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Matwin: A java tool for computing and experimenting in dynamical systems

Ehab William Aziz Rezk

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MATWIN: A JAVA TOOL FOR COMPUTING AND EXPERIMENTING IN DYNAMICAL SYSTEMS

A Project
Presented to the Faculty of California State University, San Bernardino

In Partial Fulfillment of the Requirements for the Degree Master of Science in Computer Science

by
Ehab William Aziz Rezk
September 2007
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September 2007

Approved by:

Dr. Keith Schubert, Chair, Computer Science

Date 9/12/2007

Dr. Ernesto Gomez

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ABSTRACT

The dynamic of any situation refers to how the situation changes over the course of time. A basic observation of dynamical systems is how extraordinarily complicated the motions of a system can be, even when the underlying rules are extremely simple. One of the main approaches to dynamical systems is Differential equations. Given any initial state, a differential equation, or a system of differential equations are used to describe the forces, or directions in which a system is being pushed. In many cases where exact solutions for differential equations are hard to find, approximate solutions can be calculated using numerical solution methods such as Euler and Runge Kutta.

Here, I present MATWIN, a toolkit for Windows computers, consisting of four graphics programs. It supports mathematical computation and experimentation in dynamical systems. This software was developed for a college level courses that teach dynamical systems or differential equations. Also, because one of the programs included in this software deals with basic calculus topics such as drawing curves of functions, drawing tangents at desired points, finding roots, area under
curves, maxima and minima, it can be of great help to high school calculus students too.
ACKNOWLEDGEMENTS

As I present my project, I would like to express my thanks to Dr. K. Schubert, my project supervisor, for proposing the idea of MATWIN, and for providing me with his knowledge and support. Also, I would like to thank Dr. R. Botting for being the first to introduce me to the programming world when I first started my program with C++ class. Also, special thanks to Dr. Gomez for giving me his great comments that made my project better. As the happiness fills my soul for completing this project and my degree requirements as well, a special word of thanks I would like to say to the Faculty and Staff of Computer Science department at California State University, San Bernardino, for providing this master's program in Computer Science, and for giving me this great opportunity to join and graduate from it.

Also I would like to express my deep gratitude to my dear wife, Sherein, my kids, Karen and Martin, my father, William, mother, Aida, and my beloved brothers, Mina, Sameh, and Bishoy for supporting and encouraging me at every hard time.
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The dynamic of any situation refers to how the situation changes over the course of time. A dynamical system is a physical setting together with rules for how the setting changes or evolves from one moment of time to the next. It is the tool with which scientists make mathematical models to represent the real systems. One basic goal of the mathematical theory of dynamical systems is to determine or characterize the long-term behavior of the system.

A basic observation of dynamical systems, largely due to the advent of computers and computer graphics, is how extraordinarily complicated the motions of a system can be, even when the underlying rules are extremely simple. An example for this is the Newton's description of the motion of bodies under gravity. The forces are extremely simple: bodies attract to each other by a force proportional to the product of their masses and inversely proportional to the square of the distances separating them. Yet the motions caused by these forces are
extremely complex, resulting, for instance, in the braided rings of Saturn.

One of the main approaches to dynamical systems is Differential equations. Given any initial state, a differential equation, or a system of differential equations are used to describe the forces, or directions in which a system is being pushed. Some of these differential equations are easy to solve using integration methods to get exact solutions, while in some other cases, it is hard or even impossible to find exact solutions to other system of differential equations. In such cases where exact solutions are hard to find, approximate solutions can be calculated using numerical solution methods such as Euler and Runge Kutta. These numerical methods require a huge amount of calculations that need to be repeated over and over for huge number of times, which is a waste of time plus the fact that these repeated calculations can lead to huge mistakes in the final values calculated.

In the 1940s an entirely new tool of analysis also came on the scene—the digital computer. In addition to its obvious brute-force capabilities of grinding out numerical solutions of differential equations, it
presents a flexible, interactive tool for the purpose of discoveries. This interplay between computations and analysis has proved to be of great importance, and represents one of the major methods of uncovering dynamic properties. Indeed, the area of computer science has rapidly progressed to a state in which it can now make fundamental contributions to our knowledge, rather than acting only as the servant of other methods of analysis.

Here, I present MATWIN, a toolkit for Windows computers, consisting of four graphics programs. It supports mathematical computation and experimentation in dynamical systems. This software was developed for a college level courses that teach dynamical systems or differential equations. Also, because one of the programs included in this software deals with basic calculus topics such as drawing curves of functions, drawing tangents at desired points, finding roots, area under curves, maxima and minima, it can be of great help to high school calculus students too.

Scope

MATWIN is intended to be for the college level students who study dynamical systems or differential
equations. Also it can be of great help to high school calculus students.

Purpose

The purpose of this project is to implement an integrated piece of software consisting of a number of graphics programs that support mathematical computation and experimentation in dynamical systems. These graphics programs should be able to draw the solutions to the given differential equations (automatically, using standard numerical methods) without dealing with the complications of the analytic techniques from the user side.

Project Products

Upon project completion, the following products will be delivered.

Source Code and Compiled Classes

The project consists of four applets. The source files contain the implementation of all applets, along with the parser, the methods for finding the roots, derivatives, maxima and minima, and finding the solutions to the given differential equations. Also they contain
all the comments within the source code for relevant statements and methods.

User Guide

The user guide contains documentation of this project’s products. Detailed instructions are provided for:

- Using the main applet (Analyzer).
- Using the DiffEq. applet.
- Using the Phase Plane applet.
- Using the 3Dview applet.

The user guide includes a detailed description covering all the features of the program. This guide includes also a number of examples of systems of differential equations along with their graph solutions for evaluation purposes.

Project Presentation

A presentation will be given to the public, wherein the overview of the system will be introduced. For more information, visit http://csci.csusb.edu/dick/seminar/20070628EhabAzizRezk.ppt
CHAPTER TWO
MATHEMATICAL APPROACH

Differential Equations

A differential equation is an equation involving an unknown function and its derivatives. The equation can contain only one variable and be called Ordinary, or can contain more than one variable and need to be solved along with other equations. Using analytical techniques, as direct integration and separating variables, can solve a wide range of differential equation systems. Unfortunately the equations resulting from modeling real world don’t usually fall into that category of differential equations that can be solved analytically. In such those cases, where the exact solution can’t be found by analytical methods, some well known algorithms known as numerical methods can find these solutions with a satisfied degree of accuracy.

Using MATWIN enables dealing with equations whether or not they can be solved analytically. If the equations can be stated as functions in one of the following forms:
then they can be entered in MATWIN program. Higher
order equation involving second and third derivatives can
be rewritten to fit the above formats. Once the equations
are entered, MATWIN can draw the solutions
(automatically, using standard numerical methods, to be
discussed later) without any further user calculations.

Numerical Solutions

Four numerical solutions were used in MATWIN:

Simpson’s Rule

Simpson’s rule, figure 1, is a method for finding
the definite integral, within a certain range, to a
function \( f(x) \) by dividing the area under the curve of
\( f(x) \) into a number of trapeziums and then finding the
total area by getting the sum of these areas.
Newton’s Method

Newton’s method, figure 2, is very efficient way of finding roots of complicated functions. For any function $f(x)$, the algorithm starts by guessing an initial value, $x_0$, to be the first approximation of the root. A tangent line to the curve $f(x)$ at $x = x_0$ is drawn, and then a more precise solution can be found at the intersection of this tangent line and $X$-axis. The process can be repeated for any number of times until the desired accuracy is achieved.
**Bisection Method**

This method, figure 3, is used to find the roots of a function $g(z)$ if the graph of $g(z)$ crosses $X$-axis. It is efficient in catching the crossing points that were not detected by Newton’s method. The root can be found by choosing two points $z_1$ and $z_2$. If $g(z_1) \times g(z_2) = -1$, then there is a root between $z_1$ and $z_2$. Finding the midpoint, $z_m = (z_1 + z_2)/2$, of $z_1$ and $z_2$ gives the first approximation of the root. Repeating these steps for $z_1$ and $z_m$ or $z_m$ and $z_2$ gives a better approximation and so on.
Runge Kutta Algorithm

The Runge-Kutta algorithm is the magic formula behind solving differential equations numerically. It is known to be very accurate and well behaved for a wide range of problems.

For a single variable problem \( x' = f(t, x) \),

If \( x_n \) is the value of the variable at time \( t_n \), then \( x_{n+1} \) at time, \( t_{n+h} \) is

\[
x_{n+1} = x_n + \frac{h}{6} (a + 2b + 2c + d)
\]

Where

\[
a = f(t_n, x_n),
\]

\[
b = f(t_n + \frac{h}{2}, x_n + \frac{h}{2} a),
\]

\[
c = f(t_n + \frac{h}{2}, x_n + \frac{h}{2} b), \text{ and}
\]
\[ d = f(t_n + h, x_n + h c). \]

For multi variable cases, the algorithm is still the same, with one exception. Each variable becomes a vector. Then

\[ x_{n+1} = x_n + \frac{h}{6}(\bar{a}_n + 2\bar{b}_n + 2\bar{c}_n + \bar{d}_n) \]

where

- \( \bar{a}_n = f(x_n) \)
- \( \bar{b}_n = f(x_n + \frac{h}{2}a_n) \)
- \( \bar{c}_n = f(x_n + \frac{h}{2}b_n) \)
- \( \bar{d}_n = f(x_n + hc_n) \)

Slope Fields

The slope field of a differential equation \( dy/dt = f(x,y) \) is a diagram with little line segments, drawn at the points of some grid, with the slope as given by the differential equation. These diagrams can be used to have a quick idea of what the solution may look like. Figure 4 shows the slope field diagram for \( dy/dt = x+y \), (the left one), and \( dy/dt = x^2 \), (the right shape).
MATWIN Mathematical Features

To better understand the mathematical features of MATWIN, let us discuss the following three cases:

The One-Dimensional Case

For DiffEq equation $\frac{dx}{dt} = f(t, x)$ describes a slope field in the tx-plane. For any particular point $(t, x)$ the equation tells exactly the slope of the solution through that point. The program draws the slope field by plotting, at the points of some grid, little line segments with the slope as given by the differential equation, see figure 5.
A solution $x(t)$ of the differential equation is a function of $t$, which can be most easily imagined as a graph whose slope at every point $(t,x)$ is $f(t,x)$.

If a point $(t_0,x_0)$ is chosen by the cursor, the program will draw an approximate solution through that point. The point $(t_0,x_0)$ is called the initial condition for the solution for that solution. The general idea of the approximate solution is that the computer calculates the slope at the initial condition point and increments...
the step and then calculates the slope at the new position; then it repeats incrementing the step and calculation the new slope values until it reaches the boundary of the screen. Then the program goes back to the starting point (initial condition) and repeats the process in the opposite direction. This is done by decrementing time in The Runge Kutta equations.

At the far right of the graph, figure 6, the slope field changes rapidly near the solution drawn, but it has been calculated carefully with a step size of less than one pixel, so at every point graphed it indeed has the proper slope, as if a slope mark were centered on that point. The approximate solution travels "with the flow" through the slope field.
An approximate solution made in this way can be as accurate as you want. Choosing a smaller step size will perform better if desired.

Using MATWIN you can choose any number of initial conditions (by clicking the mouse at any desired point) and have multiple solutions drawn together at the same graph, figure 7.
Now, one can have a global idea on how the solutions (at different initial points) may look like, which enable a qualitative analysis quite unavailable from non graphical numerical listings of individual solutions.

Furthermore, notice that the entire procedure of drawing an approximate solution uses only the slope, as given explicitly by the differential equation; it never makes any reference to an explicit analytic solution. The graphics program totally bypasses the need for such analytic calculation.
For the case of the above examples, \( y' = y^2 - x \) does not fit any of the special solutions for analytic solutions; moreover, it can actually be proven (with very advanced mathematics from differential Galois theory) that for this particular equation there is no formula for the solutions in terms of elementary functions (the functions we normally write, like polynomials, rational functions, trigonometric functions, and other transcendental functions). Nevertheless the solutions do exist, and still can be seen graphically.

**Raising The Number Of Dimensions.** Similar pictures can be drawn for a differential equation of higher order or with more variables. The overriding principle for changing a higher order equation in \( x \) into a system is to set \( dx/dt = y \). If a higher degree is necessary, set \( dy/dt = z \) and continue, Example 2.1.
Example 2.1

The second order differential equation $\frac{d^2x}{dt^2} = -x$ can be rewritten as a system of two first order equations. If you introduce another variable $y = \frac{dx}{dt}$, then $\frac{d^2x}{dt^2} = \frac{dy}{dt'}$, so

$$\frac{d^2x}{dt^2} = -x$$

can be rewritten as

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -x$$

which is a system of two first order differential equations.

In fact, any higher order differential equation can be rewritten as a system of first order equations, and this is what needs to be done in order to enter them in the MATWIN programs.

The Two-Dimensional Case

A two-dimensional system of differential equations for $\frac{dx}{dt}$ and $\frac{dy}{dt}$ may be autonomous, if there is no explicit dependence on $t$, or nonautonomous, if $t$ appears in the functions for $\frac{dx}{dt}$ and $\frac{dy}{dt}$.

An autonomous system describes a vector field in the $xy$-plane, the phase plane, consisting only of the dependent variables. Through any give point $(x_0, y_0)$, the
program DiffEq, Phase Plane will draw a trajectory, which is the $xy$-path determined parametrically by the solutions $x = u(t)$ and $y = v(t)$. For any particular point $(x_0, y_0)$ the equation tells exactly the slope of the trajectory through that point by

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Figure 8. Multiple Solutions In Phase Plane
Figure 8. shows the solutions of the equation \( \frac{d^2x}{dt^2} = -x \), as equivalent to the two equations: \( \frac{dx}{dt} = y \) and \( \frac{dy}{dt} = -x \).

The phase plane, however, shows how \( x \) and \( y \) interact; it is an extremely important picture. In particular, the phase plane phase plane gives crucial information about the behavior of trajectories near all possible equilibria. Also for an autonomous system of differential equations, the trajectories normally will not meet or cross each other (except at equilibria).

Equilibrium, or a singularity, is a point where the derivative of each variable is zero; for the example in figure 7, the equilibrium is at \((0,0)\). Equilibrium may be either stable or unstable, according to whether nearby trajectories approach or leave that point. The center in figure 7 was stable.

If a system is linear, with constant coefficients, then a great deal is known about the behavior of the trajectories. In fact, traditionally the only big success at finding analytic solutions was in dealing with linear equations with constant coefficients.
Nonlinear systems turn out not to be so different as one might think, now that phase plane pictures are so readily obtained. The phase plane allows us to see what happens to trajectories, especially near singularities, and to see that there the nonlinear equations behave exactly as the linear ones. So phase analysis for linear equations even serves for nonlinear ones.

Linear Equations With Constant Coefficients. In a two-dimensional system of differential equations, there are essentially four types of equilibria, named center, node, saddle, and spiral, see figure 9.
The spirals and centers correspond to solutions with trigonometric factors in sine and cosine functions (e.g., $e^{\sin t}$); these are the cases where the individual solutions will represent periodic motion.

The nodes and saddles represent the possible combinations of two principal vector directions, each of which can represent stable or unstable equilibrium.

Figure 10. represents a number of solutions to the system of two differential equations of the form $\frac{dx}{dt} = y$ and $\frac{dy}{dt} = 1 - x^2 + y$
Figure 10. Singularities In Linear Systems

The graph shows spiral singularity at (1,0) and a saddle at (-1,0).

A linear system of two first order differential equations (alternatively, a single second order equation) will exhibit a single equilibrium or singularity, showing one of six behaviors (or a degenerate case, corresponding to double and/or zero exigent values). Some degenerate possibilities are shown in figure 11.
Nonlinear Equations. A nonlinear system of two first order differential equations can have more than one equilibrium or singularity, and may exhibit a very complicated phase plane. Figure 11 shows an example of the phase plane for the system of nonlinear differential equations:

\[
\frac{dx}{dt} = \sin(x + y), \text{ And } \frac{dy}{dt} = \cos(xy)
\]

As can be easily seen in the diagram in figure 12, the phase plane exhibits spiral, saddle, and node singularities in different places. In addition, there is one another phase plane behavior a nonlinear system can exhibit, that is the limit cycle, which is shown in figure 13.
Figure 12. Nonlinear Systems.

Figure 13. Limit Cycle.
The Phase Plane could show all the singularities exhibited by the nonlinear system discussed above.

For a nonlinear system, a good approximation to a solution can be made by the process of linearization, near the singularity. The linearization formulas are not to be discussed here, but to have a little idea on this process, let us consider the system of nonlinear ordinary differential equations described by the equations

\[ x' = y(x+1), \quad \text{and} \quad y' = (3-y)x, \]

and whose phase plane is shown in figure 14.

![Figure 14. Singularities In Nonlinear Systems.](image)
Using MATWIN DiffEq, Phase Plane program, we can get graphical solutions as those of the linear systems around some points. Figure 15 shows the singularities in figure 14 around (0,0) in the left side, and (-1,3) in the right side.

![Figure 15. Separating Singularities](image)

**The Three-Dimensional Case**

The program DiffEq, 3Dview is a part of MATWIN that accepts an autonomous three-dimensional system, that is, with no explicit dependence on t:
\[ \frac{dx}{dt} = f(x, y, z), \]
\[ \frac{dy}{dt} = g(x, y, z), \]
\[ \frac{dz}{dt} = h(x, y, z). \]

In such an autonomous three-dimensional system, the critical graph that relates \( x, y, \) and \( z \) is a three-dimensional \( xyz \)-phase space. Figure 16 shows 3D-phase space for a system of three differential equations described by \( \frac{dx}{dt} = y, \) \( \frac{dy}{dx} = -x, \) and \( \frac{dz}{dt} = 1 \)
Figure 16. Three-Dimensional Phase Space

Figure 17 shows the same 3D-View of the last system after applying a compound rotation of 45 degrees around both x-axis y-axis.
Figure 17. Phase Space After Compound Rotation Around X and Y Axes
CHAPTER THREE
PROJECT DEVELOPMENT AND DESIGN

Development Tools
MATWIN was developed using the following:

- J2SE Development kit 5, update 2, (jdk-1_5_0_02-windows-i586-p).
- Java3D, (java3d-1_3_1-windows-i586-directx-sdk).

Running Requirements
In order to run MATWIN, the following requirements are needed.

- Java2 SDK 1.5.0 or later from Sun Microsystems.
- Java3d SDK 1_3_1-windows-i586-directx-sdk or later from Sun.
- Microsoft DirectX 8.0 or later, available from Microsoft.
Project Design

Sixteen classes were written to develop MATWIN.

Figure 18 shows its basic architecture.

Figure 18. MATWIN Basic Architecture
As can be seen above, the design of MATWIN consists of four main parts, the first part, the left hand side, is the core of the project. It consists of six classes that perform all the basic mathematical tasks like tokenizing and parsing inputs, finding minima and maxima, finding roots and derivatives. Also it includes the rungeKutta class, which uses the RungeKutta algorithm to integrate functions. These classes are called almost by every other class in the system.

The second set of classes in the design includes GraphArea, GraphArea2, GraphArea3, Canvas3D, and OffScreenCanvas3D. These classes implement the Graph Areas of the different parts of the software. They include methods to plot and modify the axes, methods to draw and modify the graphs of functions, the slope fields, and the trajectories of the solutions to the input equations, methods to interact with the user, as in Canvas3D, where a user can rotate, translate, and zoom the 3D view of the solution, and methods to print and save. These classes are called by the applet classes, (Main Applet, DiffEqApplet, PhasePlaneApplet, and 3DviewApplet), to draw and redraw the desired views when these applets needs to modify their contents.
The third set of classes includes Applet, DiffEqApplet, PhasePlaneApplet, and 3DviewApplet. These applets represent the interfaces through which the user will insert expressions, input data, and draw graphs and solutions. These Applets have methods to modify their contents and to call other classes to perform the needed calculations or to draw the updated view. Also they have methods to call the print and save features in other applets. Applet, (or main screen), is the main interface by which all other applets can be called. It contain all the capabilities to plot the graphs of functions, (two functions at the same time), the tangents at desired points, the slope graphs, to find the maxima and minima, and the integral within a certain range.

Quick Synopsis of The Program

MATWIN consists of four graphics programs. These programs represent the interface between the software and the user.

The Analyzer Screen

This part of MATWIN represents the main screen from which all other screens can be called. For a function \( y = f(x) \), this program draws the graph of the function and
its derivative, and the tangent to the curve at any desired point. It also finds the roots, maxima and minima (by analyzing the derivative), and numerically integrates the function within a specific region, that is determined by the user. The Analyzer can graph two functions simultaneously along with their derivatives and tangents. Analyzer is a very handy tool for dealing with awkward equations.

The Second Screen

For a first order differential equation of the form \( \frac{dy}{dx} = f(x,y) \), this program, called DiffEq Screen, displays the slope field and draws the solutions in xy-plane. The initial condition is chosen graphically with the cursor. The coordinates of the initial point are displayed. The drawing of the solution proceeds first forward in time and then backward.

The Phase Plane Screen

For an autonomous system of differential equations of the form

\[
\frac{dx}{dt} = f(x,y), \quad \frac{dy}{dt} = g(x,y)
\]

this program displays the vector field in the xy-phase plane and draws the solution in xy-plane. As the case
with DiffEq program, the initial condition is chosen graphically with the cursor and the drawing of the solution goes forward and backwards in time.

The Three-Dimensional View Screen

For autonomous systems of differential equations of the form

\[
\begin{align*}
\frac{dx}{dt} &= f(x,y,z) \\
\frac{dy}{dt} &= g(x,y,z) \\
\frac{dz}{dt} &= h(x,y,z)
\end{align*}
\]

this program draws the three-dimensional phase space. It rotates, translates, and zoom the view through user interaction.
Chapter Four

User Manual

Introduction

In this chapter, we will give a detailed description of how to run MATWIN and how use the features of every screen (Applet).

To Run MATWIN, you need a Windows computer with java and java 3D installed, see the running requirements at chapter 3.

To launch the program, first copy the class files of the program into the desired location on your hard disk. Let us say that this location is D:\MATWIN>. Now to launch the program, Move to MATWIN location and then, type the next command line and press Enter.

D:\MATWIN>java Applet

Now you should be having a screen titled MATWIN (figure 19). This is the main screen of the program from which you can launch any other screen.
The Analyzer or the program main screen is shown in figure 19. The Analyzer screen is divided into five parts. The first part is called the graph area. It is black in color and is used to display the graphs when the user performs an action to do so. X and Y-axes are already displayed, and their default upper and lower
bounds go from -100.00 to 100.00. Later we will know how to change these values.

The second part lies right above the graph area. It consists of two expression fields labeled Expression Y1, and Expression Y2. These are the places into which the expressions are to be fed to the program. The Analyzer can take two expressions at the time. One of the two expressions will be the primary one, on which all the calculations are performed while the other one is a secondary one, and only some operations will be performed on it.

The third part of the Analyzer screen is a column of buttons, expression fields and text fields, which are used to input data into or output data from the program. The features of this part work only for the primary expression (the expression on the top), You shouldn’t be expecting to apply the features of this part to the secondary expression (the lower one). These features include a calculator that can evaluate the primary expression for a given value of x, a feature to find the roots, maxima and minima, definite integral of the primary expression. Also you can show the area defined by the definite integral graphically.
The column on the right side of the Analyzer represents the fourth part of the screen. It consists of a number of buttons, data fields, and text areas. Some of these features can apply to the primary expression while the rest apply for the secondary one. It includes capabilities that can draw a function (expression) and its derivative, draw the tangent to a function at any desired point on the curve of the expression. You can undo any step at any time by pressing the appropriate clear button, as we will see later. Also this part contains some fields that can be used to input the desired lower and upper values of x and y-axes. The boundary values of the axes can be changed at any time by replacing the old value with the new one.

The last part of the analyzer screen is the row of buttons located at the bottom of the screen. It contains buttons for saving or printing the graph on the graph area, and buttons for launching the rest of the screens, DiffEq, DiffEq, Phase Plane, and DiffEq, 3Dview.

Inserting Expressions

Two expression can be inserted the expression field on the top of the graph area at any time (see figure 20). These are the only places to insert your functions
(expressions). These expression fields accept only expressions that include one variable, \( x \). You should use \( x \) always when you use this program.

![Table of expressions](image)

**Figure 20. Expression Fields**

The syntax that needs to be considered when tying the functions is easy. Table 4.10 summarizes all the functions accepted by MATWIN and the correct syntax that needs to be used.

Inserting a function into the program is very flexible. For example, if you need to get the "sin" of a variable "\( x \)" you can type \( \sin x \), \( \sin x \), or \( \sin(x) \). There is no need to use the parentheses or spaces unless you are applying the sin to a compound function as "\( x+2 \)"", "\( -x \)"", or \( (3*x) \). Both lower case and upper case letters are accepted, with no distinction between lower case and upper case. For some special values like \( \pi \) and \( e \), you can use \( \pi \) and \( e \) respectively. Example 4.1 shows how to type
two different functions which use a number of functions listed in table 1.

<table>
<thead>
<tr>
<th>Function</th>
<th>Syntax examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signed variable</td>
<td>$x, X, -x, pi, e, -pi, -e,..$</td>
</tr>
<tr>
<td>Addition &amp; Subtraction</td>
<td>$x+2, 3-x, -x-1, -(x+0.5),..$</td>
</tr>
<tr>
<td>Multiplication &amp; Division</td>
<td>$2<em>x, x/2, (x+3)/(-4</em>x)$</td>
</tr>
<tr>
<td>Raising to a power</td>
<td>$x^2, x^{(-1.5)}, (-x)^{(x/2.2)},..$</td>
</tr>
<tr>
<td>Square Root</td>
<td>$\sqrt{x}$ for $x \geq 0$, ..</td>
</tr>
<tr>
<td>Logarithmic functions</td>
<td>$\ln x, \ln(x)$ for $\log_e$, where $x &gt; 0$</td>
</tr>
<tr>
<td></td>
<td>$\log x, \log(x)$ for $\log_{10}$, where $x &gt; 0$</td>
</tr>
<tr>
<td>Exponential functions</td>
<td>$\exp x, \exp(x), \exp(x/2),..$</td>
</tr>
<tr>
<td>Trigonometric functions</td>
<td>$\sin x, \cos(-x), \atan(x),..$</td>
</tr>
<tr>
<td></td>
<td>where $x$ is in radians</td>
</tr>
</tbody>
</table>
Example 4.1.

(a) the expression \( \sin x + e^{x/2} + 2x^2 \) is typed \( \sin x + e^{(x/2)} + (2\pi x^2) \) in MATWIN.

(b) The expression \( \sqrt{x + \sin(-2.2x)/(3x/2)} \) is typed \( \text{Sqrt}(x + \sin(-2.2*\text{x})) / ((3*\text{x})/2) \) into MATWIN.

Note that, a numeral '0' (zero) is different from an 'O' (oh), and a '1' (one) is different from an 'l' (lower-case 'L'). They look similar, but using the wrong one will cause the computer to give an error message.

Using The Calculator Feature

The Analyzer includes a very handy calculator feature. It is very easy to use. Once the there is an expression in the expression field Y1, you can calculate the value of the expression for any desired value of \( x \).

to calculate the value of the expression \( \sqrt{x} \) for \( x = 2.25 \), type the expression \( \text{sqrt}(x) \) in the Y1-expression field and 2.25 in the space labeled "for \( x = \)" and press the button "Y =".

The answer is 1.5, as shown in figure 21.
The x-value and the calculated values will remain unchanged until the user changes them.

**Plotting Functions**

To draw a function on the graph area, you need to insert at least one function into the expression fields Y1 or Y2 or both, then press the Draw Y1, or Draw Y2, or both. The graph area will display the curves and their expressions, see figures 22, 23, 24.
Figure 22. Drawing The Primary Function
Figure 23. Drawing The Secondary Function
You can erase any graph at any time by pressing "Clear Y1" or "Clear Y2" buttons or both. Also you can clear everything at once by choosing "Clear" option under "Edit" menu in the menu bar.

Changing Graph Scales

You can always change the boundary values (the scales) of x and y axes by replacing the values in the fields labeled "Low x =", "High x =", "Low y =", "High y =" by the desired values. The program will read the new values and calculate the new coordinates and redraw the graph and axes. Figure 25 shows the graph area after
pressing "Draw Y2" button and changing the boundary values from -20 to 50 along X-axis and -2 to 10 along Y-axis.

The number of divisions, and the position of labels along x and y-axes can be changed by replacing the values in the fields labeled "Divide x into", "Divide y into", "Label x every", and "Label y every". It is very important to specify the number of divisions and the labels positions separately since you might need to have a more accurate idea on the curve by dividing it into a larger number of divisions and still don't want the graph to be very crowded with too many values along axes. Also sometimes numbers along x-axis can overlap, resulting in bad effects on how the graph looks. In figure 25, the graph of Y2 is displayed after changing the number of divisions along X and Y-axes. Notice that Y-axis is divided every 0.2 while its values are labeled every 1.
Figure 25. Changing The Scales And Divisions Along Axes.

Drawing Tangents And Derivatives

The Analyzer can plot the graph of the derivatives of functions. Pressing the button labeled “Draw Y1′ Curve” or “Draw Y1′ Curve” will make MATWIN calculate the values of the derivative of the corresponding function and plot it along with the function curve or by itself. Also the buttons “Clear Y1′” and “Clear Y2′” can be used to erase the corresponding curve, figure 26.

Another useful feature is plotting the tangent to a curve at a certain point. The Analyzer lets you enter the
desired points at which you like the tangents to be drawn. It reserves two fields, both labeled "For \( x = \)" for this purpose, one for Y1 and the other is for Y2. Entering a value (x-coordinate value) into any of these fields and pressing the corresponding button ("draw Y1 Tangent Line" or "Draw Y2 Tangent line" will make MATWIN draw the tangent of the corresponding curve at the chosen x-value. You can draw one tangent to any curve at the time and you still can draw the both tangent line to each curve at the same time. You can clear the tangent lines using the appropriate "Clear" buttons. Figure 26 shows the graph of y1 along with its derivative. It shows also a tangent to the curve at \( x = 69 \).
Finding Roots, Maxima, And Minima Coordinates

Two of the most important features that the analyzer provides are finding the roots of an equation and the x coordinates at which these equations have maximum or minimum values. If the equation is inserted into Y1 field as a primary one, then MATWIN can calculate the approximate values of the roots and display it in the text field that lies under the button labeled "Roots =". Also the x values corresponding the maxima and minima of
the equation will be displayed in the text field under the title "Max_Min".

To display these values, you need first to tell MATWIN for which range of the curve you need the values to calculated. Also you need to specify the step size that you desire to use in the calculations. The smaller the step size is, the more accurate the results will be, but also a longer time will be needed. MATWIN calculates the roots using Newton and Bisection methods. Figure 27 shows the roots and maxima results after specifying a range from 0 to 45 and step size of 0.1. for the expression 100* sin(x/5)* cos(x/50).
Figure 27. Roots, Maxima And Minima

**Definite Integrals**

One more thing that you can do in this part of MATWIN is to find the definite integral for a function. To do so, enter the function into expression field Y1, specify the region for which the integral is to be calculated, then press the button labeled "Def Integral =". The value will be displayed in the field below the button.

To make it more interesting, "Show" and "Clear" buttons can be used to show and clear the area of the
integral on the graph. Figure 28 shows the value and the area on the graph of the integral performed on the expression $100 \times \sin(x/5) \times \cos(x/50)$ from $x = 0$ to $x = 45$, with a step size of $0.1$.

Figure 28. Finding The Definite Integrals
Saving And Printing

Now what is left is to print or save the graph. Printing is done by pressing the "Print" button. Once the print button is pressed, a Print dialog box will appear, on which you can choose the printer, number of copies, change the orientation, quality of the printed paper. The printed paper will look exactly like the view in program, but with different colors. The graph in the printed paper is colored in black and the background is in white. this scheme is better because it will not use as much ink as if the background was in black. The Print dialog box is shown in figure 29.
Saving the graph is as easy as the printing. Once the "Save" button is pressed, MATWIN creates a name using the current date and time and attach it to the JPG file that will be used to save the graph picture. The files of each applet (screen) will be placed in a separate directory. There are four directories to store these files. These directories are located in another directory called ImageFiles located in the same directory as the program class files. The images saved from the Analyzer screen will be stored in a directory called "MainAppletImages".
Launching Other Applications

There are three other screens that can be launched from the Analyzer, DiffEq, DiffEq Phase Plane, and DiffEq 3Dview. They can be launched either, by pressing on their corresponding buttons at the bottom of the analyzer screen or, or by choosing them from the "FUNCTIONS" menu in the menu bar.

The Second And Third Screens

The DiffEq (Second), and DiffEq, Phase Plane (Third), screens are similar in their appearance and functionality. The only difference is in the number of expressions they accept. DiffEq screen accepts only one expression in the form $\frac{dy}{dx} = f(x,y)$, while DiffEq, Phase Plane screen accept two differential equations in the form of $\frac{dx}{dt} = f(x,y)$, and $\frac{dy}{dt} = g(x,y)$. Figure 30 shows the DiffEq screen.

As seen in figure 30, the DiffEq screen has capabilities to change the boundary values, number of divisions, and the labels positions along X and Y axes. One extra feature in DiffEq screen is the presence of a little white rectangle at the top left corner of the
graph area. This rectangle displays the coordinates of the point at which the cursor of the mouse is pointing at any time. This feature is very handy when the initial condition is to be selected by mouse clicks at the graph area.

Figure 30. The Second Screen
Displaying The Slope Fields And Solutions

For DiffEq program, once the expression is typed into the expression field at the top of the screen, you can display the slope field by pressing the "Draw Slope Field" button. The trajectories of solutions will need you to specify an initial point to represent the initial conditions. This point is chosen by clicking the mouse at the desired point in the graph area. Once the mouse is clicked, the program will use Runge Kutta Algorithm to calculate the coordinates of the next point, then uses the new point to calculate the following point and so on.
The program will draw the solution forward in time until the boundary of the graph area is reached, then the program will go back the starting point and draw the solution backward in the opposite direction. MATWIN uses a stepsize of 0.01. Figure 31 shows the slope field of \( \frac{dy}{dx} = y^2 - x \) and four solutions using four different initial points at \((0,0), (0,-2), (0,2), (-4.04,0)\).
The situation is very similar with DiffEq, Phase Plane program. The program draws slope field and the phase plane solutions for the system of differential equations represented by the two expression inserted into the program. The solutions can be drawn by clicking the mouse on the desired position in the graph area. Figure 32 shows a number of solutions (without the slope field lines) drawn at a number of different initial points.

![Figure 32. The Phase Plane Solution Trajectories](image)
Saving And Printing

Saving and printing are similar in functionality the Save and Print features discussed in the Analyzer section. The only difference is that the images from DiffEq program will be saved in a directory called DiffEq, and those of the DiffEq, Phase Plane in a directory called PhasePlane. Both directories are located in the ImageFiles directory.

The Three-Dimensional View Screen

DiffEq, 3Dview is very interesting tool when it comes to drawing the 3D-solution of the system of differential equations of the form \( \frac{dx}{dt} = f(x,y,z), \)
\( \frac{dy}{dt} = g(x,y,z), \) and \( \frac{dz}{dt} = h(x,y,z). \)

The DiffEq, 3Dview screen accepts three expressions for \( \frac{dx}{dt}, \frac{dy}{dt}, \) and \( \frac{dz}{dt}. \) It contains fields to input the desired values of the axes boundaries. To enable utilizing the power of 3D graphics, the program has options to rotate, translate, and zoom the view displayed in the graph area. There are three more fields labeled "_rot", "y_rot", and "z_rot". These field are used to input the desired angle values (in degrees) that the user
like to use to rotate the view around the corresponding axes. If \( x_{\text{rot}} \) is set to be 45, then pressing the button "Draw" will redraw the scene after rotating it 45 degrees around x-axis. Compound rotations around axed can be done at one time, for example a rotation of 45 degrees around x-axis followed by another rotation of 30 degrees around y-axes can be done if \( x_{\text{rot}} \) is set to 45 and \( y_{\text{rot}} \) is set to 30.
Figure 33 shows the 3D view scene of the solution of the differential equation system described by the equations $\frac{dx}{dt} = y$, $\frac{dy}{dt} = -x$, $\frac{dz}{dt} = x^2$. The coordinate of the initial value are chosen by inserting them into the fields $x_0$, $y_0$, and $z_0$ respectively. In the scene shown above, $x_0$ is set to 1, $y_0$ and $z_0$ are both zeroes, which
means the initial point is (1,0,0). Also the values of x_rot, y_rot, z_rot are all zeros, which means that there is no rotation around any axis. This is what made the scene looks like two-dimensional scene.

With a little change to the values in the x_rot and y_rot data fields, the view will change to what we see in figure 34. Notice how the view looks like a 3d-view now.
Compound Rotation Around X and Y Axes

Rotation, Translation, And Zooming

There will never be easier and more interesting than using the mouse to play with graphs. MATWIN enables this by providing features to rotate, translate, and zoom the scene using the mouse and not only the keyboard. Right clicking the mouse and moving it on the graph area will
rotate the scene around one of the axes corresponding to the motion. Also you can move (translate) the scene within the graph area boundaries by left clicking on the mouse and moving it. Also you can zoom the scene out and in by right clicking and moving the mouse while holding the "Alt" button pressed on the keyboard. Figure 35 shows the scene in figure 33, after being rotated, translated, and zoomed out using the mouse.
Figure 35. Mouse Rotation, Translation, And Zooming

Saving and Printing

Saving and printing in DiffEq3DView is similar to the saving and printing features in the rest of the programs. The printed paper will look exactly as the view on the screen. The image will be saved in a folder called 3DAppletImages in the ImageFiles folder.
CHAPTER FIVE

CONCLUSION AND FUTURE EXTENSIONS

Summary

As we have seen, MATHWIN is an integrated piece of software that consists of a number of graphics programs. These programs are very efficient in dealing graphically with certain systems of differential equations. It provides the user with the ability to compute and experiment the solutions of some complicated problems and yet it does not require a wide knowledge of Math.

We showed how MATWIN provides tools for plotting graphs of functions, graphs of derivatives, and tangents. It also provides features for solving equations using numerical methods, finding the roots, maxima and minima, and calculating integrals.

The most powerful feature of MATWIN is the ability to solve the differential equations and draw graphs of these solutions. Its ability in solving differential equations includes:

1. Solving differential equations of the form

\[
\frac{dy}{dx} = f(x,y) .
\]
2. Solving systems of two differential equations of the form \( \frac{dx}{dt} = f(x,y) \), \( \frac{dy}{dt} = g(x,y) \).

3. Solving systems of three differential equations of the form \( \frac{dx}{dt} = f(x,y,z) \), \( \frac{dy}{dt} = f(x,y,z) \), \( \frac{dz}{dt} = f(x,y,z) \).

4. Drawing the slope fields and the trajectories of the solutions of the above systems.

So MATWIN is very efficient tool that can accompany the textbook for any differential equations course. Also it can be of a great help to high school, and college students who study calculus, since large portions of calculus courses deal with roots, maxima and minima, plotting graphs and finding integrals.

Program Evaluation

The program was tested using the following procedures:

1. Inserting a special code lines during the development of the program. The purpose of these lines of code was to output certain results that were used to check the correctness of the program. This procedure has been applied to each component.
separately, then was applied one more time to the whole system after integrating its components into one program. After making sure that the program generates the expected results, these lines of code were removed and the program was compiled to generate the class files.

2. The graphs generated by MATWIN were checked against those produced by MATLAB. The results ensured that both graphs are equivalent in their outlook, the points of intersection with the axes, at the positions where the graphs show singularities.

Future Extensions

The heart of MATWIN lies in the “Equation” class. This class includes all the methods to parse and evaluate expressions. MATWIN can parse and evaluate any expression that includes real numbers or variables that represent real numbers. At this moment, it doesn’t support the complex numbers in the form of $a+bi$, where $a$ and $b$ are real numbers and $i$ is the imaginary unit with the
property $i^2 = -1$. Supporting complex numbers will have the first priority in developing MATWIN.

Extending MATWIN ability to solve a wider range of differential equations systems is another task that will be having a great deal of attention in the future.

Also, making MATWIN available online on the World Wide Web and allowing others to contribute to the development of the software will be the next step in developing MATWIN.
APPENDIX A

SAMPLE CODE
import java.util.*;

public class Equation {
    private String equation_String;
    private Vector<String> equation_Tokens; //contains the elements of equation_String

    static int convert(String s) {
        //required for function Evaluate, it converts the operator to an int representation
        if( s.equalsIgnoreCase("+")) return 1;
        if( s.equalsIgnoreCase("-")) return 2;
        if( s.equalsIgnoreCase("*")) return 3;
        if( s.equalsIgnoreCase("/")) return 4;
        if( s.equalsIgnoreCase("^")) return 5;
        if( s.equalsIgnoreCase("sin")) return 6;
        if( s.equalsIgnoreCase("cos")) return 7;
        if( s.equalsIgnoreCase("tan")) return 8;
        if( s.equalsIgnoreCase("ln")) return 9; //log base e
        if( s.equalsIgnoreCase("exp")) return 10;
        if( s.equalsIgnoreCase("asin")) return 11;
        if( s.equalsIgnoreCase("acos")) return 12;
        if( s.equalsIgnoreCase("atan")) return 13;
        if( s.equalsIgnoreCase("sqrt")) return 14;
        if( s.equalsIgnoreCase("sec")) return 15;
        if( s.equalsIgnoreCase("csc")) return 16;
        if( s.equalsIgnoreCase("cot")) return 17;
        if( s.equalsIgnoreCase("log")) return 18; //base 10
        if( s.equalsIgnoreCase("(")) return 20;
        else return -1;
    }

    static boolean isOperator(String s) {
        //return true if s is an operator
        if( s.equalsIgnoreCase("")) return true;
        else if( s.equalsIgnoreCase("+")) return true;
        else if( s.equalsIgnoreCase("-")) return true;
        else if( s.equalsIgnoreCase("*")) return true;
        else if( s.equalsIgnoreCase("/")) return true;
        else if( s.equalsIgnoreCase("^")) return true;
        else if( s.equalsIgnoreCase("(")) return true;
        else return false;
    }
}
static boolean isOperator2(String s)
    //return true for sin, cos, tan, cot, sec.
    {
        if (s.equalsIgnoreCase("sin")) return true;
        else if (s.equalsIgnoreCase("cos")) return true;
        else if (s.equalsIgnoreCase("tan")) return true;
        else if (s.equalsIgnoreCase("cot")) return true;
        else if (s.equalsIgnoreCase("sec")) return true;
        else if (s.equalsIgnoreCase("csc")) return true;
        else if (s.equalsIgnoreCase("log")) return true;
        else return false;
    } //isOperator2()

static boolean isOperator3(String s)
    //return true for ln
    {
        if (s.equalsIgnoreCase("ln")) return true;
        else return false;
    } //isOperator3()

static boolean isOperator4(String s)
    //return true for sqrt, asin, acos, atan
    {
        if (s.equalsIgnoreCase("asin")) return true;
        else if (s.equalsIgnoreCase("acos")) return true;
        else if (s.equalsIgnoreCase("atan")) return true;
        else if (s.equalsIgnoreCase("sqrt")) return true;
        else return false;
    } //isOperator4()

static boolean isNumber(char s)
    //return true for numbers
    {
        boolean returnValue=true;
        if (s !='0' && s !='1' && s !='2' && s !='3' && s !='4' && s !='5' && s !='6' && s !='7' && s !='8' && s !='9' && s !='x' && s !='X')
            returnValue = false;
        return returnValue;
    } //isNumber()

static void evaluate(Stack<Double> operand, Stack<String> operator)
    {
        double operand1, operand2; int op; double result=0.0;
        operand2 = (operand.peek()).doubleValue(); operand.pop();
        operand1 = (operand.peek()).doubleValue(); operand.pop();
        op = (convert(operator.peek())); operator.pop();
        switch (op)
        {
            case 1: result = operand1 + operand2; break;
            case 2: result = operand1 - operand2; break;
            case 3: result = operand1 * operand2; break;
        }
case 4: result = operand1 / operand2;
    break;
case 5: result = (Math.pow(operand1, operand2));
    break;
default: break;
}
operand.push(result);
//evaluate()
The following function (vectorAdjust) is added to adjust the vector coming from class String to vector tokenizer it adjust equation like \(-\sin.., -(\ldots), \ldots\) to \((-1)\times\sin..,\ldots\) this will enable the Evaluate function in the Equation class to correctly parse the expression to evaluate it It also replace \(e\) and \(\pi\) by their mathematical values */

```java
public Equation(String str) {
    equation_string = (str.trim()).concat("@"); // adding @ to the end
    StringToVectorTokenizer stv = new StringToVectorTokenizer(str);
```
Vector<String> v = vectorAdjust(stv.tokenizeToVector()); // see function vectorAdjust
for (int i = 0; i < v.size(); i++)
{
    double e = Math.E; double pi = Math.PI;
    if (v.get(i).equalsIgnoreCase("e"))
    {
        String s = Double.toString(e);
        v.set(i, s);
    }
    else if (v.get(i).equalsIgnoreCase("-e"))
    {
        String s = "-" + Double.toString(e);
        v.set(i, s);
    }
    else if (v.get(i).equalsIgnoreCase("pi"))
    {
        String s = Double.toString(pi);
        v.set(i, s);
    }
    else if (v.get(i).equalsIgnoreCase("-pi"))
    {
        String s = "-" + Double.toString(pi);
        v.set(i, s);
    }
    else{
    }
}
equation_Tokens = v;

public double Evaluate(String x, double d) // evaluate after substituting x with d
{// return the double value of the equation_string
    Vector<String> temp = equation_Tokens; // a copy of the equation vector
    for (int i = 0; i < temp.size(); i++) // replacing x with d
    {
        if ((temp.elementAt(i)).equalsIgnoreCase(x))
            temp.setElementAt(Double.toString(d), i);
        if ((temp.elementAt(i)).equalsIgnoreCase("+x") & d > 0)
            temp.setElementAt("+x + Double.toString(d)", i);
        if ((temp.elementAt(i)).equalsIgnoreCase("+x") & d == 0)
            temp.setElementAt(Double.toString(0), i);
        if ((temp.elementAt(i)).equalsIgnoreCase("-x") & d < 0)
            temp.setElementAt(Double.toString(Math.abs(d)), i);
    }
    Stack<Double> operand = new Stack<Double>();
    Stack<String> operator = new Stack<String>();
    int op; // int equivalent of operator returned from convert
    String token = temp.firstElement();
    for (int i = 0; i < temp.size(); i++)
    {
        token = temp.elementAt(i);
if(isOperator(token)||isOperator2(token)||isOperator3(token)||isOperator4(token))
{
    if (token.equalsIgnoreCase(""))
    {
        while(!oprator.peek().equalsIgnoreCase("")
            {if(isOperator(oprator.peek()))evaluate(operand, oprator);
            else{evaluate2(operand, oprator);}
        }
        oprator.pop(); //pop"(" 
    } else
    { op = convert(token);
        while(!oprator.empty()&&!oprator.peek().equalsIgnoreCase("") &&
            op<=convert(oprator.peek()))
            {if(isOperator(oprator.peek()))evaluate(operand, oprator);
            else{evaluate2(operand, oprator);}
        }
        oprator.push(token);
    } //else
} //if
else if(isNumber(token.charAt(0))) //token is a number
    operand.push(Double.valueOf(token).doubleValue());
else{operand.push(Double.valueOf(token).doubleValue());}
}//for
while (!oprator.empty())
    {if(isOperator(oprator.peek()))evaluate(operand, oprator);
    else{evaluate2(operand, oprator);}
}
return operand.peek();
}//Evaluate

public double Evaluate(double d1, double d2)
//evaluate after substituting two variables x(first), y(second) of the
//equation with d1 and d2
{
    Vector <String> temp = equation_Tokens; //a copy of the equation vector
    ///////////
    for(int i=0; i<temp.size(); i++) //replacing x with d
        if(((temp.elementAt(i)).equalsIgnoreCase("x")) temp.setElementAt( 
            Double.toString(d1),i);
    if((temp.elementAt(i)).equalsIgnoreCase("-x")&&d1>0)
        temp.setElementAt( "-"+Double.toString(d1),i);
    if((temp.elementAt(i)).equalsIgnoreCase("-x")&&d1==0)
        temp.setElementAt( Double.toString(0),i);
    if((temp.elementAt(i)).equalsIgnoreCase("-x")&&d1<0)
        temp.setElementAt( Double.toString(Math.abs(d1)),i);
    if((temp.elementAt(i)).equalsIgnoreCase("y")) temp.setElementAt( 
            Double.toString(d2),i);
    if((temp.elementAt(i)).equalsIgnoreCase("-y")&&d2>0)
        temp.setElementAt( "-"+Double.toString(d2),i);
}
if(temp.elementAt(i).equalsIgnoreCase("-y")&&d2==0)
  temp.setElementAt( Double.toString(0),i);
if(temp.elementAt(i).equalsIgnoreCase("-y")&&d2<0)
  temp.setElementAt( Double.toString(Math.abs(d2)),i);
}

Stack<Double> operand = new Stack<Double>();
Stack<String> oprator = new Stack<String>();
int op; //int equivalent of operator returned from convert
String token = temp.firstElement();
for(int i=0; i< temp.size(); i++)
{
  token = temp.elementAt(i);
  if(isOperator(token)||isOperator2(token)||isOperator3(token)||isOperator4(token))
  {
    if (token.equalsIgnoreCase (""))
    {
      while((oprator.peek()).equalsIgnoreCase("") )
      {
        if(isOperator oprator.peek()) evaluate(operand, oprator);
        else{evaluate2(operand, oprator);}
      }
      oprator.pop(); //pop"
    } else
    {
      op = convert(token);
      while(!oprator.empty()&&!oprator.peek()).equalsIgnoreCase("(") &&
      op<=convert(oprator.peek())
      {
        if(isOperator oprator.peek()) evaluate(operand, oprator);
        else{evaluate2(operand, oprator);}
      }
      oprator.push(token);
    } //else
  } //if
else if(isNumber(token.charAt(0)) ) //token is a number
  operand.push(Double.valueOf(token).doubleValue());
else{operand.push(Double.valueOf(token).doubleValue());}
}
while (!oprator.empty())
  {if(isOperator oprator.peek()) evaluate(operand, oprator);
  else{evaluate2(operand, oprator);}
  }
return operand.peek();
}

//Evaluate(double, double)
public double Evaluate(double d1, double d2, double d3)
  //evaluate after substituting two variables x(first), y(second), z or t (third)
  //of the equation with d1 and d2 and d3
Vector <String> temp = equation_Tokens; //a copy of the equation vector

///////
for(int i=0; i<temp.size(); i++) //replacing x with d
{
    if((temp.elementAt(i)).equalsIgnoreCase("x")&&!d1>0)
        temp.setElementAt( Double.toString(d1),i);
    if((temp.elementAt(i)).equalsIgnoreCase("-x")&&d1==0)
        temp.setElementAt( Double.toString(0),i);
    if((temp.elementAt(i)).equalsIgnoreCase("-x")&&d1<0)
        temp.setElementAt( Double.toString(Math.abs(d1)),i);
    if((temp.elementAt(i)).equalsIgnoreCase("y")&&!d2>0)
        temp.setElementAt( Double.toString(d2),i);
    if((temp.elementAt(i)).equalsIgnoreCase("-y")&&d2==0)
        temp.setElementAt( Double.toString(0),i);
    if((temp.elementAt(i)).equalsIgnoreCase("-y")&&d2<0)
        temp.setElementAt( Double.toString(Math.abs(d2)),i);
    if((temp.elementAt(i)).equalsIgnoreCase("z")&&!d3>0)
        temp.setElementAt( Double.toString(d3),i);
    if((temp.elementAt(i)).equalsIgnoreCase("-z")&&d3==0)
        temp.setElementAt( Double.toString(0),i);
    if((temp.elementAt(i)).equalsIgnoreCase("-z")&&d3<0)
        temp.setElementAt( Double.toString(Math.abs(d3)),i);
}

Stack<Double> operand = new Stack<Double>();
Stack<String> oprator = new Stack<String>();
int op;  //int equivalent of operator returned from convert
String token = temp.firstElement();

for(int i=0; i< temp.size(); i++)
{
    token = temp.elementAt(i);

    if(isOperator(token)||isOperator2(token)||isOperator3(token)||isOperator4(token))
    {
        if (token.equalsIgnoreCase( "\""))
        {
            while(! (oprator.peek()).equalsIgnoreCase("\"\" )
            {
                if(isOperator(oprator.peek())) evaluate(operand, oprator);
                else {evaluate2(operand, oprator)};
            }
            oprator.pop();  //pop"\"
        } else
        {
            op = convert(token);
            while(!oprator.isEmpty()&&!oprator.peek().equalsIgnoreCase("\"\" )
                && op<=convert(oprator.peek()))
            {
                if(isOperator(oprator.peek())) evaluate(operand, oprator);
            }
else {evaluate2(operand, oprator);
    oprator.push(token);
} //else

if (isNumber(token.charAt(0))){ //token is a number
    operand.push(Double.valueOf(token).doubleValue);
}

else{operand.push(Double.valueOf(token).doubleValue());}

} //for

while (!oprator.empty())
    {if(isOperator(oprator.peek())) evaluate(operand, oprator);
    else{evaluate2(operand, oprator);}
}

return operand.peek();

//Evaluate(double, double)
public void print_equation_Tokens()
{
    System.out.print("\n");
    for(int i=0; i<equation_Tokens.size()-1; i++)
    System.out.print(equation_Tokens.get(i) +", ");
    System.out.print(equation_Tokens.lastElement() +"\n");
}

public String get_equation_String()
{
    return equation_String;
}

public Vector<String> get_equation_Tokens()
{
    return equation_Tokens;
}

public static void main(String[] args)
{

} //main

}//Equation class
import java.util.*;

public class DerivativeFinder {
    private String classString; // the equation whose derivative is to be calculated
    private Vector<String> stringTokens;

    public DerivativeFinder(String s) {
        classString = s.trim();
        StringToVectorTokenizer stv = new StringToVectorTokenizer(s);
        stringTokens = stv.tokenizeToVector();
    }

    // public derivativeFinder(s)
    // the following function answer fix adjust the answer if it has term multiplied
    // or added to 0. ((ln((x-3))*(0)))-- (0))
    static String answerFix(String s) {
        String temp = s;
        int g = 0;
        while (g < s.length()) {
            if (((s.length() >= 4) && s.substring(g, g + 4).equals("\+(0)")) || ((s.length() >= 4) && s.substring(g, g + 4).equals("\-(0)"))) {
                String one = s.substring(0, g);
                String two = s.substring(g + 4);
                return one.concat(two);
            } else if (((s.length() >= 5) && s.substring(g, g + 5).equals("\*(0)"))) {
                int counter = -1;
                for (int l = g - 1; l >= 0; l--) {
                    if (s.charAt(l) == 'I') counter--;
                    else if (s.charAt(l) == '(') counter++;
                    else{}
                    if (counter == 0)
                        String one = s.substring(0, l).concat("(0)");
        }
    }
}
String two = s.substring(g+5);
    return one.concat(two);
}
}
g++;
return s;
} //answerFix

static boolean isOperator(String s)
    //return true if s is an operator
    {
        if ( s.equalsIgnoreCase(new String("\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\""))) return true;
        else if( s.equalsIgnoreCase(new String("\\+\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\\\"\n
static boolean isOperator2(String s)
    //return true for sin, cos, tan, cot, sec.
    {
        if (s.equalsIgnoreCase(new String("sin"))) return true;
        else if( s.equalsIgnoreCase(new String("cos"))) return true;
        else if (s.equalsIgnoreCase(new String("tan"))) return true;
        else if (s.equalsIgnoreCase(new String("cot"))) return true;
        else if (s.equalsIgnoreCase(new String("sec"))) return true;
        else if (s.equalsIgnoreCase(new String("csc"))) return true;
        else if (s.equalsIgnoreCase(new String("log"))) return true;
        else if (s.equalsIgnoreCase(new String("exp"))) return true;
        else return false;
    }

static boolean isOperator3(String s)
    //return true for ln
    {
        if (s.equalsIgnoreCase(new String("ln"))) return true;
        else return false;
    }

static boolean isOperator4(String s)
    //return true for sqrt, asin, acos, atan
    {
        if (s.equalsIgnoreCase(new String("asin"))) return true;
        else if (s.equalsIgnoreCase(new String("acos"))) return true;
        else if (s.equalsIgnoreCase(new String("atan"))) return true;
        else if (s.equalsIgnoreCase(new String("acot"))) return true;
        else if (s.equalsIgnoreCase(new String("asec"))) return true;
        else if (s.equalsIgnoreCase(new String("acsc"))) return true;
        else if (s.equalsIgnoreCase(new String("sqrt"))) return true;
        else return false;
    }

    static boolean isNumber(char s)
    //return true for numbers

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{  
  boolean returnValue=true;
  {  
    if (s !='.' && s !='0' && s !='1' && s !='2' && s !='3' && s !='4' && s !='5' && s !='6' && s !='7' && s !='8' && s !='9'&& s !='x'&& s !='X')
      returnValue = false;
  }
  return returnValue;
} //isPNumber()

static boolean isPNumber(String s)
{  
  s.trim();
  boolean returnValue = true;
  for(int i =0;i<s.length();i++)
  {
    if(!isNumber(s.charAt(i))) returnValue=false;
  }
  return returnValue;
}

static boolean isNNumber(String s)
{  
  s.trim();
  if(s.charAt(0)&&isPNumber(s.substring(1)))return true;
  else return false;
}

static boolean isNumber(String s)
{  
  if(isNNumber(s)||isPNumber(s)) return true;
  else return false;
}

/**
  The following function (vectorAdjust) is added to adjust the vector coming
  from class String to vector tokenizer
  it adjust equation like -sin.., -(..),... to (-1)*sin..,...
  this will enable the Evaluate function in the Equation class
to correctly parse the expression to evaluate it
  */
static Vector<String> vectorAdjust(Vector<String> vec)
{  
  Vector<String> v =new Vector<String>();
  v.add("("); v.add("-1"); v.add("") ; v.add("*");
  if(vec.get(0).equals("-"))
  
    //if the leading character is a negative sine"-", replace it
    with "(-1)*"
  vec.removeElementAt(0); vec.addAll(0,v);
  return vec;

} else{
  for(int i=0;i<vec.size()-2; i++)
  {
    if(vec.get(i).equals("(")&&vec.get(i+1).equals("-"))
{  
    //if (is followed by - change - to "(0-1)"
    vec.removeElementAt(i+1); vec.addAll(i+1,v);
    return vec;
    
}  
else{
  
return vec;
}  
//else

public void print_stringTokens()
{
    System.out.print("\n");
    for(int i=0; i<stringTokens.size()-1; i++)
        System.out.print(stringTokens.get(i) +", ");
    System.out.print(stringTokens.lastElement() +"\n");
}

public String get_equation_String()
{
    return classString;
}

public Vector<String> get_stringTokens()
{
    return stringTokens;
}

public static void printVector(Vector<String> vec)
{
    System.out.print("\n");
    for(int i=0; i<vec.size()-1; i++)
        System.out.print(vec.get(i) +", ");
    System.out.print(vec.lastElement() +"\n");
}

public static Vector<String> getVectorFrom(Vector<String> vec, int from, int until)
    //return another vector which is a part of the current vector
    //bound from "from"index to "untill" index
    Vector<String>temp = new Vector<String> ();
    for(int i = from; i<=until;i++)
    {
        temp.addElement(vec.get(i));
    }
    return temp;
}

public static String vectorToString(Vector<String> vec)
{
    String s = "";
    for(int i=0; i<vec.size(); i++)
        s=s.concat(vec.get(i));
    return s;
}  
/**  
 ***/
static public String findDerivative(Vector<String> vec) //find derivative of class string located in class vector
{
    Vector<String> vector = vectorAdjust(vec);
    if(vector.get(0).equals("(")&vector.lastElement().equals(")"))
    {
        //removing the frame (.....) or((...)...)but not the one like (..)(..)
        //by checking inside the frame if ) is found before ( or not
        int j=0; //bracket open and close counter
        boolean bracket = true;
        for(int k=1; k<vector.size()-1; k++)
        {
            if(vector.get(k).equals("(")) j+=1;
            else if(vector.get(k).equals(")")) j-=1;
            else{}
            if(j<0) bracket = false;
        }
        if(bracket)return findDerivative(getVectorFrom(vector, 1, vector.size()-2));
        else{}
    }
    if(vector.size()==0)
    {
        String ret = "there is no equation to differentiate.";
        return ret;
    }
    else if(vector.size()==1) //expression is X or constant number
    {
        if(vector.get(0).equalsIgnoreCase("x")) return "1";
        else if(vector.get(0).equalsIgnoreCase("-x")) return "-1";
        else if(isNumber(vector.get(0))) return "0";
        else
        {
            String ret = "expression not supported";
            return ret;
        }
    }
    else if(vector.size()==2) //sinX, sqrtX, sin30, cos - x, ...
    {
        if(vector.get(0).equalsIgnoreCase("sin") &&
            vector.get(1).equalsIgnoreCase("x")) return "(cos x)";
        else if(vector.get(0).equalsIgnoreCase("sin") &&
            vector.get(1).equalsIgnoreCase("-x")) return "(-cos -x)";
        else if(vector.get(0).equalsIgnoreCase("sin") &&
            isNumber(vector.get(1))) return "0";
        else if(vector.get(0).equalsIgnoreCase("cos") &&
            vector.get(1).equalsIgnoreCase("x")) return "(-sin x)";
        else if(vector.get(0).equalsIgnoreCase("cos") &&
            vector.get(1).equalsIgnoreCase("-x")) return "((sin -x))";
        else if(vector.get(0).equalsIgnoreCase("cos") &&
            isNumber(vector.get(1))) return "0";
    }
}
else if (vector.get(0).equalsIgnoreCase("tan") &&
  vector.get(1).equalsIgnoreCase("x")) return "((sec x)^2)";
else if (vector.get(0).equalsIgnoreCase("tan") &&
  vector.get(1).equalsIgnoreCase("-x")) return "(-(sec x)^2)";
else if (vector.get(0).equalsIgnoreCase("tan") &&
  isNumber(vector.get(1))) return "0";

else if (vector.get(0).equalsIgnoreCase("sec") &&
  vector.get(1).equalsIgnoreCase("x")) return "((sec x)*(tan x))";
else if (vector.get(0).equalsIgnoreCase("sec") &&
  vector.get(1).equalsIgnoreCase("-x")) return "(-(sec x)*(tan -x))";
else if (vector.get(0).equalsIgnoreCase("sec") &&
  isNumber(vector.get(1))) return "0";

else if (vector.get(0).equalsIgnoreCase("csc") &&
  vector.get(1).equalsIgnoreCase("x")) return "(-csc x)*(cot x)";
else if (vector.get(0).equalsIgnoreCase("csc") &&
  vector.get(1).equalsIgnoreCase("-x")) return "((csc -x)*(cot -x))";
else if (vector.get(0).equalsIgnoreCase("csc") &&
  isNumber(vector.get(1))) return "0";

else if (vector.get(0).equalsIgnoreCase("cot") &&
  vector.get(1).equalsIgnoreCase("x")) return "(-(csc x)^2)";
else if (vector.get(0).equalsIgnoreCase("cot") &&
  vector.get(1).equalsIgnoreCase("-x")) return "((csc x)^2)";
else if (vector.get(0).equalsIgnoreCase("cot") &&
  isNumber(vector.get(1))) return "0";

else if (vector.get(0).equalsIgnoreCase("asin") &&
  vector.get(1).equalsIgnoreCase("x")) return "(1/sqrt(1-x^2))";
else if (vector.get(0).equalsIgnoreCase("asin") &&
  vector.get(1).equalsIgnoreCase("-x")) return "(-1/sqrt(1-x^2))";
else if (vector.get(0).equalsIgnoreCase("asin") &&
  isNumber(vector.get(1))) return "0";

else if (vector.get(0).equalsIgnoreCase("acos") &&
  vector.get(1).equalsIgnoreCase("x")) return "(-1/sqrt(1-x^2))";
else if (vector.get(0).equalsIgnoreCase("acos") &&
  vector.get(1).equalsIgnoreCase("-x")) return "((1/sqrt(1-x^2)))";
else if (vector.get(0).equalsIgnoreCase("acos") &&
  isNumber(vector.get(1))) return "0";

else if (vector.get(0).equalsIgnoreCase("atan") &&
  vector.get(1).equalsIgnoreCase("x")) return "(1/(1+x^2))";
else if (vector.get(0).equalsIgnoreCase("atan") &&
  vector.get(1).equalsIgnoreCase("-x")) return "(-1/(1+x^2))";
else if (vector.get(0).equalsIgnoreCase("atan") &&
  isNumber(vector.get(1))) return "0";

else if (vector.get(0).equalsIgnoreCase("asec") &&
  vector.get(1).equalsIgnoreCase("x")) return "(1/x*sqrt((x^2)-1))";
else if (vector.get(0).equalsIgnoreCase("asec") &&
  vector.get(1).equalsIgnoreCase("-x")) return "(-1/(x*sqrt((x^2)-1)))";
else if (vector.get(0).equalsIgnoreCase("asec") &&
  isNumber(vector.get(1))) return "0";

else if (vector.get(0).equalsIgnoreCase("exp") &&
  vector.get(1).equalsIgnoreCase("x")) return "(exp(x))";
else if (vector.get(0).equalsIgnoreCase("exp") &&
  vector.get(1).equalsIgnoreCase("-x")) return "(-exp(-x))";
else if(vector.get(0).equalsIgnoreCase("exp") && isNumber(vector.get(1))) return "0";
else if(vector.get(0).equalsIgnoreCase("ln") && vector.get(1).equalsIgnoreCase("x")) return "(1/x)";
else if(vector.get(0).equalsIgnoreCase("-x")) return "(1/x)";
else if(vector.get(0).equalsIgnoreCase("ln") && isNumber(vector.get(1))) return "0";
else if(vector.get(0).equalsIgnoreCase("log") && vector.get(1).equalsIgnoreCase("x")) return "(1/(x*ln(10)))";
else if(vector.get(0).equalsIgnoreCase("log") && vector.get(1).equalsIgnoreCase("-x")) return "(1/(x*ln(10)))";
else if(vector.get(0).equalsIgnoreCase("log") && isNumber(vector.get(1))) return "0";
else if(vector.get(0).equalsIgnoreCase("sqrt") && vector.get(1).equalsIgnoreCase("x")) return "(1/(2*sqrt(x)));
//sqrt is not defined or -ve
else if(vector.get(0).equalsIgnoreCase("sqrt") && isNumber(vector.get(1))) return "0";
else{return "expression not suported";}
}
else //recursive cals
{
  boolean state=true; //for +,-
  boolean state2=true; //for */\
  boolean state3=true; //for ^
  boolean state4=true; //for sin cos,exp,...
  int i=0;///<bracket tracer
  int k = 0;///<while terminator
  while(state)
  {
    if(vector.get(k).equals("("){ i+=1; k+=1; }
    else if(vector.get(k).equals(")")} {i-=1; k+=1; }
    else if(vector.get(k).equals("+") && i==0)//....+
    {
      state2=false; //block the next while block
      state3=false; state4=false;
      String s=
      findDerivative(getVectorFrom(vector, 0, k-1)).concat("+").concat
      (findDerivative(getVectorFrom(vector, k+1, vector.size()-1)) );
      return "(".concat(s).concat("")");
    }
    else if(vector.get(k).equals("-") && i==0)//....-
    {
      state2=false; //block the next while block
      state3=false; state4=false;
      String s=
      findDerivative(getVectorFrom(vector, 0, k-1)).concat("-").concat
      (findDerivative(getVectorFrom(vector, k+1, vector.size()-1)) );
      return "(".concat(s).concat("")");
    }
    else{ k+=1; }
}
if(k==vector.size()-1) state = false;

k=0; i=0;
while(state2)//if + or - not detected in the first part
this part will work
{
    if(vector.get(k).equals("") { i+=1; k+=1; }
    else if(vector.get(k).equals("-")) { i-=1; k+=1; }
    else if(vector.get(k).equals("*"))
            { state3=false; state4=false;

            Vector<String> term1 =
            getVectorFrom(vector, 0, k-1); //first term

            Vector<String> term2 =
            getVectorFrom(vector, k+1, vector.size()-1); //second term

            String sl="(".concat(vectorToString(term1)).concat("")\).concat("*")\).concat("(")
            .concat(findDerivative(term2)).concat("")
            .concat("\(\)\).concat("\)\).concat("\)\).

            return
            "\((\".concat(sl).concat("\)\).concat("+")\).concat("\).concat(s2).concat("\)\)\)\);"
            }
    else if(vector.get(k).equals("/"))
            { state3=false; state4=false;

            Vector<String> num = getVectorFrom(vector,
            0, k-1); // numerator and denominator

            Vector<String> den = getVectorFrom(vector,
            k+1, vector.size()-1); // denominator

            String s1 =
            vectorToString(den).concat("*")\).concat(findDerivative(num))

            String s2 =
            vectorToString(num).concat("*")\).concat(findDerivative(den))

            String s3 =
            "\(".concat(vectorToString(den)).concat("\)\)\).concat("\)\)\);"

            return
            "\(((\".concat(s1).concat("\)\).concat("-"
            \)).concat("\).concat(s2).concat("\))\)\).concat(s3).concat("\)\)\);"

            } // else if
    else{ k+=1; }
    if(k==vector.size()-1) state2=false;
}

k=0; i=0;
while(state3)// if * 0 or / not detected in the first part
this part will work
{   //u^-v
    if(vector.get(k).equals("") { i+=1; k+=1; }

    "((\".concat(s1).concat("\)\).concat("+")\).concat("\).concat(s2).concat("\))\)\).concat(s3).concat("\)\)\);"

    if(k==vector.size()-1) state2=false;

    
}
else if(vector.get(k).equals("\n\n")) {i-=1; k+=1;}
else if(vector.get(k).equals("\n\n"))
{
    state4=false;
    Vector<String> term1 =
    getVectorFrom(vector, 0, k-1); //first term
    Vector<String> term2 =
    getVectorFrom(vector, k+1, vector.size()-1); //second term

    if(term2.size()==1&&isNumber(term2.get(0))&&!("x").equalsIgnoreCase(term2.get(0)))
    {
        double t2 =
        Double.valueOf(term2.get(0).trim()).doubleValue();
        String p = Double.toString(t2);
        //power
        String pml = Double.toString(t2-1);
        //power-1

        String s1 ="".concat(p).concat("\n\n").concat("\n\n").concat(vectorToString(term1))
        .concat("\n\n").concat(pml).concat("\n\n");
        String s2 = findDerivative(term1);
        return
        
        ("".concat(s1).concat("\n\n").concat(s2).concat("\n\n"));
    }
    else{ //sl=(first*second)
        String
        sl="".concat(vectorToString(term1)).concat("\n\n").concat("\n\n").concat(vectorToString(term2)).concat("\n\n");
        //s2= (second*d(first)/first)
        String s2=
        """.concat(vectorToString(term2)).concat("\n\n").concat("\n\n").concat(findDerivative(term2)).concat("\n\n");
        //s3 = ln(first)*d(second)
        String s3=
        "".concat("\n\n").concat(vectorToString(term1)).concat("\n\n");
        //s4= (second*first)".concat(findDerivative(term2)).concat("\n\n");
        String s4="".concat(s2).concat("\n\n").concat(s3).concat("\n\n");

        String strin=
        "".concat(s2).concat("\n\n").concat(s4).concat("\n\n");
        //fixing the ln part when multiplied by 0
        return strin;
    }
}

//else if
else{ k+=1;}
if(k==vector.size()-1) state3=false;
k=0;i=0;
while(state4)// if = or - not detected in the first part
this part will work
{
    if(vector.get(k).equalsIgnoreCase("sin"))/sin...
    ->cos
    {
        state4=false;
        Vector<String> term = getVectorFrom(vector,
        k+1, vector.size()-1);
        return
        "((cos(".concat(vectorToString(term)).concat("")))).concat("*")
        .concat(findDerivative(term)).concat(""));
    }
    else
    if(vector.get(k).equalsIgnoreCase("cos"))/cos...-->sin
    {
        state4=false;
        Vector<String> term = getVectorFrom(vector,
        k+1, vector.size()-1);
        return "(((-
        sin(".concat(vectorToString(term)).concat("")))).concat("*")
        .concat(findDerivative(term)).concat("")";
    }
    else
    if(vector.get(k).equalsIgnoreCase("tan"))/tan...--> sec square.
    {
        state4=false;
        Vector<String> term = getVectorFrom(vector,
        k+1, vector.size()-1);
        return
        "((sec(".concat(vectorToString(term)).concat("")))).concat("^2")).concat("*"
        .concat(findDerivative(term)).concat(""));
    }
    else
    if(vector.get(k).equalsIgnoreCase("cot"))/cot...--> csc square
    {
        state4=false;
        Vector<String> term = getVectorFrom(vector,
        k+1, vector.size()-1);
        return "((-
        (csc(".concat(vectorToString(term)).concat("")))).concat("^2")).concat("*")
        .concat(findDerivative(term)).concat("")";
    }
    else
    if(vector.get(k).equalsIgnoreCase("sec"))/sec...--> sec tan
    {
        state4=false;
        Vector<String> term = getVectorFrom(vector,
        k+1, vector.size()-1);
        return
        "((sec(".concat(vectorToString(term)).concat(""))")
        }
concat("*\(\tan(\)\).concat(vectorToString(term)).concat("))")
.concat("*\(\)\).concat(findDerivative(term)).concat("))
} else
if(vector.get(k).equalsIgnoreCase("csc")//csc...
->csc cot
{
  state4=false;
  Vector<String> term = getVectorFrom(vector,
  k+1, vector.size()-1);
  return "((-\(csc(\).concat(vectorToString(term)).concat(\)\)))")
.concat("*\(\).concat(findDerivative(term)).concat("))
} else
if(vector.get(k).equalsIgnoreCase("asin")//asin...
->1/sqrt(1-x^2)
{
  state4=false;
  Vector<String> term = getVectorFrom(vector,
  k+1, vector.size()-1);
  return "((1/(".concat("sqrt(").concat("1-
.(").concat(vectorToString(term))
.concat("\(\)\))).concat("\(\^2\)\)\))).concat("\(\)\).concat(findDerivative(term)).concat("))")
} else
if(vector.get(k).equalsIgnoreCase("acos")//acos...
->1/sqrt(1-x^2)
{
  state4=false;
  Vector<String> term = getVectorFrom(vector,
  k+1, vector.size()-1);
  return "((-1/(".concat("sqrt(").concat("1-
.(").concat(vectorToString(term))
.concat("\(\)\))).concat("\(\)\))).concat(findDerivative(term)).concat("))")
} else
if(vector.get(k).equalsIgnoreCase("atan")//atan...
->1/(1+x^2)
{
  state4=false;
  Vector<String> term = getVectorFrom(vector,
  k+1, vector.size()-1);
  return "((1/(".concat("(1+\(\)\).concat(vectorToString(term))
.concat("\(\)\))).concat("\(\)\)).concat(findDerivative(term)).concat("))")
} else
if(vector.get(k).equalsIgnoreCase("asec")//asec...
->1/x*sqrt(x^2-1)
{
  state4=false;
  Vector<String> term = getVectorFrom(vector,
  k+1, vector.size()-1);
  return "((1/(".concat(vectorToString(term)).concat(")\)*(sqrt("))"}
.concat(vectorToString(term)).concat("^2\)-1)))"

.concat("\"\".concat(findDerivative(term)).concat("\")
} else
if(vector.get(k).equalsIgnoreCase("exp")//exp -->expx....

    state4=false;
    Vector<String> term = getVectorFrom(vector,
    k+1, vector.size()-1);
    return
"((exp(".concat(vectorToString(term)).concat("\")
    .concat(findDerivative(term)).concat("\")
    )
} else
if(vector.get(k).equalsIgnoreCase("ln")//lnx -->1/x

    state4=false;
    Vector<String> term = getVectorFrom(vector,
    k+1, vector.size()-1);
    return
"((1/".concat(vectorToString(term)).concat(")*("
    .concat(findDerivative(term)).concat("\")
    )
} else
if(vector.get(k).equalsIgnoreCase("log")//log -->1/(x*ln10)

    state4=false;
    Vector<String> term = getVectorFrom(vector,
    k+1, vector.size()-1);
    return
"((1/((".concat(vectorToString(term)).concat(")\}(ln(10)))))*("
    .concat(findDerivative(term)).concat("\")
    )
} else
if(vector.get(k).equalsIgnoreCase("sqrt")//sqrt -->1/(2*sqrtx)

    state4=false;
    Vector<String> term = getVectorFrom(vector,
    k+1, vector.size()-1);
    return
"((1/(2*(sqrt(".concat(vectorToString(term)).concat(")))))")
    .concat("\")
    .concat(findDerivative(term)).concat("\")
} else{ k+=1;
    if(k==vector.size()-1) state4=false;
    }

//else
return" more rules";

}//findDerivative(..)
public static void main(String[] args) {

} // main

} // class derivativeFinder
/**********************...
* Class MaxMinFinder
* Ehab W. Aziz Rezk
* **********************
import java.util.*;

public class MaxMinFinder{
    private String expression; //the equation
    private String firstDerivative;
    private String secondDerivative;

    public MaxMinFinder(String str)
    {
        expression = str;
        DerivativeFinder df1 = new DerivativeFinder(expression);
        StringToVectorTokenizer stvl = new StringToVectorTokenizer(expression);
        Vector<String> ve = stvl.tokenizeToVector();
        firstDerivative = df1.findDerivative(ve);

        DerivativeFinder df2 = new DerivativeFinder(expression);
        StringToVectorTokenizer stv2 = new StringToVectorTokenizer(firstDerivative);
        ve = stv2.tokenizeToVector();
        secondDerivative = df2.findDerivative(ve);
    }

    public String retExpression()
    {
        return expression;
    }
    public String ret1st()
    {
        return firstDerivative;
    }
    public String ret2nd()
    {
        return secondDerivative;
    }

    /* findMaxMin(..) where from and to are the boundary of the range through which the maxima and minima are to be found. step is the amount by which the function divides the region to find the roots, eps is the accuracy of finding the roots **/
    public Vector<String> findMaxMin(double from, double to, double step, double eps)
    {
        RootFinder rf = new RootFinder(firstDerivative, eps);
Vector<String> vec = rf.findAllRoots(from, to, step);
return vec;

public static void main(String[] args)
{
    Scanner sc = new Scanner(System.in);
    System.out.print("write down the Equation ");
    String equation = sc.nextLine();
    MaxMinFinder mfl = new MaxMinFinder(equation);
    String firstd = mfl.retlst();
    String secondd = mfl.retlst();
    Equation eq1 = new Equation(firstd);
    Equation eq2 = new Equation(secondd);
    Scanner sc2 = new Scanner(System.in);
    System.out.println("for x = ");
    String input = sc2.nextLine();
    double value = Double.valueOf(input.trim()).doubleValue();
    String x = "x";
    System.out.println("\n 1st = " + eq1.Evaluate(x, value));
    System.out.println("\n 2nd = " + eq2.Evaluate(x, value));
    System.out.println("a = ");
    Scanner sc3 = new Scanner(System.in);
    String a = sc3.nextLine();
    System.out.println("b = ");
    Scanner sc4 = new Scanner(System.in);
    String b = sc4.nextLine();
    double from = Double.valueOf(a.trim()).doubleValue();
    double to = Double.valueOf(b.trim()).doubleValue();
    Vector<String> v = mfl.findMaxMin(from, to, 1, 0.0001);
    for (int i = 0; i < v.size(); i++)
        System.out.println(v.get(i) + ", ");
}

} //main
public class RootFinder {
    static double midValue(double x1, double x2) {
        //return the mid value of x1 and x2
        return (x1 + x2) / 2;
    }
    static double abs(double a) {
        //returns the absolute value of a
        if (a == 0.0) return 0.0;
        else if (a > 0.0) return a;
        else return -1 * a;
    }
    static boolean almostequals(double a, double b) {
        if (abs(a - b) < 0.009) return true;
        else return false;
    }
    private String expression; //the equation
    private double epsilon; //the accuracy in determining the root
    public RootFinder(String str, double eps) {
        expression = str;
        epsilon = eps; //the accuracy in determining the root
    }
    public double Evaluate(double a) {
        Equation e = new Equation(expression);
        return e.Evaluate("x", a);
    }
    public String firstDerivative() {
        //return first derivative of expression
        DerivativeFinder df1 = new DerivativeFinder(expression);
        StringToVectorTokenizer stvl = new StringToVectorTokenizer(expression);
        Vector<String> ve = stvl.tokenizeToVector();
        return df1.findDerivative(ve);
    }
    public String secondDerivative() {
        //returns the second derivative of expression
        DerivativeFinder df1 = new DerivativeFinder(expression);
        StringToVectorTokenizer stvl = new StringToVectorTokenizer(expression);
        Vector<String> ve = stvl.tokenizeToVector();
        String fd = df1.findDerivative(ve);
    }
DerivativeFinder df2 = new DerivativeFinder(expression);
StringToVectorTokenizer stv2 = new StringToVectorTokenizer(fd);
ve = stv2.tokenizeToVector();
return df2.findDerivative(ve);

public double fDEvaluate(double a)
{
    //return the double value of the 2nd derivative
    String firstDerivative = firstDerivative();
    Equation eq = new Equation(firstDerivative);
    return eq.Evaluate("x", a);
}

public double sDEvaluate(double a)
{
    //return the double value of the 2nd derivative
    String secondDerivative = secondDerivative();
    Equation eq = new Equation(secondDerivative);
    return eq.Evaluate("x", a);
}

boolean theFuncSlopeChange(double xl, double x2)
{
    //return true if the function slope change between xl, x2
    if ((fDEvaluate(xl) * fDEvaluate(x2)) < 0) return true;
    else return false;
}

boolean theCurveCrossX(double xl, double x2)
{
    //return true if the root lies between x1 and x2
    //i.e.. the function has different signs on both directions
    double value1 = Evaluate(xl);
double value2 = Evaluate(x2);
    if ((value1 < 0.0 && value2 > 0.0) || (value1 > 0.0 && value2 < 0.0)) return true;//they are dif in signs
    else return false;
}

//find all roots in the interval[v1, v2]taking a step Stepp
public Vector<String> findAllRoots(double v1, double v2, double stepp)
{
    Vector<String> vector = new Vector<String>();//the vector that carries the roots if found
    vector.add("");
    int n = 0; //number of times y is +ve
    int m = 0; //number of times y = -ve
    for (double k = v1; k <= v2 + 0.1; k += stepp)
    {
        if (Evaluate(k) == 0.0) vector.add(Double.toString(k));
        else if (Evaluate(k) > 0.0) n++;
        else (m++); //if (Evaluate(k) < 0.0)m++;
        if (n > 0 && m > 0) //there is crossing
        {
            String root = findRootByBisection(k-stepp,
k+stepp);
            vector.add(root);
            m = 0; n = 0;
        }
    }
}
for(double k=vl; k<=v2+0.1; k+=stepp)//only detext the touch points
{
    if(Evaluate(k-stepp)>0&&Evaluate(k+stepp)>0
        &&fDEvaluate(k-stepp)<0&&fDEvaluate(k+stepp)>0)
    {
        String root = findRootByNewton(k);
        if(!root.equals("no root exist")&&!Double.valueOf(root).doubleValue()>=vl&&
            Double.valueOf(root).doubleValue()<=v2)
            vector.add(root);
    }
    if(Evaluate(k-stepp)<0&&Evaluate(k+stepp)<0
        &&fDEvaluate(k-stepp)>0&&fDEvaluate(k+stepp)<0)
    {
        String root = findRootByNewton(k);
        if(!root.equals("no root exist")&&!Double.valueOf(root).doubleValue()>=vl&&
            Double.valueOf(root).doubleValue()<=v2)
            vector.add(root);
    }
    //adjustiing the vector to be returned
    if(vector.size()==1&&vector.get(0).equals(""))
        vector.set(0, "NaN");
    else vector.remove(0);
    Vector<Double> vect = new Vector<Double>();
    for(int l=0; l<vector.size(); l++)
    {
        if(vector.get(l).equals("no root exist")||vector.get(l).equals(""))
            vector.remove(l);
    }
    for(int l=0; l<vector.size()-1; l++)
    {
        double d1=Double.valueOf(vector.get(l)).doubleValue();
        double d2=Double.valueOf(vector.get(l+1)).doubleValue();
        if(almostequals(d1, d2))vector.remove(l);
    }
    return vector;
} //find all roots

public String findRootByNewton(double vl)
{
    double Xno =vl;
    double Xnl =Xno -(Evaluate(Xno)/fDEvaluate(Xno)); //Xno-f(Xno)/f'(Xno)
    if(Evaluate(vl)==0.0) return Double.toString(vl);
    else
    {
        double n =0; //number of iterations after which the
        fuction terminates
            //if the roots didn't show up
        while(abs(Xno-Xnl)>epsilon&n<10)
if(Evaluate(Xn1)==0) return Double.toString(Xn1);
else
{
    Xno=Xn1;
    Xn1=Xno - (Evaluate(Xno)/fDEvaluate(Xno)); //Xno-f(Xno)/f'(Xno)
    n++;
}

if(abs(Evaluate(Xn1))<=0.009) return Double.toString(Xn1);
else return "no root exist";

public String findRootByNewton()
{
    double L=Xn1;
    double R=Xno - (Evaluate(Xno)/fDEvaluate(Xno)); //Xno-f(Xno)/f'(Xno)
    n++;
    while((R-L)>epsilon)
    {
        double vo=midValue(L, R);
        if(Evaluate(vo)==0.0) return Double.toString(vo);
        else if(theCurveCrossX(L, vo)) R=vo;
        else L=vo;
    }
    double ret=midValue(L,R);
    retString= Double.toString(ret);
}

return retString;
}

public static void main(String[ ] args)
{
    //main
}

}//class
import java.util.*;
import java.awt.*;
import java.awt.event.*;
import java.applet.*;

public class Applett extends Applet {

    /** Current xValue */
    private double xValue;
    private double x_low, x_high, y_low, y_high; // the values of the axes boundaries.
    private double divx, labx, divy, laby; // the amount to divide and label axes
    private String expression, expression2;
    private final static String newline = "\n";
    /* GUI components */

    // menu items
    private static MenuItem clear = new MenuItem("Clear");
    private static MenuItem savem = new MenuItem("Save");
    private static MenuItem printm = new MenuItem("Print");
    private static MenuItem diffeq = new MenuItem("Diff.Eq");
    private static MenuItem phaseplane = new MenuItem("Phase Plane");
    private static MenuItem dview = new MenuItem("3D view");

    private TextField exprField, exprField2, xField;
    private Label expressionEquals, expressionEquals2, forX, result;
    private Button yEquals;

    private static GraphArea ga;
    private Label lowx, highx, lowy, highy, dividx, labelx, dividy, labely;
    private Label Y1SlopeAtXEquals, Y2SlopeAtXEquals;
    private TextField xlow, xhigh, ylow, yhigh, dividX, labelX, dividY, labelY;
    private TextField Y1SlopeXValue, Y2SlopeXValue;
    private Button drawY1, drawY2, clearY1, clearY2;
    private Button drawY1Slope, drawY2Slope, clearY1Slope, clearY2Slope;
    private Button drawY1SlopeCurve, drawY2SlopeCurve, clearY1SlopeCurve, clearY2SlopeCurve;

    // panel 2
    private Label RootRFrom, RootRTo, RootStep;
    private TextField RRF, RRT, RS;
    private Button FindRoots, defIntegral, showIntegral, clearIntegral;
    Label Max_Min;
    private TextArea RootsTextArea, MaxMin, integralResult;

    // panel 3
//panel 4
private Button Diff_Eq, phase_plane, DView, print, save;

public static void main(String args[]) {
    Applett a = new Applett();
a.init();
a.start();

    Frame f = new Frame("MAT WIN");
f.add("Center", a);
f.setSize(900, 590);
    Image icon =
    Toolkit.getDefaultToolkit().getImage("../icon.gif");
f.setImage(icon);

    f.addWindowListener(
        new WindowAdapter()
        {
            public void windowClosing(WindowEvent e) { System.exit(0); }
        });

    //Menue Bar
    MenuBar myMenuBar = new MenuBar();
    Menu fileMenu = new Menu("File");
    Menu editMenu = new Menu("Edit");
    Menu functionsMenu = new Menu("Functions");
    Menu helpMenu = new Menu("Help");

    fileMenu.add(savem);
    fileMenu.add(printm);

    editMenu.add(clear);

    functionsMenu.add(diffeq);
    functionsMenu.addSeparator();
    functionsMenu.add(phaseplane);
    functionsMenu.addSeparator();
    functionsMenu.add(dview);

    myMenuBar.add(fileMenu);
    myMenuBar.add(editMenu);
    myMenuBar.add(functionsMenu);
    myMenuBar.add(helpMenu);
f.setMenuBar(myMenuBar);
    f.setVisible(true);
}

public void init() {
    xValue = 10.0;
    x_low = -100; x_high = 100; y_low = -100; y_high = 100; //the default values
    divx = 10; labx = 20; divy = 10; laby = 20;
expression = "100* sin(x/5)* cos(x/50)";
expression2 = "(x/7)^2";
// add the interface components
addGUIComponents1();
}

/**
 * Creates and adds the necessary GUI components.
 */
private void addGUIComponents1() {
    setBackground(Color.white);

    GridBagLayout gridbag = new GridBagLayout();
    GridBagConstraints c = new GridBagConstraints();
    setLayout(gridbag);
    c.fill = GridBagConstraints.BOTH;
    c.weightx = 1.0;

    //Panell
    Panel panell = new Panel(gridbag);
    //the Expression textfields
    expressionEquals = new Label("Expression Y1 =", Label.LEFT);
    c.gridx=1;
    panell.add(expressionEquals, c);
    exprField = new TextField(expression, 20);//default expression = x*x
    c.gridwidth = GridBagConstraints.REMAINDER;
    panell.add(exprField, c);
    c.gridy=2;
    expressionEquals2 = new Label("Expression Y2 =", Label.LEFT);
    c.gridwidth = 1;
    panell.add(expressionEquals2, c);
    exprField2 = new TextField(expression2, 20);//default expression = x*x
    c.gridwidth = GridBagConstraints.REMAINDER;
    panell.add(exprField2, c);

    //The graph area
    c.gridx=3;
    c.insets = new Insets(0, 0, 2, 0);
    c.fill = GridBagConstraints.BOTH; c.weighty=1;
    ga = new GraphArea(x_low,x_high, y_low, y_high);
    ga.setExpression(exprField.getText());
    ga.setAxesDevisionsAndLabels(divx,labx,divy,laby);
    c.gridwidth = GridBagConstraints.REMAINDER; pane1.add(ga,c);

    //adding panell
    c.fill=GridBagConstraints.NONE;
    c.gridx=0; c.gridy=0; c.gridwidth =5;
    c.weighty=0;c.gridheight=1; add(panell,c);

    //panel2
    c.fill=GridBagConstraints.HORIZONTAL;
Panel panel2 = new Panel(gridbag);
forX = new Label("for X =", Label.LEFT);
c.gridx=0; c.gridwidth = GridBagConstraints.RELATIVE;
panel2.add(forX, c);

xField = new TextField("" + xValue,5);
c.gridx=1; c.gridy=0;
c.gridwidth = GridBagConstraints.REMAINDER;
panel2.add(xField, c);

yEquals = new Button("Y = ");
c.gridx=0; c.gridy=1;
panel2.add(yEquals, c);

result = new Label(" ", Label.LEFT);
c.gridwidth = GridBagConstraints.REMAINDER; 
c.gridy=2;
panel2.add(result, c);

RootRFrom = new Label("From ", Label.LEFT);
RootRTo = new Label("To ", Label.LEFT);
R RF  = new TextField("", 5);
R RT  = new TextField("", 5);
RS   = new TextField("", 5);
FindRoots = new Button("Roots =");
c.gridy=3; c.gridwidth = GridBagConstraints.REMAINDER;
panel2.add(RootRFrom,c);
    c.gridx=1; c.gridwidth=GridBagConstraints.REMAINDER;
panel2.add(RRF,c);
    c.gridx=0; panel2.add(RootRTo,c);
    c.gridx=1; c.gridwidth=GridBagConstraints.REMAINDER;
panel2.add(RRT,c);
    c.gridx=0; panel2.add(Rootstep,c);
    c.gridx=1; c.gridwidth=GridBagConstraints.REMAINDER;
panel2.add(RS,c);
    c.gridx=6; c.gridx=0; panel2.add(FindRoots,c);

RootsTextArea = new TextArea("",7,1);
RootsTextArea.setEditable(false);
RootsTextArea.setBackground(Color.white);
c.gridy=8;
panel2.add(defIntegral,c);

defIntegral = new Button("Def Integral =");
showIntegral = new Button("Show");
c.gridx=10; c.gridwidth=GridBagConstraints.REMAINDER;
panel2.add(defIntegral ,c);
    c.gridx=11; panel2.add(integralResult ,c);
    c.gridx=12; c.gridx=0; c.gridwidth=1;
panel2.add(showIntegral, c);
c.gridx=1; c.gridwidth=1;
panel2.add(clearIntegral, c);

//adding panel2
c.gridx=5; c.gridy=0; c.gridwidth=1; c.gridheight=1;
add(panel2,c);

//panel3
c.gridheight=1;
Panel panel3 = new Panel(gridbag);
//adding boundary buttons
lowx = new Label("Low x =", Label.LEFT);
c.gridx=0; c.gridy=0; c.gridwidth = GridBagConstraints.RELATIVE;
panel3.add(lowx,c);
xlow = new TextField("x_low",3);
c.gridx=1; c.gridy=0; c.gridwidth = GridBagConstraints.REMAINDER;
panel3.add(xlow,c);

highx = new Label("High x =", Label.LEFT);
c.gridx=0; c.gridy=1; c.gridwidth = GridBagConstraints.RELATIVE;
panel3.add(highx,c);
xhigh = new TextField("x_high",3);
c.gridx=1; c.gridy=1; c.gridwidth = GridBagConstraints.REMAINDER;
panel3.add(xhigh,c);

lowy = new Label("Low y =");
c.gridx=0; c.gridy=2; c.gridwidth = GridBagConstraints.RELATIVE;
panel3.add(lowy,c);
ylow = new TextField("y_low",3);
c.gridx=1; c.gridy=2; c.gridwidth = GridBagConstraints.REMAINDER;
panel3.add(ylow,c);

highy = new Label("High y =");
c.gridx=0; c.gridy=3; c.gridwidth = GridBagConstraints.RELATIVE;
panel3.add(highy,c);
yhigh = new TextField("y_high",3);
c.gridx=1; c.gridy=3; c.gridwidth = GridBagConstraints.REMAINDER;
panel3.add(yhigh,c);

//X and Y divisions and labeling
dividx = new Label("Divide X into", Label.LEFT);
c.gridx=0; c.gridy=4; c.gridwidth = GridBagConstraints.RELATIVE;
panel3.add(dividx,c);
dividX = new TextField("divx",3);
c.gridx=1; c.gridy=4; c.gridwidth = GridBagConstraints.REMAINDER;
panel3.add(dividX,c);

labelx = new Label("Label X every");
c.gridx=0; c.gridy=5; c.gridwidth = GridBagConstraints.RELATIVE;
panel3.add(labelx,c);
labelX = new TextField("labx",3);
c.gridx=1; c.gridy=5; c.gridwidth = GridBagConstraints.REMAINDER;
panel3.add(labelX,c);

dividy = new Label("Divide Y into", Label.LEFT);
c.gridx=0; c.gridy=6; c.gridwidth = GridBagConstraints.RELATIVE;
panel3.add(dividy,c);
dividY = new TextField("divy",3);
c.gridx=1; c.gridy=6; c.gridwidth = GridBagConstraints.REMAINDER;
panel3.add(dividY,c);

labely = new Label("Label Y every", Label.LEFT);
c.gridx=0; c.gridy=7; c.gridwidth = GridBagConstraints.RELATIVE; panel3.add(labelY, c);

labelY = new TextField("" + laby, 3);
c.gridx=1; c.gridy=7; c.gridwidth = GridBagConstraints.REMAINDER; panel3.add(labelY, c);

// Buttons draw, clear
drawYl = new Button("Draw Y1");
clearYl = new Button("Clear Y1");
c.gridx=0; c.gridy=8; c.gridwidth = GridBagConstraints.RELATIVE; panel3.add(drawYl, c);
c.gridx=1; c.gridy=8; c.gridwidth = GridBagConstraints.REMAINDER; panel3.add(clearYl, c);

drawY2 = new Button("Draw Y2");
clearY2 = new Button("Clear Y2");
c.gridx=0; c.gridy=9; c.gridwidth = GridBagConstraints.RELATIVE; panel3.add(drawY2, c);
c.gridx=1; c.gridy=9; c.gridwidth = GridBagConstraints.REMAINDER; panel3.add(clearY2, c);

// Buttons for drawing and clearing the tangent lines
YISlopeAtXEquals = new Label("For X=", Label.LEFT);
YISlopeXValue = new TextField("", 3);
drawY1Slope = new Button("Draw Y1 Tangent Line");
Y2SlopeAtXEquals = new Label("For X=", Label.LEFT);
Y2SlopeXValue = new TextField("", 3);
drawY2Slope = new Button("Draw Y2 Tangent Line");
clearY1Slope = new Button("Clear line");
clearY2Slope = new Button("Clear line");
c.gridx=0; c.gridy=10; c.gridwidth = GridBagConstraints.RELATIVE; panel3.add(YISlopeAtXEquals, c);
c.gridx=1; c.gridy=10; c.gridwidth = GridBagConstraints.REMAINDER; panel3.add(YISlopeXValue, c);
c.gridx=0; c.gridy=11; c.gridwidth = GridBagConstraints.REMAINDER; panel3.add(drawY1Slope, c);
c.gridx=1; c.gridy=11; c.gridwidth = GridBagConstraints.REMAINDER; panel3.add(clearY1Slope, c);
c.gridx=0; c.gridy=12; c.gridwidth = GridBagConstraints.REMAINDER; panel3.add(drawY2Slope, c);
c.gridx=1; c.gridy=12; c.gridwidth = GridBagConstraints.REMAINDER; panel3.add(clearY2Slope, c);

// Buttons for drawing slope curves
drawY1SlopeCurve = new Button("Draw Y1' Curve");
drawY2SlopeCurve = new Button("Draw Y2' Curve");
clearY1SlopeCurve = new Button("Clear Y1'");
clearY2SlopeCurve = new Button("Clear Y2'");
c.gridx=16; c.gridy=16; c.gridwidth = GridBagConstraints.REMAINDER; panel3.add(drawY1SlopeCurve, c);
c.gridx=17; c.gridy=17; c.gridwidth = GridBagConstraints.REMAINDER; panel3.add(clearY1SlopeCurve, c);
c.gridx=18; c.gridy=18; c.gridwidth = GridBagConstraints.REMAINDER; panel3.add(drawY2SlopeCurve, c);
c.gridy=19; c.gridwidth = GridBagConstraints.REMAINDER;
panel3.add(clearY2SlopeCurve,c);

//adding panel3
  c.fill=GridBagConstraints.NONE;
  c.gridx=6; c.gridy=0; c.gridwidth =1;c.gridheight=1;
add(panel3,c);

//panel4
GridLayout bl = new GridLayout(1,5);
Panel panel4 = new Panel(bl);
  Diff_Eq = new Button("Diff Eq");
  phase_plane = new Button("Phase Plane");
  DView = new Button("3D View");
  print = new Button("Print");
  save = new Button("Save");
  //c.gridwidth = 1; c.weightx = 0.5;
  //c.ipady = 40; //make it tall
  //c.gridx=0; c.gridy=1;
panel4.add(Diff_Eq);
  //c.gridx=1; c.gridy=1;
panel4.add(phase_plane);
  //c.gridx=2; c.gridy=1;
panel4.add(DView);
  //c.gridx=3; c.gridy=1;
panel4.add(print);
  //c.gridx=4; c.gridy=1;
panel4.add(save);

  //c.ipady = 0; //return it back
//adding panel4
  c.fill=GridBagConstraints.BOTH;
  c.gridx=0; c.gridy=1; c.gridwidth =7; add(panel4,c);

/** Adding Listeners   */
clear.addActionListener(new ActionListener(){
      public void actionPerformed(ActionEvent evt){
          clearChosen();
      }
  });
diffeq.addActionListener(new ActionListener(){
      public void actionPerformed(ActionEvent evt){
          Diff_EqPressed();
      }
  });
phaseplane.addActionListener(new ActionListener(){
      public void actionPerformed(ActionEvent evt){
          phase_planePressed();
      }
  });

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dview.addActionListener(new ActionListener(){
    public void actionPerformed(ActionEvent evt){
        DViewPressed();
    }
});
printm.addActionListener(new ActionListener(){
    public void actionPerformed(ActionEvent evt){
        ga.print();
    }
});
savem.addActionListener(new ActionListener(){
    public void actionPerformed(ActionEvent evt){
        ga.SaveGraphicsToFile();
    }
});

//when a new expression is inserted into expression field
xField.addTextListener(new TextListener(){
    public void textValueChanged(TextEvent event){
        xFieldTextValueChanged();
    }
});

//when y= button pressed
yEquals.addActionListener(new ActionListener(){
    public void actionPerformed(ActionEvent event){
        exprFieldTextValueChanged();
    }
});

//when draw buttons is pressed
drawY1.addActionListener(new ActionListener(){
    public void actionPerformed(ActionEvent event){
        drawY1ButtonPressed();
    }
});
drawY2.addActionListener(new ActionListener(){
    public void actionPerformed(ActionEvent event){
        drawY2ButtonPressed();
    }
});
clearY1.addActionListener(new ActionListener(){
    public void actionPerformed(ActionEvent event){
        clearY1ButtonPressed();
    }
});
clearY2.addActionListener(
    new ActionListener()
    {
        public void actionPerformed(ActionEvent evt) {
            clearY2ButtonPressed();
        }
    }
);}

drawY1Slope.addActionListener(
    new ActionListener()
    {
        public void actionPerformed(ActionEvent evt) {
            drawY1SlopePressed();
        }
    }
);}
clearY1Slope.addActionListener(
    new ActionListener()
    {
        public void actionPerformed(ActionEvent evt) {
            clearY1SlopePressed();
        }
    }
);}
drawY2Slope.addActionListener(
    new ActionListener()
    {
        public void actionPerformed(ActionEvent evt) {
            drawY2SlopePressed();
        }
    }
);}
clearY2Slope.addActionListener(
    new ActionListener()
    {
        public void actionPerformed(ActionEvent evt) {
            clearY2SlopePressed();
        }
    }
);}
drawY1SlopeCurve.addActionListener(
    new ActionListener()
    {
        public void actionPerformed(ActionEvent evt) {
            drawY1SlopeCurvePressed();
        }
    }
);}
clearY1SlopeCurve.addActionListener(
    new ActionListener()
    {
        public void actionPerformed(ActionEvent evt) {
            clearY1SlopeCurvePressed();
        }
    }
);}
drawY2SlopeCurve.addActionListener(
    new ActionListener()
    {
        public void actionPerformed(ActionEvent evt) {
            drawY2SlopeCurvePressed();
        }
    }
);}
clearY2SlopeCurve.addActionListener(
    new ActionListener()
    {
        public void actionPerformed(ActionEvent evt) {
            clearY2SlopeCurvePressed();
        }
    }
);
FindRoots.addActionListener(new ActionListener()
    {
       public void actionPerformed(ActionEvent evt) {
          FindRootsPressed();
       }
    });

showIntegral.addActionListener(new ActionListener()
    {
       public void actionPerformed(ActionEvent evt) {
          showIntegralPressed();
       }
    });

clearIntegral.addActionListener(new ActionListener()
    {
       public void actionPerformed(ActionEvent evt) {
          clearIntegralPressed();
       }
    });

defIntegral.addActionListener(new ActionListener()
    {
       public void actionPerformed(ActionEvent evt) {
          defIntegralPressed();
       }
    });

Diff_Eq.addActionListener(new ActionListener()
    {
       public void actionPerformed(ActionEvent evt) {
          Diff_EqPressed();
       }
    });

phase_plane.addActionListener(new ActionListener()
    {
       public void actionPerformed(ActionEvent evt) {
          phase_planePressed();
       }
    });
DView.addActionListener(new ActionListener(){
    public void actionPerformed(ActionEvent evt){
        DViewPressed();
    }
});
print.addActionListener(new ActionListener(){
    public void actionPerformed(ActionEvent evt){
        ga.print();
    }
});
save.addActionListener(new ActionListener(){
    public void actionPerformed(ActionEvent evt){
        ga.SaveGraphicsToFile();
    }
});

//addGUICmponents()
/**
 * Evaluate the current expression in the exprField.
 * this method is called every time the expression or x is changed
 */
private double evaluateExpression() {
    Equation eq = new Equation(exprField.getText());
    double x = Double.valueOf(xField.getText()).doubleValue();
    double res = eq.Evaluate("x", x);
    return res;
}
private double evaluateExpression(double x) {
    Equation eq = new Equation(exprField.getText());
    double res = eq.Evaluate("x", x);
    return res;
}
private double evaluateIntegration(double from, double to, double step) {
    double value = 0.0; //integration value
    double val1 = evaluateExpression(from);
    double currentx = from; //current x value
    for(double k=from+step; k<=to; k+=step)
{  
currentx = k;
  double val2 = evaluateExpression(k);
  double tobeadded = (0.5)*(val1+val2)*(step);
  value += tobeadded;
  val1 = val2;
}
if (currentx < to)
{
  double val2 = evaluateExpression(to);
  double tobeadded = (0.5)*(val1+val2)*(step);
  value += tobeadded;
}
return value;

private void actionPerformed(){updateResult();}
private void exprFieldTextValueChanged() {
  updateResult();
}
private void xFieldTextValueChanged() {
  try {
    xValue = Double.valueOf(xField.getText()).doubleValue();
  } catch (NumberFormatException e) {
    xValue = 0;
  }
  updateResult();
}
private void drawYlButtonPressed(){
  ga.setExpression(exprField.getText());
  double xl = Double.valueOf(xlow.getText()).doubleValue();
  double xh = Double.valueOf(xhigh.getText()).doubleValue();
  double yl = Double.valueOf(ylow.getText()).doubleValue();
  double yh = Double.valueOf(yhigh.getText()).doubleValue();
  ga.setBoundaries(xl, xh, yl, yh);
  double xd = Double.valueOf(dividX.getText()).doubleValue();
  double xlab = Double.valueOf(labelX.getText()).doubleValue();
  double yd = Double.valueOf(dividY.getText()).doubleValue();
  double ylab = Double.valueOf(labelY.getText()).doubleValue();
}
private void drawY2ButtonPressed()
{
    ga.setExpression2(exprField2.getText());
    double xl = Double.valueOf(xlow.getText()).doubleValue();
    double xh = Double.valueOf(xhigh.getText()).doubleValue();
    double yl = Double.valueOf(ylow.getText()).doubleValue();
    double yh = Double.valueOf(yhigh.getText()).doubleValue();
    ga.setBoundaries(xl, xh, yl, yh);
    double xd = Double.valueOf(dividX.getText()).doubleValue();
    double xlab = Double.valueOf(labelX.getText()).doubleValue();
    double yd = Double.valueOf(dividY.getText()).doubleValue();
    double ylab = Double.valueOf(labelY.getText()).doubleValue();
    ga.setAxesDevisionsAndLabels(xd, xlab, yd, ylab);
    ga.setFunction2Drawer(true);
    ga.repaint();
    //ga.setFunction2Drawer(false);
}

private void clearY2ButtonPressed()
{
    ga.setFunction2Drawer(false);
    ga.repaint();
}

private void drawY1SlopePressed()
{
    //ga.setFunctionDrawer(false);
    ga.repaint();
    //ga.setFunctionDrawer(true);
}
ga.setY1Slope(true);

ga.setY1Slope_x(Double.valueOf(Y1SlopeXValue.getText()).doubleValue())
;
    ga.repaint();
}
private void clearY1SlopePressed()
{
    ga.setY1Slope(false);
    ga.repaint();
}

private void drawY2SlopePressed()
{
    ga.setY2Slope(true);
    ga.setY2Slope_x(Double.valueOf(Y2SlopeXValue.getText()).doubleValue());
    ga.repaint();
}
private void clearY2SlopePressed()
{
    ga.setY2Slope(false);
    ga.repaint();
}

private void drawY1SlopeCurvePressed()
{
    ga.setY1SlopeCurve(true);
    ga.repaint();
}
private void clearY1SlopeCurvePressed()
{
    ga.setY1SlopeCurve(false);
    ga.repaint();
}
private void drawY2SlopeCurvePressed()
{
    ga.setY2SlopeCurve(true);
    ga.repaint();
}
private void clearY2SlopeCurvePressed()
{
    ga.setY2SlopeCurve(false);
    ga.repaint();
}
private void FindRootsPressed()
{
    double xfrom = Double.valueOf(RRF.getText()).doubleValue();
    double xto = Double.valueOf(RRT.getText()).doubleValue();
    double step = Double.valueOf(RS.getText()).doubleValue();

    expression = exprField.getText();
    RootsTextArea.setText('""');
    RootFinder rf = new RootFinder(expression, 0.0001);
    Vector<String> vl=rf.findAllRoots(xfrom, xto, step);
for(int j=0; j<v1.size(); j++)
RootsTextArea.append(v1.get(j) + newline);

MaxMin.setText(""");
MaxMinFinder mf = new MaxMinFinder(expression);
Vector<String> v2=mf.findMaxMin(xfrom, xto, step, 0.0001);
for(int h=0; h<v2.size(); h++)
MaxMin.append(v2.get(h) + newline);

private void defIntegralPressed()
{
    double xfrom = Double.valueOf(RRF.getText()).doubleValue();
    double xto = Double.valueOf(RRT.getText()).doubleValue();
    double step = Double.valueOf(RS.getText()).doubleValue();
    double defInt = evaluateIntegration(xfrom, xto, step);
    String stt = Double.toString(defInt);
    integralResult.setText(stt);
    ga.setIntegResult(stt);
}

private void showIntegralPressed()
{
    double xfrom = Double.valueOf(RRF.getText()).doubleValue();
    double xto = Double.valueOf(RRT.getText()).doubleValue();
    double step = Double.valueOf(RS.getText()).doubleValue();
    expression = exprField.getText();
    ga.setFromToStep(xfrom, xto, step);
    ga.setShowInteg(true);
    ga.repaint();
}

private void clearIntegralPressed()
{
    ga.setShowInteg(false);
    ga.repaint();
}

private void Diff_EqPressed()
{
    Applett2 aa = new Applett2();
    aa.init();
    aa.start();
    final Frame f = new Frame("DIFF EQ");
    f.add("Center", aa);
    f.setSize(650,540);
    Image icon = Toolkit.getDefaultToolkit().getImage("../icon.gif");
    f.setIconImage(icon);
    f.setBackground(new Color(100,100,100));
    f.addWindowListener(new WindowAdapter() { 

public void windowClosing(WindowEvent e) {
    f.dispose();
}

f.setVisible(true);

private void phase_planePressed() {
    PhasePlaneApplett aa = new PhasePlaneApplett();
    aa.init();
    aa.start();
    final Frame f = new Frame("DIFF EQ, Phase Plane");
    f.add("Center", aa);
    f.setSize(780, 490);
    Image icon = Toolkit.getDefaultToolkit().getImage("../icon.gif");
    f.setIconImage(icon);

    f.addWindowListener(
        new WindowAdapter() {
            public void windowClosing(WindowEvent e) {
                f.dispose();
            }
        });
    // f.setResizable(false);
    f.setVisible(true);
}

private void DViewPressed() {
    ThDVApplett aa = new ThDVApplett();
    aa.init();
    aa.start();
    final Frame f = new Frame("Diff. Eq. 3D. View");
    f.add("Center", aa);
    f.setSize(660, 620);
    Image icon = Toolkit.getDefaultToolkit().getImage("../icon.gif");
    f.setIconImage(icon);

    f.addWindowListener(
        new WindowAdapter() {
            public void windowClosing(WindowEvent e) {
                f.dispose();
            }
        });
    f.setResizable(false);
    f.setVisible(true);
}

private void updateResult() {
    double res;
    // Get the value
    res = evaluateExpression();
    result.setText(" " + Double.toString(res));
}
import java.util.*;
import javax.print.*;
import javax.print.attribute.*;
import java.awt.print.*;
import java.awt.geom.*;
import java.awt.image.BufferedImage;
import java.awt.image.RenderedImage;
import javax.imageio.ImageIO;
import java.io.File;
import java.io.IOException;

public class GraphArea extends Canvas implements Printable {

    Dimension dimensions; // the canvas dimensions
    private double xlow, xhigh, ylow, yhigh;
    private double DevideX, LabelX, DevideY, LabelY;
    private double y1Slope_x, y2Slope_x;
    private String expression, expression2;
    private double xScale, yScale;

    private double from, to, step;
    boolean functionDrawer, function2Drawer; // flags to determine which function to draw
    boolean y1Slope, y2Slope; // for drawing the tangent to Y1 at x=...
    boolean y1SlopeCurve, y2SlopeCurve; // for drawing the y1 slope curve
    boolean showInteg; // to show integration

    // values to be printed
    String IntegResult="";

    public GraphArea(double x_low, double x_high, double y_low, double y_high) {
        super(); // call Frame ctor, telling what to label the frame
        dimensions = getPreferredSize(); // =400,400 in this case
        Color background = new Color(0,0,0);
        setBackground(background);

        functionDrawer=false; function2Drawer=false;
        y1Slope = false; y1SlopeCurve=false;
        showInteg = false;

        xlow = x_low; xhigh = x_high; ylow = y_low; yhigh = y_high;
        DevideX = 1; LabelX = 1; DevideY = .1; LabelY = .5;
        y1Slope_x = 0.0;
    }
expression = ""; expression2="";
xScale = 1.0; yScale = 1.0;
from =0; to=0; step=0;
}

public void set_background(Color color)
{
    setBackground(color);
}

public void setIntegResult(String st)
{
    IntegResult = st;
}

public int print(Graphics g, PageFormat pageFormat, int pageIndex) {
//for printing
    int x = (int)pageFormat.getImageableX();
    int y = (int)pageFormat.getImageableY();
    g.translate(x, y);
    if (pageIndex == 0) {
        paintForPrint(g);
        return Printable.PAGE_EXISTS;
    } else {
        return Printable.NO_SUCH_PAGE;
    }
}

public void print()
{
    Graphics ggg = this.getGraphics(); //a copy of this graphics
    DocFlavor flavor = DocFlavor.SERVICE_FORMATTED.PRINTABLE;
    PrintRequestAttributeSet pras = new HashPrintRequestAttributeSet();
    Printservice defaultService
    =PrintServiceLookup.lookupDefaultPrintService();
    Printservice printservice []
    =PrintServiceLookup.lookupPrintServices(flavor, pras);
    DocPrintJob job = defaultService.createPrintJob();
    PrintJobListener pjlistener = new PrintJobAdapter()
    {
        public void printDataTransferCompleted(PrintJobEvent e) {

            }
    };
    PrintService service = ServiceUI.printDialog(null, 200, 200,
    printService, defaultService, flavor, pras);
    if (service !=null)
    {
        job.addPrintJobListener(pjlistener);
        DocAttributeSet das = new HashDocAttributeSet();
        Doc doc = new SimpleDoc(this, flavor, das);
        try {
            job.print(doc, pras);
        } catch (PrintException pe) {
            pe.printStackTrace();
        }
    }
}

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public void setExpression(String s) {
    expression = s;
}

public void setExpression2(String s) {
    expression2 = s;
}

public void setY1Slope_x(double x) // at x we will draw the slope {
    y1Slope_x = x;
}

public void setY2Slope_x(double x) // at x we will draw the slope {
    y2Slope_x = x;
}

public void setFunctionDrawer(boolean b) {
    functionDrawer = b;
}

public void setFunction2Drawer(boolean b) {
    function2Drawer = b;
}

public void setY1Slope(boolean b) {
    y1Slope = b;
}

public void setY2Slope(boolean b) {
    y2Slope = b;
}

public void setY1SlopeCurve(boolean b) {
    y1SlopeCurve = b;
}

public void setY2SlopeCurve(boolean b) {
    y2SlopeCurve = b;
}

public void setBoundaries(double x_low, double x_high, double y_low, double y_high) {
    xlow = x_low;
    xhigh = x_high;
    ylow = y_low;
    yhigh = y_high;
}

public void setAxesDevisionsAndLabels(double dx, double lx, double dy, double ly) {
    DevideX = dx; LabelX = lx; DevideY = dy; LabelY = ly;
public void setScales(double xsc, double ysc)
{
    xScale = xsc; yScale = ysc;
}

public void setFromToStep(double a, double b, double c)
{
    from = a; to=b; step=c;
}

public void setShowInteg(boolean b)
{
    showInteg=b;
}

public double plottableX(double newx)
{
    //be aware that the number of units that the origin 
    //moves along x axis equals 0-(xlow) 
    double nXUnitsTranslated = 0-xlow; //translation 
    //calculating the xscale. 
    xScale = (xhigh-xlow)/dimensions.width;
    double x = (newx + nXUnitsTranslated)/xScale;
    return x;
}

public double plottableY(double newy)
{
    //
    double nYUnitsTranslated = 0 + (yhigh);
    //calculating the Y_Scale 
    yScale = (yhigh-ylow)/dimensions.height;
    double y = (nYUnitsTranslated - newy)/yScale;
    return y;
}

static double xLowBoundFinder(double x)
{// find the lowest x boundary to be labeled 
    boolean state = true; int i=0;
    double y = Math.floor(x);
    while(state)
    {
        if(Math.abs(y)<10) state = false;
        else{
            y = Math.floor(y/10); i++;
        }
    }
    return y*(10^i);
}

public void paintAxes(Graphics2D g)
{
    //double xOrigin, yOrigin;
    Graphics2D g2d = (Graphics2D) g;
    //g2d.setColor(Color.white);
    //y-axis
    g2d.draw(new Line2D.Double(plottableX(0.0), plottableY(yhigh),

plottableX(0.0), plottableY(ylow));
g2d.drawString("Y", (float)plottableX(0.0)-10.0f, (float)plottableY(yhigh)+15.0f);

//devide y axis. there are three cases
if(ylow<0 && yhigh<0)//ylow and yhigh are both -ve
{
    for(double i = 0.0; i>ylow; i-=DevideY)
        g2d.draw(new Line2D.Double(plottableX(0.0)-2,plottableY(i),plottableX(0.0)+2,plottableY(i)));
    for(double i = 0.0; i>ylow; i-=LabelY)
    {
        g2d.draw(new Line2D.Double(plottableX(0.0)-5,plottableY(i),plottableX(0.0)+5,plottableY(i)));
        g2d.drawString(Double.toString(i), (float)plottableX(0)+10,(float)plottableY(i));
    }
}
else if(ylow>0 && yhigh>0)//both are +ve
{
    for(double i = 0.0; i<yhigh; i+=DevideY)
        g2d.draw(new Line2D.Double(plottableX(0.0)-2,plottableY(i),plottableX(0.0)+2,plottableY(i)));
    for(double i = 0.0; i<yhigh; i+=LabelY)
    {
        g2d.draw(new Line2D.Double(plottableX(0.0)-5,plottableY(i),plottableX(0.0)+5,plottableY(i)));
        g2d.drawString(Double.toString(i), (float)plottableX(0)+10,(float)plottableY(i));
    }
}
else // ylow is negative and yhigh is +ve
{
    for(double i = 0.0; i<yhigh; i+=DevideY)
        g2d.draw(new Line2D.Double(plottableX(0.0)-2,plottableY(i),plottableX(0.0)+2,plottableY(i)));
    for(double i = 0.0; i<yhigh; i+=LabelY)
    {
        g2d.draw(new Line2D.Double(plottableX(0.0)-5,plottableY(i),plottableX(0.0)+5,plottableY(i)));
        g2d.drawString(Double.toString(i), (float)plottableX(0)+10,(float)plottableY(i));
    }
}

//x-axis
    g2d.draw(new Line2D.Double(plottableX(xlow), plottableY(0.0), plottableX(xhigh),plottableY(0.0)));
g2d.drawString("X", (float)plottableX(xhigh)-10.0f, (float)plottableY(0.0)+15.0f);
//devide x axis there are 3 cases
if(xlow<0 && xhigh<0)//xlow and xhigh are both -ve
{
    for(double i = 0.0; i>xlow; i-=DevideX)
g2d.draw(new Line2D.Double(plottableX(i),plottableY(0.0)-2,plottableX(i),plottableY(0.0)+2));
    for(double i = 0; i>=xlow; i-=LabelX)
    { g2d.draw(new Line2D.Double(plottableX(i),plottableY(0.0)-5,plottableX(i),plottableY(0.0)+5));
      g2d.drawString(Double.toString(i),(float)plottableX(i),(float)plottableY(0.0)+12);
    }
}
else if(xlow>0 && xhigh>0)//xlow and xhigh are +ve
{
    for(double i = 0.0; i<xhigh; i+=DevideX)
    g2d.draw(new Line2D.Double(plottableX(i),plottableY(0.0)-2,plottableX(i),plottableY(0.0)+2));
    for(double i = 0.0; i<xhigh; i+=LabelX)
    { g2d.draw(new Line2D.Double(plottableX(i),plottableY(0.0)-5,plottableX(i),plottableY(0.0)+5));
      g2d.drawString(Double.toString(i),(float)plottableX(i),(float)plottableY(0.0)+12);
    }
}
else //xlow is -ve and xhigh is +ve
{
    for(double i = 0.0; i<xhigh ;i+=DevideX)
    g2d.draw(new Line2D.Double(plottableX(i),plottableY(0.0)-2,plottableX(i),plottableY(0.0)+2));
    for(double i = 0; i<xhigh; i+=LabelX)
    { if(i==0){} //do nothing.. don't draw 0.0..it is already drawn in y_axis
        else{
            g2d.draw(new Line2D.Double(plottableX(i),plottableY(0.0)-5,plottableX(i),plottableY(0.0)+5));
            g2d.drawString(Double.toString(i),(float)plottableX(i),(float)plottableY(0.0)+12);
        }
    }
    for(double i = 0.0; i>xlow;i-=DevideX)
    g2d.draw(new Line2D.Double(plottableX(i),plottableY(0.0)-2,plottableX(i),plottableY(0.0)+2));
    for(double i = 0; i>xlow; i-=LabelX)
    { if(i==0){} //do nothing.. don't draw 0.0..it is already drawn in y_axis
        else{
            g2d.draw(new Line2D.Double(plottableX(i),plottableY(0.0)-5,plottableX(i),plottableY(0.0)+5));
            g2d.drawString(Double.toString(i),(float)plottableX(i),(float)plottableY(0.0)+12);
        }
    }
}
}
private double evaluateExpression(double x)
{
    Equation eq = new Equation(expression);
    double res = eq.Evaluate("x", x);
    return res;
}

private double evaluateSlopeY1(double x)
{
    DerivativeFinder df = new DerivativeFinder(expression);
    Equation eq = new Equation(df.findDerivative(df.get_stringTokens()));
    String s = new String("x");
    return eq.Evaluate(s, x);
}

private double evaluateSlopeY2(double x)
{
    DerivativeFinder df = new DerivativeFinder(expression2);
    Equation eq = new Equation(df.findDerivative(df.get_stringTokens()));
    String s = new String("x");
    return eq.Evaluate(s, x);
}

private double evaluateExpression2(double x)
{
    Equation eq = new Equation(expression2);
    double res = eq.Evaluate("x", x);
    return res;
}

public void paintFunction(Graphics g)
{
    Graphics2D g2d = (Graphics2D) g;
    //g2d.setColor(Color.white);
    Point2D.Double start = new Point2D.Double(plottableX(xlow), plottableY(evaluateExpression(xlow)));

    for(double i=xlow; i<=xhigh; i+=0.01)
    {
        double x = plottableX(i);
        double y = plottableY(evaluateExpression(i));
        Point2D.Double end = new Point2D.Double(x, y);
        g2d.draw(new Line2D.Double(start, end));
        start = end;
    }
}

public void paintFunction2(Graphics g)
{
    Graphics2D g2d = (Graphics2D) g;
    //g2d.setColor(Color.red);
    Point2D.Double start = new Point2D.Double(plottableX(xlow), plottableY(evaluateExpression2(xlow)));

    for(double i=xlow; i<=xhigh; i+=0.01)
    {
    }
double x = plottableX(i);
double y = plottableY(evaluateExpression2(i));
Point2D.Double end = new Point2D.Double(x, y);
g2d.draw(new Line2D.Double(start, end));
start = end;

public void paintY1SlopeLine(Graphics g) {
    Graphics2D g2d = (Graphics2D) g;
    //g2d.setColor(Color.white);
    double x2 = y1Slope_x;
double y2 = evaluateExpression(x2);
double slope = evaluateSlopeYl(x2);
double x1 = xlow;
double y1 = slope*(x1 - x2)+y2;

double x3 = xhigh;
double y3 = slope*(x3 - x2)+y2;

    x1 = plottableX(x1); y1 = plottableY(y1);
x3 = plottableX(x3); y3 = plottableY(y3);

    Point2D.Double start = new Point2D.Double(x1, y1);
    Point2D.Double end = new Point2D.Double(x3, y3);
g2d.draw(new Line2D.Double(start, end));
}

public void paintY2SlopeLine(Graphics g) {
    Graphics2D g2d = (Graphics2D) g;
    //g2d.setColor(Color.white);
    double x2 = y2Slope_x;
double y2 = evaluateExpression2(x2);
double slope = evaluateSlopeY2(x2);
double x1 = xlow;
double y1 = slope*(x1 - x2)+y2;

double x3 = xhigh;
double y3 = slope*(x3 - x2)+y2;

    x1 = plottableX(x1); y1 = plottableY(y1);
x3 = plottableX(x3); y3 = plottableY(y3);

    Point2D.Double start = new Point2D.Double(x1, y1);
    Point2D.Double end = new Point2D.Double(x3, y3);
g2d.draw(new Line2D.Double(start, end));
}

public void paintY1SlopeCurve(Graphics g)
public void paintY2SlopeCurve(Graphics g) {
    Graphics2D g2d = (Graphics2D) g;
    //g2d.setColor(Color.white);
    Point2D.Double start = new Point2D.Double(plottableX(xlow), plottableY(evaluateSlopeY2(xlow)));
    for(double i=xlow+0.05; i<=xhigh; i+=0.05) {
        double x = plottableX(i);
        double y = plottableY(evaluateSlopeY2(i));
        Point2D.Double end = new Point2D.Double(x, y);
        g2d.draw(new Line2D.Double(start, end));
        start = end;
    }
}

public void showIntegration(Graphics g) {
    Graphics2D g2d = (Graphics2D) g;
    //g2d.setColor(Color.white);
    for(double k=from; k<=to; k+=step) {
        double val1= evaluateExpression(k);
        double val2= evaluateExpression(k+step);
        double x1 = plottableX(k); double y1 = plottableY(0);
        double x2 = plottableX(k); double y2 = plottableY(val1);
        Point2D.Double start= new Point2D.Double(x1,y1);
        Point2D.Double end = new Point2D.Double(x2,y2);
        g2d.draw(new Line2D.Double(start, end)); //vertical at k
        //now we draw thr line connecting thr vertival at k with the
        //one at k+step
        Point2D.Double end2= new Point2D.Double(plottableX(k+step), plottableY(val2));
        g2d.draw(new Line2D.Double(end, end2)); //vertical at k
    }
}

public void paintInformation(Graphics g) //add some information to the printed graphics {
    Graphics2D g2d = (Graphics2D) g;
    float ix= 5.0f, iy = 30.0f;
    if(functionDrawer==true) {
    }
g2d.drawString("Yl = " + expression, ix, iy); iy+=20.0f;

if(showInteg==true) {
    g2d.drawString("For "+Double.toString(from) + " < x < " + Double.toString(to)+", a step of " +step, ix,iy);
    iy+=20.0f;
    g2d.drawString("Integ = " +IntegResult, ix, iy);
    iy+=20.0f;
}

if(function2Drawer==true) {
    Color cc = g2d.getColor();
g2d.setColor(Color.blue);
    g2d.drawString("Y2 = " + expression2, ix,iy); iy+=20.0f;
g2d.setColor(cc);
}

public void paint(Graphics g) {
    Graphics2D g2d = (Graphics2D) g;//casting g to g2d

    //Graphics2D g2d2 = (Graphics2D) g;//casting g to g2d
    //Dimension d = this.getSize();
    //g.setColor(Color.black);
    //g.fillRect(0, 0, d.width, d.height);

    g2d.setColor(Color.white);
paintAxes(g2d);
paintInformation(g2d);
    if(functionDrawer==true) paintFunction(g2d);
    if(function2Drawer==true) {g2d.setColor(Color.yellow);
paintFunction2(g2d);}
    if(y1Slope ==true)paintY1SlopeLine(g2d);
    if(y1SlopeCurve ==true)paintY1SlopeCurve(g2d);
    if(y2Slope ==true) {g2d.setColor(Color.yellow);paintY2SlopeLine(g2d);g2d.setColor(Color.white);}
    if(y2SlopeCurve ==true) {g2d.setColor(Color.yellow);paintY2SlopeCurve(g2d);g2d.setColor(Color.white);}
    if(showInteg == true)showIntegration(g2d);
}

public void paintForPrint(Graphics g)//used for print {
    Graphics2D g2d = (Graphics2D) g;//casting g to g2d
    Color cc = g2d.getColor();
paintAxes(g2d);
paintInformation(g2d);
    if(functionDrawer==true) paintFunction(g2d);
if(function2Drawer==true){g2d.setColor(Color.blue);
paintFunction2(g2d);g2d.setColor(cc);}
if(y1Slope ==true)paintY1SlopeLine(g2d);
if(y1SlopeCurve ==true)paintY1SlopeCurve(g2d);
if(y2Slope
==true){g2d.setColor(Color.blue);paintY2SlopeLine(g2d);g2d.setColor(cc);}
if(y2SlopeCurve
==true){g2d.setColor(Color.blue);paintY2SlopeCurve(g2d);g2d.setColor(cc);}
if(showInteg == true)showIntegration(g2d);
}

public Dimension getMinimumSize() {
    return new Dimension(50, 100);
}

public Dimension getPreferredSize() {
    return new Dimension(600, 450);
}

public Dimension getMaximumSize() {
    return new Dimension(200, 400);
}

//two functions to save
public void paintForSave(Graphics2D g2d)//used for print {
{
    g2d.setColor(Color.white);
paintAxes(g2d);
paintInformation(g2d);
    if(functionDrawer==true)    paintFunction(g2d);
    if(function2Drawer==true){g2d.setColor(Color.yellow);
paintFunction2(g2d);g2d.setColor(Color.white);}
    if(y1Slope ==true)paintY1SlopeLine(g2d);
    if(y1SlopeCurve ==true)paintY1SlopeCurve(g2d);
    if(y2Slope
==true){g2d.setColor(Color.yellow);paintY2SlopeLine(g2d);g2d.setColor(Color
.white);)
    if(y2SlopeCurve
==true){g2d.setColor(Color.yellow);paintY2SlopeCurve(g2d);g2d.setColor(Color
.white);)
    if(showInteg == true)showIntegration(g2d);
}

public RenderedImage myCreateImage() {
{
    int width = 600;
    int height = 450;

    // Create a buffered image in which to draw
    BufferedImage bufferedImage = new BufferedImage(width, height, BufferedImage.TYPE_INT_RGB);

    // Create a graphics contents on the buffered image
    Graphics2D g2d = bufferedImage.createGraphics();
paintForSave(g2d);
// Graphics context no longer needed so dispose it
    g2d.dispose();

    return bufferedImage;
}

public void SaveGraphicsToFile()
{
    RenderedImage rendImage = myCreateImage();

    // Write generated image to a file
    try {
        // Save as JPEG
        String name = getDateAndTime();
        File file = new
            File("./ImageFiles/MainAppletImages/"+name+ ".jpg");
        ImageIO.write(rendImage, "jpg", file);
    } catch (IOException e) {
    }
}

//getting the date and time to name the saved image
public String getDateAndTime()
{
    /*
    ** on some JDK, the default TimeZone is wrong
    ** we must set the TimeZone manually!!
    ** Calendar cal = Calendar.getInstance(TimeZone.getTimeZone("EST"));
    */
    Calendar cal = Calendar.getInstance(TimeZone.getDefault());

    String DATE_FORMAT = "yyyy-MM-dd HH:mm:ss";
    java.text.SimpleDateFormat sdf =
        new java.text.SimpleDateFormat(DATE_FORMAT);
    /*
    ** on some JDK, the default TimeZone is wrong
    ** we must set the TimeZone manually!!
    **     sdf.setTimeZone(TimeZone.getTimeZone("EST"));
    */
    sdf.setTimeZone(TimeZone.getDefault());

    String date= sdf.format(cal.getTime());
    String year = date.substring(0,4);
    String month = date.substring(5,7);
    String day = date.substring(8,10);
    String hour = date.substring(11,13);
    String min = date.substring(14,16);
    String sec = date.substring(17);
    return year+month+day+hour+min+sec ;
}

}//class
REFERENCES

[13] Sun Microsystems, Java 3D API Tutorial