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Developing understanding and fluency with numbers

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DEVELOPING UNDERSTANDING AND FLUENCY WITH NUMBERS

A Project

Presented to the

Faculty of

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San Bernardino

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of the Requirements for the Degree

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by

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ABSTRACT

This project will provide support for teachers who have solid understanding of math as the goal for the students in their classrooms. Using the district adopted Course of Study as the foundation; this project will provide a curriculum supplement for the first grade. The curriculum will be designed around concept development, utilizing problem solving, concept games and centers, and on-going assessment. The guide provides activities and assessments designed to have students develop long-range conceptual understanding in addition to development of basic skills.

This project is composed of several chapters. Chapter 1 discusses a brief selected history of mathematics curriculum development and discusses recent reform efforts in mathematics. Chapter 2 discusses developing understanding through a look at learning theory and brain research. Chapter 3 discusses implications for effective teaching practices to facilitate understanding. Chapter 4 discusses assessment and classroom discourse management. Chapter 5 discusses the genesis of the Mathematics Chats. The Mathematics Chats comprise the bulk of the curriculum

supplement and the focus of this project. The Math Chats can be found in Appendix B

The Mathematics Chats[1] in Appendix B were envisioned to take place in a particular context. The philosophical context explored in the first four chapters of this project provides that context. The Mathematics Chats are organized around the standards and benchmarks of a Southern California School District. These content standards and benchmarks look very much like the new California Benchmarks and Standards. (A listing for reference purposes of California's Mathematics Standards can be found in Appendix F.)

Although the Mathematics Chats in this project were written to support mathematics instruction in the first grade, the concept and structure can be used at kindergarten through grade 6. Rebecca Kallinger, a Math Matters coach, workshop trainer, and math leader in our district, and myself have been working with teachers at grades k-6 using the concepts discussed in the project. We consider our techniques and our ideas under construction. We continue to research and attend conferences to improve and revise our thinking and understanding about student learning and classroom practices.

This project represents a moment in time in my research and training. I am using the Mathematics Chats herein and revising and improving them as I work with students. The concept games and centers are in a constant state of refinement as well.

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INTRODUCTION

"Almost all, who have ever fully understood arithmetic, have been obliged to learn it over again in their own way." (Colburn, 1849)

This statement is as true today as when it was published in 1849 in a mathematics text. Colburn believed that mathematics should make sense to the student. An analogy can be made to a sculptor who understands the use of the chisel and can use it in many different ways to produce many kinds of effects. The sculptor's understanding of his/her tools and materials is deep and encompasses pressure, force, angles, and reaction to the different materials it is used on.

Similarly, children who understand the use of mathematical tools can use them in many different situations to solve many different kinds of problems. Additionally, children who understand small numbers can use those numbers in many ways. They can conserve those numbers, take them apart and put them back together while being able to conceive of the parts to whole relationships between those numbers. This fluency allows them to use the numbers as tools in many different contexts.

Small number fluency was selected for this project because of its importance in mathematical development in children. Small number fluency is a critical tool for young children to develop. It is the foundation upon which the children's future ability in mathematics will rest.

Evidence of this fluency with numbers can be observed in what students are able to do with numbers. The following questions can be helpful to consider while observing students work and talk about their mathematical understanding.

- Are they able to see small numbers visually without needing to count to confirm because a sense of the number/quantity has been developed?
- Can they take numbers apart and put them back together while conserving both the parts and whole values simultaneously?
- Can they demonstrate that a number can be described in many ways?
- Can they use their number knowledge to solve problems and take skill tests?

Children demonstrate their understanding and ability to use small numbers as a tool to represent quantity and ideas through their ability to use them flexibly and fluently.

As the author reflects on literacy development in young children and compares that to mathematical development, similarities were interesting to consider. A fluent reader understands that reading will make sense, sentences make up stories, words make up sentences, and letters make up words and visa versa. Each of these components have sub meanings and structure which work together to make up the whole.

There are similarities here with mathematics. A student who is fluent with numbers understands that quantities can be represented by symbols, those symbols have names and that symbols represent quantities. These quantities and respective symbols have relationships to each other in parts and in wholes. Children who are fluent with number are able to hold and conserve quantities while dealing with the smaller number values that make up those quantities. Children who are fluent with small numbers will be able to deal with these numbers flexibly and efficiently. (See Appendix E for more thoughts on letter versus number concept development.)

The classroom teacher's philosophical structure and beliefs about mathematics guide his/her choices about classroom tasks. Those choices include, but are not limited to, whether discussion will occur, how discourse will be conducted, what tasks and materials will be used, how questions are structured, how assessment is handled informally on the spot and formally, and etc. If the philosophical construct for math is one where mathematical ideas are obvious or empirical and need to be absorbed and memorized this dictates tasks and management choices. This philosophical construct contrasts with one in which mathematical concepts are constructed within the mind through abstraction and rationalization.

In this philosophical structure the teacher helps students find information, clarify thinking, problem solve and develop an awareness of alternative problem solving possibilities. The classroom model in this paper uses mathematics chats, Problems of the Day (POD's), math centers and games, and small group and large group experiences to achieve the number fluency and flexibility described above.

Mathematics Chats use the child's senses and language abilities to practice and communicate mathematics concepts and understanding. This process relies heavily on the use of oral language and manipulatives. The process's effectiveness relies on a classroom context in which the teacher effectively manages the discourse.

The Mathematics Chats curriculum supplement supports the content standards illustrated in the 1998 State of California Content Standards and Benchmarks (Appendix G) as well as the Curriculum Standards and Benchmarks for Riverside Unified School District (Appendix F). Whole group experiences, problem solving, small group intervention and assessment will all be discussed in the chapters that follow as important components of the mathematics curriculum.

Chapter 1

A Look at Curriculum Development and Mathematics Reform

To be responsive to changes in the world, mathematics curriculum must change to keep pace. Ruth Parker (1993), in her book Mathematical Power, notes that curriculum is not keeping up with advances in technology. "While much of the mathematics used in the world today has been invented and/or extended within the past forty years (e.g., data analysis, photogrammetry, fractals and chaos, discrete mathematics), almost all of the mathematics that exists in schools is hundreds of years old" (p. 6). There is concern that course content does not reflect the dynamic nature of mathematics. "Tom Romberg, in an April 1991 address to the National Summit on Mathematics Education, addressed this issue as he described the vast discrepancy between our nation's goals for mathematics education and our current curriculum. He suggested that while we talk of America's having "world class" mathematics standards, we continue to offer a curriculum consisting of eight years of eighteenth-century arithmetic, three years of seventeenth-century arithmetic, three years of seventeenth-century algebra, and one year of third-century-B.C. geometry" (Parker, 1993, p. 6).

Tension exists not only between what to teach in mathematics, but how to teach it. The tension between differing philosophical perspectives on what to teach and how to teach it in the mathematics curriculum has resulted in a history of change. These changes are the result of different philosophical constructs rising to popularity. The pull between the seemingly opposing forces of student discovery and the teacher-directed instruction has gone on for hundreds of years.

Warren Colburn (1849) wrote mathematics texts in the 1800's. His intent was to provide mathematics lessons for children that helped them develop an understanding of the concepts being taught. He emphasizes the importance of the construction of mathematical understanding.

John Dewey, like Colburn, believed in developing understanding as he writes, "Sheer imitation, dictation of steps to be taken, mechanical drills may give results most quickly and yet strengthen traits likely to be fatal to reflective power. The pupil is enjoined to do this and that specific thing, with no knowledge of any reason except that by so doing he gets his results most speedily; his mistakes are pointed out and corrected for him, he is kept at pure repetition of certain acts till they become automatic. Later, teachers wonder why the pupil reads with so little expression, and figures with so little intelligent consideration of the terms of his problem" (Dewey, 1910, 51-52).

As industrialization went in to full swing, the schools became places where raw products were formed into compliant workers for the industrial revolution. Because of influences such as Edward L. Thorndike's (1913) Educational Psychology; W.W. Charters (1923) Curriculum Construction, and Henry Harap's (1928) The Techniques of Curriculum Making, prompted school curriculum to emphasize skills and compliance over problem solving and thinking. The call for thinking-meaning centered curriculum did not disappear, however.

In the thirties, forties and fifties, although mechanical, algorithmic learning was still in vogue, researchers like Brownell (1935) argued for the meaningfulness of learning mathematical concepts with true understanding. "…The ability merely to perform certain operations mechanically and automatically is not enough.

Children must be able to analyze real or described quantitative situations" (p. 28). Wheat (1951) Van Engen (1947) and Bruner (1960) shared Brownell's concern about true understanding of mathematical concepts.

In the last half of the 20th century, the United States began its move from an industrial-based society to one of information, technology, and service. As these changes gathered speed, ripples were sent to all facets of social, political, and economical life. Changes in educational psychology and theories of learning also reflected these changes. Changes in society at large fostered implications for changes in schools to keep pace with these changes.

Publications such as A Nation at Risk (1983), What is Still Fundamental and What is Not (1983), and Everybody Counts: A Report to the Nation on the Future of Mathematics Education (1989) called for educational reform in mathematics education. In response, the National Council of Teachers of Mathematics (NCTM) developed and published Curriculum and Evaluation Standards for School Mathematics in March of 1989. The NCTM recognizes the need for change. They have called for major curricular changes to bring instruction, objectives and assessment up to speed with the needs of society. Steen (1989) summarizes these recommendations which are paraphrased here:

- ✓ Raise expectations - the idea here is that if more is expected, more will be achieved
- ✓ Increase breadth - include more estimation, chance, measurement, symmetry, data, algorithms, visual representation
- ✓ Increase use of calculators
- ✓ Engage students - increase opportunities for experience base through active participation
- ✓ Encourage teamwork
- ✓ Assess objectives - align assessment with student objectives
- ✓ Require mathematics at all levels through each of four years of high-school
- ✓ Demonstrate connections - between different strands of mathematics, other subjects and real life
- ✓ Stimulate creativity - variety of ways and solutions to problems
- ✓ Reduce fragmentation - problem solving situations in real life may involve several different areas of mathematics and require more than one step

✓ Require writing - writing clarifies thinking and communicates understanding

✓ Encourage discussion - discussion and argument is a powerful reinforcer to mathematical reasoning

The changes described above call for increased involvement of students during math instruction. With these recommendations in place, the mathematics classroom would no longer be a place for students to parrot teacher explanations. Students would be asked to think on their own and to reason. They would be asked to communicate their reasoning and to defend it or adjust it. The student in this kind of environment can not easily just rely on the faithful hand raisers to answer the questions. They are responsible to demonstrate their thinking and be involved. Their lack of involvement is obvious in their lack of communication both verbally and non-verbally.

The NCTM set to work establishing goals and standards for students in math in their publication, <u>Curriculum and Evaluation Standards for School Mathematics</u>. The 1992 Mathematics Framework for California Public Schools was organized very strongly around the NCTM standards.

The NCTM goals for students in their 1989 publication are:

1. Learn to value mathematics.
2. Become confident in one's own ability [to do mathematics].
3. Become mathematical problem solvers.
4. Learn to communicate mathematically.
5. Learn to reason mathematically (p. 4-5).

The mathematical community was not the only organization concerned over whether or not students are prepared to live and work in the 21st Century. What skills are required of the students within a new framework of a global economy run on information and technology? In 1990, Elizabeth Dole, as the secretary of the Department of Labor, established the Secretary's Commission on Achieving Necessary Skills (SCANS). The Commission sought to find what those job related skills were.

This 31 member, multidimensional commission conducted intensive interviewing of employers and employees in varied employment sectors. Their interviews were to ascertain the skills needed of an employee in a wide variety of jobs.

Arnold Parker (1992), the Executive Directory of SCANS, reports a summary of these skills. He states that the commission identified a "three-part foundation of skills and personal qualities needed for high-performance work:

- basic skills - reading, writing, mathematics, speaking and listening;
- thinking skills - creativity, decision making, reasoning, and problem solving;
- personal qualities - individual responsibility, self-management, and integrity" (p. 27).

Parker also summarizes the commission's list of "competencies" that create a structure that rests on the above foundation skills. These are those competencies that "… are the ability to productively use:

- resources - allocating time, money, and people;
- interpersonal skills - working on teams, teaching, negotiating, and serving customers;
- information - selecting, using, and applying technology;
- systems - understanding social, organizational and technological systems (Parker, 1992, p.28).

NCTM (1992) goals recommend the kind of program that would give students the skills The SCANS research indicates as necessary skills for success in various business and industry fields. These skills and foundational qualities required by the business community are also necessary in facilitating a productive work, family, and community role/life. Productive adults in their community, job, and family need to be able to read, write, listen, and communicate their thoughts, feelings, ideas, and beliefs. They need to be able to think, make decisions, reason, and problem solve. Adults need to have a sense of responsibility, be able to manage themselves, and possess integrity. Adults need to be able to manage their money, talents and time; work with others, teach, and negotiate; acquire, evaluate, and process information; select, use and apply technology; understand social organizational and technological systems.

In the California Framework (1992) there seemed to be an unprecedented degree of consensus. This consensus existed both within and outside the educational community in terms of the kind of math that should be taught. "There is overwhelming agreement that this Framework appropriately and accurately describes the mathematics programs that should be established in our schools" (California Framework, 1992, p. ix).

"The call is to change what mathematics we teach, how we teach it, and to whom. The mathematics education that most Americans received in the past may have been good enough at the time but is unsatisfactory for today's students. The time is overdue for shifting the emphasis from an elaborate study of mechanics and procedure toward a deeper understanding of central ideas and broader study of all strands of mathematics" (California Framework, 1992, p.vii).

As a result of these recommendations, staff development in problem solving, project orientation and thinking process was put into motion. Funding was available for a broad range of teacher training through such projects as the California Math Projects, Renaissance Math, Math Matters, etc. School sites enjoyed control over who they hired to do their staff development.

Not six years later, however, the state is pulling away from the NCTM standards and rewriting the framework. Funding for the established mathematics reform programs has become sharply controlled by the Board of Education. Challenge Standards for Students Success (1995) and Content and Performance Standards (1997) have been drafted and published by the California Department of Education.

These standards and the forthcoming Mathematics Framework are a withdrawal from the NCTM standards. The focus is shifting from one of problem solving and exploring understandings to one more focused on basic skills and teacher-directed learning instead of student discovery modes of teaching and learning.

What is basic for students in the 1999 and beyond, however, is not the same as what was basic for students in the past. The California Mathematics Council describes their vision of basics in "They're Counting on Us" (1995). Basics are not only arithmetic knowledge but also a broad range of understanding of mathematical concepts including geometric ideas, data interpretation and technology use.

The California Department of Education's Mathematics Program Advisory (June 1996) looks at this issue as well. In the discussion on balance, the Advisory recommends that basic skills, development of conceptual understanding, and adeptness in problem solving coexist equally. In the Basic Skills section, four factors are identified:

1. Students must practice skills in order to become proficient. The practice should be varied and should be included in classroom activities and homework. Teachers, students, and parents should realize that

students must spend substantial time and effort to really learn a skill.

2. Basic skills are developed over time, and they increase in depth and complexity over years.
3. To maintain skills, students must use them frequently throughout their school years.
4. Students more readily learn a skill when they see how it will be useful or are intrigued by a problem that requires the skill (p.2).

In an effort to raise the standards and improve mathematics instruction, the State of California has just recently adopted The California Mathematics Academic Content Standards (1998). The first grade goal statement indicates that "By the end of first grade, students understand and use the concept of "ones" and "tens" in the place value number system. They add and subtract small numbers with ease. They measure with simple units and locate objects in space. They describe data and analyze and solve simple problems situations" (p. 22).

The California Mathematics Academic Content Standards further delineate specific skills that students are to accomplish by the end of each grade. In December 1998, the California State Board of Education adopted the Frameworks that further delineate the goals set forth in the Academic Standards.

Developing fluency with small numbers is critical is the development of student understanding in mathematics. It is a critical skill to develop. There is much concern over basic skills competency in many districts. Basic number fact tests have been re-instituted in many schools as a way to insure this proficiency. Districts have printed number fact flash cards and sent them home for parents to drill their children. With this push for skills, several concerns arise. What is meant by skills and are thinking and problem solving considered "skills"? Will a narrow definition of skills be enough for students to succeed in the future?

Heibert (1996) doesn't think a narrow definition of skills will be sufficient. "In order to take advantage of new opportunities and to meet the challenges of tomorrow, today's students need flexible approaches for defining and solving problems. They need problem solving methods that can be adapted to new situations, and they need to know-how to develop new methods for new kinds of problems. Nowhere are such approaches more critical than in the mathematics classroom. All of this means that students must learn mathematics with understanding. Understanding is crucial because things learned with understanding can be used flexibly, adapted to new situations, and used to learn new things" (p. 1).

A look back into the curriculum history and a brief look at what is happening today illustrate that the ebb and flow of control over curriculum and the tug-of-war over the purse strings that fund it are likely to continue. Among those riding on the waves of change are the teachers who must keep the boat afloat. Differing dictates come from districts and principals as to what is expected and what materials and assessments will be used. However, the classroom teacher is the person who ultimately makes the choices as to what materials will be used in the classroom.

No matter what the popular materials or philosophies are, it is the classroom teacher who makes the difference. Increasing teacher expertise is key. The State of California recognizes this. One of the new requirements of using state money for staff development is that the staff development be ongoing and be tied to the standards and benchmarks.

It is within this milieu that classroom teachers must make choices about what to teach, how to teach it and when to teach it. While their programs are evaluated by state policy, and district interpretations of those policies, the evaluation standards may or may not reflect research on what is appropriate at what grade levels and on what is known about the way in which children learn. The classroom teacher seeking answers to these questions needs information on current brain research and research on how mathematical understanding develops in children.

The intent of the Mathematics Chats in this project is to provide meaningful experiences to reinforce mathematics concepts and to support a math curriculum that includes the factors discussed in this chapter. The Mathematics Chats provide an opportunity to develop skills in the classroom and use child-centered ideology. Student thinking, communication, cooperation and accountability are all components of the Mathematics Chats.

In the next chapter, brain research and learning theory will be discussed to help build a theoretical framework for mathematics instruction.

Chapter 2

Brain Research and Learning Theory

When planning and developing math curriculum and activities in the classroom, it is helpful to have an understanding of how the brain works. Research on the brain and learning has developed significantly in recent years. Basic brain biology and memory functions are important to understand. How information is gathered, processed and stored needs to be understood. An understanding of these processes helps a teacher develop lessons compatible with this process.

Theories about how students learn have driven educational decisions. One's subscription to a particular belief system effects the curricular choices made in the classroom. What follows is a brief look at some of these theories.

Three basic groups of learning theories have evolved. These are behavioral, cognitive and developmental. Mary Conroy (1988) summarizes the essence of each of these theories:

"Behaviorists say that children learn both from what precedes and what follows their actions. Students, they say, learn a skill by watching, imitation, getting feedback, associating skills with positive consequences, associating errors to negative consequences, taking small steps, and practicing. In contrast, cognitive theorists say that children use strategies developed through self-talk to guide their performance. To learn many of these strategies children watch a trusted model who demonstrates and thinks out loud, imitate the model repeatedly saying the steps out loud, getting feedback on their performance and imitating the model while thinking the steps. Developmentalists, on the other hand, believe that what a child can learn at any given point depends on maturation and experience. Such factors as age, mobility, and facility with language influence a child's level of learning" (p. 35).

Many of current practices in education reflect one or a combination of the above learning theories however, "...researchers now accept that abilities emerge at different ages in different children and that there are rarely abrupt leaps in development. Instead transitions are subtle, gradual and tend to overlap" (Conroy, 1988 p. 35). These theories seem to be the ones favored by the back to basics movement. Educators who feel that learning is much more complex and is directed from within are resisting this movement.

Learning theories are becoming more complex and encompassing. The work of Piaget and the work of theorists and researchers since, such as Constance Kamii, have sought to construct models of how children learn. How do children construct meaning and understanding of the world around them? How does the brain process information and how are ideas, attitudes, beliefs, knowledge and understandings constructed in the learner?

Over the last 15 years research on the brain aided by advanced technology has created new theories as to how the brain works. Through the use of Magnetic Resonance Imaging (MRI) machines, researchers can watch the brain in action. They can study how different stimuli register in several different areas of the brain simultaneously. Sylwester (1994), discussing Gerald Edelman's Theory of Neuronal Group Selection [described fully in his book Bright Air, Brilliant Fire, (1992)] suggests that our brain, like a jungle, has several layers and organisms (neural networks) that are interconnected and work simultaneously. As the brain and how it works are examined closer, emotion emerges as an important element in the learning process.

Caine and several others [Ekman (1985), Clynes (1977), Holden (1979), Rosenfield (1988), and Lakoff (1987)] believe that memory (a crucial element in learning) is fueled by emotion. They agree that without emotion of some degree most memory and learning does not take place. "To teach someone any subject adequately, the subject must be embedded in all the elements that give it meaning. People must have a way to relate to the subject in terms of what is personally important and this means acknowledging both the emotional impact and their deeply held needs and drives. Our emotions are an integral part of learning. When we ignore the emotional components of any subject we teach, we actually deprive students of meaningfulness" (Caine 1991, p.58). This understanding of the brain encourages educators to seek out and find ways to teach mathematics (and all other subjects) that are enveloped with meaning and emotion.

Educational practices are based on some assumptions about how a child learns. As research continues, more and more is learned about the brain. Different theories have emerged as to what kinds of memory systems the brain possesses and how those are used. Renate and Geoffrey Caine, in their book Making Connections (1991), take a look at how the brain works. With regard to memory, natural memory versus memorization is examined. Natural memory is named local memory and memorization is handled in taxon memory systems. Memorization has been the prized aspect of memory governing teaching practices since before the industrial revolution. Over the past 50 years behaviorist thought and classical conditioning have contributed much to the classroom management and learning environment design. These practices are built on assumptions of how learning occurs in the brain. Under this umbrella, essential learning is broken into discrete skills. These skills are practiced and then reassessed to some extent for the test.

This model fails to take advantage of the brain's ability to learn. It focuses its energy on utilizing the taxon memory systems (named by O'Keefe and Nadel (1978), stemming from the word taxonomies). Taxon memory systems are characterized by the need for review. These systems utilize what Caine (1991) refers to as traditional information processing model of memory (p 38). Caine describes some basic features of the taxon memory system

1. Information that is retained is through practice and rehearsal

2. Taxon learning is linked to extrinsic motivation and is powerfully motivated by external reward and punishment.
3. Taxon memories are set in a way that makes them hard to change.
4. Items are relatively isolated.
5. Much of what is stored is not initially meaningful. (p. 39)

The taxon memory systems are only a part of the brain's multifaceted network. The taxon memory and local memory communicate with each other. Local memory unlike taxon memory is natural and unrehearsed. Life events become part of memory with little effort. These experiences have immediate meaningfulness and become memories. Some basic features of local memory are paraphrased below:

1. Every human being has a special, survival oriented memory system that is virtually unlimited.
2. Memories exist in relationship to other memories and are a record of ongoing life events.
3. Memories are formed quickly and may be brief.
4. Memories are updated continuously, instantaneously and effortlessly when new information is added.

5. Novelty, curiosity, and expectations motivate formation of these memories.
6. Sensory acuity and awareness enhance local and taxon memory.
7. Memory "maps" for specific places are relatively instant, but large maps can take considerable amounts of time to form. (p. 42)

Educational practices that only take advantage of one kind of memory system can be compared to riding on one wheel when at least two are available.

Howard Gardner has been working in the field of brain research and learning for more than 20 years at Harvard University. He provides a view of the brain and learning styles that is multifaceted. Gardner (1988) explains that the "left hemisphere is normally the dominant site for language and logical thinking while the right hemisphere is the dominant site for musical, artistic ability, creativity, and emotion" (p. 38). A learner's brain may be wired to favor one hemisphere over another or may use each equally (whole brain dominant). In whatever subject is being taught, understanding of the content is the goal. However, each learner is not predisposed to learn each subject equally well or as well as others. Pirie (1991) observes that "understanding is a whole dynamic process and not a single or multi-valued acquisition, nor is it a linear combination of knowledge categories" (p. 2).

Children engage in interpersonal and intrapersonal communication processes in which information and experiences are fit into existing taxon and local memory systems. Gardner refers to specialized learning abilities or competencies as intelligences. He proposes that people have at least seven different intelligences housed in the hemispheres of the brain. Those intelligences are briefly summarized here:

1. Linguistic: the ability to use language
2. Musical: the ability to use, enjoy and compose a musical piece
3. Logical-mathematical: the ability to explore patterns, categories, and relationships by manipulating objects or symbols, and to experiment in a controlled orderly way
4. Spatial: The ability to perceive and mentally manipulate a form or object, to perceive and create tension, balance, and composition visual or spatial display

5. Bodily-kinesthetic: the ability to use fine and gross motor skills in sports, the performing arts, or arts and crafts productions
6. Interpersonal: the ability to understand and get along with others
7. Intra-personal: the ability to gain access to and understand one's inner feelings, dreams, and ideas (Armstrong, 1994, p. 6).

There are a few key points to this theory that are important:

1. Each person possesses all seven intelligences.
2. Most people can develop their intelligences to an adequate level of competency.
3. Intelligences usually work together in complex ways.
4. There are many ways to be intelligent within each category (Armstrong, 1994, p. 12).

One's particular strengths or intelligences are related to memory. Thomas Armstrong (1994) maintains that there is not a good memory or bad memory, but rather propensities to remember different imputes and experiences better. "Thus, one may have a good memory for faces, (spatial/interpersonal intelligence) but a poor memory for names and dates (linguistic/ logical-mathematical intelligence). One may have a superior ability to recall a tune (musical intelligence) but not be able to remember the dance steps that accompanies it (bodily kinesthetic intelligence) (p. 147)."

Armstrong (1994) summarizes Multiple Intelligence theory as it relates to the classroom. "MI theory is perhaps more accurately described as a philosophy of education, an attitude toward learning, or even a meta-model of education in the spirit of John Dewey's idea on progressive education rather than a set program of fixed techniques and strategies. As such it offers educators a broad opportunity to creatively adapt its fundamental principles to any number of educational settings" (p. x).

Teachers interested in conceptual understanding look at the child's brain in terms of being under the child's construction where thinking and understanding evolve through experiences and communication. This communication both internally in thinking and processing information, and externally in the exchange of thoughts and ideas, is important for this construction. It is also important as it brings out understanding and misunderstanding for the teacher to encourage and confront. This view contrasts with one of the brain as a vat to be filled with specific types of thinking. She/he begins by examining and observing how students approach numbers, counting and problem solving situations. She/he may ask these types of questions:

- How do children look at numbers?
- Can they see any numbers visually?
- Do they see 1,2,3,4,5 visually when they look at quantities and are they able to use this information at they count and problem solve?
- Can they conserve small numbers or are they counting and recounting?
- How do they count?
- Can they count on and count back?
- Do they have any counting strategies?

Piaget examined how mathematical knowledge develops in children. "Piaget believes that within each person there is an internal self-regulation mechanism that responds to environmental stimulation by constantly fitting in new experiences into existing cognitive structures (assimilation) and revising these structures to fit the new data (accommodation). Piaget refers to these cognitive structures as schemas" (Webb 1980, p. 93).

Much of this process is assigned to internal communication. Information coming in through the senses is either discarded, or it is sorted, checked against existing frameworks, and then the old understandings are changed in some way into the new. "Piaget believes that the child first internalizes concepts from his interactions with the environment and later develops the language to labels and describe these understandings" Webb (1990, p.95).

This is illustrated with the infant who has a visual, sensation understanding of his or her world. His/her understanding of mom, for example, begins with sensations. He/she learns to recognize his/her mother by her smell, the timber of her voice, the way she looks, and her touch. Language does not exist at first, but it does develop as the child travels through the language acquisition process. He/she begins to associate the word mom with the schema filled with sensation information for mom. The category now has a label that can be communicated. Mathematics instruction, however, often gives the language labels first and then asks students to engage in problem solving. The labels are meaningless until the child has encountered contexts where the labels make sense.

Of particular concern to this project is how children develop number knowledge. Number knowledge development begins at a very young age. The development of visual and perceptual understandings of quantity without having to rely on the abstract sign for quantities is often overlooked as a developmental step in number knowledge development. As students develop understanding of number concepts they must connect quantities with names and then with the written sign for the number. Development of sophistication with quantities can be developed using visual representations of quantities using dot cards and manipulatives.

Children can look at a dot card with five dots and discuss how they knew it was five. They can also work on imagining what the number would be if one dot was added or removed. This ability to see quantities immediately without counting is called subitizing. The word subitizing "comes from a Latin word meaning suddenly, subitizing is the direct perceptual apprehension of the numerosity of a group Clements (1999). Subitizing is being able to see and recognize a quantity without using other mathematical processes (Clements, 1999, p. 400). Clements identifies two different kinds of subitizing - perceptual and conceptual.

A child sees three dots, for example, and knows that it is three without having to count them to check if he/she is right. This kind of subitizing is called perceptual subitizing. Working with dot cards, dice, and playing cards children begin to develop visual memories of number quantities. Students can use these understanding to work with more abstract representations of number quantities represented by digits.

Conceptual subitizing, on the other hand, is a little more sophisticated and leads to fluency with higher numbers. When a child can conceptually subitize the number 9 on two dice, for example, by seeing four dots and four dots make 8 plus one more dot in the middle made 9 they are demonstrating that they can see and hold onto small quantities. They also understand how to combine those quantities into larger numbers. Children who have developed conceptual subitizing are able to work flexibly and fluently with numbers.

A child is fluent with numbers when he/she can use those numbers flexibly and efficiently in a ray of problem solving situations. How do children develop small number knowledge? How can that knowledge be strengthened to the point were it can be described as fluent? Children collect, through interactions with their environment, physical knowledge about the world.

A distinction needs to be drawn between the different types of knowledge a child constructs. Children construct knowledge about their world in different ways. A child collects and stores physical knowledge. A child must construct what Piaget called logico-mathematical knowledge (Kamii, 1985). A child develops logico-mathematical knowledge through empirical and reflective abstraction.

Physical knowledge is observable knowledge about objects in the world around a child. The facts that a block is yellow or red, it has corners, it doesn't roll, it falls when you drop it is a physical fact about the block. The fact that the yellow block is different from the red block is an example of logico-mathematical knowledge. The difference between the blocks is not inherent to yellow or the red block. The difference is created inside the child's mind. This knowledge is created when the child puts the blocks in relation to each other. If the child doesn't put the two blocks into some sort of relationship, the difference does not exist. Being able to sort and pattern requires that children are able to create these relationships.

According to Piaget children construct physical and logico-mathematical knowledge by abstracting information from their environment (Kamii, 1985). Piaget distinguished two kinds of abstracting a child does as he/she process knowledge from their environment. Reflective abstraction and empirical abstraction are two different kinds of information processing processes.

Piaget believed that children abstract physical knowledge about the world around them through empirical abstraction. The knowledge the child internalizes is observable and apparent. The information is stored with little need for rehearsal and practice.

Reflective abstraction, by contrast, is knowledge constructed internally as the child puts objects and ideas in relation to each other. A child develops logico-mathematical knowledge through reflective abstraction. Reflective abstractions and empirical abstractions constructed by the child do not exist separately. The child could not construct ideas of difference and likeness (reflective abstraction) if the child could not observe properties of an object (empirical abstraction).

In school, children learn social knowledge and must develop mathematical knowledge. Some mathematical knowledge is social like arbitrary names given to numbers and concepts. A child must develop logico-mathematic knowledge. Piaget contends (Kamii, 1985) that the ultimate source for this knowledge is within the child.

Several researchers have studied how social interaction between children helps to develop logico-mathematic knowledge. Perret-Clermont (1980) found that students who engage in exchanges of ideas with each other made progress in developing higher order thinking skills. She found that higher functioning students benefited most from this kind of exchange whereas lower functioning students made little progress as a result of social interaction. Inhelder, Sinclaire, and Bovet (1974) found that when children are questioned and provided with conflicting information they often are motivated to examine their own thinking and modify it to a more logical conclusion. This conflict can be created through identification of inconsistencies in their own thinking or in opposing opinions of classmates.

As a child begins to deal with mathematical concepts, one of the first tasks is to count. Working with the child to develop subitizing abilities helps the child develop more sophisticated mathematical processes. Subitizing is instantly seeing how many. A child develops mental pictures for what Kamii (1985) calls perceptual numbers. Numbers 1-6 are perceptual numbers. Some children need help developing these mental images. Learning to see numbers and amounts quickly can be facilitated through the use of dot cards and class discussion.

Though the perception of number quantities is the development of logico-mathematical knowledge, the words we use to represent those numbers is social knowledge. Counting words are an example of social knowledge. However, the idea of quantity and that quantities have relationships to each other is an example of logico-mathematical knowledge. As children develop the language to count orally, they also need to develop the logico-mathematical concept of what Piaget called order and hierarchical inclusion (Kamii, 1985).

Looking at two different children perform counting activities helps illustrate logico-mathematical knowledge abstracted by children. The task for the children is to count a set of blocks on the table. There are 11 blocks. The first child counts the blocks and comes up with 13. The child touches the blocks as a means to establish a 1-1 relationship, but has no way of keeping track of which blocks are counted and which are not. It should be noted that this child has developed more logico-mathematical knowledge than a child who counts orally but does not match blocks to oral counting in a 1-1 relationship (one item touched to one oral expression of a number). The development of the concept of 1 to 1 matching when counting is an important beginning step in the conservation of number. The child that does this knows that he/she must count (say) one word and touch one block at a time when counting.

The second child counts the blocks and finds there are 11. This child moved the blocks and Piaget would say that this child has developed a sense of order. He/she understands that the numbers have an order and that there needs to be order in the counting process to be sure that the counting is accurate. He/she has a sense that there is a specific quantity to count and has developed a logical strategy for making sure that all the items will be counted and counted only once. This child will begin to self-correct when an error occurs. He/she will monitor the counting process to make sure that order has been maintained.

The second child discussed above also is beginning to develop a sense of what Piaget (Kamii, 1985) referred to as hierarchical inclusion. As the child counts does the term for four mean the last object counted or all the items counted into a total? A child who doesn't have a sense of inclusion in numbers doesn't understand that 1 is part of 2 and that 1 and 2 are part of three. As a child at this level of logico-mathematical development counts, the last object counted becomes the total irrespective of whether all items were counted once and only once and none were skipped. Self-monitoring and self-correcting of the mathematical mental and physical process of dealing with number and manipulatives develops as a child develops his/her logico-mathematical knowledge.

Children have different kinds of memory systems. Memory and the development of logico-mathematical knowledge can be supported through the social interactions and capitalizing on emotional involvement. Discussions of problems solving techniques help children develop more efficient and effective memories and knowledge organizational systems in their brain. The child must be actively engaged and held accountable for listening and thinking during these discussions for them to be beneficial. Through classroom discourse management and observations of children the teacher begins to understand the mathematical thinking processes the child is able to use. The teacher is then able to take the child where he/she is and stretch their thinking a little at a time with questioning that provides the child with opportunities to examine thinking. Mathematics Chats are a venue for facilitating this process. Here is should be noted that critical aspects of the classroom and feedback techniques are very important in facilitating this process. Mathematics Chats by themselves without classroom management techniques in place will not be as effective in producing metacognition. These management and teaching techniques will be discussed in subsequent chapters.

Chapter 3

Implications for Teaching

This project was motivated by the request for more direction from teachers at a Southern California elementary school. Their frustration is many fold. The current adoption materials lack direction in concept development, lack in-depth concept development within the units, and a lack of sufficient practice of concepts and skills. Often, demonstrating proficiency on skill tests is used as the measure of understanding.

Marilyn Burns (1986) comments on the narrow focus of mathematics instruction, "Elementary grade children spend an estimated 90 percent of their school mathematics time on paper and pencil computation, most often learning computational skills by rote. Many students learn the rules and are able to do the computations, but computation success often masks student's lack of understanding and reasoning skills" (p. 34). This practice runs contrary to what is known about children's oral language development and the use of communication in learning. Students are asked to use reading and writing skills in mathematics. This practice does not capitalize on their ability to listen to and talk about mathematics. It is widely accepted in other curricular areas especially reading that oral language development and schema building is a critical part of the learning process. Yet it is all but ignored in mathematics instruction.

Marilyn Burns (1988), in her book A Collection of Math Lessons, reflects on her experiences in the classroom as a teacher as well as a student. She describes her first few years as a teacher with carefully planned mathematics lessons. She comments that it was not unusual for children to come up and ask, "If you'll just tell me what to do, I'll do it," (p.2) when they had been presented with a problem solving situation. This comment indicates the student has spent much time with meaningless exercises and is not developing logical thinking processes.

What are teachers who are considered experts in this area doing to meet the needs of their students' varied learning styles? How do those techniques encourage inclusion and eliminate gender biases? As changing curriculum and restructuring efforts evolve, it is important to look at what successful teachers have been doing to meet the needs of their students' varied learning styles.

How are effective teachers constructing the learning environment to facilitate the acquisition of and ability to use and manipulate mathematics concepts at higher levels of thinking? Leinhardt (1986) and Simon (1989) looked at the practices of teachers and found some commonalties between expert teachers. They concluded: "Experts are usually good at constructing series of lessons that successfully transmit the content that needs to be learned. Their lessons are clear, accurate, and rich in examples and demonstration of particular pieces of mathematics. The expert teacher presents this new material within a coherent but flexible lesson structure. Both the lesson structure and the content presentation are critical. Further, these lessons take place in an academic environment that focuses on the specifics that students are expected to learn (Leinhardt, 1986, p.29).

Leinhardt also found that expert teachers spend more time on task, have a logical sequence to activate schema, bridge learning, teach new skills, and make content and lesson design changes wherever necessary. He found that the expert teachers read their students and establish need. Through a continual process of communication with students, expert teachers adjust instruction and provide numerous and varied types of input and modeling. This varied, flexible, and repetitious use of different strategies is in an effort to reach the strengths of all learners and help scaffold where necessary.

Simon (1986) examines behaviors of the learner that affect learning. He reinforces the idea that children journey through stages of understanding of mathematical concepts. It is the teacher's job to guide them in their development through their strengths and learning styles. He explains (citing Papert, 1972) that students need to explore and discover new [mathematical] concepts for themselves (p.41). This is in agreement with Piaget's theory of cognitive development. Simon reported that students while "...discovering new concepts for themselves have the opportunity actually to do mathematics rather that passively learn about mathematics. The guided discovery approach challenges students to think more deeply about concepts and to create representations and explanations of those concepts as they connect with their prior experiences in a personally meaningful way. As a result, children retain understanding of the concept longer than students who have only the teacher's or the textbook's explanation"

Papert, (1972), Berman (1982), cited in Simon (1986), (p.41).

Simon also recommends that while engaging in and as a follow up to discovery, students need to work together and verbalize in cooperative learning groups and work situations. Two benefits are gained from this type of interaction. "First, students are exposed to diverse thinking and problem-solving approaches. Second, they develop metacognitive skills - knowledge about their own cognitive processes and the ability to use them" (p.42).

The choices a classroom teacher makes defines for children the nature of mathematics. James Heibert, Thomas P. Carpenter, Elizabeth Fennema, Karen C. Fuson, Diana Wearne, Hanlie Murray, Alwyn Olivier, and Piet Human in their book, Making Sense (1997) examine critical aspects of the mathematics classroom environment and the teaching-learning process.

Researchers from four different universities collaborated on research designed to develop student understanding of mathematical concepts (mainly addition and subtraction concepts). The projects developed independently, but after continual collaboration, researchers on the projects began to notice that while differences existed between all of the projects, the projects had core features in common. Those features are what the researchers describe as the critical features of the classroom. These features relate to the nature of the tasks students encounter, the role of the teacher, the social culture of the classroom, the mathematical tools and learning supports provided to children, and how equitable and accessible is the program to all students.

The kinds of task students are asked to complete define for them what mathematics is. In the classroom the tasks set "the foundation for the system of instruction" (Heibert, 1997, p. 7). Heibert points out that different kinds of tasks lead to different kinds of instruction. If the goal is to generate opportunities for students to build understanding of mathematical skills through exploration, communication and debate, then the classroom tasks look very different. Heibert explains that if the task is to be problematic for the child, and encourages the child to engage in exploration and thinking, it has at least three different features. Those features are paraphrased in the bullets below:

- The task is problematic - that is to say students see the task as an interesting problem. The task is something to find out or to make sense of.
- The task must connect to where students are. Students must be able to use the skills they already have to begin developing a method for completing the task.
- The task engages students in thinking about important mathematics. The task provides students opportunities to reflect on important mathematical ideas.

Classroom discourse is managed very differently in this situation as well. This leads to the discussion of the teacher's role in selecting and managing classroom tasks. In traditional mathematics classrooms the teacher's role was one of dispenser of mathematical information. In this context, the teacher selects tasks that involve the students in listening and imitating. In facilitating the development of conceptual understanding of mathematical ideas, the teacher has a new role. Heibert (1997) describes this changing role as one where the teacher is "selecting and posing appropriate sequences of problems as opportunities for learning, sharing information when it is essential for tackling problems, and facilitating the establishment of a classroom culture in which pupils work on novel problems individually and interactively, and discuss and reflect on their answers and methods" (p. 8).

The tasks selected and the way in which these tasks and resulting discourse is managed encourages the evolution of the social culture of the classroom. The way in which students relate and interact with each other dictates the level at which collaboration, communication, and acceptance will occur. It is important to look at the ways in which social culture of the classroom can be fostered where students are reflecting on mathematical ideas and communicating with each other. This reflection and communication is what builds understanding.

To develop this kind of environment, several things have to be in place. Through their research, the authors of Making Sense (1997) identified four norms of a healthy social culture. Those norms are summarized below.

- Classroom discussions center on problem solving methods and mathematical ideas.

- Students, when presented with a problem, choose methods of solving that problem and communicate those methods to their peers. An environment of respect is necessary to encourage the taking of this risk.
- Mistakes are not ridiculed but looked at as an opportunity to examine thinking, mathematical understanding and methodology.
- Correctness is established by the recognition that mathematics has a logic and that it is not the teacher or the "smartest student" who determines that logic.

Students may find it uncomfortable when the teacher suspends the indication of correctness to encourage further discussion about mathematics or the methodology used by students. It takes a while for students to become comfortable or at least accepting of this suspended animation of correctness. Making them take a stand by voting on most probable answers and then further discussing the possibilities and then re-voting makes it necessary to reconsider the mathematics and methodology several times. In this way, students begin to see a reflecting process and begin to see the possibility of reconsidering their initial answer. Recommending this kind of process is not new. In the early part of the century, John Dewey (1929, 1933) would agree that when one experiences uncertainty and comes to revel in the problem solving pursuit the process of becoming a reflective mathematician has begun.

The fourth dimension of the classroom is the kinds of tools available to use to solve problems and how those tools are used. Tools range from the chalkboard and chalk and paper and pencil to a broad range of manipulatives. It is important to point out that tools are for use in developing understanding of mathematical concepts and communicating about mathematical ideas. The tools provide a permanent or temporary representation of understanding of concepts being explored in the task at hand. The tools used do not contain the mathematical concepts but are a conduit to developing and communicating those concepts.

The last dimension of the mathematics classroom is the question of access and equity. Do all students in the classroom have equal access to the curriculum depending on their ability? Are they provided equal opportunity to explore concepts and communicate about their ideas? The main questions to consider here is are

1. Are all students able to participate in the mathematics being explored?
2. Does everyone have a chance to explain their thinking?
3. Does everyone participate and contribute to the discourse?

If the answers to the questions are positive, then the mathematics program has elements of accessibility and equality. If the response to these questions is negative and the goal is equability and accessibility, then alternatives and modifications need to be examined.

The behavior of expert teachers has been examined to indicate the behaviors that produce successful mathematics classrooms. Expert teachers have in place critical feature of the classroom that make their mathematics program successful. The critical features of the classroom have been explored as Heibert et al. (1997) delineate them provide an interesting scaffold to construct an effective mathematics program.

Expert teachers spend more time on task, know their students, and are responsive to their needs. They adjust instruction (and tasks) to meet the ever changing needs of the students. Being able to be responsive to student needs requires knowing the students. Knowing the students requires communicating with them and getting insight into their thinking. This can be accomplished only through observation both formal and informal and through communicating. The activities described in the Mathematics Chats appendix to this project are envisioned to take place within the context described above.

In the next chapter, the use of whole class communication and management systems will be discussed briefly as it relates to ways in which ongoing communication, both verbal and non-verbally, can be managed. This discourse is critical to the ongoing assessment and feedback that is important to inform the teacher's instruction as it is unfolding. The lesson and students' development is intertwined in a dynamic and evolving process.

Chapter 4

Assessment and Management

Senate Bill 62 calls for the development of a system designed to "make assessment an integral part of the instructional process" and "to facilitate the development of each and every pupil to become a self-motivated, competent, life-long learner."

As a result of Senate Bill 62, California's educational community set out to develop performance-based assessment in several curricular areas between 1992-1994.

The tests that were developed were time intensive to administer and score. These tests came under heavy criticism and were not funded again in 1995. The state went to using the Standardized Testing and Reporting program, STAR. Districts went back to using standardized tests like the SAT 9. This kind of test was familiar to parents and scored once again by machines.

The heavy focus on assessment in the form of standardized testing has increased its presence in California's classrooms. The number of class hours students spend testing has increased. The days per year spent testing increased from 3-4 to 12-16 days per year.

Additionally to prepare students for these tests, teachers are spending more time with test preparation activities. What is concerning about the amount of time testing is that these tests are used for comparative purposes not necessarily to inform instruction. The National Research Council (1989) indicates the negative impact this process has:

- Tests become ends in themselves, not means to assess educational objectives. Knowing this, teachers often teach to the test, not to the curriculum or to the children.
- Tests stress lower - rather than higher order thinking skills.

Tests used in the above-mentioned STAR program have tremendous power over curriculum content choices made by teachers. The scores are becoming increasingly more important as they are used for evaluative purposes of not only the child, but of programs, schools, and districts. Textbooks are being written to align with the tests - when these tests do not even align well with state standards and benchmarks. State standards and benchmarks do not always agree with what researchers have found is appropriate for different developmental levels and are not, in some cases compatible with new research on how the mind works. Regardless of this research, policy makers continue to rely on these standardized tests as a barometer on student success.

Constance Kamii (1989) was concerned with whether or not students involved in traditional mathematics programs did better or worse than children in constructivist mathematics programs. She found that on the standardized tests, the traditionally taught students did a little better (79^{th} percentile versus 85^{th}). When it came to demonstrating conceptual understanding of place value with numbers and in addition and subtraction, non-traditional problem solving, and mental math activities the students in the constructivist classrooms far out performed the students in the traditional classrooms. She attributes this to the different levels of learning and thinking that occurred in the different class settings. Standardized achievement tests are constructed to test concepts of math in a framework that defines math as "a set of symbols and rules or algorithms to internalize. Because traditional mathematics educators and makers of achievement tests either do not know or do not accept the difference between social or conventional knowledge and logico-mathematical knowledge, they reduce mathematics to conventional symbols and rules" (Kamii, 1991, p.7). The concerns over the effects standardized testing have on content and curriculum existed 10 years ago as strongly as they do today.

Constance Kamii (1991) reported that the use of achievement tests in the primary classroom perpetuated lower level thinking skills. She found that students in both the traditional math program and the students in the constructivist classrooms she compared scored similarly on achievement tests. However, students in the traditional program could not explain their thinking and did not know why answers were correct only that they were. Students in a constructivist program were more able to bring problem solving skills and explanations to the problems requiring the integration of mathematics skills. The National Research Council (1989) warns that achievement "tests become ends in themselves, not means to assess educational objectives. Knowing this, teachers often teach to the tests, not to the curriculum or to the children." In addition, "Tests stress lower- rather than higher-order thinking skills" (p. 69).

The remainder of this chapter will address ongoing assessment and management techniques that can be used in the classroom to assess student progress and inform instruction. The student feedback systems that are described are designed to give the teacher immediate feedback as to understanding. This immediate feedback allows teachers to make changes in the course of a lesson in progress to respond to the understanding or misunderstand demonstrated by the students.

The management of discourse and the constant assessment of understanding are critical to the flow of the lesson. Without this information it is difficult to change a lesson or plan lessons to be responsive to the needs of the students. Using several different management techniques can facilitate assessing students at work through informal observations. Hand and finger signals, whole group responses and other response techniques increases the involvement, participation and time on task for students. These techniques go hand in hand with questioning techniques.

Student accountability and participation during the course of a lesson is the goal of the management techniques taught through the Math Matters[2] program. The Mathematics

Chats of this project are designed to take place in a classroom where ongoing assessment and student participation are essential elements of the teaching-learning process.

This process is characterized by constant information collection by the teacher through verbal and non-verbal communication and management techniques. Two key areas in Math Matters training is content and management strategies. Covering all the objectives delineated in the California Standards and Benchmarks requires time on task and effective techniques to facilitate the learning process. These are the focus of the Management, Instruction, Focus, and Feedback (MIFF) Techniques. The MIFF Techniques focus on successful management of content, involve all students and visitors, keep a focus on the curriculum and learning behaviors, and provide a feedback loop so the teacher can modify and spot teach as necessary.

The Miff Techniques are flexible and can be used across the curriculum. There are ten techniques: Modes of Response, Specific Questions, Hand and Finger Signals, Positive Reinforcement (Answers-Behavior), Space, Deliberate Mistakes, Circulation (Rapid-with hints), Involvement of Visitors, Do You Hear An Echo, and Wait

Time.

Modes of response help students know exactly how you want them to respond to a question. Specific Questions help students focus their thinking to gain insight or lead students to specific knowledge. Hand and Finger Signals give all students a way of responding to the questions. Positive Reinforcement to both behavior and to responses to the content help students learn that there are involvement expectations and that they will be consistently reinforced. Statements like the ones below tell students exactly what kind of response is expected.

Taking a look at classroom discourse during a Mathematics Chat illustrates how four of the MIFF strategies can be used simultaneously. Before showing a meaningful flash card with 5 dots the teacher might guide the discourse in this manner. "Show me on your fingers how many dots are on the card. Make sure you are showing me your answer in your super-student space." (Trace an s on your chest like Superman's reminds students where you want to see the answer.) A specific mode of response and a specific question to respond to has been given. Then wait and watch and give positive reinforcement to the appropriate behavior. "I see that John, Paul and Tiesha have ideas. I can see their thinking. Let's see what the rest of you are thinking." "Jerome, is showing me five fingers?" Write the response down and thank him for his thinking. "I also see four fingers and six fingers." Write those down too, validating that thinking happened even though it was erroneous. "Check it again and see who you agree with. Thumbs up if you think the answer is 4, 5, or 6." "Jerome can you tell us how you knew it was five?"

At this point Jerome has the floor and explains his thinking process. The teacher models active listening and might ask if any other students thought of it the same way Jerome did. S/he may also ask for another student to tell in his or her own words how Jerome figured out the problem. In this way the teacher reinforces that each student's ideas are important and are to be listed to actively. The children begin to learn that it is their job to listen and think about the ideas of their fellow classmates. The teacher has to be careful not to repeat what the child says as children will learn to wait for him or her to explain it instead of listening the first time. The above describes an implementation of the Do you hear an echo? MIFF technique. Also demonstrated above is the use of wait time. Waiting to collect answers is important in giving all children a chance to accomplish the thinking task.

Deliberate mistakes, Circulation, Involvement of visitors, and Space are four more MIFF Techniques. Deliberate mistakes set a positive environment for risk taking and help children see that it is not "bad" to make an error. Circulation through the classroom watching students work and dropping in hints helps guide students through the activity and is valuable in collecting information on how students are performing through the course of the lesson. Involving visitors helps to reduce interruptions. Using classroom space helps to reinforce positive behavior and involvement. Moving through the classroom and reducing the proximity to all students helps to reinforce and establish participation and behavioral expectations.

A snapshot into the class helps illustrate the techniques described above. A student enters the room while the class and teacher are involved with a patterning activity at the overhead. Students at their desks are looking at different patterns and reproducing those patterns at their seats. The visiting student has a note and needs a response to take back to his teacher about an upcoming event both classes are involved in later in the day. The teacher asks the visitor to make a pattern on the overhead to challenge the class while s/he responds to the note. As she responds to the note she can also circulate and respond to student work moving about the room to see that students are on task giving feedback as necessary. A student is off task and the teacher moves in to close proximity to the student encouraging appropriate behavior. The note is then finished, delivered to the visitor who now leaves, and class continues with minimal instructional time lost. The teacher responds to the task being completed and gives a new challenge: a pattern with a mistake. This pattern gives children a new challenge: make the pattern, find the mistake and fix it. Discourse continues, students notice the mistake and offer suggestions to fix it. The teacher positively reinforces their cleverness at seeing the mistake and knowing how to fix it.

Having all students involved appropriately all of the time is the goal. Using effective classroom management and discourse strategies that increase the number of students that can be involved answering questions helps to reach toward this goal. The management of discourse and the constant assessment of understanding are critical to the flow of the lesson. Without this information it is difficult to change a lesson or plan lessons to be responsive to the needs of the students.

Many different aspects of the classroom have been discussed in the previous chapters. These ideas have been developing over time with many influences from various researchers and researching practitioners in the field of mathematics and in the classroom. Some of these influential people have been widely published and others have been locally inspirational to the teachers piloting the materials and ideas in this thesis. A brief discussion of the influences that were prominent in the through the process of developing this project will be discussed in the next chapter.

Directly following the next chapter is a one-page matrix that describes a sequence of lessons that could be used to teach several different content standards and benchmarks. Mathematics Chats, problem solving and centers and games happen on a daily basis. The boxes horizontally describe learning opportunities occurring on a day in the classroom. The boxes directly below describe the next days opportunities.

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Time.

Modes of response help students know exactly how you want them to respond to a question. Specific Questions help students focus their thinking to gain insight or lead students to specific knowledge. Hand and Finger Signals give all students a way of responding to the questions. Positive Reinforcement to both behavior and to responses to the content help students learn that there are involvement expectations and that they will be consistently reinforced. Statements like the ones below tell students exactly what kind of response is expected.

Taking a look at classroom discourse during a Mathematics Chat illustrates how four of the MIFF strategies can be used simultaneously. Before showing a meaningful flash card with 5 dots the teacher might guide the discourse in this manner. "Show me on your fingers how many dots are on the card. Make sure you are showing me your answer in your super-student space." (Trace an s on your chest like Superman's reminds students where you want to see the answer.) A specific mode of response and a specific question to respond to has been given. Then wait and watch and give positive reinforcement to the appropriate behavior. "I see that John, Paul and Tiesha have ideas. I can see their thinking. Let's see what the rest of you are thinking." "Jerome, is showing me five fingers?" Write the response down and thank him for his thinking. "I also see four fingers and six fingers." Write those down too, validating that thinking happened even though it was erroneous. "Check it again and see who you agree with. Thumbs up if you think the answer is 4, 5, or 6." "Jerome can you tell us how you knew it was five?"

At this point Jerome has the floor and explains his thinking process. The teacher models active listening and might ask if any other students thought of it the same way Jerome did. S/he may also ask for another student to tell in his or her own words how Jerome figured out the problem. In this way the teacher reinforces that each student's ideas are important and are to be listed to actively. The children begin to learn that it is their job to listen and think about the ideas of their fellow classmates. The teacher has to be careful not to repeat what the child says as children will learn to wait for him or her to explain it instead of listening the first time. The above describes an implementation of the Do you hear an echo? MIFF technique. Also demonstrated above is the use of wait time. Waiting to collect answers is important in giving all children a chance to accomplish the thinking task.

Deliberate mistakes, Circulation, Involvement of visitors, and Space are four more MIFF Techniques. Deliberate mistakes set a positive environment for risk taking and help children see that it is not "bad" to make an error. Circulation through the classroom watching students work and dropping in hints helps guide students through the activity and is valuable in collecting information on how students are performing through the course of the lesson. Involving visitors helps to reduce interruptions. Using classroom space helps to reinforce positive behavior and involvement. Moving through the classroom and reducing the proximity to all students helps to reinforce and establish participation and behavioral expectations.

A snapshot into the class helps illustrate the techniques described above. A student enters the room while the class and teacher are involved with a patterning activity at the overhead. Students at their desks are looking at different patterns and reproducing those patterns at their seats. The visiting student has a note and needs a response to take back to his teacher about an upcoming event both classes are involved in later in the day. The teacher asks the visitor to make a pattern on the overhead to challenge the class while s/he responds to the note. As she responds to the note she can also circulate and respond to student work moving about the room to see that students are on task giving feedback as necessary. A student is off task and the teacher moves in to close proximity to the student encouraging appropriate behavior. The note is then finished, delivered to the visitor who now leaves, and class continues with minimal instructional time lost. The teacher responds to the task being completed and gives a new challenge: a pattern with a mistake. This pattern gives children a new challenge: make the pattern, find the mistake and fix it. Discourse continues, students notice the mistake and offer suggestions to fix it. The teacher positively reinforces their cleverness at seeing the mistake and knowing how to fix it.

Having all students involved appropriately all of the time is the goal. Using effective classroom management and discourse strategies that increase the number of students that can be involved answering questions helps to reach toward this goal. The management of discourse and the constant assessment of understanding are critical to the flow of the lesson. Without this information it is difficult to change a lesson or plan lessons to be responsive to the needs of the students.

Many different aspects of the classroom have been discussed in the previous chapters. These ideas have been developing over time with many influences from various researchers and researching practitioners in the field of mathematics and in the classroom. Some of these influential people have been widely published and others have been locally inspirational to the teachers piloting the materials and ideas in this thesis. A brief discussion of the influences that were prominent throughout the process of developing this project will be discussed in the next chapter.

Directly following the next chapter is a one-page matrix that describes a sequence of lessons that could be used to teach several different content standards and benchmarks. Mathematics Chats, problem solving and centers and games happen on a daily basis. The boxes horizontally describe learning opportunities occurring on a day in the classroom. The boxes directly below describe the next days opportunities.

Chapter 5

Process Results and Implications

The mathematics Chats that follow this chapter had a geneses several years ago as I began thinking about multiple intelligence theory, levels of questioning, and Piaget's work with young children and their thinking process. As my teaching experience increased, I learned much about student learning and my inadequacies as a teacher. Student misunderstandings taught me much more about what needed to change in my instruction that what students could do.

It was when I transferred to first grade and began working with students at the very beginning stages of literacy development in math and language that I was able to really observe the learning process as it unfolds. First graders do much of their thinking out loud giving multiple opportunities to observe and question what is going on in their heads.

At this same time I began working with Rebecca Kallinger who is currently working as a Math Matters coach in our district and is the on the California Math Council

Council board. We shared similar philosophies and engaged in many long discussions on learning and teaching techniques.

In July of 1997, Rebecca and I attended a leadership conference in Bellingham, Washington, where we worked for a week with Kathy Richardson. Kathy's books and videos were the catalyst for the development of the Mathematics Chats in this project. Kathy's number talks focused on student discussions of number problems. We have expanded these ideas to include many different mathematics strands. We came up with the name Mathematics Chats as the chats include all strands of mathematics.

Mathematics Chats provide a venue for consistent and ongoing reinforcement of mathematical concepts. When making choices on how to implement mathematics curriculum, especially in small numbers fluency development, the teacher must make philosophical decisions. These decisions may be influenced by answers to questions like these:

⇒ Do I use a discovery approach or a teacher directed approach?

⇒ What does that look like in my classroom?

⇒ What tasks and for what reasons am I selecting student tasks?

⇒ Do these tasks fit the needs of my children and are they tied to benchmarks and standards for my grade level?

⇒ Who is evaluating the choices I make?

⇒ What materials will I use to teach math concepts?

⇒ What do I want my children to know and understand after this experience?

⇒ What kind of communication do I expect to indicate concept and skill development has occurred?

⇒ How will I know they understand?

Below are uses for Mathematics Chats:

1. Develop oral math language and communication skills - clarify thinking
2. Develop math literacy
3. Develop number fluency and flexibility
4. Encourage the development of problem solving strategies
5. Provide a venue for on-going authentic informal assessment (This assessment is critical to drive instruction.)

6. Provide systematic reinforcement and review
7. Teach students to value their own thinking and that of their peers
8. Develop listening skills

Oral language development and activation of prior knowledge and experiences is a critical part in the process of teaching any curricular area. In recent years, the relationship between the child's oral language and experiences has led to the development of several theories on teaching reading. Yet this component of learning and understanding is not tapped in the mathematics classroom. When very young children begin to learn numbers, they need to make connections between arbitrary symbols and abstract quantities connected to those symbols. They must also learn how to communicate that understanding using these signs and symbols in writing and speaking. The arbitrary assignment of this squiggle, 1, is connected to the oral "one" and 2 to "two" and so on. The child is developing an understanding with * = 1 = "one" and ** = 2 = "two" and so on. These symbols and quantities are ordered. Students must also learn that these squiggles represent different quantities depending on where they appear in the number. Place value becomes important. These different aspects of number item knowledge develop through children's multiple experiences with the numbers. This knowledge must become intuitive in the child for them to be able to understand our number system.

As a learner in this area, what is produced on these pages represents a snapshot into my understanding now. This understanding is constantly changing with new research and study in learning and the brain. I will continue to follow the work of Constance Kamii, Ruth Parker, James Heibert, Kathy Richardson and their colleagues and modify my teaching practices as my theories expand. I will continue to consult resources like Kathy Richardson's Developing Number Concepts (1999), Leigh Child's (1999) Nimble with Numbers, and Constance Kamii's Young children Reinvent Arithmetic (1985) as I learn more about how children learn and develop meaningful mathematics activities for my students. Even after implementing some of the work in this project I've made changes.

The bean game, for example, where students count beans is a good activity for comparing, but I won't use it at the beginning of the year again without some work with comparing smaller numbers. Also, the spin a bingo game is too hard for many students at the beginning of first grade because the numbers are too large. The concept of more and less seemed to confuse some students. I will use it later next year. I have modified the game for use earlier, because students can work with comparing concepts with smaller numbers. The modified game is for use at the beginning of the year with numbers up to 6 instead of 11. As with the beans, numbers above 6 seemed to be too hard to compare for many of my beginning first grade students. This game will be modified using pips instead of digits at first as students develop the ability to count and see number quantities. Counting and recounting the pips (dots) on the dice and on the game board helps students develop the ability to subitize the quantities for numbers six and less.

Games and activities like these can be found in the Richardson and Childs books listed above. Both resources are full of wonderful ideas and insights for teachers interested in learning more about concept development and teaching for understanding.

Appendix A

Curriculum Matrix

This matrix shows a beginning unit for first grade. The games and centers can be modified and reused throughout the year as proficiency with number grows. Beginning number sense and addition and subtraction concepts are the focus for the activities

Standard/Bench-mark	Math Chats	Problem Solving	Centers / Games
1.1 counts, reads, and writes whole numbers to 100 (This set deals with numbers 10 and less. (Beginning weeks of 1st grade) 1.6 represents equivalent forms of the same number through the use of physical models, diagrams, and number expressions, to 20, e.g.: 8: 4+4, 5+3, 2+2+2+2, 9-1 etc.) (During this menu set, students are working on combinations to 4) 3.1 shows the meaning of addition (putting together) and subtraction (taking away, comparing, finding the difference) 3.3 knows the addition facts (sums to 10) and the corresponding subtraction facts and commits them to memory 3.4 understands the commutative property of addition 3.5 uses manipulatives to add and subtract to 20 and writes the correct equation 7.1 recognizes and understands the mathematical meaning of the symbol +, -, & = 7.2 computes equations with missing numbers using manipulatives (All these objectives are worked on in their *primary* stages using the activities and centers to the right)	Using meaningful flash cards (see Teacher Resources for sample) Show several cards to see how students are counting up the flash card shapes. Do they see any small numbers and count on?	Mental Math: Take the number of eyes you have. Add to that the number of noses you have. Then add the number of thumbs you have.	Play Snap for four. Then record the different ways to snap for four on the recording
	Use unifix cubes to do story problems totaling numbers less than 10. Do students understand the difference in stories between addition and subtraction?	Two-Problem Approach 1. 2+2=___ a. 2 b. 3 c. 4 d. 2. What are different ways 4 toys can be arranged on 2 shelves	Play Fishing for Four and then record on the recording sheet.
	Use meaningful flash cards as a means to discuss how students see numbers and how they count. Teach active listening	Problem of the Day/Week Jeremy had two dogs that got very dirty. They made muddy footprints all over. He had to clean them up. How many paws did he have to wash?	Play Spilling Four and record on the recording sheet.
	Use bears or lions to do problem solving with numbers totaling less than 10. Mix addition and subtraction. Show students how to record stories using mathematical language and symbols.	Head Problem: Take the number of paws on a dog. Take away one. Add the number of noses a dog has. What's the number?	How many beans in the Bag? Students count, order and record numbers of beans in a bag. Totals at first less than 10.
	Use meaningful flash cards as a means to discuss how students see numbers and how they count. Teach active listening. Record students problem solving on the cards in number form: "Oh, you saw 2 here and then 2 more (2+2=4)" write on the chalkboard.	Two Problem Approach (overhead or on board) 1. 1+__=3 a. 3 b. 2 c. 0 d. 5 2. Joe had three friends with bikes. How many wheels could they have altogether?	How big is your handful? Students take a handful of a manipulative count, record totals. (Use large manipulative at first to control totals under 20)
	Using the 10 frame and plastic chips, have students locate numbers. Then have them show you 1 more/less or 2 more/less than that number. Do several and discuss how they know it is one more/less or two more/less.	If a snake had 4 mice in the jar and some of them escaped, what would be some of the ways the mice could be arranged inside and outside of the jar? Draw and Show your thinking.	Play Spin a Bingo. Students spin spinner and place a chip on the number. Numbers 12 and less are used. This game is modified later on as students become more proficient.

Appendix B

Mathematics Chats

The following set of Mathematics Chats is organized with the Content Standards listed in bold. The Benchmarks are listed in blue. The Mathematics Chats are a springboard and roughly listed in order of the Content Standards and Benchmarks. However, the needs of the children as identified during Math Chat process in the classroom, through teacher observation and assessment of student behaviors, should be the driving force behind curriculum choices. Mathematics Chats should be responsive to the changing needs, understandings, or misunderstandings of the students.

The Chats are not organized in the order they should be used. As discussed above they are intended to be suggestions that can be modified and added to meet the needs of the children. The Mathematics Chats listed below are repeatable with different and increasingly more difficult numbers.

A critical aspect of the Mathematics Chat is the management and use of questioning. The goal is that all students are involved. During the course of the Math Chat students are constantly being assessed as to their understanding. Whole group responses are used as well as individual sharing of thoughts and ideas. Everyone is expected to respond to every question the best that they are able. For a more complete description of management techniques see Chapter 4 Assessment and Management.

It is important to note too that Math Chats are a daily math literacy block. They are not designed to be the math program but to provide a daily 10 or so minutes to systematically introduce, reinforce and build mathematical literacy.

Mathematics Chats follow for each benchmark. Content Standards begin each section. However, for the purpose of cross-referencing in this project and related projects still under construction, the Content Standards and Benchmarks appear numbered differently in this paper than they appear in the original form. The Strands are numbered 1-5. The Content Standards are numbered 1 - 12 and the benchmarks are numbered n.x (1.1, 1.2, 1.3 2.1, 2.2 etc.) Under each benchmark are listed different examples of Mathematics Chats that can be used to build math literacy for the benchmark listed. The different examples are cross-referenced to the benchmark and standard 1.1.1 and

1.1.2. For example Content Standard 1, Benchmark 1 and Math Chat 1 is listed as 1.1.1 the second Math Chat under this benchmark would be listed 1.1.2.

NUMBER SENSE

1. **Student understands the relationship among numbers, quantities and place value in whole numbers.**

1.1 Counts, reads, and writes whole numbers to 100

Young children need several opportunities to count and read numbers to 100. Below are several Math Chats to provide opportunities for children to construct knowledge of our counting system. The activities described in benchmark 1.2 and 1.4 also help build math literacy in this area.

1.1.1 Materials: pocket chart and index cards with 0-99 written on it

Over several days have children help construct a 0-99 chart using a pocket chart. Distribute number cards to students and have them take turns placing their cards into the pocket chart. Discuss the order and address errors. This kind of activity can be done and redone differently. The 0-99 chart can be constructed in order with guiding questions: Who has the first number? Who has the number that would go here (pointing to a space)? The chart can also be constructed randomly. Have children come up in random order and try to figure out where to place their number based on the spaces on the chart. Help children problem solve using patterns

1.1.2 Materials: passing object

Have children sit in a circle and count around the circle. Children can pass an object designating the person's turn. Each child can count two numbers as they pass the object from left hand to right hand one number for each hand as they are passing it around the circle

After several counting experiences, give students several more counting experiences (all year) asking questions like these fosters predicting/patterning and reasoning skills:

- Who will say the number 10 (or any number you choose)?
- What pattern do you notice as we count around the circle?
- After we count ten more numbers, which number will we be on?

- Can we count backward from N (any number designated)?

- Does anyone think it might be a different number than Jane suggested?

- Then ask questions such as: Do you agree/disagree…? Get students to form opinions and respond to answers with finger signals?

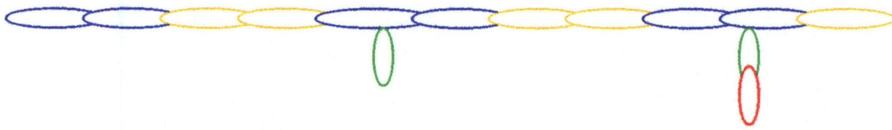
- etc.

1.2 counts by twos, fives, and tens to 100

Students can practice this skill by counting in several different situations. Below is a list of math chat ideas to help develop counting skills.

1.2.1 Materials: links

As part of your calendar wall, hang a chain of links or a bar of blocks for each day. Group the links or blocks into a pattern of two blues and two yellows for example. As the year goes, count the chain by 2's, 5's and 10's. We used links and hung one red on each 5^{th} link to designate 5's and two red links on each 10^{th} one to designate the 10's. (An example is on the next page.)



1.2.2 Materials: None

As students go to line up, you can count to XX by 2's, 5's, or 10's. Challenge them to see if all of the students can be in line (on the carpet, to their seats, have their ______book(s) out, etc.) before the class gets to NN.

1.2.3 Materials: a 3X5 card with a 2,5, or 10 written on it

As you are sitting in a circle, pass a card with a 2, 5, or 10 card. The student with the card identifies the next number in the counting pattern

1.3 reads and writes number words to ten

1.3.1 Materials: These math chats use number cards, dot cards and number word cards from 0 - 10.

Using their fingers as manipulatives have students show "x". Say, "Show me this many." Hold up a card with either the digit, dots/shapes of a quantity, or the number word on it. Students have to hold up that many fingers. For this objective, concentrate on the number words on the cards to help students have experiences with number words. You can make it harder as students gain success by asking different questions like: Can you show me one more than this "number word" (Give students a chance to read the number word. If they have trouble, ask a student if they know what the word is. Ask how could we find out or figure it out?)

1.3.2 Materials: These math chats use number cards, dot cards and number word cards from 0 - 10.

Give students number cards from 0-10 (or number word cards) Ask them to show you the card that goes with "this card" (Hold up an index card with a number word, number, or quantity of dots.)

Modify this activity for different number talks by using the different cards such as show the number word and ask for the number or dot

1.4 compares and orders numbers to 100 using the symbols for less than, equal to or greater than (<,>,=)

1.4.1 Materials: Base ten materials, digit cards 0-9, cards

>	<	=
Greater Than	Less Than	Equals

Have students build two different numbers with their base ten models. Have them compare the numbers and describe that relationship with one of the above cards. Have students consider their neighbor's ideas and see if they agree. "Do you agree with the people next to you? Why or why not? Monitor student behavior and do this over several days, as students need. Form small groups to work on the concept as you see they need it. In the beginning of the year start with smaller numbers and increase as students build facility with numbers.

1.4.2 Materials: 0-20, 0-50, 0-75, or 0-100 chart, small counters/disks or buttons to mark places.

A. Using chart (as indicated above frames for which can be found) appropriate to the needs of the students, have students place a marker on a particular number. Have students place a counter on a number less/smaller than that number. Ask for possible answers. Then pose question: "Are all these answers right? Which are not? Why?" Discuss their answers. Pose the same question but ask students to find a larger/greater number.

B. The above problem can be posed another way. Write it on the chalkboard: $35 < \boxed{}$ Then ask this question: What could go in the box?
Discuss plausibility of the answers (collect more than 1 answer.)

C. You can also pose this question on another day later in the year once students have more facility with the 100's chart $46 > \square$ How many possible answers could go in the box? Do several with < and > and explore the math question above. Pose this one: $92 = \square$ How many possible answers are there now? Why?

1.5 uses ordinal numbers to the tenth

1.5.1 Materials: none

As students line up, use this language and encourage them to use ordinals too.

1.5.2 Materials: bears, lions, Unifix cubes, or other small manipulatives, flash cards with ordinal words or ordinal numbers

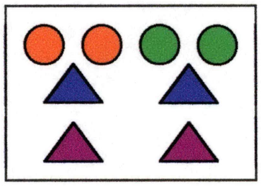
A. Have students build trains, line up bears or lions, or put blocks of one color in a row. Have them change the 1st, 2nd, 3rd, or __th one to blue for example. " Make a train of 10 red blocks. Now change the 3rd one to a blue. Change the first one to a blue or green for example"

B. Using cards with the ordinal numbers or ordinal words written on them can create another Mach Chat for this objective. Hold up a card with the ordinal word on it. "Change this one (meaning one indicated on flash card) to a different color. Monitor and assess learning.

1.6 represents equivalent forms of the same number through the use of physical models, diagrams, and number expressions, to 20, e.g.: 8: 4+4, 5+3, 2+2+2+2, 9-1 etc.)

1.6.1 Materials: Meaningful flash cards(See the samples in materials section - note that these are only a small sample of the various flash cards that could be used. Create and modify flash cards according to the needs of the children.)

Hold up a meaningful flash card and ask students what they see. Ask questions such as: How can we describe this picture?
For example:



Students could say that they saw 8. Ask the students to describe how they counted 8. You may find that students counted differently. They may have seen 2 green and 2 orange and 2 blue and 2 purple, or they may have seen 4 circles and 4 triangle or any other number of ways. Elicit from them number sentences that could describe this picture. (2+2+2+2 = 8) Is there another way? (4+4=8), (3+3+2=8) (1+1+1+1+1+1+1+1=8) (Children often do not understand that in counting it up they are actually adding.) You could do this orally and progress to having students write it on their lapboards or in journals. Do these often over time with a variety of meaningful flash cards to develop students subitizing ability. (see the Teacher Resources appendix to see a sampling of different meaningful flash cards.) Start with smaller numbers and advance to larger numbers

1.6.2 Materials: chalk boards or journals,

2. Student understands the concept of place value in the number system.

2.1 demonstrates an understanding of the place value of ones, tens, and one-hundred

Activities like those described in 2.2 encourage students to deal with ones, tens and one hundred

2.2 counts and groups objects into tens and ones (e.g., 3 groups of tens and 4 more is 34 or 30+4)

This objective is practiced well in centers where students need to count, count, and count some more. While they are working ask questions to guide thinking. How much do you have? How do you know? How did you organize your counting? Why did you do it that way? Did it help? Could it be done a different way? Do you agree with your partner? These questions can be asked during a Math Chat.

2.2.1 Materials: small cups of beans, buttons or other small objects.

Give students small cups of objects and a counting mat or yarn circle to use to define their workspace Ask children to predict the number of objects in their cup. "How could you find out?" Have students count their objects. Observe students counting. Are they developing effective strategies for counting and keeping track? Is 1-1 relationship important to them? Do they monitor their counting? After students have finished counting, discuss. Have students check their neighbor's counting. Start with small numbers at the beginning of the year move to larger and larger numbers as the year progresses.

2.2.2 Materials: linking cubes

Over several days we work on number concepts. I ask students at the beginning of the year to build different numbers and watch how they count. Do they have a strategy? Do they use tens and ones to help them count? When I ask them to build a particular number (small numbers at the beginning of the year and the numbers grow as the year progresses) I also ask if they have a strategy for figuring out quick how much they have. It takes time, at first, for children to see the need to go back and check quickly how much they have as they are counting up to the target number. They have to experience counting and loosing track and having to recount several times before they see the need to have a strategy. Talk about this. "Does anyone have a way to remember quickly how many blocks they put together to build 24?" Was your

2.2.3 Materials: base ten models, noodle sticks, number cards 0-9, or linking cubes can be used in these talks - My students each have a bag of noodle sticks and extras as a part of our math tools. They made their own. This activity in itself helped to build and reinforce tens and ones Students need many opportunities to build numbers using different models. Give them several opportunities to build different numbers using the different models above. Putting the numbers into context will give them meaning for students.

A. Using cubes that snap together engages students in problem solving situations where the numbers have meaning. For example: "______(fill with proper noun from the class or literature piece being used at the time) has NN (nn = any 2-digit number) _______ (fill with appropriate noun) Can you build that number? What would be a fast way to count that number again if you lost track of your counting? How could you organize your counting? How many 10's do you have? How many 1's?

B. Using noodle sticks (ten glued to each stick and some single noodles) have students build different 2-digit numbers then show you a number 1 more/less, 2 more or 2 less, 10 more, 10 less etc. Watch how they manage the process of completing the task.

3. Student understands and describes addition and subtraction and uses these operations to solve problems.

3.1 shows the meaning of addition (putting together) and subtraction (taking away, comparing, finding the difference)

These three forms of subtraction are not equal in difficulty for the child. Taking away is easier than comparing and finding the difference.

3.1.1 Materials: counting manipulatives (beans, blocks, buttons, bears, lions - what ever you have available as the items will represent numbers in the problems.

Over several months children, to help develop their problem solving skills, can explore these kinds of problems. Here are some examples of some problems with subtraction as taking away:

A. 5 children were left on the bus. 3 children got off at the next stop. How many children were left on the bus?
B. Jake had 8 gummy bears. He gave 3 to his friend. How many were left?
C. 6 cows were in the pen. The farmer took 4 of them to milk. How many cows were left?
D. As students become proficient, you can raise the difficulty of these problems simply by asking students to write down their answers on lap boards or in journals: Show the number sentence that tells what happened in the problem.

   As number sense grows the numbers in the problem grows. The important thing is to get students used to hearing and discussing the problems and working through them with manipulatives in a supported atmosphere. You may even challenge brighter students to make up problems for the class to solve. The development of this oral language must proceed asking students to do such problems in writing.

3.1.2 Materials manipulatives to count as in 3.1.1

Comparing numbers is more difficult than taking away and requires students to have begun developing conservation of small numbers.

3.2 uses the inverse relationship between addition and subtraction to solve problems

3.2.1 Materials: playing cards

After students have had a chance to play Fishing for 4's (or 5's, etc. on up to 10), ask them to tell you the different combinations for the number being studied. Put the cards representing those number combinations on the chalkboard. Ask children to show you (on lapboards, or journals) or tell you what they think the number sentences would be for those numbers. In the beginning if students have not had much experience recording in number sentences, you will have to guide them. Record the combinations in addition number sentences. Then show students the inverse relationship between additions and subtraction. Let's say you are working with 4's (beginning of the year) and students have discovered the combinations are 2 & 2, 3 & 1, and 4 by itself (4 & 0) 4-0=4, 3+1=4, 2+2=4. Show the corresponding subtraction facts using the cards. "If we know that 1 and 3 make four, then we can also think about what would happen to this total of four if we took a card away." Take a card away and show students how this would be recorded with a subtraction sentence. (Make sure to use mathematical language with students by using minus and subtraction not only take away as children will hear those different descriptions for the process.)

After you have developed this understanding of the inverse relationships, then present children with several problems over several days as the ones below:

1. Joe and Sam put 4 cards in a pile. Joe put 1 card in. How many did Sam put in?

2. Bonnie and Cathy had 4 flowers on their dresser and two were red and some were white. How many are white? This kind of question gets at the inverse relationship between addition and subtraction.

   In one case we can think of the problem as 2+ _ =4. It can also be conceived as 4-2=__.

3.3 knows the addition facts (sums to 10) and the corresponding subtraction facts and commits them to memory

3.3.1 Materials: playing cards

Developing this knowledge takes multiple opportunities over time. Use of the games "Fishing For X", "Spill the Beans", "Piggy Bank", "Concentration", "Building Equations", and "Snap" give student multiple opportunities to encounter combinations to 10 (sums to 10)
When children have had opportunities to work with the combinations to a certain number (say 4 or 5 in the beginning of the year) put up several number sentences on the board using those combinations. Try doing both an addition and a related subtraction problem.

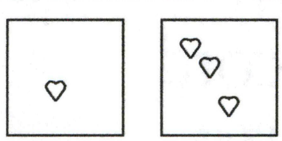
$$\begin{array}{r} 3 \\ \underline{+1} \end{array} \qquad \begin{array}{r} 4 \\ \underline{-3} \end{array}$$

Do the addition problem first and then ask: What do we know about 4 if we take one away from it? Does knowing 3+1 help us with 4-1? How does it help us? Repeating this process over time helps children with the inverse relationship understanding listed in 3.2

3.4 understands the commutative property of addition

3.4.1 Materials: chalkboard

After playing the game fishing for 4 (and after each fishing game up to 10), ask students to write down the combinations to 4, for example. (The activity can be repeated each time fishing for __ has been played for the different combinations for that number.) Ask them also to tell you what the number sentences would be for the combinations. Show both using the commutative property.



This combination can be described as 1+3=4 or 3+1=4.

3.5 uses manipulatives to add and subtract to 20 and writes the correct equation

3.5.1 Materials: any manipulative

Students need multiple opportunities to deal with numbers in problem solving situations. Over the year plan to do a number talk with numbers in problem solving situations at least once a week. Use students' names and characters from literature as the subjects of the problems. For example: You're reading Corduroy as a read aloud, and in math your Math Chats use buttons and Corduroy finds X buttons and one of the other stuffed animals on the shelf finds x buttons. How many do they have altogether. Perhaps you've just read Sione's Talo Root. Sione collected x talo roots Monday and X talo roots on Tuesday. Today is Wednesday and he digs up x more. How many talo roots does he have altogether? Another possibility is Sione collects 3 talo roots each day for a week. How many talo roots does he have at the end of the week.

3.6 identifies one more than, one less than, ten more than, ten less than a given number to 100

3.6.1 Materials: 1-20 grid, 1-50 grid 0-99 grid or the 1-100 grid, buttons or small counters

Repeat this activity over time beginning with the smaller grids and increasing as students' abilities grow.

Give students a few small counters and a grid. Have students locate a number. Then ask children to find the number 1 less than or 1 more than the target number. Do several of these. Try 2 more than and or 2 less than. Change the task and give students a target number using "Find the number that is 1 more than 15." "Find a number that is less than that number." Record the possible answers.

3.6.2 Same Materials

After students have had some experience with dimes and pennies give them tasks like: "Cover the number that tells what three dimes and two pennies is worth." "What would be 10 less than that number" "Put a chip on the number that would be a dime (10) more than that number." Discuss each "Do you agree or disagree with what your neighbor got?" (Give a mode of response such as thumbs up or thumbs down for students to indicate their thinking.)

3.7 solves addition and subtraction problems with one and two digit numbers without regrouping

3.7.1 Materials: linking cubes or unifix cubes in 10's (ten blocks linked together) and 1's (single blocks)

Show students a 1 or 2 digit subtraction problem on the board. Ask students to estimate what the answer will be. Asking Why do you think so can produce some interesting explanations of thinking (or lack there of). Then have students prove their answers with the cubes. Discuss strategies and the answers they came up with. Do several of these over time with different numbers. Smaller numbers at the beginning of the year and larger ones as the year progresses.

3.8 finds sums with 3 or more one digit addends using associative property

3.8.1 Materials: Chalkboard and manipulatives if children need them (you will know if you ask them to give you their guess without building it and see how many and how varied their responses are for the answer.) If there is a lot of variability, better have them build it and then prove their answers.

Give several opportunities to practice adding more than two numbers over the year. Begin with smaller numbers and move to larger ones. 1+1+1= on up to 9+9+9= and all the variables in between.

4. Student uses estimation strategies in computation and problem solving

4.1 estimates number of objects up to 50, (estimation jar for example)

4.1.1 Materials: two jars and manipulatives that fit into the jar.

On a weekly basis, have an estimating jar as part of the math center. Students submit estimates and then count together as a class on Friday. During the organized and shared counting experience, students can vote on ways to organize counting so as to not loose count. You can play with this a bit the first couple of times and "loose track of the number being counted" by not having a strategy to count. This will help students see the need for having some strategy for keeping tract of the count. Students may not realize that it does matter to count each object only once. The one to one correspondence of counting one number and touching one object needs to be firmly established for students to be able to count effectively and efficiently.

In one of the jars, I put 1 at the beginning of the year and a mystery number in the other. This is to encourage the use of benchmarks when counting. I also start with numbers less than 20. Numbers grow to 50 as year progresses. Later in the year, I put ten in the benchmark jar so students can make educated guesses.

4.2 makes reasonable estimates when comparing the sum of two, two-digit numbers when that sum is less than 50

4.2.1 Materials: Chalkboard, base-ten models

Show students a two-digit addition problem. Ask students to give you a good guess as to the answer. Write down their guesses then have students build the numbers (for example 22+18=_) to check their estimates. Validate the estimates that are reasonable estimates. Often students will be reluctant to estimate as they feel their estimates are not good unless they are right on. They will even lie or change their estimates to match the right answers. They have to be encouraged to take the risk of estimating in a safe and nurturing environment.

5. Student understands the relationship between a part and whole.

5.1 recognizes, understands, and uses fractional parts of a whole and a half

5.1.1 Materials: Cookies, MM's or licorice

Tell children that they may have the treat if they can find a way to share it equally with a friend. Have children discus with a friend their plan as to how to divide it equally or divide it in "half". It is interesting to discuss they ways in which they plan to share the treat. Repeat the activity with different objects.

Later, you can have them draw shapes on their chalkboard and cut them in half. Show students the fractional expression used in math to describe this part of the whole. Also check on their understanding of shape names and their ability to reproduce those shapes from the name. Can students draw a whole and label it 1? Can they divide a shape in half and label each part as $\frac{1}{2}$?

ALGEBRA AND FUNCTIONS

6. Student uses patterns to solve problems.

6.1 identifies missing objects in a simple pattern

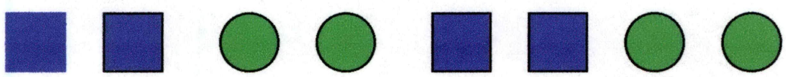
6.1.1 Materials: small manipulatives conducive to patterning (colored tiles, blocks, cubes, buttons etc.)

Draw a pattern on the board, or build one in front of the students with some parts missing. Ask students if they can reproduce this pattern and fill in the missing information. Do several different kinds and complexities of patters. Start easy and increase the complexity over time.

6.2 identifies, explains, reproduces and extends, and builds a pattern using up to three different components

6.2.1 Materials: small manipulatives conducive to patterning (colored tiles, blocks, cubes, buttons etc.)

This objective can be taught using activities as those described in 6.1.1. However, take the activity further. Show the students that the pattern below, for example, can be explained as a 1,1,2,2,1,1,2,2 pattern or a A,A,B,B,A,A,B,B pattern



Do several and have students identify, explain, build, reproduce and extend the pattern. Begin with simple patters (1,2,1,2,) and extend to more difficult patterns over time (1,2,3,3,2,1,1,2,3,3,2,1 etc.)

7. Student uses number sentences to solve problems.

7.1 recognizes and understands the mathematical meaning of the symbol +, -, & =

As you work on the many Math Chats described in this project, when an equations is involved, discuss with students what the different parts of the equations tell us to do with the numbers.

7.2 computes equations with missing numbers using manipulatives

When students play games like Fishing for __ they are working with equations with missing numbers. They have a 2 in their hand and need a total of 4 (2+__=4)

After students play the game, a Math Chat that could follow is "Boys and girls, you just played Fishing For 4. Let's take a look at some of the things you were doing. When you had a two in your hand what did you look for to get a total of four? How did you know you needed a 2 to equal four?" Do students say, "I knew that two plus two was four? Or do students say "I knew that 4 minus two equaled 2,"? Show students how their problem of finding the missing card can be described with a number sentence that looks like this 2+__=4. "Let's try some more of those to see if you can figure them out." 3+_=4 or 4+_=4 etc.

MEASUREMENT AND GEOMETRY

8. Student understands that there are properties such as length, weight, capacity, and time and that comparisons can be made using these properties

8.1 compares and measures the length and weight of two or more objects using standard or non-standard unit

8.1.1 Materials: Have students bring 4 crayons or pencils from their desk to the carpet.

Students are to measure how tall two different students are using their measuring tools (Choose two students who have obvious height differences.) Use all the shortest crayons to measure the shortest person and all the longest crayons to measure the tallest person. This will create results that are obviously anomalous and will provide interesting discussion about the size of the measuring tools and standardization of the measuring tool.

8.1.2 Materials: paper clips, snap cubes, or rulers (depending on whether you want to work on standard or non standard measure) masking tape strips placed around the room of different lengths, or objects or yarns of different lengths

Have students go and measure the strips and report back their findings. Discuss whether or not students got the same answers when they measured. Then discuss the different lengths and have students help put them in order from shortest to longest.

You can reinforce subtraction or addition concepts using information from this measuring experience on another day by comparing the lengths of the strings or finding out how long two strings would be together

Introduce algebra (string a + string b = ___)

8.2 uses standard units of inch, foot, or centimeter to measure length

8.2.1 Materials: paper slips cut in one inch, one foot, and one centimeter side strips.

Do one unit per session you can follow the same procedure for each unit. The following description is for the inch but could easily be adapted for the above measures.

Discuss the inch strip and compare it with the inch on the ruler. Have students find a place on their fingers that is one inch long. Then send them around the room to find as many things that are one inch exactly. When they have two things, come back to the carpet to report out. List their discoveries on paper

8.3 tells time to the nearest hour and half-hour and compares time related to events e.g.: before/after, shorter/longer

8.3.1 Materials: clock faces 1 for each student (in Anytime Mathematics manipulatives kit)

Begin with hours. After a discussion of the clock and its design, ask students to find various hours on the clocks and place their clock hands on the clock face to show that time. Do the same with the half-hour. Count out the minutes to "30" so children know where the 30 in 2:30, for example, comes from. This has to be done several times in different ways for some students to develop the understanding.

8.3.2 Materials: none

Draw a large clock on the board. Have the children use one arm extended for the minute hand and one bent at the elbow with the elbow kept bent for the hour hand. Have students place their hands in the right position to tell different times.

On other days, take the classroom demonstration clock and have students write down on lapboards or on journals, the times to several different hand placements on the clock (to the hour and half).

8.4 reads and uses the calendar: identifies weeks and months of the year, numbers the calendar in order, locates dates, days, months; locates today, tomorrow, and yesterday

Daily calendar and monthly calendar activities work well to reinforce this skill.

8.4.1 Materials: Blank Calendar for each student and a 16 square bingo board to play calendar bingo. (Prepare for the teacher use, days to 28, 29, 30, or 31 which ever is appropriate for the month you are in)

After the children have numbered and discussed the special days for the month, have students select 16 dates from the calendar and place those numbers on the bingo board. Then the teacher draws a number and gives students a clue as to the date. Below are some of the clues I've used:

- It's the first Monday in ----.
- It is the day after today.
- It is the 2nd Tuesday in ---.
- This date was yesterday.
- This date is Timmy's birthday.
- Valentines will be on this day.
- This date is three days from today.
- Etc.

9. Student identifies common geometric figures, classifies them by common attributes and describes their relative position/or their location in space.

9.1 identifies triangles, rectangles, square, and circles, including the faces of three dimensional objects

9.1.1 Materials: paper for recording ideas

Have students think about a triangle. Have them describe what it looks like while you draw their descriptions. Playing this literal game of tell and draw will help them define the characteristics of the shape. (Deliberate mistakes and interpretations of what they say provides lively conversation and involvement by the children in getting the teacher to draw the right shape) Once the shape is defined, invite children to find the shape in their environment. List or draw the shape in the category under the shape heading. Do a different shape each day. (This is a good beginning of the year activity)

9.1.2 Materials: 3-D shapes and play dough (Play dough: 1 cup flour, ¼ cup salt, 1 tablespoon cream of tartar, 1 cup of water, 1 tablespoon oil - cook until desired consistency. Food coloring or unsweetened powdered drink mix can be added before cooking with the water to create desired colors and/or scent. Six times this recipe makes a gallon sized zip-lock bag full.)

1. Have students make different 2-D shapes (listed above in benchmark).

2. On another day, give students the 3-D shapes. Have them press different parts of the shape into the play dough to discover the shapes of the faces. Discuss what the children discover and record on posters about the shapes to be displayed in the room. Do one shape a day to keep the activity short and provide time for thorough exploration.

9.2 classifies familiar plane and solid objects by common attributes like color, position, shape, size, roundness, and number of corners and explains which attributes are being used

9.2.1 Materials: small objects for sorting (small toys, plastic animal collections, shells, - any small assortment of things that lend themselves to sorting for the above characteristics of color, position, shape, size, roundness, corners, etc.)

Explore students' free shorts, then have them sort their collection by using a characteristic like color. Introduce children to the idea of having to sort by a sorting rule. Questions like, What would the groups look like if we sorted them by size, shape, color, numbers of feet? Do we get two groups or more than two groups? Why is that?

9.2.2 Materials: picture collections of different items. Clip art programs are good for this sort of thing.

Have students work with the 2-D objects and then glue their sorts on paper circling and writing the sorting rule for each sentence. This activity could be done whole group with an interactive writing attached to the sort. Then students could do their own sorts at a center or at their desks. The results of this activity could be used to assess the mastery of this objective.

9.3 describes and arranges objects in space in terms of proximity, position and direction, e.g., near, far, below, above, up, down, behind, in front of, next to, left/right

9.3.1 Materials: White paper, bears or lions or shells, or buttons (the manipulative you choose will dictate the picture the children draw.

Have students draw a den or cave (a shirt for the buttons or a pail on the beach for the shells). Give students a few manipulatives (bears or lions in this case) Give the students one of each color so you can use the color to talk about proximity, position and direction.

Then give students direction sentences for the location of the bear in relation to the cave or the other bears. Use the above words in the sentences. Below are a few samples of the directions I use for students to practice the above vocabulary.

- Put the blue bear near the cave.
- Put the yellow bear next to the blue bear
- Move the green bear to the right (or left) side of the cave.
- Make a line of bears with the red bear first then the green bear behind the red bear. Put the yellow bear behind the green bear. Which bear is in front on the green bear?
- Put the blue bear above the cave.
- Put the red bear below the cave.
- Put all of your bears far away from the cave looking for good things to eat.
- Etc.

10 Student identifies and knows the values and shows different combinations of money.

10.1 recognizes, counts, and combines coins to quarter

10.1.1,2,3 Materials: real or play coins and lap-boards, journals or scrap paper

At the beginning of the year use pennies then add nickels then dimes then quarters. Have students practice counting different amounts of coins. You can set up the problem differently to give different counting experiences. Below are single examples of different kinds of coin counting experiences.

1. Take four pennies. How many cents is that? Can you write it? What would it look like (using lap-boards or journals or scrap paper) When you are sure that students get the concept of recording cents, add nickels and see how they count them with the pennies. The ability to conserve that one nickel is 5 cents is hard for some students. As they count 4 coins, for example, containing 3 pennies and 1 nickel some may say it is 4 cents because as they count, they tend to count the pennies first and then the nickel. Discuss. Practice with several different amounts of coins in different combinations.

2. How many ways can you think of to make XX cents. Discuss with the children the different ways they came up with making the target amount. What coins did you use? Does that make X cents.

3 Take three (or any other designated number of coins) coins. How much is that? Could be many different answers depending on which kinds of coins are in the pile. Discuss with children the different amounts that they could have.

These different Mathematics Chats can be redone over and over as more coins are added to the pile of coins children work with.

STATISTICS, DATA ANALYSIS AND PROBABILITY

11 Student collects, records, organizes, displays and interprets data.

11.1 sorts objects and data by common attributes and describes the groups formed using categorical labels

11.1 Materials: Atrilinks, buttons, shells, bears, lions, small collections of toys etc. and sorting mats

Give students a scoop or cup of a selected manipulative as those described above. Give them a few minutes to sort their collections. Discuss the ways in which they sorted their collections. How did you sort your collection? How would you describe the different groups you made? This question gets at the attributes of the set. In sorting situations like these you can use open sorts or define attributes. You can define attributes by asking children to sort their collection such as one of the groups must be small circles (Atrilinks), or buttons with two holes, or toys with wheels etc. Have students share their thinking and label the groups. At first they will need help that the group is a group because it has definable attributes. Students can be familiarized with this kind of thinking by asking them to sort by size, shape, or color. Give several opportunities to sort groups by these different attributes and other attributes like texture, smell (depending on the kind of manipulative you decide to use - leaves, nuts, animal pictures etc. give different opportunities to sort by different attributes. Students also need to talk about how they show that things are part of a particular group (Did they group them together? Did the students put space between the groups? Can they tell the difference between their neighbors groups? Did they draw circles around their groups when they drew them on paper?).

11.2 represents and compares data, e.g., largest, smallest, most often, least often, using pictures, bar graphs, tally charts, Venn diagrams, and picture graphs and explains how the data differs

11.2.1 Materials: objects from sorts described in 11.1.1

If objects are uniform in size, students can make real bar graphs using objects used in sorting experiences. The purpose of a graph, charts and diagrams is to organize information. It is interesting to watch what children do when they create a graph without an organized mat to use. The organized graphing mat superimposes a 1 to 1 relationship between groups. Even tallying with a cross for the fifth one superimposes an organization to the counting experience that students may not use on their own. It is informative to watch what children do as they record their information. Do they use a 1 tally to 1 object in tallying and a one space to one object in graphing. If they do not understand this relationship being able to read and interpret a graph will be more difficult for them.

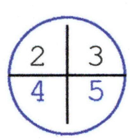
12 Student uses probability to determine the likelihood of events.

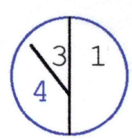
12.1 identifies the probability of an event occurring

12.1.1 Materials: brown lunch sacks with blocks or bears (blocks, buttons, or Atrilinks etc.)

The first (and possibly second) time you do this activity, do one bag as a class activity and guide students through it. Have the children reach into the bag and record what object comes out. Take 20 turns and record the objects (by color, shape, size or other attribute) Have children predict based on the information they collected what objects will likely and how many of which object there are in the bag. Record their predictions and then reveal the bag's contents. (You can start with 3-4 items in the bag to make it easier)

12.1.2 Materials: Spinners divided in different ways. Divide some in fair shares and some not fair.





Discuss the different spinners and experiment with them. Have the children take turns spinning the spinners and recording the numbers that come up. Encourage the children to explain why they think the numbers come up as they do.

MATHEMATICAL REASONING

13 Student makes decisions about how to organize and solve a problem.

13. 1decides the approach, materials, and strategies to represent problems

Through the use of centers, games, and Math Chats, students will be continually engaged in activities that will require that they make decisions about how to organize and solve problems.

14 Student solves problems in reasonable ways and justifies their reasoning.

14.1 makes calculations and checks the validity of the results from the context of the problem

Through the use of Math Chats, students are solving problems on an on going basis and are given multiple opportunities to explain their thinking and consider the thinking of others. They explain their thinking, listen to each other's explanations of thinking and make decisions as to whether or not they agree or disagree with explanations presented. Assess student progress toward this goal can be conducted through observations of participation and strategies employed during Math Chat activities, games and centers.

15 Student sorts and classifies objects.

15.1 recognizes attributes of a set

15.1.1 Materials: Atrilinks, buttons, shells, bears, lions, small collections of toys etc. and sorting mats

Give students a scoop or cup of a selected manipulative as those described above. Give them a few minutes to sort their collections. Discuss the ways in which they sorted their collections. How did you sort your collection? How would you describe the different groups you made? This question gets at the attributes of the set. In sorting situations like these you can use open sorts or define attributes. You can define attributes by asking children to sort their collection, but that one of the groups must be small circles (Atrilinks), or buttons with two holes, or toys with wheels etc. This gets them ready for using the Venn diagram described in the next objective.

15.2 uses a Venn diagram to compare two sets

15.2.1 Materials same as those described in 15.1.1, Venn diagram boards (see Teacher Resource amendment)

Begin with two circles and arrange sorts so that the two groups sorted will have items in each group so that children will have to make a choice as to which group the items will fit into. Use a sorting mat with the two circles not intersecting. Discuss the two different groups and ask students - Did you find any things, objects - buttons, bears (what every you are using) that could have gone into the other group? How did you decide where to put the "object"? Boys and girls, how could we show that the ________ could belong in both groups? This process creates a need for using the Venn diagram.

15.2.2 The use of the Venn diagram is a challenging one for students. Several opportunities to sort objects and discuss those sorts is needed before students can begin to use the Venn diagram with understanding. You can use this tool to sort words, compare how stories are alike and different, how two students are alike or different, how two characters, plants, animals, communities etc. are alike and different. Any subject area can be used to help build understanding of this sorting tool.

Teacher Resources

Materials Contents List

Materials (2-D) that were mentioned in the Mathematics Chats are listed below.

Noodle Sticks

0-9 Number Cards

0-99 Chart

1-100 Chart

1-10 frames

1-20 Chart/Frame

1-50 Chart/Frame

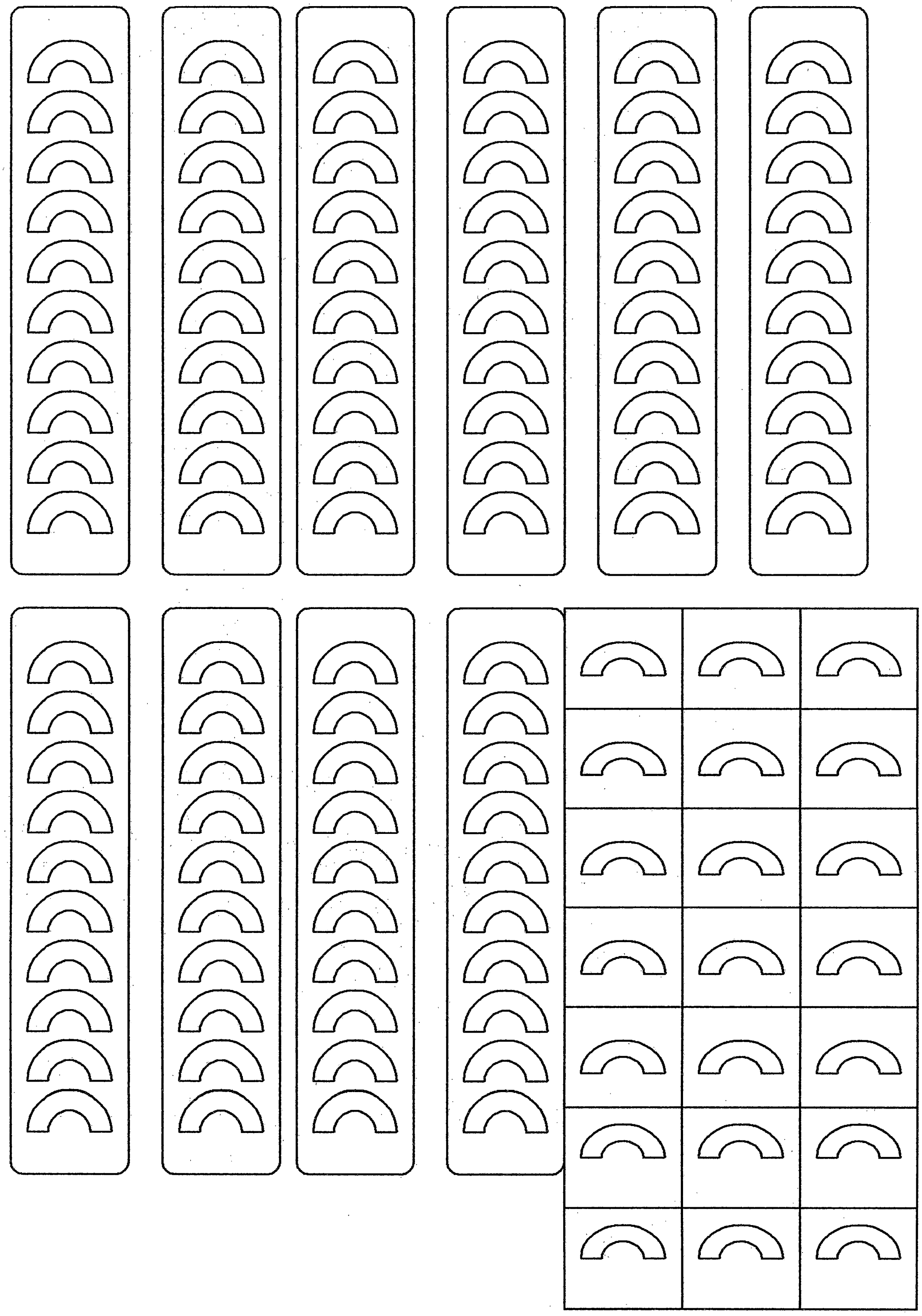
Meaningful flash cards

> It should be noted that only a two-page sample of what meaningful flash cards could look like is included.

Venn Diagram

Sorting Mat

Noodle Sticks



0-9 cards (2 sets)

0	1	2	3
4	5	6	7
8	9	0	1
2	3	4	5
6	7	8	9

0-99 Chart

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

1-100 Chart

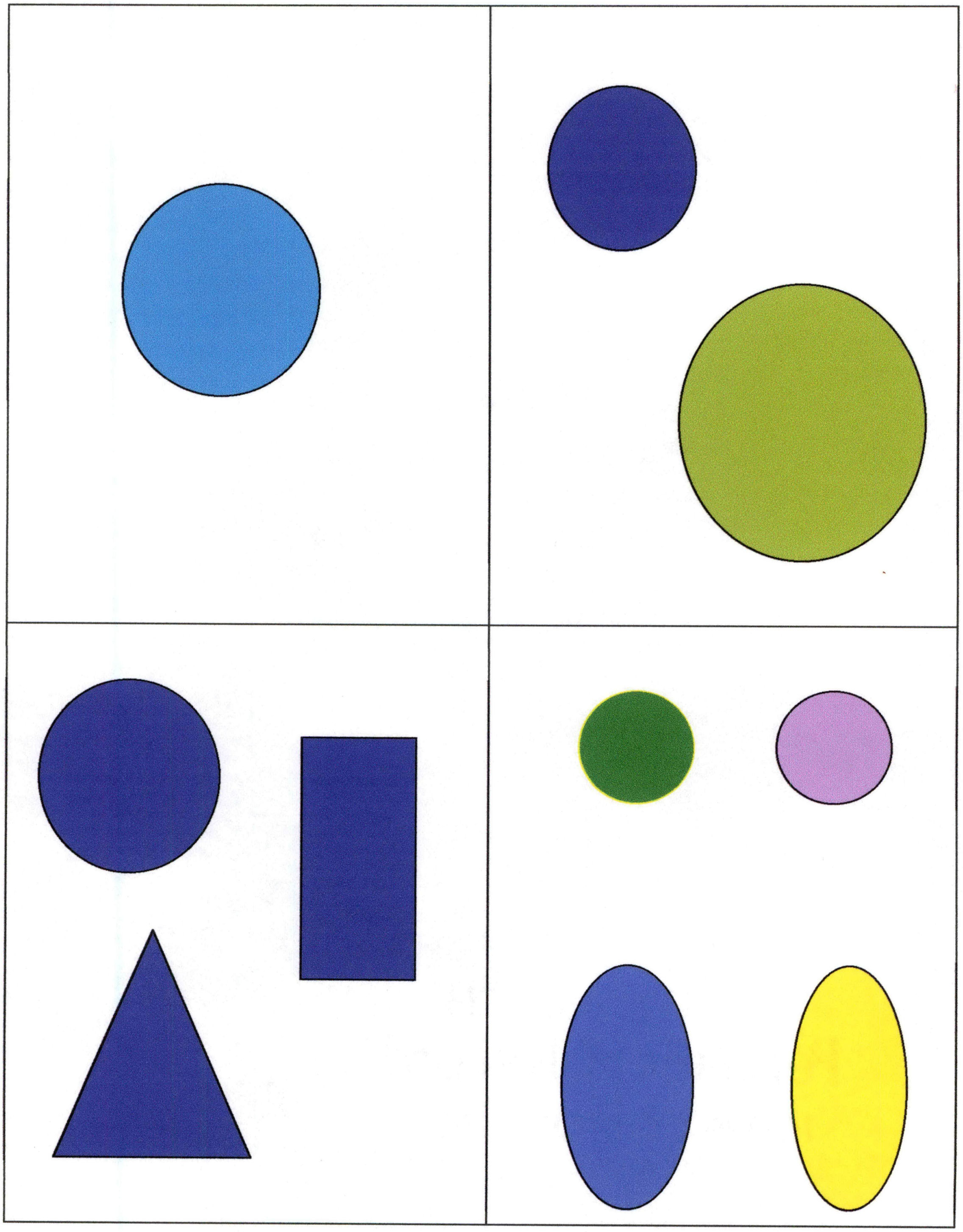
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

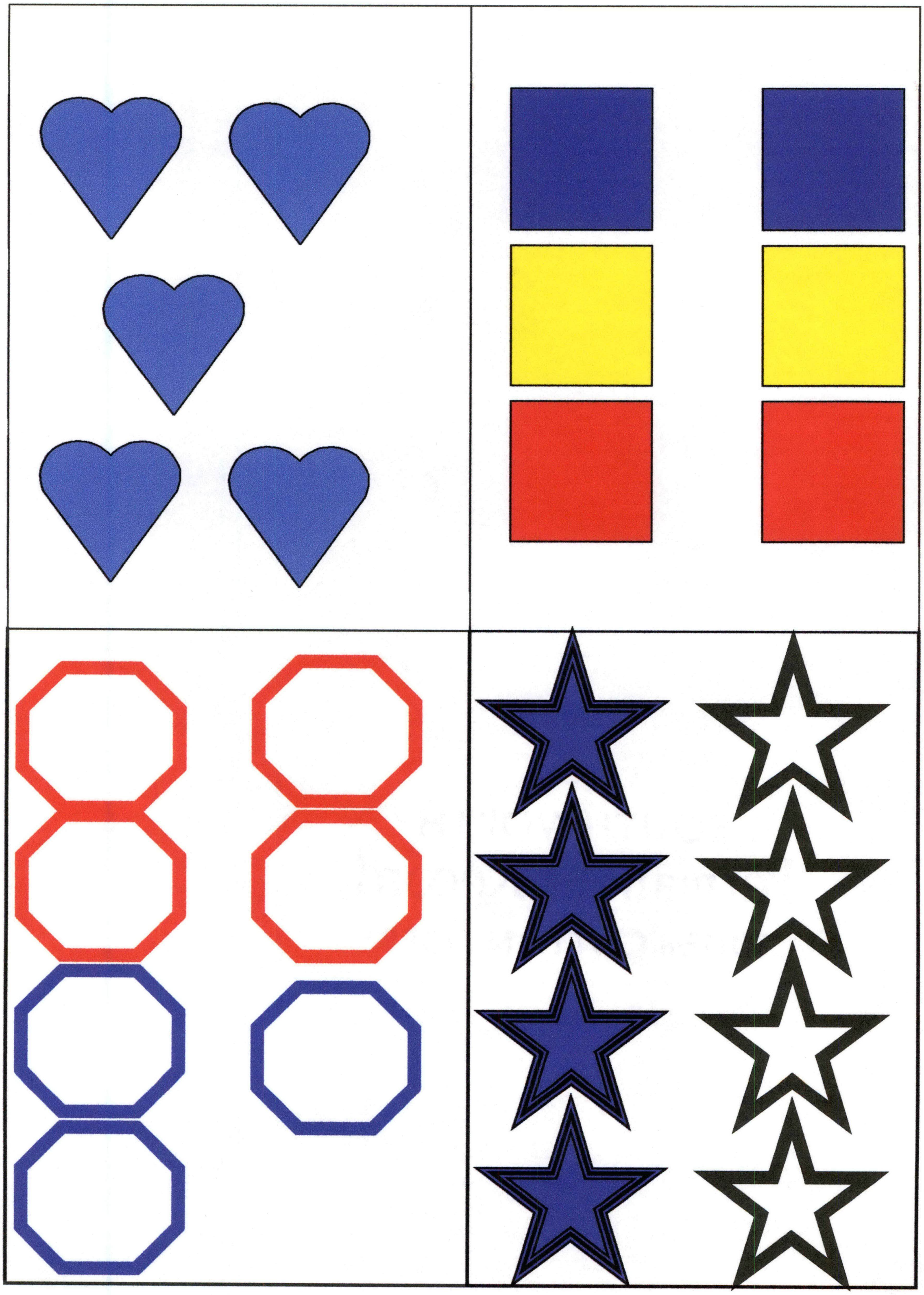
1-10 Chart

1-20 Chart

1-50 Chart (0-99 or 1-100 just use two of the 1-50 and tape)

Meaningful Flash cards





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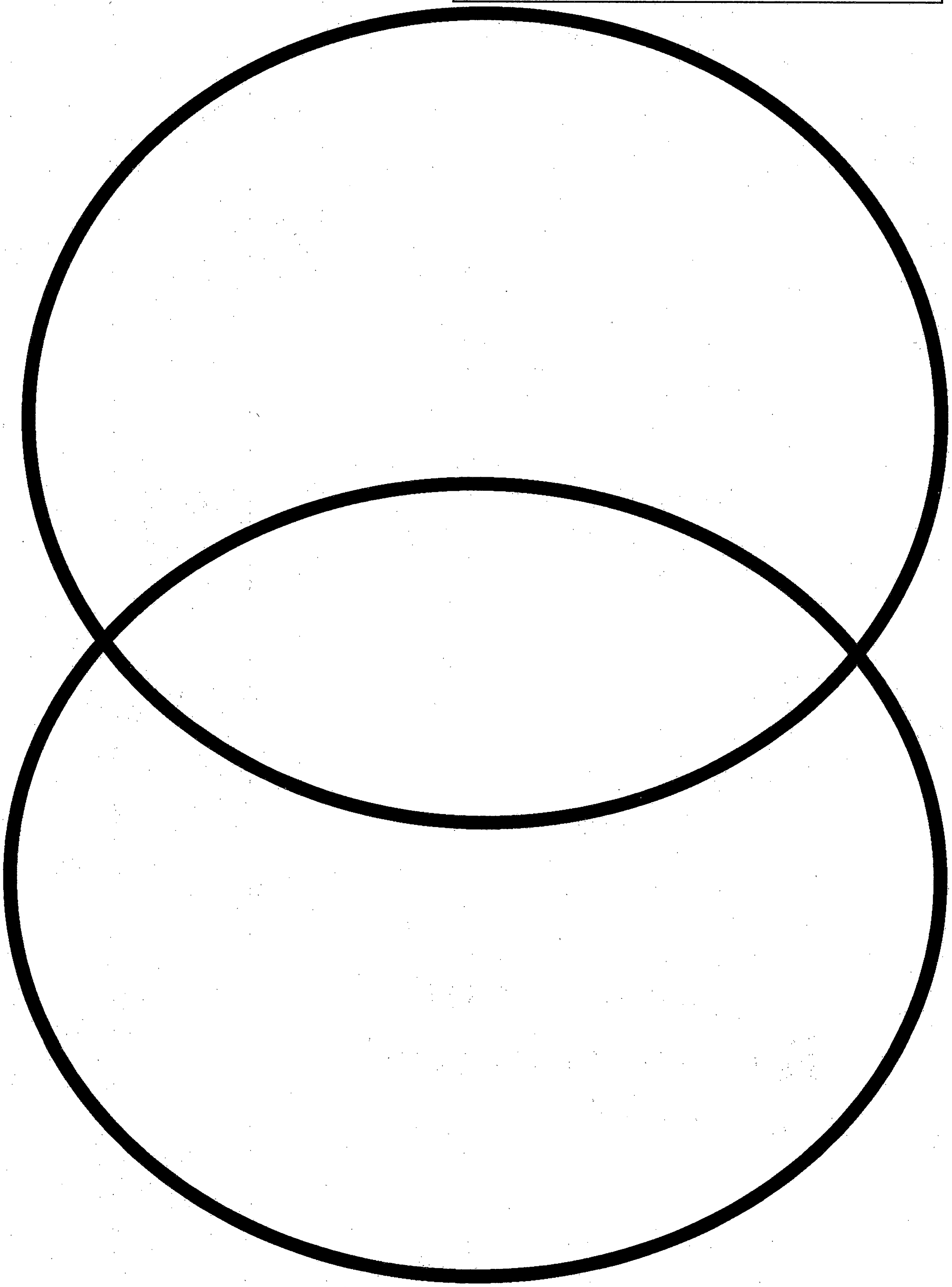

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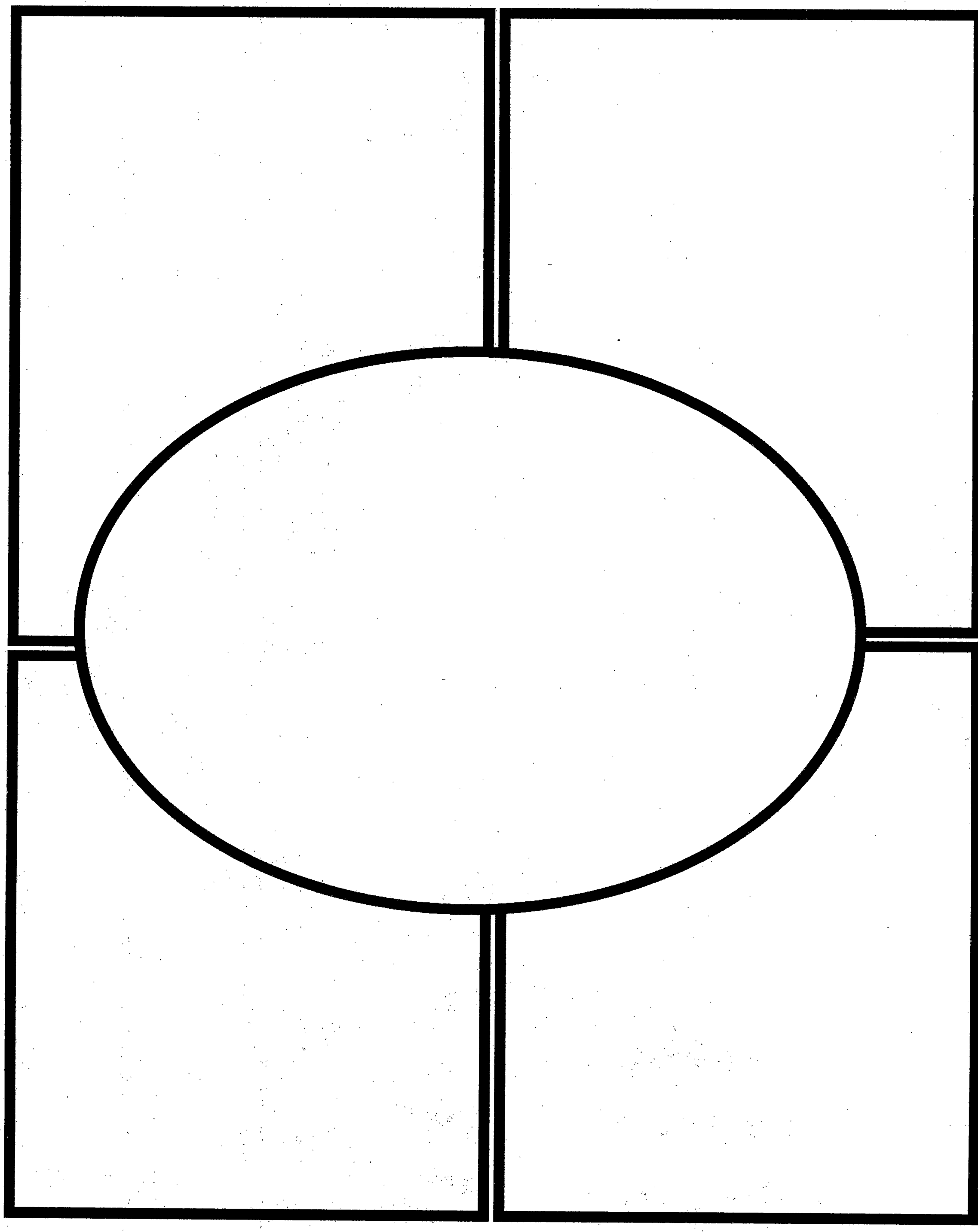


Appendix C

Mathematics Games and Center

Students need many opportunities to practice math concepts and skills in a meaningful way. A fun and motivating way to reinforce these skills is through concept games and centers. Using centers also offers the teacher a period of time where small group intervention can be facilitated.

Students are highly motivated to use math skills in game situations because they like to play and to win. It is difficult to play successfully without practicing a lot of mathematics. One advantage to using games is that the number of times that a child encounters and practices math skills is much higher than typical worksheet assignments. It is important to note that during game and center time the teacher acts as facilitator and circulating asking questions of the children as they are playing. This questioning and observation is a critical role. During these observations several guiding questions focus the observation.

- Are the children able to work cooperatively in the game situation?

- Do they understand how to play the game/do the center activities?
- How are they completing the game/task?
- Can they answer important questions about the game/task such as:
- How much do you have?
- How much do you still need to get to ___?
- Who has more?
- How do you know?
- How did you find out?
- Did you organize your counting?
- How did you keep track?
- Are they using strategies?
- -conserving small numbers
- -counting on
- -combining numbers
- -moving objects to count
- Are they confident and consistent with their answers?
- Do they see a relationship between quantity and number?
- To what number is this relationship confidently in place?
- At what point does it break down?
- Do they use tens and ones confidently and consistently?

- Do they have 1-1 in place when counting?

This is only a beginning list of the guiding questions to keep in mind while observing student behavior and while engaging in conversation with students.

Once students are working well at math menus, small groups of students can be called to work on concept development. This is an opportunity to do remedial or execrated work with students while the rest of the class is actively engaged in making sense of numbers. Independent students emerge out of a structure of teaching. Each game and center/game needs to be taught. The first week of school we learn and practice just one center a day. We discuss it together and play it and discuss it some more. The way in which a center or game is taken out, how many children can work there, how materials are handled, how clean-up will be conducted all have to be discussed and practiced so that expectations are clear.

Group process and problem solving needs to be taught. Some children do not know how to negotiate social rules in play situations. The concept of fair play and taking turns can be hard for some students. Also, being first and winning are ideas that some children have a hard time giving up to someone else without becoming emotional. These are important things to discuss and problem-solve as the teaching of the centers happens. Open discussion and problem solving of these issues helps students build the social skills they need to get along with others. Also, different ways for deciding who goes first need to be taught. We might play rock-paper-scissors on the first day just so we agree on the rules to play it and everyone knows how.

Centers can be assigned or handled in a menu form where students make choices. The children have choices about where they are going to work on a particular day with the caveat that they finish all of the menu items by the end of the cycle (which might last for two weeks or so).

A set of games comprises the rest of this appendix. These games and centers are the ones referred to in the Matrix. Recording sheets for the centers and games are listed at the end of this chapter. The recording sheets are titled identically to the title of the game or center. A sample of games is included in this project. See the matrix, Appendix A, to see how these games and centers can be used in the math program on number for first grade.

The games and centers that were chosen for inclusion were selected for their flexibility. As first graders develop their concept of small numbers and are ready to work with larger and larger number combinations, these games can be modified with little effort to meet the developing needs of the students.

Snap

Materials: plastic cubes that link together four for each student. Played in pairs

To play:
Students show the four cubes to their partner and then snap some off to hide behind their back. Their partner "guesses" how many have been snapped off. After the "guess" the blocks are shown to the partner and then the partner takes his/her turn. Students play for a while and then record all the different ways there are to snap the cubes. This game works on combinations to 4. It also works on pre-algebraic concepts (2 blocks shown plus "n" (the number of blocks behind the back) equals a total of 4 that is $2 + n = 4$).

Find 4

In this game we are playing for matches equaling 4 (or any designated total).

Materials: two decks of cards (take out all cards above 4) This game can be played with up to 4 players.

Sort the cards so that only the aces, ones, twos, threes, and fours, are in play the deck. Save the remaining cards to add in as the students work through combinations to 10. This game can be redone with the target totals 5, 6, 7, 8, 9, or 10 - just add in the cards as needed. This is the game I refer back to when showing students the connection between addition and subtraction.

To play:

The cards are placed in a pile face down in the middle of the group.

Decide who goes first and on your turn flip over the top card placing it next to the pile of cards face up. See if you can match this card to any of the cards turned up thus far to make a total of 4. If you can, take the two cards and place them in your "bank". Each combination to 4 or a four-card by itself gets a point.

Play continues around the circle until all of the cards from the pile have been turned up and matched.

To win:

The student with the most points at the end of the game wins.

Students then talk about the different ways there were to make 4. They can also record the different ways on a recording sheet.

Spill the Beans

How many ways are there to spill ___ two-color counters?

Materials: small cups, two color beans or counters, paper plates, recording sheet

To Play:

The object of this center is to find all the ways to spill 4 two color disks. This is a center not a game. Students work individually to record their work.

Students place four counters in a cup, shake and spill the beans or counters onto the plate. They record on the recording mat the different ways they found to spill 4 disks. (As students work their way through the facts, add counters to the target total being worked on).

How many beans are in the bag?

Materials: 10 quart or sandwich sized zip to close bags filled with 1-10 beans (at the beginning of the year). (If you use lima beans and label the beans in the a bag all with a's and teach children to put them back into the right bag, you will have less mess up with the numbers of beans in the bags. You can also color the beans and then code the bags)

To work at the center:

Students choose three bags and count and record the number of beans. After three bags have been counted, order the bags from smallest to largest and record on the recording sheet. Then pick three more bags to count. Count and return the beans one bag at a time. Some children will dump them all out of the three bags and then count them

How big is your handful?

Materials: nine resealable gallon sized plastic bags filled with the items to be counted, paper plates and recording sheet

To play:

Students grab 1 handful of the manipulative from the bag. They then pour their handful onto the paper plate, count, and record the number of items. Repeat for the rest of the items.

Spin a Bingo

Materials: Spinner page laminated, different colored place markers for each player, paper clip, brass paper fastener, ¼' section of soda straw for spinner (Place brass paper fastener through straw, and fasten through game board. The paper clip will spin freely around the straw section. Note: tape the back of the game board over the ends of the paper fastener so the ends do not get stuck in the carpet if children are playing on the floor.)

To play:
Decide who goes first by spinning the spinner to see who has the highest or lowest number.

On the turn, player spins both spinners and places a chip on the game board according to the results on the spinners.

Play continues until one player has four chips in a row horizontally, vertically, or diagonally.

The students can keep track and play until one player has 5 points to win. One point earned for each game won. Set the number of points based on how long you want students to play the center before they are finished.

Name ____________________

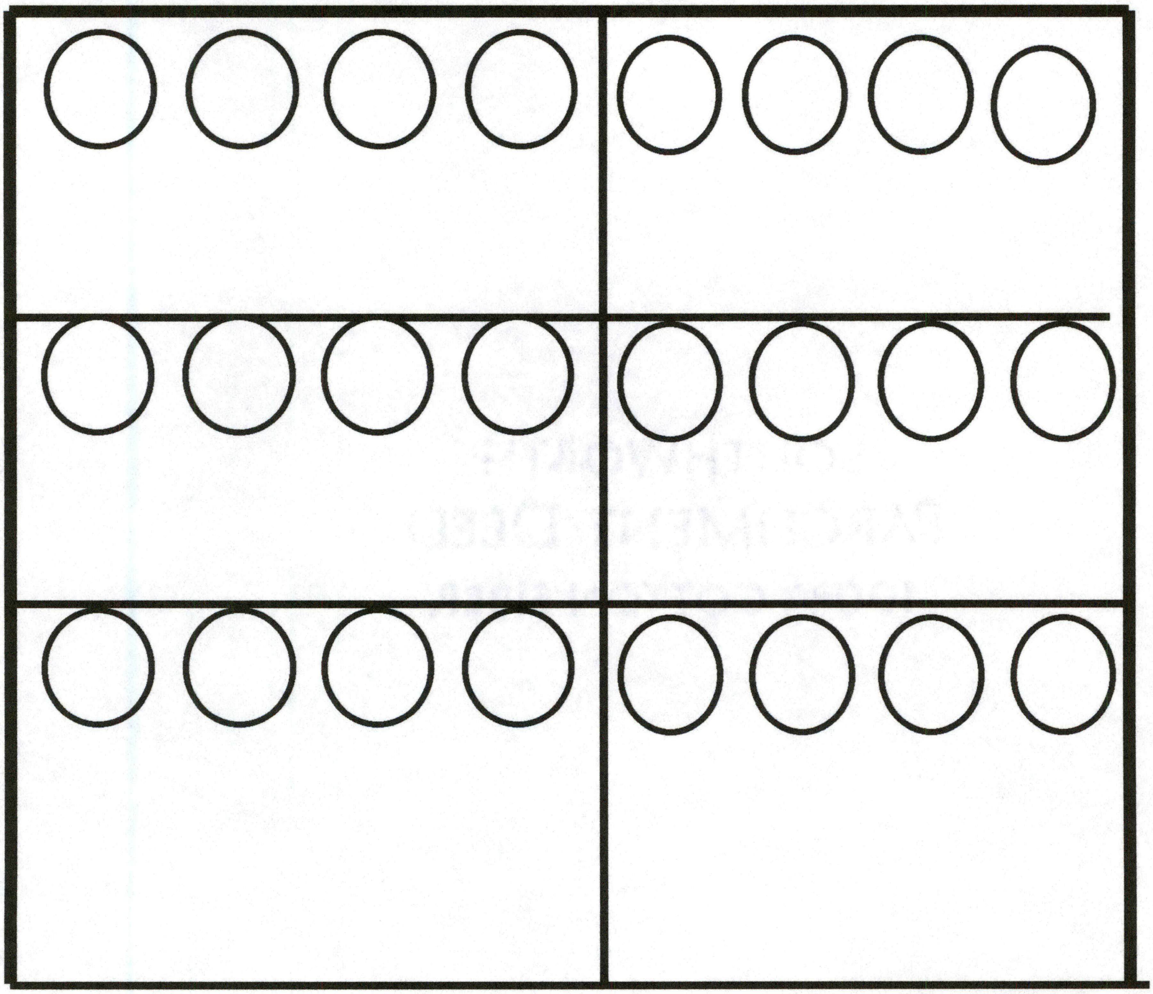
Snap

Hand	Hide	All together

Name______________________________

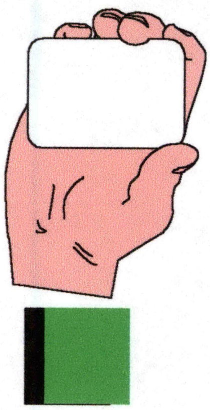
Fishing For 4

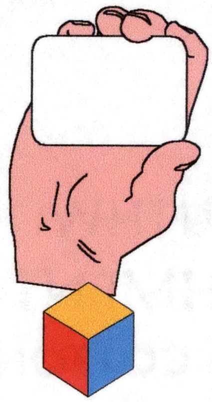
Spill the Beans

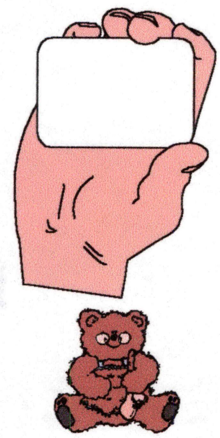


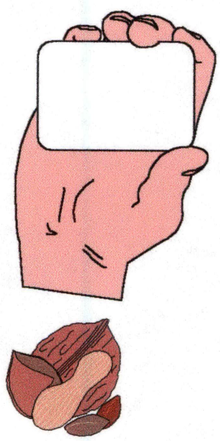
Name

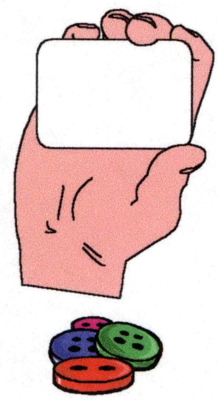
How Big is your Handful?

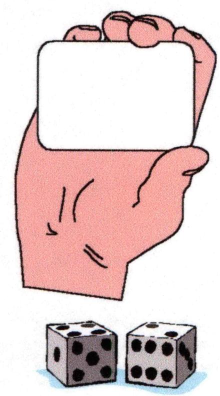


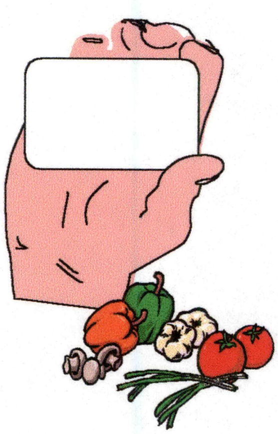


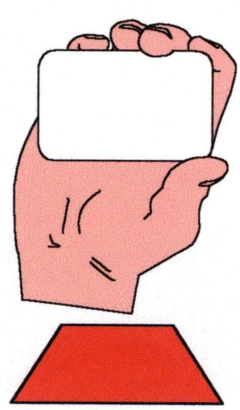


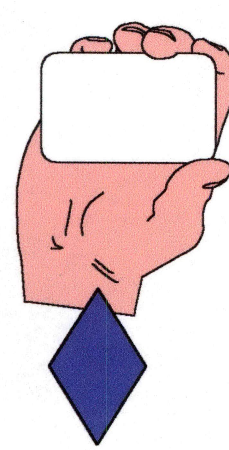










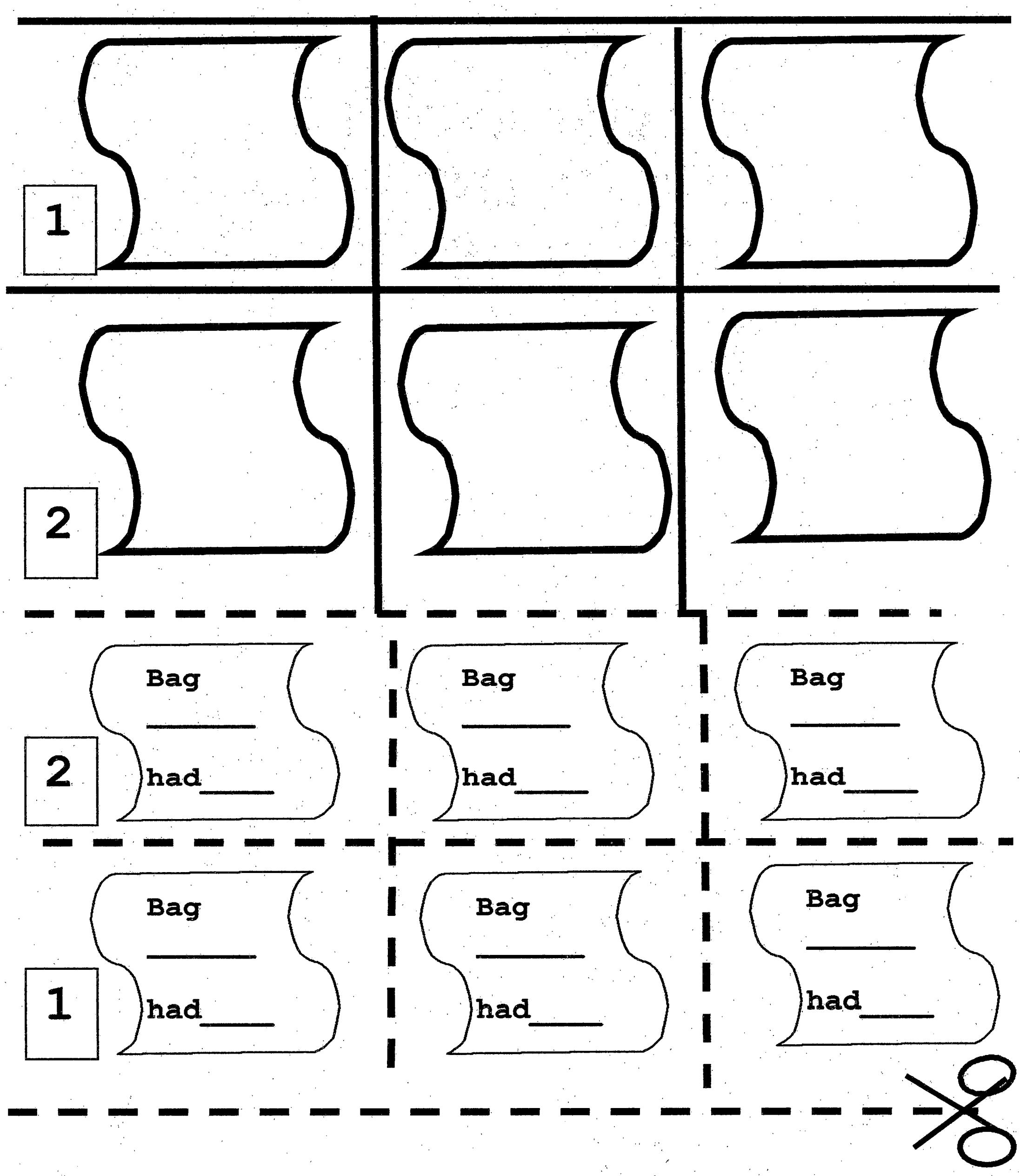


How many beans are in the bag?

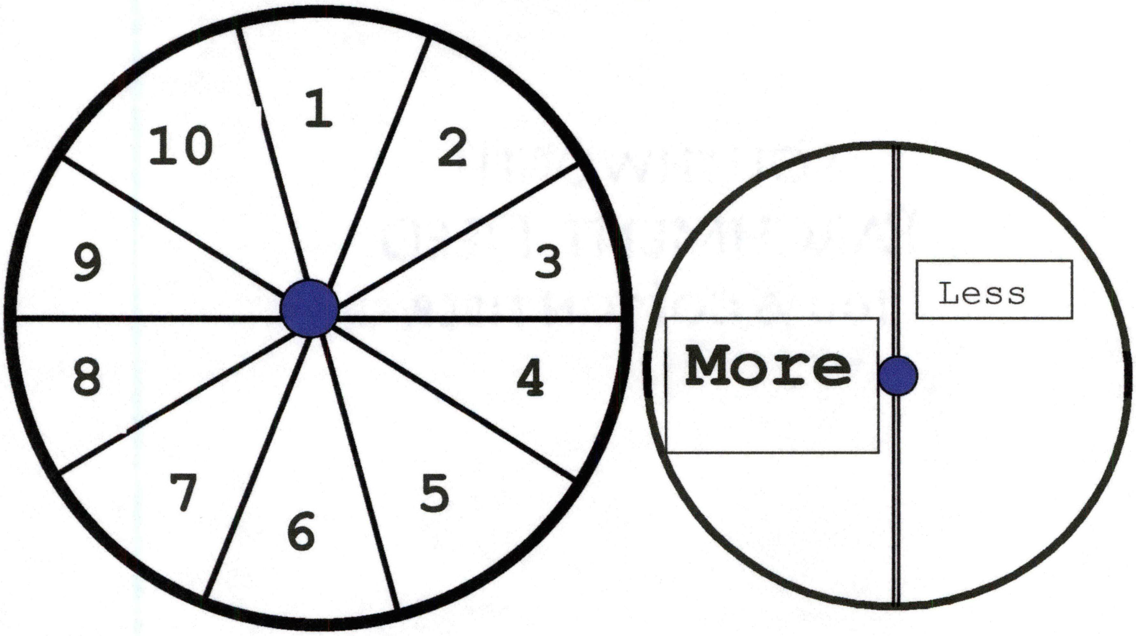
Count the beans and record the number in the bags. Then glue them in order from smallest too largest.

Smallest

Largest



Spin a Bingo 1



1	10	9	7	4	6
3	0	8	2	5	1
11	1	6	4	7	9
2	5	8	0	3	11
1	10	3	7	4	6

Appendix D

Problem Solving

Math tools are put to work in problem solving situations. Problem solving situations can take many forms to suit many purposes. The main purpose is to put a context to the numbers being used and provide opportunities for students to think about number in different ways. In this chapter three different kinds of problems solving will be discussed. Problems of the day/week (POD's and POW's), Head Problems and Two Problem Approaches are three possible ways to build math fluency. These different problem-solving techniques are used throughout the week. They are used to review and reinforce skills and provide opportunities to demonstrate and communicate understanding.

The name Head Problems is a term used in Math Matters to describe a specific process. Head problems are mental math exercises with several steps. Head problems are used on a daily basis. They are used to review and reinforce math skills previously taught. They should be easy enough to do without paper and pencil. Head Problems are comprised of a series of steps, which are given once. Students must use active listening.

Here is the head problem (mental math) that was listed in the matrix (Appendix D) with a discussion as to how it might look in a classroom.

Teacher says: "Take the number of eyes you have. Add to that the number of noses you have. Then add the number of thumbs you have." "Show me on your fingers what you think the answer is." "Who can remember what we did first?" (Generate student response after each question.) "What was that number?" "What did we do next?" "What was that number?" "What number do we have now?" "Who remembers the last step? What was it?" "Show me the number we had on step three." "Now show me the total." "Jeremy is showing a 5 who agrees with Jeremy?"

Another kind of problem solving situation is problems of the day or week. The problem-solving situation can be constructed using classroom experiences and/or make connections to literature read in class. The students' names are frequently used in the problems. The problems are read and discussed with the students.

Students begin to share their interpretations of the problems and possible solutions are discussed at the beginning, the problems are discussed extensively at first, as the goal is that students become familiar with presenting their thinking orally and in drawings, symbols, numbers and words.

Students are often engaged in oral discussion of problems. The problem of the day/week offer opportunities to see how well students can communicate their thinking in drawings and writing.

The sophistication of problem solving develops over time. Over time students take on more and more of the responsibility for re+ading and thinking through the problem solving situation. This process is a yearlong one. Young children will need modeling as to how to record their thinking on paper.

At the beginning of the year the problems are simple. After reading the story Mouse Count, Stoll Walsh (1991), I presented this problem to the children.

If a snake had 4 mice in the jar and some of them escaped, what would be some of the ways the mice could be arranged inside and outside of the jar. Draw and show as many different ways.

I used a drawing of the jar and "mice" (two color counters - any manipulative could be used to represent the mice) to have students come up and share different ways that the mice could be inside and outside of the jar. Students then is to go to their desk and record their thinking on their POD paper with the problem listed at the top. Some teachers are duplicating the problems on slips of paper to be glued into a spiral notebook at the top of the page. Other teachers are using a duplicated sheet with the problem at the top to be added to three pronged notebooks throughout the grading period. The problems are graded with the use of a rubric. A copy of this rubric can be found as Appendix H.

Once students finish recording their thinking in their journals, they are free to choose math centers and/or game like those described in the previous chapter.

Students kept track of problem solving activities in a three-pronged folder with pockets. A lesson was necessary to teach the process on how to use the three pronged folders. Students also kept track of recording sheets for math centers and games in these folders.

Students in another class were using the spiral notebooks to glue in the problem solving sheets. They also needed to be instructed as to how to glue in the sheets. Instead of using a full sheet for the problem, the teacher typed the problems on strips. Six problems fit on a sheet, which were copied, cut up into strips, and glued onto the top of pages in the students' spiral notebook. Concepts like front of book, top of page, how to use appropriate amounts of glue and where on the page to glue were all topics that needed discussion with the 5 and 6 year olds.

Students also need assistance learning to deal with standardized testing formats. The Two-Problem Approach, a Math Matters technique, offers a good way to introduce and reinforce test-taking skills as well as curriculum and skills previously taught. Two problems are written from the benchmarks and standards. The problems can be displayed on the board, overhead, or on student copies. The problems are worked out together relying heavily on student input. After using this technique for a while using blank paper, I found it helpful for students to respond on copies of the problems being worked. This gave focus to reflections on student learning. Many of the management and assessment techniques discussed in Chapter 4 are using during the course of working out these problems. Samples of two problem approaches can be found in the Matrix, Appendix A.

APPENDIX E

Similarities between letter and Number Knowledge Development

Early Number Knowledge

Three Concepts to Develop Item Knowledge

1. Name
   a. rote verbalizations - sing-song counting
   b. awareness that numbers are separate vocal units
   c. vocal units have an order

2. Shape
   a. recognition
   b. production

3. Value
   a. begins with small numbers and increases
   b. values different units
   c. conservation develops
   d. inclusion develops

Number knowledge develops through interaction with environment Awareness of patterns in numbers develops as proficiency and understanding of the number system develops.

Early Letter Knowledge

Three Concepts to Develop Item Knowledge

1. Name
   a. Rote sing-song verbalizations
   b. awareness of separate vocal units
   c. vocal units are distinct and different from one another

2. Shape
   a. recognition
   b. production

3. Sound
   a. positional variations
   b. blending/segmentati on
   c. as part of meaningful units etc.

Awareness develops for pattern seeking as letter and word knowledge develops through interaction with print.

APPENDIX F

1ST GRADE
STATE OF CALIFORNIA CONTENT STANDARDS AND BENCHMARKS

By the end of first grade, students understand and use the concept of "ones" and "tens" in the place value number system. They add and subtract small numbers with ease. They measure with simple units and locate objects in space. They describe data and analyze and solve simple problem situations.

A. NUMBER SENSE

A1. Students understand and use numbers up to 100.
- A1.1 count, read and write whole numbers to 100
- A1.2 compare and order whole numbers to 100 using the symbols for less than, equal to, or greater than (<, =, >)
- A1.3 represent equivalent forms of the same number through the use of physical models, diagrams and number expressions (to 20) (e.g., 8 can be represented as 4 + 4, 5 + 3, 2 + 2 + 2 + 2, 10 - 2, 11 - 3)
- A1.4 count and group objects into ones and tens (e.g., 3 groups of ten and 4 more is 34 or 30 + 4)
- A1.5 identify and know the value of coins and show different combinations of coins that equal the same value

A2. Students demonstrate the meaning of addition and subtraction and use these operations to solve problems.
- A2.1 know the addition facts (sums to 20) and the corresponding subtraction facts, and commit them to memory
- A2.2 use the inverse relationship between addition and subtraction to solve problems
- A2.3 identify one more than, one less than, ten more than, ten less than a given number
- A2.4 count by 2s, 5s and 10s with numbers to 100
- A2.5 show the meaning of addition (putting together, increasing) and subtraction (taking away, comparing, finding the difference)
- A2.6 solve addition and subtraction problems with one- and two-digit numbers (e.g., 5 + 58 = __)
- A2.7 find the sum of three one-digit numbers

A3. Students use estimation strategies in computation and problem solving that involve numbers that use the ones, tens, and hundreds places.
- A3.1 make reasonable estimates when comparing larger or smaller numbers

B. ALGEBRA AND FUNCTIONS

B1. Students use number sentences to solve problems.

B1.1 write and solve number sentences from problem situations that express relationships involving addition and subtraction
B1.2 understand the meaning of the symbols +, -, =
B1.3 create problem situations that could lead to given number sentences involving addition and subtraction

C. **MEASUREMENT AND GEOMETRY**
C1. Students use direct comparison and non-standard units to describe the measurements of objects.
C1.1 compare the length, weight and volume of two or more objects using direct comparison or a non-standard unit
C1.2 tell time to the nearest half hour and compare time related to events (e.g., before/after, shorter/longer)
C2. Students identify common geometric figures, classify them by common attributes and describe their relative position/or their location in space.
C2.1 identify, describe, and compare triangles, rectangles, squares and circles, including the faces of three-dimensional objects
C2.2 classify familiar plane and solid objects by common attributes like color, position, shape, size, roundness, number of corners and explain which attributes are being used for classification
C2.3 give and follow directions about location
2.4 describe and arrange objects in space in terms of proximity, position and direction (e.g., near, far, below, above, up, down, behind, in front of, next to, left/right)

D. STATISTICS, DATA ANALYSIS and PROBABILITY
D1. Students organize, represent and compare categorical data on simple graphs and charts.
D1.1 sort objects and data by common attributes and describe the groups formed using categorical labels
D1.2 represent and compare data (e.g., largest, smallest, most often, least often), using pictures, bar graphs, tally charts and picture graphs
D2. Students sort objects, and create and describe patterns involving numbers, shape, size, rhythm, or color.
D2.1 describe, extend and explain how to get to the next element in simple repeating patterns (e.g., rhythmic, numeric, color and shape patterns)

MATHEMATICAL REASONING
E1. Students make decisions about how to set up a problem.
E1.1 decide about the approach, materials and strategies to use
E1.2 use tools such as manipulatives or sketches to model problems
E2. Students solve problems and justify their reasoning.
E2.1 explain the reasoning used and justify the procedures selected
E2.2 make precise calculations and check the validity of the results from the context of the problem
E3. Students note connections between one problem and another.

APPENDIX G

Riverside Unified School District
Grade 1 Mathematics
Content Standards and Benchmarks

Note: A formating change has been made from the way in which the original Content Standards and Benchmarks was presented to teachers. Here the content standards (in bold) and the following benchmarks are listed in consecutive number order. That numbering system was used for purpose of organization in the data base - web page like design of the technology support that accompanies this project.

NUMBER SENSE

1. **Student understands the relationship among numbers, quantities and place value in whole numbers.**

   1.1 counts, reads, and writes whole numbers to 100
   1.2 counts by twos, fives, and tens to 100
   1.3 reads and writes number words to ten
   1.4 compares and orders numbers to 100 using the symbols for less than, equal to or greater than (<,>,=)
   1.5 uses ordinal numbers to the tenth
   1.6 represents equivalent forms of the same number through the use of physical models, diagrams, and number expressions, to 20, e.g.: 8: 4+4, 5+3, 2+2+2+2, 9-1 etc.)

2. **Student understands the concept of place value in the number system.**

   2.1 demonstrates an understanding of the place value of ones, tens, and one-hundred
   2.2 counts and groups objects into tens and ones (e.g., 3 groups of tens and 4 more is 34 or 30+4)

3. **Student understands and describes addition and subtraction and uses these operations to solve problems.**

   3.1 shows the meaning of addition (putting together) and subtraction (taking away, comparing, finding the difference)

3.2 uses the inverse relationship between addition and subtraction to solve problems
3.3 knows the addition facts (sums to 10) and the corresponding subtraction facts and commits them to memory
3.4 understands the commutative property of addition
3.5 uses manipulatives to add and subtract to 20 and writes the correct equation
3.6 identifies one more than, one less than, ten more than, ten less than a given number to 100
3.7 solves addition and subtraction problems with one and two digit numbers without regrouping
3.8 finds sums with 3 or more one digit addends using associative property

4. Student uses estimation strategies in computation and problem solving

4.1 estimates number of objects up to 50, (estimation jar for example)
4.2 makes reasonable estimates when comparing the sum of two, two-digit numbers when that sum is less than 50

5. **Student understands the relationship between a part and whole.**

5.1 recognizes, understands, and uses fractional parts of a whole and a half

ALGEBRA AND FUNCTIONS

6. Student uses patterns to solve problems.

6.1 identifies missing objects in a simple pattern
6.2 identifies, explains, reproduces and extends, and builds a pattern using up to three different components

7. Student uses number sentences to solve problems.

7.1 recognizes and understands the mathematical meaning of the symbol +, -, & =
7.2 computes equations with missing numbers using manipulatives

MEASUREMENT AND GEOMETRY

8. Student understands that there are properties such as length, weight, capacity, and time and that comparisons can be made using these properties

8.1 compares and measures the length and weight of two or more objects using standard or non-standard unit
8.2 uses standard units of inch, foot, or centimeter to measure length
8.3 tells time to the nearest hour and half-hour and compares time related to events e.g.: before/after, shorter/longer
8.4 reads and uses the calendar: identifies weeks and months of the year, numbers the calendar in order, locates dates, days, months; locates today, tomorrow, and yesterday

9. Student identifies common geometric figures, classifies them by common attributes and describes their relative position/or their location in space.

9.1 identifies triangles, rectangles, square, and circles, including the faces of three dimensional objects
9.2 classifies familiar plane and solid objects by common attributes like color, position, shape, size, roundness, and number of corners and explains which attributes are being used
9.3 describes and arranges objects in space in terms of proximity, position and direction, e.g., near, far, below, above, up, down, behind, in front of, next to, left/right

10 Student identifies and knows the values and shows different combinations of money.

3.1 recognizes, counts, and combines coins to quarter

STATISTICS, DATA ANALYSIS AND PROBABILITY

11 Student collects, records, organizes, displays and interprets data.

11.1 sorts objects and data by common attributes and describes the groups formed using categorical labels
11.2 represents and compares data, e.g., largest, smallest, most often, least often, using pictures, bar graphs,

tally charts, Venn diagrams, and picture graphs and explains how the data differs

12 Student uses probability to determine the likelihood of events.

12.1 identifies the probability of an event occurring

MATHEMATICAL REASONING

13 Student makes decisions about how to organize and solve a problem.

13.1 decides the approach, materials, and strategies to represent problems

14 Student solves problems in reasonable ways and justifies their reasoning.

14.1 makes calculations and checks the validity of the results from the context of the problem

15 Student sorts and classifies objects.

15.1 recognizes attributes of a set
15.2 uses a Venn diagram to compare two sets

APPENDIX H

Riverside Unified School District's

K-2 Math Rubric

4 Complete Response

- Fully completes the task or tasks
- Shows understanding of the mathematical concepts and procedures
- Satisfies all essential conditions of the problem

3 Reasonably Complete Response

- Completes most of the task or tasks
- Shows understanding of most of the mathematical concepts and procedures
- Satisfies most of the essential conditions of the problem

2 Partial Response

- Completes some of the task or tasks
- Shows understanding of some of the mathematical concepts and procedures
- Satisfies some of the essential conditions of the problem

1 Inadequate Response

- Incomplete
- Shows little understanding of the mathematical concepts and procedures
- Fails to address essential conditions of the problem

Riverside Unified School District
K-2 Math Rubric June, 1998

ENDNOTES

1. Math Chats is the term used in this thesis as it is the term used Rebecca Kallinger and Catherine Corr in their workshops on working with numbers with children. Different authors have similar processes and have given them different names.

2. Additional information on Math Matters and learning research can be found at this web address: http://www.mathmatters.net/index.htm or by contacting the organization through their e-mail: mathmatters.net The MIFF Techniques are further discussed in a 1994 Math Matters handout.

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