develop fluency with basic geometrical objects and relationships and to connect that fluency with spatial reasoning and visualization skills" (Mathematics Framework, 1992, p. 84). As students improve problem solving and relate geometry to other areas of mathematics, their development of underlying concepts grows out of their own continuing experiences. These experiences need to be structured to be developmentally appropriate and lead to an increasingly sophisticated knowledge and mastery of geometry. "Geometry should be a vehicle that challenges elementary and middle school students to observe, describe, analyze, and test hypotheses regarding shapes and forms in the world around them" (Campbell and Fey, 1988, p. 61).

While most teachers now agree that geometry is important to acquire, there is much discussion as to the how, what, and when it should be acquired (Rowan, 1990).
The geometry curriculum "must capitalize on geometry, not as a means of examining fixed line drawings for predetermined factual characteristics, but as a time to create figural models, to investigate properties, and search for essential characteristics" (Campbell and Fey, 1988, p. 61).

Research by Dutch educators Dina and Pierre van Hiele has brought specific recommendations as to the sequence of learning and teaching geometry. Students must progress through the van Hiele's model of five levels of geometric thought sequentially, acquiring specific strategies at each level (Crowley, 1989). The van Hieles believe that a student's progress through the levels depends on the type of instruction he/she has received rather than his/her age or maturation. Therefore, the method and organization of instruction, and the content and materials used, are important areas of concern (Crowley, 1989).

The question then is how to structure a sequential geometry curriculum that spirals concepts through the grade levels, rather than just repeating the same basic activities? How can geometry be taught so it is relevant
and useful to students? How do teachers provide a setting in which they "... have opportunities to explore, invent, and discuss, ... and attain a firm foundation in geometry?" (Rowan, 1990, p. 25).
SECTION 2: REVIEW OF RELATED LITERATURE

Most of the geometry taught in the elementary grades is informal geometry (Van de Walle, 1990). Informal geometry is experiential. It provides students with opportunities to explore their environment by observing, constructing relationships, and solving problems in a geometric context (Van de Walle, 1990). This literature review will focus on research about structuring such an environment, and the development of children's geometric thinking.

The NCTM Curriculum and Evaluation Standards for School Mathematics states "Evidence suggests that the development of geometric ideas progresses through a hierarchy of levels" (NCTM, 1989, p. 48). Curriculum development and instruction must consider this hierarchy as the learning of complex concepts and strategies requires a firm foundation of basic skills (NCTM, 1989).

The work of Dutch educators Pierre and Dina van Hiele has supported such a hierarchy. Their research on how children develop geometric thinking is impacting geometry instruction and curriculum in the United States
(Van de Walle, 1990). According to Pierre and Dina van Hiele's model of geometric thought, there are five levels of understanding. These levels describe characteristics of the thinking process. The learner must move through them sequentially, developing specific strategies and abilities (Van de Walle, 1990). In brief, these levels are:

**Level 0: Visualization.** Students can recognize and name figures in a gestalt manner. Students do not yet understand properties as defining characteristics. They are influenced by nonrelevant attributes such as orientation. For example, some students would not recognize a triangle that "pointed down," or that a square is also a rectangle.

**Level 1: Analysis.** Students begin to analyze various properties of shapes, and can make generalizations about classes of shapes. The same set of triangles can be sorted by the relative size of the angles or the relative lengths of the sides.

**Level 2: Informal Deduction.** Construction of relationships between classes of figures is possible. For example, rectangles are parallelograms with a right
angle. Since squares meet these criteria, all squares must be rectangles and parallelograms. At this stage formal proofs can be followed, but probably not constructed.

**Level 3: Deduction.** The significance of deduction as a way of establishing geometric theory within an axiomatic system is understood. Students can see that two different logical arguments could each be valid for the same theorem.

**Level 4: Rigor.** Students can work in a variety of axiomatic systems, and different systems can be compared. This is the level of the mathematician who is studying geometry as a mathematical science. Geometry is seen in the abstract (Van de Walle, 1990). This level is the least developed by the van Hieles.

The van Hieles also identify generalities which characterize the model. These properties help provide guidance for making instructional decisions.

1) **Sequential.** The learner must proceed through the levels in order. To be successful at each level, the learner must have acquired the strategies of preceding levels.
2) **Advancement.** Progress, or lack of it, through the levels is more dependent on content and methods of instruction than age. Some methods will enhance movement, some prevent it, but none can allow the learner to skip a level.

3) **Intrinsic and Extrinsic.** Objects inherent at one level become the objects of study at the next level.

4) **Linguistics.** Crowley quotes Pierre van Hiele as stating, "Each level has its own linguistic symbols and its own system of relations connecting these symbols" (Crowley, 1989, p. 4). A relation that is correct at one level may be modified at another level.

5) **Mismatch.** If the learner is at one level and instruction is at another level, the desired learning and progress may not occur. The learner will not be able to follow the thought processes being used (Crowley, 1989).

While the van Hieles assert that progress through the levels is not dependent on age or maturation, age does play a part in the amount and type of geometric experiences children have received. Generally, students from kindergarten through second grade are at Level 0
and most upper elementary students are at Level 1 (Van de Walle, 1990).

In the van Hiele model, method and organization of instruction, and content and materials used, are critical factors in students' progression through the levels (Crowley, 1989). Therefore, the van Hieles propose five sequential phases of learning which promote acquisition of a level:

Phase 1: **Inquiry/Information.** Teacher and students engage in conversation and activities about the objects of study for that level. Students' prior knowledge is revealed, and they know what direction further study will take.

Phase 2: **Directed Orientation.** Students explore the topic through materials carefully sequenced by the teacher. The activities should gradually reveal the structures characteristic of that level.

Phase 3: **Explication.** Building on previous experiences, students express and exchange views about observed structures. The level's system of relations becomes apparent. The teacher's role is minimal, other than to assist students in using accurate and appropriate
language.

Phase 4: Free Orientation. Students encounter more complex tasks which are multi-dimensional, and open-ended. They gain experience in finding their own solutions, and solving tasks.

Phase 5: Integration. Students review, summarize, and synthesize what they’ve learned. Summaries should not present anything new.

At completion of Phase 5, "...students have attained a new level of thought. The new domain of thinking replaces the old, and students are ready to repeat the phases of learning at the next level" (Crowley, 1989, p. 6).

Implicit to the van Hiele's model is the understanding that students should have a wide variety of exploratory geometric experiences (Crowley, 1989). Classroom teachers and researchers need to develop and implement appropriate materials and philosophies in the classroom. "Geometric thinking can be accessible to everyone" (Crowley, 1989, p. 15).

One philosophy of teaching which may be able to carry out the goals of the van Hieles' model is
constructivism. Constructivists believe that all knowledge is a product of our own cognitive acts; that we construct our understanding through our own experiences (Confrey, 1990). It assumes that all human beings are knowing subjects, their behavior is purposive, and they are adept at organizing information (Noddings, 1990).

Piaget provided insight into the constructivist perspective of teaching by demonstrating that childrens' ideas "possess a different form of argument, are built from different materials, and are based on different experiences" (Confrey, 1990, p. 109). Children will not change their ideas simply because someone furnishes them with the "correct" answer. They must be persuaded that their ideas are not effective or an alternative is preferable (Confrey, 1990).

When applying constructivism to teaching, one must reject the notion that information can be passed on and lead to understanding. The teacher must form a "model of the students' ways of viewing an idea and... assist the student in restructuring those views to be more adequate" from both student and teacher perspectives.
Two goals for mathematics instruction can be implied from the constructivist perspective. Students will:

1) develop more complex, abstract, and powerful mathematical structures for meaningful problem solving, and;

2) become autonomous and self-motivated in their mathematical activity (Kamii & Lewis, 1990).

Studies have found that students receiving constructivist teaching performed as well on standard computations as those receiving traditional teaching. They also solved problems differently, and developed a higher level of reasoning (Cobb, 1991).

Constructivism emphasizes the role of the constructive process, one's awareness of such constructions, and the ability to modify them through conscious reflection on the constructive process. It is this very act of reflection that helps to access the constructive process (Confrey, 1990).

The reflective process is essential to mathematics because "mathematics is not built from sensory data but
from human activity (mathematics is a language of human action): counting, folding, ordering, comparing, etc." (Confrey, 1990, p. 109). To create such a language, one must reflect on the activity, carry it out in the imagination, and name and represent it in symbols and images.

From a constructivist perspective, students are always constructing an understanding for their experiences. These constructions may be weak, lacking both internal consistency and explaining a limited range of phenomena. Mathematics educators must foster students' development of more powerful and effective constructions. Thought must be given to what such constructions might be, and how their promotion can be achieved (Confrey, 1990).

According to Carpenter, constructivist teachers need to follow four guidelines to be successful:

1) Problem solving is the organizing focus of all instruction.

2) Instruction should be organized so that students can actively form their own knowledge and understanding.
3) Students should relate the problem, concept, or skills, to the previous knowledge they already have.

4) The emphasis should be on learning how the learner solves problems and using that information to guide further instruction (Carpenter et al., 1989).

This constructivist view of mathematics is shared by the National Council of Teachers of Mathematics (NCTM). The Curriculum and Evaluation Standards (NCTM, 1989) state that students should be involved both mentally and physically in constructing their own mathematical knowledge. "...the curriculum should actively involve children in doing mathematics...(They should) explore, justify, represent, solve, construct, discuss, use, investigate, describe, develop, and predict" (NCTM, 1989, p. 17). Students should discover relationships by "constructing, drawing, measuring, visualizing, comparing, transforming, and classifying" (NCTM, 1989, p. 112).

The ability to visualize figures and mentally manipulate them is a basic skill in mathematics (Morrow,
1991). NCTM's *Curriculum Standards* state "Spatial understandings are necessary for interpreting, understanding, and appreciating our inherently geometric world" (1989, p. 48). In 1989, the NCTM Commission on Standards for School Mathematics defined spatial sense as, in part, "an intuitive feel for one's surroundings and objects in them" (Del Grande, 1990, p. 14).

Through their eyes, children take in patterns, shapes, location and movement of objects. As the brain processes such visual inputs, it also processes sound, touch, smell, body position, and past experiences (Del Grande, 1990). "Children's perception of their surroundings and objects in their environment incorporate all these sensory inputs to help them discover what the outer world and people are really like" (Del Grande, 1990, p. 14).

Researchers suggest that spatial perception does not consist of a single skill or ability, but rather several skills or abilities. Seven spatial abilities were selected by Del Grande as being especially relevant to the study of mathematics and geometry in particular. 1) **Eye-motor coordination.** This is the ability to
coordinate vision with the movement of the body. Students weak in this skill concentrate so much on simple motor skills, chances are they will not understand geometric ideas or concepts at the abstract level.

2) **Figure-ground perception.** This is the visual act of identifying a specific component in a situation and involves shifts in perception of figures against complex backgrounds where intersecting and "hidden" forms are used.

3) **Perceptual constancy.** This involves the recognition of geometric figures presented in a variety of sizes, shadings, textures, and positions in space and their discrimination from similar geometric figures.

4) **Position-in-space perception.** This is the ability to relate an object in space to oneself. Students with difficulty in this skill are likely to experience reversals in reading and corresponding difficulties in writing and doing arithmetic. This ability helps students to identify congruent figures, and is essential to many geometry activities.

5) **Perception of spatial relationships.** This is the
ability to see two or more objects in relation to oneself or in relation to each other.

6) Visual discrimination. This is the ability to identify the similarities and differences between or among objects. Comparing pairs of figures is an organizational strategy in problem solving that can be taught.

7) Visual memory. This last skill is the ability to recall accurately objects no longer in view and relate their characteristics to other objects either in view or not in view. A person with exceptional visual memory may have "photographic memory." To store large amounts of information for a longer period of time, they must be stored in long-term memory through abstraction and symbolic thinking (Del Grande, 1990).

Through an understanding of the relationship between spatial abilities and the development of geometric concepts, teachers will be able to identify how children learn geometry and diagnose difficulties in the learning process. An appropriate geometry for young students should develop from intuition and experimental activities involving the motion of objects in space (Del
Geometry has also been difficult for many students due to an "emphasis on the deductive aspects of the subject and a neglect of underlying spatial abilities, acquired by hands-on activities, that are necessary prerequisites for understanding and mastery of geometrical concepts" (Del Grande, 1990, p. 19).

Informal geometry can easily be taught at the elementary level, resulting in an improvement of students' spatial perception. "Since improvement in spatial abilities and the learning of geometry are interdependent,... an improvement in one leads to an improvement in the other" (Del Grande, 1990, p. 19).

The 1992 California Mathematics Framework recommends that in the strand of geometry, "...students are exposed to and investigate two-dimensional and three-dimensional space by exploring shape, area, and volume; studying lines, angles, points, and surfaces; and engaging in other visual and concrete experiences" (California Department of Education, 1992, p. 83). Older students build on this foundation of hands-on experiences, becoming familiar with properties of
geometric figures, and using them to solve problems. They also explore symmetry and proportion, and begin relating geometry to other areas of mathematics. "The goal is to develop fluency with basic geometrical objects and relationships and to connect that fluency with spatial reasoning and visualization skills" (California Department of Education, 1992, p. 84).

To accomplish this goal, the NCTM Standard for Geometry and Spatial Sense states that the mathematics curriculum for elementary students should include two and three-dimensional geometry so students can:

1) Describe, model, draw, and classify shapes.

2) Investigate and predict the results of combining, subdividing, and changing shapes.

3) Develop spatial sense.

4) Relate geometric ideas to number and measurement ideas.

5) Recognize and appreciate geometry in their world (California Department of Education, 1992, p. 106).
SECTION 3: STATEMENT OF GOALS AND OBJECTIVES

The purpose of this endeavor is to provide a manual for teaching geometry in grades kindergarten through five. The manual will include:

1) a synthesis of research on teaching geometry, including current practices and recommendations for the future;

2) lessons that meet the criteria of the van Hiele model of development of geometric thinking, and meet the criteria of the 1992 California Mathematics Framework, and;

3) additional lessons and suggestions for integrating geometry with other subject areas.

It is the hope of the author that this manual will provide a starting point for elementary grade teachers in redesigning their geometry curriculum to reflect pertinent research, and the criteria of the State Mathematics Framework and National Council of Teachers of Mathematics, and so better meet the needs of their students.
SECTION 4: LIMITATIONS

This manual is limited in its application to elementary teachers kindergarten through fifth grade. It is also intended for use in a small district set in a suburban, middle-class community. Therefore, the manual may not meet the requirements of larger districts, or schools with different demographics.

While the lessons themselves are not new to the classroom, the sequence of presentation may be different and subject to disagreement. The selection of lessons was based on the author's understanding and interpretation of the subject matter. Alterations may need to be made in the scope and sequence, and number of lessons, after the manual is used at the different grade levels.

Research showing the relationship between elementary student success and this type of geometry curriculum is also limited.
SECTION 5: IMPLICATIONS FOR TEACHING

This handbook is designed for elementary teachers in California, as it reflects the guidelines set forth in the Framework for California Public Schools in mathematics. Teachers from other states may find this project helpful, but would need to consult their own state curriculum guidelines before implementing this handbook.

The handbook’s activities are meant to be used by teachers to supplement their geometry curriculum. Many of the activities can be used as assessment tools prior to, during, and at the conclusion of, a geometry unit. The activities within each van Hiele Level are not themselves in any particular order. Teachers are encouraged to select those activities which best suit the needs of their students.

Though only activities from Levels 0 and 1 have been included, teachers should be aware of any students who appear ready for the challenge of Level 2 activities, and utilize resources in this project to provide them.
While research on the validity and reliability of utilizing the van Hiele model at the elementary level is sorely lacking, there does exist the Van Hiele Geometry Test for secondary students. This test was developed in order to test the van Hiele theory. This author suggests that the development of a similar test for elementary students is needed, along with reliable research on the success of implementing a van Hiele-based geometry curriculum at the elementary school level.
SECTION 6: CURRICULUM HANDBOOK

VAN Hiele-Based Activities

Level 0: Visualization

Geometric shapes are recognized on the basis of their physical appearance as a whole. Students should be provided with opportunities to:

1) manipulate, color, fold, and construct geometric shapes.

2) identify a shape or geometric relation -
   a) in a simple drawing
   b) in a set of cutouts, pattern blocks, or other manipulatives (i.e. sorting)
   c) in a variety of orientations
   d) involving physical objects in the classroom, home, photographs, and other places
   e) within other shapes

3) create shapes -
   a) by copying figures on dot paper, grid paper, or tracing paper, by using geoboards, or by tracing cutouts
   b) by drawing figures
   c) by constructing figures with sticks, straws, or
(Level 0 cont.)

pipe cleaners, or by tiling with pattern blocks and other manipulatives

4) describe geometric shapes and constructs verbally using appropriate standard and nonstandard language —
   a) a cube "looks like a block or box"
   b) "corners" for angles

5) work on problems that can be solved by managing shapes, measuring, and counting —
   a) find the area of a box top by tiling and counting
   b) use two triangular shapes to make a rectangle; another triangle (tangrams), (Crowley, 1987).

Sorting or classifying shapes using models is a good way to introduce geometric ideas. Names and properties of shapes can be given as students begin to recognize and discuss them in their own words.

**Activity 1: Sorting and Classifying Shapes**

1) Make collections of posterboard shapes. Include a wide assortment of shapes with curves, and straight edges and angles.

2) Have small groups of students make sets of shapes
that are alike in some way. Make sure that the concepts you want students to learn are represented by at least four or five examples.

3) Students should sort shapes indicating that they recognize a common idea. The teacher then has the opportunity to label the concept or provide the name of the shape without trying to define it formally.

When students leave out a shape, or fail to create a category, it is a clue to their perceptual thinking. Select an appropriate shape and have students discuss why they think it is the same or different as other shapes. Avoid definitions and right or wrong answers at this level.

The same activity can be done with three-dimensional shapes. Commercially-made or teacher-made solids, or real objects such as boxes, cans, balls, or styrofoam shapes are useful models.

**Activity 2: Sorting and Classifying Shapes**

1) Prepare two identical collections of shapes. Place one set in view, and the other inside a bag or box.

2) Students reach inside without looking and try to find
the matching shape from the set in view. This activity can be done with two-dimensional shapes, or three-dimensional solids. The shapes in the collection determine the concepts, and difficulty level.

A variation is to have the hidden shapes smaller versions of the visible shapes. Matching a smaller shape with a larger one provides an informal introduction to similarity: same shape, different size. Whether done in small group or whole class, this activity provides more opportunity for informal discussion of how shapes are alike and their properties.

**Activity 3: Sorting and Classifying Shapes**

1) Display a collection of shapes. Show the class a shape that has something in common with one or more of the shapes in the collection. Students select a shape that is like it and explain their choice. The shape may match, or be different with only some property in common.

**Activity 4: Shape Hunt**

1) Have students search not only for basic shapes, but also for properties of shapes. Have students search for
one specific thing, or give them a list. Different groups can hunt for different things such as:

a) parallel lines (lines "going in the same direction")
b) right angles ("square corners")
c) shapes with "dents" (concave) or without "dents" (convex)
d) shapes used over and over in a pattern (tessellation)
e) solids that are somehow like a box, cylinder, pyramid, or cone
f) shapes that are symmetrical

Activity 5: Using Tiles to Make Shapes

1) Make a specific type of shape using one or two different types of tiles.

a) Make some different triangles. Make big ones and little ones. What do you notice?

b) Make some rectangles. How many different ones can you make?

c) Make some shapes with four sides. (Try other numbers.)
d) Make some parallelograms (or trapezoids, rhombuses, squares, or hexagons.) How are your shapes alike? different?

**Activity 6: Different Shapes and Properties**

1) Make some shapes with different properties, such as:
   a) a shape that has parallel sides (or no parallel sides, or two or three sets of parallel sides)
   b) a shape with square corners. Make some with two right angles, three, four...
   c) a shape with a line of symmetry. Check it by placing a mirror on the shape.

**Activity 7: Geoboard Explorations**

1) Copy shapes, designs, and patterns from prepared cards. Begin with designs shown with dots as on a geoboard, and later have students copy designs drawn without dots.

2) Make shapes using smaller shapes.

3) Challenge students to see how many different shapes they can make of a specific type or with a specific property:
   a) make shapes with five sides (or some other
(Level 0 cont.)

b) make shapes with all square corners. Can you make one with three sides? four sides? six, seven, eight sides?
c) make trapezoids that are all different (or any other shape)
d) make a shape that has a line of symmetry

Teach students to copy their designs onto paper from the beginning. Younger students can use paper the size of the geoboard while older students can use smaller paper or centimeter dot paper.

Activity 8: Building Solids

1) Have students build skeleton models of solids or common three-dimensional shapes in their environment. Use pipe cleaners, D-Stix, or rolled up newspaper tubes. Use about three thicknesses of a large sheet, roll diagonally, and tape. Ends can be bunched together and taped. Tubes can be cut to different lengths. Pipe cleaners can not be used for a model of a cylinder, but newspaper tubes can.

2) Using wooden cubes or plastic connecting cubes, have
students find how many different rectangular solids can be made using only twelve cubes in each. Try other numbers of cubes. When are two rectangular solids congruent? How would you have to turn one solid to get it in the same orientation as another that is the same shape?

Activity 9: Geometric Puzzles

1) Tangrams. Have students make their own set before providing commercially or teacher-made sets. (Appendix VII)

2) Pentominoes. (Appendix VII) Have students find all the pentominoes on centimeter paper before moving on to specific activities (Activities 1-9: Van de Walle, 1990).

Activity 10: Congruent Polygons

This activity gives students experience with identification of congruent polygons.

For Level 0 (grades 1-2):

Directions for the teacher:

1) Provide a copy of the worksheet for each student.

(Appendix VII)
(Level 0 cont.)

2) Discuss the square and triangles shown at the top of the page. Be sure to point out their cost.

3) Discuss the cost of the first four or five examples and fill in the tags.

4) Let the students do the rest of the examples alone or in pairs.

For Level 1: (grades 3 and up):

1) Have the students study the polygons at the top of the page and complete the price tags for the other polygons alone or in pairs.

2) After they have completed the worksheet, discuss the relation between those polygons that are the same price. The area concept should come out in the discussion.

Comments: Fundamental to the development of many area concepts is the idea of conservation. In this case the area of a polygon does not change as we move it around or place it with other polygons. The use of price provides a different focus on area and forces the student to consider area in a different way. A variety of approaches to the development of a concept broadens the concept for some students and develops
understanding for students that didn't see the idea before.

Answers: A. 8  B. 2  C. 3  D. 2  E. 6  F. 8  G. 8  H. 8  I. 4  J. 8  K. 16

Anticipate both 16 and 20 as answers for L.

Activity 11: Constructing Polygons

This activity provides an opportunity to give special recognition to students who are spatially oriented.

Directions for the teacher:

1) Provide a copy of the worksheet for each student. (Appendix VII)

2) Have the student cut out the triangles at the bottom of his/her sheet.

3) Have the student form each of the polygons pictured. (The more mature students may work independently and draw line segments on the polygons to show how they constructed each.)

Comments: Several basic ideas are associated with this experience. The most important idea in preparing the student for a study of area are the facts that all
polygons can be formed from triangles and that most properties of triangles are unchanged even though the triangles are placed in different relative positions. Experiences such as these provide a very valuable change of pace for all students, and a boost in self-concept for the student who is spatially but not numerically oriented.

Activity 12: Matching Similar Polygons

This activity gives students experience in matching pictures of like shapes (similar polygons) when the pictures show different orientations of the shape.

Directions for the teacher:
1) Provide a copy of the worksheet for each student. Make a transparency for teacher use. (Appendix VII)
2) Cut up the transparency so that each polygon is on a separate piece of acetate.
3) Place the "transparent polygons" on the overhead projector one at a time. (Be sure that the student sees the image in a different orientation than that of the similar polygon on his worksheet.)
4) Have the student identify the projected image by
(Level 0 cont.)

placing his/her finger on the polygon on his/her worksheet that is "like" the one he/she sees projected on the screen.

Comments: Students need personal experience in seeing that a polygon retains many of its basic properties even after it has been rotated or flipped over. Students who have difficulty matching an image to one on their worksheet will see the situation if you physically rotate or flip over the polygon on acetate (Activities 10-12: Immerzeel, 1982).
Level 1: Analysis

Form recedes and the properties of figures emerge. Students should be provided with opportunities to:

1) measure, color, fold, cut, model, and tile in order to identify properties of figures and other geometric relationships
   a) fold a kite on a diagonal and examine the "fit."

2) describe a class of figures by its properties (on charts, verbally, with "property cards")
   a) "without using a picture, how would you describe a [figure] to someone who has never seen one?"
   b) property cards ("sides are equal", "4 right angles", etc.)

3) compare shapes according to their characterizing properties
   a) note how a square and a rhombus are alike, are different in regard to angles,... in regard to sides.

4) sort and resort shapes by single attributes
   a) sort cutouts of quadrilaterals by number of parallel sides, or number of right angles.

5) identify and draw a figure given an oral or written
(Level 1 cont.)

description of its properties

a) teacher or student describes a figure verbally and asks for all possible figures with those properties.
b) “What’s my name?” - reveal clues (properties) one by one, pausing after each, until students can accurately identify the figure. This can be done on an overhead, paper, or property cards.

6) identify a shape from visual clues

a) gradually reveal a shape, asking students to identify at each stage possible names for the shape.

7) empirically derive (from studying many examples) “rules” and generalizations

a) from tiling and measuring many rectangles, students see that “b x h” is a shortcut for adding the number of tiles.

8) identify properties that can be used to characterize or contrast different classes of figures

a) ask, “Opposite sides equal describes. . . .”
b) explore the relationship between diagonals and
figures by joining two cardboard strips. A square is generated by the end points when . . (the diagonals are congruent, bisect each other, and meet at right angles). Change the angle and the diagonals to determine . . . (a rectangle). Noncongruent diagonals generate . . .

9) discover properties of unfamiliar classes of objects
    a) from examples and nonexamples of trapezoids, determine the properties of trapezoids.

10) encounter and use appropriate vocabulary and symbols

11) solve geometric problems that require knowing properties of figures, geometric relationships, or insightful approaches

    a) without measuring, find the sum of the angles in a septagon. (Insightful students will "see" triangles, that is, relate this to known figures. Crowley, 1987).

Activities at this level begin to focus more on the properties of shapes, including some analysis of those properties. Most of the materials are the same as in Level 0. It is reasonable to expect similar investigations to take place within a classroom with
different students pursuing tasks at different levels. The van Hiele levels are descriptive of the progression in children's thinking, not what they are able to think at different ages. The passage from one level to the next is more a function of experience than maturation.

Students at Level 1 need experiences of classifying and sorting shapes by properties. Shapes should be presented so that specific properties and categorizations are evident. Be direct in pointing out specific properties or categories that students do not notice.

**Activity 1: Sorting Shapes**

Sort shapes by properties, not by names of shapes. When two or more properties are combined, sort by one property at a time.

1) "Find all of the shapes that have opposite sides parallel."
2) "Now find those that also have a right angle." (This group should include squares and nonsquare rectangles.) After sorting discuss what the name of the shape is. Sort by the same combinations of properties but in a
(Level 1 cont.)

different order.

Activity 2: Sorting Shapes

1) Use a Venn diagram on the board or paper, or with circles of string. Have students put all shapes with four congruent sides in one circle, and those with a right angle in the other. Ask, "Where do squares go?" Give them time to recognize that the two circles must overlap so the squares can be put in the intersection.

Activity 3: Classifying Solids

1) All solids: sort by

a) no edges and no vertices, edges, but no vertices, edges and vertices. Have students find real-world examples as well as make them from clay.

b) faces and surfaces. Some have all faces, all curved surfaces, some of each, with and without edges, with and without vertices.

c) parallel faces. Use solids with one or more pairs of parallel faces. Since faces are two-dimensional, students can refer to the shapes of the faces. For example, solids with two square faces that are parallel and two pairs of rectangle
(Level 1 cont.)

faces that are parallel.

2) Cylinders: Two properties separate cylinders from other solids: Cylinders have two congruent faces called bases in parallel planes. Second, all lines joining corresponding points on the bases are parallel. These lines are called elements.

   a) Right cylinders and oblique cylinders. In a right cylinder the elements are perpendicular to the bases.

   b) Prisms. If the two bases of a right cylinder are polygons, the cylinder is a prism. If the bases are rectangles, it is called a rectangular prism or rectangular solid.

   c) Cubes. A cube is the only possible solid with all square faces. A cube is a square prism with square sides.

3) Cones: A cone is a solid with at least one face called the base and a vertex that is not on the face. It is possible to draw a straight line (element) from any point on the edge of the base to the vertex. Sort by:
a) the shape of the base. If the base is a circle, the cone is a circular cone. However, the base can be any shape and the figure is still a cone.

b) pyramids. A pyramid is a special cone in which the base is a polygon. All of the faces of a pyramid are triangles except, possibly, the base.

Both cylinders and cones have straight lines called elements. A special type of both cylinders and cones occurs when the base is a polygon: a prism is a cylinder with polygon bases; a pyramid is a cone with a polygon base.

Activity 4: Construction of Shapes

Give students properties or relationships and have them construct as many shapes as possible that have these properties. Compare shapes made by different groups. For example, make:

1) a four-sided shape with two opposite sides the same length but not parallel.

2) some six-sided shapes. Make some with one, two, and then three pairs of parallel sides and some others with no parallel sides.

44
(Level 1 cont.)

3) shapes with all square corners. Can you make one with three sides? four sides? five, six, seven, or eight sides?

4) some six-sides shapes with square corners. Count how many squares are inside each. What is the distance around each?

5) five different triangles. How are they different? (This is also good for four-, five-, or six-sided shapes.)

6) some triangles with two sides equal (congruent). Make some four-sided shapes with three congruent sides. Try five-sided shapes with four congruent sides.

7) some quadrilaterals with all sides equal (or with two pairs of equal sides).

8) a shape with one or more lines of symmetry or that has rotational symmetry.

Use combinations of previously explored concepts as construction projects. Encourage students to come up with some activities. Some combinations may not be possible. Discovering why some combinations of relationships are possible and others are not is also a
valuable learning experience for students. Include concepts of perpendicular, area, perimeter, concave and convex, angle measurement, similarity, regularity, symmetry, volume, and circumferences.

**Activity 5: Shapes and Measurements**

Make shapes according to special measurement requirements.

1) Make at least five different shapes with an area of _____ (an appropriate number for your materials). What is the perimeter of each?

2) Make shapes with a fixed perimeter and examine the areas.

3) Try to make the shape with the largest area for a given perimeter, or the smallest perimeter for a given area. (For polygons, the largest area for a fixed perimeter is always regular.)

4) Make several different triangles that all have one angle the same. Next, make some different triangles with two angles the same (for example, 30 and 45 degrees). What did you notice?

5) Can you make a parallelogram with a 60 degree angle?
(Level 1 cont.)

Make several. Are they all alike? How? How are they different?

6) Make some parallelograms with a side of 5 and a side of 10. Are they all the same?

7) Draw some polygons with four, five, six, seven, and eight sides. Divide them all up into triangles, but do not let any lines cross or triangles overlap. What did you discover? Measure the angles inside each polygon.

8) Draw an assortment of rectangles and draw the diagonals in each. Measure the angles that the diagonals make with each other. Measure each part of each diagonal. What do you observe? Try this with squares, rhombuses, other parallelograms, and kites (quadrilaterals with two pairs of adjacent sides congruent).

Activity 6: Three-Dimensional Constructions

To make an easy model for exploring cylinders, cones, and prisms and pyramids, use string with poster board bases. Have students help make a wide assortment.

Materials: Soft cotton string or embroidery yarn, metal washers about 3/4 inch in diameter (about 15 per model),
(Level 1 cont.)

poster board, hole punch.

Directions:

1) Cut two identical models of the base from poster board. There are no restrictions on the shape. The size should be approximately 4 to 6 inches across.

2) Place the two bases together and punch an even number of holes around the edges, punching both pieces at the same time so the holes will line up. The holes should be about 1 cm apart.

3) Cut pieces of string about 20 inches long. You will need half as many pieces as holes.

4) Run each piece of string through a washer and thread the two ends up through two adjacent holes of both bases. Pull all the ends together directly above the base and tie a knot.

When both bases are held together, the vertex can be adjusted up, down, or sideways to produce a family of cones or pyramids with the same base. By moving one base up to the knot and adjusting the other base, you can model a family of cylinders or prisms. The bases can also be tilted (not kept parallel) and/or twisted so
that noncylindrical shapes are formed. Students should notice that for cylinders and prisms, the elements (strings) remain parallel regardless of the position of the bases. The angles that each makes with the base are the same for all elements. For cones and pyramids, examine how the the angles change relative to each other as the vertex is moved.

Activity 7: Line Symmetry

1) On a geoboard or dot or grid paper, draw a line to be a mirror line. Next draw a shape on one side that touches the line. Try to draw the mirror or symmetric image of the shape on the other side of the line. For two mirror lines, draw the reflections in stages. Mirrors can be used to test the results.

Activity 8: Line Symmetry

1) On a piece of dot paper use the technique in the previous activity to create a symmetric drawing. Fold the paper on the mirror line and notice how corresponding points on each side of the line match up. Open the paper and connect several corresponding points with a straight line. Notice that the mirror line is a
(Level 1 cont.)

perpendicular bisector of the lines joining the points.

2) Use this property to create a symmetric drawing on plane paper. Draw the line and half of the figure as before. From several critical points, draw perpendicular lines to the mirror line and extend them an equal distance beyond.

**Activity 9: Symmetry**

This activity provides a good preparation for rotational symmetry.

1) Cut out a small rectangle from paper or cardboard. Color one side and label the corners A, B, C, and D so that they have the same label on both sides. Place the rectangle on a sheet of paper and trace around it. Refer to the traced rectangle as a "box" for the cut-out rectangle. How many different ways can you flip the rectangle over so that it fits in the box?

2) Before each flip, place the rectangle in the box in the initial orientation. Each flip into the box is a flip through a line, and these lines are also lines of symmetry. For a plane shape, there are as many lines of symmetry as there are ways to flip a figure over and
3) Try with other figures: square, nonsquare rhombus, kite, parallelogram with unequal sides and angles, triangles, regular pentagons, and others.

**Activity 10: Rotational Symmetry**

If a shape will fit into its box in more than one way without flipping over, it has rotational symmetry. The order of rotational symmetry is the number of different ways it can fit into the box. A square has a rotational symmetry of order four as well as four flip lines or lines of symmetry. A parallelogram with unequal sides and angles has rotational symmetry of order two but no lines of symmetry.

1) Use tiles, geoboards, dot or grid paper to draw a shape that has rotational symmetry of a given order. Except for regular polygons, this can be very challenging. To test results, trace around it and cut out a copy of the shape. Try to rotate it on the drawing.

**Activity 11: Diagonals of Quadrilaterals**

1) Have students draw quadrilaterals of various types
using dot or grid paper. Add the diagonals. Measure and observe the relationships between the two diagonals (angles and lengths). If different groups of students investigate different types of quadrilaterals, conjectures and observations about different relationships can be made and tested. Given a particular relationship, two diagonals can be drawn and the end points connected to see what type of quadrilateral results.

**Activity 12: Diagonals of Quadrilaterals**

1) Connect two strips of tagboard and explore the different quadrilaterals that can be generated. What happens when the strips are congruent? What if they are joined at the midpoint of one but not the midpoint of the other? If they meet at right angles, do you get a special figure? Suppose one strip is longer than the other? What strips could be used and where should they be joined to form a trapezoid? What will make the trapezoid isosceles? Create other explorations for students to investigate.
Activity 13: Angles, Lines, and Planes

Students can explore the relationships between angles within a figure by tracing angles for comparisons, or comparing angles to a square corner.

1) The angles are made by intersecting lines or by lines crossing two parallel lines. Which angles are equal? Which add up to a straight angle?

2) Examine the sum of the interior angles of polygons of different types. Is there a relationship in the number of sides and the sum of the angles? What if the shape is concave?

3) Examine the exterior angles of polygons. Extend each side in the same direction (for example, clockwise) and observe the sum of these angles. How is the exterior angle related to the interior angle?

Activity 14: Geometric Problem Solving

Many of the activities already presented are explorations posed as problems. Problem solving is an important component of Level 1 activities.

1) Tessellations can be developed with the aid of geometric properties. Using grids, students can explore
which shapes will fit together in a tessellation and why. Tessellations can be described in terms of the number of lines that come together at each intersection and the properties of the tiles themselves. Through rotations and reflections, simple tiles can be altered to form tessellations similar to those of the Dutch artist M.C. Escher.

2) Properties and measurements of shapes can be used to form many problem solving opportunities:

a) Draw any triangle. Choose three measurements of either angles or lengths of sides and use only these to tell a partner how to draw a triangle that is congruent to yours. What combinations of angles and sides will work?

b) Determine the areas of odd shapes for which there are no formulas or for which dimensions are not provided.

c) If a dart lands at all points on a circular target with equal chance, is it more likely to land closer to the center or closer to the edge? (Activities 1-14: Van de Walle, 1990).
Activitv 15: Perimeter of Polygons

This activity also provides experience in identification of the edges of a solid.

Directions for the teacher:

1) Provide a copy of the worksheet for each student.
   (Appendix VII)

2) After handing out the worksheet, ask the students to fill out the price tags on each figure.

3) When the students have completed their answers, discuss the different ways the students arrived at the answers: How did you know which straws make up the sides? Did you need to measure? Which polygons have the largest perimeters? What other polygons can you make from these straws? Can you make a polygon selling for 21 cents? For 13 cents? For 28 cents? What are possible prices for polygons made from these straws?

Comments: Students confuse the perimeter concept and the area concept because they don't have enough experience where the distinction is functional. There are few places in a student's life where he/she uses perimeter and area. An occasional contact in a
classroom helps keep the distinction in mind. In many classes it would be appropriate to discuss the classification of triangles as equilateral, isosceles, or scalene. An investigation of pyramids, prisms, and other solids might also result. Students would benefit by building some of the models, using straws and tape.

Answers: A. 9  B. 15  C. 19  D. 12
E. 16  F. 24  G. 36  H. 44  I. 33
J. 65  K. 72

**Activity 16: Constructing a Square**

This activity gives students experience in constructing a square from a variety of polygons, each with at least one right angle.

**Directions for teachers:**
1) Provide a copy of the worksheet for each student.  
(Appendix VII)
2) Be sure students understand that they are to use all nine pieces in forming the two squares. (Polygon F is not one of the two squares they are to form.)

**Comments:** Students who struggle with this activity have personal experience with the basic concepts of
(Level 1 cont.)

congruence and tessellations. The fact that the solution produces two squares that are the same size has important though subtle implications for the sophisticated concept of area. In discussion, draw out many generalizations without insisting on precise language.

Activity 17: Matching Geometric Solids

This activity provides experience in matching pictures of a geometric solid when the pictures show different orientations of the solid.

Directions for the teacher:

1) Provide a copy of the worksheet for each student, and a transparency for teacher use. (Appendix VII)
2) Cut up the transparency so that each polyhedron is on a separate piece of acetate.
3) Place the “transparent solids” on the overhead projector one at a time. (Be sure that your student sees the image in a different orientation than that of the solid on his/her worksheet.)
4) Have the student identify the projected solid by placing his/her finger on the solid on his/her worksheet.
(Level 1 cont.)

that is "like" the one he/she sees projected on the screen.

5) Be sure that some of the projected images result from flipping the transparency upside-down on the projector.

Comments: This experience in visual translations is fun, and excellent for helping the student focus on the basic properties of the polyhedra. Vocabulary may be limited, but insight will be gained by having students discuss how they view the projected solid (Activities 15-17: Immerzeel, 1982).
APPENDIX I

ADDITIONAL RESOURCES

Cuisenaire Company.

Cuisenaire Company.

Exploring with Squares and Cubes. Ron Cramer. Dale
Seymour Publications.

Geometry for Grades K-6. edited by Jane M. Hill.
National Council of Teachers of Mathematics.

Gnee or Not Gnee. Sunburst Software.

Introduction To Tessellations. Dale Seymour. Dale
Seymour Publications.

Polyhedraville: Investigating Geometry. Creative
Publications.

Right Turn. Sunburst Software.

Teaching Children Mathematics. National Council of
Teachers of Mathematics.

Tessellmanial MECC, Minneapolis, Minnesota. Grades 3-12.

Tetris. Spectrum HoloByte Software.
APPENDIX II

INTEGRATING GEOMETRY

Integration of mathematics throughout the curriculum is recommended by the Curriculum and Evaluation Standards for School Mathematics [Standards], as well as the California Frameworks for Mathematics, Science, History-Social Science, and English-Language Arts. Curriculum integration provides students with a holistic view of education, allowing them to see relationships between subjects.

The Standards state, "...math should include opportunities to make connections so that students can use math in other curricular areas" (1989, p.32). Students also "...need to understand that mathematics is an integral part of real-world situations and activities in other curricular areas" (1989, p. 18). The Mathematics Framework for California Public Schools also supports an interdisciplinary focus. "As mathematics pervades all facets of society, its integration with other school subjects becomes an important goal. Over the next ten years, curriculum designers at all levels,
including teachers, can make important progress toward achieving that goal" (1992, p.101).
APPENDIX III

GEOMETRY IN CHILDREN'S LITERATURE


APPENDIX IV

GEOMETRY IN ART

Geometric Shapes

Supplies needed: paint brushes, red, blue, yellow paint, 12 x 18 white and black construction paper, newspaper to cover desks, paper towels, meat trays or pie tins to use as palettes.

1) On each palette put a small amount of yellow and red paint; don’t mix them or allow them to touch. Depending on how much you start with, more may be needed as students work.

2) Fold white paper (2 or 3 sheets per student) so there are 16 squares. Students start by painting the first square a pure yellow.

3) With brush, take a small amount of red and mix it into the yellow. Mix well so there aren’t any red streaks. Paint the next square. Paint across and down evenly so brush strokes don’t show.

4) Add the same amount of red to the yellow-red mixture. Mix well and paint another square. Emphasize putting the same amount of red paint into the mixture each time.
to get distinct changes in shade. If there is not a distinct change, the amount of red added should be increased.

5) Keep adding the same amount of red, mixing, and painting a square until you get to pure red. Compare the pure yellow and pure red squares. If not a pure red, it will look brownish.

6) Add a small amount of blue paint to the palette; keep separate from red mixture. With brush, add small amount of blue to red mixture, mix well, and paint next square. Keep adding, mixing, and painting until you progress through purple to pure blue. If the purple looks too black, add a drop of white to the mixture and mix well.

7) While students are painting, add another blotch of yellow to their palette.

8) Use brush to add a small amount of yellow to blue mixture, mix, and paint a square. Keep adding, mixing, and painting until you get back to pure yellow. Compare first and last squares.

9) After squares have dried, students cut a geometric shape from a 3 x 5 size card or scratch paper. Check that the shape fits in the painted squares. (Triangles,
diamonds, and squares work well.)

10) Trace the shape in all the squares and cut them out. Be sure to cut just inside the lines so pencil marks won’t show. (All squares painted may not be needed; choose those that show a distinct difference in shade.)

11) Arrange all the shapes on black construction paper in the same order they were painted in any design. (Students may want to number each shape as they cut it out.) Shapes should overlap each other in the same direction. Glue shapes one at a time to the paper.

Students are not making a color wheel. They are using the three primary colors and discovering all the shades in between. This is a lengthy project.

**Triangle Tricks**

Supplies needed: half sheet of black construction paper per student, other colors of construction paper, glue.

1) Students should trace a triangle pattern on scratch paper. Types and sizes of triangles can vary from student to student, or even for each student.

2) Students trace on colored construction paper; single color or a variety.
3) Cut out triangles and arrange on black construction paper so that each triangle touches another, and the spaces in-between triangles are also triangle-shaped. Glue triangles.

Name Game
Supplies needed: construction paper of contrasting colors, crayons or markers, scissors, glue.
1) Have students fold paper lengthwise and write their name in block letters along the fold.
2) Cut out around the letters, leaving the side along the fold attached. Have students open up the letters, and glue on a sheet of contracting color.
3) Students can add details to turn name into design.

Tessellated Designs
Supplies needed: per student: two-inch square of construction paper, 12 x 18 white art paper, crayons or markers.
1) Create negative space by cutting a free-form shape from one side of a two-inch square.
2) Tape that piece to the opposite side of the square to
create positive space (don’t flip cut piece).

3) Repeat steps 1 and 2 with the other sides. (The paper is no longer a square, but has the same area.)

4) Use the shape as a template and trace onto art paper. Repeat the design by fitting the template against the first outline and tracing. Continue tracing the design to fill the paper.

5) Using two or more colors, color spaces.

Triangle Tessellations

Supplies needed: pattern blocks, Triangle Tessellations recording sheets, butcher paper, crayons or markers, scissors, glue.

1) Working in groups of four, have students use pattern blocks to cover triangles on recording sheet.

2) Students should try different ways and decide as a group on one arrangement.

3) Each student should color their triangles to match the blocks in the arrangement the group selected.

4) Each student should color nine triangles, then cut them out.

5) Have groups arrange their triangles into a
tessellation. Encourage them to try different arrangements.

6) When the group has chosen one pattern, they glue triangles, matching edges, into hexagons on butcher paper.

Class Quilt
Supplies needed: pattern blocks, construction paper, 8 x 8 squares of black construction paper, butcher paper, scissors, glue.

After reading a children's literature book about quilts, and discussion of symmetrical and non-symmetrical patterns:
1) Have students trace pattern blocks on colored construction paper and cut out.
2) Students arrange cut pieces on black square to form either a symmetrical, or non-symmetrical design. Encourage students to build several different designs.
3) Have students choose one design and glue pieces to square.
4) Assemble squares quilt style on butcher paper. Space can be left between squares for butcher paper to show,
or squares can be glued edge to edge and lengths of yarn glued top to bottom and side to side separating squares.
APPENDIX V

GEOMETRY IN SCIENCE

Geo-Panes

Supplies needed: clay or gumdrops, string, toothpicks, list of polyhedrons to construct, bubble mixture (one part Dawn liquid detergent and three parts water).

Objective: Students will construct three-dimensional shapes from two-dimensional shapes. Students will learn the correct vocabulary as they build shapes. They will examine the parts of the polyhedrons for their structure and relate the shape to objects or structures found in nature.

List of Polyhedrons: triangular prism, octagonal prism, tetrahedron, pentagonal prism, hexagonal prism, pyramid, cube, octahedron.

Procedure: Students will build shapes using toothpicks and clay balls. During construction time, partners will discuss which is the appropriate term for the three-dimensional structure. All structures are started from a square or triangular base. Students should be encouraged to design other shapes. After shape is
constructed, tie string and dip into bubble mixture.

What do you see?

Extensions:

1) Examine the number of vertices, edges, and faces, in the various shapes. Estimate/ determine the degree of the different angles. Draw the shapes on paper.

2) Ancient Egypt and Mexico have magnificent pyramids. Why do you think the builders used this shape for their structures? Why are these structures so long-lasting?

Shadows

Students should start by examining their own shadow. Repeating activities at different times of the day helps students realize their shadow varies in length at different times of the day. Older students can measure their shadow, measure their height, chart the results, and explore the relationship between the height of people or objects and the lengths of their shadows.

1) Making Shadows With Four-Sided Shapes

a) Use a square cut out of tag board. Examine its shadow in sunlight. Tilt it in different ways and see how many different shadows you can make on the
ground.
b) What kinds of shapes do the shadows make? Can you make the shadow have the shape of a rectangle? a parallelogram? a trapezoid? a quadrilateral with no two sides the same length?
c) Draw the shadows you make on a large sheet of paper. See if others can make your shadows.
d) Choose another four-sided shape. Make as many different shadows as you can. Draw them.

2) Making Shapes With a Triangular Shape
a) Use a triangle cut out of tag board. Look at its shadow in sunlight. Tilt the shape in different ways. How many different shadows can you make on the ground?
b) Make the largest shadow you can. Make the smallest.
c) Make the longest shadow you can. Make the shortest.
d) Which of these shapes can you make?
   1) square
   2) right triangle
   3) obtuse triangle
4) isosceles triangle
5) scalene triangle
6) rectangle
APPENDIX VI

ADDITIONAL GEOMETRY ACTIVITIES

Pattern Blocks Recipes

Supplies needed: pattern blocks, paper and pencil.

1) Students work individually or in pairs to create designs or creatures using pattern blocks.

2) They trace around the outside perimeter of the shape.

3) Students carefully remove each block from the shape while recording the name of the pattern block piece and the number of each block used to create the shape. This list is the "recipe" for making the shape.

4) Save recipes for other students to try, or students may exchange recipes as they finish.

Geometry Game

Supplies needed: one die with a different geometric shape on each face, another die with the words go back (once), lose your turn (once), double (once), and single (three times) on the faces, a marker such as a colored chip per student, "game board."

1) The game board consists of cardboard squares on which
are drawn the shapes from the first die. These squares, or game board elements, are placed on a flat playing surface in a configuration chosen by the players. Cards with "Start" and "Finish" should be added to complete the game board. Differences in size, shape, and type of the geometric shapes can be used on the game board elements. Use a wide variety of shapes to encourage students to be flexible when thinking about geometric shapes. To prevent game board elements from being scattered, glue a piece of Velcro or flannel to the back of each piece, and cover the playing surface with a piece of flannel.

2) One student shuffles the game board elements and arranges them to form the desired game board. The same arrangement should be kept for several games and changed only when students are ready to try a different arrangement for variety. This stability will reinforce the rules and reduce the amount of time spent rearranging the game board.

3) Each player chooses a marker to keep track of her or his position. Players take turns, going clockwise, rolling the dice and moving accordingly.
4) On the game board, they move their markers to the next shape shown on die 1, either once (single), twice (double), not at all (lose your turn), or back (go back), as shown on die 2. The winner is the first one to reach the last shape on the game board. Maximum of four players.

Extensions:

As students learn the concepts and skills, more dice or game board elements could be added with more choices and attributes, such as different sizes, colors, and shapes. Words instead of geometric figures could be used on die 1. Concrete models of polygons should be available for reference while the game is played. A third die, colors, could be added for double classification.

Geometry Around Us Bulletin Board

Have students collect or draw pictures about the geometry in their world. Petal arrangements of flowers, designs in snowflakes, and architecture, are all reflective of the geometry around us. Sources such as magazines, newspapers, and photographs are useful. Have
students place the examples on pieces of colored construction paper, and label them indicating what geometric ideas they see in the picture. Place them around the words "Geometry Around Us" on a classroom bulletin board, or in a visible location in the school.

**A to Z Bulletin Board**

This activity can also be done as a class ABC book and shared with a younger "Buddies" class, or placed in the school library.

Use half sheets of white paper. Assign or allow students to choose their favorite letter of the alphabet. Students must find a geometry word starting with their letter, define it, and illustrate examples. Examples: S - symmetry, "sameness", with a drawing of a butterfly's outspread wings; T - tessellation, "repeating pattern", with a drawing of interlocking hexagons.

**Spatial Perception**

How many squares and rectangles can you form using the dots as corners?
Solution: 20

There are:
6  1 x 1 squares
2  1 x 3 rectangles
2  2 x 2 squares
1  2 x 3 rectangle
7  1 x 2 rectangles
2  special squares

These are the two special squares:
Congruent Polygons

How much are these?

A. 
B. 
C. 
D. 
E. 
F. 
G. 
H. 
I. 
J. 
K. 
L.
Cut out these pieces and put them together to make each of the figures above.
Matching Similar Polygons

Match a figure with each picture on the overhead projector.
Perimeter of Polygons

How much are these?

A. 

B. 

C. 

D. 

E. 

F. 

G. 

H. 

I. 

J. 

K. 

Name ____________________
Cut out these pieces and make two squares.
Name ____________________________

Matching Geometric Solids

Match a figure with each picture on the overhead projector.
Tangram
BIBLIOGRAPHY


