2006

The Life and Work of D.H. Hyers, 1913-1997

Brent D. Singleton

California State University, San Bernardino, bsinglet@csusb.edu

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ABSTRACT. The following is a sketch of the life and work of Donald Holmes Hyers, Professor Emeritus from the University of Southern California. The theorem put forth by Hyers in 1941 concerning linear functional equations has gained a great deal of interest over the past two decades. Hundreds of articles have been written citing his works, many of which have furthered the theorem. This paper contains a brief description of Hyers' theorem, a biographical essay and an extensive bibliography of Hyers' work and works citing the Hyers theorem or the D.H. Hyers–S.M. Ulam–Th.M. Rassias theorem or related subjects of almost the last three decades. The author of this paper is the grandson of D.H. Hyers.

1. The Theorem


S.M. Ulam in the year 1960 for very general functional equations asked the following question: “When is it true that the solution of an equation differing slightly from a given one, must of necessity be close to the solution of the given equation?” Similarly, if we replace a given functional equation by a functional inequality, when can one assert that the solutions of the inequality lie near the solutions of the strict equation? The following problem of this kind had been formulated by Ulam during a talk before a Mathematical Colloquium at the University of Wisconsin, Madison, in the year 1940. “Given a metric group $(G,\rho)$, a number $\varepsilon > 0$ and a
mapping \( f : G \to G \) which satisfies the inequality \( \rho(f(x \cdot y), f(x) \cdot f(y)) < \varepsilon \) for all \( x, y \) in \( G \), does there exist an automorphism \( a \) of \( G \) and a constant \( k > 0 \), depending only on \( G \), such that \( \rho(a(x), f(x)) \leq k \varepsilon \) for all \( x \) in \( G \) ?

If the answer is affirmative, we would call the equation \( a(x \cdot y) = a(x) \cdot a(y) \) of automorphism stable.

In trying to solve problems of this kind, most authors have considered homomorphisms between different groups or vector spaces, rather than automorphisms, for perturbations. In the year 1941, D.H. Hyers (On the stability of the linear functional equation, Proc. Nat. Acad. Sci. USA 27 (1941), 222-224) dealt with \( \varepsilon \)-additive mappings \( f : E_1 \to E_2 \) between Banach spaces, i.e. \( f \) must satisfy the inequality \( \|f(x + y) - f(x) - f(y)\| < \varepsilon \) for all \( x, y \) in \( E_1 \).

D.H. Hyers had shown that there exists an additive function \( a : E_1 \to E_2 \) satisfying the property \( \|a(x) - f(x)\| \leq \varepsilon \). Thus, in this special case, \( k = 1 \). Here the Cauchy equation \( f(x + y) - f(x) - f(y) = 0 \) is stable. Moreover, the relation between \( f \) and \( a \) is given by the formula

\[
a(x) = \lim_{n \to \infty} \frac{f(2^n x)}{2^n}.
\]

A method leading to such a formula is called a direct method.

The matter rested there until the year 1978, when Th.M. Rassias (On the stability of the linear mapping in Banach spaces, Proc. Amer. Math. Soc. 72 (1978), 297-300) considered a generalized version of Hyers’ result, which permitted the Cauchy difference to become unbounded. That is, Th.M. Rassias assumed that

\[
\|f(x + y) - f(x) - f(y)\| \leq (\|x\|^p + \|y\|^p) \cdot \varepsilon \quad (*)
\]

Key words and phrases: Donald H. Hyers, Homomorphism, functional equation, Hyers-Ulam stability,
for all \( x, y \) in \( E_1 \), where \( 0 \leq p < 1 \).

By using a direct method, he showed that, in this case too, there is an additive function \( a \) from \( E_1 \) to \( E_2 \) given by the same formula as before such that

\[
\|a(x) - f(x)\| \leq k \varepsilon \left(\|x\|^p\right),
\]

where \( k \) depends on \( p \) as well as \( \varepsilon \).

This result was later extended to all \( p \neq 1 \) by a number of mathematicians.

In 1990, Th.M. Rassias during the 27th International Symposium on Functional Equations asked the question whether a Hyers-Rassias stability theorem can also be proved for \( p \geq 1 \). In 1991, Z. Gajda (On stability of additive mappings, Internat. J. Math. and Math. Sci. 14 (1991), 431-434) following the same approach as in Th.M. Rassias’s paper, published in the year 1978, provided an affirmative solution to this question for \( p > 1 \). The inequality (*) has provided a lot of influence in the development of what is widely known today with the term Hyers-Ulam-Rassias stability of functional equations. Beginning around the year 1980 the topic of approximate homomorphisms or the Hyers-Ulam-Rassias stability of homomorphisms, was extensively studied by a number of mathematicians worldwide. For an extensive account in the subject the reader is referred to the following works and references cited there in:

During the last twenty years several results concerning the Hyers-Ulam-Rassias stability of various functional equations have been obtained, and a number of definitions of stability have been introduced. The subject has gained a lot of interest among mathematicians doing active research work in functional equations, functional analysis, algebra, topology and their various applications.

2. D.H. Hyers

Donald Holmes Hyers was the only child of Charles (1888-1929) and Faith Hyers (1887-1966), born on April 1, 1913 in Los Angeles, California. His father was an engineer, his mother a freelance fiction writer and piano teacher. From an early age his mother encouraged his academic studies, resulting in advanced reading and math skills. Upon entering formal schooling the teachers decided that Hyers would skip two grades and begin in grade 3. This was somewhat traumatic for him; the already diminutive child was dwarfed by his older classmates and taunted with the moniker, “little man.” Undeterred, he continued to excel academically and began pursuing his musical interests as well. He quickly established himself as an accomplished flutist, able to play several versions of the instrument.

In 1924, Hyers’ father was stricken with an unknown debilitating illness. The family were practicing members of Christian Science faith and refused all modern medical attention. As his condition worsened he was unable to work, forcing his wife to seek full time employment for the first time. She was able to draw upon her many years of writing experience to procure a position as publicist for the Los Angeles Public Library. Faith Hyers remained with the Los Angeles Public Library for 23 years and became well known for writing articles in various publications and hosting a radio show in the late 1930s.

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1 Personal accounts of Hyers’ life come from interviews with his daughter, Diane Kert.
2 Faith Hyers remained with the Los Angeles Public Library for 23 years and became well known for writing articles in various publications and hosting a radio show in the late 1930s.
The following fall, at the age of 16, Hyers enrolled at the University of California, Los Angeles (UCLA), joining the first class to begin studies at the newly-opened Westwood campus. He was torn between majoring in music or mathematics, however, he chose mathematics, realizing that he could pursue his musical interests outside of academia. He received his AB in 1933 and his MA in 1934. Hyers then enrolled at the California Institute of Technology (CalTech) for his doctoral studies in mathematics. There he studied under Aristotle Michal for his dissertation entitled *Integrals and Functional Equations in Linear Topological Spaces*. During his time at CalTech he often publicly played the flute in a quartet. In addition, he enjoyed camping and hiking in the national parks of southern and central California.

After graduating *summa cum laude* in 1937, Hyers took a teaching position at the University of Wisconsin, Madison. The next five years had a dramatic effect on both his personal and professional life. Early on in his stay he was introduced to his future wife, Wanda Deeming (1913-1979), a physical therapist from Massachusetts. They were married in 1940 and in the fall of that same year he met his friend and future colleague, Stanislaw M. Ulam (1909-1986). Ulam held a lecture for the University’s Mathematics Club, where he discussed many of the unsolved problems in mathematics.³ The lecture intrigued Hyers and inspired him to tackle one of Ulam’s own problems. The resulting work, *On the Stability of the Linear Functional Equation*⁴, partially solved Ulam’s problem concerning the stability of homomorphisms. Ulam later joined the faculty at Wisconsin in 1941. In his memoirs Ulam wrote that Hyers “…had persistence in thinking about problems and an ability to continue to push the train of thought on a specific problem.”⁵

In 1942 Hyers returned to California, settling in Pasadena where he was assigned as a research fellow in mechanical engineering at CalTech for the National Defense Research

Committee (NDRC). The NDRC, based in Los Alamos, New Mexico, was formed in 1940 under the direction of noted scientist, Vannevar Bush (1890-1974). Their task was “…to co-ordinate, supervise, and conduct scientific research on the problems underlying the development, production, and use of mechanisms and devices of warfare, except scientific research on the problems of flight.”

Hyers worked with other mathematicians and scientists at Los Alamos, including Ulam, but never moved to New Mexico.

Soon after his return to California Hyers’ first child, Richard, was born in 1943 and he accepted an associate professor position at the University of Southern California (USC) in 1944. He was quickly named head of the mathematics department the following year, a position he held until 1950. During his tenure he lured Ulam from Los Alamos for a short time in 1945-46.

In 1946 his second child, Diane, was born and the growing family moved to the Baldwin Hills section of Los Angeles. Hyers began to attend St. Paul’s Presbyterian Church, eventually becoming a deacon and an elder. He had turned to Presbyterianism after the anguish and bitterness he felt concerning his father’s death, having sought no attention under the precepts of Christian Science.

In 1947 Hyers was invited to join the Executive Editorial Committee of the revamped Mathematics Magazine with Glenn James of UCLA and his old advisor, A.D. Michal. Previously titled National Mathematics Magazine, the magazine had ceased publication in Nov. 1945. Hyers remained with the magazine and often contributed to it for over 15 years. In 1951 Hyers was awarded full professorship at USC as he continued his research and publishing. By 1957

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Hyers had become the principal investigator for an Office of Naval Research contract awarded to USC, a position that he held for a decade.  

Having a well-established scholarly record for over two decades, Hyers continued unabated in his research and teaching at USC. In this period he became an ardent supporter of the American Civil Liberties Union and in 1965 publicly urged his colleagues to uphold the democratic principles of the *Bill of Rights*. The second half of the decade brought dramatic changes in his personal life. He would see his children married and first grandchildren born, but it was the death of his mother in 1966 that affected him most. He was devastated by her passing and drifted into a deep depression requiring several years for recovery.

By the 1970s Hyers’ teaching career was winding down and he and his wife moved to Long Beach, CA, preparing for retirement. After 34 years of service at the University of Southern California Hyers’ retired with emeritus status in 1978. However, he was unable to enjoy this period because his wife of nearly 40 years was diagnosed with cancer and died in 1979. In order to fight off depression Hyers became active in many organizations, including his church and the USC Retired Faculty Association. In addition, he continued his involvement in the American Mathematical Society, the Mathematical Association of America, the Society for Industrial and Applied Mathematics, Phi Beta Kappa, Sigma Xi, the Caltech Alumni Association, and the USC President’s Circle.

In the early 1980s Hyers began an annual tradition of visiting Eugene, Oregon for the Oregon Bach Festival. On one of these occasions he met Roberta Spicer, a widow from Eugene and they married in the fall of 1988. During this same period Hyers began working with Dr. Themistocles M. Rassias of Athens, Greece and his interest in research was rekindled. Hyers wrote to Rassias in 1997 stating: “Themistocles, I thank you from the bottom of my heart for all

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your work and support…I may say that I have enjoyed a real stimulus to continue to do mathematics because of your correspondence including the many new problems you asked me about.”\(^\text{12}\) He began publishing more often, co-editing and co-authoring several books and articles with Themistocles M. Rassias and George Isac in the mid-1990s. In March of 1997 Hyers was hospitalized when he became suddenly ill with complications from leukemia. Within a few weeks he died on April 13 at the age of 84. He was survived by his two children, four grandchildren and two great grandchildren.

3. Works by Donald H. Hyers


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\(^{11}\) Ibid.


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Acknowledgements

My most heartfelt gratitude is extended to Dr. Themistocles M. Rassias for his patience and kindness in assisting me in bringing this paper to publication. As well, his friendship with my grandfather and kind words about him were much appreciated.

I would also like to thank Dr. Stefan Czerwik, Dr. Wang Jian, Dr. Soon-Mo Jung, Dr. Chun-Gil Park, Dr. Prasanna Sahoo, Dr. László Székelyhidi, and Dr. Tiberiu Trif, all of who graciously shared their manuscripts with me. As well, thanks to Dr. Nadia Dyakevich and Bonnie Petry for their assistance in editing the manuscript.

Brent D. Singleton
Pfau Library
California State University, San Bernardino
5500 University Parkway
San Bernardino, CA 92407
bsinglet@csusb.edu