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THE EFFECT OF NUMERACY AND MATH ANXIETY ON WHOLE NUMBER BIAS

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THE EFFECT OF NUMERACY AND MATH ANXIETY

ON WHOLE NUMBER BIAS

A Thesis

Presented to the

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In Partial Fulfillment

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Master of Arts

in

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by

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ABSTRACT

Whole number bias (WNB) has been defined as the tendency to apply natural number knowledge to rational numbers. This misapplication can often lead to erroneous responses in mathematical tasks and understanding of rational number properties. Whole number bias can be explored using Dual Processing Theories. According to Dual Processing Theory we have two types of thinking: Type I and Type II. Type I is fast, heuristic based, intuitive, and doesn't require working memory, while Type II is slow, logic based, analytical, and requires working memory. Some researchers argue that WNB is an intuitive phenomenon and occurs from a failure to activate Type II thinking. Two models explain the relationship between Type I and Type II processing. Default Interventionist (DI) model states the two types of thinking are exclusive and we first activate Type I processing, then if conflict is detected we activate Type II thinking. Hybrid model states we have two types of Type I processing: heuristic intuitions and logical intuitions. According to Hybrid model, Type II processing is only activated if a higher order of thinking is required. Individual factors such as numeracy and math anxiety could affect WNB. Attentional Control Theory states that anxiety consumes mental resources, resulting in reduction of executive functioning, including the ability to inhibit internal and external stimuli that interferes with task performance. The purpose of this study was to assess WNB from a Dual Processing perspective and examine how individual differences such as numeracy and math anxiety would affect WNB and math performance in a fraction magnitude comparison task. It was predicted that individual differences in numeracy and math anxiety would help describe WNB according to each model. The results support the notion when numeracy is low, a process similar to what is described by DI will take place whereas when numeracy is high, a process similar to what is described by Hybrid model will take place.

Keywords: Whole Number Bias, Dual Processing Theory, Numeracy, Math Anxiety

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“The heart of the prudent acquires knowledge, and the ear of the wise seeks knowledge” – Proverbs 18:15

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CHAPTER ONE

INTRODUCTION

Math and science education is important in modern society. They teach students important critical thinking and gives them problem-solving skills. There are also many jobs which require some form of Science, Technology, Engineering, and Mathematics (STEM) background, and these jobs are projected to expand in the coming years. According to the U.S. Bureau of Labor Statistics (BLS) (2021), in the United States alone, jobs in professional, business, and scientific industries are expected to grow 2.1%. Furthermore, occupations involving computers and mathematical components are expected to see fast growth in employment. This is partly due to the increased need of telework caused by the recent Covid pandemic. There is also a greater need for occupations that involve analyzing and interpreting large datasets, such as statisticians and data scientists. According to the Bureau of Labor Statistics, statisticians and data scientists and mathematical science occupations are expected to be part of the fastest growing occupations. In the next decade employment of statisticians is expected to grow 35.4% and that of data scientists and mathematical science 31.4%. With the expected employment growth in mind, understanding basic mathematical concepts, such as rational number processing, is vital. Unfortunately, people who aim to join this rapidly growing work force may have difficulties with understanding mathematical concepts or experience math anxiety.

Math anxiety has been shown to be a great impairment for students when learning about mathematical concepts. Unfortunately, students who have high math anxiety (HMA) are often more likely to have less motivation and less self-confidence which results in them avoiding mathematical majors in college, thus avoiding careers which utilize math skills (e.g., Ashcraft, 2002). In general, there is a negative relationship between math anxiety and math achievement. As math anxiety increases, math achievement decreases (e.g., Foley et al., 2017; Hembree, 1990). Students with HMA tend to take fewer math related courses and receive lower grades (e.g., Ashcraft & Krause, 2007). When students take less math courses, have poor motivation and poor performance in math, they are less likely to pursue degrees which have heavy involvement of mathematical concepts. This leads to less employment of jobs which require mathematical understanding.

Rational numbers are numbers that are represented as fractions or decimals in an infinite number of ways (e.g., McMullen & Van Hoof, 2020). Understanding rational numbers is an integral part of learning mathematics (e.g., Christou et al., 2020; Siegler et al., 2013). People often display a whole number bias (WNB) or natural number bias (NNB) toward rational numbers. WNB is the tendency to apply natural number knowledge to rational numbers which can often lead to erroneous responses in mathematical tasks and understanding of rational number properties (e.g., McMullen & Van Hoof, 2020; Ni & Zhou, 2005;

Vamvakoussi et al., 2012). This phenomenon can be seen when participants are shown decimals and ignore the placement of the decimal in the number. For example, if participants are shown 0.13 and 0.4, they might respond that 0.13 is larger, ignoring the placement of the decimal and will look only at how many digits there are. Another example of WNB is seen when participants are activating the components (numerator and denominator) of a fraction as separate whole numbers instead of one whole magnitude, combining the two components (e.g., Obersteiner et al., 2013). More examples of how WNB is measured will be discussed later. This phenomenon has also been observed among school children (e.g., Christou et al., 2020; Obersteiner et al., 2016; Rossi et al., 2019) and educated/expert adults (e.g., Christou et al., 2020; Fazio et al., 2016; Obersteiner et al., 2013, 2016; Vamvakoussi et al., 2012; Van Hoof et al., 2020) in various tasks. Whole number bias can be measured in three ways: patterns of errors, reaction times, and/or strategies employed by the participant (e.g., Alibali & Sidney, 2015). The tasks used to measure WNB can vary widely in how the stimuli are presented and which dimension the task is measuring.

Tasks to Measure Whole Number Bias

There are several tasks which are used to measure whole number bias and they can be categorized as either non-symbolic or symbolic. Non-symbolic tasks consist of shapes such as balls, dots, or lines of different colors representing the parts of a ratio (numerator and denominator; see Figure 1). These types of tasks do not use Arabic symbols to represent numbers, instead participants only see dots or lines. Participants are then tasked with estimating numerosities for each color shape (representing the numerator or denominator) and asked to determine which array of shapes is larger in magnitude. According to Matthews et al. (2016), there are two systems which are used to explain how we perceive number sets: object tracking system (OTS) and approximate number system (ANS).

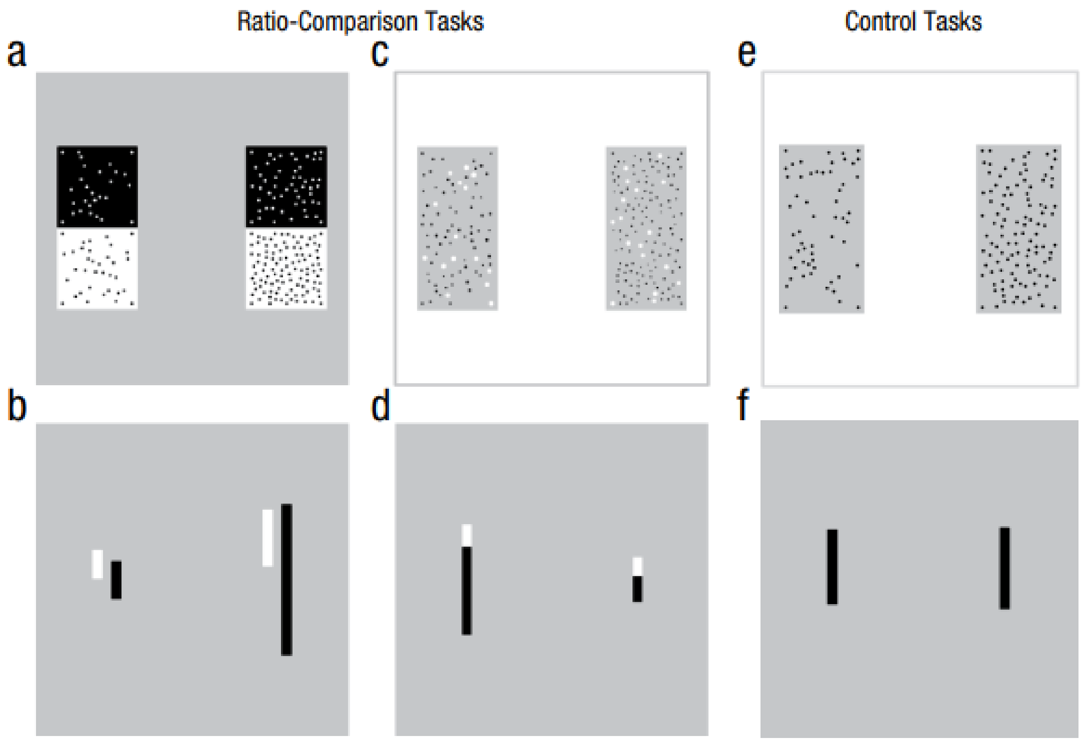

Figure 1. Sample stimuli from Matthews et al. (2016)
Participants are asked to indicate which ratio is larger, either white dots to black dots or white line lengths to black line lengths. a and b have dots or lines appearing separately whereas c and d have the dots or lines integrated. E and f are the control conditions where participants are asked to indicate which array of dots or line segment is greater.

OTS supports fast and precise enumeration of small sets which is referred to as subitizing, whereas ANS supports fast approximations of large sets (e.g., Matthews et al., 2016; Piazza, 2010). These two systems are often error-prone in fractional magnitudes because fractions can be presented in an infinite number of ways (e.g., Matthews et al., 2016). These systems allow participants to quickly respond to non-symbolic tasks without the need for counting each item. The advantages to using non-symbolic tasks is that they show the most basic form of processing numbers and therefore, are more intuition based because there is no processing of symbolic numbers. However, this kind of tasks does have limitations. For example, when numerosities get too large, estimations get worse because participants are no longer able to subitize and the ANS can only go so far in estimations.

There is literature which investigated whether non-symbolic tasks predict symbolic math performance and other research which examined how non-symbolic tasks relate to WNB. For example, Matthews et al. (2016) examined if performance in non-symbolic ratio tasks could predict performance in symbolic math tasks. They used a ratio comparison task (RCT; see Figure 1) as a non-symbolic task, and various symbolic math performance tasks including a fraction knowledge assessment, symbolic fraction comparison task, and an algebra entrance exam. In the RCT task, participants were asked to select the larger ratio from each set of dots or to indicate which line segment was larger. They found that performance on these non-symbolic RCT could predict symbolic math performance. They posit that these RCTs accessed intuitive knowledge of fractions and found participants who performed well also performed well on algebra assessments, symbolic fraction comparison task and fraction knowledge assessments. These findings suggest that participants can have intuitions about continuous numbers and not just whole numbers. Although their results suggest participants have intuitions about ratio processing, they did not examine intuitions of WNB.

Alonso-Diaz et al. (2018) examined WNB in non-symbolic and symbolic tasks across two experiments. In the first experiment participants were presented with stimuli in one of three conditions: non-symbolic (dot size equal), non-symbolic (cumulative surface area equal), or symbolic (numeral stimuli; See Figure 2). All participants were asked a ratio knowledge question. For the non-symbolic conditions (dot size equal, cumulative surface area equal), participants were asked to verbally report the proportion of white or orange balls in each urn. In the symbolic condition, participants were asked the question: “If an urn has 15 green balls and 15 red balls, what is the probability of pulling a red ball?”. They found most participants were able to state that the ratios were the same between urns. This suggests participants had a basic understanding of rational number concepts. Next, they asked participants to choose which urn they preferred if they were pull a white ball and win $100 or pull an orange ball and win $0. They found that participants chose the urn with more balls significantly more often than the urn with less balls, even though the two urns had the same ratio, regardless of stimulus size for non-symbolic stimuli. The same pattern was found for the symbolic condition, in which the larger number fraction was chosen more often than the smaller number fraction even though the ratios were the same between urns (e.g., 9/9 versus 4/4). These results showed that participants are biased toward the larger number of items even though they are aware that the ratios are the same between urns because they thought larger numbers meant greater chance of winning, thus the participants showed a WNB. In other words, these participants showed explicit knowledge of ratio understanding and they still chose the urn with the larger numbers. If there was no WNB they the urns would be no difference in which urn was chosen. In Experiment 2, the stimuli were different colored dots presented in a circle with a dotted outline (See Figure 3). Participants were asked to indicate which circle of dots had a higher winning probability. The winning probability was the chance of a participant choosing the correct color (in this case orange) to win $100 versus $0 if a green ball was chosen. They also manipulated congruency, in which larger ratio also had larger numerosity (more dots overall) in congruent trials, whereas in incongruent trials, the larger ratio had smaller numerosity (less dots overall). They found a congruency effect for accuracy on the probability distance (the distance between the ratios for each pair of urns). For congruent items, accuracy increased as the probability distance increased, and for incongruent items, accuracy increased as the probability distance increased but at a slower rate. Participants in these experiments did prefer the option with greater numerosity of winners, but it was not due to a lack of understanding of ratios, as in both experiments, participants showed explicit ratio knowledge. According to Alonso-Diaz et al. (2018) their study suggests that there is an intrinsic WNB in which participants are more focused on the number of items presented rather than the magnitude.

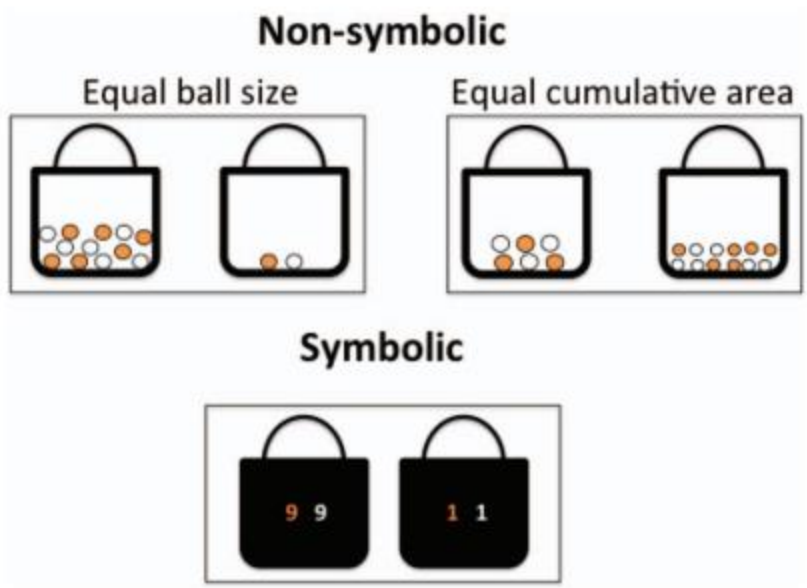

Figure 2. Sample stimuli from Alonso-Diaz et al. (2018) experiment 1. Participants were presented one of these three trials. They were asked to indicate which of the two urns they would prefer to choose form to pull the winning color (either white or orange).

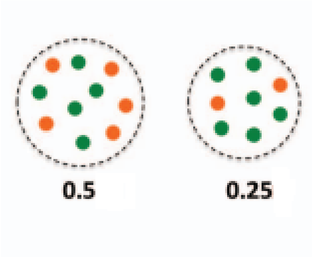

Figure 3. Sample stimuli from Alonso-Diaz et al. (2018) experiment 2. Participants chose the circle they thought would have the greater chance of winning. They were told to imagine that if they pulled an orange ball, they would win $100, but if they pulled the green ball, they would not win any money. The numbers below each circle represent the chance of winning in that circle.

WNB can be measured using symbolic tasks such as, fraction magnitude comparison, decimal comparison, algebraic equations, and density propensity. These tasks evaluate different dimensions of how rational numbers are different from natural numbers: representations, size, operations, and density (e.g., Obersteiner et al., 2016).

The fraction magnitude comparison task is one way to measure size of rational numbers in comparison to natural numbers. This task is the comparison of two fractions and participants are asked to indicate which fraction of the two is greater in magnitude (e.g., DeWolf & Vosniadou, 2015; Morales et al., 2020; Obersteiner et al., 2013, 2020; Van Hoof et al., 2020). Fraction comparison items can either be common component items in which the numerator or denominators are shared or without common components in which there are no shared components between fractions. For common component items that are congruent, the denominators are the same between fractions, whereas for incongruent items the numerators are the same between fractions (e.g., Vamvakoussi et al., 2012). For these items, congruent trials coincide with natural number rules whereas incongruent trials do not. For example, 3/5 versus 4/5, 3 is less than 4 which follows the same rules as natural numbers, however in an incongruent trial, such as 5/9 versus 5/11, the reasoning must be reversed, since 9 is less than 11, but 5/9 is larger in magnitude than 5/11. In without common component congruent items, participants might compare 5/13 versus 9/15 and are asked which is larger. For this item, the components (numerator and denominator) separately are larger in numerosity (9 > 5 and 15 > 13) and larger in magnitude (9/15 = 0.6, 5/13 = 0.38) which also follow natural number rules. For without common component incongruent items, the larger fraction in magnitude has smaller components in numerosity (e.g., 12/27 versus 15/49 in which 12 < 15 and 27 < 49 but 12/27 > 15/49) which does not follow natural number rules. In without common component items, there can also be instances that are considered neutral in which both components vary between fractions and one side is not larger in numerosity than the other (e.g., 6/14 versus 8/11; 6<8 but 14>11, but 6/14 < 8/11). An advantage to using a fraction magnitude comparison task is that it is easy to manipulate task complexity such as items having more or less digits, sharing common components or no common components as well as manipulating the distance between the two fractions. It is also possible to manipulate the familiarity of fractions. For example, it is far easier to compare fractions such as 1/4 or 1/2 compared to 4/14 or 7/34.

However, for this type of task, there are many strategies, and it is difficult to be certain which strategies are employed by participants.

The second way to measure size of rational numbers compared to natural numbers is the decimal comparison task. The decimal comparison task is the comparison of two decimals in which the length of decimals or how many decimal places are manipulated. For example, comparing 0.13 versus 0.2, participants may ignore the placement of the decimal point and compare 13 versus 2. Therefore, the WNB is evident when participants choose the longer decimal even if it is not larger in magnitude (e.g., Roell et al., 2019). An advantage of this task is its simpler presentation of rational numbers which look more like natural numbers. However, it is harder for participants to exhibit a WNB in this task because its difficulty level is low. Participants have been shown to ignore part of the decimal and to focus solely on the tenths place in this task (e.g., Dewolf et al., 2015). In this task, they can quickly determine which is larger without considering the entire number, therefore this task is not ideal for examining WNB.

A third way to measure WNB is through an algebraic equations task which measures the operation dimension of WNB. This task is used to assess misconceptions of algebraic rules, such as addition and multiplication make the result larger, and subtraction and division make it smaller. These rules are applicable to whole numbers (other than one) but might be erroneously applied to equations with rational numbers (e.g., Obersteiner et al., 2016). This misconception illustrates a bias toward whole numbers thus showing a WNB effect. Rational numbers less than one will not abide by the same rules natural numbers abide by in multiplication and division. An example of this task is when participants are presented with an equation with an unknown variable and asked to determine the validity of the statement. For true congruent and incongruent trials, the phrase "*can be"* is used, whereas for false congruent and incongruent trials, the phrase "*always"* is used. For example, in a congruent, true trial participants are presented with an equation such as 5 + 2x, and they are asked whether the result "can be" greater than 5. Here, the answer is true because the result *can be* greater than 5. However, in a congruent false trial, participants are presented with "1 + 10y is always greater than 1". Participants must respond with false, since the outcome is not *always* greater than 1. For an incongruent true trial, participants are presented with "3 + 12z *can be* smaller than 3". For this trial, a correct answer is true. However, in an incongruent false trial, participants are presented with "2 + 4y is *always* smaller than 2". The correct answer here is false since the outcome is not always smaller than 2. (e.g., Vamvakoussi et al., 2012). This task focuses more on how individuals apply natural number knowledge and evaluates whether those individuals can adapt their thinking to rational numbers. However, this task requires participants to have a much more abstract understanding of addition, multiplication, subtraction, and division when reasoning about rational numbers.

The final way WNB can be assessed is through a density propensity task which measures the density of rational numbers. This task examines the overapplication of natural number rules in which there is always a successor or antecessor to any number. However, in the case of rational numbers, there are an infinite number of numbers between any two numbers (e.g., McMullen & Van Hoof, 2020). One example of this task is when participants are given two rational numbers, such as 5/9 and 8/9, then participants are asked how many numbers exists between these two numbers. When participants display a WNB, they respond by saying there are only two numbers (6/9 and 7/9; McMullen & Van Hoof, 2020). However, the correct response would be “too many to count”, or “an infinite number”. This task examines whether participants have a conceptual understanding of rational numbers and what makes them different from natural numbers. However, this task does not examine participant’s understanding of number magnitude and is therefore limited in ways to manipulate task difficulty.

While considering all the tasks to measure WNB, the fraction magnitude comparison task seems to show the largest effect of WNB and be the most manipulatable of all the tasks. The decimal comparison task is simple, so it makes it too easy for participants to do, and participants sometimes do not consider the entire number when reasoning. The algebraic equation task appears to be too abstract for participants because it would require additional instruction on participants using rational numbers and possible clarification of what a rational number means. Finally, the density propensity task does not assess participants’ understanding of magnitude. The density propensity task is also very simple, and it is solely assessing whether participants understand there are infinite numbers between two rational numbers. These characteristics suggest the fraction magnitude comparison task as the best measure to assess participants' WNB, even though one downside is it may not be easy to determine which strategies participants may employ during the task.

Numeracy and Whole Number Bias

Alibali and Sidney (2015) argue that activation of mental representations of rational number knowledge would affect performance on tasks. Poor activation of mental representations of rational numbers would lead to poorer performance on specific tasks. There are two main strategies are often used in a fraction magnitude comparison task: componential and holistic. A componential strategy is when participants only look at the parts of a fraction (numerator and/or denominator) without considering its magnitude, whereas a holistic strategy is when participants consider the fraction as one number and consider its magnitude. Alibali and Sidney (2015) also report that a participant's level of mathematical understanding or numeracy would elicit different strategy patterns. For example, for a non-math expert group, in a fraction magnitude comparison task, participants may compare uncommon components (componential strategy) if the fractions share components for both congruent (same denominator) and incongruent (same numerator) items. This strategy often uses intuitions about natural numbers, such that larger in numerosity means larger in magnitude. However, if the fractions do not share components and are congruent, participants will choose the fraction based on how large the fractions components are in relation to each other. If the fraction does not share components and are incongruent, non-expert participants will guess. This is not the same strategy pattern seen in expert math participants. First, according to Alibali and Sidney (2015), if fractions share a component, expert math participants will use a componential strategy and compare the uncommon component (same as the non-expert group). However, if the fractions do not share components, expert math participants will estimate or compute both fractions' magnitudes (holistic strategy). If the difference between fraction magnitudes is large, they will respond with which fraction is larger by estimating magnitudes; however, if the difference between fraction magnitudes is small, expert math participants will compute each fractions magnitude or will convert the fractions to common denominators and report which fraction is larger by comparing the numerators.

It is important to understand individuals with different levels of numerosity use different strategies about rational numbers which in turn may result in differences in performance in a WNB task (e.g., Obersteiner et al., 2016; Obersteiner et al., 2020). In other words, low numeracy (LN) participants may exhibit a WNB because they are more prone to using a componential strategy applying only natural number intuitions to all problem types. On the other hand, high numeracy (HN) participants may exhibit no WNB or even a reverse congruency effect because they could be utilizing both componential and holistic strategies. However, as we learn about rational numbers, we can start developing new ways of thinking about them. For example, HN participants could develop learned intuitions about rational numbers (e.g., Van Hoof et al., 2020). This is because when we initially start learning about rational numbers, we already have learned natural number rules and these natural number rules do not apply to rational number information. For example, when we learn about algebraic equations, we learn using natural numbers and using these types of numbers always leads to the same results, such that when adding or multiplying two terms the result is always larger and when subtraction or dividing, the result is always smaller. These learned intuitions about natural numbers are then challenged when learning about rational numbers such that when multiplying a number by a rational number less than one, the result is smaller and when dividing by a number less than one, the result is larger. Therefore, while we learn about rational numbers, it is possible misconceptions may develop into intuitions about rational numbers, and they compete with natural number intuitions (e.g., Rinne et a., 2017; Van Hoof et al., 2020). Rinne et al. (2017) state when children initially learn how to reason about fractions and determine which fraction is larger, they are biased by natural number rules and choose fractions that have larger components. Then when they start understanding fractions more, they start exhibiting a bias towards fractions which have smaller components because they learned natural number rules do not always apply. In other words, in early understanding of fractions, when asked which fraction is larger, children will choose fractions that are larger in numerosity and as they learn more about fractions, they will start choosing fractions that are smaller in numerosity. These

secondary learned intuitions about rational numbers, although misconceptions, facilitate performance in incongruent items. For example, in a fraction magnitude comparison task, congruent items could be solved using intuitions about natural numbers, whereas incongruent items could
be solved using these learned intuitions about rational number (see Table 1).

This learned intuition about rational numbers is only beneficial for incongruent items because in these items, the fraction which is larger in magnitude has smaller components and is therefore smaller in numerosity. Intuitions about natural numbers are only beneficial for congruent items because in these items, the fraction which is larger in magnitude is also larger in numerosity. Therefore, when participants apply natural number intuitions in a fraction magnitude comparison task, it would result in a WNB whereas if participants only apply rational number intuitions, it would result in a reverse WNB (see Table 1).

Table 1. Examples of whole number bias and rational number bias in fraction magnitude comparison task.

	CC		WCC	
	C	IC	C	IC
	1/7 vs. 5/7	4/8 vs. 4/9	7/9 vs 3/8	5/9 vs 3/4
NNB	Correct	Incorrect	Correct	Incorrect
RNB	Incorrect	Correct	Incorrect	Correct

Natural Number strategy: Longer is larger, 3.5 < 3.42 (incorrect)
Rational Number strategy: Shorter is larger, 2.7 > 2.35 (correct), 3.4 > 3.42 (incorrect)

Note. NNB = Natural Number Bias; RNB = Rational Number Bias

Morales et al. (2020) examined participants from a highly selective university who were students from the Department of Physical and Mathematical Sciences, using a fraction magnitude comparison task. Their fraction items were both on one side of 1/2 to avoid the benchmarking strategy. Their item design was: 2 (Components: common components vs without common components) x 3 (Congruency: Congruent, Incongruent, Neutral) x 3 (Gap Thinking: leads to corrects answer, leads to incorrect answer, both fractions have the same gap). Morales et al. (2020) revealed a significant reverse congruency effect in reaction time and accuracy for items that did not share components. In other words, incongruent problems were solved more quickly and accurately than congruent ones. This suggests these high numeracy (HN) participants were not affected by WNB. It is possible these HN participants were exhibiting learned intuitions about rational numbers which resulted in a reverse WNB for accuracy and reaction time. Morales et al. (2020) also reported that gap thinking had no effect on reaction time or accuracy.

Dual Processing Theories and Whole Number Bias

Dual processing theory (DPT) is described using two types of thinking: Type I and Type II. Type I thinking is thought to be fast, automatic, and does not require working memory to respond, whereas Type II thinking is thought to be slower, more effortful, and would require working memory resources (e.g., Gawronski & Creighton, 2013; Stanovich, 2009). There are several models which aim to explain the relationship between these two types of thinking. First, the

Default Interventionist Model (DI; e.g., Evans, 2008; Evans & Stanovich, 2013) claims the two types of thinking are exclusive in which Type I is heuristic based and Type II is analytical based. According to this model, Type I always occurs first and Type II is only engaged when a conflict in the initial response is detected; and therefore, a higher order of thinking must take place. Once Type II thinking is engaged, a second, more logical response might be generated which may be different from the initial Type I response. Another model is the Parallel Model (e.g., Sloman, 1996). This model claims both types of thinking occur simultaneously. A third model is the Hybrid Model (e.g., De Neys, 2017; Pennycook, et al., 2015; Trippas et al., 2017). The Hybrid Model aims to combine aspects of both DI and Parallel processing by stating that Type I processing can be either heuristic or logic based and still be fast or automatic without the use of working memory resources. It also states that Type II processing may take place if more effortful reasoning is required (see Figure 4). It could be argued that Type I processing would facilitate a componential strategy since this strategy requires little effort and individuals can apply intuitions about number information, whereas Type II processing would facilitate a holistic strategy since this strategy requires more effort and mindware to compute or estimate a magnitude.

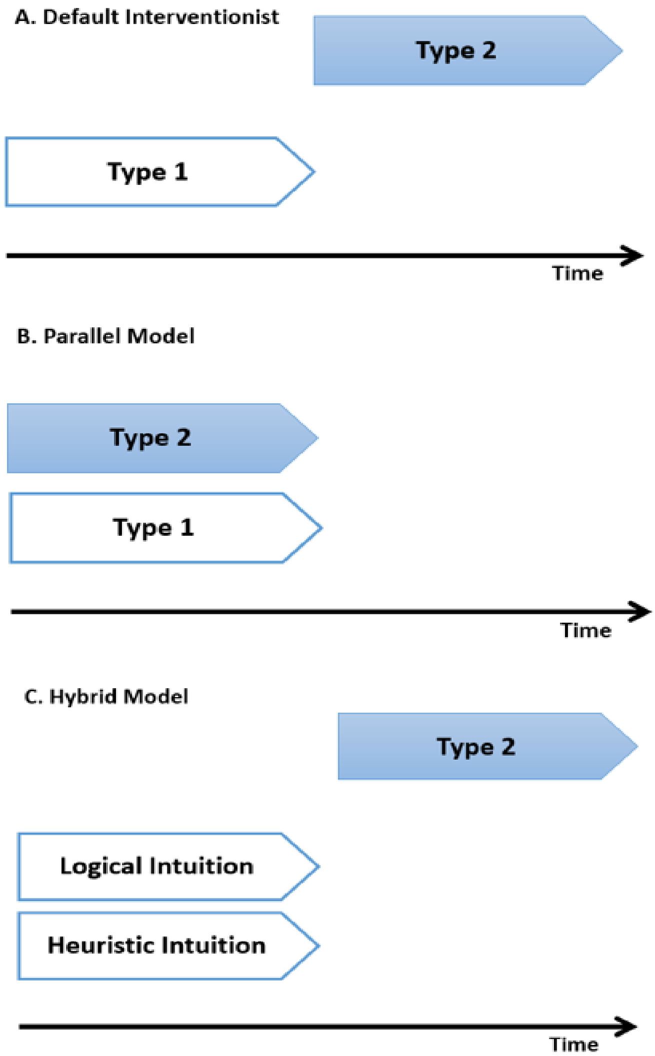

Figure 4. Three Models of Dual Processing Theory

There are several studies which examined WNB using DPT. Van Hoof et al. (2020) examined adult participants in a fraction magnitude comparison task. They tested participants at two different times. On day 1, there was no time restriction for participants to solve problems, then on day 2, a time restriction was placed based on participants median response times from day 1. They aimed to examine whether using time restriction would elicit more intuitive responses from their participants and avoid Type II thinking. Their items varied in complexity and were controlled for the distance effect such that all items were between 0.153 and 0.176 in magnitude between fractions. They also controlled for benchmarking strategy by having fractions magnitude above 0.2, or below 0.8 and having both fractions on one side of 1/2. Finally, they only included items that were less than 1 and in simplest form. The same items were administered for each testing day. They had four research questions. First, they examined whether participants would exhibit a WNB with no time restriction for both accuracy and reaction time. They found for the first test day participants were significantly more accurate and faster on congruent trials than incongruent trials; therefore, confirming traces of WNB in their participants. The second question was about the intuitive nature of WNB. On the second day of testing, with the time restriction, they found significantly lower accuracy on the congruent and incongruent items than the first day of testing in which there was no response time restriction. Also, the decrease in accuracy from day 1 to day 2 was significantly greater for incongruent items than for congruent ones, thus confirming the intuitive nature of WNB. Question three was aimed to examine whether participants exhibited conflict detection. They found reaction times were significantly shorter for correctly solved congruent trials than for incorrectly solved incongruent trials during the response time restriction day (day 2), which suggested conflict detection was present. They had a fourth question about the nature of the conflict detection. According to DI, conflict detection occurs in Type II thinking intervention whereas the Hybrid model claims conflict detection can occur intuitively from competing intuitions (Van Hoof et al., 2020). Since reaction times were restricted on day 2, Type II thinking was not activated and responses were based on intuitions which suggests conflicted detection was intuitive because reactions times were significantly shorter for correctly solved congruent trials than incorrectly solved incongruent trials, therefore, supporting the Hybrid model. Although this paper explored how Type I thinking effects performance on a fraction magnitude comparison task, it failed to consider how participants would do when they are able to activate Type II thinking.

Vamvakoussi et al., (2012) examined performance on four tasks (fraction magnitude comparison, decimal magnitude comparison, operations of addition/subtraction and multiplication/division, and density propensity of fractions and decimals) and explained their results using the dual processing theory. They hypothesized that correct incongruent responses would have longer reaction times compared to correct congruent reaction times which would provide support for DI model. Their findings were inconsistent between the two magnitude comparison tasks (fraction and decimal). For fraction comparison items that shared a common component (same numerator or same denominator), they found no difference in accuracy between congruent (same denominator) and incongruent items (same numerator). However, reaction times were significantly longer in incongruent than congruent trials, suggesting Type I responses were inhibited in these trials supporting DI model. For fractions without common components, there were no differences in reaction time or accuracy between congruent and incongruent trials which suggests, participants were not subject to larger in numerosity means larger in magnitude, rather they used holistic strategies, in which they computed the magnitude of each fraction, using Type II processing. Although there was no difference in reaction time for decimal comparison problems, there was a difference in accuracy in which incongruent trials were more accurate than congruent trials indicating a reverse congruency effect. Two possible explanations were provided. First, participants were prone to a “shorter is larger” concept sometimes seen in older children and adults. Second, participants became suspicious of the task which resulted in poorer performance in congruent items. For both operation items (addition/subtraction and multiplication/division), there was significantly lower accuracy and longer RTs for incongruent trials than congruent trials suggesting intuitions of operations (Type I thinking) were inhibited to respond correctly, and participants activated Type II thinking. For density propensity items, there was significantly higher accuracy in congruent compared to incongruent items, but no difference in reaction time, suggesting that the idea of infinite numbers existing between two rational numbers was difficult to understand overall. Their findings in the fraction magnitude comparison task in general support the DI model because in simple items (common component), Type II thinking was activated for incongruent trials but not congruent trials, resulting in a WNB in reaction time, whereas in more complex items (without common component), Type II thinking was activated for both congruent and incongruent trials resulting in no WNB in reaction time or accuracy.

Obersteiner et al. (2013) aimed to determine whether expert math level participants from the Department of Mathematics or from the Section of Applied Mathematics and Numerical Analysis of the Department of Computer Science could overcome WNB on fraction comparison problems. They examined whether participants would use a componential or holistic strategy. As previously stated, the componential strategy (Type I) is when participants ignore the magnitude of a fraction and look only at its components (numerator and/or denominator) as separate numbers. The holistic strategy (Type II) is when participants compute or estimate a magnitude of the fraction and assess the fraction as one number instead of two separate numbers. There were five types of fraction comparison pairs: two with common components and three without common components. In the common components congruent (CC-CO) items shared denominators (e.g., 7/8 versus 5/8), whereas common component incongruent items (CC-IC) shared numerators (e.g., 5/9 versus 5/7). It was expected that participants would apply a componential strategy as it is the most efficient way to complete these items. In the without common components congruent (WCC-CO) condition, each component (both numerator and denominator) of one fraction was larger than the respective component of the other fraction and larger in magnitude (e.g., 24/25 versus 11/19). In the without common components incongruent (WCC-IC) condition, the parts were smaller but the fraction was larger in magnitude (e.g., 25/36 versus 19/24). In the without common components neutral (WCC-N) condition, both parts of one fraction were not larger than the other fraction (e.g., 17/41 versus 11/57). Accuracy was not assessed because it was at ceiling level for these expert level participants, therefore only reaction time was examined. They found a WNB in CC items but not in WCC items. In other words, in CC items, reaction times were significantly faster in congruent trials than incongruent trials, whereas in WCC items, there was no difference in reaction time between congruent and incongruent trials which supports the bias is rooted in intuition and supports DI model since it was found in expert mathematicians in items that require simple straightforward processing (CC) and not more complex items (WCC). Furthermore, it was reported that for WCC items, reaction times were predicted by the distance between fraction magnitudes, but reaction time could not be predicted by the distance between numerators or distance between denominators, which suggested participants used holistic strategies rather than componential strategies. Although Obersteiner et al. (2013) was able to find WNB in expert math participants and explain their findings using DI model, they did not examine non expert participants and see how they would perform on a WNB task.

Obersteiner et al. (2016) claimed that natural numbers are included in rational numbers; for example, 1, 3, and 4 are included in 1/3, 1/4, and they automatically activate natural number knowledge. This automatization of activating natural number knowledge could also be happening in algebraic expression problems. For example, participants who are presented with algebraic expression problems may activate rule-based solutions to the problems (e.g., multiplication makes the result bigger, and division makes the result smaller). If participants are solely considering natural numbers, this may be the case; however, when they are plugging in rational numbers, these ‘rules’ could lead to incorrect answers. Experiment one examined secondary school students. Participants were presented with algebraic expressions, such as $4 * x < 4$, and asked to determine if they could be true or not. They were first presented a natural number block, then a rational number block and both block contained the same stimuli except x was different between the blocks. For the natural number block, participants were told “x” was a natural number, and the correct answer was yes for half of the items while for the other half, the correct answer was no. Also,, all items were congruent, which meant that using a natural number would always yield a correct response. For the rational number block, the same items were used but participants were told “x” represented a positive rational number. For congruent items, natural numbers would yield a correct response and for incongruent items, natural numbers would yield an incorrect response. For these items, the correct answer was always yes if participants were using a positive rational number. They found students had better accuracy on natural number block compared to rational number block overall, which suggested participants applied the same strategy and plugged-in natural numbers for both blocks. Within the rational number block, they found that accuracy was higher for congruent items than for incongruent items, suggesting a WNB. They also found that the accuracy of congruent items in the rational number block was not significantly different from the accuracy in the natural number block, possibly because their students were not aware of the differences in the task requirement between natural block and rational block. Also, the students RTs were shorter in the rational number block than in the natural number block. However, within the rational block there was no difference in reaction time between congruent and incongruent items suggesting natural number knowledge was applied, and participants did not engage in Type II processing. This may have been due to participants not understanding the differences between blocks or possible training effects from the natural number block, which always occurred first. Another explanation could be that their participants used their heuristics about operations such as multiplication makes the result larger, and division makes the result smaller which is true when natural numbers are plugged in. In a second experiment, they examined expert mathematicians in the same procedure as experiment one. All participants had a master's degree or PhD in mathematics. They expected no difference between natural and rational number blocks and no difference between congruent and incongruent items in the rational block for accuracy. Also, they predicted accuracy to be at the ceiling level and reaction time would be shorter for rational block items than for natural block items because these participants would be able to apply knowledge of the algebraic expressions solvability in which using any rational number would always yield correct results for rational block items. Results showed no difference within the rational block between congruent and incongruent items for accuracy or reaction time. However, accuracy was greater for the rational number block items than for the natural number block items suggesting these highly proficient participants were relying on their knowledge of the algebraic expression's solvability rather than general rules of multiplication and division, therefore showing no WNB. Reaction times were also significantly shorter for rational number block items than for natural number block items and they attributed this to a possible training effect since the they always performed the natural number block first. They suggested this could be due to participants relying on item structure instead of considering the problems item by item in the natural number block. These findings support the notion that WNB is affected by experience or expertise because their main finding was that their student group was affected by WNB whereas the expert group was not. It could be argued their findings gave support for a hybrid model. In other words, LN participants exhibited a traditional WNB such that they had greater accuracy on congruent than incongruent problems suggesting they did not have the necessary mindware to complete the task. HN participants (expert math group), on the other hand, showed no difference in accuracy or reaction time between congruent and incongruent items in the rational number block. Also, the finding of greater accuracy in rational number block items than natural number block items suggest their knowledge of the solvability of algebraic equations could have been a hinderance to their ability to solve natural number block items since it appears participants still tried to plug in a rational number to solve the expression even in the natural number block items. Their knowledge made them faster and more accurate on a more difficult problem, therefore, intuitive Type I logical processing may have been displayed in these participants.

Obersteiner et al. (2020) aimed to examine how college students with different math experience levels would perform in a fraction magnitude comparison task using without common component items. They examined congruency (congruent versus incongruent), benchmarking strategy (straddling, in between, and close to 0 or 1), and half of the participants received a tip on how to apply a benchmarking strategy by thinking of well-known fractions such as 1/2 or 1/4. Math experience was determined by how many calculus courses had been taken by participants. The low math experience group had less than two semesters of calculus, whereas the high experience group had two semesters of calculus or more. Overall, there was a reverse congruency effect. Accuracy was better on incongruent items compared to congruent ones. High math experience participants were more accurate than low math experience participants, but they had similar reaction times. The highest accuracy was on close-to-0-1 problems followed by straddling problems and in-between problems had the lowest accuracy. A three-way interaction was found in reaction time among congruency, problem type and mathematics experience. Low math experience participants had a reverse congruency effect for in between and straddling benchmark problems but not close to 0 or 1. High math experience participants did not show a congruency effect in any benchmark type suggesting they had a greater understanding of rational numbers. There were two possible explanations for these findings. First, this bias exists in the lower math experience participants because they relied on a gap thinking strategy that is successful on incongruent items more often than congruent items which would explain the reverse WNB. Second, they state participants may have been exhibiting a bias seen in Rinne et al. (2017) such that fractions with smaller components are larger in magnitude which facilitates performance on incongruent and not congruent items. These findings also suggest high math experience participants used a different strategy than their low math experience counterparts, possibly a holistic strategy. The tip given to half of the participants had no effect on performance. This study supported the finding that benchmarks, especially 0 and 1, are important in fraction comparison tasks and it allowed participants to overcome WNB more easily. They claim that their results of a reverse congruency effect challenge dual processing account of WNB because they did not find a WNB suggesting their participants were not affected by intuitions about natural numbers. These findings do challenge the default interventionist model but provide support for a hybrid model because their participants exhibited logical intuitions about rational numbers since performance was better on incongruent items than congruent items. It is important to note, their participants were from a highly selective university and overall had high SAT and ACT scores, and it is possible these participants may not exhibit a traditional WNB because of their expertise, therefore their results may not generalize to other populations. Although many of these papers explain WNB using dual processing theory, they did not examine how other factors would influence the bias, such as math anxiety or working memory capacity.

Math Anxiety and Whole Number Bias

Another factor that may affect WNB is math anxiety (MA). MA can be defined as feelings of apprehension, fear or tension which interferes with math performance (e.g., Ashcraft, 2002). Anxiety in general has been shown to affect performance on a variety of cognitive tasks because anxiety is known to occupy mental resources; and therefore, it would impair task performance (e.g., Beilock & Maloney, 2015). For example, Attentional Control Theory (ACT) by Eysenck (e.g., Eysenck et al., 2007) argues that anxiety occupies mental resources which reduces executive functioning. This includes the ability to inhibit responses to internal and external stimuli which could interfere with task execution. ACT also distinguishes between effectiveness and efficiency. Effectiveness is measured through accuracy rates, whereas efficiency is the amount of effort put into a task and is measured with a composite score of accuracy divided by response times. Efficiency is used because effectiveness (accuracy) alone may not measure effects of anxiety on cognition if participants put in enough effort into the task. In other words, highly anxious individuals may have the same accuracy as their low anxious counterparts but at the cost of reaction time such that highly anxious individuals would have greater reaction time than low anxious individuals.

ACT could also explain the relationship between math anxiety and mathematical tasks. Performance on math tasks is impaired when participants experience high MA. As stated before, there is a negative relationship between math anxiety and math achievement (e.g., Ashcraft, 2002; Ashcraft & Krause, 2007; Hembree, 1990). Where does this relationship stem from? Does math anxiety cause poor performance or does poor performance on mathematical tasks result in greater math anxiety? Foley (2017) argued that math anxiety has a bidirectional relationship with performance. In other words, math anxiety could cause poorer performance on math related tasks, but poorer performance could also result in higher math anxiety. Other researchers propose different accounts of the relationship. Ashcraft and Krause (2007) argued that math anxiety was learned in class settings. For example, when a student is called to the board to work a problem and they do poorly, they are embarrassed in front of their peers and teacher which could result in greater anxiety. They also stated that if students have low math aptitude or low working memory capacity, they may be at risk for developing math anxiety. There are several math anxiety measures. These measures assess a variety of aspects which involve mathematical understanding during academic situations (e.g., taking a math exam or reading a mathematics textbook), attitudes towards mathematics in everyday life (e.g., calculating a tip at a restaurant), and emotions such as nervous, anxious, confidence, and afraid (e.g., Ma, 1999). A measure widely used to assess mathematics anxiety is the Mathematics Anxiety Rating Scale (MARS) which was developed by Richardson and Suinn (1972). This is a 98-item questionnaire which used a 5-point Likert scale (1 not at all anxious to 5 very much anxious). The MARS has a test-retest reliability of $r = 0.85$ and is negatively correlated with math performance ($r = -0.64$). Plake and Parker (1982) revised MARS to a shortened 24-item questionnaire, Mathematics Anxiety Rating Scale-Revised (MARS-R). The MARS-R correlated with MARS ($r = 0.97$) and was reliable ($r = 0.98$). Hopko et al. (2003) further shortened the scale and termed it the Abbreviated Math Anxiety Scale (AMAS). AMAS is a 9-item questionnaire with good test-retest reliability ($r = 0.85$) and strong convergent validity with MARS-R with $r = 0.85$.

Sidney et al. (2019) investigated effects of math skills, math anxiety (MA), working memory (WM), and strategy variability on math performance using a fraction magnitude comparison task. They theorized strategy variability would mediate the relationship between MA and math performance and that math skills and WM would moderate the relationship between strategy variability and math performance (see Figure 5). They found that strategy variability mediated the relationship between MA and math performance, and WMC and math skills moderated the relationship between strategy variability and performance. For high math skill participants, high and low WM showed the equivalent level of performance. For low math skill students, low WM participants performance increased as strategy variability increased while there was no change in performance or strategy variability for high WM participants. High math skill participants should have greater mindware of mathematics understanding whereas low math skills should show lower mindware. Also, higher WM would show better Type II processing than low WM participants since Type II processing requires WM. Therefore, the results that showed no WM effects on math performance for high math skill participants suggest that high math skill participants had the mindware to employ efficient strategies, regardless of their WM capacity. For low math skills students, low WM participants performance increased as strategy variability increased while there was no change in performance for high WM participants. This suggests that for low WM participants they would be able to overcome difficulties in Type II thinking if they were able to better adapt their strategy use. This also suggests participants with low math skills and high WM were able to activate Type II thinking while it was more difficult for low math skills and low WM participants since they did not have same mental resources. Their findings could support a Hybrid model of DPT becasue high math skill participants could be showing intuitive logic which is why there is no difference in WM on performance. However low math skill participants' performance was dependent on WM suggesting they may have activated Type II thinking and did not have intuitive Type I logic.

MA occupies attentional resources that reduce WM resources which then reduce math performance (e.g., Ashcraft & Krause, 2007; Szczygiel et al., 2021).

In a study involving children, Szczygiel et al. (2021) examined the mediating role of WM on MA and math achievement and found that WM mediated the relationship between MA and math achievement such that as MA increased, WM resources decreased and while WM decreased so did math achievement.

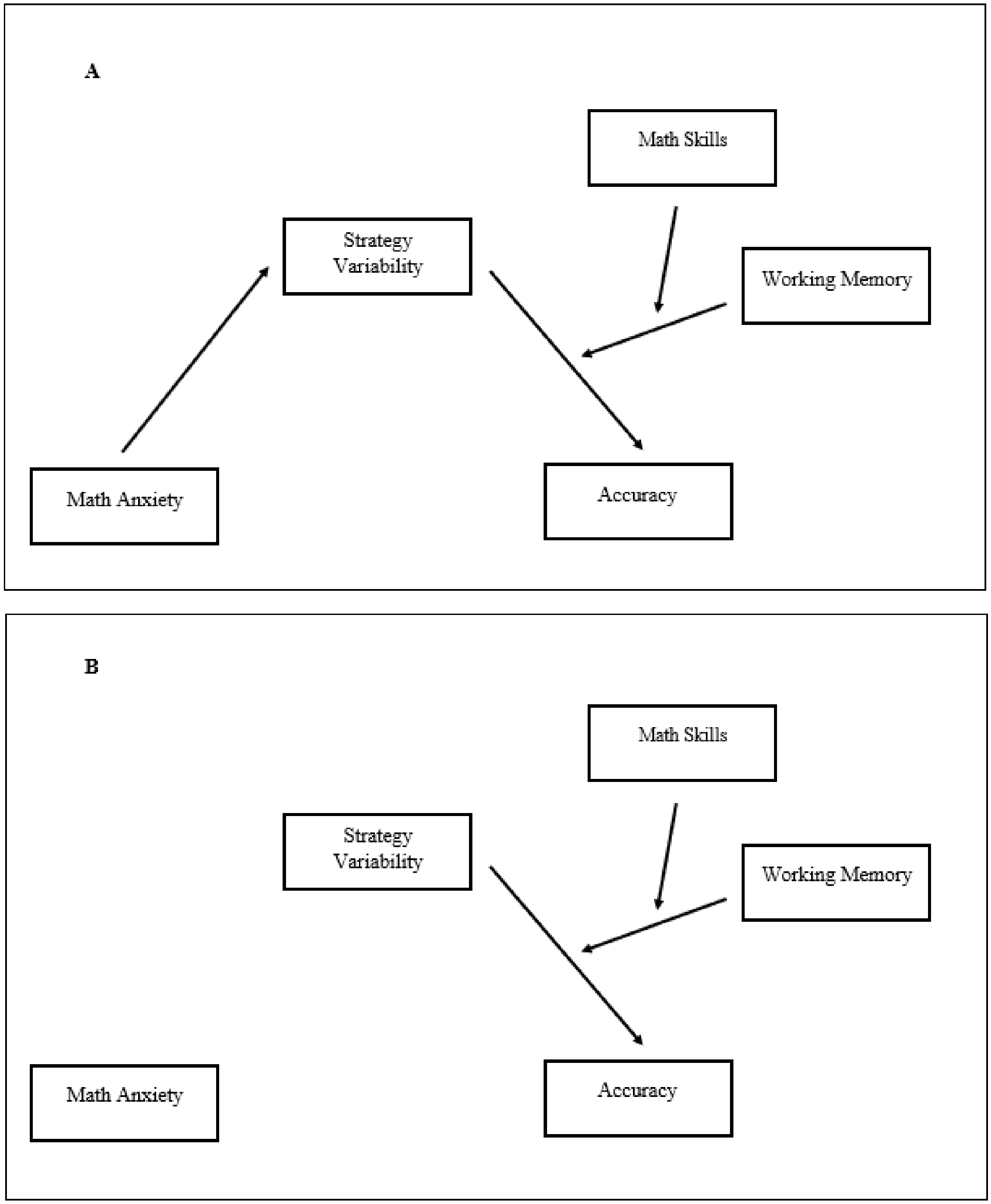

Figure 5. Experiment results from Sidney et al. (2019).
Lines indicate significant relationships between variables. A: Experiment 1 results. B: Experiment 2 results.

Summary

Whole Number Bias (WNB) can be examined from the perspective of dual processing theory (DPT). The effect of WNB has been previously explained from a Default Interventionist (DI) approach. According to the DI model, WNB occurs from a failure to activate Type II thinking and WNB is rooted in heuristic intuition. For rational number problems Type I processing is engaged when an intuitive answer, based on natural number knowledge, is easily activated. According to the DI model, for congruent problems, Type I processing would lead to correct answers whereas for incongruent problems, Type II processing must be activated to respond correctly. Furthermore, incongruent problems would take longer to respond to regardless of individual differences such as numeracy and anxiety. According to the Hybrid model, WNB would occur differently based on individual differences such as numeracy. The Hybrid model states that Type I processing can be both heuristic and logic based in which processing still occurs quickly and without working memory resources. Type II processing is still analytical based and requires working memory resources. It could be argued that HN participants may have logical Type I processing which could be examined in accuracy and reaction time data. HN participants should show greater accuracy and shorter reaction times than LN participants, thus showing a smaller WNB effect. This would suggest reasoning about rational numbers for HN participants is more intuitive because of their mindware. For LN participants, they would still have to activate their Type II thinking to perform the task which would result in longer reaction times than HN participants. In other words, DI model would explain performance in individuals with LN, whereas the Hybrid model would explain performance in individuals with HN.

Math experience (ME) has not been consistently evaluated throughout the literature and could be one reason why there are inconsistent results reported. Obersteiner et al. (2013) examined expert math level participants in a fraction magnitude comparison task and reported a congruency effect for common component items in which reaction times were shorter for congruent than incongruent problems, but no difference was found for without common component items. In other words, their high ME participants showed a WNB in fractions which shared components, but no WNB in items that do not share components. However, Morales et al. (2020) examined participants from a highly selective university and found a reverse congruency effect in both reaction time and accuracy in a fraction magnitude comparison task for items that did not share components. These findings support the notion that HN participants have intuitive logic because as item difficulty increases, participants are no longer influenced by natural number intuitions and may utilize rational number intuitions. Another reason for inconsistent results could be from differences in problem types. In addition to differences in how ME is measured, some researchers have focused more on types of strategies participants might use. Obersteiner et al. (2020) did not find the WNB in high ME participants. They examined participants from a highly selective university and separated them based on how many calculus courses they had taken. Across all participants and conditions, they found a reverse congruency effect in a fraction magnitude comparison task. They further reported that their low ME participants exhibited a reverse congruency effect on some item types (in-between and straddling) whereas high ME did not show any congruency effect on any problem types. Since these participants were from a highly selective university, their low ME participants could have misconceptions about rational numbers they never unlearned but their high ME participants were able to successfully do the task without showing a WNB suggesting they have a better understanding of rational numbers.

Anxiety would affect Type II thinking since anxiety occupies mental resources. According to Attentional Control Theory, individuals with high anxiety would take longer to respond than low anxiety individuals to maintain their accuracy. This makes high anxiety individuals less efficient than low anxiety individuals.

Although no literature exists examining the relationship between math anxiety and WNB, there is research which examines math anxiety and math performance. Szczygiel et al. (2021) found that WM mediated the relationship between MA and math achievement. In other words, as MA increased, WM resources decreased and while WM decreased, math achievement decreased. The math achievement assessment was developed around core curriculum for elementary school children and math education. It consisted of questions regarding addition, subtraction, multiplication, and division as well as clock reading and knowledge of dates and money. Sidney et al. (2019) found that strategy variability mediated the relationship between MA and math performance, and WMC and math skills moderated the relationship between strategy variability and performance. In other words, as math anxiety increased, strategy variability decreased and as strategy variability decreased, math performance decreased. Also, in low math skills, for low WM participants, as strategy variability increased, performance increased but for high WM participants strategy variability did not affect performance. In High math skills, WM did not moderate performance (see Figure 5). This finding suggests high math skills participants were showing intuitive logic since mental resources (WM) had no effect on performance however low math skills participants performance was dependent on available mental resources (WM). Unfortunately, this study did not examine reaction time, consider congruency as a factor, or speculate on WNB.

Aim and Hypothesis

The overall goal of the present study was to show how numeracy and math anxiety affect WNB in adult participants while considering DI and Hybrid models of Dual Processing Theory (DPT). To investigate this relationship, participants were divided into two groups, low numeracy (LN) group and high numeracy (HN) group based on scores in the numeracy task. Participants were also split into two groups for MA (i.e., low MA and high MA). The current study manipulated commonality of components (common components versus without common components) and congruency (congruent versus incongruent). It was predicted that the individual differences innumeracy and math anxiety would influence how WNB is presented. A congruency effect for accuracy is present if accuracy is greater for congruent items compared to incongruent items. A congruency effect for reaction time is present if reaction times are shorter for congruent compared to incongruent. A WNB is present when there is a congruency effect because application of natural number rules results in better performance on congruent than incongruent items. No congruency effect would suggest that natural number intuitions were not applied; and therefore, no WNB. However, a reverse congruency effect would suggest reverse WNB, in which intuitions of rational number rules were applied which results in better performance on incongruent than congruent items.

According to the DI model, participants would start in Type I intuitive processing and use a componential strategy applying intuitions about natural numbers. Then if participants detect conflict, they will move to Type II processing in which they would use a holistic strategy by estimating or computing a magnitude of each fraction. The Hybrid model would predict participants would have two types of intuitive Type I processing: intuition based on natural number rules and intuition based on rational number rules (e.g., Van Hoof et al., 2020). Furthermore, findings in Sidney et al. (2019) support the notion that LN participants may operate in a process described by DI model since their performance was dependent on their WM (mental resources) whereas HN participants may operate in a process described by Hybrid model since their performance was not dependent on their WM (mental resources) and could have been using logical Type I processing (see Table 1). Also, according to Attentional Control Theory (ACT) anxiety impairs mental resources which suggests participants with HMA may have longer reaction times to maintain ACC if they are using Type II processing to complete the task. Anxiety may not have as great of an effect if participants are using Type I processing since Type I does not require the same mental resources as Type II.

It was predicted the LN group would use the process similar to what is predicted by DI. They would begin with Type I processing, relying on natural number intuitions and if they detect conflict, they will move to Type II processing to complete the items. Therefore, LN participants will exhibit a WNB in ACC and in RT because they rely on Type I processing for congruent items but would need to activate Type II thinking for incongruent items thus leading to longer RT and lower ACC in incongruent items.

It was predicted the HN group would use the process similar to what is predicted by Hybrid model. They would be able to complete all items using Type I processing using either their intuitions about natural numbers which facilitates performance on congruent items or rational numbers which facilitates performance on incongruent items thus leading to no WNB in ACC or RT (see Table 1).

Finally, it was predicted math anxiety would have a greater effect on LN participants than HN because LN are predicted to use a process similar to what is predicted by DI which would require the use of Type II thinking on incongruent items; therefore, it would result in a greater WNB in HMA because it would impair performance in LN participants resulting in lower ACC compared to their LMA counterparts because anxiety occupies mental resources. Math anxiety would not have as great an effect on HN because they are predicted to operate in a process similar to Hybrid model in which there is logical Type I processing; therefore, math anxiety would not affect performance in HN participants.

Summary of Hypothesis

Hypothesis 1

LN participants performance would be similar to what is predicted by DI model in that they would be able to operate in Type I processing until they detect conflict and will then move to Type II processing. Furthermore, math anxiety would impair performance in LN because they would activate Type II processing.

Hypothesis 2

HN participants performance would be similar to what is predicted by Hybrid model in that they would be able to operate in Type I processing for all types of items. Furthermore, math anxiety would not impair performance in HN because they would be operating in Type I processing for all items.

CHAPTER TWO

METHODS

Participants

Participants were one hundred and seventy-seven undergraduate students at a public University in California. Of the 177 participants, 50 were removed from the analyses based on performance in the Fraction Magnitude Comparison Task. Participants who did not respond to 20 or more items in the task were removed from the sample. Then means and standard deviations were computed for accuracy and reaction time for each condition in the Fraction Magnitude Comparison Task. Participants were then removed if they showed 0% accuracy on any condition. Finally, participants were removed from the sample if they had less than 25% total accuracy or were +/- 2.5 standard deviations (SD) in any condition for accuracy or reaction time. The final sample size for the current study was 127. Among the participants (M_{age} = 24.9, SD_{age} = 5.72; Female = 112), 52.0% were Seniors, 37.0% were Juniors, 7.9% were Sophomores, and 2.4% were Freshman. Also 76.4% were Hispanic or Latino, 8.7% were White, 3.9% were Black or African American, 3.2% were Asian, 1.6% American Indian or Alaska Native, 1.6% were Native Hawaiian or Pacific Island, and 3.9% responded other. Participants signed an informed consent approved by the Institutional Review Board and received extra credit in their selected psychology courses upon completion of the experiment.

Overall Design

The design was: 2 (Numeracy: high, low) x 2 (Math Anxiety: high, low) x 2 (Components: common, without common) x 2 (Congruency: congruent, incongruent). Participants were divided into separate groups based on their numeracy (low versus high) and math anxiety (low versus high) scores.

Materials and Procedure

All tasks and measurements were administered in Qualtrics on participants personal computers. The Qualtrics survey link was accessed through participants SONA account. Data was analyzed using SPSS v28.

Fraction Magnitude Comparison Task

This task was adopted from Morales et al. (2020) and consisted of 180 pairs of fractions in which congruency and commonality of components were manipulated. Common component fraction pairs were separated into congruent in which fractions share denominators (i.e., 33/65 versus 49/65) and incongruent items in which fractions share numerators (i.e., 25/96 versus 25/66). Without common component fraction pairs were separated into congruent, incongruent, and neutral. Without common component congruent items are when components (numerator and denominator) of one fraction are larger in numerosity and in magnitude than components (numerator and denominator) of the comparison fraction [i.e., 34/65 (0.523) versus 57/74 (0.770)]. Without common components incongruent items are when the components of one fraction is larger in numerosity but not larger in magnitude compared to the other fraction [i.e., 39/52

(0.75) versus 45/76 (0.592)]. Without common components neutral items are when both components of one fraction are not larger in numerosity compared to the comparison fraction [i.e., 18/49 (0.367) versus 12/59 (0.203)]. In other words, the numerator of the first fraction is larger than the numerator of the second fraction, but the denominator of the first fraction is smaller than the denominator of the second fraction. All fraction pairs met specific criteria: all denominators ranged from 31 to 99, all numerators ranged from 11 up to the corresponding denominator minus 11, and each fraction pair are on the same side of 1/2. Participants were presented one fraction pair at a time and used their mouse to indicate which fraction in a pair was larger in magnitude. The fraction pair remained on the screen until the participant responded or for 10 seconds. Accuracy was computed by taking the average correct responses for each condition. Reaction time was computed by taking the average page submit output for each condition.

Numeracy

This task was adopted from Bonato et al. (2007) and is a fraction knowledge assessment task which consisted of 10 operation and 10 magnitude comparison questions. For operation problems, participants were asked to solve an equation involving fractions and all answers were to be in fraction form (see Appendix A). For magnitude comparison problems, participants were presented with two fractions with an inequality symbol (>) between the two fractions and participants were asked to indicate if the inequality was true or false. Participants for this task indicated their response using their mouse and keyboard into the provided space and were allowed to use a paper and pencil to work out the problems if needed. Participants were asked to avoid outside resources, such as calculators, and rely solely on their knowledge to complete the task. There was no time constraint.

Abbreviated Math Anxiety Scale (AMAS)

This task was adopted from Hopko et al. (2003) and consisted of 9 items (see Appendix B). Responses were on a 5-point Likert scale in which 1 (low anxiety) and 5 (high anxiety); the larger the score, the more anxious they were. Participants used their mouse to indicate their response for each item.

Demographic Survey

Demographics were collected from participants. They were questions regarding age, gender, highest level of math reached, major, mental illness, learning disabilities, stress, motivation, and their testing environment.

General Procedure

Participants accessed the Qualtrics survey from their SONA account. Participants were screened for what type of device they were using and any participant attempting to complete the study on a mobile device was automatically blocked out of the survey. After signing the informed consent, participants were provided an overview of the tasks and given instructions on what kind of environment they should complete the study in. They were asked to be in a quiet area free from distractions (e.g., phones, kids, pets, etc.) but were allowed to have a paper and pencil to perform calculations if needed. Participants then completed the tasks in the following order: informed consent, AMAS, numeracy, fraction magnitude comparison task, and demographics. Once participants complete all tasks, they were provided a debriefing statement and redirected to SONA to receive credit. The entire study took approximately 60 minutes to complete.

CHAPTER THREE

RESULTS

Participants were divided into two groups in each of the two participant factors, numeracy (low versus high) and math anxiety scores (low versus high) independently; therefore, there were a total of four groups. The means and standard deviations for each group are in Table 2. Accuracy, response time (RT) and processing efficiency coefficient (computed by Accuracy/RT; Eysenck et al., 2007) were calculated in the Fraction Magnitude Comparison Task (FMCT).

Table 2. Numeracy and Math Anxiety group means and standard deviations.

Numeracy Group	MA Group	N	NUM Score Mean	NUM Score SD	MA Score Mean	MA Score SD
Low	Low	30	0.52	0.17	2.51	0.49
	High	37	0.51	0.18	3.77	0.43
High	Low	34	0.87	0.08	2.40	0.54
	High	26	0.84	0.07	3.68	0.39

Note. MA = math anxiety; NUM = numeracy; N = number of participants in each group

Accuracy

Mean accuracy data are shown in Table 3 and were submitted to a 2(Numeracy: low vs high) x 2(Math Anxiety: low vs high) x 2[Components: common (CC) vs. without common (WCC)] x 2(Congruency: congruent vs incongruent) mixed ANOVA. There was a main effect of numeracy, $F(1, 123)$ =

13.296, $p < 0.001$, $\eta_p^2 = 0.098$, showing HN participants had greater accuracy than LN participants. There was no main effect of math anxiety $F(1, 123) = 2.454$, $p = 0.120$, $\eta_p^2 = 0.02$. There was a main effect of components, $F(1, 123) = 128.896$, $p < 0.001$, $\eta_p^2 = 0.512$, which showed significantly higher accuracy in CC items than WCC items. A significant Components x Math Anxiety interaction was found, $F(1, 123) = 4.987$, $p = 0.027$, $\eta_p^2 = 0.039$, which showed higher ACC in CC than WCC, and the difference between CC and WCC was greater for LMA than HMA. There was a significant Numeracy x Congruency interaction, $F(1, 123) = 17.067$, $p < 0.001$, $\eta_p^2 = 0.122$, in which the LN group showed the congruency effect whereas the HN group showed the reverse congruency effect. There was a significant Components x Congruency interaction, $F(1, 123) = 78.238$, $p < 0.001$, $\eta_p^2 = 0.389$, in which there was the congruency effect in the CC items whereas the reverse congruency effect in the WCC items. A marginally significant Numeracy x Components interaction was found, $F(1, 123) = 3.531$, $p = 0.063$, $\eta_p^2 = 0.028$, which showed that there was significantly greater ACC on CC than WCC and the difference between CC and WCC was greater for HN than LN. Finally a marginally significant Math Anxiety x Congruency interaction was found, $F(1, 123) = 2.838$, $p = 0.095$, $\eta_p^2 = 0.023$, in which LMA showed a marginal reverse congruency effect and HMA did not. No other main effects and two-way interactions reached statistical significance.

Table 3. Mean accuracy rates and standard deviations for each condition in the Fraction Magnitude Comparison Task.

			CC				WCC			
			C		IC		C		IC	
Numeracy	MA	N	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Low	Low	30	0.81	0.21	0.68	0.33	0.61	0.18	0.63	0.32
	High	37	0.78	0.21	0.60	0.34	0.64	0.20	0.55	0.32
High	Low	34	0.90	0.17	0.90	0.19	0.56	0.20	0.88	0.17
	High	26	0.81	0.28	0.79	0.25	0.61	0.19	0.77	0.24

Note. MA = math anxiety; CC = Common Component; WCC = Without Common Component; C = Congruent; IC = Incongruent.

More important, there was a significant Numeracy x Components x Congruency interaction, $F(1, 123) = 9.432$, $p = 0.003$, $\eta_p^2 = 0.071$, as shown in Figure 6. LN participants were significantly more accurate on congruent trials than incongruent trials (congruency effect) in CC items, $t(66) = 4.52$, $p < 0.001$, however they did not display a significant difference between congruent and incongruent trials in WCC items, $t(66) = 0.67$, $p = 0.506$, (see Figure 6). HN participants did not display a significant difference between congruent and incongruent trials in CC items, $t(59) = 0.55$, $p = 0.583$; however, they had significantly better accuracy in incongruent trials than congruent trials (reverse congruency effect) in WCC items, $t(59) = 7.35$, $p < 0.011$, (see Figure 6).

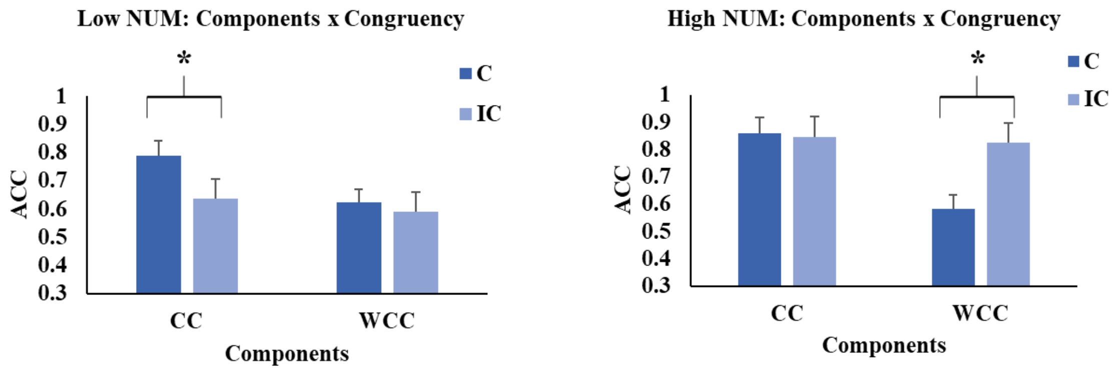

Figure 6: Accuracy Numeracy x Components x Congruency interaction.
* = p < .05; error bars are 95% CI

There was a significant Math Anxiety x Components x Congruency interaction, $F(1, 123) = 5.845$, $p = 0.017$, $\eta_p^2 = 0.045$, as shown in Figure 7. Low math anxiety (LMA) participants were significantly more accurate in congruent trials than incongruent trials (congruency effect) for CC items, $t(63) = 2.65$, $p = 0.010$, whereas they were significantly more accurate in incongruent trials than congruent trials (reverse congruency effect) for WCC items, $t(63) = 4.10$, $p < 0.001$. Also, in CC items for LMA LN participants there was a significant congruency effect $t(29) = 2.87$, $p = 0.008$, whereas LMA HN participants did not have a congruency effect $t(33) = 0.30$, $p = 0.769$. In WCC items, for LMA LN participants there was no congruency effect, $t(29) = 0.31$, $p = 0.758$, whereas LMA HN participants had a significant reverse congruency effect $t(33) = 8.44$, $p < 0.001$. HMA participants were significantly more accurate on congruent trials than incongruent trials (congruency effect) for CC items, $t(62) = 3.16$, $p = 0.002$ whereas in WCC items, there was no difference between congruent and incongruent trials, *t*(62) = 0.33, *p* = 0.745. The congruency effect for CC items, was not statistically different between the low and high MA groups, *t*(125) = 1.19, *p* = 0.237. Also, in WCC items for HMA LN participants there was a significant congruency effect *t*(36) = 3.49, *p* = 0.001, whereas LMA HN participants did not have a congruency effect *t*(25) = 0.47, *p* = 0.646. In WCC items, for HMA LN participants there was no congruency effect *t*(36) = 1.12, *p* = 0.268, whereas LMA HN participants had a significant reverse congruency effect *t*(25) = 2.78, *p* = 0.010.

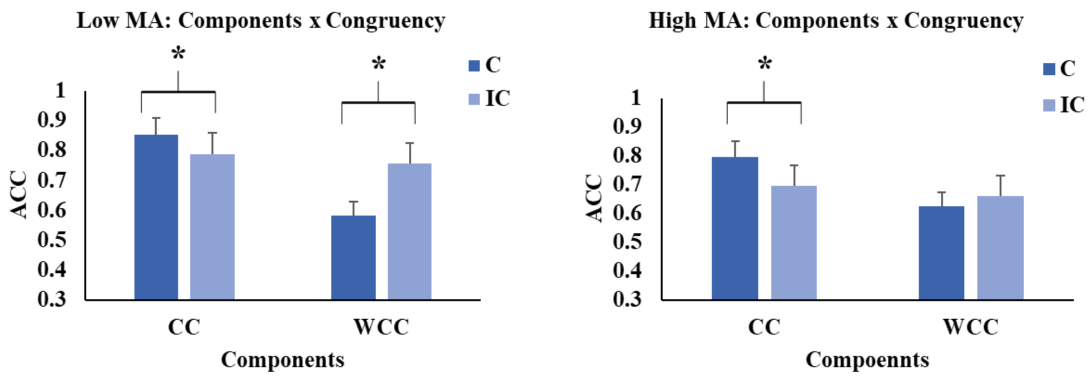

Figure 7: Accuracy Math Anxiety x Components x Congruency interaction.
* = p < .05; error bars are 95% CI

Reaction Time

Reaction time (RT) was computed by taking the average page submit data for each of the four conditions (means and standard deviations are shown in Table 4 and were submitted to a 2(Numeracy: low vs high) x 2(Math Anxiety: low vs high) x 2[Components: common (CC) vs. without common (WCC)]x

2(Congruency: congruent vs incongruent) mixed ANOVA. There was a main effect of numeracy, $F(1, 123) = 8.409$, $p = 0.004$, $\eta_p^2 = 0.064$ showing that HN participants took longer to respond than LN participants. There was also a main effect of math anxiety in RT, $F(1, 123) = 5.871$, $p = 0.017$, $\eta_p^2 = 0.046$, showing HMA participants responded faster than LMA participants. There was a main effect of components, $F(1, 123) = 55.888$, $p < 0.001$, $\eta_p^2 = 0.312$, which showed significantly longer RT in WCC items than CC items. Also a main effect of congruency was found, $F(1, 123) = 17.919$, $p < 0.001$, $\eta_p^2 = 0.127$, showing significantly longer RT in congruent trials than incongruent trials (reverse congruency effect). A Numeracy x Components interaction was significant, $F(1, 123) = 12.457$, $p < 0.001$, $\eta_p^2 = 0.092$, which showed significantly longer RT in WCC than CC and the difference was significantly greater for HN than LN. A Math Anxiety x Components interaction was significant, $F(1, 123) = 5.855$, $p = 0.017$, $\eta_p^2 = 0.045$, which showed significantly longer RT in WCC than CC and the difference between CC and WCC was greater for LMA than HMA. A Components x Congruency interaction was significant $F(1, 123) = 82.298$, $p < 0.001$, $\eta_p^2 = 0.401$, which showed that there was the congruency effect in CC but the reverse congruency effect in WCC. Finally, a marginally significant Numeracy x Math Anxiety x Congruency interaction was found, $F(1, 123) = 2.828$, $p = 0.095$, $\eta_p^2 = 0.022$. No other main effects and 2-way interactions reached statistical significance.

Table 4. Mean reaction times (RTs in msec) and standard deviations for each condition in the Fraction Magnitude Comparison Task.

			CC				WCC			
			C		IC		C		IC	
Numeracy	MA	N	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Low	Low	30	3832	1127	3963	1357	4253	1593	4000	1430
	High	37	3219	1179	3297	1155	3576	1617	3282	1305
High	Low	34	4029	1106	4199	1072	5251	1484	4527	1184
	High	26	3769	959	4121	1039	4567	1412	4006	1057

Note. MA = math anxiety; CC = Common Component; WCC = Without Common Component; C = Congruent; IC = Incongruent.

More important, there was a significant Numeracy x Components x Congruency interaction, $F(1, 123) = 13.822$, $p < 0.001$, $\eta_p^2 = 0.101$, as shown in Figure 8. LN participants had marginally longer RT in incongruent trials than congruent trials for CC items (congruency effect), $t(66) = 1.95$, $p = 0.055$ whereas they had significantly longer RT in congruent trials than incongruent trials for WCC items (reverse congruency effect), $t(66) = 4.07$, $p < 0.001$. HN participants had significantly longer RT in incongruent trials than congruent trials for common component items (congruency effect), $t(59) = 4.68$, $p < 0.001$, whereas in WCC items, they had significantly longer RT in congruent trials than incongruent trials (reverse congruency effect), $t(59) = 7.19$, $p < 0.001$. Finally, the congruency effect in CC items was marginally greater for HN participants than LN participants, $t(125) = 1.98$, $p = 0.050$. The reverse congruency effect in WCC items was significantly greater for HN participants than LN participants, $t(125) = 3.36$, $p = 0.001$.

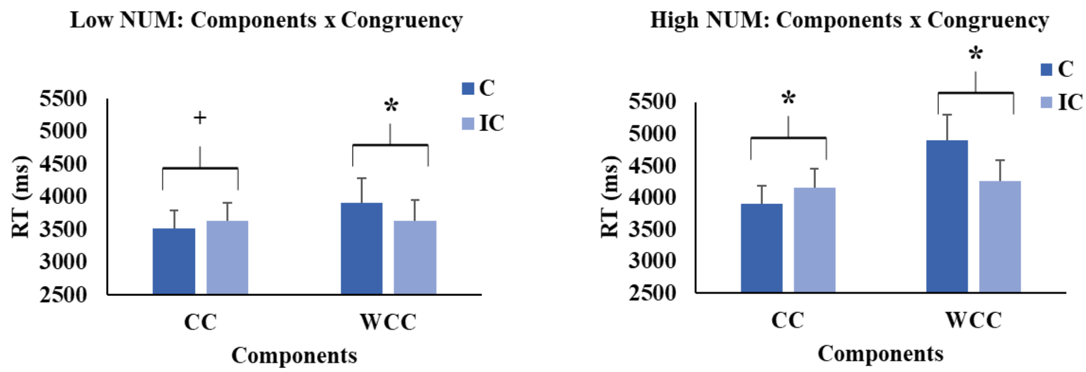

Figure 8: Reaction Time Numeracy x Components x Congruency interaction:
* = p < .05; + = p < .10; error bars are 95% CI

More important there was a marginal Numeracy x Math Anxiety x Components interaction, $F(1, 123) = 3.423$, $p = 0.067$, $\eta_p^2 = 0.027$, as shown in Figure 9. In CC items, math anxiety had an effect on LN participants, such that LMA had significantly longer RT than HMA, $t(65) = 2.20$, $p = 0.031$. However, in CC items, math anxiety did not have an effect on HN participants, $t(58) = 0.63$, $p = 0.532$. In CC items, numeracy did not have an effect on LMA participants, $t(62) = 0.75$, $p = 0.453$. However, in CC items, numeracy had an effect on HMA, such that LN had significantly shorter RT than HN, $t(61) = 2.47$, $p = 0.016$.

In WCC items, math anxiety had a marginal effect on LN participants such that LMA had marginally longer RT than HMA, $t(65) = 1.94$, $p = 0.057$. Also, math anxiety had a marginal effect on HN participants such that LMA had marginally longer RT than HMA, $t(58) = 1.84$, $p = 0.071$. For LMA, numeracy had an effect such that LN responded significantly faster than HN, t(62) =2.19, $p = 0.032$. Also for HMA, LN had significantly faster RT than HN, $t(61) = 2.49$, $p = 0.015$.

Differences were also found within each group between CC items and WCC items. In all four groups, RTs were significantly or marginally significantly shorter in CC than in WCC items, LN LMA: $t(29) = 2.37$, $p = 0.025$. LN HMA: $t(36) = 1.74$, $p = 0.090$. HN LMA: $t(33) = 7.08$, $p < 0.001$. HN HMA: $t(25) = 3.84$, $p < 0.001$.

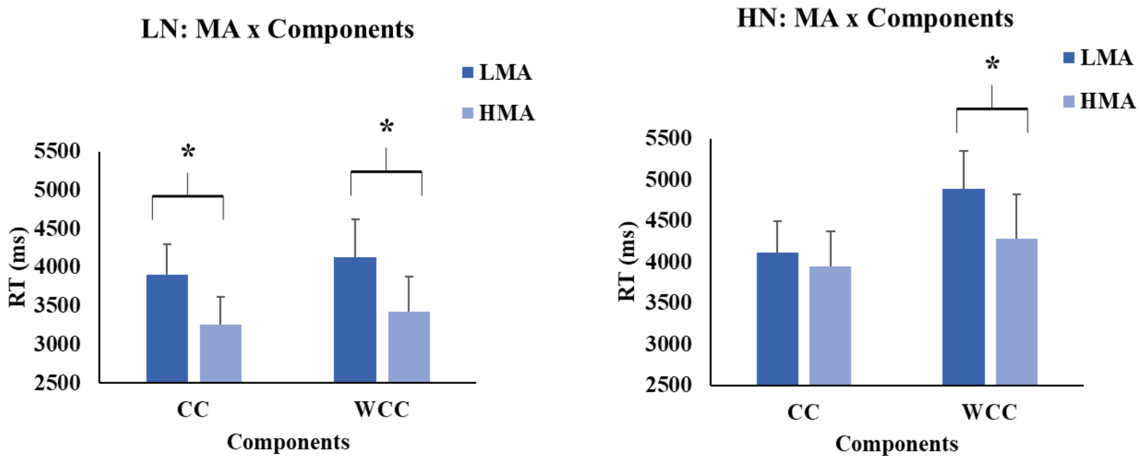

Figure 9: Reaction Time Numeracy x Math Anxiety x Components interaction: * = $p < .05$; + = $p < .10$; error bars are 95% CI

Processing Efficiency Coefficient

Processing Efficiency Coefficient (PEC) was computed by dividing accuracy by reaction time and is a measure of efficiency. An efficiency score is computed because effectiveness, measured by accuracy, may not measure effects of anxiety on cognition. In other words, if participants put enough effort into the task, there may be no difference between low and high anxiety groups regarding accuracy because one group may take longer to respond to maintain a higher accuracy score. Therefore, a composite score of accuracy and reaction time is computed to assess efficiency between groups. Means and standard deviations of PEC are shown in Table 5 and were submitted to a 2 (Numeracy: low vs high) x 2 (Math Anxiety: low vs high) x 2 [Components: common (CC) vs. without common (WCC)] x 2 (Congruency: congruent vs incongruent) mixed ANOVA. For PEC, there were no main effects of numeracy $F(1, 123) = 0.284$, $p = 0.595$, $\eta_p^2 = 0.002$, or math anxiety, $F(1, 123) = 0.592$, $p = 0.443$, $\eta_p^2 = 0.005$. There was a main effect of components, $F(1, 123) = 88.008$, $p < 0.001$, $\eta_p^2 = 0.417$, which showed participants were significantly more efficient in CC items than WCC items. There was a Numeracy x Components interaction, $F(1, 123) = 12.682$, $p < 0.001$, $\eta_p^2 = 0.093$, which showed higher efficiency in CC than WCC and the difference between CC and WCC was greater for HN than LN. There was also a Math Anxiety x Components interaction, $F(1, 123) = 4.114$, $p = 0.045$, $\eta_p^2 = 0.032$, which showed greater efficiency in CC than WCC and the difference between CC and WCC was greater for LMA than HMA. A Numeracy x Congruency interaction was found, $F(1, 123) = 14.762$, $p < 0.001$, $\eta_p^2 = 0.107$, which showed congruency effect for LN and a reverse congruency effect for HN. Finally a Components x Congruency interaction was found, $F(1, 123) = 48.213$, $p < 0.001$, $\eta_p^2 = 0.282$, which showed a congruency effect in CC but no congruency effect in WCC. No other main effects and two-way interactions reached statistical significance.

Table 5. Processing efficiency coefficient means and standard deviations for each condition in the Fraction Magnitude Comparison Task.

			CC				WCC			
			C		IC		C		IC	
Numeracy	MA	N	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Low	Low	30	0.23	0.10	0.18	0.08	0.19	0.15	0.17	0.08
	High	37	0.27	0.11	0.20	0.11	0.23	0.14	0.18	0.10
High	Low	34	0.25	0.09	0.23	0.09	0.11	0.06	0.21	0.07
	High	26	0.23	0.09	0.20	0.07	0.15	0.08	0.20	0.07

Note. CC = MA = math anxiety; Common Component; WCC = Without Common Component; C = Congruent; IC = Incongruent.

More important, there was a significant Numeracy x Components x Congruency interaction, $F(1, 123) = 15.394$, $p < 0.001$, $\eta_p^2 = 0.111$, as shown in Figure 10. LN participants were significantly more efficient in congruent trials than incongruent trials for CC items (congruency effect), $t(66) = 4.24$, $p < 0.001$, and marginally more efficient in congruent trials than incongruent trials for WCC items (congruency effect), $t(66) = 1.79$, $p = 0.078$. HN participants were significantly more efficient in CC congruent trials than CC incongruent trials (congruency effect), $t(59) = 2.87$, $p = 0.006$, however they were significantly more efficient in WCC incongruent trials than WCC congruent trials (reverse congruency effect), $t(59) = 6.78$, $p < 0.001$. The congruency effect in CC items was significantly greater in LN participants than HN participants, $t(125) = 2.55$, $p = 0.012$. In WCC items, HN participants showed a reverse congruency effect while LN participants showed a marginal congruency effect. In other words, the LN group showed the same pattern as the accuracy data in CC, and the HN group showed the same pattern as the accuracy data in WCC. However, the congruency effects in LN WCC and HN CC were driven by RT because there were no congruency effects in ACC for these items but there were congruency effects in RT.

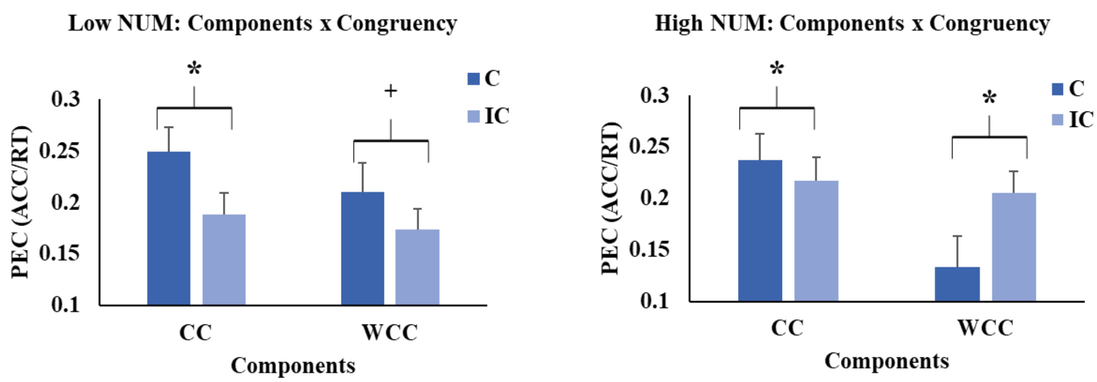

Figure 10: Processing Efficiency Coefficient Numeracy x Components x Congruency interaction:
* = p < .05; + = p < .10; error bars are 95% CI

More important there was a significant Numeracy x Math Anxiety x Components interaction, $F(1, 123) = 4.995$, $p = 0.027$, $\eta_p^2 = 0.039$, as shown in Figure 11. In CC items, math anxiety did not have an effect on LN participants, $t(65) = 1.37$, $p = 0.175$ or HN participants, $t(58) = 1.13$, $p = 0.264$. In CC items, numeracy had a marginal effect on LMA participants such that HN participants were marginally more efficient than LN participants, $t(62) = 1.71$, $p = 0.091$. In CC items, numeracy had no effect on HMA participants, $t(61) = 0.79$, $p = 0.433$. In WCC items, math anxiety did not have an effect on LN participants, $t(65)$ =

1.16, p = 0.250, or HN participants, $t(58)$ = 0.92, p = 0.362. In WCC items, numeracy did not have an effect on LMA participants, $t(62)$ = 0.96, p = 0.341, or HMA, $t(61)$ = 1.48, p = 0.144. In all four groups, participants showed higher PEC in CC than in WCC trials. LN LMA: $t(29)$ = 2.59, p = 0.015. LN HMA: $t(36)$ = 3.00, p = 0.005. HN LMA: $t(33)$ = 10.12, p < 0.001. HN HMA: $t(25)$ = 4.31, p < 0.001.

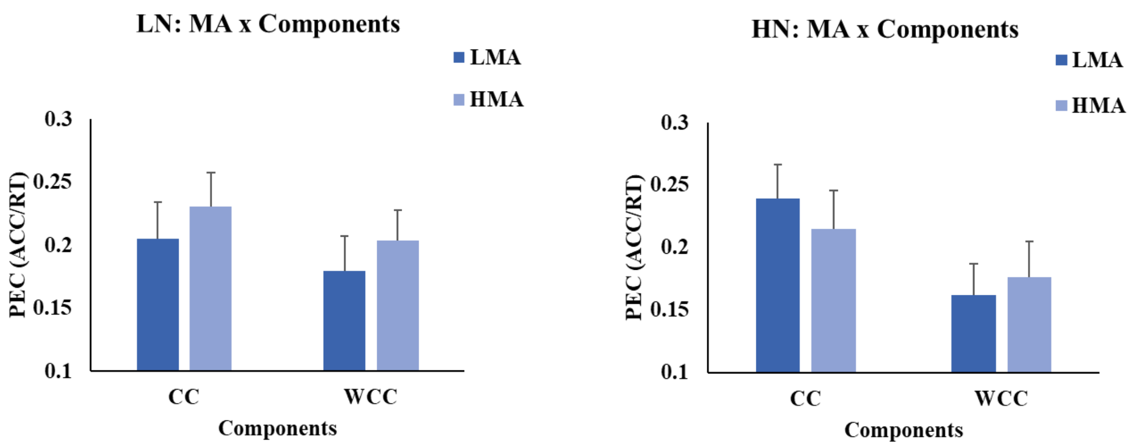

Figure 11: Processing Efficiency Coefficient Numeracy x Math Anxiety x Components interaction:
* = p < .05; + = p < .10; error bars are 95% CI

CHAPTER FOUR

DISCUSSION

The current study aimed to show how individual differences (numeracy and math anxiety) would influence the Whole number bias (WNB) in undergraduate participants. The current study also examined which model of Dual Processing Theory (DPT), the DI model or Hybrid model, would explain the WNB. In the fraction magnitude comparison task, commonality of components (CC vs WCC) and congruency (congruent vs incongruent) were manipulated. Application of natural number intuitions (larger in numerosity means larger in magnitude) would lead to correct responses in congruent but not incongruent items which also results in WNB (congruency effect). Application of rational number intuitions (larger in numerosity means smaller in magnitude) leads to the correct response in incongruent but not congruent items which also results in a reverse WNB (reverse congruency effect).

There are two strategies which are often employed in a fraction comparison task. The first, a componential strategy, is when participants compare the components (numerator and denominator) of a fraction to the comparison fractions' components without considering the magnitude of each fraction. This strategy requires little effort (Type I) and often leads to incorrect responses depending on which intuition (natural number or rational number) is used, since natural number intuitions facilitate performance on congruent items, whereas rational number intuitions facilitate performance on incongruent items.

Between the two intuitions, students typically acquire the natural number intuition first, and the rational number intuition might be obtained after some practice and experience in math. In other words, students with LN may possess only the natural number intuition, whereas students with HN may possess the rational number intuition as well. The second, a holistic strategy, is when participants compute or estimate the magnitude of each fraction and compare these magnitudes to determine which fraction is larger. Although this strategy is more reliable, it requires more effort and knowledge about fractions which every individual may not have.

DI model would predict participants would operate first in Type I processing relying only on natural number intuitions. Then, if conflict is detected, participants would move to Type II processing and use the holistic strategy to solve the problems. Hybrid model would predict participants would have two types of Type I intuitions: natural number and rational number, and these intuitions would be used to solve the items. LN participants would follow a similar process described by DI model whereas, HN participants would follow a similar process described by Hybrid model. Furthermore, math anxiety would have an effect on LN participants since they would activate Type II processing similar to what is predicted by DI model, but math anxiety would have no effect on HN participants because they would be operating in Type I processing similar to what is predicted by Hybrid model.

Effects of Numeracy on Components and Congruency

A 3-way interaction: Numeracy x Components x Congruency was found in ACC, RT, and PEC. First, the effects of numeracy were found in the accuracy data, as shown in Figure 6, that LN showed the congruency effect, whereas HN did not in CC items. However, HN showed the reverse congruency effect, but LN did not in WCC items.

In RT (see Figure 8), in CC items, both LN and HN participants showed congruency effects, however HN participants showed a greater magnitude of the congruency effect. In WCC items, both numeracy groups showed a reverse congruency effect, however the magnitude of the reverse congruency effect was greater for HN than LN participants.

In the processing efficiency (see Figure 10), both LN and HN participants exhibited a congruency effect in CC items, however the magnitude was greater for LN than HN participants. In WCC items, LN participants had a marginal congruency effect whereas, HN participants showed a reverse congruency effect.

These results showed that LN exhibited the congruency effect in CC, whereas HN exhibited the reverse congruency effect in WCC across the three measures. This suggests in CC items, LN participants relied on the natural number intuitions by using a componential strategy as predicted by the DI model, whereas HN relied on both natural number intuitions and rational number intuitions, by applying componential strategies as predicted by the Hybrid model which resulted in no WNB in ACC and but a WNB in RT. It appears HN participants applied their strong intuitions of natural numbers to CC congruent items and a secondary intuition about rational number in CC incongruent problems which resulted in longer RT in incongruent items, further supporting that they operated in a process similar to one described by Hybrid model. In WCC items, LN participants showed lower ACC and shorter RTs than HN participants, suggesting they guessed on these items because they either lacked the mindware to activate Type II processing or lacked motivation to complete the task. In WCC items, it appears HN participants applied rational number intuitions which resulted in a reverse WNB in both ACC and RT because rational number intuitions facilitate performance in incongruent but not congruent items. It is also possible HN participants detected conflict in WCC congruent items and tried to calculate the magnitudes, which resulted in significantly longer RT; however, they were not able to overcome their intuitions about rational numbers or activated Type II thinking. The PEC further supports the notion that LN participants utilized natural number intuitions (Type I processing) because they showed better efficiency in congruent than incongruent items for CC and WCC. For HN participants, in CC items, they had greater efficiency in congruent than incongruent items, but in WCC they had greater efficiency in incongruent than congruent items. This suggests in CC items, HN participants applied natural number intuitions resulting in greater efficiency in congruent than incongruent condition, whereas in WCC items, they applied rational number intuitions resulting in greater efficiency in incongruent than congruent items. These results seem to suggest that the LN group relied on the natural number intuition only, supporting the DI model, whereas the HN group used the natural number and rational number intuitions, supporting the hybrid model.

Effects of Math Anxiety on Components and Congruency

A 3-way interaction: Math Anxiety x Components x Congruency was found in ACC (see Figure 7). It revealed both LMA and HMA participants exhibited the same magnitude of congruency effects in CC items, whereas in WCC items, LMA participants exhibited a reverse congruency effect, and HMA did not show any effect. These findings suggest in CC items, MA did not influence the occurrence of WNB, and this could be due to the simplicity of these items, whereas WCC items are more complex and would require a greater understanding of rational number information. In other words, CC items, may facilitate Type I processing, whereas WCC items may facilitate Type II processing which made it difficult for HMA participants to do well. Also, these findings are similar to ones in ACC for numeracy, such that LMA are comparable to HN whereas HMA are comparable to LN. Further analysis revealed that for CC items for both LMA and HMA, LN participants exhibited a congruency effect and HN did not, therefore LN participants seem to have driven this effect. In WCC items further analysis revealed in LMA and HMA participants, HN exhibited a reverse congruency effect whereas LN had no effect, therefore the effect in LMA was driven by HN participants but the effect was not great enough in HMA participants. Therefore, for CC items it appears DI model would better explain results because these items facilitated the use of Type I processing and results were driven by LN participants who applied natural number intuitions leading to WNB for LMA and HMA. In WCC items, it appears Hybrid model would better explain the results because in LMA participants, there was a reverse WNB driven by HN participants who applied rational number intuitions, however HMA participants had overall low ACC suggesting they may have been avoidant of the task and guessed.

Effects of Numeracy and Math Anxiety on Components

Finally, a 3-way interaction: Numeracy x Math Anxiety x Components, was found in RT (see Figure 9) and PEC (see Figure 11). In the RT data (Figure 9) HMA group showed faster RTs than LMA group except for HN in CC items, which suggests that for HN participants, CC items were easy enough that anxiety would not affect performance as their accuracy rates were close to 90% (see Figure 6). This finding also suggests, in the other three conditions, that either HMA participants gave up on the task, and/or LMA participants tried harder to solve problems.

For PEC, LMA should show higher efficiency than HMA if anxiety consumes working memory resources, as suggested by Eysenck's ACT model. However, as shown in Figure 11, only HN in CC condition show a trend that LMA has higher efficiency than HMA. In all other conditions, there are trends in the other direction, that HMA tend to show higher efficiency than LMA, however they are not statistically significant. A significant effect was found in LMA between HN and LN in CC condition as shown in Figure 11 A. These data seem to suggest that PEC in the present study was driven by shorter RTs in the HMA and LN participants. As was discussed above, the HMA and LN participants tended to give up on tasks, whereas LMA and HN participants tended to try harder, taking longer time.

Conclusion

Overall, numeracy seems to have stronger effects than anxiety on the performance of the fraction comparison task. Also, the present study results seem to suggest that LN participants relied on the natural number intuitions, resulting in a WNB in CC items, supporting the DI model. They were not able to successfully activate Type II processing in WCC because they lacked the mindware or motivation which led to them guessing on these items also resulting in lower ACC and relatively shorter RTs. The data from HN participants, on the other hand, appear to support the Hybrid model because they did not exhibit a WNB in ACC for CC items, whereas they showed the reverse congruency effect in WCC items, suggesting that they have both natural number intuitions and rational number intuitions. However, it is unlikely HN participants activated Type II thinking in WCC congruent items because although their RTs were significantly longer, suggesting they detected conflict, their ACC in congruent items was relatively low implying they were not successful in overcoming their intuitions about rational numbers.

The effect of math anxiety on components and congruency revealed that anxiety did not influence how WNB would be presented in simple items (CC), however, as complexity increased (WCC items), anxiety had a greater effect. In other words, there was no difference in the presence of WNB in CC items between LMA and HMA participants, suggesting mental resources were not affected by anxiety, which might also imply that participants were not engaged in Type II processing. However, in WCC items, anxiety had an effect in LMA participants, who showed a reverse WNB, and further analysis revealed this finding was driven by HN participants; however, in HMA participants, for WCC items, there was no congruency effect suggesting that in HMA participants mental resources were occupied or participants were unmotivated or avoidant of the task which resulted in relatively low ACC.

In terms of RT, the effect of math anxiety seems to be greater for LN than HN because, in LN participants, RTs in CC and WCC items were significantly longer for LMA than HMA; however, in HN participants, math anxiety only had an effect on WCC items, such that LMA participants had significantly longer RT than HMA suggesting when numeracy is high, math anxiety only has an effect on more complex items. Finally, math anxiety did not significantly affect efficiency. This may have been due to HMA and LN participants who seemed to have given up on the task, whereas LMA and HN participants appeared to have tried harder.

Limitations and Future Directions

There were some limitations in the current study. First, data was collected online using Qualtrics which may make findings in RT less reliable than other forms of data collection. Second, the measure of PEC assumes that efficiency increases if participants can solve problems in a shorter period of time. However, current data shows that greater PEC may not reflect greater task efficiency, since some participants may have given up on task performance and responded quickly. In other words, PEC may not be a reliable measure of efficiency when levels of motivation vary across participants.

Future research should examine how participants reasoned during a fraction magnitude comparison task by asking them to write down how they would determine which fraction is larger on a set number of items. This can be used to better understand how participants reason about fraction information and may unveil possible misconceptions some participants have about rational numbers depending on their complexity and individual differences. Finally, further research examining different biases in rational number information may help instructors understand how misconceptions are developed and better aid them in helping students overcome these misconceptions during learning.

APPENDIX A

NUMERACY TASK

Please make sure you have a sheet of paper and a pen or pencil to complete the task.

Calculators or other electronic devices are not permitted to solve any of the questions. We are interested in your performance on the next task without the use of these devices.

There are two parts which will be randomly presented to you.

In one part, there are 10 questions, and you will be asked to indicate the answer to an equation. All answers for this part must be in fraction form and in simplest form. Any other form will be considered wrong.

In the other part, there are 10 questions you will be asked to indicate whether the inequality is true or false.

Operation Items:
1/4 + 3/8 =
3/5 – 1/3 =
3/10 + 5/6 =
5/8 – 3/4 =
3/4 X 1/6 =
25/3 X 7/10 =
1/2 ÷ 3/4 =
5/6 ÷ 21/3 =
(1/2)(2/3) + (5/6)(2/5) =
(1/4)(3 -3/5) =

Magnitude Comparison Items:
1/1 > 1/5 True – False
1/4 > 1/5 True – False
1/6 > 1/5 True – False
1/8 > 1/5 True – False
1/5 > 1 True – False
3/5 > 1 True – False
7/5 > 1 True – False
3/7 > 3/9 True – False
8/6 > 6/4 True – False
7/8 > 2/3 True - False

Adopted from Bonato et al. (2007)

APPENDIX B

THE ABBREVIATED MATH ANXIETY SCALE (AMAS)

Please rate each item below in terms of how anxious you would feel during the event specified.

1 = Low Anxiety, 2 = Some Anxiety, 3 = Moderate Anxiety, 4 = Quite a bit of Anxiety, 5 = High Anxiety

1. Having to use the tables in the back of a mathematics book.
2. Thinking about an upcoming math test one day before.
3. Watching a teacher work an algebraic equation on the blackboard.
4. Taking an examination in a mathematics course.
5. Being given a homework assignment of many difficult problems which is due the next class meeting.
6. Listening to a lecture in mathematics class.
7. Listening to another student explain a mathematics formula.
8. Being given a “pop” quiz in a mathematics class.
9. Starting a new chapter in a mathematics book.

Adopted from Hopko et al. (2003)

APPENDIX C

IRB APPROVAL

April 26, 2022

CSUSB INSTITUTIONAL REVIEW BOARD
Expedited Review
IRB-FY2022-293
Status: Approved

Hideya Koshino Jasmine Bonsel
Department of CSBS - Psychology
California State University, San Bernardino
5500 University Parkway
San Bernardino, California 92407

Dear Hideya Koshino Jasmine Bonsel:

Your application to use human subjects, titled "A Comparison Study" has been reviewed and approved by the Institutional Review Board (IRB). The informed consent document you submitted is the official version of your study and cannot be changed without prior IRB approval. A change in your informed consent (no matter how minor the change) requires re-submission of your protocol as amended using the Cayuse Human Ethics (IRB) system protocol change form.

Your IRB proposal (IRB-FY2022-293) is approved. You are permitted to collect information from **[270]** participants for **[2 SONA credits]**.
Your application is approved for one year from April 26, 2022 through --.

This approval notice does not replace any departmental or additional campus approvals which may be required including access to CSUSB campus facilities and affiliate campuses. Investigators should consider the changing COVID-19 circumstances based on current CDC, California Department of Public Health, and campus guidance and submit appropriate protocol modifications to the IRB as needed. CSUSB campus and affiliate health screenings should be completed for all campus human research related activities. Human research activities conducted at off-campus sites should follow CDC, California Department of Public Health, and local guidance. See CSUSB's COVID-19 Prevention Plan for more information regarding campus requirements.

If your study is closed to enrollment, the data has been de-identified, and you're only analyzing the data - you may close the study by submitting the closure form through the Cayuse Human Ethics (IRB) system. The Cayuse system automatically reminders you at 90, 60, and 30 days before the study is due for renewal or submission of your annual report (administrative check-in). The modification, renewal, study closure, and unanticipated/adverse event forms are located in the Cayuse system with instructions provided on the IRB Applications, Forms, and Submission Webpage. Failure to notify the IRB of the following requirements may result in disciplinary action. Please note a lapse in your approval may result in your not being able to use the data collected during the lapse in the application's approval period.

You are required to notify the IRB of the following as mandated by the Office of Human Research Protections (OHRP) federal regulations 45 CFR 46 and CSUSB IRB policy.

- **Ensure your CITI Human Subjects Training is kept up-to-date and current throughout the study for all investigators.**
- **Submit a protocol modification (change) if any changes (no matter how minor) are proposed in your study for review and approval by the IRB before being implemented in your study.**
- **Notify the IRB within 5 days of any unanticipated or adverse events are experienced by subjects during your research.**
- **Submit a study closure through the Cayuse IRB submission system once your study has ended.**

The CSUSB IRB has not evaluated your proposal for scientific merit, except to weigh the risk to the human participants and the aspects of the proposal related to potential risk and benefit. This approval notice does not replace any departmental or additional approvals which may be required. If you have any questions regarding the IRB decision, please contact Dr. Jacob Jones, Assistant Professor of Psychology. Dr. Jones can be reached by email at Jacob.Jones@csusb.edu. Please include your application approval identification number IRB-FY2022-293 in all correspondence.

Best of luck with your research.

Sincerely,

Nicole Dabbs

Nicole Dabbs, Ph.D., IRB Chair
CSUSB Institutional Review Board

ND/MG

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