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Naomi Regina Warder

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HOW CAN TEACHERS SUPPLEMENT THEIR GIVEN PROGRAMS IN AN
EFFORT TO PROVIDE MEANINGFUL MATHEMATICS ACTIVITIES IN
ACCORDANCE WITH THE STANDARDS?

A Project
Presented to the
Faculty of
California State University,
San Bernardino

In Partial Fulfillment
of the Requirements for the Degree
Master of Arts
In
Education

by
Naomi Regina Warder

June 1999

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Approved by:


Dr. James Mason, First Reader

6/12/99
Date


Dr. Ina Katz, Second Reader

ABSTRACT

This project is a resource book for kindergarten teachers. There are many lessons written into this handbook. These lessons are designed to be independently run by the students. There are also assessment lessons for the teachers in this book. Finally, there are many mathematics books listed that give the teacher ideas for upcoming lessons or units. These mathematics lessons are to be used as a supplement and should not be used in place of the adopted text or series of kindergarten.

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Section 1: Introduction

As the twenty-first century approaches, education in the nation, and in the state, is in the midst of many changes (NCTM, 1998). Mathematics is one area that is being impacted by a plethora of changes. The National Council of Teachers of Mathematics is on the verge of adopting new nationwide standards (NCTM, 1998). California, as well, is making changes in their mathematics standards (CMACS, 1998). At every grade level the standards are becoming more articulated with expectations becoming clearer. The strands or content of mathematics is organized for the schools by grade level. There is no exception for kindergarten.

The overriding goal for students in mathematics is to become competent citizens who are mathematically literate, and have the ability to collect data, communicate, reason, and also to become problem solvers (NCTM, 1998). This is especially true in this new technological era that places world-wide information in everyone's possession, at the push of a button. Mathematical and technological literacy need to be at the forefront of educational goals (NCTM, 1998).

Formal education beginning in kindergarten is the first step toward achieving these goals. More specifically speaking, in kindergarten students need to understand the consistency of small numbers, quantities and simple shapes in their everyday environment. They need to count, compare, describe and sort objects, and develop a sense about properties and patterns as stated in The California Mathematics Academic Content Standards (CMACS, 1998). As one can surmise by this list, these are the first steps toward the goal.

The manner in which teachers address these concepts is a matter of national, state, and local concern. The teacher may use a variety of methods and activities to introduce and provide learning opportunities for his/her students (CMACS,1998). However, the teacher should not rely solely on the given textbook to create and implement an effective mathematics program. They should search for whatever resources are available to them. An important aspect of an effective mathematics program is that children are able to take an active role in learning mathematics. In addition, the learning that occurs should be meaningful to them (Kamii,1991).

In order to consider what is meaningful to kindergartners, learning theories must be known and considered. It is important to realize that the new standards have also considered this, and The National Council of Teachers of Mathematics have based their decisions on the constructivist view of learning (Price,1995). Therefore, all teachers who teach mathematics should become familiar with this theory as well as the standards.

The organization of an effective mathematics program is of the utmost importance. Whole group lessons, small group and independent activities, as well as games should be included. Literature can be integrated as well, not to mention regular assessment. The organization should also allow the teacher to act as a facilitator of the on-going activities. These activities should provide a non-threatening method that will give the students an opportunity to feel successful, and a chance to enhance their understanding of mathematics. More importantly, the students can begin to become independent thinkers. Therefore, the question addressed by this project is:

How can teachers supplement their given programs in an effort to provide meaningful mathematics activities in accordance with the standards?

Section 2: Literature Review

Children learn mathematics in various ways (NCTM, 1998). However, this process may vary depending on the theory to which one subscribes. Different learning theories will be introduced as part of this literature review. The contrasts between the theories of developmentalists such as Jean Piaget, and behaviorists such as Robert Gagné will be discussed, along with the theories of constructivists such as Constance Kamii. In addition, the beliefs of the founding fathers of early childhood education such as Pestalozzi will be included in this discussion. The second portion will review the Mathematics Framework for California Public Schools and the Principles and Standards for School Mathematics: Standards 2000, as educators need to be familiar with the trends at the national and state levels. Finally, some ideas from several authors describing effective teaching methods in kindergarten will be included.

Philosophies On Learning

According to Jean Piaget, a developmentalist, children's mental processes must be taken into account when

presenting them with new cognitive concepts. He believes children cannot learn the same content as an adult, and there are developmental stages in the ability of children to think logically or mathematically (Copeland, 1984). As children pass through these stages their mental processes develop and they can begin to grasp the more abstract concepts.

At the sensorimotor stage the infant or toddler uses his/her senses to understand the world. Although this is an important period in the child's life, the focus of this review will be on the two stages that follow this one.

The preoperational stage is next. It lasts approximately from ages two through seven. During this stage children are characterized as being semi-logical. Their mathematical expressions can be thought of as one-way functions, or as lacking reversibility of thought (Lovell, 1971). An example of this concept is a simple equation. Young children may learn that $3+1=4$. They may also learn that $4-1=3$. Although they may learn both facts separately, there is no reversibility, or no connection made between the two facts. They see each equation as a one-way expression only.

Children at this age level cannot consistently keep in mind more than one relationship at a time (Lovell,1971). If sorting a group of blocks, they may say that these blocks are green, or that those blocks are red. However, they are much less likely to say that these are the green plastic blocks, or those are the red wooden blocks. They will usually refer to the objects by one attribute only. Considering two dimensions simultaneously is still too difficult at this stage.

They also do not stick to one opinion, even over short periods of time, and their judgements are based on perceptions (Lovell, 1971). An example of this is Piaget's conservation test where he lines up two rows of objects by one-to-one correspondence. He then asks the child to state whether the rows are equal or whether one has more than the other. When the child responds, he then spreads out one of the rows. Although there are an equal amount of objects in each row, one row appears longer than the other. He then again asks the child if they are equal or if one row has more than the other. A child at this stage usually perceives that the longer row has more, even though nothing was added or subtracted. Moreover, if the child is asked

again, his/her opinion may change which is a common occurrence at this level.

The concrete operational stage lasts approximately from ages seven through nine. This period marks the beginning of logical thinking in children, which is a characteristic that developmentalists believe is necessary to understand number (Copeland, 1984). Piaget demonstrates proof of this with the same conservation test described above. At this stage the children develop the ability to conserve. The children respond that the longer row is equal to the shorter row because the tester did not add or subtract any object when elongating the row. Although one row appeared longer, they were equal. This is the ability to conserve. Moreover, if they are questioned again, they maintain their answer, and they can explain their reasons. Reversibility is another concept that children at this stage can understand. If $3+1=4$, then the opposite must also be true, $4-1=3$. For any operation there is a reverse operation which cancels it.

It is at this stage that children can begin to learn about number in other than a rote fashion, according to Copeland (1984). At this level children should use

concrete objects as a basis for grasping abstract, mathematical ideas, just as the name of this stage suggests. These children need repeated experiences with concrete materials in order to gain concepts.

Developmentalists are not as concerned with subject matter, basic facts, or answers, as they are with the process of understanding the concepts of the operations involved in the learning (Copeland, 1984). Furthermore, they also subscribe to the idea that children cannot be taught mathematics concepts if they are not developmentally ready. Therefore, teachers should understand these stages and plan their mathematics instruction accordingly.

Copeland (1984) suggests that the goals of mathematics education for young children should be to encourage them to compare and form relationships of all kinds, and also to encourage them to think independently.

Proponents of constructivism share similar beliefs in so far as they refer to children as meaning-makers rather than passive recipients of information (Riley, Cobb, Gordon, Howard, 1993). Children must be encouraged to do their own thinking rather than be taught shortcuts because it interferes with their natural way of constructing ideas (Kamii,

Lewis, Jones, 1991). Furthermore, Kamii and associates maintain that educators should study children's process of learning and facilitate their process of construction instead of continuing to teach in ways that seem efficient for adults. Instruction must enhance, rather than undermine, children's own construction of mathematics ideas (Kamii, Lewis, Jones, 1991). In this view of thinking, children take very active roles in whatever they learn. Their learning should be meaningful, and directly related to their own experiences and backgrounds.

The forefathers to modern kindergartens shared some of the same philosophies with developmentalists and constructivists. The Swiss educator Johann Heinrich Pestalozzi (1746-1827) believed that learning was best accomplished through experience, observation, and the handling of actual objects, rather than by books, or blind memorization (Downs, 1975). "Either we go from words to things, or we go from things to words," wrote Pestalozzi (Downs, 1975). Furthermore, he contended that learning must begin with the real and the concrete. Young children are unable to understand the remote, the abstract, and the ideal. The young learner should never be introduced to concepts with

definitions, rules, and ideas that cannot be supported by direct observation and activities that are interactive.

Another predecessor to modern kindergarten, Friedrich Froebel (1782-1852), from Germany, like Pestalozzi, advocated that kindergarten children need freedom of movement to explore the world. They also agreed that children were continually gaining new perceptual awareness through their manipulation of objects (Ornstein, Levine 1994). Hence the name Object Lesson, a teaching method he extended from Pestalozzi's idea. Froebel would give his students surprises that were actually simple objects designed to encourage spontaneous and independent activities (Ornstein, Levine, 1994). He is best known for the kindergarten curriculum he developed which was based on the technique described above. He believed that students needed to be given manipulatives that were appropriate to their stage of development. Kindergartens in the United States, beginning in the mid 1800s, were influenced by Froebels theories that the child's first formal learning should be based on self-activity, in games, in songs, in crafts, and in play (Ornstein, Levine, 1994).

Influenced by her predecessors, Maria Montessori (1870-1952) molded her program to the interests and needs of her students. She also agreed that children should have the freedom to explore and teach themselves. She designed a "specially prepared environment" where each child was allowed to work independently at his/her own pace (Ornstein, Levine 1994). The materials she included were specific in so far as they stimulated the child's senses. Numbers were made of sandpaper and other sensory textures. The idea of quantity was also always a part of arithmetic in her program. Through manipulation of the materials Montessori maintained that the children would grasp the abstract concepts. "When the figures are known, they will serve the very purpose in the abstract which the rods serve in the concrete (Hainstock, 1978)."

In contrast, the behaviorist's approach to learning is based on the premise, "Provide the proper conditioning and you can get human beings to behave in almost any way you want (Copeland 1984)." Hence the name behaviorist. Two well-known psychologists who represent the behaviorist point of view are B. F. Skinner and Robert Gagné.

Their approach involves analyzing the task to be taught, isolating all the necessary components and then placing them in a sequence for the learner to follow. By isolating each element or step, the idea is that there is no room for error. This system is also known as guided learning. An example of this is a simple addition problem such as $3+2=5$. The teacher demonstrates exactly how to do it step-by-step using counters, and explains what the plus and the equal signs mean. The students miss no necessary step and therefore should not fail if the instruction has been devised correctly. If the students miss any component they go back and repeat the sequence leading to that component. The repetition also becomes the corrective method. The premise is that if they repeat the sequence, sooner or later they will remember it. It is also known as drill and practice (Copeland,1984).

As part of the "proper conditioning" referred to in the Copeland's (1984) book, rewards or recognition for the appropriate responses are given to students. The students associate the proper response with a reward so they are encouraged to respond correctly again. Hence the term association which refers to the "stimulus-response

mechanism" (Schulman,1974). This is the same psychology applied to teaching rats (Copland,1984). The idea is to control the stimulus to obtain the desired response. Rewards and positive reinforcements are used to obtain appropriate responses. The objectives or capabilities are always stated in specific, behavioral terms. This approach historically began in the early 1900's with Pavlov's work on classical conditioning. In the 1960's B.F. Skinner extended Pavlov's work by focusing on operant conditioning. Age and stages of development are not important in this type of learning according to Gagné and Skinner.

It is along these lines that there is a great deal of controversy. What kind of, and how much, instruction or directions should be given to young children? This continues to be the subject of much debate. The behaviorist position is more traditional on what should be taught. The trend "back to basics" usually aligns itself with the behaviorist's school of thought (Copeland,1984). Copeland(1984), in How Children Learn Mathematics: Teaching Implications of Piaget's Research, realizes that because of the stress on objectives and standards, teachers may find it easier to conform to the behaviorist method of teaching

rather than the developmental approach because it is easier to "show and tell," rather than allowing the student time and freedom to explore and develop an understanding. This is especially true when the learning is computation and facts. However, for problem solving, applying their knowledge, and making generalizations, the developmentalists approach may be more appropriate. It seems apparent that there must be some compromise among educators when applying the various learning theories to teaching.

Teachers understand that it is difficult to allow students the time to explore and develop understanding while trying to address all of the strands and/or concepts that are prescribed at the given grade levels. This is especially true when there are standardized tests, other assessments, and the state levels, not to mention national standards. The entire idea can be somewhat overwhelming. However, the future of mathematics is changing and educators should keep themselves abreast of these developments.

Principles and Standards for School Mathematics

The National Council of Teachers of Mathematics (NCTM) is in the process of writing the new Principles and Standards for School Mathematics: Standards 2000 which is a revision of the Curriculum and Evaluation Standards for School Mathematics. They have published a working draft, and although it does not represent official policy, it does represent the draft in its final stages. It also demonstrates the trends at the national level.

In the Principles and Standards for School Mathematics: Standards 2000 (henceforth called the Standards) NCTM has set six principles that describe basic tenets about high-quality mathematics instructional programs. These principles offer educators a guide for making decisions that influence students' learning opportunities. They are as follows:

- **Equity Principle:** Mathematics instructional programs should promote the learning of all students.
- **Mathematics Curriculum Principle:** Mathematics instructional programs should emphasize important and meaningful mathematics through curricula that are coherent and comprehensive.
- **Teaching Principle:** Mathematics instructional programs depend on competent and caring teachers who teach all students to understand and use mathematics.

- Learning Principle: Mathematics instructional programs should enable all students to understand and use mathematics.
- Assessment Principle: Mathematics instructional programs should include assessment to monitor, enhance and evaluate the mathematics learning of all students and to inform teaching.
- Technology Principle: Mathematics instructional programs should use technology to help all students understand mathematics and should prepare them to use mathematics in an increasingly technological world.
(pg. 22)

These principles apply at many levels in the educational system. At the school level they are the basis for an effective teaching program. They are also the foundation for the ten standards focused upon in this document. The Standards are divided into two areas. The first five describe the mathematical content that students should learn:

- 1) Number and Operation
- 2) Patterns, Functions, and Algebra
- 3) Geometry and Spatial Sense
- 4) Measurement
- 5) Data and Analysis, Statistics, and Probability

The next five standards describe the mathematical processes through which students should acquire and use their mathematical knowledge:

- 6) Problem Solving
- 7) Reasoning and Proof
- 8) Communication
- 9) Connections
- 10) Representation (NCTM pg.46)

The Standards address both mathematical content and mathematical processes. Roughly speaking, the content standards represent what students should know, and the process standards represent ways of acquiring and using that knowledge (pg.47). They are also inclusive of grades Kindergarten through twelve.

More specifically speaking, at the kindergarten level, the Standards state that developing a solid mathematics foundation means "nurturing children's confidence that mathematics learning is within their reach and also facilitating their success" (pg. 105). Furthermore, high quality early childhood mathematics programs:

- Build upon and extend children's intuitive and informal mathematics knowledge
- Are grounded in knowledge of child development
- Provide environments that encourage children to be active learners who are eager for new challenges
- Develop a strong conceptual framework that provides an anchoring for skill acquisition
- Nurture and develop children's natural inclination to solve problems (pg.105)

It seems that the new Standards are quite compatible with the constructivist's and developmentalist's

theories about how children learn mathematics. Their common idea that "young children are active, resourceful individuals who construct meaning, modify, and integrate ideas by interacting with the physical world and with other children and adults" (pg. 106) is reiterated many times throughout the Standards.

As NCTM discusses nurturing the student's confidence, the 1992 Mathematics Framework for California Public Schools (henceforth called the Framework) asserts the goal of mathematical power for all students (pg.2) The Framework defines mathematically powerful students as children who "think and communicate, drawing on mathematical ideas and using mathematical tools and techniques." (pg.3) Furthermore, the Framework envisions its overall educational mission as being, "to equip students with the reasoning tools they need as good citizens; to prepare students for successful work lives; and to develop students' personal capacities to enjoy and appreciate mathematics." (pg.2)

The Framework identifies ten elements, that when combined, are the basis of an empowering mathematics program for all grade levels:

1. All students participate fully.
2. Students take responsibility for their learning; they question, create, and help decide what to do.
3. Teachers are facilitators of learning rather than imparters of information.
4. All students regularly use manipulatives, calculators and computers.
5. All students frequently work together, sharing and discussing ideas.
6. All students frequently reflect their thinking orally and in writing.
7. Assessment is integrated with instruction; it focuses on what students understand and can do rather than on what they don't know or can't do.
8. The Program is appropriate to the maturity and development of the students as it meets other goals.
9. The program develops every students positive disposition toward mathematics in several ways.
10. The program usually introduces computational procedures only when students need them. (pp.40-43)

By integrating the elements listed here, the Framework reiterates that children will learn a "comprehensive body of meaningful mathematics." (pg.40) These qualities are the basis for an effective mathematics program and may be utilized as a check list for presenting and teaching the different mathematical concepts.

As the Framework is in its final year, The California State Board of Education has started to circulate a new document regarding the new standards. This document, named The California Mathematics Academic Content Standards (CMACS), discusses five mathematical strands that organize the content in kindergarten through seventh grades. These strands are, 1) number sense, 2) algebra and functions, 3) measurement and geometry, 4) statistics, data, analysis, and probability and, 5) mathematical reasoning (Pg.1). They also state that the goals for California students are as follows:

- develop fluency with basic computational skills;
- develop understanding of mathematical concepts;
- become mathematical problem-solvers who can recognize and solve routine problems readily and can find ways to reach a solution or goal where no routine path is apparent;
- communicate precisely about quantities, logical relationships and unknowns via the use of signs, symbols, models, graphs and mathematical terms;
- gather data analyze evidence and build arguments using mathematical ideas and between mathematics and other disciplines (pg. 1).

Although these newly adopted California standards have set the goals and organized the content areas, it also states

that there is "local flexibility" and that the decisions about how best to teach them are left to the teachers, the schools, and the districts (pg. 6). It should be noted that the Mathematics Framework for California Public Schools and The California Mathematics Academic Content Standards are based on the Curriculum and Evaluation Standards For School Mathematics, published by NCTM in 1989 and 1998 respectively.

In addition, these new standards state that by the end of kindergarten, students should understand small numbers, quantities, and simple shapes in their everyday environment. They should also be able to count, compare, describe and sort objects. Furthermore, they should be developing sense about properties and patterns (CMACS,1998).

As far as kindergarten is concerned the Standards and the Framework have guided teachers to consider the children's development when designing a "meaningful" (Framework,pg.40) mathematics program for all. Therefore, considering learning theories and studying standards and frameworks are not enough. The next step is to apply what has been learned using a variety of methods.

Effective Mathematics in Kindergarten

Teachers realize that mathematical abilities develop at different ages, and at different rates for each child. All children need time and opportunities to develop, construct ideas, and understand mathematical concepts. Kindergarten education should build on the idea that all children can learn mathematics, and an effective kindergarten program should take the responsibility for that learning (NCTM,1998). Furthermore, an effective mathematics program will provide the opportunity for all children to gain confidence and become competent at their grade level.

As reiterated throughout this review, in an effective program, the students should be actively involved in a variety of meaningful mathematics learning experiences that will allow each of them to construct their own ideas in their own ways (Kamii, 1985). Furthermore, Kamii states, "Instruction must enhance, rather than undermine, children's own construction of mathematics." Many of the experts discussed in this review agree that children come to kindergarten with much knowledge and understanding of number. Moreover, they believe that educators should build

on this foundation rather than "unteach" what the students already understand (Kamii, 1991).

NCTM (1998) states that number and geometry are at the core of primary mathematics, but that each of the other standards, including patterns, measurement, data and the process standards are interwoven, and are learned along with the first two (pg. 107). This document also states that technology is an important feature of any mathematics program. Therefore, an effective program will continuously include all elements prescribed by the Standards or the Framework.

In addition, an effective mathematics program should include activities that allow students to work independently at times, and in small groups at others. This is not to say that whole group lessons are inappropriate. However, an effective program will include independent activities, cooperative learning, direct lessons, small group activities, sharing, and communicating with peers and teacher (California Framework, 1992).

In conjunction with a variety of lesson and activity styles, various materials should be employed as well (Jenson, Spector, 1984). The textbook can be used as part

of a comprehensive mathematics program that can include a variety of materials, manipulatives, games, technological tools, and other resources as well. This is not to say that textbooks are not valuable teaching tools. However, teachers should use all resources available (California Framework, 1992). There are various alternatives listed in the Framework and the Standards as well.

Literature is another area that can be used as a valuable tool to teach mathematics concepts (Midkiff, Cramer, 1993). There are many books at the kindergarten level that can enhance children's mathematical understanding. Counting is a group of books that has numerous titles from which to choose. Size and classification, measurement, money, and problem solving are other genres, just to mention a few (Midkiff, Cramer, 1993). Furthermore, Midkiff and Cramer state that children's literature can serve as "logical stepping stones" to mathematical understanding and illustrate the "natural connection between language and mathematics" (pg.305).

As it is another key element, assessment should be an integral part of any effective mathematics program (Joyner,

1995). NCTM (1995) discusses four broad purposes for assessment:

- Monitoring students' progress
- Making instructional decisions
- Evaluating students' achievement
- Evaluating programs

Furthermore, articulating the purpose for assessment determines the manner of gathering information and the decisions made based on that information.

Moreover, assessment should to be interwoven with the instruction, the learning, and the curriculum to create a well rounded program (Joyner,1995). It is also important that the organization of the mathematics program permits the kindergarten teacher to observe individual youngsters at work and to assess them in an informal and formal manner (Sgoi,Gropper,Kilker,Semonite,1995). Assessment incorporated into everyday routines allows the teacher to monitor individual growth constantly, and plan for consequent activities. This is called Cognitive Guided Instruction by some experts (Chambers,1994). However, NCTM (1995) concedes that "assessing students' learning is a complex process made more intricate by demands on teachers'

time, diverse student populations, community expectations, and needs for greater personal expertise." As one can surmise, assessment is a complex and vital part of an effective mathematics program. Assessment is also the element that bonds instruction and learning together (NCTM, 1995).

In conclusion, the literature reviewed has discussed the importance of being knowledgeable about learning theories, instructional theories, and standards. Finally, aspects of an effective kindergarten program have been reviewed for the purpose of this project which is to create a resource or handbook that can supplement an existing mathematics program, at the kindergarten level.

Section 3: Goals and Limitations

The first goal of this project is to provide a Resource or handbook of individual and small group activities and games that are meaningful and highly interactive for kindergartners. These activities will include some ideas that will provide remediation for the slower learner, and also ideas that will challenge the advanced students. These games and activities will help to create a non-threatening climate where all of the students can gain confidence, experience success, and learn mathematics during this very crucial year.

The second goal is to discuss methods which integrate on-going assessment as an integral part of the program using a format which will be discussed in the assessment section. It will make the teachers more effectively plan and implement their programs with the use of an assessment tool.

A final goal is to provide a list of literature appropriate for this grade level. This list will include books that teach mathematical concepts. They can be used as valuable teaching tools, and they can serve as vehicles for

introducing concepts and making learning mathematics more exciting and meaningful for kindergartners.

However, there are limitations in this project design. The purpose of this handbook is to serve as a supplement to the mathematics program prescribed by the school, and not as its replacement. It is also specifically designed for schools in the urban areas of California, and not other states.

Section 4: Project Design

The project is in the form of a resource or handbook for kindergarten teachers. It is divided into three sections: Activities and Games, Assessment Ideas, and Mathematics Literature.

Each activity is in the form of a lesson plan. The necessary reproducible parts that are needed to implement the plans are included. Also included are directions for games and worksheets that must be completed. All parts are reproducible. Finally, ideas for remedial students and challenges for advanced students are an important part of this section.

The assessment ideas included are intended to be easy to use and very flexible. These ideas include informal observations and making anecdotal notes that can be placed into a portfolio. They may also be kept near the plan book so that these assessments can be used as a kind of guide when planning future lessons and activities. Also included are several assessment sheets that can be adapted for a variety of the activities. However, decisions on how they are used can be made by classroom teachers.

The literature list includes a variety of titles. The mathematics related topics presented in the books is discussed. Authors and ISBN numbers are given, in case a teacher would like to purchase any one of the books.

Section 5: Interactive Lessons

As this project is designed for California schools, the state standards for kindergarten will be listed here so that the readers may see how the following activities align themselves with The California Mathematics Academic Content Standards. At the top of each activity the standard will be listed in an abbreviated form, and the list that follows can serve as a reference. The standards for kindergarten are as follows:

Kindergarten

By the end of kindergarten, students understand the consistency of small numbers, quantities and simple shapes in their everyday environment. They count, compare, describe and sort objects, and develop a sense about properties and patterns.

Number Sense

1. Students understand the relationship between numbers and quantities, i.e., that a set of objects has the same number of objects in different situations, regardless of its position or arrangement.
 - 1.1 compare two or more sets (up to 10 objects in each group), and identify which set is equal to, more than, or less than the other
 - 1.2 count, recognize, represent, name and order numbers (to 30) using objects
 - 1.3 know that the larger number describes sets with more objects in them than smaller numbers

2. Students understand and describe simple addition and subtraction situations.

2.1 use concrete objects to determine the answers to addition and subtraction problems (for two numbers each less than 10)

3. Students use estimation strategies in computation and problem solving that involve numbers that use the ones and the tens places.

3.1 recognize when an estimate is reasonable

Algebra and Functions

1. Students sort and classify objects.

1.1 identify, sort and classify objects by attribute and identify objects that do not belong to a particular grouping (e.g., all these balls are green, those are red)

Measurement and Geometry

1. Students understand that there are properties such as length, weight, capacity and time and that comparisons can be made by using these properties.

1.1 compare the length, weight, and capacity of objects by making direct comparisons or using reference objects (e.g., shorter/longer/taller, heavier/lighter, which holds more?)

1.2 demonstrate understanding of concepts of time (e.g., morning, afternoon, evening, day, yesterday, tomorrow, week, year) including tools that measure time (e.g., clock, calendar)

1.3 name the days of the week

1.4 identify the time (to the nearest hour) of everyday events (e.g., lunch time is 12 o'clock, bedtime is 8 o'clock at night)

2. Students identify common geometric objects in their environment and describe their features.

2.1 identify and describe common geometric objects (e.g., circle, triangle, square, rectangle, cube, sphere, cone)

2.2 compare familiar plane and solid objects by common attributes (e.g., position, shape, size, roundness, number of corners)

Statistics, Data Analysis and Probability

1. Students collect information about objects and events in their environment.

1.1 pose information questions, collect data and record the results using objects, pictures and picture graphs

1.2 identify, describe, and extend simple patterns involving shape, size, or color such as a circle, triangle, or red, blue

Mathematical Reasoning

1. Students make decisions about how to set up a problem.

1.1 decide about the approach, materials and strategies to use

1.2 use tools and strategies such as manipulatives or sketches to model problems

2. Students solve problems in reasonable ways and justify reasoning.

2.1 explain the reasoning used with concrete objects and pictorial representations

2.2 make precise calculations and check the validity of the results from the context of the problem. (CMACS Pps. 2-3)

The first eight activities listed can be used as frequent or permanent centers, especially at the beginning of the year. There are a variety of other activities that will be introduced. They can be used to enrich and reinforce the basic centers. Most of the activities described in this project can be made from school materials so that the teacher may implement them with minimal expense.

Also included in most of these activity lessons are remediation and extension suggestions. The extension ideas for the advanced students who need to be challenged and the remedial ideas are for the students who have problems with the given activities. Many lessons will also have figures that illustrate the activities. As assessment is an integral part of this project it will be discussed in the following section. Several ideas on how to make assessment fulfill the teacher's purpose and making it convenient will be discussed.

Numeral Writing

- Content Standard: Number Sense 1.2
- Time required: 10-15 minutes (frequently)
- Grouping: individually or in small groups
- Materials: laminated $\frac{1}{2}$ sheet grids, made from cardstock, with numbers modeled at the top. erasers made from carpet swatches or old socks.

At first sight this may seem like a trivial activity. However, students can practice a variety of the skills that are listed in the kindergarten standards. These grids can go higher in number as the student progresses at his or her own rate. It is also suggested that hand written instead of typed written numbers be the models as some fonts are different than the way people write. This can cause confusion for the student.

Object of Activity: To practice writing numerals in order.
To practice numeral formation and recognition.

Instructions: 1. Students will copy or trace given numerals four or five times depending on the size of the grid sheet. See figure 1.

Remediations: The teacher can decrease the numbers to smaller increments. For example from 0 to 3 or, 0 to 5 depending on what the child can handle (remember feeling successful is also important). The models may also be written on every other line so that the student can copy from a closer model, so that he does not have to reference to the top of the page. For some students this may not be enough. Many kindergarteners enter school while they are still at the tracing stage. Therefore, tracing cards may be necessary at first.

Extensions: As mentioned earlier, numeral writing can go as high as the student can write. However, kindergarten standards state that students should know numbers to 30.

Figure 1 Number Grid Sample.

0	1	2	3	4	5

Figure 2 Number Grid for Remediation.

0	1	2	3	4	5
0	1	2	3	4	5

Figure 3 Tracing Card Sample.

0	1	2	3	4	5
0	1	2	3	4	5
0	1	2	3	4	5

Free Exploration

- Content Standard: Algebra and Functions 1.1
Measurement and Geometry 1.1, 2.1, 2.2
Mathematical reasoning 1.1, 1.2
- Time Required: 15-30 minutes
- Grouping: Individually, pairs or small group
- Materials: blocks, pattern blocks, counters
geoboards, puzzles, etc.
- Object of Activity: To provide many experiences with manipulatives using their own ideas and problem solving.
- Instructions:
1. It is a good idea for the teacher to always have a variety of tubs filled with manipulatives and a free exploration area in the classroom. This type of center also helps to keep things manageable, especially at the beginning of the year.
 2. Students will choose a tub and work with its contents.
- Note: Most of the lessons included in this project will contain ideas for remediation and extension as well. However, at this center the students will naturally modify their activities to suit them.

Computer Activities

Content Standards: All standards (depending on Program)

Time Required: 15-30 minutes

Grouping: individually or in pairs

Materials: computer, various software programs

Object of Activity: To provide the opportunity to gain computer literacy as well as work on different mathematic concepts that are presented.

Instructions:

1. The teacher should introduce all programs to the whole class via a large monitor or television set that can be adapted to be used as the monitor of a computer. The computer specialist at the school site will be able to help or the district specialist may be of some help.
2. All students should be allowed to work on the computer weekly if not daily.
3. Set up a schedule so that everyone will have their turn.

Remediations: This is an activity that the students will do at their own rate and work at levels that are comfortable to them. If the teacher finds that the whole program is too difficult he/she should have programs with easier concepts on hand.

Extensions: The same is true for the advanced students. Have higher level programs on hand for them.

Note: Software can be expensive. It may be economical to organize a software pool at your school so that there will be a wider variety to work with. Below are the names of several software companies that have software at reasonable prices.

Smart Kids Software
P.O. Box 590464
Houston TX 77259-0464

888-881-6001
www.smartkidssoftware.com/kinder1.htm

Knowledge Adventure, Inc.
1311 Grand Central Ave.
Glendale CA 91201

818-246-4400
www.adventure.com

Sunburst Communication Inc.
101 Castleton St.
Pleasantville NY 10570

1-800-321-7511
www.sunburstdirect.com

Lattice Work Software
P.O. Box 362
Lafayette Hill PA 19444

www.latticeworksw.com/rox_math.htm

Tactile Numbers

- Content Standard: Number Sense 1.1
- Time Required: 15-20 minutes
- Grouping: Individually
- Materials: beans, rice, macaroni, beads
cotton balls, yarn, stickers,
etc., glue, construction paper
- Object of Activity: To make tactile numbers and
practice numeral recognition
or counting.
- Instructions:
1. This activity can focus on one number at a time or a 0-10 number line. Both ways can be addressed at different times during the year.
 2. The teacher will set out items for gluing. Cut paper to appropriate size and write number(s) on them. (some copy machines will take construction paper).
 3. The students will glue the given item on the number(s) to create tactile numbers.
 4. Run their fingers along number to feel the numbers.
- Remediations: The teacher may help with the gluing for this activity.
- Extensions: There are no systems to make this easier.

Making Sets

- Content Standard: Number Sense 1.1, 1.2, 1.3
- Time Required: 10-20 minutes (weekly)
- Grouping: Individually
- Materials: Paper plates, cups, jars, paper mats or designed by theme, or any container that will hold objects, Small objects that are easy to count and handle such as, beads, buttons, or marbles, numeral cards about 3x3
- Object of Activity: To practice numeral recognition and making sets by placing the correct quantity of objects with the given numerals.
- Note: In the beginning making sets with manipulatives is the most interactive method. However, as the students begin to master this skill the teacher may want to do it as a stamping activity, or by coloring and circling the correct quantities on a worksheet. See figure 4 and 5.
- Instructions: 1. Place numerals in front of or in container or plate. Line up the containers so that the numbers are in random order.
2. The students will read the numeral and place the correct quantity in each container.
3. In kindergarten the main focus should be on numbers 1-10.
- Remediation: A number line or card may be placed at the center so that the child who needs a reference guide can count to the number that he/she is working on. For some students placing just a few numbers at a time, or placing the

numbers in order may be the best way at first.

Extensions:

For the more advanced students using large 12 x 18 sheet of paper the teacher can create pens, cages, or other enclosures with a colored marker and have the students make sets with the quantities of their choice. For example, the teacher could make four "animal pens" for farm animals and the students can stamp, or draw as many of that specific animal in their pen as they wish. They can then count and write the corresponding numeral in or next to the "animals pen". Next, the student can draw the appropriate environment for the animal such as mud, grass or hay.

The teacher may change the set making centers according to the theme or topic of the week, for example from farm to zoo animals, or even cars. See figures 6.

Figure 4 Making Sets.



Figure 5 Using Strings and Beads.

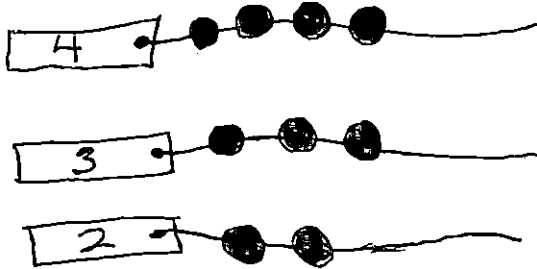
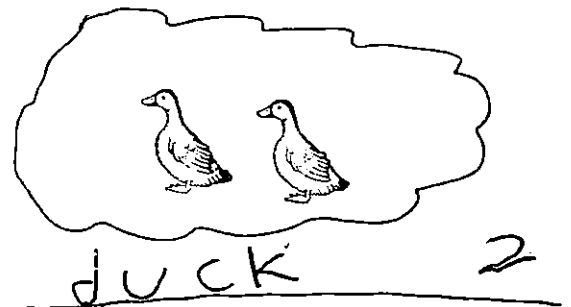
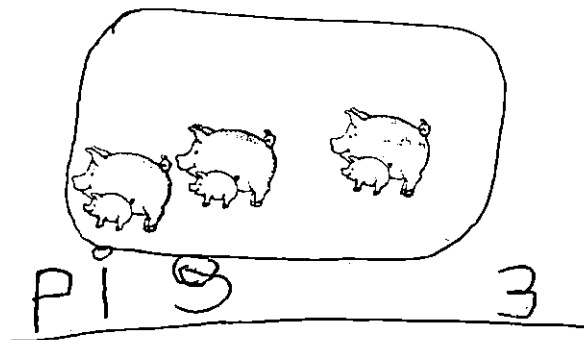
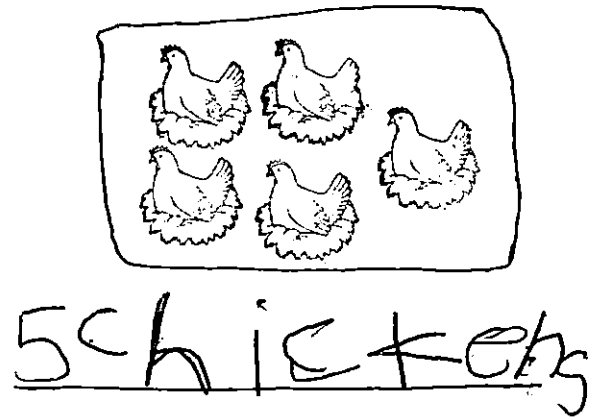
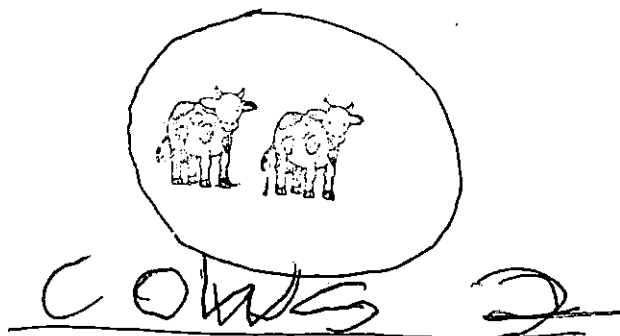


Figure 6 Animal Pens.



Object Count

Content Standard: Number Sense 1.2, 1.3, 3.1

Time Required: 10-15 minutes

Grouping: Individually or in pairs

Materials: clothesline, clothespins, index cards to be used as number cards, 8½ X 11 sheets of paper cut into fourths, tape and crayons

Object of Activity: To practice counting and writing numbers. They will also estimate quantities.

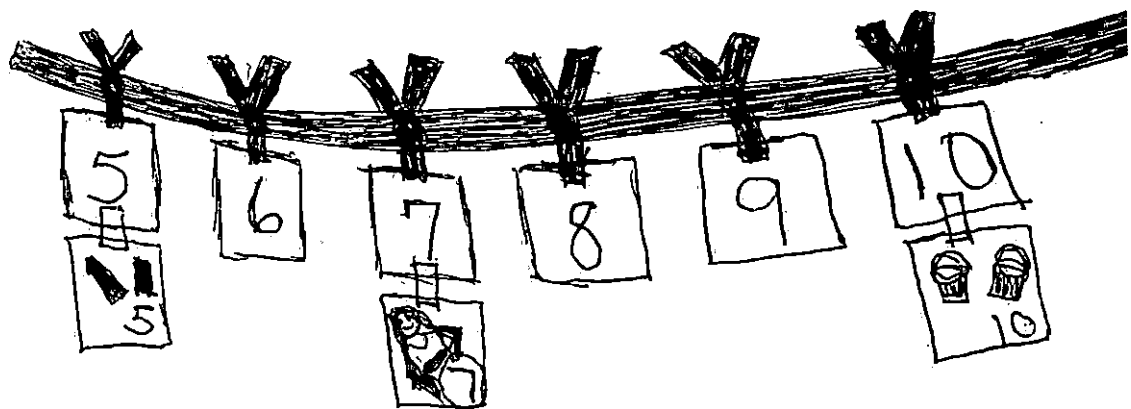
Instructions:

1. Teacher will set up a number line using the clothesline, pins, and number cards(0-20 in the beginning may be sufficient. Later up to a 100 may be more challenging).
2. The students will be instructed to decide what objects in the classroom they want to count (1-3 objects is enough).
3. First the students will estimate how many objects they think there are.
4. They will then count the object and illustrate it on the ¼ sheet of paper.
5. They will also write the quantity on their paper.
6. They will repeat this for each separate object they have chosen.
7. They will then place it with tape to the matching numeral on the number line.

Remediations: The teacher may want to designate the objects of lower quantities to students who have problems. Another way these students can participate is to assign them the part of illustrator.

Extensions: The teacher may assign the objects of higher quantities to advanced students.

Figure 7 Object Count.



Number Order

- Content Standard: Number Sense 1.2, 1.3
- Time Required: 5-10 minutes (weekly)
- Grouping: Individually or in Pairs
- Materials: number cards with numbers written on one side and the equivalent quantity (in dots) on the other, buttons, counters, or small numbered tiles
- Object of Activity: To place numbers in ordinal fashion.
- Instructions: 1. Students will place given cards in order. Depending on their ability they may work up to different numbers.
- Remediations: The students with lower abilities may need a number line for matching or self-checking. They should also begin with lower numbers and smaller amounts of cards, for example 0-5. On the opposite side of the number card the quantity may be shown with dots so that these students can use this as another reference as well.
- Extensions: The advanced student may be able to work with higher numbers or more cards 0-30 for example.

Classification

- Content Standard: Algebra and Functions
Mathematical Reasoning 2.1
- Time Required: 10-20 minutes
- Grouping: individually or in pairs
- Materials: buttons, nuts and bolts, crayons,
plastic counter type toys, rocks
junk box items, or anything else that
can be sorted or grouped by attribute
- Object of Activity: To sort and classify objects by
attribute, and to identify objects that
do not belong to a particular grouping.
- Instructions:
1. Give student a pile of objects to classify.
 2. Student will make as many groups as he/she sees.
 3. Make sure there is enough room to make the groups.
 4. Draw a picture and describe the method of grouping.
- Remediations: For the child who has problems with this activity, the teacher may limit the amount of objects and the grouping possibilities. A sheet of paper with circles or paper plates may help them make more distinct divisions. The groupings can be chosen by the teacher and the students can be given illustrated labels to match to the appropriate groups.
- Extensions: For the advanced child the teacher may make groups that do not actually fit together and instruct the students to make more appropriate groupings. The teacher may also want to set the amount of groups that should be made.

Patterning

- Content Standard: Statistics, Data Analysis and Probability 1.2
- Time Required: 10-15 minutes
- Grouping: Individually or in pairs
- Materials: linker cubes, pattern blocks counting bears, any manipulative that can be used for patterning by color or attribute
- Object of Activity: To reproduce, extend or create a pattern.
- Instructions:
 1. Place items at center.
 2. Depending on student ability the instructions may be to reproduce, extend or create a pattern with manipulatives.
- Remediation: Students who have problems with this task should be limited to reproducing before moving on to extending. Using concrete objects for this activity is very important, especially in the beginning. It is also important that these students work with ABAB patterns as this will make the task easier.
- Extensions: To make this task more challenging advanced students may do more complex patterns such as ABCABC, AABBAABB, or ABBABBA. Creating their own patterns is another good idea.

Squares, Circles, Triangles, and Rectangles
Shape Collages

- Content Standard: Measurement and Geometry 2.0, 2.1
- Time Required: 15-20 minutes
- Grouping: Small Group
- Materials: traceable geometric shapes of various sizes, 12x18 sheet of paper, scraps of construction paper for tracing, crayons, scissors and glue
- Object of Activity: To create a collage focusing on one shape.
- Instructions:
 1. Teacher will provide traceable shapes of various sizes. The activity should focus on only one form at a time.
 2. The students will trace between 4-8 forms or shapes on the scraps of construction paper.
 3. Next, students will cut, color, and glue the shapes to their large sheet of paper to create their shape collage.
- Remediation: For the children who have trouble with this activity, ready made shapes may be used, or lines for cutting may be drawn for them.
- Extensions: The advanced students may be asked to create specific designs in their collages.

Figure 8 Shape Collages.

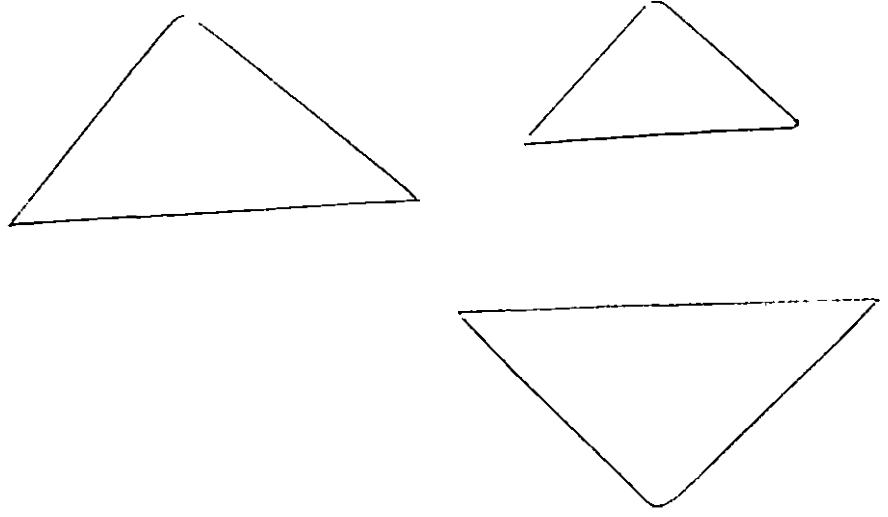
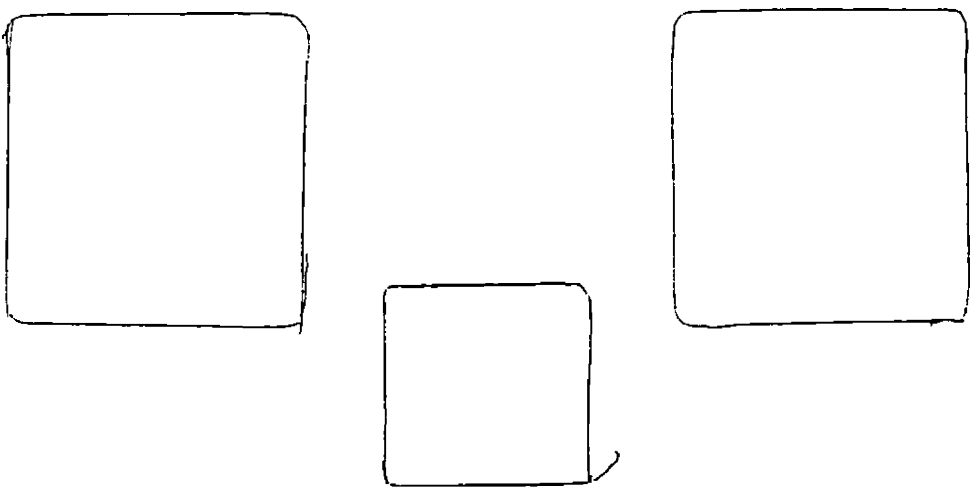
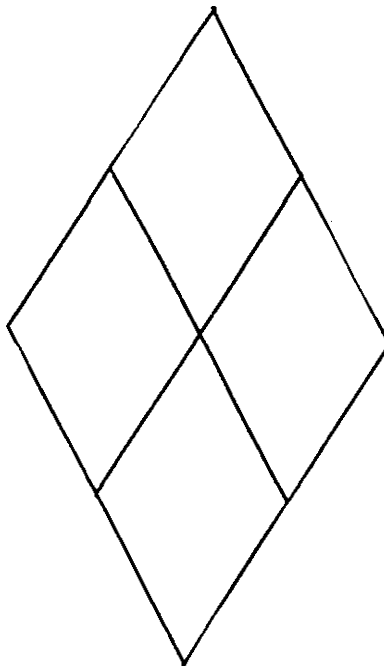
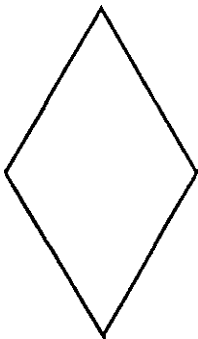
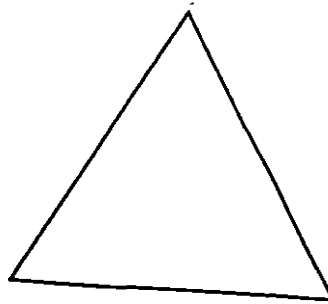
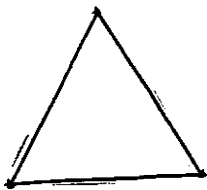
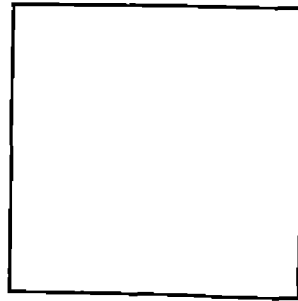
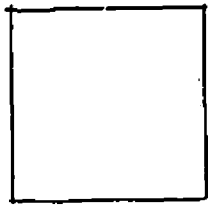


Figure 9 Traceable Shapes.


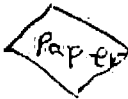
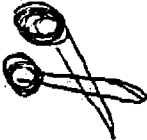













Comparing Weights

- Content Standard: Measurement and Geometry 1.1
Statistics, Data Analysis and Probability 1.1
- Time Required: 10-15 minutes
- Grouping: Individually, Pairs, Small Groups
- Materials: balance scale (may need to check in resource room or with upper grade teachers for this item), objects to weigh, teacher made worksheet for comparing weights
- Object of Activity: To compare weights of objects, and practice recording information.
- Instructions:
1. Teacher will set up the scales and place all objects to be weighed and compared at the center. The worksheet will consist of a pair of objects that the students need to compare.
 2. The students will place the objects from the worksheet on the scales and circle the object that weighs the most or the least depending on teacher instructions.
- Remediations: It is quite easy to weigh objects on the balance scale. The difficult part is making the decision about which one is heavier or lighter. It may be necessary for the teacher to make the comparison objects very obvious. Another Suggestion may be to forget about the worksheet the first few times until the child makes some discoveries about weights and the scales.
- Extensions: Comparing the weight of three or more objects and ranking them in order of their weight may be a challenge to the

advanced students. Finding their own objects and predicting which is the heavier or lighter is another suggestion for these students. They could even make up their own worksheet using drawings.

Figure 10 Balance Scales Worksheet Sample.


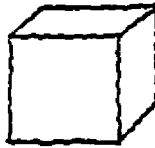

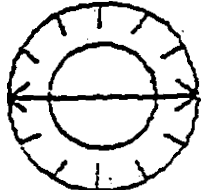
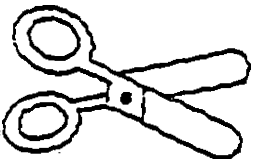



				
				
				
				
				
				
				

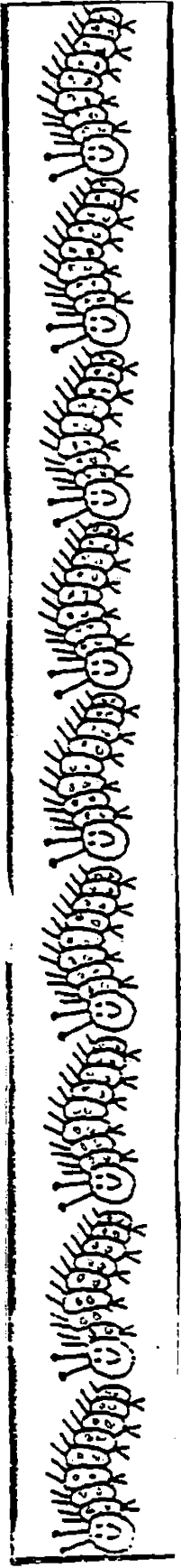
Measuring Lengths

- Content Standard:** Measurement and Geometry 1.1
Statistics, Data Analysis and Probability 1.1
Number Sense 1.2, 3.1
- Time Required:** 15-20 minutes
- Grouping:** individually or Pairs
- Materials:** measuring device (teacher designed) a sample will be included, a pictorial list of items including a space to record their measurements (see figures 11 and 12)
- Object of activity:** To practice measuring lengths of objects and to record findings.
to estimate lengths.
- Instructions:**
1. Give Students list of things to measure.
 2. Give students "rulers." Teacher may have students cut out their own rulers.
 3. Students estimate length, measure each item and record measurement.
- Remediations:** A number line may help the student who does not know or remember how to write the specific numbers. It may be helpful to give the slower student several objects with the same measurement.
- Extensions:** In addition to measuring the objects the advanced student may compare the items and put them in order by length. He/she may make a pictograph from this activity by cutting the items out and gluing them to construction paper. The student may also wish to generate their own list of objects to measure.

Figure 11 Measuring Device.

Figure 12 Length Worksheet.

	<input type="checkbox"/>		<input type="checkbox"/>
your hand		block	
	<input type="checkbox"/>		<input type="checkbox"/>
pencil		paper plate	
	<input type="checkbox"/>		<input type="checkbox"/>
scissors		your foot	
	<input type="checkbox"/>		<input type="checkbox"/>
crayon		marking pen	



Dice Roll

- Content Standard: Statistics, Data Analysis and Probability 1.1
Number Sense 1.2
- Time Required: 15-20 minutes (several times during Year)
- Grouping: Individually, small group
- Materials: dice (one for every child playing), laminated (or not) number grid (see figure 13)
- Object of Activity: To record on a grid or graph the results of each die throw.
- Instructions: 1. Place one die and one number grid sheet at center for each child.
2. The student throws the die and marks the appropriate square, in the appropriate number column.
3. The student continues these steps until one of the number columns is completely marked.
- Remediations: This is a fun game but it can be confusing to some students. But, it may be less difficult if they do not have as many numbers to concern themselves with, so the teacher may want to block out several of the numbers with stickers or masking tape. Starting with just two or three numbers may be helpful. Before the slower students start with dice the teacher may want to try a two sided coin with a number on each side. In this way the students only have to worry about two columns. The student can also practice numeral writing by filling each square with the numeral instead of coloring or putting an "x".

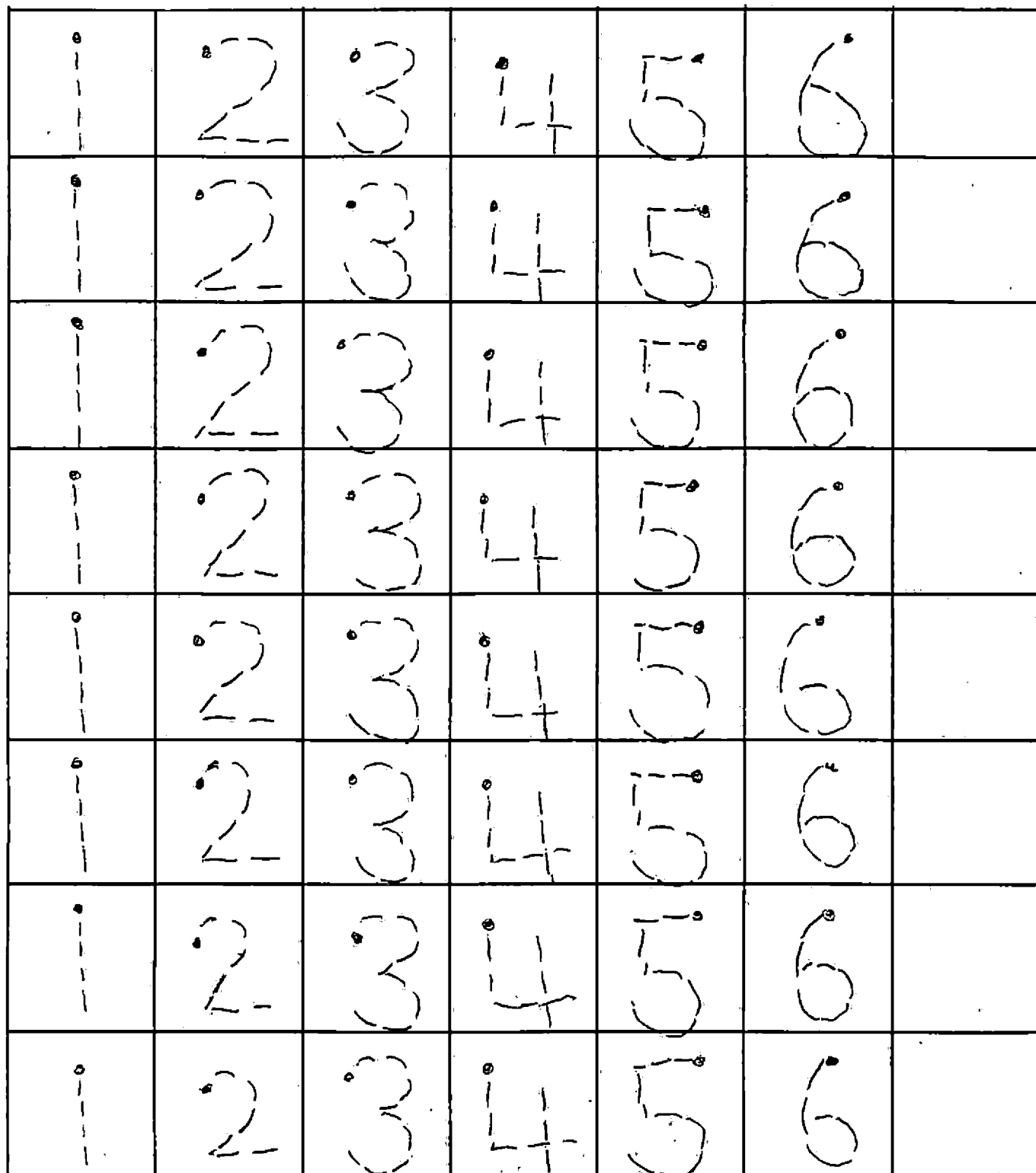
Extensions:

The advanced student may find it more challenging to use two, three or even four dice simultaneously. The teacher will need to modify the number grid accordingly.

Figure 13 Number Grid or Graph Sheet.

1	2	3	4	5	6

Figure 14 Remediation Example.

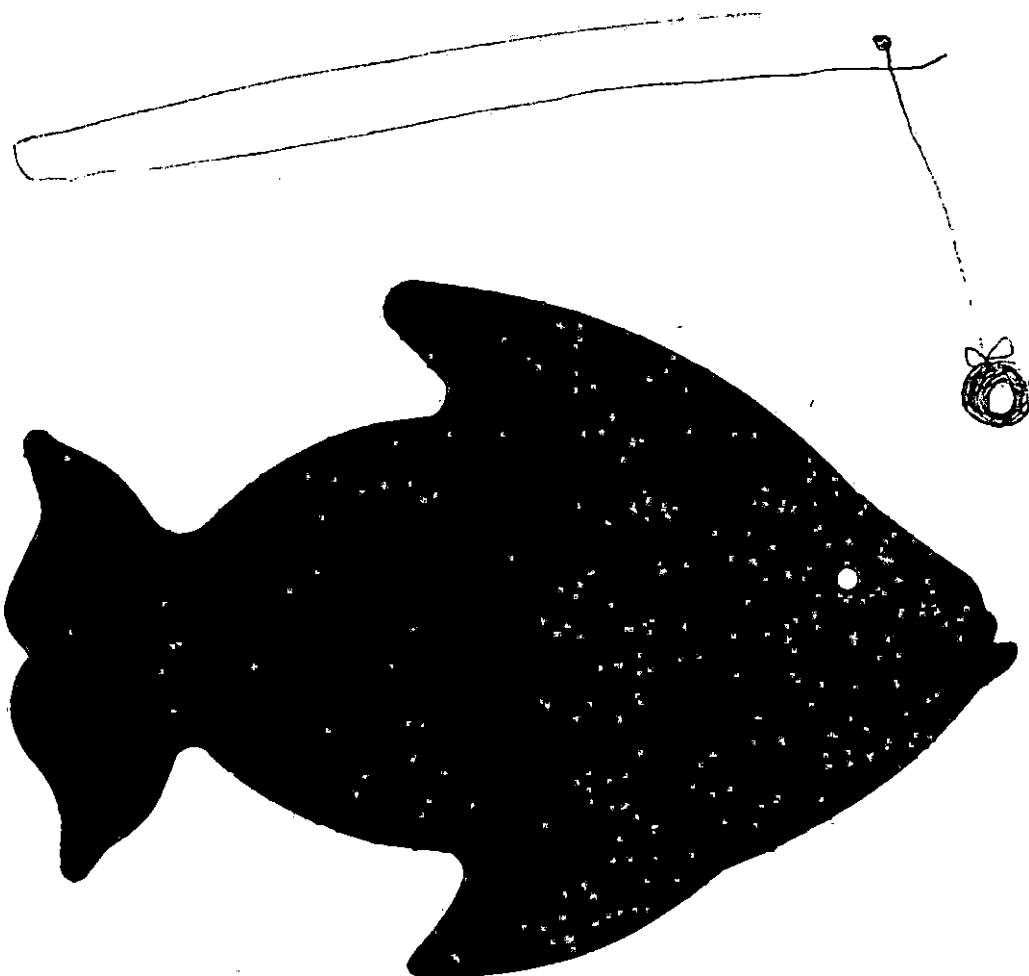


Gone Fishing

- Content Standard: Number Sense 1.1, 1.2, 1.3
- Time Required: 10-20 minutes
- Grouping: pair or small groups
- Materials: large box (larger than a shoe box) covered with blue paper, different colored fish with numbers printed on one side (and the quantity in dots on the other if needed), paper clips, 2-4 sticks, and string and ring shaped magnets to tie on the string.
- Object of activity: To recognize and compare numbers and their quantities.
- Instruction:
1. The teacher will put fish with paperclips in lake (box) and assemble fishing poles.
 2. Students will take turns fishing. They may only catch one fish at a time.
 3. If they can identify the number or the quantity he or she may keep the fish. If not the must throw it back.
 4. Students will continue until all fish are caught. The winner is the student who ends up with the most fish.
- Remediations: Those students who have trouble with numeral recognition should use the dot side for counting instead. A number line, for counting, at this center may also be helpful.
- Extensions: Larger numbers may be more challenging for the advanced child. Adding a comparing aspect to the game may make

it more exciting. In this game the students would all catch a fish and when everyone has their fish they could compare numbers. The person with the highest (or the lowest) number would win that round and everyone else would have to throw their fish back into the "lake." The game ends when all fish are caught, and the person with the most fish wins.

Figure 15 Fish Shape Pattern and Pole.



Candy Graphs

- Content Standard:** Statistics, Data Analysis, and Probability 1.1
Mathematical Reasoning 1.1
Number Sense 1.1, 1.2
- Time Required:** 20-30 minutes
- Grouping:** individually or small groups
- Materials:** graph sheets - these will vary depending on theme, candy such as jelly beans, talking hearts, m&ms, or gummi bears, crayons or colored pencils, sandwich bags
- Object of Activity:** To practice graphing and comparing information.
- Instructions:**
1. The teacher will place graphs at the center and sandwich bags filled with the candy, no more than twenty pieces in each and no more than 5 or 6 of each color (some colors more than others).
 2. The students will record the amount of each color on their graph.
 3. They will then compare their graphs with someone else's.
 4. Eat candy.
- Remediations:** The students who have trouble with this can work with fewer colors and fewer quantities. They may also need a model exactly like theirs in the beginning. It may also help to make the quantities of each color the same.
- Extensions:** To make this more challenging some students can receive a blank grid and they can fill in the squares with any information they think is pertinent.

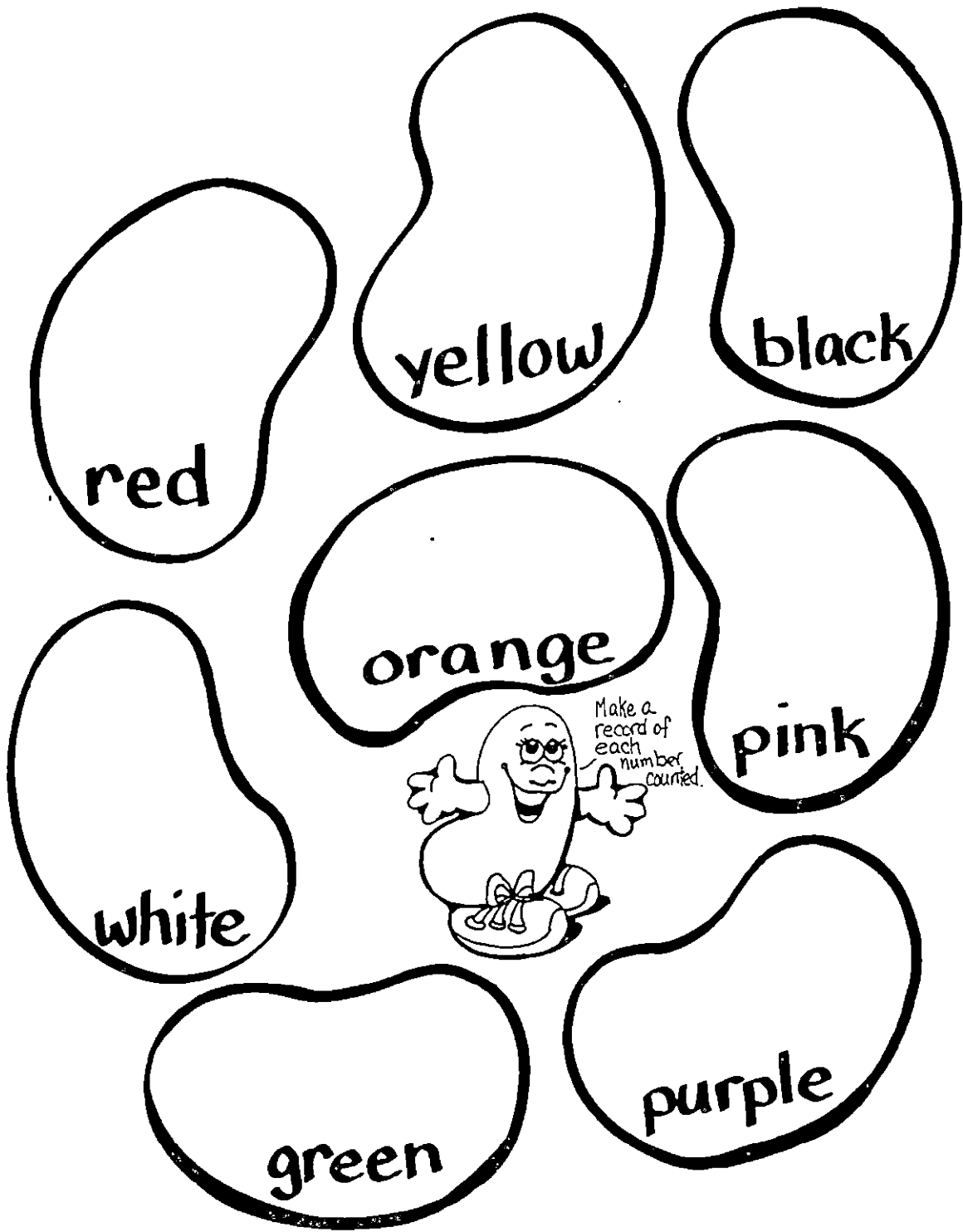
Figure 16 Basic Candy Graph.

(Type of candy)	Name	red	orange	yellow	green	purple	pink	white	black

Candy Sorting

- Content Standard: Algebra and Functions 1.1
Mathematical Reasoning 2.1
- Time Required: 10-20 minutes
- Grouping: individually or in small groups
- Materials: Candy- gummi bears, m&ms, talking hearts, etc., sorting sheets, sandwich baggies
- Object of activity: To practice sorting by attributes.
To compare findings.
- Instructions:
1. Give students baggies filled with candy and sorting sheets or cups.
 2. Students will sort candy into groups.
 3. When finished they will compare findings with others at center.
 4. Students can either glue candy to the worksheet or eat candy. However, if they eat it they will have to record findings on worksheet by drawing the quantity or writing the number.
- Remediations: Sorting by color is the easiest method. The teacher may want to instruct the slower students to group by color and also tell them where each color should go. Limiting the number of colors to sort may also make this assignment easier.
- Extensions: The advanced student may find it more challenging to organize his/her own sorting sheet on blank paper.

Figure 17 Sorting Sample.



Estimation

- Content Standard: Number Sense 1.2, 3.1
- Time Required: 10-20 minutes
- Grouping: individually
- Materials: buttons, beads, marbles and any other small items that are easy to count, baggies or clear containers that can hold these items, paper to write estimate, stamps and pad, laminated number line with corresponding boxes for each number
- Object of activity: To practice making reasonable estimates and to practice counting.
- Instructions:
1. Teacher will place objects in various baggies or containers (all items in baggie should be same, but may vary from baggie to baggie) and cards or sheets of paper to write estimates.
 2. Paper and baggies should be in pairs so there is no confusion.
 3. Student will pick up bag and estimate quantity and write it on the paper (or pinpoint it on number line).
 4. They will then open the container and count to see how many there actually are.
 5. If they were close or exact they should put a stamp on their paper.
- Note: This exercise may need some monitoring by an adult at the beginning.
- Remediations: A number line with boxes may help all students. After examining the container (without taking out its con-

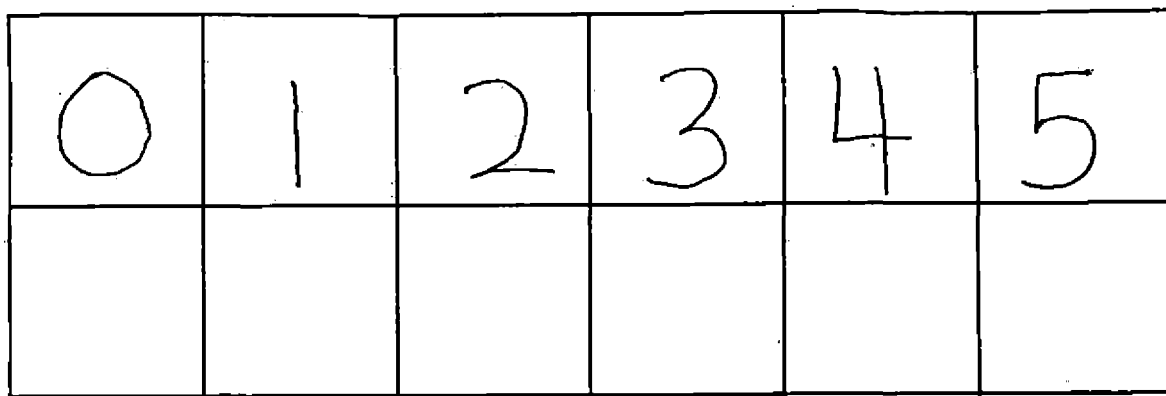
tents) the students will place an "X" in the appropriate box on the number line.

Then they will take out the contents, count and mark the exact amount on the number line. Last they should compare the difference visually. Using small quantities can also make this task easier.

Extensions:

Containers with larger quantities to estimate may make this activity more challenging. The teacher may also ask the advanced student to find the differences in their estimations mathematically.

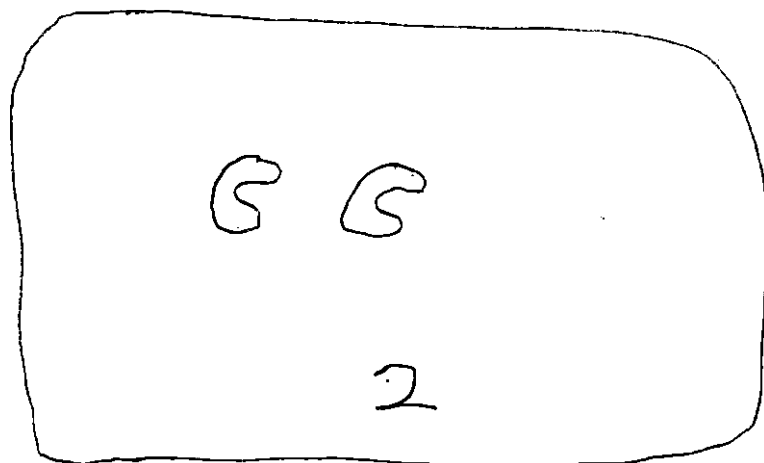
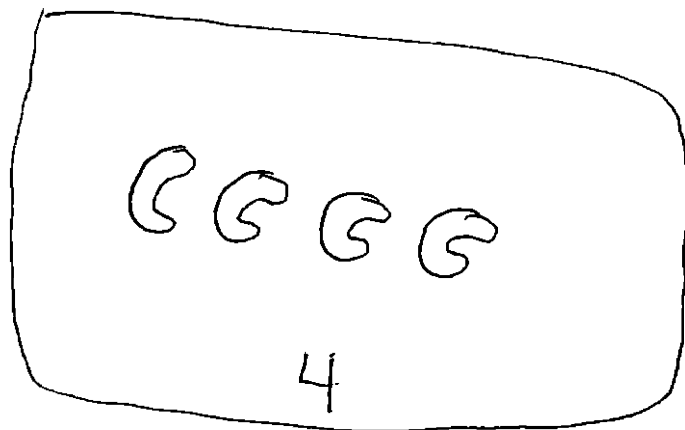
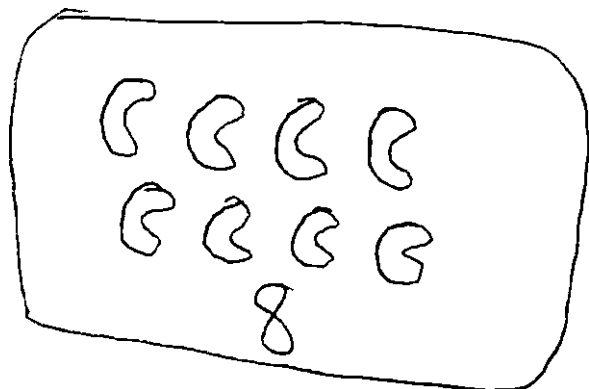
Figure 18 Number Line With Corresponding Boxes.



More Sets

- Content Standard: Number Sense 1.1, 1.2, 1.3
- Time Required: 10-15 minutes
- Grouping: Individually
- Materials: Cotton balls, beans, pieces of pasta, popcorn, or cheerios, styrofoam meat trays or paper plates, markers, construction paper, glue
- Object of Activity: To match numeral to quantity and provide counting practice.
- Instructions:
1. Give each student a 9X12 sheet of construction paper that is already divided into four sections.
 2. Student will cut sheet into four sections.
 3. Have the students write a number of their choice in 1 corner of each sheet (but the teacher can set the choice range).
 4. Place the trays of the different objects at the table.
 5. Select the corresponding number of objects from the trays and glue to paper. For example, on the number 5 the student would glue five pieces of macaroni. Display sets of students' choice.
- Remediations: Putting the correct number of boxes on the paper to correspond with the quantity may help make this task easier (glue items in each square). Smaller numbers may also help.
- Extensions: Doing this project using larger numbers will make it more challenging.

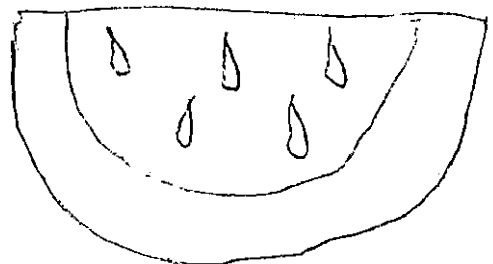
Figure 19 Making More Sets.



Watermelon Seeds

- Content Standard: Number Sense 1.1, 1.2, 1.3
- Time Required: 15-20 minutes
- Grouping: individually
- Materials: beans or watermelon seeds, paper plates cut in $\frac{1}{2}$, red and green markers or paint, glue
- Object of Activity: To practice numeral recognition and counting.
- Instructions:
1. Student should make watermelon slice. Color or paint the rim of the plate green and the center red.
 2. Teacher writes a numeral on the slice.
 3. The student reads the numeral and places that many Seeds (beans) on the slice.
 4. If the student does more than one then he/she may put them in order.
- Remediations: The teacher can draw seed place holders so that the student who cannot count objects will know how many seeds to glue. Staying with lower numbers is also a good idea.
- Extensions: The advanced student can do both sides (or two) and show which side has more, or he/she may find the difference.

Figure 20 Watermelon Seeds.

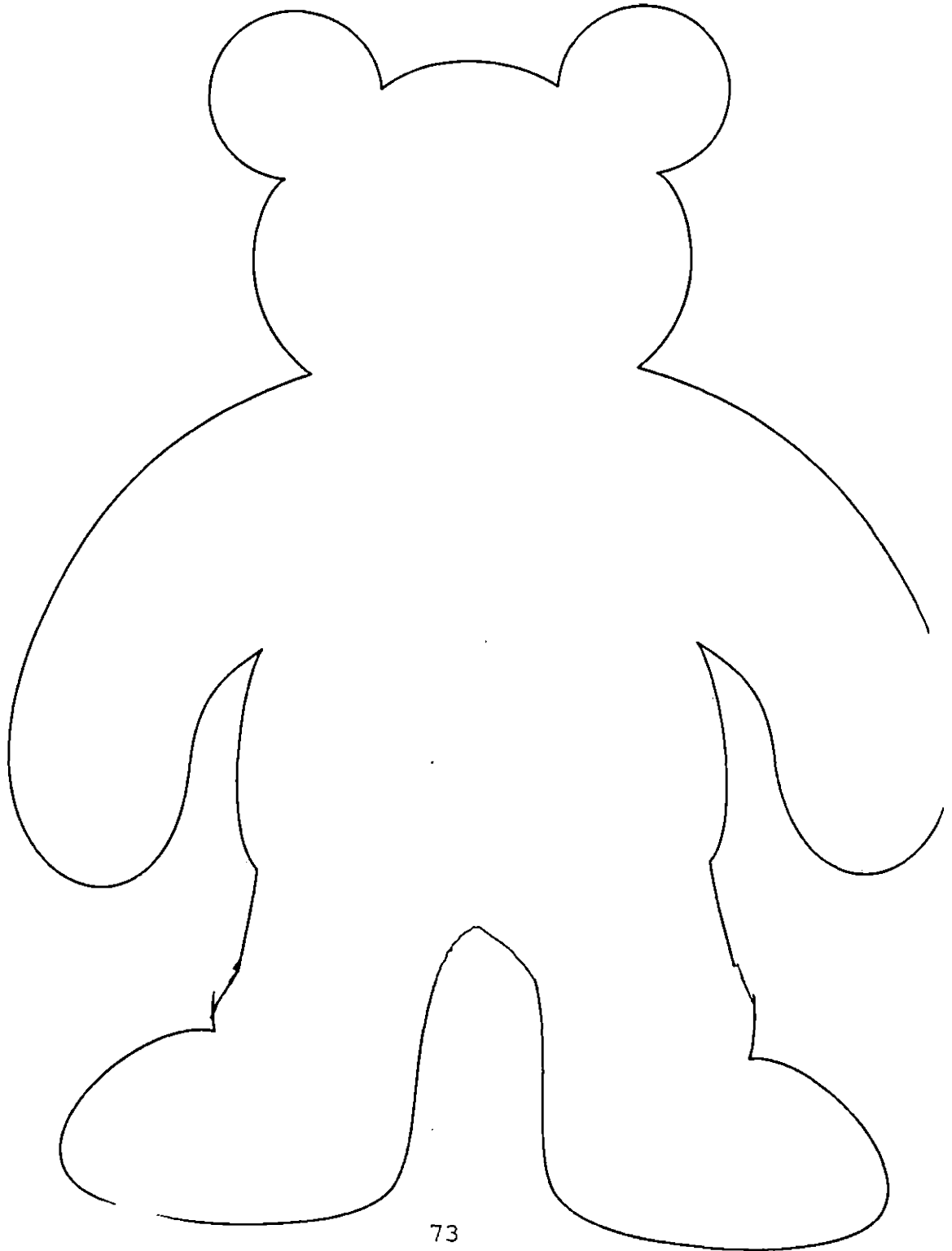


Seasonal Counting Activities

- Content Standard: Number Sense 1.1, 1.2, 3.1
- Time Required: 10-20 minutes
- Grouping: individually
- Materials: holiday patterns, decorations
made from stickers or cut-outs, glue,
scissors, crayons
- Object of Activity: To provide practice estimating and counting.
- Instructions:
1. Teacher will make 10 decorated trees. Write the number of ornaments on the trees on the back.
 2. Students will estimate how many ornaments on the tree (can use number line from figure 18).
 3. They will count the ornaments and then flip the tree to check their answer.
 4. As an optional activity the students may make their own trees with given numbers on each and they must match the quantity by gluing the items.
- Remediations: The students who have problems with this activity may find it easier to glue and count their ornaments then write the matching number at the bottom. They may also write the numbers on the ornaments for extra practice writing and counting.
- Extensions: Advanced students may want to find the difference between their estimation and the actual quantity by counting on a number line or subtracting if they can.

Note: This activity can be adapted to fit any theme, ghosts, hearts, bears, etc.

Figure 21 Theme Pattern to Enlarge.



Shape Sponge Painting

- Content Standard: Measurement and Geometry
- Time Required: 5-10 minutes
- Grouping: individually
- Materials: shape sponges of different sizes, pie tins, paint, construction paper
- Object of Activity: To focus on geometric shape while painting.
- Instructions:
1. Teacher will place sponges and pie tins with paint and paper at center. At the beginning of the year it may be best to focus on only one shape at a time. Combining shapes in one painting can be saved for later in the year.
 2. Students will dip sponges into the paint and press on the construction paper.
 3. They will repeat this several times or until their painting is complete.
- Remediations: Limit students who have problems with this to only one shape.
- Extensions: The advanced students can be asked to create an object or an animal using the sponge paints.
- Note: Shape sponge painting is also excellent for a patterning activity (circle-square-circle-square).

Category Chart With Shapes

Content Standards: Measurement and Geometry 2.1

Time Required: 10-15 minutes

Grouping: individually

Materials: various shapes traced and ready to cut out (limit shapes to circles, squares, triangles, rectangles) 12X18 construction paper, scissors, glue

Object of Activity: To provide practice sorting by shape.

Instructions:

1. Give each student three or four of each shape and a sheet of construction paper divided into four sections.
2. Students will cut out all of their shapes.
3. Have students group them by shape and glue each set in a common section on the construction paper.
4. Copy shape word in each section.

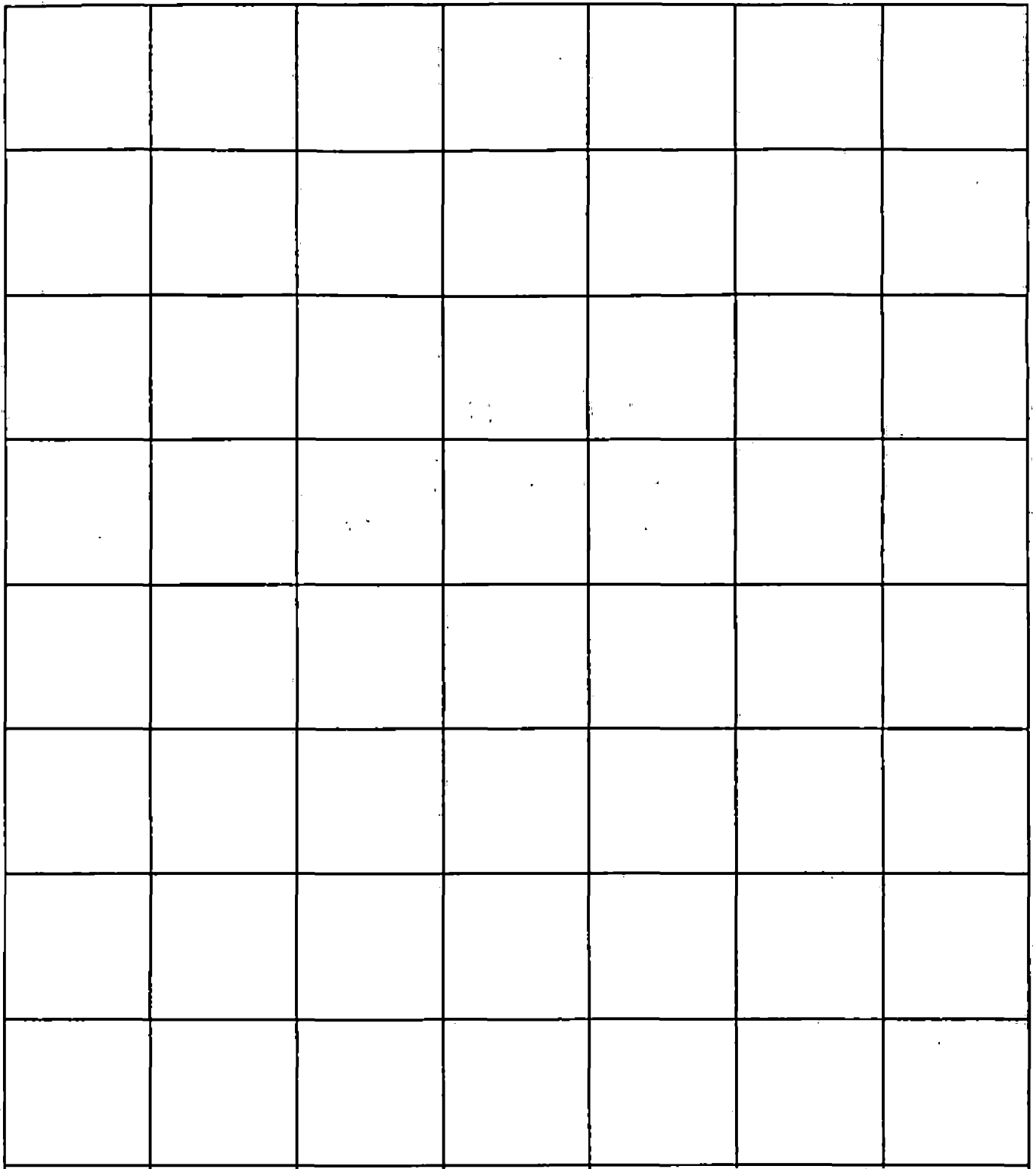
Remediations: It would be helpful to mark each section with the appropriate shape so that the students know where to glue their shapes.

Extensions: The advanced students could turn their chart into a graph by placing their shapes in a linear fashion within each section, and writing 1,2,3,4 on each one to show quantity.

More Graphing

- Content Standard:** Algebra and Functions 1.1
Statistics, Data Analysis and
Probability 1.1
Mathematical Reasoning 1.1, 1.2, 2.1
- Time required:** 5-15 minutes
- Grouping:** individually or in pairs
- Materials:** Macaroni, beans, buttons, any other
small items that come in mass
quantities, graphing worksheet, glue
- Object of Activity:** To provide practice sorting, recording
data, and analyzing information.
- Instructions:**
1. Students should have various items to sort, and graph.
 2. Students will sort by attribute and glue items on graph sheet by group.
- Remediations:** The sorting portion of this activity can be completed using paper plates marked with an illustration that tells which items to place in each plate. Then the items can be placed into the columns or rows that are also clearly marked.
- Extensions:** Have students design their own graphs.

Figure 22 Graphing Grids.



Cage the Animals

- Content Standard: Algebra and Functions 1.1
Mathematical Reasoning 2.1
- Time required: 10 minutes
- Grouping: individually or in pairs
- Materials: large sorting mat made from fabric or paper (cages), or boxes can serve this purpose, stuffed animals everyone has brought from home
- Object of Activity: To provide practice sorting.
- Instructions: 1. Teacher will make "cage" mat or decorate boxes.
2. Students will place all the animal that are running around the zoo back into the cages they belong.
- Remediations: The teacher can place picture cards in the appropriate cages so the children who have problems with this task will have a guide.
- Extensions: The advanced students can write the animal name and the quantity on a card they place in the cages.

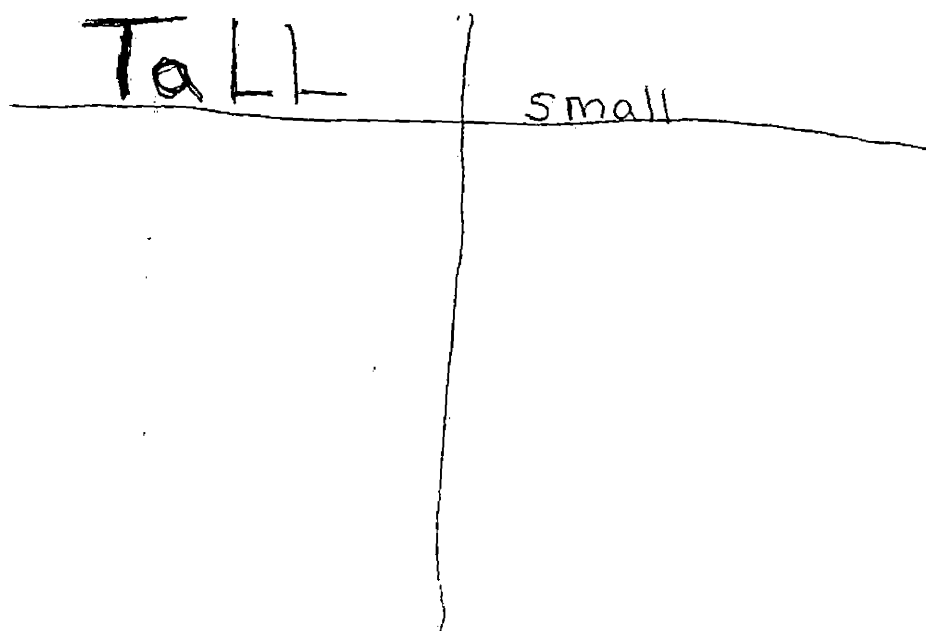
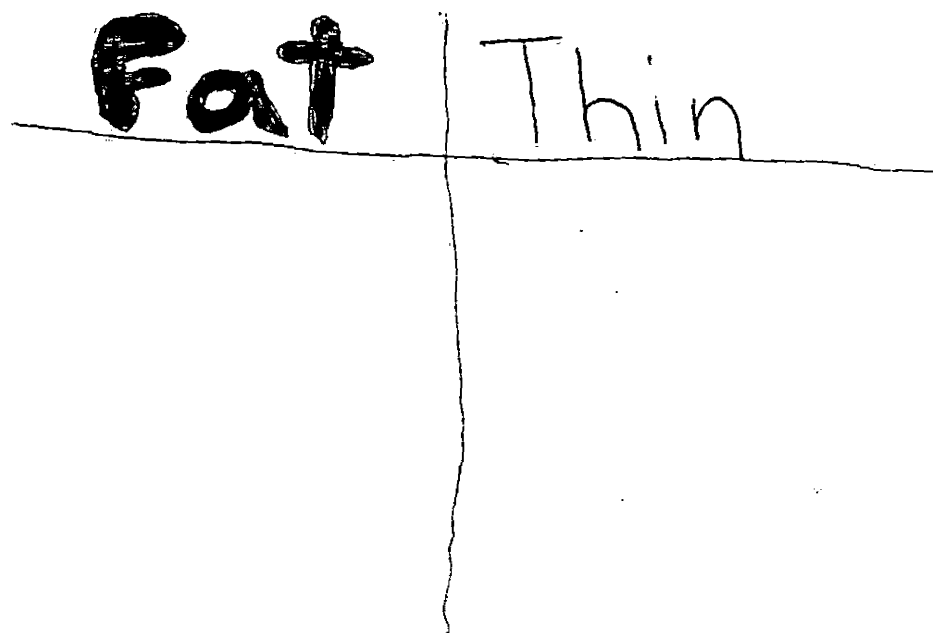
Domestic or Wild?

- Content Standard: Algebra and Functions 1.1
Mathematical Reasoning 2.1
- Time Required: 10-15 minutes
- Grouping: individually, pairs or small group
- Materials: large mat or chart with two sides,
stuffed animals brought from home
- Object of Activity: To provide practice sorting
and graphing animals.
- Instructions:
1. Place mat in middle of group on the floor.
 2. Each child takes turn deciding whether their animal is domestic or wild, and places it on the appropriate side.
 3. Repeat until all animals have been placed.
- Remediations: A similar completed picture chart may help children who have problems with this task.
- Extensions: These students can tally their results, make comparisons and report them to the class.

Measuring, Comparing and Sorting

- Content Standard: Measurement and Geometry 1.1
Algebra and Functions 1.1
- Time Required: 10-20 minutes
- Grouping: individually or in pairs
- Materials: Sorting mats (see directions)
items to sort - small toys or
stuffed animals, yarn or string
measuring devices such as string,
stick, balance scales
- Object of Activity: To provide practice comparing,
measuring and sorting items.
- Instructions:
1. Teacher will make labeled mats. These mats can be labeled fat/thin or tall/short, etc. See figure 23.
 2. A piece of yarn can be stapled to the middle on one side, and can be stretched out to form the columns.
 3. Children will work alone or in pairs comparing their toys or stuffed animals using the mats to sort or graph them.
 4. This activity can be repeated with other categories too - heavy/light or short/long for example.
- Remediations: All students may need measuring devices such as sticks, string, balance scales to help them make their comparisons. Showing them how to compare items to each other is also necessary. Sample picture graphs may be helpful too.
- Extensions: Some students may find it more of a challenge to make up their own categories. They can also make up a report or tally sheet to show their findings.

Figure 23 Labeled Mats.



Stuffed Animal Patterning

Content Standard: Statistics, Data Analysis and Probability 1.2

Time Required: 10-15 minutes

Grouping: individually or in pairs

Materials: Stuffed animals students have brought from home, large floor area

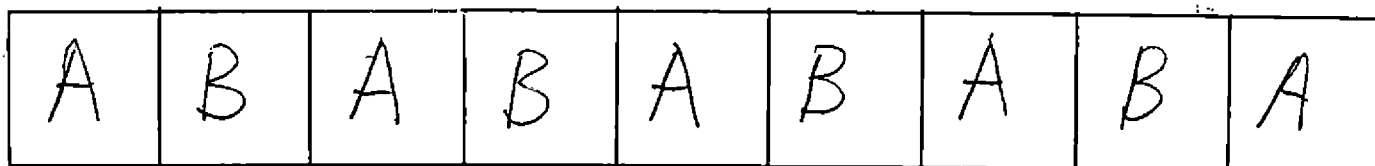
Object of activity: To provide practice patterning.

Instructions: 1. Students will line the stuffed animals into a pattern (by color, size shape) that they can describe or explain. Teacher may want to give a few ideas.

Remediations: A simple ABAB pattern marked on a mat with big and little circles to show what comes next may work for the student who does not understand this activity.

Extensions: The advanced students may find it more challenging to design their own (more complex) patterns. The teacher may suggest a few patterns and the students can choose from them.

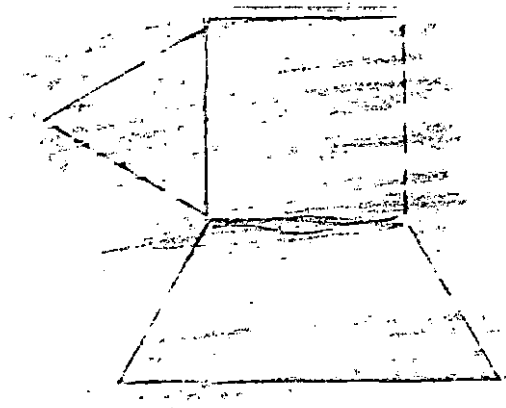
Figure 24 Remedial Pattern Mat.



Pattern Block Rubbings

- Content Standard: Measurement and Geometry 2.1
- Time Required: 15-20 minutes
- Grouping: individually
- Materials: pattern blocks, paper, crayons
pattern die cuts (most schools or districts have these machines and patterns), or Creative Publications
pattern stickers, paper, glue
- Instructions:
1. The children will make designs with pattern blocks.
 2. Record the designs on paper with the pattern stickers or paper shapes (glue).
 3. Make rubbings of the sticker recordings.
 4. Recreate other students designs by putting blocks on the rubbings.
- Remediations: Have several patterns already glued to paper for the students who have trouble recording their own patterns.
- Extensions: The advanced students should be encouraged to make more complex designs. The teacher may want to let them glance at one or two samples for ideas.

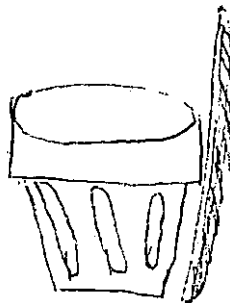
Figure 25 Pattern Rubbing.



Strings and Things

- Content Standard: Measurement and Geometry 1.1
Number Sense 3.1
- Time Required: 20-30 minutes
- Grouping: pairs
- Materials: yarn, scissors, paper cut in half, crayons, tape, clothesline, clothespins
- Object of Activity: To provide practice estimating length measuring items.
- Instructions:
1. Teacher will hang clothesline and pins.
 2. Students will measure several items that can be found in the classroom with yarn.
 3. They will cut yarn same size as item.
 4. Draw a picture of that item, tape to yarn and hang on clothesline.
- Remediations: Students who have problems with this can "help" someone who knows how and do the sections that are comfortable to them.
- Extensions: In addition to measuring, the advanced students can take everything that has been measured and put them into order by size or classify them.

Figure 26 Measurement Sample.



Geoboards

- Content Standard: Measurement and Geometry 2.1
- Time Required: 10-15 minutes
- Grouping: individually
- Materials: geoboards, rubberbands
crayons, paper
- Object of Activity: To provide opportunities to focus on geometric shapes.
- Instructions:
1. Before starting let the students know on what shape they will be focusing.
 2. Students will make various sizes of that shape.
 3. Try to form an object using only the given shape (or a combination of two shapes).
 4. Copy on paper what they have created.
 5. Teacher should have shape word at center so that students can copy it.
 6. Display work.
- Remediations: The students who have trouble with this activity may need pattern cards to copy.
- Extensions: These students can create more complex items using more than one shape as long as they know what shapes they are.
- Note: This activity can also focus on forming numbers and letters.

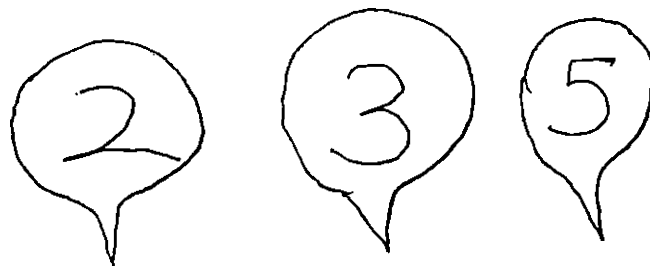
How Many Are In My Other Hand?

- Content Standard: Number Sense 2.1, 3.1
- Time Required: 5-10 minutes
- Groupings: pairs
- Materials: Small counters such as beans, beads, counting bears or linker cubes
- Object of Activity: To provide opportunities to see that number combinations will equal bigger numbers.
- Instructions:
1. Teacher will pick a number such as three and have each pair of students take that many counters.
 2. The first student will put some counters in each hand.
 3. That student will open one hand and show how many are in it.
 4. The other child will guess how many counters are in the other hand.
 5. The first student will then open the remaining hand and show the counters.
 6. Together the students will say, "3 is 1 and 2."
 7. They will take turns repeating these steps several times.
 8. Next lesson go up to next number. Do this activity many times.
- Remediations: The students who have problems with this activity may find it helpful to use number cards. For the number three they will have several cards that demonstrate how the numbers can be combined. See the following figure.

Extensions:

The advanced students may be able to start with larger numbers. They may also write or illustrate the different number combinations.

Figure 27 Number Combination Sample.



Beginning Addition and Subtraction

- Content Standard: Number sense 2.1
- Time Required: 20-30 minutes
- Grouping: Small group
- Materials: counters or linker cubes
problem worksheets
- Object of Activity: To provide practice adding or subtracting to sums and differences less than ten.
- Instructions:
1. Explain what the + (or -) and = signs mean.
 2. Give everyone ten counters. Then walk through a problem verbally and demonstrate what it looks like.
 3. The teacher will say $3+2=5$ and show it with the counters.
 4. The next time the students will do it with the teacher. This may need to be repeated three or four times.
 5. When the students are ready they can be shown how to do it with a written problem, see next figure. It may take several lessons before they are able to work with written problems.
- Remediations: The students who have problems do not need to move into the written problems. They should stay with simple verbal instructions such as one bean and two beans are three beans. Give them lots of time to count and be comfortable with these number sentences and their meanings.
- Extensions: Before moving on to larger numbers let the advanced students work with written problems using manipulatives or dots.

Do not rush them. Bigger numbers are not necessarily better. Do this activity many times.

Figure 28 Beginning Addition Worksheet.

$$2 + 3 =$$

$$4 + 2 =$$

$$3 + 1 =$$

$$5 + 1 =$$

Ten by Ten

- Content Standard: Number Sense 1.1, 1.2, 1.3
- Time Required: 20-30 minutes
- Grouping: individually
- Materials: small glueable objects(mass quantities)
cereal, beans, macaroni, construction
paper (charted by tens), glue, yarn
- Object of Activity: To provide opportunities to count by
ones and tens.
- Instructions: 1. Place item and worksheet or yarn at
table.
2. Have students glue items in rows of
ten or string cereal in groups of ten
until they have made ten rows or groups
of ten.
- Remediations: Have rows clearly marked and items
piled into groups of ten. This should
make this activity easier to do.
- Extensions: Let the advanced students design their
own rows or groups of ten. Then they
can count their groups by tens, 10, 20,
30, etc.

Figure 29 Tens Chart.

Collection of 100

	Name									
	1	2	3	4	5	6	7	8	9	10
I										
11	12	13	14	15	16	17	18	19	20	
21	22	23	24	25	26	27	28	29	30	
31	32	33	34	35	36	37	38	39	40	
41	42	43	44	45	46	47	48	49	50	
51	52	53	54	55	56	57	58	59	60	
61	62	63	64	65	66	67	68	69	70	
71	72	73	74	75	76	77	78	79	80	
81	82	83	84	85	86	87	88	89	90	
91	92	93	94	95	96	97	98	99	100	

Section 6: Assessment

The aim of assessment should be to monitor students' progress, make instructional decisions and evaluate students' achievement (NCTM, 1995). In order for the program to be effective the information gathered during assessment should be used for planning subsequent activities. It must also be an on going activity that is convenient for the teacher to manage. Included here are assessment ideas that are easy to use. A 0-3 rubric, student profiles, and authentic assessment will be discussed.

The first method includes a 0-3 rubric that teachers can use to find out what the students' general level of understanding is. This simple chart describes the four levels. They are as follows:

Levels of Response

0 - Student cannot do any part of task. Cannot communicate concept involved.

1 - Student cannot do the task with out step-by-step guidance. Communication of concept is vague.

2 - Student can complete the task with some help. Communication of concept is limited but understandable.

3 - Student can accomplish the task without help. Communication of concept is clear.

This chart is an easy and quick way for the teacher to get a sense of how well the student is learning the given concepts. Below is an assessment sheet that incorporates the rubric discussed here.

Figure 30 assessment sheet.

0- Cannot do any part of task. Cannot communicate concept.		
1- Cannot do task without step-by-step guidance. Communication is vague.		
2- Can complete the task with some help. Communication of concept is limited but understandable.		
3- Can accomplish task without help. Communication of concept is clear.		
Date _____	Activity _____	
Name	Level	Notes
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____

In order to complete the preceding assessment sheet the teacher must give the student a 2-3 question interview.

The first question may focus on the student's attitude toward the task at hand. The next two questions should focus on the student's knowledge. How will you arrange these? His/her ability to communicate the ideas presented will also be questioned. Why does it have to go this way? The following are a few tips that may help with this assessment:

- 1) Try to keep the interview as informal as possible so that the child is not self-conscious.
- 2) Allow plenty of response-time to questions.
- 3) Do not correct student misunderstandings during assessment. Avoid letting children know whether they are right or wrong. Do activities several times before assessing them.

Authentic assessment is a key element in all of the ideas suggested in this project. This term refers to the assessment of a student under natural circumstances as opposed to being a formal testing exercise that is not part of a routine activity. The teacher may also collect finished products from the student and save them in a portfolio. These products can be used to track the progress of the student throughout the year. Anecdotal notes are also important. The next sample is a profile sheet that can be kept on each student. This profile contains the date, level of understanding (0-3), task, and anecdotal

notes. The level of understanding rubric discussed previously should be used with this profile sheet.

These are just a few suggestions for assessment. There are various other methods and the teacher should feel free to try several before choosing the most comfortable and convenient ways.

In kindergarten, students learn and develop at amazing rates. It is the responsibility of the teacher to facilitate that learning through assessment and deliberate planning. There should be a variety of activities included in the centers portion of the mathematics program to accommodate the various learning styles. Above all, kindergarten mathematics should be interactive and fun so that the students learn and develop a good attitude toward mathematics.

Section 7: Mathematics through Literature

Literature plays an important part in kindergarten mathematics. A variety of concepts can be introduced using interesting stories. In this section books will be listed by various mathematical concepts. The titles, authors, and ISBN will be included here so that teachers can purchase these books.

Predictable Patterns:

Martin, Bill Jr. *Brown Bear, Brown Bear, What Do you See?* Henry Holt and Company, 1983. ISBN: 0-8050-0201-4

McQueen, Lucinda. *The Little Red Hen*. Scholastic, 1985. ISBN: 0-590-41145-4

Rosen, Michael. *We're Going On a Bear Hunt*. McElderry Books, 1989. ISBN: 0-689-81581-6

Wood, Audrey. *The Napping House*. Harcourt, Brace, Jovanovich, 1984. ISBN: 0-152-01062-9

Beginning Numbers:

Galdone, Paul. *The Three Bears*. Clarion Books, 1972. ISBN: 0-395-28811-8

Galdone, Paul. *The Three Little Kittens*. Houghton Mifflin & Co. 1986. ISBN: 0-899-19426-5

Marshall, James. *Three Little Pigs*. Turtleback Books 1996. ISBN: 0-606-09966-2

Stevens, Janet. *Three Billy Goats Gruff*. Harcourt, Brace, Jovanovich, 1990. ISBN: 0-152-86397-4

_____. *Walt Disney's Snow White and the Seven Dwarfs*.
Random House, 1973. ISBN: 0-736-40122-9

Sorting and Classifying

Hoberman, Mary Ann. *Here is a House for Me*. Viking
Penguin, 1978. ISBN: 0-823-69838-9

Slobodkina, Esphyr. *Caps for Sale*. Addison-Wesley,
1940. ISBN: 0-590-71742-1

Zolotow, Charlotte. *Mr. Rabbit and the Lovely Present*.
Harper and Row, 1962. ISBN: 0-060-26945-6

Counting Books:

Boegehold, Lindley. *The Gummi Bear Counting Book*.
Lorenz Books, 1997. ISBN: 1-85967-601-4

Crews, Donald. *Ten Black Dots*. Scholastic, 1986.
ISBN: 0-590-46479-5

Hammond, Franklin. *Ten Little Ducks*. Scholastic, 1987.
ISBN: 0-590-73339-7

And many more...

Shapes:

Ehlert, Lois. *Color Farm*. Harper Collins Juvenile Books
1997. ISBN: 0-694-01066-9

Engel, Diana. *Circle Song*. Marshall Cavendish Corp.
1999. ISBN: 0-761-45040-8

Grover, Max. *Circles and Squares Everywhere*. Browndee
Press, 1996. ISBN: 0-152-00091-7

Hoban, Tana. *Shapes, Shapes, Shapes*. William Morrow &
Company, 1996. ISBN: 0-688-05832-9

Measuring:

Hoban, Tana. *Is It Larger? Is It Smaller?* Turtleback Books 1997. 0-606-11510-2

Lionni, Leo. *Inch by Inch.* Mulberry Books 1995. ISBN: 0-688-13283-9

King, Andrew. *Measuring Sizes.* Copper Beech Books, 1998. ISBN: 7-061-30853-9

There are of course many other books that could be listed here, but that are not. Teachers should look in there school or public libraries for some of these titles. The stores are the last resort. The teacher may also look in the school book orders forms for these.

Evaluation of Project

1. Clarity of the project _____

2. Practicality of assessment used in this project _____

3. Suggestions for improving the project _____

4. Which lessons did you like and why? _____

5. Which lessons did you dislike and why? _____

Bibliography

- Andrews, Angela. (1995, October). The role of self-directed discovery time in the development of mathematics concepts. Teaching Children Mathematics, 2, pp. 116-120.
- The California mathematics academic content standards.
California State Board of Education. 1998, February.
- Chambers, Donald. (1994, October). Cognitively guided instruction. Teaching Children Mathematics, 1, pg. 116.
- Copeland, Richard W. (1984). How children learn mathematics: teaching implications of piaget's research. New York: Macmillan Publishing Company.
- Corneille, Birgitta. (1997, October). Recognizing the mathematics. Teaching Children Mathematics, 4, pp. 112-115.
- Corwin, Rebecca. (1996). Talking mathematics: supporting children's voices. Portsmouth, New Hampshire: Heinemann.
- Curcio, Frances & Folkson, Susan. (1996, February). Exploring data: kindergarten children do it their way. Teaching Children Mathematics, 2, pp. 382-385.
- Downs, Robert E. (1978). Friedrich Froebel. Boston, Massachusetts: Twayne Publishers.
- Dumas, Enoch. (1971). Math activities for child involvement. Boston, Massachusetts: Ally and Bacon, Inc.
- Gelman, Rachel & Gallistel, C.R. (1986). The Child's Understanding of Number. Cambridge, Massachusetts: Harvard University Press.
- Jensen, Rosalie & Spector, Deborah. (1984). Teaching mathematics to young children. Englewoods Cliffs,

New Jersey: Prentice-Hall, Inc.

- Joyner, Jeane M. (1995, September). NCTM's assessment standards: a document for all educators. Teaching Children Mathematics, 2, pp. 20-22.
- Kamii, Constance, Lewis, Barbara A. & Jones, Sally. (1991, Fall). Reform in primary mathematics education: a Constructivist view. Educational Horizons, 70, pp. 19-26.
- Kamii, Constance & Georgia De Clark. (1985). Young Children reinvent arithmetic: implications of Piaget's theory. New York: Teacher's College Press.
- Lovell, Kenneth. (1971). The growth of understanding in mathematics. New York: Holt, Rinehart and Winston, Inc.
- Midkiff, Ruby B. & Cramer, Mary M. (1993, February). Stepping stones to mathematical understanding. Arithmetic Teacher, 40, 303-305.
- Ornstein, Allan C. (1992, Summer). The textbook curriculum. Educational Horizons, 70, pp. 167-169.
- Price, Jack. (1995, September). Selling and buying reforms: If we build it will they come? Teaching Children Mathematics, 2, pp. 62-64.
- Principles and standards for school mathematics. National Council of Teacher of Mathematics. 1998, October.
- Riley, Keohane R. & Cobb, Catherine & Marquis, Sue & Howard, Carol. (1993, Summer). Walking the talk: putting constructivist thinking into practice in classrooms, 71, pp. 187-196.
- Science framework for California public schools, Kindergarten through grade twelve. Sacramento California Department of Education, 1992.

Sgroi, Laura A., Gropper, Nancy, Kilker, Mary, Rambusch, Nancy & Semonite, Barbara. (1995, January). Assessing young children's mathematical understandings. Teaching Children Mathematics, 1, 275-277.