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A curriculum of non-routine problems in the middle school

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A CURRICULUM OF NONROUTINE PROBLEMS IN THE MIDDLE SCHOOL

A Project

Presented to the

Faculty of

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In Partial Fulfillment

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in

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ABSTRACT

This project is a study to determine if a group of middle school students can improve their problem solving ability by means of a curriculum of nonroutine problems that was presented over a six-month period. It explains the problem solving process and the concept of nonroutine problems as a curriculum. This project was conducted in an eighth grade AVID class at Jehue Middle School in Rialto, California. This project explains the nonroutine problems presented to the students and their solutions. The proposed idea is based upon the review of literature stating that math classrooms are typically an arena for merely going over answers and mistakes and that the use of classroom discussion to express mathematical ideas beyond that of just content is limited. This study includes a comparison to a similar group of students not exposed to the curriculum to determine if the students' problem solving ability had improved. The results of the evaluation indicate that the curriculum was effective in improving the students' problem solving abilities.

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INTRODUCTION

Background

The typical middle school math classroom today is a procedural-driven environment. Students are taught methods and formulae to achieve results without necessarily requiring the development of content-based, problem solving skills. In this experimenter's nine years of teaching middle school math, the absence of transferable problem solving ability has been evident. Students are able to learn their rote math curriculum, but remain unable to apply this learning to any related situations.

The experimenter determined that students could learn to be effective problem-solvers if given the opportunity and guidance necessary. Wanting students to leave his class with more than just the ability to solve a text-book equation, the experimenter chose the following project for further exploration.

Reinforcing the beliefs of the experimenter, the National Council of Teachers of Mathematics (NCTM) has made problem solving a mandatory component of the national curriculum (NCTM, 1989). Despite this mandate, research shows that current teaching methods have not produced adequate problem solvers in schools. To improve students' problem solving abilities, some educators, such as Schoenfeld (1992), Bottage (2001) and London (1995) have implemented a different approach to the teaching of problem solving. This project will highlight those ideas, as well as present the results of a similar curriculum implemented in the experimenter's classroom.

Purpose of the Study

This project focuses on the integration of nonroutine problems for math students at the middle school level. The intent is that through a curriculum of nonroutine problems, students will become better problem solvers. Every nonroutine problem presented would be centered on three key components: Problem Recognition, Trying Something, and Persistence. With these three steps in mind, students completed a series of 8-10 problems throughout the school year that at first dealt directly with mathematics while eventually evolving into topics that did not solely relate to mathematics. As the concept of problem solving progressed, its three components: problem recognition, trying something, and persistence remained the focal point throughout.

As an introduction to this curriculum, students expressed their personal ideas on what it means to be a good problem solver. Their initial perception of what is necessary was later compared with what they expressed at the end of the curriculum. Through this, the experimenter was able to show growth and maturity in their perception of what problem solving truly is. Since the idea of nonroutine problems was a fairly new concept, the experimenter shared careful and meaningful examples from real-life situations. In all examples, the three steps of problem recognition, trying something, and persistence were discussed in a manner that was obvious to the students.

Once clear examples were given and students had time to discuss them in context to the three steps of problem solving, students were then introduced to nonroutine problems of their own to solve. Although the problems were nonroutine in nature, students' methods for evaluation became somewhat routine. With every problem, whether worked individually or in groups, students were evaluated by four components: (1) the quality or accuracy of their answer, (2) the quality of their written explanation which included alternative methods, (3) the quality of their oral report and (4) an evaluation of the work on the identified skills. As the literature shows, evaluation of their written and oral explanations is the most critical element throughout this entire process. The way students expressed themselves both in writing and orally, along with class discussions directed by the experimenter was the means by which students moved themselves toward becoming more sophisticated in their problem solving ability. This project devotes most of its efforts to describing this exchange amongst what students write, say, and their class discussions of how each nonroutine problem addressed the three steps along with their implementation of those steps.

In testing the curriculum of nonroutine problems, an ongoing, qualitative approach was used. All student work was kept throughout, in order to monitor their progress as well as what they had done. As said, special note was made and discussed as students related the curriculum to the three steps of problem solving. What must not be overlooked is the objective for the students to become better problem solvers. With this in mind, students were given opportunities to see their early attempts at the three steps of problem solving by looking back and commenting on their previous solutions to their nonroutine problems. From this, students' self discovery of their growth was attained.

As a final evaluation, the experimenter gave a couple of exiting nonroutine questions: one question to be solved individually and the other to be solved in groups. The experimenter also gave these same questions to another group of students who had not been exposed to this curriculum of nonroutine problems to compare the development of answers. Since the same Algebra teacher teaches both groups of students (the control group and the students doing the curriculum), the experimenter believed the end results held more credibility. Also, this control group had no interaction with the experimenter as their instructor. The development of answers proved essential for explaining if a curriculum of nonroutine problems really does have an effective influence in improving problem solving skills.

The following chapters of this project are spent exploring the implementation of a curriculum of nonroutine problems, how students adapted to this curriculum, and the results of such an experiment. In Chapter Two, current literature pertinent to this project is explored and discussed; this section highlights the work done by leading authors in the field of nonroutine problem solving curriculum. Chapter Three describes the methodology used in this project, including the development of a curriculum and questions posed to the students. In Chapter Four, all nonroutine problems and the students' results are presented and analyzed for their effectiveness in teaching problem solving. Chapter Five summarizes the project in its entirety and offers the conclusions of the experimenter, as well as the project's limitations and recommendations for future research.

CHAPTER TWO

LITERATURE REVIEW

In this review of the literature, a basis for the curricular approach implemented in this project is provided by first discussing pedagogical approaches to teaching problem solving and second, discussing approaches to implementing a curriculum of nonroutine problems - the focus of this project.

Problem Solving Pedagogy

The typical math classroom is filled with textbooks that are a source of questions and answers with little else to offer. These texts focus on being content driven with little emphasis on strategy and metacognitive understanding. However, the National Council of Teachers of Mathematics (NCTM, 1989) has set as Standard 2 of the ten standards for effective math instruction, the requirement for students to "be able to reflect upon and clarify their thinking about mathematical ideas and relationships, express mathematical ideas orally and in writing, and ask clarifying and extending questions related to mathematics they have read or heard about" (p. 140). The NCTM reflects the importance of classroom discussion to promote understanding beyond that of just content. Frequently, though, classroom discussions are limited to merely going over answers and mistakes.

There are presently many articles and case studies dealing with teachers that make discussion in the classroom a focal point for learning. The effort to implement an effective math discussion activity to achieve Standard 2 of the NCTM has recently flooded the literature of mathematics education. Learning how to perform such tasks has been the topic of these articles. The changing role of the teacher from a more traditional sense has been clearly articulated by a number of authors (e.g., Romberg & Carpenter, 1986; Stigler & Stevenson, 1991).

Wood, Cobb, and Yackel (1990) describe a case study of one teacher who was "no longer the authority and sole source of knowledge whose role was to transmit information, but instead was actively involved with students' learning by negotiating meaning with them" (p. 20). Mathematics was seen as a "community project." The obligations the teacher attempted to negotiate included respecting each other's thinking and working collaboratively on the instructional activities. The teacher came to see herself as the facilitator of learning and encouraged students to take greater responsibility for their own learning. The concept of students and teacher as coworkers, or fellow players, is evident in her descriptions of the classroom climate.

Similarly, Lampert (1990) aimed to create "a community of discourse." In her classroom, students' ideas were brought into the public forum, arguments were refereed by her, and she attempted to sanction students' intuitive use of mathematical principles. Great importance is placed on the teacher modeling an approach to problem solving.

The Cognitively Guided Instruction (CGI) program is about teachers making instructional decisions based on their knowledge of individual children's thinking. Children in CGI classrooms spend most of their time solving problems. Fennema, Carpenter, and Franke (1992) claim that "the climate in a CGI classroom is one in which each person's thinking is important and respected by peers and teachers" (p.1). Much time is given to students' sharing their strategies for problems and teachers' attempting to build on the mathematical knowledge of their students.

Schoenfeld (1987) has developed a variety of techniques in his problem solving class at the college level, which he describes as "a kitchen sink approach to developing metacognitive skills" (p. 198). Schoenfeld sees himself as a role model for students, and, in solving problems with the whole group, he attempts to make the "struggle" obvious to them. During whole-class discussions, Schoenfeld takes the role of "scribe and orchestrator of the students' suggestions, [with his aim being] to help the students make the most of what they themselves generate and to help them reflect on how they did it" (p. 201). One of Schoenfeld's major aims is to create a "microcosm of mathematical culture" (p. 205).

Tanner and Casados (1988) took a group of high school math students and introduced discussions as an integral weekly component to learning. In the beginning, students were uneasy with discussing math in a group forum and as a result, very few students participated. Part of the reason students were uneasy to participate could be directly attributed to the newness of the idea. These students were used to having a teacher-centered, lecture format for learning. However, with time, students participated very effectively. In Dillon's (1984) *Using Discussion in the Classroom,* readers are given useful formats and tips for classroom discussion, including educating teachers on how to conduct discussions in the classroom is critical for students to have success in their learning.

The Socratic Seminar (Gray, 1989; Overholser, 1992) promotes focused classroom discussion. The Socratic Seminar offers students a new technique to articulate their math learning strategies while demonstrating to students the effectiveness of articulating their understanding. A useful component of the Socratic Seminar is that it allows the teacher some important opportunities. During a Socratic Seminar, teachers can focus the discussions on major steps and strategies in order and relation, point out previous work the students may have done with their relation to the current problem and most importantly, fill in gaps or paraphrase assumptions made by students to describe strategies.

There is a defined decorum with the Socratic Seminar; as students learn it, their discussions take on more effective and direct meaning to the topic of discussion. Tanner and Casados (1988) learned through video taping of these discussions, that even students recognized the improved focus of their discussions when reviewing themselves. By the end of the 18-week semester, all students were involved in the discussions and testified that discussing their understanding in this format allowed them to understand what they were learning. Students found they learned more from their discussions than when left alone with the responsibility to find out something they did not understand. Students reported that questions they may never have thought to ask in class would naturally come up during the discussions.

Another key component to the Socratic Seminar is that it provides the teacher with a means of evaluation. By listening and documenting the students' discussions, teachers, through use of student language, can determine to what depth students understand a particular concept or strategy. This format elicits more student response and gives the teacher the opportunity to listen to students and their explanations in order to evaluate the extent of attainment. By documenting such discussions, teachers can compare the growth from the beginning of a semester, for example, to the type of discussions and explanations given at the end of a semester.

The need to not only understand mathematical content, but also make connections to other concepts cannot be overstated. The importance of problem solving, and its ability to make these connections, is vital for a more thorough understanding. According to Schoenfeld (1992), problem solving curricula allow students the opportunity to "study mathematics as an exploratory, dynamic, evolving discipline rather than as a rigid, absolute, closed body of laws to be memorized" (p.12). Problem solving differentiates rote learning from meaningful learning. Richard

Mayer defines these terms in his essay "The Psychology of Mathematical Problem Solving." Mayer (1982) states that rote learning occurs when the student gives a memorized response without understanding. He defines meaningful learning as the act of problem solving by correlating it with other knowledge and gaining an understanding of the material. The main goal of any educational instruction is to "develop skills, knowledge, and abilities that transfer to tasks not explicitly covered in the curriculum" (Fernandez, Hadaway, and Wilson, 1993). According to recent literature on mathematics education, particularly on teaching mathematics, as individuals increase connections among mathematical ideas, they can do some or all of the following: (a) relate a given mathematical idea to a greater number or variety of contexts, (b) relate a given problem to a greater number of mathematical ideas implicit in it, or (c) construct relationships among the various mathematical ideas embedded in a problem (Lester et al., 1994; Schroeder and Lester, 1989). Although this was referring to teachers of mathematics from elementary to the university level, this relation to students is similar. The leap to this level of understanding is bridged by the ability to be an effective problem solver.

Nonroutine Problems

There is the prevalent belief amongst students that mathematics in the classroom consists of mastering formulas and therefore students do not understand how mathematics can be meaningful. Schoenfeld (1997) believes in the creation of a mathematical culture in the classroom in order to increase the potential of finding meaningfulness of mathematics. Schoenfeld believes that a "microcosm of mathematical culture" would encourage students to think of mathematics as an integral part of their everyday lives, promote the possibility of students making connections between mathematical concepts in different contexts, and build a sense of a community of learners working out the intricacies of mathematics together.

Some educators have taken an approach to implementing a problem solving curriculum consistent with Schoenfeld's concept, which permits all students to use what prerequisite skills they have to enhance their reasoning. For example, from 1981 to 1995, London (1995) implemented a curriculum of what he calls nonroutine problems in a high school setting. The process of problem solving versus mathematical content is what he believes to be most essential in mathematics education.

London describes nonroutine problems in many works (1976, 1989, 1993 and 1995). A nonroutine problem contains four components: (1) The problem requires three steps to complete: problem recognition, trying something and persistence. (2) The problem allows for various solutions while requiring the students to consider one or more methods to complete. (3) A good solution requires that students use some sort of problem solving technique such as pattern finding, generating and organizing data, manipulating numbers or reducing a problem to an easier equivalent problem. (4) Every problem must be solvable by every student. Although varying degrees of student ability will generate different solutions, each student will be able to use his or her ability to create a solution consistent with their capabilities. In short, prerequisite skills are not necessary to finding a solution.

Unfortunately, problem solving curricula in many school texts translate to word problems that require a student to have a predetermined reading ability. This is understandable, but it impedes those students who do not meet that reading requirement. These students are left with little opportunity to develop their problem solving ability given the textbooks in their possession. A characteristic of a problem solving curriculum with an emphasis on nonroutine problems, is that every student is able to solve any given problem, although the quality of the solution may vary (London, 1995). When students are not able to solve problems, they do not reach their potential as problem solvers. Social cognitive theorists have hypothesized that students' self-efficacy beliefs, that is, their judgments of their capability to accomplish specific academic tasks, are important determinants of academic motivation, choices, and performance (e.g., Bandura, 1986, 1997; Pajares, 1996; Schunk, 1991). So it naturally follows that a curriculum that does not allow all students to fully participate is not sufficient in guiding students to develop their problem solving capabilities.

Having a better definition of what problem solving really is can serve to improve students' abilities to improve such skills. Problem solving is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation (Krulik and Rudnick, 1993). In order for a situation to be considered a problem situation, it must be an unfamiliar one (Richichi, 1997). This is the major difference between exercises and questions that are found in most classroom texts. When students enter the outside world of their classroom, they will not see exact replicas of theorems and algorithms they studied in their class. In reality, students will face a variety of problems which will require them to analyze and develop a plan of attack for their problem. After trying their strategy, they will attempt a new one if their previous attempt has failed or extend and reflect upon the strategy itself. "We must prepare students for these types of confrontations they will face out in the 'real world.' Repetitive mathematical exercises will not give them the preparation they need. Problem solving activities, on the other hand, will." (Richichi, 1997, p.1).

In a case study of middle school students (Bottage, 2001) an "engaging and interesting" format of problems was implemented in order to determine if students from special education to the gifted class were able to retain and apply math content taught in a non-traditional manner. Students were to predict natural world occurrences related to the functions of distance, rate, and time. In this problem, students had to calculate the distance they could release a wooden car (that they had to design) from a 6-foot ramp to achieve a desired speed on a 10-foot straightway, while negotiating varied courses of loops, banks and humps. Students were given time to test speeds from different heights and then graph their findings in order to use their data when presented with unknown courses.

Bottage (2001) found that differentiating the work of remedial students and regular math students was difficult. But most importantly, when given post tests on the topics taught, these students scored significantly higher than students taught these topics with traditional instruction. In addition, these students were able to complete tasks with greater success on applications of these topics. This study also found that students using this non traditional approach to learning had better retention of what was taught.

A typical nonroutine problem often times will be reality based and gives the student the opportunity to apply learned skills in the classroom to real life situations outside the classroom. In Anita Benna Tepper's "Designing a City Park" (Tepper, 1999), a fifth-grade classroom developed a conceptual understanding of geometry in order to create a city park for their town. There were three components in this project. First, students needed to divide their knowledge into two categories: what they knew about park design, and what they needed to know about park design. Secondly, by individual investigations with manipulatives, students had to learn geometric concepts for later use in actual park-design layout and lastly, students had to work in cooperative groups to address the park-design problem.

In the first component, the known and what needs to be known, students soon realized that they did not know what the community needs were when it came to a city park. Not knowing how to acquire this information was a potential stumbling block for the students, but their teacher invited a statistics professor who gave the students a lesson with M&M's. The professor told them that it would be impossible to determine the number of yellow M&M's in the world, but taking a sample of the jar she had in front of her would give an accurate indication. So the concept of a sample survey was developed. Students went door-to-door and collected 345 surveys of the 3500 population. Having gathered their data of what the community needed, the students presented their findings to city council who in turn expressed the need for this information to be used in the city's long-term planning. With the city's interest now obvious, the students were then left with the task of actually designing the city park to present to the city council six weeks later.

In order to accomplish designing a city park, students needed to have a working knowledge of perimeter, area, symmetry, circles, triangles, angles and solid figures, which is typical firth-grade curriculum, as well as an understanding and ability to draw to scale while using blueprint techniques. Again, more community involvement was used as a local architect showed students how to put their conceptual understanding to work, thus allowing the students to put their designs on blueprints. These designs were then proposed at a joint meeting of city council and the Parks and Recreation Board, and the designs were kept on file to be used when funding became available for such a park.

The teacher found that this type of nonroutine curriculum allowed students to connect mathematical concepts and apply them to real-world experiences. What was evident in this "Designing a City Park" problem was that students had to master conceptual skills and understand their relevance in order to effectively complete their task.

Implementing a curriculum of nonroutine problems offers its challenges, but sometimes finding them can be just as difficult. Sometimes knowing where to look means only looking as close as one's own community. One middle school teacher learned that as students solved more nonroutine problems, they developed a better understanding of mathematical concepts (Olson, 2003). She discovered that her students could solve and understand real-world problems by inventing mathematical procedures for them. She then set out to find problems so her students could explore mathematical ideas at varied levels of abstraction. This led to the creation of "Community Problems."

"Community Problems" began with students brainstorming to determine the problems they saw in their community. The defined problems were posed as questions:

Do students exhibit racist behavior?

Where do students cross the street to get to school? How safe are they?

Why do people litter so much in the park?

How crowded are the school buses?

Where can kids go to use their skateboards?

Teams of students studied their problems directly in the environment and used this to develop ideas to collect data to explore the nature of the problem, analyze their data and make recommendations. An example of this is the pedestrian safety problem question.

A group of students observed a busy street next to their school where a majority of students crossed daily. They made the following observations: students ignored the painted crosswalks or ran across the street in the middle of traffic; at the four-way stop sign, many drivers did not yield to pedestrians; and lastly, as students were dropped off by cars, traffic around the school became a problem. Coincidentally, street improvements had been scheduled by the city transportation department in an attempt to alleviate these problems; however, nothing to date had changed the situation of pedestrian safety. As a result, like in the Creating a City Park Problem, the city took a keen interest in the students' observations and recommendations.

Students collected a variety of data to display in tables and stem-and-leaf plots. In a twenty-four hour period, using automated traffic counters and recording the number of students crossing the street at different times, students recorded that nearly 800 cars crossed their street in the 7AM hour and that 1700 cars passed their street in the 7AM to 9AM hours. Students also recorded traffic speed, as well as traffic volume and traffic gap for the twenty-four hour period. With confirmation that traffic safety was indeed a concern, they suggested two solutions to the city's pedestrian committee: install a traffic light at the four-way stop, or construct a drop-off site next to the school.

City officials agreed with the students' findings, but encouraged them on their second suggestion of constructing a drop-off zone because it would be more cost-effective. Now with the task of finding an optimum drop-off site, student teams observed and recorded where most parents dropped off their children and the result of such actions. Again, using graphs, students presented their findings to the school board and found as a students explained, "Sixty-seven percent of people who drop off children use the busy street. Children getting out of cars cause a lot of cars to stop, making a traffic jam. The traffic jam causes unsafe crossing for children because cars just want to get out to the area around the school." (Olson, 2003, p.264). After hearing the student presentations, funding for the drop-off site was approved the following board meeting and the drop-off site became reality. It was found that as a result of the drop-off site, 84% of parents used the drop-off site, which meant an increase in traffic flow around the school while vehicles reduced their speed around the school.

The teacher found that the flexibility of the problem allowed all the students to be successful because they could build on their own strengths and choose how to communicate their ideas. Students became empowered and took pride in what they were doing while making a curricular connection to what they were learning in school and influencing public policy in their community. For all the students involved, math was not created in the classroom, it was real and connected to daily life and was found to be a necessity to solve real-world problems.

For teachers that are concerned with State Standards Based instruction and specific timelines that are so prevalent in this profession, the teacher of "Community Problems" said,

> Gradually, my practice and beliefs about teaching have changed. My role as a teacher is not confined by curricula or benchmarks. I have found that benchmarks will be met when students are engaged in their learning and I give them time to construct meaning. When I covered everything in the textbook, students did not retain the topics taught. As I shifted my focus toward developing understanding, student achievement increased. With increased achievement, I spent little time reviewing or practicing previously learned skills, which, in turn, freed time to explore additional topics or work on projects. By focusing on developing student's thinking and reallocating classroom time, the changes in my practice helped students gain mathematical

> insights and perceive many connections in their world. (Olson, 2003, p.265)

The benefits and advantages of a curriculum of non-routine problems are evident in her classroom.

In a classroom of nonroutine curriculum, students are engaged and interested in what they are doing. But what about those classrooms whose focus is mostly on formulas and algorithms? What about their success? Unfortunately, there is no such thing as a crystal ball to look into the students' future, but we do have the luxury of looking at the past and how it has affected the present. In a front page story of the Los Angeles Times (Leovy, 15 March, 1999) titled: "Math Equals Fear at 2-Year Colleges", nearly 47% of students do not pass the required math courses at community colleges in order to either graduate or transfer to a four-year university. State community colleges contain about 75% of post-secondary students and are filled with students who are repeating classes or dropping them while losing time and money because they cannot pass Algebra. Professors say that "students are trying so hard, but nothing is making any sense." There are numerous factors for the lack of success by many in mathematics, but traditional curriculum is one to take the blame. Math, to these students, does not make any sense and a curriculum of merely memorizing facts and using theorems and algorithms has served little purpose in making math understandable and most importantly, meaningful. This trend, of course, can easily continue, but it is evident that a curriculum on nonroutine problems offers an alternative for success in the classroom.

The examples of nonroutine problems that will be presented in this project comprise a curriculum that is not typical in the average mathematics classroom. On the surface, these problems are often viewed to not have a simple solution. They are not meant to. The steps of problem recognition, trying something and persistence are mandatory to adequately complete these tasks. As students become aware of this, their skills in problem solving become keener. As a result, students are better equipped to apply their problem solving ability to situations in their lives. It is these types of nonroutine problems and others similar in nature that will be implemented as curriculum for this study. As mentioned earlier, the goal of this is to discover whether the implementation of such a curriculum can help students develop their thinking, over time, to apply solid problem solving strategies to situations of importance. The answer to this question will be addressed to the fullest in this project.

CHAPTER THREE

METHODOLOGY

This project is a study to determine if a group of middle school students could improve their problem solving ability by means of a curriculum of nonroutine problems. This chapter will describe the methodology employed, consisting of a pre and post test of students' perception of what problem solving is; an exploration of the students' living environment as it potentially relates to their concept of problem solving; the methodology for implementing the curriculum of nonroutine problems; and comparing the performance of a control group with the experimental group on two nonroutine problems.

Pretest

Since one intent of this field test was to help students become better problem solvers through recognition of the steps of problem solving, the experimenter wanted to get an idea of students' perceptions of what problem solving meant to them. To do this, students were given a simple writing prompt that asked: *What does it mean to be a good problem solver? What components are necessary?*

It was hoped that this would give insight as to what students defined as problem solving (in the beginning) to what they would later define at the conclusion of the six-month field test.

It was hypothesized by the experimenter that since these students were being presented a unit on problem solving by a math teacher, their comments on what it takes to be a good problem solver would lean heavily toward being able to solve word problems and similar ideas related solely to academics.

Students' Neighborhood

The development of an appropriate nonroutine problem solving curriculum, depends in part on discovering the existing academic and comprehension levels of the audience. In order to determine this about his classroom, the experimenter gave the students a short writing prompt: Describe your Neighborhood. The following are students' responses to this prompt.

Student:

> My neighborhood is dirty and trashy. Gangs live across the street. Sometimes things happen. There are stores on the same side and across the street. The houses and apartments are dirty on the outside. It's ghetto.

Student:

> My neighborhood has apartments. There are security watching us. There is a basketball court where we play basketball. They called the place The Zoo. There are Black people who smoke weed. But I don't do none of that stuff. Where I live it is quiet. If I just mind

my own business, then no one will ever come up to me and offer weed. People say that my zoo place was very dangerous but since I have lived there nothing has happened yet and I hope that nothing will ever happen.

Student:

My neighborhood has a lot of drug dealing and gang affiliated stuff such as shooting and stealing, but that is usually at night, they are sneaky, but the guy at the corner got locked up for selling drugs and other stuff. The house next to us got broken in to and got stuff stolen. We also have drive by's in the apartments next to us. A lot of people get hurt in that place such as stabbing and shooting, etc.

Student:

My neighborhood is not very peaceful. There are all kinds of little kids running around getting in trouble. Also, there are four to six drug houses in my park which is not very good.

Student:

My neighborhood is very nice. The cops and ambulance almost never come. There are never any fights at our neighborhood. The neighborhood people never tag or break into houses. Each 4th of July we show our fireworks and display them.

Based on the students' descriptions of their neighborhoods, it became clear why their definitions of problem solving turned directly toward social issues. Conflict is no stranger to them and for some, unfortunately, it is commonplace. With this in mind, the experimenter thought it would be interesting to see if students' perceptions of problem solving would alter after participating in a curriculum of nonroutine problems.

Curriculum of Nonroutine Problems

To develop the curriculum used within this project, the experimenter reviewed current literature to determine the needs of the students and to find suitable materials from which to generate appropriate nonroutine problems. Based on this literature review the experimenter produced a tentative curriculum as described in Chapter Four. After each problem was presented to the students, the findings were recorded and analyzed by the experimenter and his advisor. The ensuing problem to be given to the students was formed based upon the cumulative results of the previous problems. The initial tentative curriculum was constantly revised through this process.

Control Group vs. Experimenter's Group

Once the students had gone through a curriculum of nonroutine problems for six months, the experimenter wanted to compare the students' ability to problem solve against a like group of students (control group) who had not been exposed to a curriculum of nonroutine problems. The final evaluation would take two class periods, giving all participants three problems to perform. Of the three problems, students were required to choose two, however, of the three, the lake problem was mandatory to be solved.

The Control Group:

For the final evaluation, there would be 50 participants: 25 in the control group and 25 from the field test. These two groups could be considered comparable when described. These 50 students were AVID students. AVID stands for Advancement Via Individual Determination. This program targets students who would be considered "in the middle"; meaning that they were not the typical "A" students, but had the potential for college if given the opportunity. AVID seeks out students who get "B" and "C" grades and do not come from families with college backgrounds. The students are given the most rigorous curriculum available and are required to take Cornell notes for all classes. In addition, they are not permitted any typical elective class such as music, art, yearbook, etc. Instead, they must take the AVID elective. It is this AVID elective in which students study self-esteem and preparedness for college, are made to organize their study habits, and turn in their class notes for weekly evaluation. In return, they are given the opportunity for extra help in what are called tutorials twice a week. This program has been successful, with over 92% of AVID graduates attending and finishing college. It is a collaboration of a variety of people working together. In order to be in

AVID, students, parents and teachers sign a contract with a list of responsibilities to which they will adhere. Parents are a vital role in this process and are in a sense, forced to take an active role in their child's education.

The students of AVID travel together throughout the day to the same teachers for English, Social Studies, Science and Algebra I. The only time they do not have the same teacher is for their AVID Elective. It is here that the two classes are held simultaneously with 25 going to one teacher and the other 25 going to another teacher. Because the Elective class is similar, the students do some of the same projects and are very familiar with what the other class is doing. However, the real difference between these two classes was that the experimenter's group went through a curriculum of nonroutine problems while the control group did not. It is this single difference that separates these two groups and establishes the opportunity to test the hypothesis that a group exposed to a curriculum of nonroutine problems will show an increased ability to problem solve versus a group that has not been exposed to the curriculum.

Post Test

On the penultimate day of school, the experimenter gave the students a final writing prompt on what it meant to be a "good problem solver." They were also asked what components are necessary as well as inquiring whether or not they felt their own problem solving ability had improved.

CHAPTER FOUR

RESULTS AND EVALUATION

Introduction

This project is a study to determine if a group of middle school students can improve their problem solving ability by means of a curriculum of nonroutine problems. To evaluate this project, a pre and post test assessment of students' perception of problem solving skills was given; a sequentially developed curriculum of nonroutine problems was presented to the students over a six month period; and a final evaluation using nonroutine problems was performed, utilizing a control group, as well as the experimenter's group. This chapter will report the results of implementing the methodology.

Curriculum of Nonroutine Problems

The nonroutine problem curriculum was not developed in advance. Rather, it was tentatively developed with an advisor, Dr. London of California State University, San Bernardino. Each nonroutine problem was implemented over a three to four week period. The next nonroutine problem was developed based upon the results of the previous non-routine problem, taking into consideration the students' collective development of the three steps of problem solving. Problems were designed to expand the students' perception of what type of problems could be solved using their own abilities.

Five of the following nonroutine problems were written by Dr. London, Associate Professor, College of Education, California State University, San Bernardino. Dr. London developed these problems to improve the problem solving skills of high school students. The selection of problems used within this project was chosen for their proven ability to enhance problem solving by means of a curriculum of nonroutine problems.

The remaining three nonroutine problems were written by the experimenter with assistance from Dr. London. These problems were created based on the students' progress during the project, and with the intent of making the three-step problem solving model understandable for the students.

The Expensive Tape Problem:

The Expensive Tape Problem was the first opportunity for students to use the three steps of problem solving in solving a nonroutine problem. In this problem, students were given the task of creating three boxes that held the most volume; however, 30 inches of "expensive tape" had to be used to seal each box (for packaging and mailing purposes). Every box had to be taped in a particular fashion with one length and one girth (two widths and heights). Another stipulation demanded that two of the three dimensions (length, width, and height) had to be whole numbers. Since this was the first attempt at solving a nonroutine problem using the three steps, the experimenter provided the students with a format to organize their data that would help calculate volumes.

Although most nonroutine problems should not require much prerequisite knowledge, it was important that the students knew what volume meant and the method (Length x Width x Height) to calculate it for a rectangular solid.

To demonstrate how to create the proper measurements, the experimenter offered this initial box: length 6 inches, width 6 inches and height 6 inches. Figure 1 demonstrates the format that was shown to the students.

Length	Width	Height	Volume
6	6	6	216

Figure 1. The Expensive Tape Format

Maximizing Volume:

In this problem, students were given the task of constructing the largest box (by volume) possible when being restricted to the amount of material that could be used. Students worked in cooperative groups and were given the same size piece of construction paper with which to construct their box. Students were allowed to come up with any shape they wanted as long as their box did not have a lid and that the lid area was flat. Students were informed that the lid would be made of another material. It was explained that each group had to construct their three best designs. However, each design had to be a different shape, meaning, for example, that three rectangular solids would not be allowed. Students had to submit a written report in addition to an oral report explaining how they came up with their designs. Finally, to test their volumes, each container constructed was filled with rice to see how much volume their design held.

The Most Pleasing Rectangle:

Students were given the task of constructing the rectangle that would be the most pleasing to other people. The students were immediately left with the impression that this was basically an unsolvable problem. However, what intrigued them was the assurance by the experimenter that if they employed the three steps of problem solving, they could find the most pleasing rectangle. The experimenter told them that there is a rectangle "out there" that is the most pleasing to other people, but it was up to the students to find it.

Making Predictions:

The task for this nonroutine problem was to predict the outcome of certain events involving a bag of 35 marbles of three different colors of unequal amounts. For this problem, students used 20 green marbles, 10 red marbles, and 5 blue marbles. Students needed to predict the probability of these five events: (1) the probability of picking two consecutive green marbles; (2) the probability of picking two consecutive red marbles; (3) the probability of picking two consecutive blue marbles; (4) the probability of picking three consecutive green marbles; (5) the probability of picking ten consecutive green marbles. Students had to express all answers as decimals and were allowed to round off answers to the nearest hundredth when necessary.

Predicting the Number of Student Lunches:

This nonroutine problem required students to predict the number of student lunches that would be served on a pre-designated day. To make this problem more challenging, students were not allowed to ask any cafeteria staff for assistance and were limited to gathering data solely from the student body. Students were put in groups of two or three and given two weeks to devise a plan, have it approved by the experimenter and implement the plan on the designated day. Reports on each group's findings would be presented afterwards.

Predicting the Number of Student Lunches Part II:

In this attempt at predicting student lunches, the students worked collectively. The reason for this was that their ideas that they had discussed after their first attempt seemed to indicate that they had one central idea and working as a class they could incorporate their plan effectively. It was felt that this additional work on the problem would be particularly beneficial for the students.

The Decision of Street Names:

The nonroutine question presented to students was quite simple on the surface: "Why was your street given its name?" Initially, some students thought this would be an easy problem. One student said, "That's easy, my street is Jefferson and it is named after the President." More explanation of this problem was necessary for the students to understand what was being asked.

Five Calculations:

This was the last nonroutine problem the students would attempt before their final evaluation. In this nonroutine problem, students were to create and implement a means to quickly reduce any given number to zero in five or fewer arithmetic computations. Of course, certain limitations were in place to make the task more challenging.

The class would be given 20 numbers between 100 and 900 that needed to be reduced to zero using only addition, subtraction, multiplication and/or division with the numbers 1 to 9 only. An example would be the number 299, one solution would be a) subtract 9: 299-9= 290, b) divide by 5: 290/5= 58, c) subtract by 4: 58-4= 54, d) divide by 9: 54/9= 6, and e) subtract by 6: 6-6= 0. This would be a timed test and students would be allowed to work in pairs. In addition, students would be allowed only two calculators, pencil and paper and one 3 x 5 card with notes on one side only.

Data was collected for each of the above nonroutine problems, analyzed and used for the development of the subsequent problem. Results of students' work on each problem are presented later in this chapter.

Findings

Pretest

The students were asked the following questions "*What does it mean to be a good problem solver? What components* *are necessary?"* The majority of students' answers associated more to personal relations than to academics. The following are excerpts of what students believed to be the definition of a good problem solver.

Student 1:

> Being a good problem solver means that you have a way of solving things that need fixing or when somebody has their feelings hurt or makes them angry or are flustered over a problem.

Student 2:

> There are many ways a person could be a good problem solver. First of all I think a problem is something people have when things go wrong, jealousy occurs or that person is simply angry. If more people set their minds to solving problems in a non-fighting manner the world would be very different.

Student 3:

> It means that someone is willing to sit down with the victims and listen to their conflicts. A problem solver would listen to both sides of the story and would have to put in a lot of their time. They would have to do research on both families and see if there are any related problems that have happened in the past of the families.

Student 4:

> To be a good problem solver you cannot have a temper or an attitude. I mean if you have a big attitude with that person that you are trying to help then they might not want to talk to you anymore because you are giving an attitude.

Student 5:

> I see a problem solver as being a friend, a person who can give advice and mediate issues among their peers.

Student 6:

> To me, being a good problem solver is being able to solve problems on your own that help you make the right decision. For example, if someone offers you drugs, you have two choices, walk away and not take the drug, or think to yourself 'I'll just try it once, I won't get addicted to it.' A good problem solver can make split second decisions, or just solve problems in his/her life.

The majority of students responded to this question in a similar fashion to the above statements. For them, problem solving at this stage meant to resolve personal conflicts amongst friends and family. The social implications in the students' responses intrigued the experimenter. It was obvious that these students were no strangers to conflicts in their lives and had many experiences with them based upon their responses. With these responses in mind, the experimenter wanted to find out more about their environment without getting personal or risk offending any of the students. Trying to find out what shapes their experiences is not an easy task, but the experimenter decided to simply inquire about their neighborhood. The neighborhood does not necessarily describe the students or their background, but does give some indication of their life experiences.

The Gobi Desert Expedition:

For an introduction to the three steps of problem solving, students were read the story *The Gobi Desert*

Expedition: A Fuller Account. The purpose of this story was to give students an example of a real life problem that from the onset had no clear solution, yet would clearly provide an example of what could be accomplished when incorporating the steps of problem solving. Another purpose in using this story as an introduction to problem solving was to show that problem solving does not just pertain to solving math problems, but can be used to solve, in addition, the problems in their lives.

The Problem:

In this story, a group of explorers envisioned crossing a portion of the Gobi Desert that at that time, 1898, had been untraveled.

Step 1: Problem Recognition

The clear problem was that under ordinary circumstances, crossing this portion of the desert would be impossible and ultimately fatal. Three critical problems they faced were not being able to progress during sandstorms, finding food, and transporting all their supplies that would be needed to accomplish such an expedition.

Step 2: Trying Something

It was decided that the members of the expedition would take a month to individually explore various solutions to the clear problems that the group would have to overcome. To combat the lack of food, one explorer experimented with using sand in a mixture to create food for the pack animals, while another experimented with stilts to walk clear above sandstorms that would normally impede any sort of progress.

Step 3: Persistence

Each person in their research for the period of a month demonstrated persistence in solving their part of the problem. For example, in working with the sand, it was reported:

> I bought two camels, two yaks, two horses, two mules, two asses, ten sheep, ten goats, ten dogs and ten Kerishkis cats, and keeping them hungry, I began little by little introducing into their food this sand which I had prepared in various ways. For the first few days of my experiments, none of the animals would eat any of these mixtures. But when I began to prepare this sand in an entirely new way, after only a week's trial the sheep and goats suddenly began to eat it with great pleasure. I then concentrated all my attention on these animals. In two days I completely convinced myself that the sheep and goats had already begun to prefer this mixture to all other kinds of food. It was composed of seven and a half parts of sand, two parts of ground mutton and one-half of ordinary salt. (Gurdjieff, 1963, pp.168-169)

In this example, the explorer continued to persist despite the initial discouraging results. After the first two reports, it was evident that they had discovered a solution and only needed to work out some details on how to finish the problem. They finalized that the sheep could carry the stilts when not in use, which also afforded the explorers the ability to be carried by the animals simultaneously. Once they were adept at using stilts, they were on their way to a successful trip across the once untraveled portion of the Gobi Desert.

Since the students had been introduced to the three steps of problem solving, they were required to take notice when they heard the steps being utilized. Discussion followed the reading of the story to ensure that students made the correct correlation to each step as it happened in the story.

Nonroutine Problem Data Results

With the advisor's guidance and analysis of the results, subsequent nonroutine problems were appropriately defined and presented to the students, with the intent of developing and reinforcing their problem solving ability. Below are the results of these nonroutine problems being presented to the students:

The Expensive Tape Problem:

Step One: Understanding the Problem

At first students thought that this example only used 18 inches of tape, but it had to be pointed out that both width and height represented two heights and two widths (girth). So in reality, 6 inches was used to tape one length, 6 inches for one width, 6 inches for the other width, 6 inches for one height and 6 inches for the other height for a total of 30 inches of tape. Once students understood this concept, they were left to work in groups to generate the three largest boxes by volume while using exactly 30 inches of tape.

Step Two: Trying Something

Students did not have much difficulty in generating volumes, however, the number of attempts at this early stage in problem solving indicated that students were not generating multiple solutions in attempting to find the three largest boxes. In the experimenter's opinion, students were content in finding three large volumes. At this stage, students were not aware if their answers represented the three highest volumes. What was clear to most was that the largest volume that could be generated was a box with length of 10" and width and height of 5". This indeed is true. When the girths are as close to squares and the length is close to 10", this produces the higher volumes. Figure 2 demonstrates how this works. Notice that when girths are close to square and the length close to 10", the volume increases.

Length	Width	Height	Volume
10	5	5	250
10	6	4	240
10	7	3	210
10	8	2	160
10	9	1	90
16	4	3	192
14	4	4	224
12	5	4	240
8	6	5	240
6	6	6	216
4	7	6	168

Figure 2. The Expensive Tape Partial Solutions

Step Three: Persistence

In the Expensive Tape Problem, students were able to perform the second step of trying something (generate various volumes), but they did not search out other options (persistence) that might leave them convinced that they had not omitted possible better solutions while finding the three largest boxes. An example of persistence would be looking at lengths such as 9" and 11". Most students kept to using even numbers for lengths and never even considered trying odd numbered lengths. If students had attempted to use lengths of 9" or 11" they could have come across the other stipulation that at least two lengths must be whole numbers. For example, a length of 9" and a width of 5" would force a height of 5 ½" and a final volume of 247.5 in^3. The idea behind persistence in this example would lead a student to potentially realize that 9" and 11" is closer to 10" than 8" and 12". Recall that 8" produced a maximum volume of 240 in^3. After 250 in^3, 240 in^3 was the next highest volume that was attained by most students.

In one case, a group did use 11" and found a volume of 247.5 in^3, but did not carry it out further to see if 9" would produce the same result. In this particular group, 250 in^3, 247.5 in^3 and 240 in^3 were the three highest volumes that the group found.

It was hoped that the students would not easily find the three highest volumes in this exercise. The true intent of this opening nonroutine problem was to have a discussion about the entire process that would clearly demonstrate the use of the three steps with the most emphasis on the final step of persistence. The solution of this nonroutine problem was well within every student's ability to understand. The value of the problem became evident when showing students that had they thought of other ways,

ways that were clear to them after processing (using lengths of 9 and 11), they would have been able to solve this problem well on their own. There is no greater example of persistence than when students realize that the solution was within their ability all along; they just did not search long enough or explore enough avenues to find the solution on their own.

This learning process is what is being allowed to develop in this curriculum of nonroutine problems.

Maximizing Volume:

Step One: Problem Recognition

In this problem, the concept is quite clear: make the box that holds the most. As in some nonroutine problems, the students had little trouble understanding how to get started. Students struggled in the area of persistence. This will be addressed at the appropriate time.

Step Two: Trying Something

It is the experimenter's opinion that the time spent on this problem was not adequate for the students to really try something to the point that they were convinced that they had the largest box by volume. Although this task is valuable in practicing a nonroutine problem, it requires too much class time for its effectiveness to really take place. To design a box, the student has to cut and tape to construct. This process alone takes about 20 minutes; this does not take into consideration the time it takes to plan the design that is going to be cut and taped. It took most groups an average of a class period to design and construct a single box. Since trying something in this problem means potentially making and planning several designs, the time it takes to allow the problem solving process to happen is not time effective in relation to the structure of the school schedule.

Obviously, students being able to work together outside of class would have facilitated the process. Since these students were in middle school and did not necessarily live close to each other, the experimenter felt that this would not have been a possibility for all groups to meet with each other for the time necessary to let this problem develop to its fullest potential. Nevertheless, in its weakened form, this problem was valuable. However, in the future, the experimenter would omit this problem from the curriculum.

Step Three: Persistence

As was stated earlier, students were not given enough class time to develop many designs. Nevertheless, the experimenter did observe students making attempts to create different shapes that would improve upon their previous designs. What was good about this problem was the requirement that all three shapes had to be different. In this problem, all students naturally constructed the rectangular prism first. In measuring these boxes, the rectangular prism offered students the highest volumes. However, since students made these rectangular prisms first, they were determined that their next two boxes would have to be of equal volume or greater.

In early attempts at their second designs, many volumes dropped by 40%. What the experimenter found effective was the desire of students to "go back to the drawing board" to create a better box. In fact, many students never gave themselves the opportunity to even make the third box, because they felt their second design was not adequate enough. In the experimenter's opinion, this is a good example of students applying persistence in a situation in which the students felt they could improve upon something, even though the answer was not clear. The fact that they did not just settle for a design so they could get to the third design showed evidence that the students were beginning to understand the concept of persistence. At this point it is irrelevant if students find the three largest boxes. What is relevant is if students are experiencing the steps of problem solving. It was clear that in this problem, even in its truncated form, students began to better understand and incorporate the three steps of problem solving.

The Most Pleasing Rectangle:

Step One: Problem Recognition

Certain restrictions were set to give parameters that would make the number of possibilities more controllable. These restrictions were also set to ensure that only the shape of the rectangle was the contributing factor to deciding if the rectangle was the most pleasing. The restrictions were as follows: one side had to be six inches and they would all be using the same kind of green construction paper. That was it. How they found the most pleasing rectangle was then left completely up to them. In this early stage of performing nonroutine problems, however, the experimenter offered some advice and suggestions on how they might start finding the most pleasing rectangle.

Step Two: Trying Something

It was suggested that students try cutting out various rectangles that met the requirements and ask friends and family which one was the most pleasing, noticing which rectangles consistently evoked the most approval.

Step Three: Persistence

It was stressed that if students created six different rectangles and the same one was always selected as the most pleasing, it would not necessarily indicate that they had indeed constructed the most pleasing rectangle. Even within the set parameters of this problem, there are numerous potential solutions, and therefore the problem would require a certain amount of persistence to find the solution to this problem.

After one week, students turned in their rectangles and the following day the experimenter taped them to the white board for other classes to identify which rectangles were the most pleasing. Four classes were given the opportunity to make their selection and consistently the same three rectangles were given the highest rank. The top three rectangles were very similar in dimension with two of them being nearly identical. In the three examples, the ratio of the long side to the short side was reasonably close to the Golden Mean with the short side equal to six inches. Typically, rectangles with ratios of adjacent sides similar to the Golden Mean prove to be the most pleasing.

In this nonroutine problem it is not necessary for students to understand the Golden Mean, but it is a good reference point for when it is formally introduced later in their studies. What is important, however, is the approach to finding the most pleasing rectangle. It was this topic that was discussed in length after a short discussion of why the Golden Mean usually indicates the most pleasing rectangle. It is important to note that finding the correct solution is secondary to understanding the process of getting to the solution.

The experimenter and students discussed the experience that the students went through to find what they believed to be the most pleasing rectangle. It was concluded that it seemed impossible to find the most pleasing rectangle. But the main point that came across from the discussion was that even if it were impossible, it was of utmost importance that the person that constructed the rectangle had to be convinced that he or she had singled out the most pleasing rectangle, while being assured that he or she had eliminated other candidates that could be more pleasing. In this problem, when the students are convinced they have found the most pleasing rectangle, there is nothing left to be done. The "being convinced" is the most difficult part of the problem, but is the whole point of using the three steps of problem solving. One student said it best when he said, "I am convinced that this is the most pleasing rectangle because I made it from my own judgment, and I just have to be right." The experimenter was convinced that the student used the three steps to the best of his ability.

Making Predictions:

Step One: Problem Recognition

This problem proved to be quite difficult for students to come up with a uniform answer. What the problem did do, however, was give credence that some problems require persistence in order to feel comfortable with the final results. But to get students to that point, the experimenter had to intervene often to keep students from inadvertently wandering off course. This problem caused confusion for students that had little experience with probability. These students' experiences had been basically limited to flipping coins and rolling number cubes. What made this problem more of a challenge was the fact that students needed to predict the outcome of two occurrences coming together to form one desired outcome. For example, knowing that the odds of rolling a one on a number cube is a one-in-six chance, while rolling two consecutive ones is a one-in-thirty-six chance was within the students' ability. But the fact that there were not an equal amount of colors served to make the problem less understandable.

Step Two: Trying Something

In initial attempts, it was noticed that students did not understand that, for example, picking three consecutive green marbles was one event. It was evident that students were merely drawing 100 times in a row and counting how many times they got at least three green marbles to come up in a row. The experimenter had to stress that picking three consecutive marbles was one event. In other words, if one could only pull out three marbles, what was the probability that all three would be green? This understanding of the problem did not come easily and took some persistence from the experimenter to help the students figure this out, but once students started understanding this concept, the manner in which they attempted to find the probability altered considerably.

One group stated:

> The probability of picking two green marbles in a row was 45%. We came up with this estimate by trying to pick two green marbles in a row for 100 times. We picked two in a row forty-five times out of 100. We feel that 45% is the probability of picking two consecutive green marbles because we did them out of 100 times.

This group essentially drew 200 marbles to create the 100 events and showed evidence that they understood the problem, tried many times and felt that they had persisted to get an answer that made sense.

Another group used a similar format, but decided that they would create 200 possible events instead of 100. Both groups generated similar results. Another group took a less redundant approach and tested their results 20 times in groups of ten. For example, in trying to find the probability of pulling 2 consecutive green marbles, this group did it 3 times out of 10 and then 5 times out of 10. They combined this data and determined that the result happened 8 times out of 20 which gave them a 40% probability. This result was similar to the groups who tried this 100 times and 200 times. This offered the opportunity to discuss that persistence does not necessarily mean trying as many times as possible, but continuing to try until it is believed that one has accomplished the task that he or she has set out to complete.

Step Three: Persistence

While teaching persistence, this difficult problem also offered other opportunities to expand on topics that are in line with state standards in the area of statistics. What was interesting was the discussions about probability. It was discussed that probability does not necessarily mean that an event is going to happen, only that the likelihood of that event occurring has those particular odds. One idea arose that the probability of flipping heads on a coin is 50/50. However, it does not guarantee that a person ever flip a heads on a coin. The class realized that it is nearly impossible for a person to never flip a heads on a coin in a lifetime, but at the same time it is a possible event. But what students learned the most was the continued importance of persistence when attempting a nonroutine problem. It was evidenced in this problem.

By this point, students were getting a strong sense of what it meant to persist. Students were able to point out areas of persistence and knew when they were not persisting, or rather had not persisted enough. It is this growth that was now starting to be realized by the group and would prove effective in the completion of the next scheduled nonroutine problem.

Predicting the Number of Student Lunches:

Step One: Problem Recognition

By this point of the field testing, students were aware of the three steps of problem solving. However, awareness and incorporation can have different meanings. It was during this nonroutine problem that the experimenter finally became convinced that the students knew what it meant to put into effect the three steps of problem solving, especially the third step of persistence. As will be described, it took a second opportunity at this same problem for students to reach this level.

Besides the fact that students could only generate data from the student body, this problem offered another twist or obstacle that the students would have to face. Because the school had over 1300 students, lunch was held twice daily. The participants only had access to one of the lunches. It was this obstacle that would be their stumbling block in acquiring accurate data. Unfortunately, at the time, the students did not completely recognize it as such and in the experimenter's opinion, did not show enough persistence in getting an accurate count of lunches served.

Step Two: Trying Something

Nevertheless, students gave reports concerning what they did and despite the fact that they could not accurately account for one of the lunch servings, their methods for acquiring data were interesting and showed use of the first two steps of problem solving. Group A decided to stand in the cafeteria and count individuals as they came through the lunch line. They did this process for four consecutive days with the exception of Wednesday.

This group determined that since Wednesday was a minimum day, those students who normally ate lunch, might not that day. To accommodate the fact that they could not be present for first lunch, Group A "hired" a younger sibling that had first lunch to count students who went through the lunch line for them. ("Hired" is used because group A told the experimenter that they paid the sibling a dollar a day to help them count.) Group A averaged the number of lunches from the four-day count and used this data to predict the number of lunches for the pre-determined day. Since their numbers for the four days were fairly consistent, they felt assured that their prediction was accurate.

Group B altered their plan to predict the number of lunches. Since their data could only be student generated, they tried asking a number of students how many lunches they thought would be served. They quickly determined that this method did not leave them convinced that their final answer would be accurate. Therefore, Group B decided to count the number of students going through the line during their lunch. Since Group B could only count the 8th graders going through the lunch line, they came up with their own solution for predicting the total lunches for the school. Their group agreed that since they were not able to count the 6^{th} and 7^{th} graders going through first lunch that they would simply double the amount of lunches they counted from 8^{th} grade. They figured that there were twice as many 6^{th} and 7^{th} graders as there were 8^{th} graders. By taking the lunch count from 8^{th} grade lunch and multiplying that by three, they arrived at their prediction for the total lunch count.

Another group looked to the Internet for answers. Although students in this problem were limited to getting data only from students, their efforts were applauded for trying something different. This group maintained that since they were the ones looking up the data, they were the ones directly generating the data for their predictions. This group looked up the home page for the school district and found it to be of no use. They continued looking up many websites (persistence) and found one called "School Wise Press" that they thought would be of most help. This website contained the percent of students who qualified for free lunch at middle schools in the area. Unfortunately for them, since their school was less that a year old, there was no data for their school. To remedy this, they decided to take the data from the five other middle schools in the surrounding area and calculated the average of free lunches for those schools.

Their average was 59.8%. They rounded this to 60%. Knowing they had 1300 students in their school, they simply took 60% of that total to predict their final count of student lunches that would be served that day (780).

The remaining reports that followed had similar strategies. Most groups stood outside of the lunch line and attempted to count the number of students who had school lunches, while using this data to estimate the number of lunches that were served in the earlier 6th and 7th grade lunch period. When all reports were given, a experimenter led discussion was held.

This discussion proved to be the turning point in how this group attempted future nonroutine problems. The first issue that was brought up in discussion was the discrepancy of the final tally of lunches by each group. Every group that presented indicated that they had persisted on this problem and felt assured they had done everything possible to get the correct answer, yet many predictions were no where near each other. The question that naturally followed from this was "Why?" This also led to a discussion of what they could have done differently as well as what things they overlooked. What the experimenter did at this point was draw out the flaws and oversights of their strategies and showed how had these items been considered, the students would have had a difficult time feeling assured of their final predictions.

It should be stated that when students presented their formal plans for determining the number of student lunches served that these holes in their strategies were evident. There would have been little use in pointing them out initially; rather, it was more valuable for the students to realize this themselves in order to find ways to continue to persist.

Step Three: Persistence

During the discussion, as obstacles of this problem were brought up, students continued to ask themselves, "Why didn't I think of that?" What was obvious to the experimenter was not so obvious to the students initially. Students began to see what real perseverance and persistence meant, especially for this problem. Students who were once convinced they had done and thought of everything were left a little dumbfounded by their effort. When asked at this point of the discussion to give a percentage to the accuracy of their predictions, the class felt that they were about 55% sure their answers were accurate. This did not leave them feeling good about their final reports and by this point the experimenter was feeling really good about the discussion.

The discussion naturally evolved into "What could we have done better?" It was here that really good ideas were generated on how they could have done this problem differently and more effectively. As more ideas were built off one another, students sighed in agreement wishing they had thought of these ideas from the onset. Once the discussion was finished, the experimenter explained that this is what was meant by really incorporating the three components of problem solving.

A hush fell over the class until one student said, "I wish we could do this problem again." A few others immediately agreed and it became evident that this was the feeling of the class. The experimenter, without hesitation asked, "Why not?" Excitedly the students asked "Really?!" Obviously it was never the experimenter's intention to do this problem twice; the point of it was to demonstrate the steps of problem solving. However, the students were demonstrating their own persistence in wanting to do this problem again and this looked like a golden opportunity for students to really incorporate the three steps and honestly know that they had done it successfully. As a result, part II of Predicting Student Lunches began.

Predicting the Number of Student Lunches Part II:

After the first attempt, students realized what they could have done better to solve this problem. As a result, students requested a second opportunity in which to solve this problem.

Step One: Problem Recognition

The issue that they had to overcome, which was overlooked the first time, was accounting for every student. In their first attempt, most students were only able to account for one grade level let alone the entire student body. To remedy this, the students knew they needed a way to determine "yes" or "no" if students were eating school lunch that day. The students needed a way to talk to every student and find out definitively whether they were eating lunch that day or had eaten lunch that day. Either way, making contact with each student was vital in the minds of the students to ensure success.

Step Two: Trying Something

The idea that the students proposed was quite simple: create a letter asking all teachers at a designated time to ask if students had eaten school lunch that day. These letters would be collected the following period and the results tallied. To make sure the letter suited their needs, all students drafted their own letter that night to bring back the following day. The letters were read aloud and the best components of each letter were agreed upon and formed together to make the final letter that would be distributed to teachers.

Step Three: Persistence

Even though this appeared to be an effective way to determine the number of lunches, students now faced some new obstacles. The greatest one was how to get a staff of 60 teachers to remember to take time out of their class to do this survey. The students thought it necessary that in order to ensure all teachers remembered, they would need advance warning and frequent reminders. The students made copies of the letter and had a student that worked in the office to put them in all the teachers' boxes on a Monday. The students then put a daily bulletin in the morning announcements reminding teachers that on Friday the surveys needed to be given; this announcement was read over the intercom everyday that week.

Having taken those steps, the students felt assured that they had taken the steps necessary to accurately determine as best as possible the number of student lunches served that day. In addition, the daily announcements and letter to all the staff created great interest concerning what was happening in the classroom. From this problem, word got out about what was happening, which allowed the experimenter to share the type of curriculum that was going on in the classroom and sparked interest in the other types of nonroutine problems that had taken place previously.

Once the survey had taken place, teachers began returning them and the excitement built. With all the work that had taken place, the pay-off, in students' minds, was about to happen. Students used a staff checklist to determine which teachers responded. When all the surveys were turned in, students were only missing 3 teacher surveys. These few omissions were due to substitute teachers that day. However, students felt very comfortable in taking a very educated estimate as to what those three classes might have submitted based on all the data from the entire school's submissions.

Students were both relieved and elated when they found out the accuracy of their prediction. This once again led to another conversation on the importance of problem solving and being persistent. By now, students experienced the fruits of their labor and showed appreciation for persistence while recognizing it as mandatory for an effective problem solving strategy. As stated earlier, this was the nonroutine problem that proved to be the turning point in students' understanding and acceptance of the entire process of problem solving.

From this point on, the experimenter was confident that the students would approach the remainder of nonroutine problems differently and more completely than the ones they had already completed. Previously, students needed much direct guidance from the experimenter to understand the problems as well as guidance during the solving to navigate through the problem. It was thought that with their newly acquired understanding of the problem solving strategy, students would be more independent in the approach and solution of each nonroutine problem presented to them. The remaining few explanations of the nonroutine problems presented in this field study hope to support this belief.

The Decision of Street Names:

Step One: Problem Recognition

The experimenter explained that having a street named Jefferson or Washington is obvious for knowing its origin. However, the real question is, "Why was your street named that and not the street next to you? Who was responsible for giving that location the name; who decided that it was going to be called that?" From this discussion, students understood their task and most figured that it was not a simple one. They were given a week to research and present their answers. Looking back, though, it would have been better to allow for more time for this problem. During this field study, the school year was rapidly coming to a close and the element of time was a limiting factor. However, this problem proved valuable for the experimenter in realizing the extent that the students understood the three steps of problem solving.

Step Two: Trying Something

In attempting to answer the question, many students went to considerable lengths in their research. Many students also learned the frustration of trying to contact city government and being put on hold or not getting the desired response and being transferred to someone else only to get the same response. Some also learned that sometimes there are no definitive answers available amongst the resources used by them. This idea would have greater meaning on their own assessment of how much they persisted on the task.

Nevertheless, the experimenter felt that the strategies used in their limited time working on the problem were impressive. Many students searched libraries, city hall and of course, the Internet. Students did become frustrated with the answers they were receiving. For example, students who talked to the city planner department got trivial answers like, "I think at the time your area had a lot of vineyards so that's why your street is called Vine Street." Answers like these left the students not very confident, while letting them know that the true answer may be difficult if not impossible to find.

Some students did have some success by calling the builder of their homes to find out how their streets were named. But the answers once again were generic in nature, "We thought of a theme of trees and just started to name a bunch of trees and that's how your street was named Cluster Pine Road." Despite the fact that their answers were not 100% convincing, students felt that they had put forth a good effort in order to find some solution.

Step Three: Persistence

What was interesting in the whole process was the students' final evaluations of themselves and their persistence. Students were asked to rate how well they persisted on finding their answers. And despite everyone having some sort of explanation of why their street was named what it was, they felt that they could have done better (with more time) than they did. Despite all the frustration of continuous dead ends and faulty leads, they believed they still had it within themselves to find better answers than what they could provide at the time. To the experimenter, this was a good sign that although the students did not necessarily persist to their full potential, at least the students really recognized when they felt they had not done so. This problem provided an arena for recognizing growth within the students as they evaluated themselves. Students were now aware what it felt to know when they had put forth their best effort while also knowing when they could still push themselves further.

This concept of understanding one's own persistence was something that was developed and observed over time while incorporating this curriculum of nonroutine problems. By now, students had a working knowledge of the three steps of problems solving. The timing of all of this was perfect as the field study was drawing to a close and leading to the final evaluation of the students in comparison to a control group. At this stage, there was only one nonroutine problem left to present before the final evaluation was given. It would be math based and would really test if the students would use persistence to determine a solution that would effectively work for this problem.

Five Calculations:

Step One: Problem Recognition

In this problem it is important that students understand what to do. Therefore, using smaller examples to illustrate was necessary to ensure students understood their task. To illustrate, the experimenter used a couple numbers below 100 and one above 100 to make sure they understand while not revealing any strategies that they would need to determine on their own. Once everyone understood completely, they were given class time and a week at home to develop their strategies to bring for their test the following week. It was stressed that their methods had to be quick and effective to reduce any number to zero. Since the 20 numbers selected by the experimenter would not be known to them until the test, it was important that they worked out the best method to handle any number that would be presented to them. Students were then responsible to give an oral presentation along with a written report of what they did to get their method and why they felt it would work for any number.

For numbers 100-819, there is a solution that works every time: if the number is not divisible by 9: a) subtract a number (less than 10) that will make it divisible by 9, b) divide by 9 to get a number less than 82, c) if it is not divisible by 9 then subtract a number that will make it divisible by 9, d) divide it by a 9 to get a number less than 10 and then, e) subtract that number to equal 0. In the case of numbers 810 to 900, it is necessary to find two larger factors to divide into the number to get it to a single digit number after four calculations. For example, the numbers from 855 to 873 require that one first get the number to 864 by adding or subtracting the appropriate number and then dividing by 9 and then 8. Finding these intervals or even being aware of such intervals was difficult for students to recognize. One note: there is no solution for the numbers 851 and 853. In this nonroutine problem, this was not brought up until after the problem was finished and the post discussion took place. This left the opportunity for students to potentially figure this out on their own. Also, it let students believe that any number from 100 to 900 could be reduced in five calculations or less.

Step Two: Trying Something

After a week, students submitted their reports. Many students caught on to the method necessary for numbers 100 to 810. For example, in one report, the students said:

> To calculate a number to zero in five or less calculations, we found out that there is an easy way to start. When you have a number no matter how many digits you

need to know what it is divisible by the sum of it. For example, 652, the sum of the digits are 13 and 13 is a prime number so you could not divide in into a big number. The main focus of this method is to make the original number to divide the biggest number that is given and it is 9. To make the number divide into 9 and equal an even number, you need to add or subtract so we subtracted it by four. The number now equals 648, its sum is 18 and 18 is divisible by 3,6 and 9. Now divide it by 9 and that equals 72 and 9 could go into 72 so it equals 8 and subtract that which equals 0. This method is successful all the time when you want to reduce the number to its lowest using the highest number given.

With minor variations, this was the typical report that students gave. In some cases, students understood that they needed to divide, but looked to divide by 2 because the number was even or by 5 when the number ended in 5 or 0. However, some groups became frustrated when dividing by 5 or 2 left them needing more than 5 calculations to reduce their number to zero.

Step Three: Persistence

The frustration that students felt was the topic of the discussion after the nonroutine problem was finished. Despite the progress that students had felt about the process of problem solving, students were not left with the idea that they had conquered it and could handle any problem presented. The experimenter left them with the idea that despite feeling some failure in not being able to find every solution, they could be assured that they had used some of the skills necessary to try to solve their problem. Just practicing good methods develops their thinking and gives them more experience in handling a variety of situations presented to them. The experimenter shared with them that even when less than successful than desired, learning and growth can take place and makes one more aware of the possibility to persist further than originally thought possible. The students agreed with this and shared that had they put more time into it, they felt that they would have created a better method of calculation.

This need for "more time" was a common statement made by the students, especially in the last couple of nonroutine problems presented. In their defense, in the perfect scenario, more time would have been allotted. In addition, this was the end of their 8th grade year and with that came a busier than normal schedule for them, with the demands of finishing their projects from their other classes as well as studying for finals. So it was understandable that their keenest efforts may not have been used when solving these problems. Despite this external influence, growth and understanding was still achieved and the opportunity to develop better problem solving strategies was clear.

Control vs. Experimenter Problems

At this point all nonroutine problems had been given, leaving just the final evaluation, which was also given to a control group. In addition, the participants completed a post test on what it meant to be a good problem solver.

The Lake question:

For this question, an asymmetrical lake was drawn on graph paper and students were asked to create as many strategies as possible to find the area of the lake. They then had to use one of their solutions to calculate their answer. In addition, they were asked to explain their strategies.

Scenario questions:

Since the Lake problem dealt with mathematics, the experimenter wanted to offer the students the opportunity to solve a problem that had a more personal impact. Unlike the Lake problem in which students had to work individually, this time students were allowed to work in pairs to come up with their solution.

Students were given the choice of two scenarios to address. Again, as in the Lake problem, students were asked to explain their answers as well as come up with as many solutions as possible.

The Real Life question - Scenario 1:

You notice your best friend has not been acting as usual for the last couple of months. Your friend's grades have dropped, he/she has gotten in trouble in school and lately has missed much school. Your friend confides in you that he or she has been drinking, "now and then, but it's no big deal." What do you do?

The Real Life question - Scenario 2:

Your family needs to cut expenses at home in order to save for something important. What do you suggest to your family to save money and also keep them on track for the next two years?

Control vs. Experimenter Results

The Lake question - Control Group:

Of the students in the control group, only two offered more than one strategy. Every student submitted the idea of counting all the whole squares and adding up the half squares to make a whole square. This is how the entire class generated their solution for the area of the lake. This was completely consistent across the entire class, with the exception of one student who did not offer any solution. Of the two students that offered a second solution, one stated to use a string to put around the lake to calculate the area. That was the extent of her explanation. The second student offered three solutions. After counting whole squares and estimating the partial squares, he offered the idea of multiplying the width and height. That was his complete explanation. His third idea was, "You could make it a rectangle and multiply the bottom with the wall. Then subtract the ones outside from the score." Although his explanation was vague, he did draw a rectangle around the figure and counted the whole squares outside the lake. I believe that perhaps this is what he meant by subtracting this from the total length times width. It was this student that offered the most strategies of anyone in the control group. Again, though, on the whole, these students only offered one solution to this problem. And as one would expect, this would be the most common solution to this problem. In total, the control group of 25 students submitted 28 solutions for this problem with four solutions being different.

The Lake question - Experimenter's Group:

For the experimenter's group, every student but one offered more than one solution for the problem. In all, 73 solutions were offered compared to the 28 that were offered by the control group. One note: only 23 students were there in the experimenter's group that day making the average of solutions per student approximately 3.2 for the

6. "You could cut into squares 4 x 4. There are only 7 whole complete 4 x 4 squares and 45 extra normal squares. That's a total of 167 plus the incomplete squares which makes about 20+-."
7. "Another one would be to notice the unfinished squares by shading the finished ones. This way you could add together the fractured ones."
8. "Count up all the whole squares. Count up all the half squares, divide the sum of the half squares by 2. Then add all the 1/3 squares and divide the sum by 3. When you get all the totals, add them up to get the answer."

The experimenter liked this answer because it was a more complete version of the typical answer of counting the whole squares along with the partial squares.

There were more suggestions, but they were altered variations of the above examples. These examples clearly show more depth in explanation and more creativity for the same task than the control group.

Obviously, the time factor has to be a consideration. In the future, in creating a final examination with a control group, the experimenter would design a problem where both groups could have a week to develop strategies for solving a nonroutine problem, similar to how the experimenter's students had done with the curriculum. The experimenter believes that this would truly create a disparity between the control groups' answers and the experimenter's group. Nevertheless, it was obvious that the students who had gone through a curriculum of nonroutine problems had demonstrated a higher ability to offer more strategies than the group that had not. The fact that within a 45 minute time period students completing the nonroutine curriculum were able to generate three times the amount of solutions is clear evidence that this curriculum is effective in developing good problem solving ability.

The following two scenarios are more difficult to assess. Unlike many mathematical problems there is not one objective solution. Also, deciding which solution is best can be difficult to determine. Another obstacle in assessing performance is how fair it is to only permit 45 minutes for an issue that can take weeks to potentially solve. All these factors limit the effectiveness of the examination in the experimenter's opinion. Regardless, the results will be shared in an attempt to analyze the complexity of answers given by the control group as compared to the experimenter's group.

The Real Life question: Scenario 1: Best Friend is Drinking

Control Group:

Group 1:

First I'll talk to him/her. I'll come up with some plan to keep them from doing that. If that doesn't help, I'll probably convince her to call Alcoholics Anonymous. Then tell her to go with a counselor and I will go with her.

Group 2:

If my best friend was in trouble I would go over his house and talk to him. I would tell him the dangers of drinking and driving. If he has a girlfriend she might leave him for another boy. Drinking does not calm you down, it gets you hyper and you do things you wouldn't normally do to people or things. Drinking is bad because you get queasy and throw up. You miss school and your friends don't hang out with you anymore because they don't want to get involved with drugs. I would talk to him and tell him to get alcoholic help (Alcoholics Anonymous).

Group 3:

One thing we don't want to do is tell their parents. You don't want them to turn away from you. You want to approach them when they are sober and alone. You talk about all the bad things that can happen to her. Someone could take advantage of you when you are drunk. You can get caught by the police. You can either get a big fine, or even go to jail. You don't want anything happening to her. Anything can happen when you are drunk. And the things it does to your body, kills brain cells and gets you out of good shape.

Group 4:

If my friend was drinking I would try to tell her all the bad things that could happen to her. Like when she went to a party she could get drunk with a guy because the guy would go get the beers. Then when they are drunk they might not know what they are doing and they might have sex and she could have a baby and the guy might deny that they ever did anything. Another thing that might happen is after the party is over she might drink and drive and if she crashes she might go to jail and might have to pay for the insurance and medical bill.

Most of the answers were along this line. These students definitely had some good insight into the problem. Obviously finding a solution is not easy, but these students do offer explanations and mostly consequences of the situations. The focus will now turn to what the experimenter's students said for their solutions.

The Real Life question: Scenario 1: Best Friend is Drinking

Experimenter's Group:

Group 1:

The first thing I would do would be to find information on recovering from drinking that would change my friend's attitude toward alcohol. I would go to the library and gather some information on the addiction of alcohol. I would also call alcoholics' anonymous numbers and ask them how to help my friend. With the pamphlets and books, this would be my first step. My second step would be to actually go about helping my friend. If he refused my offer for help, I would involve his parents. With his parents and my help I think he would respond.

Group 2:

We would talk to him and say that drinking is bad for him. If it doesn't work, we might threaten to not be his friend any more. We will then call the drinking and drugs hotline and let these people give us advice to deal with the situation. If that doesn't work will research the rate of people who die because of drinking. If that doesn't work, we will talk to the school counselor and tell her that our best friend is drinking. The next thing we will do is tell his parents to take him to a psychologist and that should help. The next thing we would do is tell him that we would be there for him 100% no matter what happens.

Group 3:

If one of our best friends had a drinking problem, we would try to help the person. The problem with alcoholics is that when you try to help them, it could either help them quit, or it could push them in an even more dangerous situation. But we choose to take the risk of trying to lead them to quit. We would try to explain or give them living proof of what happens to people who drink too much. We would also explain to her/him about alcohol poisoning, which is deadly. If he/she didn't listen, we would have no choice but to leave that person alone, but before we do that we would tell someone close to them (parents, grandparents, aunts, uncles, etc.) in hopes that that person can get through to them. But there really isn't anything we can do if the person doesn't want help. If they do want help, their parents could send them to a rehab center. That is pretty much all we can do.

Group 4:

I. We first would recognize the situation.
   a) this friend has a problem
   b) this friend needs help, but might not want it
   c) is telling authority the right thing to do?
   d) Should we approach her will she want to be approached or the flip side?
   e) Does she want to drink or is it stress relief?

II. What is our opinion?
   Since she came to us we believe that she doesn't know what to do- We feel drinking isn't just her only problem, but that she shouldn't drink to solve her problem.

III. Plan- Researching the problem
   a) Ask her why she feels she drinks; whether or not she could get upset.

b) Ask her if she wants help. Explain that we want to help. If she wants help, explain she needs to talk to her parents or older family member.
c) Call seminars
d) Call programs
e) Call rehabilitation centers and help her to stay away from alcohol
f) Help her to explain her problem

IV. Flip Side- If she doesn't want help
a) Call rehab center anonymously to see how we should deal with the problem.
b) Tell parents
c) Introduce her to the effects of alcohol
d) Introduce her to possible help
e) If the person doesn't want to be confronted, you ask someone else to talk to them.

Like the control group, these students offer great insight in solving this problem. However, the real difference is that their solutions are more in depth and again, as in the lake problem, more potential solutions were explored in the process. In addition, the solutions that the experimenter's students put forth focused more on actions and answers rather than on consequences.

It is the hope that these 13-year-old students have not had to deal with alcoholism on any sort of personal level, but their ability to offer solutions and suggestions are definitely on the level that most adults would attempt. This experimenter claims that based on the answers presented that the students who went through the curriculum of nonroutine problems used a more complete method of approaching this question from that of the control group. Their ability to offer more solutions while looking at more potential obstacles was obvious from their responses when compared to the control group. It offers evidence that a curriculum of nonroutine problems does strengthen the problem solving ability of students and develops their minds more rapidly than if they are not being exposed to this type of curriculum.

The Real Life question - Scenario 2 Family Needs to Save Money

Control Group:

None of the students from the control group even attempted to answer this question. The reasons for this are unclear. Speculation, at this point, would certainly be inconclusive.

The Real Life question - Scenario 2 Family Needs to Save Money

Experimenter's Group:

Two groups from the experimenter's class made an attempt at the problem. Here is what the two groups offered.

Group 1:

> If our family needs to save money for something important then we have some suggestions for them. They need to stop wasting money on decorating the house such as expensive furniture. They need to stop spending money on unwanted food or wine. Less time spending on shopping at malls, stores or markets. They need to spend money on stuff that they need but stop spending money

on junk. Less time on going out eating, entertainment such as movies, music and games. When they buy cars they don't have to buy new cars like '99's or 2000's, they just need a car to go to work. When they buy food they have to make a list of what they are going to buy. Never buy anything you see that your friends have and you don't. These are the savings that could save the family up to 40% of the uses of money.

Group 2:

Here are a few suggestions that we can give our families to save money and keep them on track for the next couple of years. The suggestions all range from how much money they are going to give to the children as well as how they are going to spend it.

1. Make a list of groceries, or for anything else you need to buy, and buy only what's on the list and nothing else that tempts someone in your family.
2. Save up your coupons for things you need to get a discount.
3. Try to buy store brand things instead of name brand things. The things will be cheaper and they'll be the same.
4. Go to cheap convenient stores for clothes and shoes. Ex. Payless.
5. Try to get your children in free or reduced lunch, give them $1.10 a day, which is $5.50 a week for the meal deal lunch. For extra expenses, like little sibling's baby sitter, try to get someone who'll work for free. You could try getting a neighbor or a family member to do the job.

Those things were all to cut expenses from home. The next few things are if our parents would like to make extra money on the side.

1. Try to get a bank account with a high interest, so that you can get some extra money a year.
2. Invest on things that you know will be worth it, and you'll know you're getting more money back.
3. Work overtime, a couple of hours, to get paid extra, if the job that the parent has can be worked overtime.
4. Get an extra job or get your spouse working if they don't work already.

It is difficult to state that these groups would have answered this question the same way at the beginning of the implementation of the nonroutine curriculum. However, the solutions presented were quite reasonable for students who do not necessarily have much to do with the family budget at this age. Each group showed the ability to offer multiple solutions, which is an indication of good problem solving technique. Again, it is unfortunate that no groups from the control group attempted to answer this question. Nevertheless, it seems too reasonable to claim that the experimenter's group exhibited skill in their approach to this problem.

Post Test

At this stage, the field test was essentially complete and the experimenter felt confident that the students' problem solving ability had improved over the course of the curriculum. What remained, however, were students' thoughts on the definition of a problem solver. This gave the students a final opportunity to reflect upon their own growth over the past six months. It also allowed the experimenter to compare their answers to what they had previously offered at the beginning of the field test.

In the beginning of this field study, the experimenter asked the question concerning what it meant to be a good problem solver. Recall that back then, students' answers dealt more with social issues than with academic issues. The idea that "problem" meant turmoil in their life rather than something of less significance was so commonplace in their answers it made the experimenter curious to see if students would approach the question the same way that they had done nearly six months prior. It would be obvious, that although problem solving still potentially meant dealing with hardships in life, it took on a more widespread significance to different parts of life.

The following are a few excerpts from the post test of what the students thought a good problem solver was:

Student 1:

> To be a good problem solver takes time, patience and independence in your work. It takes knowledge in reading the problem and understanding it, trying something and of course persistence. Also finding ways or using anything to solve it. It means concentrating on the task and being organized with any data you receive.

Student 2:

> A good problem solver means that person knows what steps should be done in order to solve a problem. There are many steps that the person needs to learn and know to use to solve the problem. First the person needs to recognize and find out what is going on. Second, the person needs to try to solve the problem many times, there are so many solutions that the person could think of and that will help the solving strategy easier.

Once the person finds more solutions, he/she should choose the quickest and easiest way to solve the problem. The most important plan that the person needs is 'persistence.' This means the person needs to be patient and keep trying to find another and another solution.

Student 3:

I think that being a good problem solver means that you're a person who has patience and a desire to solve the problem. The components that are necessary are very simple. The first step to solving a problem is actually recognizing the problem, because you can't solve a problem if you don't know what the problem is! (That would be a big problem.) The second step is to try something. If you already recognize the problem, you should have several ideas for solving the problem. You pick one idea and try to solve the problem. If that doesn't solve the problem, you go on to the last step, which is persistence. To persist means to keep on trying different ideas until you solve the problem.

Student 4:

This year I have to say that those problem solving projects were annoying, but they totally taught me so many lessons and I know that these tools will come in handy in the near future. I recently used them for solving a problem. Yesterday my friends and I were in a huge argument. Instead of blowing up and totally hurting their feelings I recognized the problem and cooled down. I left and thought about what needed to be said and approached the problem, which didn't work, but I tried to approach it again differently and we worked things out. My problem solving ability has helped me in many ways.

The second part of the writing prompt asked the students if they thought their own problem solving ability had improved. Here are some of their thoughts:

Student 5:

I think my problem solving skills did improve. I was able to improve by learning how to keep on trying. I always would end giving up after I've gone so far. I think I've learned to break that bad habit. This class really prepared me for problem solving in life.

Student 6:

I think my problem solving ability has improved in some ways. What I mean by that is that I learned how to persist, or not give up after my first idea didn't solve the problem; with persistence, I learned to keep trying new things and that even though an idea might be 'stupid' it still might work. The idea of persistence will help me in the future, in school and in life. So I am very glad to have learned how to persist.

Student 7:

I do believe that my problem solving ability has improved. Compared to what I knew when I started problem solving to right now, I have no doubt that I know a lot more at this moment. Due to all the problems we've all gone through we've all come to be better problem solvers. Also, because of all of the experience we've all had this past year, of solving almost any type of problem, I think we are capable of solving more various types of problems than before. We've all solved problems ranging from area problems to word problems. We've all solved problems dealing with mass and volume. Now, maybe, we'll be able to solve more of our own problems with a notable answer of our own.

Student 8:

I would have to say that my problem solving ability has improved in the last year. With the projects of persistence that we have completed, I have learned more and more different techniques of problem solving. When I first started the year, I hadn't the clue of the key components of problem solving. Now, not only do I know them, but also I know how to carry them out and in different ways. With each technique, I learn a little more.

It was evident from the students' comments that the component of persistence was understood by them. The students clearly learned that being a good problem solver meant that one had to persist. When given the same question six months ago, it was mentioned that their answers were more social in nature and not really academically oriented. Notice that when given the same question at the end of the curriculum, that the students did not necessarily relate problem solving ability to academics again. In fact, the idea that problem solving meant dealing with issue of life and not academics held true again this time, yet their definitions of what a problem solver really is had a more complex meaning the second time. This understanding and growth was evident as the curriculum progressed and students were allowed to practice the steps of problem solving. Being allowed to recognize the steps as they were happening or being allowed to recognize how they could have been incorporated after the fact allowed the students a better opportunity to understand for themselves what it meant to be a good problem solver. This, in the experimenter's observation, really shaped their definition as it evolved throughout the curriculum. It also gave credence to the fact that students' problem solving ability can improve over time with a curriculum of nonroutine problems. Having the belief in it and support of the teacher and students is the means to make it a reality.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

Introduction

This project was initiated to explore the use of a nonroutine problem solving curriculum as a means of improving the problem solving abilities of students. To accomplish this, relevant literature on the topic was reviewed and from this review, appropriate methodology was created and used. In order to see students' improvement, a flexible curriculum of nonroutine problems was developed. Cumulative results collected from previously presented problems facilitated the creation of each ensuing problem to be used.

This sequentially developed curriculum of nonroutine problems was used in combination with pre and post test assessments, and a control group. Results of each stage were used to hypothesize that useful problem solving ability could be enhanced using an effective curriculum of nonroutine problems.

At the end of the project, conclusions were drawn from the results and applied to the experimenter's original hypothesis. These conclusions are discussed below.

Research Questions and Hypothesis

This project asked: Can students develop their thinking, over time, to apply solid problem solving strategies to any and all situations of importance? This project was designed to give students that opportunity.

Ultimately, the idea is to offer students an alternative means to approach challenges as it has been described: nonroutinely. All people face challenges throughout their lives. The manner in which people handle those situations determines their success and sometimes even their happiness in life. Many students in the classroom give up too easily. There are so many reasons for this, many of which are external, but the one reason they can control is within themselves. When students' environments do not support their academic and social successes, nonroutine problem solving can equip them with a means to combat these external influences that would otherwise make them fail. Many students do not realize this. In fact, some teachers do not realize this. Sometimes the self is the only thing a person can rely upon. But many students have not discovered this and live on believing they have little opportunity to do the things that on the surface appear to be insurmountable. Anyone we can think of that has done something of any significance did not just happen upon it by luck. It is evident that what they did can usually be attributed to the three steps of problem solving.

Outcome

As a result of completing a six-month curriculum of nonroutine problems, students clearly enhanced their ability to recognize what it takes to become an effective problem solver, as well as become one themselves.

Students initially defined problem solving in socially related terms, rather than academic ones. The post test results did not generate a shift from this trend. After the six-month period, students still related problem solving to the social aspects of their lives. However, the difference after the curriculum was that the students were now able to recognize the problem, transfer their new skills to it and effectively solve it. The curriculum of nonroutine problems equipped these students with problem solving tools that could be used in many challenging situations.

The data results from the nonroutine problem curriculum show a progression of enhanced problem solving ability among the students. However, the most evident results came from the Control vs. Experimenter's group test.

Students in the control group were not equipped with the problem solving tools necessary to offer even one solution in response to the real-life scenarios. In sharp contrast, the experimenter's group offered several suitable solutions, indicating their matured problem solving skills and their retention of such learning.

Limitations

This hypothesis is limited in the real world by the typical classroom structure, which does not allow the time necessary to fully develop the three-step model of problem solving adequately. By using an elective class to implement this study, the experimenter was able to devote the time necessary to implement a reasonable curriculum that the typical classroom cannot support.

Another limitation lies in the creation of personalized nonroutine problems that fit the dynamics of each class. Each class has unique life experiences that influence their learning parameters. Meeting those parameters will always be a challenge. Specifically, the sequence of problems that a class in another context should use would probably be different than the sequence the experimenter used.

Future Research and Recommendations

To overcome these restrictions, the experimenter recommends placing less emphasis on a content-driven curriculum and instead, committing to implementing several nonroutine problems during the academic year. In implementing these nonroutine problems, the teacher should also present the three-step problem solving model. Although this is not a fully-developed curriculum, it is a positive step toward the development of nonroutine problem solving skills. The results of such implementation should prove valuable to the students.

The development of a more thorough nonroutine problem library will offer teachers the resources necessary to put into practice a "mini" nonroutine problem solving curriculum. This will minimize the creation aspect and anxiety of developing a nonroutine problem curriculum, easing the path for teacher use in the classroom.

Incorporating a curriculum of nonroutine problems in the classroom would take a lot of understanding, the ability to try different things with much modification, and most importantly, the desire to persist and believe that in time, despite the struggle as it is taking place, that students' ability to solve problems academically or socially will improve. The potential benefits for the students are significant.

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