# Use of the multiplication chart in solving problems with fractions

Adan Espinosa

# USE OF THE MULTIPLICATION CHART IN SOLVING PROBLEMS WITH FRACTIONS

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A Thesis

Presented to the

Faculty of

California State University,

San Bernardino

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In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

in

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by

Adan Espinosa

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## ABSTRACT

This project will explore the effectiveness of Brad Fulton paper on Maximum Math From the Multiplication Table. Maximum Math From the Multiplication Table has nine separate topics: simplify fractions, find equivalent fractions, add and subtract fractions of unlike denominators, multiply fractions, divide fractions, help students understand fractions procedures better, solve proportions, explore algebraic proofs and explore quadratic functions.

The participants (Bear High School) in this study are students in a public high school in a large urban area of the southeast area of California. Bear High School (pseudonym) has a high number of students who are taking Algebra 1. This study was conducted during the 2007-2008 school year. Bear High School students often complain that they cannot do fractions. The results of this study reveal students overall lack of experience with basic fractions concepts. Even with the gain in the post-test students are still averaging 53%. Only 22% of the students scored higher than 70%. These numbers show my students lack of understanding with basic fractions.

## ACKNOWLEDGMENTS

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# BACKGROUND

## Introduction

Fractions are often a difficult concept for students to grasp. One factor lies in that fractions are comprised of multifaceted constructs (Brusseau, Brousseau, and Warfield, 2004). In the mid 1970's Kieren (1976) proposed that the concept of fractions is multifaceted and that it consists of five interrelated subconstructs: part-whole, ratio, operator, quotient and measure. The part-whole subconstruct of fractions is a situation in which a continuous set of objects is partitioned into parts of equal size (Lamon, 1999). The ratio subconstruct views fractions as a relationship between two quantities (Lamon, 1999). In the operator subconstructs "rational numbers are regarded as functions applied to some number, object, or set" (Lamon, 1999, p. 9). The quotient subconstruct of fractions uses the numerator to define the quantity to be shared and the denominator to partition the quantity (Marshall, 1993). The measure subconstruct conveys the idea that a fraction is a number (Marshall, 1993). The federally sponsored National Assessment of Educational Progress

(Mullis, Dossey, Owen, and Phillips, 1991) report indicates that fractions are exceedingly difficult for children to master. In 1973 NAEP (National Assessment of Educational Progress) reported that only 42% of the 13 year olds and 60% of the 17 year olds in the sample could correctly add 1/2 + 1/3 (Carpenter, Coburn, Reyes, and Wilson, 1976). The most common error among students is how they interchange the algorithms for addition and multiplication (Bernadette, Russell, Douglas, 1990). The 1986 NEAP finding concluded that, "older students' difficulties with fractions, decimals and percents reflected serious gaps in their knowledge of basic fractions, decimal and percent concepts" (NCTM, 1988, p. 16). When a teacher first introduces fractions, keeping the concepts concrete is a great first step. The teacher should continue to use concrete models, area models or number line models, until students can understand that fractions really just mean part of the whole. Teachers should also include examples of improper fractions. For example 3/2, which can be explained by breaking the improper fraction into two parts, the whole part and the part of a whole. Students' understanding and ability to reason with fractions will grow as they represent fractions and decimals with physical materials and on number lines and as they learn to generate equivalent representations of fractions and decimals (NCTM, 1989).

## Patterns in the Multiplication Chart

This project will explore the effectiveness of Brad Fulton paper on Maximum Math From the Multiplication Table (2005). Maximum Math From the Multiplication Table (Fulton, 2005) has nine separate topics: simplify fractions, find equivalent fractions, add and subtract fractions of unlike denominators, multiply fractions, divide fractions, help students understand fractions procedures better, solve proportions, explore algebraic proofs and explore quadratic functions. Brad Fulton did not name his method. This project will use the term multiplication method when he uses the multiplication table to model a problem and box method when he used a four by four square. This project will analyze the effectiveness of how Brad Fulton's (2005) multiplication table and box method to add, subtract, divide and multiply fractions.

## Purpose of the Study

The purpose of this study is to analyze whether or not using the Multiplication Table when teaching fractions will improve students' problem-solving skills and accuracy. The major goal of this project is to explore whether students will become more effective mathematical problem solvers with fractions if they use the multiplication table method or box method.

## Selection of School Site

I selected Bear High School in Rialto to complete the study. The participants in this study are students in a public high school in a large urban area of the southeast area of California. Bear High School (pseudonym) has a high number of students who are taking Algebra 1. The data is summarized in Appendix A. This study was conducted during the 2007-2008 school year.

I introduced the Multiplication Chart Method to ninety-three students enrolled in my Algebra 1 and Geometry class in grades 9-12. All 93 students receive mathematics instruction in a regular education classroom, and therefore no resource aide is assigned to these classes. The students range in age between 14 and 18 years. With the help of the Multiplication Chart Method students improved their ability to solve problems that involve fractions as measured by the post-test.

## Justification of the Study

This is an important project because it will evaluate a resource to help their students add, subtract, multiply and divide fractions. Many students have a hard time solving higher math problems because they do not have the basic math skills. The Multiplication Chart Method, with all its steps is a tool that assists in the teaching and learning of fractions.

## Research Hypothesis

The hypothesis of this project is that the Multiplication Chart Method will improve students' math problem-solving skills and accuracy with fractions.

## Project Overview

This project will examine the effects of the Multiplication Chart Method or The Box Method on students' understanding of fractions. It also provides an error analysis on students work with the Multiplication Chart Method or the Box Method. It was conducted at Bear High School in Rialto, California, with students from all academic levels.

The proposed objective of this research is based on my personal observation of students' math learning needs. The goal of this project is to utilize the Multiplication Chart Method to create and develop a unit lesson on the Multiplication Chart Method.

# CHAPTER TWO

# REVIEW OF THE LITERATURE

This review of the literature will focus on four areas: 1) when are fractions taught; 2) the importance of learning fractions; 3) common mistakes when working with fractions; and 4) proportional reasoning.

## When are Fractions Taught?

When are students ready for fractions? Piaget's theory of cognitive development (Wadsworth, 1996) concludes that, in general, school-age children are either in the concrete operational stage at ages 7–11 of development or in the stage of formal operation at ages 11–16. The child in the concrete stage "must deal with each problem in isolation" (p. 112) and is unable to construct new knowledge from internal reflection alone. Formal thinkers are able to generalize and use internal reflection that "can result in new knowledge – new construction" (p. 118). Too often the algorithm for solving fractions has simply been taught, providing no connections for understanding, and leaving the student clinging to a prescribed step-by-step set of instructions. Based on Piaget's theory of cognitive development, teacher must use appropriate instructional strategies at different stages of development.

Anecdotal evidence from classroom observations indicate that adding, subtracting, multiplying and dividing fractions can be confusing and even intimidating to many students. Math Framework for California Public School (2006), defines fractions, as concrete objects that are important as a conceptual foundation so that students can make the transition to the generalized definition of rational numbers and their operations. The California Framework (2006) suggests using concrete models and placing fractions in context of children's life experiences to help students grasp the concept of fraction and decrease students' confusion with fractions. Common models for fractions are the division of a set or an area, and the points on a number line (Math Framework for California Public School, 2006). Young children, students in elementary, can be encouraged to understand and represent commonly used fractions in context, such as 1/2 or 1/8 of a pie, and to see fractions as part of a unit whole or a collection. Start by establishing the unit underlying fractions. Beginning with a unit or unit whole, the teacher divides the unit into *b* equal parts and takes one or more of them calling that number of parts *a*. A visual model of the fraction a/b is the area of a single part or segment when the whole area of the unit is partitioned into *b* parts of equal area, as shown below (see Figure 1) (Mathematics Framework, for California Public School, 2006).

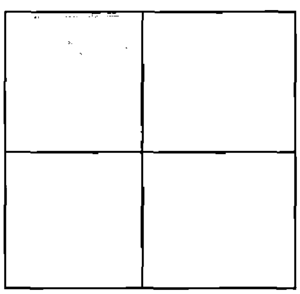


Figure 1. One Square Divided into Equal Parts

Teachers should help students develop an understanding of fractions as division of numbers and represent numbers with various physical materials (NCTM, 1989). For example, 1/2 of the ten-team members are girls.

The National Council of Teachers of Mathematics (NCTM) contains various national standards related to fractions instruction (NCTM, 2000). Prekindergarten to second grade students should have some experience with simple commonly used fractions, such as 1/4, 1/3 and 1/2, through connections to everyday events and language, according to the NCTM (2000, p. 215) Standards. The NCTM also suggests that in grades 3-5 students should develop understanding of "fractions as parts of a unit whole, use models and equivalent forms to judge the size of fractions and recognize and generate equivalent forms of commonly used fractions, decimals, and percents". In grades 6-8 students should work with fractions, decimals, and percents to solve problems, and compare and order fractions, decimals and percents efficiently and be able to represent fraction in a various way, so that they can see that 1/4, 25%, and 0.25 are all different representations of the same number. Lastly the NCTM Standards stress the importance of providing a variety of fraction models and connecting fraction problems to real-life situations. Additionally, when working with proportionality, students need to solidify their understanding of fractions as numbers (NCTM, 2000). Students need to be able to use proportionality to form ratios between measurements of the same kind: length-to-length, time-to-time, dollars to dollars, and so on.

The Multiplication/Box Method is not intended to replace concrete examples as outlined by the NCTM. The multiplication/box method could be used after students can connect fractions to real-life situations.

## The Importance of Learning Fractions

Knowledge of common fractions is important. A wide variety of occupations (carpenter, plumber, cook) depend on a working knowledge of addition, subtraction, multiplication, division of fractions, and of common fractions. For example, a cook may need to prepare a dish for twice as many people. This would require him/her to double all the ingredients. Any teacher who fails to teach common fractions to their students is guilty of blocking students' development and limiting career choices (Peck, Jencks, 1979). The ability to use different representations of fractions is important as a foundation for later work in algebra (Mathematics Framework for California Public School, 2006). In algebra students need to solve and simplify rational expressions. California's Standardized Tests (CST) in mathematics is laden with items that measure fraction knowledge as shown in Appendix B.

Based on the number of questions released by the state of California to review for the California Standards Test, teachers in their lectures must emphasize fractions. In addition to the CST Release Questions, the state of California has outlined six volumes of recommendations for mathematic instruction: Place Value and Basic Number Skills, Fraction and Decimal, Ratios Rates and Percents, The Core Processes of Mathematics, Functions and Equations and Measurement. In Volume II, Fraction and Decimal, of the Mathematics Framework for California Public School (2006), 18 out of the 23 standards are related to the concept of fractions and decimals. The volumes are designed to serve strategically for students in grades four through seven so that they can learn efficiently from basic grade-level instructional materials. The National Council of Teachers of Mathematics is in favor of teaching fractions in a "systematic and direct way" (NCTM 1989) and even argues that time must be found to give increased attention to fractions (Bezuk & Cramer, 1989).

The Mathematics Framework for California Public Schools (2006) also outlines an Algebra 1 Readiness Program that includes nine topics. The Algebra 1 Readiness Program is designed for students who have trouble passing Algebra 1 in high school (grades 9-12). This program outlines sixteen standards that students should master before enrolling into Algebra 1. These topics are also generally used as guidelines for designing a focused curriculum for struggling students in grades eight or above. Five out of the sixteen standards are related to rational expressions and fractions (Mathematics Framework, for California Public School, 2006).

## Common Mistake when Working with Fractions

Do children who use concrete representations or manipulative automatically understand fraction concepts? No, this is not necessarily the case (Thompson & Lambdin, 1994). For example, some students define a fraction as "a piece of pie to eat," because they have only had circle diagrams examples (Niemi, 1996). Providing a variety of representations will encourage students develop a deeper understanding of fractions, as long as educators help connect the students' understanding of concepts to the different representations and not just the circle diagrams.

According to McLeod and Armstrong (1982) "teaching and learning fractions has traditionally been one of the most problematic areas in school mathematics". A teacher needs to specify "the whole" explicitly before discussing "parts" of the whole (Math Framework for California Public School, 2006). For example, if the teacher uses a circle to illustrate fractions, then he or she should specify clearly how to divide a circle into equal parts. Otherwise, students may divide a circle into parts that are not equal, for example, three subsets of equal width (see Figure 2) and claim that each subset is 1/3, (see Figure 2).

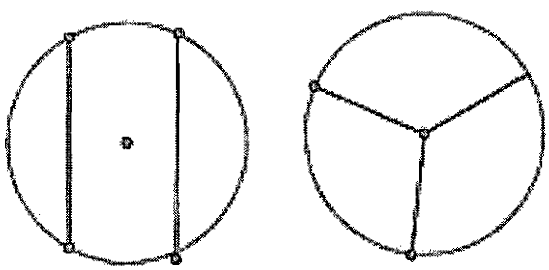


Figure 2. Circle Divide Into Equal Parts

Algorithms that are confusing to students cause many of the errors students make with fractions. "Also, errors are caused by students applying algorithms inappropriately" (Jencks, Peck & Chatterley, 1980; Lankford 1974, p. 39), for example, 2/3 x 3/5 = 10/15 x 9/15 = 90/15. The common mistake a student would make is that she would find a common denominator even though this is a multiplication problem, and therefore unnecessary. Then the student would multiply the numerator only (Lankford 1974). Students making this mistake have learned the method of finding common denominators but they do not understand when it needs to be applied (Carpenter et al., 1976). Unit design is often the problem with students who tended to miss apply the algorithms for addition and multiplication of fractions (Bernadette, Russell & Douglas, 1990). For example, teaching fractions in isolation, where one day the instructor teaches students how to add fractions and on the next day they learn how to subtract fractions without connecting the two lessons together.

Peck and Jencks (1979) state that the difficulties children have with fractions are conceptual. Children appear to be going through the motions of operations of fractions but they have not been exposed to the kinds of experiences that could provide them with the necessary understanding. Multiplication is also an addition problem. Start by introducing them to examples that reinforces that idea. For example, $3 \cdot 1/2$, one gets three groups and in each group it has one half of a whole. How many one-halves do you have? You have three one-halves (3/2). Peck and Jencks (1979) suggest that mathematics educators and curriculum writers need to shift emphasis from the learning of rules for operations of fractions to the unveiling of a conceptual basis for fractions.

Ginther, Ng and Begle's (1976) study gather information about how much average students can learn about fractions under the best conditions. The results showed that in well-to-do (areas where the average household income is above the national average) middle schools, instruction, even under most favorable conditions, did not provide students with the necessary fractional skills. While students understood the fraction concept, they showed a poor understanding of the structure of the rational number system (Ginther, Ng & Begle, 1976). Rational number system is the set off all possible rational numbers. In some classes teacher models this on a number line. The study found that only 30% of the students were able answer the question correctly (Ginther et al., 1976, p. 4). Additionally, the results of the computation test found that students did poorly on simple word problems that involved fractions. This study suggests that without understanding structure students will not be able to manipulate fractions. Since current instruction in a high school depends too much on learning algorithms, students make little sense of fractions and often misapplied the algorithm. Is this because the concepts are being presented too early in a child's cognitive development? The study does not attempt to answer this question, but concludes: "Much of the work on fractions should be postponed to secondary school" (p. 9).

## Proportional Reasoning

Proportional reasoning is a fundamental mathematical process. An important application of proportional reasoning is the construction of equivalent fractions. Equivalent fractions are proportionality statements that play a key role in operations such as the addition and subtraction of fractional numbers. Addition and subtraction of fractions require a common denominator.

The chapter, "Number Concepts and Operations in the Middle Grades" (NCTM, 2000), stresses proportional reasoning as a pivotal concept. Proportional reasoning is the capstone of children's elementary school arithmetic. Proportional reasoning is also the cornerstone of all that is to follow. "It therefore occupies a critical position in school mathematics" (NCTM, 2000, p. 95). Given that proportional reasoning plays a central role in mathematics, how can we best nurture proportional reasoning? In every opportunity where proportional reasoning is involved pointing out its role, discussing how it helps will build understanding.

Reliance on the multiplication table as an aid to avoid mastering basic multiplication facts is discouraged. The proportional nature of the multiplication table becomes even more apparent using the next approach. In this instance, a rectangle is drawn anywhere in the table with sides parallel to the table boundaries, as shown below (see Figure 3).

| * | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 A | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 B | 21 | 24 | 27 | 30 | 33 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 64 | 70 | 77 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 |

Figure 3. Multiplication Table and Proportional Reasoning Rectangle

Study the four numbers in the corners of Rectangle A (1, 4, 5, 20) 1/4 and 5/20 can be viewed as the equivalent fractions. They can also be viewed as another set of equivalent fractions: 1/5 and 4/20. The relationship can be expressed as proportions in several ways (Wiebe, 1998):

1 : 5 = 4 : 20

1 : 4 = 5 : 20

Finally, the cross products are equal (1 x 20 = 4 x 5). Note in particular the multiplicative relationships that permeate this array. Five-twentieths is derived from 1/4 by multiplying its numerator and denominator by 5, 4/20 is derived from 1/5 by multiplying by 4, and the cross products are equal. These relationships are the hallmark of all proportions (Wiebe, 1998).

Rectangle B has a different orientation. In this instance the numbers in the corner are:

| | |
|---|---|
| 18 | 24 |
| 48 | 64 |

The fractional numbers 18/48 and 24/64 are equivalent and both are equivalents of 3/8; further, 18/24 and 48/64 are equivalent and both are equivalents of 3/4. The fraction 24/64 is derived from 18/48 through multiplication.

However, if both numerator and denominator of 18/48 are multiplied by 4/3, the result is 24/64. Similarly, 48/64 is the result of multiplying the numerator and denominator of 18/24 by 8/3. The multiplicative relationships in proportions should be point out constantly (Wiebe, 1998).

The Multiplication Table Method and the Box Method

During the 2007 CMC conference in Palm Springs, speaker Brad Fulton presented strategies on how to add, subtract, divide and multiply fractions using the multiplication table, which he called Maximum Math From the Multiplication Table (2005). During the presentation I asked Brad Fulton if any research had been conducted on this method. Brad Fulton replied that to his knowledge, no study has been conducted on the use of the multiplication table or box method to reduce, add, subtract, multiply and divide fractions.

Prior to the conference Dolores Jones a consultant from the Education Testing Service working with Rialto Unified School District math coaches, had a warm-up where she placed a four by four square on the board and had us multiply the numbers. She then asked us to find patterns. Working with other Rialto Unified School Coaches we found patterns for adding, subtracting, dividing and multiplying fractions. In 2008, I emailed Dolores, asking her for any information about the multiplication table/box method. She informed me that she had not conducted any research on this topic and that she did not remember where she first seen it used. My advisors and I spent countless hours searching for research on this topic. We did not uncover any research on this method. To my knowledge, my thesis is the first attempt to research this method.

Teaching that allows students to construct their own understanding of fractions can be powerful. This project will use student's prior knowledge of the multiplication table to simplify fractions. Kieren (1980, p. 102) asserts that "the number of disjointed protocols a learner must control to form the rational number concept is extensive". Henry Margenau (1961) defines protocols as a "collection of facts and related experiences that an individual brings to bear upon a problem". If the facts and related experiences can be connected effectively, then the individual is able to construct their own knowledge.

When algorithms are beyond the learner's cognitive development. The learner is force to abandon their own thinking and resort to memorization and doing without understanding. Learners who forget the algorithm retreat back to familiar procedures (protocols) and apply it to the given situation. Learners may try to apply a natural number protocols for addition of fractions and adding both numerators and denominators. Since addition of natural numbers arise from the natural activity of children (Kieran, 1980). The multiplication table/box method allows learners to retreat to familiar protocol if they get confused.

The multiplication table/box method will focus on seven protocols: 1) equivalent fractions; 2) reducing fractions; 3) adding and subtraction fractions using equivalent fractions and reducing fractions; 4) Addition and subtraction of fractions using the multiplication table method or the box method; 5) multiplication of fractions using the multiplication table/box method; 6) division of fractions using the multiplication table/box method; 7) solving equations using the multiplication table/box method.

Equivalent Fractions

Brad Fulton (2005) does not explain or talk about equivalent fraction in his paper Maximum Math From the Multiplication Table. Equivalent fractions are an important concept in this topic. They are the building block to addition and subtraction of fractions.

Proportional reasoning involves a purely multiplicative relationship. For example, $x/4 = 8/3$ can be rewritten in the form $1/4 \cdot x = 8/3$ where you are multiplying x to 1/4. It should come as no great surprise that the multiplication table is a table of proportions. Since equivalent fractions are statements of proportionality, it is reasonable to expect that the multiplication table is a set of equivalent fractions. For example 1/3 and all its equivalent forms can be found on the multiplication chart ($1/3 = 2/6 = 3/9$ ...). Equivalent fractions can be generated by multiplying the given fraction by $1/1 = \ldots b/b$ (top row). Any two rows would generate equivalent fractions, as shown below (Forsten, 2005). This protocol allows students to compare fractions and is the building block for addition and subtractions of fractions (see figure 4).

| * | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 |

Figure 4. Multiplication Table Equivalent Fractions

<u>Reducing Fractions</u>

Brad Fulton (2005) does explain how to reduce fraction but he calls it simplifying fractions on his paper Maximum Math from the Multiplication Table. I have provided his example below. The second protocol is reducing fractions. Fractions can be reduced by locating the fraction vertically in the multiplication chart and by reading the simplified value from the left-hand column. For example, 12/21 (see Figure 5) can be reduced to 4/7 (Forsten, 2005).

| * | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 48 | 56 | 63 | 70 | 77 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 |

Figure 5. Multiplication Table Reducing Fractions

<u>Adding and Subtraction Fractions using Equivalent Fractions and Reducing Fractions.</u>

Next, the students can move on to addition and subtraction of fractions. During this protocol students will practice finding equivalent fractions and reducing fraction.

Brad Fulton (2005) models how to add and subtract fraction using the multiplication table (see Figure 6). He calls this section adding and subtracting fractions of unlike denominators. He used 2/7 + 3/5 as shown below. He starts by finding the first fraction on the left side of the multiplication table and the second at the top of the multiplication table. His next step is to multiply the denominator as shown below. This will be your denominator of your answer (see Figure 6).

| * | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 48 | 56 | 63 | 70 | 77 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 |

Figure 6. Add and Subtract Fraction Using the Multiplication Table A

His next step is to multiply the numerator of the first fraction with the denominator of the second and the denominator of the first fraction with the numerator of the second, as shown below (see Figure 7).

| * | 1 | 2 | **3** | 4 | **5** | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **2** | 2 | 4 | 6 | 8 | **10** | 12 | 14 | 16 | 18 | 20 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| **7** | 7 | 14 | **21** | 28 | **35** | 42 | 48 | 56 | 63 | 70 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |

Figure 7. Add and Subtract Fraction Using the Multiplication Table B

The last step is to add the products. This is the numerator of your answer. This method was not model to my students. I felt that it did not provide any new conceptual understanding of fractions to students. This project combined equivalent fraction protocol and reducing fractions protocol to add and subtract fractions (see Figure 8). For example, 1/3 + 1/2. First, have students find equivalent forms of 1/3 and 1/2.

1/3=2/6=3/9 …

| * | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 |

AND

1/2= 2/4 = 3/6 …

| * | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 |

Figure 8. Add and Subtract Fraction Using the Multiplication Table C

Adding and subtracting fractions requires the use of common denominators. 1/3 and 1/2 have common denominators at 6, 12,18 and so on.

1/3 + 1/2

2/6 + 3/6 = 5/6

4/12 + 6/12 = 10/ 12 (reduce using the Box Method) = 5/6

6/18 + 9/18 = 15/18 (reduce using the Box Method) = 5/6

<u>Addition and Subtraction of Fractions using the Multiplication Table Method or the Box Method (a Two by Two Grid)</u>

Other proportional reasoning patterns in the multiplication table (see Figure 9) are the classic algorithm for addition, subtraction, division and multiplication of fractions. Below you will see the multiplication table (Forsten, 2005).

| * | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | -- | c | d |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1c | 1d |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 2c | 2d |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 3c | 3d |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 4c | 4d |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 5c | 5d |
| : | | | | | | | | | | | |
| a | 1a | 2a | 3a | 4a | 5a | 6a | 7a | 8a | 9a | ac | ad |
| b | 1b | 2b | 3b | 4b | 5b | 6b | 7b | 8b | 9b | bc | bd |

Figure 9. Multiplication Table

When working with fractions students can generate the multiplication table or isolate the fraction a/b and c/d.

The fraction a/b will be located vertically on the first column of the multiplication table and the fraction c/d will be located horizontally on the first row of the multiplication table. Write the fractions a/b and c/d on its own two by two grid, as shown below (see Figure 10). I will this square the box method (Fulton, 2005).

a/b, c/d

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | bd |

Figure 10. The Box Method

Brad Fulton (2005) refers to this section as: fraction operations without a multiplication table. His example is 2/3 + 1/4 and 2/3 - 1/4 (see Figure 11).

| | 1 | 4 |
|---|---|---|
| 2 | 2 | **8** |
| 3 | **3** | **12** |

Figure 11. Brad Fulton Example 1

He then said that to find the sum it is necessary to add 8 and 3 and write the answer over 12 and to find the difference, subtract 8 from 3 and write the answer over 12. No other explanation was given. After reviewing his paper he does not reference any articles or journals on this method (The Box Method).

In this project I used his method (The Box Method) for addition and subtraction of fractions. When this method was introduced in class I used the multiplication table to explain where the numbers were coming from. I also explained that they were finding equivalent fractions. Students were asked to rewrite their two new fractions and then add the numerator.

8/12+3/12 = 11/12

8/12-3/12 = 5/12

As the Math Coach at Bear High school, the box method has been an established instructional strategy for the last three years. I have been using the box method to multiply binomials for ten years. Holt Algebra 1 (2007) California edition had an activity on the use of the box method for multiplying binomials. The first binomial is placed on the top row and the second binomial is placed on its side vertically (see Figure 12).

For example (x-2)(x+1)

| * | x | -2 |
|---|---|---|
| x | $X^2$ | -2x |
| +1 | +1x | -2 |

$x^2 - 2x + x - 2$

Figure 12. The Box Method Multiplying Binomials

Holts Algebra 1 (2007) California edition modeled the same problems using algebra tiles to multiply binomials (see Figure 13), for example, (x-2)(x+1).

$x^2 - 2x + x - 2$

| X | x | -1 | -1 |
|---|---|---|---|
| +x | $X^2$ | -x | -x |
| 1 | +x | -1 | -1 |

Figure 13. Holt Algebra 1 the Box Method

When multiplication is being modeled with algebra tiles, the dimensions of a rectangle represent the factor, and the area of the rectangle represents the product. The area of the rectangle can sometimes be simplified by removing zero pairs. The Box Method also represents the same information.

The use of the Box Method to solve problems with fractions is a new idea. After careful research, I found no study on the use of the Box Method to solve problems with fractions.

The classic algorithm for adding and subtracting fractions requires the use of common denominators. Common denominators ensure that the quantities being added or subtracted are based on equal parts of a whole. Addition and subtraction of fractions is often taught with an emphasis on finding the least common denominator (LCD) rather than any common denominator. It is important for students to understand that there is not one unique denominator. The Box Method finds a common denominator (see Figure 14).

a/b and c/d

a/b = ac/bc = ad/bd are equivalent fraction and

c/d = ac/ad = cd/bd are equivalent fractions.

Fraction ad/bd and cd/bd have common denominators.

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | bd |

Figure 14. Box Method Common Denominator

When the LCD is not easily identifiable, multiplying the denominators may be a more efficient method of finding a common denominator. The classic algorithm for adding and subtracting any two fractions a/b and c/d, when b and d are not equal to zero, is as shown bellow, (see Figure 15).

$a/b \pm c/d$

$(a/b)(d/d) \pm (c/d)(b/b)$

$((ad)/(bd)) \pm ((bc)/(bd))$

$(ad \pm bc)/bd$ (The classic algorithm).

Figure 15. The Classic Algorithm

The multiplication table and the Box Method also has the same pattern embedded in it (see Figure 16). Write the first fraction on the left of the grid. Write the second fraction on the top of the grid (The Box Method). Multiply the digits to complete the grid (Fulton, 2005).

a/b , c/d

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | bd |

Identify each piece of the classic algorithm (ad, cb, bd)

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | bd |

ad ± cb numerator

bd denominator

(ad ÷ cd)/bd (The classic algorithm derived from the Box Method)

Figure 16. The Box Method and the Classic Algorithm

Students who have forgotten the algorithm can resort back to addition and subtraction of fractions using the multiplication table/box method or adding and subtraction fractions using equivalent fractions and reducing fractions.

Multiplication of Fractions using the Multiplication Table Method or the Box Method

The underlying concept of multiplying fractions may not be easy to grasp. The product of (a/b)(c/d) = ac/bd can be defined as the area of a rectangle (see Figure 17) with side lengths a/b and c/d. This approach is called the area model. Consider the problem of multiplying 3/4 and 2/3. Draw a rectangle with dimension 3/4 and 2/3 unit inside a square with side lengths of 1 unit.

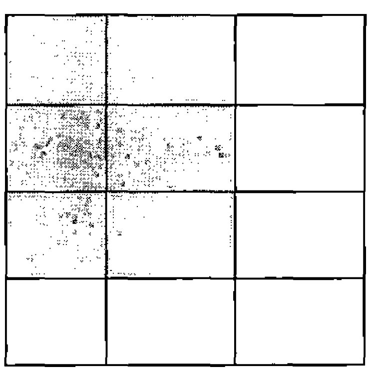


Figure 17. Area of a Rectangle

The diagram above demonstrates that the area of the rectangle is 6/12. The product of the denominators is equal to the total number of parts into which the square is divided and that the product of the numerator is equal to the number of shaded parts ((a/b)(c/d) = (ac/bd)). When multiplying fractions the classic algorithm tells us to multiply straight across numerator with numerator and denominator with denominator, ((a/b)(c/d) = (ac/bd)), where b and d are nonzero, and the area model supports or explains that result. Students can also use the last step of the Multiplication Table Method or The Box Method to multiply fractions. Multiplication of fractions with the square grid, The Box Method, also multiply numerator with numerator and denominator with denominator like in the classical algorithm.

When multiplying fractions, Brad Fulton (2005), first identify the one of the fractions on the left hand side of the multiplication table. Then, identify the second fraction at the top of the multiplication table. Multiply as shown. The upper number is the numerator of your answer and the lower number is the denominator (see Figure 18).

Example: 4/5 • 3/4

| * | 1 | 2 | ***<u>3</u>*** | ***<u>4</u>*** | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | **3** | **4** | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 2 | 4 | **6** | **8** | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 3 | 6 | **9** | **12** | 15 | 18 | 21 | 24 | 27 | 30 |
| ***<u>4</u>*** | 4 | **8** | **<u>12</u>** | **16** | 20 | 24 | 28 | 32 | 36 | 40 |
| ***<u>5</u>*** | **5** | **10** | **15** | **<u>20</u>** | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 48 | 56 | 63 | 70 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |

Figure 18. Brad Fulton Example 2

Using the same box from addition and subtraction identify ac and bd (Fulton, 2005). Brad Fulton also modeled another example (see Figure 19) using the box method, as shown below [(2/3) times (1/4)].

| | 1 | 4 |
|---|---|---|
| 2 | <u>2</u> | 8 |
| 3 | 3 | <u>12</u> |

Figure 19. Brad Fulton Example 3

He then said that the product is found diagonally as shown above, 2/12. Again no reference was provided (see Figure 20).

a/b , c/d

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | bd |

(ac = numerator and bd = denominator)

Figure 20. The Box Method Multiplying Fractions

<u>Understanding why Multiplication Works in the Box Method?</u>

First, prove that the elements inside the box are unique and not random numbers. The three numbers inside the box are proportional and are located on a multiplication table. Next, prove that the fourth vertex is the solution to the proportion. Brad Fulton (2005) called this section solving proportions, an algebraic proof (see Figure 21).

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | x |

Figure 21. Algebraic Proof A

If x is the solution to the proportion, then the left column is equal to the right column (ac/bc = ad/x).
Applying the cross product rule shows, (ac)x=(ad)(bc).
Divide by (ac) to solve for x.
x = ((ad)(bc)/ac
Canceling the common factor leaves, x = bd (see Figure 22).

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | bd |

Then ac/bd = a/b • c/d
Figure 22. Algebraic Proof B

When presenting this section to my class I first proved that the classic algorithm was embed in the box method. Students are familiar with the classic algorithm from prior years. The box method tries to build on students' prior knowledge as much as possible. Students' work shows that students remember which diagonal represented the product of two fractions.

Division of Fractions using the Multiplication Table Method or the Box Method

The concept of whole number can be used to explain division of fractions. For example, the problem 1/2 ÷ 1/8 can be solved by considering the question, how many groups of 1/8 are in 1/2? These problems can be illustrated by using geometry models as shown below (see Figure 23).

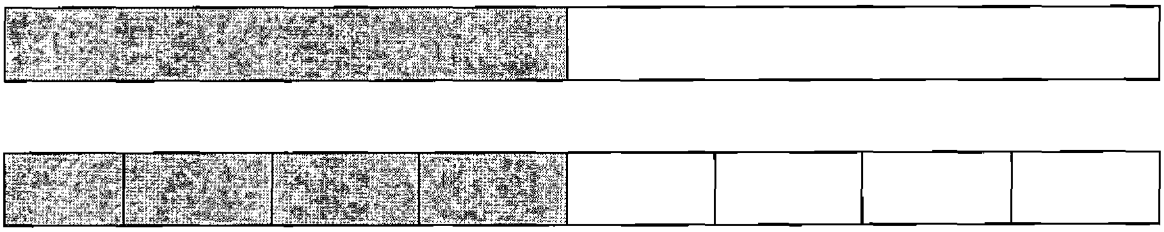


Figure 23. Division of Fractions Geometry Models

There are 4 groups of 1/8 in 1/2, so 1/2 ÷ 1/8 = 4. The classical algorithm for division of fraction tells us to "invert-and-multiply", a/b ÷ c/d = a/b x d/c, (1/2 ÷ 1/8 = 1/2 x 8/1). First let's answer the question: why do we invert-and-multiply? In any fraction a/b where a and b are both non-zero we have that a/b x b/a = 1. If we have (a/b)/(c/d) and we multiply the denominator and numerator by d/c we then get ((a/b)(d/c))/((c/d)(d/c) = (a/b)(d/c). You can also derive division of fractions by applying the definition of division, c ÷ b = a means that a x b = c (b≠0). Now let's apply it to fractions a/b ÷ c/d = m/n means m/n x c/d = a/b. Multiplying both sides of the latter equation by d/c gives m/n x c/d x d/c = a/b x d/c, which simplifies to m/n = a/b x d/c. By substitution, a/b ÷ c/d = a/b x d/c showing that the invert-and-multiply rule is valid and make it clear which fraction needs to be inverted. When using the last step to the Box Method to solve problem of division of fractions you still multiply numerator with denominator and denominator with numerator.

When dividing fractions, Brad Fulton (2005), find the first fraction on the left hand side of the multiplication table. Then, find the second fraction at the top of the multiplication table. Multiply as shown. The upper number is the numerator of your answer and the lower number is the denominator (see Figure 24). Example: 4/5 ÷ 3/4 (Fulton, 2005).

| * | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | **3** | **4** | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 2 | 4 | **6** | **8** | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 3 | 6 | **9** | **12** | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | **4** | **8** | **12** | **<u>16</u>** | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | **5** | **10** | **<u>15</u>** | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 48 | 56 | 63 | 70 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |

Figure 24. Dividing Fractions using the Multiplication Table

Brad Fulton (2005) provided an example (see Figure 25) on division of fraction using the box method, (2/3 ÷ 1/4).

| | 1 | 4 |
|---|---|---|
| 2 | 2 | 8 |
| 3 | 3 | 12 |

Figure 25. Brad Fulton Example 4

The quotient is found the diagonal as shown (8/3). Brad Fulton (2005) did not provide any reference to this method (see Figure 26).

a/b , c/d

(ad = numerator

bc = denominator)

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | bd |

Figure 26. The Box Method Dividing Fractions

We already proved that the elements inside the box are unique; see section: multiplication of fractions using the multiplication table method or the box method. When this method was introduced in my class I first proved that ad/bc was the classic algorithm for division of fractions. Start with a true statement, ad/bc = a/b x d/c, and prove that left diagonal is the solution when dividing fractions. Multiply ad/bc = a/b x d/c the numerator and denominator by c/d.

ad/bc = a/b x d/c

a/b x (d/c)/1 x (c/d)/(c/d)

a/b x ((dc/cd)=1)/(c/d)

a/b x (1)/(c/d) = (a/b)/(c/d)

a/b x d/c = (a/b) ÷ (c/d)

Which is the definition for classic algorithm for dividing fractions.

### Solving Equations using the Multiplication Table Method or the Box Method

Brad Fulton (2005) called this section solving proportions using the multiplication table (see Figure 27); first find the three known numbers as vertices of a rectangle, for example 12/30=16/x (as showed below). The missing vertex is the solution (see Figure 27) to the proportion (x = 40).

| * | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 2 | 4 | 6 | 8 | 10 | **12** | 14 | **16** | 18 | 20 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 5 | 10 | 15 | 20 | 25 | **30** | 35 | **40** | 45 | 50 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 48 | 56 | 63 | 70 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |

Figure 27. Solving Proportions using the Multiplication Table

Brad Fulton (2005) did not provide any reference to this method. This works because 12 is the product of 6 times 2, the product of 16 is 8 times 2, and the product of 30 is 6 times 5. We can rewrite the proportion as (6x2)/(6x5)=(8x2)/X. Using the cross product rule gives us (6x2)(X) = (6x5)(8x2). The associative and commutative property gives us (6x2)(X)=(6x2)(8x5). Canceling the common factor leaves us with (X)=(8x5) (Fulton, 2005).

Brad Fulton (2005) did not provide any extension to the box method. He did show that you can introduce quadratic equations (exploring quadratic functions) to students using the multiplication table. These next examples were not presented to my classes during this project. I did present them to my AP Calculus class and my Algebra 2 class. I did not collect any student work. For students who are in Algebra 1 or 2 the box method can also aid in solving equations that involve proportions. If 2/7=x/14 we can multiply a/a = 1 (multiplication property of equality) to the right or left side of the equation and still have an equivalent equation. Then 2/7 x a/a = X/14. Now substitute this information into the Box Method (see Figure 28).

| * | a | a |
|---|---|---|
| 2 | x | |
| 7 | | 14 |

Figure 28. Solving Proportions using the Box Method A

First solve the equation 7a = 14, a = 2, then rewrite the box with a = 2 (see Figure 29).

| * | 2 | 2 |
|---|---|---|
| 2 | x | |
| 7 | | 14 |

Figure 29. Solving Proportions using the Box Method B

Now solve for x, 2 x 2 = x (4 = x). We can also use the Box Method to simplify rational expressions, (2-X)/7 + X/14. Fill in the box as seen below (see Figure 30)

| * | x | 14 |
|---|---|---|
| 2-x | $2x-x^2$ | 28-14x |
| 7 | 7x | 98 |

Figure 30. Simplify Rational Expressions with the Box Method A

For addition we only need the right diagonal and the bottom right corner (see Figure 31).

| * | X | 14 |
|---|---|---|
| 2-x | + | 28-14x |
| 7 | 7x | 98 |

(2-x)/7 + x/14

((28-14x)+(7x))/98

(28-7x)/98

Figure 31. Simplify Rational Expressions with the Box Method B

# CHAPTER THREE

# METHODOLOGY

## Introduction

There are many strategies used by teachers to solve problems with fractions. The use of physical materials and other representations to help children develop there understanding of fraction concepts is recommended by the *NCTM Curriculum and Evaluation Standards* (1989). The three commonly used representations are area models, linear models, and discrete models. Fraction circles, paper folding and geoboards are commonly used with area models. Linear models are taught using fraction strips, Cuisenaire rods and number lines. Discrete models use counters, sets. The goal of my project is to use the Multiplication Table or the Box Method to solve problems with fractions.

## Description of the Research Design

The overall design of this project consists of the following two stages: 1) pre-project stage 2) classroom stage. The following paragraphs of this chapter discuss each stage.

## Conduction of the Project Class

### Pre-Project Stage

Three major tasks were performed in the pre-project stage. The first was the determination of the school site in which the project would be carried out. To determine the school site, a review of the Algebra 1 and Geometry CST (California Standards Test) data was conducted on the three high schools at Rialto Unified School District. Bear High School (pseudonym) was selected to be the test site of this project. For information regarding Bear High School, please see Chapter 1.

Secondly, the pre and posttests were made using Exam View Test Generator. Exam View is a test generator that came with the new book adoption approved by the state of California (Prentice Hall Mathematics California Algebra Readiness). The pretest will be given one week before the lesson is presented to students and the posttest will be given the day following the completion of the lesson. All items are aligned to the California State Standards. Pretest and the Posttest will have the same type of problems and will be aligned to the targeted standards for Algebra 1 readiness from the Mathematics Framework, for California Public School (2006). The test will measure each student's knowledge and understanding of fraction concepts and procedures. The pretest-posttest will be the primary way to measure success. The test will be administered to all students during one 50-minute regular scheduled mathematics period. Students will be encouraged to try their best, and directions will be reread or restated as needed.

Finally, the target students for this study were selected. These students were all enrolled in my Algebra 1 and Geometry class. The demographics are as follow:

a. Age range: 15–18 years old

b. Sex distribution: 63% female, 37% male

c. Ethnic breakdown: 13% white, 67% Hispanic, 15% black, 5% Asian

The project was conducted in one week. During the week students had to complete a pre-test and a post-test, as well as all assigned homework.

<u>Classroom Stage</u>

Three major tasks were performed in the classroom stage. The first was the determination of the standards to use. To determine the standards a review of Mathematics Framework for California Public School (2006) was conducted. After the review the standards were selected from Appendix E chapter on Algebra Readiness Program. Here is a list of the standards selected. Algebra readiness program (Mathematics Framework for California Public School, 2006) standards selected for this study (p. 365)

- CA 6.NS.2.1 - Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.
- CA 6.NS.2.2 - Explain the meaning of multiplication and division of positive fractions and perform the calculations.
- CA 6.NS.2.4 - Determine the least common multiple and the greatest common divisor of whole numbers. Use them to solve problems with fractions.
- CA 7.NS.1.2 - Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.
- CA 7.NS.1.3 - Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.
- CA 7.NS.1.5 - Know that every rational number is either a terminating or a repeating decimal and be able to

convert terminating decimals into reduced fractions.

Secondly, the lesson will have three main components: teacher's demonstration, guided practice and independent practice. During the teacher demonstration, the teacher will first demonstrate the use of the multiplication table to solve problems with fractions while describing aloud the steps. During this time students will copy examples and take notes using Cornell style. During the guided practice, the teacher will give prompts and cues as students solve problems together. As students gain independence, the teacher will monitor students and assist only as needed. During independent practice, the students will solve problems independently using the skills that had been taught. The teacher will not provide assistance, but students will be encouraged to help each other.

Finally, the lesson was written using the Three-Phase Model (Mathematics Framework for California Public School, 2006). In the first phase the teacher introduces, demonstrates, or explains the new concept while asking questions and checking for understanding. The second phase, an intermediate step designed to result in the independent application of the new concept. Students gradually make the transition from teachers regulated to self-regulated. In the third phase students work independently. This phase often serves as part of an assessment where students will use their knowledge or skill (Mathematics Framework for California Public School, 2006). To see a copy of the lesson plan see Appendix A. The lesson also outlines five specific learning objectives. Here is a list of the learning objectives.

- Students will learn how to reduce fractions using the multiplication chart.
- Students will be able to find equivalent fractions using multiplication chart.
- Students will be able to derive the classic algorithm using the multiplication chart.
- Students will learn how to add, subtract, multiply and divide fractions using the multiplication chart or the box method.

# CHAPTER FOUR

# RESULTS AND DISCUSSION

## Introduction

A close examination of the student's errors made on the pre and posttest will help teachers understand how students think about fractions and what they have learned and what they have not. It is essential to note that each of the nineteen problems on the pre and posttests was selected for specific reasons. The questions have been broken into three general categories: algorithmic applications, specific arithmetic skills that are prerequisite to algebra, and application of basic fraction concepts in word problems. Some of the problems could fit into more than one category, but each problem was assigned to a single category. Examples of common errors and unique errors are discussed.

## Analysis of Students' Error

For the first part of the study, students were asked to take a pre-test. For the pre-test, students were told a day ahead of the test but no study guide or lecture was presented prior to the test. For the post-test, students were told to study the multiplication table method and the box method and practice their multiplication table. There was evidence of the multiplication/box method in all post-test student work. Students had four different approaches on how they used the multiplication/box method. In their first approach, students combination of the multiplication/box method and the classic algorithm (as shown below). Fifty eight percent of the students work had evidence of this approach (Appendix G).

In their second approach, students only filled in the potion of the multiplication/box needed to answer the question (as shown below). Sixty four percent of the students work had evidence of this approach (Appendix G).

In their next approach, students filled in the entire box and then identified the numbers needed to answer the question (as shown below). Seventy three percent of the students work had evidence of this approach (Appendix G).

In the last approached students wrote all four algorithms that are derived by the multiplication/box method (as shown below). Only 2 students (less than two percent) used this approach (Appendix G).

Pre-Test Results (Appendix E) indicates that my students found most of the items difficult to solve. The mean score was 41.1%, with a median of 38.9% and a standard deviation of 4.21. Item 3 was the highest percent correct at 68%, and item 17 and 18 were the most difficult with 13%. Post-Test Results (Appendix E) had a mean score of 53%, with a median of 52.6% and a standard deviation of 4.09. The post-test data also had 6 items with a score of 70% or higher compared to none in the pre-test. Item 14 was the most difficult with only 26 out of 93 students answered it correct (13%). When comparing the pre-test and post-test we found that students' improved their median score (38.9% to 52.6%), mean score (41.4% to 53.0%), and highest score (16 to 18). Students also lowered their standard deviation from 4.21 to 4.09.

Category I: Algorithmic Applications

The following eight examples were selected to check the algorithms that students use for finding sums, products, quotients, differences, and for reducing fractions to lowest terms. In the post-test students were able to find a common denominator, for addition and subtraction of fraction, and apply the standard algorithm, but then students were unable to reduce the fraction. Students' work clearly illustrates this problem.

What is 11/6 divided by 1 and 2/3? Forty eight percent of the students answered this problem correct in the pre test compared to forty seven percent in the post-test. There was a twenty one percent decrease in accuracy (Appendix G).

The problem above demonstrates how students are flexible. This student was able to use the multiplication/box method and the classic algorithm in the same problem. During the week, I instructed my students to use any method (classic algorithm or the multiplication/box Method. I walked around my class to provide feedback and support to my students regardless of the method they chose.

Find the quotient: 2/3 divided by 1/9? Fifty three percent of the students answered the above problem correct in the pre-test compared to seventy-two percent in the post-test. There was a nineteen percent increase in accuracy (see Appendix G example 6).

The problem above demonstrates that even after a week of instruction on the Multiplication Table (Box Method) thirty one percent of the students continued to use the classic algorithm for multiplication and simplifying fractions. Even after I walked around my class to provide feedback and support to my students regardless of the method they chose. No student solely used the classic algorithm during the post-test. All students were able to apply both methods when necessary.

Find the quotient: 1/5 divided by 1/4? Fifty one percent of the students answered the above problem correct in the pre test compared to seventy seven percent in the post-test. There was a twenty six percent increase in accuracy (see Appendix G example 7).

Seventy percent of the students who answered the problem correct used the Multiplication Table (Box) Method. As illustrated above

Subtract, 5/6 minus 3/4? Fifty three percent of the students answered the above problem correct in the pre test compared to seventy three percent in the post-test. There was a twenty percent increase in accuracy (see Appendix G example 8).

Fifty two percent of the students work had this type of approach to the solution. This student work does not allow a teacher to see if the student finished the problem correctly. During the week lesson I instructed my students to show all their work. This students is missing one step (20+18)/24 = 38/24).

What is 65/104 in simplest form? Thirty six percent of the students answered the above problem correct in the pre test compared to forty two percent in the post-test. There was a six percent increase in accuracy (see Appendix G example 9).

Only twelve percent of the students showed any work. It was not evident whether or not they used the multiplication table directly to answer this problem.

Solve 4/5 divided by 2/3? Thirty nine percent of the students answered the above problem correct in the pre-test compared to seventy one percent in the post-test. There was a thirty nine percent increase in accuracy (see Appendix G example 10).

Twenty three percent of the students who answered the problem correct used the classic algorithm compared to thirty nine percent of the students who answer the problem correct used the multiplication/box Method. That leaves thirty eight percent of the students who used another method like mental math.

Solve, 7 and 1/3 minus 4 and 1/4? Fifty two percent of the students answered the above problem correct in the pre-test compared to seventy percent in the post-test. There was an eighteen percent increase in accuracy. The most common approach to the solution is as follows (see Appendix G example 11).

Solve 11.75 – 6(1/2)? Thirteen percent of the students answered the above problem correct in the pre test compared to thirty five percent in the post-test. There was a twenty two percent increase in accuracy (see Appendix G example 12).

Only thirty percent of all students had any work shown for this problem. Above you can see that this student combined two of the classical algorithms into one problem (addition and multiplication of fractions).

Any re-teaching in this category should include visual and verbal reasoning activities to build conceptual understanding of rational numbers.

Category II: Specific Arithmetic Skills that is Prerequisite to Algebra 1

Rotman (1991) believed that the foundation for understanding algebra is laid in the understanding of arithmetic that student encounter before they reach algebra courses.

The following five examples were selected to check the problem solving skills to use in Algebra students finding sums, products, quotients, differences, reducing fractions to lowest terms. After looking at the distracters in this section of the test, one can see a common error. I found similar errors as in category I. One significant difference between category I and II is the use of vocabulary, quotient, expression, equivalent, and simplest form. Bear High School has a high level of EL (English Learners) students. The use of vocabulary in this section would pose a significant problem for our EL students. Students' error would indicate that more time should be spent on reducing fraction using the Multiplication Table Method. One can see in the student's work shown below, that he or she was able to get the correct solution but again failed to reduce.

Which expression can be used to find the quotient 21/13 ÷ 1/4? Sixty eight percent of the students answered the above problem correct in the pre-test compared to sixty one percent in the post-test. There was a seven percent decrease in accuracy (see Appendix G example 13).

This was one of the three items that had a negative growth from the pre to post test. As shown this student is able to divide fractions. His mistake was computational. This student also did not indicate his choice.

Which expression below will result in a quotient of 10? Forty eight percent of the students answered the above problem correct in the pre-test compared to sixty eight percent in the post-test. There was a twenty percent increase in accuracy (see Appendix G example 14).

Even with a twenty percent increase the average was still below 70%. This student was applying the multiplication/box method correctly. His mistake was that he reduced 3/30 to 10.

In simplest form, 2/5 – 4/10? Forty six percent of the students answered the above problem correct in the pre-test compared to eighty-two percent in the post-test. There was a thirty six percent increase in accuracy (see Appendix G example 15). Sixty percent of the student that had the correct answer had evidence of the multiplication/box method.

Which of the following is equivalent to 14/5? Eleven percent of the students answered the above problem correct in the pre-test compared to fifty two percent in the post-test. There was a thirty nine percent increase in accuracy (see Appendix G example 16). There was no evidence that students used the multiplication/box method. As part of the lesson students were told to write the multiplication table every day. The extra practice with the multiplication table improved students' accuracy.

Which fraction is the same as 2.02? Thirty six percent of the students answered the above problem correct in the pre-test compared to forty-two percent in the post-test. There was a six percent increase in accuracy (see Appendix G example 17).

The algorithm employed by students during the posttest is dependent upon their ability or comfort level. Students who were very proficient with the classic algorithm continue to use it and students who were unsure used the Box Method.

Category III: Application of Basic Fraction Concepts in Word Problem

The next five problems are word problems involving basic fraction concepts and simple fraction computations. The object was to determine if students could recognize what operation with fractions should be used in each of the five contexts and then correctly apply the operation. Encouraging students to use pictorial representations will address some of these errors and provide a variety of partitioning experiences (see Lamon,1999).

What is the perimeter of Mr. MacDonald's cow pasture pictured below (see Figure 32)?

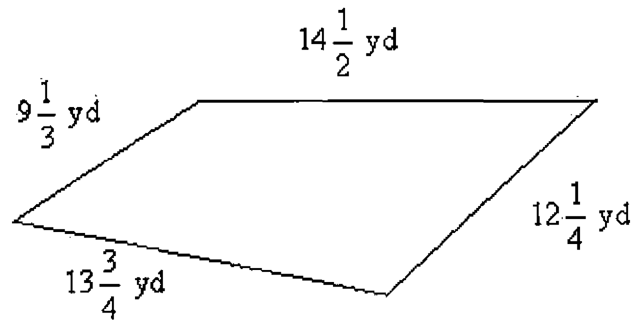



Figure 32. Perimeter of Mr. MacDonald's Cow Pasture

Thirty nine percent of the students answered the above problem correct in the pre-test compared to twenty eight percent in the post-test. There was a eleven percent decrease in accuracy (see Appendix G example 18).

When working with word problems students need to have good problem solving skills. In the student work shown above, he or she understood that finding the perimeter meant that he or she had to add all the sides of the quadrilateral, but he or she did not have a good strategy for working with fractions (see Appendix G example 19).

On the other hand this student had a good strategy. He or she added the whole number then rewrote all fractions to have a common denominator. Only two students tried using the multiplication/box method three times to solve this problem. As you can see this student went back to prior strategies they had learned.

Mr. Henry drove 2/5 of the distance to his grandmother's house this morning and another 3/10 of the distance this afternoon. What fraction of the total distance does Mr. Henry have left to drive to get to his grandmother's house? Thirty nine percent of the students answered the above problem correct in the pre-test compared to forty four percent in the post-test. There was a five percent increase in accuracy. The following illustrates the most used strategy of solution (see Appendix G example 20)

Tina ate 2.5 pieces of pizza. This represented 1/4 of the entire pizza. How many pieces were in the pizza? Forty three percent of the students answered the above problem correct in the pre-test compared to fifty four percent in the post-test. There was a nine percent increase in accuracy (see Appendix G example 21).

Twenty percent of the students were able to apply the Box Method for addition correctly, but they added the wrong numbers. After reviewing other students' work this was a common mistake. Student just used the numbers given and then applied one of the algorithms.

Caroline baked cookies from a recipe that called for 3/4 cup of sugar. She planned to triple the recipe. How much sugar did she need? Forty four percent of the students answered the above problem correct in the pre-test compared to fifty nine percent in the post-test. There was a fifteen percent increase in accuracy (see Appendix G example 22).

Eighty five percent of the students' work who answered this problem correct used repeated addition and the other fifteen percent used multiplication of fractions. No student used the multiplication/box method (see Appendix G example 23).

A hospital-parking garage has 1/10 of its spaces reserved for handicap parking. The garage has 2 floors with 220 spots on each floor. What is the total number of handicap spaces in the garage? Thirty two percent of the students answered the above problem correct in the pre-test compared to twenty seven percent in the post-test. There was a four percent decrease in accuracy. No Student Work.

## Discussion

Bear High School students often complain that they cannot do fractions. Only twenty one percent students used pictorial representations to help them answer some of the questions. Twenty three percent of students were able to apply the concept directly and provide the correct answer without resorting to an algorithm. For most of the students the strategy of choice was to select an algorithm or The Box Method and then use it. The results also reveal an overall lack of experience with basic fractions concepts. Even with the gain in the post-test students are still averaging 53%. Only 22% of the students scored higher than 70%. These numbers show my students lack of understanding with basic fractions.

## Project Constraints

The constraints of this project is summarized as the following:

<u>Limitations on Project: Sample of Project</u>

In this project, there was no comparison group. This is due to the fact that the purpose of the project is solely to identify the positive effects of the Multiplication Table Method or Box Method on high school students. At this point, only my students score have been obtained and reviewed on the pre-test and post-test. More studies need to take place in order to generalize any effects of the Multiplication Table Method or Box Method.

Limitations on Project: How Much Time for Presentation

The Project lasted eleven months from the preparation phase. Yet only one week was spent on the classroom time phase. How much time should be spent on the Multiplication Table Method or Box Method? I assumed one week would be enough time. Spending a different number of weeks on the Multiplication Table Method or Box Method may change the results. It is expected that further studies would be carried over on a continuous basis, which would provide more findings on the effectiveness of the Multiplication Table Method or Box Method.

## Conclusion

There is no escaping fractions. The state of California has embedded in all its CST test fractions. At the same time the State of California has developed an Algebra Readiness program that has fractions as a core requirement, and Mathematics Framework for California Public School (2006) has outline Volume II (Fractions and Decimals) as an essential component in all California Schools. You will find examples that are directly and indirectly related to fractions in linear equations to completing the square, from solving systems of linear equations to solving rational equations, and from simple probabilities to the binomial theorem. Much of the basis for algebraic thought rests on a clear understanding of rational number concepts (Kieren, 1980; Driscoll, 1982) and the ability to manipulate common fractions.

Bezuk and Cramer (1989) offer a few general recommendations for teaching fraction concepts. First the use of manipulatives is fundamental in developing students' understanding of fractions. Next, before grade six students should be committed to developing a conceptual base of fraction relationships such as equivalent fractions. Bezuk and Cramer (1989) also suggest that operations on fractions be delayed until students have a solid understanding of order and equivalence of fractions.

Implementing a balanced curriculum with fractional concepts should first have children in the early primary grades develop whole number concepts and whole number operations informally with abundant concrete referents. Lamon (1999) claims that studies have shown that "if children are given the time to develop their own reasoning for at least three years without being taught standard algorithms for operations with fractions and ratios, then a dramatic increase in their reasoning abilities occurred, including their proportional thinking" (p. 5).

The next step is to give upper primary students experiences that extend the whole number concept towards algebra. Fractions should include manipulation of concrete objects and the use of pictorial representations (Lamon, 1999; Huinker, 1998). The goal at this level is to give learners lots of experience that will be the foundation for a more formal approach to learning fractions. Algorithms should be postponed during this time (Lamon, 1999).

Finally, fraction operations as an extension of whole number operations should be taught in middle school (Lamon, 1999). Students should be provided experiences that guide and encourage them to construct their own algorithm (Lappan & Bouck, 1998; Sharp, 1998). More time is needed to allow students to discover the way to operate on fractions rather than memorizing a procedure (Huinker, 1998). This development should lead to a more formal definition of fractions. Operations and algorithms prepare students for the abstract that arise later in the study of algebra (Wu, 2001). Much of the foundation for algebraic thinking rests on a clear understanding of rational number concepts (Kieren, 1980; Driscoll, 1982) and the ability to manipulate common fractions. Rational number concepts and whole number concepts build fraction concepts, which can be extended to form algebraic concepts. The lack of experience is a problem that will not be resolved unless the philosophy of American mathematics education undergoes a dramatic reformation (Lamon, 1999).

# APPENDIX A

## BEAR HIGH SCHOOL 2006 AND 2007 ALGEBRA 1 CALIFORNIA STANDARDS TEST SCORE

2007 Rialto High School Algebra 1 California Standards Test Score

| 2007 CST Algebra I | 9th | 10Th | 11th | EOC |
|---|---|---|---|---|
| Students Tested | 611 | 244 | 119 | 974 |
| % Of Enrollment | 65.4 % | 28.6 % | 15.5 % | |
| Students with Scores | 611 | 243 | 119 | 973 |
| Mean Scale Score | 277.5 | 273.1 | 270.2 | 275.5 |
| % Advanced | 0 % | 0 % | 0 % | 0 % |
| % Proficient | 3 % | 2 % | 3 % | 3 % |
| % Basic | 22 % | 16 % | 18 % | 20 % |
| % Below Basic | 48 % | 55 % | 46 % | 50 % |
| % Far Below Basic | 27 % | 27 % | 34 % | 28 % |

(Education, California Standardized Testing and Reporting, 2007)

2006 Rialto High School Algebra 1 California Standards Test Score

| 2006 CST Algebra I | 9th | 10Th | 11th | EOC |
|---|---|---|---|---|
| Students Tested | 616 | 402 | 155 | 1173 |
| % Of Enrollment | 64.4 % | 48.5 % | 21.3 % | |
| Mean Scale Score | 270.3 | 272.7 | 278.3 | 272.2 |
| % Advanced | 0 % | 0 % | 1 % | 0 % |
| % Proficient | 3 % | 3 % | 6 % | 3 % |
| % Basic | 15 % | 18 % | 16 % | 16 % |
| % Below Basic | 50 % | 46 % | 46 % | 48 % |
| % Far Below Basic | 32 % | 33 % | 30 % | 32 % |

(Education, California Standardized Testing and Reporting, 2006)

APPENDIX B

NUMBER OF QUESTIONS THAT INCLUDE FRACTIONS, DECIMALS OR PERCENTS FROM THE CALIFORNIA STANDARDS TEST RELEASE QUESTIONS, 2008

NUMBER OF QUESTION THAT INCLUDE FRACTIONS, DECIMALS OR PERCENTS FROM THE CALIFORNIA STANDARDS TEST RELEASE QUESTIONS, 2008

| Grade | Number of items | Percent |
|---|---|---|
| $2^{nd}$ | 7 | 7/80 = 8.75% |
| $3^{rd}$ | 6 | 6/80 = 7.5% |
| $4^{th}$ | 5 | 5/80 = 6.25% |
| $5^{th}$ | 21 | 21/80 = 26.25% |
| $6^{th}$ | 26 | 26/80 = 32.5% |
| $7^{th}$ | 14 | 14/80 = 17.5% |
| Algebra1 | 17 | 17/80 = 21.25% |
| Geometry | 5 | 5/80 = 6.25% |
| Algebra II | 21 | 21/80 = 26.25% |

(Education, 2003 Through 2008 CST Release Test Questions, 2008)

APPENDIX C

THE BOX METHOD LESSON

Algebra readiness program (Mathematics Framework for California Public School, 2006) standards selected for this study (p. 365)

- CA 6.NS.2.1 - Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.
- CA 6.NS.2.2 - Explain the meaning of multiplication and division of positive fractions and perform the calculations.
- CA 6.NS.2.4 - Determine the least common multiple and the greatest common divisor of whole numbers. Use them to solve problems with fractions.
- CA 7.NS.1.2 - Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.
- CA 7.NS.1.3 - Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.
- CA 7.NS.1.5 - Know that every rational number is either a terminating or a repeating decimal and be able to convert terminating decimals into reduced fractions.

Objectives

- Students will learn how to reduce fraction using the multiplication chart (Box Method).
- Students will be able to find equivalent fractions using multiplication chart (Box Method)

- Students will be able to derive the classic algorithm using The Box Method.
- Students will learn how to add, subtract, multiply and divide using The Box Method.

Warm-up

Students will walk in and fill in a twelve by twelve multiplication chart.

| * | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 |

Then students fill out the box chart below (The Box Method).

(P1)

| * | 5 | 8 |
|---|---|---|
| 1 | 5 | 8 |
| 3 | 15 | 24 |

P-2

| * | 3 | 5 |
|---|---|---|
| 7 | 21 | 35 |
| 9 | 27 | 45 |

P-3

| * | 3 | 7 |
|---|---|---|
| 11 | 33 | 77 |
| 12 | 36 | 84 |

Lesson – Guided Practice

Students reduce these fractions and answer this question.

What is the greatest common factor of 2 and 3?

1. 5/10 GCF = 5 (1/2)
2. 3/9 GCF = 3 (1/3)

Now model 5/10 and 3/9 using the multiplication chart

First find 5/10 vertically in the multiplication chart and read its simplified value from the left-hand column.

| * | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 |

1/2 = 5/10

The top row gives the GCF and the left-hand column gives the simplest form. Now have the students explain 3/9 to their neighbor using the multiplication chart and answer this question, What other fractions would also have 1/2 as their simplest form? Model their solution on the multiplication chart.

| * | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 |

Addition and subtraction of fractions with like denominators requires students to combine only the numerator. Remind students of the process for finding equivalent fractions and review the fact that multiplying the numerator and denominator by the same number does not change the value. Also remind students that in order to add or subtract fractions we must have common denominators. One can find a common denominator by multiplying the denominators or finding the LCM (least common multiple) of the denominators. Once students have fractions with like denominators, they can add or subtract the numerators, and then simplify. The classic algorithm assumes that a and d have no common factors (a/b + c/d).

Now write a/b + c/d on the board and ask students to find a common denominator.

$$\frac{ad}{bd} \pm \frac{cb}{bd}$$

$$\frac{ad \pm cb}{bd}$$ (The classic algorithm)

Now have students draw a two by two grid. Write the first fraction on the left of the grid. Write the second fraction on the top of the grid. Multiply the digits to complete the grid.

$\frac{a}{b}, \frac{c}{d} \dashrightarrow$

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | bd |

Ask students to identify each piece of the classic algorithm (ad, cb, bd).

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | bd |

Ask students to answer this question.

What direction do you need to move to generate the classic algorithm for addition and subtraction of fractions?

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | bd |

| | |
|---|---|
| $ad \pm cb$ | numerator |
| bd | denominator |

$$\frac{ad \pm cb}{bd}$$

Have students solve these problems using the classic algorithm

$$\frac{7}{9} + \frac{3}{5} = \qquad -> \frac{35 + 27}{45} = \frac{62}{45}$$

$$\frac{11}{12} \div \frac{3}{7} = \qquad -> \frac{77 + 36}{84} = \frac{113}{84}$$

Now have students use the Box Method

$$\frac{7}{9} + \frac{3}{5} --> \frac{35 + 27}{45} = \frac{62}{45}$$

| * | 3 | 5 |
|---|---|---|
| 7 | + | 35 |
| 9 | 27 | 45 |

$$\frac{a}{b} + \frac{c}{d} \Longrightarrow \frac{ac+bc}{bd} \to$$

$$\frac{11}{12} \div \frac{3}{7} \Longrightarrow \frac{77+36}{84} = \frac{113}{84}$$

| * | 3 | 7 |
|---|---|---|
| 11 | | 77 |
| 12 | 36 | 84 |

$$\frac{a}{b} \div \frac{c}{d} \Longrightarrow \frac{ac-bc}{bd} \to$$

Now have students use the Box method to solve this problem

$$\frac{7}{9} - \frac{3}{5} \Longrightarrow \frac{35-27}{45} = \frac{8}{45}$$

| * | 3 | 5 |
|---|---|---|
| 7 | – | 35 |
| 9 | 27 | 45 |

$$\frac{a}{b} + \frac{c}{d} \Longrightarrow \frac{ac-bc}{bd} \to$$

Ask students to explain the process of finding a common denominator. (LCD and product of denominators)

Independent Practice

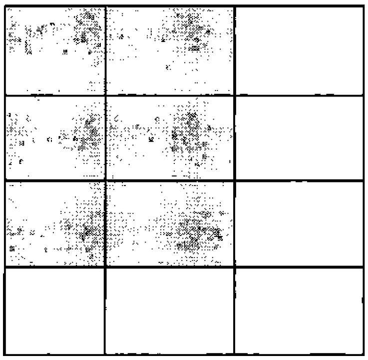
1. $\frac{1}{3} - \frac{7}{10} =$

2. $1\frac{2}{5} + \frac{7}{9} =$

3. $1\frac{2}{5} - 2\frac{7}{9} =$

Multiplication of fractions

Consider the problem $\frac{3}{4}$ times $\frac{2}{3}$. Draw a rectangle with dimension $\frac{3}{4}$ and $\frac{2}{3}$ unit inside a square with side lengths of 1 unit. Now shade in 3 row and 2 columns.



The underlying concept may not be easy to grasp. The product of $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}$ can be defined as the area of a rectangle with side lengths $\frac{a}{b}$ and $\frac{c}{d}$. The diagram above shows that the area of the rectangle is $\frac{6}{12}$. The product of the denominators is equal to the total number of parts into which the square is divided and that the product of the numerator is equal to the number of shaded parts ( $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}$ ).

Method 1: Classical Algorithm ( $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}$ )

$$\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) = \frac{3 \bullet 2}{4 \bullet 3} = \frac{6}{12}$$

Using the same box from addition and subtraction have the students identify the ac and bd.

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | bd |

ac = numerator

bd = denominator

Method 2: The Box Method

$$\left(\frac{2}{3}\right)\left(\frac{1}{4}\right) = \frac{2}{12}$$

| * | 1 | 4 |
|---|---|---|
| 2 | 2 | |
| 3 | | 12 |

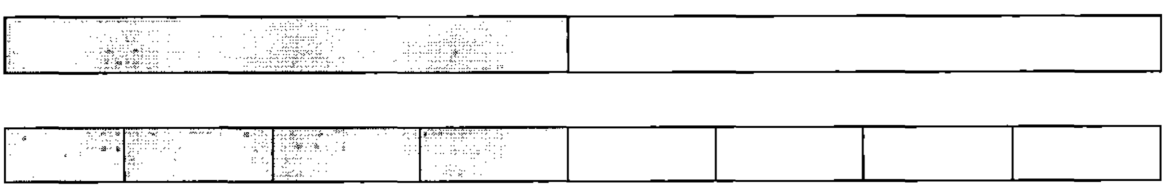
2 = numerator

12 = denominator

$\left(\frac{2}{3}\right)\left(\frac{1}{4}\right) = \frac{2}{12}$ Simplify as necessary. $\frac{2}{12} = \frac{1}{6}$

Division of fractions

Consider the problem $\frac{1}{2} \div \frac{1}{8}$ . The problem cam be represented by the question, how many groups of $\frac{1}{8}$ are $\frac{1}{2}$ in? Illustrated it using geometry models as shown below.



There are 4 groups of $\frac{1}{8}$ in $\frac{1}{2}$, so $\frac{1}{2} \div \frac{1}{8} = 4$

The classical algorithm for division of fraction tells us to "invert-and-multiply", $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$ ( $\frac{1}{2} \div \frac{1}{8} = \frac{1}{2} \bullet \frac{8}{1} = \frac{8}{2} = 4$).

Method 1: Classic Algorithm

$\frac{2}{3} \div \frac{1}{4} =$

$\frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \bullet \frac{4}{1}$

$= \frac{8}{3}$

Method 2: The Box Method

Using the same box have students identify the ab and bc.

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | bd |

ad = numerator

bc = denominator

| * | 1(n) | 4(d) |
|---|---|---|
| 2(n) | | 8 |
| 3(d) | 3 | |

$\frac{8}{3} = \frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \bullet \frac{4}{1}$ Or $\frac{8}{3} = 2\frac{2}{3}$

8 = numerator

3 = denominator

| * | c | d |
|---|---|---|
| a | ac | ad |
| b | bc | bd |

Independent Practice

1. $\frac{2}{3} \bullet \frac{3}{10} =$

2. $2\frac{2}{5} \div \frac{1}{9} =$

4. $\frac{7}{5} \div \frac{8}{9} =$

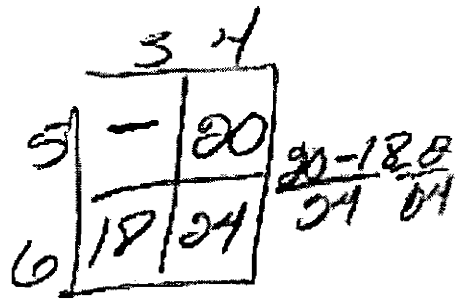
APPENDIX D

STUDENT WORK

Analysis of Students' Error

**Example 1**

9. Subtract.
$\frac{5}{6} - \frac{3}{4}$




**Example 2**

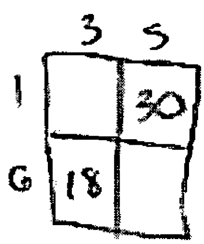
Which expression below will result in a quotient of 10?

a. $\frac{3}{5} \div 6$

b. $\frac{5}{3} \div \frac{1}{6}$

c. $6 \div \frac{5}{3}$

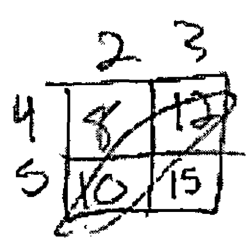
d. $\frac{1}{6} \div \frac{3}{5}$



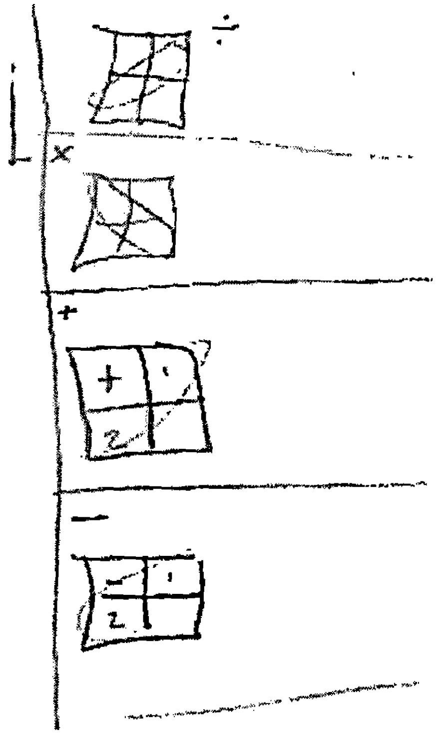

$\frac{30}{18} = \frac{5}{3} = 1\frac{2}{3}$

Example 3

12. $\frac{4}{5} \div \frac{2}{3}$

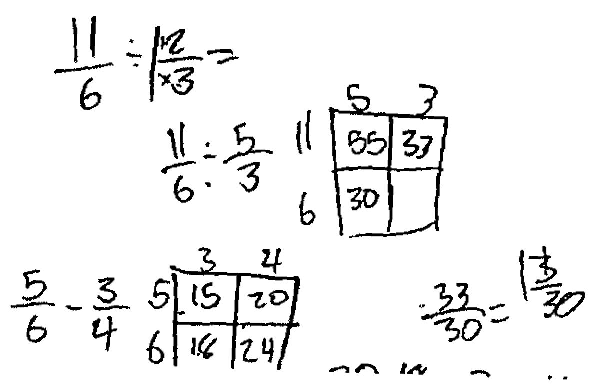


Example 4



<u>Category I: Algorithmic applications</u>

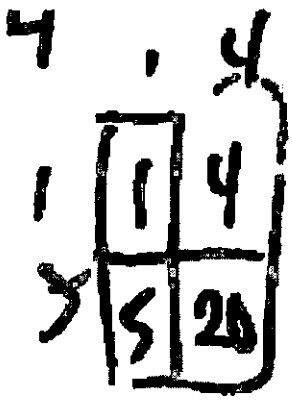
Example 5



Example 6

$$12)\ \frac{2}{3}\cdot\frac{9}{1}\quad\frac{18}{3}\quad\frac{6}{1}=\boxed{6}$$

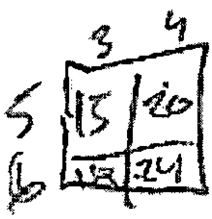
Example 7




$\frac{4}{5}$

Example 8

$\frac{5}{6}$ $\frac{3}{4}$




Example 9

10. What is $\frac{65}{104}$ in simplest form?

$\frac{65}{104}$

a. $\frac{1}{12}$

b. $\frac{5}{13}$

c. $\frac{5}{8}$

d. $\frac{13}{14}$

Example 10

_12. $\frac{4}{5} \div \frac{2}{3}$ $\quad \frac{4}{5} \cdot \frac{3}{2} = \frac{12}{10}$

a. $\frac{3}{4}$

b. $\frac{5}{6}$

c. $\frac{6}{5}$

d. $\frac{15}{8}$

Example 11

$2.\ 7\frac{1}{\times 3} - 4\frac{1}{\times 4} =$

$\frac{22}{3} - \frac{17}{4} =$

| | 17 | 4 |
|---|---|---|
| 22 | – | 88 |
| 3 | 61 | 12 |

$\frac{88-61}{12} = \frac{27}{12} = 3\frac{1}{12}$

3.

Example 12

$\frac{11.75}{1} - 6\frac{1}{2}$ $\quad \frac{2}{13}$ $\quad \frac{22.75}{13}$

---

Category II: Specific arithmetic skills that is prerequisite to algebra 1.

Example 13

____ 10. Which expression can be used to find the quotient $\frac{12}{13} \div \frac{1}{4}$?

$\frac{12}{4}$ / 46

| | 1 | 4 |
|---|---|---|
| 12 | 12 | 46 |
| 13 | 13 | 52 |

$\frac{46}{13}$ $\quad \frac{13}{4}$ / 52

a. $\frac{1}{4} \div \frac{12}{13}$

b. $\frac{12}{13} * \frac{1}{4}$

c. $\frac{12}{13} * \frac{4}{1}$

d. $\frac{1}{4} * \frac{12}{13}$

Example 14

b 1. Which expression below will result in a quotient of 10?

a. $\frac{1}{6} \div \frac{3}{5}$ $\frac{5}{18}$

**(b)** $\frac{3}{5} \div \frac{6}{1}$

c. $\frac{5}{3} \div \frac{1}{6}$

d. $6 \div \frac{5}{3}$

Example 15

____ 5. In simplest form, $\frac{2}{5} - \frac{4}{10} = ?$

a. 0 $\frac{40}{50}$

b. $\frac{1}{5}$

c. $\frac{1}{10}$

d. $\frac{2}{5}$

Example 16

$$\begin{array}{r} 2.8 \\ 5\overline{)14.0} \\ 10\phantom{.0} \\ \hline 04.0 \\ 40 \\ \hline 00 \end{array}$$

Example 17

____19. Which fraction is the same as 2.02?

a. $\frac{51}{25}$

b. $\frac{101}{50}$

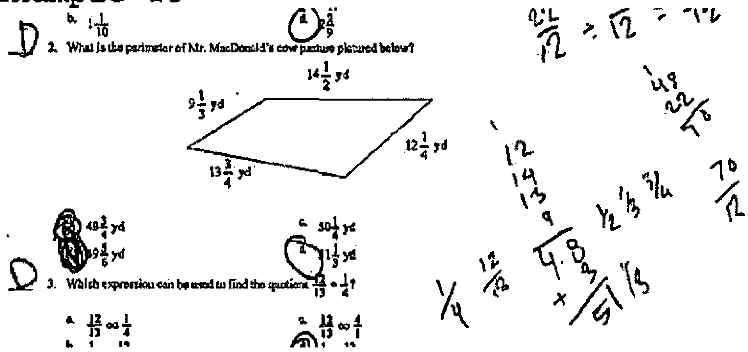
c. $\frac{11}{5}$

d. $\frac{81}{40}$

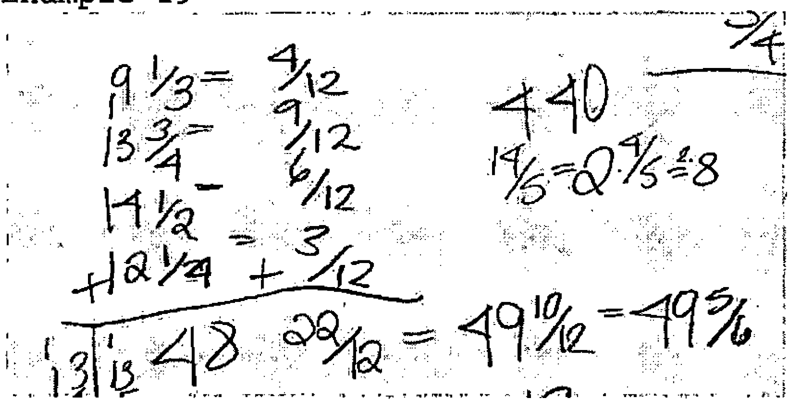
c 2.2
5 ⟌ 11.0
10
1 0

Category III: Application of basic fraction concepts in word problem

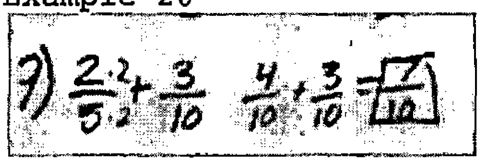
Example 18



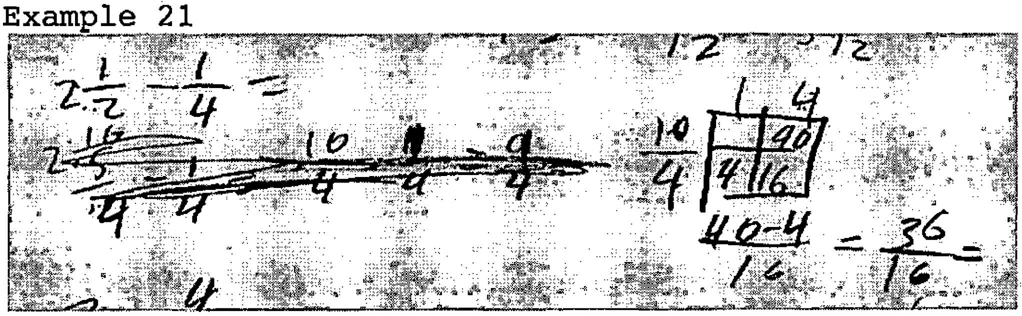
Example 19



Example 20



Example 21



Example 22

8) $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4}$

Example 23

$$\frac{3}{4} \times 3 = \frac{9}{12}$$

APPENDIX E

PRE TEST – STANDARDS ITEM ANALYSIS

| | | | | |
|---|---|---|---|---|
| Total Possible | 19.0 | Median Score | 7.0 | 38.9% |
| Question Count | 19 | Mean Score | 7.5 | 41.4% |
| Tests Scored | 87 | Highest Score | 16.0 | 88.9% |
| Standard Deviation | 4.21 | Lowest Score | .0 | .0% |

**Responses**

| | | | | |
|---|---|---|---|---|
| 1 | A (10, 11%) | ***B (42, 48%)** | C (19, 22%) | D (10, 11%) |
| 2 | A (38, 44%) | ***B (34, 39%)** | C (6, 7%) | D (6, 7%) |
| 3 | A (11, 13%) | B (4, 5%) | ***C (59, 68%)** | D (9, 10%) |
| 4 | A (6, 7%) | B (19, 22%) | C (10, 11%) | ***D (46, 53%)** |
| 5 | A (16, 18%) | ***B (42, 48%)** | C (8, 9%) | D (16, 18%) |
| 6 | A (7, 8%) | B (17, 20%) | C (20, 23%) | ***D (38, 44%)** |
| 7 | A (8, 9%) | ***B (44, 51%)** | C (12, 14%) | D (18, 21%) |
| 8 | A (10, 11%) | ***B (34, 39%)** | C (25, 29%) | D (11, 13%) |
| 9 | A (15, 17%) | B (9, 10%) | C (11, 13%) | ***D (46, 53%)** |
| 10 | A (9, 10%) | B (22, 25%) | ***C (31, 36%)** | D (16, 18%) |
| 11 | ***A(40, 46%)** | B (10, 11%) | C (8, 9%) | D (23, 26%) |
| 12 | A (6, 7%) | B (20, 23%) | ***C (34, 39%)** | D (18, 21%) |
| 13 | A (15, 17%) | B (19, 22%) | ***C (37, 43%)** | D (9, 10%) |
| 14 | A (13, 15%) | B (11, 13%) | ***C (38, 44%)** | D (16, 18%) |
| 15 | A (10, 11%) | ***B (45, 52%)** | C (10, 11%) | D (14, 16%) |
| 16 | A (18, 21%) | B (15, 17%) | **C (28, 32%)** | D (18, 21%) |
| 17 | ***A(33, 38%)** | B (23, 26%) | C (16, 18%) | **D (4, 5%)** |
| 18 | A (14, 16%) | **B (12, 14%)** | C (23, 26%) | *D (28, 32%) |
| 19 | A (9, 10%) | B (22, 25%) | ***C (31, 36%)** | D (16, 18%) |

APPENDIX F

POST TEST — STANDARDS ITEM ANALYSIS

| | | | | |
|---|---|---|---|---|
| Total Possible | 19.0 | Median Score | 10.0 | 52.6% |
| Question Count | 19 | Mean Score | 10.1 | 53.0% |
| Tests Scored | 93 | Highest Score | 18.0 | 94.7% |
| Standard Deviation | 4.09 | Lowest Score | .0 | .0% |

**Responses**

| | | | | |
|---|---|---|---|---|
| 1 | A (7, 8%) | B (13, 14%) | ***C(63, 68%)** | D (7, 8%) |
| 2 | A (3, 3%) | B (16, 17%) | ***C(65, 70%)** | D (5, 5%) |
| 3 | ***A(50, 54%)** | B (13, 14%) | C (17, 18%) | D (8, 9%) |
| 4 | A (10, 11%) | B (30, 32%) | ***C(39, 42%)** | D (8, 9%) |
| 5 | ***A(76, 82%)** | B (6, 6%) | C (1, 1%) | D (7, 8%) |
| 6 | A (6, 6%) | B (7, 8%) | ***C(66, 71%)** | D (11, 12%) |
| 7 | A (4, 4%) | ***B(72, 77%)** | C (6, 6%) | D (8, 9%) |
| 8 | A (6, 6%) | B (14, 15%) | C (15, 16%) | ***D(55, 59%)** |
| 9 | A (8, 9%) | B (15, 16%) | C (21, 23%) | ***D(41, 44%)** |
| 10 | **A (8, 9%)** | B (15, 16%) | ***C(57, 61%)** | D (8, 9%) |
| 11 | A (6, 6%) | B (26, 28%) | ***C(44, 47%)** | D (10, 11%) |
| 12 | ***A(68, 73%)** | B (4, 4%) | C (8, 9%) | D (6, 6%) |
| 13 | ***A(67, 72%)** | B (10, 11%) | C (7, 8%) | D (2, 2%) |
| 14 | A (26, 28%) | B (14, 15%) | C (20, 22%) | ***D(26, 28%)** |
| 15 | ***A(60, 65%)** | B (10, 11%) | C (10, 11%) | D (4, 4%) |
| 16 | A (19, 20%) | B (16, 17%) | C (27, 29%) | ***D(27, 29%)** |
| 17 | A (3, 3%) | B (13, 14%) | ***C(48, 52%)** | D (19, 20%) |
| 18 | A (11, 12%) | B (25, 27%) | C (13, 14%) | ***D(33, 35%)** |
| 19 | A (5, 5%) | B (20, 22%) | ***C(40, 43%)** | D (16, 17%) |

## REFERENCES


Bernadette, K., Russell, G., Douglas, C. (1990). Student error patterns as a function of curriculum design: Teaching fractions to remedial high school students and high school students with learning disabilities. *Journal of Learning Disabilities*, 23(1), 23-29. Retrieved from EBSCOhost.

Bezuk, N. , & Cramer, K. (1989). Teaching about fractions: What, when, and how?. *Yearbook (National Council of Teachers of Mathematics) V. 1989 (1989) P. 156-67, 1989*, 156-167.

Brusseau, G., Brousseau, N. and Warfield, V. (2004).Rationales and decimals as required in the school curriculum. part 1: *Rationals as measurements'. Journal of Mathematics Bevavior* , 1-20.

Carpenter, T. P., & And, O. (1976). Using Research in Teaching. Notes from National Assessment: Addition and Multiplication with Fractions. Arithmetic Teacher. Retrieved from EBSCOhost.

Driscoll, M., & CEMREL, I. e. (1982). Research Within Reach: Secondary School Mathematics. A Research-Guided Response to the Concerns of Educators. Retrieved from EBSCOhost.

Education, C. D. (2006). *California Standardized Testing and Reporting*. Retrieved January 11, 2011, from California Standards Test Scores: http://star.cde.ca.gov/star2006/viewreport.asp

Education, C. D. (2007). *California Standardized Testing and Reporting*. Retrieved January 11, 2011, from California Standards Test Scores: http://star.cde.ca.gov/star2007/viewreport.asp

Education, C. D. (2008). *2003 Through 2008 CST Release Test Questions*. Retrieved January 11, 2011, from Algebra 1: http://www.cde.ca.gov/ta/tg/sr/css05rtq.asp

Forsten, Char. 2005. Math strategies you can count on: Tools & Activities to Build Math Appreciation, Understanding & Skills. *Crystal Springs Books,* 2005. ERIC, EBSCOhost (accessed March 6, 2011).

Fulton, B. (2005). *Teacher to Teacher Press.* Retrieved Nov 14, 2010, from HANDOUT, MAXIMUM MATH FROM THE MULTIPLICATION TABLE: http://www.tttpress.com/pdf/Maximum-Math-from-the-Multiplication-Table-2006.pdf

Ginther, J., Stanford Univ., C. p., & And, O. (1976). A survey of student achievement with fractions. SMESG Working Paper No. 20. Retrieved from EBSCOhost.

Hillen, Judith. (2000). Fabulous Fractions. *AIMS Education Foundation.* Fresno, Ca.

Huinker, D. (1998). Letting fraction algorithms emerge through problem solving. *Yearbook (National Council of Teachers of Mathematics) V. 1998 (1998) P. 170-82, 1998,* 170-182.

Jenks, S.M, Peck, D.M.,& Chatterley, L.T. (1980). Why blame the kids? We teach mistakes!. *The Arithmetic Teacher,* 28, 33-42.

Kieren, T. E. (1976). On the mathematical, cognitive, and instructional fundations of rational number. In R. Lesh (Ed.), Number and measurement: *Paper from a research workshop* (pp. 101-144). Columbus, OH: ERIC/SMEAC

Kieren, T. E. (1980). The rational number construct: Its elements and mechanisms. In T. E. Kieren (Ed.), *Recent Research on Number Learning* (pp. 125–149). Columbus: Ohio State University. (ERIC Document Reproduction Service No. ED 212 463).

Lamon, S. J. (1999). Teaching fractions and ratios for understanding: *Essential Knowledge and Instructional Strategies for Teachers.* Mahwah, NJ: Lawrence Erlbaum Associates.

Lankford, F. r. (1974). What can a teacher learn about a pupil's thinking through oral interviews?. *Arithmetic Teacher*. Retrieved from EBSCOhost.

Lappan, G. & Bouck, M. K. (1998). Developing algorithms for adding and subtracting fractions. In L. J. Morrow & M. J. Kenny (Eds), *The Teaching and Learning of Algorithms in School Mathematics*(pp. 183—197). Reston, VA: National Council of Teachers of Mathematics.

Margenau, H. (1961). Bacon and modern physics: A confrontation. *Proceedings of the American Philosophical Society, 105*(5), 487-492.

Marshall, S.P. (1993). Assessment of rational number understanding: A schema-based approach. In T.P. Carpenter, E. Fennema, T.A. Romberg(Eds.), Rational number: *An integration of research*, (pp. 261-288). New NJ: Lawrence Erlbaum Associates.

McLeod,T. M., & Armstrong, S. W. (1982). *Learning disabilities in mathematics: Skill deficits and remedial approaches at the intermediate and secondary level. Learning Disability Quarterly*, 5, 305-310.

Mullis, I. S., Dossey, J. A., Owen, E. H., Phillips, G. W., National Assessment of Educational Progress, P. J., & Educational Testing Service, P. J. (1991). The state of mathematics achievement: NAEP's 1990 Assessment of the Nation and the Trial Assessment of the States. Retrieved from EBSCOhost.

National Council for Teachers of Mathematics.Commission on Standards for School Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, Va.: National Council of Teachers of Mathematics.

National Council for Teachers of Mathematics.Commission on Standards for School Mathematics. (2000). *Curriculum and evaluation standards for school mathematics.* Reston, Va.: National Council of Teachers of Mathematics.

Niemi, D. (1996). A Fraction is not a piece of pie: Assessing Exceptional Performance and Deep Understanding in Elementary School Mathematics. *Gifted Child Quarterly*, 40(2), 70-80. Retrieved from EBSCOhost.

Peck, D. M., & Jencks, S. M. (1979). Are common fractions out-of-date?. *Elementary School Journal*, Retrieved from EBSCOhost.

Rotman, J. W. (1991). Arithmetic: Prerequisite to Algebra?. Retrieved from EBSCOhost.

Thompson, P. W., & Lambdin, D. (1994). Research into practice: Concrete Materials and Teaching for Mathematical Understanding. *Arithmetic Teacher*, 41(9), 556-58. Retrieved from EBSCOhost.

Wadsworth, B. J. (1996). *Piaget's theory of cognitive and affective development (5th ed.). White Plain*, NY: Longman Publishers.

Wiebe, P., Arthur, D. (1998). *Actions with fractions!. AIMS Education Foundation*. Fresno, Ca.

Wu, H. H. (2001). How to prepare students for algebra. *American Educator*, 25(2), 10-17. Retrieved from EBSCOhost.