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Kristin Marie Dillard

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STEINER SYSTEMS OF THE MATHIEU GROUP $M_{12}$

A Project
Presented to the
Faculty of
California State University,
San Bernardino

In Partial Fulfillment
of the Requirements for the Degree
Master of Arts
in
Mathematics

by
Kristin Marie Dillard
June 2000
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ABSTRACT

A Steiner system $T$ with parameters $(5,6,12)$ is a collection of 6-element sets, called hexads, of a 12-element set $Q$, such that any 5 of the 12 elements belong to exactly one hexad.

In this project we construct a graph whose vertices are the orbits of $S_{12}$ on $T \times T$, where $T$ is the set of all Steiner systems $S(5,6,12)$. Two vertices are joined if an orbit is taken into another under the action of a transposition. The number of hexads common to two Steiner systems are also given. We also prove that any two Steiner systems with parameters $(5,6,12)$ can intersect only in 0, 12, 24, 36, or 60 hexads.
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INTRODUCTION

The Mathieu group $M_{24}$ appears to occupy a unique position among the simple sporadic groups. Quintuply transitive on 24 objects, it contains the Mathieu group $M_{12}$, itself quintuply transitive on 12 objects. These together with the alternating groups constitute the complete list of quintuply transitive simple groups. Moreover, their point-stabilizing subgroups $M_{23}$ and $M_{11}$ are the only quadruply but not quintuply transitive groups other than the obvious ones. $M_{24}$ is involved in 20 of the 26 sporadic simple groups and played a dominant role in the discovery of the largest Fischer and Conway groups. The importance of $M_{24}$ in the search for a better understanding of the sporadic groups is evident. Many attempts have been made to find geometrical justification for the group, none of which have proved to be completely satisfactory. As a result of the immense importance of $M_{24}$, its geometrical structure needs to be thoroughly considered. This project is a step in that direction.

The Steiner system $S(5,6,12)$ is a collection of 6-element subsets, called hexads, of a 12-element set, $\Omega$, such that any 5 of the 12 elements belong to exactly one hexad.
There are \(12C5/6C5 = 132\) hexads in a Steiner system of this type. It has been shown by Witt (1938) that a Steiner system \(S(5,6,12)\) exists and is unique up to a relabelling of the 12 points. Thus any two Steiner systems with parameters \((5,6,12)\) are isomorphic in the sense that if \(S_A\) and \(S_B\) are two Steiner systems then there exists a permutation in \(S_{12}\), the symmetric group of degree 12, which maps the hexads of \(S_A\) onto the hexads of \(S_B\). Since the full automorphism group of \(M_{12}\), the Mathieu group on 12 elements, and the centralizer and normalizer of \(M_{12}\) in \(S_{12}\) are given by the identity and \(M_{12}\) respectively (Wielandt 1964), the total number of Steiner systems is

\[
[S_{12}:M_{12}] = 2^4 \cdot 3^2 \cdot 5 \cdot 7 \quad \text{(Isaacs 1976)}.
\]

There is a natural one-to-one correspondence between the orbits of \(M_{12}\) on \(T\), the set of Steiner systems with parameters \((5,6,12)\), and the double cosets \(M_{12} \times M_{12}\) in \(S_{12}\), since \(\{Sg \mid g \in M_{12} \times M_{12}, S \in T\}\) is an orbit of \(M_{12}\) on \(T\), and different double cosets give rise to different orbits of \(M_{12}\) on \(T\).
DEFINITIONS AND EXAMPLES

Orbit
Let G be a permutation group on Ω and let α ∈ Ω. Then

\[ \text{Orb}(G) = \{ g(\alpha) \mid g \in G \} \].

Example
Let k be a field, then \( S_n \) acts on \( k[x_1, \ldots, x_n] \), by \( g^\sigma = g^\sigma \),

where \( g^\sigma (x_1, \ldots, x_n) = (x_{\sigma_1}, \ldots, x_{\sigma_n}) \). Then let \( X = k[x_1, \ldots, x_n] \)

and \( G = S_n \). If \( g \in k[x_1, \ldots, x_n] \), then \( \text{Orb}(g) \) is the set of

distinct polynomials of the form \( g^\sigma \).

Quadratic residues
Let p be an odd prime and \( \gcd(a, p) = 1 \). If \( x^2 \equiv a \pmod{p} \) has a solution, then \( a \) is said to be a quadratic residue of \( p \).

Transitive
Let G be a permutation group on the set Ω. G is transitive on Ω iff Ω is the only orbit of G, that is for all \( x, y \in G \) there exists a \( \sigma \in G \) such that \( \sigma x = y \).

k-transitive
Let G be a permutation group on Ω. G is said to be k-transitive on Ω iff for every pair of k-tuples having distinct entries in G, say \( (x_1, \ldots, x_k) \) and \( (y_1, \ldots, y_k) \), there exists a \( \sigma \in G \) with \( \sigma x_i = y_i \) for \( i = 1, \ldots, k \).
Example

Let $X = \{1, \ldots, k\}$ and $G = S_k$. Clearly, $X$ is a $k$-transitive $S_k$-set.

Mathieu Group $M_{12}$

The Mathieu group $M_{12}$ is a quintuply transitive permutation group on $\Omega$ which has order $12 \cdot 11 \cdot 10 \cdot 9 \cdot 8$ and preserves the hexads of $S(5, 6, 12)$.

THEOREMS

Theorem 1

There exists a sharply 5-transitive group $M_{12}$ of degree 12 and order $95,040 = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 = 2^6 \cdot 3^3 \cdot 5 \cdot 11$ such that the stabilizer of a point is $M_{11}$, the Mathieu group on 11 elements (Rotman 1994).

Theorem 2

The Mathieu groups $M_{11}$ and $M_{12}$ are simple (Rotman 1984).

DEVICE FOR CONSTRUCTING HEXADS

We obtain $M_{12}$ as a permutation group acting on the 12 points of the projective line

$\text{PG}(1, 11) = \{\infty, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, X\}$ (it is convenient to write $X$ for 10, 0 for 11 and $\infty$ for 12) as
follows.

Let $Q = \{0, 1, 3, 4, 5, 9\}$ be the subset of quadratic residues together with 0, and $L$ be the linear fractional group that consists of all permutations of the form

$$y \rightarrow \frac{ay + b}{cy + d}, \quad ad - bc = 1.$$ 

Note:

$$L = \langle \alpha, \gamma; \alpha: y \rightarrow y + 1, \gamma: y \rightarrow -1/y = -y^{-1} \rangle$$

$= \text{PSL}_2(11)$, where

$$\alpha: y \rightarrow y + 1 = (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, X)(\infty)$$ and

$$\gamma: y \rightarrow -1/y = -y^{-1} = (0\infty)(1X)(25)(37)(48)(69).$$

Now $S = \{Q^x; x \in L\}$ consists of 132 subsets of size 6, called hexads, which form the Steiner system $S(5, 6, 12)$. We are now able define:

$$M_{12} = \langle g \in S_{12}; S^g = S \rangle,$$ therefore, the permutations of $M_{12}$ preserve the Steiner system $S(5, 6, 12)$ and it turns out that $M_{12}$ is generated by $\alpha, \beta, \gamma, \delta$ where $\beta = (\infty)(0)(13954)(267X8)$ and $\delta = (\infty)(0)(1)(2X)(34)(59)(67)(8)$ (Atkinson, 1984).

There is a simple device produced by R.T. Curtis (see Atkinson 1984) in which a hexad containing a given five points can be produced. The basis for the construction of this device is as follows. We start with any three points.
which together with the set \( \{0, 1\} \) form a hexad. It turns out that these hexads will form the Steiner system \( S(2,3,9) \) which turns out to be the affine plane. Since there is only one such plane, its lines are determined to be the rows, columns and generalized diagonals shown in Fig. 1.

\[
\begin{array}{ccc}
  x & x & x \\
  \circ & \circ & \circ \\
  \circ & \circ & \circ \\
\end{array}
\]

(a)

\[
\begin{array}{ccc}
  x & \circ & \circ \\
  \circ & x & \circ \\
  \circ & \circ & x \\
\end{array}
\]

(b)

\[
\begin{array}{ccc}
  x & \circ & \circ \\
  \circ & \circ & x \\
  \circ & x & \circ \\
\end{array}
\]

(c)

\[
\begin{array}{ccc}
  \circ & \circ & x \\
  \circ & x & \circ \\
  \circ & \circ & x \\
\end{array}
\]

(d)

Fig. 1

There are \( 9C2/3C2 = 12 \) such lines whose rows, columns, and diagonal are considered to be perpendicular. The diagonals in (c) and (d) are considered to be perpendicular as well. The union of two perpendicular lines are known as crosses. There are \( 12 \cdot 3/2 = 18 \) of these crosses as shown in Fig. 2.
The compliment of a cross is known to be a square as shown in Fig. 3.
We continue by taking three points-at-∞ together with the array of points in the plane corresponding to each. A more detailed description of this device can be found in Atkinson (1984). We show this correspondence in Fig. 4.

\[
\begin{array}{ccc}
6 & X & 3 \\
2 & 7 & 4 \\
5 & 9 & 8 \\
\end{array}
\]

∞-picture

\[
\begin{array}{ccc}
5 & 7 & 3 \\
6 & 9 & 4 \\
2 & X & 8 \\
\end{array}
\]

0-picture

\[
\begin{array}{ccc}
5 & 7 & 3 \\
9 & 4 & 6 \\
8 & 2 & X \\
\end{array}
\]

1-picture

Fig. 4

The hexads are then determined as follows:

1. \{∞,0,1\} ∪ \{any line\}

2. the union of two parallel lines

3. a point-at-∞ together with a cross in the corresponding picture

4. two points-at-∞ and a square in the picture that corresponds to the omitted point-at-∞.

Now let's consider some examples.

1. To find the hexad containing \{0,1,3,5,6\} we look at the ∞-picture and find that 8 completes the square. Therefore, \{0,1,3,5,6,8\} is the desired hexad.

2. To find the hexad containing \{∞,2,3,5,6\} we look at the ∞-picture and determine that X completes the
cross. Thus, the hexad containing \( \{\infty, 2, 3, 5, 6\} \) is 
\( \{\infty, 2, 3, 5, 6, X\} \).

3. To find the hexad containing the points \( \{3, 4, 5, 6, 7\} \) we look at the 1-picture, finding that 9 completes the union of two parallel lines. Thus, the desired hexad is \( \{3, 4, 5, 6, 7, 9\} \).

The complete list of 132 hexads can be found in appendix A.

**DOUBLE COSET DECOMPOSITION OF \( S_{12} \) OVER \( M_{12} \)**

There are 8 distinct double cosets of \( S_{12} \) over \( M_{12} \), which we will describe below. Let \( M = M_{12} \) and \( M \times M = \{Mxm \mid m \in M\} = \{Mmm^{-1}xm \mid m \in M\} = \{Mx^m \mid m \in M\} \).

1. \( MeM = M \), where \( e \) is the identity of \( S_{12} \).

2. \( MxM \), where \( x \) is a transposition. There is only one double coset of this type since \( M \) is quintuply transitive on \( \Omega \). This orbit consists of \( 12 \times 11/2 = 66 \) single cosets.

3. \( MxM \), where \( x \) is the product of two disjoint transpositions. Again, \( M \) is quintuply transitive on \( \Omega \) and there is only one orbit of this type and this double coset consists of

\( (((12 \times 11/2) \times (10 \times 9/2))/2)/3 = 495 \) single cosets.

4. \( MxM \), where \( x \) is a 3-cycle. In this case 5-
transitivity holds as well. There is only one orbit containing 3-cycles and it contains 
\((12 \times 11 \times 10)/3 = 440\) single cosets.

5. \textit{MxM}, where \(x\) is the product of a 3-cycle and a disjoint transposition. This will be the only orbit of this type as well since \(M\) is 5-transitive. This double coset consists of
\(((12 \times 11/2) \times (10 \times 9 \times 8/3))/12 = 1320\) single cosets.

6. \textit{MxM}, where \(x\) is a 4-cycle. 5-Transitivity applies here as well. Here we have \((12 \times 11 \times 10 \times 9/4)/3 = 990\) single cosets in the orbit.

7. \textit{MxM}, where \(x\) is a 5-cycle. Again, \(M\) is 5-transitive and there is only one double coset of this type. It contains \((12 \times 11 \times 10 \times 9 \times 8/5)/12 = 1584\) single cosets.

8. \textit{MxM}, where \(x\) is a 6-cycle. Since \(M\) is only quintuply transitive on the 12-element set \(Q\), there are 3 orbits of this type namely 1320, 990, and 144 orbits. There are 36 6-cycles in each single coset of the 1320 orbit and 48 6-cycles in each single coset of the 990 orbit and 110 6-cycles in each single coset in the 144 orbit. This calculation is as follows:
\[((12 \times 11 \times 10 \times 9 \times 8 \times 7)/6 -47520-47520))/110 = 144.\)
Upon further investigation, we also note, for example, that 24 elements of the type \( M(1234)(57)M \) and 96 elements of the type \( M(1\infty)(37)(5X8)M \) are contained in each single coset of the 495 orbit. There are 54 elements of the types \( M(23)(5978)M \) and \( M(273)(45)(6\infty)M \) in each single coset of the 440 orbit. 144 elements of the type \( M(16490)(37)M \) reside in each single coset of the 1320 orbit and 192 elements of the type \( M(16490)(23)M \) are located in each single coset of the 990 orbit. In the 1584 orbit there are 30 elements of the type \( M(1\infty)(5X98)M \) and 60 elements of the type \( M(172)(38)(9X)M \) in each single coset. Finally, in the 144 orbit there are 132 elements of the type \( M(1X)(3\infty794)M \) contained in each single coset. Thus, continuing in this manner, every permutation of \( S_{12} \) can be accounted for.

**Graph of \( S_{12} \) on \( T \times T \)**

Since \( S_{12} \) acts transitively on the set of all Steiner systems \( T \), in order to study the action of \( S_{12} \) on cosets of \( M_{12} \) in \( S_{12} \) the ideal would be to study a graph of \( S_{12} \) on \( T \times T \). For any \( x \in S_{12} \), we have \( MxM = \{Mxm \mid m \in M\} = \{Mm^{-1}xm \mid m \in M\} = \{M^{-1}xm \mid m \in M\} = \{Mx^m \mid m \in M\} \). For this reason the orbits of \( S_{12} \) on \( T \times T \) are in one-to-one correspondence
with the orbits of $M$, the stabilizer of $S \in T$, on $T$. The number of orbits of $M$ on $T$ is given by the length of permutation character $1_m \uparrow S_{12}$ (the identity of $M$ induced on $S_{12}$), which turns out to be 8. We also note that two Steiner systems $S_A$ and $S_B$ intersecting $S$ in different numbers $N_1, N_2$, respectively, of hexads, cannot lie in the same orbit of $M$ on $T$, since, if $S_A$ and $S_B$ are in the same orbit of $M$ on $T$ and we let $S \cap S_A = A$, and $S \cap S_B = B$, where $N_1 \neq N_2$, and $A$ and $B$ contain $N_1$ and $N_2$ hexads respectively, then there exists an $m \in M$ such that $(S_A)^m = S_B$ and $(S \cap S_A)^m = A^m$, which implies $S^m \cap (S_A)^m = A^m$, which implies $S \cap S_B = A^m$, but $|A^m| = |A| = N_1$.

Fig. 5 contains the complete graph of $S_{12}$ on $T \times T$. 

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Fig 5.
We fix the Steiner system $S(5,6,12)$, call it $S$ and represent it as 1. We can go from one orbit into another by the action of a transposition and there are $(12 \times 11)/2 = 66$ transpositions in $S_{12}$. Thus, each of these transpositions takes $S$ to $S^x$, where $x \in S_{12}$. The number of Steiner systems in the orbit $MxM$, where $x$ is a transposition is 66. It turns out that $S \cap S^x = 60$. Thus, every Steiner system $S^x$ of the orbit $MxM$ must intersect $S$ in exactly 60 hexads.

The number of single cosets in the double coset $MxM$, where $x$ is the product of two disjoint transpositions turns out to be $(((12 \times 11)/2) \times (10 \times 9/2))/2)/3 = 495$ since $MxM = \{M^m | m \in M\}$ and $Mx = My = Mz$ for exactly three different products $x, y, z$, of two transpositions. For example, in this case, if $x = (12)(34)$, $y = (57)(X\infty)$, and $z = (60)(89)$ where $xy \in M$, $xz \in M$, and $yz \in M$, then $M(12)(34) = M(57)(X\infty) = M(60)(89)$. There are 36 hexads common to the Steiner systems of this orbit and the original fixed Steiner system $S(5,6,12)$.

For the orbit $MxM$ where $x$ is a 3-cycle, there are exactly $(12 \times 11 \times 10)/3 = 440$ single cosets in the double coset and $S$ intersects the Steiner systems of this orbit in 24 hexads. The orbit $MxM$, where $x$ is the product of a transposition and a 3-cycle (both disjoint) contains
\[(12 \times 11/2) \times (10 \times 9 \times 8/3))/12 = 1320\] Steiner systems. Here, \(M \times M = \{M \times m \mid m \in M\}\) and each single coset contains precisely 12 unique disjoint products of a transposition and a 3-cycle. A list of these products can be found in appendix A. The number of hexads in the intersection of the Steiner systems of this double coset and \(S\) is 24.

The orbit \(M \times M\), where \(x\) is a 4-cycle has
\[
(12 \times 11 \times 10 \times 9/4)/3 = 990\] Steiner systems. In this case \(M \times M = \{M \times m \mid m \in M\}\), where there are exactly 3 different 4-cycles in each single coset of this double coset. This list is also included in appendix A. The number of hexads in the intersection of this Steiner system and \(S\) is 12.

There are \((12 \times 11 \times 10 \times 9 \times 8/5)/12 = 1584\) single cosets in the double coset \(M \times M\), where \(x\) is a 5-cycle. \(M \times M = \{M \times m \mid m \in M\}\) and in this case we have precisely 12 different 5-cycles in each single coset, which are also contained in appendix A. The Steiner systems of this orbit contain 12 hexads in common to the Steiner system \(S(5,6,12)\).

Finally, the double coset \(M \times M\), where \(x\) is a 6-cycle contains 144 single cosets. The calculation for the number of single cosets in this orbit is more difficult since 5-transitivity no longer holds. We start in the usual manner with \((12 \times 11 \times 10 \times 9 \times 8 \times 7)/6 = 110,880\). Next we note that
there are 48 6-cycles living in the 990 orbit and 36 6-cycles living in the 1320 orbit which gives a total of 95,040 6-cycles that we must subtract from 110,880 leaving us with 15,840. Now, \( M \times M = \{ M^m | m \in M \} \) and there exactly 110 distinct 6-cycles in each single coset leaving us with \( 15,840/110 = 144 \). This orbit does not contain any hexads in the intersection of its Steiner systems with \( S \).

**DESCRIPTION OF THE GRAPH**

The numbers in the ovals shown in Fig. 5 represent the number of Steiner systems in the corresponding orbits. The numbers 1, 20 and 45 near the 66 orbit imply that one of the 66 transpositions takes a Steiner systems of the 66 orbit back to the fixed Steiner system \( S \), 20 of the remaining transpositions take one of the 66 Steiner systems into the 440 orbit, and the remaining 45 transpositions take one of the 66 Steiner systems into the 495 orbit. The bracketed numbers represent the number of hexads in common with the original fixed Steiner system. The transpositions near the edges of the graph represent the action that moves one orbit into another. Finally, the cosets located to the left or right of each orbit are the coset representatives for each orbit.
APPENDIX A

THE HEXADS OF THE STEINER SYSTEM S(5,6,12)

\{∞,0,1\} ∪ \{any line\}: 12 such

\{∞,0,1,3,6,X\}, \{∞,0,1,2,4,7\}, \{∞,0,1,5,8,9\},
\{∞,0,1,3,5,7\}, \{∞,0,1,4,6,9\}, \{∞,0,1,2,8,X\},
\{∞,0,1,2,5,6\}, \{∞,0,1,7,9,X\}, \{∞,0,1,3,4,8\},
\{∞,0,1,6,7,8\}, \{∞,0,1,4,5,X\}, \{∞,0,1,2,3,9\}.

The union of two parallel lines: 12 such

\{2,3,4,6,7,X\}, \{3,5,6,8,9,X\}, \{2,4,5,7,8,9\},
\{3,4,5,6,7,9\}, \{2,3,5,7,8,X\}, \{2,4,6,8,9,X\},
\{2,5,6,7,9,X\}, \{2,3,4,5,6,8\}, \{3,4,7,8,9,X\},
\{4,5,6,7,8,X\}, \{2,3,6,7,8,9\}, \{2,3,4,5,9,X\}.

A point-at-∞ together with a cross in the corresponding picture: 3 × 18 = 54 such

\{∞,2,3,5,6,X\}, \{∞,3,6,7,9,X\}, \{∞,3,4,6,8,X\},
\{∞,2,6,7,8,X\}, \{∞,4,6,7,8,9\}, \{∞,2,4,5,6,7\},
\{∞,2,4,7,9,X\}, \{∞,2,3,4,7,8\}, \{∞,3,4,5,7,X\},
\{∞,2,3,5,7,9\}, \{∞,2,5,6,8,9\}, \{∞,5,7,8,9,X\},
\{∞,3,4,5,8,9\}, \{∞,3,5,6,7,8\}, \{∞,4,5,6,9,X\},
\{∞,2,3,4,6,9\}, \{∞,2,3,8,9,X\}, \{∞,2,4,5,8,X\},
\{0,2,3,5,6,7\}, \{0,3,5,7,9,X\}, \{0,3,4,5,7,8\},
\{0,5,6,7,8,9\}, \{0,4,5,8,9,X\}, \{0,2,4,5,6,9\},
\{0,4,6,7,9,X\}, \{0,3,4,6,8,9\}, \{0,2,3,4,7,9\},
\{0,2,3,6,9,X\}, \{0,2,5,6,8,X\}, \{0,2,7,8,9,X\},
\{0,2,3,4,8,X\}, \{0,2,3,5,8,9\}, \{0,2,4,5,7,X\},
\{0,2,3,6,9,X\}, \{0,2,7,8,9,X\}, \{0,2,6,7,9,X\}. 

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\{0,3,4,5,6,X\}, \{0,3,6,7,8,X\}, \{0,2,4,6,7,8\},
\{1,3,5,7,8,9\}, \{1,2,3,4,5,7\}, \{1,3,5,6,7,X\},
\{1,4,5,7,9,X\}, \{1,2,4,5,6,X\}, \{1,4,5,6,8,9\},
\{1,2,4,6,7,9\}, \{1,3,4,6,9,X\}, \{1,3,4,6,7,8\},
\{1,2,3,4,8,9\}, \{1,2,5,8,9,X\}, \{1,2,4,7,8,X\},
\{1,2,3,6,8,X\}, \{1,3,4,5,8,X\}, \{1,2,5,6,7,8\},
\{1,2,3,5,6,9\}, \{1,2,3,7,9,X\}, \{1,6,7,8,9,X\}.

Two points-at-\(\infty\) and a square in the picture corresponding to
the omitted point-at-\(\infty\):

\{\infty,0,2,4,6,X\}, \{\infty,0,6,8,9,X\}, \{\infty,0,2,4,8,9\},
\{\infty,0,2,3,6,8\}, \{\infty,0,3,7,8,9\}, \{\infty,0,2,3,7,X\},
\{\infty,0,3,5,8,X\}, \{\infty,0,2,5,7,8\}, \{\infty,0,2,5,9,X\},
\{\infty,0,5,6,7,X\}, \{\infty,0,2,6,7,9\}, \{\infty,0,3,4,9,X\},
\{\infty,0,4,7,8,X\}, \{\infty,0,4,5,6,8\}, \{\infty,0,2,3,4,5\},
\{\infty,0,3,4,6,7\}, \{\infty,0,3,5,6,9\}, \{\infty,0,4,5,7,9\},
\{\infty,1,4,8,9,X\}, \{\infty,1,2,4,6,8\}, \{\infty,1,2,6,9,X\},
\{\infty,1,2,3,4,X\}, \{\infty,1,2,3,6,7\}, \{\infty,1,3,7,8,X\},
\{\infty,1,2,3,5,8\}, \{\infty,1,2,5,7,X\}, \{\infty,1,5,6,8,X\},
\{\infty,1,4,5,7,8\}, \{\infty,1,4,6,7,X\}, \{\infty,1,3,6,8,9\},
\{\infty,1,2,7,8,9\}, \{\infty,1,2,4,5,9\}, \{\infty,1,3,5,9,X\},
\{\infty,1,3,4,7,9\}, \{\infty,1,3,4,5,6\}, \{\infty,1,5,6,7,9\},
\{0,1,4,7,8,9\}, \{0,1,2,4,5,8\}, \{0,1,2,5,7,9\},
\{0,1,3,4,5,9\}, \{0,1,2,3,5,X\}, \{0,1,3,8,9,X\},
\{0,1,3,5,6,8\}, \{0,1,5,6,9,X\}, \{0,1,2,6,8,9\},
\{0,1,4,6,8,X\}, \{0,1,2,4,9,X\}, \{0,1,2,3,7,8\},
\{0, 1, 5, 7, 8, X\}, \{0, 1, 4, 5, 6, 7\}, \{0, 1, 3, 6, 7, 9\},
\{0, 1, 3, 4, 7, X\}, \{0, 1, 2, 3, 4, 6\}, \{0, 1, 2, 6, 7, X\}.
APPENDIX B

TRANSPOSITIONS

Transpositions taking the 1 orbit to the 66 orbit:

(12), (13), (14), (15), (16), (17), (18), (19), (1X),
(10), (1∞), (23), (24), (25), (26), (27), (28), (29),
(2X), (20), (2∞), (34), (35), (36), (37), (38), (39),
(3X), (30), (3∞), (45), (46), (47), (48), (49), (4X),
(40), (4∞), (51), (56), (57), (58), (59), (5X), (50),
(5∞), (67), (68), (69), (6X), (60), (6∞), (78), (79),
(7X), (70), (7∞), (89), (8X), (80), (8∞), (9X), (90),
(9∞), (X0), (0∞).

Transpositions taking the 66 orbit to the 1 orbit:

(12).

Transpositions taking the 66 orbit to the 495 orbit:

(34), (35), (36), (37), (38), (39), (3X), (30), (3∞),
(45), (46), (47), (48), (49), (4X), (40), (4∞), (56),
(57), (58), (59), (5X), (50), (5∞), (67), (68), (69),
(6X), (60), (6∞), (78), (79), (7X), (70), (7∞), (89),
(8X), (80), (8∞), (9X), (90), (9∞), (X0), (X∞), (0∞).

Transpositions taking the 495 orbit to the 66 orbit:

(12), (34), (57), (X∞), (60), (89).

Transpositions taking the 66 orbit to the 440 orbit:

(13), (14), (15), (16), (17), (18), (19), (1X), (10),
(1∞), (23), (24), (25), (26), (27), (28), (29), (2X),
(20), (2∞).
Transpositions taking the 440 orbit to the 66 orbit:
(12), (13), (2,3).

Transpositions taking the 440 orbit to the 990 orbit:
(14), (15), (16), (17), (18), (19), (1X), (10), (1∞),
(24), (25), (26), (27), (28), (29), (2X), (20), (2∞),
(34), (35), (36), (37), (38), (39), (3X), (30), (3∞).

Transpositions taking the 990 orbit to the 440 orbit:
(12), (14), (23), (34), (58), (59), (6X), (6∞), (78),
(79), (X0), (0∞).

Transpositions taking the 440 orbit to the 1320 orbit:
(45), (46), (47), (48), (49), (4X), (40), (4∞), (56),
(57), (58), (59), (5X), (50), (5∞), (67), (68), (69),
(6X), (60), (6∞), (78), (79), (7X), (70), (7∞), (89),
(8X), (80), (8∞), (9X), (90), (9∞), (X0), (X∞), (0∞).

Transpositions taking the 1320 orbit to the 440 orbit:
(16), (1∞), (28), (20), (39), (3X), (45), (47), (57),
(6∞), (80), (9X).

Transpositions taking the 495 orbit to the 1320 orbit:
(15), (16), (17), (18), (19), (10), (1∞), (25), (26),
(27), (28), (29), (2X), (20), (2∞), (36), (37), (38),
(39), (3X), (30), (3∞), (45), (46), (47), (48), (49),
(4X), (40), (4∞), (56), (58), (59), (50), (67), (6X),
(6∞), (78), (79), (70), (8X), (8∞), (9∞), (X0), (0∞).

Transpositions taking the 1320 orbit to the 495 orbit:
(12), (13), (17), (23), (27), (37), (49), (40), (4∞),
(56), (58), (5X), (68), (6X), (8X), (90), (9∞), (0∞).

Transpositions taking the 495 orbit to the 990 orbit:
(13), (14), (23), (24), (5X), (5∞), (68), (69), (7X),
(7∞), (80), (90).

Transpositions taking the 990 orbit to the 495 orbit:
(13), (24), (57), (60), (89), (X∞).

Transpositions taking the (990) orbit to the 1584 orbit:
(15), (16), (17), (18), (19), (1X), (10), (1∞), (25),
(26), (27), (28), (29), (2X), (20), (2∞), (35), (36),
(37), (3∞), (46), (47), (48), (49), (4X), (40), (4∞),
(39), (38), (3X), (30), (3∞), (56), (5X), (50), (5∞),
(67), (68), (69), (7X), (70), (7∞), (8X), (80), (8∞),
(9X), (90), (9∞).

Transpositions taking the 1584 orbit to the 990 orbit:
(12), (15), (16), (18), (10), (23), (26), (28), (2∞),
(34), (36), (3X), (3∞), (45), (46), (49), (4X), (56),
(59), (50), (78), (79), (7X), (70), (7∞), (80), (8∞),
(9X), (90), (X∞).

Transpositions taking the 1584 orbit to the 1320 orbit:
(13), (14), (17), (19), (1∞), (24), (25), (27), (2X),
(20), (35), (37), (38), (39), (47), (40), (4∞), (57),
(58), (5X), (68), (69), (6X), (60), (6∞), (89), (8X),
(9∞), (X0), (0∞).

Transpositions taking the 1320 orbit to the 1584 orbit:
(15), (18), (19), (1X), (10), (14), (24), (25), (26),
(29), (2X), (2∞), (34), (35), (36), (38), (30), (3∞),
(46), (48), (4X), (59), (50), (5∞), (67), (69), (60),
(78), (79), (7X), (70), (7∞), (89), (8∞), (X0), (X∞).

Transpositions taking the 1584 orbit to the 144 orbit:
(1X), (29), (30), (48), (5∞), (67).

Transpositions taking the 144 orbit to the 1584 orbit:
(12), (13), (14), (15), (16), (17), (18), (19), (1X),
(10), (1∞), (23), (24), (25), (26), (27), (28), (29),
(2X), (20), (2∞), (34), (35), (36), (37), (38), (39),
(3X), (30), (3∞), (45), (46), (47), (48), (49), (4X),
(40), (4∞), (51), (56), (57), (58), (59), (5X), (50),
(5∞), (67), (68), (69), (6X), (60), (6∞), (78), (79),
(7X), (70), (7∞), (89), (8X), (80), (8∞), (9X), (90),
(9∞), (X0), (0∞).
APPENDIX C

COSET REPRESENTATIVES

Coset representative for the 440 orbit:

(123).

Coset representative for the 495 orbit:

(12)(34), (57)(X∞), (60)(89).

Coset representatives for the 1320 orbit:

(123)(45), (16)(490), (47)(68X), (28)(4∞9), (137)(80),
(1∞)(5X8), (57)(9∞0), (3X)(40∞), (39)(586), (273)(6∞),
(20)(56X), (172)(9X).

Coset representatives for the 990 orbit:

(1234), (5978), (6∞0X).

Coset representatives for the 1584 orbit:

(12345), (16490), (2∞X46), (287X3), (15978), (26508),
(18∞36), (107∞2), (809X∞), (4X705), (3X956), (3∞794).

Coset Representatives for the 144 orbit:

(12345X), (16490X), (2∞8459), (2∞6570), (106259),
(182935), (147X68), (1425∞7), (16X275), (34∞8X7),
(206∞34), (1∞0294), (2839∞6), (269587), (20X786),
(18X042), (36∞759), (254083), (253X79), (1X462∞),
(1X3287), (4X∞760), (2569X∞), (27964∞), (15978X),
(486∞X9), (3X5∞64), (246098), (204975), (10X538),
(27X430), (1∞4X85), (1∞7830), (350∞86), (139X20),
(3∞589X), (2X5748), (285X90), (143692), (18749∞),
(173X06), (130465), (13∞248), (1637∞4), (150372),
(5\times 087), (194576), (247356), (192708), (2\times 3605),
(386904), (1\times 5326), (370\times 9\times), (18536X), (16598),
(290763), (27\times 385), (15\times 6\times 0), (168023), (12\times 973),
(237894), (29\times 654), (6897\times 0), (1983X4), (148950),
(175843), (140\times 3), (398057), (107\times 2X), (3594X0),
(39746X), (50967X), (1\times 809), (1\times 69X7), (349685),
(1790\times 5), (3765X8), (4079X8), (1084\times 6), (3\times 0745),
(298\times 7X), (26X0\times 9), (280\times 47), (308X69), (45806X),
(23095\times), (19\times 52), (4678\times 5), (1X7054), (364870),
(24\times 50X), (1357X\times), (176\times 82), (2\times 493), (1X9563),
(126740), (23\times 67), (2\times 03X8), (154\times 39), (185607),
(109347), (1724X9), (138679), (12\times 896), (12058\times),
(19603\times), (47\times 95X), (26384X), (152864), (40569\times).
APPENDIX D

THE CONJUGACY CLASSES OF S_{12} AND M_{12}

Conjugacy Classes of group S_{12}

[1]  Order 1  Length 1  Rep Id(s_{12})
[2]  Order 2  Length 66  Rep (1, 2)
[3]  Order 2  Length 1485  Rep (1, 2)(3, 4)
[4]  Order 2  Length 10395  Rep (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)
[5]  Order 2  Length 13860  Rep (1, 2)(3, 4)(5, 6)
[6]  Order 2  Length 51975  Rep (1, 2)(3, 4)(5, 6)(7, 8)
[7]  Order 2  Length 62370  Rep (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)
[8]  Order 3  Length 440  Rep (1, 2, 3)
[9]  Order 3  Length 36960  Rep (1, 2, 3)(4, 5, 6)
[10] Order 3  Length 246400  Rep (1, 2, 3)(4, 5, 6)(7, 8, 9)(10, 11, 12)
[11] Order 3  Length 492800  Rep (1, 2, 3)(4, 5, 6)(7, 8, 9)
[12] Order 4  Length 2970  Rep (1, 2, 3, 4)
[13] Order 4  Length 83160  Rep (1, 2, 3, 4)(5, 6)
[14] Order 4  Length 311850  Rep (1, 2, 3, 4)(5, 6)(7, 8)(9, 10)(11, 12)
[15] Order 4  Length  623700  
   Rep (1, 2, 3, 4)(5, 6, 7, 8)  

[16] Order 4  Length  623700  
   Rep (1, 2, 3, 4)(5, 6)(7, 8)  

[17] Order 4  Length  1247400  
   Rep (1, 2, 3, 4)(5, 6, 7, 8)(9, 10, 11, 12)  

[18] Order 4  Length  1247400  
   Rep (1, 2, 3, 4)(5, 6)(7, 8)(9, 10)  

[19] Order 4  Length  1871100  
   Rep (1, 2, 3, 4)(5, 6, 7, 8)(9, 10)(11, 12)  

[20] Order 4  Length  3742200  
   Rep (1, 2, 3, 4)(5, 6, 7, 8)(9, 10)  

[21] Order 5  Length  19008  
   Rep (1, 2, 3, 4, 5)  

[22] Order 5  Length  4790016  
   Rep (1, 2, 3, 4, 5)(6, 7, 8, 9, 10)  

[23] Order 6  Length  15840  
   Rep (1, 2, 3)(4, 5)  

[24] Order 6  Length  110880  
   Rep (1, 2, 3, 4, 5, 6)  

[25] Order 6  Length  166320  
   Rep (1, 2, 3)(4, 5)(6, 7)  

[26] Order 6  Length  415800  
   Rep (1, 2, 3)(4, 5)(6, 7)(8, 9)(10, 11)  

[27] Order 6  Length  554400  
   Rep (1, 2, 3)(4, 5, 6)(7, 8)(9, 10)(11, 12)  

[28] Order 6  Length  554400  
   Rep (1, 2, 3)(4, 5, 6)(7, 8)  

[29] Order 6  Length  554400  
   Rep (1, 2, 3)(4, 5, 6)(7, 8)(9, 10)  

[30] Order 6  Length  1478400  
   Rep (1, 2, 3)(4, 5, 6)(7, 8, 9)(10, 11)  

[31] Order 6  Length  1663200
Rep (1, 2, 3, 4, 5, 6) (7, 8) (9, 10) (11, 12)

[32] Order 6 Length 1663200
Rep (1, 2, 3, 4, 5, 6) (7, 8)

[33] Order 6 Length 1663200
Rep (1, 2, 3) (4, 5, 6) (7, 8) (9, 10)

[34] Order 6 Length 4435200
Rep (1, 2, 3, 4, 5, 6) (7, 8, 9) (10, 11, 12)

[35] Order 6 Length 4435200
Rep (1, 2, 3, 4, 5, 6) (7, 8, 9)

[36] Order 6 Length 4989600
Rep (1, 2, 3, 4, 5, 6) (7, 8) (9, 10)

[37] Order 6 Length 6652800
Rep (1, 2, 3, 4, 5, 6) (7, 8, 9, 10, 11, 12)

[38] Order 6 Length 13305600
Rep (1, 2, 3, 4, 5, 6) (7, 8, 9) (10, 11)

[39] Order 7 Length 570240
Rep (1, 2, 3, 4, 5, 6, 7)

[40] Order 8 Length 2494800
Rep (1, 2, 3, 4, 5, 6, 7, 8)

[41] Order 8 Length 7484400
Rep (1, 2, 3, 4, 5, 6, 7, 8) (9, 10) (11, 12)

[42] Order 8 Length 14968800
Rep (1, 2, 3, 4, 5, 6, 7, 8) (9, 10, 11, 12)

[43] Order 8 Length 14968800
Rep (1, 2, 3, 4, 5, 6, 7, 8) (9, 10)

[44] Order 9 Length 8870400
Rep (1, 2, 3, 4, 5, 6, 7, 8, 9)

[45] Order 9 Length 17740800
Rep (1, 2, 3, 4, 5, 6, 7, 8, 9) (10, 11, 12)

[46] Order 10 Length 399168
Rep (1, 2, 3, 4, 5) (6, 7)

[47] Order 10 Length 1995840
Rep (1, 2, 3, 4, 5)(6, 7)(8, 9)(10, 11)

[48] Order 10  Length 1995840
Rep (1, 2, 3, 4, 5)(6, 7)(8, 9)

[49] Order 10  Length 4790016
Rep (1, 2, 3, 4, 5)(6, 7, 8, 9, 10)(11, 12)

[50] Order 10  Length 23950080
Rep (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)(11, 12)

[51] Order 10  Length 23950080
Rep (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)

[52] Order 11  Length 43545600
Rep (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11)

[53] Order 12  Length 332640
Rep (1, 2, 3, 4)(5, 6, 7)

[54] Order 12  Length 3326400
Rep (1, 2, 3, 4)(5, 6, 7)(8, 9, 10)(11, 12)

[55] Order 12  Length 3326400
Rep (1, 2, 3, 4)(5, 6, 7)(8, 9, 10)

[56] Order 12  Length 3326400
Rep (1, 2, 3, 4)(5, 6, 7)(8, 9)

[57] Order 12  Length 4989600
Rep (1, 2, 3, 4)(5, 6, 7, 8)(9, 10, 11)

[58] Order 12  Length 4989600
Rep (1, 2, 3, 4)(5, 6, 7, 8)(9, 10, 11)

[59] Order 12  Length 9979200
Rep (1, 2, 3, 4, 5, 6)(7, 8, 9, 10)(11, 12)

[60] Order 12  Length 9979200
Rep (1, 2, 3, 4, 5, 6)(7, 8, 9, 10)

[61] Order 12  Length 39916800
Rep (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)

[62] Order 14  Length 5702400
Rep (1, 2, 3, 4, 5, 6, 7)(8, 9)

[63] Order 14  Length 8553600
Rep (1, 2, 3, 4, 5, 6, 7)(8, 9)(10, 11)
Conjugacy Classes of group $M_{12}$

[1] Order 1 Length 1
Rep $Id(m)$
[2] Order 2 Length 396
   Rep (1, 11) (2, 6) (3, 12) (4, 5) (7, 8) (9, 10)

[3] Order 2 Length 495
   Rep (1, 4) (2, 9) (3, 8) (6, 7)

[4] Order 3 Length 1760
   Rep (1, 10, 3) (2, 7, 9) (6, 12, 11)

[5] Order 3 Length 2640
   Rep (1, 7, 2) (3, 5, 12) (4, 10, 6) (8, 11, 9)

[6] Order 4 Length 2970
   Rep (1, 8, 4, 3) (2, 7, 9, 6)

[7] Order 4 Length 2970
   Rep (1, 9) (2, 11, 12, 3) (4, 5) (6, 10, 8, 7)

[8] Order 5 Length 9504
   Rep (1, 12, 7, 10, 5) (3, 8, 9, 4, 11)

[9] Order 6 Length 7920
   Rep (1, 4, 7, 10, 2, 6) (3, 11, 5, 9, 12, 8)

[10] Order 6 Length 15840
    Rep (1, 11, 10, 6, 3, 12) (2, 9, 7) (5, 8)

    Rep (1, 2, 8, 7, 4, 9, 3, 6) (5, 10)

[12] Order 8 Length 11880
    Rep (1, 7, 12, 10) (2, 11, 3, 6, 5, 9, 8, 4)

[13] Order 10 Length 9504
    Rep (1, 9, 12, 4, 7, 11, 10, 3, 5, 8) (2, 6)

[14] Order 11 Length 8640
    Rep (2, 12, 9, 5, 10, 4, 3, 7, 8, 11, 6)

    Rep (2, 9, 10, 3, 8, 6, 12, 5, 4, 7, 11)

[1] Order 1 Length 1
   Rep Id(m)

[2] Order 2 Length 396
   Rep (1, 11) (2, 6) (3, 12) (4, 5) (7, 8) (9, 10)

[3] Order 2 Length 495
Rep (1, 4) (2, 9) (3, 8) (6, 7)
[4] Order 3 Length 1760
Rep (1, 10, 3) (2, 7, 9) (6, 12, 11)
[5] Order 3 Length 2640
Rep (1, 7, 2) (3, 5, 12) (4, 10, 6) (8, 11, 9)
[6] Order 4 Length 2970
Rep (1, 8, 4, 3) (2, 7, 9, 6)
[7] Order 4 Length 2970
Rep (1, 9) (2, 11, 12, 3) (4, 5) (6, 10, 8, 7)
[8] Order 5 Length 9504
Rep (1, 12, 7, 10, 5) (3, 8, 9, 4, 11)
[9] Order 6 Length 7920
Rep (1, 4, 7, 10, 2, 6) (3, 11, 5, 9, 12, 8)
[10] Order 6 Length 15840
Rep (1, 11, 10, 6, 3, 12) (2, 9, 7) (5, 8)
Rep (1, 2, 8, 7, 4, 9, 3, 6) (5, 10)
[12] Order 8 Length 11880
Rep (1, 7, 12, 10) (2, 11, 3, 6, 5, 9, 8, 4)
[13] Order 10 Length 9504
Rep (1, 9, 12, 4, 7, 11, 10, 3, 5, 8) (2, 6)
[14] Order 11 Length 8640
Rep (2, 12, 9, 5, 10, 4, 3, 7, 8, 11, 6)
Rep (2, 9, 10, 3, 8, 6, 12, 5, 4, 7, 11)
APPENDIX E

COMPUTER PROGRAMS

Computes the orbits of $S_{12}$

12:=SymmetricGroup(12);
a:=s12!(1,2,3,4,5,6,7,8,9,10,11,12);
b:=s12!(1,2);
s:=sub<s12|a,b>;
c:=s!(1,2,3,4,5,6,7,8,9,10,11);
d:=s!(11,12)(2,6)(7,9)(3,8);
m:=sub<s|c,d>;
T:=Transversal(s,m);
  temp:=2;
  st:=
  for v in m do
    for l:= 1 to 5039 do
      if T[temp]^v*T[1]^(-1) in m then
        if l notin st then st:=Include(st,l); end if;
      end if;
    end for;
  end for;
print #st;

Computes the number of coset representatives for each orbit

s12:=Sym(12);
a:=s12!(1,2,3,4,5,6,7,8,9,10,11,12);
b:=s12!(1,2);
s:=sub<s12|a,b>;
c:=s!(1,2,3,4,5,6,7,8,9,10,11);
d:=s!(11,12)(2,6)(7,9)(3,8);
m:=sub<s|c,d>;
T:=Transversal(s,m);
a:= sl2!(1,2,3,4,10);
C:=Class(sl2,a);
D1:=[Id(sl2): i in [1..6888]];
j:=0;
for x in m do
  if Order(x*a) eq 6 then
    j:=j+1;
    D1[j]:= x*a;
  end if;
end for;
D:= [Id(sl2): i in [1..6888]];
k:=0;
for i:= 1 to 6888 do
if D[i] in C then
k:=k+1;
D[k]:=D[i];
end if

Computes the number of Steiner systems in $S \cap S^x$

\[
s_{12}:=\text{SymmetricGroup}(12);
\]
\[
a:=s_{12}!(1,2,3,4,5,6,7,8,9,10,11,12);
b:=s_{12}!(1,2);
s_{12}:=(s_{12}|a,b);
\]
\[
aaa:=s_{12}!(1,2,3,4,5,6,7,8,9,10,11,12);
bbb:=s_{12}!(1,3,9,5,4)(2,6,7,10,8);
ccc:=s_{12}!(11,12)(1,10)(2,5)(3,7)(4,8)(6,9);
ddd:=s_{12}!(2,10)(3,4)(5,9)(6,7);
ml_{2t}:=\text{sub}(s_{12}|aaa,bbb,ccc,ddd);
h:=\text{sub}(ml_{2t}|aaa,ccc);
\]
\[
\text{str}_{56}:=[];
q:=\{1,3,4,5,9,11\};
\text{for } x \text{ in } h \text{ do}
\text{str}_{56}:=(\text{Include} (\text{str}_{56}, q^x));
\text{end for};
d:=s_{12}!(1,11)(2,3);
\text{str}_{56111}:=\text{str}_{56}^d;
l:=0;
\text{for } v \text{ in } \text{str}_{56} \text{ do}
\text{if } v \text{ in } \text{str}_{56111} \text{ then } l:=l+1;
\text{end if};
\text{end for};
\text{print } l;

Computes the coset representatives for each orbit

\[
s_{12}:=\text{Sym}(12);
a:=s_{12}!(1,2,3,4,5,6,7,8,9,10,11,12);
b:=s_{12}!(1,2);
s:=\text{sub}(s_{12}|a,b);
c:=s!(1,2,3,4,5,6,7,8,9,10,11);
d:=s!(11,12)(2,6)(3,8);
m:=\text{sub}(s|c,d);
T:=\text{Transversal}(s,m);
D_1:=\{\text{Id}(s_{12}) : i \text{ in } [1..6888]\};
j:=1;
r_1:=s_{12}!(1,2,3);
\text{for } x \text{ in } m \text{ do}
\text{if } \text{Order} (x*r_1) \text{ eq } 4 \text{ then } D_1[j]:=x*r_1;
\text{end if};
\text{end for};
C1:=Class(sl2, rl);
for i:= 1 to 6888 do
if D1[i] in C1 then print D1[i];
end if;
end for;

Computes the transpositions that will move one orbit into another

s12:=Sym(12);
a:=s12!(1,2,3,4,5,6,7,8,9,10,11,12);
b:=s12!(1,2);
s:=sub<s12|a,b>;
c:=s!(1,2,3,4,5,6,7,8,9,10,11);
d:=s!(11,12) (2,6) (7,9) (3,8);
m:=sub<s|c,d>;
T:=Transversal(s,m);
D1:=[Id(sl2):i in [1..6888]];
j:=1;
rl:=s12!(1,2,3);
D:=[Id(sl2): i in [1..2]];
for x in m do
if Order(x*rl) eq 3 then D1[j]:=x*rl;
j:=j+1;
end if;
end for;
C1:=Class(sl2, rl);
D:=[Id(sl2): i in [1..2]];
F:=[Id(sl2): i in [1..66]];
k:=1;
for i:= 1 to 6888 do
if D1[i] in C1 then D[k]:=D1[i];
k:=k+1;
end if; end for;
r:=s12!(1,2);
C:=Class(sl2, r);
E:=[Id(sl2):i in [1..2]];
EE:=[[Id(sl2): i in [1..2]]: j in [1..66]];
l:=1; ll:=1;
for x in C do
E:=D;
F[ll]:=x;
ll:=ll+1;
for k:= 1 to 2 do
E[k]:=D[k]*x;
end for;
EE[l]:=E;
l:=l+1;
end for;
BIBLIOGRAPHY


