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On a symmetric presentation of the double cover of $M_{22}: 2$

Gabriela Laura Maerean

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ON A SYMMETRIC PRESENTATION OF THE DOUBLE COVER OF $M_{22} : 2$

A Thesis
Presented to the
Faculty of
California State University,
San Bernardino

In Partial Fulfillment
of the Requirements for the Degree
Master of Arts
in
Mathematics

by
Gabriela Laura Maerean

June 2009
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Abstract

The main purpose of this project is to construct finite homomorphic images of infinite semi-direct products. We will construct two finite homomorphic images, $L_2(8)$ and $PGL_2(9)$ of the infinite semi-direct product $2^{*3} : S_3$. The main part of this project is to construct the double cover $2 \cdot M_{22} : 2$ and the automorphism group $M_{22} : 2$ of the Mathieu sporadic group $M_{22}$ as a homomorphic image of the progenitor $2^{*7} : L_3(2)$. The by hand constructions given in this thesis, up to our knowledge, have not been proved elsewhere.
Acknowledgements

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Chapter 1

Introduction

Permutations and matrices are two methods for working with groups but they are inconvenient or unmanageable for large finite groups and in particular for the larger sporadic groups. The main purpose of this paper is to give an alternative and more efficient method for working with groups, which is symmetric presentation. Symmetric presentation provides a uniform way of constructing finite groups. We are particularly interested in symmetric presentations of finite simple groups, since all finite groups can be constructed from these. Double coset enumeration of a finite simple group $G$ over a group of permutation on $n$ letters, $N$, provides a method of conveniently representing $G$ as a group of permutations. Double coset enumeration can be performed on groups that possess generating sets of involutions and any finite group generated by a conjugacy class of involutions. All finite non-abelian simple groups have symmetric generating sets of involutions (see [Bra97]). It is this technique of double coset enumeration that allows us to write elements in a much more concise manner. For example, the smallest Janko group $J_1$ has a maximal subgroup, $L_2(11)$, of lowest index 266. Thus $J_1$ is represented as a permutation group on 266 letters. Double coset enumeration of $J_1$ over $L_2(11)$ enables us to represent every element of $J_1$ as a permutation of $L_2(11)$ on 11 letters followed by a word of length at most 4 in the symmetric generators (see [CH96]).

1.1 Symmetric Generation of a Group

Consider a group $G$ and $T = \{t_1, t_2, \ldots, t_n\} \subseteq G$. Define $T = \{T_1, T_2, \ldots, T_n\}$, where $T_i = \langle t_i \rangle$, $i = 1, 2, \ldots, n$, which is the cyclic subgroup generated by $t_i$. Let $N = \ldots$
$N_G(T)$, a subgroup of $G$ called the control subgroup. $T$ is a symmetric generating set for $G$ if and only if

(i) $G = \langle T \rangle$, and

(ii) $N$ acts transitively on $T$.

(i) and (ii) imply that $G$ is a homomorphic image of the (infinite) progenitor $m^* n : N$, where $m^* n$ represents a free product of $n$ copies of the cyclic group $C_m$, $m$ being the order of $t_i$, and $N$ is a group of automorphisms of $m^* n$ which permutes the $n$ cyclic subgroups by conjugation. If $m = 2$ then $N$ will simply act by conjugation as permutation of the $n$ involutory symmetric generators. Thus, elements of $N$ can be gathered on the left and every element of the progenitor can be represented as $pw$, where $p \in N$ and $w$ is a word in the symmetric generators. This representation is unique provided that $w$ is simplified so that adjacent symmetric generators are distinct. Thus, any additional relation by which we factor the progenitor to obtain $G$ must have the form $pw(t_1, t_2, \ldots, t_n)$.

Let $N$ be a group of permutations on $n$ letters. Then $2^* n : N$ means $2^* n$ extended by $N$ acting as automorphism (by conjugation). The objective here is to factor $2^* n : N$ by relations that equate elements of $N$ to the product of $t_i$'s that give finite homomorphic images. For example, the progenitor $2^* 6 : S_6$ is the free product of 6 copies of the cyclic group $C_2$ of order 2 extended by $S_6$, the symmetric group of order 6. As mentioned above, in order to find finite homomorphic images of the infinite progenitor $2^* 6 : S_6$, we must divide by suitable relations.

1.2 Manual Double Coset Enumeration

Since we are only concerned with involutory symmetric generators, we restrict our attention to the case $m = 2$ Therefore, we seek homomorphic images of the progenitor $2^* n : N$. In order to construct the factor group $G$ by hand, we use the method called manual double coset enumeration. By constructing $G$ with this method we can determine the index of $N$ in $G$ and also the order of $G$, which is the number of distinct right cosets of $N$ in $G$. The procedure for manual double coset enumeration involves the following steps:

1. First consider the double coset $NeN$ denoted by $[\ast]$ which is characterized by a word $w_0 = e$ of length zero. The number of distinct right cosets of $N$ in $G$ in $[\ast]$ is
given by $|N : N^{(w_0)}|$. Since $w_0$ is a word of length zero, the number of right cosets in $[s]$ is equal to $|N : N| = 1$.

2. Next, we determine the orbits of $N$ on $T$. Since $N$ is transitive on $T = \{t_0, t_1, t_2, \ldots, t_n\}$, we take a representative $t_i$ from each orbit and multiply $N$ on the right by it, to get the right coset $Nt_i$. Using the relations that we factored $G$ by, we determine whether the double coset $Nt_iN$ is new or not. If $Nt_iN$ is new, then we proceed to the next step.

3. Next, we consider the double coset $Nt_iN$ denoted by $[i]$, characterized by a reduced word $w_1 = t_i$ of length one. The number of right cosets in $[i]$ is equal to $|N : N^{(i)}|$, where $N^{(i)} = \{\pi \in N | Nt_i^\pi = Nt_i\}$ is the coset stabilizer. Then, we determine the orbits of $N^{(i)}$ on $T$ and we take one representative from each of the orbits, say $t_j$, $t_h$ and so on, and we multiply $Nt_i$ on the right by it, to get the right cosets $Nt_it_j$, $Nt_it_h$ and so on. Again using the relators that we factor $G$ by, we determine whether the double cosets $Nt_it_jN$, $Nt_it_hN$ and so on, are new or not. If any of this double cosets are new, we proceed to the next step.

4. We now consider the double cosets characterized by the reduced words of length two and we repeat the step (3) for each one of the distinct double coset $Nt_it_jN$, $Nt_it_hN$ and so on. We keep repeating the process until we get to the point where the relations indicate that there are no new double cosets and we conclude that right multiplication on the right cosets of $N$ in $G$ is closed. At this time our manual double coset enumeration is complete.

In addition to constructing $G$ using the method of manual double coset enumeration we also need to determine the permutation representation of the generators of $G$ and finally, show that $G$ is isomorphic to a group generated by these permutation representations.
Chapter 2

$L_2(8)$ as a Homomorphic Image of the Progenitor $2^*^3 : S_3$

2.1 Introduction

Consider the progenitor $2^*^3 : S_3$ which is given by

$$\langle x, y, t | x^3 = y^2 = (xy)^2 = e = t^2 = [t, y], x \sim (0,1,2), y \sim (1,2), \text{ and } t \sim t_0. \text{ We factor the progenitor by the following relations: } [(0,1,2)t_0]^7 = e \text{ and } [(0,1)t_0]^9 = e. \text{ The first relation } [\pi t_0]^7 = e, \text{ where } \pi = (0,1,2), \text{ implies}

$$
\pi t_0 \pi t_0 \pi t_0 \pi t_0 \pi t_0 \pi t_0 \pi t_0 = e
\Rightarrow \pi^7 t_0^7 t_0^6 t_0^4 t_0^3 t_0^2 t_0^1 t_0 = e
\Rightarrow \pi t_0 t_2 t_1 t_0 t_2 t_1 t_0 = e
\Rightarrow (0,1,2)t_0 t_2 t_1 t_0 = t_0 t_1 t_2 \tag{2.1}
$$

The second relation $[\pi t_0]^9 = e$, where $\pi = (0,1)$, implies

$$
\pi t_0 \pi t_0 \pi t_0 \pi t_0 \pi t_0 \pi t_0 \pi t_0 \pi t_0 = e
\Rightarrow \pi^9 t_0^9 t_0^8 t_0^7 t_0^6 t_0^4 t_0^3 t_0^2 t_0^1 t_0 = e
\Rightarrow \pi t_0 t_1 t_0 t_1 t_0 t_1 t_0 t_1 t_0 t_0 = e
\Rightarrow (0,1)t_0 t_1 t_0 t_1 t_0 t_0 t_1 t_0 = t_0 t_1 t_0 t_1 \tag{2.2}
$$

Let $G$ denote the progenitor factored by the above relations, that is $G \cong \frac{2^*^3 S_3}{[(0,1,2)t_0]^7 = e = [(0,1)t_0]^9}$ and we will show that $G$ is isomorphic to $L_2(8)$. 
2.2 Manual Double Coset Enumeration of $G$ Over $S_3$

Now we will perform manual double coset enumeration of $G$ over $N \cong S_3$ to find the index of $N$ in $G$. In fact we will show that $|G|$ is at most $6 \times 84 = 504.$

We start with the double coset $NeN$ denoted by $[e]$. $[e]$ consists of the single coset $N$. Now $N$ is transitive on $T = \{0, 1, 2\}$, so we consider the double coset $Nt_0N$ denoted by $[0]$. Since $N^{(0)} = N^0 = \{e, (1, 2)\}$, the number of single cosets in $[0]$ is equal to $|N|/|N^{(0)}| = 3$.

Since $N^{(0)}$ has orbits $\{0\}$ and $\{1, 2\}$, we must consider the double cosets $Nt_0t_0N$ and $Nt_0t_2N$. However, $Nt_0t_0N = NeN = [e]$, but $Nt_0t_2N$ denoted by $[01]$ is a new double coset. The number of single cosets in $[01]$ equals to $|N|/|N^{(01)}| = 6/1 = 6$, since $N^{(01)} = \{e\}$.

The orbits of $N^{(01)}$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_0N$, $Nt_0t_1t_1N$, and $Nt_0t_1t_2N$. However, $Nt_0t_1t_1N = Nt_0N = [0]$. But $Nt_0t_1t_0N$ and $Nt_0t_1t_2N$ are new. Now $N^{(010)} = N^{(012)} = \{e\}$, so the number of single cosets in $[010]$ and $[012]$ equals to $|N|/|N^{(010)}| = 6/1 = 6$.

The orbits of $N^{(010)}$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_0t_0N$, $Nt_0t_1t_0t_1N$, and $Nt_0t_1t_0t_2N$. However, $Nt_0t_1t_0t_0N = Nt_0t_1t_1N = [01]$. But $Nt_0t_1t_0t_1N$ and $Nt_0t_1t_0t_2N$ are new double cosets.

The orbits of $N^{(012)}$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_2t_0N$, $Nt_0t_1t_2t_1N$, and $Nt_0t_1t_2t_2N$. However, $Nt_0t_1t_2t_2N = Nt_0t_1N = [01]$. Also, from the relation $(0, 1, 2)t_0t_2t_1t_0 = t_0t_1t_2$ we have:

\[
[(0, 1, 2)t_0t_2t_1t_0]^{(1, 2)} = [t_0t_1t_2]^{(1, 2)}
\]

\[
\implies (0, 1, 2)t_0t_1t_2t_0 = t_0t_1t_2
\]

\[
\implies N(0, 2, 1)t_0t_1t_2t_0 = Nt_0t_2t_1
\]

\[
\implies Nt_0t_1t_2t_0N = Nt_0t_2t_1N
\]

Thus, $[0120] = [012]$ and the only new double coset is $Nt_0t_1t_2t_1N$.

The number of single cosets in $Nt_0t_1t_0t_1N$ is $|N|/|N^{(0101)}| = 6/1 = 6$, since $N^{(0101)} = N^{(01)} = \{e\}$. The orbits of $N^{(0101)}$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_0t_0N$, $Nt_0t_1t_0t_1N$, and $Nt_0t_1t_0t_2N$. However, $Nt_0t_1t_0t_1N = Nt_0t_1t_0N = [010]$. From the relation $(0, 1)t_0t_1t_0t_0 = t_0t_1t_0t_1$, we have
\( N_{t_0 t_1 t_0 t_1 t_0} N = N_{t_0 t_1 t_0 t_1 t_0} N = [0101] \). So the only new double coset is \( N_{t_0 t_1 t_0 t_1 t_2} N \) denoted by [01012].

The number of single cosets in \( N_{t_0 t_1 t_0 t_2} N \) is \( |N|/|N^{(0102)}| = 6/1 = 6 \), since \( N^{(0102)} = N^{(012)} = \{ e \} \). The orbits of \( N^{(012)} \) on \( T \) are \( \{0\} \), \( \{1\} \), and \( \{2\} \). We must consider the double cosets \( N_{t_0 t_1 t_0 t_2} N \), \( N_{t_0 t_1 t_0 t_1} N \), and \( N_{t_0 t_1 t_0 t_2} N \). However, \( N_{t_0 t_1 t_0 t_2} N = N_{t_0 t_1 t_0} N = [010] \). From the relation \( (0, 1, 2)_{t_0 t_2 t_1 t_0} = t_0 t_1 t_2 \), we have

\[
[(0, 1, 2)_{t_0 t_2 t_1 t_0}]^{(0, 1, 2)} = [t_0 t_1 t_2]^{(0, 1, 2)}
\]

\[
\implies (1, 2, 0)_{t_1 t_0 t_2 t_1} = t_1 t_0 t_2
\]

\[
\implies (0, 2, 1)_{t_2 t_1 t_2 t_0} = (0, 2, 1)_{t_2 t_1 t_2 t_0} (1, 2, 0)_{t_1 t_0 t_2 t_1}
\]

\[
\implies (0, 2, 1)_{t_2 t_1 t_2 t_0} = t_0 t_1 t_0 t_2 t_1
\]

\[
\implies N_{t_0 t_1 t_0 t_2 t_1} = N(0, 2, 1)_{t_2 t_1 t_2 t_0}
\]

\[
\implies N_{t_0 t_1 t_0 t_2 t_1} N = N_{t_2 t_1 t_2 t_0} N = [0102].
\]

So the only new double coset is \( N_{t_0 t_1 t_0 t_2} N \) denoted by [01020].

The number of single cosets in \( N_{t_0 t_1 t_2 t_1} N \) is \( |N|/|N^{(0121)}| = 6/1 = 6 \), since \( N^{(0121)} = N^{(012)} = \{ e \} \). The orbits of \( N^{(0121)} \) on \( T \) are \( \{0\} \), \( \{1\} \), and \( \{2\} \). We must consider the double cosets \( N_{t_0 t_1 t_2 t_1 t_0} N \), \( N_{t_0 t_1 t_2 t_1} N \), and \( N_{t_0 t_1 t_2} N \). However, \( N_{t_0 t_1 t_2 t_1 t_0} N = N_{t_0 t_1 t_2} N = [012] \). But [01210] and [01212] are new double cosets.

The number of single cosets in \( N_{t_0 t_1 t_0 t_1} N \) is \( |N|/|N^{(01221)}| = 6/1 = 6 \), since \( N^{(01221)} = N^{(012)} = \{ e \} \). The orbits of \( N^{(01221)} \) on \( T \) are \( \{0\} \), \( \{1\} \), and \( \{2\} \). We must consider the double cosets \( N_{t_0 t_1 t_0 t_1 t_2} N \), \( N_{t_0 t_1 t_0 t_2} N \), and \( N_{t_0 t_1 t_0 t_1} N \). However, \( N_{t_0 t_1 t_0 t_2} N = N_{t_0 t_1 t_0} N = [0101] \). From the relation \( (0, 1, 2)_{t_0 t_2 t_1 t_0} = t_0 t_1 t_2 \) we have:

\[
[(0, 1, 2)_{t_0 t_2 t_1 t_0}]^{(1, 2)} = [t_0 t_1 t_2]^{(1, 2)}
\]

\[
\implies (0, 2, 1)_{t_0 t_2 t_1 t_0} = t_0 t_1 t_2
\]

\[
\implies (0, 1, 2)_{t_1 t_0 t_2 t_1} = (0, 1, 2)_{t_1 t_0 t_2 t_1} (0, 2, 1)_{t_0 t_1 t_2 t_0}
\]

\[
\implies (0, 1, 2)_{t_1 t_0 t_2 t_1} = t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_2 t_0
\]

\[
\implies N_{t_0 t_1 t_0 t_2 t_1} N = N(0, 1, 2)_{t_1 t_2 t_0 t_2 t_1} N
\]

\[
\implies N_{t_0 t_1 t_0 t_2 t_1} N = N_{t_1 t_2 t_0 t_2 t_1} N = [01210] .
\]
Thus, \([010120] = [01210]\) and the only new double coset is \(N t_0 t_1 t_0 t_1 t_2 t_1 N = [010121]\).

The number of single cosets in \(N t_0 t_1 t_0 t_2 t_0 N\) is \(|N|/|N^{(01210)}| = 6/1 = 6\), since \(N^{(01210)} = N^{(012)} = \{e\}\). The orbits of \(N^{(01210)}\) on \(T\) are \(\{0\}, \{1\}, \text{ and } \{2\}\). We must consider the double cosets \(N t_0 t_1 t_0 t_2 t_0 N, N t_0 t_1 t_0 t_2 t_0 t_1 N, \text{ and } N t_0 t_1 t_0 t_2 t_0 t_2 N\). However, \(N t_0 t_1 t_0 t_2 t_0 N = N t_0 t_1 t_0 t_2 N = [0102]\). From the relation \((0, 1, 2)t_0 t_2 t_1 t_0 = t_0 t_1 t_2\) we have:

\[
[(0, 1, 2)t_0 t_2 t_1 t_0]^{(0, 1, 2)} = [t_0 t_1 t_2]^{(0, 1, 2)}
\]

\[
\Rightarrow (1, 2, 0)t_1 t_0 t_2 t_1 = t_1 t_2 t_0
\]

\[
\Rightarrow (1, 2)t_2 t_0 t_2 t_1 t_2 t_0 = (1, 2)t_2 t_0 t_2 t_1(1, 2, 0)t_1 t_0 t_2 t_1
\]

\[
\Rightarrow (1, 2)t_2 t_0 t_2 t_1 t_2 t_0 = (1, 2)(1, 2, 0)t_0 t_1 t_0 t_0 t_2 t_1
\]

\[
\Rightarrow (1, 2)t_2 t_0 t_2 t_1 t_2 t_0 = (1, 0)t_0 t_1 t_0 t_0 t_2 t_1
\]

\[
\Rightarrow (1, 2)t_2 t_0 t_2 t_1 t_2 t_0 = t_0 t_1 t_0 t_1 t_2 t_1
\]

\[
= (2, 0, 1)t_2 t_1 t_0 t_2 = t_2 t_0 t_1
\]

\[
\Rightarrow (0, 2, 1)t_2 t_0 t_2 t_0 t_1 = (0, 2, 1)t_2 t_0 (2, 0, 1)t_2 t_1 t_0 t_2
\]

\[
\Rightarrow (0, 2, 1)t_2 t_0 t_2 t_0 t_1 = (0, 2, 1)(2, 0, 1)t_0 t_1 t_2 t_1 t_0 t_2
\]

\[
\Rightarrow (0, 2, 1)t_2 t_0 t_2 t_0 t_1 = t_0 t_1 t_2 t_1 t_0 t_2
\]

\[
\Rightarrow N(0, 2, 1)t_2 t_0 t_2 t_0 t_1 = N t_0 t_1 t_2 t_1 t_0 t_2
\]

\[
\Rightarrow N t_0 t_1 t_2 t_1 t_0 t_2 N = N t_2 t_0 t_2 t_0 t_1 N = [01012].
\]
From the relation $(0, 1, 2)t_0t_1t_0 = t_0t_1t_2$ we have:

$$[(0, 1, 2)t_0t_1t_0]^{(0, 2, 1)} = [t_0t_1t_2]^{(0, 2, 1)}$$

$$
\implies (2, 0, 1)t_2t_1t_0t_2 = t_2t_0t_1
\implies (0, 1)t_2t_0t_1t_0t_2 = (0, 1)(2, 0, 1)t_2t_1t_0t_2t_0
\implies (0, 1)t_2t_0t_1t_0t_2 = (0, 2)t_2t_1t_0t_2t_0
\implies (0, 1)t_2t_0t_1t_0t_2 = (0, 2)t_2t_1t_0t_2t_0t_2t_0$$

$$\implies (0, 1)t_2t_0t_1t_0t_2 = (0, 2)t_2t_1t_0t_2t_0t_2t_0$$

$$\implies (0, 1)t_2t_0t_1t_0t_2 = (0, 2)t_2t_1t_0t_2t_0t_2$$

$$\implies (2, 0, 1)t_2t_1t_0t_2 = t_2t_0t_1$$

$$\implies (0, 1)t_2t_0t_1t_0t_2 = (0, 2)t_2t_1t_0t_2t_0t_2t_0$$

But, $Nt_2t_0t_1t_0t_2t_0 \in [012101]$ and $Nt_0t_1t_0t_0t_2 \in [010202]$, thus $[012101] = [010202]$

The number of single cosets in $Nt_0t_1t_0t_1t_2N$ is $|N|/|N^{(01212)}| = 6/1 = 6$, since $N^{(01212)} = N^{(012)} = \{e\}$. The orbits of $N^{(01212)}$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_2t_1t_2t_0N$, $Nt_0t_1t_2t_1t_2t_2N$, and $Nt_0t_1t_2t_2t_2N$. However, $Nt_0t_1t_2t_1t_2t_2N = Nt_0t_1t_2t_1N = [0121]$. From the relation (2.2) we have:

$$*[0, 1, 2)t_0t_1t_0t_1t_0]^{(0, 1, 2)} = [t_0t_1t_0t_1]^{(0, 1, 2)}$$

$$\implies (1, 2)t_1t_2t_1t_2t_1 = t_1t_2t_1t_2$$

$$\implies (1, 2)t_0t_1t_0t_1t_0 = (1, 2)t_0(1, 2)t_0t_1t_2t_1t_2t_1$$

$$\implies (1, 2)t_0t_1t_0t_1t_2 = t_0t_1t_0t_1t_2t_1$$

$$\implies N(1, 2)t_0t_1t_0t_1t_2 = Nt_0t_1t_0t_1t_2t_1$$

$$\implies Nt_0t_1t_0t_1t_2t_1N = Nt_0t_1t_2t_2t_2N = [01212].$$

Thus, the only new double coset is $Nt_0t_1t_2t_1t_2t_0N = [012120]$.

The number of single cosets in $Nt_0t_1t_0t_1t_2t_1N$ is $|N|/|N^{(010121)}| = 6/1 = 6$, since $N^{(010121)} = N^{(012)} = \{e\}$. The orbits of $N^{(010121)}$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_0t_1t_2t_0N$, $Nt_0t_1t_0t_1t_2t_1t_1N$, and $Nt_0t_1t_0t_1t_2t_1t_2N$. However, $Nt_0t_1t_0t_1t_2t_1t_1N = Nt_0t_1t_0t_1t_2N = [010212]$. Also, from (2.3) we have

$$t_0t_1t_0t_1t_2t_1t_0 = (1, 2)t_2t_0t_2t_1t_2t_0t_0$$
Thus the only new double coset is $N_0t_0t_1t_0t_2t_1t_2N$.

The number of single cosets in $N_0t_0t_0t_2t_0t_2N$ is $|N|/|N^{(010202)}| = 6/1 = 6$, since $N^{(010202)} = N^{(012)} = \{e\}$. The orbits of $N^{(010202)}$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $N_0t_0t_0t_2t_0t_0N$, $N_0t_0t_0t_0t_2t_1N$, and $N_0t_0t_0t_0t_2t_2N$. However, $N_0t_0t_0t_0t_2t_2N = N_0t_0t_0t_0t_03N = [010202]$. Also, from (2.4) we get

$$t_0t_1t_0t_2t_0t_2t_0 = (0,1)t_2t_0t_1t_0t_2t_0$$

$$\Rightarrow t_0t_1t_0t_2t_0t_2t_0 = (0,1)t_2t_0t_1t_0t_2$$

$$\Rightarrow N_0t_0t_1t_0t_2t_0t_2t_0 = N(0,1)t_2t_0t_1t_0t_2$$

$$\Rightarrow N_0t_0t_1t_0t_2t_0t_2t_0 = Nt_2t_0t_1t_0t_2 \in [01210]$$

Thus the only new double coset is $N_0t_0t_1t_0t_2t_0t_2t_1N$.

Consider the double coset $[012120]$ and note that $t_0t_1t_2t_1t_2t_0 = t_0t_1t_2(1,2,0)t_1t_0t_2t_1 = (1,2,0)t_1t_2t_0t_1t_0t_2t_1 = (0,2,1)t_1t_0t_2t_0t_2t_1 \Rightarrow N_0t_1t_2t_1t_2t_0 = N(0,2,1)t_1t_2t_0t_2t_0t_2 = Nt_1t_0t_2t_0t_2t_1$. Similarly, $N_0t_2t_1t_2t_1t_0 = Nt_2t_0t_1t_0t_1t_2$ and $Nt_1t_2t_0t_2t_0t_1 = Nt_2t_1t_0t_1t_0t_2$. Thus, the number of single cosets in $[012120]$ is 3. We now consider the double cosets $N_0t_1t_2t_1t_2t_0t_0N$, $N_0t_1t_2t_1t_2t_0t_1N$, and $N_0t_1t_2t_1t_2t_0t_2N$. However, $N_0t_0t_1t_2t_1t_2t_0N = N_0t_0t_1t_2t_1t_2N = [01210]$. From the relation (2.1) we get:

$$[(0,1,2)t_0t_2t_1t_0]^{(0,1)} = [t_0t_1t_2]^{(0,1)}$$

$$\Rightarrow (1,0,2)t_1t_2t_0t_1 = t_1t_0t_2$$

$$\Rightarrow (0,2,1)t_1t_0t_2t_0t_2 = (0,2,1)(1,0,2)t_1t_2t_0t_1t_0t_2$$

$$\Rightarrow (0,2,1)t_1t_0t_2t_0t_2 = (0,1,2)t_1t_2t_0t_1t_0t_2$$

$$\Rightarrow (0,2,1)t_1t_0t_2t_0t_2 = (0,1,2)t_1t_2t_0(1,0,2)t_1t_2t_0t_1$$

$$\Rightarrow (0,2,1)t_1t_0t_2t_0t_2 = (0,1,2)(1,0,2)t_0t_1t_2t_1t_2t_0t_1$$

$$\Rightarrow (0,2,1)t_1t_0t_2t_0t_2 = t_0t_1t_2t_1t_2t_0t_1$$
From (2.2) we have:
\[ [(0,1)t_0t_1t_0t_1t_0]^{(0,2)} = [t_0t_1t_0t_1]^{(0,2)} \]
\[ \iff (2,1)t_2t_1t_2t_1t_2 = t_2t_1t_2t_1 \]
\[ \implies t_0t_1t_0t_2t_0t_1t_2t_1t_0t_2t_0t_1 = t_0t_1t_0t_2t_0(2,1)t_2t_1t_0t_2t_0t_1 = (2,1)t_0t_2t_0(0,2,1)t_1t_2t_1t_0t_2t_0t_1 = (2,1)(0,2,1)(2,1)t_2t_1t_2t_1t_0t_2t_1t_0t_2t_0t_1 = (2,0,1)t_2t_1t_0t_2t_0t_1 = (2,0,1)(0,2,1)t_1t_0t_1t_0t_1t_0 = e \implies t_0t_1t_0t_2t_0t_1 = t_1t_0t_2t_0t_1 \implies Nt_0t_1t_0t_2t_0t_1 = Nt_1t_0t_2t_0t_2. \text{ Since } Nt_0t_1t_0t_2t_0t_1 \in [0102021] \text{ and } Nt_1t_0t_2t_0t_2t_1t_2 \in [0121202], \text{ we have } [0102021]= [0121202]. \]

Consider the double coset [0102021] in which we have the following:
\[ t_0t_1t_0t_2t_0t_1 = t_0t_1t_0t_2(0,2,1)t_0t_1t_2t_0 = (0,2,1)t_0t_2t_0t_1t_0t_2t_0 = \]
\[ (0,1,2)t_1t_0t_1t_2t_0t_1 = Nt_0t_1t_0t_2t_0t_2 = N(0,1,2)t_1t_0t_1t_0t_2t_0t_1 = Nt_0t_1t_0t_2t_0t_1. \]
By conjugation of the above relation we get $Nt_1t_2t_1t_0t_1t_0t_2 = Nt_2t_1t_0t_2t_0t_1$ and $Nt_0t_2t_0t_1t_0t_1 = Nt_2t_0t_1t_0t_2t_1t_0$. Thus in the double coset [0102021] are three distinct single cosets. We must consider the double cosets $Nt_0t_1t_0t_2t_0t_2t_1t_0N$, $Nt_0t_1t_0t_2t_0t_2t_1N$, and $Nt_0t_1t_0t_2t_0t_2t_1t_2N$. However, $Nt_0t_1t_0t_2t_0t_2t_1t_2N = Nt_0t_1t_0t_2t_0t_2N = [010202].$ Also,
\[ (0,1,2)t_1t_0t_1t_2t_1t_2 = (0,1,2)t_1(0,1,2)t_0t_2t_1t_0t_1t_2 \]
\[ \implies (0,1,2)t_1t_0t_1t_2t_1t_2 = (0,2,1)t_2t_0t_2t_1t_0t_1t_2 \]
\[ \implies (0,1,2)t_1t_0t_1t_2t_1t_2 = (0,2,1)t_2t_0t_2t_1(0,1,2)t_0t_2t_1t_0 \]
\[ \implies (0,1,2)t_1t_0t_1t_2t_1t_2 = t_0t_1t_0t_2t_0t_2t_1t_0 \]
\[ \implies Nt_0t_1t_0t_2t_0t_2t_1t_0 = Nt_1t_0t_1t_2t_1t_2 \in [010202] \]
\[ \implies Nt_0t_1t_0t_2t_0t_2t_1t_0N = [010202]. \]

Now,
\[ t_0t_1t_0t_2t_0t_2t_1t_2t_1t_0t_2t_0t_1 = t_0t_1t_0t_2(2,1)t_2t_1t_0t_2t_0t_1 \]
\[ \implies t_0t_1t_0t_2t_0t_2t_1t_2t_1t_0t_2t_0t_1 = (2,1)t_0t_2t_0t_1t_0t_2t_1t_0t_2t_0t_1 \]
\[ \implies t_0t_1t_0t_2t_0t_2t_1t_2t_1t_0t_2t_0t_1 = (2,1)t_0t_2(0,2,1)t_3t_1t_2t_0t_1t_0t_2t_0t_1 \]
\[ \Rightarrow t_{01t0t1t2t1t2t1t2t0t2t0t1} = (2,0)t_{2t1t2t1t2t0t2t1t0t2t0t1} \]
\[ \Rightarrow t_{01t0t1t2t1t2t1t2t0t2t0} = (2,0)(2,1)t_{2t1t2t1t0t2t0t1} \]
\[ \Rightarrow t_{01t0t1t2t1t2t1t2t0t2t0} = (0,1,2)t_{2t1t2t1}(0,2,1)t_{0t1t2t0t1} \]
\[ \Rightarrow t_{01t0t1t2t1t2t1t2t0t2t0t1} = (0,1,2)(0,2,1)t_{1t0t1t0t1t0t1} \]
\[ \Rightarrow t_{01t0t2t0t1t2t1t2t0t2t0} = t_{1t0t2t0t1} \]
\[ \Rightarrow N_{t0t1t0t2t0t1t0} = N_{t0t1t0t2t0t1t0} \in [012120] \]
\[ \Rightarrow N_{t0t1t0t2t0t2t1t2} = N_{01220} \]

Consider the double coset [0101212] and we note:

- \( t_{0t1t0t1t2t1t2t1t2t0t1} = (0,1)t_{0t1t0t1t2t1t2t1t2t0t1} = (0,1)t_{0t1t0t1t2t1t2t1t2t0t1} = (1,2)t_{t0t1t2t0t1t2t0t1t2t0t1} = (1,2)t_{t0t1t2t0t1t2t0t1t2t0t1} = (1,2)t_{t2t1t2t0t1t2t0t1t2t0t1} = (0,2)t_{t0t1t2t0t1t2t0t1t2t0t1} = (0,2)t_{t0t1t2t0t1t2t0t1t2t0t1} = (0,2)t_{t0t1t2t0t1t2t0t1t2t0t1} = (0,2)t_{t0t1t2t0t1t2t0t1t2t0t1} = (0,1)t_{t2t1t2t0t1t2t0t1t2t0t1} = t_{2t1t2t0t1t2t0t1t2t0t1} \]

Thus, \( N_{t0t1t0t1t2t1t2t1t2t0t1} = N_{t0t1t0t1t2t1t2t1t2t0t1} \). Similarly, \( N_{t0t2t0t2t1t2t1t2t0t1} = N_{t0t1t0t2t1t2t1t2t0t1t2t0t1} \). Therefore, \([0101212]\) has only two distinct single cosets. Now we must consider the double cosets \( N_{t0t1t0t1t2t1t2t0t2t0t1} \), \( N_{t0t1t0t1t2t1t2t0t2t0t1} \), and \( N_{t0t1t0t1t2t1t2t0t2t0t1} \). However, \( N_{t0t1t0t1t2t1t2t0t2t0t1} = N_{t0t1t0t1t2t1t2t0t2t0t1} \), \( N_{t0t1t0t1t2t1t2t0t2t0t1} \), and \( N_{t0t1t0t1t2t1t2t0t2t0t1} \). Also, 

\[ t_{0t1t0t1t2t1t2t0t2t0t1t2t1} = t_{0t1t0t1t2t1t2t0t2t0t1t2t1} \]
\[ \Rightarrow t_{0t1t0t1t2t1t2t0t2t0t1t2t1} = (2,0)t_{t2t1t2t1t0t2t0t1t2t0t1t2t1} \]
\[ t_{0t1t0t1t2t1t2t0t2t0t1t2t1} = (2,0)t_{t2t1t2t1t0t2t0t1t2t0t1t2t1} \]
\[ \Rightarrow t_{0t1t0t1t2t1t2t0t2t0t1t2t1} = (2,0)(0,1,2)t_{0t2t0t2t0t1t2t0t1t2t1} \]
\[ \Rightarrow t_{0t1t0t1t2t1t2t0t2t0t1t2t1} = (2,0)(0,1,2)t_{0t2t0t2t0t1t2t0t1t2t1} \]
\[ t_0 t_0 t_0 t_0 t_1 t_2 t_1 t_2 t_0 t_2 t_0 t_2 t_1 t_2 t_1 = (0, 2, 1) t_0 t_2 t_0 t_1 t_2 t_0 t_0 t_2 t_1 \]
\[ t_0 t_0 t_0 t_0 t_1 t_1 t_2 t_0 t_2 t_1 t_2 t_1 t_2 t_1 = (0, 2, 1) t_0 t_1 t_1 t_2 t_0 t_1 t_1 t_2 t_1 \]
\[ t_0 t_0 t_0 t_0 t_1 t_2 t_1 t_2 t_0 t_2 t_0 t_2 t_0 t_2 t_1 t_2 t_1 t_1 = (0, 2, 1) t_1 t_0 t_0 t_0 t_2 t_1 \]
\[ t_0 t_0 t_1 t_1 t_2 t_1 t_2 t_0 t_0 t_2 t_1 t_2 t_1 t_1 = e \]
\[ t_0 t_0 t_1 t_1 t_2 t_1 t_2 t_0 t_0 t_2 t_1 t_2 t_1 t_1 = t_1 t_2 t_0 t_2 t_1 \]
\[ N t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 = N t_1 t_2 t_1 t_2 t_0 t_2 \in [010121] \]
\[ N t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 N = [010121]. \]

And,
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_1 t_0 t_0 t_0 t_2 t_0 t_2 = t_0 t_1 t_0 (1, 2) t_1 t_2 t_1 t_2 t_1 t_0 t_0 t_0 t_0 t_2 t_0 t_2 \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_1 t_0 t_2 t_0 t_2 = (1, 2) t_0 t_2 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 t_0 t_2 t_0 t_2 \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_1 t_0 t_2 t_0 t_2 = (1, 2) t_0 t_2 t_0 t_1 (0, 1, 2) t_1 t_0 t_2 t_0 t_2 t_0 t_2 t_0 t_2 \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_1 t_0 t_0 t_2 t_0 t_2 = (0, 1) t_1 t_0 t_1 t_2 t_1 t_0 t_0 t_2 t_0 t_2 t_0 t_2 t_0 t_2 \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_1 t_0 t_2 t_0 t_2 = (0, 1) t_1 t_0 t_1 t_2 t_1 t_0 t_2 t_0 t_1 t_2 t_0 t_2 t_0 t_2 \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_1 t_0 t_0 t_2 t_0 t_2 = (0, 1, 2) t_1 t_2 t_1 t_0 t_2 t_1 t_2 t_0 t_2 t_0 t_2 \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_1 t_0 t_2 t_0 t_2 = (0, 1) t_1 t_2 t_1 t_0 t_2 t_1 t_2 t_0 t_2 t_0 t_2 t_0 t_2 \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_1 t_0 t_0 t_2 t_0 t_2 = (0, 1, 2) t_1 t_2 t_1 t_0 t_2 t_1 t_2 t_0 t_2 t_0 t_2 \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_1 t_0 t_0 t_2 t_0 t_2 = (0, 1, 2) t_1 t_2 t_1 t_0 t_2 t_1 t_2 t_0 t_2 t_0 t_2 \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_1 t_0 t_0 t_2 t_0 t_2 = (0, 1) t_1 t_0 t_1 t_2 t_1 t_0 t_0 t_2 t_0 t_2 t_0 t_2 \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_1 t_0 t_0 t_2 t_0 t_2 = (0, 1, 2) t_1 t_2 t_1 t_0 t_2 t_1 t_2 t_0 t_2 t_0 t_2 \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_1 t_0 t_0 t_2 t_0 t_2 = t_0 t_1 t_0 t_0 t_1 t_2 t_2 t_0 \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_0 t_2 t_0 t_2 = t_0 t_1 t_0 t_0 t_1 t_2 t_2 t_0 \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_1 t_0 t_2 t_0 t_2 = e \]
\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 t_0 t_1 t_0 t_2 t_0 t_2 = t_2 t_0 t_2 t_0 t_1 t_0 \]
\[ N t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 = N t_2 t_0 t_2 t_1 t_0 \in [010121] \]
\[ N t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_1 = [010121]. \]

Therefore, we have found all double cosets, and the result of the double coset enumeration is shown in the Cayley diagram in figure 2.1.
Figure 2.1: Cayley Diagram of $L_2(8)$ over $S_3$
2.3 Action of the Symmetric Generators and the Generators of $S_3$ on the Right Cosets of $G$ Over $S_3$

We found that $|G| \leq 6 \times 84 = 504$. We now show that $|G| = 504$.

Let $X$ denote the set of all (84) distinct right cosets of $N$ in $G$ that we have found in the previous section and we consider $G$ as a subgroup of the symmetric group on 84 symbols, where the 84 symbols are the elements of $X$. The following is the labeling of the 84 right cosets:

<p>| | | |</p>
<table>
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<td>$Nt_0t_0$</td>
</tr>
<tr>
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<td>$Nt_0t_1$</td>
</tr>
<tr>
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<td>$Nt_2$</td>
<td>$Nt_0t_2$</td>
</tr>
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The action of N and the action of t's is as follows: $G \times X \rightarrow X$ defined by $(g,x) \mapsto gx$, and $\hat{\alpha} : G \xrightarrow{\text{hom}} S_X$.

$x = (0,1,2)$

$\alpha(x) = (1)(2,3,4)(5,8,9)(6,7,10)(11,14,15)(12,13,16)(17,20,21)(18,19,22)(23,26,27)$
$(47,50,51)(48,49,52)(53,55,57)(54,55,58)(59,62,63)(60,61,64)(65,68,69)$
$(66,67,70)(71,74,75)(72,73,76)(77,79,78)(80)(81)(82,83,84)$

$y = (1,2)$

$\alpha(y) = (1)(2,3,4)(5,6)(7,9)(8,10)(11,12)(13,15)(14,16)(17,18)(19,21)(20,22)(23,24)$
$(68,70)(71,72)(73,75)(74,76)(77,78)(79)(80,81)(82,84)(83)$

$(59,77)(60,78)(61)(63)(68,80)(70,81)(73,82)(75,84)(79,83)$

It is readily verified that $|\langle x, y, t_0 \rangle| = 504$. Note that $t_0$ has exactly three conjugates under conjugation by $\alpha(x), \alpha(y)$ and $\alpha(x), \alpha(y)$ acts as the group $S_3$ on $T = \{t_0, t_1, t_2\}$ by conjugation:

$\alpha(x) = (\alpha(t_0), \alpha(t_1), \alpha(t_2))$
\[ \hat{a}(y) = (\hat{a}(t_1), \hat{a}(t_2)). \]

\[ t_1 = [\hat{a}(t_0)]^{\hat{a}(x)} = (1,3)(4,10)(2,5)(8,14)(7,13)(9,21)(6,18) (16,28)(15,33)(11,23)(12,30)(20,19) \]
\[ (63,78)(64,79)(65,80)(67,81)(72,84)(74,83)(77,82) \]

\[ t_2 = \hat{a}(t_2) = [\hat{a}(t_1)]^{\hat{a}(x)} = (1,4)(2,6)(3,8)(9,15)(10,16)(5,17)(7,19)(12,24) (11,29)(14,26)(13,31)(21,22) \]
\[ (63,78)(64,79)(65,80)(67,81)(72,84)(74,83)(77,82) \]

Now we need to check the additional relations. First we need to check if \( \hat{a}(0,1,2) = \hat{a}(x) = \)
\[ t_0t_1t_2t_0t_1t_2t_0 = (1)(2,3,4)(5,8,9)(6,7,10)(11,14,15)(12,13,16)(17,20,21) \]
\[ (18,19,22)(23,26,27)(24,25,28)(29,32,33)(30,31,34)(35,38,39)(36,37,40) \]
\[ (41,44,45)(42,43,46)(47,50,51)(48,49,52)(53,56,57)(54,55,58)(59,62,63) \]
\[ (60,61,64)(65,68,69)(66,67,70)(71,74,75)(72,73,76)(77,79,78)(80)(81) \]
\[ (82,83,84) \]

Next we check if \( \hat{a}(0,1) = t_0t_1t_0t_1t_0t_1t_0. \)
\[ \hat{a}(0,1) = (1)(2,3)(4)(5,7)(6,8)(9,10)(11,13)(12,14)(15,16)(17,19) \]
\[ (63,64)(65,67)(66,68)(69,70)(71,73)(72,74)(75,76)(77)(78,79)(80,81)(82)(83,84) \]
\[ =t_0t_1t_0t_1t_0t_1t_0t_1t_0. \]

We now have that \( G/Ker\hat{a} \cong \langle x,y,t_0 \rangle \implies |G| \geq 504. \) But earlier we found that \( |G| \leq 504, \) so we conclude that \( |G| = 504. \) In fact, in this case it is easy to show that \( G \cong L_2(8). \)
2.4 Proof of the Isomorphism between $G$ and $L_2(8)$

We start by constructing a homomorphism $\Phi$ from the progenitor $2^{*3} : S_3$ to $L_2(8)$ by defining

$\Phi(x) = \frac{6x+7}{2x+7} = (1, 2, 0)(3, 6, \infty)(4, 5, 7)$ and $\Phi(y) = \frac{y}{4x+1} = (1, 2)(3, 5)(4, 6)(7, \infty)$, since the orders of $\Phi(x)$, $\Phi(y)$, and $\Phi(x)\Phi(y)$ are 3, 2 and 2 respectively, $N = \langle \Phi(x), \Phi(y) \rangle \cong S_3$.

We now let $\Phi(t_0) = \frac{y}{6x+1} = (1, 4)(2, 6)(3, \infty)(5, 7)$. It is readily verified that $L_2(8) \cong \langle \Phi(x), \Phi(y), \Phi(t_0) \rangle$. Next we show that $\Phi$ preserves the operation of $2^{*3} : S_3$.

We find that $|\Phi(t_0)| = 3$ and $\Phi(t_0) = \Phi(t) = (1, 4)(2, 6)(3, \infty)(5, 7)$, $\Phi(t_1) = \Phi(t_0^3) = (2, 5)(0, \infty)(6, 3)(7, 4)$, $\Phi(t_2) = \Phi(t_0^2) = (0, 7)(1, 3)(\infty, 6)(4, 5)$, and that $N$ permutes the three images of $t_0$ by conjugation as the group $S_3$ does. $\Phi(x) : \langle \Phi(t_0), \Phi(t_1), \Phi(t_2) \rangle$ and $\Phi(y) : \langle \Phi(t_0), \Phi(t_1), \Phi(t_2) \rangle$. Thus, $\Phi(2^{*3} : S_3) \cong L_2(8)$.

Now the additional relation given by $[xt_0]^7 = e \iff t_0t_2t_1t_0t_2t_1t_0 = (0, 2, 1)$ is satisfied in $L_2(8)$ because $\Phi(t_0)\Phi(t_2)\Phi(t_1)\Phi(t_0)\Phi(t_2)\Phi(t_1)\Phi(t_0) = (1, 0, 2)(3, \infty, 6)(4, 7, 5)$ acts as $\langle \Phi(t_1), \Phi(t_0), \Phi(t_2) \rangle$ by conjugation on the images of the three symmetric generators. This shows that $L_2(8)$ is an image of $G$. Thus, $|G| \geq |L_2(8)|$; but $|G| \leq 504 = |L_2(8)|$, and so the equality holds and $G \cong L_2(8)$. 
Chapter 3

\( PGL_2(9) \) as a Homomorphic Image of the Progenitor \( 2^*3 : S_3 \)

3.1 Introduction

Consider the progenitor \( 2^*3 : S_3 \). We factor this by \([(0,1,2)t_0]^8 = e \) and \([(0,1)t_0t_2]^5 = e \). The progenitor \( 2^*3 : S_3 \) is given by
\[ x, y, t | x^3 = y^2 = (xy)^2 = e = t^2 = [t, y], \]
where \( x \sim (0,1,2) \), \( y \sim (1,2) \), and \( t \sim t_0 \).

The first relation \([\pi t_0]^8 = e \), where \( \pi = (0,1,2) \), implies
\[ \pi t_0 \pi t_0 \pi t_0 \pi t_0 \pi t_0 \pi t_0 \pi t_0 \pi t_0 = e \]
\[ \implies \pi^8t_0^7t_0^6t_0^5t_0^4t_0^3t_0^2t_0^t_0 = e \]
\[ \implies \pi t_1t_0t_1t_0t_2t_1t_0 = e \]
\[ \implies (0,2,1)t_1t_0t_2t_1 = t_0t_1t_2t_0 \quad \text{(3.1)} \]

The second relation \([\pi t_0 t_2]^5 = e \), where \( \pi = (0,1) \), implies
\[ \pi t_0 t_2 \pi t_0 t_2 \pi t_0 t_2 \pi t_0 t_2 \pi t_0 t_2 = e \]
\[ \implies \pi^5(t_0 t_2)\pi^4(t_0 t_2)\pi^3(t_0 t_2)\pi^2(t_0 t_2)\pi t_0 t_2 = e \]
\[ \implies \pi t_0 t_2 t_1 t_0 t_2 t_1 t_2 t_2 = e \]
\[ \implies (0,1)t_0 t_2 t_1 t_0 t_2 = t_0 t_2 t_1 t_2 \quad \text{(3.2)} \]

Let \( G \) denote the progenitor factored by the above relations, that is
\[ G \cong \frac{2^*3:S_3}{[(0,1,2)t_0]^8 = e = [(0,1)t_0t_2]^5} \]
and we will show that \( G \) is isomorphic to \( PGL_2(9) \).
3.2 Manual Double Coset Enumeration of $G$ Over $S_3$

Now we will perform manual double coset enumeration of $G$ over $N \cong S_3$ to find the index of $N$ in $G$. In fact we will show that $|G|$ is at most $6 \times 120 = 720$.

We start with the double coset $NeN$ denoted by $[\ast]$. $[\ast]$ consists of the single coset $N$. Now, $N$ is transitive on $T = \{0,1,2\}$, so we consider the double coset $Nt_0N$ denoted by $[0]$. Since $N^{(0)} \geq N^0 = \{e, (1,2)\}$, the number of single cosets in $[0]$ is equal to $|N|/|N^{(0)}| \leq 6/2 = 3$.

Since $N^{(0)}$ has orbits $\{0\}$ and $\{1,2\}$ on $T$, we must consider the double cosets $Nt_0t_0N$ and $Nt_0t_1N$. However, $Nt_0t_0N = NcN = [\ast]$, but $Nt_0t_1N$ denoted by $[01]$ is a new double coset. The number of single cosets in $[01]$ equals to $|N|/|N^{(01)}| = 6/1 = 6$ since $N^{(01)} = \{e\}$.

The orbits of $N^{(01)}$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_0N$, $Nt_0t_1t_1N$, and $Nt_0t_1t_2N$. However, $Nt_0t_1t_1N = Nt_0N = [0]$. But $Nt_0t_1t_0N$ and $Nt_0t_1t_2N$ are new. Now $N^{(010)} = N^{(012)} = \{e\}$, so the number of single cosets in $[010]$ and $[012]$ equals to $|N|/|N^{(010)}| = 6/1 = 6$.

The orbits of $N^{(010)}$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_0t_0N$, $Nt_0t_1t_0t_1N$, and $Nt_0t_1t_0t_2N$. However, $Nt_0t_1t_0t_0N = Nt_0t_1N = [01]$. But $Nt_0t_1t_0t_1N$ and $Nt_0t_1t_0t_2N$ are new double cosets.

The orbits of $N^{(012)}$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_2t_0N$, $Nt_0t_1t_2t_1N$, and $Nt_0t_1t_2t_2N$. However, $Nt_0t_1t_2t_2N = Nt_0t_1N = [01]$, but $Nt_0t_1t_2t_0N = [0120]$ and $Nt_0t_1t_2t_1N = [0121]$ are new double cosets.

The number of single cosets in $Nt_0t_1t_0t_1N$ is $|N|/|N^{(0101)}| = 6/1 = 6$, since $N^{(0101)} = N^{(01)} = \{e\}$. The orbits of $N^{(0101)}$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_0t_1t_0N$, $Nt_0t_1t_0t_1t_1N$, and $Nt_0t_1t_0t_1t_2N$. However, $Nt_0t_1t_0t_1t_1N = Nt_0t_1t_0N = [010]$, but $Nt_0t_1t_0t_1N = [01010]$ and $Nt_0t_1t_0t_2N = [01012]$ are new.

The number of single cosets in $Nt_0t_1t_0t_2N$ is $|N|/|N^{(0102)}| = 6/1 = 6$, since $N^{(0102)} = N^{(012)} = \{e\}$. The orbits of $N^{(0102)}$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_0t_2t_0N$, $Nt_0t_1t_0t_2t_1N$, and $Nt_0t_1t_0t_2t_2N$. However, $Nt_0t_1t_0t_2t_2N = Nt_0t_1t_0N = [010]$, but $Nt_0t_1t_0t_2t_0N = [01020]$ and $Nt_0t_1t_0t_2t_1N = [01021]$ are new.
The number of single cosets in $[0120]$ is 3 since we have the relation:

$$(0,2,1)t_1t_0t_2t_1 = t_0t_1t_2t_0$$

$$\implies N(0,2,1)t_1t_0t_2t_1 = Nt_0t_1t_2t_0$$

$$\implies Nt_1t_0t_2t_1 = Nt_0t_1t_2t_0.$$

Also, from the same relation (3.1) and by conjugation with $(0,2)$ and $(1,0,2)$, we get

$Nt_1t_2t_0t_1 = Nt_2t_1t_0t_2$ and $Nt_0t_2t_1t_0 = Nt_2t_0t_1t_2$ respectively. Thus, there are only three distinct single cosets in $[0120]$. $N(0120) = \{e, (0,1)\}$, and the orbits of $N(0120)$ are $\{0,1\}$ and $\{2\}$. We must consider the double cosets $Nt_0t_1t_2t_0t_0N$ and $Nt_0t_1t_2t_0t_2N$. However, $Nt_0t_1t_2t_0t_0N = Nt_0t_1t_2N = [012]$ and the only new double coset is $Nt_0t_1t_2t_0t_2N = [01202]$.

The number of single cosets in $Nt_0t_1t_2t_1N$ is $|N|/|N(0121)| = 6/1 = 6$, since $N(0121) = N(012) = \{e\}$. The orbits of $N(0121)$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_2t_1t_0N$, $Nt_0t_1t_2t_1t_1N$, and $Nt_0t_1t_2t_1t_2N$. However, $Nt_0t_1t_2t_1t_1N = Nt_0t_1t_2N = [012]$. From (3.2) conjugated by $(1,2)$ we get

$$(0,2)t_0t_1t_2t_1t_0 = t_1t_0t_1t_2t_1$$

$$\implies N(0,2)t_0t_1t_2t_1t_0 = Nt_1t_0t_1t_2t_1$$

$$\implies Nt_0t_1t_2t_1t_0 = Nt_1t_0t_1t_2t_1 \in [01202].$$

So, $Nt_0t_1t_2t_1t_0 = [01210] = [01202]$ and the only new double coset is $Nt_0t_1t_2t_1t_2N = [01212]$.

The number of single cosets in $Nt_0t_1t_0t_0t_0N$ is $|N|/|N(0101)| = 6/2 = 3$, since $N(0101) = \{e, (0,1)\}$. The orbits of $N(0101)$ on $T$ are $\{0,1\}$ and $\{2\}$. We must consider the double cosets $Nt_0t_1t_0t_1t_0N$, $Nt_0t_1t_0t_1t_1N$, and $Nt_0t_1t_0t_1t_0t_2N$. However, $Nt_0t_1t_0t_1t_0t_0N = Nt_0t_1t_0t_1N = [0101]$ and the only new double coset is $Nt_0t_1t_0t_1t_0t_2N = [010102]$.

The number of single cosets in $Nt_0t_1t_0t_1t_2N$ is $|N|/|N(01012)| = 6/1 = 6$, since $N(01012) = N(012) = \{e\}$. The orbits of $N(01012)$ on $T$ are $\{0\}$, $\{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_0t_1t_2t_0N$, $Nt_0t_1t_0t_1t_2t_1N$, and $Nt_0t_1t_0t_1t_2t_2N$. However, $Nt_0t_1t_0t_1t_2t_2N = Nt_0t_1t_0t_1N = [0101]$ but $Nt_0t_1t_0t_1t_2t_0N = [010120]$ and $Nt_0t_1t_0t_1t_2t_1N = [010121]$ are new.
The number of single cosets in \( N_{0102}t_0t_1t_0t_2t_0N \) is \(|N|/|N^{(1020)}| = 6/1 = 6\), since \( N^{(1020)} = N^{(012)} = \{e\} \). The orbits of \( N^{(1020)} \) on \( T \) are \{0\}, \{1\}, and \{2\}. We must consider the double cosets \( N_{0102}t_0t_1t_0t_2t_0N \), \( N_{0102}t_1t_0t_2t_0t_1N \), and \( N_{0102}t_1t_0t_2t_0t_2N \). However, \( N_{0102}t_0t_1t_0t_2t_0N = N_{0102}t_0t_2t_0N \). From the relation (3.2) conjugated by \( (0,1,2) \) we have:

\[
(1,2)t_1t_0t_2t_0t_1 = t_0t_1t_0t_2t_0 \Rightarrow (1,2)t_1t_0t_2t_0 = t_0t_1t_0t_2t_0t_1
\]

\[
\Rightarrow N_{0102}t_1t_0t_2t_0 t_1 = N_{0102}t_0t_0t_2t_0 \in [0121]
\]

\[
\Rightarrow N_{0102}t_1t_0t_2t_0 t_1N = [01201] = [0121].
\]

Also,

\[
(0,2)t_0t_2t_0t_2t_1t_0t_2t_0t_2t_0t_1t_0 = (0,2)t_0t_2(0,1,2)t_2t_0t_1t_0t_2t_0t_2t_0t_1t_0
\]

\[
\Rightarrow (0,2)t_0t_2t_0t_1t_0t_2t_0t_1t_0 = (1,2)t_1t_2t_0t_1t_0t_2t_0t_1t_0
\]

\[
\Rightarrow (0,2)t_0t_2t_0t_1t_0t_2t_0t_1t_0 = t_0t_1t_0t_2t_0t_2t_0t_1t_0
\]

\[
\Rightarrow (0,2)t_0t_2t_0t_1t_0t_2t_0t_1t_0 = e
\]

\[
\Rightarrow (0,2)t_0t_2t_0t_1t_0t_2t_0t_2t_0t_1t_0 = t_0t_1t_0t_2t_0t_2
\]

\[
\Rightarrow N_{0102}t_0t_2t_0t_2t_0t_1N = [01020] \in [010212] = [01201].
\]

The number of single cosets in \( N_{0102}t_0t_1t_0t_2t_1N \) is \(|N|/|N^{(0120)}| = 6/2 = 3\), since \( N^{(0120)} = \{e, (0,1)\} \). The orbits of \( N^{(0120)} \) on \( T \) are \{0,1\} and \{2\}. We must consider the double cosets \( N_{0102}t_0t_1t_0t_2t_1t_1N \) and \( N_{0102}t_0t_1t_0t_2t_1t_2N \). However, \( N_{0102}t_0t_1t_0t_2t_1t_1N = N_{0102}t_0t_1t_0t_2t_2N \), but \( N_{0102}t_0t_1t_0t_2t_1t_2N \) is a new double coset.

The number of single cosets in \( N_{0102}t_0t_1t_0t_2t_2N \) is \(|N|/|N^{(0122)}| = 6/2 = 3\), since \( N^{(0122)} = \{e, (0,1)\} \). The orbits of \( N^{(0122)} \) on \( T \) are \{0,1\} and \{2\}. We must consider the double cosets \( N_{0102}t_0t_1t_2t_0t_2t_0N \) and \( N_{0102}t_0t_1t_2t_0t_2t_2N \). However, \( N_{0102}t_0t_1t_2t_0t_2t_2N \) is a new double coset.

\[
(0,2)t_0t_2t_0t_2t_1t_0t_2t_0t_2t_0t_1t_0 = (0,2)t_0t_2(0,1,2)t_2t_0t_1t_0t_2t_0t_2t_0t_1t_0
\]

\[
\Rightarrow (0,2)t_0t_2t_0t_1t_0t_2t_0t_2t_0t_1t_0 = (0,2,1)t_1t_2t_0t_2t_1t_0t_2t_1t_0
\]

\[
\Rightarrow (0,2)t_0t_2t_0t_1t_0t_2t_0t_2t_0t_1t_0 = (0,2,1)t_1t_0t_2t_1t_0t_2t_1t_0
\]
\[ \rightarrow (0,2)t_0t_2t_0t_2t_1t_2t_0t_0t_2t_1t_0 = (0,2,1)t_1t_0t_2t_1(0,1,2)t_2t_0t_1t_2 \]
\[ \rightarrow (0,2)t_0t_2t_0t_2t_1t_2t_0t_2t_1t_0 = t_2t_1t_0t_2t_0t_1t_2 \]
\[ \rightarrow (0,2)t_0t_2t_0t_2t_1t_2t_0t_2t_1t_0 = e \]
\[ \rightarrow (0,2)t_0t_2t_0t_2t_1t_2 = t_0t_1t_0t_2t_0 \]
\[ \rightarrow N_{t_0t_1t_2t_0t_2} = N_{t_0t_2t_0t_2t_1} \in [010121] \]
\[ \rightarrow N_{t_0t_1t_2t_0t_2}N = [012020] = [010121]. \]

The number of single cosets in \( N_{t_0t_1t_2t_1t_2N} \) is \( |N|/|N^{(01212)}| = 6/1 = 6 \), since \( N^{(01212)} = N^{(012)} = \{e\} \). The orbits of \( N^{(01212)} \) on \( T \) are \( \{0\}, \{1\}, \) and \( \{2\} \). We must consider the double cosets \( N_{t_0t_1t_2t_1t_2N}, N_{t_0t_1t_2t_1t_2N}, \) and \( N_{t_0t_1t_2t_1t_2N} \). However, \( N_{t_0t_1t_2t_1t_2N} = N_{t_0t_1t_2t_1t_2N} = [0121]. \) Also we have:

\[ (1,2)t_1t_2t_1t_2t_1t_0t_2t_1t_2t_1t_0 = (1,2)t_1t_2(0,2)t_2t_1t_0t_2t_1t_2t_1t_0 \]
\[ \rightarrow (1,2)t_1t_2t_1t_2t_1t_0t_2t_1t_2t_1t_0 = (0,2,1)t_1t_0t_2t_1t_0t_2t_1t_0 \]
\[ \rightarrow (1,2)t_1t_2t_1t_2t_1t_0t_2t_1t_2t_1t_0 = (0,2,1)(0,1,2)t_0t_1t_2t_0t_2t_1t_0 \]
\[ \rightarrow (1,2)t_1t_2t_1t_2t_1t_0t_2t_1t_2t_1t_0 = e \]
\[ \rightarrow (1,2)t_1t_2t_1t_2t_1t_0t_2t_1t_2t_1t_0 = t_0t_1t_2t_1t_2t_1 \]
\[ \rightarrow N_{t_0t_1t_2t_1t_2} = N_{t_1t_2t_1t_2N} \in [010121] \]
\[ \rightarrow N_{t_0t_1t_2t_1t_2}N = [012121] = [010121]. \]

Thus, the only new double coset is \( N_{t_0t_1t_2t_1t_2N} = [012120]. \)

The number of single cosets in \( N_{t_0t_1t_0t_1t_0t_2N} \) is \( |N|/|N^{(010102)}| = 6/2 = 6 \), since \( N^{(010102)} = \{e, (0,1)\} \). The orbits of \( N^{(010102)} \) on \( T \) are \( \{0,1\} \) and \( \{2\} \). We must consider the double cosets \( N_{t_0t_1t_0t_1t_0t_2N}, N_{t_0t_1t_0t_1t_0t_2N}, \) and \( N_{t_0t_1t_0t_1t_0t_2N} \). However, \( N_{t_0t_1t_0t_1t_0t_2N} = N_{t_0t_1t_0t_1t_0t_2N} = [01010]. \) Also we have:

\[ (0,1)t_2t_0t_1t_0t_1t_2t_0t_1t_0t_1t_0 = (0,1)t_2t_0t_1t_0t_1(1,2)t_2t_1t_0t_2t_1t_0 \]
\[ \rightarrow (0,1)t_2t_0t_1t_0t_1t_2t_0t_1t_0t_1t_0 = (0,2,1)t_1t_0t_2t_0t_2t_1t_0t_2t_1t_0 \]
\[ \rightarrow (0,1)t_2t_0t_1t_0t_1t_2t_0t_1t_0t_1t_0 = (0,2,1)t_1t_0t_2t_1t_0t_2t_1t_0 \]
\[ \rightarrow (0,1)t_2t_0t_1t_0t_1t_2t_0t_1t_0t_1t_0 = (0,2,1)t_1(0,1,2)t_2t_0t_1t_2t_1t_0 \]
The number of single cosets in $Nt_0t_1t_0t_0t_2t_0N$ is $|N|/|N^{(010120)}| = 6/1 = 6$, since $N^{(010120)} = N^{(012)} = \{e\}$. The orbits of $N^{(010120)}$ on $T$ are $\{0\}, \{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_0t_0t_2t_0N$, $Nt_0t_0t_0t_1t_2t_1N$, and $Nt_0t_1t_0t_0t_2t_0N$. However, $Nt_0t_1t_0t_0t_2t_0N = Nt_0t_0t_1t_2t_0N = [01012]$. We also have:

$$\Rightarrow (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 1)t_0t_0t_1t_2t_0t_0t_1t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 2)t_1t_0t_2t_1t_0t_2t_0t_1t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 2)t_1t_0t_1t_0t_1t_0t_1t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 2)t_1t_0t_1t_0t_1t_0t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 2)t_1t_0t_0t_1t_0t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 1)t_0t_2t_0t_1t_0t_0t_0t_1t_0t_0$$

Thus, the only new double coset is $Nt_0t_1t_0t_1t_2t_0t_2N = [0101202]$. The number of single cosets in $Nt_0t_1t_0t_1t_2t_1N$ is $|N|/|N^{(010121)}| = 6/1 = 6$, since $N^{(010121)} = N^{(012)} = \{e\}$. The orbits of $N^{(010121)}$ on $T$ are $\{0\}, \{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_0t_1t_2t_1N$, $Nt_0t_0t_0t_1t_2t_1N$, and $Nt_0t_1t_0t_1t_2t_1N$. However, $Nt_0t_1t_0t_1t_2t_1N = Nt_0t_0t_1t_2t_1N = [01012]$. Also,

$$\Rightarrow (0, 1)t_0t_2t_1t_0t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 1)t_0t_2t_1t_0t_1t_2t_1t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_1t_0t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 1)t_0t_2t_1t_0t_1t_2t_1t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_1t_0t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 1)t_0t_2t_1t_0t_1t_2t_1t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_1t_0t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 1)t_0t_2t_1t_0t_1t_2t_1t_0t_0$$
$$\Rightarrow (0, 1)t_0t_2t_1t_0t_0t_1t_0t_0t_0t_1t_0t_0 = (0, 1)t_0t_2t_1t_0t_1t_2t_1t_0t_0$$

Thus, the only new double coset is $Nt_0t_1t_0t_1t_2t_0t_2N = [0101202]$. The number of single cosets in $Nt_0t_1t_0t_1t_2t_1N$ is $|N|/|N^{(010121)}| = 6/1 = 6$, since $N^{(010121)} = N^{(012)} = \{e\}$. The orbits of $N^{(010121)}$ on $T$ are $\{0\}, \{1\}$, and $\{2\}$. We must consider the double cosets $Nt_0t_1t_0t_1t_2t_1N$, $Nt_0t_0t_0t_1t_2t_1N$, and $Nt_0t_1t_0t_1t_2t_1N$. However, $Nt_0t_1t_0t_1t_2t_1N = Nt_0t_0t_1t_2t_1N = [01012]$. Also,
Thus, the only new double coset is \( N_{t_0t_1t_0t_1t_2t_1t_2N} = [0101212] \).

The number of single cosets in \( N_{t_0t_1t_0t_1t_2t_1t_2N} \) is \( |N|/|N^{(010212)}| = 6/2 = 3 \), since \( N^{(010212)} = \{e, (0,1)\} \). The orbits of \( N^{(010212)} \) on \( T \) are \{0, 1\} and \{2\}. We can consider the double cosets \( N_{t_0t_1t_0t_2t_2t_0N} \) and \( N_{t_0t_1t_0t_2t_2t_2N} \). However, \( N_{t_0t_1t_0t_2t_2t_2N} = N_{t_0t_1t_0t_2t_1t_2N} = [010211] \), but \( N_{t_0t_1t_0t_2t_2t_0N} = [0102120] \) is new.

The number of single cosets in \( [0121201] \) is \( |N|/|N^{(012120)}| = 6/1 = 6 \), since \( N^{(012120)} = \{e\} \). The orbits of \( N^{(012120)} \) on \( T \) are \{0\}, \{1\}, and \{2\}. We can consider the double cosets \( N_{t_0t_1t_2t_2t_0t_0N} \), \( N_{t_0t_1t_2t_2t_0t_1N} \), and \( N_{t_0t_1t_2t_2t_0t_2N} \). However, \( N_{t_0t_1t_2t_2t_0t_0N} = N_{t_0t_1t_2t_1t_2N} = [0121201] \). We also have:

\[
(1,2)t_0t_1t_0t_1t_2t_1t_0t_2t_1t_2t_2t_1t_2t_1t_0 = (1,2)t_0t_1t_0t_1t_2t_1(0,1,2)t_1t_2t_0t_1t_1t_2t_1t_0
\]

\[
\Rightarrow (1,2)t_0t_1t_0t_1t_2t_1t_0t_2t_1t_2t_1t_0 = (0,1)t_1t_2t_1t_2t_0t_1t_2t_1t_0
\]

\[
(1,2)t_0t_1t_0t_2t_1t_2t_1t_2t_1t_0t_2t_1t_0 = (0,1)t_1t_2t_1t_2t_0t_1t_2t_2t_1t_0
\]

\[
\Rightarrow (1,2)t_0t_1t_0t_1t_2t_1t_2t_1t_0t_2t_1t_0 = e
\]

\[
\Rightarrow N_{t_0t_1t_2t_1t_2t_0t_1} = N_{t_0t_1t_0t_1t_2t_1t_2} \in [0101212]
\]

\[
\Rightarrow N_{t_0t_1t_2t_1t_2t_0t_1} = [0121201] = [0101212].
\]

And

\[
(0,1)t_1t_0t_1t_2t_0t_1t_2t_0t_1t_2t_1t_0 = (0,1)t_1t_0t_1t_2t_0t_2(0,1)t_1t_2t_0t_2t_1t_0
\]

\[
\Rightarrow (0,1)t_1t_0t_1t_2t_0t_1t_2t_0t_1t_2t_1t_0 = t_0t_1t_0t_2t_1t_2t_1t_1t_2t_0t_2t_1t_0
\]
\[ (0,1) t_1 t_0 t_1 t_2 t_0 t_2 t_1 t_2 t_0 t_1 t_0 = e \]

\[ (0,1) t_1 t_0 t_1 t_2 t_0 t_2 = t_0 t_1 t_2 t_1 t_2 t_0 \]

\[ N t_0 t_1 t_2 t_0 t_2 = N t_1 t_0 t_1 t_2 t_0 t_2 \in [0102120] \]

\[ N t_0 t_1 t_2 t_0 t_2 N = [0121202] = [0120212]. \]

The number of single cosets in [0101202] is \( |N|/|N^{(0101202)}| = 6/1 = 6 \). The orbits of \( N^{(0101202)} \) on \( T \) are \( \{0\}, \{1\}, \) and \( \{2\} \). We must consider the double cosets \( N t_0 t_1 t_0 t_1 t_2 t_0 t_0 N, N t_0 t_1 t_0 t_1 t_2 t_0 t_2 N \), and \( N t_0 t_1 t_0 t_1 t_2 t_0 t_2 N \). However, \( N t_0 t_1 t_0 t_1 t_2 t_0 t_2 \) \( N t_0 t_1 t_0 t_1 t_2 t_0 t_2 N = [0101202] \), but \( N t_0 t_1 t_0 t_1 t_2 t_0 t_0 N = [0101202] \) and \( N t_0 t_1 t_0 t_1 t_2 t_0 t_1 N = [0101202] \) are new double cosets.

The number of single cosets in [0101212] is \( |N|/|N^{(0101212)}| = 6/1 = 6 \). The orbits of \( N^{(0101212)} \) on \( T \) are \( \{0\}, \{1\}, \) and \( \{2\} \). We must consider the double cosets \( N t_0 t_1 t_0 t_1 t_2 t_0 t_0 N, N t_0 t_1 t_0 t_1 t_2 t_0 t_1 N \), and \( N t_0 t_1 t_0 t_1 t_2 t_0 t_2 N \). However, \( N t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 = N t_0 t_1 t_0 t_1 t_2 t_0 \) \( = [010121] \) Also we have:

\[ t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 t_0 t_1 t_2 t_1 t_2 t_0 t_1 = t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 (0,1,2) t_0 t_1 t_2 t_0 t_2 t_1 \]

\[ \implies t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 (0,1,2) t_1 t_2 t_1 t_2 t_0 t_1 t_0 (0,1,2) t_0 t_1 t_2 t_0 t_2 t_1 t_0 t_2 t_1 \]

\[ \implies t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 (0,1,2) t_1 t_0 t_2 t_0 t_1 t_0 (0,1,2) t_0 t_1 t_2 t_0 t_2 t_1 t_0 t_2 t_1 \]

\[ \implies t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 (0,1,2) t_1 t_0 t_2 t_0 t_1 t_0 (0,1,2) t_0 t_1 t_2 t_0 t_2 t_1 t_0 t_2 t_1 \]

\[ \implies t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 (0,1,2) t_1 t_0 t_2 t_0 t_1 t_0 (0,1,2) t_0 t_1 t_2 t_0 t_2 t_1 t_0 t_2 t_1 \]

\[ \implies t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 (0,1,2) t_1 t_0 t_2 t_0 t_1 t_0 (0,1,2) t_0 t_1 t_2 t_0 t_2 t_1 t_0 t_2 t_1 \]

\[ \implies t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 (0,1,2) t_1 t_0 t_2 t_0 t_1 t_0 (0,1,2) t_0 t_1 t_2 t_0 t_2 t_1 t_0 t_2 t_1 \]

\[ \implies t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 (0,1,2) t_1 t_0 t_2 t_0 t_1 t_0 (0,1,2) t_0 t_1 t_2 t_0 t_2 t_1 t_0 t_2 t_1 \]

\[ \implies t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_0 (0,1,2) t_1 t_0 t_2 t_0 t_1 t_0 (0,1,2) t_0 t_1 t_2 t_0 t_2 t_1 t_0 t_2 t_1 \]

\[ \implies N t_0 t_1 t_0 t_1 t_2 t_1 t_2 t_0 = N t_1 t_2 t_1 t_2 t_0 t_1 t_0 \in [0101202] \]
\[ \Rightarrow N t_0 t_1 t_0 t_1 t_2 t_1 t_0 N = [01012120] = [01012020]. \]

And
\[ (1, 2) t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = (1, 2) t_0 t_1 t_2 t_1 (0, 2, 1) t_0 t_2 t_1 t_0 t_1 t_2 t_1 t_0 t_1 t_0 \]
\[ \Rightarrow (1, 2) t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = (0, 2) t_2 t_0 t_1 t_0 t_2 t_1 t_2 t_1 t_0 t_1 t_0 \]
\[ \Rightarrow (1, 2) t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = (0, 2) t_2 t_0 t_1 t_0 t_1 t_2 t_1 t_0 t_1 t_0 \]
\[ \Rightarrow (1, 2) t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = (0, 2) t_2 t_0 t_1 (0, 2) t_1 t_2 t_1 t_0 t_1 t_0 t_1 t_0 \]
\[ \Rightarrow (1, 2) t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = t_0 t_2 t_1 t_2 t_0 \]
\[ \Rightarrow (1, 2) t_0 t_1 t_2 t_1 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = t_0 t_1 t_2 t_1 t_0 t_1 t_2 t_1 \]
\[ \Rightarrow N t_0 t_1 t_0 t_1 t_2 t_1 N = N t_0 t_1 t_0 t_2 t_1 N = [01012120] = [01012020]. \]

The number of single cosets in \([0102120]\) is \(|N|/|N^{(0102120)}| = 6/1 = 6\), since \(N^{(0102120)} = \{\epsilon\}\). The orbits of \(N^{(0102120)}\) on \(T\) are \(\{0\}\), \(\{1\}\), and \(\{2\}\). We must consider the double cosets \(N t_0 t_1 t_0 t_2 t_1 t_2 t_0 N\), \(N t_0 t_1 t_0 t_2 t_1 t_2 t_0 t_1 N\), and \(N t_0 t_1 t_0 t_2 t_1 t_2 t_0 t_2 N\). However, \(N t_0 t_1 t_0 t_2 t_1 t_2 t_0 t_0 N = N t_0 t_1 t_0 t_2 t_1 t_2 N = [0102120]\). Also we have:
\[ (0, 1) t_2 t_1 t_2 t_1 t_0 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = (0, 1) t_2 t_1 t_2 t_1 t_0 t_2 t_0 t_2 t_1 t_1 t_1 t_0 \]
\[ \Rightarrow (0, 1) t_2 t_1 t_2 t_1 t_0 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = t_2 t_0 t_2 t_0 t_1 t_2 t_1 t_2 t_0 t_2 t_1 t_0 \]
\[ \Rightarrow (0, 1) t_2 t_1 t_2 t_1 t_0 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = t_2 t_0 (0, 2, 1) t_0 t_2 t_1 t_2 t_0 t_2 t_1 t_2 t_0 t_1 t_0 \]
\[ \Rightarrow (0, 1) t_2 t_1 t_2 t_1 t_0 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = (0, 2, 1) t_1 t_2 t_0 t_1 t_2 t_0 t_2 t_0 t_2 t_1 t_0 \]
\[ \Rightarrow (0, 1) t_2 t_1 t_2 t_1 t_0 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = (0, 2, 1) t_1 t_2 t_0 t_1 t_2 t_0 t_2 t_0 t_2 t_1 t_0 \]
\[ \Rightarrow (0, 1) t_2 t_1 t_2 t_1 t_0 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = (0, 2) t_2 t_1 t_2 t_0 t_0 t_2 t_1 t_2 t_0 t_2 t_1 t_0 \]
\[ \Rightarrow (0, 1) t_2 t_1 t_2 t_1 t_0 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = (0, 1) t_1 t_0 t_2 t_0 t_1 t_0 t_2 t_0 t_2 t_1 t_0 \]
\[ \Rightarrow (0, 1) t_2 t_1 t_2 t_1 t_0 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = (0, 1) t_1 t_0 t_2 t_0 t_2 t_0 t_2 t_0 t_2 t_1 t_0 \]
\[ \Rightarrow (0, 1) t_2 t_1 t_2 t_1 t_0 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = (0, 1) t_2 t_1 t_2 t_0 t_2 t_0 t_2 t_0 t_2 t_1 t_0 \]
\[ \Rightarrow (0, 1) t_2 t_1 t_2 t_1 t_0 t_2 t_0 t_1 t_2 t_1 t_0 t_1 t_0 = t_0 t_1 t_0 t_0 t_0 t_2 t_1 t_2 t_0 t_2 t_1 t_0 \]
\[\Rightarrow (0,1)t_1t_2t_1t_0t_2t_0t_1t_0 = t_0t_1t_0t_2t_1t_2t_0t_1t_0\]
\[\Rightarrow Nt_0t_1t_0t_2t_1t_0t_1 = Nt_2t_1t_2t_1t_0t_2t_0t_2 \in [01012020]\]
\[\Rightarrow Nt_0t_1t_0t_2t_1t_0t_1N = [01021201] = [01012020].\]

And
\[\[(0,1)t_1t_0t_2t_0t_2t_1t_0t_2t_0t_1t_0 = (0,1)t_1t_0t_2(0,1)t_2t_0t_2t_1t_2t_1t_2t_0t_1t_0\]
\[\Rightarrow (0,1)t_1t_0t_2t_0t_2t_1t_0t_2t_1t_0t_1t_0 = t_0t_1t_0t_2t_0t_2t_1t_2t_0t_1t_0\]
\[\Rightarrow (0,1)t_1t_0t_2t_0t_2t_1t_2t_0t_1t_0t_1t_0 = t_0t_1t_0t_2t_1t_2t_0t_1t_0\]
\[\Rightarrow (0,1)t_1t_0t_2t_0t_2t_1t_2t_0t_1t_0t_1t_0 = \epsilon\]
\[\Rightarrow (0,1)t_1t_0t_2t_0t_2t_1 = t_0t_1t_0t_2t_1t_2t_0t_2\]
\[\Rightarrow Nt_0t_1t_0t_2t_1t_0t_2t_0 = Nt_1t_0t_2t_0t_2t_1 \in [012120]\]
\[\Rightarrow Nt_0t_1t_0t_2t_1t_0t_2N = [01021202] = [012120].\]

The number of single cosets in \([01012020]\) is \(|N|/|N^{(01012020)}| = 6/1 = 6\), since \(N^{(01012020)} = \{\epsilon\}\). The orbits of \(N^{(01012020)}\) on \(T\) are \(\{0\}\), \(\{1\}\), and \(\{2\}\). We must consider the double cosets \(Nt_0t_1t_0t_2t_0t_2t_0t_0N\), \(Nt_0t_1t_0t_1t_2t_0t_2t_0t_1N\), and \(Nt_0t_1t_0t_2t_0t_2t_0t_2N\). However, \(Nt_0t_1t_0t_1t_2t_0t_2N = Nt_0t_1t_0t_1t_2t_0t_2N = [0101202].\) Also we have:

\[(1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = t_1t_2t_0t_2t_1t_0t_1t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[\Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0 = (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_2t_1t_0t_1t_0\]
\[ \Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2t_0t_1t_0t_1t_0 = e \]
\[ \Rightarrow (1,2)t_2t_1t_2t_0t_1t_0t_2 = t_0t_1t_0t_2t_0t_1t_0t_1t_0 \]
\[ \Rightarrow Nt_0t_1t_0t_1t_2t_0t_1t_0t_1\in[0102120] \]
\[ \Rightarrow Nt_0t_1t_0t_1t_2t_0t_1N = [010120201] = [0101202] \]

And
\[ t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0 = t_0t_1(0,2,1)t_1t_0t_2t_1t_2t_0t_1t_0t_1t_0t_2t_0t_2 \]
\[ \Rightarrow t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = (0,2,1)t_2t_1t_0t_2t_1t_2t_0t_1t_0t_1t_0t_2t_0t_2 \]
\[ \Rightarrow t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = (0,2,1)t_2t_1t_0t_2t_0t_1t_0t_1t_0t_2t_0t_2 \]
\[ \Rightarrow t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = (0,2,1)t_2t_1t_0t_2t_0(1,2)t_0t_2t_0t_1t_0t_2t_0t_2 \]
\[ \Rightarrow t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = (0,1,2)t_1t_2t_0t_1t_0t_0t_2t_0t_1t_2 \]
\[ \Rightarrow t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = (0,1,2)t_1t_2t_0t_1t_2t_0t_1t_2 \]
\[ \Rightarrow t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = (0,1,2)t_1t_2t_0t_1t_2t_1t_2 \]
\[ \Rightarrow t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = (0,1,2)t_1t_2t_0t_1t_2t_1t_2 \]
\[ \Rightarrow t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = (0,1,2)t_1t_2t_0t_1t_2t_1t_2 \]
\[ \Rightarrow t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = (0,1,2)t_1t_2t_0t_1t_2t_1t_2 \]
\[ \Rightarrow t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 \]
\[ \Rightarrow t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 \]
\[ \Rightarrow t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = t_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 \]
\[ \Rightarrow Nt_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = Nt_2t_0t_2t_0t_1t_0t_1t_0t_2t_0t_2 \]
\[ \Rightarrow Nt_0t_1t_0t_1t_2t_0t_2t_1t_0t_1t_0t_2t_0t_2 = Nt_2t_0t_2t_0t_1t_0t_1t_0t_2t_0t_2 \]

The number of single cosets in \([01012021]\) is \(|N|/|N^{01012021}| = 6/3 = 2\), since \(N^{01012021} = \{e, (0,1,2), (0,2,1)\}\). The orbit of \(N^{01012021}\) on \(T\) is \(\{0,1,2\}\) and we only consider the double cosets \(Nt_0t_1t_0t_1t_2t_0t_2t_1t_1\). However, \(Nt_0t_1t_0t_1t_2t_0t_2t_1t_1N = Nt_0t_1t_0t_1t_2t_0t_2t_1t_1N\). Therefore, we have found all double cosets. The result of the double coset enumeration is shown in the Cayley diagram in figure 3.1.
Figure 3.1: Cayley Diagram of $PGL_2(9)$ over $S_3$
### 3.3 Action of the Symmetric Generators and the Generators of $S_3$ on the Right Cosets of $G$ Over $S_3$

We found that $|G| \leq 6 \times 120 = 720$. We now show that $|G| = 720$.

Let $X$ denote the set of all (120) distinct right cosets of $N$ in $G$ that we have found in the previous section and we consider $G$ as a subgroup of the symmetric group on 120 symbols, where the 120 symbols are the elements of $X$. The following is the labeling of the 120 right cosets:

<table>
<thead>
<tr>
<th>1</th>
<th>$N$</th>
<th>25</th>
<th>$N_{t_1 t_0 t_1 t_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$N_{t_0}$</td>
<td>26</td>
<td>$N_{t_1 t_0 t_1 t_0}$</td>
</tr>
<tr>
<td>3</td>
<td>$N_{t_1}$</td>
<td>27</td>
<td>$N_{t_2 t_0 t_2 t_0}$</td>
</tr>
<tr>
<td>4</td>
<td>$N_{t_2}$</td>
<td>28</td>
<td>$N_{t_2 t_1 t_2 t_1}$</td>
</tr>
<tr>
<td>5</td>
<td>$N_{t_0 t_1}$</td>
<td>29</td>
<td>$N_{t_0 t_1 t_0 t_2}$</td>
</tr>
<tr>
<td>6</td>
<td>$N_{t_0 t_2}$</td>
<td>30</td>
<td>$N_{t_0 t_2 t_0 t_1}$</td>
</tr>
<tr>
<td>7</td>
<td>$N_{t_1 t_0}$</td>
<td>31</td>
<td>$N_{t_1 t_0 t_1 t_2}$</td>
</tr>
<tr>
<td>8</td>
<td>$N_{t_1 t_2}$</td>
<td>32</td>
<td>$N_{t_1 t_2 t_1 t_0}$</td>
</tr>
<tr>
<td>9</td>
<td>$N_{t_2 t_0}$</td>
<td>33</td>
<td>$N_{t_2 t_0 t_2 t_1}$</td>
</tr>
<tr>
<td>10</td>
<td>$N_{t_2 t_1}$</td>
<td>34</td>
<td>$N_{t_2 t_1 t_2 t_0}$</td>
</tr>
<tr>
<td>11</td>
<td>$N_{t_0 t_1 t_0}$</td>
<td>35</td>
<td>$N_{t_0 t_1 t_2 t_0} = N_{t_1 t_0 t_2 t_1}$</td>
</tr>
<tr>
<td>12</td>
<td>$N_{t_0 t_2 t_0}$</td>
<td>36</td>
<td>$N_{t_1 t_2 t_0 t_1} = N_{t_2 t_1 t_0 t_2}$</td>
</tr>
<tr>
<td>13</td>
<td>$N_{t_1 t_0 t_1}$</td>
<td>37</td>
<td>$N_{t_0 t_2 t_1 t_0} = N_{t_2 t_0 t_1 t_2}$</td>
</tr>
<tr>
<td>14</td>
<td>$N_{t_1 t_2 t_1}$</td>
<td>38</td>
<td>$N_{t_0 t_1 t_2 t_1}$</td>
</tr>
<tr>
<td>15</td>
<td>$N_{t_2 t_0 t_2}$</td>
<td>39</td>
<td>$N_{t_0 t_2 t_1 t_2}$</td>
</tr>
<tr>
<td>16</td>
<td>$N_{t_2 t_1 t_2}$</td>
<td>40</td>
<td>$N_{t_0 t_1 t_2 t_0}$</td>
</tr>
<tr>
<td>17</td>
<td>$N_{t_0 t_1 t_2}$</td>
<td>41</td>
<td>$N_{t_1 t_2 t_0 t_2}$</td>
</tr>
<tr>
<td>18</td>
<td>$N_{t_0 t_2 t_1}$</td>
<td>42</td>
<td>$N_{t_2 t_0 t_1 t_0}$</td>
</tr>
<tr>
<td>19</td>
<td>$N_{t_1 t_0 t_2}$</td>
<td>43</td>
<td>$N_{t_2 t_1 t_0 t_1}$</td>
</tr>
<tr>
<td>20</td>
<td>$N_{t_1 t_2 t_0}$</td>
<td>44</td>
<td>$N_{t_0 t_1 t_0 t_1 t_0} = N_{t_1 t_0 t_1 t_0 t_1}$</td>
</tr>
<tr>
<td>21</td>
<td>$N_{t_2 t_0 t_1}$</td>
<td>45</td>
<td>$N_{t_0 t_2 t_0 t_2 t_0} = N_{t_2 t_0 t_2 t_2 t_0}$</td>
</tr>
<tr>
<td>22</td>
<td>$N_{t_2 t_1 t_0}$</td>
<td>46</td>
<td>$N_{t_1 t_2 t_1 t_2 t_1} = N_{t_2 t_1 t_2 t_1 t_2}$</td>
</tr>
<tr>
<td>23</td>
<td>$N_{t_0 t_1 t_0 t_1}$</td>
<td>47</td>
<td>$N_{t_0 t_1 t_0 t_1 t_2}$</td>
</tr>
<tr>
<td>24</td>
<td>$N_{t_0 t_2 t_0 t_2}$</td>
<td>48</td>
<td>$N_{t_0 t_2 t_0 t_2 t_1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td>49</td>
<td>(Nt_1t_0t_1t_0t_2)</td>
<td>81</td>
<td>(Nt_0t_2t_0t_2t_1t_2)</td>
</tr>
<tr>
<td>50</td>
<td>(Nt_1t_2t_1t_2t_0)</td>
<td>82</td>
<td>(Nt_1t_0t_1t_0t_2t_0)</td>
</tr>
<tr>
<td>51</td>
<td>(Nt_2t_0t_2t_0t_1)</td>
<td>83</td>
<td>(Nt_1t_2t_2t_2t_0)</td>
</tr>
<tr>
<td>52</td>
<td>(Nt_2t_1t_2t_1t_0)</td>
<td>84</td>
<td>(Nt_2t_0t_2t_0t_1t_0)</td>
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The action of \( N \) and the action of \( t \)'s is as follows: \( G \times X \rightarrow X \) defined by
\[(g, x) \mapsto gx, \text{ and } \alpha : G \xrightarrow{\text{hom}} S_X.\]

\[x = (0, 1, 2)\]
\[y = (1, 2)\]
\[t_0 = \alpha(t_0) = (1, 2, 3, 7, 4, 9, 5, 11)\]

It is readily verified that \(|\langle x, y, t_0 \rangle| = 720\). Note that \( t_0 \) has exactly three conjugates under conjugation by \( \langle \alpha(x), \alpha(y) \rangle \).
Now we need to check the additional relations. First we need to check if \( \tilde{\alpha}(0, 2, 1) = \tilde{\alpha}(t_0 t_1 t_2 t_0 t_1 t_2 t_0 t_1) \).

\[ \tilde{\alpha}(0, 2, 1) = \tilde{\alpha}(x^3) = (\tilde{\alpha}(x))^3 \]

= \((1, 2, 4, 3)(5, 9, 8)(6, 10, 7)(11, 15, 14)(12, 16, 13)(17, 21, 20)
(62, 64, 63)(65, 69, 68)(66, 70, 67)(71, 72, 73)(74, 78, 77)(75, 79, 76)(80, 84, 83)
(81, 85, 82)(86, 87, 88)(89, 93, 92)(90, 94, 91)(95, 100, 98)(96, 99, 97)(101, 105, 104)

= \tilde{\alpha}(t_0)\tilde{\alpha}(t_1)\tilde{\alpha}(t_2)\tilde{\alpha}(t_0)\tilde{\alpha}(t_1)\tilde{\alpha}(t_2)\tilde{\alpha}(t_0)\tilde{\alpha}(t_1)\)

Next we check if \( \tilde{\alpha}(0, 1) = \tilde{\alpha}(t_2 t_0 t_2 t_1 t_2 t_0 t_1 t_2 t_0) \).

\[ \tilde{\alpha}(0, 1) = (1)(2,3)(4,5)(7,6)(9,10)(11,13)(12,14)(15,16)(17,19)
(62,63,64)(65,67)(66,68)(69,70)(71,72,73)(74,76)(75,77)(78,79)(80,82)(81,83)
(84,85)(86,87,88)(89,91)(90,92)(93,94)(95,97)(96,98)(99,100)(101,103)
(119,120)\)
We now have that $G/Ker\alpha \cong \langle x, y, t_0 \rangle \implies |G| \geq 720$. But earlier we found that $|G| \leq 720$, so we conclude that $|G| = 720$. In fact, in this case it is easy to show that $G \cong PGL_2(9)$.

### 3.4 Proof of the Isomorphism between $G$ and $PGL_2(9)$

We start by constructing a homomorphism $\Phi$ from the progenitor $2^3 : S_3$ to $PGL_2(9)$ by defining $\Phi(x) = \frac{\eta + (1+1)(1+\eta)}{\eta + 1} = (1, 2, 8)(3, 4, 6)(5, 0, \infty)$ and $\Phi(y) = \frac{-\eta - 1}{\eta + 1} = (1, \infty)(2, 0)(3, 6)(5, 8)$. Since the orders of $\Phi(x)$, $\Phi(y)$, and $\Phi(x)\Phi(y)$ are 3, 2 and 2 respectively, $N = \langle \Phi(x), \Phi(y) \rangle \cong S_3$.

We now let $\Phi(t_0) = -\frac{\eta - 2(1+1)}{\eta + 1} = (1, 3)(2, 8)(4, 7)(5, 0)(6, \infty)$. It is readily verified that $PGL_2(9) \cong \langle \Phi(x), \Phi(y), \Phi(t_0) \rangle$. Next we show that $\Phi$ preserves the operation of $2^3 : S_3$. We find that $|\Phi(t_0)^N| = 3$ and $\Phi(t_0) = \Phi(t) = (1, 3)(2, 8)(4, 7)(5, 0)(6, \infty)$, $\Phi(t_1) = \Phi(t_2) = \Phi(t_2^3) = (2, 4)(8, 1)(6, 7)(0, \infty)(3, 5)$, $\Phi(t_2) = \Phi(t_2^3) = (8, 6)(1, 2)(3, 7)(\infty, 5)(4, 0)$, and that $N$ permutes the three images of $t_0$ by conjugation as the group $S_3$ does. $\Phi(x) : (\Phi(t_0), \Phi(t_1), \Phi(t_2))$ and $\Phi(y) : (\Phi(t_0))(\Phi(t_1), \Phi(t_2))$. Thus, $\Phi(2^3 : S_3) = PGL_2(9)$.

Now the additional relation given by $[xt_0]^8 = e \iff t_1t_0t_2t_1t_0t_2t_1t_0 = (0, 1, 2)$ is satisfied in $PGL_2(9)$ because $\Phi(t_1)\Phi(t_0)\Phi(t_2)\Phi(t_1)^3\Phi(t_0)\Phi(t_2)^3\Phi(t_1)^3\Phi(t_0) = (0, \infty, 5)(1, 2, 8)(3, 4, 6)$ acts as $\Phi(t_0), \Phi(t_1), \Phi(t_2)$ by conjugation on the images of the three symmetric generators. This shows that $PGL_2(9)$ is an image of $G$. Thus, $|G| \geq |PGL_2(9)|$; but $|G| \leq 720 = |PGL_2(9)|$, and so the equality holds and $G \cong PGL_2(9)$. 

Chapter 4

The Double Cover $2 \cdot M_{22} : 2$

4.1 Introduction

$M_{22}$ is one of the five sporadic simple groups discovered by the French mathematician Emile Leonard Mathieu. Simple groups are groups which have no nontrivial proper normal subgroups, meaning that they cannot be broken down into products of smaller groups. $M_{22}$ is a group of order 443,520 and it can be thought of as a permutation group on 22 letters. Moreover it is 3-fold transitive, meaning that any triplet of symbols can be carried to any other by a suitable permutation in the group. We know that $M_{22} : 2$ and $2 \cdot M_{22} : 2$ are perfect groups and $2 \cdot M_{22} : 2 / Z(2 \cdot M_{22} : 2) \cong M_{22} : 2$ thus, $2 \cdot M_{22} : 2$ is a covering of $M_{22} : 2$. Moreover, since the order of the center $Z(2 \cdot M_{22} : 2)$ is 2, $2 \cdot M_{22} : 2$ is called a double cover of $M_{22} : 2$.

Consider the progenitor:

$2^*7 : L_3(2) \cong \langle x, y, t | x^7 = y^2 = (xy)^3 = [x, y]^4 = e = t^2 = [xy, t^x] = [y, t^x] \rangle$, where $x \sim (0, 1, 2, 3, 4, 5, 6)$ and $y \sim (2, 6)(4, 5)$. We factor $2^*7 : L_3(2)$ by the following relations:

$[(1, 2, 4)(3, 6, 5)t_3]^8$, $[(1, 3)(2, 4, 6, 5)t_2]^{10}$, and $[(2, 6)(4, 5)t_0t_2]^4$. The first relation $[\pi t_3]^8 = e$, where $\pi = (1, 2, 4)(3, 6, 5)$ implies

$$\pi t_3 \pi t_3 \pi t_3 \pi t_3 \pi t_3 \pi t_3 \pi t_3 = e$$

$$\Rightarrow \pi^8 t_3^\pi t_3^6 t_3^\pi t_3^4 t_3^\pi t_3^2 t_3^\pi t_3 = e$$

$$\Rightarrow (1, 4, 2)(3, 5, 6) t_6 t_3 t_5 t_0 t_3 t_0 t_3 t_0 t_3 = e$$

$$\Rightarrow (1, 4, 2)(3, 5, 6) t_6 t_3 t_5 t_6 = t_3 t_6 t_5 t_3$$  \hspace{1cm} (4.1)
The second relation \([\pi t_2]^{10} = e\), where \(\pi = (1, 3)(2, 4, 6, 5)\) implies

\[
\pi t_2 \pi t_2 \pi t_2 \pi t_2 \pi t_2 \pi t_2 \pi t_2 \pi t_2 e = e
\]

\[
\Rightarrow \pi^{10} t_2^7 t_2^6 t_2^5 t_2^4 t_2^3 t_2^2 t_2^1 t_2^0 = e
\]

\[
\Rightarrow (2, 6)(4, 5)t_4 t_2 t_5 t_4 t_2 t_5 t_4 t_2 = e
\]

\[
\Rightarrow (2, 6)(4, 5)t_4 t_2 t_5 t_4 = t_2 t_4 t_6 t_5 t_2
\]

The third relation \([\pi t_0 t_2]^{4} = e\), where \(\pi = (2, 6)(4, 5)\) implies

\[
\pi t_0 t_2 \pi t_0 t_2 \pi t_0 t_2 e = e
\]

\[
\Rightarrow \pi^4 (t_0 t_2)^7 (t_0 t_2)^6 (t_0 t_2)^5 t_0 t_2 e = e
\]

\[
\Rightarrow t_0 t_5 t_0 t_2 t_6 t_0 t_2 = e
\]

\[
\Rightarrow t_0 t_5 t_0 t_2 = t_2 t_0 t_5 t_0
\]

(4.3)

4.2 Manual Double Coset Enumeration of \(G\) Over \(L_3(2)\)

Let \(G \cong \frac{2^{27}:L_3(2)}{[(1,2,4)(3,6,5)t_0]^6;[(1,3)(2,4,8,5)t_0]^4[(4,5)(4,5)t_0 t_2]^2}\) and let \(N = L_3(2)\). In order to find the index of \(N\) in \(G\) we will perform manual double coset enumeration of \(G\) over \(N\). Thus, we will find all double cosets and determine how many single cosets each one of them contains. In fact we will show that \(|G|\) is at most \(168 \times 10560 = 1774080\).

We start we the double coset \(N e N\) denoted by \([*]\) which consists of the single coset \(N\). Since \(N\) is transitive on \(T = \{0, 1, 2, 3, 4, 5, 6\}\), \(N_{t_0} N = \{N t_0^0 | \pi \in N\} = \{N t_0, N t_1, N t_2, N t_3, N t_4, N t_5, N t_6\}\) and we denote the double coset \(N t_0 N\) by \([0]\). Thus, there are seven single cosets in \([0]\). Now we determine the orbits of \(N(0) = N^0 = \{(2, 6)(4, 5), (2, 5)(4, 6), (1, 5, 2)(3, 4, 6)\}\) on \(T\) which are \([0]\) and \([1, 2, 3, 4, 5, 6]\), and consider the double cosets \(N t_0 t_i N\) for one \(t_i\) from each orbit of \(N^0\). First we find that \(N t_0 t_0 = N \in NeN\), so \(t_0\) takes us back to \([*]\). Next we consider \(t_1\), a representative from the orbit \([1, 2, 3, 4, 5, 6]\) and find that \(N t_0 t_1 N\) denoted by \([01]\) is a new double coset. The number of single cosets in \([01]\) equals to \(|N|/|N(01)| \leq 168/4 = 42\), since \(N(01) \geq N^{01} = \{(2, 6)(4, 5), (2, 5)(4, 6)\}\) and \(N^{01}\) has order 4.

The orbits of \(N^{01}\) on \(T\) are \([0], [1], [3]\), and \([2, 4, 5, 6]\). We must consider the double cosets \(N t_0 t_1 t_0 N, N t_0 t_1 t_1 N, N t_0 t_1 t_2 N, N t_0 t_1 t_3 N\). However, \(N t_0 t_1 t_1 N = \)
$N_{t_0} = [0]$, but $N_{t_0t_1t_0N}, N_{t_0t_1t_2N}$, and $N_{t_0t_1t_3N}$ are new. Now, with the help of MAGMA we find that

$$t_0t_1t_0 = t_{10}t_1$$

Thus, we have $N_{t_0t_1t_0}(1,0)(2,4,5,6) = N_{t_0t_1}t_1 = N_{t_0t_10} \Rightarrow (1,0)(2,4,5,6) \in N^{010}$ and $N^{010} = \langle (2,6)(4,5), (2,5)(4,6), (1,0)(2,4,5,6) \rangle$ is of order 8, thus the number of single cosets in [010] is equal to $|N|/|N^{010}| = 168/8 = 21$. Also, $N^{012} = \{e\}$ is of order 1, therefore there are 168 single cosets in [012], and $N^{013} = \langle (2,6)(4,5), (2,5)(4,6) \rangle$ is of order 4 which implies that the number of single cosets in [013] is equal to $|N|/|N^{013}| = 168/4 = 42$.

The orbits of $N^{010}$ on $T$ are $\{0\}, \{1\}, \{2,4,5,6\}, \{3\}$. So we consider the double cosets $N_{t_0t_1t_0t_0N}, N_{t_0t_1t_0t_1N}, N_{t_0t_1t_0t_2N}, N_{t_0t_1t_0t_3N}$. $N_{t_0t_10t_0N} = N_{t_0t_1} = [01]$. Since $t_0t_1t_0 = t_{10}t_1 \Rightarrow t_0t_1t_0 = t_{10}t_1 = (t_{01})^{(1,0)(2,4,5,6)}$, $N_{t_0t_1t_0t_1N} = N_{t_0t_1N} = [01]$. But the other two double cosets are new. We note that $N^{012} = \{e\}$, but $N_{t_0t_1t_0t_2(1,0)(4,6)} = N_{t_0t_1t_0} = N_{t_0t_1t_0t_2}, by (4.4) \Rightarrow (1,0)(4,6) \in N^{012}$. Therefore, the number of single cosets in [012] is $|N|/|N^{012}| = 168/2 = 84$ since $N^{010} = \{e, (1,0)(4,6)\}$. Also, $N_{t_0t_1t_0t_3(1,0)(2,4,5,6)} = N_{t_0t_1t_0t_3} = N_{t_0t_1t_0t_3}, by (4.4) \Rightarrow (1,0)(2,4,5,6) \in N^{013}$, thus $N^{013} = \langle (2,6)(4,5), (2,4)(5,6), (1,0)(2,4,5,6) \rangle$ is of order 8 which implies that the number of single cosets in [013] is equal to $|N|/|N^{013}| = 168/8 = 21$.

The orbits of $N^{012}$ on $T$ are $\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}$, and $\{6\}$. So we consider the double cosets $N_{t_0t_1t_2t_0N}, N_{t_0t_1t_2t_1N}, N_{t_0t_1t_2t_2N}, N_{t_0t_1t_2t_3N}, N_{t_0t_1t_2t_4N}, N_{t_0t_1t_2t_5N}$ and $N_{t_0t_1t_2t_6N}$. However, $N_{t_0t_1t_2t_2N} = N_{t_0t_1N} = [01]$. But the other six double cosets are new. Now $N_{t_0t_1t_2t_0(1,0)(4,6)} = N_{t_0t_1t_2} = N_{t_0t_1t_2t_0}, by relation (4.1) conjugated by (1,4,3,0,5,2,6) \Rightarrow (1,0)(4,6) \in N^{0120}$. The number of single cosets in [0120] is $|N|/|N^{0120}| = 168/2 = 84$ since $N^{010} = \{e, (1,0)(4,6)\}$. Also, $N_{t_0t_1t_2t_1(1,2)(3,6)} = N_{t_0t_1t_2t_3} = N_{t_0t_1t_2t_1}, by (4.4) conjugated with (2,0)(3,4). Thus, (1,2)(3,6) $\in N^{0121}$ and $N^{0121} = \{e, (1,2)(3,6)\}$ implies that the number of single cosets in [0121] is equal to $|N|/|N^{0121}| = 168/2 = 84$. For the other four double cosets: [0123], [0124], [0125], [0126], we have that $N^{0123} = N^{0124} = N^{0125} = N^{0126} = \{e\}$, so each one contains 168 single cosets.

The orbits of $N^{013}$ on $T$ are $\{0\}, \{1\}, \{2,4,5,6\}$, and $\{3\}$. So we consider the double cosets $N_{t_0t_1t_3t_0N}, N_{t_0t_1t_3t_1N}, N_{t_0t_1t_3t_2N}, N_{t_0t_1t_3t_3N}$. $N_{t_0t_1t_3t_3N} = \{e\}$, but $N_{t_0t_1t_3t_1N}, N_{t_0t_1t_3t_2N}$, and $N_{t_0t_1t_3t_3N}$ are new. Now, with the help of MAGMA we find that
Now, we have the following:

\[(t_{0}t_{1}t_{0}t_{3})^{(2,6,4,5)(3,0)} = t_{3}t_{1}t_{3}t_{0}\]

\[\Rightarrow (t_{0}t_{1}t_{0}t_{3})^{(2,6,4,5)(3,0)} = t_{1}t_{3}t_{1}t_{0}, \text{by (4.4)}\]

\[\Rightarrow (t_{0}t_{1}t_{0}t_{3})^{(2,6,4,5)(3,0)} = t_{0}t_{1}t_{3}t_{1}, \text{by (4.3)}\]

\[\Rightarrow N_{t_{0}t_{1}t_{0}t_{3}} = N_{t_{0}t_{1}t_{3}t_{1}},\]

\[\Rightarrow N_{t_{0}t_{1}t_{3}t_{1}N} = N_{t_{0}t_{1}t_{0}t_{3}N} = [0103].\]

But the other two double cosets are new. Now, \(N_{t_{0}t_{1}t_{2}t_{0}N} = N_{t_{1}t_{3}t_{0}N,}\) by relation (4.1) \(\Rightarrow (1, 0)(4, 5, 6) \in N^{(0130)}\). The number of single cosets in \([0130]\) is \(|N|/|N^{(0130)}|=168/12=14\) since \(N^{(0130)}=\langle (2, 6)(4, 5), (2, 4)(5, 6), (1, 3)(0)(4, 5, 6) \rangle\) has order 12. Also, \(N^{(0132)}=\{e\}\) thus, the number of single cosets in \([0132]\) is 168.

The orbits of \(N^{(0132)}\) on \(T\) are \(\{0, 1\}, \{2\}, \{3\}, \{4, 6\}\) and \(\{5\}\). So we consider the double cosets \(N_{t_{0}t_{1}t_{0}t_{2}t_{0}N}, N_{t_{0}t_{1}t_{0}t_{2}t_{2}N}, N_{t_{0}t_{1}t_{0}t_{3}t_{3}N}\) and \(N_{t_{0}t_{1}t_{0}t_{3}t_{5}N}.\) However, \(N_{t_{0}t_{1}t_{0}t_{2}t_{2}N}=N_{t_{0}t_{1}t_{0}t_{3}N}=[010].\) But the other four double cosets are new. We note that \(N_{t_{0}t_{1}t_{0}t_{2}t_{3}(1, 0)(4, 6)}=N_{t_{1}t_{0}t_{1}t_{2}t_{3}=N_{t_{0}t_{1}t_{0}t_{2}t_{3}}, \text{by (4.4)} \Rightarrow (1, 0)(4, 6) \in N^{(01023)}\). Similarly, \((1, 0)(4, 6) \in N^{(01020)},\) and \((1, 0)(4, 6) \in N^{(01025)}.\) Thus the number of single cosets in each one of the following double cosets: \([01020],[01023],[01025]\) is \(|N|/|N^{(01020)}|=168/2=84\) since \(N^{(01020)}=N^{(01023)}=N^{(01025)}=\{e, (1, 0)(4, 6)\}.\) Also, \(N^{(01024)}=\{e\}\) thus, the number of single cosets in \([01024]\) is 168.

The orbits of \(N^{(0103)}\) on \(T\) are \(\{0, 1\}, \{2, 4, 5, 6\},\) and \(\{3\}\). So we consider the double cosets \(N_{t_{0}t_{1}t_{0}t_{3}t_{0}N}, N_{t_{0}t_{1}t_{0}t_{3}t_{2}N}, N_{t_{0}t_{1}t_{0}t_{3}t_{3}N}.\) \(N_{t_{0}t_{1}t_{0}t_{3}t_{3}N}\) = \(N_{t_{0}t_{1}t_{0}N}=[010]\) and since relation (4.3) conjugated by \((1, 2, 3, 6)(4, 6)\) implies that \(t_{3}t_{0}t_{1} = t_{0}t_{1}t_{0}t_{3}t_{0},\) we have that \(N_{t_{0}t_{1}t_{0}t_{3}t_{0}} = N_{t_{2}t_{0}t_{1}} \in [013] \Rightarrow N_{t_{0}t_{1}t_{0}t_{3}t_{0}N} = [01030] = [013].\) So the only new double coset is \([01032]\) and it contains \(|N|/|N^{(01032)}|=168/2=84\) since \(N_{t_{0}t_{1}t_{0}t_{3}t_{2}(1, 0)(4, 6)}=N_{t_{0}t_{1}t_{0}t_{3}t_{2}=N_{t_{0}t_{1}t_{3}t_{3}t_{2}, \text{by (4.4)} \Rightarrow (1, 0)(4, 6) \in N^{(01032)}\Rightarrow N^{(01032)}=\{e, (1, 0)(4, 6)\}}.\)

Now, we are going to look at the orbits of \(N^{(0120)}\) on \(T\) which are \(\{0, 1\}, \{2\}, \{3\}, \{4, 6\}\) and \(\{5\}.\) So we consider the double cosets \(N_{t_{0}t_{1}t_{2}t_{0}t_{0}N}, N_{t_{0}t_{1}t_{2}t_{0}t_{1}N}, N_{t_{0}t_{1}t_{2}t_{0}t_{2}N}, N_{t_{0}t_{1}t_{2}t_{0}t_{3}N}, N_{t_{0}t_{1}t_{2}t_{0}t_{4}N},\) and \(N_{t_{0}t_{1}t_{2}t_{0}t_{5}N}.\) However, \(N_{t_{0}t_{1}t_{2}t_{0}t_{0}N}=
\[ N_{t_0t_1t_2N} = [012] \text{ and } N_{t_0t_1t_2t_0t_1N} = N_{t_0t_1t_2N} = [012] \text{ since } (4.1) \text{ conjugated by } (1,6,0,5,2,4,3) \text{ implies } \\
(4.5) \]

\[ t_1t_0t_2 = (6,3,4)(1,2,0)t_0t_1t_2t_0t_1 \]

\[ \implies (1,0,2)t_1t_0t_2 = (1,0)(2)(3,6,4)(6,3,4)(1,2,0)t_0t_1t_2t_0t_1 \]

\[ \implies (1,0,2)t_1t_0t_2 = (1,0,2)(3,6,4)(6,3,4)(1,2,0)t_0t_1t_2t_0t_1 \]

\[ \implies (1,0,2)t_1t_0t_2 = t_0t_1t_2t_0t_1 \]

\[ \implies N_{t_0t_1t_2t_0t_1} = N_{t_1t_0t_2} \in [012] \]

\[ \implies N_{t_0t_1t_2t_0t_1N} = [01201 = [012]. \]

Also, \( N_{t_0t_1t_2t_0t_2N} = N_{t_0t_1t_0t_2t_0N} = [01020] \) since

\[ (1,0,2)(3,6,4)t_1t_0t_1t_2t_1 = (1,0,2)(3,6,4)t_1t_0t_1t_2t_2, \text{ by } (4.4) \]

\[ \implies (1,0,2)(3,6,4)t_1t_0t_1t_2t_1 = (1,0,2)(3,6,4)(6,3,4)(1,2,0)t_0t_1t_2t_0t_2, \text{ by } (4.5) \]

\[ \implies N_{t_0t_1t_2t_0t_2} = N_{t_1t_0t_1t_2} \in [01020] \]

\[ \implies N_{t_0t_1t_2t_0t_2N} = [01202] = [01020]. \]

But the other three double cosets are new. Now, \( N_{t_0t_1t_2t_0t_3t_1}(1,0)(4,6) = N_{t_1t_0t_2t_1t_3} = N_{t_0t_1t_2t_0t_3} \) since \( t_1t_0t_2t_1t_3 = (6,3,4)(1,2,0)t_0t_1t_2t_0t_3 \), by (4.1) conjugated by \( (1,6,0,5,2,4,3) \). Thus, \( (1,0)(4,6) \in N^{(01203)} \). Also \( N_{t_0t_1t_2t_0t_5t_1}(1,0)(4,6) = N_{t_1t_0t_2t_1t_5} = N_{t_0t_1t_2t_0t_5} \) since \( t_1t_0t_2t_1t_5 = (6,3,4)(1,2,0)t_0t_1t_2t_0t_5 \) \( \implies (1,0)(4,6) \in N^{(01205)} \). Thus, \( N^{(01203)} = N^{(01205)} = \{e, (1,0)(4,6)\} \), so the number of single cosets in the double cosets: [01203], [01205] is \(|N| / |N^{(01203)}| = 168/2 = 84 \). Also, \( N^{(01204)} = \{e\} \) thus the number of single cosets in [01204] is 168.

The orbits of \( N^{(0121)} \) on \( T \) are \{0\}, \{1, 2\}, \{3, 6\}, \{4\}, and \{5\}. So we consider the double cosets \( N_{t_0t_1t_2t_1t_0N} \), \( N_{t_0t_1t_2t_1t_1N} \), \( N_{t_0t_1t_2t_1t_3N} \), \( N_{t_0t_1t_2t_1t_4N} \), and \( N_{t_0t_1t_2t_1t_5N} \). However, \( N_{t_0t_1t_2t_1t_1N} = N_{t_0t_1t_2N} = [012] \) and \( N_{t_0t_1t_2t_1t_4N} = N_{t_0t_1t_2t_0t_5N} = [01205] \) since \( t_0t_1t_2t_1t_4 = (1,4,2)(3,0,5)(t_0t_1t_2t_0t_5)(1,3,4)(2,0,6) \). But the other three double cosets are new. Now, \( N_{t_0t_1t_2t_1t_0}(1,0)(4,6) = N_{t_1t_0t_2t_0t_1} = N_{t_0t_1t_2t_1t_0} \) since

\[ t_1t_0t_2t_0t_1 = (1,2,0)(3,4,6)t_0t_1t_2t_0t_1t_0t_1, \text{ by } (4.5) \]
\[ t_1 t_0 t_2 t_0 = (1, 2, 0)(3, 4, 6) t_0 t_1 t_2 t_0 t_1 t_1, \text{ by (4.4)} \]
\[ t_1 t_0 t_2 t_0 t_1 = (1, 2, 0)(3, 4, 6) t_0 t_1 t_2 t_0 \]
\[ N t_1 t_0 t_2 t_0 t_1 = N t_0 t_1 t_2 t_0, \]
and \( N t_0 t_1 t_2 t_0 (1, 0, 2)(3, 6, 4) = N t_2 t_0 t_1 t_0 t_2 = N t_0 t_1 t_2 t_0 \) since
\[ t_2 t_0 t_1 t_0 t_2 = (1, 0, 2)(3, 6, 4) t_0 t_2 t_1 t_0 t_2 t_2, \text{ by (4.5)} \]
\[ t_2 t_0 t_1 t_0 t_2 = (1, 0, 2)(3, 6, 4) t_0 t_2 t_1 t_0 t_2 t_2, \text{ by (4.4)} \]
\[ t_2 t_0 t_1 t_0 t_2 = (1, 0, 2)(3, 6, 4) t_0 t_2 t_1 t_0 \]
\[ N t_2 t_0 t_1 t_0 t_2 = N t_0 t_2 t_1 t_0. \]
Thus, \((1, 0)(4, 6)\) and \((1, 0, 2)(3, 6, 4)\) are in \( N^{(01210)} \), and the number of single cosets in the double coset \([01210]\) is equal to \(|N|/|N^{(01210)}| = 168/6 = 28\) since \( N^{(01210)} = \{e, (1, 0)(4, 6), (1, 0, 2)(3, 6, 4)\} \). \( N^{(01213)} = \{e\} \) thus, the number of single cosets in \([01213]\) is 168. Also, \( N t_0 t_1 t_2 t_4 (1, 2)(3, 6) = N t_0 t_2 t_1 t_2 t_5 = N t_0 t_1 t_2 t_1 t_5 \), by (4.4) \( (1, 2)(3, 6) \) \( \in N^{(01215)} \), so \( N^{(01215)} = \{e, (1, 2)(3, 6)\} \). Thus, the number of single cosets in \([01215]\) is \( 168/2 = 84 \).

The orbits of \( N^{(0123)} \) on \( T \) are \( \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \) and \( \{6\} \). So we consider the double cosets \( N t_0 t_1 t_2 t_3 t_0 N, N t_0 t_1 t_2 t_3 t_1 N, N t_0 t_1 t_2 t_3 t_2 N, N t_0 t_1 t_2 t_3 t_3 N, N t_0 t_1 t_2 t_3 t_4 N, N t_0 t_1 t_2 t_3 t_5 N, N t_0 t_1 t_2 t_3 t_6 N \). However, \( N t_0 t_1 t_2 t_3 t_3 N = N t_0 t_1 t_2 N = [012] \) and \( N t_0 t_1 t_2 t_3 t_6 N = N t_0 t_1 t_2 t_4 N = [01024] \) since \( t_0 t_1 t_2 t_3 = (1, 3, 2)(4, 0, 5)(t_0 t_1 t_0 t_2 t_4)^{(1,4,2)(3,5,6)} = (1, 3, 2)(4, 0, 5) t_4 t_0 t_1 t_2 \). But the other five double cosets are new. There are 168 single cosets in the double cosets \([01230], [01232], [01234], \) and \([01235]\) since \( N^{(01230)} = N^{(01232)} = N^{(01234)} = N^{(01235)} = \{e\} \). Now, \( N t_0 t_1 t_2 t_3 t_1 (1, 2)(5, 0) = N t_5 t_2 t_1 t_3 t_2 = N t_0 t_1 t_2 t_3 t_1 \) since
\[ t_5 t_2 t_1 t_3 t_2 = t_5 t_2 (5, 0, 4)(1, 2, 3) t_3 t_1 t_2 t_3 t_1, \text{ by (4.5)} \]
\[ t_5 t_2 t_1 t_3 t_2 = (5, 0, 4)(1, 2, 3) t_0 t_3 t_1 t_2 t_3 t_1 \]
\[ t_5 t_2 t_1 t_3 t_2 = (5, 0, 4)(1, 2, 3) t_0 t_1 t_2 t_3 t_1 \]
\[ (1, 2)(5, 0) \in N^{(01231)} \Rightarrow N^{(01231)} = \{e, (1, 2)(5, 0)\} \]. Thus there are 84 single cosets in \([01231]\).
The orbits of $N^{(0124)}$ on $T$ are $\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\},$ and $\{6\}$. So we consider the double cosets $Nt_{0t}t_{2t}t_{4t}oN$, $Nt_{0t}t_{1t}t_{2t}t_{4t}oN$, $Nt_{0t}t_{1t}t_{2t}t_{4t}t_{N}$, $Nt_{0t}t_{1t}t_{2t}t_{4t}t_{3t}N$, $Nt_{0t}t_{1t}t_{2t}t_{4t}N$, $Nt_{0t}t_{1t}t_{2t}t_{5t}N$, and $Nt_{0t}t_{1t}t_{2t}t_{4t}t_{6t}N$. However, $Nt_{0t}t_{1t}t_{2t}t_{4t}oN = Nt_{0t}t_{1t}t_{2t}N = [012]$, and $Nt_{0t}t_{1t}t_{2t}t_{2t}N = Nt_{0t}t_{1t}t_{2t}t_{5t}N = [01235]$ since $t_{0t}t_{1t}t_{2t}t_{2t} = (1,2,4)(3,5,0)(t_{0t}t_{1t}t_{2t}t_{2t})(1,5,3)(2,0,6) = (1,2,4)(3,5,0)t_{6t}t_{5t}t_{0t}t_{6t}$. Also, $Nt_{0t}t_{1t}t_{2t}t_{4t}t_{5t}N = Nt_{0t}t_{1t}t_{2t}t_{3t}t_{2t}N = [01235]$ since $t_{0t}t_{1t}t_{2t}t_{3t} = (1,4,0,6)(2,5)(t_{0t}t_{1t}t_{2t}t_{3t})(3,0,2,5)(3,6) = (1,4,0,6)(2,5)t_{0t}t_{3t}t_{2t}$. But the other four double cosets are new. There are 168 single cosets in the double cosets $[01240]$, [01243], and [01246] since $N^{(01240)} = N^{(01243)} = N^{(01246)} = \{e\}$. $Nt_{0t}t_{1t}t_{2t}t_{4t}(1,4,2)(3,5,6) = Nt_{0t}t_{4t}t_{2t}t_{4t} = Nt_{0t}t_{1t}t_{2t}t_{4t} \Rightarrow (1,4,2)(3,5,6) \in N^{(01241)} \Rightarrow N^{(01241)} = \{e,(1,4,2)(3,5,6),(1,2,4)(3,6,5)\}$. Thus, there are $|N|/|N^{(01241)}| = 168/3 = 56$ single cosets in $[01241]$.

The orbits of $N^{(0125)}$ on $T$ are $\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\},$ and $\{6\}$. So we consider the double cosets $Nt_{0t}t_{1t}t_{2t}t_{6t}oN$, $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{1t}N$, $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{2t}N$, $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{3t}N$, $Nt_{0t}t_{1t}t_{2t}t_{6t}N$, $Nt_{0t}t_{1t}t_{2t}t_{5t}N$, and $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{N}$. However, $Nt_{0t}t_{1t}t_{2t}t_{6t}N = Nt_{0t}t_{1t}t_{2t}N = [012]$, and $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{1t}N = Nt_{0t}t_{1t}t_{2t}t_{6t}t_{5t}N = [01235]$ since $t_{0t}t_{1t}t_{2t}t_{6t}t_{1t} = (2,0,5)(3,6,4)(t_{0t}t_{1t}t_{2t}t_{6t}t_{1t})(1,5,3)(2,0,6) = (2,0,5)(3,6,4)t_{3t}t_{5t}t_{0t}t_{6t}$. Also, $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{5t}N = Nt_{0t}t_{1t}t_{2t}t_{6t}t_{3t}N = [01235]$ since $t_{0t}t_{1t}t_{2t}t_{6t}t_{3t} = (1,6,2,3,5,0)t_{0t}t_{1t}t_{2t}t_{6t}t_{3t} = (1,6,2,3,5,0)t_{0t}t_{1t}t_{2t}t_{6t}t_{3t}$. Also, $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{5t}N = Nt_{0t}t_{1t}t_{2t}t_{6t}t_{3t}N$ since $t_{0t}t_{1t}t_{2t}t_{6t}t_{5t} = (1,0,5)(3,4,6)(t_{0t}t_{1t}t_{2t}t_{6t}t_{5t})(2,6,0)(3,4,5) = (1,0,5)(3,4,6)t_{2t}t_{6t}t_{4t}$, and $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{6t}N = Nt_{0t}t_{1t}t_{2t}t_{6t}t_{4t}N = [01234]$ since $t_{0t}t_{1t}t_{2t}t_{6t} = (2,0,5)(3,6,4)(t_{0t}t_{1t}t_{2t}t_{6})t_{4t}(2,0,5)(3,4) = \{1,4,0,6\}(2,5)t_{0t}t_{3t}t_{2t}$. But the other two double cosets are new. $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{1t}(0,1)(2,5) = Nt_{1t}t_{0t}t_{2t}t_{6t}t_{2t} = Nt_{0t}t_{1t}t_{2t}t_{6t}t_{0t}$ since (4.2) conjugated by $(1,3,4,0,6,5,2) \Rightarrow t_{0t}t_{1t}t_{2t}t_{6t}t_{2t} = (1,5)(0,2)t_{6t}t_{5t}t_{0t}t_{6t}$. Therefore, $N^{(01250)} = \{e,(1,0)(2,5)\}$. Also, $Nt_{0t}t_{1t}t_{2t}t_{6t}(2,5)(4,6) = Nt_{0t}t_{1t}t_{2t}t_{6t} = Nt_{0t}t_{1t}t_{2t}t_{5t}$, by (4.4) $N^{(01252)} = \{e,(2,5)(4,6)\}$. Thus, the number of single cosets in $[01250]$ and $[01252]$ is 84.

The orbits of $N^{(0126)}$ on $T$ are $\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\},$ and $\{6\}$. So we consider the double cosets $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{0t}N$, $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{1t}N$, $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{2t}N$, $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{3t}N$, $Nt_{0t}t_{1t}t_{2t}t_{6t}N$, and $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{N}$. However, $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{6t}N = Nt_{0t}t_{1t}t_{2t}N = [012]$, and $Nt_{0t}t_{1t}t_{2t}t_{6t}t_{3t}N = Nt_{0t}t_{1t}t_{2t}t_{6t}t_{3t} = (2,0,5)(3,6,4)(t_{0t}t_{1t}t_{2t}t_{4t}t_{3t})(2,0,3,4) = (2,0,5)(3,6,4)t_{2t}t_{6t}t_{3t}$. But the other three double cosets are new.
\[ N_{t_0 t_1 t_2 t_0 t_0 (1,5)(4,0)} = N_{t_4 t_5 t_2 t_0 t_4} = N_{t_0 t_1 t_2 t_0} \Rightarrow N^{(01)260} = \{ e, (1,5)(4,0) \} \]
\[ N_{t_0 t_1 t_2 t_0 t_0 N (2,6)(4,5)} = N_{t_0 t_1 t_2 t_0} N \Rightarrow N^{(01)262} = \{ e, (2,6)(4,5) \}. \] Thus, the number of single cosets in [01260] and [01262] is 84. The number of single cosets in [01261] is 168 since \( N^{(01)260} = \{ e \} \).

The orbits of \( N^{(01)30} \) on \( T \) are \( \{ 0,1,3 \}, \{ 2,4,5,6 \} \). So we only consider the double cosets \( N_{t_0 t_1 t_3 t_0 t_0 N} \) and \( N_{t_0 t_1 t_3 t_0 t_2 N} \). However, \( N_{t_0 t_1 t_3 t_0 t_0 N} = N_{t_0 t_1 t_3 N} = [013] \), but the [01302] is a new double coset. \( N_{t_0 t_1 t_3 t_0 t_2} (1,0,3)(4,6,5) = N_{t_3 t_0 t_1 t_3} = N_{t_0 t_1 t_3 t_0 t_2} \Rightarrow N^{(01)302} = \{ e, (1,0,3)(4,6,5), (4,5,6)(1,3,0) \}. \] Thus, the number of single cosets in [01302] is \( |N|/|N^{(01)302}| = 168/3 = 56. \)

The orbits of \( N^{(01)32} \) on \( T \) are \( \{ 0 \}, \{ 1 \}, \{ 2 \}, \{ 3 \}, \{ 4 \}, \{ 5 \}, \) and \( \{ 6 \} \). So we consider the double cosets \( N_{t_0 t_1 t_3 t_2 t_0 N}, N_{t_0 t_1 t_3 t_2 t_1 N}, N_{t_0 t_1 t_3 t_2 t_2 N}, N_{t_0 t_1 t_3 t_2 t_3 N}, N_{t_0 t_1 t_3 t_2 t_4 N}, \) and \( N_{t_0 t_1 t_3 t_2 t_6 N} \). However, \( N_{t_0 t_1 t_3 t_2 t_2 N} = N_{t_0 t_1 t_3 N} = [013] \), and \( N_{t_0 t_1 t_3 t_2 t_1 N} = N_{t_0 t_1 t_2 t_6 t_1 N} = [01261] \) since relation (4.5) conjugated by \( (2,3,4,0)(5,6) \) gives:

\[
(1,2,3)(4,5,0) t_4 t_3 t_1 t_2 t_3 = (1,2,3)(4,5,0) t_4 t_3 (5,4,0)(1,3,2) t_2 t_1 t_3 t_2 t_1
\]

\[ \Rightarrow (1,2,3)(4,5,0) t_4 t_3 t_1 t_2 t_3 = (1,2,3)(4,5,0)(5,4,0)(1,3,2) t_0 t_2 t_1 t_3 t_2 t_1 \]

\[ \Rightarrow (1,2,3)(4,5,0) t_4 t_3 t_1 t_2 t_3 = t_0 t_1 t_3 t_2 t_1 \]

\[ \Rightarrow N_{t_0 t_1 t_3 t_2 t_1} = N_{t_4 t_3 t_1 t_2 t_3} \in [01261] \]

\[ \Rightarrow (1,2,3)(4,5,0) t_4 t_3 t_1 t_2 t_3 \in [01321] = [01261]. \]

Also, \( N_{t_0 t_1 t_3 t_2 t_3 N} = N_{t_0 t_1 t_2 t_3 t_2 N} = [01232] \) since \( t_0 t_1 t_3 t_2 t_3 = t_0 t_1 t_2 t_3 t_2 \), by (4.4); \( N_{t_0 t_1 t_3 t_2 t_4 N} = N_{t_0 t_1 t_3 t_2 t_4} = [01243] \) since \( t_0 t_1 t_3 t_2 t_4 t_4 = (1,0,5,2)(3,4)(t_0 t_1 t_2 t_4 t_3)^{(1,2,3)(4,5,0)} = (1,0,5,2)(3,4)(t_4 t_3 t_5 t_4 t_1, N_{t_0 t_1 t_3 t_2 t_5 N} = N_{t_0 t_1 t_2 t_4 t_6 N} = [01246] \) since \( t_0 t_1 t_3 t_2 t_5 = (1,0,6,2,3)(5,0)(t_0 t_1 t_2 t_4 t_0)^{(1,2,0)(3,4,6)} = (1,0,6,2,3)(5,0)(t_1 t_2 t_4 t_3, N_{t_0 t_1 t_3 t_2 t_6 N} = [01302] \) since \( t_0 t_1 t_3 t_2 t_6 = (1,6)(2,0,3,4)(t_0 t_1 t_3 t_2 t_0)^{(1,2,0)(4,6,5,3)} = (1,6)(2,0,3,4)(t_4 t_2 t_1 t_4 t_0}. \] The only new double coset is [01320]. \( N_{t_0 t_1 t_3 t_2 t_0 (3,5)(6,0)} = N_{t_4 t_1 t_5 t_2 t_0} = N_{t_0 t_1 t_3 t_2 t_0} \Rightarrow N^{(01)320} = \{ e, (3,5)(6,0) \}. \) Thus, [01320] contains 84 single cosets.

The orbits of \( N^{(01)220} \) on \( T \) are \( \{ 0,1 \}, \{ 2 \}, \{ 3 \}, \{ 4,6 \}, \) and \( \{ 5 \} \). So we consider the double cosets \( N_{t_0 t_1 t_0 t_2 t_0 t_0 N}, N_{t_0 t_1 t_0 t_2 t_0 t_1 N}, N_{t_0 t_1 t_0 t_2 t_0 t_2 N}, N_{t_0 t_1 t_0 t_2 t_0 t_4 N}, \) and \( N_{t_0 t_1 t_0 t_2 t_0 t_5 N} \). However, \( N_{t_0 t_1 t_0 t_2 t_0 t_0 N} = N_{t_0 t_1 t_0 t_2 N} = [0102] \), and \( N_{t_0 t_1 t_0 t_2 t_0 t_2 N} =
\[ N_{t_0t_1t_2t_0N} = [0120] \] since by (4.4) \( t_0t_1t_0t_2t_0t_2 = t_0t_1t_2t_0t_2t_2 = t_0t_1t_2t_0. \)

The only new double cosets are \([010203], [010204], \) and \([010205].\) \( N_{t_0t_1t_2t_0t_3(1,0)(4,6)} = N_{t_0t_1t_2t_1t_3} = N_{t_0t_1t_0t_2t_0t_3}, \) since

\[
t_1t_0t_1t_2t_1t_3 = t_0t_1t_0t_2t_1t_3, \text{ by (4.4)}
\]

\[ \Rightarrow \quad t_1t_0t_1t_2t_1t_3 = t_0t_1(6,4,3)(2,0,1)t_2t_0t_1t_2t_0t_3, \text{ by (4.5)} \]

\[ \Rightarrow \quad t_1t_0t_1t_2t_1t_3 = (6,4,3)(2,0,1)t_1t_2t_0t_1t_2t_0t_3 \]

\[ \Rightarrow \quad t_1t_0t_1t_2t_1t_3 = (6,4,3)(2,0,1)t_1t_0t_1t_2t_0t_3, \] and similarly, \( N_{t_0t_1t_0t_2t_3(1,0)(4,6)} = N_{t_0t_1t_2t_1t_5} = N_{t_0t_1t_0t_2t_0t_5} \]

\[ \Rightarrow \quad N^{(010203)} = N^{(010205)} = \{e, (1,0)(4,6)\}. \] Thus, there are 84 single cosets in \([010203]\) and \([010205].\)

The orbits of \( N^{(010223)} \) on \( T \) are \{0,1\}, \{2,3\}, \{4,6\}, and \{5,\}. So we consider the double cosets \( N_{t_0t_1t_0t_2t_3t_0N}, N_{t_0t_1t_0t_2t_3t_2N}, N_{t_0t_1t_0t_2t_3t_3N}, N_{t_0t_1t_0t_2t_3t_4N}, \) \( N_{t_0t_1t_0t_2t_3t_5N}. \) However, \( N_{t_0t_1t_0t_2t_3t_3N} = N_{t_0t_1t_0t_2t_2N} = [0102].\) The other four double cosets are new. \( N_{t_0t_1t_0t_2t_3t_2(1,0)(4,6)} = N_{t_0t_1t_0t_2t_3t_2} = N_{t_0t_1t_0t_2t_3t_2}, \) since \( t_1t_0t_1t_2t_3t_2 = t_0t_1t_0t_2t_3t_2 \) by (4.4) and \( N_{t_0t_1t_0t_2t_3t_5(1,0)(4,6)} = N_{t_0t_1t_0t_2t_3t_5} = N_{t_0t_1t_0t_2t_3t_5} \) since \( t_1t_0t_1t_2t_3t_5 = t_0t_1t_0t_2t_3t_5.\) Thus, \( N^{(010223)} = N^{(010235)} = \{e, (1,0)(4,6)\}. \)

So, there are 84 single cosets in \([010232] \) and \([010235].\) On the other hand, there are 168 single cosets in \([010230] \) and \([010234]\) since \( N^{(010230)} = N^{(010234)} = \{e\}. \)

The orbits of \( N^{(010242)} \) on \( T \) are \{0,\}, \{1,\}, \{2,\}, \{3,\}, \{4,\}, \{5\} and \{6,\}. So we consider the double cosets \( N_{t_0t_1t_0t_2t_4t_0N}, N_{t_0t_1t_0t_2t_4t_1N}, N_{t_0t_1t_0t_2t_4t_2N}, N_{t_0t_1t_0t_2t_4t_3N}, N_{t_0t_1t_0t_2t_4t_4N}, N_{t_0t_1t_0t_2t_4t_5N}, \) \( N_{t_0t_1t_0t_2t_4t_6N}. \) However, \( N_{t_0t_1t_0t_2t_4N} = N_{t_0t_1t_0t_2N} = [0102]; N_{t_0t_1t_0t_2t_4t_5N} = N_{t_0t_1t_2t_3N} = [0123], \) since \( t_0t_1t_0t_2t_4t_5 = (1,3,0)(2,6,5)(t_0t_1t_2t_3(1,2,4)(3,6,5) = (1,3,0)(2,6,5)t_0t_2t_4t_6; N_{t_0t_1t_0t_2t_4t_6N} = N_{t_0t_1t_0t_2t_3t_4N} = [010234] \) since \( t_0t_1t_0t_2t_4t_6 = (1,5,2)(3,4,6)(t_0t_1t_0t_2t_3t_4)^{(1,0,2)(3,6,4)} = (1,5,2)(3,4,6)t_2t_0t_2t_1t_4t_6. \) The other four double cosets are new. There are 168 single cosets in \([010240]\) since \( N^{(010240)} = \{e\}. \) \( N_{t_0t_1t_0t_2t_4t_1(1,6)(4,0)} = N_{t_0t_1t_0t_2t_4t_5} \Rightarrow N^{(010241)} = \{e, (1,6)(4,0)\}. \) \( N_{t_0t_1t_0t_2t_4t_2(2,4)(5,6)} = N_{t_0t_1t_0t_4t_4t_4} = N_{t_0t_1t_0t_2t_2t_2}, \) by (4.4) \( \Rightarrow N^{(010242)} = \{e, (2,4)(5,6)\}. \) Thus there are 84 single cosets in \([010241] \) and \([010242].\) Also, \( N_{t_0t_1t_0t_2t_3t_1(2,0)(3,4,6)} = N_{t_1t_2t_1t_0t_2t_4t_3} = N_{t_0t_1t_0t_2t_4t_3} \Rightarrow N^{(010243)} = \{e, (1,2,0)(3,4,6), (1,0,2)(3,6,4)\}, \) so \([010243]\) contains \( |N| / |N^{(010243)}| = 168/3 = 56 \) single cosets.

The orbits of \(N^{(01025)}\) on \(T\) are \(\{0,1\}, \{2\}, \{3\}, \{4,6\}, \) and \(\{5\}\). So we consider the double cosets \(Nt_0t_1t_0t_2t_5t_0N, Nt_0t_1t_0t_2t_5t_2N, Nt_0t_1t_0t_2t_5t_3N, Nt_0t_1t_0t_2t_5t_4N, \) and \(Nt_0t_1t_0t_2t_5t_5N. \) However, \(Nt_0t_1t_0t_2t_5t_3N = Nt_0t_1t_0t_2N = [0102] \) and \(Nt_0t_1t_0t_2t_5t_4N = Nt_0t_1t_0t_2t_4t_1N = [010241] \) since \(t_0t_1t_0t_2t_5t_3 = (1,6,2,3)(5,0)(t_0t_1t_0t_2t_4t_1)(1,6,4,0)(3,5) = (1,6,2,3)(5,0)t_1t_0t_2t_5t_6. \) The other three double cosets are new. There are 168 single cosets in [010250] and [010254] since \(N^{(010250)} = N^{(012025)} = \{e\}. \) \(Nt_0t_1t_0t_2t_5t_2(2,5)(4,6) = Nt_0t_1t_0t_5t_2t_5 = Nt_0t_1t_0t_2t_5t_2, by (4.4) \) and \(Nt_0t_1t_0t_2t_3t_2(1,5,0,2)(4,6) = Nt_2t_5t_2t_0t_1 = Nt_0t_1t_0t_2t_5t_2 = N^{(012022)} = ((2,5)(4,6), (1,5,0,2)(4,6)). \) Thus, there are \(|N|/|N^{(012025)}| = 168/8 = 21 \) single cosets in [010252].

The orbits of \(N^{(01302)}\) on \(T\) are \(\{0,1\}, \{2\}, \{3\}, \{4,6\}, \) and \(\{5\}\). So we consider the double cosets \(Nt_0t_1t_0t_3t_2t_0N, Nt_0t_1t_0t_3t_2t_2N, Nt_0t_1t_0t_3t_2t_3N, Nt_0t_1t_0t_3t_2t_4N, \) and \(Nt_0t_1t_0t_3t_2t_5N. \) However, \(Nt_0t_1t_0t_3t_2t_2N = Nt_0t_1t_0t_3N = [0103]; \) \(Nt_0t_1t_0t_3t_2t_3N = Nt_0t_1t_0t_3t_2N = [010232] \) since \(t_0t_1t_0t_3t_2t_3 = t_0t_1t_0t_2t_3, by (4.4); \) \(Nt_0t_1t_0t_3t_2t_4N = Nt_0t_1t_2t_5N = [0125] \) since \(t_0t_1t_0t_3t_2t_4 = (1,3,0)(2,6,5)(t_0t_1t_2t_5)(1,4,3)(2,4,0) = (1,3,0)(2,6,5)t_2t_4t_5. \) Thus the only new double cosets are [013020] and [013025]. There are 168 single cosets in [013020] since \(N^{(013020)} = \{e\}. \) \(Nt_0t_1t_0t_3t_2t_5(1,0)(4,6) = Nt_1t_0t_1t_3t_2t_5 = Nt_0t_1t_0t_3t_2t_5, by (4.4) \Rightarrow \(N^{(013025)} = \{e, (1,0)(4,6)\}. \) Thus, there are \(|N|/|N^{(013025)}| = 168/2 = 84 \) single cosets in [013025].

The orbits of \(N^{(01203)}\) on \(T\) are \(\{0,1\}, \{2\}, \{3\}, \{4,6\}, \) and \(\{5\}\). So we consider the double cosets \(Nt_0t_1t_2t_0t_3t_0N, Nt_0t_1t_2t_0t_3t_2N, Nt_0t_1t_2t_0t_3t_3N, Nt_0t_1t_2t_0t_3t_4N, \) and \(Nt_0t_1t_2t_0t_3t_5N. \) However, \(Nt_0t_1t_2t_0t_3t_3N = Nt_0t_1t_2t_0N = [0120]. \) The other four double cosets are new. There are 168 single cosets in [012030] and [012034] since \(N^{(012030)} = N^{(012034)} = \{e\}. \) \(Nt_0t_1t_2t_0t_4t_2(1,0)(4,6) = Nt_1t_0t_2t_1t_3t_2 = Nt_0t_1t_2t_0t_3t_2, \) since (4.1) conjugated with \((1,6,0,5,2,4,3)\) yields \(t_1t_0t_2t_1t_3t_2 = (6,3,4)(1,2,0)t_0t_1t_2t_0t_3t_2 \) and similarly, \(Nt_0t_1t_2t_0t_3t_5(1,0)(4,6) = Nt_1t_0t_2t_1t_3t_5 = Nt_0t_1t_2t_0t_3t_5 \Rightarrow \) \(N^{(012032)} = N^{(012035)} = \{e, (1,0)(4,6)\}. \) Thus there are \(|N|/|N^{(012032)}| = 168/2 = 84 \) single cosets in [012032] and [012035].

The orbits of \(N^{(01204)}\) on \(T\) are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4,5\}, \) and \(\{6\}\). So we consider the double cosets \(Nt_0t_1t_2t_0t_4t_0N, Nt_0t_1t_2t_0t_4t_1N, Nt_0t_1t_2t_0t_4t_2N, Nt_0t_1t_2t_0t_4t_3N, \) \(Nt_0t_1t_2t_0t_4t_4N, Nt_0t_1t_2t_0t_4t_5N, \) and \(Nt_0t_1t_2t_0t_4t_6N. \) However, \(Nt_0t_1t_2t_0t_4t_4N = Nt_0t_1t_2t_0N = [0120]. \) Also, \(Nt_0t_1t_2t_0t_4t_0N = Nt_0t_1t_0t_2t_4t_0N = [012040] \) since \(t_0t_1t_2t_0t_4t_0 = (1,6,2)(4,5,0)(t_0t_1t_0t_2t_4t_0)(1,0,5,3,4,2,6) = (1,6,2)(4,5,0)t_5t_0t_5t_6t_2t_5; \)
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= (1,2,3,6)2t2t4t5t6; Nt0t1t2t0t4t5N = Nt0t1t2t034N = [012034] since
t0t1t2t04t6 = (1,5)(2,0)t0t1t2t03. The other two double cosets are new. There are 168 single cosets in [012043] since N(012043) = \{e\}. Nt0t1t2t04t5(2,0,5)(3,6,4) = Nt5t1t05t3t2 \Rightarrow N(012045) = \{e,(2,5,0)(3,4,6),(2,0,5)(3,6,4)\}. Thus there are |N|/|N(012045)| = 168/3 = 56 single cosets in [012045].

The orbits of N(01205) on T are \{0,1\}, \{2\}, \{3\}, \{4,6\}, and \{5\}. So we consider the double cosets Nt0t1t2t05t0N, Nt0t1t2t05t2N, Nt0t1t2t05t3N, Nt0t1t2t05t4N, and Nt0t1t2t05t5N. However, Nt0t1t2t05t5N = Nt0t1t2t00N = [0120]. Also, Nt0t1t2t05t0N = Nt0t1t05t0t6N = [012050] since t0t1t2t05t0 = (2,0,5)(3,6,4)(t0t1t2t05t6)(1,0,2,5)(3,6) = (2,0,5)(3,6,4)t2t2t5t1t2; Nt0t1t2t05t2N = Nt0t1t2t1N = [0121] since t0t1t2t05t3 = (1,5,2)(3,4,6)(t0t1t2t1)(1,6)(2,4,3,6) = 1,5,2(3,4,6)t2t2t4t6; Nt0t1t2t05t4N = Nt0t1t2t4N = [0124] since t0t1t2t05t4 = (1,5,2)(3,4,6)(t0t1t2t4)(1,3,5,0,2,4,6) = 1,5,2(3,4,6)t2t2t4t6. The only new double coset is [012052]. Nt0t1t2t05t2(1,0,2)(3,6,4) = Nt2t0t1t2t1 = Nt0t1t2t05t2 \Rightarrow N(012052) = \{e,(2,0)(3,4),(1,2)(3,6),1,0,4,6),(1,0,2)(3,6,4),(1,2,0)(3,4,6)\}. Thus, [012052] contains |N|/|N(012052)| = 168/6 = 28 single cosets.

The orbits of N(01210) on T are \{0,1,2\}, \{3,4,6\}, and \{5\}. So we consider the double cosets Nt0t1t2t1t0t0N, Nt0t1t2t1t0t3N, and Nt0t1t2t1t0t5N. However,
\[Nt0t1t2t1t0t0N = Nt0t1t2t1N = [0121].\] The other two double cosets are new.
\[Nt0t1t2t1t0t3(1,0)(4,6) = Nt1t0t2t01t3 = Nt0t1t2t1t0t3 \Rightarrow N(012103) = \{e,(1,0)(4,6)\}\] Thus, there are |N|/|N(012103)| = 168/2 = 84 single cosets in [012103]. Also,
\[Nt0t1t2t1t0t5(1,2,0)(3,4,6) = Nt1t0t2t02t5 = Nt0t1t2t1t0t5 \text{ and } Nt0t1t2t1t0t5(1,0)(4,6) = Nt1t0t2t01t5 = Nt0t1t2t1t0t5 \Rightarrow N(012105) = \{e,(1,0,2)(3,6,4),(1,2,0)(3,4,6),(2,0)(3,4),(1,0)(4,6),(1,2)(3,6)\}.\] Thus, there are |N|/|N(012105)| = 168/6 = 28 single cosets in [012105].

The orbits of N(01213) on T are \{0\}, \{1\}, \{2\}, \{3\}, \{5\}, and \{6\}. So we consider the double cosets Nt0t1t2t1t3t0N, Nt0t1t2t1t3t1N, Nt0t1t2t1t3t2N, Nt0t1t2t1t3t3N, Nt0t1t2t1t3t4N, Nt0t1t2t1t3t5N, and Nt0t1t2t1t3t6N. However, Nt0t1t2t1t3t3N = Nt0t1t2t1N = [0121]. Also, Nt0t1t2t1t3t0N = Nt0t1t2t04t3N = [012043] since
\[ t_0 t_1 t_2 t_3 t_0 = (1, 4, 2)(3, 5, 6)(t_0 t_1 t_2 t_3 t_4 t_5)^{(1,2,3,6)(4,5)} = (1, 4, 2)(3, 5, 6)t_0 t_2 t_3 t_0 t_5 t_6; \]
\[ N t_0 t_1 t_2 t_3 t_1 N = N t_0 t_1 t_2 t_3 t_0 N = [010203] \text{ since } t_0 t_1 t_2 t_3 t_1 = (2, 3, 4, 0)(5, 6)(t_0 t_1 t_2 t_3 t_0)(1,5,3,4)(2,6) = (2, 3, 4, 0)(5, 6)t_0 t_3 t_0 t_0 t_4 t_1; \]
\[ N t_0 t_1 t_2 t_3 t_2 t_2 N = N t_0 t_1 t_2 t_3 t_0 N = [010320] \text{ since } t_0 t_1 t_2 t_3 t_2 = (1, 2, 3)(4, 5, 0)(t_0 t_1 t_2 t_3 t_0)(0,1,2,3,4,5,6) = (1, 2, 3)(4, 5, 0)t_1 t_2 t_3 t_4 t_3 t_1. \]

The double cosets \([012134], [012135], [012136]\) are new and each one contains 168 single cosets since \(N^{(012134)} = N^{(012135)} = N^{(012136)} = \{e\} \]

The orbits of \(N^{(012115)}\) on \(T\) are \(\{0\}, \{1, 2\}, \{3, 6\}, \{4\}, \{5\}. \)
So we consider the double cosets \(N t_0 t_1 t_2 t_1 t_5 t_0 N, N t_0 t_1 t_2 t_1 t_5 t_1 N, N t_0 t_1 t_2 t_1 t_5 t_2 N, N t_0 t_1 t_2 t_1 t_5 t_4 N, \) and \(N t_0 t_1 t_2 t_1 t_5 t_5 N. \)

However, \(N t_0 t_1 t_2 t_1 t_5 t_6 N = N t_0 t_1 t_2 t_1 t_5 t_3 N = [010325] \text{ since } t_0 t_1 t_2 t_1 t_5 t_1 = (2, 0, 5)(3, 6, 4)(t_0 t_1 t_0 t_3 t_5)^{(1,3,4)(2,6)} = (2, 0, 5)(3, 6, 4)t_0 t_3 t_0 t_4 t_6 t_5 t_3; \]
\(N t_0 t_1 t_2 t_1 t_5 t_3 N = N t_0 t_1 t_0 t_3 t_4 N = [010234] \text{ since } t_0 t_1 t_2 t_1 t_5 t_3 = (1, 2, 4)(5, 6, 0)(t_0 t_1 t_2 t_3 t_4 t_5)^{(2,3,6)(4,0,5)} = (1, 2, 4)(5, 6, 0)t_3 t_1 t_5 t_3 t_6 t_0; \]
\(N t_0 t_1 t_2 t_1 t_5 t_4 N = N t_0 t_1 t_2 t_3 t_5 t_3 = N t_0 t_1 t_2 t_3 t_5 t_6 N = [012035] \text{ since } t_0 t_1 t_2 t_1 t_5 t_4 = (1, 2, 3)(0, 5, 6)(t_0 t_1 t_2 t_3 t_5)^{(1,3,4)(2,6)} = (1, 2, 3)(0, 5, 6)t_3 t_0 t_4 t_6 t_5 t_3. \)
The only new double coset is \([012150]. \)

\(N t_0 t_1 t_2 t_1 t_5 t_0(1, 2)(3, 6) = N t_0 t_1 t_2 t_1 t_5 t_0 = N t_0 t_1 t_2 t_1 t_5 t_0, \text{by } (4, 4) \Rightarrow N^{(012150)} = \{e, (1, 2)(3, 6)\}. \)

Thus \([012150]\) contains 84 single cosets.

The orbits of \(N^{(012120)}\) on \(T\) are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \) and \(\{6\}. \)
So we consider the double cosets \(N t_0 t_1 t_2 t_3 t_0 t_0 N, N t_0 t_1 t_2 t_3 t_0 t_1 N, N t_0 t_1 t_2 t_3 t_0 t_2 N, N t_0 t_1 t_2 t_3 t_0 t_3 N, \) and \(N t_0 t_1 t_2 t_3 t_0 t_4 N, \) and \(N t_0 t_1 t_2 t_3 t_0 t_5 N. \)

However, \(N t_0 t_1 t_2 t_3 t_0 t_0 N = N t_0 t_1 t_2 t_3 t_0 N = [01203] \text{ since } t_0 t_1 t_2 t_3 t_0 t_1 = (1, 6)(2, 0, 3, 4)(t_0 t_1 t_0 t_2 t_3 t_0)(2,4)(3,6,0,5) = (1, 6)(2, 0, 3, 4)t_3 t_1 t_5 t_4 t_6 t_0; \]
\(N t_0 t_1 t_2 t_3 t_0 t_2 N = N t_0 t_1 t_2 t_3 t_0 t_0 N = [010320] \text{ since } t_0 t_1 t_2 t_3 t_0 t_0 = (1, 5, 6)(2, 0, 3)(t_0 t_1 t_0 t_3 t_2 t_0)^{(1,5,6)(2,0,3)} = (1, 5, 6)(2, 0, 3)t_3 t_5 t_3 t_2 t_0 t_3; \]
\(N t_0 t_1 t_2 t_3 t_0 t_3 N = N t_0 t_1 t_2 t_3 t_0 t_3 = N t_0 t_1 t_2 t_3 t_0 t_5 N = N t_0 t_1 t_2 t_3 t_0 t_0 N = [012030] \text{ since } t_0 t_1 t_2 t_3 t_0 t_5 = (1, 2, 6, 0, 4, 3, 5)(t_0 t_1 t_2 t_3 t_0)^{(1,5,6,2,4,0)} = (1, 2, 6, 0, 4, 3, 5)t_1 t_5 t_4 t_6 t_3. \)
The only new double coset is \([012304] \) and it contains 168 single cosets since \(N^{(012304)} = \{e\}. \)

The orbits of \(N^{(012131)}\) on \(T\) are \(\{0, 5\}, \{1, 2\}, \{3\}, \{4\}, \) and \(\{6\}. \)
So we consider the double cosets \(N t_0 t_1 t_2 t_3 t_0 t_0 N, N t_0 t_1 t_2 t_3 t_0 t_1 N, N t_0 t_1 t_2 t_3 t_0 t_2 N, N t_0 t_1 t_2 t_3 t_0 t_3 N, \) and \(N t_0 t_1 t_2 t_3 t_0 t_4 N. \)

However, \(N t_0 t_1 t_2 t_3 t_0 t_0 N = N t_0 t_1 t_2 t_3 t_0 N = [012043] \text{ since } t_0 t_1 t_2 t_3 t_0 t_0 = (1, 6, 3, 2)(4, 5)(t_0 t_1 t_2 t_3 t_0)^{(0,1,2,3,4,5,6)} = (1, 6, 3, 2)(4, 5)t_1 t_3 t_1 t_4 t_5 t_4; \]
\(N t_0 t_1 t_2 t_3 t_0 t_2 N = N t_0 t_1 t_2 t_3 t_0 N = [010203] \text{ since } t_0 t_1 t_2 t_3 t_1 t_3 = (1, 4, 6, 0)(2, 3)(t_0 t_1 t_2 t_3 t_0)^{(1,0,5,3,4,2,6)} = (1, 4, 6, 0)(2, 3)t_5 t_0 t_0 t_5 t_4; \)
\[ N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4} = N_{t_0t_1t_2t_3t_5t_0} = [012150] \] since \( t_0t_1t_2t_3t_4 = (1, 2, 6, 3)(4, 0)(t_0t_1t_2t_3t_4) \) 
\( = (1, 2, 6, 3)(4, 0)t_5t_1t_2t_3t_4t_6; \)

\[ N_{t_0t_1t_2t_3t_4t_5} = [010242] \] since \( t_0t_1t_2t_3t_4t_5 = (2, 6, 3)(4, 5, 0)(t_0t_1t_0t_2t_4t_2) \) 
\( = [1, 4, 0, 3, 6, 2, 5] = (2, 6, 3)(4, 5, 0)t_3t_4t_5t_6t_0. \)

The orbits of \( N^{[01232]} \) on \( T \) are \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, and \{6\}. So we consider the double cosets \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \), \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \), \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \), and \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \). However, \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \). Also, \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \). The only new double coset is \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \). So \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \). Thus, \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) contain 84 single cosets.

The orbits of \( N^{[01234]} \) on \( T \) are \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}. So we consider the double cosets \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \), \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \), \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \), and \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \). However, \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \). Also, \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \). The only new double coset is \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \). So \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \). Thus, \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) contain 84 single cosets.

The orbits of \( N^{[01235]} \) on \( T \) are \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}. So we consider the double cosets \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \), \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \), \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \), and \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \). However, \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \). Also, \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \). The only new double coset is \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \). So \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \) 
\( + N_{t_0t_1t_2t_3t_4t_5} \). Thus, \( N_{t_0t_1t_2t_3t_4t_5} = N_{t_0t_1t_2t_3t_4t_5} \) contain 84 single cosets.
\[ t_{0t1t2t3t4t5} = (1, 3, 2, 4, 6, 0, 5)(t_{0t1t2t1t3t5}(1, 6, 4, 0)(3, 5) = (1, 3, 2, 4, 6, 0, 5)t_{1t6t2t6t3t3}; \]
\[ N_{t_{0t1t2t3t4t5}} = N_{t_{0t1t2t0t4t3}} = [012043] \text{ since } t_{0t1t2t3t5t1} = (1, 5, 0, 3, 2, 6, 4)(t_{0t1t2t0t4t3}(1, 5, 0, 3, 2, 6, 4) = (1, 5, 0, 3, 2, 6, 4)t_{0t5t1t0t3t4}; \]
\[ N_{t_{0t1t2t3t5t2}} = N_{t_{0t1t2t4t2t3}} = [0124] \text{ since } t_{0t1t2t3t5t2} = (1, 4, 5, 3, 2, 6, 4)(t_{0t1t2t4t2t3}(1, 4, 5, 3, 2, 6, 4) = (1, 4, 5, 3, 2, 6, 4)t_{0t1t2t4t2t3}. \]

The only new double coset is [012354]. \( N_{t_{0t1t2t3t4t1}}(1, 5, 6)(2, 3, 4) = N_{t_{0t1t2t3t5t4}} = [012354] \) contains \( |N|/|N| = 168/3 = 56 \) single cosets.

The orbits of \( N^{(01240)} \) on \( T \) are \( \{0\}, \{1\}, \{2, 4\}, \{3\}, \{5\}, \text{ and } \{6\} \). So we consider the double cosets \( N_{t_{0t1t2t4t0t0}}, N_{t_{0t1t2t4t0t1}}, N_{t_{0t1t2t4t0t2}}, N_{t_{0t1t2t4t0t3}}, N_{t_{0t1t2t4t0t4}}, \text{ and } N_{t_{0t1t2t4t0t6}} \). However, \( N_{t_{0t1t2t4t0t0}} = N_{t_{0t1t2t4t0t1}} = [010243] \text{ since } t_{0t1t2t3t5t6} = (1, 5)(2, 4, 0, 3)(t_{0t1t2t3t5t6}(1, 5)(2, 4, 0, 3) = (1, 5)(2, 4, 0, 3)t_{2t4t2t3t1}. \]

The only new double coset is [012354]. \( N_{t_{0t1t2t3t4t1}}(1, 5, 6)(2, 3, 4) = N_{t_{0t1t2t3t5t4}} \) contains \( |N|/|N| = 168/3 = 56 \) single cosets.

The orbits of \( N^{(01241)} \) on \( T \) are \( \{0\}, \{1, 2, 4\}, \text{ and } \{3, 5, 6\} \). So we consider the double cosets \( N_{t_{0t1t2t4t1t0}}, N_{t_{0t1t2t4t1t1}}, N_{t_{0t1t2t4t1t2}}, N_{t_{0t1t2t4t1t3}} \). However, \( N_{t_{0t1t2t4t1t1}} = N_{t_{0t1t2t4t1t2}} = [012403] \text{ since } t_{0t1t2t4t0t0} = (1, 2, 4)(3, 5, 0)(t_{0t1t2t4t0t0}(1, 2, 4)(3, 5, 0) = (1, 2, 4)(3, 5, 0)t_{3t4t0t6t5}. \]

The only new double coset is [012403]. \( N_{t_{0t1t2t4t0t0t0}} = N_{t_{0t1t2t4t0t0t1}} = [012403] \text{ since } t_{0t1t2t4t0t0} = (1, 5)(2, 3, 0, 4)(t_{0t1t2t4t0t0}(1, 5)(2, 3, 0, 4) = (1, 5)(2, 3, 0, 4)t_{4t4t0t1t3t4}. \]

The orbits of \( N^{(01243)} \) on \( T \) are \( \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \text{ and } \{6\} \). So we consider the double cosets \( N_{t_{0t1t2t4t3t0}}, N_{t_{0t1t2t4t3t1}}, N_{t_{0t1t2t4t3t2}}, N_{t_{0t1t2t4t3t3}} \).
N_{t0t1t2t4t3t4N}, N_{t0t1t2t4t3t5N}, \text{ and } N_{t0t1t2t4t3t6N}. \text{ However, } N_{t0t1t2t4t3t3N} = N_{t0t1t2t4t3t4N} = [0124]. \text{ Also, } N_{t0t1t2t4t3t0N} = N_{t0t1t3t2N} = [0132] \text{ since } t_{0t1t2t4t3t0} = (1,4,5,3)(2,o)(t_{0t1t3t2})(1,3,2)(4,0,5) = (1,4,5,3)(2,o)t_{t5t3t2t1}; N_{t0t1t2t4t3t1N} = N_{t0t1t0t2t3t5N} = [010235] \text{ since } t_{0t1t2t4t3t1} = (1,6,4,5,3,0,2)(t_{0t1t0t2t3t5})^{(1,6)(4,0)} = (1,6,4,5,3,0,2)t_{t4t6t4t3t5}; N_{t0t1t2t4t3t2N} = N_{t0t1t2t0t4t5N} = [012045] \text{ since } t_{0t1t2t4t3t2} = (1,5,2,4,0,6,3)(t_{0t1t0t2t4t5})^{(1,6,2,5,0,3,4)} = (1,5,2,4,0,6,3)t_{3t6t5t3t1}; N_{t0t1t2t4t3t4N} = N_{t0t1t2t6N} = [0126] \text{ since } t_{0t1t2t4t3t4} = (2,0,5)(3,6,4)(t_{0t1t2t6})^{(2,0,5,3,6,4)} = (2,0,5)(3,6,4)t_{2t1t0t6}; N_{t0t1t2t4t3t5N} = N_{t0t1t2t5t6N} = [012354] \text{ since } t_{0t1t2t4t3t5} = (1,6,3,2)(4,5)(t_{0t1t2t4t5})^{(1,5)(2,3,4)} = (1,6,3,2)(4,5)t_{t6t1t2t4t5}; N_{t0t1t2t4t3t6N} = N_{t0t1t0t2t4t5N} = [010234] \text{ since } t_{0t1t2t4t3t6} = (2,4)(3,5,0,6)(t_{0t1t0t2t3t4})^{(0,1,2,3,4,5,6)} = (2,4)(3,5,0,6)t_{1t2t1t3t4t5}.

The orbits of N_{(01246)} on T are \{0\}, \{1,2\}, \{3\}, \{4\}, \{5\}, and \{6\}. So we consider the double cosets N_{t0t1t2t4t3t0N}, N_{t0t1t2t4t3t1N}, N_{t0t1t2t4t3t2N}, N_{t0t1t2t4t3t3N}, N_{t0t1t2t4t3t4N}, N_{t0t1t2t4t3t5N}, and N_{t0t1t2t4t3t6N}. \text{ However, } N_{t0t1t2t4t3t6N} = N_{t0t1t2t4t3t4N} = [0124]. \text{ Also, } N_{t0t1t2t4t3t0N} = N_{t0t1t2t0t3t4N} = [012034] \text{ since } t_{0t1t2t4t3t0} = (1,2,4)(3,6,5)(t_{0t1t2t0t3t4})^{(1,4,3,5)(2,6)} = (1,2,4)(3,6,5)t_{4t0t0t5t3}; N_{t0t1t2t4t3t1N} = N_{t0t1t0t2t0t5N} = [012025] \text{ since } t_{0t1t2t4t3t1} = (1,0)(2,6,5,4)(t_{0t1t0t2t0t5})^{(1,4,5,2,0,3,6)} = (1,0)(2,6,5,4)t_{3t4t3t0t3t2}; N_{t0t1t2t4t6t2N} = N_{t0t1t2t3t0t4N} = [012304] \text{ since } t_{0t1t2t4t6t2} = (3,5)(0,6)(t_{0t1t2t3t0t4})^{(2,6,4,5)(3,0)} = (3,5)(0,6)t_{3t1t4t3t0t5}; N_{t0t1t2t4t6t3N} = N_{t0t1t2t0t3t5N} = [012035] \text{ since } t_{0t1t2t4t6t3} = (2,4)(3,5,0,6)(t_{0t1t2t0t3t5})^{(1,4,0,3,6,2,5)} = (2,4)(3,5,0,6)t_{3t4t3t5t1t1}; N_{t0t1t2t4t6t4N} = N_{t0t1t0t2t4t5N} = [010254] \text{ since } t_{0t1t2t4t6t4} = (2,5,0)(3,4,0)(t_{0t1t0t2t4t5})^{(1,0)(2,3,6,4)} = (2,5,0)(3,4,0)t_{2t0t3t1t5t3}; N_{t0t1t2t4t6t5N} = N_{t0t1t3t2N} = [0132] \text{ since } t_{0t1t2t4t6t5} = (1,4,0,6)(2,5)(t_{0t1t3t2})^{(1,6,2)(3,6,4)} = (1,4,0,6)(2,5)t_{2t0t3t1}.

The orbits of N_{(01250)} on T are \{0,1\}, \{2,5\}, \{3\}, \{4\}, and \{6\}. So we consider the double cosets N_{t0t1t2t5t0t0N}, N_{t0t1t2t5t0t2N}, N_{t0t1t2t5t0t3N}, N_{t0t1t2t5t0t4N}, and N_{t0t1t2t5t0t6N}. \text{ However, } N_{t0t1t2t5t0t0N} = N_{t0t1t2t5t5N} = [0125]. \text{ Also, } N_{t0t1t2t5t0t2N} = N_{t0t1t0t2t5t0N} = [010250] \text{ since } t_{0t1t2t5t0t2} = (2,0,5)(3,6,4)(t_{0t1t0t2t5t0})^{(2,6)(5,0)} = (2,0,5)(3,6,4)t_{5t1t5t2t5t5}; N_{t0t1t2t5t0t3N} = N_{t0t1t0t2t5t5N} = [010235] \text{ since } t_{0t1t2t5t0t3} = (1,0,2)(3,6,4)(t_{0t1t0t2t5t5})^{(1,2,6)(4,0,3)} = (1,0,2)(3,6,4)t_{5t2t6t3t4}; N_{t0t1t2t5t0t4N} = N_{t0t1t0t2t4t1N} = [010241] \text{ since } t_{0t1t2t5t0t4} = (1,0)(2,6,5,4)(t_{0t1t0t2t4t1})^{(2,6)(3,4,5)} = (1,0)(2,6,5,4)t_{2t1t2t6t5t1}; N_{t0t1t2t5t0t6N} =
$$N_{t_0 t_1 t_2 t_3 t_2 N} = [010232] \text{ since } t_0 t_1 t_2 t_3 t_0 t_0 = (1, 0, 2)(3, 6, 4)(t_0 t_1 t_0 t_2 t_3 t_2)^{(1,5,6)(2,4,0)} = (1, 0, 2)(3, 6, 4)t_2 t_3 t_2 t_4 t_3 t_4.$$

The orbits of $N^{(01232)}$ on $T$ are $\{0\}, \{1\}, \{2, 5\}, \{3\},$ and $\{4, 6\}$. So we consider the double cosets $N t_0 t_1 t_2 t_3 t_2 N, N t_0 t_1 t_2 t_3 t_4 N, N t_0 t_1 t_2 t_3 t_2 N, N t_0 t_1 t_2 t_3 t_2 N,$ and $N t_0 t_1 t_2 t_3 t_2 t_4 N$. However, $N t_0 t_1 t_2 t_3 t_2 t_4 N = N t_0 t_1 t_2 t_3 t_2 t_4 N = [012150] \text{ since } t_0 t_1 t_2 t_3 t_2 t_4 = (t_0 t_1 t_2 t_3 t_0)^{(1,2,5,0)(3,4)} = t_1 t_2 t_3 t_2 t_4 t_1; N t_0 t_1 t_2 t_3 t_4 N = N t_0 t_1 t_2 t_3 t_2 t_4 t_5 N = [012105] \text{ since } t_0 t_1 t_2 t_3 t_2 t_1 = (t_0 t_1 t_2 t_1 t_0 t_5)^{(1,5,0)(3,6,4)} = t_1 t_2 t_3 t_4 t_0; N t_0 t_1 t_2 t_3 t_2 t_3 N = N t_0 t_1 t_2 t_3 t_2 t_4 t_5 N = [010205] \text{ since } t_0 t_1 t_2 t_3 t_2 t_3 = (1, 0, 2)(2, 5, 3)(t_0 t_1 t_0 t_2 t_3)^{(1,5,2)(4,5,0)} = (1, 0, 6)(2, 5, 3)t_0 t_1 t_0 t_2 t_3 t_5; N t_0 t_1 t_2 t_3 t_2 N = N t_0 t_1 t_2 t_3 t_2 t_4 N = [012134] \text{ since } t_0 t_1 t_2 t_3 t_2 t_4 = (1, 2, 3, 6)(4, 5)(t_0 t_1 t_2 t_1 t_3 t_4)^{(1,0)(2,6,5,4)} = (1, 2, 3, 6)(4, 5)t_1 t_6 t_0 t_3 t_2.$$

The orbits of $N^{(01260)}$ on $T$ are $\{0\}, \{4\}, \{1, 6\}, \{2\}, \{3\},$ and $\{5\}$. So we consider the double cosets $N t_0 t_1 t_2 t_3 t_2 N, N t_0 t_1 t_2 t_3 t_0 N, N t_0 t_1 t_2 t_3 t_2 t_0 N, N t_0 t_1 t_2 t_3 t_2 t_0 t_3 N,$ and $N t_0 t_1 t_2 t_3 t_0 t_5 N$. However, $N t_0 t_1 t_2 t_3 t_0 t_5 N = N t_0 t_1 t_2 t_3 t_0 t_4 N = [010204] \text{ since } t_0 t_1 t_2 t_3 t_0 t_1 = (1, 4, 5, 3)(2, 0)(t_0 t_1 t_0 t_2 t_4)^{(1,4,2)(3,5,0)} = (1, 4, 5, 3)(2, 0)t_0 t_4 t_3 t_2 t_1; N t_0 t_1 t_2 t_3 t_0 t_2 N = N t_0 t_1 t_2 t_3 t_0 t_4 t_1 N = [012401] \text{ since } t_0 t_1 t_2 t_3 t_0 t_2 = (t_0 t_1 t_2 t_3 t_4)^{(1,0)(4,6)} = t_1 t_0 t_1 t_2 t_3 t_0; N t_0 t_1 t_2 t_3 t_0 t_3 N = N t_0 t_1 t_2 t_3 t_0 t_4 N = [010235] \text{ since } t_0 t_1 t_2 t_3 t_0 t_5 = (1, 5, 2, 0)(3, 6)(t_0 t_1 t_0 t_3 t_2)^{(2,6,5,0)} = (1, 5, 2, 0)(3, 6)t_6 t_0 t_3 t_2 t_3.$$

The orbits of $N^{(01261)}$ on $T$ are $\{0\}, \{1\}, \{2, 5\}, \{3\}, \{4\}, \{5\},$ and $\{6\}$. So we consider the double cosets $N t_0 t_1 t_2 t_3 t_2 t_0 N, N t_0 t_1 t_2 t_3 t_2 t_0 t_1 N, N t_0 t_1 t_2 t_3 t_2 t_0 t_2 N, N t_0 t_1 t_2 t_3 t_2 t_0 t_3 N,$ $N t_0 t_1 t_2 t_3 t_0 t_6 N, N t_0 t_1 t_2 t_3 t_2 t_0 t_5 N,$ and $N t_0 t_1 t_2 t_3 t_2 t_0 t_5 N$. However, $N t_0 t_1 t_2 t_3 t_2 t_0 t_5 N = N t_0 t_1 t_2 t_3 t_2 t_0 t_4 N = [010216] \text{ since } t_0 t_1 t_2 t_3 t_2 t_0 t_1 = (1, 2, 3, 4, 5, 6, 7)(t_0 t_1 t_2 t_4 t_3)^{(1,6)(2,0,3,4)} = (1, 2, 3, 4, 5, 6, 7)t_3 t_6 t_0 t_2 t_3 t_4; N t_0 t_1 t_2 t_3 t_2 t_0 t_3 N = N t_0 t_1 t_2 t_3 t_2 t_0 t_4 N = \ [0132] \text{ since } t_0 t_1 t_2 t_3 t_2 t_2 = (1, 6, 2)(4, 5, 0)(t_0 t_1 t_3 t_2)^{(4,0)} = (1, 6, 2)(4, 5, 0)t_4 t_2 t_1 t_6; N t_0 t_1 t_2 t_3 t_2 t_3 N = N t_0 t_1 t_2 t_3 t_0 t_4 N = [010204] \text{ since } t_0 t_1 t_2 t_3 t_0 t_3 = (1, 4, 6)(2, 3, 5)(t_0 t_1 t_0 t_2 t_4)^{(1,3,5)(2,0,6)} = (1, 4, 6)(2, 3, 5)t_0 t_3 t_4 t_0 t_4 t_4; N t_0 t_1 t_2 t_3 t_0 t_4 N = N t_0 t_1 t_2 t_3 t_0 t_4 N = [010234] \text{ since } t_0 t_1 t_2 t_3 t_0 t_3 = (1, 3, 5)(2, 6, 0)(t_0 t_1 t_3 t_0 t_4)^{(1,4,2)(3,0,5)} = (1, 3, 5)(2, 6, 0)t_3 t_4 t_0 t_3 t_2; N t_0 t_1 t_2 t_3 t_0 t_4 N = N t_0 t_1 t_2 t_3 t_0 t_4 N = [010320] \text{ since } t_0 t_1 t_2 t_3 t_0 t_5 = (1, 6, 2)(4, 5, 0)(t_0 t_1 t_3 t_0 t_4)^{(2,6,0)(3,4,5)} = (1, 6, 2)(4, 5, 0)t_3 t_2 t_4 t_6 t_2.$
168 single cosets since $N^{(012615)} = \{e\}$.

The orbits of $N^{(01262)}$ on $T$ are $\{0\}$, $\{1\}$, $\{2,6\}$, $\{3\}$, and $\{4,5\}$. So we consider the double cosets $Nt_{0t_1t_2t_3t_4t_0}N$, $Nt_{0t_1t_2t_3t_2t_1}N$, $Nt_{0t_1t_2t_3t_2t_2}N$, $Nt_{0t_1t_2t_3t_2t_3}N$, and $Nt_{0t_1t_2t_3t_2t_4}N$. However, $Nt_{0t_1t_2t_3t_2t_0}N = Nt_{0t_1t_2t_3t_4t_0}N = [010242]$ since $t_{0t_1t_2t_3t_2t_0} = (t_{0t_1t_2t_3t_2t_0})^{(1,0)(2,6,5,4)}$; $Nt_{0t_1t_2t_3t_2t_1}N = Nt_{0t_1t_2t_3t_2t_1}N = [012321]$ since $t_{0t_1t_2t_3t_2t_1} = (1,2,6)(4,0,5)(t_{0t_1t_2t_3t_2t_1})^{(1,6,3)(4,5)} = (1,2,6)(4,0,5)t_{0t_1t_2t_3t_2t_1}$. 

The orbits of $N^{(01320)}$ on $T$ are $\{0,1,3\}$, $\{2\}$, and $\{4,5,6\}$. So we consider the double cosets $Nt_{0t_1t_3t_0t_2t_0}N$, $Nt_{0t_1t_3t_0t_2t_0}N$, $Nt_{0t_1t_3t_0t_3t_2}N$, and $Nt_{0t_1t_3t_0t_3t_3}N$, and $Nt_{0t_1t_3t_0t_4t_0}N$ and $Nt_{0t_1t_3t_0t_3t_4}N$. However, $Nt_{0t_1t_3t_0t_2t_0}N = Nt_{0t_1t_3t_0t_2t_0}N = [013032]$ since $t_{0t_1t_3t_0t_2t_0} = (1,0,2)(3,6,4)(t_{0t_1t_3t_0t_2t_0})^{(1,6,3,2)(4,5)} = (1,0,2)(3,6,4)t_{0t_1t_3t_0t_2t_0}$. 

The orbits of $N^{(01203)}$ on $T$ are $\{0,6\}$, $\{1\}$, $\{2\}$, $\{3,5\}$, and $\{4\}$. So we consider the double cosets $Nt_{0t_1t_3t_0t_2t_0}N$, $Nt_{0t_1t_3t_0t_2t_0}N$, $Nt_{0t_1t_3t_0t_3t_2}N$, and $Nt_{0t_1t_3t_0t_3t_3}N$, and $Nt_{0t_1t_3t_0t_4t_0}N$ and $Nt_{0t_1t_3t_0t_3t_4}N$. However, $Nt_{0t_1t_3t_0t_2t_0}N = Nt_{0t_1t_3t_0t_2t_0}N = [012032]$ since $t_{0t_1t_3t_0t_2t_0} = (1,0,2)(3,6,4)(t_{0t_1t_3t_0t_2t_0})^{(1,6,3,2)(4,5)} = (1,0,2)(3,6,4)t_{0t_1t_3t_0t_2t_0}$. 

The orbits of $N^{(01202)}$ on $T$ are $\{0,1\}$, $\{2\}$, $\{3\}$, $\{4,6\}$, and $\{5\}$. So we consider the double cosets $Nt_{0t_1t_3t_0t_2t_0}N$, $Nt_{0t_1t_3t_0t_2t_0}N$, $Nt_{0t_1t_3t_0t_3t_2}N$, and $Nt_{0t_1t_3t_0t_3t_3}N$, and $Nt_{0t_1t_3t_0t_4t_0}N$ and $Nt_{0t_1t_3t_0t_3t_4}N$. However, $Nt_{0t_1t_3t_0t_2t_0}N = Nt_{0t_1t_3t_0t_2t_0}N = [01020]$ since $t_{0t_1t_3t_0t_2t_0} = (1,3,2)(4,0,5)(t_{0t_1t_3t_0t_2t_0})^{(1,0,2,6,3,5)(4,5)} = (1,3,2)(4,0,5)t_{0t_1t_3t_0t_2t_0}$; $Nt_{0t_1t_3t_0t_3t_2}N = Nt_{0t_1t_3t_0t_3t_2}N = [01230]$ since $t_{0t_1t_3t_0t_3t_2} = (1,4)(3,6,5,0)(t_{0t_1t_3t_0t_3t_2})^{(1,2,3,4,5,6)(0,5)} = (1,4)(3,6,5,0)t_{0t_1t_3t_0t_3t_2}$.
\[(1, 2)(3, 5, 6, 0)(t_0 t_1 t_2 t_3 t_4)1,4,3,5(2,6) = (1, 2)(3, 5, 6, 0)t_0 t_1 t_2 t_3 t_4; N t_0 t_1 t_0 t_2 t_0 t_3 t_5 N = N t_0 t_1 t_2 t_3 t_1 N = [01231] \text{ since } t_0 t_1 t_0 t_2 t_0 t_3 t_5 = (1, 2)(3, 5, 6, 0)(t_0 t_1 t_2 t_3 t_1)1,4,3,5(2,6) = (1, 2)(3, 5, 6, 0)t_0 t_1 t_2 t_3 t_4.

The orbits of } N^{0102040} \text{ on } T \text{ are } \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \text{ and } \{6\}. \text{ So we consider the double cosets } N t_0 t_1 t_0 t_2 t_0 t_4 N, N t_0 t_1 t_0 t_2 t_0 t_4 t_1 N, N t_0 t_1 t_0 t_2 t_0 t_4 t_2 N, N t_0 t_1 t_0 t_2 t_0 t_4 t_3 N, N t_0 t_1 t_0 t_2 t_0 t_4 t_4 N, N t_0 t_1 t_0 t_2 t_0 t_4 t_5 N, \text{ and } N t_0 t_1 t_0 t_2 t_0 t_4 t_6 N. \text{ However, } N t_0 t_1 t_0 t_2 t_0 t_4 t_4 N = N t_0 t_1 t_0 t_2 t_0 t_4 t_5 N = [01202040]. \text{ Also, } N t_0 t_1 t_0 t_2 t_0 t_4 t_4 N = N t_0 t_1 t_0 t_2 t_0 t_4 t_4 N = [012040] \text{ since }
\begin{align*}
t_0 t_1 t_0 t_2 t_0 t_4 t_1 & = (2, 4)(3, 6, 0, 5)(t_0 t_1 t_2 t_3 t_4)2,4(5,6) = (2, 4)(3, 6, 0, 5)t_0 t_1 t_4 t_2 t_3; \\
N t_0 t_1 t_0 t_2 t_0 t_4 t_2 N & = N t_0 t_1 t_0 t_2 t_0 t_4 t_0 N = [01260] \text{ since } t_0 t_1 t_0 t_2 t_0 t_4 t_2 = (1, 0, 2)(3, 6, 4)(t_0 t_1 t_2 t_3 t_4)1,0,2,4,5,6 = (1, 0, 2)(3, 6, 4)t_1 t_4 t_2 t_4; \ N t_0 t_1 t_0 t_2 t_0 t_4 t_5 N = N t_0 t_1 t_0 t_2 t_0 t_4 t_6 N = [01261] \text{ since } t_0 t_1 t_0 t_2 t_0 t_4 t_5 = (1, 5, 6)(3, 0, 4)(t_0 t_1 t_2 t_3 t_4)1,3,5(2,6,0) = (1, 5, 6)(3, 0, 4)t_2 t_4 t_3 t_3. \text{ The other three double cosets are new. Now, } N t_0 t_1 t_0 t_2 t_0 t_4 t_0(1, 4)(6, 0) = N t_0 t_1 t_0 t_2 t_0 t_4 t_6 = N t_0 t_1 t_0 t_2 t_0 t_4 t_0 = (1, 4)(6, 0) \in N^{0102040} = \{e, (1, 4)(6, 0)\} \text{ and therefore, there are } |N|/|N^{0102040}| = 168/2 = 84 \text{ single cosets in } [0102040]. \text{ On the other hand } [0102043] \text{ and } [0102046], \text{ both contain 168 single cosets since } N^{0102043} = N^{0102046} = \{e\}.

The orbits of } N^{0102040} \text{ on } T \text{ are } \{0\}, \{1\}, \{2\}, \{3\}, \{4, 6\}, \text{ and } \{5\}. \text{ So we consider the double cosets } N t_0 t_1 t_0 t_2 t_0 t_3 t_5 N, N t_0 t_1 t_0 t_2 t_0 t_3 t_5 N, N t_0 t_1 t_0 t_2 t_0 t_3 t_5 N. \text{ However, } N t_0 t_1 t_0 t_2 t_0 t_3 t_5 N = N t_0 t_1 t_0 t_2 t_0 t_3 t_5 N = [01202040]. \text{ Also, } N t_0 t_1 t_0 t_2 t_0 t_3 t_5 N = N t_0 t_1 t_0 t_2 t_0 t_3 t_5 N = [01252] \text{ since } t_0 t_1 t_0 t_2 t_0 t_3 t_3 = (1, 5, 2)(3, 4, 6)(t_0 t_1 t_2 t_3 t_4)1,2,4(5,6,0) = (1, 5, 2)(3, 4, 6)t_3 t_4 t_3 t_4; \ N t_0 t_1 t_0 t_2 t_0 t_3 t_5 N = N t_0 t_1 t_0 t_2 t_0 t_3 t_5 N = [01246] \text{ since } t_0 t_1 t_0 t_2 t_0 t_3 t_4 = (1, 6, 2)(4, 5, 0)(t_0 t_1 t_2 t_3 t_4)1,4,0,2,5,6,3 = (1, 6, 2)(4, 5, 0)t_2 t_4 t_3 t_3. \text{ The other two double cosets are new. There are 168 single cosets in } [0102050] \text{ since } N^{0102050} = \{e\}. \text{ N t_0 t_1 t_0 t_2 t_0 t_3 t_2(1, 0)(4, 6) = N t_1 t_0 t_2 t_1 t_5 t_2 = N t_0 t_1 t_0 t_2 t_0 t_5 t_2. \text{ Then, } N^{0102052} = \langle (1, 0)(4, 6), (1, 2)(3, 6) \rangle \text{ which is of order 6, thus the number of single cosets in } [0102052] \text{ equals } |N|/|N^{0102052}| = 168/6 = 28.

The orbits of } N^{0102203} \text{ on } T \text{ are } \{0\}, \{1, 3\}, \{2\}, \{4\}, \{5\}, \text{ and } \{6\}. \text{ So we consider the double cosets } N t_0 t_1 t_0 t_2 t_3 t_0 t_6 N, N t_0 t_1 t_0 t_2 t_3 t_0 t_1 N, N t_0 t_1 t_0 t_2 t_3 t_0 t_2 N, N t_0 t_1 t_0 t_2 t_3 t_0 t_4 N, N t_0 t_1 t_0 t_2 t_3 t_0 t_5 N, \text{ and } N t_0 t_1 t_0 t_2 t_3 t_0 t_6 N. \text{ However, } N t_0 t_1 t_0 t_2 t_3 t_0 t_6 N = N t_0 t_1 t_0 t_2 t_3 t_0 N = [010230]. \text{ Also, } N t_0 t_1 t_0 t_2 t_3 t_0 t_1 N = N t_0 t_1 t_0 t_2 t_3 t_0 N = [01230] \text{ since }
\begin{align*}
t_0 t_1 t_0 t_2 t_3 t_0 t_1 & = (1, 5, 4, 2, 3, 6, 7)(t_0 t_1 t_2 t_3 t_0)1,0,4,2,6,3,5 = (1, 5, 4, 2, 3, 6, 7)t_4 t_0 t_6 t_3 t_4; \end{align*}
\[ N_{t_0 t_1 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_2 t_3 t_4 t_5} = [01320] \text{ since } t_0 t_1 t_2 t_3 t_4 t_5 = (1,0,2,5)(3,6)(t_0 t_1 t_2 t_3 t_4 t_5) = (1,0,2,5)(3,6) t_1 t_5 t_6 t_7 t_8; \]
\[ N_{t_0 t_1 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_2 t_3 t_4 t_5} = [012043] \text{ since } t_0 t_1 t_2 t_3 t_4 t_5 = (1,5)(3,4)(t_0 t_1 t_2 t_3 t_4 t_5) = (1,5)(3,4) t_2 t_3 t_4 t_5 t_6 = N_{t_0 t_1 t_2 t_3 t_4 t_5} = [0102050] \text{ since } t_0 t_1 t_2 t_3 t_4 t_5 = (1,5)(3,4) t_4 t_5 t_6 t_7; \]

The only new double coset is [0102304] and it contains 168 single cosets since \( N'_{0102304} = \{e\}. \)

The orbits of \( N'_{010223} \) on \( T \) are \( \{0,1\}, \{2\}, \{3\}, \{4,6\}, \) and \( \{5\}. \) So we consider the double cosets \( N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}, N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}, N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}, \) and \( N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}. \) However, \( N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = [010233]. \) Also, \( N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = [0102046] \text{ since } t_0 t_1 t_0 t_2 t_3 t_4 t_5 = (1,0,3)(4,6,5)(t_0 t_1 t_2 t_3 t_4 t_5) = (1,0,3)(4,6,5) t_6 t_7 t_8 t_9 t_{10}; \]
\[ N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = [01032] \text{ since } t_0 t_1 t_0 t_2 t_3 t_4 t_5 = t_0 t_1 t_0 t_2 t_3 t_4 t_5; \]
\[ N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = [01250] \text{ since } t_0 t_1 t_0 t_2 t_3 t_4 t_5 = (1,6,4)(2,5,3)(t_0 t_1 t_2 t_3 t_4 t_5) = (1,6,4)(2,5,3) t_6 t_7 t_8; \]

The only new double coset is [0102324] and it contains 168 single cosets since \( N'_{0102324} = \{e\}. \)

The orbits of \( N'_{010234} \) on \( T \) are \( \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \) and \( \{5\}. \) So we consider the double cosets \( N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}, N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}, N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}, \) and \( N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}. \) However, \( N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = [0102304] \text{ since } t_0 t_1 t_0 t_2 t_3 t_4 t_5 = (1,3,6,5,2,0,4)(t_0 t_1 t_0 t_2 t_3 t_4 t_5) = (1,3,6,5,2,0,4) t_6 t_7 t_8 t_9 t_{10}; \]
\[ N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = [0102324] \text{ since } t_0 t_1 t_0 t_2 t_3 t_4 t_5 = (1,5)(2,3,0,4)(t_0 t_1 t_0 t_2 t_3 t_4 t_5) = (1,5)(2,3,0,4) t_6 t_7 t_8 t_9 t_{10}; \]
\[ N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = [01215] \text{ since } t_0 t_1 t_0 t_2 t_3 t_4 t_5 = (1,5,6)(3,0,4)(t_0 t_1 t_0 t_2 t_3 t_4 t_5) = (1,5,6)(3,0,4) t_6 t_7 t_8 t_9 t_{10}; \]
\[ N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = [01024] \text{ since } t_0 t_1 t_0 t_2 t_3 t_4 t_5 = (2,0,5)(3,6,4)(t_0 t_1 t_0 t_2 t_3 t_4 t_5) = (2,0,5)(3,6,4) t_6 t_7 t_8 t_9 t_{10}; \]
\[ N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = [01243] \text{ since } t_0 t_1 t_0 t_2 t_3 t_4 t_5 = (1,3)(2,5,6,4)(t_0 t_1 t_0 t_2 t_3 t_4 t_5) = (1,3)(2,5,6,4) t_6 t_7 t_8 t_9 t_{10}; \]

The only new double coset is [0102346] and it contains 84 single cosets since \( N'_{0102346} = \{e, (1,2)(3,6)\}. \)

The orbits of \( N'_{010235} \) on \( T \) are \( \{0,1\}, \{2\}, \{3\}, \{4,6\}, \) and \( \{5\}. \) So we consider the double cosets \( N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}, N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}, N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}, \) and \( N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}. \) However, \( N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = \)
[01023]. Also, \( N_{t_0 t_1 t_2 t_3 t_5 t_0} = N_{t_0 t_1 t_2 t_3 t_4} = [0102043]\) since \( t_{01} t_0 t_2 t_3 t_5 t_0 = (0, 3, 2, 4, 6, 0, 5) t_{01} t_0 t_2 t_3 t_5 t_0 \) and \( N_{t_0 t_1 t_2 t_3 t_5 t_0} = [01260]\) since \( t_{01} t_0 t_2 t_3 t_5 t_2 = (1, 4, 5, 3)(2, 0)(t_{01} t_1 t_2 t_0 t_4)(2, 5, 3)(4, 6) = (1, 4, 5, 3)(2, 0) t_{01} t_1 t_2 t_0 t_4; N_{t_0 t_1 t_2 t_3 t_5 t_3} = N_{t_0 t_1 t_2 t_5 t_0} = [01250]\) since \( t_{01} t_0 t_2 t_3 t_5 t_2 = (2, 3, 5)(4, 0, 6)(t_{01} t_1 t_2 t_5 t_0)(1,4,5)(2,0,6) = (2, 3, 5)(4, 0, 6) t_0 t_4 t_0 t_1 t_6; N_{t_0 t_1 t_2 t_3 t_5 t_4} = N_{t_0 t_1 t_2 t_4 t_3 t_5} = [01243]\) since \( t_{01} t_0 t_2 t_3 t_5 t_4 = (1, 0, 4, 2, 6, 3, 5) t_0 t_4 t_0 t_1 t_3. \)

The orbits of \( N_{[010240]} \) on \( T \) are \{0\}, \{1\}, \{2, 3\}, \{4\}, \{5\}, and \{6\}. So we consider the double cosets \( N_{t_0 t_1 t_0 t_2 t_4 t_0 t_2 N}, N_{t_0 t_1 t_0 t_2 t_4 t_0 t_2 N}, N_{t_0 t_1 t_0 t_2 t_4 t_0 t_2 N}, \) and \( N_{t_0 t_1 t_0 t_2 t_4 t_0 t_2 N} = [01204]. \) Also, \( N_{t_0 t_1 t_0 t_2 t_4 t_0 t_1} = N_{t_0 t_1 t_2 t_4 t_0} = [01204]\) since \( t_{01} t_0 t_2 t_4 t_0 t_1 = (1, 0, 3)(2, 4, 0)(t_{01} t_1 t_2 t_0 t_4)(1,6,2,4,3,5) = (1, 0, 3)(2, 4, 0) t_0 t_4 t_1 t_3; N_{t_0 t_1 t_0 t_2 t_4 t_0 t_2} = N_{t_0 t_1 t_0 t_2 t_4 t_0 t_2} = [01240]\) since \( t_{01} t_0 t_2 t_4 t_0 t_2 = (1, 6, 5)(2, 0, 4)(t_0 t_1 t_2 t_4 t_0)(1,6,4) = (1, 6, 5)(2, 0, 4) t_0 t_4 t_1 t_3; N_{t_0 t_1 t_0 t_2 t_4 t_0 t_3} = N_{t_0 t_1 t_0 t_2 t_4 t_0 t_3} = [012040] since \( t_{01} t_0 t_2 t_4 t_0 t_3 = t_0 t_1 t_0 t_2 t_4 t_0 t_0; N_{t_0 t_1 t_0 t_2 t_4 t_0 t_4} = N_{t_0 t_1 t_0 t_2 t_4 t_0 t_4} = [0102040]\) since \( t_{01} t_0 t_2 t_4 t_0 t_4 = (3, 6)(5, 0)(t_0 t_1 t_0 t_2 t_4 t_0)(1,3,5,6,4,0,2) = (3, 6)(5, 0) t_0 t_4 t_2 t_1 t_3 t_2 t_0. \) The only new double coset is \([012406]. \) 

\( N_{t_0 t_1 t_0 t_2 t_4 t_0 t_6}(1, 6, 4, 2, 5, 3) = N_{t_0 t_0 t_1 t_0 t_4} = N_{t_0 t_1 t_0 t_2 t_4 t_0 t_6} \Rightarrow (1, 6, 4)(2, 5, 3) \in N_{[0102406]} \) Thus, \( N_{[0102406]} = \{e, (1, 6, 4)(2, 5, 3), (1, 4, 6)(2, 3, 5)\} \) and the number of single cosets in \([012046] \) is \( |N| / |N_{[0102406]}| = 168/3 = 56. \)

The orbits of \( N_{[010241]} \) on \( T \) are \{0\}, \{1\}, \{2\}, \{3\}, and \{5\}. So we consider the double cosets \( N_{t_0 t_1 t_0 t_2 t_4 t_1 t_0 N}, N_{t_0 t_1 t_0 t_2 t_4 t_1 t_1 N}, N_{t_0 t_1 t_0 t_2 t_4 t_1 t_2 N}, N_{t_0 t_1 t_0 t_2 t_4 t_1 t_3 N}, \) and \( N_{t_0 t_1 t_0 t_2 t_4 t_1 t_5 N}. \) However, \( N_{t_0 t_1 t_0 t_2 t_4 t_1 t_1 N} = N_{t_0 t_1 t_0 t_2 t_4 t_1 N} = [01024]. \) Also, \( N_{t_0 t_1 t_0 t_2 t_4 t_1 t_0 N} = N_{t_0 t_1 t_0 t_2 t_4 t_1 t_0} = (1,2)(3,0,6,5)(t_0 t_1 t_2 t_0 t_4)(1,6,5)(2,0,4) = (1,2)(3,0,6,5) t_0 t_2 t_4 t_2 t_1 t_2 N = N_{t_0 t_1 t_0 t_2 t_4 t_1 t_0 N} = [01260]\) since \( t_{01} t_0 t_2 t_4 t_1 t_2 = (t_0 t_1 t_2 t_0)(1,0)(4,6) = t_0 t_2 t_4 t_1; N_{t_0 t_1 t_0 t_2 t_4 t_1 t_3 N} = N_{t_0 t_1 t_0 t_2 t_4 t_1 t_3 N} = N_{t_0 t_1 t_0 t_2 t_4 t_1 t_3 N} = N_{t_0 t_1 t_0 t_2 t_4 t_1 t_3 N} = N_{t_0 t_1 t_0 t_2 t_4 t_1 t_5 N} = [01025]\) since \( t_{01} t_0 t_2 t_4 t_1 t_5 = (1, 0, 5, 2)(3, 4)(t_0 t_1 t_0 t_2 t_5)(1,4)(3,5) = (1, 0, 5, 2)(3, 4) t_0 t_4 t_0 t_2 t_3. \)

The orbits of \( N_{[010242]} \) on \( T \) are \{0\}, \{1\}, \{2, 4\}, \{3\}, and \{5, 6\}. So we consider
the double cosets $N_{t_0}t_1t_0t_2t_4t_2t_0N$, $N_{t_0}t_1t_0t_2t_4t_2t_1N$, $N_{t_0}t_1t_0t_2t_4t_2t_2N$,
$N_{t_0}t_1t_0t_2t_4t_2t_3N$, and $N_{t_0}t_1t_0t_2t_4t_2t_5N$. However, $N_{t_0}t_1t_0t_2t_4t_2t_2N = N_{t_0}t_1t_0t_2t_4t_2N = [01024]$. Also, $N_{t_0}t_1t_0t_2t_4t_2t_0N = N_{t_0}t_1t_0t_2t_4t_2t_0 = (2,5)(4,6)(t_0t_1t_0t_2t_4t_2)^{(1,4,2),(3,0,5)} = (2,5)(4,6)t_5t_4t_5t_1t_5t_2t_5; N_{t_0}t_1t_0t_2t_4t_2t_1N = N_{t_0}t_1t_0t_2t_4t_2N = [01262]$ since $t_0t_1t_0t_2t_4t_2t_1 = (t_0t_1t_0t_2t_4)^{(1,0),(4,6)} = t_1t_0t_2t_4t_2$;
$N_{t_0}t_1t_0t_2t_4t_2t_3N = N_{t_0}t_1t_2t_3t_1N = [01231]$ since $t_0t_1t_0t_2t_4t_2t_3 =
(1,2,4)(3,5,0)(t_0t_1t_2t_3t_4)^{(1,3,6,2,5,4)} = (1,2,4)(3,5,0)t_2t_6t_5t_3$. The only new double
coset is $[0102425]$. $N_{t_0}t_1t_0t_2t_4t_2t_5(2,3,5)(4,0,6) = N_{t_0}t_1t_0t_2t_4t_2t_5 
\Rightarrow (2,3,5)(4,0,6) \in N^{(0102425)}$. Thus, there are $|N|/|N^{(0102425)}| = 168/3 = 56$ single
cosets in $[0102425]$ since $N^{(0102425)} = \{e, (2,3,5)(4,7,6), (2,5,3)(4,6,7)\}$.

The orbits of $N^{(010243)}$ on $T$ are $\{0, 1, 2\}, \{3, 4, 6\}$, and $\{5\}$. So we consider the
double cosets $N_{t_0}t_1t_0t_2t_4t_3t_0N$, $N_{t_0}t_1t_0t_2t_4t_3t_3N$, and $N_{t_0}t_1t_0t_2t_4t_3t_5N$. However,
$N_{t_0}t_1t_0t_2t_4t_3t_3N = N_{t_0}t_1t_0t_2t_4t_3t_4N = [01024]$. Also, $N_{t_0}t_1t_0t_2t_4t_3t_0N = N_{t_0}t_1t_0t_2t_4t_3t_5N =
[01235]$ since $t_0t_1t_0t_2t_4t_3t_0 = (1,6,5)(2,0,4)(t_0t_1t_2t_3t_4)^{(1,4,2),(3,6,0)}$
$\Rightarrow (1,6,5)(2,0,4)t_3t_1t_6t_5$. The only new double coset is $[0102435]$.
$N_{t_0}t_1t_0t_2t_4t_3t_5(2,5,3)(4,6,0) = N_{t_4t_1t_4t_5t_0t_2t_3} = N_{t_0}t_1t_0t_2t_4t_3t_5$ and
$N_{t_0}t_1t_0t_2t_4t_3t_5(1,4,2)(3,0,5) = N_{t_3t_4t_1t_2t_0t_3} = N_{t_0}t_1t_0t_2t_4t_3t_5 
\Rightarrow (2,5,3)(4,6,0) \in N^{(0102435)} \Rightarrow N^{(0102435)} = ((2,5,3)(4,6,0), (1,4,2)(3,0,5))$, which is a
group of order 21, thus the number of single cosets in $[0102435]$ is equal to
$|N|/|N^{(0102435)}| = 168/21 = 8$.

The orbits of $N^{(010250)}$ on $T$ are $\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}$, and $\{6\}$. So we consider the
double cosets $N_{t_0}t_1t_0t_2t_5t_0t_0N$, $N_{t_0}t_1t_0t_2t_5t_0t_1N$, $N_{t_0}t_1t_0t_2t_5t_0t_2N$,
$N_{t_0}t_1t_0t_2t_5t_0t_3N$, $N_{t_0}t_1t_0t_2t_5t_0t_4N$, $N_{t_0}t_1t_0t_2t_5t_0t_5N$, and $N_{t_0}t_1t_0t_2t_5t_0t_6N$. However,
$N_{t_0}t_1t_0t_2t_5t_0t_0N = N_{t_0}t_1t_0t_2t_5t_0N = [01025]$. Also, $N_{t_0}t_1t_0t_2t_5t_0t_1N = N_{t_0}t_1t_0t_2t_5t_1N =
[01205]$ since $t_0t_1t_0t_2t_5t_0t_1 = (1,5)(2,0)(t_0t_1t_2t_0)^{(2,0,5),(3,6,4)} = (1,5)(2,0)t_5t_1t_0t_5t_2;\nN_{t_0}t_1t_0t_2t_5t_0t_2N = N_{t_0}t_1t_2t_5t_0N = [01250]$ since $t_0t_1t_0t_2t_5t_0t_2 =
(1,0,2)(3,6,4)(t_0t_1t_2t_0)^{(1,5,2,0),(3,6)} = (1,0,2)(3,6,4)t_3t_5t_0t_2; N_{t_0}t_1t_0t_2t_5t_0t_3N =
N_{t_0}t_1t_0t_2t_5t_0t_4N = [01205]$ since $t_0t_1t_0t_2t_5t_0t_3 = t_0t_1t_0t_2t_5t_0$. The other three double
cosets are new. There are 168 single cosets in $[0102503]$ and in $[0102504]$ since $N^{(0102503)} =
N^{(0102504)} = \{e\}$. But $N_{t_0}t_1t_0t_2t_5t_0t_6(1,4,2)(3,5,6) = N_{t_0}t_1t_0t_2t_5t_0t_3 = t_0t_1t_0t_2t_5t_0 \Rightarrow
(1,4,2)(3,5,6) \in N^{(0102506)} \Rightarrow N^{(0102506)} = \{e, (1,4,2)(3,5,6), (1,2,4)(3,6,5)\}$; thus the number of single cosets in $[0102506]$ is equal to $|N|/|N^{(0102506)}| = 168/3 = 56$. 
The orbits of $N^{(010252)}$ on $T$ are $\{0, 1, 2, 5\}$, $\{3\}$, and $\{4, 6\}$. So we consider the double cosets $Nt_{01}t_{02}t_{3}t_{2}t_{2}N$, $Nt_{01}t_{02}t_{3}t_{2}t_{3}t_{3}N$, and $Nt_{01}t_{02}t_{3}t_{2}t_{4}t_{2}N$. However, $Nt_{01}t_{02}t_{3}t_{2}t_{2}N = Nt_{02}t_{01}t_{2}t_{5} = [01025]$. The other two double cosets are new. $Nt_{01}t_{02}t_{3}t_{2}t_{3}(1,0)(4,6) = Nt_{1}t_{0}t_{1}t_{2}t_{3} = Nt_{01}t_{02}t_{3}t_{2}t_{3}$ and $Nt_{01}t_{02}t_{3}t_{2}t_{3}(1,2)(5,0) = Nt_{2}t_{0}t_{2}t_{1}t_{3} = Nt_{01}t_{02}t_{3}t_{2}t_{3} \Rightarrow (1,0)(4,6)$ and $(1,2)(5,0) \in N^{(0102523)} \Rightarrow N^{(0102523)} = \{(1,0)(4,6), (1,2)(5,0)\}$, which is a group of order 8, thus the number of single cosets in $[0102523]$ is equal to $|N|/|N^{(0102523)}| = 168/8 = 21$. Also, $Nt_{01}t_{02}t_{3}t_{2}t_{4}(1,0)(2,5) = Nt_{1}t_{0}t_{1}t_{2}t_{4}t_{2} = Nt_{01}t_{02}t_{3}t_{2}t_{4}$ and $Nt_{01}t_{02}t_{3}t_{2}t_{4}(1,2)(5,0) = Nt_{2}t_{0}t_{2}t_{1}t_{4} = t_{01}t_{02}t_{3}t_{2}t_{4} \Rightarrow (1,2)(5,0)$ and $(1,0)(2,5) \in N^{(0102524)} \Rightarrow N^{(0102524)} = \{(1,2)(5,0), (1,0)(2,5)\}$, which is a group of order 4, thus the number of single cosets in $[0102524]$ is equal to $|N|/|N^{(0102524)}| = 168/4 = 42$. The orbits of $N^{(010254)}$ on $T$ are $\{0\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, and $\{6\}$. So we consider the double cosets $Nt_{01}t_{02}t_{3}t_{2}t_{0}N$, $Nt_{01}t_{02}t_{3}t_{2}t_{1}N$, $Nt_{01}t_{02}t_{3}t_{2}t_{2}N$, $Nt_{01}t_{02}t_{3}t_{4}t_{2}t_{3}N$, $Nt_{01}t_{02}t_{3}t_{4}t_{2}t_{4}N$, $Nt_{01}t_{02}t_{3}t_{4}t_{2}t_{5}N$, and $Nt_{01}t_{02}t_{3}t_{4}t_{2}t_{6}N$. However, $Nt_{01}t_{02}t_{3}t_{4}t_{2}t_{5}t_{2}N = Nt_{01}t_{02}t_{3}t_{4}t_{2}t_{6}N = [01025]$. Also, $Nt_{01}t_{02}t_{3}t_{4}t_{2}t_{5}t_{2}t_{6}N = Nt_{01}t_{02}t_{3}t_{4}t_{2}t_{0}t_{4}N$ $= [0102504]$ since $t_{01}t_{02}t_{3}t_{4}t_{2} = (1,4,5,2,0,3,6)(t_{01}t_{02}t_{3}t_{4}t_{2}t_{0})^{(1,3,0)(2,4,6)} = (1,4,5,2,0,3,6)t_{1}t_{2}t_{3}t_{4}t_{5}$ and $Nt_{01}t_{02}t_{3}t_{4}t_{2}t_{5}t_{2}t_{6}t_{0}t_{4} = Nt_{01}t_{02}t_{3}t_{4}t_{2}t_{5}t_{2}t_{6}t_{0} = [01234]$ since $t_{01}t_{02}t_{3}t_{4}t_{2} = (1,0,5)(3,4,6)(t_{01}t_{02}t_{3}t_{4})^{(1,2)(3,6)} = (1,0,5)(3,4,6)t_{0}t_{2}t_{3}t_{4}$. The other two double cosets are new. $Nt_{01}t_{02}t_{3}t_{4}t_{2}t_{0}(1,2,5)(5,6,0) = Nt_{02}t_{02}t_{3}t_{4}t_{2}t_{0} = Nt_{01}t_{02}t_{3}t_{4}t_{2}t_{0} \Rightarrow (1,2,4)(5,6,0) \in N^{(0102540)} \Rightarrow N^{(0102540)} = \{(1,2,4)(5,6,0), (1,4,2)(5,0,6)\}$, thus the number of single cosets in $[0102540]$ is equal to $|N|/|N^{(0102540)}| = 168/3 = 56$. On the other hand there are 168 single cosets in $[0102541]$ since $N^{(0102541)} = \{e\}$.

The orbits of $N^{(010320)}$ on $T$ are $\{0\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, and $\{6\}$. So we consider the double cosets $Nt_{01}t_{02}t_{3}t_{2}t_{0}t_{0}N$, $Nt_{01}t_{02}t_{3}t_{2}t_{0}t_{1}N$, $Nt_{01}t_{02}t_{3}t_{2}t_{0}t_{2}N$, $Nt_{01}t_{02}t_{3}t_{2}t_{0}t_{3}N$, $Nt_{01}t_{02}t_{3}t_{2}t_{0}t_{4}N$, $Nt_{01}t_{02}t_{3}t_{2}t_{0}t_{5}N$, and $Nt_{01}t_{02}t_{3}t_{2}t_{0}t_{6}N$. However, $Nt_{01}t_{02}t_{3}t_{2}t_{0}N = Nt_{01}t_{02}t_{3}t_{2}t_{0} \Rightarrow (1,2,4)(5,6,0) \in N^{(0102540)} \Rightarrow N^{(0102540)} = \{(1,2,4)(5,6,0), (1,4,2)(5,0,6)\}$, thus the number of single cosets in $[0102540]$ is equal to $|N|/|N^{(0102540)}| = 168/3 = 56$. On the other hand there are 168 single cosets in $[0102541]$ since $N^{(0102541)} = \{e\}$.
\[ (1, 0, 2)(3, 6, 4)(t_0 t_1 t_2 t_3 t_4)_{(2,0,6)(3,5,4)} = (1, 0, 2)(3, 6, 4)t_5 t_1 t_0 t_2 t_4; \quad N t_0 t_1 t_2 t_3 t_0 t_3 N = N t_0 t_1 t_2 t_3 t_0 N = [01230] \text{ since } t_0 t_1 t_0 t_3 t_2 t_0 = (1, 0, 2)(3, 6, 4)(t_0 t_1 t_2 t_3 t_4)_{(1,4,5)(2,3,0)} = (1, 0, 2)(3, 6, 4)t_2 t_0 t_0 t_1 t_0 t_2 t_0 t_2. \]

Also, \( N t_0 t_1 t_0 t_2 t_3 t_0 N = N t_0 t_1 t_0 t_2 t_0 t_3 N = [01240] \text{ since } t_0 t_1 t_0 t_3 t_2 t_0 t_6 = (1, 0, 2)(3, 6, 4)(t_0 t_1 t_2 t_3 t_4)_{(1,6,4,5,3,0,2)} = (1, 0, 2)(3, 6, 4)t_2 t_0 t_1 t_3 t_2. \]

The only new double coset is \([013204]. \quad N t_0 t_1 t_0 t_3 t_2 t_0 t_4(1, 5, 4)(2, 6, 0) = N t_2 t_5 t_3 t_4 t_4 t_2 t_3 t_2 t_1 = N t_0 t_1 t_0 t_3 t_2 t_0 t_4 \Rightarrow (1, 5, 4)(2, 6, 0) \in N^{(013204)} \Rightarrow N^{(013204)} = \{ e, (1, 5, 4)(2, 6, 0), (1, 4, 5)(2, 6, 0) \}, \]

and the number of single cosets in \([013204] \) is equal to \( \| N \| / \| N^{(013204)} \| = 168/3 = 56. \]

The orbits of \( N^{(013204)} \) on \( T \) are \( \{ 0, 1 \}, \{ 2 \}, \{ 3 \}, \{ 4, 6 \}, \text{ and } \{ 5 \} \). So we consider the double cosets \( N t_0 t_1 t_0 t_3 t_2 t_3 t_0 N, N t_0 t_1 t_0 t_3 t_2 t_5 t_2 N, N t_0 t_1 t_0 t_3 t_2 t_5 t_3 N, N t_0 t_1 t_0 t_3 t_2 t_5 t_4 N, \text{ and } N t_0 t_1 t_0 t_3 t_2 t_5 t_5 N. \text{ However, } N t_0 t_1 t_0 t_3 t_2 t_5 t_5 N = N t_0 t_1 t_0 t_3 t_2 N = [013204]. \] Also, \( N t_0 t_1 t_0 t_3 t_2 t_5 t_0 N = N t_0 t_1 t_0 t_3 t_2 t_5 t_4 N = [012046] \text{ since } t_0 t_1 t_0 t_3 t_2 t_5 t_0 = (1, 6, 0, 4)(2, 5)(t_0 t_1 t_0 t_3 t_2 t_5 t_0)_{(1,4,6)(2,3,5)} = (1, 6, 0, 4)(2, 5)t_0 t_0 t_3 t_2 t_5 t_0 t_1; \quad N t_0 t_1 t_0 t_3 t_2 t_5 t_2 N = N t_0 t_1 t_0 t_3 t_2 t_5 t_3 N = [012523] \text{ since } t_0 t_1 t_0 t_3 t_2 t_5 t_0 = t_0 t_1 t_0 t_3 t_2 t_5 t_3; \quad N t_0 t_1 t_0 t_3 t_2 t_5 t_3 N = N t_0 t_1 t_0 t_3 t_2 t_5 t_4 N = [01260] \text{ since } t_0 t_1 t_0 t_3 t_2 t_5 t_3 = (1, 5)(2, 3, 0, 4)(t_0 t_1 t_0 t_3 t_2 t_5 t_0)_{(1,0,4,6)(3,5)} = (1, 5)(2, 3, 0, 4)t_0 t_1 t_0 t_3 t_2 t_5 t_4; \quad N t_0 t_1 t_0 t_3 t_2 t_5 t_4 N = N t_0 t_1 t_0 t_3 t_2 t_5 t_4 = (1, 3, 0)(2, 6, 5)(t_0 t_1 t_2 t_1 t_5)_{(1,0)(2,4,3,0)} = (1, 3, 0)(2, 6, 5)t_2 t_6 t_4 t_5. \]

The orbits of \( N^{(012304)} \) on \( T \) are \( \{ 0 \}, \{ 1 \}, \{ 2 \}, \{ 3 \}, \{ 4 \}, \{ 5 \}, \text{ and } \{ 6 \}. \) So we consider the double cosets \( N t_0 t_1 t_0 t_2 t_3 t_0 t_0 N, N t_0 t_1 t_0 t_2 t_3 t_0 t_1 N, N t_0 t_1 t_0 t_2 t_3 t_0 t_2 N, N t_0 t_1 t_0 t_2 t_3 t_0 t_3 N, N t_0 t_1 t_0 t_2 t_3 t_0 t_4 N, N t_0 t_1 t_0 t_2 t_3 t_0 t_5 N, \text{ and } N t_0 t_1 t_0 t_2 t_3 t_0 t_6 N. \text{ However, } N t_0 t_1 t_0 t_2 t_3 t_0 t_6 N = N t_0 t_1 t_0 t_2 t_3 t_0 N = [01203]. \] Also, \( N t_0 t_1 t_0 t_2 t_3 t_0 t_1 N = N t_0 t_1 t_0 t_2 t_3 t_0 t_4 N = [0102304] \text{ since } t_0 t_1 t_0 t_2 t_3 t_0 t_1 = (1, 2, 4)(3, 0, 6)(t_0 t_1 t_0 t_2 t_3 t_0 t_4)_{(1,4,6)(2,3,5)} = (1, 2, 4)(3, 0, 6)t_0 t_4 t_0 t_3 t_2 t_0 t_6; \quad N t_0 t_1 t_0 t_2 t_3 t_0 t_2 N = N t_0 t_1 t_0 t_2 t_3 t_0 t_4 N = [012046] \text{ since } t_0 t_1 t_0 t_2 t_3 t_0 t_2 = (1, 3, 5, 4)(2, 0)(t_0 t_1 t_0 t_2 t_3 t_0)_{(1,4,3,2,0,5,0)} = (1, 3, 5, 4)(2, 0)t_1 t_4 t_1 t_4 t_1 t_3 t_5; \quad N t_0 t_1 t_0 t_2 t_3 t_0 t_3 N = N t_0 t_1 t_0 t_2 t_3 t_0 N = [01230] \text{ since } t_0 t_1 t_0 t_3 t_0 t_3 = t_0 t_1 t_2 t_3 t_0; \quad N t_0 t_1 t_0 t_2 t_3 t_0 t_4 N = [01234] \text{ since } t_0 t_1 t_2 t_3 t_0 t_4 = (1, 3, 2)(4, 0, 5)(t_0 t_1 t_2 t_3 t_4)_{(1,4,5,2,0,3,6)} = (1, 3, 2)(4, 0, 5)t_3 t_4 t_0 t_6 t_5; \quad N t_0 t_1 t_2 t_3 t_0 t_5 N = N t_0 t_1 t_2 t_3 t_0 t_6 N = [0102050] \text{ since } t_0 t_1 t_2 t_3 t_0 t_5 = (1, 3, 5, 4)(2, 0)(t_0 t_1 t_0 t_2 t_3 t_0)_{(2,4)(5,6)} = (1, 3, 5, 4)(2, 0)t_0 t_1 t_0 t_4 t_0 t_0; \quad N t_0 t_1 t_2 t_3 t_0 t_5 N = N t_0 t_1 t_2 t_3 t_0 t_6 N = [0120503] \text{ since } t_0 t_1 t_2 t_3 t_0 t_6 = (1, 0)(2, 4, 5, 6)(t_0 t_1 t_0 t_2 t_3 t_0)_{(1,4)(3,5)} = (1, 0)(2, 4, 5, 6)t_0 t_4 t_0 t_2 t_3 t_5. \]
The orbits of \( N^{(012302)} \) on \( T \) are \( \{0, 1\}, \{2, 3\}, \{4, 6\}, \) and \( \{5\} \). So we consider the double cosets \( Nt_0t_1t_2t_0t_3t_2t_0N, Nt_0t_1t_2t_0t_3t_2t_0N, Nt_0t_1t_2t_0t_3t_2t_0N, \) \( Nt_0t_1t_2t_0t_3t_2t_4N, \) and \( Nt_0t_1t_2t_0t_3t_55N \). However, \( Nt_0t_1t_2t_0t_3t_2N = Nt_0t_1t_2t_0t_3N = [01203] \). Also, \( Nt_0t_1t_2t_0t_3t_2t_0N = Nt_0t_1t_2t_0t_3N = (1, 6, 5)(3, 2, 0)(t_0t_1t_3t_0t_2) \) \( (1, 2, 3, 6)(4, 5) = (1, 6, 5)(3, 2, 0)t_0t_2t_0t_3; Nt_0t_1t_2t_0t_3t_2t_3N = Nt_0t_1t_2t_0t_3t_55N = [01320] \) since \( t_0t_1t_2t_0t_3t_2t_3 = (1, 4)(3, 6, 5, 0)(t_0t_1t_3t_2t_0) \) \( (1, 2, 3, 4, 5, 6, 0) = (1, 4)(3, 6, 5, 0)t_1t_2t_4t_3t_1; Nt_0t_1t_2t_0t_3N = [0102046] \) since \( t_0t_1t_2t_0t_3t_4t_2 = (1, 0, 4)(2, 3, 5)(t_0t_1t_0t_2t_0t_4t_6) \) \( (1, 5, 4, 3)(6, 0) = (1, 0, 4)(2, 3, 5)t_0t_5t_0t_3t_0t_3t_0; Nt_0t_1t_2t_0t_3t_5N = Nt_0t_1t_0t_2t_0t_4t_6N = [0102346] \) \( Nt_0t_1t_2t_0t_3t_4t_2N = [012032] \) since \( t_0t_1t_2t_0t_3t_4t_2 = (1, 6, 5)(2, 4, 3)(t_0t_1t_2t_4t_6) \) \( (1, 5, 3, 4)(2, 6) \) \( Nt_0t_1t_2t_0t_3t_4t_2N = (1, 6, 5)(2, 4, 3)t_0t_5t_0t_5t_6t_5; Nt_0t_1t_2t_0t_3t_4t_2N = [0120503] \) since \( t_0t_1t_2t_0t_3t_4t_2 = (1, 5, 6)(2, 3, 4)(t_0t_1t_0t_5t_0t_5) \) \( (1, 0, 2, 4, 5, 3, 6) = (1, 5, 6)(2, 3, 4)t_0t_2t_0t_3t_2t_6; Nt_0t_1t_2t_0t_3t_4t_3N = Nt_0t_1t_0t_2t_0t_4t_6N = [0102324] \) since \( t_0t_1t_2t_0t_3t_4t_3 = (1, 0, 4, 6)(3, 5)(t_0t_1t_0t_2t_3t_4t_6) \) \( (2, 6, 0, 3, 4, 5) = (1, 0, 4, 6)(3, 5)t_0t_1t_2t_0t_3t_4t_5N = [01232] \) since \( t_0t_1t_2t_0t_3t_4t_3 = (1, 2)(3, 5, 6, 0)(t_0t_1t_3t_2) \) \( (1, 0, 2, 4, 5, 3, 6) = (1, 2)(3, 5, 6, 0)t_0t_2t_0t_4t_6; Nt_0t_1t_2t_0t_3t_4t_6N = Nt_0t_1t_0t_2t_0t_4t_6N = [01204] \) since \( t_0t_1t_2t_0t_3t_4t_6 = (1, 5)(2, 0)t_0t_1t_2t_4t_0. \) The only new double coset is \([0120341]\). \( Nt_0t_1t_2t_0t_3t_4t_1(1, 2, 4)(5, 6, 0) = Nt_0t_2t_3t_5t_1t_2 = Nt_0t_1t_2t_0t_3t_4t_1 \Rightarrow (1, 2, 4)(5, 6, 0) \in N^{(0123041)} \). Then, \( N^{(0123041)} = \{e, (1, 2, 4)(5, 6, 0), (1, 4, 2)(5, 0, 6)\} \), and there are \(|N|/|N^{(0123041)}| = 168/3 = 56 \) single cosets in \([0120341]\).

The orbits of \( N^{(012305)} \) on \( T \) are \( \{0, 1\}, \{2, 3\}, \{4, 6\}, \) and \( \{5\} \). So we consider the double cosets \( Nt_0t_1t_2t_0t_3t_5t_0N, Nt_0t_1t_2t_0t_3t_5N, Nt_0t_1t_2t_0t_3t_5t_3N, \) \( Nt_0t_1t_2t_0t_3t_5t_1N, \) and \( Nt_0t_1t_2t_0t_3t_5t_5N \). However, \( Nt_0t_1t_2t_0t_3t_5N = Nt_0t_1t_2t_0t_3N = [01203] \). Also, \( Nt_0t_1t_2t_0t_3t_5N = Nt_0t_1t_2t_0t_3N = (1, 0, 2, 4, 3)(5, 0, 6) \) \( (1, 2, 4)(5, 6, 0)t_0t_1t_2t_2t_0t_3t_5t_2N = Nt_0t_1t_2t_0t_3t_5t_2N = [01262] \) \( Nt_0t_1t_2t_0t_3t_5t_2 = (1, 5, 2, 0)(3, 6)(t_0t_1t_2t_3t_2) \) \( (1, 5, 2, 0)(3, 6)t_0t_1t_0t_3t_3N = Nt_0t_1t_2t_0t_3t_5N = [01215] \) since \( t_0t_1t_2t_0t_3t_5t_3 \)
\[ (1,5,2)(3,4,6)(t_0t_1t_2t_3t_4t_5)^{(1,6)(2,4,3,0)} = (1,5,2)(3,4,6)t_2t_0t_4t_5t_4N = N_{t_0t_1t_2t_3t_4} = [01234] \text{ since } t_0t_1t_2t_3t_4 = (1,2)(3,5,6,0)(t_0t_1t_2t_3t_4)^{(1,0,2,4,5,3,6)} = (1,2)(3,5,6,0)t_2t_0t_4t_6t_5. \]

The orbits of \( N^{(012043)} \) on \( T \) are \( \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \) and \( \{6\} \). So we consider the double cosets \( N_{t_0t_1t_2t_0t_4t_3t_0}N, N_{t_0t_1t_2t_0t_4t_3t_1}N, N_{t_0t_1t_2t_0t_4t_3t_2}N, N_{t_0t_1t_2t_0t_4t_3t_3}N, N_{t_0t_1t_2t_0t_4t_3t_4}N, N_{t_0t_1t_2t_0t_4t_3t_5}N, \) and \( N_{t_0t_1t_2t_0t_4t_3t_6}N. \) However, \( N_{t_0t_1t_2t_0t_4t_3t_3}N = N_{t_0t_1t_2t_0t_4t_3t_4}N = [01204]. \) Also, \( N_{t_0t_1t_2t_0t_4t_3t_0}N = N_{t_0t_1t_2t_1t_3}N = [01213] \) since
\[
t_0t_1t_2t_0t_4t_3t_0 = (1,5,6)(2,3,4)(t_0t_1t_2t_3t_4)^{(1,6,3,2,4,5)} = (1,5,6)(2,3,4)t_0t_1t_6t_7t_2; \]
\[
N_{t_0t_1t_2t_0t_4t_3t_1}N = N_{t_0t_1t_2t_0t_4t_6t_6}N = [012046] \text{ since } t_0t_1t_2t_0t_4t_8t_1 = (2,5)(4,6)(t_0t_1t_2t_0t_4t_6)^{(1,2,6,4,0,3,5)} = (2,5)(4,6)t_2t_3t_4t_5t_4t_4; N_{t_0t_1t_2t_0t_4t_3t_2}N = N_{t_0t_1t_2t_0t_4t_3t_4}N = [012056] \text{ since } t_0t_1t_2t_0t_4t_3t_2 = (1,6,5)(3,4,0)(t_0t_1t_2t_0t_4t_3t_0)^{(1,6,3,4)} = (1,6,5)(3,4,0)t_0t_5t_6t_7t_1t_3; N_{t_0t_1t_2t_0t_4t_3t_3}N = N_{t_0t_1t_2t_0t_4t_3t_4}N = [012024] \text{ since } t_0t_1t_2t_0t_4t_3t_4 = (1,0,4,6)(3,5)(t_0t_1t_2t_3t_4)^{(2,6,0,3,4,5)} = (1,0,4,6)(3,5)t_2t_1t_2t_4t_6t_5; \]
\[
N_{t_0t_1t_2t_0t_4t_3t_5}N = N_{t_0t_1t_2t_3t_5}N = [01235] \text{ since } t_0t_1t_2t_0t_4t_3t_5 = (1,5,3,6,2,4,0)(t_0t_1t_2t_3t_5)^{(1,5,3,4)} = (1,5,3,6,2,4,0)t_0t_5t_6t_4t_4; N_{t_0t_1t_2t_0t_4t_3t_6}N = N_{t_0t_1t_2t_3t_1}N = [01231] \text{ since } t_0t_1t_2t_0t_4t_3t_6 = (2,5)(4,6)(t_0t_1t_2t_3t_1)^{(2,0,4,3,5,6)} = (2,5)(4,6)t_4t_1t_0t_2t_1. \]

The orbits of \( N^{(012046)} \) on \( T \) are \( \{0,2,5\}, \{1\}, \) and \( \{3,4,6\} \). So we consider the double cosets \( N_{t_0t_1t_2t_0t_4t_5t_5}N, N_{t_0t_1t_2t_0t_4t_5t_1}N, \) and \( N_{t_0t_1t_2t_0t_4t_5t_3}N. \) However, \( N_{t_0t_1t_2t_0t_4t_5t_5}N = N_{t_0t_1t_2t_0t_4t_5t_4}N = [01204]. \) Also, \( N_{t_0t_1t_2t_0t_4t_5t_1}N = N_{t_0t_1t_2t_0t_4t_5t_3}N = [0120341] \text{ since } t_0t_1t_2t_0t_4t_5t_1 = (1,2,3,0,6,4,5)(t_0t_1t_2t_0t_4t_5t_1)^{(1,3,2,4,6,5)} = (1,2,3,0,6,4,5)t_0t_4t_4t_5t_6t_6; N_{t_0t_1t_2t_0t_4t_5t_3}N = N_{t_0t_1t_2t_0t_4t_5t_2}N = [01243] \text{ since } t_0t_1t_2t_0t_4t_5t_3 = (1,4,0,6)(2,5)(t_0t_1t_2t_4t_5t_3)^{(1,6,2,3)} = (1,4,0,6)(2,5)t_0t_4t_5t_4t_2. \]

The orbits of \( N^{(012052)} \) on \( T \) are \( \{0,1,2\}, \{3,4,6\}, \) and \( \{5\} \). So we consider the double cosets \( N_{t_0t_1t_2t_0t_5t_2t_2}N, N_{t_0t_1t_2t_0t_5t_3t_2}N, \) and \( N_{t_0t_1t_2t_0t_5t_3}N. \) However, \( N_{t_0t_1t_2t_0t_5t_2}N = N_{t_0t_1t_2t_0t_5}N = [01205]. \) Also, \( N_{t_0t_1t_2t_0t_5t_2t_3}N = N_{t_0t_1t_2t_0t_4t_0}N = [012040] \text{ since } t_0t_1t_2t_0t_5t_2t_3 = (1,5,2,6,3,0,4)(t_0t_1t_2t_0t_4t_0)^{(1,0,4,2,3,5)} = (1,5,2,6,3,0,4)t_0t_4t_3t_4t_1t_4; N_{t_0t_1t_2t_0t_5t_2t_5}N = N_{t_0t_1t_2t_0t_5t_2}N = [012052] \text{ since } t_0t_1t_2t_0t_5t_2 = t_0t_1t_2t_0t_5t_2. \]

The orbits of \( N^{(0120103)} \) on \( T \) are \( \{0,1\}, \{2\}, \{3\}, \{4,6\}, \) and \( \{5\} \). So we consider the double cosets \( N_{t_0t_1t_2t_1t_0t_5t_0}N, N_{t_0t_1t_2t_1t_0t_3t_2}N, \) and \( N_{t_0t_1t_2t_1t_0t_3}N. \)


\( N_{0t_{1210}} \) \( t_{0t_{1210}} t_{0t_{5t_{5}}N} \) and \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \) However, \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = [01210] \). Also, \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = [01210] \) since \( t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = (1, 2, 4)(3, 0, 6)(t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N})^{(1, 4, 6)(2, 3, 5)} = (1, 2, 4)(3, 0, 6)t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \) \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = [01210] \) since \( t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = (1, 2, 4)(3, 0, 6)(t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N})^{(1, 4, 6)(2, 3, 5)} = (1, 2, 4)(3, 0, 6)t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \) \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = [01210] \) since \( t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = (1, 2, 4)(3, 0, 6)(t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N})^{(1, 4, 6)(2, 3, 5)} = (1, 2, 4)(3, 0, 6)t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \) 

The orbits of \( N^{(01210)} \) on \( T \) are \{0, 1, 2\}, \{3, 4, 6\}, and \{5\}. So we consider the double cosets \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \) \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \), and \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \). However, \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = [01210] \) \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = [01210] \) since \( t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = (1, 2, 4)(3, 0, 6)(t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N})^{(1, 4, 6)(2, 3, 5)} = (1, 2, 4)(3, 0, 6)t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \) \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = [01210] \) since \( t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = (1, 2, 4)(3, 0, 6)(t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N})^{(1, 4, 6)(2, 3, 5)} = (1, 2, 4)(3, 0, 6)t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \) 

The orbits of \( N^{(012124)} \) on \( T \) are \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, and \{6\}. So we consider the double cosets \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \) \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \), \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \), \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \), \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \), \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \), and \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \). However, \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = [01210] \) \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = [01210] \) since \( t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} = (1, 2, 4)(3, 0, 6)(t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N})^{(1, 4, 6)(2, 3, 5)} = (1, 2, 4)(3, 0, 6)t_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \) 

The orbits of \( N^{(012135)} \) on \( T \) are \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, and \{6\}. So we consider the double cosets \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \) \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \), \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \), \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \), \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \), \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \), and \( N_{0t_{1210}} t_{0t_{1210}} t_{0t_{5t_{5}}N} \). However,
\[ N_{t0t1t2t1t3t5t6} = N_{t0t1t2t1}t3t5t6 = [01213]. \] Also, \[ N_{t0t1t2t1t3t5t0} = N_{t0t1t2t1t4t1} = \]
\[ = [0102541] \] since \[ t0t1t2t1t3t5t0 = (1, 2, 4)(3, 0, 6)(t0t1t0t2t3t4) = \]
\[ (1, 2, 4)(3, 0, 6)t0t2t0t4t1t2; \]
\[ N_{t0t1t2t1t3t5t1} = (1, 6, 3, 2)(4, 5)(t0t1t2t4t0) = \]
\[ (1, 6, 3, 2)(4, 5)t2t5t4t2; \]
\[ N_{t0t1t2t1t3t5t2} = N_{t0t1t2t1t5t0} = [0102504] \] since \[ t0t1t2t1t3t5t2 = \]
\[ (1, 4, 5, 3)(2, 0)(t0t1t2t5t0t4) = (1, 4, 5, 3)(2, 0)t0t2t0t5t0t4; \]
\[ = [0102304] \] since \[ t0t1t2t1t3t5t3 = (1, 4, 6)(2, 3, 5)(t0t1t0t2t3t4) = \]
\[ (1, 4, 6)(2, 3, 5)t0t3t0t6t0t4; \]
\[ N_{t0t1t2t1t3t5t4} = N_{t0t1t2t3t5} = [01235] \] since \[ t0t1t2t1t3t5t4 = (1, 6, 2, 5, 0, 3, 4)(t0t1t2t3t5) = \]
\[ (1, 6, 2, 5, 0, 3, 4)t4t0t2t5t3; \]
\[ N_{t0t1t2t1t3t5t6} = [01234] \] since \[ t0t1t2t1t3t5t6 = \]
\[ (1, 2, 4)(3, 0, 6)(t0t1t2t3t4) = (1, 2, 4)(3, 0, 6)t0t1t3t5t6. \]

The orbits of \( N_{\{01213\}} \) on \( T = \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \) and \( \{6\} \). So we consider the double cosets \( N_{t0t1t2t1t3t5t6} = N_{t0t1t2t1t3t5t6} = \)
\[ \] Since \( t0t1t2t1t3t5t6 = (1, 0, 2, 3, 6, 5, 4)t1t5t0t5t0t4; \]
\[ = [0102043] \] since \[ t0t1t2t1t3t6 = (1, 4)(3, 0, 5, 6)(t0t1t0t2t0t4) = (1, 4)(3, 0, 5, 6)t1t4t1t5t0t1; \]
\[ N_{t0t1t2t1t3t5t6} = N_{t0t1t0t2t3t0t4} = [0102245] \] since \[ t0t1t2t1t3t5t6 = \]
\[ (1, 0, 5)(3, 4, 6)(t0t1t0t2t5t0t4) = (1, 0, 5)(3, 4, 6)t0t5t0t1t5t0t5; \]
\[ = [01262] \] since \[ t0t1t2t1t3t5t6 = (1, 3, 5, 4)(2, 0)(t0t1t2t3t6) = \]
\[ (1, 2, 4)(5, 6, 0)(t0t1t0t2t3t4) = (1, 2, 4)(5, 6, 0)t5t5t3t5t0t4. \]
\[ N_{t_{01}t_{2}t_{3}t_{4}t_{5}} = [01231] \] since \( t_{01}t_{2}t_{3}t_{4}t_{5}t_{64} = (1,2,5,0)(3,4)(t_{01}t_{2}t_{3}t_{1})^{(1,2)(3,5,6,0)} = (1,2,5,0)(3,4)t_{3}t_{2}t_{1}t_{5}t_{6}t_{2} \); \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}} = [01252] \) since \( t_{01}t_{2}t_{3}t_{5}t_{6}t_{5} = (t_{01}t_{2}t_{3}t_{1})^{(1,0,5)(3,4,6)} = t_{3}t_{4}t_{2}t_{1}t_{2} \).

The orbits of \( N^{(012304)} \) on \( T \) are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \) and \(\{6\}. \) So we consider the double cosets \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{0}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{1}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{2}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{3}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{4}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{5}} \), and \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{6}} \). However, \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{0}} = [01230] \) since \( t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{2} = (2,3,6)(t_{01}t_{2}t_{3}t_{4}t_{5}t_{6})^{(1,2,0,3,5,4,6)} = (1,6,5)(2,3,0)(t_{01}t_{2}t_{3}t_{4}t_{5}t_{6})^{(1,2,0,3,5,4,6)} = [01240] \) since \( t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{3} = (1,5,6)(2,4,0)(t_{01}t_{2}t_{3}t_{4}t_{5})^{(1,6,3,0,2,5,4)} = [01246] \) since \( t_{01}t_{2}t_{3}t_{4}t_{5}t_{6} = (2,3,4,0,4,0)(t_{01}t_{2}t_{3}t_{4}t_{5}t_{6})^{(2,5,4,6,3,0)} = [01232] \) since \( t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{6} = (1,0,2,5)(3,6)(t_{01}t_{2}t_{3}t_{4}t_{5}t_{6})^{(1,3,4,0,6,5,2)} = (1,0,2,5)(3,6)_{6}t_{01}t_{2}t_{3}t_{4}t_{5}t_{6} \).

The orbits of \( N^{(012321)} \) on \( T \) are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \) and \(\{6\}. \) So we consider the double cosets \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{0}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{1}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{2}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{3}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{4}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{5}} \), and \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{6}} \). However, \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{0}} = [01232] \) since \( t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{5} = (2,5,0)(3,4,6)(t_{01}t_{2}t_{3}t_{4}t_{5}t_{6})^{(1,2,0,5)(4,6)} = (2,5,0)(3,4,6)t_{5}t_{2}t_{3}t_{4}t_{0}t_{5}t_{6}; N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{5}} = [01262] \) since \( t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{3} = (1,2,3)(4,5,0)(t_{01}t_{2}t_{3}t_{4}t_{5}t_{6})^{(1,3,6,2)(4,6)} = (1,2,3)(4,5,0)t_{4}t_{3}t_{2}t_{1}t_{6}t_{6}t_{0}; N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{6} = [01234] \) since \( t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{6} = (1,2,5,0)(3,4)(t_{01}t_{2}t_{3}t_{4}t_{5}t_{6})^{(1,0,6)(2,5,3)} = (1,2,5,0)(3,4)_{5}t_{01}t_{2}t_{3}t_{4}t_{5}t_{6} \).

The orbits of \( N^{(012354)} \) on \( T \) are \(\{0\}, \{1\}, \{5,6\}, \) and \(\{2,3,4\}. \) So we consider the double cosets \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{0}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{1}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{2}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{3}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{4}} \), \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{5}} \), and \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{6}} \). However, \( N_{t_{01}t_{2}t_{3}t_{4}t_{5}t_{6}t_{0}} = [01235] \) since \( t_{01}t_{2}t_{3}t_{5}t_{4}t_{0} = (t_{01}t_{2}t_{3}t_{5}t_{4}t_{0})^{(1,5,2,6,3,0,4)} = t_{4}t_{5}t_{4}t_{6}t_{2}t_{1}t_{4} \); \( N_{t_{01}t_{2}t_{3}t_{5}t_{4}t_{1}t_{4}t_{0}} = [01243] \) since \( t_{01}t_{2}t_{3}t_{5}t_{4}t_{1}t_{4} = \)}
(1, 3)(2, 5, 6, 4)(t_0 t_1 t_2 t_4 t_3)\((1,5,6)(2,4,3) = (1, 3)(2, 5, 6, 4)t_0 t_5 t_4 t_3 t_2.

The orbits of \(N^{(012403)\)} on \(T\) are \(\{0, 2, 6\}, \{1\}, \text{ and } \{3, 4, 5\}. \) So we consider the double cosets \(N t_0 t_1 t_2 t_4 t_0 t_3 t_4, N t_0 t_1 t_2 t_4 t_0 t_3 t_1\), and \(N t_0 t_1 t_2 t_4 t_0 t_3 t_2.\)

However, \(N t_0 t_1 t_2 t_4 t_0 t_3 t_0 N = N t_0 t_1 t_2 t_4 t_0 t_3 t_0 = [01240].\) Also, \(N t_0 t_1 t_2 t_4 t_0 t_3 t_0 N = N t_0 t_1 t_2 t_6 t_1 N = [01261]\) since \(t_0 t_1 t_2 t_4 t_0 t_3 t_0 = (1, 5)(2, 4, 0, 3)(t_0 t_1 t_2 t_6 t_1)\((1,2,3,6)(4,5) = (1, 5)(2, 4, 0, 3)t_0 t_2 t_3 t_1; N t_0 t_1 t_2 t_4 t_0 t_1 N = N t_0 t_1 t_2 t_4 t_1 N = [01241]\) since \(t_0 t_1 t_2 t_4 t_0 t_3 t_1 = (1, 4, 6, 0)(2, 3)(t_0 t_1 t_2 t_4 t_1)\((1,0)(4,6) = (1, 4, 6, 0)(2, 3)t_0 t_1 t_0 t_2 t_6 t_0.\)

The orbits of \(N^{(012615)\)} on \(T\) are \(\{0, 4\}, \{1\}, \{2\}, \{3\}, \{5\}, \text{ and } \{6\}. \) So we consider the double cosets \(N t_0 t_1 t_2 t_6 t_1 t_5 t_0 N, N t_0 t_1 t_2 t_6 t_1 t_5 t_1 N, N t_0 t_1 t_2 t_6 t_1 t_5 t_2 N, N t_0 t_1 t_2 t_6 t_1 t_5 t_3 N, N t_0 t_1 t_2 t_6 t_1 t_5 t_5 N, N t_0 t_1 t_2 t_6 t_1 t_5 t_6 N.\)

However, \(N t_0 t_1 t_2 t_6 t_1 t_5 t_0 N \neq N t_0 t_1 t_2 t_6 t_1 t_5 t_1 N = [01261].\) Also, \(N t_0 t_1 t_2 t_6 t_1 t_5 t_0 N = N t_0 t_1 t_2 t_6 t_1 t_5 t_1 N = [0102541]\) since \(t_0 t_1 t_2 t_6 t_1 t_5 t_0 = (1, 0, 2, 5)(3, 6)(t_0 t_1 t_0 t_2 t_6 t_1)\((1,3,5)(2,6,0) = (1, 0, 2, 5)(3, 6)t_2 t_3 t_2 t_6 t_1 t_4 t_3; N t_0 t_1 t_2 t_6 t_1 t_5 t_1 N = N t_0 t_1 t_2 t_6 t_1 t_5 t_2 N = N t_0 t_1 t_2 t_6 t_1 t_5 t_3 N = N t_0 t_1 t_2 t_6 t_1 t_5 t_5 N = [0102046]\) since \(t_0 t_1 t_2 t_6 t_1 t_5 t_1 = (1, 3)(2, 5, 6, 4)(t_0 t_1 t_0 t_2 t_6 t_4 t_6)\((2,4)(3,5,6,0) = (1, 3)(2, 5, 6, 4)t_6 t_1 t_4 t_6 t_2 t_3; N t_0 t_1 t_2 t_6 t_1 t_5 t_2 N = N t_0 t_1 t_2 t_6 t_1 t_5 t_3 N = [0102324]\) since \(t_0 t_1 t_2 t_6 t_1 t_5 t_2 = (1, 6, 5)(2, 0, 4)(t_0 t_1 t_0 t_2 t_3 t_4 t_6)\((1,3,0)(2,4,6) = (1, 6, 5)(2, 0, 4)t_1 t_3 t_1 t_4 t_0 t_4 t_6; N t_0 t_1 t_2 t_6 t_1 t_5 t_3 N = N t_0 t_1 t_2 t_6 t_1 t_5 t_5 N = [0120341]\) since \(t_0 t_1 t_2 t_6 t_1 t_5 t_3 = (1, 0, 6, 5, 4, 3, 2)(t_0 t_1 t_2 t_6 t_1 t_5 t_4)\((1,5,2,4,6,3) = (1, 0, 6, 5, 4, 3, 2)t_6 t_5 t_4 t_5 t_1 t_0 t_5; N t_0 t_1 t_2 t_6 t_1 t_5 t_6 N = N t_0 t_1 t_2 t_6 t_1 t_5 t_8 N = [0120346]\) since \(t_0 t_1 t_2 t_6 t_1 t_5 t_6 = (1, 4, 2)(3, 0, 5)(t_0 t_1 t_0 t_2 t_3 t_4 t_6)\((1,2,6)(4,0,5) = (1, 4, 2)(3, 0, 5)t_5 t_2 t_5 t_6 t_3 t_0 t_1.\)

The orbits of \(N^{(010240)\)} on \(T\) are \(\{0, 6\}, \{1, 4\}, \{2\}, \{3\}, \text{ and } \{5\}. \) So we consider the double cosets \(N t_0 t_1 t_0 t_2 t_4 t_0 t_3 N, N t_0 t_1 t_0 t_2 t_4 t_0 t_1 N, N t_0 t_1 t_0 t_2 t_4 t_0 t_2 N, N t_0 t_1 t_0 t_2 t_4 t_0 t_3 N, \) and \(N t_0 t_1 t_0 t_2 t_4 t_0 t_5 N.\)

However, \(N t_0 t_1 t_0 t_2 t_4 t_0 t_6 N = N t_0 t_1 t_0 t_2 t_4 t_0 t_7 N = [010204].\) Also, \(N t_0 t_1 t_0 t_2 t_4 t_0 t_4 t_0 N = N t_0 t_1 t_0 t_2 t_4 t_0 t_8 N = [010240]\) since \(t_0 t_1 t_0 t_2 t_4 t_0 t_1 = (t_0 t_1 t_0 t_2 t_4)\((1,4)(6,0) = t_0 t_4 t_0 t_2 t_1 t_0; N t_0 t_1 t_0 t_2 t_4 t_0 t_2 N = N t_0 t_1 t_0 t_2 t_4 t_0 t_2 = [012052]\) since \(t_0 t_1 t_0 t_2 t_4 t_0 t_2 = (1, 6, 4, 2, 0, 5, 3)(t_0 t_1 t_0 t_2 t_4 t_0 t_2 = (1, 6, 4, 2, 0, 5, 3)\((2,5,3)(4,6,0) = (1, 6, 4, 2, 0, 5, 3)t_4 t_1 t_5 t_4 t_3 t_5; N t_0 t_1 t_0 t_2 t_4 t_0 t_3 N = N t_0 t_1 t_0 t_2 t_4 t_2 N = [010242]\) since \(t_0 t_1 t_0 t_2 t_4 t_0 t_3 t_0 = (1, 6)(4, 0)(t_0 t_1 t_0 t_2 t_4 t_2)\((1,2)(3,5,6,0) = (1, 6)(4, 0)t_4 t_6 t_4 t_2 t_0 t_2; N t_0 t_1 t_0 t_2 t_4 t_0 t_5 N = N t_0 t_1 t_0 t_2 t_4 t_3 N = [012103]\) since \(t_0 t_1 t_0 t_2 t_4 t_0 t_5 = (1, 5, 6)(2, 4, 0)(t_0 t_1 t_2 t_4 t_3)\((1,6,4)(2,5,3) = (1, 5, 6)(2, 4, 0)t_2 t_5 t_4 t_3 t_2 t_3.\)

The orbits of \(N^{(010243)\)} on \(T\) are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \text{ and } \{6\}. \) So we consider the double cosets \(N t_0 t_1 t_0 t_2 t_4 t_3 t_0 N, N t_0 t_1 t_0 t_2 t_4 t_3 t_1 N, N t_0 t_1 t_0 t_2 t_4 t_3 t_2 N, \)
\[ N_{0010203046t4t6t5} \]

However, \( N_{0010203046t4t6t5} = N_{0010203046t4t6} \) and \( N_{0010203046t4t6t5} = N_{0010203046t4t6} \). Also, \( N_{0010203046t4t6t5} = N_{0010203046t4t6} \) since \( t_{0010203046t4t6t5} = (1,0,2,4,5,6)(t_{0010203046t4t6})^{(2,3,5)} \) = (1,0,2,4,5,6)(t_{0010203046t4t6t5}). \( N_{0010203046t4t6t5} = N_{0010203046t4t6} \) since \( t_{0010203046t4t6t5} = (1,2)(3,5,6)(t_{0010203046t4t6t5})^{(2,3,5)} \) = (1,2)(3,5,6)(t_{0010203046t4t6t5}). \( N_{0010203046t4t6t5} = N_{0010203046t4t6} \) since \( t_{0010203046t4t6t5} = (1,2)(3,5,6)(t_{0010203046t4t6t5})^{(2,3,5)} \) = (1,2)(3,5,6)(t_{0010203046t4t6t5}).

The orbits of \( N^{(0102046)} \) on \( T \) are \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, and \{6\}. So we consider the double cosets \( N_{0010203046t4t6t5}, N_{0010203046t4t6t5}, N_{0010203046t4t6t5}, N_{0010203046t4t6t5}, \) and \( N_{0010203046t4t6t5} \).

However, \( N_{0010203046t4t6t5} = N_{0010203046t4t6} \) is \( [0102046] \). Also, \( N_{0010203046t4t6t5} = N_{0010203046t4t6} \) since \( t_{0010203046t4t6t5} = (1,0,2,4,5,6)(t_{0010203046t4t6})^{(2,3,5)} \) = (1,0,2,4,5,6)(t_{0010203046t4t6t5}). \( N_{0010203046t4t6t5} = N_{0010203046t4t6} \) since \( t_{0010203046t4t6t5} = (1,2)(3,5,6)(t_{0010203046t4t6t5})^{(2,3,5)} \) = (1,2)(3,5,6)(t_{0010203046t4t6t5}). \( N_{0010203046t4t6t5} = N_{0010203046t4t6} \) since \( t_{0010203046t4t6t5} = (1,2)(3,5,6)(t_{0010203046t4t6t5})^{(2,3,5)} \) = (1,2)(3,5,6)(t_{0010203046t4t6t5}).

The only new coset is \( [01020462] \). \( N_{0010203046t4t6t5} \) is \( [01020462] \). Also, \( N_{0010203046t4t6t5} = N_{0010203046t4t6} \) since \( t_{0010203046t4t6t5} = (1,2)(3,5,6)(t_{0010203046t4t6t5})^{(2,3,5)} \) = (1,2)(3,5,6)(t_{0010203046t4t6t5}).

The orbits of \( N^{(01020300)} \) on \( T \) are \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, and \{6\}. So we
consider the double cosets $Nt_0t_1t_0t_2t_0t_5t_0t_0N$, $Nt_0t_1t_0t_2t_0t_5t_0t_1N$, $Nt_0t_1t_0t_2t_0t_5t_0t_2N$, $Nt_0t_1t_0t_2t_0t_5t_0t_3N$, $Nt_0t_1t_0t_2t_0t_5t_0t_4N$, $Nt_0t_1t_0t_2t_0t_5t_0t_5N$, and $Nt_0t_1t_0t_2t_0t_5t_0t_6N$.

However, $Nt_0t_1t_0t_2t_0t_5t_0t_2N = Nt_0t_1t_2t_1t_0t_0N = [012150]$ since $t_0t_1t_2t_0t_5t_0t_2 = (2, 0, 5)(3, 6, 4)(t_0t_1t_2t_1t_0t_0)^{[1220][346]}$ = $(2, 0, 5)(3, 6, 4)(t_0t_1t_2t_1t_0t_0)(1, 2, 0)(3, 4, 6)$ = $(2, 0, 5)(3, 6, 4)(t_0t_1t_2t_0t_5t_0t_3N = Nt_0t_1t_0t_2t_0t_3t_0N = [012034]$ since $t_0t_1t_0t_2t_0t_3t_0t_3 = (2, 0, 5)(3, 6, 4)(t_0t_1t_2t_0t_3t_0)^{[1230][4]}(2, 3, 4) = (2, 0, 5)(3, 6, 4)(t_0t_1t_2t_0t_3t_0t_4; Nt_0t_1t_0t_2t_0t_3t_0t_4N = Nt_0t_1t_0t_2t_0t_3t_0N = [012030]$ since $t_0t_1t_0t_2t_0t_5t_0t_4 = (3, 6)(5, 0)(t_0t_1t_0t_2t_3t_0)^{[1235][642]} = (3, 6)(5, 0)(t_0t_1t_0t_2t_3t_0)$ = $Nt_0t_1t_0t_2t_0t_3t_0N = [012050]$ since $t_0t_1t_0t_2t_0t_3t_0N = Nt_0t_1t_0t_2t_0t_3t_0N = Nt_0t_1t_0t_2t_0t_3t_0N = [012030]$ since $t_0t_1t_0t_2t_0t_3t_0 = (1, 2, 6, 3)(4, 0)(t_0t_1t_0t_2t_3)^{[245][56]} = (1, 2, 6, 3)(4, 0)(t_0t_1t_0t_2t_3t_0$. The only new double coset is $[0120501]$. $Nt_0t_1t_0t_2t_0t_3t_0t_1(1,3,4,5) = Nt_0t_0t_2t_0t_3t_0t_1$ and $Nt_0t_1t_0t_2t_0t_3t_0t_1(1,4,3,5) = Nt_0t_1t_0t_2t_0t_3t_0t_1(t_0t_1t_0t_2t_0t_3t_0t_1(3,4,5) = (1, 3, 4, 5)$ and $(1, 4, 3, 5) \in N([0120501]) \Rightarrow N([0120501]) = \{e, (1, 3)(4, 5), (1, 4)(3, 5), (1, 5)(3, 4)\}$ thus, there are $|N| / |N([0120501])| = 168 / 4 = 42$ single cosets in $[0120501]$. The orbits of $N([0120502])$ on $T$ are $\{0, 1, 2\}$, $\{3, 4, 6\}$, and $\{5\}$. So we consider the double cosets $Nt_0t_1t_0t_2t_0t_3t_0t_5t_2t_2N$, $Nt_0t_1t_0t_2t_0t_3t_0t_5t_3N$, and $Nt_0t_1t_0t_2t_0t_3t_0t_5t_4N$. However, $Nt_0t_1t_0t_2t_0t_3t_0t_5t_2t_2N = Nt_0t_1t_0t_2t_0t_3t_0N = [012050]$. Also, $Nt_0t_1t_0t_2t_0t_3t_0t_5t_3N = Nt_0t_1t_0t_2t_0t_3t_0t_5t_2t_2N = [012321] since t_0t_1t_0t_2t_0t_3t_0t_5 = (1, 0, 5, 2)(3, 4)(t_0t_1t_0t_2t_0t_3)^{[245][6][54]} = (1, 0, 5, 2)(3, 4)(t_0t_1t_0t_2t_0t_3). The only new double coset is $[012052]$. The orbits of $N([012030])$ on $T$ are $\{0, 1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, and $\{6\}$. So we consider the double cosets $Nt_0t_1t_0t_2t_0t_3t_0t_4t_0N$, $Nt_0t_1t_0t_2t_0t_3t_0t_4t_1N$, $Nt_0t_1t_0t_2t_0t_3t_0t_4t_2N$, $Nt_0t_1t_0t_2t_0t_3t_0t_4t_3N$, $Nt_0t_1t_0t_2t_0t_3t_0t_4t_4N$, $Nt_0t_1t_0t_2t_0t_3t_0t_4t_5N$, and $Nt_0t_1t_0t_2t_0t_3t_0t_4t_6N$. However, $Nt_0t_1t_0t_2t_0t_3t_0t_4t_4N = Nt_0t_1t_0t_2t_0t_3t_0N = [012030]. Also, $Nt_0t_1t_0t_2t_0t_3t_0t_4t_1N = Nt_0t_1t_0t_2t_0t_3t_0t_4t_2N = [012135] since t_0t_1t_0t_2t_0t_3t_0t_4t_1 = (1, 6, 5)(2, 4, 3)(t_0t_1t_0t_2t_0t_3)^{[12][6]} = (1, 6, 5)(2, 4, 3)t_0t_1t_0t_2t_0t_3t_0t_4t_2N = Nt_0t_1t_0t_2t_0t_3t_0t_4t_2N = [012136] since t_0t_1t_0t_2t_0t_3t_0t_4t_2 = (1, 3, 6, 5, 2, 0, 4)(t_0t_1t_0t_2t_0t_3)^{[12][6][3]} = (1, 3, 6, 5, 2, 0, 4)t_0t_1t_0t_2t_0t_3t_0t_4t_2 = (1, 5)(3, 4)(t_0t_1t_0t_2t_0t_3)^{[12][6][53]} = (1, 5)(3, 4)t_0t_1t_0t_2t_0t_3t_0t_4t_2. The only new double coset is $[012053]$. $Nt_0t_1t_0t_2t_0t_3t_0t_4t_5N = Nt_0t_1t_0t_2t_0t_3t_0t_4t_5N = [012034] since t_0t_1t_0t_2t_0t_3t_0t_4t_5 = (1, 4, 6, 2, 3, 0, 5)(t_0t_1t_0t_2t_0t_3)^{[12][6][40]} = (1, 4, 6, 2, 3, 0, 5)t_0t_1t_0t_2t_0t_3t_0t_4t_5N.$
\[ N = N_{t_0 t_1 t_2 t_3 t_4 t_5} = [010254] \] since \( t_0 t_1 t_2 t_3 t_4 t_5 = (1,3,6,5,2,0,4)(t_0 t_1 t_2 t_3 t_4 t_5) \).

The only new double coset is \( N_{t_0 t_1 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_2 t_3 t_4 t_5} = (1,3,6,5,2,0,4)(t_3 t_0 t_3 t_5 t_2). \)

The only new double coset is \( N_{t_0 t_1 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_2 t_3 t_4 t_5} \).

\[ N_{t_0 t_1 t_2 t_3 t_4 t_5} = \{ e, (1,3,2)(4,0,5), (1,2,3)(4,5,0) \} \] thus, there are \( |N|/|N_{t_0 t_1 t_2 t_3 t_4 t_5}| = 168/3 = 56 \) single cosets in \( [01023040]. \)

The orbits of \( N_{t_0 t_1 t_2 t_3 t_4 t_5} \) on \( T \) are \( \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \) and \( \{6\}. \) So we consider the double cosets \( N_{t_0 t_1 t_2 t_3 t_4 t_5}, N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5}, \)

\[ N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} \]

However, \( N_{t_0 t_1 t_2 t_3 t_4 t_5} = [0102320] \) since \( t_0 t_1 t_2 t_3 t_4 t_5 = (2,5,3)(4,6,0)(t_0 t_1 t_2 t_3 t_4 t_5) \).

\[ N_{t_0 t_1 t_2 t_3 t_4 t_5} = [010234] \] since \( t_0 t_1 t_2 t_3 t_4 t_5 = (1,5)(2,3,0,4)(t_0 t_1 t_2 t_3 t_4 t_5) \).

\[ N_{t_0 t_1 t_2 t_3 t_4 t_5} = [010234] \] since \( t_0 t_1 t_2 t_3 t_4 t_5 = (1,5)(2,3,0,4)(t_0 t_1 t_2 t_3 t_4 t_5) \).

\[ N_{t_0 t_1 t_2 t_3 t_4 t_5} = [01023040] \) since \( t_0 t_1 t_2 t_3 t_4 t_5 = (1,5)(2,3,0,4)(t_0 t_1 t_2 t_3 t_4 t_5) \).

The orbits of \( N_{t_0 t_1 t_2 t_3 t_4 t_5} \) on \( T \) are \( \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \) and \( \{6\}. \) So we consider the double cosets \( N_{t_0 t_1 t_2 t_3 t_4 t_5}, N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} \),

\[ N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} \]

However, \( N_{t_0 t_1 t_2 t_3 t_4 t_5} = [0102320] \) since \( t_0 t_1 t_2 t_3 t_4 t_5 = (2,5,3)(4,6,0)(t_0 t_1 t_2 t_3 t_4 t_5) \).

\[ N_{t_0 t_1 t_2 t_3 t_4 t_5} = [010234] \] since \( t_0 t_1 t_2 t_3 t_4 t_5 = (1,5)(2,3,0,4)(t_0 t_1 t_2 t_3 t_4 t_5) \).

\[ N_{t_0 t_1 t_2 t_3 t_4 t_5} = [01023040] \) since \( t_0 t_1 t_2 t_3 t_4 t_5 = (1,5)(2,3,0,4)(t_0 t_1 t_2 t_3 t_4 t_5) \).

The orbits of \( N_{t_0 t_1 t_2 t_3 t_4 t_5} \) on \( T \) are \( \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \) and \( \{6\}. \) So we consider the double cosets \( N_{t_0 t_1 t_2 t_3 t_4 t_5}, N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} \),

\[ N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} N_{t_0 t_1 t_2 t_3 t_4 t_5} \]
double cosets $N_{t0t1t0t2t4t0t0N}$, $N_{t0t1t0t2t4t0t0N}$, and $N_{t0t1t0t2t4t0t0N}$. However, $N_{t0t1t0t2t4t0t0N} = N_{t0t1t0t2t4t0t0N} = N_{t0t1t0t2t4t0t0N} = [010240]$. Also, $N_{t0t1t0t2t4t0t0N} = N_{t0t1t0t2t4t0t0N} = N_{t0t1t0t2t4t0t0N} = (1,0,2)(3,6,4)(t0t1t0t2t4t0t3t1)^{(1,5,2,4,3,0,4)} = (1,0,2)(3,6,4)t0t4t5t4t0t1t0t5$; $N_{t0t1t0t2t4t0t0N} = N_{t0t1t0t2t4t0t0N} = [010243]$ since $t_0t_1t_0t_2t_4t_0t_6t_2 = (1,4,3,0,5,2,6)(t_0t_1t_2t_4t_0t_3)^{(1,2,4)(3,0,6)} = (1,4,3,0,5,2,6)t_0t_4t_5t_0t_1t_0$.

The orbits of $N^{(0102425)}$ on $T$ are $\{0,4,6\}$, $\{1\}$, and $\{2,3,5\}$. So we consider the double cosets $N_{t0t1t0t2t4t2t5N}$, $N_{t0t1t0t2t4t2t5N}$, and $N_{t0t1t0t2t4t2t5N}$. However, $N_{t0t1t0t2t4t2t5N} = N_{t0t1t0t2t4t2t5N} = [010242]$. Also, $N_{t0t1t0t2t4t2t5N} = N_{t0t1t0t2t4t2t5N} = N_{t0t1t0t2t4t2t5N} = [012136]$ since $t_0t_1t_0t_2t_4t_2t_5t_0 = (1,2,4)(3,6,5)(t_0t_1t_2t_3t_4)^{(1,5,2,0)} = (1,2,4)(3,6,5)t_2t_5t_3t_2t_5t_6$. The only new double coset is $[01024251]$. $N_{t0t1t0t2t4t2t5t1,4,6}(2,3,5) = N_{t0t4t0t3t4t5t4} = N_{t0t1t0t2t4t2t5t1} and N_{t0t1t0t2t4t2t5t1,0,4,6} = N_{t1t0t1t2t4t2t5t0} = N_{t0t1t0t2t4t2t5t1} \Rightarrow (1,4,6)(2,3,5) and (1,0)(4,6) \in N^{(01024251)} = \{(1,4,6)(2,3,5), (1,0)(4,6)\}$ is a group of order 12 $\Rightarrow$ there are 168/12 = 14 single cosets in $[01024251]$.

The orbit of $N^{(0102435)}$ on $T$ is $\{0,1,2,3,4,5,6\}$, so we only consider the double coset $N_{t0t1t0t2t4t3t5t5N} = N_{t0t1t0t2t4t3N} = [010243]$.

The orbits of $N^{(0102503)}$ on $T$ are $\{0\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, and $\{6\}$. So we consider the double cosets $N_{t0t1t0t2t5t0t0N}$, $N_{t0t1t0t2t5t0t1N}$, $N_{t0t1t0t2t5t0t2N}$, $N_{t0t1t0t2t5t0t3N}$, $N_{t0t1t0t2t5t0t4N}$, $N_{t0t1t0t2t5t0t5N}$, and $N_{t0t1t0t2t5t0t6N}$. However, $N_{t0t1t0t2t5t0t3N} = N_{t0t1t0t2t5t0t0N} = [010250]$. Also, $N_{t0t1t0t2t5t0t0N} = N_{t0t1t0t2t5t0t4N} = [012034] since t_0t_1t_0t_2t_5t_0t_3t_0 = (2,5,0)(3,4,6)(t_0t_1t_2t_3t_4)^{(1,6,3,5,4,2,0)} = (2,5,0)(3,4,6)t_1t_6t_1t_4t_2t_2; N_{t0t1t0t2t5t0t1N} = N_{t0t1t0t2t5t0t4N} = [012304] since t_0t_1t_0t_2t_5t_0t_3t_1 = (1,2,0)(3,4,6)(t_0t_1t_2t_3t_4)^{(1,6,4,5,3,0,2)} = (1,2,0)(3,4,6)t_2t_4t_1t_0t_5; N_{t0t1t0t2t5t0t3N} = N_{t0t1t0t2t5t0t6N} = [0102462] since t_0t_1t_0t_2t_5t_0t_3t_2 = (3,5)(6,0)(t_0t_1t_0t_2t_4t_6t_2)^{(1,5,4,2,3,6,0)} = (3,5)(6,0)t_1t_5t_1t_3t_1t_2t_3; N_{t0t1t0t2t5t0t4N} = N_{t0t1t0t2t5t0t3t4N} = [012134] since t_0t_1t_0t_2t_5t_0t_3t_4 = (1,0,2)(3,6,4)(t_0t_1t_2t_3t_4)^{(1,2,5)(3,6,4)} = (1,0,2)(3,6,4)t_0t_2t_5t_3t_2t_3; N_{t0t1t0t2t5t0t3N} = N_{t0t1t0t2t0t4t3t2} = [0102432] since t_0t_1t_0t_2t_5t_0t_3t_5 = (1,3,2)(4,0,5)(t_0t_1t_0t_2t_4t_3t_2)^{(1,5,2,0)(3,6)} = (1,3,2)(4,0,5)t_1t_5t_1t_4t_6t_0; N_{t0t1t0t2t0t3t0N} = N_{t0t1t0t2t0t3t0N} = [012030] since t_0t_1t_0t_2t_5t_0t_3t_6 = (1,2,6,3)(4,0)(t_0t_1t_2t_3t_4)^{(1,4)(3,5)} = (1,2,6,3)(4,0)t_0t_4t_2t_0t_5t_0$.

The orbits of $N^{(0102504)}$ on $T$ are $\{0\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, and $\{6\}$. So we
The double cosets \( Nt_0t_1t_0t_2t_3t_4t_0N, Nt_0t_1t_0t_2t_3t_4t_1N, Nt_0t_1t_0t_2t_3t_4t_2N, Nt_0t_1t_0t_2t_3t_4t_3N, Nt_0t_1t_0t_2t_3t_4t_4N, Nt_0t_1t_0t_2t_3t_4t_5N, \) and \( Nt_0t_1t_0t_2t_3t_4t_6N. \) However, \( Nt_0t_1t_0t_2t_3t_4t_0N = Nt_0t_1t_2t_3t_4t_3N = [012103] \) since \( t_0t_1t_2t_3t_4t_0 = (1,5,6)(2,4,0)(t_0t_1t_2t_3t_4)_{(1,6,4)(2,5,3)} = (1,5,6)(2,4,0)ot_0t_1t_2t_3t_4t_0; \) \( Nt_0t_1t_0t_2t_3t_4t_1N = Nt_0t_1t_2t_3t_4t_5N = [012135] \) since \( t_0t_1t_2t_3t_4t_1 = (1,0)(2,6,5,4)(t_0t_1t_2t_3t_5)^{(1,2,3)(3,6)} = (1,0)(2,6,5,4)ot_0t_1t_2t_3t_5t_5; \) \( Nt_0t_1t_0t_2t_3t_4t_2N = Nt_0t_1t_2t_3t_4t_3N = [012043] \) since \( t_0t_1t_0t_2t_3t_4t_2 = (1,6,5)(3,4,0)(t_0t_1t_2t_3t_4)^{(1,5,3)(3,4)} = (1,6,5)(3,4,0)ot_0t_1t_2t_3t_4t_2; \) \( Nt_0t_1t_0t_2t_3t_4t_3N = Nt_0t_1t_2t_3t_4t_5N = [012136] \) since \( t_0t_1t_0t_2t_3t_4t_3 = (1,0,2)(3,6,4)(t_0t_1t_2t_3t_5)^{(1,2,5)(3,6,4)} = (1,0,2)(3,6,4)ot_0t_1t_2t_3t_5t_5; \) \( Nt_0t_1t_0t_2t_3t_4t_4N = Nt_0t_1t_2t_3t_4t_6N = [012030] \) since \( t_0t_1t_0t_2t_3t_4t_4 = (1,5,6)(2,4,0)(t_0t_1t_2t_3t_5)^{(1,6,4)(2,5,3)} = (1,5,6)(2,4,0)ot_0t_1t_2t_3t_5t_5. \)

The orbits of \( N(1012506) \) on \( T \) are \( \{0\}, \{1,2,4\}, \) and \( \{3,5,6\} \). So we consider the double cosets \( Nt_0t_1t_0t_2t_3t_4t_5t_0N, Nt_0t_1t_0t_2t_3t_4t_5t_1N, \) and \( Nt_0t_1t_0t_2t_3t_4t_5t_6N. \) However, \( Nt_0t_1t_0t_2t_3t_4t_5t_6N = Nt_0t_1t_0t_2t_3t_4t_0N = [012050]. \) Also, \( Nt_0t_1t_0t_2t_3t_4t_5t_6 = Nt_0t_1t_0t_2t_3t_4t_6N = [01203040] \) since \( t_0t_1t_0t_2t_3t_4t_6 = (t_0t_1t_0t_2t_3t_4)^{(1,6,0,4)(2,5)} = t_0t_1t_0t_2t_3t_4t_6t_6; \) \( Nt_0t_1t_0t_2t_3t_4t_5N = Nt_0t_1t_0t_2t_3t_4t_1N = [012134] \) since \( t_0t_1t_0t_2t_3t_4t_1 = (1,5,2)(3,4,6)(t_0t_1t_0t_2t_3t_4)^{(1,3,2)(2,4,5,6)} = (1,5,2)(3,4,6)ot_0t_1t_0t_2t_3t_4t_6t_6. \)

The orbits of \( N(10125032) \) on \( T \) are \( \{0,1,2,5\}, \{3\}, \) and \( \{4,6\} \). So we consider the double cosets \( Nt_0t_1t_0t_2t_3t_4t_5t_0N, Nt_0t_1t_0t_2t_3t_4t_5t_1N, \) and \( Nt_0t_1t_0t_2t_3t_4t_5t_2N. \) However, \( Nt_0t_1t_0t_2t_3t_4t_5t_6N = Nt_0t_1t_0t_2t_3t_4t_5t_0N = [012052]. \) Also, \( Nt_0t_1t_0t_2t_3t_4t_5t_0N = Nt_0t_1t_0t_2t_3t_4t_5t_2N = [010325] \) since \( t_0t_1t_0t_2t_3t_4t_0 = (t_0t_1t_0t_2t_3t_4)^{(1,2,0,5)(4,6)} = t_0t_1t_0t_2t_3t_4t_0; \) \( Nt_0t_1t_0t_2t_3t_4t_6N = Nt_0t_1t_0t_2t_3t_4t_1N = [0120501] \) since \( t_0t_1t_0t_2t_3t_4t_6 = (1,4,0)(2,5,3)(t_0t_1t_0t_2t_3t_4)^{(2,4,3,5,6)} = (1,4,0)(2,5,3)ot_0t_1t_0t_2t_3t_4t_5t_5. \)

The orbits of \( N(10125024) \) on \( T \) are \( \{0,1,2,5\}, \{3\}, \{4\}, \) and \( \{6\} \). So we consider the double cosets \( Nt_0t_1t_0t_2t_3t_4t_5t_0N, Nt_0t_1t_0t_2t_3t_4t_5t_1N, Nt_0t_1t_0t_2t_3t_4t_5t_2N, \) and \( Nt_0t_1t_0t_2t_3t_4t_5t_6N. \) However, \( Nt_0t_1t_0t_2t_3t_4t_5t_2N = Nt_0t_1t_0t_2t_3t_4t_5t_0N = [012052]. \) Also, \( Nt_0t_1t_0t_2t_3t_4t_5t_0N = \) \( Nt_0t_1t_0t_2t_3t_4t_5t_2N = [012136] \) since \( t_0t_1t_0t_2t_3t_4t_0 = (t_0t_1t_0t_2t_3t_4)^{(1,5,3,0,4,6,2)} = t_0t_1t_0t_2t_3t_4t_0; \) \( Nt_0t_1t_0t_2t_3t_4t_5t_3N = Nt_0t_1t_0t_2t_3t_4t_5t_1N = [01024251] \) since \( t_0t_1t_0t_2t_3t_4t_3 = (1,4)(3,6,5,0)(t_0t_1t_0t_2t_3t_4t_1)^{(1,0,5,3,4,2,6)} = (1,4)(3,6,5,0)ot_0t_1t_0t_2t_3t_4t_5t_5; \) \( Nt_0t_1t_0t_2t_3t_4t_6N = Nt_0t_1t_0t_2t_3t_4t_5t_1N = [01020501] \)
since \( t_0t_1t_0t_2t_5t_2t_4t_6 = (1, 5, 3)(2, 0, 6)(t_0t_1t_0t_2t_5t_0t_1) = (1, 5, 3)(2, 0, 6)t_4t_5t_3t_4t_1t_4t_5.

The orbits of \( N^{(0102540)} \) on \( T \) are \{0, 5, 6\}, \{1, 2, 4\}, and \{3\}. So we consider the double cosets \( Nt_0t_1t_0t_2t_5t_4t_0t_0N, Nt_0t_1t_0t_2t_5t_4t_0t_1N, \) and \( Nt_0t_1t_0t_2t_5t_4t_0t_3N. \) However, \( Nt_0t_1t_0t_2t_5t_4t_0t_0N = Nt_0t_1t_0t_2t_5t_4t_0t_1N = [010254]. \) Also, \( Nt_0t_1t_0t_2t_5t_4t_0t_1N = Nt_0t_1t_0t_2t_5t_4t_0t_1N = [012103] \) since \( t_0t_1t_0t_2t_5t_4t_0t_1N = (1, 6, 4, 0)(3, 5)(t_0t_1t_2t_1t_3t_3)(1, 0, 3, 4, 2, 6) = (1, 6, 4, 0)(3, 5)t_5t_0t_5t_4; \) \( Nt_0t_1t_0t_2t_5t_4t_0t_3N = Nt_0t_1t_2t_3t_4N = [012354] \) since \( t_0t_1t_0t_2t_5t_4t_0t_3 = (t_0t_1t_2t_3t_4t_2)(1, 4, 0, 3, 6, 2, 5) = t_5t_4t_5t_6t_1t_0.

The orbits of \( N^{(0102541)} \) on \( T \) are \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, and \{5, 6\}. So we consider the double cosets \( Nt_0t_1t_0t_2t_5t_4t_0t_0N, Nt_0t_1t_0t_2t_5t_4t_1t_1N, Nt_0t_1t_0t_2t_5t_4t_1t_2N, Nt_0t_1t_0t_2t_5t_4t_1t_3N, Nt_0t_1t_0t_2t_5t_4t_1t_4N, \) and \( Nt_0t_1t_0t_2t_5t_4t_1t_5N. \) However, \( Nt_0t_1t_0t_2t_5t_4t_1t_1N = Nt_0t_1t_0t_2t_5t_4t_0N = [010254]. \) Also, \( Nt_0t_1t_0t_2t_5t_4t_1t_0N = Nt_0t_1t_0t_2t_5t_4t_0N = [012135] \) since \( t_0t_1t_0t_2t_5t_4t_0t_0N = (1, 4, 2)(3, 0, 5)(t_0t_1t_2t_3t_5t_5)(1, 4, 2)(3, 0, 5)t_5t_4t_4t_0t_3; \) \( Nt_0t_1t_0t_2t_5t_4t_1t_2N = Nt_0t_1t_0t_2t_5t_4t_1t_3N = [012134] \) since \( t_0t_1t_0t_2t_5t_4t_1t_2 = (1, 4, 6, 0)(2, 3)(t_0t_1t_2t_3t_4t_4)(1, 4, 0, 2, 5, 6, 3) = (1, 4, 6, 0)(2, 3)t_2t_4t_3t_1t_0; \) \( Nt_0t_1t_0t_2t_5t_4t_1t_3N = Nt_0t_1t_0t_2t_3t_0t_4N = [012304] \) since \( t_0t_1t_0t_2t_5t_4t_1t_3 = (t_0t_1t_2t_3t_0t_4)(1, 4, 0, 3, 6, 2, 5) = t_3t_4t_5t_6t_1t_0; \) \( Nt_0t_1t_0t_2t_5t_4t_1t_4N = Nt_0t_1t_0t_2t_5t_4t_0t_0N = [0123040] \) since \( t_0t_1t_0t_2t_5t_4t_1t_4 = (t_0t_1t_0t_2t_3t_0t_4t_0)(1, 4, 0, 6)(2, 3) = t_6t_4t_6t_5t_3t_6t_0t_0; \) \( Nt_0t_1t_0t_2t_5t_4t_1t_5N = Nt_0t_1t_0t_2t_6t_1t_5N = [012615] \) since \( t_0t_1t_0t_2t_5t_4t_1t_5 = (1, 0)(2, 5)(t_0t_1t_2t_6t_1t_5)(1, 3)(4, 5) = (1, 0)(2, 5)t_0t_3t_2t_6t_4.

The orbits of \( N^{(0103204)} \) on \( T \) are \{0, 2, 6\}, \{1, 4, 5\}, and \{3\}. So we consider the double cosets \( Nt_0t_1t_0t_3t_2t_0t_4t_4N, Nt_0t_1t_0t_3t_2t_0t_4t_4N, \) and \( Nt_0t_1t_0t_3t_2t_0t_4t_3N. \) However, \( Nt_0t_1t_0t_3t_2t_0t_4t_4N = Nt_0t_1t_0t_3t_2t_0t_4t_0N = [010320]. \) Also, \( Nt_0t_1t_0t_3t_2t_0t_4t_0N = Nt_0t_1t_0t_3t_2t_0t_4t_0N = [012321] \) since \( t_0t_1t_0t_3t_2t_0t_4t_0 = (1, 2, 0)(3, 4, 6)(t_0t_1t_2t_3t_2t_1)(2, 4)(5, 6) = (1, 2, 0)(3, 4, 6)t_0t_1t_4t_3t_4t_1; \) \( Nt_0t_1t_0t_3t_2t_0t_4t_3N = Nt_0t_1t_0t_3t_2t_0t_4t_3N = [01020432] \) since \( t_0t_1t_0t_3t_2t_0t_4t_3 = (2, 6, 0)(3, 4, 5)(t_0t_1t_0t_2t_0t_4t_3t_2)(1, 5, 0, 3, 2, 6, 4) = (2, 6, 0)(3, 4, 5)t_3t_5t_3t_6t_3t_2t_2t_6.

The orbits of \( N^{(0120341)} \) on \( T \) are \{0, 5, 6\}, \{1, 2, 4\}, and \{3\}. So we consider the double cosets \( Nt_0t_1t_2t_0t_3t_4t_0t_0N, Nt_0t_1t_2t_0t_3t_4t_1t_1N, \) and \( Nt_0t_1t_2t_0t_3t_4t_1t_3N. \) However, \( Nt_0t_1t_2t_0t_3t_4t_1t_1N = Nt_0t_1t_2t_0t_3t_4N = [012034]. \) Also, \( Nt_0t_1t_2t_0t_3t_4t_1t_0N = Nt_0t_1t_2t_0t_3t_4t_1t_0N = [012615] \) since \( t_0t_1t_2t_0t_3t_4t_1t_0 = \)
(1, 2, 3, 4, 5, 6, 0)\( t_0 t_1 t_2 t_3 t_4 t_5 \) = (1, 2, 3, 4, 5, 6, 0)\( t_1 t_3 t_2 t_4 t_5 t_3 \); \( N t_0 t_1 t_2 t_3 t_4 t_5 N \) = \( [012045] \) since \( t_0 t_1 t_2 t_3 t_4 t_5 t_3 = (1, 6, 5)\( t_0 t_1 t_2 t_0 t_4 t_5 \) = (1, 6, 5)\( (2, 0, 4) t_0 t_1 t_2 t_0 t_4 t_5 \).

The orbits of \( N^{(10120431)} \) on \( T \) are \{0, 2, 4\}, \{1, 5, 6\}, and \{3\}. So we consider the double cosets \( N t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_1 t_0 N \), \( N t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_1 t_1 N \), and \( N t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_1 t_3 N \).

However, \( N t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_1 t_1 N = N t_0 t_1 t_0 t_2 t_0 t_4 t_3 N = [0102043] \). Also, 
\( N t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_1 t_0 N = N t_0 t_1 t_0 t_2 t_3 t_2 t_4 N = [0102324] \) since \( t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_1 t_0 = (1, 5, 4, 2, 3, 6, 0) (t_0 t_1 t_0 t_3 t_2 t_3 t_4) = (1, 5, 4, 2, 3, 6, 0) t_3 t_0 t_3 t_1 t_3 t_2 ; \)
\( N t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_1 t_3 N = N t_0 t_1 t_0 t_2 t_4 t_6 N = [0102406] \) since \( t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_1 = (1, 5, 4, 2, 3, 6, 0) (t_0 t_1 t_0 t_2 t_4 t_6) (1, 0, 3) (2, 5, 4) = (1, 5, 4, 2, 3, 6, 0) t_3 t_0 t_3 t_0 t_2 t_4 .

The orbits of \( N^{(10120432)} \) on \( T \) are \{0\}, \{1, 4, 6\}, and \{2, 3, 5\}. So we consider the double cosets \( N t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_2 t_0 N \), \( N t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_2 t_1 N \), and \( N t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_2 t_2 N \).

However, \( N t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_2 t_2 N = N t_0 t_1 t_0 t_2 t_0 t_4 t_3 N = [0102043] \). Also, 
\( N t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_2 t_0 N = N t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_2 t_0 N = [0102304] \) since \( t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_2 t_0 = (1, 6, 0)(2, 3, 5)(t_0 t_1 t_0 t_3 t_2 t_4) = (1, 6, 0)(2, 3, 5) t_3 t_0 t_3 t_5 t_0 t_3 ; \)
\( N t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_2 t_1 N = N t_0 t_1 t_0 t_2 t_5 t_0 t_3 N = [0102503] \) since \( t_0 t_1 t_0 t_2 t_0 t_4 t_3 t_2 t_1 = (1, 2, 4)(5, 6, 0)(t_0 t_1 t_0 t_2 t_0 t_3) (1, 0, 2, 5) (3, 6) = (1, 2, 4)(5, 6, 0) t_2 t_0 t_2 t_5 t_1 t_2 t_6 .

The orbits of \( N^{(10120462)} \) on \( T \) are \{0, 2, 6\}, \{1, 4, 5\}, and \{3\}. So we consider the double cosets \( N t_0 t_1 t_0 t_2 t_0 t_4 t_5 t_2 t_2 N \), \( N t_0 t_1 t_0 t_2 t_0 t_4 t_5 t_2 t_1 N \), and \( N t_0 t_1 t_0 t_2 t_0 t_4 t_5 t_2 t_3 N \).

However, \( N t_0 t_1 t_0 t_2 t_0 t_4 t_5 t_2 t_2 N = N t_0 t_1 t_0 t_2 t_0 t_4 t_6 N = [0102046] \). Also, 
\( N t_0 t_1 t_0 t_2 t_0 t_4 t_5 t_2 t_1 N = N t_0 t_1 t_0 t_2 t_5 t_0 t_3 N = [0102503] \) since \( t_0 t_1 t_0 t_2 t_0 t_4 t_5 t_2 t_1 = (1, 5, 3, 0, 4, 6, 2)(t_0 t_1 t_0 t_2 t_5 t_0 t_3) (1, 2)(3, 6) = (1, 5, 3, 0, 4, 6, 2) t_0 t_2 t_0 t_1 t_0 t_6 . \) But [010204623] is a new double coset. There are \( |N|/|N^{(101204623)}| = 168/21 = 8 \) single cosets in [010204623].

The orbits of \( N^{(10120501)} \) on \( T \) are \{0\}, \{1, 3, 4, 5\}, \{2\} and \{6\}. So we consider the double cosets \( N t_0 t_1 t_0 t_2 t_0 t_5 t_0 t_1 t_0 N \), \( N t_0 t_1 t_0 t_2 t_0 t_5 t_0 t_1 t_1 N \), \( N t_0 t_1 t_0 t_2 t_0 t_5 t_0 t_1 t_2 N \), and \( N t_0 t_1 t_0 t_2 t_0 t_5 t_0 t_1 t_6 N \). However, \( N t_0 t_1 t_0 t_2 t_0 t_5 t_0 t_1 t_1 N = N t_0 t_1 t_0 t_2 t_0 t_5 t_0 t_1 t_1 N = [0102050] \). Also, 
\( N t_0 t_1 t_0 t_2 t_0 t_5 t_0 t_1 t_2 N = N t_0 t_1 t_0 t_2 t_0 t_5 t_0 t_1 t_2 N = [0102503] \) since \( t_0 t_1 t_0 t_2 t_0 t_5 t_0 t_1 t_2 = (1, 5, 2)(3, 4, 6)(t_0 t_1 t_0 t_2 t_0 t_5 t_0 t_3) (1, 4, 0, 3, 6, 5) = (1, 5, 2)(3, 4, 6) t_3 t_1 t_3 t_5 t_1 t_6 ; \)
\( N t_0 t_1 t_0 t_2 t_0 t_5 t_0 t_1 t_6 N = N t_0 t_1 t_0 t_2 t_0 t_5 t_0 t_4 N = [0102524] \) since \( t_0 t_1 t_0 t_2 t_0 t_5 t_0 t_4 t_6 = (1, 5, 2)(3, 4, 6)(t_0 t_1 t_0 t_2 t_0 t_4) (2, 3)(4, 0) = (1, 5, 2)(3, 4, 6) t_4 t_1 t_4 t_3 t_5 t_3 t_0 . \) But [010205010] is a new double coset. There are \( |N|/|N^{(101205010)}| = 168/24 = 7 \) single cosets in [010205010].
The orbits of \( N^{(01023040)} \) on \( T \) are \{0, 4, 5\}, \{1, 2, 3\}, and \{6\}. So we consider the double cosets \( Nt_0t_1t_0t_2t_3t_4t_5t_6t_0N \), \( Nt_0t_1t_0t_2t_3t_4t_5t_6t_1N \), and \( Nt_0t_1t_0t_2t_3t_4t_5t_6t_2N \). However, \( Nt_0t_1t_0t_2t_3t_4t_0t_0N = Nt_0t_1t_0t_2t_3t_0t_1N \) and \( Nt_0t_1t_0t_2t_3t_0t_1t_1N \). Also, \( Nt_0t_1t_0t_2t_3t_4t_1N = Nt_0t_1t_0t_2t_3t_4t_2t_1N \) and \( Nt_0t_1t_0t_2t_3t_4t_3t_1N \). Therefore, we have found all double cosets, and the result of the double coset enumeration is shown in the Cayley diagram in figure 4.1.

4.3 Proof of the Isomorphism between \( G \) and \( 2 \cdot M_{22} : 2 \)

In order to prove that \( G \cong 2 \cdot M_{22} : 2 \), we need to show first that 
\[
\langle \Phi(x), \Phi(y), \Phi(t) \rangle
\]
is a homomorphic image of \( G \) and that \(|G| = |\langle \Phi(x), \Phi(y), \Phi(t) \rangle| = 1,774,080\). Second, we must show that \( \langle \Phi(x), \Phi(y), \Phi(t) \rangle \cong 2 \cdot M_{22} : 2 \), from which we can conclude that \( 2 \cdot M_{22} : 2 \) is a homomorphic image of \( G \) and \( G \cong 2 \cdot M_{22} : 2 \).

In the previous section we constructed \( G \) using manual double coset enumeration and we concluded that \( G \) defined by the symmetric presentation must contain a homomorphic image of \( N = L_3(2) \). The index \(|G : N|\) is at most 10,560, since
Figure 4.1: Cayley Diagram of $2 \cdot M_{22} : 2$ over $L_3(2)$
\[ [G : N] = \frac{|N|}{|N^{(0)}|} + \frac{|N|}{|N^{(010)}|} + \frac{|N|}{|N^{(012)}|} + \frac{|N|}{|N^{(013)}|} + \frac{|N|}{|N^{(0120)}|} + \frac{|N|}{|N^{(0121)}|} + \frac{|N|}{|N^{(0123)}|} + \frac{|N|}{|N^{(0124)}|} + \cdots\]

\[ \leq \frac{168}{168} + \frac{168}{24} + \frac{168}{8} + \frac{168}{168} + \frac{168}{2} + \frac{168}{4} + \frac{168}{2} + \frac{168}{1} + \frac{168}{1} + \cdots + \frac{168}{168} = 10,560.\]

That is, \([G : N] = \frac{|G|}{|N|} \leq 10,560\). Since the index of \(N\) in \(G\) is at most 10,560 and \(|G| = \frac{|G|}{|N|} \cdot |N|\), the order of the homomorphic image group \(G\) is at most 1,774,080.

\[ |G| = \frac{|G|}{|N|} \cdot |N| \leq 10,560 \cdot |N| = 10,560 \cdot 168 = 1,774,080 \Rightarrow |G| \leq 1,774,080.\]

We now consider \(\langle \Phi(x), \Phi(y), \Phi(t) \rangle\). We note that \(\langle \Phi(x), \Phi(y), \Phi(t) \rangle\) is the group generated by the action of the generators \(x, y\) and \(t\) on the 10,560 cosets that we have found and it is a subgroup of the symmetric group \(S_{10560}\). Let \(G_1 = \langle \Phi(x), \Phi(y), \Phi(t) \rangle\). It is easily verified that \(|G_1| = 1,774,080\). Then, \(G_1\) is a homomorphic image of \(G\) since:

1. \(\Phi(N)\) acts as the group \(N\) on the symmetric generators, that is
   \[\Phi(x) : (\Phi(t_0), \Phi(t_1), \Phi(t_2), \Phi(t_3), \Phi(t_4), \Phi(t_5), \Phi(t_6))\]
   \[\Phi(y) : (\Phi(t_2), \Phi(t_6))(\Phi(t_4), \Phi(t_5))\]

2. \(t_0\) has exactly seven conjugates under conjugation by \(\Phi(N)\):
   \[t_0^{\Phi(N)} = \{t_0, t_1, t_2, t_3, t_4, t_5, t_6\}\]

3. Additional relations hold:
   \[\Phi(t_3)\Phi(t_6)\Phi(t_5)\Phi(t_6)\Phi(t_5)\Phi(t_6)\Phi(t_5)\Phi(t_6)\Phi(t_6) = (\Phi(t_1), \Phi(t_4), \Phi(t_2))\Phi(t_3), \Phi(t_5), \Phi(t_6))\]
   \[\Phi(t_2)\Phi(t_4)\Phi(t_6)\Phi(t_5)\Phi(t_2)\Phi(t_4)\Phi(t_5)\Phi(t_6)\Phi(t_5)\Phi(t_4) = (\Phi(t_2), \Phi(t_6))(\Phi(t_4), \Phi(t_5))\]
   \[\Phi(t_0)\Phi(t_6)\Phi(t_0)\Phi(t_2)\Phi(t_0)\Phi(t_6)\Phi(t_2)\Phi(t_2) = e\]

Therefore, we have that \(G/Ker(\Phi) \cong G_1 \Rightarrow |G| \geq |G_1|\). Thus, we proved that \(|G| = 1,774,080\).

Now, with the help of MAGMA we find that the elements
\[c = (1, 6, 5)(2, 0, 4)t_4t_2t_0t_2t_4\]
\[d = (1, 4, 5, 3)(2, 0)t_2t_3t_0t_3t_5\]
in \(G_1\) satisfy the following presentation of \(2 \cdot M_{22} : 2\):
\[\langle c, d | c^2 = d^4 = (cd)^{11} = (cd)^6 = [c, d]^4 = (cdcdcdcd)^3 = e \rangle.\]
Now, $c, d \in G_1$, thus $2 \cdot M_{22} : 2 = \langle c, d \rangle \leq G_1$. But $|2 \cdot M_{22} : 2| = |G_1| = 1,774,080$. Therefore, $G_1 \cong 2 \cdot M_{22} : 2$. Finally, since $G \cong G_1$, we conclude that $G \cong 2 \cdot M_{22} : 2$.

Table 4.1 lists the symmetric representation of the representatives of the 37 conjugacy classes of $2 \cdot M_{22} : 2$.

<table>
<thead>
<tr>
<th>Class</th>
<th>Representative</th>
<th>Class</th>
<th>Representative</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\text{Id}(N)$</td>
<td>6E</td>
<td>$(2,0,6)(3,5,4)t_5t_4t_0t_6t_0t_6t_1t_6$</td>
</tr>
<tr>
<td>2A</td>
<td>$(1,2,5)(3,6,4)t_0t_1t_0t_0t_5t_0t_1t_2$</td>
<td>7A</td>
<td>$(1,6,2)(4,5,0)t_0t_4t_0t_0$</td>
</tr>
<tr>
<td>2B</td>
<td>$(1,2,3)(4,5,0)t_1t_3t_1t_0t_2t_1t_6$</td>
<td>7B</td>
<td>$(1,6,4)(2,5,3)t_4t_4$</td>
</tr>
<tr>
<td>2C</td>
<td>$(1,2,3)(4,5,0)t_0t_4t_0t_0t_4t_0t_5$</td>
<td>8A</td>
<td>$(1,0,4,6)(3,5)t_2t_1t_0t_5$</td>
</tr>
<tr>
<td>2D</td>
<td>$(1,3,5,0,2,4,6)t_0t_5t_6t_1t_0t_4$</td>
<td>8B</td>
<td>$(1,0,4,2,6,3,5)t_6t_5t_0t_5$</td>
</tr>
<tr>
<td>2E</td>
<td>$(1,4,0,2,5,6,3)t_4t_5t_1t_3t_6t_0t_6$</td>
<td>10A</td>
<td>$(1,6)(2,3)t_0t_5t_0t_0t_4t_4$</td>
</tr>
<tr>
<td>2F</td>
<td>$(2,6)(4,5)t_4t_2t_5t_0t_5$</td>
<td>10B</td>
<td>$(1,6,4,2,0,5,3)t_6t_4t_4t_0t_1$</td>
</tr>
<tr>
<td>3</td>
<td>$(2,6,0)(3,4,5)t_3t_1t_4t_3t_0t_2t_2t_3$</td>
<td>10C</td>
<td>$(1,6,4,2,0,5,3)t_6t_5t_6t_5t_3$</td>
</tr>
<tr>
<td>4A</td>
<td>$(1,0,3)(2,4,5)t_1t_4t_5$</td>
<td>11</td>
<td>$(1,2,5,6,0,4,3)t_1t_5t_1t_5t_0t_3$</td>
</tr>
<tr>
<td>4B</td>
<td>$(1,2,4)(5,6,0)t_4t_3t_4t_4t_4t_3t_4$</td>
<td>12A</td>
<td>$(1,3,4)(2,0,6)t_4t_4t_6t_0t_0t_1$</td>
</tr>
<tr>
<td>4C</td>
<td>$(3,6)(5,0)t_0t_0t_3t_0t_3t_4$</td>
<td>12B</td>
<td>$(1,3,4)(2,0,6)t_0t_3t_3$</td>
</tr>
<tr>
<td>4D</td>
<td>$(1,4,6,5,0,2,3)t_0t_4t_0t_3t_0t_5$</td>
<td>14A</td>
<td>$(1,5,0)(3,6,4)t_4t_4t_4t_4t_4t_4$</td>
</tr>
<tr>
<td>4E</td>
<td>$(1,2,4)(3,6,5)t_1t_4t_0t_5t_1$</td>
<td>14B</td>
<td>$(1,6,2)(4,5,0)t_0t_2t_2t_2t_0t_0$</td>
</tr>
<tr>
<td>4F</td>
<td>$(1,3)(2,4,6,5)t_5t_3t_5t_4t_5$</td>
<td>14C</td>
<td>$(1,6,4,2,0,5,3)t_0t_1t_0t_5t_0t_5$</td>
</tr>
<tr>
<td>5</td>
<td>$(1,4,2)(3,5,6)t_2t_4t_5t_6$</td>
<td>14D</td>
<td>$(1,4)(3,5)t_5t_0t_5t_5t_1t_2$</td>
</tr>
<tr>
<td>6A</td>
<td>$(1,0,2)(3,6,4)t_5t_1t_4t_5t_1$</td>
<td>14E</td>
<td>$(1,3,6,5,2,0,4)t_2t_1t_5t_3t_1$</td>
</tr>
<tr>
<td>6B</td>
<td>$(1,2)(3,6)t_1t_3t_5t_3t_2t_4$</td>
<td>14F</td>
<td>$(1,4,0,3,6,2,5)t_2t_2t_0t_7t_7t_1$</td>
</tr>
<tr>
<td>6C</td>
<td>$(2,5,0)(3,4,6)t_4t_3t_4t_5$</td>
<td>22</td>
<td>$(1,2,5,6,0,4,3)t_4t_4t_4t_2t_3$</td>
</tr>
<tr>
<td>6D</td>
<td>$(1,4,3,2,6,5,0)t_3t_4t_2t_0t_5$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.1: Representatives of the Conjugacy Classes of $2 \cdot M_{22} : 2$. 
Chapter 5

The Automorphism Group of $M_{22}$

5.1 Introduction

In the last chapter we proved that $2 \cdot M_{22} : 2 \cong 2^* : L_3(2)$. We now factor this by $Z(2 \cdot M_{22} : 2) = \langle (1,2,5)(3,6,4)t_0t_1t_2t_0t_3t_1t_2 \rangle$. Thus, $2^* : L_3(2)$ is factored by the additional relation $(1,2,5)(3,6,4)t_0t_1t_2t_0t_3t_1t_2 = e$ and we are going to perform manual double coset enumeration of $G$ over $L_3(2)$ to construct $M_{22} : 2$.

5.2 Manual Double Coset Enumeration

We start we the double coset $N \in N\mathbb{N}$ denoted by $[s]$ which consists of the single coset $N$. Since $N$ is transitive on $T = \{0,1,2,3,4,5,6\}$, $Nt_0N = \{Nt_0^\sigma | \sigma \in N\} = \{Nt_0, Nt_1, Nt_2, Nt_3, Nt_4, Nt_5, Nt_6\}$ and we denote the double coset $Nt_0N$ by $[0]$. Thus, there are seven single cosets in $[0]$. Now we determine the orbits of $N^0 = \langle (2,6)(4,5), (2,5)(4,6), (1,5,2)(3,4,6) \rangle$ on $T$ which are $\{0\}$ and $\{1,2,3,4,5,6\}$, and consider the double cosets $Nt_0t_iN$ for one $t_i$ from each orbit of $N^0$. First we find that $Nt_0t_0 = N \in NeN$, so $t_0$ takes us back to $[s]$. Next we consider $t_1$, a representative from the orbit $\{1,2,3,4,5,6\}$ and find that $Nt_0t_1N$ denoted by $[01]$ is a new double coset. The number of single cosets in $[01]$ equals to $|N|/|N^{(01)}| = 168/4 = 42$, since $N^{(01)} = \langle (2,6)(4,5), (2,5)(4,6) \rangle$ has order 4.

The orbits of $N^{(01)}$ on $T$ are $\{0\}, \{1\}, \{3\}$, and $\{2,4,5,6\}$. We must consider the double cosets $Nt_0t_1t_0N, Nt_0t_1t_1N, Nt_0t_1t_2N$, and $Nt_0t_1t_3N$. However, $Nt_0t_1t_1N = Nt_0 = [0]$, but $Nt_0t_1t_0N, Nt_0t_1t_2N$, and $Nt_0t_1t_3N$ are new. Now, we have that $t_0t_1t_0 =$
Thus, \(Nt_0t_1t_0(1,0)(2,4,5,6) = Nt_1t_0t_1 = Nt_0t_1t_0 \Rightarrow (1,0)(2,4,5,6) \in N^{(010)}\) and \(N^{(010)} = \{(2,6)(4,5), (2,5)(4,6), (1,0)(2,4,5,6)\}\) is of order 8, thus the number of single cosets in \([010]\) is equal to \(|N|/|N^{(010)}| = 168/8 = 21\). Also, \(N^{(010)} = \{e\}\) is of order 1, therefore there are 168 single cosets in \([012]\), and \(N^{(013)} = \{(2,6)(4,5), (2,5)(4,6)\}\) is of order 4 which implies that the number of single cosets in \([013]\) is equal to \(|N|/|N^{(013)}| = 168/4 = 42\).

The orbits of \(N^{(010)}\) on \(T\) are \{0\}, \{1\}, \{2, 4, 5, 6\}, and \{3\}. So we consider the double cosets \(Nt_0t_1t_0t_0N, Nt_0t_1t_0t_1N, Nt_0t_1t_0t_2N, \) and \(Nt_0t_1t_0t_3N\). \(Nt_0t_1t_0t_0N = Nt_0t_1N = [01]\). Since \(t_0t_1t_0 = t_1t_0t_1 \Rightarrow t_0t_1t_0t_1 = t_1t_0 = (t_0t_1)(1,0)(2,5,4,6)\), \(Nt_0t_1t_0t_1N = Nt_0t_1N = [01]\). But the other two double cosets are new. We note that \(N^{(0102)} = \{e\}\), but \(Nt_0t_1t_0t_2(1,0)(4,6) = Nt_0t_1t_0t_2 = Nt_0t_1t_0t_2 \Rightarrow (1,0)(4,6) \in N^{(0102)}\). Therefore, the number of single cosets in \([0102]\) is \(|N|/|N^{(0102)}| = 168/2 = 84\) since \(N^{(0102)} = \{e, (1,0)(4,6)\}\). Also, \(Nt_0t_1t_0t_2(1,0)(2,4,5,6) = Nt_1t_0t_2(1,0)(2,4,5,6) \in N^{(0103)}\), thus \(N^{(0103)} = \{(2,6)(4,5), (2,5)(4,6), (1,0)(2,4,5,6)\}\) is of order 8 which implies that the number of single cosets in \([013]\) is equal to \(|N|/|N^{(013)}| = 168/8 = 21\).

The orbits of \(N^{(012)}\) on \(T\) are \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, and \{6\}. So we consider the double cosets \(Nt_0t_1t_2t_0N, Nt_0t_1t_2t_1N, Nt_0t_1t_2t_2N, Nt_0t_1t_2t_3N, Nt_0t_1t_2t_4N, Nt_0t_1t_2t_5N, Nt_0t_1t_2t_6N, Nt_0t_1t_2t_7N, Nt_0t_1t_2t_8N, \) and \(Nt_0t_1t_2t_9N, Nt_0t_1t_2t_{10}N, Nt_0t_1t_2t_{11}N, Nt_0t_1t_2t_{12}N, \) and \(Nt_0t_1t_2t_{13}N, Nt_0t_1t_2t_{14}N, Nt_0t_1t_2t_{15}N, Nt_0t_1t_2t_{16}N, \) and \(Nt_0t_1t_2t_{17}N\). However, \(Nt_0t_1t_2t_0N = Nt_0t_1N = [01]\). But the other six double cosets are new. Now, \(Nt_0t_1t_2t_0(1,0)(4,6) = Nt_0t_1t_0t_2t_0, \) by relation (1) \(\Rightarrow (1,0)(4,6) \in N^{(0120)}\). The number of single cosets in \([0120]\) is \(|N|/|N^{(0120)}| = 168/2 = 84\) since \(N^{(0120)} = \{e, (1,0)(4,6)\}\). Also, \(Nt_0t_1t_2t_1(1,2)(3,6) = Nt_0t_2t_1t_2 = Nt_0t_1t_2t_1, \) since \(t_0t_1t_0 = t_1t_0t_1 \Rightarrow t_0t_1t_0(1,2)(3,6) = (t_0t_1)(1,2)(3,6) \Rightarrow t_2t_1t_2 = t_1t_2t_1. \) Thus, \((1,2)(3,6) \in N^{(0121)}\) and \(N^{(0121)} = \{e, (1,2)(3,6)\}\) implies that the number of single cosets in \([0121]\) is equal to \(|N|/|N^{(0121)}| = 168/2 = 84\). For the other four double cosets: \([0123], [0124], [0125], [0126], \) we have that \(N^{(0123)} = N^{(0124)} = N^{(0125)} = N^{(0126)} = \{e\}\), so each one contains 168 single cosets.

The orbits of \(N^{(013)}\) on \(T\) are \{0\}, \{1\}, \{2, 4, 5, 6\}, and \{3\}. So we consider the double cosets \(Nt_0t_1t_3t_0N, Nt_0t_1t_3t_1N, Nt_0t_1t_3t_2N, \) and \(Nt_0t_1t_3t_3N, \) \(Nt_0t_1t_3t_4N = Nt_0t_1N = [01]\). Now, we show that \(t_0t_1t_3t_1 = (t_0t_1t_0)(2,6,4,5)(3,0)\). \((t_0t_1t_0)(2,6,4,5)(3,0) = t_0t_1t_0t_0 = t_1t_0t_1t_0 = Nt_0t_1t_3t_1N = Nt_0t_1t_3t_2N = [0103]\). But the other two double cosets are new. Now \(Nt_0t_1t_3t_0(1,3,0)(4,5,6) = Nt_1t_3t_0t_1 = Nt_0t_1t_3t_0, \) by relation (1) \(\Rightarrow (1,3,0)(4,5,6) \in N^{(0130)}\). The number of single cosets in \([0130]\) is \(|N|/|N^{(0130)}| = \)
$168/12 = 14$ since \(N^{(0130)} = \{(2,6)(4,5),(2,4)(5,6),(1,3,0)(4,5,6)\}$ has order 12. Also, \(N^{(0132)} = \{e\}\) thus, the number of single cosets in [0132] is 168.

The orbits of \(N^{(0102)}\) on \(T\) are \(\{0,1\}, \{2\}, \{3\}, \{4,6\}\) and \(\{5\}\). So we consider the double cosets \(N t_0 t_1 t_0 t_2 t_0 N, N t_0 t_1 t_0 t_2 t_2 N, N t_0 t_1 t_0 t_2 t_3 N, N t_0 t_1 t_0 t_2 t_4 N\) and \(N t_0 t_1 t_0 t_2 t_5 N\). However, \(N t_0 t_1 t_0 t_2 t_2 N = N t_0 t_1 t_0 t_2 t_3 N = N t_0 t_1 t_0 t_2 t_3 \Rightarrow (1,0)(4,6) \in N^{(01023)}\). Similarly, \((1,0)(4,6) \in N^{(01020)}, \text{ and } (1,0)(4,6) \in N^{(01023)}\). Thus the number of single cosets in each one of the following double cosets: \([01020], [01023], \text{ and } [01025]\) is \(\lvert N \rvert / \lvert N^{(01020)} \rvert = 168/2 = 84\) since \(N^{(01020)} = N^{(01022)} = N^{(01023)} = \{e, (1,0)(4,6)\}\). Also, \(N^{(01024)} = \{e\}\) thus, the number of single cosets in \([01024]\) is 168.

The orbits of \(N^{(0103)}\) on \(T\) are \(\{0,1\}, \{2,4,5,6\}\), and \(\{3\}\). So we consider the double cosets \(N t_0 t_1 t_0 t_3 t_0 N, N t_0 t_1 t_0 t_3 t_2 N, \text{ and } N t_0 t_1 t_0 t_3 t_3 N\). \(N t_0 t_1 t_0 t_3 t_3 N = N t_0 t_1 t_0 N = [010]\) Also, \(N t_0 t_1 t_0 t_3 t_0 N = N t_0 t_1 t_3 N = [013]\) since \(t_0 t_1 t_0 t_3 t_0 = (1,2,0)(3,4,6)t_0 t_1 t_3; N t_0 t_1 t_0 t_3 t_2 N = N t_0 t_1 t_2 t_5 N = [01025]\) since \(t_0 t_1 t_0 t_3 t_0 = (t_0 t_1 t_3)^{(1,6)(4,0)} = t_4 t_1 t_3\).

Now we are going to look at the orbits of \(N^{(0120)}\) on \(T\) which are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4,6\}\) and \(\{5\}\). So we consider the double cosets \(N t_0 t_1 t_2 t_0 t_0 N, N t_0 t_1 t_2 t_0 t_1 N, N t_0 t_1 t_2 t_0 t_2 N, N t_0 t_1 t_2 t_0 t_3 N, N t_0 t_1 t_2 t_0 t_4 N, \text{ and } N t_0 t_1 t_2 t_0 t_5 N\). However, \(N t_0 t_1 t_2 t_0 t_0 N = N t_0 t_1 t_2 N = [012]\) and \(N t_0 t_1 t_2 t_0 t_1 N = N t_0 t_1 t_2 N = [012]\) since \(t_0 t_1 t_2 t_0 t_1 = (1,0,2)(3,6,4)(t_0 t_1 t_2)^{(1,0)(4,6)} = (1,0,2)(3,6,4)t_1 t_2 t_0\). Also, \(N t_0 t_1 t_2 t_0 t_2 N = N t_0 t_1 t_2 t_0 N = [0120]\) since \(t_0 t_1 t_2 t_0 t_2 = (1,0,2)(3,6,4)(t_0 t_1 t_2 t_0)^{(1,0)(4,6)} = (1,0,2)(3,6,4)t_1 t_2 t_0 t_1.\) But the other three double cosets are new. Now, \(N t_0 t_1 t_2 t_0 t_3(1,0)(4,6) = N t_1 t_2 t_0 t_3 N = N t_0 t_1 t_2 t_3 N \Rightarrow (1,0)(4,6) \in N^{(01203)}\). Also, \(N t_0 t_1 t_2 t_0 t_5(1,0)(4,6) = N t_1 t_2 t_0 t_5 N = N t_0 t_1 t_2 t_5 N \Rightarrow (1,0)(4,6) \in N^{(01205)}\). Thus, \(N^{(01203)} = N^{(01205)} = \{e, (1,0)(4,6)\}\), so the number of single cosets in the double cosets: \([01203], [01205]\) is \(\lvert N \rvert / \lvert N^{(01203)} \rvert = 168/2 = 84\). Also, \(N^{(01204)} = \{e\}\) thus, the number of single cosets in \([01204]\) is 168.

The orbits of \(N^{(0121)}\) on \(T\) are \(\{0\}, \{1,2\}, \{3,6\}, \{4\}, \text{ and } \{5\}\). So we consider the double cosets \(N t_0 t_1 t_2 t_1 t_0 N, N t_0 t_1 t_2 t_1 t_1 N, N t_0 t_1 t_2 t_1 t_3 N, N t_0 t_1 t_2 t_1 t_4 N, \text{ and } N t_0 t_1 t_2 t_1 t_5 N\). However, \(N t_0 t_1 t_2 t_1 t_1 N = N t_0 t_1 t_2 N = [012]; N t_0 t_1 t_2 t_1 t_4 N = N t_0 t_1 t_2 t_0 t_5 N = [01205]\) since \(t_0 t_1 t_2 t_1 t_4 = (1,4,2)(3,0,5)(t_0 t_1 t_2 t_0 t_5)^{(1,3,4)(2,0,6)} = (1,4,2)(3,0,5)t_0 t_3 t_0 t_6 t_5\). \(N t_0 t_1 t_2 t_1 t_5 N = N t_0 t_1 t_2 t_0 N = [0120]\) since \(t_0 t_1 t_2 t_1 t_5 = \ldots\)
\[(1, 0, 5)(3, 4, 6)(t_0 t_1 t_2 t_0) = (1, 0, 5)(3, 4, 6)t_2 t_1 t_2 t_0 t_2.\] But the other two double cosets are new. Now, \(N t_0 t_1 t_2 t_0 (1, 0, 4, 6) = N t_1 t_0 t_2 t_0 t_1 = N t_0 t_1 t_2 t_0 t_1\) and \(N t_0 t_1 t_2 t_0 (1, 0, 2)(3, 6, 4) = N t_2 t_0 t_1 t_0 t_2 = N t_0 t_1 t_2 t_1 t_0 \Rightarrow (1, 0)(4, 6)\) and \((1, 0, 2)(3, 6, 4) \in N^{(01210)}\), and the number of single cosets in the double cosets \(01210\) is equal to \(|N|/|N^{(01210)}| = 168/6 = 28\) since \(N^{(01210)} = \{e, (1, 0)(4, 6), (1, 0, 2)(3, 6, 4)\}.\) \(N^{(01213)} = \{e\}\) thus, the number of single cosets in \(01213\) is 168.

The orbits of \(N^{(0123)}\) on \(T\) are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\},\) and \(\{6\}.\) So we consider the double cosets \(N t_0 t_1 t_2 t_3 t_0 N, N t_0 t_1 t_2 t_3 t_1 N, N t_0 t_1 t_2 t_3 t_2 N, N t_0 t_1 t_2 t_3 t_3 N, N t_0 t_1 t_2 t_3 t_4 N, N t_0 t_1 t_2 t_3 t_5 N,\) and \(N t_0 t_1 t_2 t_3 t_6 N.\) However, \(N t_0 t_1 t_2 t_3 t_3 N = N t_0 t_1 t_2 t_2 N = [012].\) Also, \(N t_0 t_1 t_2 t_3 t_1 N = N t_0 t_1 t_2 t_0 t_2 N = [01203]\) since \(t_0 t_1 t_2 t_3 t_1 = (1, 6, 2, 3)(5, 0)(t_0 t_1 t_2 t_3 t_0)^{(1, 2, 6, 5, 3, 4, 0)} = (1, 6, 2, 3)(5, 0)t_0 t_1 t_2 t_3 t_4.\) But \(N t_0 t_1 t_2 t_3 t_4 N = N t_0 t_1 t_2 t_3 t_0 N = [01230]\) since \(t_0 t_1 t_2 t_3 t_4 = (1, 6, 2, 3)(5, 0)(t_0 t_1 t_2 t_2 t_3)^{(1, 2, 6, 5, 3, 4, 0)} = (1, 6, 2, 3)(5, 0)t_0 t_1 t_2 t_3 t_1 t_4.\) \(N t_0 t_1 t_2 t_3 t_6 N = N t_0 t_1 t_2 t_4 t_4 N = [01204]\) since \(t_0 t_1 t_2 t_3 t_6 = (1, 3, 2)(4, 0, 5)(t_0 t_1 t_0 t_2 t_4)^{(1, 4, 2)(3, 5, 6)} = (1, 3, 2)(4, 0, 5)t_0 t_1 t_0 t_2 t_6.\) But the other three double cosets are new. There are 168 single cosets in the double cosets \([01230],[01232],\) and \([01235]\) since \(N^{(01230)} = N^{(01232)} = N^{(01235)} = \{e\}.\)

The orbits of \(N^{(0124)}\) on \(T\) are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\},\) and \(\{6\}.\) So we consider the double cosets \(N t_0 t_1 t_2 t_4 t_0 N, N t_0 t_1 t_2 t_4 t_1 N, N t_0 t_1 t_2 t_4 t_2 N, N t_0 t_1 t_2 t_4 t_3 N, N t_0 t_1 t_2 t_4 t_4 N, N t_0 t_1 t_2 t_4 t_5 N,\) and \(N t_0 t_1 t_2 t_4 t_6 N.\) However, \(N t_0 t_1 t_2 t_4 t_4 N = N t_0 t_1 t_2 t_2 N = [0122]\) since \(t_0 t_1 t_2 t_4 t_2 = (1, 2, 4)(3, 5, 0)(t_0 t_1 t_2 t_4 t_0)^{(1, 5, 3, 2, 0, 6)} = (1, 2, 4)(3, 5, 0)t_0 t_1 t_2 t_4 t_5.\) \(N t_0 t_1 t_2 t_4 t_5 N = N t_0 t_1 t_2 t_3 t_5 N = [01235]\) since \(t_0 t_1 t_2 t_4 t_5 = (1, 4, 0, 6)(2, 5)(t_0 t_1 t_2 t_3 t_5)^{(1, 2, 0, 5, 3, 6)} = (1, 4, 0, 6)(2, 5)t_0 t_1 t_2 t_4 t_5 t_1.\) \(N t_0 t_1 t_2 t_4 t_0 N = N t_0 t_1 t_2 t_1 t_3 N = [01213]\) since \(t_0 t_1 t_2 t_4 t_6 = (1, 4, 0, 6)(2, 5)(t_0 t_1 t_2 t_4 t_3)^{(1, 3, 2, 4, 0, 5)} = (1, 4, 0, 6)(2, 5)t_0 t_3 t_4 t_5 t_6 t_1.\) But the other three double cosets are new. There are 168 single cosets in the double cosets \([01240],[01243]\) since \(N^{(01240)} = N^{(01243)} = N^{(01246)} = \{e\}.\) \(N t_0 t_1 t_2 t_4 t_1 (1, 4, 2)(3, 5, 6) = N t_0 t_4 t_1 t_2 t_4 = N t_0 t_1 t_2 t_4 t_1 \Rightarrow (1, 4, 2)(3, 5, 6) \in N^{(01241)} \Rightarrow N^{(01241)} = \{e, (1, 4, 2)(3, 5, 6), (1, 2, 4)(3, 6, 5)\}.\) Thus, there are \(|N|/|N^{(01241)}| = 168/3 = 56\) single cosets in \([01241].\)

The orbits of \(N^{(0125)}\) on \(T\) are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\},\) and \(\{6\}.\) So we consider the double cosets \(N t_0 t_1 t_2 t_5 t_0 N, N t_0 t_1 t_2 t_5 t_1 N, N t_0 t_1 t_2 t_5 t_2 N, N t_0 t_1 t_2 t_5 t_3 N, N t_0 t_1 t_2 t_5 t_4 N, N t_0 t_1 t_2 t_5 t_5 N,\) and \(N t_0 t_1 t_2 t_5 t_6 N.\) However, \(N t_0 t_1 t_2 t_5 t_5 N = N t_0 t_1 t_2 N = [01253] = \{e\}.\)
and \( N_{t_0 t_1 t_2 t_3 t_1} = N_{t_0 t_1 t_3 t_2 t_0} = [01032] \) since \( t_0 t_1 t_2 t_3 t_1 = (2, 0, 5)(3, 6, 4)(t_0 t_1 t_0 t_3 t_2)^{(1, 0)}(2, 0, 3, 4) \). Also, \( N_{t_0 t_1 t_2 t_3 t_2} = N_{t_0 t_1 t_2 t_0 t_5} = [01205] \) since \( t_0 t_1 t_2 t_3 t_2 = (1, 0, 2)(3, 6, 4)(t_0 t_1 t_2 t_0 t_5)^{(1, 2, 0)}(5, 0) = (1, 0, 2)(3, 6, 4)t_5 t_2 t_3 t_0 t_0; N_{t_0 t_1 t_2 t_3 t_3} = N_{t_0 t_1 t_2 t_3 t_1} = [01241] \) since \( t_0 t_1 t_2 t_3 t_3 = (1, 6, 2, 3)(5, 0)(t_0 t_1 t_2 t_4 t_1)^{(1, 2, 0)}(3, 4, 6) = (1, 6, 2, 3)(5, 0)t_1 t_2 t_0 t_2; N_{t_0 t_1 t_2 t_3 t_4} = N_{t_0 t_1 t_2 t_3 t_5} = [01235] \) since \( t_0 t_1 t_2 t_3 t_4 = (1, 0, 5)(3, 4, 6)(t_0 t_1 t_2 t_3 t_5)^{(2, 6, 0)}(3, 4, 5) = (1, 0, 5)(3, 4, 6)t_5 t_0 t_2 t_4 t_1 \) and \( N_{t_0 t_1 t_2 t_3 t_6} = N_{t_0 t_1 t_2 t_3 t_2} = [01232] \) since \( t_0 t_1 t_2 t_3 t_6 = (1, 0, 5)(3, 4, 6)(t_0 t_1 t_2 t_3 t_2)^{(2, 3, 4, 0)}(5, 6) = (1, 0, 5)(3, 4, 6)t_2_1 t_3 t_4 t_3 \). The only new double coset is \([01250] \). \( N_{t_0 t_1 t_2 t_3 t_6} = (1, 0)(2, 5) = N_{t_0 t_1 t_2 t_5 t_1} = N_{t_0 t_1 t_2 t_5 t_0} \Rightarrow N^{[01250]} = \{ e, (1, 0)(2, 5) \}.

The orbits of \( N^{[01262]} \) on \( T \) are \( \{ 0 \}, \{ 1 \}, \{ 2 \}, \{ 3 \}, \{ 4, 5 \}, \) and \( \{ 6 \} \). So we consider the double cosets \( N_{t_0 t_1 t_2 t_3 t_0} \), \( N_{t_0 t_1 t_2 t_3 t_1} \), \( N_{t_0 t_1 t_2 t_3 t_2} \), \( N_{t_0 t_1 t_2 t_3 t_3} \), \( N_{t_0 t_1 t_2 t_3 t_4} \), and \( N_{t_0 t_1 t_2 t_3 t_5} \). However, \( N_{t_0 t_1 t_2 t_3 t_6} = N_{t_0 t_1 t_2 t_3 t_2} = [0126] \) since \( t_0 t_1 t_2 t_3 t_6 = (1, 0, 3)(4, 6, 5) = N_{t_0 t_1 t_2 t_3 t_3} \) \( N_{t_0 t_1 t_2 t_3 t_2} \Rightarrow N^{[01262]} = \{ e \} \). Thus, the number of single cosets in \([01262] \) is 84. The number of single cosets in \([01261] \) is 168 since \( N^{[01260]} = \{ e \} \).

The orbits of \( N^{[0130]} \) on \( T \) are \( \{ 0 \}, \{ 1 \}, \{ 2, 4, 5, 6 \} \). So we only consider the double cosets \( N_{t_0 t_1 t_3 t_0 t_0} \) and \( N_{t_0 t_1 t_3 t_0 t_2} \). However, \( N_{t_0 t_1 t_3 t_0 t_0} = N_{t_0 t_1 t_3 N_{t_0 t_1 t_3 t_3} = [0132] \), but the \( [01302] \) is a new double coset. \( N_{t_0 t_1 t_3 t_3 t_2} = (1, 0, 3)(4, 6, 5) = N_{t_0 t_1 t_3 t_3} \) \( N_{t_0 t_1 t_3 t_2} \Rightarrow N^{[01302]} = \{ e, (1, 0, 3)(4, 6, 5), (4, 5, 6)(1, 3, 0) \} \). Thus, the number of single cosets in \([01302] \) is \( |N| / |N^{[01302]}| \) = 168/3 = 56.

The orbits of \( N^{[0132]} \) on \( T \) are \( \{ 0 \}, \{ 1 \}, \{ 2 \}, \{ 3 \}, \{ 4 \}, \{ 5 \}, \) and \( \{ 6 \} \). So we consider the double cosets \( N_{t_0 t_1 t_3 t_3 t_0} \), \( N_{t_0 t_1 t_3 t_3 t_1} \), \( N_{t_0 t_1 t_3 t_3 t_2} \), \( N_{t_0 t_1 t_3 t_3 t_3} \), \( N_{t_0 t_1 t_3 t_3 t_4} \), and \( N_{t_0 t_1 t_3 t_3 t_5} \). However, \( N_{t_0 t_1 t_3 t_3 t_6} = N_{t_0 t_1 t_3 t_3 t_2} = [01262] \) since \( t_0 t_1 t_3 t_2 t_6 = (1, 6)(2, 0, 3, 4)(t_0 t_1 t_2 t_6)^{(1, 4, 5, 6, 0, 2, 3)} = (1, 6)(2, 0, 3, 4)t_2 t_4 t_3 t_5 t_3; N_{t_0 t_1 t_3 t_3 t_1} = N_{t_0 t_1 t_3 t_3 t_6} = [01261] \) since \( t_0 t_1 t_3 t_2 t_1 = (1, 2, 3)(4, 5, 0)(t_0 t_1 t_2 t_6 t_1)^{(1, 3, 6, 2, 4)} = (1, 2, 3)(4, 5, 0)t_4 t_3 t_1 t_2 t_3 \). Also, \( N_{t_0 t_1 t_3 t_3 t_3} = N_{t_0 t_1 t_2 t_3 t_2} = [01232] \) since \( t_0 t_1 t_3 t_3 t_3 = (1, 2, 3)(4, 5, 0)t_4 t_3 t_1 t_2 t_3 \).
\[ t_{01}t_{23}t_{24} = N_{t_{01}t_{23}t_{24}}N = [01243] \] since \( t_{01}t_{23}t_{24} = (1, 0, 5, 2)(3, 4)(t_{01}t_{23}t_{24})(1, 2, 3)(4, 5, 0) = (1, 0, 5, 2)(3, 4)t_{4}t_{23}t_{5}t_{t_{1}} \); \( N_{t_{01}t_{23}t_{24}}N = \]
\[ N_{t_{01}t_{23}t_{24}}N = [01246] \] since \( t_{01}t_{23}t_{24} = (1, 0, 5, 2)(3, 4)(t_{01}t_{23}t_{24})(1, 2, 3)(4, 5, 0) = (1, 0, 5, 2)(3, 4)t_{4}t_{23}t_{5}t_{t_{1}} \); \( N_{t_{01}t_{23}t_{24}}N = \]
\[ N_{t_{01}t_{23}t_{24}}N = [01320] \] since \( t_{01}t_{23}t_{24} = (1, 0, 2)(3, 4)(t_{01}t_{23}t_{24})(1, 2, 3)(4, 5, 0) = (1, 0, 2)(3, 4)t_{4}t_{23}t_{5}t_{t_{1}} \).

The orbits of \( N^{(01020)} \) on \( T \) are \( \{0, 1\}, \{2\}, \{3, 4, 6\}, \) and \( \{5\}. \) So we consider the double cosets \( N_{t_{01}t_{23}t_{24}}N, N_{t_{01}t_{23}t_{24}}N, N_{t_{01}t_{23}t_{24}}N, N_{t_{01}t_{23}t_{24}}N, \) and \( N_{t_{01}t_{23}t_{24}}N. \) However, \( N_{t_{01}t_{23}t_{24}}N = N_{t_{01}t_{23}t_{24}}N = [0102], \) and \( N_{t_{01}t_{23}t_{24}}N = \]
\[ N_{t_{01}t_{23}t_{24}}N = [0120] \] since \( t_{01}t_{23}t_{24} = (1, 0, 2)(3, 4)(t_{01}t_{23}t_{24})(1, 2, 3)(4, 5, 0) = (1, 0, 2)(3, 4)t_{4}t_{23}t_{5}t_{t_{1}} \).

The only new double cosets are \( [010203] \) and \( [010204]. \) \( N_{t_{01}t_{23}t_{24}}N = (1, 0, 2)(3, 4)(t_{01}t_{23}t_{24})(1, 2, 3)(4, 5, 0) = (1, 0, 2)(3, 4)t_{4}t_{23}t_{5}t_{t_{1}} \). Since \( t_{01}t_{23}t_{24} = (1, 0, 2)(3, 4)(t_{01}t_{23}t_{24})(1, 2, 3)(4, 5, 0) = (1, 0, 2)(3, 4)t_{4}t_{23}t_{5}t_{t_{1}} \), the only new double cosets are \( [010203] \) and \( [010204]. \) \( N_{t_{01}t_{23}t_{24}}N = (1, 0, 2)(3, 4)(t_{01}t_{23}t_{24})(1, 2, 3)(4, 5, 0) = (1, 0, 2)(3, 4)t_{4}t_{23}t_{5}t_{t_{1}} \).

The orbits of \( N^{(01022)} \) on \( T \) are \( \{0\}, \{1, 2\}, \{3, 4\}, \{5\}, \) and \( \{6\}. \) So we consider the double cosets \( N_{t_{01}t_{23}t_{24}}N, N_{t_{01}t_{23}t_{24}}N, N_{t_{01}t_{23}t_{24}}N, N_{t_{01}t_{23}t_{24}}N, \) and \( N_{t_{01}t_{23}t_{24}}N. \) However, \( N_{t_{01}t_{23}t_{24}}N = N_{t_{01}t_{23}t_{24}}N = [0102], \) and \( N_{t_{01}t_{23}t_{24}}N = \]
\[ N_{t_{01}t_{23}t_{24}}N = [0120] \] since \( t_{01}t_{23}t_{24} = (1, 0, 2)(3, 4)(t_{01}t_{23}t_{24})(1, 2, 3)(4, 5, 0) = (1, 0, 2)(3, 4)t_{4}t_{23}t_{5}t_{t_{1}} \).

The only new double cosets are \( [010203] \) and \( [010204]. \) \( N_{t_{01}t_{23}t_{24}}N = (1, 0, 2)(3, 4)(t_{01}t_{23}t_{24})(1, 2, 3)(4, 5, 0) = (1, 0, 2)(3, 4)t_{4}t_{23}t_{5}t_{t_{1}} \).

The orbits of \( N^{(01024)} \) on \( T \) are \( \{0\}, \{1, 2\}, \{3, 4\}, \{5\}, \) and \( \{6\}. \) So we consider the double cosets \( N_{t_{01}t_{23}t_{24}}N, N_{t_{01}t_{23}t_{24}}N, N_{t_{01}t_{23}t_{24}}N, N_{t_{01}t_{23}t_{24}}N, \) and \( N_{t_{01}t_{23}t_{24}}N. \) However, \( N_{t_{01}t_{23}t_{24}}N = N_{t_{01}t_{23}t_{24}}N = [0102], \) and \( N_{t_{01}t_{23}t_{24}}N = \]
\[ N_{t_{01}t_{23}t_{24}}N = [0120] \] since \( t_{01}t_{23}t_{24} = (1, 0, 2)(3, 4)(t_{01}t_{23}t_{24})(1, 2, 3)(4, 5, 0) = (1, 0, 2)(3, 4)t_{4}t_{23}t_{5}t_{t_{1}} \).

The only new double cosets are \( [010203] \) and \( [010204]. \) \( N_{t_{01}t_{23}t_{24}}N = (1, 0, 2)(3, 4)(t_{01}t_{23}t_{24})(1, 2, 3)(4, 5, 0) = (1, 0, 2)(3, 4)t_{4}t_{23}t_{5}t_{t_{1}} \).

The orbits of \( N^{(01024)} \) on \( T \) are \( \{0, 1\}, \{2\}, \{3, 4\}, \{5\}, \) and \( \{6\}. \) So we consider the double cosets \( N_{t_{01}t_{23}t_{24}}N, N_{t_{01}t_{23}t_{24}}N, N_{t_{01}t_{23}t_{24}}N, N_{t_{01}t_{23}t_{24}}N, \) and \( N_{t_{01}t_{23}t_{24}}N. \) However, \( N_{t_{01}t_{23}t_{24}}N = N_{t_{01}t_{23}t_{24}}N = [0102], \) and \( N_{t_{01}t_{23}t_{24}}N = \]
\[ N_{t_{01}t_{23}t_{24}}N = [0120] \] since \( t_{01}t_{23}t_{24} = (1, 0, 2)(3, 4)(t_{01}t_{23}t_{24})(1, 2, 3)(4, 5, 0) = (1, 0, 2)(3, 4)t_{4}t_{23}t_{5}t_{t_{1}} \).

Thus there are 84 single cosets in \([010232]\) and \([010235].\)
\[ N_{t_1t_2t_3t_4} = N_{t_0t_1t_0t_2t_3t_4} \implies N^{(010243)} = \{e, (1,2,0)(3,4,6), (1,0,2)(3,6,4)\}, \]
so \([010243]\) contains \(|N|/|N^{(010243)}| = 168/3 = 56\) single cosets.

The orbits of \(N^{(01025)}\) on \(T\) are \(\{0, 1\}, \{2\}, \{3\}, \{4, 6\}, \{5, \}\). So we consider the double cosets \(N_{t_0t_1t_0t_2t_3t_0N}, N_{t_0t_1t_0t_2t_3t_2N}, N_{t_0t_1t_0t_2t_3t_3N}, N_{t_0t_1t_0t_2t_3t_4N}, \) and \(N_{t_0t_1t_0t_2t_3t_5N}\). However, \(N_{t_0t_1t_0t_2t_3t_2N} = N_{t_0t_1t_0t_2t_3t_3N} = [0102] \) and \(N_{t_0t_1t_0t_2t_3t_0N} = N_{t_0t_1t_0t_2t_3t_5N} = [0125]\) since \(t_0t_1t_0t_2t_3t_0 = (1, 5, 2)(3, 4, 6)(t_0t_1t_2t_2) = (1, 5, 2)(3, 4, 6)t_2t_0t_1t_5. \)
\(N_{t_0t_1t_0t_2t_3t_2N} = [0103]\) since \(t_0t_1t_0t_2t_3t_2 = (t_0t_1t_2t_3)(1, 4)(6, 0) = t_6t_4t_6t_3. \)
\(N_{t_0t_1t_0t_2t_3t_3N} = N_{t_0t_1t_0t_2t_4t_1N} = [010241]\) since \(t_0t_1t_0t_2t_3t_3 = (1, 6, 2, 3)(5, 0)(t_0t_1t_0t_2t_4t_1)^{(1, 6, 4, 0)}(3, 5) = (1, 6, 2, 3)(5, 0)t_1t_0t_2t_0t_6. \)

The only new double cosets is \([010254]\) and it contains \(168\) single cosets since \(N^{(010254)} = \{e\} \).

The orbits of \(N^{(01203)}\) on \(T\) are \(\{0, 1\}, \{2\}, \{3\}, \{4, 6\}, \{5, \}\). So we consider the double cosets \(N_{t_0t_1t_0t_2t_3t_0N}, N_{t_0t_1t_0t_2t_3t_2N}, N_{t_0t_1t_0t_2t_3t_3N}, N_{t_0t_1t_0t_2t_3t_4N}, \)
and \(N_{t_0t_1t_0t_2t_3t_5N}. \) However, \(N_{t_0t_1t_0t_2t_3t_3N} = N_{t_0t_1t_0t_2t_0} = [0120]. \) Also,
\(N_{t_0t_1t_0t_2t_3t_3N} = \{0123\} \) since \(t_0t_1t_0t_2t_3t_0 = (1, 2, 0, 5)(4, 6)(t_0t_1t_2t_3) = \{01242\} \) since \(t_0t_1t_0t_2t_3t_2 = (1, 5, 2, 0)(3, 6)(t_0t_1t_2t_4) = (1, 5, 2, 0)(3, 6)t_3t_5t_4t_6t_7. \)
\(N_{t_0t_1t_0t_2t_3t_5N} = \{01203\} \) since \(t_0t_1t_0t_2t_3t_5 = (1, 6, 2, 3)(t_0t_1t_0t_2t_3t_3)^{(1, 6, 4, 0)}(3, 5) = (1, 6, 2, 3)(4, 6)t_4t_6t_3t_4t_3t_3. \) Hence new double coset is \([012034]\) and it contains \(168\) single cosets since \(N^{(012034)} = \{e\} \).

The orbits of \(N^{(01204)}\) on \(T\) are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \) and \(\{6\}. \) So we consider the double cosets \(N_{t_0t_1t_2t_0t_4t_0N}, N_{t_0t_1t_2t_0t_4t_1N}, N_{t_0t_1t_2t_0t_4t_2N}, N_{t_0t_1t_2t_0t_4t_3N}, \)
\(N_{t_0t_1t_2t_0t_4t_4N}, N_{t_0t_1t_2t_0t_4t_5N}, \) and \(N_{t_0t_1t_2t_0t_4t_6N}. \) However, \(N_{t_0t_1t_2t_0t_4} = [0120]. \) Also,
\(N_{t_0t_1t_2t_0t_4t_0N} = [01240] \) since \(t_0t_1t_2t_0t_4t_0 = (1, 6, 2, 4)(5, 0)(t_0t_1t_0t_4t_0)^{(1, 6, 5, 3, 4, 2, 6)} = (1, 6, 2, 4)(5, 0)t_3t_5t_6t_5t_5. \)
\(N_{t_0t_1t_2t_0t_4t_1N} = [01240] \) since \(t_0t_1t_2t_0t_4t_1 = (2, 4)(3, 6, 0, 5)(t_0t_1t_0t_2t_4t_0)^{(2, 4)}(5, 6) = (2, 4)(3, 6, 0, 5)t_0t_1t_0t_4t_0t_2; \)
\(N_{t_0t_1t_2t_0t_4t_2N} = [01241] \) since \(t_0t_1t_2t_0t_4t_2 = (1, 2, 3, 6, 4)(t_0t_1t_0t_4t_1)^{(1, 6, 5, 2, 4, 0)} = (1, 2, 3, 6, 4)(t_0t_1t_0t_4t_1)^{(2, 5, 4, 6)} = (1, 0, 2)(3, 6, 4)t_6t_1t_5t_6t_6. \)
\(N_{t_0t_1t_2t_0t_4t_3N} = [012034] \) since \(t_0t_1t_2t_0t_4t_3 = (1, 5, 2)(3, 4, 6)t_2t_0t_3t_3t_6; \)
\(N_{t_0t_1t_2t_0t_4t_4N} = [012034] \) since \(t_0t_1t_2t_0t_4t_4 = (1, 5)(2, 0)t_0t_1t_2t_0t_4t_4; \)
\(N_{t_0t_1t_2t_0t_4t_5N} = [012045] \) since \(t_0t_1t_2t_0t_4t_5 = (2, 0, 5)(3, 6, 4)t_5t_1t_0t_5t_3t_2. \)

Thus there are
\[ |N| / \left| N^{(012045)} \right| = 168/3 = 56 \text{ single cosets in } [012045]. \]

The orbits of \( N^{(01205)} \) on \( T \) are \{0, 1\}, \{2\}, \{3\}, \{4, 6\}, and \{5\}. So we consider the double cosets \( N t_0 t_1 t_2 t_0 t_5 t_0 N, N t_0 t_1 t_2 t_3 t_2 t_2 N, N t_0 t_1 t_2 t_0 t_5 t_3 N, N t_0 t_1 t_2 t_0 t_5 t_4 N, \) and \( N t_0 t_1 t_2 t_0 t_5 t_5 N. \) However, \( N t_0 t_1 t_2 t_0 t_5 t_5 = N t_0 t_1 t_2 t_0 t_5 t_0 = [0120] \). Also, \( N t_0 t_1 t_2 t_0 t_5 t_0 N = N t_0 t_1 t_2 t_0 t_5 t_0 = [010250] \) since \( t_0 t_1 t_2 t_0 t_5 t_0 = (2, 0, 5)(3, 6, 4)(t_0 t_1 t_2 t_0 t_5 t_0) = (1, 2, 5)(3, 6, 4)(t_0 t_1 t_2 t_0 t_5 t_0) \)

\[ (2, 0, 5)(3, 6, 4), t_2 t_0 t_5 t_5 t_1 t_2; N t_0 t_1 t_2 t_0 t_5 t_5 N = N t_0 t_1 t_2 t_0 t_5 t_1 = [0121] \] since \( t_0 t_1 t_2 t_0 t_5 t_3 = (1, 5, 2)(3, 4, 6)(t_0 t_1 t_2 t_0 t_5 t_3) = (1, 5, 2)(3, 4, 6)(t_0 t_1 t_2 t_0 t_5 t_4) \)

\[ N t_0 t_1 t_2 t_0 t_5 t_4 = [0124] \) since \( t_0 t_1 t_2 t_0 t_5 t_4 = (1, 5, 2)(3, 4, 6)(t_0 t_1 t_2 t_0 t_5 t_4) \) \[ (1, 5, 2)(3, 4, 6)(t_0 t_1 t_2 t_0 t_5 t_4) \] The only new double coset is \([012052] \). \( N t_0 t_1 t_2 t_0 t_5 t_0 = (1, 0, 2)(3, 6, 4) = N t_0 t_1 t_2 t_0 t_5 t_1 = N t_0 t_1 t_2 t_0 t_5 t_2 \Rightarrow \) \( N^{(012052)} = \{e, (2, 0)(3, 4), (1, 0)(3, 5, 6), (1, 0, 2)(3, 4, 6), (1, 0, 2)(3, 4, 6), (1, 0, 2)(3, 4, 6), \} \) Thus, \([012052] \) contains \( |N| / |N^{(012052)}| = 168/6 = 28 \) single cosets.

The orbits of \( N^{(01210)} \) on \( T \) are \{0, 1, 2\}, \{3, 4, 6\}, and \{5\}. So we consider the double cosets \( N t_0 t_1 t_2 t_0 t_0 t_0 N, N t_0 t_1 t_2 t_0 t_0 t_3 N, \) and \( N t_0 t_1 t_2 t_0 t_0 t_5 N. \) However, \( N t_0 t_1 t_2 t_0 t_0 t_5 N = N t_0 t_1 t_2 t_0 t_0 = [0121] \). \( N t_0 t_1 t_2 t_0 t_0 t_5 N = N t_0 t_1 t_2 t_0 t_0 t_5 = [012052] \) since \( t_0 t_1 t_2 t_0 t_5 t_0 = (1, 0)(2, 5)(t_0 t_1 t_2 t_0 t_5) = (1, 2, 5)(3, 6, 4) \) \( t_0 t_1 t_2 t_0 t_5 t_1 = \) \( t_0 t_1 t_2 t_0 t_5 t_2 = \) \( t_0 t_1 t_2 t_0 t_5 t_3 \) The only new double coset is \([012103] \). \( N t_0 t_1 t_2 t_0 t_0 = (1, 0)(3, 5, 6) = N t_0 t_1 t_2 t_0 t_0 t_5 \Rightarrow \) \( N^{(012103)} = \{e, (1, 0)(3, 5, 6) \} \) Thus, there are \( |N| / |N^{(012103)}| = 168/2 = 84 \) single cosets in \([012103] \).

The orbits of \( N^{(01213)} \) on \( T \) are \{0, \} \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, and \{6\}. So we consider the double cosets \( N t_0 t_1 t_2 t_0 t_3 t_0 N, N t_0 t_1 t_2 t_0 t_3 t_1 N, N t_0 t_1 t_2 t_0 t_3 t_2 N, N t_0 t_1 t_2 t_0 t_3 t_3 N, \) \( N t_0 t_1 t_2 t_0 t_3 t_4 N, N t_0 t_1 t_2 t_0 t_3 t_5 N, \) and \( N t_0 t_1 t_2 t_0 t_3 t_6 N. \) However, \( N t_0 t_1 t_2 t_0 t_3 t_6 N = N t_0 t_1 t_2 t_0 t_3 t_1 N = [0121] \). Also, \( N t_0 t_1 t_2 t_0 t_3 t_0 N = N t_0 t_1 t_2 t_0 t_3 t_2 = [010243] \) since \( t_0 t_1 t_2 t_0 t_3 t_0 = (1, 4, 2)(3, 5, 6)(t_0 t_1 t_2 t_0 t_3) = (1, 4, 2)(3, 5, 6)(t_0 t_1 t_2 t_0 t_3) = (1, 4, 2)(3, 5, 6)(t_0 t_1 t_2 t_0 t_3) \) \( N t_0 t_1 t_2 t_0 t_3 t_1 N = N t_0 t_1 t_2 t_0 t_3 t_2 N = [010203] \) since \( t_0 t_1 t_2 t_0 t_3 t_1 = (2, 3, 4, 0)(5, 6)(t_0 t_1 t_2 t_0 t_3) = (1, 2, 3)(4, 5, 0)(t_0 t_1 t_2 t_0 t_3) = (1, 2, 3)(4, 5, 0)(t_0 t_1 t_2 t_0 t_3) = (1, 2, 3)(4, 5, 0)(t_0 t_1 t_2 t_0 t_3) = (1, 2, 3)(4, 5, 0)(t_0 t_1 t_2 t_0 t_3) \) \( N t_0 t_1 t_2 t_0 t_3 t_2 N = [010320] \) since \( t_0 t_1 t_2 t_0 t_3 t_2 = (1, 2, 3)(4, 5, 0)(t_0 t_1 t_2 t_0 t_3) = (1, 2, 3)(4, 5, 0)(t_0 t_1 t_2 t_0 t_3) = (1, 2, 3)(4, 5, 0)(t_0 t_1 t_2 t_0 t_3) = (1, 2, 3)(4, 5, 0)(t_0 t_1 t_2 t_0 t_3) \) \( N t_0 t_1 t_2 t_0 t_3 t_4 N = [0124] \) since \( t_0 t_1 t_2 t_0 t_3 t_4 = (2, 3, 4, 0)(5, 6)(t_0 t_1 t_2 t_0 t_3) = (2, 3, 4, 0)(5, 6)(t_0 t_1 t_2 t_0 t_3) = (2, 3, 4, 0)(5, 6)(t_0 t_1 t_2 t_0 t_3) = (2, 3, 4, 0)(5, 6)(t_0 t_1 t_2 t_0 t_3) \) \( N t_0 t_1 t_2 t_0 t_3 t_6 N = [0132] \) since \( t_0 t_1 t_2 t_0 t_3 t_6 = (t_0 t_1 t_2 t_0 t_3) = (t_0 t_1 t_2 t_0 t_3) = (t_0 t_1 t_2 t_0 t_3) = (t_0 t_1 t_2 t_0 t_3) \) The double coset \([012135] \) is the only new one and it contains \( 168 \) single cosets since \( N^{(012135)} = \{e\} \).

The orbits of \( N^{(01230)} \) on \( T \) are \{0, 1\}, \{2\}, \{3\}, \{4\}, and \{5, 6\}. So we con-
sider the double cosets \(Nt_0t_1t_2t_3t_0t_0N, Nt_0t_1t_2t_3t_0t_1N, Nt_0t_1t_2t_3t_0t_2N, Nt_0t_1t_2t_3t_0t_3N, Nt_0t_1t_2t_3t_0t_4N, \) and \(Nt_0t_1t_2t_3t_0t_5N.\) However, \(Nt_0t_1t_2t_3t_0t_0N = Nt_0t_1t_2t_3N = [0123].\) Also, \(Nt_0t_1t_2t_3t_0t_1N = Nt_0t_1t_2t_3t_0t_3N = [010203]\) since \(t_0t_1t_2t_3t_0t_1 = (1,6)(2,0,3,4)(t_0t_1t_0t_2t_0)(2,4)(3,6,0,5) = (1,6)(2,0,3,4)t_5t_1t_5t_0t_5t_6; Nt_0t_1t_2t_3t_0t_2N = Nt_0t_1t_0t_3t_0N = [010320]\) since \(t_0t_1t_2t_3t_0t_2 = (1,5,6)(2,0,3)(t_0t_1t_0t_3t_2)(1,5,6)(2,0,3) = (1,5,6)(2,0,3)t_3t_5t_3t_2t_0; Nt_0t_1t_2t_3t_0t_3N = Nt_0t_1t_2t_3t_0t_4N = [012030]\) since \(t_0t_1t_2t_3t_0t_3 = t_0t_1t_2t_0t_4; Nt_0t_1t_2t_3t_0t_4N = Nt_0t_1t_2t_1t_5N = [012135]\) since \(t_0t_1t_2t_3t_0t_4 = (1,6,3)(4,0,5)(t_0t_1t_2t_3t_0)(1,5,6)(2,6,4) = (1,6,3)(4,0,5)t_3t_0t_0t_1t_5; Nt_0t_1t_2t_3t_0t_5N = Nt_0t_1t_0t_2t_30N = [010230]\) since \(t_0t_1t_2t_3t_0t_5 = (1,2,6,0,3,5)(t_0t_1t_0t_2t_3)(1,5,6)(3,4,6,2,4,0) = (1,2,6,0,3,5)t_1t_5t_1t_4t_6t_1.\)

The orbits of \(N^{[01232]}\) on \(T\) are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\},\) and \(\{6\}.\) So we consider the double cosets \(Nt_0t_1t_2t_3t_0N, Nt_0t_1t_2t_3t_1N, Nt_0t_1t_2t_3t_2N, Nt_0t_1t_2t_3t_3N, Nt_0t_1t_2t_3t_4N, Nt_0t_1t_2t_3t_5N,\) and \(Nt_0t_1t_2t_3t_6N.\) However, \(Nt_0t_1t_2t_3t_2N = Nt_0t_1t_2t_3N = [0123].\) Also, \(Nt_0t_1t_2t_3t_0N = [012034]\) since \(t_0t_1t_2t_3t_0t_0 = (1,0)(2,6,5,4)(t_0t_1t_2t_3t_0t_4)(1,2,5,6,0,4,3) = (1,0)(2,6,5,4)\(t_4t_5t_1t_4t_3; Nt_0t_1t_2t_3t_1N = (1,2,3,6)(4,5)(t_0t_1t_2t_3t_0t_4)(2,6,0)(3,4,5) = (1,2,3,6)(4,5)t_5t_1t_6t_1t_4t_2; Nt_0t_1t_2t_3t_2N = Nt_0t_1t_3t_2N = [0132]\) since \(t_0t_1t_2t_3t_2t_3 = t_0t_1t_3t_2; Nt_0t_1t_2t_3t_4N = Nt_0t_1t_2t_3t_0t_4N = [012034]\) since \(t_0t_1t_2t_3t_2t_4 = (1,3,4,5)(6,0)(t_0t_1t_2t_3t_4)(1,6,3,5,4,2,0) = (1,3,4,5)(6,0)\(t_6t_1t_4t_3t_2; Nt_0t_1t_2t_3t_6N = Nt_0t_1t_2t_5N = [0125]\) since \(t_0t_1t_2t_3t_6t_5 = (1,6,4)(2,5,3)(t_0t_1t_2t_5)(2,0,4,3,5,6) = (1,6,4)(2,5,3)t_4t_5t_6; Nt_0t_1t_2t_3t_6N = Nt_0t_1t_0t_2t_3t_4N = [010254]\) since \(t_0t_1t_2t_3t_2t_6 = (1,6,3)(4,0,5)(t_0t_1t_0t_2t_3)(1,4,2,3,5,6) = (1,6,3)(4,0,5)t_0t_4t_1t_6t_2.\)

The orbits of \(N^{[01235]}\) on \(T\) are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\},\) and \(\{6\}.\) So we consider the double cosets \(Nt_0t_1t_2t_3t_0t_0N, Nt_0t_1t_2t_3t_0t_1N, Nt_0t_1t_2t_3t_0t_2N, Nt_0t_1t_2t_3t_0t_3N, Nt_0t_1t_2t_3t_0t_4N, Nt_0t_1t_2t_3t_0t_5N,\) and \(Nt_0t_1t_2t_3t_0t_6N.\) However, \(Nt_0t_1t_2t_3t_5N = Nt_0t_1t_2t_3N = [0123].\) Also, \(Nt_0t_1t_2t_3t_0t_4N = Nt_0t_1t_2t_1t_3t_5N = [012135]\) since \(t_0t_1t_2t_3t_0t_0 = (1,3,2,4,6,0,5)(t_0t_1t_2t_3t_0)(1,6,4,0)(3,5) = (1,3,2,4,6,0,5)t_4t_6t_2t_3t_3; Nt_0t_1t_2t_3t_5t_1N = Nt_0t_1t_2t_0t_4t_3N = [012043]\) since \(t_0t_1t_2t_3t_5t_1 = (1,5,0,3,2,6,4)(t_0t_1t_2t_4t_3)(1,5,4)(3,4) = (1,5,0,3,2,6,4)\(t_0t_5t_2t_3t_4; Nt_0t_1t_2t_3t_5t_2N = Nt_0t_1t_2t_4N = [0124]\) since \(t_0t_1t_2t_3t_5t_2 = (1,4,5,3)(2,0)(t_0t_1t_2t_4)(1,5,2,0)(3,6) = (1,4,5,3)(2,0)t_5t_0t_4; Nt_0t_1t_2t_3t_5t_3N = Nt_0t_1t_2t_5N = [0125]\) since \(t_0t_1t_2t_3t_5t_3 = \)
\[(1, 4, 6)(2, 3, 5)(t_0 t_1 t_2 t_3 t_5)^{(2, 0, 6)(3, 5, 4)} = (1, 4, 6)(2, 3, 5)t_0 t_1 t_2 t_3 t_5 = N t_0 t_1 t_2 t_3 t_5 t_6 N = N t_0 t_1 t_0 t_2 t_3 t_4 t_5 = \{012043\} \text{ since } t_0 t_1 t_2 t_3 t_4 t_5 = (1, 5)(2, 4, 0, 3)(t_0 t_1 t_0 t_2 t_4 t_3)^{(1, 4, 3)(2, 6, 0)} = (1, 5)(2, 4, 0, 3)t_2 t_4 t_2 t_3 t_1. \text{ The only new double coset is } [012354].

N t_0 t_1 t_2 t_3 t_4 t_5(1, 5, 6)(2, 3, 4) = N t_0 t_5 t_3 t_4 t_5 t_2 = N t_0 t_1 t_2 t_3 t_5 t_4 \Rightarrow N^{(012354)} = \{e, (1, 6, 5)(2, 4, 3), (1, 5, 6)(2, 3, 4)\}. \text{ Thus, } [012354] \text{ contains } |N|/|N^{(012354)}| = 168/3 = 56 \text{ single cosets.}

The orbits of } N^{(01240)} \text{ on } T \text{ are } \{0\}, \{1\}, \{2, 4\}, \{3\}, \{5\}, \text{ and } \{6\}. \text{ So we consider the double cosets } N t_0 t_1 t_2 t_4 t_0 t_0 N, N t_0 t_1 t_2 t_4 t_0 t_1 N, N t_0 t_1 t_2 t_4 t_0 t_2 N, N t_0 t_1 t_2 t_4 t_0 t_3 N, N t_0 t_1 t_2 t_4 t_0 t_5 N, \text{ and } N t_0 t_1 t_2 t_4 t_0 t_6 N. \text{ However, } N t_0 t_1 t_2 t_4 t_0 t_0 N = N t_0 t_1 t_0 t_3 t_2 t_0 N = \{013204\} \text{ since } t_0 t_1 t_2 t_4 t_0 t_1 = (1, 4, 5, 3)(2, 0)(t_0 t_1 t_0 t_3 t_2 t_0)^{(1, 2, 0, 3, 5, 4, 6)} = (1, 4, 5, 3)(2, 0)t_3 t_2 t_3 t_5 t_0 t_3; N t_0 t_1 t_2 t_4 t_0 t_2 N = N t_0 t_1 t_0 t_2 t_4 t_0 N = \{012043\} \text{ since } t_0 t_1 t_2 t_4 t_0 t_2 = (1, 4, 5, 3)(2, 0, 4)(t_0 t_1 t_0 t_4 t_0)^{(1, 6)(4, 0)} = (1, 4, 5, 3)(2, 0, 4)t_4 t_0 t_4 t_0 t_2 t_4 t_0; N t_0 t_1 t_2 t_4 t_0 t_3 N = N t_0 t_1 t_2 t_1 t_3 t_0 t_5 N = \{012135\} \text{ since } t_0 t_1 t_2 t_4 t_0 t_5 = (1, 4)(3, 6, 5, 0)(t_0 t_1 t_2 t_3 t_5 t_3 t_5) = (1, 4)(3, 6, 5, 0)t_3 t_5 t_3 t_2 t_3 t_5 = N t_0 t_1 t_2 t_3 t_4 t_0 N = \{012034\} \text{ since } t_0 t_1 t_2 t_4 t_0 t_5 = (1, 2, 4)(3, 5, 0)(t_0 t_1 t_2 t_3 t_4)^{(1, 4, 5, 2, 0, 3, 6)} = (1, 2, 4)(3, 5, 0)t_3 t_4 t_0 t_5 t_3 t_5; \text{ The only new double coset is } [012403].

N t_0 t_1 t_2 t_4 t_0 t_3(2, 0, 6)(3, 5, 4) = N t_0 t_1 t_0 t_3 t_6 t_5 = N t_0 t_1 t_2 t_4 t_0 t_3 \Rightarrow N^{(012403)} = \{e, (2, 6, 0)(3, 4, 5), (2, 0, 6)(3, 5, 4)\}. \text{ Thus, } [012403] \text{ contains } |N|/|N^{(012403)}| = 168/3 = 56 \text{ single cosets.}

The orbits of } N^{(01241)} \text{ on } T \text{ are } \{0\}, \{1, 2, 4\}, \text{ and } \{3, 5, 6\}. \text{ So we consider the double cosets } N t_0 t_1 t_2 t_4 t_0 t_0 N, N t_0 t_1 t_2 t_4 t_0 t_1 N, N t_0 t_1 t_2 t_4 t_0 t_2 N, N t_0 t_1 t_2 t_4 t_0 t_3 N, \text{ and } N t_0 t_1 t_2 t_4 t_0 t_6 N. \text{ However, } N t_0 t_1 t_2 t_4 t_0 t_1 N = N t_0 t_1 t_2 t_4 t_0 N = \{012403\} \text{ since } t_0 t_1 t_2 t_4 t_0 t_1 = (1, 5)(2, 3, 0, 4)(t_0 t_1 t_2 t_4 t_0)^{(1, 0, 4, 2, 5, 6, 2)} = (1, 5)(2, 3, 0, 4)t_4 t_1 t_3 t_4 t_5 t_4 \text{ and } N t_0 t_1 t_2 t_4 t_0 t_3 N = N t_0 t_1 t_2 t_3 t_5 N = \{0125\} \text{ since } t_0 t_1 t_2 t_4 t_1 t_3 = (1, 6)(2, 0, 3, 4)(t_0 t_1 t_2 t_5)^{(1, 0)(2, 4, 5, 6)} = (1, 6)(2, 0, 3, 4)t_1 t_0 t_4 t_6.

The orbits of } N^{(01243)} \text{ on } T \text{ are } \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \text{ and } \{6\}. \text{ So we consider the double cosets } N t_0 t_1 t_2 t_4 t_3 t_0 N, N t_0 t_1 t_2 t_4 t_3 t_1 N, N t_0 t_1 t_2 t_4 t_3 t_2 N, N t_0 t_1 t_2 t_4 t_3 t_3 N, N t_0 t_1 t_2 t_4 t_3 t_4 N, N t_0 t_1 t_2 t_4 t_3 t_5 N, \text{ and } N t_0 t_1 t_2 t_4 t_3 t_6 N. \text{ However, } N t_0 t_1 t_2 t_4 t_3 t_3 N = N t_0 t_1 t_2 t_4 t_3 t_4 N = \{0124\} \text{ since } t_0 t_1 t_2 t_4 t_3 t_0 = (1, 4, 5, 3)(2, 0)(t_0 t_1 t_3)^{(1, 3, 2)(4, 0, 5)}; N t_0 t_1 t_2 t_4 t_3 t_4 N = N t_0 t_1 t_0 t_2 t_3 t_6 N = \{012035\} \text{ since } t_0 t_1 t_2 t_3 t_4 t_3 t_1 = (1, 6, 4, 5, 3, 0, 2)(t_0 t_1 t_0 t_2 t_3 t_5)^{(1, 6)(4, 0)} = (1, 6, 4, 5, 3, 0, 2)t_4 t_6 t_4 t_2 t_3 t_5; \text{ and } N t_0 t_1 t_2 t_4 t_3 t_2 N = N t_0 t_1 t_0 t_4 t_5 N = \{012045\} \text{ since } t_0 t_1 t_2 t_4 t_3 t_2 =
The orbits of $N^{(01250)}$ on $T$ are $\{0, 1, 2, 5, 3, 4, 6\}$. So we consider the double cosets $Nt_0t_1t_2t_3t_0t_0N$, $Nt_0t_1t_2t_3t_0t_2N$, $Nt_0t_1t_2t_3t_0t_3N$, $Nt_0t_1t_2t_3t_0t_4N$, and $Nt_0t_1t_2t_3t_0t_5N$. However, $Nt_0t_1t_2t_3t_0t_0N = Nt_0t_1t_2t_3t_0t_5N = Nt_0t_1t_2t_3t_0t_4N = Nt_0t_1t_2t_3t_0t_3N = Nt_0t_1t_2t_3t_0t_2N = Nt_0t_1t_2t_3t_0t_1N = Nt_0t_1t_2t_3t_0N = Nt_0t_1t_2t_3t_0^6N = \{0, 1, 2, 5, 3, 4, 6\}$. We consider the double cosets $Nt_0t_1t_2t_3t_0t_1N$, $Nt_0t_1t_2t_3t_0t_2N$, $Nt_0t_1t_2t_3t_0t_3N$, $Nt_0t_1t_2t_3t_0t_4N$, and $Nt_0t_1t_2t_3t_0t_5N$. However, $Nt_0t_1t_2t_3t_0t_1N = Nt_0t_1t_2t_3t_0t_6N = Nt_0t_1t_2t_3t_0t_4N = Nt_0t_1t_2t_3t_0t_3N = Nt_0t_1t_2t_3t_0t_2N = Nt_0t_1t_2t_3t_0t_1N = Nt_0t_1t_2t_3t_0N = Nt_0t_1t_2t_3t_0^6N = \{0, 1, 2, 5, 3, 4, 6\}$.
\[ = N_{0,1,2,3,4,5,6} \] since \( t_{0,1,2,3,4,5,6} \) of cosets \( \{0,1,2,3,4,5,6\} \) of \( N_{0,1,2,3,4,5,6} \) are \( (1,2,3,4,5,6) \) since \( t_{0,1,2,3,4,5,6} \) of cosets \( \{0,1,2,3,4,5,6\} \).}

However, \( N_{0,1,2,3,4,5,6} = N_{0,1,2,3,4,5,6} = [01230] \). So we consider the double cosets \( N_{0,1,2,3,4,5,6} \) since \( t_{0,1,2,3,4,5,6} \) of cosets \( \{0,1,2,3,4,5,6\} \).}

The orbits of \( N_{0,1,2,3,4,5,6} \) on \( T \) are \( \{0,1,2,3,4,5,6\} \). So we consider the double cosets \( N_{0,1,2,3,4,5,6} \) since \( t_{0,1,2,3,4,5,6} \) of cosets \( \{0,1,2,3,4,5,6\} \) of \( N_{0,1,2,3,4,5,6} \).}

The orbits of \( N_{0,1,2,3,4,5,6} \) on \( T \) are \( \{0,1,2,3,4,5,6\} \). So we consider the double cosets \( N_{0,1,2,3,4,5,6} \) since \( t_{0,1,2,3,4,5,6} \) of cosets \( \{0,1,2,3,4,5,6\} \) of \( N_{0,1,2,3,4,5,6} \).}

The orbits of \( N_{0,1,2,3,4,5,6} \) on \( T \) are \( \{0,1,2,3,4,5,6\} \). So we consider the double cosets \( N_{0,1,2,3,4,5,6} \) since \( t_{0,1,2,3,4,5,6} \) of cosets \( \{0,1,2,3,4,5,6\} \) of \( N_{0,1,2,3,4,5,6} \).}

The orbits of \( N_{0,1,2,3,4,5,6} \) on \( T \) are \( \{0,1,2,3,4,5,6\} \). So we consider the double cosets \( N_{0,1,2,3,4,5,6} \) since \( t_{0,1,2,3,4,5,6} \) of cosets \( \{0,1,2,3,4,5,6\} \) of \( N_{0,1,2,3,4,5,6} \).}

The orbits of \( N_{0,1,2,3,4,5,6} \) on \( T \) are \( \{0,1,2,3,4,5,6\} \). So we consider the double cosets \( N_{0,1,2,3,4,5,6} \) since \( t_{0,1,2,3,4,5,6} \) of cosets \( \{0,1,2,3,4,5,6\} \) of \( N_{0,1,2,3,4,5,6} \).}

The orbits of \( N_{0,1,2,3,4,5,6} \) on \( T \) are \( \{0,1,2,3,4,5,6\} \). So we consider the double cosets \( N_{0,1,2,3,4,5,6} \) since \( t_{0,1,2,3,4,5,6} \) of cosets \( \{0,1,2,3,4,5,6\} \) of \( N_{0,1,2,3,4,5,6} \).}
$(1,5,6)(3,0,4)t_2t_3t_6t_0t_3$. The only new double coset is $[0102040]$. 
$Nt_0t_1t_0t_4t_0t_3(1,4)(6,0) = Nt_0t_0t_2t_3t_4t_6 = Nt_0t_1t_0t_2t_0t_40 \Rightarrow (1,4)(6,0) \in N^{[0102040]} = \{e, (1,4)(6,0)\}$ and therefore, there are $|N|/|N^{[0102040]}| = 168/2 = 84$ single cosets in $[0102040]$.

The orbits of $N^{[010232]}$ on $T$ are $\{0, 1\}$, $\{2\}$, $\{3\}$, $\{4, 6\}$, and $\{5\}$. So we consider the double cosets $Nt_0t_1t_0t_2t_3t_2t_0N$, $Nt_0t_1t_0t_2t_3t_2N$, $Nt_0t_1t_0t_2t_3t_2t_3N$, $Nt_0t_1t_0t_2t_3t_2t_4N$, and $Nt_0t_1t_0t_2t_3t_2t_5N$. However, $Nt_0t_1t_0t_2t_3t_2t_3N = Nt_0t_1t_0t_2t_3t_4N$ and $Nt_0t_1t_0t_2t_3t_2t_5N$. However, $Nt_0t_1t_0t_2t_3t_2t_3N = Nt_0t_1t_0t_2t_3t_4N$ since $t_0t_1t_0t_2t_3t_2t_0 = (1, 0, 3)(4,6,5)(t_0t_1t_0t_2t_0t_4t_0)^{(1,4,6,0)(2,5)} = (1,0,3)(4,6,5)t_0t_1t_0t_2t_0t_4t_0; Nt_0t_1t_0t_2t_3t_2t_3N = Nt_0t_1t_0t_2t_3t_4N$ since $t_0t_1t_0t_2t_3t_2t_5 = (1,3)(2,5,6,4)t_0t_1t_0t_2t_3t_2t_5; Nt_0t_1t_0t_2t_3t_2t_5N = Nt_0t_1t_0t_2t_3t_4N = [01204]$ since $t_0t_1t_0t_2t_3t_2t_4 = (1,3)(2,5,6,4)t_0t_1t_0t_2t_3t_2t_4; Nt_0t_1t_0t_2t_3t_2t_5N = [01250]$ since $t_0t_1t_0t_2t_3t_2t_5 = (1,6,4)(2,5,3)t_0t_1t_0t_2t_3t_2t_5.$

The orhbits of $N^{[010235]}$ on $T$ are $\{0, 1\}$, $\{2\}$, $\{3\}$, $\{4, 6\}$, and $\{5\}$. So we consider the double cosets $Nt_0t_1t_0t_2t_3t_2t_0N$, $Nt_0t_1t_0t_2t_3t_2N$, $Nt_0t_1t_0t_2t_3t_2t_3N$, $Nt_0t_1t_0t_2t_3t_2t_4N$, and $Nt_0t_1t_0t_2t_3t_2t_5N$. However, $Nt_0t_1t_0t_2t_3t_2t_3N = Nt_0t_1t_0t_2t_3t_4N$ since $t_0t_1t_0t_2t_3t_2t_3 = (0,3,2,4,6,0,5)(t_0t_1t_0t_2t_0t_3)^{(1,5,0)(3,6,4)} = (0,3,2,4,6,0,5)t_0t_1t_0t_2t_3t_0; Nt_0t_1t_0t_2t_3t_2t_3t_2t_2t_3N = [01260]$ since $t_0t_1t_0t_2t_3t_2t_2 = (1,4,5,3)(2,0)(t_0t_1t_0t_2t_0)^{(2,3,5)(4,6,0)} = (1,4,5,3)(2,0)t_1t_0t_2t_3t_0t_4; Nt_0t_1t_0t_2t_3t_2t_3t_2t_4N = Nt_0t_1t_0t_2t_3t_2t_3t_3N = [01250]$ since $t_0t_1t_0t_2t_3t_2t_3t_2t_2t_3t_2t_3t_3N = Nt_0t_1t_0t_2t_3t_2t_3t_3t_4N = Nt_0t_1t_0t_2t_3t_2t_3t_3t_4N = [01250]$ since $t_0t_1t_0t_2t_3t_2t_3t_2t_4 = (2,3,5)(4,0,6)t_0t_1t_0t_2t_3t_2t_3t_2t_4N = Nt_0t_1t_0t_2t_3t_2t_3t_2t_4N = [01243]$ since $t_0t_1t_0t_2t_3t_2t_3t_2t_4 = (1,0,3,2,5,3,4,6,3,5)t_0t_1t_0t_2t_3t_2t_3t_2t_4t_0t_3.$

The orbits of $N^{[010240]}$ on $T$ are $\{0\}$, $\{1\}$, $\{2, 3\}$, $\{4\}$, $\{5\}$, and $\{6\}$. So we consider the double cosets $Nt_0t_1t_0t_2t_4t_0t_0N$, $Nt_0t_1t_0t_2t_4t_0t_1N$, $Nt_0t_1t_0t_2t_4t_0t_2N$, $Nt_0t_1t_0t_2t_4t_0t_4N$, $Nt_0t_1t_0t_2t_4t_0t_5N$, and $Nt_0t_1t_0t_2t_4t_0t_6N$. However, $Nt_0t_1t_0t_2t_4t_0t_0N = Nt_0t_1t_0t_2t_4t_0N = [01024]$. Also, $Nt_0t_1t_0t_2t_4t_0t_1N = Nt_0t_1t_0t_2t_4t_0N = [01024]$ since $t_0t_1t_0t_2t_4t_0t_1 = (1,0,3)(2,4,6)(t_0t_1t_0t_2t_0)^{(1,6,2,4,3,5,0)} = (1,0,3)(2,4,6)t_0t_1t_0t_2t_4t_0t_3t_6; Nt_0t_1t_0t_2t_4t_0t_2N = Nt_0t_1t_0t_2t_4t_0N = [01240]$ since $t_0t_1t_0t_2t_4t_0t_2 = (1,6,5)(2,0,4)(t_0t_1t_0t_2t_0)^{(1,6,4)(4,0)} = (1,6,5)(2,0,4)t_0t_1t_0t_2t_4t_0t_4; Nt_0t_1t_0t_2t_4t_0t_4N = Nt_0t_1t_0t_2t_4t_0t_0N = [0102040]$ since $t_0t_1t_0t_2t_4t_0t_4 = t_0t_1t_0t_2t_4t_0t_4; Nt_0t_1t_0t_2t_4t_0t_5N = Nt_0t_1t_0t_2t_4t_0t_4N = [0102304]$ since $t_0t_1t_0t_2t_4t_0t_5 = (3,6)(5,0)(t_0t_1t_0t_2t_3t_0)^{(1,3,5,6,4,0,2)}$
\[
(3,6)(5,0)_{t_0}t_3t_2t_1t_5t_2t_0. \quad \text{The only new double coset is } [0102406].
\]

\[N_{t_0t_1t_0t_2t_4t_0t_1}(1,6,4)(2,5,3) = N_{t_0t_1t_0t_2t_4t_0} = N_{t_0t_1t_0t_2t_4t_0t_6} \Rightarrow (1,6,4)(2,5,3) \in N^{(0102406)}. \quad \text{Thus, } N^{(0102406)} = \{e, (1,6,4)(2,5,3), (1,4,6)(2,3,5)\} \quad \text{and the number of single cosets in } [0102406] \text{ is } |N| / |N^{(0102406)}| = 168/3 = 56.

The orbits of } N^{(0102422)} \text{ on } T \text{ are } \{0\}, \{1\}, \{2,4\}, \{3\}, \text{ and } \{5,6\}. \text{ So we consider the double cosets } N_{t_0t_1t_0t_2t_4t_0t_2t_0}, N_{t_0t_1t_0t_2t_4t_0t_2t_1N}, N_{t_0t_1t_0t_2t_4t_0t_2N}, \text{ and } N_{t_0t_1t_0t_2t_4t_0t_3N}. \text{ However, } N_{t_0t_1t_0t_2t_4t_0t_2} = N_{t_0t_1t_0t_2t_4t_0t_2N} = [010242]. \text{ Also, } N_{t_0t_1t_0t_2t_4t_0t_3N} = N_{t_0t_1t_0t_2t_4t_0t_3N} = (2,5)(4,6)(t_0t_1t_0t_2t_4t_0)(3,4,2)(3,0,5) = (2,5)(4,6)t_3t_4t_5t_1t_5t_2t_3; \text{ and } N_{t_0t_1t_0t_2t_4t_0t_1N} = N_{t_0t_1t_0t_2t_4t_0t_1N} = [01262] \text{ since } t_0t_1t_0t_2t_4t_0t_1 = (t_0t_1t_0t_2t_4t_0)(1,0)(4,6) = t_0t_1t_0t_2t_42;

\begin{align*}
N_{t_0t_1t_0t_2t_4t_0t_2N} &= N_{t_0t_1t_0t_2t_4t_0t_2N} = N_{t_0t_1t_0t_2t_4t_0t_2N} = [01231] \text{ since } t_0t_1t_0t_2t_4t_0t_2 = (1,2,4)(3,5,0)(t_0t_1t_0t_2t_3t_1)(1,6,3,0,2,5,4) = (1,2,4)(3,5,0)t_3t_5t_0t_6; \text{ and } N_{t_0t_1t_0t_2t_4t_0t_3N} = N_{t_0t_1t_0t_2t_4t_0t_3N} = [01302] \text{ since } t_0t_1t_0t_2t_4t_0t_3 = (1,0,2,5)(3,6)(t_0t_1t_0t_2t_3)(1,3,2)(4,0,5) = (1,0,2,5)(3,6)t_5t_3t_2t_5t_1.
\end{align*}

The orbits of } N^{(010243)} \text{ on } T \text{ are } \{0,1,2\}, \{3,4,6\}, \text{ and } \{5\}. \text{ So we consider the double cosets } N_{t_0t_1t_0t_2t_4t_0t_4N}, N_{t_0t_1t_0t_2t_4t_0t_4N}, \text{ and } N_{t_0t_1t_0t_2t_4t_0t_3N}. \text{ However, } N_{t_0t_1t_0t_2t_4t_0t_4} = N_{t_0t_1t_0t_2t_4t_0t_4N} = [010243]. \text{ Also, } N_{t_0t_1t_0t_2t_4t_0t_3N} = N_{t_0t_1t_0t_2t_4t_0t_3N} = [01235] \text{ since } t_0t_1t_0t_2t_4t_0t_3 = (1,6,5)(2,0,4)(t_0t_1t_0t_2t_3)(1,4,2)(3,6,0) = (1,6,5)(2,0,4)t_3t_4t_5t_6. \text{ The only new double coset is } [0102435].

\begin{align*}
N_{t_0t_1t_0t_2t_4t_0t_4t_3} &= N_{t_0t_1t_0t_2t_4t_0t_4t_3N} = N_{t_0t_1t_0t_2t_4t_0t_4t_3N} = (2,5,3)(4,6,0)(1,4,2)(3,0,5) = (2,5,3)(4,6,0)N_{t_0t_1t_0t_2t_4t_0t_5} \Rightarrow (2,5,3)(4,6,0) \in N^{(0102435)} \Rightarrow N^{(0102435)} = \{(2,5,3)(4,6,0), (1,4,2)(3,0,5)\}, \text{ which is a group of order } 21, \text{ thus the number of single cosets in } [0102435] \text{ is equal to } |N| / |N^{(0102435)}| = 168/21 = 8.
\end{align*}

The orbits of } N^{(010254)} \text{ on } T \text{ are } \{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \text{ and } \{6\}. \text{ So we consider the double cosets } N_{t_0t_1t_0t_2t_5t_4t_0N}, N_{t_0t_1t_0t_2t_5t_4t_1N}, N_{t_0t_1t_0t_2t_5t_4t_2N}, \text{ and } N_{t_0t_1t_0t_2t_5t_4t_3N}, \text{ and } N_{t_0t_1t_0t_2t_5t_4t_4N}, \text{ and } N_{t_0t_1t_0t_2t_5t_4t_5N}, \text{ and } N_{t_0t_1t_0t_2t_5t_4t_6N}, \text{ and } N_{t_0t_1t_0t_2t_5t_4t_7N}. \text{ However, } N_{t_0t_1t_0t_2t_5t_4t_0N} = [01025]. \text{ Also, } N_{t_0t_1t_0t_2t_5t_4t_1N} = N_{t_0t_1t_0t_2t_5t_4t_2N} = [01240] \text{ since } t_0t_1t_0t_2t_5t_4t_1 = (1,4,3,5)(2,6)(t_0t_1t_0t_2t_4t_0)(2,6,0)(3,4,5) = (1,4,3,5)(2,6)t_0t_1t_0t_2t_5t_4; \text{ and } N_{t_0t_1t_0t_2t_5t_4t_3N} = N_{t_0t_1t_0t_2t_5t_4t_3N} = [012304] \text{ since } t_0t_1t_0t_2t_5t_4t_2 = (1,4,5,2,0,3,6)(t_0t_1t_0t_2t_3t_4t_0)(1,3,0)(2,4,6) = (1,4,5,2,0,3,6)t_1t_3t_1t_4t_0t_1t_6; \text{ and } N_{t_0t_1t_0t_2t_5t_4t_3N} = N_{t_0t_1t_0t_2t_5t_4t_3N} = [01234] \text{ since }

\begin{align*}
N_{t_0t_1t_0t_2t_5t_4t_0N} &= N_{t_0t_1t_0t_2t_5t_4t_0N} = (3,6)(5,0)t_0t_3t_2t_1t_5t_2t_0. \quad \text{The only new double coset is } [0102406].
\end{align*}
\[ t_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = (1, 0, 5)(3, 4, 6)(t_{t_0 t_1 t_2 t_3 t_4})^{(1,2)(3,6)} = (1, 0, 5)(3, 4, 6)_{t_0 t_2 t_1 t_0 t_4}; \]
\[ N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5 N} = N_{t_0 t_1 t_2 t_3 t_4 t_5} = [01232] \] since \[ t_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = (1, 3, 0)(2, 6, 5)(t_{t_0 t_1 t_2 t_3 t_4})^{(1,2)(3,6)} = (1, 3, 0)(2, 6, 5)_{t_0 t_2 t_4 t_0 t_4}; \]
\[ N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = [01246] \] since \[ t_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = (1, 5, 0)(3, 6, 4)(t_{t_0 t_1 t_2 t_4})^{(1,2)(3,6)} = (1, 5, 0)(3, 6, 4)_{t_1 t_2 t_0 t_3 t_4 t_5}. \] The only new double coset is \([012540].\)
\[ N_{t_0 t_1 t_0 t_2 t_3 t_4 t_0 (1, 2, 4)(5, 0, 0)} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_0} ⇔ (1, 2, 4)(5, 0, 0) \in N^{0(102540)} \Rightarrow N^{0(102540)} = \{ (1, 2, 4)(5, 0, 0), (1, 4, 2)(5, 0, 6) \}, \text{ thus the number of single cosets in } [0102540] \text{ is equal to } |N|/|N^{0(102540)}| = 168/3 = 56.

The orbits of \(N^{0(102340)}\) on \(T\) are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \text{ and } \{6\}. \) So we consider the double cosets \(N_{t_0 t_1 t_0 t_2 t_3 t_4 t_0}, N_{t_0 t_1 t_0 t_2 t_3 t_4 t_1}, N_{t_0 t_1 t_0 t_2 t_3 t_4 t_2},\)
\(N_{t_0 t_1 t_0 t_2 t_3 t_4 t_3}, N_{t_0 t_1 t_0 t_2 t_3 t_4 t_4}, N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}, \text{ and } N_{t_0 t_1 t_0 t_2 t_3 t_4 t_6}. \) However, \(N_{t_0 t_1 t_0 t_2 t_3 t_4 t_1} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = [01203]. \) Also, \(N_{t_0 t_1 t_0 t_2 t_3 t_4 t_0} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_6} = [01246] \) since \(t_{t_0 t_1 t_0 t_2 t_3 t_4 t_0} = (1, 6, 5)(2, 4, 3)(t_{t_0 t_1 t_0 t_2 t_4})^{(1,2)(3,4)(2,6)} = (1, 6, 5)(2, 4, 3)t_{0 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_4 t_0} = [0102406] \) since \(t_{t_0 t_1 t_0 t_2 t_3 t_4 t_1} = (1, 3, 6, 2)(4, 0)(t_{t_0 t_1 t_0 t_2 t_4})^{(1,0,2)(3,4)(2,6)} = (1, 3, 6, 2)(4, 0)_{t_0 t_0 t_3 t_4 t_5 t_6}; \)
\(N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_4 t_5 t_0} = [0102503] \) since \(t_{t_0 t_1 t_0 t_2 t_3 t_4 t_2} = (1, 5, 6)(2, 3, 4)(t_{t_0 t_1 t_0 t_2 t_3 t_4})^{(1,0,2,3,4,5,6)} = (1, 5, 6)(2, 3, 4)_{t_0 t_0 t_3 t_4 t_6}; \)
\(N_{t_0 t_1 t_0 t_2 t_3 t_4 t_3} = N_{t_0 t_1 t_0 t_2 t_3 t_4 t_4} = [012324] \) since \(t_{t_0 t_1 t_0 t_2 t_3 t_4 t_3} = (1, 0, 4, 6)(3, 5)(t_{t_0 t_1 t_0 t_2 t_3 t_4})^{(2,6)(3,4,5)} = (1, 0, 4, 6)(3, 5)_{t_0 t_1 t_0 t_2 t_4 t_5 t_6}; \)
\(N_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = N_{t_0 t_1 t_0 t_2 t_4 t_5 t_0} = [01232] \) since \(t_{t_0 t_1 t_0 t_2 t_3 t_4 t_5} = (1, 2)(3, 5, 6, 0)(t_{t_0 t_1 t_0 t_2 t_3 t_4})^{(1,0,2,3,4,5,6)} = (1, 2)(3, 5, 6, 0)_{t_0 t_0 t_3 t_4 t_5 t_6}; \)
\(N_{t_0 t_1 t_0 t_2 t_3 t_4 t_6} = N_{t_0 t_1 t_0 t_2 t_4 t_5 t_0} = [01204] \) since \(t_{t_0 t_1 t_0 t_2 t_3 t_4 t_6} = (1, 5, 6)(2, 0)_{t_0 t_1 t_0 t_4 t_5 t_6}. \)

The orbits of \(N^{0(102945)}\) on \(T\) are \(\{0, 2, 5\}, \{1\}, \text{ and } \{3, 4, 6\}. \) So we consider the double cosets \(N_{t_0 t_1 t_0 t_2 t_4 t_5 t_3}, N_{t_0 t_1 t_0 t_2 t_4 t_5 t_1}, \) and \(N_{t_0 t_1 t_0 t_2 t_4 t_5 t_3}. \) However, \(N_{t_0 t_1 t_0 t_2 t_4 t_5 t_3} = N_{t_0 t_1 t_0 t_2 t_4 t_5} = [01204]. \) Also, \(N_{t_0 t_1 t_0 t_2 t_4 t_5 t_1} = N_{t_0 t_1 t_0 t_2 t_4 t_5 t_1} = [0120341] \) since \(t_{t_0 t_1 t_0 t_2 t_4 t_5 t_1} = (1, 2, 3, 0, 6, 4, 5)(t_{t_0 t_1 t_0 t_2 t_4 t_5})^{(1,3)(2,4,5,6)} = (1, 2, 3, 0, 6, 4, 5)_{t_0 t_1 t_0 t_2 t_3 t_4 t_5}; \)
\(N_{t_0 t_1 t_0 t_2 t_4 t_5 t_3} = N_{t_0 t_1 t_0 t_2 t_4 t_5 t_3} = [01243] \) since \(t_{t_0 t_1 t_0 t_2 t_4 t_5 t_3} = (1, 4, 0, 6)(2, 5)(t_{t_0 t_1 t_2 t_4 t_5})^{(1,0,2,3,4,5,6)} = (1, 4, 0, 6)(2, 5)_{t_0 t_2 t_3 t_4 t_5}.

The orbits of \(N^{0(10292)}\) on \(T\) are \(\{0, 1, 2\}, \{3, 4, 6\}, \text{ and } \{5\}. \) So we consider the double cosets \(N_{t_0 t_1 t_0 t_2 t_5 t_2 t_3}, N_{t_0 t_1 t_0 t_2 t_5 t_2 t_5}, \) and \(N_{t_0 t_1 t_0 t_2 t_5 t_2 t_5}. \) However, \(N_{t_0 t_1 t_0 t_2 t_5 t_2 t_5} = N_{t_0 t_1 t_0 t_2 t_5 t_2} = [01205]. \) Also, \(N_{t_0 t_1 t_0 t_2 t_5 t_2 t_3} N_{t_0 t_1 t_0 t_2 t_5 t_0} = [012040] \) since \(t_{t_0 t_1 t_0 t_2 t_5 t_2 t_3} = (1, 5, 2, 6, 3, 0, 4)(t_{t_0 t_1 t_0 t_2 t_5 t_0})^{(1,0,2,3,4,5,6)} = \)}
\[(1,5,2,6,3,0,4)t_4t_4t_4t_4t_4; \quad Nt_0t_1t_2t_3t_5t_5N = Nt_0t_1t_0t_2t_0t_5t_2N = [0102052] \text{ since } t_0t_1t_2t_0t_5t_5t_5 = t_0t_1t_0t_2t_0t_5t_2.\]

The orbits of \(N^{(0,1,2,10,3)}\) on \(T\) are \(\{0,1\}, \{2\}, \{3\}, \{4,6\}, \text{ and } \{5\}. \) So we consider the double cosets \(Nt_0t_1t_2t_1t_0t_5t_0N, Nt_0t_1t_2t_1t_0t_5t_2N, Nt_0t_1t_2t_1t_0t_3t_3N, \quad Nt_0t_1t_2t_1t_0t_3t_4N, \quad \text{and } Nt_0t_1t_2t_1t_0t_5t_5N. \) However, \(Nt_0t_1t_2t_1t_0t_3t_3N = Nt_0t_1t_2t_1t_0N = [01210].\) Also, \(Nt_0t_1t_2t_1t_0t_3t_0N = Nt_0t_1t_0t_2t_5t_0N = [0102054] \text{ since } t_0t_1t_2t_1t_0t_3t_0 = (1,2,4)(3,0,6)(t_0t_1t_0t_2t_5t_0t_4)(1,4,6)(2,3,5) = (1,2,4)(3,0,6)t_0t_4t_0t_3t_3t_6; \quad Nt_0t_1t_2t_1t_0t_3t_2N = Nt_0t_1t_2t_0t_4t_0N = [0102040] \text{ since } t_0t_1t_2t_1t_0t_3t_2 = (1,2,4)(3,0,6)(t_0t_1t_0t_2t_4t_0)(1,4,6)(2,3,5) = (1,2,4)(3,0,6)t_0t_4t_0t_3t_3t_6; \quad Nt_0t_1t_2t_1t_0t_3t_4N = Nt_0t_1t_2t_0t_4t_0N = [0102040] \text{ since } t_0t_1t_2t_1t_0t_3t_4 = (1,5,6)(2,0,3)(t_0t_1t_0t_2t_4t_0)(1,3,5,0,2,4,6) = (1,5,6)(2,0,3)t_2t_3t_4t_0t_6t_2; \quad Nt_0t_1t_2t_1t_0t_3t_5N = Nt_0t_1t_3t_2t_0N = [01320] \text{ since } t_0t_1t_2t_1t_0t_3t_5 = (1,6,5)(2,4,3)(t_0t_1t_3t_2t_0)(1,2,3)(4,5,0) = (1,6,5)(2,4,3)t_4t_2t_1t_3t_4.\]

The orbits of \(N^{(0,1,2,13,5)}\) on \(T\) are \(\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \text{ and } \{6\}. \) So we consider the double cosets \(Nt_0t_1t_2t_1t_3t_5t_0N, Nt_0t_1t_2t_1t_3t_5t_1N, Nt_0t_1t_2t_1t_3t_5t_2N, \quad Nt_0t_1t_2t_1t_3t_5t_3N, \quad Nt_0t_1t_2t_1t_3t_5t_4N, \quad Nt_0t_1t_2t_1t_3t_5t_5N, \quad Nt_0t_1t_2t_1t_3t_5t_6N. \) However, \(Nt_0t_1t_2t_1t_3t_5t_5N = Nt_0t_1t_2t_1t_3t_5N = [01213].\) Also, \(Nt_0t_1t_2t_1t_3t_5t_0N = Nt_0t_1t_0t_2t_5t_1N = [0102541] \text{ since } t_0t_1t_2t_1t_3t_5t_0 = (1,2,4)(3,0,6)(t_0t_1t_0t_2t_5t_4t_1)(1,2,4)(3,0,6) = (1,2,4)(3,0,6)t_0t_4t_0t_3t_3t_6; \quad Nt_0t_1t_2t_1t_3t_5t_2N = Nt_0t_1t_0t_2t_5t_1N = [010240] \text{ since } t_0t_1t_2t_1t_3t_5t_2 = (1,4,5,3)(2,0)(t_0t_1t_0t_2t_3t_4t_0)(1,2,3)(3,6) = (1,4,5,3)(2,0)t_0t_2t_0t_1t_3t_4; \quad Nt_0t_1t_2t_1t_3t_5t_3N = Nt_0t_1t_0t_2t_3t_5t_4N = [010230] \text{ since } t_0t_1t_2t_1t_3t_5t_3 = (1,4,6)(2,3,5)t_0t_0t_1t_3t_4t_0; \quad Nt_0t_1t_2t_1t_3t_5t_4N = Nt_0t_1t_2t_3t_5t_3N = [01235] \text{ since } t_0t_1t_2t_1t_3t_5t_4 = (1,6,2,5,0,3,4)(t_0t_1t_2t_3t_5)(1,0,4,6)(3,5) = (1,6,2,5,0,3,4)t_4t_0t_2t_5t_3; \quad Nt_0t_1t_2t_1t_3t_5t_6N = Nt_0t_1t_2t_3t_4N = [01234] \text{ since } t_0t_1t_2t_1t_3t_5t_6 = (1,2,4)(3,0,6)(t_0t_1t_2t_3t_4)(2,3,5)(4,0,6) = (1,2,4)(3,0,6)t_6t_1t_3t_5t_0.\]

The orbits of \(N^{(0,1,2,13,5,4)}\) on \(T\) are \(\{0\}, \{1,5,6\}, \text{ and } \{2,3,4\}. \) So we consider the double cosets \(Nt_0t_1t_2t_3t_5t_4t_0N, Nt_0t_1t_2t_3t_5t_4t_1N, \text{ and } Nt_0t_1t_2t_3t_5t_4t_4N. \) However, \(Nt_0t_1t_2t_3t_5t_4t_4N = Nt_0t_1t_2t_3t_5t_3N = [01235].\) Also, \(Nt_0t_1t_2t_3t_5t_4t_0N = Nt_0t_1t_0t_2t_5t_4t_0N = [0102540] \text{ since } t_0t_1t_2t_3t_4t_0 = (t_0t_1t_0t_2t_5t_4t_0)(1,5,2,6,3,0,4) = t_4t_5t_4t_6t_2t_1; \quad Nt_0t_1t_2t_3t_5t_4t_1N = Nt_0t_1t_2t_3t_4N = [01243] \text{ since } t_0t_1t_2t_3t_5t_4t_1 = ...
\[(1, 3)(2, 5, 6, 4)(t_0 t_1 t_2 t_4 t_3)^{(1, 5, 0)(2, 4, 3)} = (1, 3)(2, 5, 6, 4)t_0 t_5 t_4 t_3 t_2.\]

The orbits of \(N^{(012409)}\) on \(T\) are \(\{0, 2, 6\}, \{1\}\), and \(\{3, 4, 5\}\). So we consider the double cosets \(N t_0 t_1 t_2 t_4 t_3 t_0 t_3 N, N t_0 t_1 t_2 t_4 t_3 t_0 t_4 t_1 N, \) and \(N t_0 t_1 t_2 t_3 t_4 t_5 t_3 N\). However, \(N t_0 t_1 t_2 t_4 t_3 t_3 N = N t_0 t_1 t_2 t_4 t_0 t_4 t_0 t_4 t_0 t_4 t_0 t_4 t_0 N = [012409]\). Also, \(N t_0 t_1 t_2 t_4 t_0 t_0 t_3 N = N t_0 t_1 t_2 t_4 t_0 t_1 N = [01261]\) since \(t_0 t_1 t_2 t_4 t_0 t_3 t_0 = (1, 5)(2, 4, 0, 3)(t_0 t_1 t_2 t_4 t_1)^{(1, 2, 3, 5)(4, 5)} = (1, 5)(2, 4, 0, 3)t_0 t_2 t_3 t_1 t_2; N t_0 t_1 t_2 t_4 t_0 t_3 t_1 N = N t_0 t_1 t_2 t_4 t_1 N = [012411]\) since \(t_0 t_1 t_2 t_4 t_0 t_3 t_1 = (1, 4, 6, 0)(2, 3)(t_0 t_1 t_2 t_4 t_1)^{(1, 0)(4, 6)} = (1, 4, 6, 0)(2, 3)t_1 t_0 t_2 t_6 t_0.\]

The orbits of \(N^{(0102040)}\) on \(T\) are \(\{0, 6\}, \{1, 4\}, \{2\}, \{3\}, \) and \(\{5\}\). So we consider the double cosets \(N t_0 t_1 t_0 t_2 t_4 t_0 t_0 t_0 N, N t_0 t_1 t_0 t_2 t_4 t_0 t_1 N, \) and \(N t_0 t_1 t_0 t_2 t_4 t_0 t_2 N, \) \(N t_0 t_1 t_0 t_2 t_3 t_0 t_3 N, \) and \(N t_0 t_1 t_0 t_2 t_4 t_0 t_6 N.\) However, \(N t_0 t_1 t_0 t_2 t_4 t_0 t_0 = N t_0 t_1 t_0 t_2 t_4 t_0 t_4 = [010204].\) Also, \(N t_0 t_1 t_0 t_2 t_4 t_0 t_0 t_1 N = N t_0 t_1 t_0 t_2 t_4 t_0 N = [010240]\) since \(t_0 t_1 t_0 t_2 t_4 t_0 t_0 t_1 = (t_0 t_1 t_0 t_2 t_4 t_0)^{(1, 4, 0)(6, 0)} = t_0 t_4 t_0 t_2 t_1 t_6; N t_0 t_1 t_0 t_2 t_4 t_0 t_2 N = N t_0 t_1 t_0 t_2 t_4 t_0 t_2 N = [012052]\) since \(t_0 t_1 t_0 t_2 t_4 t_0 t_0 t_0 t_2 = (1, 6, 4, 2, 0, 5, 3)(t_1 t_0 t_0 t_5 t_3 t_2)^{(2, 5, 3)(4, 0)} = (1, 6, 4, 2, 0, 5, 3)t_1 t_0 t_5 t_3 t_2; N t_0 t_1 t_0 t_2 t_4 t_0 t_3 N = N t_0 t_1 t_0 t_2 t_4 t_0 = [010242]\) since \(t_0 t_1 t_0 t_2 t_4 t_0 t_3 = (1, 6)(4, 0)(t_0 t_1 t_0 t_2 t_4 t_2)^{(1, 2)(3, 5, 6, 0)} = (1, 6)(4, 0)t_3 t_2 t_3 t_1 t_4 t_1; N t_0 t_1 t_0 t_2 t_4 t_0 t_5 N = N t_0 t_1 t_0 t_2 t_3 N = [012103]\) since \(t_0 t_1 t_0 t_2 t_4 t_0 t_5 = (1, 5, 6)(2, 4, 0)(t_0 t_1 t_2 t_4 t_0)^{(1, 6, 0)(2, 5, 3)} = (1, 5, 6)(2, 4, 0)t_0 t_0 t_5 t_0 t_0 t_2.\]

The orbits of \(N^{(0102406)}\) on \(T\) are \(\{0\}, \{1, 4, 6\}, \) and \(\{2, 3, 5\}\). So we consider the double cosets \(N t_0 t_1 t_0 t_2 t_4 t_0 t_0 t_0 N, N t_0 t_1 t_0 t_2 t_4 t_0 t_1 N, \) and \(N t_0 t_1 t_0 t_2 t_4 t_0 t_2 N.\) However, \(N t_0 t_1 t_0 t_2 t_4 t_0 t_0 t_0 N = N t_0 t_1 t_0 t_2 t_4 t_0 t_0 N = [010240].\) Also, \(N t_0 t_1 t_0 t_2 t_4 t_0 t_0 = N t_0 t_1 t_0 t_2 t_4 t_0 t_0 t_0 = [012043]\) since \(t_0 t_1 t_0 t_2 t_4 t_0 t_0 = (1, 0, 2)(3, 6, 4)(t_0 t_1 t_0 t_2 t_4 t_3 t_2)^{(1, 5, 2, 6, 3, 4)} = (1, 0, 2)(3, 6, 4)t_4 t_4 t_4 t_4 t_0 t_5; N t_0 t_1 t_0 t_2 t_4 t_0 t_2 N = N t_0 t_1 t_0 t_2 t_4 t_0 t_3 = [012043]\) since \(t_0 t_1 t_0 t_2 t_4 t_0 t_2 = (1, 4, 3, 0, 5, 2, 6)(t_0 t_1 t_0 t_2 t_4 t_3)^{(1, 2, 4)(3, 0, 6)} = (1, 4, 3, 0, 5, 2, 6)t_0 t_2 t_0 t_1 t_0.\]

The orbit of \(N^{(0102435)}\) on \(T\) is \(\{0, 1, 2, 3, 4, 5, 6\}\), so we only consider the double coset \(N t_0 t_1 t_0 t_2 t_4 t_3 t_5 t_5 N = N t_0 t_1 t_0 t_2 t_4 t_3 N = [010243].\]

The orbits of \(N^{(0102540)}\) on \(T\) are \(\{0, 5, 6\}, \{1, 2, 4\}, \) and \(\{3\}\). So we consider the double cosets \(N t_0 t_1 t_0 t_2 t_3 t_4 t_0 N, N t_0 t_1 t_0 t_2 t_3 t_4 t_0 t_1 N, \) and \(N t_0 t_1 t_0 t_2 t_3 t_4 t_0 t_3 N.\) However, \(N t_0 t_1 t_0 t_2 t_3 t_4 t_0 t_4 N = N t_0 t_1 t_0 t_2 t_3 t_4 N = [010254].\) Also, \(N t_0 t_1 t_0 t_2 t_3 t_4 t_0 t_1 N = N t_0 t_1 t_0 t_2 t_3 t_4 t_0 t_3 N = [012103]\) since \(t_0 t_1 t_0 t_2 t_3 t_4 t_0 t_1 = (1, 6, 4, 0)(3, 5)(t_0 t_1 t_2 t_4 t_0)^{(1, 0, 5, 3, 4, 2, 6)} = (1, 6, 4, 0)(3, 5)t_3 t_0 t_0 t_5 t_4; N t_0 t_1 t_0 t_2 t_3 t_4 t_0 t_3 N = N t_0 t_1 t_0 t_2 t_3 t_4 N = [012354]\) since \(t_0 t_1 t_0 t_2 t_3 t_4 t_0 t_3 = (t_0 t_1 t_2 t_3 t_5 t_4)^{(1, 4, 0, 3, 5, 2, 3)} = t_3 t_4 t_0 t_1 t_0.\)
Therefore, we have found all double cosets. The result of the double coset enumeration is shown in the Cayley diagram in figure 3.1.

5.3 Proof of the Isomorphism between $G$ and $M_{22} : 2$

In order to prove that $G \cong M_{22} : 2$, we need to show first that 
\[
\langle \Phi(x), \Phi(y), \Phi(t) \rangle
\]

is a homomorphic image of $G$ and that $|G| = |\langle \Phi(x), \Phi(y), \Phi(t) \rangle| = 887,040$. Second, we must show that $\langle \Phi(x), \Phi(y), \Phi(t) \rangle \cong M_{22} : 2$, from which we can conclude that $M_{22} : 2$ is a homomorphic image of $G$ and $G \cong M_{22} : 2$.

In the previous section we constructed $G$ using manual double coset enumeration and we concluded that $G$ defined by the symmetric presentation must contain a homomorphic image of $A = 1/3(2)$. The index $[G : A]$ is at most 5,280, since

\[
[G : N] = \frac{|N|}{|N(x)| - |N(0)|} + \frac{|N|}{|N(010)|} + \frac{|N|}{|N(012)|} + \frac{|N|}{|N(013)|} + \frac{|N|}{|N(015)|} + \frac{|N|}{|N(017)|} + \frac{|N|}{|N(019)|} + \frac{|N|}{|N(020)|} + \frac{|N|}{|N(021)|} + \frac{|N|}{|N(023)|} + \frac{|N|}{|N(024)|} + \cdots + \frac{|N|}{|N(02540)|} \\
\leq \frac{168}{168} + \frac{168}{24} + \frac{168}{4} + \frac{168}{8} + \frac{168}{8} + \frac{168}{2} + \frac{168}{2} + \frac{168}{2} + \frac{168}{1} + \frac{168}{1} + \cdots + \frac{168}{3} = 5,280.
\]

That is, $|G : N| = \frac{|G|}{|N|} \leq 5,280$. Since the index of $N$ in $G$ is at most 5,280 and $|G| = |G : N| \cdot |N|$, the order of the homomorphic image group $G$ is at most 887,040.

$|G| = \frac{|G|}{|N|} \cdot |N| \leq 5,280 \cdot 168 = 887,040 \Rightarrow |G| \leq 887,040.$

We now consider $\langle \Phi(x), \Phi(y), \Phi(t) \rangle$. We note that $\langle \Phi(x), \Phi(y), \Phi(t) \rangle$ is the group generated by the action of the generators $x$, $y$ and $t$ on the 5,280 cosets that we have found and it is a subgroup of the symmetric group $S_{5280}$. Let $G_1 = \langle \Phi(x), \Phi(y), \Phi(t) \rangle$. It is easily verified that $|G_1| = 887,040$. Then, $G_1$ is a homomorphic image of $G$ since:

1. $\Phi(N)$ acts as the group $N$ on the symmetric generators, that is

$\Phi(x) : (\Phi(t_0), \Phi(t_1), \Phi(t_2), \Phi(t_3), \Phi(t_4), \Phi(t_5), \Phi(t_6))$ and

$\Phi(y) : (\Phi(t_2), \Phi(t_6))(\Phi(t_4), \Phi(t_5))$.

2. $t_0$ has exactly seven conjugates under conjugation by $\Phi(N)$:

$\Phi(N)^{t_0} = \{t_0, t_1, t_2, t_3, t_4, t_5, t_6\}$. 
Figure 5.1: Cayley Diagram of the Automorphism Group $M_{22} : 2$ over $L_3(2)$
3. Additional relations hold:

(i) \( \Phi(t_2)\Phi(t_4)\Phi(t_5)\Phi(t_3)\Phi(t_6)\Phi(t_2)\Phi(t_4)\Phi(t_5)\Phi(t_6)\Phi(t_2)\Phi(t_4) = (\Phi(t_2), \Phi(t_4), \Phi(t_2))(\Phi(t_3), \Phi(t_5), \Phi(t_6)) \)

(ii) \( \Phi(t_2)\Phi(t_4)\Phi(t_6)\Phi(t_5)\Phi(t_2)\Phi(t_4)\Phi(t_5)\Phi(t_6)\Phi(t_2)\Phi(t_4) = (\Phi(t_2), \Phi(t_6))(\Phi(t_4), \Phi(t_5)) \)

(iii) \( \Phi(t_0)\Phi(t_6)\Phi(t_2)\Phi(t_0)\Phi(t_6)\Phi(t_2)\Phi(t_0) = e \)

(iv) \( \Phi(t_2)\Phi(t_0)\Phi(t_1)\Phi(t_0)\Phi(t_2)\Phi(t_0)\Phi(t_1)\Phi(t_0) = (\Phi(t_2), \Phi(t_0), \Phi(t_5))(\Phi(t_3), \Phi(t_6), \Phi(t_4)) \)

Therefore, we have that \( G/Ker(\Phi) \cong G_1 \Rightarrow |G| \geq |G_1| \). Thus, we proved that \( |G| = 887,040 \).

Now, with the help of MAGMA we find that the elements \( c = (1, 3)(4, 5)t_3t_1t_3 \) and \( d = (1, 3)(2, 5, 6, 4)t_2t_4t_1t_0t_1t_3 \) in \( G_1 \) satisfy the following presentation of \( M_{22} : 2 \):

\[ \langle c, d | c^2 = d^4 = (cd)^{11} = (cd^2)^6 = [c, d]^4 = (cdcdcdcd^2cd^2)^6 = e \rangle. \]

Now, \( c, d \in G_1 \). Thus \( M_{22} : 2 = \langle c, d \rangle \leq G_1 \). But \( |M_{22} : 2| = |G_1| = 887,040 \). Therefore, \( G_1 \cong M_{22} : 2 \). Finally, since \( G \cong G_1 \), we conclude that \( G \cong M_{22} : 2 \).

Table 5.1 lists the symmetric representation of the representatives of the 21 conjugacy classes of the automorphism group \( M_{22} : 2 \).

<table>
<thead>
<tr>
<th>Class</th>
<th>Representative</th>
<th>Class</th>
<th>Representative</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Id(N)</td>
<td>6B</td>
<td>(1, 5, 0, 6, 4, 2, 3)t_3t_4t_6t_2</td>
</tr>
<tr>
<td>2A</td>
<td>(1, 0, 3)(2, 6, 4)t_3t_4t_6t_0</td>
<td>7A</td>
<td>(1, 2, 5)(3, 6, 4)t_1t_3t_0</td>
</tr>
<tr>
<td>2B</td>
<td>(1, 3)(4, 5)t_3t_4t_6t_4</td>
<td>7B</td>
<td>(1, 2, 3)(4, 5, 0)t_4t_6t_2</td>
</tr>
<tr>
<td>2C</td>
<td>(1, 3)(2, 6)t_5</td>
<td>8A</td>
<td>(1, 5)(2, 3, 0, 4)t_3t_5t_6t_2</td>
</tr>
<tr>
<td>3</td>
<td>(1, 4, 6, 2, 3, 0, 5)t_3t_4t_6t_5t_3</td>
<td>8B</td>
<td>(1, 5, 3, 0, 4, 6, 2)t_4t_5t_6t_3</td>
</tr>
<tr>
<td>4A</td>
<td>(1, 4, 5, 3)(2, 0)t_3t_4t_6t_3t_0t_6</td>
<td>10</td>
<td>(1, 2, 5, 0)(3, 4)t_4t_3t_2t_6</td>
</tr>
<tr>
<td>4B</td>
<td>(2, 0, 6)(3, 5, 4)t_3t_6t_3t_0</td>
<td>11</td>
<td>(1, 5, 4)(2, 6, 0)t_3t_1t_3t_0</td>
</tr>
<tr>
<td>4C</td>
<td>(1, 4)(3, 0, 5, 6)t_3t_4t_6t_5t_3t_6</td>
<td>12</td>
<td>(1, 2, 5, 0)(3, 4)t_6t_3t_2t_6</td>
</tr>
<tr>
<td>4D</td>
<td>(1, 6, 5)(2, 0, 4)t_3t_4t_6t_1t_2t_6</td>
<td>14A</td>
<td>(1, 6)(4, 0)t_4t_2t_4t_3t_6t_4</td>
</tr>
<tr>
<td>5</td>
<td>(1, 3, 4, 5)(6, 0)t_4t_1t_3t_0</td>
<td>14B</td>
<td>(1, 2, 0, 3, 5, 4, 6)t_2t_1t_2t_4t_3t_6t_1</td>
</tr>
<tr>
<td>6A</td>
<td>(1, 4, 2)(3, 0, 5)t_3t_3t_1t_0t_4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: Representatives of the Conjugacy Classes of \( M_{22} : 2 \)
Appendix A

MAGMA Code for Constructing $2 \cdot M_{22} : 2$

N:=Sym(7);
x:=N!(7,1,2,3,4,5,6);
y:=N!(2,6)(4,5);
N:=sub<N|xx,yy>;
G<x,y,t>:=Group<x,y,t|x^7,y^2,(x*y)^3,(x,y)^4, t^2,
(x*y,t^-(x^4)),(y,t^-(x^3)), (y * x^3 * y * x^2 * y*t^-(x^3))^8,
(x^-2 * y * x^-2 * y * x^-2*t^-2(x^2))^10,
(y*t*t^-(x^2))^4>;
Index(G,sub<G|x,y>);
f,G1,k:=CosetAction(G,sub<G|x,y>);
IN:=sub<G1|f(x),f(y)>;
ts:=[f(t^-i) : i in [1..7]];
prodim := function(pt, Q, I)
  /*
  Return the image of pt under permutations Q[I] applied sequentially.
  */
  v := pt;
  for i in I do
    v := v^Q[i];
  end for;
  return v;
end function;
cst := [null : i in [1 .. 10560]] where null is [Integers() | ];
  for i := 1 to 7 do
    cst[prodim(1, ts, [i])] := [i];
  end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N7 := Stabiliser(N, 7);
N71 := Stabiliser(N7, 1);
T71 := Transversal(N, N71);
for i in [1..#T71] do
  ss := [7, 1] ~ T71[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N71s := N71;
for g in N do if 7^g eq 1 and 1^g eq 7 then N71s := sub<N|N71s, g>; end if; end for;
T71s := Transversal(N, N71s);
for i in [1..#T71s] do
  ss := [7, 1, 7] ~ T71s[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N712 := Stabiliser(N71, 2);
T712s := Transversal(N, N712);
for i in [1..#T712s] do
  ss := [7, 1, 2] ~ T712s[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N713 := Stabiliser(N71, 3);
T713 := Transversal(N, N713);
for i in [1..#T713] do
  ss := [7, 1, 3] ~ T713[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N712 := Stabiliser(N71, 2);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 then
N7172s := sub<N|N712, g>; end if; end for;
T7172s := Transversal(N, N7172s);
for i in [1..#T7172s] do
ss := [7, 1, 7, 2] ^ T7172s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;
N713 := Stabiliser(N71, 3);
for g in N do if 7 ^ g eq 1 and 1 ^ g eq 7 and 3 ^ g eq 3 then
N7173s := sub<N|N713, g>; end if; end for;
T7173s := Transversal(N, N7173s);
for i in [1..#T7173s] do
ss := [7, 1, 7, 3] ^ T7173s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;
N712 := Stabiliser(N71, 2);
for g in N do if 7 ^ g eq 1 and 1 ^ g eq 7 and 2 ^ g eq 2 then
N7127s := sub<N|N712, g>; end if; end for;
T7127s := Transversal(N, N7127s);
for i in [1..#T7127s] do
ss := [7, 1, 2, 7] ^ T7127s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;
N712 := Stabiliser(N71, 2);
for g in N do if 7 ^ g eq 7 and 1 ^ g eq 7 and 2 ^ g eq 2 then
N7127s := sub<N|N712, g>; end if; end for;
T7127s := Transversal(N, N7127s);
for i in [1..#T7127s] do
ss := [7, 1, 2, 1] ^ T7127s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;
N7123 := Stabiliser(N712, 3);
T7123:=Transversal(N,N7123);
for i in [1..#T7123] do
ss:=[7,1,2,3]^T7123[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7124:=Stabiliser(N712,4);
T7124:=Transversal(N,N7124);
for i in [1..#T7124] do
ss:=[7,1,2,4]^T7124[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7125:=Stabiliser(N712,5);
T7125:=Transversal(N,N7125);
for i in [1..#T7125] do
ss:=[7,1,2,5]^T7125[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7126:=Stabiliser(N712,6);
T7126:=Transversal(N,N7126);
for i in [1..#T7126] do
ss:=[7,1,2,6]^T7126[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N713:=Stabiliser(N71,3);
for g in N do if 7^g eq 1 and 1^g eq 3 and 3^g eq 7 then
N713s:=sub<N|N713,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 7 and 3^g eq 1 then
N713s:=sub<N|N713s,g>; end if; end for;
T713s:=Transversal(N,N713s);
for i in [1..#T713s] do
ss:=[7,1,3,7]^T713s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7132:=Stabiliser(N713,2);
T7132:=Transversal(N,N7132);
for i in [1..#T7132] do
  ss:=[7,1,3,2]~T7132[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N712:=Stabiliser(N71,2);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 then
  N71727s:=sub<N[N712,g]>; end if; end for;
T71727s:=Transversal(N,N71727s);
for i in [1..#T71727s] do
  ss:=[7,1,7,2,7]~T71727s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 3^g eq 3 then
  N71723s:=sub<N[N7123,g]>; end if; end for;
T71723s:=Transversal(N,N71723s);
for i in [1..#T71723s] do
  ss:=[7,1,7,2,3]~T71723s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7124:=Stabiliser(N712,4);
T71724:=Transversal(N,N7124);
for i in [1..#T71724] do
  ss:=[7,1,7,2,4]~T71724[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N7125 := Stabiliser(N712, 5);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 5^g eq 5 then
N7125s := sub<N|N7125, g>; end if; end for;
T7125s := Transversal(N, N7125s);
for i in [1..#T7125s] do
ss := [7, 1, 7, 2, 5] ^ T7125s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N7123 := Stabiliser(N712, 3);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 3^g eq 3 then
N7123s := sub<N|N7123, g>; end if; end for;
T7123s := Transversal(N, N7123s);
for i in [1..#T7123s] do
ss := [7, 1, 7, 3, 2] ^ T7123s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N7124 := Stabiliser(N712, 4);
T7124 := Transversal(N, N7124);
for i in [1..#T7124] do
ss := [7, 1, 2, 7, 4] ^ T7124[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N7125 := Stabiliser(N712, 5);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 5^g eq 5 then
N71275s := sub<N|N7125,g>; end if; end for;
T71275s := Transversal(N,N71275s);
for i in [1..#T71275s] do
  ss := [7,1,2,7,5] ~ T71275s[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do cst[i] ne [] then m := m+1; end if; end for; m;

N712 := Stabiliser(N71,2);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 then
  N71217s := sub<N|N712,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 7 and 2^g eq 1 then
  N71217s := sub<N|N71217s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 2 and 2^g eq 7 then
  N71217s := sub<N|N71217s,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 1 and 2^g eq 7 then
  N71217s := sub<N|N71217s,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 2 and 2^g eq 1 then
  N71217s := sub<N|N71217s,g>; end if; end for;
T71217s := Transversal(N,N71217s);
for i in [1..#T71217s] do
  ss := [7,1,2,1,7] ~ T71217s[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do cst[i] ne [] then m := m+1; end if; end for; m;

N7123 := Stabiliser(N71,3);
T71213 := Transversal(N,N7123);
for i in [1..#T71213] do
  ss := [7,1,2,1,3] ~ T71213[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do cst[i] ne [] then m := m+1; end if; end for; m;

N7125 := Stabiliser(N71,5);
for g in N do if 7^g eq 7 and 1^g eq 2 and 2^g eq 1 and 5^g eq 5 then
  N71215s := sub<N|N7125,g>; end if; end for;
T71215s := Transversal(N,N71215s);
for i in [1..#T71215s] do
  ss := [7,1,2,1,5] ~ T71215s[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N7123 := Stabiliser(N712, 3);
T71237 := Transversal(N, N7123);
for i in [1..#T71237] do
  ss := [7, 1, 2, 3, 7] - T71237[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N7123 := Stabiliser(N712, 3);
for g in N do if 7^-g eq 5 and 1^-g eq 2 and 2^-g eq 1 and 3^-g eq 3 then
  N71231s := sub(N, g); end if; end for;
T71231s := Transversal(N, N71231s);
for i in [1..#T71231s] do
  ss := [7, 1, 2, 3, 1] - T71231s[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N7123 := Stabiliser(N712, 3);
T71232 := Transversal(N, N7123);
for i in [1..#T71232] do
  ss := [7, 1, 2, 3, 2] - T71232[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N71234 := Stabiliser(N7123, 4);
T71234 := Transversal(N, N71234);
for i in [1..#T71234] do
  ss := [7, 1, 2, 3, 4] - T71234[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N71235 := Stabiliser(N7123, 5);
T71235 := Transversal(N, N71235);
for i in [1..#T71235] do
  ss:=[7,1,2,3,5]~T71235[i];
  cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7124:=Stabiliser(N712,4);
T71247:=Transversal(N,N7124);
for i in [1..#T71247] do
  ss:=[7,1,2,4,7]~T71247[i];
  cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7124:=Stabiliser(N712,4);
for g in N do if 7~g eq 7 and 1~g eq 4 and 2~g eq 1 and 4~g eq 2 then
  N7124is:=sub<N|N7124,g>; end if; end for;
for g in N do if 7~g eq 7 and 1~g eq 2 and 2~g eq 4 and 4~g eq 1 then
  N7124is:=sub<N|N7124is,g>; end if; end for;
T7124is:=Transversal(N,N7124is);
for i in [1..#T7124is] do
  ss:=[7,1,2,4,1]~T7124is[i];
  cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71234:=Stabiliser(N7123,4);
T71243:=Transversal(N,N71234);
for i in [1..#T71243] do
  ss:=[7,1,2,4,3]~T71243[i];
  cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71236:=Stabiliser(N7123,6);
T71246:=Transversal(N,N71236);
for i in [1..#T71246] do
  ss:=[7,1,2,4,6]~T71246[i];
  cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do cst[i] ne [] then m:=m+1; end if; end for; m;

N7125:=Stabiliser(N712,5);
for g in N do 7^g eq 1 and 1^g eq 7 and 2^g eq 5 and 5^g eq 2 then
N7125s:=sub<N\N7125,g>; end if; end for;
T7125s:=Transversal(N,N7125s);
for i in [1..#T7125s] do ss:=[7,1,2,5,7]^-T7125s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do cst[i] ne [] then m:=m+1; end if; end for; m;

N7125:=Stabiliser(N712,5);
for g in N do 7^g eq 7 and 1^g eq 1 and 2^g eq 5 and 5^g eq 2 then
N7125s:=sub<N\N7125,g>; end if; end for;
T7125s:=Transversal(N,N7125s);
for i in [1..#T7125s] do ss:=[7,1,2,5,7]^-T7125s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do cst[i] ne [] then m:=m+1; end if; end for; m;

N7126:=Stabiliser(N712,6);
for g in N do 7^g eq 4 and 1^g eq 6 and 2^g eq 2 and 6^g eq 1 then
N7126s:=sub<N\N7126,g>; end if; end for;
T7126s:=Transversal(N,N7126s);
for i in [1..#T7126s] do ss:=[7,1,2,6,7]^-T7126s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do cst[i] ne [] then m:=m+1; end if; end for; m;

N7126:=Stabiliser(N712,6);
T71261:=Transversal(N,N7126);
for i in [1..#T71261] do ss:=[7,1,2,6,1]^-T71261[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do cst[i] ne [] then m:=m+1; end if; end for; m;
N7126:=Stabiliser(N712,6);
for g in N do if 7^g eq 7 and 1^g eq 1 and 2^g eq 6 and 6^g eq 2 then
N7126s:=sub<N|N7126,g>; end if; end for;
T7126s:=Transversal(N,N7126s);
for i in [1..#T7126s] do
ss:=[7,1,3,2,7]~T7126s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 3 and 1^g eq 7 and 3^g eq 1 and 2^g eq 2 then
N7137s:=sub<N|N7123,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 3 and 3^g eq 0 and 2^g eq 2 then
N7137s:=sub<N|N7137s,g>; end if; end for;
T7137s:=Transversal(N,N7137s);
for i in [1..#T7137s] do
ss:=[7,1,3,7,2]~T7137s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 6 and 1^g eq 1 and 3^g eq 5 and 2^g eq 2 then
N71327s:=sub<N|N7123,g>; end if; end for;
T71327s:=Transversal(N,N71327s);
for i in [1..#T71327s] do
ss:=[7,1,3,7,2]~T71327s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 1 and 1^g eq 7 and 3^g eq 3 and 2^g eq 2 then
N717273s:=sub<N|N7123,g>; end if; end for;
T717273s:=Transversal(N,N717273s);
for i in [1..#T717273s] do
ss:=[7,1,7,2,7,3]~T717273s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7124:=Stabiliser(N712,4);
T717274:=Transversal(N,N7124);
for i in [1..#T717274] do
    ss:=[7,1,7,2,7,4]~T717274[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7125:=Stabiliser(N712,5);
for g in N do if 7^g eq 1 and 1^g eq 7 and 5^g eq 5 and 2^g eq 2 then
    N717275s:=sub<N|N7125,g>; end if; end for;
T717275s:=Transversal(N,N717275s);
for i in [1..#T717275s] do
    ss:=[7,1,7,2,7,5]~T717275s[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
T717237:=Transversal(N,N7123);
for i in [1..#T717237] do
    ss:=[7,1,7,2,3,7]~T717237[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 1 and 1^g eq 7 and 3^g eq 3 and 2^g eq 2 then
    N717232s:=sub<N|N7123,g>; end if; end for;
T717232s:=Transversal(N,N717232s);
for i in [1..#T717232s] do
    ss:=[7,1,7,2,3,2]~T717232s[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71234:=Stabiliser(N712,4);
T717234:=Transversal(N,N71234);
for i in [1..#T717234] do
    ss:=[7,1,7,2,3,4]~T717234[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;

m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for;

N71235:=Stabiliser(N7123,5);
for g in N do if 7~g eq 7 and 2~g eq 2 and 5~g eq 5 and 3~g eq 3 and
    1~g eq 1 then N717235s:=sub<N|N71235,g>; end if; end for;
T717235s:=Transversal(N,N717235s);
for i in [1..#T717235s] do
    ss:=[7,1,7,2,3,5]~T717235s[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;

m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for;

N7124:=Stabiliser(N712,4);
T717247:=Transversal(N,N7124);
for i in [1..#T717247] do
    ss:=[7,1,7,2,4,7]~T717247[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;

m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for;

N7124:=Stabiliser(N712,4);
for g in N do if 7~g eq 4 and 1~g eq 2 and 4~g eq 7 and 2~g eq 2 then
    N717241s:=sub<N|N7124,g>; end if; end for;
T717241s:=Transversal(N,N717241s);
for i in [1..#T717241s] do
    ss:=[7,1,7,2,4,1]~T717241s[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;

m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for;

N7124:=Stabiliser(N712,4);
for g in N do if 7~g eq 7 and 1~g eq 1 and 4~g eq 2 and 2~g eq 4 then
    N717242s:=sub<N|N7124,g>; end if; end for;
T717242s:=Transversal(N,N717242s);
for i in [1..#T717242s] do
ss := [7, 1, 7, 2, 4, 2]^T717242s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m+1; end if; end for; m;

N71234 := Stabiliser(N7123, 4);
for g in N do if 7^g eq 1 and 1^g eq 2 and 4^g eq 6 and 2^g eq 7 and
3^g eq 4 then N717243s := sub<N|N71234, g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 7 and 4^g eq 3 and 2^g eq 1 and
3^g eq 6 then N717243s := sub<N|N717243s, g>; end if; end for;
T717243s := Transversal(N, N717243s);
for i in [1..#T717243s] do
ss := [7, 1, 7, 2, 4, 3]^T717243s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m+1; end if; end for; m;

N7125 := Stabiliser(N712, 5);
T717257 := Transversal(N, N7125);
for i in [1..#T717257] do
ss := [7, 1, 7, 2, 5, 7]^T717257[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m+1; end if; end for; m;

N7125 := Stabiliser(N712, 5);
for g in N do if 7^g eq 2 and 1^g eq 5 and 5^g eq 1 and 2^g eq 7 then
N717252s := sub<N|N7125, g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 7 and 5^g eq 2 and 2^g eq 5 then
N717252s := sub<N|N717252s, g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 2 and 5^g eq 1 and 2^g eq 7 then
N717252s := sub<N|N717252s, g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 5^g eq 2 and 2^g eq 5 then
N717252s := sub<N|N717252s, g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 2 and 5^g eq 1 and 2^g eq 7 then
N717252s := sub<N|N717252s, g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 5^g eq 7 and 2^g eq 1 then
N717252s := sub<N|N717252s, g>; end if; end for;
T717252s := Transversal(N, N717252s);
for i in [1..\#T717252s] do
    ss:=\[7,1,7,2,5,2\]~T717252s[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;

m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71245:=Stabiliser(N7124,5);
T717254:=Transversal(N,N71245);
for i in [1..\#T717254] do
    ss:=\[7,1,7,2,5,4\]~T717254[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;

m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
T717327:=Transversal(N,N7123);
for i in [1..\#T717327] do
    ss:=\[7,1,7,3,2,7\]~T717327[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;

m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71235:=Stabiliser(N7123,5);
for g in N do if 7^g eq 1 and 1^g eq 7 and 3^g eq 3 and 5^g eq 5 and 2^g eq 2 then N717325s:=sub<N\N71235,g>; end if; end for;
T717325s:=Transversal(N,N717325s);
for i in [1..\#T717325s] do
    ss:=\[7,1,7,3,2,5\]~T717325s[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;

m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
T712737:=Transversal(N,N7123);
for i in [1..\#T712737] do
    ss:=\[7,1,2,7,3,7\]~T712737[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;

m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N7123 := Stabiliser(N7123,3); 
for g in N do if 7^g eq 1 and 1^g eq 7 and 3^g eq 3 and 2^g eq 2 then 
N712732s := sub<N\{N7123, g>; end if; end for; 
T712732 := Transversal(N, N712732s); 
for i in [1..#T712732] do 
ss := [7,1,2,7,3,2]^-T712732[i]; 
cst[prodim(1, ts, ss)] := ss; 
end for; 
m := 0; 
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m; 

N71234 := Stabiliser(N7123,4); 
T712734 := Transversal(N, N71234); 
for i in [1..#T712734] do 
ss := [7,1,2,7,3,4]^-T712734[i]; 
cst[prodim(1, ts, ss)] := ss; 
end for; 
m := 0; 
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m; 

N71235 := Stabiliser(N7123,5); 
for g in N do if 7^g eq 1 and 1^g eq 7 and 3^g eq 3 and 2^g eq 2 and 
5^g eq 5 then N712735s := sub<N\{N71235, g>; end if; end for; 
T712735s := Transversal(N, N712735s); 
for i in [1..#T712735s] do 
ss := [7,1,2,7,3,5]^-T712735s[i]; 
cst[prodim(1, ts, ss)] := ss; 
end for; 
m := 0; 
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m; 

N71245 := Stabiliser(N7124,5); 
for g in N do if 7^g eq 2 and 1^g eq 1 and 4^g eq 6 and 2^g eq 5 and 
5^g eq 7 then N712745s := sub<N\{N71245, g>; end if; end for;
for $g$ in $N$ do if $7^g \equiv 5$ and $1^g \equiv 1$ and $2^g \equiv 3$ and $2^g \equiv 7$ and $5^g \equiv 2$ then $N712745s := \text{sub}<N|N712745s,g>$; end if; end for;
$T712745s := \text{Transversal}(N,N712745s)$;
for $i$ in $[1..\#T712745s]$ do
$ss := [7,1,2,7,4,5]^{T712745s}[i]$;
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for $i$ in $[1..10560]$ do if cst[i] ne [] then $m := m + 1$; end if; end for; m;

$N71246 := \text{Stabiliser}(N712,6)$;
$T712746 := \text{Transversal}(N,N71246)$;
for $i$ in $[1..\#T712746]$ do
$ss := [7,1,2,7,4,6]^{T712746}[i]$;
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for $i$ in $[1..10560]$ do if cst[i] ne [] then $m := m + 1$; end if; end for; m;

$N7125 := \text{Stabiliser}(N712,5)$;
for $g$ in $N$ do if $7^g \equiv 2$ and $1^g \equiv 1$ and $2^g \equiv 7$ and $5^g \equiv 5$ then
$N712752s := \text{sub}<N|N7125,g>$; end if; end for;
for $g$ in $N$ do if $7^g \equiv 1$ and $1^g \equiv 2$ and $2^g \equiv 7$ and $5^g \equiv 5$ then
$N712752s := \text{sub}<N|N712752s,g>$; end if; end for;
for $g$ in $N$ do if $7^g \equiv 2$ and $1^g \equiv 7$ and $2^g \equiv 1$ and $5^g \equiv 5$ then
$N712752s := \text{sub}<N|N712752s,g>$; end if; end for;
for $g$ in $N$ do if $7^g \equiv 1$ and $1^g \equiv 2$ and $2^g \equiv 7$ and $5^g \equiv 5$ then
$N712752s := \text{sub}<N|N712752s,g>$; end if; end for;
for $g$ in $N$ do if $7^g \equiv 2$ and $1^g \equiv 7$ and $2^g \equiv 1$ and $5^g \equiv 5$ then
$N712752s := \text{sub}<N|N712752s,g>$; end if; end for;
$T712752s := \text{Transversal}(N,N712752s)$;
for $i$ in $[1..\#T712752s]$ do
$ss := [7,1,2,1,7,3]^{T712752s}[i]$;
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for $i$ in $[1..10560]$ do if cst[i] ne [] then $m := m + 1$; end if; end for; m;

$N7123 := \text{Stabiliser}(N712,3)$;
for $g$ in $N$ do if $7^g \equiv 1$ and $1^g \equiv 7$ and $2^g \equiv 2$ and $3^g \equiv 3$ then
$N712173s := \text{sub}<N|N7123,g>$; end if; end for;
$T712173s := \text{Transversal}(N,N712173s)$;
for $i$ in $[1..\#T712173s]$ do
$ss := [7,1,2,1,7,3]^{T712173s}[i]$;
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N7125:=Stabiliser(N712,5);
for g in N do if 7^g eq 2 and 1^g eq 7 and 2^g eq 1 and 5^g eq 5 then
N712175s:=sub<N|N7125,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 2 and 2^g eq 7 and 5^g eq 5 then
N712175s:=sub<N|N712175s,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 1 and 2^g eq 7 and 5^g eq 5 then
N712175s:=sub<N|N712175s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 2 and 2^g eq 7 and 5^g eq 5 then
N712175s:=sub<N|N712175s,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 2 and 2^g eq 1 and 5^g eq 5 then
N712175s:=sub<N|N712175s,g>; end if; end for;
T712175s:=Transversal(N,N712175s);
for i in [1..#T712175s] do
ss:=[7,1,2,1,7,5]*T712175s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N71234:=Stabiliser(N7123,4);
T712134:=Transversal(N,N71234);
for i in [1..#T712134] do
ss:=[7,1,2,1,3,4]*T712134[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N71235:=Stabiliser(N7123,5);
T712135:=Transversal(N,N71235);
for i in [1..#T712135] do
ss:=[7,1,2,1,3,5]*T712135[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N71236:=Stabiliser(N7123,6);
T712136:=Transversal(N,N71236);
for i in [1..#T712136] do 
ss:=[7,1,2,1,3,6] T712136[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7125:=Stabiliser(N712,5);
for g in N do if 7\g eq 7 and 1\g eq 2 and 2\g eq 1 and 5\g eq 5 then 
N712157s:=Sub<N|N7125,g>; end if; end for;
T712157s:=Transversal(N,N712157s);
for i in [1..#T712157s] do 
ss:=[7,1,2,1,5,7] T712157s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71234:=Stabiliser(N7123,4);
T712374:=Transversal(N,N71234);
for i in [1..#T712374] do 
ss:=[7,1,2,3,7,4] T712374[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
for g in N do if 7\g eq 5 and 1\g eq 2 and 2\g eq 1 and 3\g eq 3 then 
N712321s:=Sub<N|N7123,g>; end if; end for;
T712321s:=Transversal(N,N712321s);
for i in [1..#T712321s] do 
ss:=[7,1,2,3,2,1] T712321s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N712345:=Stabiliser(N71234,5);
for g in N do if 7\g eq 7 and 1\g eq 6 and 2\g eq 4 and 3\g eq 2 and 
5\g eq 1 and 4\g eq 3 then N712354s:=Sub<N|N712345,g>; end if; end for;
for g in N do if 7\g eq 7 and 1\g eq 5 and 2\g eq 3 and 3\g eq 4 and 
5\g eq 6 and 4\g eq 2 then N712354s:=Sub<N|N712354s,g>; end if; end for;
T712354s:=Transversal(N,N712354s);
for i in [1..#T712354s] do
ss:=[7,1,2,3,5,4]*T712354s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71234:=Stabiliser(N7123,4);
for g in N do if 7^g eq 2 and 1^g eq 1 and 2^g eq 6 and 4^g eq 5 and
7^g eq 2 and 3^g eq 4 then N712473s:=sub<N|N71234,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 1 and 2^g eq 7 and 4^g eq 3 and
7^g eq 6 and 3^g eq 5 then N712473s:=sub<N|N712473s,g>; end if; end for;
T712473s:=Transversal(N,N712473s);
for i in [1..#T712473s] do
ss:=[7,1,2,4,7,3]*T712473s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71256:=Stabiliser(N7125,6);
T712615:=Transversal(N,N71256);
for i in [1..#T712615] do
ss:=[7,1,2,6,1,5]*T712615[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7124:=Stabiliser(N712,4);
for g in N do if 7^g eq 6 and 1^g eq 4 and 2^g eq 2 and 4^g eq 1 then
N7172747s:=sub<N|N7124,g>; end if; end for;
T7172747s:=Transversal(N,N7172747s);
for i in [1..#T7172747s] do
ss:=[7,1,7,2,7,4,7]*T7172747s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71234:=Stabiliser(N7123,4);
T7172743:=Transversal(N,N71234);
for i in [1..#T7172743] do
ss:=[7,1,7,2,7,4,3]*T7172743[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for;

N71246:=Stabiliser(N7124,6);
T7172746:=Transversal(N,N71246);
for i in [1..#T7172746] do
ss:=[7,1,7,2,7,4,6]^T7172746[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for;

N7125:=Stabiliser(N712,5);
T7172757:=Transversal(N,N7125);
for i in [1..#T7172757] do
ss:=[7,1,7,2,7,5,7]^T7172757[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for;

N7125:=Stabiliser(N712,5);
for g in N do if 7^g eq 2 and 1^g eq 7 and 2^g eq 1 and 5^g eq 5 then
N7172752s:=sub<N|N7125,g>;
end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 2 and 2^g eq 7 and 5^g eq 5 then
N7172752s:=sub<N|N7172752s,g>;
end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 1 and 2^g eq 7 and 5^g eq 5 then
N7172752s:=sub<N|N7172752s,g>;
end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 5^g eq 5 then
N7172752s:=sub<N|N7172752s,g>;
end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 2 and 2^g eq 1 and 5^g eq 5 then
N7172752s:=sub<N|N7172752s,g>;
end if; end for;
T7172752s:=Transversal(N,N7172752s);
for i in [1..#T7172752s] do
ss:=[7,1,7,2,7,5,2]^T7172752s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for;

N71234:=Stabiliser(N7123,4);
T7172374:=Transversal(N,N71234);
for i in [1..#T7172374] do
    ss:=[7,1,7,2,3,7,4]~T7172374[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N71234:=Stabiliser(N7123,4);
T7172324:=Transversal(N,N71234);
for i in [1..#T7172324] do
    ss:=[7,1,7,2,3,2,4]~T7172324[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N712346:=Stabiliser(N7123,6);
for g in N do if 7^g eq 7 and 1^g eq 2 and 2^g eq 1 and 3^g eq 6 and 4^g eq 4 and 6^g eq 3 then N7172346s:=sub<N|N712346,g>; end if; end for;
T7172346s:=Transversal(N,N7172346s);
for i in [1..#T7172346s] do
    ss:=[7,1,7,2,3,4,6]~T7172346s[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N71246:=Stabiliser(N7124,6);
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 5 and 4^g eq 1 and 6^g eq 4 then N7172476s:=sub<N|N71246,g>; end if; end for;
T7172476s:=Transversal(N,N7172476s);
for i in [1..#T7172476s] do
    ss:=[7,1,7,2,3,4,6]~T7172476s[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N71245:=Stabiliser(N7124,5);
for g in N do if 7^g eq 6 and 1^g eq 1 and 2^g eq 3 and 4^g eq 7 and 5^g eq 2 then N7172425s:=sub<N|N71245,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 1 and 2^g eq 5 and 4^g eq 6 and
n := 0;
for i in [1..#T7172425s] do
    ss := [7, 1, 7, 2, 4, 2, 5] \cdot T7172425s[i];
    cst[prodim(1, ts, ss)] := ss;
end for;
end if; end for;

N712345 := Stabiliser(N7124, 5);
for g in N do if 7 \cdot g eq 1 and 1 \cdot g eq 5 and 2 \cdot g eq 4 and 4 \cdot g eq 7 and 3 \cdot g eq 6 and 5 \cdot g eq 3 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 4 and 1 \cdot g eq 1 and 2 \cdot g eq 5 and 4 \cdot g eq 6 and 3 \cdot g eq 2 and 5 \cdot g eq 3 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 5 and 1 \cdot g eq 4 and 2 \cdot g eq 1 and 4 \cdot g eq 2 and 3 \cdot g eq 7 and 5 \cdot g eq 3 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 7 and 1 \cdot g eq 3 and 4 \cdot g eq 2 and 3 \cdot g eq 4 and 5 \cdot g eq 6 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 2 and 1 \cdot g eq 5 and 2 \cdot g eq 6 and 4 \cdot g eq 1 and 3 \cdot g eq 3 and 5 \cdot g eq 4 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 5 and 1 \cdot g eq 6 and 2 \cdot g eq 2 and 4 \cdot g eq 7 and 3 \cdot g eq 1 and 5 \cdot g eq 4 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 3 and 1 \cdot g eq 7 and 2 \cdot g eq 5 and 4 \cdot g eq 4 and 3 \cdot g eq 1 and 5 \cdot g eq 6 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 2 and 1 \cdot g eq 4 and 2 \cdot g eq 3 and 4 \cdot g eq 6 and 3 \cdot g eq 1 and 5 \cdot g eq 7 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 6 and 1 \cdot g eq 4 and 2 \cdot g eq 7 and 4 \cdot g eq 5 and 3 \cdot g eq 3 and 5 \cdot g eq 1 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 1 and 1 \cdot g eq 2 and 2 \cdot g eq 7 and 4 \cdot g eq 6 and 3 \cdot g eq 4 and 5 \cdot g eq 5 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 6 and 1 \cdot g eq 1 and 2 \cdot g eq 3 and 4 \cdot g eq 7 and 3 \cdot g eq 5 and 5 \cdot g eq 2 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 4 and 1 \cdot g eq 7 and 2 \cdot g eq 6 and 4 \cdot g eq 2 and 3 \cdot g eq 5 and 5 \cdot g eq 1 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 6 and 1 \cdot g eq 2 and 2 \cdot g eq 5 and 4 \cdot g eq 3 and 3 \cdot g eq 7 and 5 \cdot g eq 4 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 5 and 1 \cdot g eq 3 and 2 \cdot g eq 7 and 4 \cdot g eq 1 and 3 \cdot g eq 2 and 5 \cdot g eq 6 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 3 and 1 \cdot g eq 6 and 2 \cdot g eq 1 and 4 \cdot g eq 5 and 3 \cdot g eq 4 and 5 \cdot g eq 2 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 7 and 1 \cdot g eq 6 and 2 \cdot g eq 4 and 4 \cdot g eq 3 and 3 \cdot g eq 2 and 5 \cdot g eq 1 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for g in N do if 7 \cdot g eq 1 and 1 \cdot g eq 3 and 2 \cdot g eq 6 and 4 \cdot g eq 4 and 3 \cdot g eq 7 and 5 \cdot g eq 2 then N7172435s := sub<N|N7172435s, g>; end if; end for;
for $g$ in $N$ do if $7^g \equiv 4$ and $1^g \equiv 3$ and $2^g \equiv 2$ and $4^g \equiv 5$ and $3^g \equiv 6$ and $5^g \equiv 7$ then $N_7172435s := \text{sub}<N|N_7172435s,g>$; end if; end for;
for $g$ in $N$ do if $7^g \equiv 3$ and $1^g \equiv 2$ and $2^g \equiv 4$ and $4^g \equiv 1$ and $3^g \equiv 5$ and $5^g \equiv 7$ then $N_7172435s := \text{sub}<N|N_7172435s,g>$; end if; end for;
for $g$ in $N$ do if $7^g \equiv 2$ and $1^g \equiv 4$ and $2^g \equiv 1$ and $4^g \equiv 3$ and $3^g \equiv 6$ and $5^g \equiv 5$ then $N_7172435s := \text{sub}<N|N_7172435s,g>$; end if; end for;
$T_{7172435s} := \text{Transversal}(N,N_7172435s)$;
for $i$ in $[1..\#T_{7172435s}]$ do
    ss := $[7,1,7,2,4,3,5]^T_{7172435s}[i]$;
    cst[prodim(1, ts, ss)] := ss;
end for;
$m := 0$;
for $i$ in $[1..10560]$ do if cst$[i]$ ne $[]$ then $m := m+1$; end if; end for; $m$;

$N_{71235} := \text{Stabiliser}(N_{7123},5)$;
$T_{7172573} := \text{Transversal}(N,N_{71235})$;
for $i$ in $[1..\#T_{7172573}]$ do
    ss := $[7,1,7,2,5,7,3]^T_{7172573}[i]$;
    cst[prodim(1, ts, ss)] := ss;
end for;
$m := 0$;
for $i$ in $[1..10560]$ do if cst$[i]$ ne $[]$ then $m := m+1$; end if; end for; $m$;

$N_{71245} := \text{Stabiliser}(N_{7124},5)$;
$T_{7172574} := \text{Transversal}(N,N_{71245})$;
for $i$ in $[1..\#T_{7172574}]$ do
    ss := $[7,1,7,2,5,7,4]^T_{7172574}[i]$;
    cst[prodim(1, ts, ss)] := ss;
end for;
$m := 0$;
for $i$ in $[1..10560]$ do if cst$[i]$ ne $[]$ then $m := m+1$; end if; end for; $m$;

$N_{71256} := \text{Stabiliser}(N_{7125},6)$;
for $g$ in $N$ do if $7^g \equiv 7$ and $1^g \equiv 4$ and $2^g \equiv 1$ and $5^g \equiv 6$ and $6^g \equiv 3$ then $N_{7172576s} := \text{sub}<N|N_{71256},g>$; end if; end for;
for $g$ in $N$ do if $7^g \equiv 1$ and $1^g \equiv 2$ and $2^g \equiv 4$ and $4^g \equiv 3$ and $3^g \equiv 6$ and $5^g \equiv 5$ then $N_{7172576s} := \text{sub}<N|N_{7172576s},g>$; end if; end for;
$T_{7172576s} := \text{Transversal}(N,N_{7172576s})$;
for $i$ in $[1..\#T_{7172576s}]$ do
    ss := $[7,1,7,2,5,7,6]^T_{7172576s}[i]$;
    cst[prodim(1, ts, ss)] := ss;
end for;
$m := 0$;
for $i$ in $[1..10560]$ do if cst$[i]$ ne $[]$ then $m := m+1$; end if; end for; $m$;
N71235 := Stabiliser(N7123,5);
for g in N do if 7°g eq 7 and 1°g eq 1 and 2°g eq 5 and 5°g eq 2 and 3°g eq 3 then N7172523s := sub<N|N712523,g>; end if; end for;
for g in N do if 7°g eq 1 and 1°g eq 7 and 2°g eq 2 and 5°g eq 5 and 3°g eq 3 then N7172523s := sub<N|N7172523,g>; end if; end for;
for g in N do if 7°g eq 5 and 1°g eq 2 and 2°g eq 1 and 5°g eq 7 and 3°g eq 3 then N7172523s := sub<N|N7172523,g>; end if; end for;
for g in N do if 7°g eq 2 and 1°g eq 5 and 2°g eq 1 and 5°g eq 7 and 3°g eq 3 then N7172523s := sub<N|N7172523,g>; end if; end for;
for g in N do if 7°g eq 2 and 1°g eq 5 and 2°g eq 7 and 5°g eq 1 and 3°g eq 3 then N7172523s := sub<N|N7172523,g>; end if; end for;
T7172523s := Transversal(N,N7172523s);
for i in [1..#T7172523s] do
ss := [7,1,7,2,5,2,3] ^ T7172523s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N71245 := Stabiliser(N7124,5);
for g in N do if 7°g eq 1 and 1°g eq 7 and 2°g eq 5 and 5°g eq 2 and 4°g eq 4 then N712524s := sub<N|N71245,g>; end if; end for;
for g in N do if 7°g eq 2 and 1°g eq 5 and 2°g eq 7 and 5°g eq 1 and 4°g eq 4 then N712524s := sub<N|N712524s,g>; end if; end for;
for g in N do if 7°g eq 5 and 1°g eq 2 and 2°g eq 1 and 5°g eq 7 and 4°g eq 4 then N712524s := sub<N|N712524s,g>; end if; end for;
T7172524s := Transversal(N,N712524s);
for i in [1..#T7172524s] do
ss := [7,1,7,2,5,2,4] ^ T7172524s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N71245 := Stabiliser(N7124,5);
for g in N do if 7°g eq 5 and 1°g eq 2 and 2°g eq 4 and 5°g eq 6 and 4°g eq 1 then N712547s := sub<N|N71245,g>; end if; end for;
for g in N do if 7°g eq 6 and 1°g eq 4 and 2°g eq 1 and 5°g eq 7 and 4°g eq 2 then N712547s := sub<N|N712547s,g>; end if; end for;
T7172547s := Transversal(N, N7172547s);
for i in [1..#T7172547s] do
ss := [7, 1, 7, 2, 5, 4, 7] \* T7172547s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N71245 := Stabiliser(N7124, 5);
T7172541 := Transversal(N, N71245);
for i in [1..#T7172541] do
ss := [7, 1, 7, 2, 5, 4, 1] \* T7172541[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N71234 := Stabiliser(N7123, 4);
for g in N do if 7 \^ g eq 2 and 1 \^ g eq 5 and 3 \^ g eq 3 and 2 \^ g eq 6 and 4 \^ g eq 1 then N7173274s := sub\langle N | N71234, g\rangle; end if; end for;
for g in N do if 7 \^ g eq 6 and 1 \^ g eq 4 and 3 \^ g eq 3 and 2 \^ g eq 7 and 4 \^ g eq 5 then N7173274s := sub\langle N | N7173274s, g\rangle; end if; end for;
T7173274s := Transversal(N, N7173274s);
for i in [1..#T7173274s] do
ss := [7, 1, 3, 2, 7, 4] \* T7173274s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N71234 := Stabiliser(N7123, 4);
for g in N do if 7 \^ g eq 6 and 1 \^ g eq 4 and 2 \^ g eq 1 and 3 \^ g eq 3 and 4 \^ g eq 2 then N7127341s := sub\langle N | N71234, g\rangle; end if; end for;
for g in N do if 7 \^ g eq 5 and 1 \^ g eq 2 and 2 \^ g eq 4 and 3 \^ g eq 3 and 4 \^ g eq 1 then N7127341s := sub\langle N | N7127341s, g\rangle; end if; end for;
T7127341s := Transversal(N, N7127341s);
for i in [1..#T7127341s] do
ss := [7, 1, 2, 7, 3, 4, 1] \* T7127341s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..10560] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N71234 := Stabiliser(N7123, 4);
for g in N do if 7·g eq 2 and 1·g eq 5 and 2·g eq 4 and 4·g eq 7 and 3·g eq 3 then N71727431s:=sub<N|N71234,g>; end if; end for;
for g in N do if 7·g eq 4 and 1·g eq 6 and 2·g eq 7 and 4·g eq 2 and 3·g eq 3 then N71727431s:=sub<N|N71727431s,g>; end if; end for;
T71727431s:=Transversal(N,N71727431s);
for i in [1..#T71727431s] do
ss:=[7,1,7,2,7,4,3,1]·T71727431s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71234:=Stabiliser(N7123,4);
for g in N do if 7·g eq 7 and 1·g eq 6 and 2·g eq 5 and 4·g eq 1 and 3·g eq 2 then N71727432s:=sub<N|N71234,g>; end if; end for;
for g in N do if 7·g eq 7 and 1·g eq 4 and 2·g eq 3 and 4·g eq 6 and 3·g eq 5 then N71727432s:=sub<N|N71727432s,g>; end if; end for;
T71727432s:=Transversal(N,N71727432s);
for i in [1..#T71727432s] do
ss:=[7,1,7,2,7,4,3,2]·T71727432s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71246:=Stabiliser(N7124,6);
for g in N do if 7·g eq 6 and 1·g eq 4 and 2·g eq 7 and 4·g eq 5 and 6·g eq 2 then N71727462s:=sub<N|N71246,g>; end if; end for;
for g in N do if 7·g eq 2 and 1·g eq 5 and 2·g eq 6 and 4·g eq 1 and 6·g eq 7 then N71727462s:=sub<N|N71727462s,g>; end if; end for;
T71727462s:=Transversal(N,N71727462s);
for i in [1..#T71727462s] do
ss:=[7,1,7,2,7,4,6,2]·T71727462s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71245:=Stabiliser(N7124,5);
for g in N do if 7·g eq 7 and 1·g eq 3 and 2·g eq 2 and 5·g eq 4 then N71727571s:=sub<N|N71245,g>; end if; end for;
for g in N do if 7·g eq 7 and 1·g eq 4 and 2·g eq 2 and 5·g eq 3 then N71727571s:=sub<N|N71727571s,g>; end if; end for;
for g in N do if 7·g eq 7 and 1·g eq 5 and 2·g eq 2 and 5·g eq 1 then
N71727571s:=sub<N\N71727571s,g>; end if; end for;
T71727571s:=Transversal(N,N71727571s);
for i in [1..#T71727571s] do
ss:=[7,1,7,2,7,5,7,1]-T71727571s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N71234:=Stabiliser(N7123,4);
for g in N do if 7^g eq 5 and 1^g eq 3 and 2^g eq 1 and 3^g eq 2 and
4^g eq 7 then N71723747s:=sub<N\N71234,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 2 and 2^g eq 3 and 3^g eq 1 and
4^g eq 5 then N71723747s:=sub<N\N71723747s,g>; end if; end for;
T71723747s:=Transversal(N,N71723747s);
for i in [1..#T71723747s] do
ss:=[7,1,7,2,3,7,4,7]-T71723747s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N71245:=Stabiliser(N7124,5);
for g in N do if 7^g eq 7 and 1^g eq 4 and 2^g eq 3 and 4^g eq 6 and
5^g eq 2 then N71724251s:=sub<N\N71245,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 6 and 2^g eq 3 and 4^g eq 4 and
5^g eq 2 then N71724251s:=sub<N\N71724251s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 4^g eq 6 and
5^g eq 5 then N71724251s:=sub<N\N71724251s,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 1 and 2^g eq 3 and 4^g eq 7 and
5^g eq 2 then N71724251s:=sub<N\N71724251s,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 7 and 2^g eq 3 and 4^g eq 1 and
5^g eq 2 then N71724251s:=sub<N\N71724251s,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 6 and 2^g eq 2 and 4^g eq 7 and
5^g eq 5 then N71724251s:=sub<N\N71724251s,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 1 and 2^g eq 5 and 4^g eq 6 and
5^g eq 3 then N71724251s:=sub<N\N71724251s,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 5 and 4^g eq 1 and
5^g eq 3 then N71724251s:=sub<N\N71724251s,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 4 and 2^g eq 1 and 4^g eq 6 and
5^g eq 5 then N71724251s:=sub<N\N71724251s,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 7 and 2^g eq 5 and 4^g eq 4 and
5^g eq 3 then N71724251s:=sub<N\N71724251s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 4 and 2^g eq 5 and 4^g eq 7 and
5^g eq 3 then N71724251s:=sub<N|N71724251s,g>; end if; end for;
T71724251s:=Transversal(N,N71724251s);
for i in [1..#T71724251s] do
ss:=[7,1,7,2,4,2,5,1]^T71724251s[i];
cst[prodimd(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..10560] do if cst[i] ne {} then m:=m+1; end if; end for; m;
N712346:=Stabiliser(N71234,6);
for g in N do if 7^g eq 7 and 1^g eq 2 and 2^g eq 4 and 4^g eq 1 and
6^g eq 5 and 3^g eq 6 then N717274623s:=sub<N|N712346,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 5 and 2^g eq 2 and 4^g eq 3 and
6^g eq 1 and 3^g eq 7 then N717274623s:=sub<N|N71274623,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 7 and 2^g eq 5 and 4^g eq 4 and
6^g eq 1 and 3^g eq 2 then N717274623s:=sub<N|N71274623,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 7 and 2^g eq 6 and 4^g eq 2 and
6^g eq 4 and 3^g eq 1 then N717274623s:=sub<N|N71274623,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 7 and 2^g eq 3 and 4^g eq 1 and
6^g eq 2 and 3^g eq 4 then N717274623s:=sub<N|N71274623,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 4 and 2^g eq 7 and 4^g eq 5 and
6^g eq 2 and 3^g eq 3 then N717274623s:=sub<N|N71274623,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 6 and 2^g eq 2 and 4^g eq 7 and
6^g eq 1 and 3^g eq 4 then N717274623s:=sub<N|N71274623,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 3 and 2^g eq 5 and 4^g eq 2 and
6^g eq 7 and 3^g eq 6 then N717274623s:=sub<N|N71274623,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 2 and 3^g eq 4 then N717274623s:=sub<N|N71274623,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 5 and 2^g eq 4 and 4^g eq 7 and
6^g eq 3 and 3^g eq 1 then N717274623s:=sub<N|N71274623,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 3 and 2^g eq 2 and 4^g eq 1 and
6^g eq 7 and 3^g eq 3 then N717274623s:=sub<N|N71274623,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 2 and 3^g eq 7 and 4^g eq 3 and
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7125:=Stabiliser(N712,5);
for g in N do if 7^g eq 4 and 1^g eq 6 and 2^g eq 2 and 5^g eq 5 then N717275717s:=sub<N|N717275717s,g> end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 3 and 2^g eq 2 and 5^g eq 6 then N717275717s:=sub<N|N717275717s,g> end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 7 and 2^g eq 2 and 5^g eq 4 then N717275717s:=sub<N|N717275717s,g> end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 7 and 2^g eq 2 and 5^g eq 1 then N717275717s:=sub<N|N717275717s,g> end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 7 and 2^g eq 2 and 5^g eq 3 then
m:=0;
for i in [1..10560] do if cst[i] ne [] then m:=m+1; end if; end for; m;

T7172757172s:=Transversal(N,N);
for i in [1..#T7172757172s] do
ss:=[7,1,7,2,7,5,7,1,7,2] T7172757172s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
Appendix B

MAGMA Code for Constructing $M_{22} : 2$

N:=Sym(7);
xx:=N!(7,1,2,3,4,5,6);
yy:=N!(2,6)(4,5);
N:=sub<N|xx,yy>;
G<x,y,t>:=Group<x,y,t|x^7,y^2,(x*y)^3, (x,y)^4, t^2, (x*y, t^-(x^4)), (y, t^-(x^3)), (y * x^-3 * y * x^-2 * y*t^-3(x^-3))^-8, (x^-2 * y * x^-2 * y * x^-2*t^-(x^-2))^-10, (y*t*t^-2*(x^-2))^4, x^-3 * y * x^-2 * y * t*t^-x*t* (x^-2)*t*t^-x*(x^-5)*t*t^-x*t*(x^-2)>;
Index(G,sub<G|x,y>);
f,G1,k:=CosetAction(G,sub<G|x,y>);
IN:=sub<G1|f(x),f(y)>;
ts:=[f(t^-((x^-i))): i in [1..7]];
prodim := function(pt, Q, I)
    /*
    Return the image of pt under permutations Q[I] applied sequentially.
    */
    v := pt;
    for i in I do
        v := v^Q[i];
    end for;
    return v;
end function;
cst := [null : i in [1 .. 5280]] where null is [Integers() | ];
for i := 1 to 7 do
cst[prodim(i, ts, [i])] := [i];
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7:=Stabiliser(N,7);
N71:=Stabiliser(N7,1);
T71:=Transversal(N,N71);
for i in [1..#T71] do
  ss:=[7,1]^T71[i];
  cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

for g in N do if 7^g eq 1 and 1^g eq 7 then N71s:=sub<N\N71,g>; end if; end for;
T71s:=Transversal(N,N71s);
for i in [1..#T71s] do
  ss:=[7,1,7]^T71s[i];
  cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N712:=Stabiliser(N71,2);
T712s:=Transversal(N,N712);
for i in [1..#T712s] do
  ss:=[7,1,2]^T712s[i];
  cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N713:=Stabiliser(N71,3);
T713:=Transversal(N,N713);
for i in [1..#T713] do
  ss:=[7,1,3]^T713[i];
  cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N712:=Stabiliser(N71,2);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 then
  N7172s:=sub<N\N712,g>; end if; end for;
T7172s := Transversal(N, N7172s);
for i in [1..#T7172s] do
  ss := [7, 1, 7, 2] ~ T7172s[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..5280] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N713 := Stabiliser(N71, 3);
for g in N do if 7~g eq 1 and 1~g eq 7 and 3~g eq 3 then
  N7173s := sub<N|N713, g>; end if; end for;
T7173s := Transversal(N, N7173s);
for i in [1..#T7173s] do
  ss := [7, 1, 7, 3] ~ T7173s[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..5280] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N712 := Stabiliser(N71, 2);
for g in N do if 7~g eq 1 and 1~g eq 7 and 2~g eq 2 then
  N7172s := sub<N|N712, g>; end if; end for;
T7172s := Transversal(N, N7172s);
for i in [1..#T7172s] do
  ss := [7, 1, 2, 7] ~ T7172s[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..5280] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N712 := Stabiliser(N71, 2);
for g in N do if 7~g eq 7 and 1~g eq 2 and 2~g eq 1 then
  N7121s := sub<N|N712, g>; end if; end for;
T7121s := Transversal(N, N7121s);
for i in [1..#T7121s] do
  ss := [7, 1, 2, 1] ~ T7121s[i];
  cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..5280] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N7123 := Stabiliser(N712, 3);
T7123 := Transversal(N, N7123);
for i in [1..#T7123] do
ss:=[7,1,2,3]^{-T7123[i]};
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7124:=Stabiliser(N712,4);
T7124:=Transversal(N,N7124);
for i in [1..#T7124] do
ss:=[7,1,2,4]^{-T7124[i]};
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7125:=Stabiliser(N712,5);
T7125:=Transversal(N,N7125);
for i in [1..#T7125] do
ss:=[7,1,2,5]^{-T7125[i]};
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7126:=Stabiliser(N712,6);
T7126:=Transversal(N,N7126);
for i in [1..#T7126] do
ss:=[7,1,2,6]^{-T7126[i]};
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N713:=Stabiliser(N71,3);
for g in N do if 7^g eq 1 and 1^g eq 3 and 3^g eq 7 then
N713s:=sub<N|N713,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 7 and 3^g eq 1 then
N713s:=sub<N|N713s,g>; end if; end for;
T7137s:=Transversal(N,N7137s);
for i in [1..#T7137s] do
ss:=[7,1,3,7]^{-T7137s[i]};
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7132:=Stabiliser(N713,2);
T7132:=Transversal(N,N7132);
for i in [1..#T7132] do
  ss:=[7,1,3,2]~T7132[i];
  cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N712:=Stabiliser(N71,2);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 then
  N71727s:=sub<N|N712,g>; end if; end for;
T71727s:=Transversal(N,N71727s);
for i in [1..#T71727s] do
  ss:=[7,1,7,2,7]~T71727s[i];
  cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 3^g eq 3 then
  N71723s:=sub<N|N7123,g>; end if; end for;
T71723s:=Transversal(N,N71723s);
for i in [1..#T71723s] do
  ss:=[7,1,7,2,3]~T71723s[i];
  cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7124:=Stabiliser(N712,4);
T71724:=Transversal(N,N7124);
for i in [1..#T71724] do
  ss:=[7,1,7,2,4]~T71724[i];
  cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7125:=Stabiliser(N712,5);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 5^g eq 5 then
N71725s:=sub<N|N7125,g>; end if; end for;
T71725s:=Transversal(N,N71725s);
for i in [1..#T71725s] do
ss:=[7,1,7,2,5]~T71725s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
T71273s:=Transversal(N,N71273s);
for i in [1..#T71273s] do
ss:=[7,1,2,7,3]~T71273s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7124:=Stabiliser(N712,4);
T71274:=Transversal(N,N7124);
for i in [1..#T71274] do
ss:=[7,1,2,7,4]~T71274[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7125:=Stabiliser(N712,5);
T71275s:=Transversal(N,N71275s);
for i in [1..#T71275s] do
ss:=[7,1,2,7,5]~T71275s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N712:=Stabiliser(N71,2);
for g in N do if 7\~ g eq 1 and 1\~ g eq 7 and 2\~ g eq 2 and 5\~ g eq 5 then
N71275s:=sub<N|N712,g>; end if; end for;
T71275s:=Transversal(N,N71275s);
for i in [1..#T71275s] do
ss:=[7,1,2,7,5]~T71275s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;
for \( g \) in \( N \) do if \( 7g \equiv 1 \) and \( 1g \equiv 2 \) and \( 2g \equiv 7 \) then
N71217s:=sub\( <N|N71217s,g> \); end if; end for;
for \( g \) in \( N \) do if \( 7g \equiv 2 \) and \( 1g \equiv 1 \) and \( 2g \equiv 7 \) then
N71217s:=sub\( <N|N71217s,g> \); end if; end for;
for \( g \) in \( N \) do if \( 7g \equiv 7 \) and \( 1g \equiv 2 \) and \( 2g \equiv 7 \) then
N71217s:=sub\( <N|N71217s,g> \); end if; end for;
T71217s:=Transversal\( (N,N71217s) \);
for \( i \) in \([1..#T71217s]\) do
ss:=\([7,1,2,1,7]\) ~ T71217s\([i]\);
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for \( i \) in \([1..5280]\) do if \( \text{cst}[i] \neq \emptyset \) then \( m:=m+1 \); end if; end for; m;
N7123:=Stabiliser\( (N712,3) \);
T71213:=Transversal\( (N,N7123) \);
for \( i \) in \([1..#T71213]\) do
ss:=\([7,1,2,1,3]\) ~ T71213\([i]\);
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for \( i \) in \([1..5280]\) do if \( \text{cst}[i] \neq \emptyset \) then \( m:=m+1 \); end if; end for; m;
N7123:=Stabiliser\( (N712,3) \);
T71237:=Transversal\( (N,N7123) \);
for \( i \) in \([1..#T71237]\) do
ss:=\([7,1,2,3,7]\) ~ T71237\([i]\);
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for \( i \) in \([1..5280]\) do if \( \text{cst}[i] \neq \emptyset \) then \( m:=m+1 \); end if; end for; m;
N7123:=Stabiliser\( (N712,3) \);
T71232:=Transversal\( (N,N7123) \);
for \( i \) in \([1..#T71232]\) do
ss:=\([7,1,2,3,2]\) ~ T71232\([i]\);
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for \( i \) in \([1..5280]\) do if \( \text{cst}[i] \neq \emptyset \) then \( m:=m+1 \); end if; end for; m;
N71235:=Stabiliser\( (N7123,5) \);
T71235:=Transversal\( (N,N71235) \);
for i in [1..#T71235] do
  ss:=[7,1,2,3,5]~T71235[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N7124:=Stabiliser(N712,4);
T71247:=Transversal(N,N7124);
for i in [1..#T71247] do
  ss:=[7,1,2,4,7]~T71247[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N7124:=Stabiliser(N712,4);
for g in N do if 7~g eq 7 and 1~g eq 4 and 2~g eq 1 and 4~g eq 2 then
  N7124is:=sub<N|N7124, g>; end if; end for;
for g in N do if 7~g eq 7 and 1~g eq 2 and 2~g eq 4 and 4~g eq 1 then
  N7124is:=sub<N|N7124is, g>; end if; end for;
T7124is:=Transversal(N,N7124is);
for i in [1..#T7124is] do
  ss:=[7,1,2,4,1]~T7124is[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N71234:=Stabiliser(N7123,4);
T71243:=Transversal(N,N71234);
for i in [1..#T71243] do
  ss:=[7,1,2,4,3]~T71243[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N7125:=Stabiliser(N712,5);
for g in N do if 7~g eq 1 and 1~g eq 7 and 2~g eq 5 and 5~g eq 2 then
  N71257s:=sub<N|N7125, g>; end if; end for;
T71257s:=Transversal(N,N71257s);
for i in [1..#T71257s] do
  ss:=[7,1,2,5,7]~T71257s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7126:=Stabiliser(N712,6);
T71261:=Transversal(N,N7126);
for i in [1..#T71261] do
  ss:=[7,1,2,6,1]^T71261[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7126:=Stabiliser(N712,6);
for g in N do if 7~g eq 7 and 1~g eq 1 and 2~g eq 6 and 6~g eq 2 then
  N71262s:=sub<N|N7126,g>; end if; end for;
T71262s:=Transversal(N,N71262s);
for i in [1..#T71262s] do
  ss:=[7,1,2,6,2]^T71262s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
for g in N do if 7~g eq 3 and 1~g eq 7 and 3~g eq 1 and 2~g eq 2 then
  N71372s:=sub<N|N7123,g>; end if; end for;
T71372s:=Transversal(N,N71372s);
for i in [1..#T71372s] do
  ss:=[7,1,3,7,2]^T71372s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
for g in N do if 7~g eq 1 and 1~g eq 7 and 3~g eq 3 and 2~g eq 2 then
  N717273s:=sub<N|N7123,g>; end if; end for;
T717273s:=Transversal(N,N717273s);
for i in [1..#T717273s] do
  ss:=[7,1,7,2,7,3]^T717273s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7124:=Stabiliser(N712,4);
T717247:=Transversal(N,N7124);
for i in [1..#T717247] do
ss:=[7,1,7,2,4,7]^{T717247[i]};
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 1 and 1^g eq 7 and 3^g eq 3 and 2^g eq 2 then
N717232s:=sub<N|N7123,g>; end if; end for;
T717232s:=Transversal(N,N717232s);
for i in [1..#T717232s] do
ss:=[7,1,7,2,3,2]^{T717232s[i]};
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71235:=Stabiliser(N712,5);
for g in N do if 7^g eq 1 and 1^g eq 7 and 5^g eq 5 and 3^g eq 3 and 2^g eq 2 then
N717235s:=sub<N|N71235,g>; end if; end for;
T717235s:=Transversal(N,N717235s);
for i in [1..#T717235s] do
ss:=[7,1,7,2,3,5]^{T717235s[i]};
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7124:=Stabiliser(N712,4);
T717247:=Transversal(N,N7124);
for i in [1..#T717247] do
ss:=[7,1,7,2,4,7]^{T717247[i]};
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N7124:=Stabiliser(N712,4);
for g in N do if 7~g eq 7 and 1~g eq 1 and 4~g eq 2 and 2~g eq 4 then
N717242s:=sub<N|N7124,g>; end if; end for;
T717242s:=Transversal(N,N717242s);
for i in [1..#T717242s] do
ss:=[7,1,7,2,4,2]~T717242s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71234:=Stabiliser(N7123,4);
for g in N do if 7~g eq 1 and 1~g eq 2 and 4~g eq 6 and 2~g eq 7 and
3~g eq 4 then N717243s:=sub<N|N71234,g>; end if; end for;
for g in N do if 7~g eq 2 and 1~g eq 7 and 4~g eq 3 and 2~g eq 1 and
3~g eq 6 then N717243s:=sub<N|N717243s,g>; end if; end for;
T717243s:=Transversal(N,N717243s);
for i in [1..#T717243s] do
ss:=[7,1,7,2,4,3]~T717243s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71245:=Stabiliser(N7124,5);
T712734:=Transversal(N,N71245);
for i in [1..#T712734] do
ss:=[7,1,7,2,5,4]~T712734[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71234:=Stabiliser(N7123,4);
T712734:=Transversal(N,N71234);
for i in [1..#T712734] do
ss:=[7,1,2,7,3,4]~T712734[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71245:=Stabiliser(N7124,5);
for g in N do if 7\textsuperscript{g} eq 2 and 1\textsuperscript{g} eq 1 and 4\textsuperscript{g} eq 6 and 2\textsuperscript{g} eq 5 and 5\textsuperscript{g} eq 7 then N\textsubscript{712745s}:=\text{sub}\langle N|N\textsubscript{71245},g\rangle; end if; end for;
for g in N do if 7\textsuperscript{g} eq 5 and 1\textsuperscript{g} eq 1 and 4\textsuperscript{g} eq 3 and 2\textsuperscript{g} eq 7 and 5\textsuperscript{g} eq 2 then N\textsubscript{71245s}:=\text{sub}\langle N|N\textsubscript{71245s},g\rangle; end if; end for;
T\textsubscript{71245s}:=\text{Transversal}(N,N\textsubscript{71245s});
for i in [1..#T\textsubscript{71245s}] do
ss:=[7,1,2,7,4,5]\textsuperscript{-}T\textsubscript{71245s}[i];
cst[\text{prodim}(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N\textsubscript{7125}:=\text{Stabiliser}(N\textsubscript{712},5);
for g in N do if 7\textsuperscript{g} eq 2 and 1\textsuperscript{g} eq 1 and 2\textsuperscript{g} eq 7 and 5\textsuperscript{g} eq 5 then
N\textsubscript{712752s}:=\text{sub}\langle N|N\textsubscript{7125},g\rangle; end if; end for;
for g in N do if 7\textsuperscript{g} eq 7 and 1\textsuperscript{g} eq 2 and 2\textsuperscript{g} eq 1 and 5\textsuperscript{g} eq 5 then
N\textsubscript{712752s}:=\text{sub}\langle N|N\textsubscript{712752s},g\rangle; end if; end for;
for g in N do if 7\textsuperscript{g} eq 1 and 1\textsuperscript{g} eq 7 and 2\textsuperscript{g} eq 2 and 5\textsuperscript{g} eq 5 then
N\textsubscript{712752s}:=\text{sub}\langle N|N\textsubscript{712752s},g\rangle; end if; end for;
for g in N do if 7\textsuperscript{g} eq 2 and 1\textsuperscript{g} eq 7 and 2\textsuperscript{g} eq 1 and 5\textsuperscript{g} eq 5 then
N\textsubscript{712752s}:=\text{sub}\langle N|N\textsubscript{712752s},g\rangle; end if; end for;
for g in N do if 7\textsuperscript{g} eq 1 and 1\textsuperscript{g} eq 2 and 2\textsuperscript{g} eq 7 and 5\textsuperscript{g} eq 5 then
N\textsubscript{712752s}:=\text{sub}\langle N|N\textsubscript{712752s},g\rangle; end if; end for;
T\textsubscript{712752s}:=\text{Transversal}(N,N\textsubscript{712752s});
for i in [1..#T\textsubscript{712752s}] do
ss:=[7,1,2,7,5,2]\textsuperscript{-}T\textsubscript{712752s}[i];
cst[\text{prodim}(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N\textsubscript{7123}:=\text{Stabiliser}(N\textsubscript{712},3);
for g in N do if 7\textsuperscript{g} eq 1 and 1\textsuperscript{g} eq 7 and 2\textsuperscript{g} eq 2 and 3\textsuperscript{g} eq 3 then
N\textsubscript{712173s}:=\text{sub}\langle N|N\textsubscript{7123},g\rangle; end if; end for;
T\textsubscript{712173s}:=\text{Transversal}(N,N\textsubscript{712173s});
for i in [1..#T\textsubscript{712173s}] do
ss:=[7,1,2,1,7,3]\textsuperscript{-}T\textsubscript{712173s}[i];
cst[\text{prodim}(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N\textsubscript{71235}:=\text{Stabiliser}(N\textsubscript{7123},5);
T\textsubscript{712135}:=\text{Transversal}(N,N\textsubscript{71235});
for i in [1..#T712135] do
    ss:=[7,1,2,1,3,5]~T712135[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N712345:=Stabiliser(N71234,5);
for g in N do if 7~g eq 7 and 1~g eq 6 and 2~g eq 4 and 3~g eq 2 and
5~g eq 1 and 4~g eq 3 then N712354s:=sub<N|N712345,g>; end if; end for;
for g in N do if 7~g eq 7 and 1~g eq 5 and 2~g eq 3 and 3~g eq 4 and
5~g eq 6 and 4~g eq 2 then N712354s:=sub<N|N712354s,g>; end if; end for;
T712354s:=Transversal(N,N712354s);
for i in [1..#T712354s] do
    ss:=[7,1,2,3,5,4]~T712354s[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N71234:=Stabiliser(N71234,5);
for g in N do if 7~g eq 7 and 1~g eq 6 and 2~g eq 4 and 3~g eq 2 and
5~g eq 1 and 4~g eq 3 then N712473s:=sub<N|N71234,g>; end if; end for;
for g in N do if 7~g eq 7 and 1~g eq 5 and 2~g eq 3 and 3~g eq 4 and
5~g eq 6 and 4~g eq 2 then N712473s:=sub<N|N712473s,g>; end if; end for;
T712473s:=Transversal(N,N712473s);
for i in [1..#T712473s] do
    ss:=[7,1,2,4,7,3]~T712473s[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N7124:=Stabiliser(N712,4);
for g in N do if 7~g eq 2 and 1~g eq 1 and 2~g eq 6 and 4~g eq 5 and
7~g eq 2 and 3~g eq 4 then N7172747s:=sub<N|N712473s,g>; end if; end for;
for g in N do if 7~g eq 6 and 1~g eq 1 and 2~g eq 7 and 4~g eq 3 and
7~g eq 6 and 3~g eq 5 then N712473s:=sub<N|N712473s,g>; end if; end for;
T7172747s:=Transversal(N,N7172747s);
for i in [1..#T7172747s] do
    ss:=[7,1,7,2,7,4,7]~T7172747s[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N71246 := Stabiliser(N7124, 6);
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 5 and 4^g eq 1 and 6^g eq 4 then N7172476s := sub<N|N71246, g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 5 and 2^g eq 4 and 3^g eq 3 then N7172476s := sub<N|N712476s, g>; end if; end for;
T7172476s := Transversal(N, N7172476s);
for i in [1..#T7172476s] do
ss := [7, 1, 7, 2, 4, 7, 6] ~ T7172476s[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m := 0;
for i in [1..5280] do if cst[i] ne [] then m := m + 1; end if; end for; m;

N712345 := Stabiliser(N7124, 5);
for g in N do if 7^g eq 1 and 1^g eq 6 and 2^g eq 4 and 4^g eq 7 and 3^g eq 6 and 5^g eq 3 then N7172435s := sub<N|N712345, g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 3 and 4^g eq 2 and 3^g eq 4 and 5^g eq 1 then N7172435s := sub<N|N712435s, g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 3 and 4^g eq 2 and 3^g eq 4 and 5^g eq 1 then N7172435s := sub<N|N712435s, g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 3 and 4^g eq 2 and 3^g eq 4 and 5^g eq 1 then N7172435s := sub<N|N712435s, g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 3 and 4^g eq 2 and 3^g eq 4 and 5^g eq 1 then N7172435s := sub<N|N712435s, g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 3 and 4^g eq 2 and 3^g eq 4 and 5^g eq 1 then N7172435s := sub<N|N712435s, g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 3 and 4^g eq 2 and 3^g eq 4 and 5^g eq 1 then N7172435s := sub<N|N712435s, g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 2^g eq 4 and 3^g eq 3 and 4^g eq 1 and 5^g eq 6 then N7172435s := sub<N|N712435s, g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 2^g eq 4 and 3^g eq 3 and 4^g eq 1 and 5^g eq 6 then N7172435s := sub<N|N712435s, g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 2^g eq 4 and 3^g eq 3 and 4^g eq 1 and 5^g eq 6 then N7172435s := sub<N|N712435s, g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 2^g eq 4 and 3^g eq 3 and 4^g eq 1 and 5^g eq 6 then N7172435s := sub<N|N712435s, g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 2^g eq 4 and 3^g eq 3 and 4^g eq 1 and 5^g eq 6 then N7172435s := sub<N|N712435s, g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 2^g eq 4 and 3^g eq 3 and 4^g eq 1 and 5^g eq 6 then N7172435s := sub<N|N712435s, g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 2^g eq 4 and 3^g eq 3 and 4^g eq 1 and 5^g eq 6 then N7172435s := sub<N|N712435s, g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 1 and 2^g eq 3 and 4^g eq 7 and 3^g eq 5 and 5^g eq 2 then N7172435s:=sub<N|N7172435s,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 7 and 2^g eq 6 and 4^g eq 2 and 3^g eq 5 and 5^g eq 1 then N7172435s:=sub<N|N7172435s,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 2 and 2^g eq 5 and 4^g eq 3 and 3^g eq 7 and 5^g eq 4 then N7172435s:=sub<N|N7172435s,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 3 and 2^g eq 7 and 4^g eq 1 and 3^g eq 2 and 5^g eq 6 then N7172435s:=sub<N|N7172435s,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 6 and 2^g eq 1 and 4^g eq 5 and 3^g eq 4 and 5^g eq 2 then N7172435s:=sub<N|N7172435s,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 4 and 4^g eq 3 and 3^g eq 2 and 5^g eq 1 then N7172435s:=sub<N|N7172435s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 3 and 2^g eq 6 and 4^g eq 4 and 3^g eq 7 and 5^g eq 2 then N7172435s:=sub<N|N7172435s,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 3 and 2^g eq 2 and 4^g eq 5 and 3^g eq 6 and 5^g eq 7 then N7172435s:=sub<N|N7172435s,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 2 and 2^g eq 4 and 4^g eq 1 and 3^g eq 5 and 5^g eq 7 then N7172435s:=sub<N|N7172435s,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 7 and 2^g eq 1 and 4^g eq 3 and 3^g eq 6 and 5^g eq 5 then N7172435s:=sub<N|N7172435s,g>; end if; end for;
T7172435s:=Transversal(N,N7172435s);
for i in [1..#T7172435s] do
ss:=[7,1,1,7,2,4,3,5]^T7172435s[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;
N71245:=Stabiliser(N7124,5);
for g in N do if 7^g eq 5 and 1^g eq 2 and 2^g eq 4 and 5^g eq 6 and 4^g eq 1 then N7172547s:=sub<N|N71245,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 4 and 2^g eq 1 and 5^g eq 7 and 4^g eq 2 then N7172547s:=sub<N|N7172547s,g>; end if; end for;
T7172547s:=Transversal(N,N7172547s);
for i in [1..#T7172547s] do
    ss:=[7,1,7,2,5,4,7]~T7172547s[i];
    cst[prodim(1, ts, ss)]:=ss;
end for;

m:=0;
for i in [1..5280] do if cst[i] ne [] then m:=m+1; end if; end for; m;
Appendix C

Permutation to Symmetric Representation for $2 \cdot M_{22} : 2$

\begin{verbatim}
N:=Sym(7);
xx:=N!(7,1,2,3,4,5,6);
yy:=N!(2,6)(4,5);
N:=sub<N|xx,yy>;
G<x,y,t>:=Group<x,y,t|x^7,y^2,(x*y)^3,(x,y)^4, t^2,
(x*y,t^-(x^4)),(y,t^-(x^3)), (y * x^-3 * y * x^2 * y*t^-(x^3))^8,
(x^-2 * y * x^-2 * y * x^-2*t^-(x^2))^10, 
(y*t*t^-(x^-2))^4>;
Index(G,sub<G|x,y>);
f,G1,k:=CosetAction(G,sub<G|x,y>);
IN:=sub<G1|f(x),f(y)>;
ts:=[f(t^-(x^-i)) : i in [1..7]];
prodim := function(pt, Q, I)
/*Return the image of pt under permutations Q[I] applied sequentially. */
v := pt;
for i in I do
  v := v^(Q[i]);
end for;
return v;
end function;

per2sym := function(G1, p)
  ww := cst[1^p];
  tt := p * &*[G1|ts[ww[#ww - 1 + 1]]: 1 in [1 .. #ww]];
  zz := N![rep{j: j in [1..7] | (1^-ts[i])^tt eq 1^-ts[j]}: i in [1..7]];
\end{verbatim}
return <zz, ww>
end function

sym2per := function(G1, x)
xx:=N!x[1]; uu:=x[2];
p := [1 : i in [1 . 10560]];
for i := 1 to 7 do
p[prodim(1, ts, [i])] := prodim(1, ts, [i]^xx);
end for;
N7:=Stabiliser(N,7);
N71:=Stabiliser(N7,1);
T71:=Transversal(N,N71);
for i in [1..#T71] do
ss:=[7,1,7]~T71[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N71s:=N71;
for g in N do if 7^g eq 1 and 1^g eq 7 then N71s:=sub<N|N71s,g>; end if; end for;
T71s:=Transversal(N,N71s);
for i in [1..#T71s] do
ss:=[7,1,7]~T71s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N712:=Stabiliser(N71,2);
T712s:=Transversal(N,N712);
for i in [1..#T712s] do
ss:=[7,1,2]~T712s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N713:=Stabiliser(N71,3);
T713:=Transversal(N,N713);
for i in [1..#T713] do
ss:=[7,1,3]~T713[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N712:=Stabiliser(N71,2);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 then
N7172s:=sub<N|N712,g>; end if; end for;
T7172s:=Transversal(N,N7172s);
for i in [1..#T7172s] do
ss:=[7,1,7,2]~T7172s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
\[ \text{N713:=Stabiliser(N71,3);} \]

for g in N do if \( 7^g \equiv 1 \mod 1^g \) eq 7 and \( 3^g \equiv 3 \mod 3^g \) eq 3 then
\[ \text{N7173s:=sub<N|N713,g>}; \] end if; end for;
\[ \text{T7173s:=Transversal(N,N7173s);} \]

for i in [1..#T7173s] do
\[ \text{ss:=[7,1,7,3]~T7173s[i];} \]
\[ \text{p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);} \]
end for;
\[ \text{N712:=Stabiliser(N71,2);} \]

for g in N do if \( 7^g \equiv 1 \mod 1^g \) eq 7 and \( 2^g \equiv 2 \mod 2^g \) eq 2 then
\[ \text{N7127s:=sub<N|N712,g>}; \] end if; end for;
\[ \text{T7127s:=Transversal(N,N7127s);} \]

for i in [1..#T7127s] do
\[ \text{ss:=[7,1,2,7]~T7127s[i];} \]
\[ \text{p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);} \]
end for;
\[ \text{N712:=Stabiliser(N71,2);} \]

for g in N do if \( 7^g \equiv 7 \mod 7^g \) eq 7 and \( 2^g \equiv 2 \mod 2^g \) eq 1 then
\[ \text{N7121s:=sub<N|N712,g>}; \] end if; end for;
\[ \text{T7121s:=Transversal(N,N7121s);} \]

for i in [1..#T7121s] do
\[ \text{ss:=[7,1,2,3]~T7121s[i];} \]
\[ \text{p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);} \]
end for;
\[ \text{N712:=Stabiliser(N71,2);} \]

T7123:=Transversal(N,N7123);

for i in [1..#T7123] do
\[ \text{ss:=[7,1,2,3]~T7123[i];} \]
\[ \text{p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);} \]
end for;
\[ \text{N712:=Stabiliser(N71,2);} \]

T7124:=Transversal(N,N7124);

for i in [1..#T7124] do
\[ \text{ss:=[7,1,2,4]~T7124[i];} \]
\[ \text{p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);} \]
end for;
\[ \text{N712:=Stabiliser(N71,2);} \]

T7125:=Transversal(N,N7125);

for i in [1..#T7125] do
\[ \text{ss:=[7,1,2,5]~T7125[i];} \]
\[ \text{p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);} \]
end for;
\[ \text{N712:=Stabiliser(N71,2);} \]

T7126:=Transversal(N,N7126);
for i in [1..#T7126] do
  ss:=[7,1,2,6]^T7126[i];
  p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N713:=Stabiliser(N71,3);
for g in N do if 7^g eq 1 and 1^g eq 3 and 3^g eq 7 then
  N7137s:=sub<N|N713,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 7 and 3^g eq 1 then
  N7137s:=sub<N|N7137s,g>; end if; end for;
T7137s:=Transversal(N,N7137s);
for i in [1..#T7137s] do
  ss:=[7,1,3,7]^T7137s[i];
  p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N7132:=Stabiliser(N713,2);
T7132:=Transversal(N,N7132);
for i in [1..#T7132] do
  ss:=[7,1,3,2]^T7132[i];
  p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N712:=Stabiliser(N71,2);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 then
  N71727s:=sub<N|N712,g>; end if; end for;
T71727s:=Transversal(N,N71727s);
for i in [1..#T71727s] do
  ss:=[7,1,7,2,7]^T71727s[i];
  p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 3^g eq 3 then
  N71723s:=sub<N|N7123,g>; end if; end for;
T71723s:=Transversal(N,N71723s);
for i in [1..#T71723s] do
  ss:=[7,1,7,2,3]^T71723s[i];
  p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N7124:=Stabiliser(N712,4);
T71724:=Transversal(N,N7124);
for i in [1..#T71724] do
  ss:=[7,1,7,2,4]^T71724[i];
  p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N7125:=Stabiliser(N712,5);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 5^g eq 5 then
N71725s:=sub<N|N7125,g>; end if; end for;
T71725s:=Transversal(N,N71725s);
for i in [1..#T71725s] do
ss:=[7,1,7,2,5]*T71725s[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;

N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 3^g eq 3 then
N71732s:=sub<N|N7123,g>; end if; end for;
T71732s:=Transversal(N,N71732s);
for i in [1..#T71732s] do
ss:=[7,1,7,3,2]*T71732s[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;

N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 3^g eq 3 then
N71273s:=sub<N|N7123,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 3^g eq 3 then
N71275s:=sub<N|N7125,g>; end if; end for;
T71275s:=Transversal(N,N71275s);
for i in [1..#T71275s] do
ss:=[7,1,2,7,5]*T71275s[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;

N7124:=Stabiliser(N712,4);
T71274:=Transversal(N,N7124);
for i in [1..#T71274] do
ss:=[7,1,2,7,4]*T71274[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;

N7125:=Stabiliser(N712,5);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 5^g eq 5 then
N71275s:=sub<N|N7125,g>; end if; end for;
T71275s:=Transversal(N,N71275s);
for i in [1..#T71275s] do
ss:=[7,1,2,7,5]*T71275s[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;

N712:=Stabiliser(N71,2);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 then
N71217s:=sub<N|N712,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 7 and 2^g eq 1 then
N71217s:=sub<N|N71217s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 2 and 2^g eq 7 then
N71217s:=sub<N|N71217s,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 1 and 2^g eq 7 then
N71217s:=sub<N|N71217s,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 2 and 2^g eq 1 then
N71217s:=sub<N|N71217s,g>; end if; end for;
T71217s:=Transversal(N,N71217s);
for i in [1..#T71217s] do
ss:=[7,1,2,1,7]~T71217s[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^x); end for;
N7123:=Stabiliser(N712,3);
T71213:=Transversal(N,N7123);
for i in [1..#T71213] do
ss:=[7,1,2,1,3]~T71213[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^x); end for;
N7125:=Stabiliser(N712,5);
for g in N do if 7^g eq 7 and 1^g eq 2 and 2^g eq 1 and 5^g eq 5 then
N71215s:=sub<N|N7125,g>; end if; end for;
T71215s:=Transversal(N,N71215s);
for i in [1..#T71215s] do
ss:=[7,1,2,1,5]~T71215s[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^x); end for;
N7123:=Stabiliser(N712,3);
T71237:=Transversal(N,N7123);
for i in [1..#T71237] do
ss:=[7,1,2,3,7]~T71237[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^x); end for;
N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 5 and 1^g eq 2 and 2^g eq 1 and 3^g eq 3 then
N71231s:=sub<N|N7123,g>; end if; end for;
T71231s:=Transversal(N,N71231s);
for i in [1..#T71231s] do
ss:=[7,1,2,3,1]~T71231s[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^x); end for;
N7123:=Stabiliser(N712,3);
T71232:=Transversal(N,N7123);
for i in [1..#T71232] do
ss:=[7,1,2,3,2]~T71232[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^x); end for;
N71234:=Stabiliser(N7123,4);
T71234:=Transversal(N,N71234);
for i in [1..#T71234] do
ss:=[7,1,2,3,4]~T71234[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss~xx);
end for;

N71235:=Stabiliser(N7123,5);
T71235:=Transversal(N,N71235);
for i in [1..#T71235] do
ss:=[7,1,2,3,5]~T71235[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss~xx);
end for;

N7124:=Stabiliser(N712,4);
T71247:=Transversal(N,N7124);
for i in [1..#T71247] do
ss:=[7,1,2,4,7]~T71247[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss~xx);
end for;

N7124:=Stabiliser(N712,4);
for g in N do if 7~g eq 7 and 1~g eq 4 and 2~g eq 1 and 4~g eq 2 then
N71241s:=sub<N|N7124,g>; end if; end for;
for g in N do if 7~g eq 7 and 1~g eq 2 and 2~g eq 4 and 4~g eq 1 then
N71241s:=sub<N|N71241s,g>; end if; end for;
T71241s:=Transversal(N,N71241s);
for i in [1..#T71241s] do
ss:=[7,1,2,4,1]~T71241s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss~xx);
end for;

N71234:=Stabiliser(N7123,4);
T71243:=Transversal(N,N71234);
for i in [1..#T71243] do
ss:=[7,1,2,4,3]~T71243[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss~xx);
end for;

N71236:=Stabiliser(N7123,6);
T71246:=Transversal(N,N71236);
for i in [1..#T71246] do
ss:=[7,1,2,4,6]~T71246[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss~xx);
end for;

N7125:=Stabiliser(N712,5);
for g in N do if 7~g eq 1 and 1~g eq 7 and 2~g eq 5 and 5~g eq 2 then
N71257s:=sub<N|N7125,g>; end if; end for;
T71257s:=Transversal(N,N71257s);
for i in [1..#T71257s] do
ss:=[7,1,2,5,7]~T71257s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7125 := Stabiliser(N712, 5);
for g in N do if 7^-g eq 7 and 1^-g eq 1 and 2^-g eq 5 and 5^-g eq 2 then
N7125s := sub<N|N7125, g>; end if; end for;
T7125s := Transversal(N, N7125s);
for i in [1..#T7125s] do
ss := [7, 1, 2, 5, 2]^-T7125s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7126 := Stabiliser(N712, 6);
for g in N do if 7^-g eq 4 and 1^-g eq 6 and 2^-g eq 2 and 6^-g eq 1 then
N7126s := sub<N|N7126, g>; end if; end for;
T7126s := Transversal(N, N7126s);
for i in [1..#T7126s] do
ss := [7, 1, 2, 6, 7]^-T7126s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

T71261 := Transversal(N, N7126);
for i in [1..#T71261] do
ss := [7, 1, 2, 6, 1]^-T71261[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7126 := Stabiliser(N712, 6);
for g in N do if 7^-g eq 7 and 1^-g eq 1 and 2^-g eq 6 and 6^-g eq 2 then
N7126s := sub<N|N7126, g>; end if; end for;
T7126s := Transversal(N, N7126s);
for i in [1..#T7126s] do
ss := [7, 1, 2, 6, 2]^-T7126s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7123 := Stabiliser(N712, 3);
for g in N do if 7^-g eq 3 and 1^-g eq 7 and 3^-g eq 1 and 2^-g eq 2 then
N7137s := sub<N|N7123, g>; end if; end for;
for g in N do if 7^-g eq 1 and 1^-g eq 3 and 3^-g eq 0 and 2^-g eq 2 then
N7137s := sub<N|N7137s, g>; end if; end for;
T7137s := Transversal(N, N7137s);
for i in [1..#T7137s] do
ss := [7, 1, 3, 7, 2]^-T7137s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7123 := Stabiliser(N712, 3);
for g in N do if 7^-g eq 6 and 1^-g eq 1 and 3^-g eq 5 and 2^-g eq 2 then
N71327s := sub<N|N7123, g>; end if; end for;
T71327s := Transversal(N, N71327s);
for i in [1..#T71327s] do
    ss := [7, 1, 3, 2, 7] * T71327s[i];
    p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7123 := Stabiliser(N712, 3);
for g in N do if 7^g eq 1 and 1^g eq 7 and 3^g eq 3 and 2^g eq 2 then
    N717273s := sub<N|N7123, g>; end if; end for;
T717273s := Transversal(N, N717273s);
for i in [1..#T717273s] do
    ss := [7, 1, 7, 2, 7, 3] * T717273s[i];
    p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7124 := Stabiliser(N712, 4);
T717274 := Transversal(N, N7124);
for i in [1..#T717274] do
    ss := [7, 1, 7, 2, 7, 4] * T717274[i];
    p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7125 := Stabiliser(N712, 5);
for g in N do if 7^g eq 1 and 1^g eq 7 and 5^g eq 5 and 2^g eq 2 then
    N717275s := sub<N|N7123, g>; end if; end for;
T717275s := Transversal(N, N717275s);
for i in [1..#T717275s] do
    ss := [7, 1, 7, 2, 7, 5] * T717275s[i];
    p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7123 := Stabiliser(N712, 3);
T717273 := Transversal(N, N7123);
for i in [1..#T717273] do
    ss := [7, 1, 7, 2, 3, 7] * T717273[i];
    p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7123 := Stabiliser(N712, 3);
for g in N do if 7^g eq 1 and 1^g eq 7 and 3^g eq 3 and 2^g eq 2 then
    N717232s := sub<N|N7123, g>; end if; end for;
T717232s := Transversal(N, N717232s);
for i in [1..#T717232s] do
    ss := [7, 1, 7, 2, 3, 2] * T717232s[i];
    p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N71234 := Stabiliser(N7123, 4);
T717234 := Transversal(N, N71234);
for i in [1..#T717234] do
    ss:=[7,1,7,2,3,4]~T717234[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss~xx);
end for;
N71235:=Stabiliser(N7123,5);
for g in N do if 7~g eq 1 and 1~g eq 7 and 5~g eq 5 and 3~g eq 3 and 2~g eq 2 then N717235s:=sub<N|N71235,g>; end if; end for;
T717235s:=Transversal(N,N717235s);
for i in [1..#T717235s] do
    ss:=[7,1,7,2,3,5]~T717235s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss~xx);
end for;

N7124:=Stabiliser(N712,4);
T717247:=Transversal(N,N717247);
for i in [1..#T717247] do
    ss:=[7,1,7,2,4,7]~T717247[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss~xx);
end for;
N7124:=Stabiliser(N712,4);
for g in N do if 7~g eq 4 and 1~g eq 6 and 4~g eq 7 and 2~g eq 2 then N717241s:=sub<N|N7124,g>; end if; end for;
T717241s:=Transversal(N,N717241s);
for i in [1..#T717241s] do
    ss:=[7,1,7,2,4,1]~T717241s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss~xx);
end for;
N7124:=Stabiliser(N712,4);
for g in N do if 7~g eq 1 and 1~g eq 7 and 5~g eq 5 and 3~g eq 3 and 2~g eq 2 then N717243s:=sub<N|N71234,g>; end if; end for;
T717243s:=Transversal(N,N717243s);
for i in [1..#T717243s] do
    ss:=[7,1,7,2,4,3]~T717243s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss~xx);
end for;
N7125:=Stabiliser(N712,5);
T717257:=Transversal(N,N7125);
for i in [1..#T717257] do
  ss:=[7,1,7,2,5,7]^T717257[i];
  p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N7125:=Stabiliser(N712,5);
for g in N do if 7^g eq 2 and 1^g eq 5 and 5^g eq 1 and 2^g eq 7 then
  N717252s:=sub<N|N7125,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 7 and 5^g eq 5 and 2^g eq 2 then
  N717252s:=sub<N|N717252s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 7 and 5^g eq 2 and 2^g eq 5 then
  N717252s:=sub<N|N717252s,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 2 and 5^g eq 1 and 2^g eq 7 then
  N717252s:=sub<N|N717252s,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 1 and 5^g eq 2 and 2^g eq 5 then
  N717252s:=sub<N|N717252s,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 5^g eq 7 and 2^g eq 1 then
  N717252s:=sub<N|N717252s,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 2 and 5^g eq 7 and 2^g eq 1 then
  N717252s:=sub<N|N717252s,g>; end if; end for;
T717252s:=Transversal(N,N717252s);
for i in [1..#T717252s] do
  ss:=[7,1,7,2,5,4]^T717252s[i];
  p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N71245:=Stabiliser(N7124,5);
T717254:=Transversal(N,N71245);
for i in [1..#T717254] do
  ss:=[7,1,7,2,5,2]^T717254[i];
  p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N7123:=Stabiliser(N712,3);
T717327:=Transversal(N,N7123);
for i in [1..#T717327] do
  ss:=[7,1,7,3,2,7]^T717327[i];
  p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N71235:=Stabiliser(N7123,5);
for g in N do if 7^g eq 1 and 1^g eq 7 and 3^g eq 3 and 5^g eq 5 and
  2^g eq 2 then N717325s:=sub<N|N71235,g>; end if; end for;
T717325s:=Transversal(N,N717325s);
for i in [1..#T717325s] do
  ss:=[7,1,7,3,2,5]^T717325s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7123:=Stabiliser(N712,3);
T712737:=Transversal(N,N7123);
for i in [1..#T712737] do
ss:=[7,1,2,7,3,7]^T712737[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 1 and 1^g eq 7 and 3^g eq 3 and 2^g eq 2 then
N71232s:=sub<N|N7123,g>; end if; end for;
T712732:=Transversal(N,N71232s);
for i in [1..#T712732] do
ss:=[7,1,2,7,3,2]^T712732[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7123:=Stabiliser(N7123,4);
T712734:=Transversal(N,N71234);
for i in [1..#T712734] do
ss:=[7,1,2,7,3,4]^T712734[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7123:=Stabiliser(N7123,5);
for g in N do if 7^g eq 1 and 1^g eq 7 and 3^g eq 3 and 2^g eq 2 and
5^g eq 5 then N712735s:=sub<N|N71235,g>; end if; end for;
T712735s:=Transversal(N,N712735s);
for i in [1..#T712735s] do
ss:=[7,1,2,7,3,5]^T712735s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7123:=Stabiliser(N7123,4);
T712743:=Transversal(N,N71234);
for i in [1..#T712743] do
ss:=[7,1,2,7,4,3]^T712743[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N71245:=Stabiliser(N7124,5);
for g in N do if 7^g eq 2 and 1^g eq 1 and 4^g eq 6 and 2^g eq 5 and
5^g eq 7 then N712745s:=sub<N|N71245,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 1 and 4^g eq 3 and 2^g eq 7 and
5^g eq 2 then N712745s:=sub<N|N712745s,g>; end if; end for;
T712745s:=Transversal(N,N712745s);
for i in [1..#T712745s] do
ss:=[7,1,2,7,4,5]^T712745s[i];
\[
p(\text{prodim}(1, ts, ss)) := \text{prodim}(1, ts, ss^{\text{xx}});
\]
end for;

\[
N71246 := \text{Stabiliser}(N7124, 6);
T712746 := \text{Transversal}(N, N71246);
\]
for i in [1..#T712746] do
ss := [7, 1, 2, 7, 4, 6] \cdot T712746[i];
\[
p(\text{prodim}(1, ts, ss)) := \text{prodim}(1, ts, ss^{\text{xx}});
\]
end for;

\[
N7125 := \text{Stabiliser}(N712, 5);
\]
for g in N do if 7 \cdot g \equiv 2 \text{ and } 1 \cdot g \equiv 1 \text{ and } 2 \cdot g \equiv 7 \text{ and } 5 \cdot g \equiv 5 \text{ then }
\[
N712752s := \text{sub}(N|N7125, g);
\]
end if; end for;

\[
N712752 := \text{Transversal}(N, N712752);
T712752 := \text{Transversal}(N, N712752);
\]
for i in [1..#T712752s] do
ss := [7, 1, 2, 7, 5, 2] \cdot T712752s[i];
\[
p(\text{prodim}(1, ts, ss)) := \text{prodim}(1, ts, ss^{\text{xx}});
\]
end for;

\[
N7123 := \text{Stabiliser}(N712, 3);
\]
for g in N do if 7 \cdot g \equiv 1 \text{ and } 1 \cdot g \equiv 7 \text{ and } 2 \cdot g \equiv 2 \text{ and } 3 \cdot g \equiv 3 \text{ then }
\[
N712173s := \text{sub}(N|N7123, g);
\]
end if; end for;

\[
T712173s := \text{Transversal}(N, N712173);
\]
for i in [1..#T712173s] do
ss := [7, 1, 2, 1, 7, 3] \cdot T712173s[i];
\[
p(\text{prodim}(1, ts, ss)) := \text{prodim}(1, ts, ss^{\text{xx}});
\]
end for;

\[
N7125 := \text{Stabiliser}(N712, 5);
\]
for g in N do if 7 \cdot g \equiv 2 \text{ and } 1 \cdot g \equiv 7 \text{ and } 2 \cdot g \equiv 1 \text{ and } 5 \cdot g \equiv 5 \text{ then }
\[
N712175s := \text{sub}(N|N7125, g);
\]
end if; end for;

\[
T712175s := \text{Transversal}(N, N712175);
\]
for i in [1..#T712175s] do
ss:=[7,1,2,1,7,5]~T712175s[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N71234:=Stabiliser(N7123,4);
T712134:=Transversal(N,N71234);
for i in [1..#T712134] do
ss:=[7,1,2,1,3,4]~T712134[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N71235:=Stabiliser(N7123,5);
T712135:=Transversal(N,N71235);
for i in [1..#T712135] do
ss:=[7,1,2,1,3,6]~T712135[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N7125:=Stabiliser(N7125,4);
T712374:=Transversal(N,N71234);
for i in [1..#T712374] do
ss:=[7,1,2,3,7,4]~T712374[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 7 and 1^g eq 2 and 2^g eq 1 and 5^g eq 5 then
N712157s:=sub<N|N7125,g>; end if; end for;
T712157s:=Transversal(N,N712157s);
for i in [1..#T712157s] do
ss:=[7,1,2,1,5,7]~T712157s[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N71234:=Stabiliser(N7123,4);
T712134:=Transversal(N,N71234);
for i in [1..#T712134] do
ss:=[7,1,2,1,3,4]~T712134[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N71235:=Stabiliser(N7123,5);
T712135:=Transversal(N,N71235);
for i in [1..#T712135] do
ss:=[7,1,2,1,3,6]~T712135[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N7125:=Stabiliser(N7125,4);
T712374:=Transversal(N,N71234);
for i in [1..#T712374] do
ss:=[7,1,2,3,7,4]~T712374[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N7123:=Stabiliser(N712,3);
for g in N do if 7^g eq 5 and 1^g eq 2 and 2^g eq 1 and 3^g eq 3 then
N712321s:=sub<N|N7123,g>; end if; end for;
T712321s:=Transversal(N,N712321s);
for i in [1..#T712321s] do
ss:=[7,1,2,3,2,1]~T712321s[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N712345:=Stabiliser(N71234,5); for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 4 and 3^g eq 2 and 5^g eq 1 and 4^g eq 3 then N712354s:=sub<N|N712345,g>; end if; end for; for g in N do if 7^g eq 7 and 1^g eq 5 and 2^g eq 3 and 3^g eq 4 and 5^g eq 6 and 4^g eq 2 then N712354s:=sub<N|N712354s,g>; end if; end for; T712354s:=Transversal(N,N712354s);
for i in [1. #T712354s] do ss:=[7,1,2,3,5,4]~T712354s[i]; p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx); end for;

N71234:=Stabiliser(N7123,4); for g in N do if 7^g eq 2 and 2^g eq 6 and 4^g eq 5 and 7^g eq 2 and 3^g eq 4 then N712473s:=sub<N|N71234,g>; end if; end for; for g in N do if 7^g eq 6 and 2^g eq 7 and 4^g eq 3 and 7^g eq 6 and 3^g eq 5 then N712473s:=sub<N|N712473s,g>; end if; end for; T712473s:=Transversal(N,N712473s);
for i in [1. #T712473s] do ss:=[7,1,2,4,7,3]~T712473s[i]; p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx); end for;

N71256:=Stabiliser(N7125,6); T712615:=Transversal(N,N71256);
for i in [1. #T712615] do ss:=[7,1,2,6,1,5]~T712615[i]; p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx); end for;

N7124:=Stabiliser(N7124,6);
for g in N do if 7^g eq 6 and 1^g eq 4 and 2^g eq 2 and 4^g eq 1 then N7172747s:=sub<N|N7124,g>; end if; end for;
T7172747s:=Transversal(N,N7172747s);
for i in [1. #T7172747s] do ss:=[7,1,7,2,7,4,7]~T7172747s[i]; p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx); end for;

N71234:=Stabiliser(N7123,4); T7172743:=Transversal(N,N71234);
for i in [1. #T7172743] do ss:=[7,1,7,2,7,4,3]~T7172743[i]; p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx); end for;

N71246:=Stabiliser(N7124,6); T7172746:=Transversal(N,N71246);
for i in [1. #T7172746] do ss:=[7,1,7,2,7,4,6]~T7172746[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7125:=Stabiliser(N712,5);
T712757:=Transversal(N,N7125);
for i in [1..#T712757] do
ss:=[7,1,7,2,7,5,7]^T712757[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N7125:=Stabiliser(N712,5);
for g in N do if 7^g eq 2 and 1^g eq 7 and 2^g eq 1 and 5^g eq 5 then
N712752s:=sub<N|N7125,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 2 and 2^g eq 7 and 5^g eq 5 then
N712752s:=sub<N|N712752s,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 1 and 2^g eq 7 and 5^g eq 5 then
N712752s:=sub<N|N712752s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 5^g eq 5 then
N712752s:=sub<N|N712752s,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 2 and 2^g eq 1 and 5^g eq 5 then
N712752s:=sub<N|N712752s,g>; end if; end for;
T712752s:=Transversal(N,N712752s);
for i in [1..#T712752s] do
ss:=[7,1,7,2,7,5,2]^T712752s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N71234:=Stabiliser(N7123,4);
T712374:=Transversal(N,N71234);
for i in [1..#T712374] do
ss:=[7,1,7,2,3,7,4]^T712374[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N71234:=Stabiliser(N7123,4);
T712324:=Transversal(N,N71234);
for i in [1..#T712324] do
ss:=[7,1,7,2,3,2,4]^T712324[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;

N712346:=Stabiliser(N71234,6);
for g in N do if 7^g eq 7 and 1^g eq 2 and 2^g eq 1 and 3^g eq 6 and
and 6^g eq 3 then N712346s:=sub<N|N712346,g>; end if; end for;
T712346s:=Transversal(N,N712346s);
for i in [1..#T712346s] do
ss:=[7,1,7,2,3,4,6]^T712346s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N71246 := Stabiliser(N7124, 6);
for g in N do if $7^g \text{eq } 7$ and $1^g \text{eq } 6$ and $2^g \text{eq } 5$ and $4^g \text{eq } 1$ and $6^g \text{eq } 4$ then $N7172476s := \text{sub}<N|N71246, g>$; end if; end for;
for g in N do if $7^g \text{eq } 7$ and $1^g \text{eq } 4$ and $2^g \text{eq } 3$ and $4^g \text{eq } 6$ and $6^g \text{eq } 1$ then $N7172476s := \text{sub}<N|N7172476s, g>$; end if; end for;
T7172476s := \text{Transversal}(N, N7172476s);
for i in [1..#T7172476s] do
  ss := [7, 1, 7, 2, 4, 7, 6] \text{~T7172476s}[i];
  p[\text{prodim}(1, ts, ss)] := \text{prodim}(1, ts, ss\text{xx});
end for;
N71245 := Stabiliser(N7124, 5);
for g in N do if $7^g \text{eq } 6$ and $1^g \text{eq } 1$ and $2^g \text{eq } 3$ and $4^g \text{eq } 7$ and $5^g \text{eq } 2$ then $N7172425s := \text{sub}<N|N71245, g>$; end if; end for;
for g in N do if $7^g \text{eq } 4$ and $1^g \text{eq } 5$ and $2^g \text{eq } 5$ and $4^g \text{eq } 6$ and $5^g \text{eq } 3$ then $N7172425s := \text{sub}<N|N7172425s, g>$; end if; end for;
T7172425s := \text{Transversal}(N, N7172425s);
for i in [1..#T7172425s] do
  ss := [7, 1, 7, 2, 4, 2, 5] \text{~T7172425s}[i];
  p[\text{prodim}(1, ts, ss)] := \text{prodim}(1, ts, ss\text{xx});
end for;
N712345 := Stabiliser(N7124, 5);
for g in N do if $7^g \text{eq } 1$ and $1^g \text{eq } 5$ and $2^g \text{eq } 4$ and $4^g \text{eq } 7$ and $3^g \text{eq } 6$ and $5^g \text{eq } 3$ then $N7172435s := \text{sub}<N|N712345, g>$; end if; end for;
for g in N do if $7^g \text{eq } 4$ and $1^g \text{eq } 1$ and $2^g \text{eq } 5$ and $4^g \text{eq } 6$ and $3^g \text{eq } 2$ and $5^g \text{eq } 3$ then $N7172435s := \text{sub}<N|N7172435s, g>$; end if; end for;
for g in N do if $7^g \text{eq } 5$ and $1^g \text{eq } 4$ and $2^g \text{eq } 1$ and $4^g \text{eq } 2$ and $3^g \text{eq } 7$ and $5^g \text{eq } 3$ then $N7172435s := \text{sub}<N|N7172435s, g>$; end if; end for;
for g in N do if $7^g \text{eq } 7$ and $1^g \text{eq } 5$ and $2^g \text{eq } 3$ and $4^g \text{eq } 2$ and $3^g \text{eq } 4$ and $5^g \text{eq } 6$ then $N7172435s := \text{sub}<N|N7172435s, g>$; end if; end for;
for g in N do if $7^g \text{eq } 2$ and $1^g \text{eq } 5$ and $2^g \text{eq } 6$ and $4^g \text{eq } 1$ and $3^g \text{eq } 3$ and $5^g \text{eq } 4$ then $N7172435s := \text{sub}<N|N7172435s, g>$; end if; end for;
for g in N do if $7^g \text{eq } 6$ and $2^g \text{eq } 2$ and $4^g \text{eq } 7$ and $3^g \text{eq } 1$ and $5^g \text{eq } 4$ then $N7172435s := \text{sub}<N|N7172435s, g>$; end if; end for;
for g in N do if $7^g \text{eq } 3$ and $1^g \text{eq } 7$ and $2^g \text{eq } 5$ and $4^g \text{eq } 4$ and $3^g \text{eq } 1$ and $5^g \text{eq } 6$ then $N7172435s := \text{sub}<N|N7172435s, g>$; end if; end for;
for g in N do if $7^g \text{eq } 2$ and $1^g \text{eq } 4$ and $2^g \text{eq } 3$ and $4^g \text{eq } 6$ and $3^g \text{eq } 1$ and $5^g \text{eq } 7$ then $N7172435s := \text{sub}<N|N7172435s, g>$;
end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 4 and 2^g eq 7 and 4^g eq 5 and
3^g eq 3 and 5^g eq 1 then N7172435s:=sub<N|N7172435s,g>;
end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 2 and 2^g eq 7 and 4^g eq 6 and
3^g eq 4 and 5^g eq 5 then N7172435s:=sub<N|N7172435s,g>;
end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 1 and 2^g eq 3 and 4^g eq 7 and
3^g eq 5 and 5^g eq 2 then N7172435s:=sub<N|N7172435s,g>;
end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 7 and 2^g eq 6 and 4^g eq 2 and
3^g eq 5 and 5^g eq 1 then N7172435s:=sub<N|N7172435s,g>;
end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 2 and 2^g eq 5 and 4^g eq 3 and
3^g eq 7 and 5^g eq 4 then N7172435s:=sub<N|N7172435s,g>;
end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 3 and 2^g eq 7 and 4^g eq 1 and
3^g eq 2 and 5^g eq 6 then N7172435s:=sub<N|N7172435s,g>;
end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 6 and 2^g eq 1 and 4^g eq 5 and
3^g eq 4 and 5^g eq 2 then N7172435s:=sub<N|N7172435s,g>;
end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 4 and 4^g eq 3 and
3^g eq 2 and 5^g eq 1 then N7172435s:=sub<N|N7172435s,g>;
end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 3 and 2^g eq 6 and 4^g eq 4 and
3^g eq 7 and 5^g eq 2 then N7172435s:=sub<N|N7172435s,g>;
end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 3 and 2^g eq 2 and 4^g eq 5 and
3^g eq 6 and 5^g eq 7 then N7172435s:=sub<N|N7172435s,g>;
end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 2 and 2^g eq 4 and 4^g eq 1 and
3^g eq 5 and 5^g eq 7 then N7172435s:=sub<N|N7172435s,g>;
end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 7 and 2^g eq 1 and 4^g eq 3 and
3^g eq 6 and 5^g eq 5 then N7172435s:=sub<N|N7172435s,g>;
end if; end for;
T7172435s:=Transversal(N,N7172435s);
for i in [1..#T7172435s] do
ss:=[7,1,7,2,4,3,5]~T7172435s[i];
p[prodim(1, ss, ss)]:= prodim(1, ts, ss^xx);
end for;
N71235:=Stabiliser(N71235,5);
T7172573:=Transversal(N,N71235);
for i in [1..#T7172573] do
ss:= [7,1,7,2,5,7,3]^T7172573[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N71245:= Stabiliser(N7124,5);
T7172574:= Transversal(N,N71245);
for i in [1..#T7172574] do
ss:= [7,1,7,2,5,7,6]^T7172574[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N71256:= Stabiliser(N7125,6);
for g in N do if 7^g eq 7 and 1^g eq 4 and 2^g eq 1 and 5^g eq 6 and
6^g eq 3 then N7172576s:= sub<N|N71256,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 2 and 2^g eq 4 and 5^g eq 3 and
6^g eq 5 then N7172576s:= sub<N|N71256,g>; end if; end for;
T7172576s:= Transversal(N,N712576s);
for i in [1..#T7172576s] do
ss:= [7,1,7,2,5,7,6]^T7172576s[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N71235:= Stabiliser(N7123,5);
for g in N do if 7^g eq 7 and 1^g eq 1 and 2^g eq 5 and 5^g eq 2 and
3^g eq 3 then N7172523s:= sub<N|N71235,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 5^g eq 5 and
3^g eq 3 then N7172523s:= sub<N|N71235,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 5 and 5^g eq 2 and
3^g eq 3 then N7172523s:= sub<N|N71235,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 2 and 2^g eq 1 and 5^g eq 7 and
3^g eq 3 then N7172523s:= sub<N|N71235,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 2^g eq 7 and 5^g eq 1 and
3^g eq 3 then N7172523s:= sub<N|N71235,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 2 and 2^g eq 7 and 5^g eq 1 and
3^g eq 3 then N7172523s:= sub<N|N71235,g>; end if; end for;
T7172523s:= Transversal(N,N7172523s);
for i in [1..#T7172523s] do
ss:= [7,1,7,2,5,2,3]^T7172523s[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss^xx);
end for;
N71245:= Stabiliser(N7124,5);
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 5 and 5^g eq 2 and
4^g eq 4 then N7172524s:= sub<N|N71245,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 2^g eq 7 and 5^g eq 1 and
4^g \text{ eq } 4 \text{ then } N7172524s:=\text{sub}<N|N7172524s,g> \text{; end if; end for;}
for \ g \text{ in } N \text{ do if } 7^g \text{ eq } 5 \text{ and } 1^g \text{ eq } 2 \text{ and } 2^g \text{ eq } 1 \text{ and } 5^g \text{ eq } 7 \text{ and }
4^g \text{ eq } 4 \text{ then } N7172524s:=\text{sub}<N|N7172524s,g> \text{; end if; end for;}
T7172524s:=\text{Transversal}(N,N7172524s);
for \ i \text{ in } [1..#T7172524s] \text{ do }
ss:=[7,1,7,2,5,2,4] ^T7172524s[i];
p[\text{prodim}(1, ts, ss)]:=\text{prodim}(1, ts, ss \cdot xx); 
end for;
N71245:=\text{Stabiliser}(N7124,5);
for \ g \text{ in } N \text{ do if } 7^g \text{ eq } 5 \text{ and } 1^g \text{ eq } 2 \text{ and } 2^g \text{ eq } 4 \text{ and } 5^g \text{ eq } 6 \text{ and } 4^g \text{ eq } 1 \text{ then } N7172547s:=\text{sub}<N|N71245,g> \text{; end if; end for;}
for \ g \text{ in } N \text{ do if } 7^g \text{ eq } 6 \text{ and } 1^g \text{ eq } 4 \text{ and } 2^g \text{ eq } 1 \text{ and } 5^g \text{ eq } 7 \text{ and } 4^g \text{ eq } 2 \text{ then } N7172547s:=\text{sub}<N|N7172547s,g> \text{; end if; end for;}
T7172547s:=\text{Transversal}(N,N7172547s);
for \ i \text{ in } [1..#T7172547s] \text{ do }
ss:=[7,1,7,2,5,4,7] ^T7172547s[i];
p[\text{prodim}(1, ts, ss)]:=\text{prodim}(1, ts, ss \cdot xx); 
end for;
N71234:=\text{Stabiliser}(N7123,4);
for \ g \text{ in } N \text{ do if } 7^g \text{ eq } 2 \text{ and } 1^g \text{ eq } 5 \text{ and } 3^g \text{ eq } 3 \text{ and } 2^g \text{ eq } 6 \text{ and } 4^g \text{ eq } 1 \text{ then } N7173274s:=\text{sub}<N|N71234,g> \text{; end if; end for;}
for \ g \text{ in } N \text{ do if } 7^g \text{ eq } 6 \text{ and } 1^g \text{ eq } 3 \text{ and } 2^g \text{ eq } 7 \text{ and } 4^g \text{ eq } 5 \text{ then } N7173274s:=\text{sub}<N|N7173274s,g> \text{; end if; end for;}
T7173274s:=\text{Transversal}(N,N7173274s);
for \ i \text{ in } [1..#T7173274s] \text{ do }
ss:=[7,1,7,3,2,7,4] ^T7173274s[i];
p[\text{prodim}(1, ts, ss)]:=\text{prodim}(1, ts, ss \cdot xx); 
end for;
N71234:=\text{Stabiliser}(N7123,4);
for \ g \text{ in } N \text{ do if } 7^g \text{ eq } 6 \text{ and } 1^g \text{ eq } 4 \text{ and } 2^g \text{ eq } 1 \text{ and } 3^g \text{ eq } 3 \text{ and } 4^g \text{ eq } 2 \text{ then } N7127341s:=\text{sub}<N|N71234,g> \text{; end if; end for;}
for \ g \text{ in } N \text{ do if } 7^g \text{ eq } 5 \text{ and } 1^g \text{ eq } 2 \text{ and } 2^g \text{ eq } 4 \text{ and } 3^g \text{ eq } 3 \text{ and } 4^g \text{ eq } 1 \text{ then } N7127341s:=\text{sub}<N|N7127341s,g> \text{; end if; end for;}
T7127341s:=\text{Transversal}(N,N7127341s);
for \ i \text{ in } [1..#T7127341s] \text{ do }
ss:=[7,1,2,7,3,4,1] ^T7127341s[i];
p[\text{prodim}(1, ts, ss)]:=\text{prodim}(1, ts, ss \cdot xx); 
end for;
N71234:=Stabiliser(N7123,4);
for g in N do if 7^g eq 2 and 1^g eq 5 and 2^g eq 4 and 4^g eq 7 and 3^g eq 3 then N71727431s:=sub<N\N71234,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 6 and 2^g eq 7 and 4^g eq 2 and 3^g eq 3 then N71727431s:=sub<N\N71727431s,g>; end if; end for;
T71727431s:=Transversal(N,N71727431s);
for i in [1..#T71727431s] do
ss:=[7,1,7,2,7,4,3,1]^T71727431s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N71234:=Stabiliser(N7123,4);
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 5 and 4^g eq 1 and 3^g eq 2 then N71727432s:=sub<N\N71234,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 4 and 2^g eq 3 and 4^g eq 6 and 3^g eq 5 then N71727432s:=sub<N\N71727432s,g>; end if; end for;
T71727432s:=Transversal(N,N71727432s);
for i in [1..#T71727432s] do
ss:=[7,1,7,2,7,4,3,2]^T71727432s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N71246:=Stabiliser(N7124,6);
for g in N do if 7^g eq 6 and 1^g eq 4 and 2^g eq 7 and 4^g eq 5 and 6^g eq 2 then N71727462s:=sub<N\N71246,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 2^g eq 6 and 4^g eq 1 and 6^g eq 7 then N71727462s:=sub<N\N71727462s,g>; end if; end for;
T71727462s:=Transversal(N,N71727462s);
for i in [1..#T71727462s] do
ss:=[7,1,7,2,7,4,6,2]^T71727462s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N7125:=Stabiliser(N712,5);
for g in N do if 7^g eq 7 and 1^g eq 3 and 2^g eq 2 and 5^g eq 4 then N71727571s:=sub<N\N7125,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 4 and 2^g eq 2 and 5^g eq 3 then N71727571s:=sub<N\N71727571s,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 5 and 2^g eq 2 and 5^g eq 1 then N71727571s:=sub<N\N71727571s,g>; end if; end for;
T71727571s:=Transversal(N,N71727571s);
for i in [1..#T71727571s] do
ss:=[7,1,7,2,7,5,7,1]^T71727571s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss^xx);
end for;
N71234:=Stabiliser(N7123,4);
for g in N do if 7^g eq 5 and 1^g eq 3 and 2^g eq 1 and 3^g eq 2 and
4^g eq 7 then N71723747s:=sub<N|N71234,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 2 and 2^g eq 3 and 3^g eq 1 and
4^g eq 5 then N71723747s:=sub<N|N71723747s,g>; end if; end for;
T71723747s:=Transversal(N,N71723747s);
for i in [1..#T71723747s] do
  ss:=[7,1,7,2,3,7,4,7]"T71723747s[i];
p[prodim(1, ts, ss)]:=prodim(1, ts, ss"xx);
end for;
N71245:=Stabiliser(N71245,5);
for g in N do if 7^g eq 7 and 1^g eq 4 and 2^g eq 3 and 4^g eq 6 and
5^g eq 2 then N71724251s:=sub<N|N71245,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 6 and 2^g eq 3 and 4^g eq 4 and
5^g eq 2 then N71724251s:=sub<N|N71724251s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 7 and 2^g eq 2 and 4^g eq 6 and
5^g eq 5 then N71724251s:=sub<N|N71724251s,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 1 and 2^g eq 3 and 4^g eq 7 and
5^g eq 2 then N71724251s:=sub<N|N71724251s,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 7 and 2^g eq 3 and 4^g eq 1 and
5^g eq 2 then N71724251s:=sub<N|N71724251s,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 6 and 2^g eq 2 and 4^g eq 7 and
5^g eq 5 then N71724251s:=sub<N|N71724251s,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 1 and 2^g eq 5 and 4^g eq 6 and
5^g eq 3 then N71724251s:=sub<N|N71724251s,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 6 and 2^g eq 5 and 4^g eq 1 and
5^g eq 3 then N71724251s:=sub<N|N71724251s,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 4 and 2^g eq 1 and 4^g eq 6 and
5^g eq 5 then N71724251s:=sub<N|N71724251s,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 7 and 2^g eq 5 and 4^g eq 4 and
5^g eq 3 then N71724251s:=sub<N|N71724251s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 4 and 2^g eq 5 and 4^g eq 7 and
5^g eq 3 then N71724251s:=sub<N|N71724251s,g>; end if; end for;
T71724251s:=Transversal(N,N71724251s);
for i in [1..#T71724251s] do
  ss:=[7,1,7,2,4,2,5,1]"T71724251s[i];
p[prodim(1, ts, ss)]:=prodim(1, ts, ss"xx);
end for;
N712346:=Stabiliser(N71234,6);
for g in N do if and 1^g eq 2 and 2^g eq 4 and 4^g eq 1 and 6^g eq 5
and 3^g eq 6 then N717274623s:=sub<N|N712346,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 5 and and 4^g eq 3 and 6^g eq 1
and 3^g eq 7 then N717274623s:=sub<N|N717274623s,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 7 and 2^g eq 5 and and 6^g eq 1
and 3^g eq 2 then N717274623s:=sub<N|N717274623s,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 7 and 2^g eq 6 and 4^g eq 2 and
6^g eq 4 and 3^g eq 1 then N717274623s:=sub<N|N717274623s,g>;
end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 7 and 2^g eq 3 and 4^g eq 1 and
6^g eq 2 and 3^g eq 4 then N717274623s:=sub<N|N717274623s,g>;
end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 4 and 2^g eq 7 and 4^g eq 5 and
6^g eq 2 and 3^g eq 3 then N717274623s:=sub<N|N717274623s,g>;
end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 6 and 2^g eq 2 and 4^g eq 7 and
6^g eq 5 and 3^g eq 4 then N717274623s:=sub<N|N717274623s,g>;
end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 3 and 2^g eq 5 and 4^g eq 2 and
6^g eq 7 and 3^g eq 6 then N717274623s:=sub<N|N717274623s,g>;
end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 1 and 2^g eq 5 and 4^g eq 6 and
6^g eq 3 and 3^g eq 4 then N717274623s:=sub<N|N717274623s,g>;
end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 5 and 2^g eq 4 and 4^g eq 7 and
6^g eq 3 and 3^g eq 1 then N717274623s:=sub<N|N717274623s,g>;
end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 5 and 2^g eq 6 and 4^g eq 1 and
6^g eq 7 and 3^g eq 3 then N717274623s:=sub<N|N717274623s,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 1 and 2^g eq 7 and 4^g eq 3 and
6^g eq 4 and 3^g eq 6 then N717274623s:=sub<N|N717274623s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 6 and 2^g eq 3 and 4^g eq 4 and
6^g eq 7 and 3^g eq 5 then N717274623s:=sub<N|N717274623s,g>; end if; end for;
for g in N do if 7^g eq 4 and 1^g eq 2 and 2^g eq 3 and 4^g eq 5 and
6^g eq 6 and 3^g eq 1 then N717274623s:=sub<N|N717274623s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 4 and 2^g eq 6 and 4^g eq 3 and
6^g eq 5 and 3^g eq 2 then N717274623s:=sub<N|N717274623s,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 3 and 2^g eq 1 and 4^g eq 7 and
6^g eq 6 and 3^g eq 2 then N717274623s:=sub<N|N717274623s,g>; end if; end for;
for g in N do if 7^g eq 2 and 1^g eq 6 and 2^g eq 1 and 4^g eq 5 and
6^g eq 4 and 3^g eq 7 then N717274623s:=sub<N|N717274623s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 3 and 2^g eq 4 and 4^g eq 6 and
6^g eq 2 and 3^g eq 7 then N717274623s:=sub<N|N717274623s,g>; end if; end for;
for g in N do if 7~g eq 7 and 1~g eq 4 and 2~g eq 1 and 4~g eq 2 and 6~g eq 3 and 3~g eq 5 then N717274623s:=sub<N|N717274623s,g>; end if; end for;
for g in N do if 7~g eq 3 and 1~g eq 2 and 2~g eq 7 and 4~g eq 6 and 6~g eq 1 and 3~g eq 5 then N717274623s:=sub<N|N717274623s,g>; end if; end for;
T717274623s:=Transversal(N,N717274623s);
for i in [1..#T717274623s] do
ss:=[7,1,7,2,7,4,6,2,3]~T717274623s[i];
p[prodim(1, ts, ss)]:= prodim(1, ts, ss~xx); end for;
N7125:=Stabiliser(N712,5);
for g in N do if 7~g eq 4 and 1~g eq 6 and 2~g eq 2 and 5~g eq 5 then N717275717s:=sub<N|N7125,g>; end if; end for;
for g in N do if 7~g eq 1 and 1~g eq 3 and 2~g eq 2 and 5~g eq 6 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7~g eq 3 and 1~g eq 7 and 2~g eq 2 and 5~g eq 4 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7~g eq 5 and 1~g eq 7 and 2~g eq 2 and 5~g eq 1 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7~g eq 4 and 1~g eq 7 and 2~g eq 2 and 5~g eq 3 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7~g eq 6 and 1~g eq 3 and 2~g eq 2 and 5~g eq 1 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7~g eq 3 and 1~g eq 4 and 2~g eq 2 and 5~g eq 7 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7~g eq 4 and 1~g eq 3 and 2~g eq 2 and 5~g eq 7 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7~g eq 7 and 1~g eq 5 and 2~g eq 2 and 5~g eq 1 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7~g eq 1 and 1~g eq 7 and 2~g eq 2 and 5~g eq 5 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7~g eq 7 and 1~g eq 3 and 2~g eq 2 and 5~g eq 4 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7~g eq 5 and 1~g eq 4 and 2~g eq 2 and 5~g eq 6 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7~g eq 3 and 1~g eq 1 and 2~g eq 2 and 5~g eq 6 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7~g eq 6 and 1~g eq 4 and 2~g eq 2 and 5~g eq 5 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7^g eq 7 and 1^g eq 4 and 2^g eq 2 and 5^g eq 3 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 6 and 2^g eq 2 and 5^g eq 4 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7^g eq 1 and 1^g eq 6 and 2^g eq 2 and 5^g eq 3 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 1 and 2^g eq 2 and 5^g eq 3 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7^g eq 3 and 1^g eq 6 and 2^g eq 2 and 5^g eq 1 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7^g eq 5 and 1^g eq 1 and 2^g eq 2 and 5^g eq 7 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
for g in N do if 7^g eq 6 and 1^g eq 1 and 2^g eq 2 and 5^g eq 4 then N717275717s:=sub<N|N717275717s,g>; end if; end for;
T717275717s:=Transversal(N,N717275717s);
for i in [1..#T717275717s] do
  ss:=[7,1,7,2,7,5,7,1,7]~T717275717s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss~xx);
end for;
T7172757172s:=Transversal(N,N);
for i in [1..#T7172757172s] do
  ss:=[7,1,7,2,7,5,7,1,7,2]~T7172757172s[i];
p[prodim(1, ts, ss)] := prodim(1, ts, ss~xx);
end for;
return (G1 ! p) * &*[G1|ts[uu[j]]: j in [1 .. #uu]];
end function;
Bibliography


