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Assessing the three-squares model for teaching algebra

Stephen Guy Richardson

ASSESSING THE THREE-SQUARES MODEL
FOR TEACHING ALGEBRA

A Project

Presented to the

Faculty of

California State University,

San Bernardino

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

in

Teaching:

Mathematics

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March 2012

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ABSTRACT

The purpose of this study was to assess the effectiveness of the three-squares model of teaching algebra on adult students enrolled in Math 101 classes at Platt College in Ontario, California. Students enrolled in Math 101 were invited to participate in the study that was designed to show contrast between two models of teaching, a traditional textbook driven approach versus the three-squares model. Four sections of Math 101 were included in the study, with two sections functioning under each of the two models of instruction employed in the study. The control group, or traditional textbook approach group, consisted of 27 students and the experimental group, or three-squares group, consisted of 22 students. Data were analyzed using ANCOVA with the covariant being an arithmetic/algebra diagnostic. The research hypothesis that predicted a difference in student performance between the two groups was not supported. There were no differences in the mean midterm and final exam scores between the control group (traditional text book approach) and the experimental group (three-squares approach).

ACKNOWLEDGMENTS

I would like to express sincere thanks to Dr. Davida Fischman, the director of the MAT program, for her excellent instruction in teaching geometry and her massive efforts toward the enhancement of mathematics education. Additionally, I would like to thank my project committee for their help and input. Thank you all for your patience and efforts.

DEDICATION

I dedicate all the effort put forth is this research project and my degree to my wife Shalee and my three children, Sabrina, Cayley, and Evan. Thank you for loving me and supporting all my endeavors in teaching math and science. Thank you for helping me put things back together when it felt like everything had fallen apart. Thank you kids for the time with me that you sacrificed so I could complete this work. I will repay every minute.

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CHAPTER ONE

THE THREE-SQUARES MODEL

Research Problem

Essential to learning advanced mathematics is a strong conceptual understanding of algebra. Over years of teaching algebra, the author has noticed that students that succeed in advanced courses have a firm grasp of concepts in addition to skills in computing. Thus the author has developed a model for teaching algebra that is designed to build a strong conceptual understanding of algebra. The three-squares model was developed by the author to meet the learning needs of the students enrolled in Math 101 at Platt College and prepare them for future academic challenges in mathematics. The model has been informally assessed but not quantitatively assessed. The focus of this research is to look deeper into student performance under the three-squares model.

Introduction to the Three-Squares Model

In an effort to improve the effectiveness of teaching algebra to adult students at Platt College, a model for designing activities and presenting lessons was developed; the model is called three-squares. The model is designed around three teaching elements: *algebra habits of mind*, *proof or justification*, and *active learning.* The model can be visually depicted as three squares that overlap in a Venn diagram as shown in Figure 1. The squares are arranged in Figure 1 with the purpose of depicting the overlapping of two or three teaching elements. The model has been used to determine the teaching value of potential activities/resources or for the development of teaching activities and lessons. If an activity or teaching idea falls in at least two of the squares, then it should be designated a good activity or teaching idea; if one falls in all three squares, then it is designated an excellent activity or teaching idea. Additionally, the model can be used as a framework to create lecture notes.

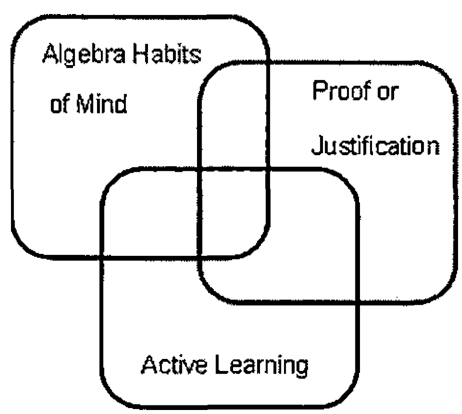


Figure 1. Three-Squares Model

All lecture notes created under the model should address at least two of the three squares with ideal lecture notes addressing all three elements. Further, the model can be used to guide student and teacher discourse; student and teacher discourse may include responding to student questions, probing questions, and teacher generated questions that are designed to move students to delve deeper into content. Essentially all normal teaching practices are filtered through the three-squares model and, if needed, modified to include at least two of the teaching elements of the model. Each square in the figure is described below.

Algebra Habits of Mind Square

The algebra habits of mind square represents a set of thought processes that can be a routine part of an effective algebra curriculum. The *algebra habits of mind* are explained by Mark Driscoll in his book Fostering Algebraic Thinking, A Guide for Teachers, Grades 6 -10. The habits of mind include, *building rules to represent functions, doing and undoing, and abstraction from computation*. Mark Driscoll's book is a research based framework for teaching algebra to students grades six through ten; however, the *algebra habits of mind* have been adapted to fit well into the Math 101 curriculum for adult students at Platt College. The *algebra habits of mind* alone are potentially a pedagogical framework for teaching algebra.

The *building rules to represent functions* habit of mind entails recognition of patterns in calculations or given data and the organization of data with an input and output relationship; that is, the development of a function from data and calculations. The *doing and undoing* habit of mind looks at the reversibility of a process in algebra; students should not just be concerned with a calculation or process, they should additionally be concerned with undoing a calculation or process. Thirdly, *abstraction from computation* is the ability to think of computations independently of any particular numbers being used; this habit of mind lends itself well to the process of generalization much like *building rules to represent functions*. The *algebra habits of mind* work well together to help students progress through an algebra class and lead students to higher levels of mathematical activities such as writing a formal proof.

Proof or Justification Square

Educators and mathematics majors understand the roll of proof or justification in algebra. Students in introductory algebra classes do not always see a need for formal proof, but they will usually concede that there is some need for justification. Many adult students at Platt College have not experienced a formal geometry class where formal proof is taught and expected as a regular mode of operation. Adult students with poor experiences in algebra may not have the expectation that they will have to justify, or in some cases prove, their mathematical conclusions. The *proof or justification square* is designed to bring out the importance of proof or justification in an introductory algebra course and highlight the role of proof in mathematics in general. It is important to note that proof can always function as justification; however, justification does not always involve a formal mathematical proof. For example, the proof of the fact that a negative real number multiplied by another negative real number will always give a positive result is one that is beyond the scope of an introductory algebra class. However, patterns and the properties of real numbers can be used to provide a convincing argument that the product of two negative real numbers is positive. Additionally, proof or justification can be used as explanation for a particular mathematical concept within a lecture or activity and is most effective if the students are involved in the formation of the explanation.

Active Learning Square

Active learning can be applied in many contexts in a classroom. For the three-squares model, active learning means the involvement of the students in the process of the development of the mathematical concepts; the involvement may be deductive or inductive in nature. In the active learning square, activities are used in a way that places the responsibility of the development or discovery of mathematics content on the student. Great care is needed to make sure lessons and activities are designed to promote inquiry and discovery. The *active learning square* should work in concert with the *algebra habits of mind* and *proof or justification squares*.

CHAPTER TWO

LITERATURE REVIEW

Current Research

The three-squares model is the synthesis of twelve years of teaching experience and the core content offered in teaching algebra and geometry in the MAT program at California State University; specifically, three-squares is believed by the author to be a unique framework for structuring the teaching of algebra. Thus there is no literature that addresses the model specifically; however, each square is a topic of concern for many mathematics educators and professional journals such as *Mathematics Teacher* by the National Council of Teachers of Mathematics, and in particular the work in Mark Driscoll's books Fostering Algebraic Thinking, A Guide for Teachers, Grades 6 -10 and Fostering Geometric Thinking, A Guide for Teachers, Grades 5-10. Driscoll's books are the culmination of several teacher-enhancement projects by the Education Department Center (EDC); namely, the Linked Learning in Mathematics project, the Leadership for Urban Mathematics Reform (LUMR), and the Assessment Communities of Teachers (ACT). The work in Driscoll's books and the MAT classroom experiences in Math 631 and 632 (teaching algebra and teaching geometry) derived from Driscoll's books are the inspiration for including the *algebra habits of mind square* in the three-squares model. The algebra habits of mind lend themselves nicely to giving students a complete mathematical experience that includes formal and informal proof.

The inclusion of formal proof in lower math classes, such as courses in arithmetic, pre-algebra, and introductory algebra, can provide rich and valuable learning experiences, and it allows students to understand the role of proof and justification in mathematics in general. Research in the area of student views of mathematics has shown that students see mathematics as a collection of processes and rules (Otten, Herbel-Eisenmann, and Males, 2010), a view that is perpetuated by many of the more mundane direct instruction approaches commonly provided in math education at lower levels. According to Otten et al (2010)

> ...reasoning and proof provide us with a means of working against the kinds of beliefs that disempower students as mathematical thinkers. When students are engaged in proof or proof like activities across and throughout their mathematics courses, they are more likely to realize that mathematics is a state of mind characterized by inquiry and a thirst for justification. (p. 518)

Including logical arguments and proof in pre-algebra and algebra classes is appropriate; students, as mentioned above, need to understand the significance of proof and argumentation in mathematics and see proof as a regular mode of operation. Further, according to Barnes and Hamon (2010), requiring "...students to communicate clearly the reasoning behind their solutions, with appropriate mathematical language and notation, helps lay the groundwork for future, proof-based mathematics classes." (p. 597) There are strong pedagogical reasons for including proof or justification in the three-squares model; additionally, there is reason for promoting student engagement in the discovery and development of algebra content.

In many classroom learning activities, when given freedom and appropriate experiences, students can discover important properties, postulates, or theorems with little instruction from the teacher. Further, students may travel down a path of discovery that may be unexpected. Students should be on the lookout for patterns and be willing to try to explain what they notice. Such authentic learning experiences empower students and provide a high level of efficacy within the students. The teacher role in the activities mentioned above, written or spoken, is to facilitate and refine student thinking. Teachers can provide motivating and/or guiding questions at the right time during an activity in order to keep up the momentum of inquiry. This is the role of the teacher in the *active learning square*. The role of the students is to compute, inquire, conjecture, and possibly prove. Teaching by questioning, or the Socratic Method, has long been an important part of any teaching; certainly, it should have a permanent role in the teaching of algebra and is, thusly, placed as the third element of the three-squares model. Suitable teacher and student discourse can give students ownership of mathematics content and place the responsibility of the development of mathematics content on the student (Otten et al, 2010). The literature referred to above provide viable rationale for the implementation of the three-squares model, with all aspects of the three-square model given credence. The authors experience and the literature above provide rationale for the formulation of the hypothesis for this study. Between the control group (traditional textbook approach) and experimental group (three-squares approach), there is a difference between the mean performance on both the midterm and final exam.

CHAPTER THREE

STUDY DESIGN AND DATA ANALYSIS

Methods

Participants

The research participants in this study consisted of career oriented students enrolled in Math 101 classes at Platt College; in many cases the students in this study have a weak algebra background. Additionally, since the assessment of the three-squares model is centered on adult students, students were only selected if they met an age requirement of eighteen years or older. Math 101 at Platt College is an introductory algebra class that includes the review of arithmetic and builds up to linear equation solving, polynomials, and a variety of applications. Both the control group and the research group were selected based on a questionnaire administered in the first class meeting after the presentation of informed consent. The questionnaire included a series of questions designed to bring out the past algebra experience of each student. Additionally, the students were given the opportunity to opt to not be a part of the study on the questionnaire. Only Math 101 students were admitted into either the control group or experimental group. Specifically, two classes were admitted into the control group and two classes where included in the experimental group. Each of the groups were presented Math 101 content using different instructional approaches with common assessments throughout the course.

Measures

Three assessment tools were employed in this study: an arithmetic/algebra diagnostic, a midterm exam, and a final exam. The diagnostic includes a series of thirty arithmetic questions and ten basic algebra questions. The arithmetic questions include topics such as computing with fractions and decimals, computing with integers, and applying the order of operations. The algebra section is brief including topics in simplifying expressions using the properties of the real numbers and applying some of the basic properties of exponents. The diagnostic has been used by the author as a pre-assessment tool in order to gauge student levels and adjust instruction early in Math 101 courses. The problems are scored as being either correct or incorrect.

The midterm and final exams are standard tests used in Math 101 courses at Platt College. The exams were written by the author and include three sections: free response, matching, and a series of arithmetic questions. The arithmetic and matching portions on both the midterm and final exam include problems that are similar and are designed to measure improvement. The matching questions require students to identify the properties of the real numbers in action; that is, the students are given an identity equation and they must choose the property that best justifies the equation. Lastly, the free response questions are graded using a 5-point rubric and are representative of the topics covered prior to the exam. The points on the exam are partitioned as follows:

free response-100 points, matching-10 points, and arithmetic diagnostic-30 points.

Procedures

In order to assess the three-squares model of teaching algebra, the content in Math 101 was presented to the students using two different curricula that were presented to a control group and an experimental group. The curriculum for the control group included the content, examples, and exercises exactly as organized in Marvin Bittinger's text Introductory Algebra, 11th Edition. Bittinger's text is the chosen text for Math 101 and 102 classes at Platt College. Every included concept and example was presented to the students in a lecture format. Each lecture was followed by the completion of the exercises in each relevant section of Bittinger's text. In order to insure a consistent presentation of content to both classes in the control group, the textbook examples and concepts were presented to the students as shown in the textbook; that is, Bittinger's text was used as a script for teaching Math 101 content. The control group was provided a seamless coverage of Math 101 content in what might be considered a traditional manner.

In contrast, the experimental group was presented a curriculum designed around a series of lectures and activities that have been selected or designed using the three-squares model as a guide. All the lectures and activities utilized in class for the experimental group fall in at least two of the squares of the three-squares model. In the majority of the meetings with the research group, at least one activity was completed. In this study, for the experimental group, all teaching practices, including lecture notes, learning activity design, or instruction decisions were made using the three-squares model as a framework. All student and teacher discourse where governed by the three-squares model. Most instruction allowed for student driven inquiry and involved teacher questioning matched to the algebra *habits of mind square* as provided by Mark Driscoll's book Fostering Algebraic Thinking, A Guide for Teachers, Grades 6 -10. All students in the experimental group were trained to apply the algebra habits of mind. It is important to note that, in this study, a textbook driven approach was contrasted with the three-squares model and any differences in student performances under each approach can only be attributed to the three teaching elements in the three-squares model working together and not attributed to any one of the three elements in the model alone.

A series of nine activities were used to help present content to the experimental group. A complete list of the activities is shown in Table 1 along with a listing of each teaching element addressed by the activity. Note that some activities work together to meet at least two of the teaching elements of the three-squares model.

The activities *A Little Number Theory for Math 101* and *Discovering Facts about the GCD and LCM* work together to meet the requirement of addressing two of the squares of the three-squares model. The first activity introduces basic sets, prime factorization, the greatest common divisor, and the least common

Table 1. Learning Activities

Learning Activity	Three-Square Elements
A Little Number Theory for Math 101 and Discovering Facts About the GCD and LCM	Algebra Habits of Mind Active Learning
Writing One (Unity)	Proof or Justification Active Learning
Exponent Play-An Investigation of Decimals	Algebra Habit of Mind Proof or Justification Active Learning
Scale Factors and Area and Perimeter	Algebra Habits of Mind Proof or Justification Active Learning
Proof with Whole Numbers and Natural Numbers	Algebra Habits of Mind Proof or Justification Active Learning
Exploring Inequalities Part 1-A Puzzling Property of Inequalities Exploring Inequalities Part 2-Exploring the Triangle Inequality Theorem	Algebra Habits of Mind Proof or Justification Active Learning
Guess and Check Tables and Writing Equations	Algebra Habits of Mind Active Learning
The Area Model for Multiplication and Identity Equations	Algebra Habits of Mind Proof or Justification Active Learning

multiple; while the activity addresses only the *active learning square* directly, it does prepare students to complete the *Discovering Facts about the GCD and LCM* activity that involves *abstraction through calculation* and *active learning.* In the GCD and LCM activity, students work out a seriesof calculations and induce some computational facts about the GCD and LCM; namely, if gcd(a,b)=a, then the lcm(a,b)=b and if the gcd(a,b)=1, then lcm(a,b)=ab. The activity is well suited for the application of the algebra habits of mind and placing mathematical development into the hands of the students.

Second in the series of activities is the *Writing One* activity that introduces the identity property of one and the inverse property of multiplication. The activity works students through several situations in fractions, percent, and the division of decimals where properties are used to justify all computations. The Writing One activity fits into all three of the squares of the three-squares model. The beginning of the activity works through writing equivalent fractions, followed by the use of the *doing and undoing* habit of mind to undo writing equivalent fractions; that is, simplify fractions. The *proof or justification square* is addressed throughout the activity with the inclusion of arguments for both the process of dividing fractions and how to handle a particular case of dividing decimals.

The third activity, *Exponent Play-An Exploration of Decimals*, serves two purposes, the development and justification of the properties of exponents and the development of the skill of writing numbers from decimal form to expanded form. The activity relies heavily on the habit of mind *abstraction through calculation* along with expansion of powers of ten. All the properties of exponents needed to write decimals in expanded form are discovered and justified in the activity with an emphasis on the interpretation of negative exponents and the fact that any real number, except zero, raised to the zero power is one. The activity focuses on the explanation, *proof or justification*, of basic properties and moves students to explore more of the properties of exponents. Hence, the activity addresses the *active learning square*.

The fourth activity in the sequence, *Scale Factors and Area and Perimeter*, is based on geometric content, in particular perimeter and area. The focus of the activity is to look into scale factors and what effect scaling the dimensions of a shape has on perimeter, area, and volume. Essentially, the activity addresses the concept of direct variation where perimeter varies directly to the scale factor, area varies directly to the square of the scale factor, and volume varies directly to the cube of the scale factor. The *Scale Factors and Area and Perimeter* activity addresses all three squares of the three-squares model with the inclusion of two formal proofs that demonstrate the effect of scaling on perimeter and area.

The *Proof with Whole Numbers and Natural Numbers* activity is designed to move students to apply the properties of real numbers and practice using variable notation. The students revisit some set concepts and apply the commutative, associative, and distributive properties along with the closure properties of addition and multiplication to prove fundamental propositions pertaining to addition and multiplication. For example, the sum of two odd numbers is even and the sum of three consecutive odd numbers is odd. The students first use *abstraction through calculation* to verify each proposition and then they proceed to prove it. The students organize their arguments in a twocolumn format where they actively work through the proofs and address all three of the squares in the three-squares model.

The remaining activities utilized by this study address more advanced topics such as solving inequalities, using algebra to solve word problems, and computing with polynomials. *Exploring Inequalities Part 1* and *Part 2* provide both an algebraic and geometric approach to solving and applying inequalities. Both activities utilize *abstraction through calculation*, *proof or justification*, and *active learning*. Part 1 addresses the common error made by students involving negative constants and the multiplication property of inequalities. In Part 2, the triangle inequality theorem is justified, but not formally proven, by a series of geometric constructions that lead to the discovery of the theorem. The objective of Part 2 is to give students some real applications of inequalities with other tertiary objectives involving formal proof and the learning of a new geometry theorem. The remaining two activities additionally fit well into the three-squares model of teaching algebra; the activities are explained below.

Solving word problems, for many students, has typically been the least favorite portion of an introductory algebra class. Students often openly express their aversion to word problems and often criticize the authenticity of a word problem or application problem. In some cases the student's criticisms are warranted, at least in regard to the contrived nature of some word problems. Some of the content of Math 631, a MAT course that focuses on teaching algebra, provides teachers with a method of building student efficacy toward solving word problems. *Guess and Check Tables* provide students an opportunity to use *abstraction through calculation* to develop equations or functions that solve particular word problems. Often, students have little problem with solving an equation that describes a situation, but they are challenged by and struggle with the task of developing the equation that models the situation. The *Guess and Check* method promotes success within the content of developing equations or functions and serves as a scaffolding that students may later abandon when they develop a more refined approach that leads to the equations or functions with less effort. The *Guess and Check Tables* include both *abstraction through calculation* and *active learning*.

The final activity, *The Area Model for Multiplication and Identity Equations,* uses a little math history, at least in regard to methodology, to multiply and factor polynomials. Polynomials are compared to the dimensions of rectangles and the notion of base times height is adapted to multiply polynomials. The terms of polynomials correspond to segment lengths resulting in the partitioning of "rectangles" into regions that have monomials for dimensions. Essentially, the area model for multiplying polynomials reduces the multiplication of polynomials to monomial multiplication and organizes the terms that are to be combined in a rectangular array. The model increases student computational accuracy and makes the multiplication of higher order polynomials fairly routine. More importantly, the activity leads students to discover identity equations for special products such a squaring a binomial and multiplying conjugates. The area model serves as justification for the special products and it can be reversed and used to factor or divide polynomials. The area model activity applies the *doing and undoing* algebra habit of mind and it addresses the *active learning* and *proof or justification* learning elements.

With the instructional methodology and associated activities identified, the statistical treatment of data can now be explained.

Analysis

The data collected for this research project was analyzed using analysis of variance with one covariate (ANCOVA). This research project was a quasi-experimental design (assignment to the groups was not random) utilizing one covariate to adjust the criterion of two groups, (control group n = 27 and an experimental group n = 22). The sample size of the experimental group was smaller than desired since many potential participants opted to not be included in the research project after the administration of informed consent. Additionally, multiple students in the experimental group were dropped from the class or went on leave and thus they gave no data for the midterm and final exams. One participant was treated as an outlier and removed from the study since the midterm score for the participant was more than three z-scores from the mean. The results of the midterm and final exam were tested separately with a diagnostic test acting as the covariate. The purpose of a covariate is to remove possible variance within groups that can be explained by individual differences (in this case, differences in arithmetic/algebra aptitude identified by the diagnostic). Also valuable in a quasi-experimental design, the covariate can remove pre-existing differences between the control group and experimental group in order to provide a refined comparison between the mean scores on the midterm and final exam scores between both groups. The descriptive statistics are summarized in Tables 2 and 3 below. The scores on the covariate, midterm, and final were normally distributed for both the control group and experimental group (see Appendix A).

ANCOVA was appropriate for the data in this research since the slopes of the regression lines describing midterm scores as function of the covariant scores are nearly the same for both groups; that is, there is homogeneity of slope. The regression model gave similar results for the final exam. The results are summarized in the Figures 2 and 3.

Table 2. Control Group

	Covariate	Midterm	Final Exam
Mean	18.56	107.74	106.70
Median	18.00	109.00	110.00
Std. Deviation	9.40	21.97	20.47
Minimum	3.00	65.00	58.00
Maximum	37.00	136.00	133.00

Table 3. Experimental Group

	Covariate	Midterm	Final Exam
Mean	18.05	105.32	104.68
Median	18.00	107.50	106.50
Std. Deviation	8.76	19.17	22.05
Minimum	2.00	55.00	60.00
Maximum	34.00	133.00	138.00

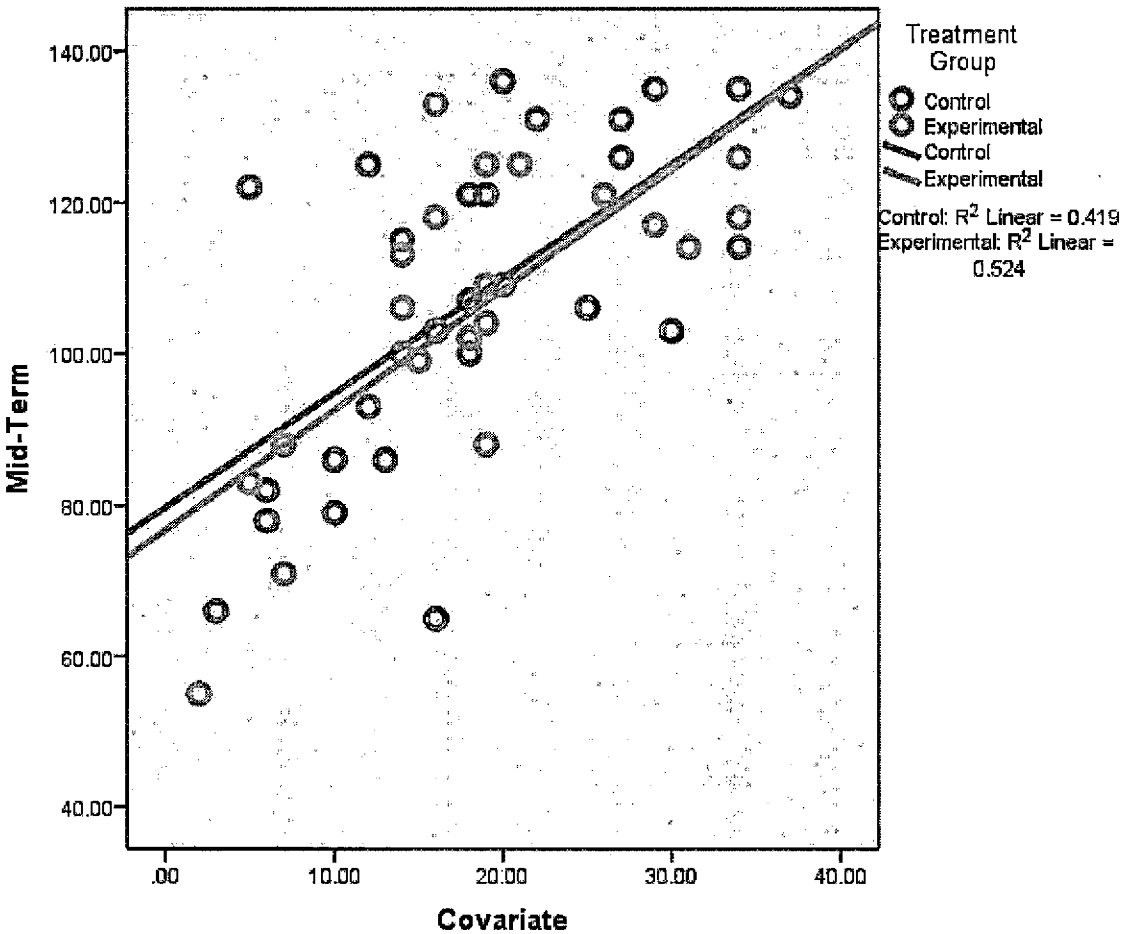


Figure 2. Homogeneity of Slope (Midterm)

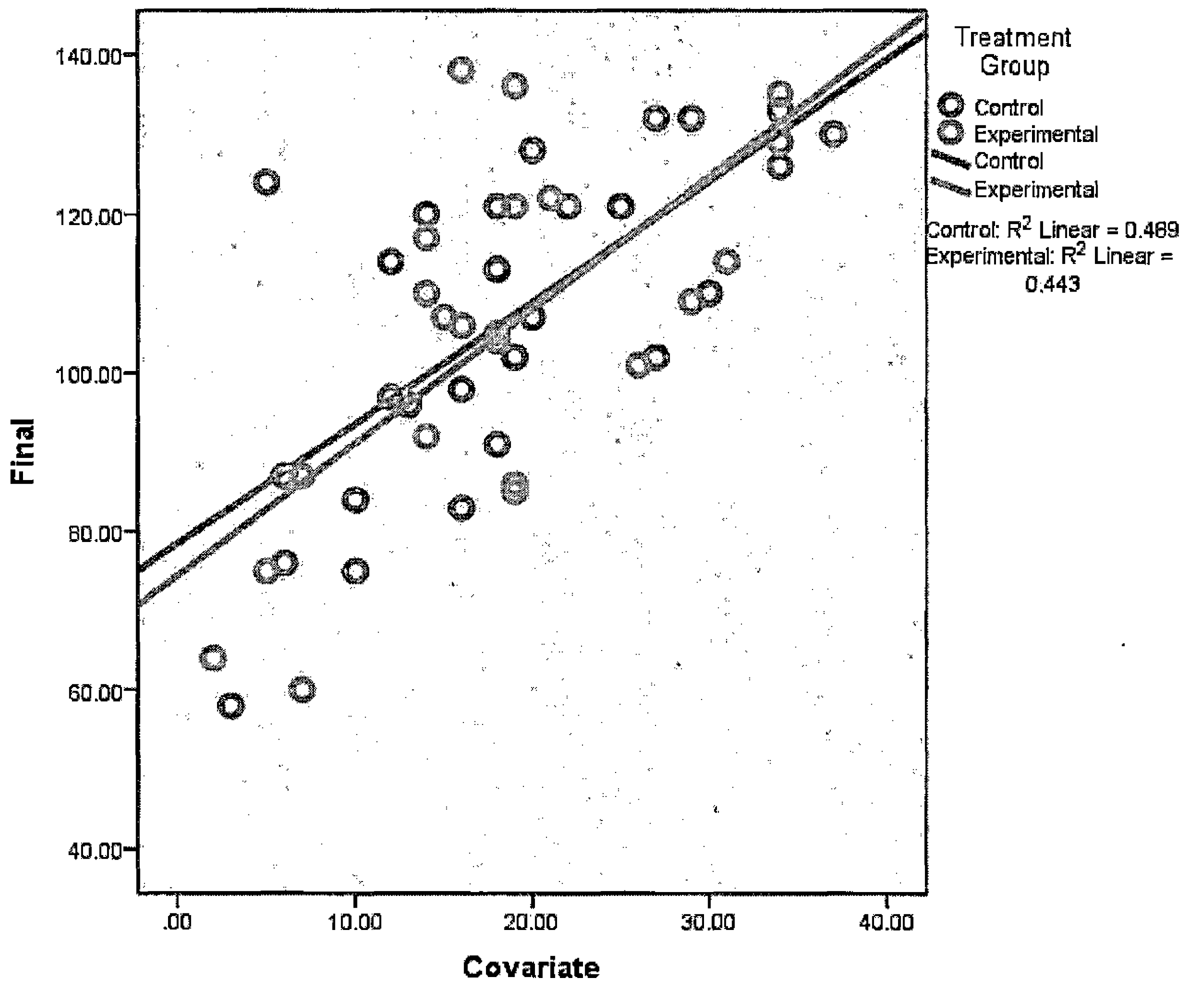


Figure 3. Homogeneity of Slope (Final Exam)

With the homogeneity of slope for both the midterm and final exams verified, possible variance between groups can now be discussed.

The control group and experimental group were matched well together. There were no statistical differences between groups on the pre-treatment diagnostic. This was verified with a t-test ($t(47)=0.20$ and $p=0.846$). Nearly all the variance explained by the covariate was within groups and not between the control group and experimental group. The diagnostic is a good predictor of how students will perform on the midterm and final exams. As mentioned earlier, the diagnostic scores were normally distributed. Tables 4 and 5 below give a summary of the ANCOVA results. The effect of the covariate was statistically significant and large in magnitude for both the mid-term and final exam criteria. The effects of the treatment, however, were insignificant and very small in magnitude.

Table 4. Test Between-Subjects Effects (Midterm)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Total Model	9371.86	2	4685.929	19.651	.00	.46
Diagnostic	9300.71	1	9300.71	39.00	.00	.46
Treatment Effect	32.40	1	32.40	.14	.71	.00
Error	10969.24	46	238.46	----------	----------	----------
Corrected Total	20341.10	48	----------	----------	----------	----------

The graphs in Appendix B summarize the results of this study. The graphs display error bars that give a 95% confidence interval around the actual mean midterm and final exams. The first graphs in each pair, midterm and final exam, have not had the variance explained by the covariate removed (either within or between the groups).

Table 5. Test Between-Subjects Effects (Final Exam)

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Total Model	9879.65	2	4939.82	20.14	.00	.47
Diagnostic	9830.09	1	9830.09	40.09	.00	.47
Treatment Effect	17.82	1	17.82	.07	.79	.00
Error	11280.314	46	245.22	----------	----------	----------
Corrected Total	21159.959	48	----------	----------	----------	----------

The second graphs have had variance explained removed. The mean scores for both groups on the midterm and final exams shifted little after the variance explained was removed. The original and covariate-adjusted mean scores are summarized in Table 6. The table and graphs clearly show that there is no difference between each group's performance on both the midterm and final exams; thus, the research hypothesis was not supported. There was no difference in student performance under a traditional direct instruction approach compared to performance using the three-squares model for teaching algebra. The findings of this study are diametric to what the author expected, they were not met without some disappointment. A discussion of potential reasons for these results follows. This discussion includes the consideration of some qualitative evidence observed by the researcher during the administration of the procedures.

Table 6. Summary of Means

	Control Group	Experimental Group
Midterm	107.741	105.318
Midterm Var. Removed	107.387	105.752
Final Exam	106.704	104.682
Final Exam Var. Removed	106.340	105.128

CHAPTER FOUR

DISCUSSION

Qualitative Explanation for the Quantitative Results

The results of this study are disappointing for a proponent of the three-squares model; all three elements of the three-squares model are grounded in solid mathematics pedagogical theory. The expectations of the author preceding this study where high in regard to student performance; there was an expected difference in the mean scores on the midterm and final exams between both the control group and the experimental group. The discrepancy between expectations and the actual data of this research project may be attributed to a high level of apathy among one of the classes in the research group.

Many of the students at Platt College come to class with various challenges or stressors that affect learning. An article by Foster Walsh, a specialist in secondary level education, identifies a number of factors that may contribute to disengagement among middle school students. Like the algebra habits of mind, the insights of Walsh are believed to be applicable to adult students enrolled at Platt College. Among the many factors in Walsh's list, four may be particularly applicable to the adult students in this research: interest in subject matter, past school or subject experience, current emotional state, and task complexity.

During the author's nearly four years of teaching experience at Platt College, many students have openly expressed their disappointment in their past experiences in arithmetic and algebra classes. Many students share their disappointing grades in mathematics and often explain that their teacher did not meet their expectations. There does seem to be a propensity for students to shift the responsibility of their success to the performance of their instructor, a tendency that is evident in both adolescent and adult students. Regardless of who may be to responsible for the unsatisfactory progress, many students come into Math 101 with an aversion to mathematics that is associated with past experiences, which may contribute to apathy among some students.

Similarly, students often give the author a fair warning before class or during introductions; they often claim that they have some strange disposition and that they will not perform well in math class. The author contends that such comments may be addressing their past poor learning experiences in mathematics. Some students have claimed that they can do mathematics, but they do not enjoy the subject. Platt College students are career-driven students that often do not see the value of their general education program. The author does believe that the past subject experiences of the students and disinterest in subject matter did contribute to the apparent apathy or disengagement of some students in the experimental group. There were other apparent contributors to apathy in the experimental group.

Many students that attend Platt College have factors that affect their emotional state while attending their classes. A number of students in the research group openly shared family health issues, living situation concerns, and stressors associated with unemployment. Possibly, such factors contributed negatively to the attitude of the students towards learning Math 101 content and working through some of chosen activities. While students are expected to balance their personal lives with their academic lives, the author sources some of the apparent apathy in the experimental group back to the emotional state of the students. As students deal with challenges that disrupt their lives, their attention to learning Math 101 is disrupted; that is, life stressors disrupt the process outlined by the three-squares model. An additional contributor to the disengagement of the experimental group in this research may be, as identified by Walsh, task complexity.

The activities utilized in this research all address some of the higher levels of Bloom's Taxonomy with most of them beginning with knowledge and spanning through synthesis. The participants of this study may not have had past educational experiences that included inquiry based activities, and they may not have had a chance to move through the process of inductive and deductive thinking in an organized fashion. Such a lack of meaningful learning experiences addresses again some of the earlier factors that contribute to disengagement, namely, past school or subject experience. The complexity of the tasks (activities) included in this study could have contributed to the apparent apathy in the students. Many of the tasks given to the students challenged them to discover some of the mathematics with little to no direct instruction; such challenges may have been perceived as a threat to efficacy. Reported in the study by Meyer, Schweinle, and Turner,"Striking the Right Balance: Student's Motivation and Affect in Elementary Mathematics", tasks and challenges were perceived as a threat on the efficacy of the students. The author observed cautious comments in class regarding the challenging exploratory nature of some activities, and many statements similar in form to, "Why doesn't he just teach us the material?" were expressed between students. Some students openly questioned the author on the inclusion of the activities in Math 101; they were met with a brief pedagogical explanation as given earlier in the text of this report. Of course the students were aware that they were cooperating in a research project; thus, they accepted the rationale and returned to the task. With some of the disengagement of the research group sourced to Walsh's insights, the author can take great comfort in the comparison of the mean midterm and final exam scores.

CHAPTER FIVE

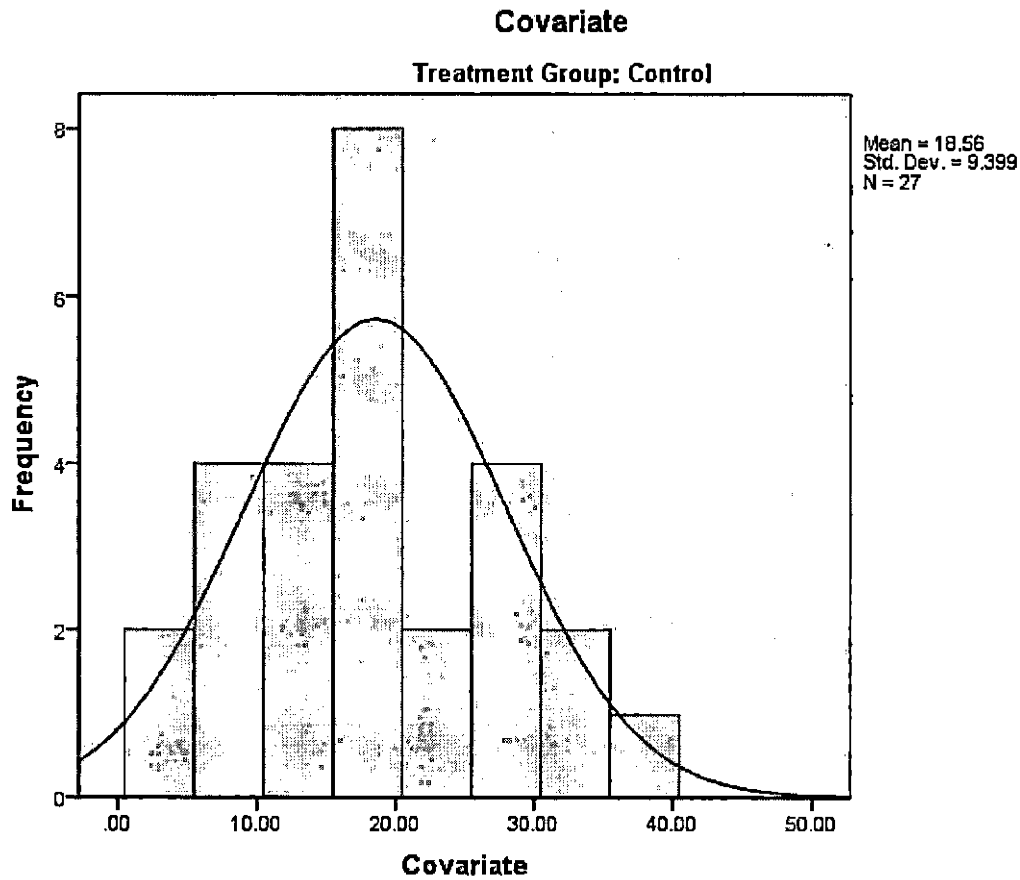
CONCLUSION

The Future of the Three-Squares Model

While the findings of this study were unanticipated, the author can take comfort in the fact that the data shows that, in comparison to a traditional textbook driven approach, the three-squares model does the students no harm. Consistent to the claims included in the informed consent from of this study, it seems that there were no apparent risks connected to participating in this research, at least in terms of academic performance. The author has been developing the three-squares model of teaching algebra informally for over three years. The development of the model is the synthesis of content from the MAT program at California State University, San Bernardino, years of teaching experience, and the training associated with most mathematics degrees. Regardless of the anticlimactic results of this research, the author contends that the three-squares model is an effective framework for teaching algebra. There is clearly room for refinement and further assessment of the model. Some of the activities utilized in the assessment of the three-squares model need improvement. The author looks forward to making improvements and becoming skilled in the presentation of the activities to students at Platt College. Perhaps this study should be repeated in the future with the goal of validating the three-square model; thus validating the professional practices of the author.

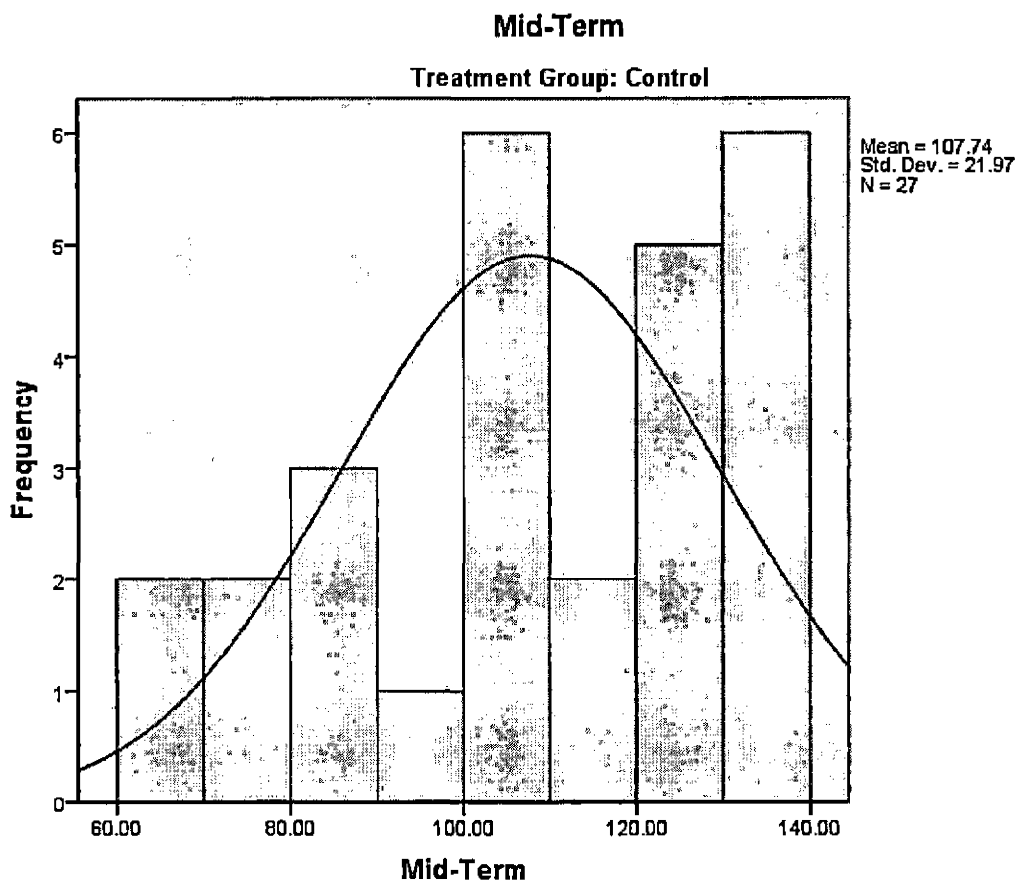
APPENDIX A

NORMAL DISTRIBUTION OF MEASURES



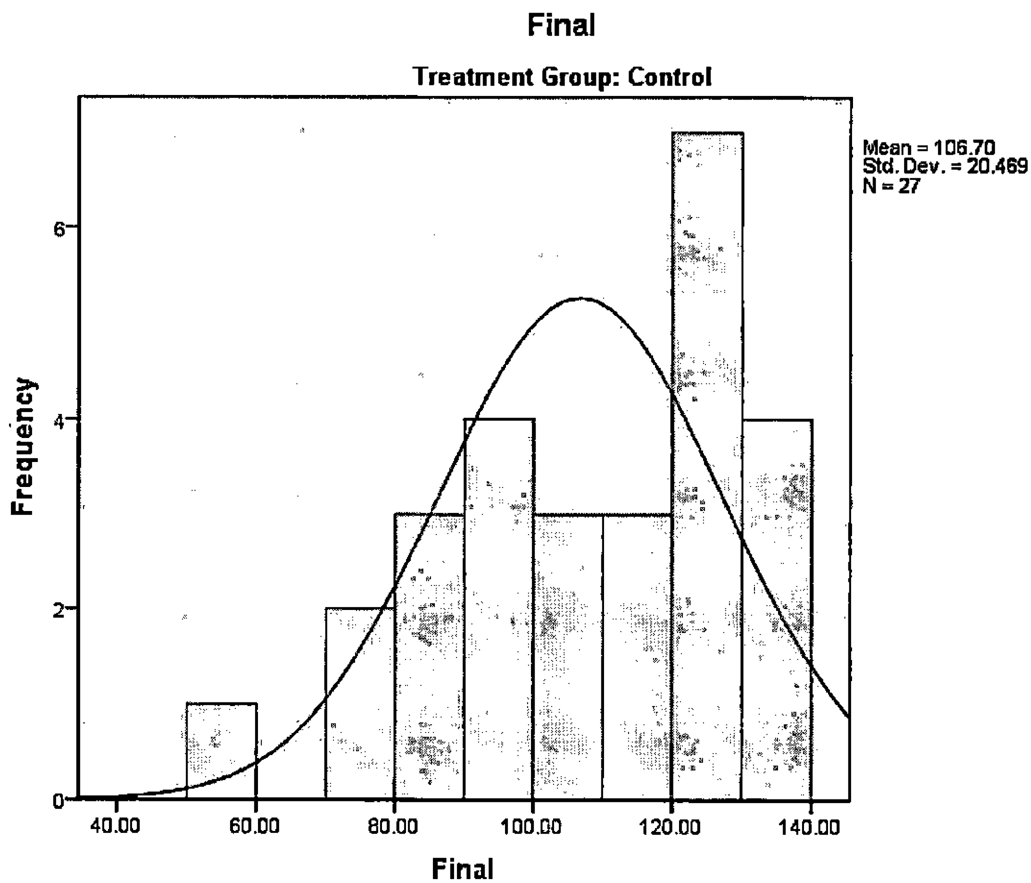
Normal Distribution of Diagnostic Scores

Control Group



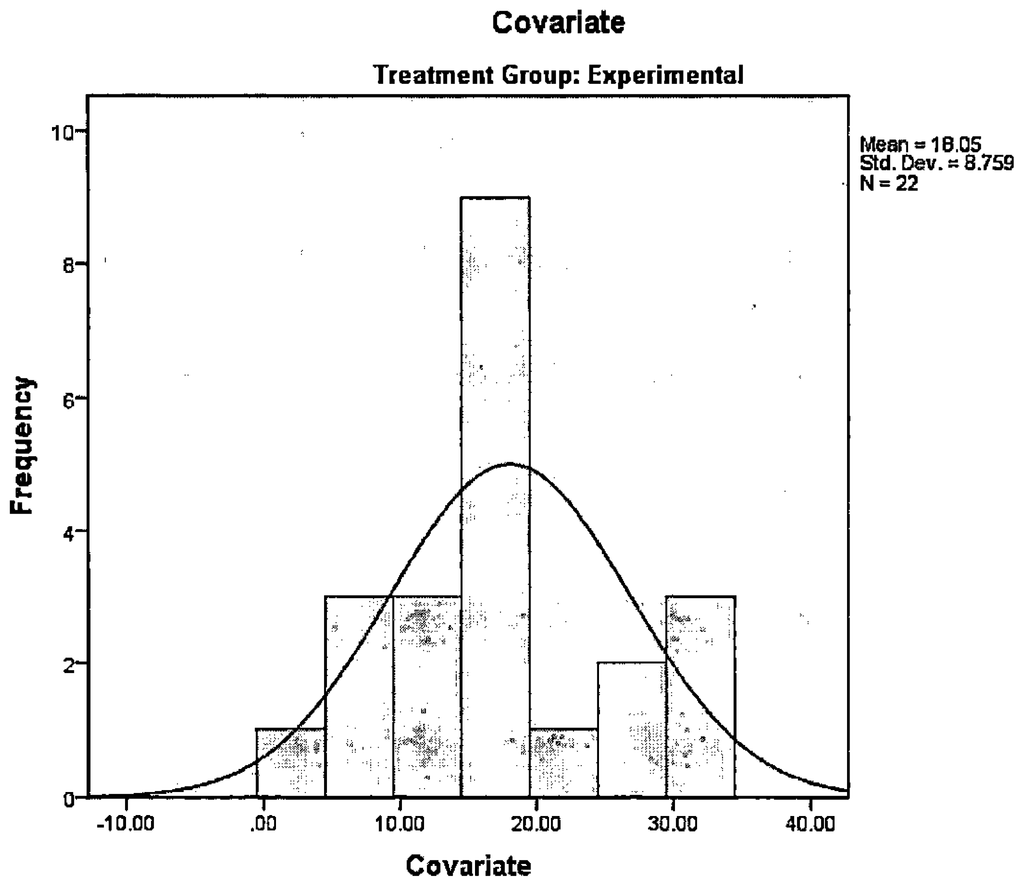
Normal Distribution of Midterm Scores

Control Group



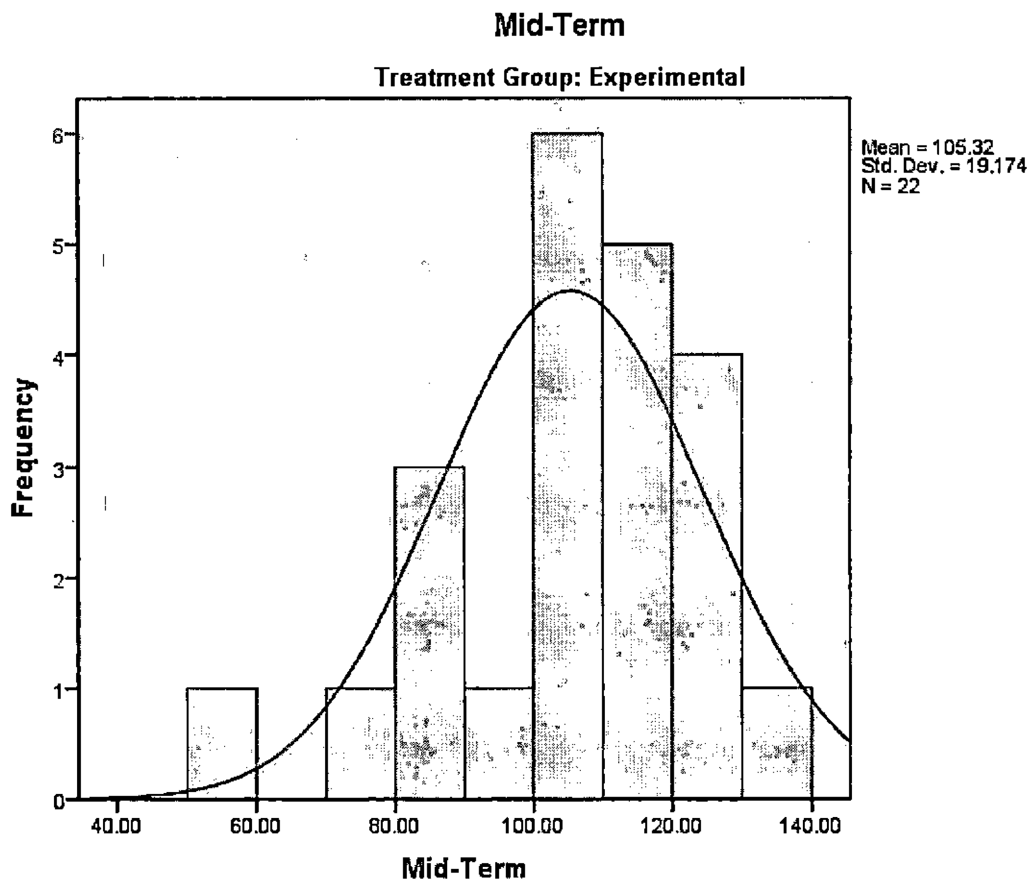
Normal Distribution of Final Exam Scores

Control Group



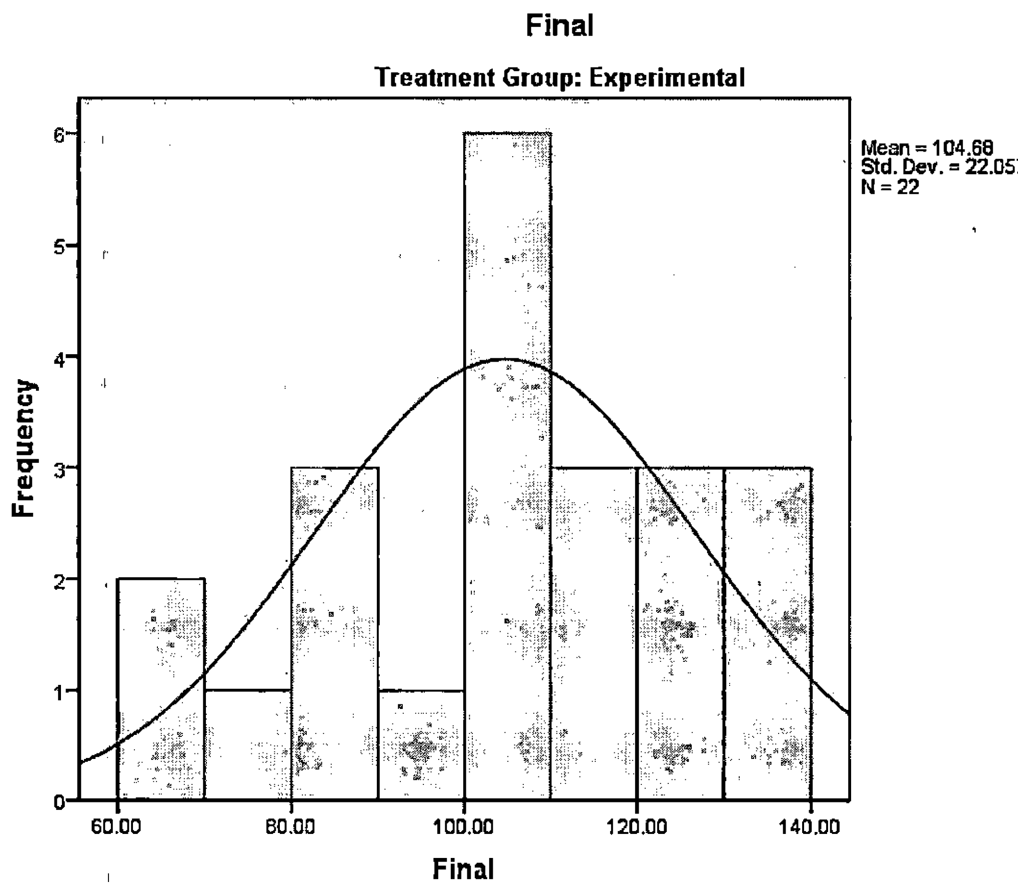
Normal Distribution of Diagnostic Scores

Experimental Group



Normal Distribution of Midterm Scores

Experimental Group

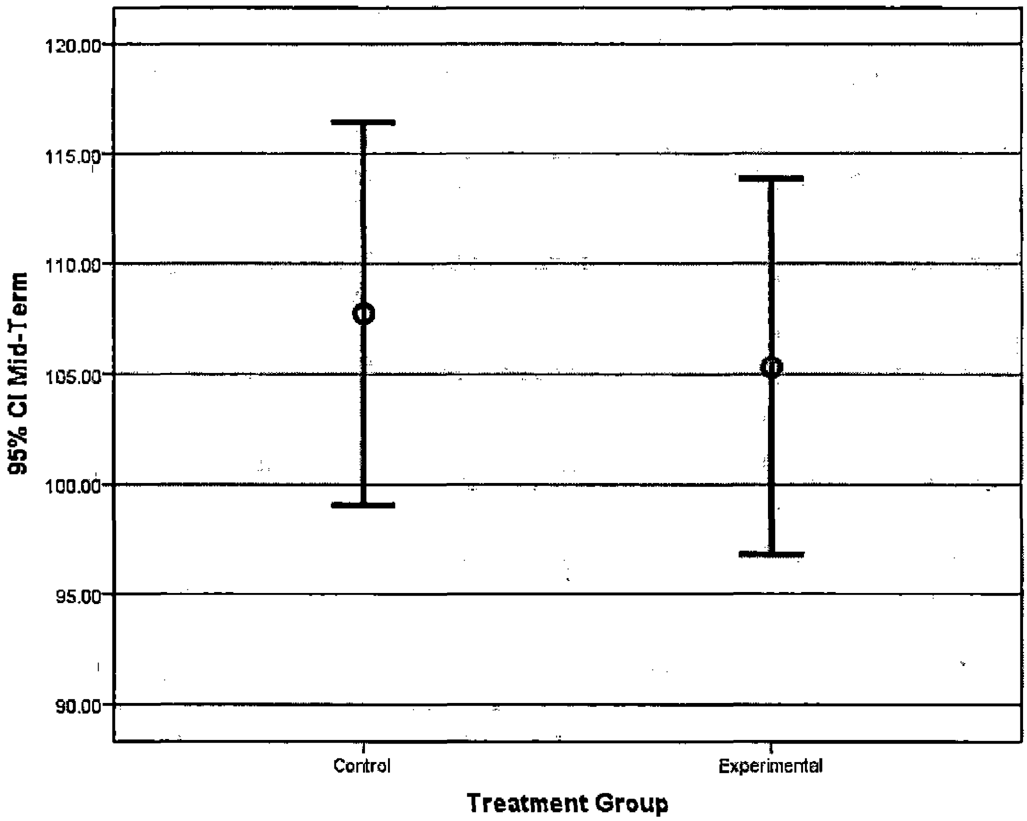


Normal Distribution of Final Exam Scores

Experimental Group

APPENDIX B

COMPARISON OF MEANS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95% CI Mid-Term
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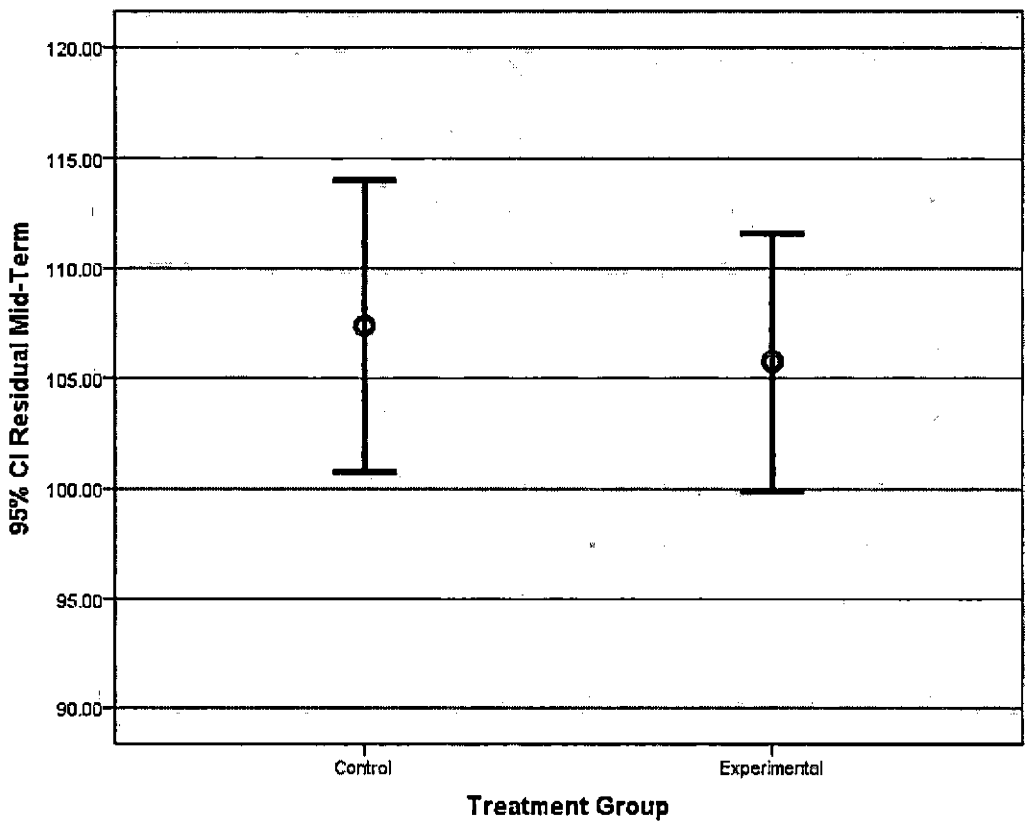
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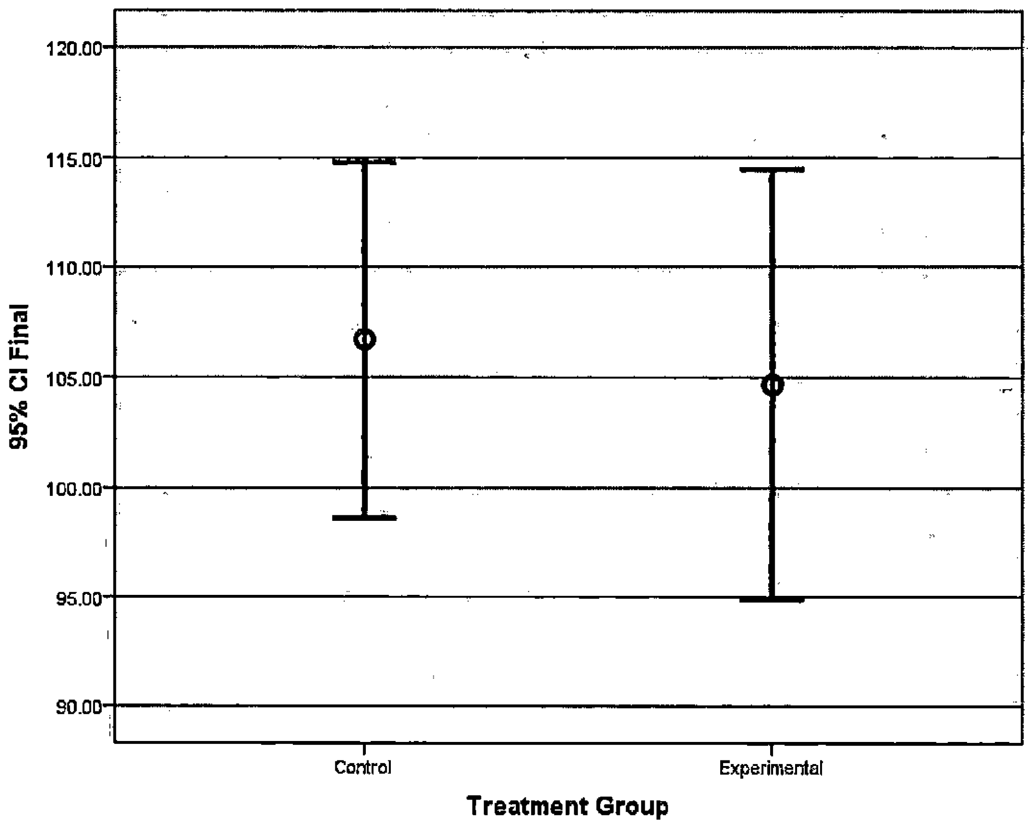
95% CI Residual Mid-Term
Control
Experimental
Treatment Group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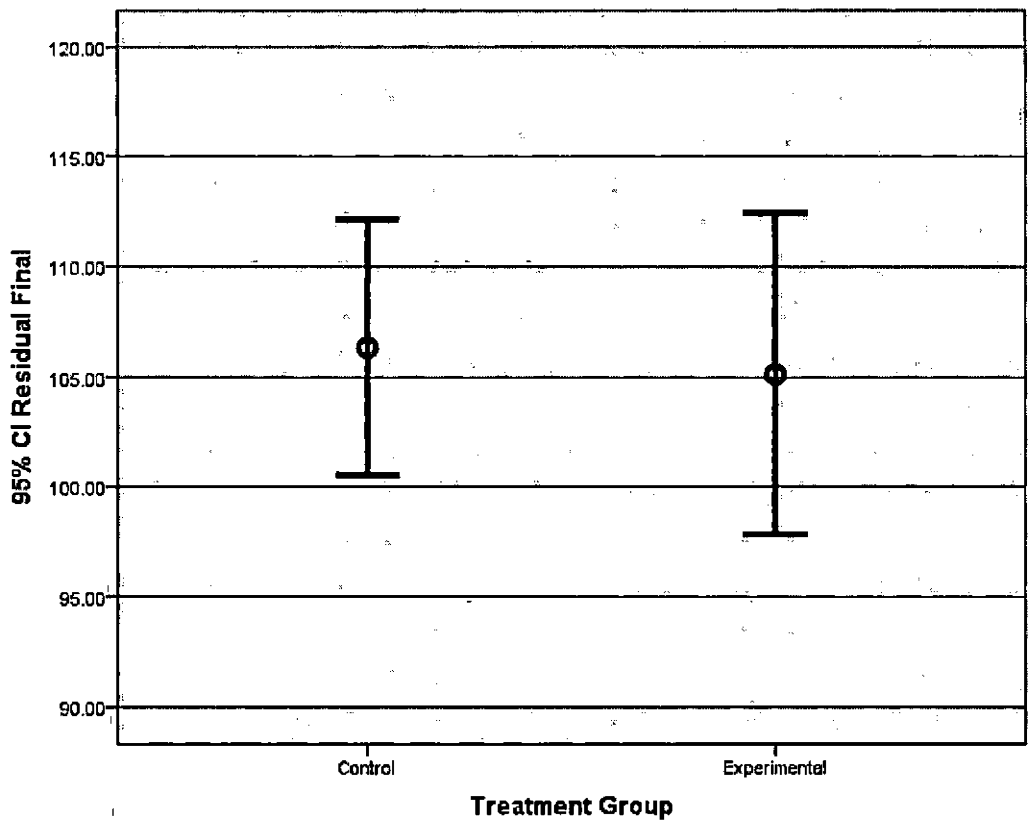
Treatment Group



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95% CI Residual Final
Control
Experimental
Treatment Group

APPENDIX C

INSTITUTIONAL REVIEW BOARD APPROVAL

CALIFORNIA STATE UNIVERSITY
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Academic Affairs
Office of Academic Research • Institutional Review Board

June 02, 2011

CSUSB INSTITUTIONAL REVIEW BOARD
Expedited Review
IRB# 10086
Status
APPROVED

Mr. Stephen Richardson
c/o: Prof. Shawnee McMurran
Department of Mathematics
California State University
5500 University Parkway
San Bernardino, California 92407

Dear Mr. Richardson:

Your application to use human subjects, titled "Assessing the Three Squares Model for Teaching Algebra to Adult Students" has been reviewed and approved by the Institutional Review Board (IRB). The attached informed consent document has been stamped and signed by the IRB chairperson. All subsequent copies used must be this officially approved version. A change in your informed consent (no matter how minor the change) requires resubmission of your protocol as amended. **Your application is approved for one year from June 02, 2011 through June 01, 2012. One month prior to the approval end date you need to file for a renewal if you have not completed your research. See additional requirements (Items 1 – 4) of your approval below.**

Your responsibilities as the researcher/investigator reporting to the IRB Committee include the following 4 requirements as mandated by the Code of Federal Regulations 45 CFR 46 listed below. Please note that the protocol change form and renewal form are located on the IRB website under the forms menu. Failure to notify the IRB of the above may result in disciplinary action. You are required to keep copies of the informed consent forms and data for at least three years.

1) Submit a protocol change form if any changes (no matter how minor) are made in your research prospectus/protocol for review and approval of the IRB before implemented in your research.
2) If any unanticipated/adverse events are experienced by subjects during your research,
3) Too renew your protocol one month prior to the protocols end date;
4) When your project has ended by emailing the IRB Coordinator/Compliance Analyst.

The CSUSB IRB has not evaluated your proposal for scientific merit, except to weigh the risk to the human participants and the aspects of the proposal related to potential risk and benefit. This approval notice does not replace any departmental or additional approvals which may be required.

If you have any questions regarding the IRB decision, please contact Michael Gillespie, IRB Compliance Coordinator. Mr. Michael Gillespie can be reached by phone at (909) 537-7588, by fax at (909) 537-7028, or by email at mgillesp@csusb.edu. Please include your application approval identification number (listed at the top) in all correspondence.

Best of luck with your research.

Sincerely,

Sharon Ward, Ph.D

Sharon Ward, Ph.D., Chair
Institutional Review Board

SW/mg

cc: Prof. Shawnee McMurran, Department of Mathematics

909.537.7588 • fax: 909.537.7028 • http://irb.csusb.edu/
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