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A play on touching

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A PLAY ON TOUCHING

A Project

Presented to the

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in

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by

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ABSTRACT

The purpose of the present study is to examine what type of mathematic program best teaches students with mental retardation how to compute basic addition facts. It was hypothesized that the Touch Math program would be used as a strategy more effectively and with greater speed by students with mental retardation when solving addition problems, than other non-Touch Math strategies. The participant completed Touch Math probes (treatment) as well as non-Touch Math probes when she was given the option to choose any mathematic strategy, other than Touch Math (non-treatment). The alternate mathematic strategies suggested were tally marks, use of manipulatives (blocks), or finger counting. State standard comparison and standardized testing was completed before and after all curriculum-based assessments to further validate progress made. Results showed that the student completed many more problems and with greater accuracy when completing the Touch Math probes versus the non-Touch Math probes. Results of the pre-intervention and post-intervention testing showed that growth over the eight week testing period was limited; however a few factors may have contributed to the lack of growth. A very short testing period of only eight weeks,

increased seizure activity and poorer attendance at the end of post-intervention testing may partially have resulted in lack of growth. Aside from growth, the Touch Math program did however demonstrate to be a more effective mathematic program used by the participant than the other, non-Touch Math program.

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CHAPTER ONE

INTRODUCTION

Strategy use is critical in the process of solving mathematic functions when one has trouble retrieving basic facts (Kerkman & Siegler, 1997). Some researchers have demonstrated that children with mental retardation are at a disadvantage to strategizing and learning basic mathematic functions compared to typically developing children. Organizational strategies involved in memorization have been thought to be a problem for children with mental retardation (Byrnes & Spitz, 1977). It is also thought that they demonstrate limitations in the areas of logic, strategy, and foresight (Byrnes & Spitz, 1977). Conversely, it also has been suggested that these children are able to learn similar to typically developing children (Berdine & Blackhurst, 1981; Burns, Roe & Ross, 1988; Katims, 1996; Kavale & Forness, 1992). The purpose of the present study is to look at the Touch Math program in comparison to other, non-Touch Math programs in regards to how well students with mental retardation are able to effectively compute basic addition problems.

Mathematic Strategies Utilized by
Children with Mental Retardation

Mathematics

Mathematics is a fundamental piece of education that is taught in elementary school and provides a highly functional skill commonly used outside of the school setting. Strategies involved in learning mathematics can range from fairly simple to extremely complex, making the complexity of mathematics and its need for the use of a strategy to solve mathematics problems a fairly complicated subject for children to comprehend. Children must rely on the use of strategies when they have trouble retrieving basic facts in various mathematical processes (Kerkman & Siegler, 1997). In early elementary school, children must strategize and devise ways to solve basic math facts in order to successfully complete higher forms of mathematics. Children with disabilities have more difficulty strategizing and devising ways to solve mathematical problems. The present study will examine how well children with mental retardation are able to solve mathematic problems when they are and are not required to strategize using their own method to solving mathematical problems.

Learning Mathematics

It has been thought that children as young as pre-toddlers are beginning to understand some of the basic facts of mathematics including the understanding that one item and another item equals two items (Gelman & Gallistel, 1978). It has more recently been suggested that young children are not yet able to distinguish between quantities or collections of items; however this skill may be a step towards learning one-to-one correspondence (Mix, Levine & Huttenlocher, 2001) as one-to-one correspondence is one of the earliest forms of mathematics that young children experience. One-to-one correspondence includes the understanding that items in a collection can be matched with items in another collection (Baroody & Benson, 2001). Furthermore, it is thought that children can point to each item in a collection in a one-to-one fashion before they can actually count the number of items in the collection using number words (Beckwith & Restle, 1966). The development of a basic number sense typically occurs during kindergarten and first grade (Correa, Nunes & Bryant, 1999). Number counting is another fairly early step in the learning process of mathematical constructs.

According to Gelman and Gallistel (1978), the first of five principles of learning to count includes the comprehension of one-to-one correspondence where the child is able to identify each item as a single object when counting, occupying one number. The second principal involves the child's understanding that counting occurs in a stable order. A child at this stage would understand that when counting one always counts in the same direction (i.e., three always comes before four and after two). The third principal of counting is cardinality where the child understands that the last number said when counting represents the total number of items present. A child who understands that all like items of one kind can be counted together, has an understanding of the next, fourth principal of counting, abstraction. Lastly, the fifth principal of counting involves order irrelevance, where the child understands that he or she can count items in any sequence. These five basic principles of learning to count are essential for children to understand the concept of counting (Gelman & Meck, 1983), and therefore to understand and comprehend the concept of other basic mathematical processes.

Once basic number sense and counting is learned, the next step would be to learn the four basic mathematical operations which are addition, subtraction, multiplication and division. Knowledge of these operations play an important role in understanding and learning more advanced forms of mathematics (Mercer & Miller, 1992; Van Luit & Naglieri, 1999). According to Simon and Hanrahan (2004) these operations are the very basic of fundamental mathematic operations. With focus on addition as the most basic and fundamental, there are steps to learning new addition strategies (Carpenter & Moser, 1984). The first step of learning addition is the concept of concrete referents, such as using finger counting or manipulatives to represent numbers. A child in this stage would use physical items to represent numbers, in order to count and add these items to solve addition facts. During the second stage children typically would begin using the counting-all strategy. The counting-all procedure consists of counting the total number of items in each group to add two sums together. For example, a child who is adding 5+5 would demonstrate the counting-all procedure if they counted five fingers and an additional five fingers, totaling ten (i.e., one, two, three, four… ten). The next stage would include the counting-on procedure where children would say the total number of items in the first group, and count additional items from other groups onto the first number. Using the example, 5+5, a child who is demonstrating the counting-on procedure would say "five" and count five items onto that number (i.e., five… six, seven, eight, nine, ten). The final stage of learning addition is obviously the most advanced, and hardest for some to reach. This stage includes the memorization of addition facts, which may actually be a result of the counting procedures themselves (Siegler & Shrager, 1984). Memorization and similar difficult stages of addition can be a challenge for many children.

Mental Retardation

Many children have difficulties understanding some of the key concepts in mathematics. Therefore, it is apparent that children with mental disabilities, such as mental retardation, may not have the capacity to think in abstract terms even with great support, and are at a great disadvantage to learning such complex mathematical terms.

Mental retardation is a neurological disability that results in cognitive delays. According to the American Association of Intellectual and Developmental Disabilities

(AAIDD), in order for a child to be diagnosed with Mental Retardation, he or she must demonstrate significant limitations in cognitive functioning defined as the intelligence quotient (IQ), below average adaptive behaviors and the onset of the disorder must occur within the developmental period, between birth and 16-21 years of age (AAIDD, 2007). Significantly below average cognitive functioning, used to diagnose mental retardation, includes an intelligence quotient (IQ) of 70 points or less. Battery tests usually make up the clinical judgment portion of diagnoses including intelligence and adaptive behavior assessments.

The AAIDD specifies that in addition to significantly below average IQ and onset age, there must also be two or more deficits in the adaptive functioning components. Adaptive behaviors are those skills necessary to interact with the child's environment. Skills measured on adaptive behavior scales include communication, self-help, functional academics, home and living skills, social skills, work and leisure, community use, health and safety, and self-direction (Accardo & Capute, 1998). MacMillan, Gresham, and Siperstein (1993) believe the adaptive behavior component within the diagnosis of mental retardation is extremely important due to the flexible range in the IQ cutoff for mental retardation diagnosis. Discrepancies in diagnosis exist when the onset of delay and the IQ score falls within the mentally retarded range, but adaptive behavior skills fall within the normal range. This discrepancy has been thought to be attributed to the insensitivity of the IQ test identifying a learning or communication disorder (Accardo & Capute, 1998).

Levels of Mental Retardation were previously based upon IQ scores by the AAIDD (2007) as follows: mild (IQ 50-70), moderate (IQ 35-50), severe (IQ 20-35) and profound (IQ 20 and below). The AAIDD (2007) has revised their definition to categorize the same levels of mental retardation, now based upon the level of needed support to achieve independence rather than by degree of cognitive severity, or IQ (Luckasson, et al. 2002). Areas of need are identified based on adaptive skills needed to function in the individual's environment. Based on these areas of need, supports are then determined and may include teaching, befriending, financial planning, employee assistance, behavioral support, in-home living assistance, community access and use, and health assistance (Luckasson, et al. 2002).

Mental Retardation and Math Difficulty

It is apparent that children with mental retardation learn differently and at a different pace than typically developing children. Students with learning disabilities as well as mental retardation have more difficulties with the complexities of mathematics than typically developing children. Students with mathematical learning disabilities often make more common errors in counting than those typically developing children (Geary, Hoard & Hamson, 1999; Geary, Hamson & Hoard, 2000), have more trouble with the basic math process of counting (Geary, 2004), and show greater difficulty understanding Gellman and Gallistel's (1978) fifth principal, order irrelevance (Geary, 2004). It was also found that these children, compared to typically developing children, were more likely to use the more immature, counting-all procedure, relying mostly on finger counting (Fuson, 1982; Geary et al., 1999; Geary et al., 2000; Groen & Parkman, 1972).

Children with mental retardation likely would have similar or perhaps greater difficulties than students with learning disabilities when learning mathematic processes. According to Geary, Bow-Thomas and Yao (1992) children who had problems and were stuck using the counting-all procedure, did so because they did not truly understand the concept of order irrelevance, where they could begin counting in any sequence. Also, it has been found that deficits in visual-spatial skills can contribute to arithmetic problems due to the inability to align numerals when computing multi digit addition problems, misreading numeral signs and rotating numbers (Sousa, 2001). Children with mental retardation demonstrate more limits than children without mental retardation in the areas of logic, strategy and foresight (Byrnes & Spitz, 1977), all skills necessary to effectively compute mathematics.

Mathematically learning disabled children are thought to demonstrate working memory difficulties (Siegel & Ryan, 1989), which likely contributes to their difficulties in mathematics. Bull & Scerif (2001) found that children who have lower mathematic abilities, tended to demonstrate poorer working memory skills, demonstrating some bi-directionality between the two. Although working memory deficits are thought to be a result of left hemisphere dysfunction (Sousa, 2001), it is still unclear what is causing memory problems for people with mental retardation (Vakil, Shelef-Reshef & Levy-Shiff, 1997).

While typically developing children benefit from the use of organizational strategies to aid in memorization tasks, children with mental retardation typically do not benefit from such strategies (Byrnes & Spitz, 1977). Geary (2004) found that these children demonstrated difficulty holding information in working memory, which could play a role in the tendency of children to undercount or over count, an obvious source of counting errors (Geary, 1990). This deficit also helps explain the use of finger-counting. Finger-counting reduces the information that must be held in working memory when attempting to solve basic addition facts, explaining its common use in children (Geary, 1990). Children with mental retardation often lack simple math skills due to problems with working memory, creating a situation where information is lost during the solving process. Not only can working memory contribute to difficulties in solving mathematic problems, but long-term memory would seem to also create problems. Long-term memory deficits could create difficulties for children as previously learned skills would need to be constantly re-taught. There are similarities and differences between children with learning disabilities and children with mental retardation in respect to the problems each experience when computing mathematic problems (Table 1).

Table 1. Mathematical Difficulties for Children with Disabilities

	Conceptual	Intelligence Quotient	Memory	Assumption
Learning Disabled	Yes	No	Yes	Unexpected Failure
Mental Retardation	Yes	Yes	Yes	Expected Failure

According to Geary (1994) children with mild mental retardation can have an especially difficult time when learning even the most basic math skills, consequently requiring teachers to evaluate their teaching strategies to make sure students fully understand the difficult and complex process.

There are some discrepancies between research findings looking at how well students with disabilities are able to learn mathematics. It has been suggested that children with mathematical learning disorders are in fact able to compute basic arithmetic processes and even eventually do move beyond the finger-counting method to more advanced forms of the arithmetic process (Geary & Brown, 1991). Although the learning process is undoubtedly delayed for children with mental retardation, it is thought by some that children with mental retardation are actually able to learn the academic processes fairly similar to that of normally developing children (Berdine & Blackhurst, 1981; Burns, Roe & Ross, 1988; Katims, 1996; Kavale & Forness, 1992). In fact, Kavale and Forness (1992) found that children with mild disabilities, learning disabilities and mental retardation learn similarly enough that they could benefit from somewhat similar types of instruction. With appropriate training, children with mental retardation are able to learn some math strategies and perform them well (Baroody, 1988; Bray & Turner, 1986). It is suggested that students with mental disabilities must first learn the basic concrete facts of mathematics and then will later be able to move into an attempt at more abstract processes (Van Luit & Naglieri, 1999). The child's severity and/or level of mental retardation are likely a critical factor in whether or not the child has the ability to learn various forms of mathematics as well.

Approaches to Learning

A factor contributing to the discrepancies of how students with disabilities learn is the manner in which these skills are taught. Two largely different teaching strategies are the student-led and the teacher-led approach. The student-led approach is characterized by a student having the chance to become creative and play a role in the design of the lesson, demonstrating more of a constructivist learning approach, whereas the teacher-led approach includes the students being taught the direct lesson by the teacher.

The constructivist approach has been suggested to be quite advantageous for students' learning by allowing the child to construct his or her own knowledge by assessing ideas based on their previous knowledge. In fact, children performing at or above average in mathematics have been found to use more flexible strategies when solving math problems when taught from a constructivist approach (Klein, 1998). Some children have the capability to initiate some form of their learning process, which is also called active learning (Baroody & Ginsburg, 1986; Resnick & Ford, 1981; Siegler & Jenkins, 1989). Bonwell and Eison (1991) defined active learning as "instructional activities involving students in doing things and thinking about what they are learning". Similar to active learning, the student-led instruction provides opportunities for the child to determine some of the strategies to be used (Jones, Wilson & Bhojwani, 1997).

The constructivist teaching approach, student-led approach, and active learning approach would seem to be quite beneficial for average or above average students who would be able to integrate their knowledge of mathematics into different domains. However, lower performing students would likely not be able to do this as effectively (Geary, Brown & Samaranayake, 1991).

Some research suggests that children with mental retardation do not play an active role in their learning process and demonstrate more of a passive learning process (Cherkes-Julkowski & Gertner, 1989; Ferretti & Cavalier, 1991). Bellamy, Greiner & Buttars (1974) found that mentally retarded children do not show any evidence of the ability to invent addition strategies past rote counting. In fact, it is thought that children with mental retardation benefit from strategy or rehearsal training (Belmont & Butterfield, 1973; Brown, 1974). Contradictory to this research, Baroody (1996) found that children with mental retardation were, in fact, able to demonstrate some capabilities of self-initiated learning or active learning abilities. The level and/or severity of Mental Retardation would likely play a role in how well the child is able to learn these mathematic strategies and how well he or she would be able to use them.

Regardless of differing results in response to whether or not children with mental retardation learn mathematics similar to normal developing children, perhaps what is needed is an approach that guarantees a better understanding to those who otherwise have difficulty with some of these basic mathematical skills.

Touch Math

If children with mental retardation are in fact able to aquire the academic processes fairly similar to that of typically developing children (Berdine & Blackhurst, 1981; Bernes, Roe & Ross, 1988; Katims, 1996) but are delayed and should learn the basic and concrete facts first, we must focus on the type of mathematics curriculum that would be most beneficial for this group. One approach, given the passive learning style combined with remedial efforts that could be especially beneficial to students with mental retardation, is Touch Math. Touch Math is a mathematical curriculum that has various characteristics which seem to be quite concrete and structured, perhaps offering a strategy involving passive learning, which may be more beneficial for students with mental retardation.

Touch Math consists of Touch Points on each number one through nine, which aids children in counting strategies involving these numbers. See Figure 1 for an example of touch points on numbers one through nine.

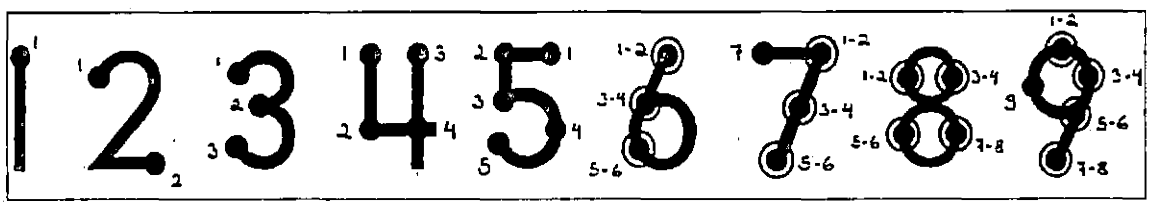


Figure 1. Touch Points Placed on Numbers One Through Nine.

This method of mathematical strategy can be used beginning with basic addition, subtraction, multiplication and division. The first steps of Touch Math begin with the process of learning the positions of the points on the numbers. Children learn exactly where the points lie on each number, and through this, learn the counting procedure of these points. The next step includes basic addition of single digit numbers where children count all of the touch points on each number, continuing to count in order to reach a sum. This child is demonstrating the counting-all procedure as described by Fuson (1982) and Groen and Parkman (1972). For example, a child computing the addition problem of 3+2 would count the touch points on the number 3 and continue counting touch points on the number 2 for a total sum, "1,2,3…4,5". Later, the touch points are removed from one number, where children must say that number and count from there. For example, a child with the addition problem of 3+2 would say 3 then add each of the two touch points on the number 2, "3…4,5". This would demonstrate the counting-on procedure (Fuson, 1982; Groen & Parkman, 1972). Finally, all dots are removed from the numbers in the addition problem, requiring the student to recognize the larger number, say that number and count the imaginary touch points on the remaining numbers.

Although these are just the beginning steps of Touch Math as a strategy for computing mathematical terms, this example demonstrates some interesting characteristics. Touch Math is simple and concrete which is perhaps necessary for children with mental retardation. Kramer and Krug (1973) first began the idea of the dot-notation method (Touch Math) with students with special needs. As Siegel and Ryan (1989) suggested, students with learning disabilities have a difficult time in tasks requiring use of working memory. Touch Math teaches addition strategies such as the above explained counting-all and counting-on procedures, but does not require the skill of retrieving information from memory, which has proven to be difficult for children with learning disabilities (Miller & Mercer, 1997). In other words, Touch Math allows the child with a disability to bypass the memory processing deficit, because of the lack of reliance on memory within the Touch Math system.

Touch Math has specific touch points from which children memorize where the points are, and the order in which they are counted, requiring less information in working memory compared to the other methods, where one would have to remember where he or she left off in counting. Touch Math also poses a potential benefit to a limitation in Simon and Hanrahan's three basic steps to solving addition problems (2004). The first of the three basic steps to addition is the use of concrete referents, such as finger-counting (Simon & Hanrahan, 2004). There is an obvious limitation to this first step where children may run out of fingers if using the finger-counting method to add sums greater than ten. Also, perhaps the child may not have manipulatives available, such as blocks, to represent these numbers. Furthermore, students using Touch Math as their procedure for solving an addition problem would not be faced with such a dilemma. The student can simply imagine the touch points on the numbers when counting, therefore eliminating the reliance on such concrete referents.

An important characteristic of Touch Math is that it is multisensory, (Dev, Doyle & Valente, 2002). Multisensory methods of learning mathematics have been thought to increase achievement for students who have difficulties with such processes (Stern, 1999; Thorton, Jones & Toohey, 1983; Scott, 1993). Multisensory methods include auditory, visual and tactile information, which has been suggested to be beneficial to students when being introduced to basic number concepts (Thorton, et al., 1983). The auditory aspect of Touch Math includes the counting procedure where the child will hear him or herself counting each touch point. The visual aspect of Touch Math includes seeing the actual touch point or visualizing where they should be when math problems do not include the touch points. The tactile aspect of Touch Math includes the actual touching of each touch point as they are counted.

Research on touch math is limited. Simon and Hanrahan (2004) looked at 10 year old students with learning disabilities and found that these students actually were able to effectively use the Touch Math, dot-notation method to solve up to three-row, double-digit addition problems requiring regrouping. In fact, these students were also able to generalize this information and solve similar addition problems never seen before. When given the choice of various mathematic methods these learning disabled children chose the Touch Math method for computations. Yet to date, no research has examined the efficacy of Touch Math with children with other cognitive impairments.

As limited published research has investigated the Touch Math system with children, the Touch Math system needs further investigation. Various systems of teaching mathematics should be analyzed with mentally retarded children to determine which would be most beneficial for this population. Given the characteristics of children with mental retardation, a logical assumption is that the Touch Math system, demonstrating a more passive learning approach, would be a more beneficial system of teaching such processes.

The present study examines the effectiveness of the Touch Math system as a mathematic strategy to aid in addition solving for children with mental retardation by comparing Touch Math to other non-Touch Math strategies using single-case methodologies. A logical assumption is that the Touch Math system, demonstrating a more passive learning approach, would be a more beneficial system of teaching mathematical processes, particularly to students who are mentally retarded. The difference in how well the children learn and use the mathematic strategy of the Touch Math system in comparison to other, non-Touch Math strategies is analyzed using a within-variations single-case design measured by a curriculum-based assessment tool.

<u>Curriculum-Based Assessments</u>

Curriculum-based assessments (CBA) are an ongoing method of assessment which can target many different areas to be examined. This type of assessment can be used to gather data about some behavior or ability academically, and then to monitor and evaluate the changes once an intervention has been implemented. A CBA is an assessment technique that connects assessment to curriculum where educational success is evaluated and when the assessment purpose is to determine instructional needs (Shapiro & Derr, 1990).

Furthermore, CBA can be used to examine individual student progress on material that a student is expected to learn (Shapiro & Derr, 1990). These findings would help teachers decide where in the curriculum the child should begin, based on how they performed on the CBA, measuring student's level in that particular area. Through such detailed examination, teachers can repeat the testing probes without practice effects and with a high sensitivity to short term growth within the child.

The CBA possess an advantage over standardized, norm-referenced assessment measures. Standardized, norm-referenced assessment measures are academic assessment tools used to determine current academic level among students. However, there are some problems with using these standardized tests. For example, it is thought that there is very little overlap between what is taught in the classroom and what is being tested when using standardized tests (Jenkins & Pany, 1978) therefore these tests may actually be testing how well a student is able to generalize and relate information that was taught, with other material.

Standardized tests also are not sensitive enough to demonstrate short term growth within an educational domain (Deno, 1985), whereas the CBA is highly sensitive to short term effects (Shapiro & Derr, 1990). Therefore, CBA is sufficient for this study as it not only collects data but provides the opportunity to use this information to apply and develop an intervention strategy. This study gathers information about effective strategies for solving addition problems which can then be applied, helping to determine successful curriculum to be used in the classroom.

In all CBAs the first step is to interview the participating teacher to determine student level and instructional environment in the area to be studied (Shapiro & Derr, 1990). The next step is to sequence curriculum-based probes that will be used based on computational difficulty. These steps will be utilized in the present study for the student on an individualized basis.

Single-Case Studies

Single-case methodologies look at a single participant only in comparison to him or herself, making it extremely important to understand the degree of variability within the participant in the specific area being observed. This is done by making determinations about the current level of the participant and then making predictions about the future level of the participant, given the treatment (Hayes, 1981).

In most single-case designs, it is beneficial to first establish a baseline of the target behavior. A more accurate baseline can be established by measuring over various periods of measurements. However, shorter baselines do not pose a problem in a study where the treatment is going to be later removed (Hayes, 1981), or if the study has multiple baselines. In a study where progress is expected regardless of treatment, it might be common to see a slow rising baseline between the points measured. It is appropriate to have a rising baseline, as long as progress is expected to be much greater when the treatment is given (Hayes, 1981).

Design

Most within-series, simple phase change studies are an A/B design, looking at the behavior or skill during the first phase, then looking at the same participant's behavior or skill during the second phase, however this time with the treatment (Johnston & Pennypacker, 1993). If the behavior found in A changes when B is present, the likelihood that the change was due to B has increased. After giving the treatment, there are various possibilities of findings. One possibility is no improvement, which after given the treatment, could be explained by either no influence at all from the treatment or a delayed effect, as the effect of the treatment may take time to surface (Johnston & Pennypacker, 1993). Another possibility is deterioration after treatment, which then should be immediately withdrawn, as it may actually be causing a negative effect. Negative findings in any area other than the baseline might suggest a detrimental impact from the treatment (Johnston & Pennypacker, 1993).

On the other hand, one might find improvement, and there are different possibilities for the next step. A placebo can be given, treatment can continue, or treatment can be applied to other problem areas. In order to increase the likelihood that the study is correctly viewing B to cause change in A, one might then want to repeat the phase change by withdrawing the treatment (A/B/A), hopefully to find the improvement will slow, demonstrating a treatment effect (Johnston & Pennypacker, 1993). One might also find no change or continued improvement upon withdrawing the treatment, which would demonstrate less effect from the treatment. If one did find deterioration upon withdrawing the treatment, they then can re-implement the treatment hopefully to find progress once again (A/B/A/B). Repeated design, as just described, is intended to control for any possible non-treatment changes during a simple A/B design (Johnston & Pennypacker, 1993).

Validating Mathematic Progress

The purpose of the present study is to examine, via single case methodology, the effectiveness of mathematic strategies to aid in addition solving for children with mental retardation. Specifically, this study will compare math scores when using the Touch Math program and other, non-Touch Math programs. A logical assumption is that the Touch Math system, demonstrating a more passive learning approach, would be a more beneficial system of teaching mathematical processes to students who are mentally retarded. The difference in how well the children learn and use the mathematic strategy of the Touch Math system in comparison to other, non-Touch Math strategies will be analyzed using a within-variations single-case design measured by a CBA tool.

As test results rely on the validity and reliability of the assessment procedures, the assessment results of the CBA tool along with assessment results of standardized testing and state standards will be compared. Furthermore, the CBA tool will primarily look at growth among the participant and in order to further validate progress made, a standardized test of mathematical achievement will be used before and after all CBA tools. Additionally, Regular Education Standards and the California Alternate Performance Assessment (CAPA) standards for special education will be examined before and after all CBA tools are used, to determine if the student has met the standard or made progress towards the standard.

According to the California State Board of Education (http://www.cde.ca.gov), "Content standards were designed to encourage the highest achievement of every student, by defining the knowledge, concepts, and skills that students should acquire at each grade level." The Regular Education standards that align with the mathematic skills being measured for the present study fall within Kindergarten through Grade 2 standards. The mathematic section of the standards was adopted in December of 1997 by the California

Department of Education (Table 2). Retrieved June 8, 2008, from http://www.cde.ca.gov.

Table 2. Regular Education Standards

Grade Level	Standard	Description of Standard
Kindergarten	2.0	"Students understand and describe simple additions and subtractions."
	2.1	"Use concrete objects to determine the answers to addition and subtraction problems (for two numbers that are each less than 10)."
Grade 1	2.0	"Students demonstrate the meaning of addition and subtraction and use these operations to solve problems."
	2.6	"Solve addition and subtraction problems with one- and two-digit numbers (e.g., 5 + 58 = __)."
	2.7	"Find the sum of three one-digit numbers."
Grade 2	2.0	"Students estimate, calculate, and solve problems involving addition and subtraction of two- and three-digit numbers."

2.2 "Find the sum or difference of two whole numbers up to three digits long."

Each of these Regular Education standards includes an understanding of basic addition skills which aligns with the skill that is being analyzed for the present study.

In addition, the special education alternative set of standards designed for students with severe cognitive disabilities, the CAPA, will be examined before and after all CBA tools are used. CAPA standards were adopted as part of accountability among special education teachers as an alternate set of state standards. The CAPA standards were designed to align with the Regular Education standards; however they are designed to be more appropriate for students with severe cognitive delays. According to the California Department of Education (http://www.cde.ca.gov), these standards define what this group of students' skill levels should be in relation to the Regular Education state standards.

In order to meet the requirements of the Individuals with Disabilities Education Act

(IDEA) and the No Child Left Behind Act (NCLB), California must show evidence that all students are included in our statewide assessment and accountability system. The California Department of Education (CDE) is required to develop and implement an alternate assessment for children with disabilities who cannot take part in general statewide assessment programs. The California Alternate Performance Assessment (CAPA) is the alternate assessment for the California Standards Tests.

These CAPA standards were adopted by the State Board of Education, in March of 2006. The CAPA assessment is divided into five levels based on the student's grade and/or cognitive level (Table 3). Retrieved June 8, 2008, from http://www.cde.ca.gov.

Table 3. California Alternate Performance Assessment Levels

CAPA Level	Participants
Level 1	"Grades 2-11 with cognitive developmental abilities of 24 months or below."
Level 2	"Grades 2 and 3."
Level 3	"Grades 4 and 5."
Level 4	"Grades 6, 7 and 8."
Level 5	"Grades 9, 10 and 11."

For purposes of the present study, the CAPA standards that will be used for comparison of students' ability level will include CAPA Level 3 which falls within the mathematic standards as related to solving basic addition facts. These are also the CAPA standards which fall within the grade level of the participating student (Table 4). Retrieved June 8, 2008, from http://www.cde.ca.gov.

Table 4. California Alternate Performance Assessment Standards

CAPA Level	Grade Level	CAPA Standards
Level 3	Grade 4	2.0 "Students extend their use and understanding of whole numbers to the addition and subtraction of simple decimals." 2.1 "Estimate and compare the sum or difference of whole numbers and positive decimals to two places."
	Grade 5	2.0 "Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals." 2.1 "Add, subtract, multiply and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results. Add whole numbers with sums up to 50 and subtract single digit numbers."

2.3 "Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form. Solve simple problems with sums up to 20, including ones arising in concrete situations, involving the addition and subtraction of whole numbers."

The student's skill level in relation to the CAPA standards will be examined before and after all CBA measurement tools are used, to analyze and compare ability levels to determine if the student has met the standard or made progress towards the standard.

Again, the assessment results of the CBA tool are paired with the Regular Education and CAPA standards, and the Brigance CIBS-R (1999) standardized testing for comparison.

The Brigance CIBS-R (1999) will be used as a standardized test from which to measure progress. This standardized test was designed for use with students within elementary and middle school age groups. It also is especially useful for teachers who serve students with special needs (Brigance CIBS-R, 1999). Although the Brigance CIBS-R (1999) manual indicates this can be used in a variety of ways, for the purposes of this study it will be used as a tool to aid in the development of present levels to observe progress made as a form of standardized testing.

The Brigance CIBS-R describes its features as being comprehensive in various levels and is criterion and text-book referenced. The test has been validated and includes two forms of some of the assessments. It is designed to be easy to administer and does not require specialized training or materials to administer the test (Brigance CIBS-R, 1999). A combination of each of these assessment tools are compared in order to aid in the convergent and discriminate validity of Touch Math as a viable means of instruction.

Present Study

The current study follows an applied approach to research as it is attempting to solve a practical issue of whether Touch Math deems as a more beneficial mathematic strategy to be used by students who are mentally retarded, than other, non-Touch Math strategies to solve mathematic problems. More specifically, the study looks at whether or not Touch Math is a more beneficial addition strategy by means of efficiency and speed to solve addition problems, as opposed to other methods. It is an attempt to improve the current methods that special education teachers are using when teaching this population by giving them a clear vision of the type of strategy that would be most beneficial for a child with mental retardation.

This single-case experimental design will utilize a within-series design that, over several time periods, will compare the progress on a student's addition skills. The participant in this study will receive all levels of the independent variable, or the various strategies of learning addition facts, making it a within-series design.

In this type of study, it would not be uncommon to observe some progress made during the baseline because students should be maturing and learning greater mathematic skills during their educational instruction. The current study is looking at simple phase change, examining whether or not some type of treatment works, in this case, the strategy of using Touch Math to solve addition problems.

Although these students are expected to progress due to maturation and instruction, treatment in the current study is expected to increase the progress much more substantially making it clear when progress is a result of the mathematic strategy being used. The current study begins using an A/B design; however the participants' skill levels will again be compared by repeating the treatment after the withdraw in order to determine if improvement is greatest during treatment (A/B/A). By withdrawing the treatment, the major threats to the internal validity are controlled. For instance, coincidental changes among the participant might demonstrate some progress in the observed area, which are changes that cannot be attributed to the treatment.

In order to control for such changes in the participant's ability level, the treatment is withdrawn, or the Touch Math procedure is removed and other non-Touch Math procedures are used. In addition to maturation, some threats to internal validity might include history, a tendency to regress towards the mean, and selection. By removing the treatment, these major threats would be controlled for, therefore any deterioration after withdrawal would increase the confidence that the treatment is causing an effect, or that using Touch Math as a strategy to complete the math problems is more beneficial than using other non-Touch Math strategies.

The dependent variable (DV) in this study includes the test scores as measured by number of digits correctly answered as well as percentage of digits correctly answered when given addition problems. The quasi-independent variables are the Touch Math and other, non-Touch Math related mathematic problems, which are examined based on the effect of the dependent variable, or mathematic measurement tool.

<u>Hypothesis</u>

It is hypothesized that children with Mental Retardation will complete addition problems more efficiently and with greater speed when using the Touch Math method to complete worksheets to compute addition problems than when using other, non-Touch Math strategies to complete addition problems. Convergent and discriminate validity will be examined by comparing CBA probes with other measurement tools. Results may also show that participants will make progress on the skill level as measured by the Brigance CIBS-R (1999) standardized test. Furthermore, results may also show that progress will be made toward the Regular Education and CAPA standards, if standards are not completely met. Progress made on the standardized test and the state standards will further validate findings that general progress is made on mathematic ability. However, more significant growth within the Touch Math section of the CBA tool rather than the non-Touch Math section will demonstrate that Touch Math is in fact a more successful strategy to use for children with mental retardation.

CHAPTER TWO

METHOD

Participants

Participants include one child with the secondary diagnosis of mental retardation, as determined by her latest Individualized Education Plan (IEP). The participant was recruited with written and verbal permission by her parent. The participant receives her primary instruction from a special day classroom and was chosen for the study based on her qualification to fit the study, based on the secondary diagnosis, and the potential to benefit from the current research. The parent of the student was given detailed information describing the study and its purposes, the confidentiality of students' scores and the right to withdraw from the study at any time without any repercussions. The parent was also given a permission form, which she signed, indicating that her child was permitted to participate in this study.

Participant A, "Brittney", has a primary diagnosis of Multiple Disabled and a secondary diagnosis of Mental Retardation as determined on the latest Individualized Education Plan. Brittney is an 11 year old female, who is in the sixth grade. Brittney is Caucasian and her primary language is English. The participant was treated in accordance with the Ethical Principles of Psychologists and Code of Conduct (American Psychological Association, 1992).

Procedure

The consent form was collected from the participant's parent indicating her understanding of what the study would entail and approval of her child's participation in the study. All math work sessions were conducted in the office of the teacher's special education classroom. In this classroom office, all distracters were minimized, such as no mathematic posters of any kind were posted in view of the participating student. At the beginning of each work session, the participant was given a new pencil with a full eraser and scratch paper from which to work and/or re-align horizontally written problems to vertical format.

Pre-Intervention Data Collection

The participants' skill level was compared with the standardized testing using the Brigance CIBS-R (1999), the Regular Education standards and the CAPA standards. The scores on these assessments were recorded and compared to the scores that the participant received when re-tested using the same Brigance CIBS-R (1999) and same standards after all CBAs were completed. Comparison of scores for all procedures was completed in order to validate progress made.

Baseline Assessment

For the CBA, the participant first was tested in order to establish a baseline for mathematic ability. The participant was given two worksheets containing mathematic addition problems individualized to meet the student's ability level, two times per week, for two weeks using non-Touch Math worksheets.

Curriculum-Based Assessment

Once a baseline was established, Touch Math mathematic probes, treatment, and non-Touch Math mathematic probes, non-treatment, were alternated every two weeks, given two times per week, for a total of eight weeks. Given the math level of the participant, the difficulty of the probes varied between single digit mathematic problems with Touch Math strategy clues, Touch Points, to double digit addition, with regrouping, without touch points. On problems requiring regrouping, a box was present above the left column of the problem, providing a place for the re-grouped number. The non-treatment probes included the same problems; however none of the problems had any Touch Math strategy clues, or Touch Points, as Touch Math was not used for this portion.

When given the Touch Math probes the participant was reminded to use Touch Math as she has been taught in her curricular instruction. When given the non-Touch Math probes, the participant was given manipulatives and was verbally reminded of the optional finger counting method or tally mark method to encourage the use of a non-Touch Math strategy. Furthermore, encouragement of any other strategy use other than the Touch Math strategy was given during the non-Touch Math mathematic probes.

The number of problems attempted, the number of problems correct, the number of digits correct and the percentage correct on each probe was calculated by one grader, the Special Education teacher of the participant. Correctly regrouped numbers placed on top of addition columns were also counted as a correct digit. Changes in difficulty among the treatment and non-treatment probes occurred during each week of testing.

Post-Intervention Data Collection

Following the eight week interventions, the participant was given the Brigance CIBS-R (1999) standardized test again from which to compare scores to the scores recorded prior to the CBAs. This analysis was completed in order to further validate any progress demonstrated from the CBA tools.

The participant was also assessed using Regular Education and CAPA standards again. These results were analyzed in comparison to the standards analysis completed prior to the CBAs. This comparison was again completed in order to further validate any progress demonstrated on the CBA.

Materials

The Brigance CIBS-R (1999) test sheets that were used are as follows: page 344 test O-1, addition facts; page 354, test P-2, addition of whole numbers; page 326, test M-1 form A, computation skills and grade placement test where only addition problems were completed; page 328, test M-1 form B, computation skills and grade placement test where only addition problems were completed. These test sheets were completed in order to compare student's skill level before and after probe sheets were given in order to further validate progress made.

Regular Education Standards used for comparison included Kindergarten Standards 2.0 and 2.1, Grade One Standards 2.0, 2.6 and 2.7, and Grade Two standards 2.0 and 2.2. These standards will be analyzed from which to compare the participant's ability level in order to determine progress toward the standards. CAPA Standards will include Level 3, Grade Four Standards 2.0 and 2.1, Level 3, Grade Five Standards 2.0, 2.1 and 2.3, and Level 4, Grade Six Standards 3.0 and 3.1. These standards will be analyzed in order to compare students' ability level before and after probe sheets are given in order to further validate progress made.

Using CBA procedures for the baseline and treatment portion of the study, the participant was given a set of probes which included more mathematic problems than the student would have been able to complete in the allotted 3 minute timed session allowing the opportunity to complete as many problems as possible, without running out of mathematic problems. During each CBA session, the participating student was given two carefully selected mathematic worksheets individualized based on the students' current mathematic level; therefore over the total of 8 weeks that probes were administered, the student was given two worksheets per session, with two sessions per week, totaling 32 pages of mathematic probes (see Appendices B and C).

CHAPTER THREE

RESULTS

Pre-Intervention

Brigance

On the dates of 9/8, 9/10, 9/16, and 9/18, the standardized testing portion was administered to Brittney in order to establish her present level of addition functioning. The post-intervention Brigance scores were utilized at the end of the intervention in order to further validate progress made over time.

Testing sheet O-1, addition facts, on page 344 was completed with a score of 98/116, or 84%. This sheet consisted of horizontal problems, single and double digit, totaling no more than 19. On incorrect problems, it appeared that the student tried to subtract the numbers, one problem was skipped over, and the other incorrect problems appeared to be simple addition errors. Brittney combined Touch Math and using her fingers on various problems. She physically added Touch Points to the numbers, using the Touch Math method as previously taught, however, she did not consistently place the Touch Points on the lower of the two numbers as taught in the Touch Math program. Brittney was observed to use her fingers on some problems and did not align horizontal problems to vertical format in order to align columns. The participant also made a couple of comments regarding the number 4 which looked different than she was used to seeing (closed top versus open top). She also was observed to count out loud the entire time when completing problems.

Using testing sheet M-1 form A, Computational Skills and Grade Placement Test (on which only addition problems were completed), was completed with a score of 4/14 possible digits correct, or 29%. This sheet consisted of single, double and triple digit problems, with and without regrouping. Regrouped numbers were counted as a digit correct, in addition to digits within the answer. Although, the participant was observed either physically putting Touch Points on numbers, or imagining where the Touch Points would be, demonstrating use of the Touch Math procedure, this method was used ineffectively on most problems on this worksheet. The participant also was observed to not consistently add each column individually. Also, she was observed to add numbers horizontally across all three columns on one problem. On other problems she did attempt to add each column separately. The inconsistency in adding columns individually appeared to be a problem for the participant during this worksheet, resulting in a poor score. Lastly, the participant was prompted to keep going when she was observed to sit without working for more than 5 seconds. After one prompt she replied "I am just meditating".

Testing sheet M-1 form B, Computational Skills and Grade Placement Test (on which only addition problems were completed), was completed with a score of 6/14 possible digits correct, or 43%. This sheet consisted of single, double and triple digit problems, with and without regrouping. Again, regrouped numbers were counted as a digit correct, in addition to digits within the answer. Before beginning, the participant made a comment that "these are big ones". However, when looking at a triple digit problem, she indicated that "This one's smaller than the big ones", demonstrating that she did not understand the number concepts of a 3 digit problem, being larger than a 2 or 1 digit problem. The participant was observed to count out loud the entire time while completing the problems.

Testing sheet P-2, addition of whole numbers, was completed with a score of 61/99, or 62%. This sheet consisted of vertical problems with 2, 3, 4 and 5 digit answers. Again, regrouped numbers were counted as a digit correct, in addition to digits within the answer. The participant inconsistently carried the 1 over the appropriate columns, and when she did, she did not always add the extra one when adding that column. Regrouping and column confusion appeared to create some difficulties for the participant. The participant skipped an entire column during one of the problems. She again, counted out loud during the addition of all problems.

The above scores are consistent with the teacher reports of her math skills. She has demonstrated some column confusion in her class work, confusion on where to place the regrouped one and remembering to add this one when adding the next column. The participant seems to do a little better when completing class work, perhaps because class work typically includes much larger numbers, making it easier to see the columns and leaving more room for Touch Points, if used.

State Standards

On 9/19, the state standard comparison was completed by the participant's special education teacher in addition to the standardized testing in order to establish her present level of addition functioning. Various standards were used for comparison to the present level of the participant and were determined to be met, partially met, or not met. For all standards, the focus remained on the addition portion of the standards.

The regular education standard kindergarten, 2.0 was determined to be met, as the participant had shown signs understanding simple addition problems. Kindergarten 2.1 was determined to be met as the participant is able to use concrete objects to complete addition problems with sums less than 10. Grade one 2.0 also focused on a basic understanding of the meaning of addition, and therefore was marked as met. Grade one, 2.6 was marked as met as the participant is able to complete one and two digit addition problems. Grade one, 2.7 was marked as partially met as the participant has shown some confusion when counting three single digit numbers, however is able to complete with some degree of accuracy. Grade two, 2.0 and 2.2 were marked as not met, because the participant has shown some organization problems and confusion when adding three or more digit numbers. For instance, during the Brigance CIBS-R (1999) testing, the participant was not able to effectively complete three digit problems, demonstrating some column confusion. See Table 5 for a visual inspection of the standard comparison to the participant's skill level before all CBA tools were used.

Table 5. Pre-Intervention Skill Level Comparison to Regular Education Standards

Grade	Standard	Met	Partially Met	Not Met
Kindergarten	2.0	X		
	2.1	X		
Grade 1	2.0	X		
	2.6	X		
	2.7		X	
Grade 2	2.0			X
	2.2			X

The CAPA standard comparison was also completed by the participant's special education teacher in order to establish her present level of addition skills. Again, various standards were used for comparison to the present level of the participant and were determined to be met, partially met, or not met. Comparison was also completed at the end of all CBA tools in order to further validate progress made over time. CAPA level III, grade four, 2.0 and 2.1 was marked as not met because the participant has not shown signs of an understanding of simple decimals. Grade five, 2.0 was also marked as not met because the participant has not shown any signs of an understanding of fractions. Grade five, 2.1 was marked as partially met as the participant is able to compute whole numbers with sums up to 50, but is not able to compute decimals or integers. Grade five, 2.3 was also marked as partially met as the participant does have some ability to solve problems with sums up to 20 given concrete situations, but is not able to compute fractions and mixed numbers. See Table 6 for a visual inspection of the CAPA standard comparison to the participant's skill level before all CBA tools were used.

Table 6. Pre-Intervention Skill Level Comparison to California Alternate Performance Assessment Standards

Level III	Standard	Met	Partially Met	Not Met
Grade 4	2.0			X
	2.1			X

Grade 5	2.0		X
	2.1	X	
	2.3	X	

Baseline

On the dates of 9/23, 9/25, 9/30, and 10/2, the baseline portion of the CBA was administered to the participant in order to determine where the participant's present skill level was, by establishing consistent scores. During each session, the participant received two worksheets, each containing 12 problems, totaling 24 problems. Each session was completed in a 3 minute time allotment, giving more than enough problems to be completed during the 3 minutes. During all 4 sessions of baseline assessment, the participant was observed to count out loud during all addition counting.

During session 1, the participant completed 13/14 attempted problems correctly, or 92.86%. The number of digits correct was 21, and 1 digit incorrect. During session 2, the participant completed 12/13 attempted problems correctly, or 92.31%. The number of digits correct was 20, and 1 digit incorrect. During session 3, the participant completed 12/14 attempted problems correctly, or 85.71%. The number of digits correct was 18, with 2 digits incorrect. During session 4, the participant completed 13/13 attempted problems correctly, or 100%. The number of digits correct was 23, with 0 digits incorrect. On most problems, the participant was observed to count imaginary Touch Points, or use the Touch Math program, by placing her pencil in specific areas on each number as she counted.

Curriculum-Based Assessment

Over a two week period beginning on 10/6, the Touch Math mathematic system was used to analyze the participant's ability level when using this program, (A). For each session, the participant was given two worksheets consisting of 24 single digit problems, to be completed within a three-minute time period. Week one consisted of two sessions of adding numbers zero through nine, single digit, with one Touch Point given on each problem. During session one, the participant completed 16 problems, with all 16 being correct, and 23 digits correct. During session two, the participant completed 17 problems, with all 17 being correct, and 30 digits correct.

Week two consisted of two sessions of adding double digit numbers, with no regrouping required, and with one Touch Point given on each column. During session three, the participant completed 9.5 problems, with all 9.5 problems correct, and 18 digits correct. During session four, the participant completed 10 problems, with 9 problems being correct, and 19 digits correct (see Figure 2).

Over the following two week period beginning on 10/20, the non-treatment, or any other mathematic system not including Touch Math (tallies, finger counting, manipulatives), was used to analyze the participant's ability level when using any strategy other than the Touch Math program, (B). For each session, the participant was given two worksheets consisting of 24 single digit problems to be completed within a three-minute time period. Week three consisted of 2 sessions of adding numbers zero through nine, single digit. During session 5, the participant completed five problems, with 3 of the problems being correct and 4 digits correct. During session 6, the participant completed 8 problems, with 3 of the problems being correct, and 7 digits correct.

Week 4 consisted of two sessions of adding double digit numbers, with no regrouping required. During session seven, the participant completed 3 problems, with 1 problem being correct, and 4 digits correct. During session 8, the participant completed 4.5 problems, with 2 problems being correct, and 6 digits correct (see Figure 2).

Over the following two week period beginning on 11/3, the treatment, or Touch Math mathematic system, was again used to analyze the participant's ability level when using this program, (A). For each session, the participant was given two worksheets consisting of 24 single digit problems to be completed within a 3 minute time period. Week 5 consisted of double digit problems, with no regrouping required, and no Touch Points given on any of the numbers. During session nine the participant completed 7.5 problems, with 5 of the problems being correct, and 20 digits correct. During session ten, the participant completed 7 problems, with 5 problems being correct, and 19 digits correct.

Week 6 consisted of double digits problems, with and without regrouping required, with no Touch Points given on any of the numbers. During session 11, the participant completed 7 problems, with 7 problems being correct, and 21 digits correct. During session 12, the participant completed 6 problems, with 5 problems being correct, and 19 digits correct (see Figure 2).

Over the following two week period beginning on 11/17, the non-treatment, or any other mathematic system not including Touch Math (tallies, finger counting, manipulatives), was again used to analyze the participant's ability level when using any strategy other than the Touch Math program, (B). For each session, the participant was given two worksheets consisting of 24 single digit problems to be completed within a three-minute time period. Week 7 consisted of double digit problems with regrouping required. During session 13, the participant completed 2 problems, with 1 problem being correct, and 4 digits correct. During session 14, the participant completed 2.5 problems, with 1 problem being correct, and 7 digits correct.

Week 8 consisted of double digit problems, with and without regrouping required. During session 15, the participant completed 2.5 problems, with 0 problems correct, and 5 digits correct. During session 16, the participant completed 1.5 problems, with 1 problem being correct, and 4 digits correct (see Figure 2).

Figure 2 presents a visual inspection of all data and progress made on the curriculum-based measurement tool for the participant over the baseline and eight week CBA period.

Post-Intervention

Brigance

Following the completion of the CBA, the Brigance CIBS-R (1999), was again used for standardized testing in order to determine the new skills level of mathematics and to further validate findings and progress made on mathematic skills. During the week of 12/8, Brigance testing began, again with completion of the same testing sheets as those completed during the pre-intervention testing. Brigance testing was completed over approximately a month time period due to increased seizure activity and poorer school attendance.

Testing sheet O-1, addition facts, on page 344 was completed with a score of 102/116, or 88%. This sheet consisted of horizontal problems, single and double digit, totaling no more than 19. Testing sheet M-1 form A, Computational Skills and Grade Placement Test (which only addition problems were completed), on page 326 was completed with a score of 11/14 possible digits correct, or

79%. This sheet consisted of single, double and triple digit problems, with and without regrouping. Regrouped numbers were counted as a digit correct, in addition to digits in the answer. Testing sheet M-1 form B, Computational Skills and Grade Placement Test (which only addition problems were completed), on page 328 was completed with a score of 8/14 possible digits correct, or 57%. This sheet consisted of single, double and triple digit problems, with and without regrouping. Regrouped numbers were counted as a digit correct, in addition to digits in the answer. Testing sheet P-2, addition of whole numbers, on page 354 was completed with a score of 65/99, or 66%. This sheet consisted of vertical problems with 2, 3, 4 and 5 digit answers. Regrouped numbers were counted as a digit correct, in addition to digits in the answer.

State Standards

On the date of 12/19, the state standard comparison was again completed by the participant's special education teacher in order to establish an idea of the new present level of addition functioning and to further validate findings and progress made. Various standards were used for comparison to the present level of the participant and were determined to be met, partially met, or not met. For all standards, focus remained on the addition portion of the standards.

Results of the state standard comparison were very similar to those findings from before the treatment. Each of the standards were marked with the same ratings (met, partially met, not met) as the previous rating date. On CAPA standard grade one, 2.6, it was noted that the participant is able to complete the single and double digit addition problems, however it was found through the treatment that this is done best when using the Touch Math procedure versus the participant's choice of another method. It was also noted on CAPA standard grade 5, 2.3 that the participant is able to relate addition problems to concrete situations, but when using concrete items, such as manipulatives, the participant performs more poorly than when using the Touch Math procedure. Regular Education standard grade two, 2.0 demonstrated some growth in the area of column organization, as the participant did better in this area, however still has some confusion when aligning columns. See Table 5 and Table 6 for a visual inspection of the standard comparison to the participant's skill level after all CBA tools were used, and note that the same tables are used for pre- and post- intervention data collection, as the results did not change.

The results of the standards comparison were quite similar, demonstrating an accurate assessment of the present mathematic skill level for the participant, but limited growth over the eight week period.

CHAPTER FOUR

DISCUSSION

Summary

The purpose of the present study was to examine the effectiveness of Touch Math mathematic strategies in comparison to other, non-Touch Math mathematic strategies for a child with mental retardation, when computing addition problems. Results showed that the participant was able to compute mathematic addition problems with significantly greater speed and accuracy when utilizing Touch Math than using other, non-Touch Math strategies for solving the same problems. Pre and post-intervention assessments on the standardized testing portion showed that there was growth in skill level due to increased scores across all four testing sheets. On the other hand, when compared to Regular Education and CAPA standards, there was no significant growth in basic mathematic skills during the 8 week intervention period.

Limitations

Limitations within this study include the possibility that results from one participant may not demonstrate generalized information in regards to mathematic performance among all students with mental retardation. It is also important to examine the amount of time spent in the classroom practicing various mathematic programs in order to compare them fairly. Although in the current study, the participant has practiced both Touch Math and manipulative math in the classroom, it would have been beneficial to have an idea of how much time was spent practicing each of the strategies, which may contribute to differences in performance among the programs. Extending the intervention may show more of an accurate assessment of skill differences, over a longer period of time. It also would be interesting to see how well the participant is able to generalize the information by solving actual real life simple addition problems, and if she would be able to apply the Touch Math method to problem solving.

One specific limitation concerning the participant includes poor health and attendance. The participant experienced increased seizure activity during the latter portion of the testing, resulting in poorer school attendance and an expected regression in overall skills. Although increased seizure activity is expected to negatively affect basic skills, the participant still showed significantly higher scores among the Touch Math program versus the non-Touch Math program. Seizure activity may partially explain the consistency among pre-intervention and post-intervention assessments of state standards. However, improved scores on standardized testing demonstrate some growth. This information should be considered when interpreting the scores and her growth.

Comparison to Literature

Results of the present study are consistent with previous literature which showed that Touch Math is in fact a successful mathematic program that children with special needs can use to solve basic mathematic functions (Kramer and Krug, 1973). Simon and Hanrahan (2004) found that children with learning disabilities were able to effectively use the program, and it appears that children with mental retardation are as well. Touch Math is an effective mathematic program for children with mental retardation at least partially due to one or more of the characteristics of the program. There are characteristics of mathematics that pose difficulty for children with mental retardation which Touch Math relieves. Touch Math does not require active learning, but rather passive learning, it does not require reliance on working memory, and it is multisensory, which includes auditory, visual and tactile components (Dev, Doyle & Valente, 2002).

Implications

Professionals working with children should make educational decisions based on ongoing assessments and what works for the students. This study demonstrates that the Touch Math program is a mathematic program that has worked for children with mental retardation to solve basic addition facts. Information from this study should demonstrate to teachers that providing strategies to children who are not able to establish strategies themselves, allows children a chance to solve problems while alleviating various difficult areas of such processes. This body of work expands the growing literature on data driven decision making when working with children in special education. Results from this study and future studies should improve current methods that special education teachers are using when teaching students with mental retardation.

Future Research

As this was the first research study examining Touch Math specifically with children with mental retardation, additional replication studies are needed. Future research should further study the mathematic program of Touch Math and how well students with various disabilities are able to benefit from using the program. More importantly, future research should also assess how well these students are able to generalize the basic skill of addition and subtraction into real life situations, for problem solving. A similar design study would demonstrate how well students are able to perform when using Touch Math in comparison to other mathematic programs; however, a larger sample over a longer period of time would further validate findings that Touch Math is a more beneficial program.

APPENDIX A

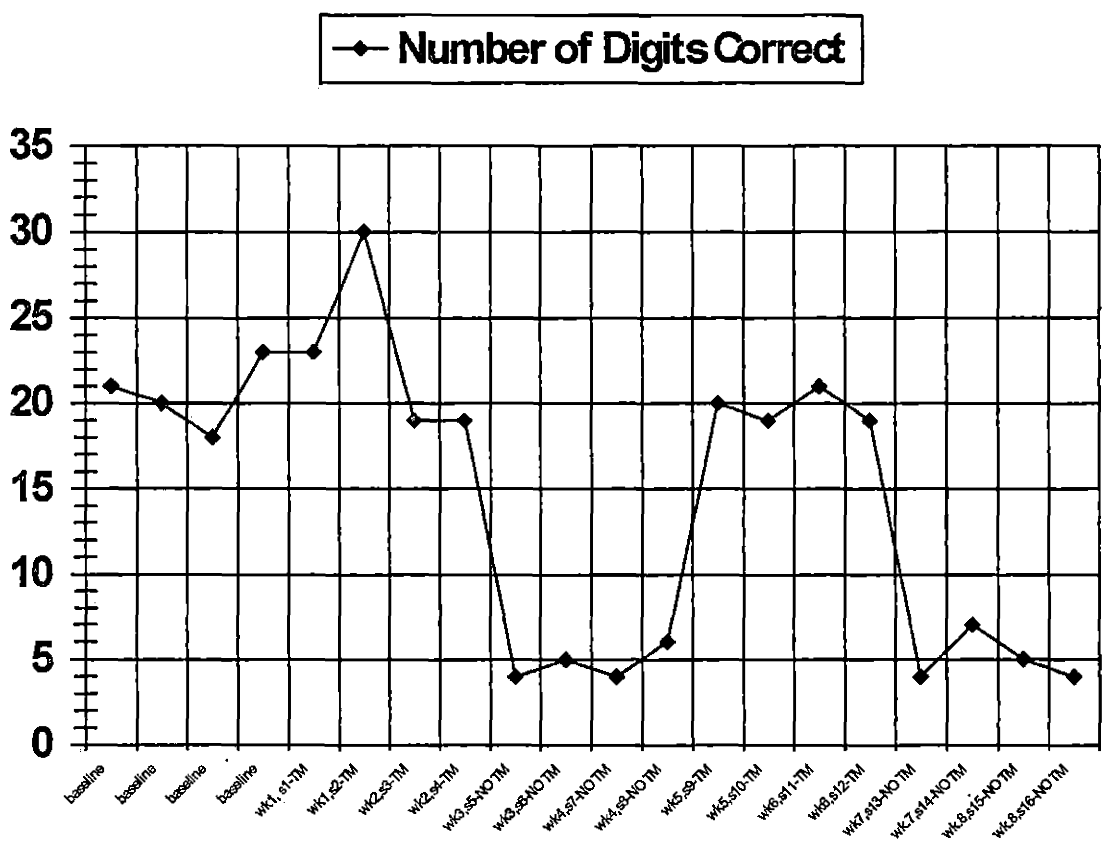
GRAPH DEPICTING SCORES ON VARIOUS

MATHEMATIC PROBES

APPENDIX A

Graph Depicting Scores on Various Mathematic Probes

Figure 1.



APPENDIX B

BASELINE MATHEMATIC PROBES

APPENDIX B

Baseline Mathematic Probes

$\begin{array}{r} 4 \\ +4 \\ \hline 8 \end{array}$	$\begin{array}{r} 6 \\ +3 \\ \hline 9 \end{array}$	$\begin{array}{r} 4 \\ +9 \\ \hline 10 \end{array}$	$\begin{array}{r} 6 \\ +9 \\ \hline 15 \end{array}$
$\begin{array}{r} 2 \\ +8 \\ \hline 10 \end{array}$	$\begin{array}{r} 3 \\ +5 \\ \hline 8 \end{array}$	$\begin{array}{r} 2 \\ +4 \\ \hline 6 \end{array}$	$\begin{array}{r} 9 \\ +6 \\ \hline 15 \end{array}$
$\begin{array}{r} 5 \\ +6 \\ \hline 11 \end{array}$	$\begin{array}{r} 5 \\ +9 \\ \hline 14 \end{array}$	$\begin{array}{r} 7 \\ +2 \\ \hline 9 \end{array}$	$\begin{array}{r} 5 \\ +8 \\ \hline 13 \end{array}$

$\begin{array}{r} 4 \\ +4 \\ \hline 8 \end{array}$	$\begin{array}{r} 8 \\ +3 \\ \hline 11 \end{array}$	$\begin{array}{r} 5 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +9 \\ \hline \end{array}$
$\begin{array}{r} 2 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ +0 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +4 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +6 \\ \hline \end{array}$
$\begin{array}{r} 5 \\ +6 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +9 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ +8 \\ \hline \end{array}$

6	1	3	8
+5	+9	+2	+0
11	10	5	8

5	9	5	3
+5	+7	+8	+5
10	16	13	8

3	4	7	6
+6	+8	+4	+3
9	12	11	9

5	7	2	3
+9	+6	+8	+3
10			
7	9	8	9
+0	+9	+5	+7
6	3	7	6
+4	+9	+7	+2

$\begin{array}{r} 1 \\ +8 \\ \hline 9 \end{array}$	$\begin{array}{r} 6 \\ +3 \\ \hline 9 \end{array}$	$\begin{array}{r} 4 \\ +9 \\ \hline 13 \end{array}$	$\begin{array}{r} 7 \\ +1 \\ \hline 8 \end{array}$
$\begin{array}{r} 6 \\ +6 \\ \hline 12 \end{array}$	$\begin{array}{r} 3 \\ +5 \\ \hline 9 \end{array}$	$\begin{array}{r} 2 \\ +4 \\ \hline 6 \end{array}$	$\begin{array}{r} 9 \\ +7 \\ \hline 15 \end{array}$
$\begin{array}{r} 8 \\ +4 \\ \hline 12 \end{array}$	$\begin{array}{r} 5 \\ +9 \\ \hline 14 \end{array}$	$\begin{array}{r} 7 \\ +2 \\ \hline 9 \end{array}$	$\begin{array}{r} 6 \\ +2 \\ \hline 8 \end{array}$

9 +3 12	4 +5 9	0 +8	6 +6
7 +5	9 +9	7 +1	5 +7
2 +6	8 +4	3 +4	8 +9

$\begin{array}{r} 7 \\ +6 \\ \hline 13 \end{array}$	$\begin{array}{r} 3 \\ +9 \\ \hline 12 \end{array}$	$\begin{array}{r} 6 \\ +4 \\ \hline 10 \end{array}$	$\begin{array}{r} 0 \\ +7 \\ \hline 7 \end{array}$
$\begin{array}{r} 9 \\ +1 \\ \hline 10 \end{array}$	$\begin{array}{r} 7 \\ +8 \\ \hline 15 \end{array}$	$\begin{array}{r} 5 \\ +5 \\ \hline 10 \end{array}$	$\begin{array}{r} 9 \\ +9 \\ \hline 18 \end{array}$
$\begin{array}{r} 4 \\ +7 \\ \hline 11 \end{array}$	$\begin{array}{r} 3 \\ +6 \\ \hline 9 \end{array}$	$\begin{array}{r} 8 \\ +9 \\ \hline 17 \end{array}$	$\begin{array}{r} 2 \\ +8 \\ \hline 10 \end{array}$

$$\begin{array}{r} 1 \\ +8 \\ \hline 9 \end{array} \qquad \begin{array}{r} 2 \\ +9 \\ \hline \end{array} \qquad \begin{array}{r} 3 \\ +4 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ +6 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ +6 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ +3 \\ \hline \end{array} \qquad \begin{array}{r} 8 \\ +2 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ +8 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ +4 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ +5 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ +4 \\ \hline \end{array} \qquad \begin{array}{r} 4 \\ +7 \\ \hline \end{array}$$

APPENDIX C

CURRICULUM-BASED ASSESSMENT PROBES

APPENDIX C

Curriculum-Based Assessment Probes

1	6	4	7
+ 8	+ 3	+ 9	+ 1
9	9	13	8

6	3	2	9
+ 6	+ 5	+ 4	+ 7
12	8	6	16

8	5	7	6
+ 4	+ 9	+ 2	+ 2
12	14	9	8

$\begin{array}{r} 9 \\ +\,3 \\ \hline 12 \end{array}$	$\begin{array}{r} 4 \\ +\,5 \\ \hline 9 \end{array}$	$\begin{array}{r} 0 \\ +\,8 \\ \hline 8 \end{array}$	$\begin{array}{r} 6 \\ +\,6 \\ \hline 12 \end{array}$
$\begin{array}{r} 7 \\ +\,5 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ +\,9 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ +\,1 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ +\,7 \\ \hline \end{array}$
$\begin{array}{r} 2 \\ +\,6 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +\,4 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ +\,4 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +\,9 \\ \hline \end{array}$

$\begin{array}{r} 7 \\ +\,6 \\ \hline 13 \end{array}$	$\begin{array}{r} 3 \\ +\,9 \\ \hline 12 \end{array}$	$\begin{array}{r} 6 \\ +\,4 \\ \hline 10 \end{array}$	$\begin{array}{r} 0 \\ +\,7 \\ \hline 7 \end{array}$
$\begin{array}{r} 9 \\ +\,1 \\ \hline 10 \end{array}$	$\begin{array}{r} 7 \\ +\,8 \\ \hline 15 \end{array}$	$\begin{array}{r} 5 \\ +\,5 \\ \hline 10 \end{array}$	$\begin{array}{r} 9 \\ +\,9 \\ \hline 18 \end{array}$
$\begin{array}{r} 4 \\ +\,7 \\ \hline 11 \end{array}$	$\begin{array}{r} 3 \\ +\,6 \\ \hline 9 \end{array}$	$\begin{array}{r} 8 \\ +\,9 \\ \hline 17 \end{array}$	$\begin{array}{r} 2 \\ +\,8 \\ \hline 10 \end{array}$

1	2	3	5
+ 8	+ 9	+ 4	+ 6
9	11	7	11

6	6	8	7
+ 6	+ 3	+ 2	+ 8
12			

8	4	7	4
+ 4	+ 5	+ 4	+ 7

$$\begin{array}{r} 55 \\ +\,23 \\ \hline 78 \end{array} \qquad \begin{array}{r} 33 \\ +\,13 \\ \hline 46 \end{array} \qquad \begin{array}{r} 15 \\ +\,84 \\ \hline 99 \end{array} \qquad \begin{array}{r} 86 \\ +\,11 \\ \hline 97 \end{array}$$

$$\begin{array}{r} 44 \\ +\,33 \\ \hline 77 \end{array} \qquad \begin{array}{r} 21 \\ +\,63 \\ \hline 84 \end{array} \qquad \begin{array}{r} 79 \\ +\,10 \\ \hline 89 \end{array} \qquad \begin{array}{r} 23 \\ +\,56 \\ \hline 79 \end{array}$$

$$\begin{array}{r} 34 \\ +\,23 \\ \hline 57 \end{array} \qquad \begin{array}{r} 78 \\ +\,20 \\ \hline 8 \end{array} \qquad \begin{array}{r} 34 \\ +\,23 \\ \hline \end{array} \qquad \begin{array}{r} 78 \\ +\,20 \\ \hline \end{array}$$

$$\begin{array}{r} 11 \\ +77 \\ \hline \end{array} \quad \begin{array}{r} 30 \\ +67 \\ \hline \end{array} \quad \begin{array}{r} 25 \\ +14 \\ \hline \end{array} \quad \begin{array}{r} 63 \\ +32 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ +35 \\ \hline \end{array} \quad \begin{array}{r} 71 \\ +10 \\ \hline \end{array} \quad \begin{array}{r} 82 \\ +12 \\ \hline \end{array} \quad \begin{array}{r} 31 \\ +45 \\ \hline \end{array}$$

$$\begin{array}{r} 56 \\ +42 \\ \hline \end{array} \quad \begin{array}{r} 46 \\ +41 \\ \hline \end{array} \quad \begin{array}{r} 15 \\ +84 \\ \hline \end{array} \quad \begin{array}{r} 86 \\ +11 \\ \hline \end{array}$$

$\begin{array}{r} 24 \\ +\ 12 \\ \hline 38 \end{array}$	$\begin{array}{r} 25 \\ +\ 32 \\ \hline 57 \end{array}$	$\begin{array}{r} 81 \\ +\ 17 \\ \hline 98 \end{array}$	$\begin{array}{r} 41 \\ +\ 38 \\ \hline \end{array}$
$\begin{array}{r} 21 \\ +\ 47 \\ \hline 68 \end{array}$	$\begin{array}{r} 19 \\ +\ 30 \\ \hline 49 \end{array}$	$\begin{array}{r} 50 \\ +\ 47 \\ \hline 97 \end{array}$	$\begin{array}{r} 32 \\ +\ 55 \\ \hline 87 \end{array}$
$\begin{array}{r} 64 \\ +\ 21 \\ \hline 85 \end{array}$	$\begin{array}{r} 44 \\ +\ 45 \\ \hline 89 \end{array}$	$\begin{array}{r} 34 \\ +\ 23 \\ \hline 57 \end{array}$	$\begin{array}{r} 78 \\ +\ 20 \\ \hline \end{array}$

$$\begin{array}{r} 25 \\ +30 \\ \hline \end{array} \quad \begin{array}{r} 62 \\ +10 \\ \hline \end{array} \quad \begin{array}{r} 14 \\ +34 \\ \hline \end{array} \quad \begin{array}{r} 50 \\ +25 \\ \hline \end{array}$$

$$\begin{array}{r} 46 \\ +22 \\ \hline \end{array} \quad \begin{array}{r} 12 \\ +17 \\ \hline \end{array} \quad \begin{array}{r} 40 \\ +36 \\ \hline \end{array} \quad \begin{array}{r} 84 \\ +15 \\ \hline \end{array}$$

$$\begin{array}{r} 44 \\ +52 \\ \hline \end{array} \quad \begin{array}{r} 36 \\ +43 \\ \hline \end{array} \quad \begin{array}{r} 55 \\ +23 \\ \hline \end{array} \quad \begin{array}{r} 33 \\ +13 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ +8 \\ \hline 9 \end{array} \qquad \begin{array}{r} 6 \\ +3 \\ \hline 9 \end{array} \qquad \begin{array}{r} 4 \\ +9 \\ \hline 13 \end{array} \qquad \begin{array}{r} 7 \\ +1 \\ \hline 7 \end{array}$$

$$\begin{array}{r} 6 \\ +6 \\ \hline 7 \end{array} \qquad \begin{array}{r} 3 \\ +5 \\ \hline \end{array} \qquad \begin{array}{r} 2 \\ +4 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ +7 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ +4 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ +9 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ +2 \\ \hline \end{array} \qquad \begin{array}{r} 6 \\ +2 \\ \hline \end{array}$$

$\begin{array}{r} 9 \\ \underline{+3} \end{array}$	$\begin{array}{r} 4 \\ \underline{+5} \end{array}$	$\begin{array}{r} 0 \\ \underline{+8} \end{array}$	$\begin{array}{r} 6 \\ \underline{+6} \end{array}$
$\begin{array}{r} 7 \\ \underline{+5} \end{array}$	$\begin{array}{r} 9 \\ \underline{+9} \end{array}$	$\begin{array}{r} 7 \\ \underline{+1} \end{array}$	$\begin{array}{r} 5 \\ \underline{+7} \end{array}$
$\begin{array}{r} 2 \\ \underline{+6} \end{array}$	$\begin{array}{r} 8 \\ \underline{+4} \end{array}$	$\begin{array}{r} 3 \\ \underline{+4} \end{array}$	$\begin{array}{r} 8 \\ \underline{+9} \end{array}$

$\begin{array}{r} 7 \\ +6 \\ \hline 9 \end{array}$	$\begin{array}{r} 3 \\ +9 \\ \hline 11 \end{array}$	$\begin{array}{r} 6 \\ +4 \\ \hline 9 \end{array}$	$\begin{array}{r} 0 \\ +7 \\ \hline 7 \end{array}$
$\begin{array}{r} 9 \\ +1 \\ \hline 10 \end{array}$	$\begin{array}{r} 7 \\ +8 \\ \hline 16 \end{array}$	$\begin{array}{r} 5 \\ +5 \\ \hline 11 \end{array}$	$\begin{array}{r} 9 \\ +9 \\ \hline 18 \end{array}$
$\begin{array}{r} 4 \\ +7 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ +6 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +9 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ +8 \\ \hline \end{array}$

$\begin{array}{r} 1 \\ +8 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ +9 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ +4 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ +6 \\ \hline \end{array}$
$\begin{array}{r} 6 \\ +6 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ +3 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ +2 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ +8 \\ \hline \end{array}$
$\begin{array}{r} 8 \\ +4 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ +5 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ +4 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ +7 \\ \hline \end{array}$

55	33	15	86
+23	+13	+84	+11
88	46	910	
44	21	79	23
+33	+63	+10	+56
34	78	34	78
+23	+20	+23	+20

11	30	25	63
<u>+77</u>	<u>+67</u>	<u>+14</u>	<u>+32</u>

23	71	82	31
<u>+55</u>	<u>+10</u>	<u>+12</u>	<u>+45</u>

56	46	15	86
<u>+42</u>	<u>+41</u>	<u>+84</u>	<u>+11</u>

24	25	81	41
+12	+32	+17	+38
36	47	97	79

21	19	50	32
+47	+30	+47	+55
7			

64	44	34	78
+21	+45	+23	+20

25	62	14	50
+30	+10	+34	+25
46	12	40	84
+22	+17	+36	+15
44	36	55	33
+52	+43	+23	+13

$$\begin{array}{rrrr}
\boxed{1}\downarrow & \boxed{1}\downarrow & \boxed{1}\downarrow & \boxed{1}\downarrow \\
36 & 56 & 26 & 77 \\
+\,48 & +\,37 & +\,65 & +\,19 \\
\hline
84 & 93 & 91 & 85 \\
\\
\boxed{1}\downarrow & \boxed{1}\downarrow & \boxed{1}\downarrow & \boxed{1}\downarrow \\
18 & 54 & 17 & 25 \\
+\,38 & +\,28 & +\,75 & +\,58 \\
\hline
56 & 72 & 92 & 3 \\
\\
\boxed{}\downarrow & \boxed{}\downarrow & \boxed{}\downarrow & \boxed{}\downarrow \\
55 & 45 & 78 & 69 \\
+\,27 & +\,15 & +\,16 & +\,28 \\
\hline
 & & &
\end{array}$$

$$\begin{array}{r} \square\downarrow \\ 45 \\ +\,45 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 66 \\ +\,18 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 26 \\ +\,65 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 77 \\ +\,19 \\ \hline \end{array}$$

$$\begin{array}{r} \square\downarrow \\ 33 \\ +\,47 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 48 \\ +\,26 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 17 \\ +\,75 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 25 \\ +\,58 \\ \hline \end{array}$$

$$\begin{array}{r} \square\downarrow \\ 59 \\ +\,22 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 34 \\ +\,17 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 78 \\ +\,16 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 69 \\ +\,28 \\ \hline \end{array}$$

1	1	1	1
29	65	34	19
+ 38	+ 16	+ 47	+ 55
67	81	89	74
1	1	1	
36	29	14	56
+ 28	+ 23	+ 66	+ 16
64	92	810	
58	46	78	53
+ 22	+ 47	+ 12	+ 38

$$\begin{array}{r} \square\downarrow \\ 33 \\ +\,47 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 48 \\ +\,26 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 34 \\ +\,47 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 19 \\ +\,55 \\ \hline \end{array}$$

$$\begin{array}{r} \square\downarrow \\ 36 \\ +\,28 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 29 \\ +\,23 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 59 \\ +\,22 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 34 \\ +\,17 \\ \hline \end{array}$$

$$\begin{array}{r} \square\downarrow \\ 58 \\ +22 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 46 \\ +47 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 78 \\ +12 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 53 \\ +38 \\ \hline \end{array}$$

$$\begin{array}{rrrr}
\boxed{1} & & \boxed{1} & \\
38 & 44 & 69 & 56 \\
+47 & +45 & +53 & +72 \\
\hline
85 & 89 & 122 & 128
\end{array}$$

$$\begin{array}{rrrr}
\boxed{1} & & \boxed{1} & \boxed{} \\
73 & 77 & 26 & 62 \\
+37 & +12 & +47 & +58 \\
\hline
110 & 89 & 73 &
\end{array}$$

$$\begin{array}{rrrr}
 & \boxed{} & & \boxed{} \\
14 & 49 & 24 & 94 \\
+45 & +82 & +84 & +19 \\
\hline
 & & &
\end{array}$$

$$\begin{array}{r} \square \\ 39 \\ +85 \\ \hline \end{array} \qquad \begin{array}{r} \square \\ 73 \\ +48 \\ \hline \end{array} \qquad \begin{array}{r} \\ 65 \\ +54 \\ \hline \end{array} \qquad \begin{array}{r} \square \\ 28 \\ +88 \\ \hline \end{array}$$

$$\begin{array}{r} \\ 64 \\ +44 \\ \hline \end{array} \qquad \begin{array}{r} \square \\ 53 \\ +97 \\ \hline \end{array} \qquad \begin{array}{r} \\ 75 \\ +33 \\ \hline \end{array} \qquad \begin{array}{r} \square \\ 35 \\ +69 \\ \hline \end{array}$$

$$\begin{array}{r} \square \\ 54 \\ +46 \\ \hline \end{array} \qquad \begin{array}{r} \square \\ 69 \\ +55 \\ \hline \end{array} \qquad \begin{array}{r} \square \\ 69 \\ +54 \\ \hline \end{array} \qquad \begin{array}{r} \square \\ 97 \\ +48 \\ \hline \end{array}$$

1	1		
25	56	33	38
+ 59	+ 98	+ 19	+ 81
124	154	52	119

1	1		
46	64	57	78
+ 29	+ 99	+ 32	+ 41
75	163		

37	78	56	89
+ 24	+ 46	+ 63	+ 95

$\begin{array}{r} 64 \\ +44 \\ \hline \end{array}$	$\begin{array}{r} \square \\ 53 \\ +97 \\ \hline \end{array}$	$\begin{array}{r} \square \\ 69 \\ +53 \\ \hline \end{array}$	$\begin{array}{r} 56 \\ +72 \\ \hline \end{array}$
$\begin{array}{r} \square \\ 73 \\ +37 \\ \hline \end{array}$	$\begin{array}{r} 77 \\ +12 \\ \hline \end{array}$	$\begin{array}{r} \square \\ 54 \\ +46 \\ \hline \end{array}$	$\begin{array}{r} \square \\ 69 \\ +55 \\ \hline \end{array}$
$\begin{array}{r} 65 \\ +54 \\ \hline \end{array}$	$\begin{array}{r} \square \\ 28 \\ +88 \\ \hline \end{array}$	$\begin{array}{r} 24 \\ +84 \\ \hline \end{array}$	$\begin{array}{r} \square \\ 94 \\ +19 \\ \hline \end{array}$

$$\begin{array}{r} \square\downarrow \\ 36 \\ +\,48 \\ \hline 84 \end{array} \qquad \begin{array}{r} \square\downarrow \\ 56 \\ +\,37 \\ \hline 24 \end{array} \qquad \begin{array}{r} \square\downarrow \\ 26 \\ +\,65 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 77 \\ +\,19 \\ \hline \end{array}$$

$$\begin{array}{r} \square\downarrow \\ 18 \\ +\,38 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 54 \\ +\,28 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 17 \\ +\,75 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 25 \\ +\,58 \\ \hline \end{array}$$

$$\begin{array}{r} \square\downarrow \\ 55 \\ +\,27 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 45 \\ +\,15 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 78 \\ +\,16 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 69 \\ +\,28 \\ \hline \end{array}$$

$$\begin{array}{r} \square\downarrow \\ 45 \\ +\,45 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 66 \\ +\,18 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 26 \\ +\,65 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 77 \\ +\,19 \\ \hline \end{array}$$

$$\begin{array}{r} \square\downarrow \\ 33 \\ +\,47 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 48 \\ +\,26 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 17 \\ +\,75 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 25 \\ +\,58 \\ \hline \end{array}$$

$$\begin{array}{r} \square\downarrow \\ 59 \\ +\,22 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 34 \\ +\,17 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 78 \\ +\,16 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 69 \\ +\,28 \\ \hline \end{array}$$

$\begin{array}{r} \boxed{1}\downarrow \\ 29 \\ +38 \\ \hline 67 \end{array}$ $\quad$ $\begin{array}{r} \boxed{1}\downarrow \\ 65 \\ +16 \\ \hline 121 \end{array}$ $\quad$ $\begin{array}{r} \boxed{1}\downarrow \\ 34 \\ +47 \\ \hline 1 \end{array}$ $\quad$ $\begin{array}{r} \boxed{}\downarrow \\ 19 \\ +55 \\ \hline \end{array}$

$\begin{array}{r} \boxed{}\downarrow \\ 36 \\ +28 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} \boxed{}\downarrow \\ 29 \\ +23 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} \boxed{}\downarrow \\ 14 \\ +66 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} \boxed{}\downarrow \\ 56 \\ +16 \\ \hline \end{array}$

$\begin{array}{r} \boxed{}\downarrow \\ 58 \\ +22 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} \boxed{}\downarrow \\ 46 \\ +47 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} \boxed{}\downarrow \\ 78 \\ +12 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} \boxed{}\downarrow \\ 53 \\ +38 \\ \hline \end{array}$

$$\begin{array}{r} \square\downarrow \\ 33 \\ +47 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 48 \\ +26 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 34 \\ +47 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 19 \\ +55 \\ \hline \end{array}$$

$$\begin{array}{r} \square\downarrow \\ 36 \\ +28 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 29 \\ +23 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 59 \\ +22 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 34 \\ +17 \\ \hline \end{array}$$

$$\begin{array}{r} \square\downarrow \\ 58 \\ +22 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 46 \\ +47 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 78 \\ +12 \\ \hline \end{array} \qquad \begin{array}{r} \square\downarrow \\ 53 \\ +38 \\ \hline \end{array}$$

$\begin{array}{r} \boxed{1} \\ 38 \\ +\,47 \\ \hline 87 \end{array}$	$\begin{array}{r} 44 \\ +\,45 \\ \hline 8\,\vert\vert\vert \end{array}$	$\begin{array}{r} \boxed{1} \\ 69 \\ +\,53 \\ \hline 2 \end{array}$	$\begin{array}{r} 56 \\ +\,72 \\ \hline \end{array}$
$\begin{array}{r} \boxed{} \\ 73 \\ +\,37 \\ \hline \end{array}$	$\begin{array}{r} 77 \\ +\,12 \\ \hline \end{array}$	$\begin{array}{r} \boxed{} \\ 26 \\ +\,47 \\ \hline \end{array}$	$\begin{array}{r} \boxed{} \\ 62 \\ +\,58 \\ \hline \end{array}$
$\begin{array}{r} 14 \\ +\,45 \\ \hline \end{array}$	$\begin{array}{r} \boxed{} \\ 49 \\ +\,82 \\ \hline \end{array}$	$\begin{array}{r} 24 \\ +\,84 \\ \hline \end{array}$	$\begin{array}{r} \boxed{} \\ 94 \\ +\,19 \\ \hline \end{array}$

$\begin{array}{r} \square \\ 39 \\ +85 \\ \hline \end{array}$ $\begin{array}{r} \square \\ 73 \\ +48 \\ \hline \end{array}$ $\begin{array}{r} 65 \\ +54 \\ \hline \end{array}$ $\begin{array}{r} \square \\ 28 \\ +88 \\ \hline \end{array}$

$\begin{array}{r} 64 \\ +44 \\ \hline \end{array}$ $\begin{array}{r} \square \\ 53 \\ +97 \\ \hline \end{array}$ $\begin{array}{r} 75 \\ +33 \\ \hline \end{array}$ $\begin{array}{r} \square \\ 35 \\ +69 \\ \hline \end{array}$

$\begin{array}{r} \square \\ 54 \\ +46 \\ \hline \end{array}$ $\begin{array}{r} \square \\ 69 \\ +55 \\ \hline \end{array}$ $\begin{array}{r} \square \\ 69 \\ +54 \\ \hline \end{array}$ $\begin{array}{r} \square \\ 97 \\ +48 \\ \hline \end{array}$

$$\begin{array}{r} \boxed{1} \\ 25 \\ +59 \\ \hline 84 \end{array} \qquad \begin{array}{r} \boxed{1} \\ 56 \\ +98 \\ \hline 5 \end{array} \qquad \begin{array}{r} \boxed{} \\ 33 \\ +19 \\ \hline \end{array} \qquad \begin{array}{r} 38 \\ +81 \\ \hline \end{array}$$

$$\begin{array}{r} \boxed{} \\ 46 \\ +29 \\ \hline \end{array} \qquad \begin{array}{r} \boxed{} \\ 64 \\ +99 \\ \hline \end{array} \qquad \begin{array}{r} 57 \\ +32 \\ \hline \end{array} \qquad \begin{array}{r} 78 \\ +41 \\ \hline \end{array}$$

$$\begin{array}{r} \boxed{} \\ 37 \\ +24 \\ \hline \end{array} \qquad \begin{array}{r} \boxed{} \\ 78 \\ +46 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ +63 \\ \hline \end{array} \qquad \begin{array}{r} \boxed{} \\ 89 \\ +95 \\ \hline \end{array}$$

$\begin{array}{r} \\ 64 \\ +\ 44 \\ \hline \end{array}$	$\begin{array}{r} \square \\ 53 \\ +\ 97 \\ \hline \end{array}$	$\begin{array}{r} \square \\ 69 \\ +\ 53 \\ \hline \end{array}$	$\begin{array}{r} \\ 56 \\ +\ 72 \\ \hline \end{array}$
$\begin{array}{r} \square \\ 73 \\ +\ 37 \\ \hline \end{array}$	$\begin{array}{r} \\ 77 \\ +\ 12 \\ \hline \end{array}$	$\begin{array}{r} \square \\ 54 \\ +\ 46 \\ \hline \end{array}$	$\begin{array}{r} \square \\ 69 \\ +\ 55 \\ \hline \end{array}$
$\begin{array}{r} \\ 65 \\ +\ 54 \\ \hline \end{array}$	$\begin{array}{r} \square \\ 28 \\ +\ 88 \\ \hline \end{array}$	$\begin{array}{r} \\ 24 \\ +\ 84 \\ \hline \end{array}$	$\begin{array}{r} \square \\ 94 \\ +\ 19 \\ \hline \end{array}$

APPENDIX D

INFORMED CONSENT FORM

APPENDIX D

Informed Consent Form

CALIFORNIA STATE UNIVERSITY
SAN BERNARDINO

College of Social and Behavioral Sciences
Department of Psychology

If you have any questions regarding this study you may contact me or my advisor at (909)537-7304.

Thank you for considering this important research,

Sincerely,

Lindsay Rodriguez

CALIFORNIA STATE UNIVERSITY, SAN BERNARDINO
PSYCHOLOGY INSTITUTIONAL REVIEW BOARD SUB-COMMITTEE
APPROVED 8/19/08 VOID AFTER 8/18/09
IRB# H-08SU03 CHAIR

Yes, I agree to volunteer my child's math probe sheets and test scores for the above explained research.

Katherine J. Dittes Aug 28, 08
Parent Signature Date

909.537.5570 • 909.537.7003 • http://www.psychology.csusb.edu
5500 UNIVERSITY PARKWAY, SAN BERNARDINO, CA 92407-2393

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