1991

Effects of CAI on the achievement and attitudes of high school geometry students

Evelyn A. O'Prey

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San Bernardino

EFFECTS OF CAI ON THE ACHIEVEMENT
AND ATTITUDES OF HIGH SCHOOL GEOMETRY STUDENTS

A Project Submitted to
The Faculty of the School of Education
In Partial Fulfillment of the Requirements of the
Degree of

Master of Arts
in
Education: Secondary Option

By

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San Bernardino, California
1991
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Abstract

The purpose of this study was to determine if the use of computer-assisted instruction as a supplement to the traditional teaching of plane geometry would produce greater performance in achievement and enhance the mathematical attitudes of plane geometry students at a local high school. For a period of 15 weeks, 22 students from one class used the software The Geometric Supposer to investigate geometric shapes and to make conjectures about the relationships observed in their investigations. Inductive reasoning was emphasized. Another class of 27 students was used as a control group and were instructed using only the traditional teaching method. Findings indicated that the scores on the geometry achievement test of the group using CAI were significantly higher at the .05 level. There was no significant difference in the mathematical attitudes between the two classes.
Acknowledgements

My deepest thanks goes to my advisor, Dr. Alvin Wolf, who has been a thoughtful and caring source of guidance and assistance throughout my graduate study at California State University, San Bernardino. He was always there when I needed help or advice. I would like to thank Dr. Charles Funkhouser, the second reader, for his support throughout this project. His answers to all my questions were greatly appreciated. I am grateful to Mr. William Klein, Assistant Principal for counseling at Redlands High School, for providing me with two plane geometry classes and for hand scheduling the students second semester. Without his help, this project could not have been completed. I would like to thank the students in both geometry classes. They were cooperative and positive subjects throughout the study. My thanks to my children and my friends for their encouragement and support. Finally, to my husband, Bernie, my thanks for his constant support in the final hours of this project.
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Effects of CAI on the Achievement and Attitudes of High School Geometry Students

Over the past two and one-half decades the use of computer-assisted instruction (CAI) as a supplement to or replacement of traditional instruction has become very popular. In the mathematics classroom, CAI can remove the drudgery from drill and practice, be used as a tutor for learning new skills, provide simulation exercises, retain the student's attention and put the student in charge of his own learning through a discovery approach. In addition, it fosters a spirit of cooperative learning and communication.

Statement of the Problem

There have been mixed results on the effectiveness of the computer as an aid to instruction. In my review of the literature, few studies were found that involved the use of CAI in high school geometry classes throughout the year. Some mentioned the use of the computer for individual topics. One, involving a year-long study, was written by the co-author of the software used. The sample size in the experimental groups was less than half the size of my current classes. Having had some success with technology in
pre-calculus classes, I wondered about the effectiveness of using CAI as a supplement to traditional methods in plane geometry classes. The primary purpose of this study was to determine if the inclusion of computers and appropriate software in the plane geometry class would yield significant differences in the learning outcomes when compared to the traditional approach. The secondary purpose was to investigate the effect of CAI on the mathematical attitudes of plane geometry students.
Review of the Literature

Discovery Learning

Long before Papert's (1980) dream of a computer for every classroom or for every student, Bruner (1961) stated that learning that has come about by active participation and discovery is of a most personal nature and indeed the most useful and powerful in subsequent problem solving situations. He placed on teachers the responsibility to assist students to become independent thinkers and to enable them to become discoverers.

Polya (1954, 1981) stated that learners should be active rather than passive, and that the most beneficial learning is attained when the learner discovers a large portion of it. He believed that guessing based on observation, inductive reasoning, and conjecturing, which he called plausible reasoning, play a large part in mathematical discovery.

Brown (1982) advocated students' active participation in the learning process by means of discovery. He claimed that educated guesses or conjectures can be formulated through inductive
reasoning, a procedure requiring numerous examples. Flit (1983) indicated that computers can bring a variety of experiences to the classroom including discovery. More recently NCTM (1989) in the Curriculum and Evaluation Standards for School Mathematics envisioned students exploring, discovering, conjecturing, and confirming.

Computers and Mathematics Instruction

Niemic and Walberg (1987) stated that when computers first appeared as a means of instruction almost three decades ago, they created great excitement among educational psychologists. However, their effectiveness did not meet the expectations of educators and the high cost of the technology made them impracticable. With the emergence of the microcomputer in the 1970s, there was greater use of the computer in education.

Taylor classified the instructional use of the computer as tutor, tool and tutee. Computer programs that teach new skills or concepts or remediate tutor the student. When the student programs the computer, the computer becomes the tutee. A program that is used to perform a task such as word processing or The
Geometric Supposer is a tool. Fey and Held (1984) stated that initially, the role of tutee was predominant as it was felt that the students would have a deeper understanding of mathematics through programming. With the advent of educational software, the role of tutor became more prevalent. More recent developments focus on the role as a tool, which allows the student to take on more of a discovery role. Cuban (1989) indicated that computer instruction accounts for only 5% of all instruction. Niemic and Walberg's statement that 90% of American schools use computers for instruction (1987) is misleading. While 90% of the schools may do some CAI, this researcher's feeling in reading the literature is that the extent of that type of instruction is minor. Certainly Papert's (1980) goal of a computer for every student has not been reached.

For the past two decades mathematics educators have been concerned with having the mathematics curriculum respond to the influence of computer technology. The National Council of Teachers of Mathematics' 1984 yearbook dealt exclusively with computers and mathematics instruction. At the 1984
NCTM conference, The Impact of Computing on School Mathematics, it was suggested that content priorities in all mathematics courses be adjusted in light of computer graphics and technology. Furthermore, it was suggested technology would offer enriched curriculum for students with limited abilities or interest in mathematics (Corbitt, 1985). The NCTM’s Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) for grades K-12 calls for computers to be integrated into mathematics instruction and the use of computers for investigations by individuals and groups of students.

Kulik, Bangert and Williams (1983) used a meta-analysis to integrate 51 studies about computer-based instruction in grades 6-12 that used treatment and control groups of similar aptitudes. The studies involved using the computer for drill, tutoring, simulation, and programming the computer to solve problems. In some cases the computer was a substitute for traditional teaching, while in others it was a supplement. Duration of the studies varied from one week to one semester. According to the analysis, computer-based instruction raised scores from the 50th
to the 63rd percentile on final examinations and in follow-up tests there was a measurable gain. In addition, students who had used the computer had more positive attitudes toward the computer, enjoyed their mathematics courses more, and spent less time in the learning process.

In a more recent review of the literature, Niemiec and Walberg (1987) concluded that CAI used in mathematics instruction moderately raised the achievement levels of the students. They also concluded that secondary and college students did not benefit as much from CAI as did elementary students. However, when CAI was used at upper levels, it decreased the learning time and achieved a higher rate for course completion. Another conclusion was that special populations, such as learning disabled tended to receive the greatest effect from CAI. The authors suggested that CAI was less threatening than classroom recitation. Niemiec and Walberg cited the fact that studies did not address the possibility of the Hawthorne effect of being present in novel use of computers. The Hawthorne effect alone could account for enhanced learning. They suggested that one of the
benefits of CAI to all students was the positive effect of students' attitudes toward the mathematics they were studying.

**Geometry and Computers**

While there are serious limitations on the availability of quality software to use in geometry instruction, *The Geometric Supposer* and Logo are two programs which are used for guided inquiry in geometry classes. It is suggested that they allow for flexibility in structuring learning environments that are challenging to students. Battista (1988) stated that this software encourages students to explore significant problems.

Papert (1980), the developer of Logo, maintained that through active participation in the programming approach of Logo, students could learn powerful mathematics in an informal manner. He claimed by using Logo students would think about thinking, be given experiences to close the gap between the Piagetian stages of concrete and formal operations, and become better problem solvers. Although Logo was originally developed for younger children, Kenney (1987) and Battista and Clements (1988) supported its use at the
secondary level. Kenney suggested that it can extend informal knowledge, promote conjecturing and discovery learning and increase problem solving skills. Battista and Clements believed Logo would help high school geometry students progress in van Hiele’s hierarchy of geometric thinking from visual, to descriptive, to theoretical. They claimed that the theoretical level is a necessary requirement for proof-oriented geometry classes.

Research on the cognitive benefits of using Logo as an instructional strategy in mathematics education is conflicting. Turner and Land (1988) reported on a study that used Logo with one group and traditional instruction with another to teach mathematical concepts about geometric shapes, coordinate systems, negative numbers, and variables. The experimental Logo group showed no significant increase in achievement or in cognitive development. A further result suggested that the Logo approach was even less effective for low achieving concrete-operational students. This was explained by stating that many of the processes involved abstract concepts. Gallini (1987) investigated the use of Logo and CAI to enhance the
direction following and formulating skills of two
groups of students. The results indicated that the
more learner directed Logo group achieved significantly
higher performance than the programmed approach.
Clements and Battista (1990) examined the use of Logo
as a supplement to traditional instruction to aid in
the movement of children from the visual to the
descriptive level of thinking about angles and
polygons. The control group spent an equal amount of
time using word processors to minimize the Hawthorne
effect. The Logo group developed more mathematical
ideas about the concepts being taught.

Yerushalmy (1986), Schwartz (1989) and Yerushalmy
(1990), the developers of The Geometric Supposer,
promoted its use as a means for students to create
mathematics rather than passively learn geometry in a
teacher centered environment. They suggested that
creativity takes place when the students use The
Geometric Supposer to explore shapes and their
geometrical relationships and to make conjectures
through inductive processes. They envisioned a
classroom where students communicate their findings in
a seminar-like environment. The Geometric Supposer
provides visual and numerical data without interpretation, allowing the student to form his own conjectures and arrive at generalizations through inductive reasoning. Schwartz and Yerushalmy stated that the pedagogy used in the development of The Geometric Supposer is similar to that in a science lab; that is, data are gathered, conjectures or hypotheses are formed and generalized, and conclusions are reached either proving the hypotheses as theorems or rejecting them by finding counter examples. Troutner (1988) encouraged teachers to have students use the computer to discover geometric concepts and supported the use of The Geometric Supposer for this purpose. Chazan and Houde (1989) and Chazan (1990) explained how to use The Geometric Supposer for conjecturing. They discussed the inquiry method and its necessary skills, which included verifying, conjecturing, generalizing, proving and communicating. They stated that the speed of the program, its ability to make any Euclidian construction and its repeat feature provide the many examples needed to arrive at a conjecture.

A single piece of research by Yerushalmy, Chazan and Gordon (1987) was found using The Geometric
Supposer as a tool. In a yearlong study conducted by the authors at three separate high schools, there was an experimental and a comparable control group at each site. The experimental groups used a guided inquiry approach with emphasis on lab work and classroom discussions in which students took more responsibility for their learning. Discussions concentrated on the sharing of students' conjectures based on data collected using inductive reasoning. The students using The Geometric Supposer learned at least as much geometry as the control group. On a test administered to both groups the experimental group was able to produce higher level generalizations and could produce more arguments about abstract topics. In addition, the experimental group demonstrated comprehension and skills that were required for students to take an active role in learning mathematics. When the computers were in the classrooms, teachers and students felt that the use of CAI was more readily integrated into the curriculum.

Trueman (1981) reported in a study involving a lesson on transformational geometry compared the achievement levels between a group taught using a
traditional Socratic method and a group that used CAI. The results showed that the guided inquiry method using CAI was more beneficial for average and above average students. The below average students showed little enthusiasm for either approach.

Related Mathematical Research

Some recent research studies on the effectiveness of computer aided instruction in mathematics in middle schools presented a variety of results. In a study of CAI immersion in a sixth grade mathematics class, Ferrell (1986) found a small amount of statistically significant difference in achievement for those students using CAI as compared to a control group. However, in spite of observed high levels of motivation and enthusiasm on the part of the experimental group, no difference in attitude toward mathematics was found.

Another study involved 117 eighth grade students learning to compute area of a circle by means of mastery learning using traditional or computer-assisted instruction. Instruction and remediation, when needed, were given in a variety of teacher and computer combinations. Dalton and Hannafin (1988) concluded that while there was no significant effect on
achievement for computers versus traditional methods, changing the means of remediation showed higher performances. The importance of varied learning opportunities was supported. Computers and traditional instruction can complement one another.

In a further study Zehavi (1988) suggested that students are not ready for the abstract concepts involved in graphing linear equations and can be helped in their understanding by a more informal approach using computer software. The experimental group used the software for four days prior to graphing instruction. When tested after the topic was completed, the experimental group showed significant achievement over the control group. The study was repeated on a group of seventh graders who would be enrolling in algebra the following year. This time the control group was given worksheets and board games that dealt with the same topic in a similar informal approach. Although there were no significant differences in achievement following this treatment, in a follow-up test given 8 months later, just prior to the graphing instruction, only the experimental group showed significant amounts of retention of the graphing
concept. It was implied that the software activity filled a cognitive gap and aided the students' intuitive ideas about graphing (Zehavi, 1988).

Compared to middle schools, fewer studies involving high school mathematics curriculum could be located. Using computers to supplement the normal curriculum, Damarin, Dziak, Stull and Whiteman (1988) found that the estimation skills of 108 high school students enrolled in classes from general mathematics to trigonometry were substantially improved. In fifteen minute sessions throughout a period of eight weeks, each student received approximately four hours of instruction using six computer discs that were programmed to accept a range of acceptable answers and limit the response time to discourage paper and pencil calculations. The only teacher time required was for initial introduction to estimation and the computer software.

A computer-intensive algebra curriculum was field tested at two Maryland high schools. Students used computers to solve real-world problems that involved algebra before learning the skill of manipulating algebraic symbols. Teachers involved in the field
test, Lynch, Fischer, and Green (1989) reported that the students developed an understanding of the algebraic concepts and at the same time increased their problem solving skills. Through the shared use of computers, they learned to communicate mathematically and to take on a greater responsibility for their own learning.

Waits and Demana (1989, 1990) advocated the appropriate use of micro-computers and hand held computers to enhance understanding of algebraic concepts especially functions and their graphs. They stated that the use of computers will eliminate contrived problems and replace them with realistic and more difficult problems. The speed of the computer might allow for the solution of many problems in a short time.

In searching the literature, studies involving university students were more available. In a study using CAI as a supplement to the traditional approach of teaching statistics, Varnhagen and Zumbo (1990) found there was no direct positive effect on student achievement. However, there was a significant positive
effect on students' attitudes toward the instruction and subject matter.

In a subsequent study by Marcoulides (1990) two types of software were used. One was a program using self-evaluation, simulation, and tutorial strategies. The other was a program to help the students understand and use statistical analysis. A control group used neither program. The results showed the computer use improved the performance of the students.

Another study by MacGregor, Shapiro, and Niemic (1988) involved developmental education students in an algebra class. The students were tested for field-dependence and independence. In addition to the lecture class, there was an hour spent each week in a computer lab or problem solving lab. The authors reported that while there was no significant differences in achievement for the groups, field-dependent students enrolled in the computer lab out performed the field-dependent students in the problem solving lab. The study suggested that students with different learning styles benefit from a variety of instructional strategies.
The search for literature in mathematical and computer journals investigating mathematical achievement as a result of the use of CAI reveals insufficient and inconclusive research in this field in the past ten years. Moreover, literature is severely limited for studies involving computers and geometry instruction. In spite of the availability of technology, Day (1987) found that few teachers incorporated it into their classroom instruction and researchers have reported difficulty in finding teachers to match their research criteria (Day, 1987).

Cuban (1989) stated that computer use places a great burden on the ordinary teacher. Flake (1990) indicated that there is a considerable amount of time invested by teachers using computers. Bork (1984) cited lack of teacher training and resistance. Hatfield (1984) stressed a need for a plan to implement computers into the curriculum. Fey and Heid (1984) and Cuban (1989) implied that without a change in mathematics curriculum traditional instruction will continue to dominate. Johanson (1988) warned that educational use of computers is in its infancy and that perhaps impatience pervades the literature. Even
though there are inadequate computers and software for mathematics instruction, Battista (1988) urges educators not to poison their attitudes toward the future use of computers.
Purpose and Hypotheses

Purpose of the Study

In spite of all the discussions, research and suggestions for improvement in the last twenty years, United States students' scores on standardized mathematics test have been below the expectations of many educators. Many educators believe that students will learn and have a better understanding, if the students are provided with learning situations in which mathematical meanings and concepts are discovered by the students. The Geometric Supposer is software that allows students to discover.

This discovery approach raises the following questions:

Will geometry students be more successful if they use selected computer programs to investigate and discover certain geometric concepts?

Will geometry students have more positive attitudes towards mathematics if they use selected computer programs to investigate and discover certain geometric concepts?
These questions and the review of the literature helped formulate the research hypotheses which state the expected outcome of the study.

**Hypotheses**

1. The use of *The Geometric Supposer* as a supplement in teaching geometry to high school students does produce higher achievement in learning outcomes than using traditional methods.

2. The use of *The Geometric Supposer* as a supplement in teaching geometry to high school students does produce a more positive attitude towards mathematics.
Method

Subjects

Subjects in this study were 57 students enrolled in the researcher's comparable first and sixth period geometry classes at Redlands High School in the 1990-1991 school year. This large high school with almost 3000 students is located in southern California. As the study went into second semester, there was a loss of eight students due to moving, dropping the class, or changing schedules. The subjects used in data gathering for achievement were only the students who were enrolled in the class from the beginning to the end of the study. Because the attitude surveys did not have the students' names on them, all 57 were used in the pretest survey analysis, while only 49 were used in posttest survey analysis.
The final makeup of students in the experimental and control groups was the following:

Table 1

Subjects in the Study With and Without CAI Treatment

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<th>Grade</th>
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<tr>
<td>Group</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Girls</td>
<td></td>
<td></td>
</tr>
<tr>
<td>With</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>Without</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>Boys</td>
<td></td>
<td></td>
</tr>
<tr>
<td>With</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>Without</td>
<td>14</td>
<td>4</td>
</tr>
</tbody>
</table>

The classes included a broad range of abilities. The only prerequisite to enroll in plane geometry is that students have passed Algebra I with a D. There were eleven plane geometry classes at the high school. The students were assigned to their respective classes by means of computer generated scheduling. This is not random selection in the strictest terms. However,
Campbell and Stanley (1968) state that in large school settings where students sign up for a specific course and are then assigned to specific sections by some process, selection comes close to randomization.

The two groups in the study were shown to be comparable in three ways. First, their final Algebra I grades were used to find the mean grade of each group. On a 4.0 scale, the control group had a mean of 3.17 with a standard deviation of .80 and the treatment group had a mean of 3.27 with a standard deviation of .87. Using the t-test to compare mean scores, the t-test statistic was 0.484 which indicates no significant difference.

Second, a chapter test with a total value of 70 points, given by this researcher to both groups prior to treatment, yielded a mean of 56.28 with a standard deviation of 10.18 for the control group and a mean of 56.32 with a standard deviation of 7.23 for the treatment group. Again, using the t-test, the t-test statistic was 0.587 which indicates no significant difference.

Finally, the mathematics attitude survey pretest showed a marginally significant difference for only one
item from a group of ten items. That was "I look forward to coming to school". The t-test statistic was 2.777 which was greater than the critical value at the .05 probability level.

Materials

The Geometric Supposer, a computer program that was developed to help students use an inquiry method to discover geometric concepts through inductive reasoning. This tool allows the student to perform any construction normally completed with a straight edge and compass, find measurements, repeat the process on other figures of the same class, make conjectures, and arrive at generalizations about the class of figures. The Geometric Supposer provides information without interpretation.

16 Apple IIE Computers, located in the classroom were used by the students.

Instruments

A ten-item survey, designed by the researcher, reflecting students' attitudes towards mathematics was administered at the beginning and at the end of the treatment to both the control and the experimental groups. Each statement was accompanied by a Likert
response scale with categories ranging from 1 (strongly disagree) to 5 (strongly agree).

A 50 question geometry final, developed by geometry teachers in the mathematics department, was administered at the end of the third quarter. All items that pertained to chapters in the geometry text that had not been covered were deleted to avoid guessing. Content validity was established by having three other geometry teachers review the instrument. Internal-consistency reliability was determined by a split-half reliability test using an odd-even division. This method was supported by McMillan and Schumacher (1989). The Pearson Product-Moment Correlation (r) was found using the pairs of scores, $r = .68$. Since this value estimates the reliability of only half the test, the value was corrected for the whole test using the Spearman-Brown Formula, $r_{xx} = .81$.

**Procedure**

The traditional approach to teaching geometry is generally taught in a lecture format presenting key concepts through deductive reasoning. Students are usually passive learners in this setting. In addition to this approach, the experimental group used The
Geometric Supposer once a week for 15 weeks during the second and third quarters of the school year. Each session at the computer lasted about 35 minutes. The remainder of the period was used to report the students' geometric discoveries. The students worked in groups of two at each computer. A guided inquiry approach was used. At the beginning of the study the students used The Geometric Supposer to write their own definitions of such terms as median, altitude, and angle bisector. In subsequent sessions students explored open ended problems. At first they needed more guidance to formulate conjectures. Worksheets that paralleled the content being taught to the control group were used at each lab. The students' investigations usually resulted in producing more geometric ideas than were found in the textbook for the same content. Students were instructed to use the computer program to make certain constructions and find measurements of segments, angles, and areas and often ratios of measurements. After making drawings and collecting and analyzing data, the students used inductive reasoning to make conjectures. At times investigations led to counterexamples and rejection of
the original conjectures. The repeat option allowed them to perform the same constructions on different figures of the same class so that they could generalize their conjectures. Statements were not accepted as theorems until they were proved using deductive reasoning. Students in the control group were encouraged to participate in the development of the geometry theorems that the teacher was presenting. In addition to the teacher centered instruction, the control group spent more time on compass and straight edge construction.

Research Design

The research design used in this study to investigate achievement was Posttest-Only Control Group Design as shown in Figure 1.

Figure 1. Posttest-Only Control Group Design where the treatment X is given to group 1.
Campbell and Stanley (1968) support the use of this design for the introduction of new subject matter for which pretests are impractical or unavallable. The plane geometry curriculum is predominantly new material for the students. According to Algebra I final grades and a geometry chapter test given before treatment, the groups were comparable in mathematical abilities at the start of the experiment. The dependent variable was the students' scores on the achievement test administered at the end of treatment. The independent variable was the use of the computer program as a supplement in the experimental groups' instruction. This design controls reactive effect of pretesting and allows experimental evidence when it is not possible to give a pretest. Furthermore, it controls history and maturation.

The research design used in this study for the attitude survey was a Pretest-Posttest Control Group Design as shown in Figure 2.
The assignment of the treatment to one group was selected by the researcher before meeting either class. Threats to history are usually controlled as events outside of the study will affect both groups in the same way. However, it is possible for an unusual event to happen to one of the groups. This design controls statistical regression as both groups are affected by the same factors.

Limitations to the Study

1. The two geometry classes in the study met at different times of the school day. The treatment group met the last period of each day and the control group met the first period of each day. This could affect their attention spans.

2. The researcher was also the instructor for both groups. The teacher could be biased toward one group.
3. The attitude survey used was designed by the researcher and has been validated.

4. The achievement test used to measure differences in the group were mathematics department instruments and might not be valid for schools using different texts.

5. The attitude scale administered at the beginning of treatment showed a significant difference between the groups in only one item.

6. As attitudes are a personal and subjective matter, it is difficult to determine how honestly they are reported. Perhaps some students inflate the responses while others deflate them.

7. The results of the study are significant for plane geometry classes at large high schools.

8. The students in both groups were aware that they were being used in an experiment.

9. There was a loss of five students from the control group and three from the treatment group.

10. The control group was 66.6% boys, while the experimental group was 60% girls. While both groups were predominantly students in the tenth grade, 18% of
the control group and 9% of the experimental were in the eleventh grade.

11. The N for each group was below 30. This could distort statistical analysis.

12. The scale on the attitude survey was somewhat ambiguous as the middle three descriptors were not shown.
Results

Descriptive Statistics for Geometry Achievement Test

The means and standard deviations for the geometry achievement test administered to both groups at the end of the study are given in Table 2.

Table 2.
Mean and Standard Deviation Scores on the Geometry Achievement Test

<table>
<thead>
<tr>
<th></th>
<th>CAI</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>37.00 a</td>
<td>34.26</td>
</tr>
<tr>
<td>Std</td>
<td>(5.15) b</td>
<td>(7.65)</td>
</tr>
</tbody>
</table>

Note.  a = mean    b = standard of deviation

Inferential Statistics for Geometry Achievement Test

As the major focus of this study was to determine whether there would be significantly increased levels of achievement in the CAI group as compared to the control, the mean score of experimental group was compared to the mean score of the control using the t-test to determine the level of significance. This test is very often used in educational research to determine the probability that the mean scores are
different. The null hypothesis that the means are the same is stated: \( H_0: \bar{X}_1 = \bar{X}_2 \)

The \( t \)-test statistic was 2.057. From the \( t \)-distribution table the critical value \( t_p > \) with 47df at the .05 level of significance is 2.012. Since \( p < t \), this \( t \) value is significant beyond the .05 level and the null hypothesis concerning achievement will be rejected. The results showed that CAI using The Geometric Supposer produced higher achievement in learning outcomes.

**Descriptive Statistics for Attitude Survey**

For the ten-item attitude survey the means and standard deviations were found for each item on both the pretest and posttest for the CAI group and the control group. These are reported in Table 3. See Appendix A for complete items.
Table 3.

Means and Standard Deviations of the Mathematics Attitude Survey

<table>
<thead>
<tr>
<th>Item</th>
<th>CAI N = 25</th>
<th>Control N = 27</th>
<th>CAI N = 32</th>
<th>Control N = 22</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pretest</td>
<td>Post</td>
<td>Pretest</td>
<td>Post</td>
</tr>
<tr>
<td>1.</td>
<td>3.60(^{a})</td>
<td>3.50</td>
<td>3.50</td>
<td>4.15</td>
</tr>
<tr>
<td></td>
<td>(0.97)(^{b})</td>
<td>(1.37)</td>
<td>(1.30)</td>
<td>(0.97)</td>
</tr>
<tr>
<td>2.</td>
<td>4.00</td>
<td>4.04</td>
<td>3.66</td>
<td>4.24</td>
</tr>
<tr>
<td></td>
<td>(0.85)</td>
<td>(0.98)</td>
<td>(1.02)</td>
<td>(1.07)</td>
</tr>
<tr>
<td>3.</td>
<td>3.60</td>
<td>3.59</td>
<td>3.41</td>
<td>3.33</td>
</tr>
<tr>
<td></td>
<td>(1.09)</td>
<td>(1.12)</td>
<td>(1.09)</td>
<td>(1.15)</td>
</tr>
<tr>
<td>4.</td>
<td>3.44</td>
<td>3.14</td>
<td>2.63</td>
<td>2.85</td>
</tr>
<tr>
<td></td>
<td>(1.02)</td>
<td>(1.10)</td>
<td>(1.11)</td>
<td>(1.21)</td>
</tr>
<tr>
<td>5.</td>
<td>3.40</td>
<td>3.22</td>
<td>3.03</td>
<td>3.33</td>
</tr>
<tr>
<td></td>
<td>(1.26)</td>
<td>(1.31)</td>
<td>(1.33)</td>
<td>(1.31)</td>
</tr>
<tr>
<td>6.</td>
<td>3.84</td>
<td>3.68</td>
<td>3.59</td>
<td>3.77</td>
</tr>
<tr>
<td></td>
<td>(1.01)</td>
<td>(1.10)</td>
<td>(0.96)</td>
<td>(1.20)</td>
</tr>
<tr>
<td>7.</td>
<td>3.80</td>
<td>3.95</td>
<td>3.34</td>
<td>3.41</td>
</tr>
<tr>
<td></td>
<td>(0.98)</td>
<td>(0.98)</td>
<td>(1.31)</td>
<td>(1.34)</td>
</tr>
<tr>
<td>8.</td>
<td>3.60</td>
<td>3.86</td>
<td>3.47</td>
<td>3.74</td>
</tr>
<tr>
<td></td>
<td>(0.74)</td>
<td>(0.97)</td>
<td>(1.17)</td>
<td>(1.24)</td>
</tr>
</tbody>
</table>
Table 3. (continued)

<table>
<thead>
<tr>
<th>Item</th>
<th>CAI</th>
<th>Control</th>
<th>t-statistic</th>
<th>table-t</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>4.16</td>
<td>4.00</td>
<td>4.09</td>
<td>4.07</td>
</tr>
<tr>
<td>10</td>
<td>3.92</td>
<td>4.14</td>
<td>3.88</td>
<td>4.04</td>
</tr>
</tbody>
</table>

Note.  
a = mean score on each item of the mathematical attitude survey.  
b = the standard of deviation for each item.

The mean scores were used to perform t-tests to determine if there was any significant difference in attitude scores by comparing each item with regards to CAI and control groups' pretest, their posttests, CAI's pretest and posttest, and control group's pretest and posttest. These findings are reported in Tables 4, 5, 6, and 7.

Table 4.  
Means, t statistic, and table t for Attitude Pretest

<table>
<thead>
<tr>
<th>Item</th>
<th>CAI</th>
<th>Control</th>
<th>t-statistic</th>
<th>table-t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>3.60</td>
<td>3.50</td>
<td>0.3178</td>
<td>&lt; 2.0040</td>
</tr>
<tr>
<td>2.</td>
<td>4.00</td>
<td>3.66</td>
<td>1.3199</td>
<td>&lt; 2.0040</td>
</tr>
<tr>
<td>3.</td>
<td>3.60</td>
<td>3.41</td>
<td>0.6415</td>
<td>&lt; 2.0040</td>
</tr>
</tbody>
</table>
Table 4. (continued)

<table>
<thead>
<tr>
<th>Item</th>
<th>CAI</th>
<th>Control</th>
<th>t-statistic</th>
<th>table-t</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3.44</td>
<td>2.63</td>
<td>2.7773</td>
<td>&gt; 2.0040</td>
</tr>
<tr>
<td>5</td>
<td>3.40</td>
<td>3.03</td>
<td>0.6979</td>
<td>&lt; 2.0040</td>
</tr>
<tr>
<td>6</td>
<td>3.84</td>
<td>3.59</td>
<td>0.9359</td>
<td>&lt; 2.0040</td>
</tr>
<tr>
<td>7</td>
<td>3.80</td>
<td>3.34</td>
<td>1.0970</td>
<td>&lt; 2.0040</td>
</tr>
<tr>
<td>8</td>
<td>3.60</td>
<td>3.47</td>
<td>0.4744</td>
<td>&lt; 2.0040</td>
</tr>
<tr>
<td>9</td>
<td>4.16</td>
<td>4.09</td>
<td>0.2429</td>
<td>&lt; 2.0040</td>
</tr>
<tr>
<td>10</td>
<td>3.92</td>
<td>3.88</td>
<td>0.2497</td>
<td>&lt; 2.0040</td>
</tr>
</tbody>
</table>

Note. * indicates that the mean is significantly different at p<.05.

Table 5.
Means, t statistic, and table t for Attitude Posttest

<table>
<thead>
<tr>
<th>Item</th>
<th>CAI</th>
<th>Control</th>
<th>t-statistic</th>
<th>table-t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.50</td>
<td>4.15</td>
<td>1.8968</td>
<td>&lt; 2.0117</td>
</tr>
<tr>
<td>2</td>
<td>4.04</td>
<td>4.24</td>
<td>0.5246</td>
<td>&lt; 2.0117</td>
</tr>
<tr>
<td>3</td>
<td>3.59</td>
<td>3.33</td>
<td>0.7799</td>
<td>&lt; 2.0117</td>
</tr>
<tr>
<td>4</td>
<td>3.14</td>
<td>2.85</td>
<td>0.8521</td>
<td>&lt; 2.0117</td>
</tr>
<tr>
<td>5</td>
<td>3.22</td>
<td>3.33</td>
<td>0.2867</td>
<td>&lt; 2.0117</td>
</tr>
<tr>
<td>6</td>
<td>3.68</td>
<td>3.77</td>
<td>0.2656</td>
<td>&lt; 2.0117</td>
</tr>
<tr>
<td>7</td>
<td>3.95</td>
<td>3.41</td>
<td>1.5464</td>
<td>&lt; 2.0117</td>
</tr>
</tbody>
</table>
Table 5. (continued)

<table>
<thead>
<tr>
<th></th>
<th>Pretest</th>
<th>Posttest</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>3.86</td>
<td>3.74</td>
<td>0.3644</td>
<td>&lt; 2.0117</td>
</tr>
<tr>
<td>9</td>
<td>4.00</td>
<td>4.07</td>
<td>0.1928</td>
<td>&lt; 2.0117</td>
</tr>
<tr>
<td>10</td>
<td>4.14</td>
<td>4.04</td>
<td>0.3703</td>
<td>&lt; 2.0117</td>
</tr>
</tbody>
</table>

Note. Means are not significantly different at \( p < .05 \)

Table 6.

Means, t statistic, and table t for CAI’s Attitude Surveys

<table>
<thead>
<tr>
<th>Item</th>
<th>Pretest</th>
<th>Posttest</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.60</td>
<td>3.50</td>
<td>0.2835</td>
<td>&lt; 2.0141</td>
</tr>
<tr>
<td>2</td>
<td>4.00</td>
<td>4.04</td>
<td>0.1471</td>
<td>&lt; 2.0141</td>
</tr>
<tr>
<td>3</td>
<td>3.60</td>
<td>3.59</td>
<td>0.0303</td>
<td>&lt; 2.0141</td>
</tr>
<tr>
<td>4</td>
<td>3.44</td>
<td>3.14</td>
<td>0.9479</td>
<td>&lt; 2.0141</td>
</tr>
<tr>
<td>5</td>
<td>3.40</td>
<td>3.22</td>
<td>0.4681</td>
<td>&lt; 2.0141</td>
</tr>
<tr>
<td>6</td>
<td>3.84</td>
<td>3.68</td>
<td>0.5085</td>
<td>&lt; 2.0141</td>
</tr>
<tr>
<td>7</td>
<td>3.80</td>
<td>3.95</td>
<td>0.5133</td>
<td>&lt; 2.0141</td>
</tr>
<tr>
<td>8</td>
<td>3.60</td>
<td>3.86</td>
<td>1.0145</td>
<td>&lt; 2.0141</td>
</tr>
<tr>
<td>9</td>
<td>4.16</td>
<td>4.00</td>
<td>0.4967</td>
<td>&lt; 2.0141</td>
</tr>
<tr>
<td>10</td>
<td>3.92</td>
<td>4.14</td>
<td>0.9468</td>
<td>&lt; 2.0141</td>
</tr>
</tbody>
</table>

Note. Means are not significantly different at \( p < .05 \).
Table 7.

Means, t statistic, and table t for Control’s Attitude Surveys

<table>
<thead>
<tr>
<th>Item</th>
<th>Pretest</th>
<th>Posttest</th>
<th>t-statistic</th>
<th>table-t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>3.50</td>
<td>4.15</td>
<td>2.1074</td>
<td>&gt; 2.0025*</td>
</tr>
<tr>
<td>2.</td>
<td>3.66</td>
<td>4.24</td>
<td>1.7185</td>
<td>&lt; 2.0025</td>
</tr>
<tr>
<td>3.</td>
<td>3.41</td>
<td>3.33</td>
<td>0.2692</td>
<td>&lt; 2.0025</td>
</tr>
<tr>
<td>4.</td>
<td>2.63</td>
<td>2.85</td>
<td>0.7155</td>
<td>&lt; 2.0025</td>
</tr>
<tr>
<td>5.</td>
<td>3.03</td>
<td>3.33</td>
<td>0.8542</td>
<td>&lt; 2.0025</td>
</tr>
<tr>
<td>6.</td>
<td>3.59</td>
<td>3.77</td>
<td>0.6289</td>
<td>&lt; 2.0025</td>
</tr>
<tr>
<td>7.</td>
<td>3.34</td>
<td>3.41</td>
<td>0.1598</td>
<td>&lt; 2.0025</td>
</tr>
<tr>
<td>8.</td>
<td>3.47</td>
<td>3.74</td>
<td>0.8453</td>
<td>&lt; 2.0025</td>
</tr>
<tr>
<td>9.</td>
<td>4.09</td>
<td>4.07</td>
<td>0.0628</td>
<td>&lt; 2.0025</td>
</tr>
<tr>
<td>10.</td>
<td>3.88</td>
<td>4.04</td>
<td>1.0104</td>
<td>&lt; 2.0025</td>
</tr>
</tbody>
</table>

Note. * indicates that this mean is significantly different at p<.05.

Inferential Statistics for Attitude Survey

In the t-tests, a significant difference was found twice. In the pretest comparisons, the CAI group’s response to the statement, "I look forward to coming to
math class", showed a significant difference for p<.05, but not in the posttest comparisons.

The other significant difference for p<.05 was a positive gain for the control group from pretest to posttest on the item, "One of my best subjects is math".

The null hypothesis that the means are the same is stated: $H_0: \bar{x}_1 = \bar{x}_2$

The overwhelming evidence shows that the null hypothesis cannot be rejected. It must be concluded that using CAI and more specifically The Geometric Supposer did not produce more positive attitudes towards mathematics for the students in the experimental group.
Discussion

Conclusions

This study investigated the effectiveness of the inclusion of computer-assisted instruction in a plane geometry course. The use of The Geometric Supposer to explore geometric concepts has been in some ways beneficial to the treatment group. Consistent with the findings of some studies, but in contrast with others, the inclusion of CAI in the learning process was shown to have a positive effect on the achievement of the plane geometry students. The results of the geometry achievement test administered at the end of treatment to both groups indicated that scores for the CAI group were significantly higher at the .05 level. The students became actively involved in their own learning through the discovery process. Cooperative learning was fostered by having students work together at the computers. Furthermore, the students communicated mathematical ideas by reporting their findings to the class.

The use of CAI did not appear to enhance the attitudes of the students toward mathematics. This was a surprising result since the majority of studies
Involving mathematics and CAI reported that the students had more positive attitudes toward mathematics. In spite of this unexpected result, the students in the CAI group indicated that they enjoyed their experiences using the computer and looked forward to each lab day. Were the study to be repeated a standardized survey of mathematical attitude should be used.

Implications for Education

The use of The Geometric Supposer will allow students to become active participants in the learning process. Computer-assisted instruction should be used as a supplement to plane geometry instruction. This suggests that plane geometry curriculum and textbooks, based on the power of technology, must be created and adopted by mathematics educators. This is necessary to effectively integrate CAI into the geometry curriculum and to facilitate its use by sometimes reluctant teachers. Tests should be developed that reflect the inclusion of CAI in the geometry course. Furthermore, teachers must be trained and provided the necessary time to incorporate technology into their lessons. This researcher believes that the active learning in
which the students become involved is well worth the loss of some teacher centered learning.

The sparse amount of research on plane geometry and computer-assisted instruction found in the literature invites further research in this area. In addition, no objective study on the use of The Geometric Supposer was found.

This researcher feels that more studies, involving large numbers of students in various school settings and compared to a variety of "textbook" approaches, should be undertaken before the evidence is conclusive. While the computer has been used in mathematics instruction for over fifteen years, CAI is still a relatively new approach and must be investigated by further research.
Appendix A

Mathematics Attitude Survey
### Mathematics Attitude Survey

Place an X in the appropriate space.

1 = strongly disagree 5 = strongly agree

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. One of my best subjects is math.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. I feel comfortable in math class.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. I am satisfied with the work I do in math.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. I look forward to coming to math class.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. One of my favorite subjects is math.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. If I cannot solve a problem at first, I keep trying.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. I will raise my hand to ask a question in math class.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8. I am confident when I take a math test.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9. Math is valuable in the real world.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10. When the teacher explains a math problem, I understand it as well as others.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix B

Sample Computer Lab Worksheet Using

The Geometric Supposer
Appendix B

Sample Computer Lab Worksheet Using

The Geometric Supposer

Task: To investigate the midsegments of a triangle.


Define: Midsegment.

Drawings and Data:

Conjectures:

Procedure continued: Draw the other midsegments in the same triangle. Measure any new segments or angles formed.

Conjectures:

Perform the constructions and investigations on another type triangle by using the repeat key.
Appendix C

Plane Geometry - Third Quarter Cumulative Test
Appendix C

Plane Geometry - Third Quarter Cumulative Test

1. For the diagram at the right, what is \( m \angle ECD \)?
   a. 50     b. 60     c. 70     d. 65

2. If \( \sin A = \frac{3}{5} \), which of the following is true?
   I. \( \sin B = \frac{3}{5} \)     II. \( \cos C = \frac{3}{5} \)     III. \( \tan A = \frac{3}{4} \)
   a. I only     b. II only     c. I and II only     d. II and III only

3. If \( m \angle A = 24 \) and \( AB = 20.44 \), find \( BC \) to the nearest tenth.
   a. 9.1     b. 4.1     c. 4.5     d. 2.5

4. Which angle is an exterior angle of \( \triangle BCE \)?
   a. \( \angle ECD \)     b. \( \angle ABE \)     c. \( \angle AED \)     d. \( \angle BEA \)
5. The Triangle Inequality Theorem states that the sum of the lengths of two sides of a triangle is ______ the length of the third side.
   a. less than    b. greater than    c. equal to    d. twice

6. In \( \triangle PET \), if PE = 18 and ET = 10, PT can be which of the following?
   a. 27    b. 7    c. 28    d. 8

7. In \( \triangle XYZ \), if \( m \angle X = 35 \) and \( m \angle Z = 50 \), which of the following is the shortest side?
   a. \( \overline{XY} \)    b. \( \overline{YZ} \)    c. \( \overline{XZ} \)    d. none of these

8. In \( \triangle ABC \), if AB = 16, BC = 20, and AC = 17, which of the following is true?
   a. \( m \angle A < m \angle B < m \angle C \)    b. \( m \angle B < m \angle C < m \angle A \)
   c. \( m \angle C < m \angle B < m \angle A \)    d. \( m \angle C < m \angle A < m \angle B \)

9. Which of the following do not represent the measures of the sides of a triangle?
   a. 5, 6, 7    b. 43, 89, 133
   c. 24, 57, 80    d. 20, 20, 30

10. In \( \triangle TJM \), what is the measure of \( \angle a \) ?
    a. 103    b. 32
    c. 148    d. 58
11. In $\triangle MKR$, what is the measure of $\angle b$?
   a. 25  b. 50  c. 130  d. 60

12. Two lines are parallel if they
   a. have no points in common
   b. are not skew lines
   c. are not intersecting lines
   d. are coplanar and do not intersect

13. Which of the following represents the distance between a point and a line?
   a. The length of any segment from the point to the line
   b. The length of any segment perpendicular to the line
   c. The length of the segment parallel to the line from the point
   d. The length of the segment from the point perpendicular to the line

14. Which type of angles are $\angle 2$ and $\angle 6$?
   a. alternate interior angles
   b. alternate exterior angles
   c. consecutive interior angles
   d. corresponding angles

15. If $m\angle 3 = 62$, which of the following is true?
   a. $m\angle 13 = 118$  b. $m\angle 14 = 118$
   c. $m\angle 15 = 118$  d. $m\angle 16 = 62$
16. If \( \angle 1 = 50 \), which of the following is true?
   a. \( \angle 2 = 50 \)  
   b. \( \angle 14 = 50 \)  
   c. \( \angle 15 = 50 \)  
   d. \( \angle 16 = 50 \)

17. If \( \angle 2 = 2x + 30 \), and \( \angle 16 = 3x - 10 \), what is \( \angle 3 \)?
   a. 28  
   b. 86  
   c. 110  
   d. 130

18. Which of the following is NOT a characteristic of ALL parallelograms?
   a. Diagonals are congruent
   b. Diagonals bisect each other
   c. A diagonal separates the parallelogram into two congruent
   d. Consecutive angles are supplementary

19. In parallelogram ABCD, \( AB = 3x - 4 \), \( BC = x + 5 \), and \( CD = 2x + 10 \). What is \( AD \)?
   a. 14  
   b. 19  
   c. 38  
   d. cannot be determined

20. Which of the following is NOT a characteristic of ALL rhombi?
   a. Diagonals bisect each other
   b. Diagonals are perpendicular
   c. Each diagonal bisects a pair of opposite angles
   d. Diagonals are equal

21. Which of the following is NOT a characteristic of ALL rectangles?
   a. Opposite angles are congruent
   b. Diagonals are perpendicular
   c. Diagonals are congruent
   d. Diagonals bisect each other
22. What is the width of a rectangle with perimeter 24 cm and length 8 cm?
   a. 2 cm  b. 4 cm  c. 8 cm  d. 16 cm

23. Which of the following is not a proportion?
   a. \( \frac{4}{6} = \frac{8}{12} \)  b. \( \frac{7}{3} = \frac{21}{9} \)  c. \( \frac{8}{18} = \frac{4}{9} \)  d. \( \frac{5}{7} = \frac{8}{10} \)

24. Which value of \( x \) satisfies the proportion \( \frac{4}{12} = \frac{x + 2}{2x + 5} \)?
   a. 0.75  b. -1  c. 1  d. 11

25. A building casts a 90 foot shadow. Nearby a 6 foot man casts a shadow 9 feet long. What is the height of the building?
   a. 135 ft.  b. 54 ft.  c. 60 ft.  d. 5760 ft.

26. If \( \triangle ABC \sim \triangle DEF \), \( AB = 5 \), \( AC = 8 \), \( BC = 6 \), and \( DE = 2 \), what is \( DF \)?
   a. 1.6  b. 2.4  c. 3.2  d. 20

27. Using the figure at the right, what is the value of \( x \)?
   a. 20.6  b. 20  c. 9.6  d. 14
Given: \( \angle BAF = \angle DCE \)
\( AB = CD \)
\( AF = CE \)

Prove: \( \Box ABCD \) is a parallelogram

<table>
<thead>
<tr>
<th>STATEMENTS</th>
<th>REASONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle BAF = \angle DCE ) ( AB = CD ) ( AF = CE )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \triangle ABF \cong \triangle CDE )</td>
<td>2. CPCTC</td>
</tr>
<tr>
<td>3. ( \angle ABF = \angle CDE )</td>
<td>3.</td>
</tr>
<tr>
<td>4. ( \overline{AB} \parallel \overline{CD} )</td>
<td>4.</td>
</tr>
<tr>
<td>5. ( \Box ABCD ) is a parallelogram</td>
<td>5.</td>
</tr>
</tbody>
</table>

28. Reason 2 in the proof above is
   a. SAS   b. SSS   c. AAS   d. HL

29. Reason 4 in the proof above is
   a. Definition of parallel lines
   b. Definition of parallelogram
   c. Alternate Interior Angle Postulate
   d. Corresponding Angle Postulate

30. Reason 5 in the proof above is
   a. Definition of a parallelogram
   b. Definition of a polygon
   c. If both pairs of opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram
   d. If two sides of a quadrilateral are parallel and equal, then the quadrilateral is a parallelogram
31. If EF = 6, FA = 9, and ED = 4, what is EC?
   a. 6   b. 12   c. 8   d. 10

32. KB || MT. MT = ___.
   a. 28   b. 25   c. 23.3   c. 9.3

33. ST || PR. TR = ___.
   a. 6   b. 2   c. 1.5   d. 2.6

34. The ratio of the sides of two similar triangles is 2:3. If the area of the smaller triangle is 16, what is the area of the larger?
   a. 24   b. 81   c. 36   d. 28

35. The perimeters of two triangles have measures 24.4 and 100. A side of the smaller triangle has measure 6.1. Which is the measure of the corresponding side of the larger triangle?
   a. 4.1   b. 10.2   c. 25   d. 24.4
36. Using the figure at the right, determine the length of the lake?
   a. 2.4 km   b. 3.6 km
   c. 4.8 km   d. 6.4 km

37. What is \( \sqrt{80} \) in simplified form?
   a. \( 2 \sqrt{20} \)   b. \( 5 \sqrt{4} \)   c. \( 4 \sqrt{5} \)   d. \( 2 \sqrt{5} \)

38. If \( x^2 = 72 \), what is the value of \( x \) in simplified form?
   a. \( 6 \sqrt{3} \)   b. \( 9 \sqrt{8} \)   c. \( 6 \sqrt{2} \)   d. \( 2 \sqrt{6} \)

39. What is the geometric mean between 16 and 9 in simplified form?
   a. 12.5   b. 12   c. \( 4 \sqrt{9} \)   d. \( \frac{4}{3} \)

40. If \( AD = 8 \), and \( DC = 4 \), what is \( BD \) to the nearest tenth?
   a. 32.0   b. 22.6
   c. 5.7   d. 11.3

41. If \( AB = 10 \) and \( AD = 5 \), what is \( DC \) to the nearest tenth?
   a. 15.0   b. 7.1
   c. 15.8   d. 20.0
42. A right triangle has a leg of length 9 feet and a hypotenuse of length 15 feet. What is the measure of the other leg?
   a. 17   b. 7   c. 9   d. 12

43. What is the measure of the hypotenuse of a right triangle, if the measures of the legs of the triangle are 6 and 5?
   a. √11   b. 11   c. 1   d. √61

44. If QR = 8, what is PR in simplified form?
   a. 4   b. 4√3
   c. 8√3   d. 16

45. If PR = 9, what is QR in simplified form?
   a. 4.5   b. 3√3
   c. 6√3   d. 6

46. If the diagonal of a square has a measure of 8, what is the measure of the side of the square?
   a. 4   b. 4√2   c. 8√2   d. 4√3
47. What is the perimeter of an equilateral triangle with an altitude of $4\sqrt{3}$?
   a. 12 b. 24 c. 12 3 d. 24

48. What is the area of the parallelogram shown at the right?
   a. $96\sqrt{2}$ b. $48\sqrt{2}$
   c. 96 d. $48\sqrt{3}$

49. What is the area of a trapezoid having bases, 2 and 3 and having a height of 10?
   a. 25 b. 50 c. 12.5 d. 30

50. What is the area of an isosceles triangle whose base is 24, and whose legs are 13?
   a. 156 b. 60 c. 120 d. 30
References


