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A unit on proportional relationships: A preparation for algebra

Jennifer Virginie Pidgeon
Katherine Anne Yule

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A UNIT ON PROPORTIONAL RELATIONSHIPS:
A PREPARATION FOR ALGEBRA

A Project
Presented to the
Faculty of
California State University,
San Bernardino

In Partial Fulfillment
of the Requirements for the Degree
Master of Arts
in
Education: Middle Grades Option

by
Jennifer Virginie Pidgeon
Katherine Anne Yule
September 1998
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Approved by:

Irvin Howard, First Reader

Alvin Wolf, Second Reader
ABSTRACT

A 7th grade curriculum for the period of one half of a trimester with an emphasis on proportional representations for bilingual immersion students will be presented. This course will prepare those subject students to matriculate into Algebra 1 as 8th graders at Los Alisos Intermediate School. The unit will present activities, teacher instruction, a student needs assessment, and final assessment for use with the class of approximately 30 students. It will be shown that there is no present curriculum which presently fulfills the needs of these students. The new California Framework and Standards have emphasized the need for students at all levels of learning capabilities to be enrolled in an algebra course in the 8th grade. Proportional reasoning with problem solving is one of the most essential unifying ideas which supports readiness for algebra. Different learning styles will be addressed in this curriculum which will basically be of a progressive style integrating a substantial review of the basic arithmetic skills with hands on projects and assignments.
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Katherine Anne Yule
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CHAPTER 1

General Introductory Remarks

Mathematics curriculum has been undergoing revision and reform for the last six years. Teachers are finding that students learn better when their mathematical experience takes into consideration how students learn. When the students experience a curriculum where they are thoroughly engaged with the mathematics and the environment into which this mathematics takes place, their knowledge is more meaningful, and thus retained for a longer period of time. Although new mathematics programs have been introduced into the school and the district within the last two years, the material does not prepare students to complete preparation for algebra in the span of one year. The present mathematics materials do not have intense focus on algebraic preparatory concepts, rather they repeat elementary concepts which have been spiraled throughout the curriculum for many years.

The project will contain a complete half-trimester of mathematics curriculum for entering seventh grade students. The main theme of this curriculum will be preparation for algebra using the unifying mathematical idea of proportional relationships. The curriculum will consist of a unit for students who will experience projects and activities that strengthen previously learned mathematical ideas and skills and blend them with new concepts. The integration of those concepts into meaningful activities will be the main emphasis of this project.

The project will be used for 30 students who will be
entering Los Alisos Intermediate School in September, 1998, who have been taught using a bilingual immersion program since they were in kindergarten. These students have a wide range of mathematical capabilities and proficiency, and thusly require a curriculum which addresses various learning styles and interests. The languages in which they have been instructed are Spanish and English.

The authors of this project are qualified to write this material, as one is presently the teacher of the ESL mathematics classes at Los Alisos and the summer school teacher for the academy to which these 30 students have been attending. The other author is the algebra teacher at Los Alisos and a former cluster leader for the Mathematics Renaissance Reform Movement. Both authors collaborated completely with the creation of this project from the conception of the idea to the writing of the lessons and the final typing and revisions.

Significance Of The Project

An immediate need exists at Los Alisos Intermediate School for an algebra preparatory course offered to seventh grade students in September of 1998. These seventh grade students are part of a Spanish/English bilingual-immersion program and have been together since kindergarten. This class of approximately 30 students is made up of 50% native Spanish speakers and 50% native English speakers. It is the desire of both parents and school officials to advance these students to a college preparatory course upon entering high school, and thusly, it is essential that all these students
complete Algebra I in the eighth grade year. Although there is a current plan for students to accomplish this, the program in place is designed for a homogeneous group of mathematically gifted students. Because the immersion student class contains a wide spectrum of mathematical abilities and interests, a new program needs to be designed to accommodate this heterogeneous group.

The State of California and the Saddleback Valley Unified School District has written new mathematics standards which will be implemented within the next three years, and these standards state that the curriculum for all eighth grade students will be a first year algebra course. In line with this, our project will create a curriculum which could be used to accomplish this demanding goal for all of our eighth grade students.

In addition to the documents regarding the new curriculum is the disclosure of the mathematical scores of eighth grade students throughout the world via the Third International Mathematics and Science Study (TIMSS). This international report found that the eighth grade students in the United States were remarkably lacking in pre-algebra skills and problem solving strategies. The countries that tested the highest on this internationally normed test were countries where all eighth grade students were taught basic algebra.

This project could lead to the creation of a year-long curriculum which would prepare all students for a thorough knowledge which would guide them on the road to algebra.
Statement Of Needs

In order to delineate the exact needs of this algebra preparatory course we not only need to look at the content for this course but also examine the inadequacies of Los Alisos' current mathematical program. There are basically two problems. The first is that the curriculum is extremely broad and shallow. Throughout the year, a very large number of unconnected mathematical concepts are introduced with very little time or effort allowed for students to fully understand and digest mathematical significance. Dr. William Schmidt refers to this as "a mile wide and an inch deep" curriculum (TIMSS 1997.) It is our attempt to write curriculum for the purpose of mathematical depth and thorough understanding. In other countries of the world where students are successfully learning mathematics, very few, yet related concepts are taught each year. This is done successfully because each year the students encounter new and higher level mathematical concepts than the previous year. Remediation and spiraling, which is an integral part of the United States curriculum, is nonexistent in those countries.

The second problem exists because, ironically, mathematical education leaders tried to solve the first problem of a shallow, wide curriculum by instituting a program of several six week units for a year program. In the early 1990's Saddleback Valley Unified School District wanted to make a systemic change in the way mathematics was taught, so it became part of the Mathematics Renaissance Program, whose goal was to help teachers engage students to become
active participants in their learning of mathematics. What was meant to be a new style of introducing and exploring mathematical concepts with students became a dispensary of replacement units. Although these units were mathematically sound and engaged students in their own learning, teachers were not willing to make the pedagogical changes that would allow or empower students to construct their own mathematical meanings. Teachers had a hard time breaking away from the comfort, power, and control of imparting knowledge to students and becoming facilitators of knowledge. Basically, they had a hard time letting go of their control on how the students would learn mathematics. Many teachers felt they already knew everything that there was to know about mathematics and they continued to teach the way they were taught.

In addition to the teacher drawbacks, the units themselves were never put in a year's curriculum in a definitive scope and sequence. The seventh and eighth grade units were "cobbled" together to create a curriculum without regard to a central unifying theme. Our project is an attempt to write curriculum that is a mathematically sound, cohesive unit with a central, unifying theme of proportional relationships focusing on algebra readiness. This unit needs to be relevant to the adolescent's world and provide opportunities for reflections and connections to other subjects within and outside of school situations. Students need to be taught to think critically, and be taught to learn as well as to test successfully (Carnegie 1989). Presently,
the standards for mathematical education in the state and the district require all eighth grade students to ultimately be enrolled in algebra. The present curriculum's scope and sequence requires two years of pre-algebra to accomplish this algebra readiness for ninth grade students. So our challenge is to condense, narrow, and sort out the most necessary concepts and skills of pre-algebra and place them in a year's study for seventh graders so that they might take algebra in the eighth grade. We contend that a unit on proportional relationships is an important element in algebra readiness and that students of this age group are cognitively ready to explore a deeper look at the components entailed in proportional thinking activities.

The students being addressed in this project are coming to Los Alisos with a large range of cognitive levels and mathematical abilities. The group of bilingual-immersion students have come from a very diversified experiential home life. Thusly, the mathematics that the students will experience must address many learning styles and cognitive levels of development.

Upon entering middle school students tend to be somewhere within Piaget's concrete operational stage. (Santrock 1998) At this stage, intuition is replaced with logical reasoning as long as this reasoning can be applied to specific or concrete examples. With most students at this stage the curriculum should be filled with many hands-on, relevant, and visually perceptive activities which are easily found in the unifying idea of proportionality. The concrete
operational child has experienced proportional relationships as they relate a set of tangible objects to another set of tangible objects, such as in figure 1, where the ratio of dotted squares to striped circles is 2:3.

FIGURE 1

The purpose of a unit on proportional reasoning is to take the concrete operational learner to the early operational thinker, where relationships are no longer concrete, but hypothetical. Engaging students in activities which involve measuring distances that are concrete and then extending them to those which cannot be directly measured, will move the students to a higher cognitive stage.

Since some students will be further along on the trek to formal operational thought, those who have already devised hypotheses can tie their conjectures to concrete proportional examples.

The students in this program have been together for seven years, and have learned each other's strengths and weaknesses. The curriculum needs to pull upon these differences and offer students opportunities to work collaboratively, and in concert with each other's gifts to assist each other in successfully completing this algebra readiness course. It is our intent that all students in the immersion program remain together in this heterogeneous
class, and culminate their year being prepared for algebra by raising their level of mathematical understanding. The unit that is contained within this project will attempt to obtain this goal.

The students in this project consist of native Spanish speakers and native English speakers and have been instructed in both languages since kindergarten. Because their educational emphasis has been on reading and language development in an attempt to become fluently bilingual most students' mathematical skills are below grade level. The students were involved in a mathematical institute called Mathmania in the summer of 1997 at Los Alisos Intermediate School between their fifth and sixth grade year. This institute was established because it was found that there were deficiencies in their mathematical backgrounds. The curriculum that we are developing must narrow their mathematical gaps. This can be done by eliminating topics that have already been taught and concentrating on those topics that promote algebraic or abstract thinking. It has been found that high expectations in students encourages students to higher achievement.

Program Plan

Goal #1: Prepare students for algebra in grade 8

Objective #1: Raise the Piagetian cognitive level of all students

Title: Raising Cognitive Levels

Strategies:

1. Use of manipulatives
for mathematical activities which move students from concrete to abstract understanding

2. Gather and create activities designed to elevate cognitive stages of all students

Measure: Piaget testing of students after each unit

Objective #2: Create connections between and among math concepts

Title: Making Mathematical Connections

Strategies:

1. Units relative to adolescent world
2. Journals and/or reflections
3. Cross-curricular activities
4. Multi-media activities and investigations which connect to the world outside the classroom

Measure: Students will successfully complete a project at the end of the unit which links math concepts to a situation which involves the concepts and techniques previously encountered in the curriculum.
Objective #3: Raise student levels of proportional reasoning skills

Title: Experiencing Proportionality

Strategies:
1. Experience activities and investigations relative to proportional thinking
2. Relate proportional experiences to algebraic concepts

Measure: Students will be able to solve mathematical problems using proportional strategies at end of unit

Limitations And Delimitations

The major concern of most teachers is that all students will not be successful. This is also a major concern of the authors, as our experience tells us that not all students accept the responsibility for their educational progress, nor do they all have the support and encouragement from their parents or guardians. Since the goal of this project is to prepare students to be enrolled in algebra at the eighth grade level, the success of the project will be determined by the relative success of the students as they matriculate to the eighth grade algebra course. Even when students have been accelerated through many math courses, there is a small rate of failure. To expect this to not be repeated for students who are being accelerated in only one year of
preparation, would be naive. Although it is the opinion of the authors that a specialized curriculum will prepare a student for the mathematical understanding necessary for the undertaking of algebra, the cognitive levels of the students may not be ready to accept this abstract thinking even when pushed to do so (Santrock 1998).

It is a possibility that this project will create a program that no teacher is willing to teach. As this creates a unique class with new and creative pedagogy, with new lessons for which the teacher must prepare, no teacher may be willing to expend the extra time and effort that this will entail.

A further limitation is that the course will be offered after this project has been completed, and hence examples of student work will not be included. Student work is an invaluable tool to assess the success of any curriculum.

The basic delimitation of this project is that time does not allow for the creation of mathematical units of work for a complete school year. It is the desire of the authors that further curriculum for students in this program will be created to complete this project before the end of the students' seventh grade year. Although the unifying idea of proportional relationships is a very important preparation for algebra, other important strands should be included in the curriculum (California Math Framework 1992). An additional delimitation in the project is the absence of assigned drill-type homework activities, and daily quizzes or assessments. It is left to the individual teachers to decide
the needs of their classes as to what kind of reinforcement, evaluation, and extensions are necessary.

Only seventh grade students will be the subject of this project, and there is no one year class for eighth grade students in place whose statistics could be used for comparison or control. There is an accelerated class for seventh grade students so they may take algebra as eighth graders, but the curriculum is very different than the one proposed in this project. Therefore, algebra readiness can not be assessed on equal levels. In addition, the algebra readiness exam currently used and accepted as a verifying statistic is not aligned to this curriculum.

**Assumptions**

It is assumed that all students will raise their cognitive levels with the activities presented. The activities are designed to be apropos to the age group, and therefore each student will increase in abstract thinking and thus reach a higher cognitive level.

The success of this project is also built on the assumption that support will come from the administration. At the district level, the board of education must agree that this course will meet the needs of the subject group. At site level, support is necessary to the degree that class scheduling will allow the subject students to be placed in the class; funds will be allocated to purchase supplies necessary for the teaching of the course; and the teacher or teachers will be given release time to plan the unit set forth here, as well as further units to complete the year.
It is also assumed that the teacher or teachers will have philosophies and goals that are consistent with the ideals of the program. It is necessary for the teachers to have the following qualities: openness to change, willing to accept challenges, willingness to collaborate with other teachers, and a nurturing, caring nature (Inter-American Magnet School 1997).

It is further assumed that the parents of the subject students will be supportive to a program which will be a creative and innovative approach to the acquisition of mathematics skills.

Another assumption that the authors hold is that most students will be successful. That is, they will bridge the gaps in their mathematical education and raise their cognitive levels and progress with a passing grade to algebra as eighth graders.

If the implementation of the California Math Standards, whose guidelines recommend the teaching of algebra to all eighth graders, does in fact become a reality, this project will effectively create curriculum that will move students through seventh grade to algebra in one year. It is assumed therefore that this project will provide a basis for curriculum that could be used by all teachers in the next few years.

**Definition Of Terms**

For this project the following definitions apply, as many
mathematical and educational terms could be defined in various ways outside of this context.

1. In mathematics an **abstract** concept is theoretical and usually makes no connection to the real world.

2. Students can be evaluated for entrance into an algebra program by examining their cognitive development as well as their arithmetic skills. This determines their **algebra readiness**.

3. A **bilingual immersion** program exists when students are taught in two languages throughout the entire curriculum. This program is started in kindergarten with 90% of instruction in Spanish and 10% in English. Each year there is a 10% increase of English instruction until instruction is 50% English and 50% Spanish.

4. **Cobbling** a curriculum occurs when unrelated units of instruction are placed in a year's curriculum without regard to a central theme.

5. Students understand mathematical concepts that are **concrete** because they are understood through the senses and are related directly to objects in the real world.

6. **Cooperative learning** involves the use of collaborative teaching strategies designed to help students learn to relate positively to each other and to work together in groups designed to achieve specific learning objectives.

7. **Early adolescence** is used to describe the initial phase
of the transition which humans experience between childhood and maturity.

9. In mathematics, a function is a rule that relates one number to another distinct number. It is often written as an algebraic equation.

10. Hands-on mathematics are methods that require students to participate more actively in their learning. Students are encouraged to explore a wide variety of mathematical concepts in ways that preserve their natural curiosity as they develop higher order thinking skills. Hands-on instruction requires extensive use of manipulatives and experiential learning.

11. Heterogeneous grouping is the practice of organizing classroom instruction without direct reference to differences in academic ability among students as measured by standardized tests, teacher observation, or other comparable criteria.

12. The term, higher-level thinking skills is used to distinguish between basic cognitive abilities and those which require more abstract thinking processes.

13. Mathematical investigations occur when students are presented with an open-ended problem situation without direct teacher instruction as to how to find its solution. The students' solutions are not necessarily all the same, nor do they have a unique and singular solution.

14. When students are working with manipulatives, they are working with tangible objects to aid them in concrete mathematical understanding, such as blocks, straws, dice, etc.
15. A student is said to **matriculate** when they have completed a course of study and move to the next higher level of education.

16. **Multi-media** presentations use varied forms of technology to relate information to the audience, such as television, computer generated display, movies, etc.

17. The **Piaget stages of cognitive development** include the following which apply to the age group in this project:
   - In the **concrete operational** stage the child has the ability to reason logically about concrete events and classify objects into different sets.
   - In the **formal operational** stage, the adolescent reasons in more abstract and logical ways. Thought is more idealistic.

18. A test which follows a unit of instruction is a **post assessment**, where students can demonstrate what they have learned, such as a objective test, or a culminating project.

19. A test which precedes a unit of instruction is a **pre assessment**, which establishes students' prior knowledge, such as a investigative project or a objective test.

20. A mathematical **proportion** is an equation in the form \(a/b = c/d\), where \(a, b, c,\) and \(d\) are numbers, \(b\) and \(d\) not equal to zero. A **proportion** relates one thing to another with respect to size, number, amount or degree.

21. A change that is **systemic** takes place at the lowest level of organization, and moves up and through the system to affect a difference which occurs from within, instead of the desired change being imposed from a higher authority.
22. The pedagogical technique of **spiraling** is introducing mathematical concepts at a low cognitive level and cyclically revisiting that concept to expand on its meaning months or perhaps years later.

23. **Tracking** is the practice of identifying and grouping students for instructional purposes according to presumed ability and demonstrated academic achievement.

24. In mathematics the term **variable** refers to a symbol which can be replaced by a number. Variables are usually letters of the alphabet.
CHAPTER 2

Review Of Related Literature

In an article by Katherine K. Merseth a discussion ensues regarding the way students learn. Cognitive science has found students do not come to teachers with "blank slates", nor as "empty vessels". Children and adults have personally constructed theories on what they have learned. Scientists believe that students construct their own meanings and understandings. They reflect upon what's been offered or told to them and make connections relative to what they already know and understand. These self-constructed theories are called "naive" theories and many people hold on to them even when not appropriate or correct (Merseth 1993).

Teachers must be aware of this as they work to develop curriculum. Changes must be made that create an atmosphere where the previously passive learner now becomes an active participant by gathering, collecting, sorting and generalizing information as they are engaged in activities relative to their known world (Merseth 1993).

Instruction must also change. Multiple opportunities must be provided to make the necessary connections that students need for mathematical engagement. Multiple representations and explanations of concepts must be presented as the construction of knowledge and understanding can be triggered in many ways (Merseth 1993).

Issues of equity have always been at the heart of math reform. Beliefs are finally changing as to who can do mathematics. Acquarelli and Mumme, in their roles of math
reformers, stress that students can and will be engaged in the classroom if worthwhile mathematical experiences are offered. Their work with the Middle School Math Renaissance in California have offered insight into what needs to be done to create curriculum which, in fact, accomplishes this. The reform program with which they were involved attempted to systemically alter the way that teachers teach. In effect what was accomplished was the distribution of mathematical units for teachers to use with students to "practice" what was learned in the training sessions. The teachers placed more importance on the units than on the reflective, interactive engagement of the students, and they assumed that the units would automatically do that job for them. The experience did have an effect in getting teachers to think about pedagogy that would engage all students. Hence, the curriculum for many schools became a "cobbled" one of five or six of these units placed one after the other into the school year. Often the chunks of material did not go together to create a cohesive school year's curriculum, and so a whole integrated program was not created (Acquarelli & Mumme 1996).

The results of the Renaissance confirmed the fact that a change to a more interactive and engaging curriculum for middle school students had a positive effect on the students. More young adolescent students were being successful and becoming engaged in complex mathematical thinking (Acquarelli & Mumme 1996).

The NCTM Standards (1989) recommends a three year middle school curriculum where topics are fundamentally the
same for all students and that the differences in student
ability is reconciled with appropriate variation in "the
depth and breadth of the treatment of topics by the nature of
applications". Every student has the right to a mathematics
education that ensures that he or she achieves mathematical
literacy and develops the concepts, skills, and dispositions
necessary for a meaningful and productive life. Middle
school students particularly, are at a critical stage, as the
attitudes which are developed have severe impacts upon their
chances for success in high school and on life choices.
Therefore it is essential that students experience
mathematics in a personally meaningful and worthwhile style,
and that they see math as a powerful and useful tool in all
aspects of their present and future lives (NCTM 1998).

One of the most successful bilingual immersion programs
exists in Chicago. A huge component of their success is
attributed to the use of heterogeneous grouping. Their
students are grouped heterogeneously in classrooms from
preschool through eighth grade without regard to language
dominance, ethnicity, gender, or academic ability. Students
in heterogeneous groups learn to prize diversity, rather than
segregating by differences and proves to be more beneficial
than current tracking practices. The aim of their program is
to produce proficient, literate bilingual students who meet
high academic standards (Coronado-Greeley 1997).

Another important component to the Chicago dual
immersion program is it's highly qualified staff that is
dedicated to the philosophy and goals of the dual language
program. In order for any program to be successful and useful this component is a must. It is believed that all children can achieve their fullest potential in all areas of curriculum given the appropriate instruction and the necessary home/school support and that caring, accepting, and cooperative behavior on the part of the staff, parents, and students promotes the development of the whole child (Coronado-Greeley 1997).

Cooperative learning is yet another component for the Chicago dual immersion program. This method allows for maximum flexibility; as students are grouped based on the activity and or subject, and are frequently regrouped according to student needs. When learning cooperatively, students have ample opportunities to participate more actively in the learning process. The cooperative learning method has transformed the social organization of the classroom, moving the emphasis away from competition and toward the true competence, proficiency, and literacy that students need to become productive citizen in the twenty-first century. It is an appropriate method which coincides with the multicultural ambiance that exits in the world (Coronado-Greeley 1997).

In terms of their mathematics curriculum they use the hands-on approach to the teaching and learning of mathematics. This method requires that students participate more actively in their learning. Students are encouraged to explore a wide variety of mathematical concepts in ways that produce their natural curiosity as they develop higher order
thinking skills. Hands on instruction requires extensive use of manipulatives and experimental learning (Coronado-Greeley 1997).

Lynn Arthur Steen said in "Teaching Mathematics for Tomorrow's World" (1989) that in order to prepare students for the future, mathematics teachers must change their curriculums, teaching methods, and techniques. Mathematics is the key to opportunities for jobs. Presently, jobs require much more of workers. Employers need people who are prepared to absorb new ideas, to perceive patterns and to solve unconventional problems. Mathematics is essential for everyone due to technology in the workplace.

According to Steen, the National Council of Teachers of Mathematics (1989) there are five broad goals required to meet students' mathematical needs for the twenty-first century:

• To reason mathematically. This helps students to clarify complex situations.
• To value mathematics. Students need experiences that bring them to believe that mathematics has value for them.
• To communicate mathematics. There is no better way to learn mathematics than by working in groups, by teaching mathematics to one another, by arguing about strategies, and by expressing arguments in written form.
• To solve problems. Students need to learn how to analyze problems, select appropriate strategies to solve problems, recognize and formulate several solutions, and
to work with others in reaching consensus on solutions that are effective as well as logical.

- To develop confidence. Mathematics can neither be used nor learned unless it is supported by self confidence built on success. Parents also need to stop saying that they are not mathematically competent.

When compared to the NCTM's five goals, today's curriculum is totally inadequate. The new curriculum standards (NCTM 1989) make clear that the whole mathematical environment of learning must change, not only what is taught but also how it is taught and how it is assessed. The following actions need to be taken:

- Raise expectation. If more is expected of our students more will be achieved.
- Increase breadth.
- Use of calculators.
- Engage students. Students need to be active participants in learning rather than passive receivers of knowledge.
- Encourage teamwork.
- Access objectives. We need to move away from multiple choice standardized test.
- Require mathematics. Students need to take mathematics every year.
- Demonstrate connections. Showing mathematical connections motivates learning and reinforces ideas arising in different contexts.
- Stimulate creativity. In the computer age, one needs to use one's imagination as much as one's intellect and
one's judgment as much as one's memory.

- Reduce fragmentation. Real problems don't come in compartmentalized form.
- Require writing. Writing helps to clarify understanding.
- Encourage discussion. Learning is done by doing and not by listening.

"Curriculum, teaching, and testing must change together to improve mathematics education. Unless all improve in concert, nothing will change. We know what needs to be done, and we know how to do it. What is required now is a commitment to action" (Steen 1989).

Most mainstream educators already agree that American mathematics instruction needs a drastic overhaul, with more emphasis on group problem solving and creative thinking rather than repetitive drills. Educators say too many children are wasting time practicing adding, subtracting, multiplying and dividing, when they could be moving on to more interesting and challenging mathematics. Students need to talk and write about mathematics and work with real life examples like students do in Japan (Kantrowitz and Wingert 1991).

In the recent Third International Mathematics and Science Study (TIMSS), William H. Schmidt summarizes the findings of the largest, most comprehensive and most reliable comparison of education ever undertaken. Half of a million eighth graders were tested from 41 countries on their knowledge of math and science. The United States was below the international average in mathematics (TIMSS 1997).
Schmidt (1997) claims that the good news is that the answer to the problem lies in the classrooms of the country. He cites four significant differences which can be altered in the education of United States students.

- Algebra and geometry are studied by most eighth graders around the world.
- United States mathematics teachers teach students how to do something, while the goal of a country such as Japan, whose students scored at the top of the study, is to help the students understand mathematical concepts.
- United States eighth graders are exposed to 35-40 different and often unrelated topics, while in Japan, for instance, students delve deeply into 5-7 major topics in a year. We, in the United States do not have a focus about what to teach (Schmidt 1997).
- Students in other countries who competed successfully were not tracked into ability groups. All students were engaged in the same curriculum and it was expected that all students be successful (Schmidt 1997).

Upon examination of teachers, classrooms and lessons, the curricula of the United States was found to be scattered and splintered, as are the textbooks which drive the courses. There is no cohesiveness in the curricula as there is in other counties who do better at teaching students mathematics. In addition, low expectations may insufficiently challenge our students. They may be capable of learning more than is presently offered (Schmidt 1997).

Curricula, textbooks and teaching should be part of an
integrated solution united around common goals, approaches, and measurements. Educators must consider what is basic at every level and make sure all students receive such instruction. Our diversity must become part of our solution (Schmidt 1997).

In "Learning for the 21st Century" (1992) Glen Thomas also focuses on why our current system of educating must change. The first is we are currently preparing students for a manufacturing economy. Instead we should be preparing them for the information age. Students must know how to think and reason, analyze data, solve complex problems and communicate well. The second is there is a shift in student composition moving from white to ethnic. Thirdly, a new understanding on how children learn due to recent cognitive and constructivist research has come to the fore. Children need to and only truly learn when the information relates to them and their own experiences.

In "Multiplication with Fractions: A Piagetian, Constructivist Approach", Warrington and Kamii (1998) state that whereas traditional instruction is based upon the assumption that children must digest the results of centuries of adult mathematicians, children will go further with depth pleasure, and confidence if they are allowed to construct their own mathematics; that which makes sense to them.

The issue of tracking takes a political, as well as pedagogical role in the education of today's middle school students. The California Mathematics Framework (1992) recommends that all students study a common core curriculum
from kindergarten through grade eight in a heterogeneously
grouped environment. Working in special interest groups,
students with special talents or interests can dive more
deeply into some investigations. Student experiencing
difficulty could get assistance before and/or after
school (Framework 1992).

Tracking filters out those student who despair at low
grades and those students do not continue with math past what
is minimally required. "The filter does not distribute
misery evenly across the student population. African
American and Latinos are under represented in higher
education in general and in mathematically based programs of
study in particular" (Framework 1992).

Paul S. George in his article in the Middle School
Journal lists ten tentative truths regarding tracking, which
he would hope would encourage classes at this level to be
taught in a heterogeneous setting.

• Identification and placing students into ability groups
  is more difficult to accomplish fairly and accurately
  than often thought to be the case.
• Students are increasingly unlikely to be moved to a
  supposed higher group once placed in one of a lower
  standing.
• The importance of student, teacher, and parent effort is
downplayed, and the the centrality of the individual
  student is overemphasized.
• Ability grouping seems to be related to substantial
differences in self-esteem.
• Academic achievement does not appear to improve with the use of ability grouping.
• Racial, ethnic and income isolation may occur as a byproduct of ability grouping.
• The sense of community in and out of school is destroyed.
• Middle school resources are delivered in fundamentally unfair and inequitable ways.
• Some middle school ability grouping practices may be illegal.
• All middle school students, including gifted students, deserve to receive effective instruction in a challenging curriculum (George 1993).

Glen Thomas (1992) suggests that curriculum frameworks focus on student understanding and engagement. There needs to be an emphasis on student thinking and conceptual understanding. "Less is more" needs to be instituted. Teachers need to cover fewer topics and the topics they do cover are covered in depth. Students need to be presented with complex problem solving situations based on real life situations. Active learning and activity based instruction needs to be used in the classroom. Mathematical connections need to be made by the students based upon their own experiences and interest. There needs to be extensive use of students collaboratively working on problems. Teachers need to use assessment that features application and use of students' knowledge and problem solving skills in multiple settings. Lastly, instructional materials need to be viewed
as a resource rather than as the curriculum itself and to integrate the use of technology in the classroom.

Patrick W. Thompson's "Concrete Materials and Teaching for Mathematical Understanding" (1994) stresses that mathematical understanding is what students need, not just mathematical learning. The use of concrete materials will aid in mathematical understanding and lead to mathematical learning. Just using concrete materials is not enough to guarantee success. We must look at the total instructional environment to understand the effective use of concrete materials, especially teachers' images of the activities of what they intend to teach and students' images of the activities in which they are asked to engage.

Concrete materials do not automatically carry mathematical meaning for students. It is not easy to use concrete materials well and it is easy to misuse them. The materials need to be examined by the students, broken down into components and put back together again for complete understanding (Ozmon & Craver 1995). Concrete materials serve two purposes.

1. They enable the teacher and the student to have grounded conversations about something concrete. This includes how to think about the materials and the meaning of various actions with the materials.

2. Concrete materials furnish something on which students can act. Also it is important that students can create multiple interpretations of materials. Students are empowered when they recognize the multiplicity of viewpoints from which
valid interpretations can be made. (Thompson 1994).

Mathematical investigations using concrete materials and concepts take place when students explore through inquiry, examination, and research mathematically complex situations. The depth and breadth of these investigations are affected by the students' interests, backgrounds and willingness to dig deeply as well as the topic itself. It is a multi-dimensional exploration of a meaningful topic with the goal being to discover ways to think about math, rather than just "get the answers". Students must speculate, conjecture and generalize as they work through these investigations (Chapin 1998).

The topics of an investigation are often brought to the students by some interesting question of theirs or the teachers'. It may be in the form of a why, or how, or how much, which leads to more questions that require the gathering of data and the conducting of research in order to complete the task. The mathematics is driven by the questions that are asked (Chapin 1998).

Investigations help students to make connections among different areas of math (algebra, geometry, statistics) as they are bumping into them as questions arise from the investigation. This "bumping into" process enables the teacher to pursue related topics, thus making the necessary and interesting connections. In addition, this form of student work assists in creating an environment of inquiry, where student realize they are important in questioning, responding, and reasoning (Chapin 1998).

Among the list of several suggestions for meaningful
units of study which prepare middle school students for algebraic thought, is the concept of proportional relationships (California Mathematics Framework 1992). The idea of proportional relationships is broader than the topics of ratio, proportion and percent which are typically in the middle school math texts. More widely, they are the basic ideas in the quantitative understandings of the world. Students need to explore a variety of situations in which proportional reasoning plays a major role and work to develop a broad sense of proportionality (Framework 1992).

The representation of one quantity as a proportion of another is the key concept, and this concept plays a key role in rate, ratio, percent quantities, per-unit quantities, proportional parts, slope, similarity, scale relationships, linear functions, and probability, all of which involve proportional relationships. "When students complete eighth grade, they should have a solid background of experiences with similarity and simple linear functions" (Framework 1992). Proportional scaling (enlarging and shrinking by use of a scale factor) is especially important as it connects similarity and linearity (Framework 1992).

A description of a suggested unit on proportional relationships is found in the California Math Framework (1992). In general, this unit should take the student from his/her intuitively embedded knowledge of proportional relationships to an expression of these relationships as functions. The students need to learn to use functional expressions to investigate and compare
those relationships with which they are already familiar. The "basic library" of functions are as follows:

- pure linear functions of the form $y = kx$ which expresses a relationship between $y$ and $x$
- offset linear functions of the form $y = y_0 + kx$ which expresses relationships between $(y - y_0)$ and $x$
- inverse functions of the form $y = k/x$

The framework is adamant in its views of what needs to be done in the middle schools. "Curriculum developers will need to develop units that give students appropriate experiences with the range of issues described in the standards, emphasizing unifying ideas", one of which is proportional relationships.

The desire to have algebra placed in the eighth grade curriculum has been a source of discussion for many years. It is a concern to educators and psychologists that many students at the middle school are not cognitively ready to tackle the abstract nature of algebra. A look at what algebra is and is not is important in making the decision about offering algebra to these students. At the National Council of Teachers of Mathematics (NCTM) Annual Convention in Indianapolis in 1994, president, Mary M. Lindquist stated that a course in first year algebra needs to be reconceptualized. She stated that there were two guiding principles in this reconceptualization.

1. There needs to be a goal to develop confidence and facility in using variables and functions to model
relationships both abstractly and in relation to worldly settings.

2. The use of technology allows the focus to be on modeling more complex concepts which are attainable for students of a very broad interest and ability range. Students are in possession of powerful learning tools and strategies (Lindquist 1994).

The learning of algebra before 9th grade is common in other countries of the world (Wirszup & Streit 1987). A strong seventh grade course would prepare students to complete algebra in the eighth grade. In addition, the present seventh and eighth grade curriculum, based upon textbooks used throughout the United States, contains only 33% new material in the seventh grade and 29% in the eighth grade (Flanders 1987). The algebra course and text offered at Los Alisos is one whose curriculum addresses these concerns. Author, Zalman Usiskin (1995) states that unless algebra is taught before ninth grade, the broad curriculum recommended by the NCTM Standards document will not be able to be taught in the limited time of four years.

200 teachers using the University of Chicago School Mathematics Project (UCSMP) algebra program attended a conference, and more teachers reported using the text with their eighth grade students than with ninth graders; and these were not honors students (Hirschhorn, Thompson, Usiskin, & Senk 1995). In the UCSMP texts, students become accustomed to dealing with real world
applications daily, become competent with a scientific calculator and graphing calculator, and have had quite a few geometry experiences (Hirschhorn et al. 1995).

In summary, we have found that all related literature substantiates the need for a heterogeneous group of middle school students to be taught a cohesive unit which prepares them to take algebra as eighth graders. These students need to experience a hands-on type of curriculum, which raises their cognitive levels of understanding as it strengthens their mathematical concepts and skills. Further the unifying concept of proportional relationships is a strong introduction and preparation for the more abstract study of algebra.
CHAPTER 3

A Unit On Proportional Reasoning: A Preparation For Algebra

This unit is intended to introduce 7th grade students to algebraic thinking. Although this unit is focused on proportions and ratios, those concepts are treated via the mathematical strand of "measurement". The unit starts with several lessons on ratio, where the students explore the comparison of two numbers via hands-on measuring experiences. Students will be exposed to the special ratio called the Golden Ratio or Golden Rectangle.

There are lessons related to scale factors and similarity, as they relate to proportionality. The students will perform simulations and physics experiments to bring more meaning to the concept of proportion.

In an attempt to move the students to a higher level of thinking, they will discover the algebraic relationships \( \frac{Y}{X} = K \) and \( XY = K \), also known as direct and inverse variations. They will be asked to measure objects remotely and to graph their results, thus using multiple representations of the concepts. The trigonometric function of tangency will also be presented as students use this ratio to find unknown quantities (heights).

Finally, the students will incorporate all of their experiences into a culminating project that demonstrates their proportional knowledge.

The lessons, teacher plans, and instructions will be found in this Chapter 3. There is a brief description of each lesson at the beginning of each lesson plan describing
intended activity. Also in quotation marks are the journal writes. Things teachers need to say to the students are in quotations as well. All worksheets which accompany the lessons will be referenced in the lesson plans and found in the appendix. No attempt has been made to incorporate daily homework of the drill or practice variety, nor have daily quizzes been created. The individual teacher needs to look at the skill needs of the class to more accurately determine what kind of practice or assessment is necessary. Homework that is directly related to and upon which the next lesson is contingent are also included in the appendix.

**Multiple Intelligence Test**

**GOAL:** Determine the various learning modalities of the individual students for the purposes of grouping.

**TIME REQUIRED:** 20-30 minutes for the test, and 10-15 minutes for processing.

**MATERIALS NEEDED:** Class set of Multiple Intelligence tests (Appendix A).

**PROCEDURE:**

**Set Up:** Distribute tests and have students work individually.

**Implementation:** Allow the students 30 minutes to take the test and create a bar graph displaying their results.

**Processing:** Have students report the intelligence on which they scored the highest, and create a class bar graph recording that data. Have the students create a Journal Write. “Did you feel that the test truly indicates the way you learn? Were you surprised by the results. Give
Piaget Test For Cognitive Development Stages

GOAL: To determine students' cognitive development stage individually and as a group.

TIME REQUIRED: 20-30 minutes.

MATERIALS NEEDED: Class set of four index cards with the numbers 7, 1, 9, and 4; clock for timing; Recording Sheet for each student (Appendix B).

PROCEDURE:

Set Up: Pass out set of four cards to each student. Pass out the recording sheet to each student.

Implementation: Instruct all students as follows:

"Here are four digits; 7, 1, 9, and 4. These four digits can make the number 7194. I want to see how many different numbers you can make using the four digits: 7, 1, 9, and 4. Another number using the digits would be 4719, for example. See how many members you can make using the four digits. Remember, you must use all four digits, and each digit can be used only once in any number. Write down all the numbers on the recording sheet." Allow the student a maximum of five minutes or until he says he is finished.

Processing: After the student has written each number on his recording sheet, have him write a short sentence to explain how he went about generating the list. Have students share their problem solving strategies.

Mathematical Assessment Test

GOAL: To determine the mathematical skills of entering 7th graders. Ultimately to assess the mathematical growth at the
end of the year.

**TIME REQUIRED:** 45-50 minutes.

**MATERIALS NEEDED:** Copies of the test (Appendix C) for each student. Scan-tron answer sheet for each student. Scratch paper for students to do their work.

**PROCEDURE:**

**Set Up:** Copy test for each individual student. Procure scan-trons for each student. Gather up scratch paper.

**Implementation:** Distribute copy of test and one scan-tron answer sheet to each student. Have students write their names in the appropriate place on the scan-tron and instruct them to not write on the test.

**Processing:** Only teacher processing is required as a thorough examination of each student score will indicate the strengths and weaknesses of the student, individually, and of the group as a whole.

**Lesson #1: An Introduction To Ratio**

In this activity, students will examine different geometric shapes and find ratios of their properties.

**GOAL:** To introduce the student to the meaning of "ratio" and learn the 3 different ways to write a ratio. The students will discover (review) the decimal equivalences of each, and also review measuring with a ruler using their choice of metric or standard units to find the lengths of sides of geometric shapes and their perimeters and areas.

**TIME REQUIRED:** 1 hour.

**MATERIALS NEEDED:** Copy of the worksheets (Appendix D & E) for each student, a ruler with centimeter and standard units.
PROCEDURE:

Set Up: Copy worksheets (Appendices D & E) for each student and gather rulers for each.

Implementation: With the class discuss their pre-learned meanings of ratio, leading them to the idea that ratio is the comparing of one number to another. You might have the students give the ratio of boys to girls and ask a student to write that on the board. He/she will use one of the forms, either B:G, B/G, or B to G, and then illicit from the group the other two ways to write the ratio. Ask the class if they know another way to write the fractional form, thus getting to the fact that often ratios can be written as a decimal number. Pass out the worksheet and have the students proceed to work.

Processing: Have students work in their groups to determine if they all agree upon the correct numbers, and discuss the differences between those that used metric versus standard units. Discuss the ratio that exists between one metric and one standard measure (1 lin : 2.54 cm). The students should be able to see that the inch measure compares to the centimeter measure as approximately 1 to 2.5. Discuss as a class the similarities and differences between the opposite ratios, that is A:B and B:A. Discuss the differences in their decimal equivalences, and what that is called (reciprocal). Ask the students to examine their calculator keys to find one that will do the calculations for them (1/x). Discuss the ratios of the perimeters of the figures compared with the sides, and then to the areas of the
figures. They may deduce that the ratios of the sides and the perimeters are the same, and the ratios of the sides and the areas are a lot different. Some may see that the ratio of the side to the area is the side to the square of the side. Discuss whether or not this is a coincidence, by testing on the overhead or board another example of a rectangle or triangle.

Lesson #2: The Golden Rectangle And Ratio

In this activity, the students will construct a rectangle with the properties of the Golden Ratio and find the decimal equivalent for the ratio.

GOAL: Students will discover the ratio of a very special rectangle, which they will construct, to be the Golden Ratio. The Golden Ratio, which is the ratio found in much architecture, often in nature, and in many abstract mathematical concepts, is equal to the ratio of the length of this very special rectangle to its width and is equal to 1.618. Students will examine and discern, by looking at many pictures, if the golden ratio exists in them, and then examine various parts of the body in which the ratio exists. The students will review using a compass as they construct the rectangle.

TIME REQUIRED: 2 hours.

MATERIALS NEEDED: Transparency of Golden Rectangle Construction (Appendix F); rulers; compasses; transparency of God and Goddess with ratios shown (Appendix G); transparency and classroom set of worksheet of Animals (Appendix H), Shapes (Appendix I) and the Parthenon (Appendix J); overhead
or board compass and ruler; worksheet for homework on "Measuring Parts of the Human Body" (Appendix K).

PROCEDURE:

   Set Up: Make transparencies, gather compasses and rulers, copy worksheets of animals and shapes (Appendices G & H).

   Implementation: Pass out needed supplies and paper on which to do construction. Go through instructions on overhead transparency with class, and then instruct them to proceed using any side square they wish. Have students measure the length and width of their rectangles and write the ratio as a fraction and a decimal. As students complete the calculations, have them come to the board and record on a scatter plot the lengths and widths of their individual rectangles.

   Pass out worksheets of the animals and objects to each student and ask them to measure the various parts of the figures, trying to locate measurements which yield the golden ratio. They need to complete this for homework if it is not completed in class. On the following day, do the processing on the first two parts of the activity, and proceed with the human body ratios. Place the transparencies on the board and have a student come up to the board and measure the indicated distances on the bodies. You may wish to cover the "private parts" up with stickers or clothes. Have the students at their desks calculate the golden ratios as they are measured. The students should then go around the room with a ruler and try to find objects that illustrate the golden ratio. Make a
list on the board of the objects and their ratios as they are found by the students. (Make a large poster of these objects and they will be a fine bulletin board.) Pass out the worksheet on the human body measurements, and allow the students to start measuring themselves and recording the data. Have them finish the sheet for homework.

**Processing:** Discuss with the class the results of the different squares with which they started and the results of the scatter plot. "Why are the points of the scatter plot on the same line?" "Can you write an equation for the rule that takes the width of the rectangle to its length?" Have a transparency of the homework on the overhead and have individual students come up and show where they found the ratio. As the students measure the body parts of the classical sculptures of the god and goddess, ask "Do you believe that there is a difference between the male and female idea of classic beauty, and why?" "What is the golden section of each?" After completing the homework, ask "Did you find anyone on your list with classic beauty?" "Did you find a Golden Section different than the ones on the sculptures?"

**Lesson #3: Scaling To Proportion**

In this activity, students sample and explore figures to be enlarged and get a taste of cartography in scaling up a map. They have the opportunity to work cooperatively as a small group to produce a large product.

**GOAL:** Students will understand and use a scale factor to draw a scale drawing in proportion.
TIME REQUIRED: 1 hour.
MATERIALS NEEDED: Worksheet (Appendix L) with cartoon to be enlarged as a pre-*homework* assignment; small pieces of a map, which have been cut to identical size pieces; plain white drawing paper; rulers; colored pencils or crayons; tape to connect final enlargements together.

PROCEDURE:

Set Up: Have students complete the small cartoon enlargement prior to this lesson. (Appendix L) With a copy machine, copy a relatively large (as large as your copy machine will accommodate - 11"x14") map of the United States, Europe, or other region of the world or interest (perhaps a continent or country presently being studied in Social Studies class). Divide the copy into equal sized pieces, so that each student has one piece. You may need to have several different large maps for each class depending upon the number of students in the class.

Implementation: Pass out one piece of the map to each student. Have the students grid their map piece in any way they wish, and duplicate that grid, making it 2 or 3 times larger on the plain piece of white paper. The class needs to determine whether they will all use the scale factor of 2 or 3. Make sure you use that term as you direct them in the enlargement of the grid. (They may want to use inches or centimeters, and as long as the grids are in the same relative positions on the map as they are on the plain paper, it's all right.) The students then take each grid of the original map and duplicate what they see on the map to the
enlargement on the white paper. Emphasize that every thing will be enlarged; roads, border lines, rivers, words, rivers, etc. After they finish their drawings, they need to color their part of the map. You may have the students work in groups and coordinate the colors used, or just use whatever color they wish. This part of the project may need to be finished at home for homework. Upon returning to class the next day, have the students get together and decide which part of the map goes where, and tape together to form a map 2 or 3 times larger than the original. This also makes an impressive bulletin board.

Processing: Have a copy of the original map near the new enlargement, and ask the students the following questions, either as a classroom discussion or as a Journal Write: "What was the scale factor that the class used for the project, and what does that mean?" "If the distance between two cities was X miles on the original, how many miles does the enlargement represent?" "If the distance between two cities was X centimeters on the original, how many centimeters does the enlargement represent?" "What happened to the distance on the map?" "If a park measured 2" by 4" on the original map, what were the measurements on the enlargement?" "What was the area on the original map; the enlarged map; the park in real life, if the original scale was 1" to 10 mi.?"

Lesson #4: Drawing And Graphing On Distorted Grids

In this activity, students will connect mathematics to art by investigating and replicating drawings and designs on
various non-routine grids and surfaces.

**GOAL:** Students will investigate the results of graphing onto grids that are distorted (morphing), and be introduced to "Anamorphic Art". Students will explore the reflection properties of the mirrored cylinder, by seeing how the cylinder distorts basic line relationships. They will also gain experience in locating reflections in the mirrored cylinder.

**TIME REQUIRED:** 1 - 2 hours.

**MATERIAL NEEDED:** Copies of the worksheets for each student. (Appendices N, O, P, and Q); tape; reflective mylar; cardboard tubes; colored pens or pencils. The mirrored cylinders needed for the activity can be made by taping the reflective mylar or reflective origami paper around the outside of a cardboard tube, such as a paper-towel or bath-tissue roll. Do not remove the adhesive backing on the mylar if it comes that way.

**PROCEDURE:**

**Set Up:** Copy worksheets (Appendices N, O, P, and Q) for all students; gather cardboard rolls; purchase mylar or reflective origami paper.

**Implementation:** Read the story on the following page(s) about Anamorphic Art, and show photos and pictures of mirrored cylinder art (Appendix M). Have students complete the first two worksheets (Appendices N & O). Have students construct the mirrored cylinders by taping vertically the overlap of the reflective material to the cardboard tubes. Then have them do the last two worksheets (Appendices P & Q).
On the last worksheet "Creating an Anamorphic Art Piece", the students create an easy monogram or an easy geometric design on the standard grid, and then transfer this drawing to the circular grid. Once they have transferred the sketches to the circular grid, have them cut along the heavy horizontal line. The resulting sheet will display their anamorphic artwork with no hint of how they drew the sketch. More talented or artistically advanced students can go further than the monogram or design, creating pictures, etc. They may need to finish the project as homework if you don't wish to use two class periods.

**Processing:** Have students present their anamorphic art to the class and see if they can predict what the design would be on the cylinder. An extension would be to use the mylar to make a reflective cone, and then replicate the activities. (When viewed from above the vertex of the cone, the mirror reflects the full 360 degrees of an anamorphic artwork. Class discussion could center on the following questions: "Where do you think this type of distortion might occur?" "What relationship does this have to scale factor and proportionality?"

**The Story Of Anamorphic Art**

It is 1655 and you are living in England, a supporter of the royal family who just got kicked off the throne. If anyone were to know of your allegiance to this now deposed family, your life would definitely be in danger. But, being the loyal subject that you are, you carry around a picture of
your revered and executed king, Charles I, with you at all times. You are confident that you will not be arrested because, although everyone knows what the king looked like, your picture will not give away your affection. Why not? The portrait of your king was done in an anamorphic-art style, and it cannot be recognized by anyone who does not know the secret of this art style.

Anamorphic art refers to artwork that is indistinct when viewed from a normal perspective, but becomes recognizable when the image is viewed from a different perspective or reflection. The term is derived from two Greek words meaning to change again. Leonardo da Vinci was an early experimenter with anamorphic art. He produced black and white sketches that were distorted from a normal view, but formed human faces when viewed from the extreme edge of the canvas.

Anamorphic art which uses reflective or mirrored cylinders was thought to have originated in China about 700 years ago. Although there are some examples dating back to 1575, the secret was not divulged until around 1630. It soon became the rage, and many examples were found in wealthy estates throughout Europe, and it soon became as common as murals are today. By 1860, the fascination for this art style died out as people focused on the photograph instead to show pictures of life of that time.

Two main techniques are available for anamorphic art. The Chinese artists apparently drew their pictures while concentrating on the reflection in a polished cylinder or sphere. In contrast, the European artists used the grid
system which is presented today in this activity. Soon you will be able to draw an anamorphic art piece that may not be able to be recognized unless you use a mirrored cylinder (Johnson & Dean 1998).

Lesson #5: Building Similar Figures

In this activity, students will build similar shapes with pattern blocks to see similarity, scale factors, and ratios of the sides, areas, and perimeters of the figures. GOAL: Students will use pattern blocks to form similar figures of differing sizes. They will investigate the relationships between the scale factors of similar figures and the areas and perimeters of those figures.

TIME REQUIRED: 2 hours.

MATERIALS NEEDED: Set of pattern blocks for each group of four students; measuring tape or meter stick; copies of the lesson (Appendix R) for each group.

PROCEDURE:

Set Up: Gather sets of pattern blocks for each group after removing the hexagon blocks as they do not tessellate to similar figures. Run off copies of the lesson instructions (Appendix R) for each group of students.

Implementation: Distribute the pattern blocks and the worksheet. Read the instructions over with the class, before setting them to work. Emphasize that each group member should have a different block, and that they must use all of that same block for building their figures, and that the final figure must look exactly like the original block. Encourage students to create a recording sheet to sketch each figure.
they construct, recording how many blocks they used, and the
perimeters and areas of each. Demonstrate how to align the
individual figure over the constructed similar figure, as
well as how to measure the distances of eye to piece and eye
to figure. For some of the larger figures, they may have to
set the piece on the floor. (But don’t tell them that unless
they worry about it as they are doing the activity.) As they
begin to measure perimeter and area, they may become
concerned with what unit of measure to use. Let them use what
ever unit standard, metric, or even the side of the block as
their unit of measure.

Processing: Have each group report on their discoveries
about the various shapes. You should ask the following
questions either as a class discussion or as a Journal
Write. “What is the ratio of the eye measures?” “What is the
ratio of the size of the sides of the block to the size of
the sides of the constructed figure?” “What is the ratio of
the area of the block to the area of the figure?” “What is
the ratio of the two perimeters?” “What is the ratio of the
eye measure to the areas of the figures?” “Can you make a
rule that will work for all figures?”

Lesson #6: Sharks In A Box: A Tagging And Recapturing
Simulation (Curcio & McNeece 1994)

In this activity, students appreciate the “power” of a
simulation in which sharks are represented by cubes and the
lagoon is represented by a shoe box. They will “tag” sharks
and test the population of the lagoon and ultimately use
proportions to estimate the total number of sharks in the
lagoon.

GOAL: Students will estimate the number of cubes in a box using data which they have organized and collected by using the mathematical concept of proportion. They will use equivalent fractions to represent proportions and discover how to solve a proportion by seeing the equivalency which must result in the simulation.

TIME REQUIRED: 1 hour.

MATERIALS NEEDED: One shoe box per group of students; 150-250 centimeter cubes or other easily accessible and taggable object; "Sharks" video clip from Estimation: Am I Close? (The Challenge of the Unknown 1986); video playback equipment; activity Sheet (Appendix H); one package of 450 self-adhesive circular color-coding labels (6mm in diameter).

PROCEDURE:

Set Up: Label each shoe box with a different letter, and fill each with around 150-250 centimeter cubes; run off the recording sheet (Appendix S) for each student; set up the video machine to the proper place on the tape.

Implementation: Give the following background information for viewing the four-minute video clip on estimating the number of sharks in Bimini Lagoon: Samuel Gruber is a marine biologist at the University of Miami who studies lemon sharks. In the film Gruber struggles with the problem of determining how to estimate the number of sharks in Bimini Lagoon (Maddux 1986). Pose this question, "What are some ways that you think he can estimate the number of sharks in the lagoon?" Then show the video clip. Discuss the
methods used by Gruber. Discuss also the concept of simulation emphasizing the need to do so when expense or size or time is unreasonable to hold an actual test. Ask the students to explain a way to use the box and the cubes to simulate what they saw in the film. Explain how they will take out a sample number of cubes and "tag" them with the circular adhesive discs, "throw" them back into the "lagoon" and have the "sharks" "swim" around (shake up the box). Then they will take out another sample and see the ratio of tagged to untagged "sharks" and record all data on the recording sheet they are given. Distribute the boxes ("lagoons") filled with cubes ("sharks"), and the recording sheet and have them do the simulation five times, completing the recording sheet only up through #4. Emphasize that the tagging is done only once, before all student do their "shark fishing". Have students complete the worksheet only after the first paragraph of the Processing is completed with the class.

Processing: This is the meat of the lesson, and may require a few preemptive questions on arithmetic. Ask the students to quickly find N in each of the following equations:

\[ \frac{3}{N} = \frac{1}{2}, \frac{7}{N} = \frac{2}{8}, \frac{12}{N} = \frac{25}{100}, \text{ and } \frac{3}{N} = \frac{17}{38} \]

Spend time finding out what the students know about equivalency of fractions, and how they go about multiplying and/or dividing to find the answers. Get them to the point where they see that cross-multiplication to solve for N will work for all situations, especially those where it doesn’t work out by simplification of fractions. Discuss ideas for
using information about \( T \) (the number of sharks tagged), \( t \) (the number of tagged sharks recaptured), \( n \) (the total number of sharks captured), and \( N \) (the total number of sharks in the lagoon). Ask students to describe the many different ratios that exist within the data, and see if they can see words and numbers that would make two ratios that mean almost the same thing: \( t/n \) and \( T/N \). Ask the students if that relationship makes sense and why or why not. Gather data from one of the groups and plug in the number \( t \), \( n \), and \( T \), and calculate with the class the number for \( N \), the total number of "sharks in the lagoon", which, of course, is the number of cubes in the shoe box. Ask: "Does this number seem reasonable?" Do another example from the same group and see if this is a number that is also reasonable for the previous example.

After the students have continued to calculate all the data on the recording sheet and answered the questions contained therein, ask the following questions either as a classroom discussion or a Journal Write or as a homework assignment: "What is a proportion?" "How are proportions related to equivalent fractions?" "How can proportions be used to estimate the size of a population?"

**EXTENSION:** Students may wish to make a simulation similar to this by counting and recording the number of dogs and cats in their neighborhoods. A report on what the forest services do to count animals could be an interesting extra credit assignment.
Lesson #7: Exploring Light And Proportion With Pinhole Viewers

In this activity, students will explore the physics of light while making a pinhole viewer and using proportions to find a indirect distance from a light source (Lambertson, 1997).

GOAL: Students will learn a scientific reason for using proportion and discover the way to find distances that cannot be physically measured.

TIME REQUIRED: 1 hour.

MATERIALS NEEDED: Cardboard paper towel or toilet tissue roll for each student; small piece of waxed paper (5cm x 5 cm) for each student; small piece of aluminum foil (5cm x 5 cm) for each student; 2 rubber bands for each student; push pin per group; overhead projector; recycled manila folder for each group; one pair of scissors for each group; metric measuring tape or meter stick for each group; masking tape for each group; lab directions and recording sheet for each student (Appendix T).

PROCEDURE:

Set Up: Gather all equipment above and put enough of the equipment into a box for each group. Talk about what they know about light and how it travels, emphasizing the fact that it travels in a straight line, always. Set up the overhead projector in the corner or front of the room.

Implementation: Pass out the supply box to each group and have them distribute the equipment to each member. Have the students read and do the list of instructions on the Lab
Recording Sheet (Appendix T). After the students have built their individual pinhole viewers, cut out a triangle (around 5cm for the height) from the middle of the manila folder, and place the folder over the glass of the overhead projector, so that the only light escaping is the image of the triangle. Turn on the projector and focus so that you have a clear image of a triangle. First ask the students what they think they will see as they look through their pinhole viewers. Then, ask the students to describe the projected image as to its shape and estimated height. Have the students do the experiment following the directions on the Instruction and Recording Sheet (Appendix T). Circulate around the room to assure all are on task and measuring the correct distances.

Processing: The following questions will naturally arise as the experiment is taking place. "What happened to the image on the waxed paper?" (It is reversed - up side down.) Have some one come up to the board and draw a figure which explains what and why this happens. It should look something like this:

![Diagram](image-url)

- Waxed Paper Image
- PINHOLE
- Projected Overhead Image
A discussion of similar triangles could take place at this point, noting that the vertical angles are formed at the pinhole, making similar triangles on both sides of the pinhole. Questions to ask the class: "What happens to the image on the waxed paper as you move closer or farther away from the projected image?" "What happened to the distance to the projected image when a member of your group had a tube smaller or larger than yours?" "Did your distances which you calculated and then measured come close to each other? Why or why not?" "What is meant by reasonable error, human error, expected outcome?"

Lesson #8: Color Mixing With Your Pinhole Viewer

In this activity, the students use the pinhole viewer to extend the previous lesson on light, and examine what happens when colors are mixed and observed by the human eye. This is more an art/physics lesson, but uses the previous tools.

GOAL: The students will explore what happens when primary colors are mixed together, and how the human eye works.

TIME REQUIRED: 15-20 minutes

MATERIALS NEEDED: Pinhole viewers for each student; a pushpin for each group; a power strip; 3 light sockets which can be plugged into the power strip; 3 "party" bulbs, one of each color: red, blue and green.

PROCEDURE:

Set Up: Gather the supplies, and screw the bulbs into the sockets. Make sure that there are still enough pinhole viewers for each student.

Implementation: Distribute the supplies to each student.
and group. Place one socket and bulb into the power strip and turn off the room lights. Ask the students what they think they will see when they look through the pinhole viewer. Have the students look through the pinhole viewer, holding the tube about 30 cm. away from their eyes, with the waxed paper side toward their eye and the aluminum side toward the light source. Ask the students what they think they will see when a second color is placed into the power strip. Then place a second light into the power strip. Hold the bulbs vertically and ask the students to look at the strip without the pinhole viewer and then through the viewer and describe what they see. (The top light will be on the bottom and vice-versa.) Have the students make a second hole in the viewer using the pushpin, and observe the two lights again. Now add the third light and ask what they think they will see before they actually observe the result. (The three lights will be discernible and in reverse order.) Make a pattern of pinholes in the viewer and look at the lights again. Have them use their little finger and make a hole that large. Ask them what they notice when they look at the lights. (They will see different colors than the three original ones.) Secondary colors are formed where the two primary colors are overlap. Ask the students what colors are being mixed to form what other colors.

Processing: Ask the students if they know how the eye works at sorting out these colors. You might assign a report or talk with the health teacher to integrate this activity with the study of cones and receptors of the eye. "What
happens when we mix all three primary colors of light?" (White light.) The cones of our eyes respond only to red, blue and green, and when all three are firing simultaneously, the color is white.

**Lesson #9: Ratios Of Afterimages Of Light**

In this activity, the students will stare at a colored square and measure area and distance of the afterimage created by the eye.

**GOAL:** Students will see yet another use for ratio and proportion and practice this concept in the context of the physics of light.

**TIME REQUIRED:** 50 minutes.

**MATERIALS NEEDED:** Brightly colored paper, scissors, and tape OR brightly colored pencils for each student; pencil; 1" grid paper for each student; a brightly lighted room or area; measuring tape for each pair of students.

**PROCEDURE:**

**Set Up:** Gather the materials, and cut out a 2" square of the brightly colored paper for each student, or get some very brightly colored pens. Make sure all the lights are working in the class room or arrange to do the experiment outside on a very sunny day.

**Implementation:** Pass out the materials to each student. Have the students glue or tape the colored paper in the center of the white side of the 1" grid paper (or draw a two inch square in the middle of the white side of the grid paper) and have them place a pencil dot in the center of the square. The students should hold the paper in their hands,
perpendicular to the floor, colored square facing up, and stare at the center of the colored square for about 30 seconds (using the dot to help them focus.) After about 30 seconds, have them QUICKLY turn over the paper, and move the paper grid further from their eyes. Move it toward the eyes again. Ask them what they notice. Have the partner hold the paper for them this time. Before they stare at the square again, measure the distance their eyes are away from the paper. Record this data. Stare again for 30 seconds and have the partner quickly flip the paper over and move the paper away from them until the "after image" square has sides that are twice the length of the original square. Measure the distance from the paper's new position to the eye and record that data. Ask the students to comment on the following question. "If the original square has a side equal to 2 in. (and covers 4 sq. in.), and the afterimage of the square has a side equal to 4 in. (and covers 16 sq. in.), what do you think the relationship between the two distances from the paper to your eye might be?"

Now, have the students try this again, only this time move the grid closer to your eye after you have stared at the square for 30 seconds. What happens to the afterimage? Stop moving the grid when the afterimage appears to fill one square inch on the grid. Have the students hold the paper at this position while their partner measures the distance from the paper to the eye and the data is recorded.

Processing: Ask the following questions: "Can you guess the relationship between the sizes of the images and the
distances between the paper and your eye?" Have the students fill in the following chart.

<table>
<thead>
<tr>
<th>Distance</th>
<th>Height of Square in inches</th>
<th>Ratio of Ht. of Paper square to Ht. of image</th>
<th>Area of square in square inches</th>
<th>Ratio of paper square to area of image</th>
</tr>
</thead>
<tbody>
<tr>
<td>(initial distance)</td>
<td>2 inches</td>
<td>2/2=1</td>
<td>4 sq. in.</td>
<td>4/4=1</td>
</tr>
<tr>
<td>4 inches</td>
<td>4 sq. in.</td>
<td>4/4=1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 inch</td>
<td>1 inch</td>
<td>1/1=1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ask the following questions: "What color did you see for the afterimage?" "What is the relationship of the afterimage size when the distance is twice as far as the original distance, half the distance?" "What proportion can you write to find the distance the images are away from the eye?" "What other ratios do you find in the chart that you can use in other proportions?"

**Lesson #10: Proportions And Projections With Lenses And Lanterns**

In this interactive physics activity, the students will be using light, lenses, and a projection screen to investigate proportional relationships of the distances between them.

**GOAL:** Students will gain experience using proportions in the scientific context of the physics of light, as well as learning about physics concepts concerning lenses and projections.

**TIME REQUIRED:** 1 hour.

**MATERIALS NEEDED:** Light source (single 2.3 light cut from a string of mini tree lights wired to two 1.4 V. batteries);
foil lantern (10 cm square of aluminum foil, from which a triangle of altitude 1 inch has been cut and covered with paper; 3" x 5" card with a binder clip stand; 2" "bug box" (acrylic box with a built-in lens); masking tape; metric measuring tape or meter stick; and worksheets for each student (Appendix U).

PROCEDURE:

Set Up: Gather all equipment and cut up the tree lights; make the lanterns (or save for students to do, if time allows). Run off individual worksheets (Appendix U).

Implementation: Distribute materials to each group as well as direction/worksheet (Appendix U). Have the students follow along on their instruction sheet up to the point where they start investigating distances on their own. Make sure students are on task and that each group has set up the experiment correctly.

Processing: Have the students write a report using these questions as a guide, or have them do these questions as a Journal Write or homework. "What happened as you moved the lantern away from the lens?" "Was your measured Hi exactly the same as your calculated Hi?" "If it was not, what do you think accounted for the difference?"

Lesson #11: "Only The Shadow Knows"

In this activity, students will use proportions to find the height of an object by using the length of shadows.

GOAL: Students will experience a practical use for proportions and see how similar triangles are formed by looking at the height of an object and its shadow. They will
measure objects, and shadows, and work cooperatively with each other to find a good answer.

**TIME REQUIRED:** 1 hour on a sunny day (preferably not at high noon).

**MATERIALS NEEDED:** Meter sticks for each pair of students; calculators; copy of directions for each student (Appendix V).

**PROCEDURE:**

**Set Up:** Gather meter sticks and run off the worksheet for the lesson (Appendix V).

**Implementation:** Discuss with the class when and where shadows are seen. Have several students come to the board and draw objects with their shadows. Discuss the similar triangles that are drawn. Ask the class what the time of the day has to do with shadows. Explain to the class that they will be using proportions of similar triangles to measure the height of an object that is too tall for them to measure. Discuss what proportions will look like for similar triangles. For example:

\[
\frac{\text{Height of object}}{\text{Length of shadow of object}} = \frac{\text{Height of friend}}{\text{Length of shadow of friend}}
\]

Go over the directions with the class and have them go out with recording materials and do the experiment.

**Processing:** Have students share the heights they found as they measured. Compare the discrepancies which may occur because of human error, etc. Have the students explain reasons why they may all get a different answer. Talk about reasonable answers, and how to tell which measurement is not reasonable.
Lesson #12: A Special Ratio For Circles

In this activity, the students will measure many circles and discover the ratio of Pi and the resulting equation of $Y/X = K$.

**GOAL:** Students will explore the relationship of the circumference and diameter of circles and the ratio which exists, and that it is equal to $\pi$ (Pi). Students will measure with either standard or metric measures and ultimately graph the relationship on a scatter plot. They will extend that concept to the $Y/X = K$ family of graphs.

**TIME REQUIRED:** 1 hour.

**MATERIALS NEEDED:** A circular object for each student in the class to measure (preferable all of different dimensions); string for each student to encircle their circular objects; tape measures or meter sticks for each group; newsprint paper for each group; calculators for each student.

**PROCEDURE:**

**Set Up:** Gather all the circular objects you can find that are measurable by the students (plastic plates, paper plates, trays, etc.) and put a variety of sizes out for each group. Measure out a large piece of string for each group, making sure that it is as long as the longest circumference. Make sure there are enough meter sticks and/or measuring tapes for each group. Place a large first quadrant graph on the board, either with chalk or on butcher paper, leaving the scale of the numbers off until the data is collected by the students.

**Implementation:** Pass out all the materials. Direct the
students to draw on the newsprint paper each circle and then measure the diameter and circumference of each, recording those dimensions next to their circle. In addition have them set up a chart for all of their group's data with the following calculations: circumference + diameter, circumference - diameter, circumference x diameter, and circumference ÷ diameter. After they have completed the measurements and calculations, proceed with the processing of the activity, which is the meat of the lesson.

**Processing:** Ask the student what in general they found as they measured the circles. Ask them what they found in the chart of calculations they did with the circumference and diameter. They should see only that things got bigger and smaller depending upon the operation. List each group's calculation data in a chart on the board or overhead. Hopefully their last calculations of circumference ÷ diameter will all come close to 3.14. At this point a discussion about $\pi$ should occur, allowing the class to tell what they already know about this number. "Did the circumference depend upon the diameter or vice-versa?" Since the independent measure is the diameter, we will put those measurements across the x-axis, and the circumference along the y-axis. Ask the students to determine the scale of those axes by illiciting from the group what the smallest and largest numbers they found to be. Have each student come up to the board and place a data point on the graph to show their circumference and diameter. Ask the questions: "What do you observe on the graph?" (dots almost form a straight line)
"What is the ratio that is illustrated on the graph?" \( \frac{c}{d} = \pi \) "Substitute the axis names for the c and d, and state the resulting sentence." \( \frac{Y}{X} = \pi \) Have the students write either as a Journal Write or as homework what they discovered about this lesson.

Lesson #13: Perspective Drawing And Scatter Plot

In this activity, students will explore the concept that the farther away an object is the smaller it's apparent size. GOAL: Students will create a perspective drawing and graph the ratio of the distance away from the vanishing point verses the height of the segment, which will turn out to be linear.

TIME REQUIRED: 50 minutes.

MATERIALS NEEDED: Construction paper; graph paper; rulers; Perspective Drawing picture in transparency form (Appendix W); and student copies of Perspective Drawing Directions (Appendix X).

PROCEDURE:

Set Up: Distribute one piece of graph paper, one piece of construction paper, the Perspective Drawing directions (Appendix X) and a ruler to each student.

Implementation: Conduct a discussion regarding the perspective drawing pictures in transparency form. While showing drawing A (Appendix W), discuss with the students that when you are creating a perspective drawing you need to locate a vanishing point on the horizon. The vanishing point corresponds to the spot where the eye is focused when looking at the real situation. As the objects in the picture are
placed closer to the vanishing point, their size is shrunken proportionately.

Show drawing B (Appendix W) and prompt the students with the following question, "What is wrong with this picture?" In this picture the length of the poles are correct but the spacing is wrong. Once they see that then discuss with your students the idea that when the real poles are equally spaced then the drawing of the poles must get closer together as they get nearer to the vanishing point in order to look real.

Next have students create their own perspective drawing by following the Perspective Drawing directions (Appendix X) and graph the results. Where x = the height of the line segment, y = distance away the endpoint of the segment is from the vanishing point.

Processing: In order to compare results have a few of the students share their drawings and graphs with the class. Put the following Journal Write on the board: "Describe what your graph look like?" "Why do you think your graph looks like what you described?" "Explain the relationship between the x and y coordinates."

Lesson #14: Investigating Scatter Plots For The Family Y/X=K

In this activity, students will investigate the geometric shape of scatter plots arising from the algebraic relationship Y/X=K, which is called direct variation.

GOAL: Students will create graphs for the Y/X=K family and formulate connections between X, Y, and K.

TIME REQUIRED: 3 hours.

MATERIALS NEEDED: Investigating Scatter Plots For The Family
Y/X=K student worksheet (Appendix Y); graph paper; poster board; rulers; markers; and calculators.

**PROCEDURE:**

**Set Up:** Put students into pairs or groups of three. Pass out the worksheets (Appendix Y), rulers, graph paper and poster board to each group.

**Implementation:** Read through the Investigating Scatter Plots For The Family $Y/X=K$ directions (Appendix Y) with the students. Do the following sample problem with them. Have each group come up with a pair of numbers that divide to give 2. ($Y/X=2$) When students give your their pairs, they need to be sure to list the x coordinate first and the y coordinate second, such as (5,10) and not (10,5) because division is not commutative. If they graphed (10,5) that would be for $Y/X=0.5$, which is not the same thing.

**Processing:** Have the students present their poster and give their oral reports to the class about their investigations of $Y/X=K$. Discuss and come to a consensus about the answers to the report questions. **Homework:** Have the students find their K for their perspective drawing.

**Lesson #15: Sighting Your Partner**

In this activity, students are introduced to the use of the centimeter ruler held at arm’s length to measure the apparent height of objects.

**GOAL:** Students will explore the relationship between the apparent height of a person and that person’s distance away.

**TIME REQUIRED:** 50 minutes.

**MATERIALS NEEDED:** Sighting Your Partner directions (Appendix
Z); graph paper; centimeter rulers; and a large, long open space such as a hallway or gym.

PROCEDURE:

Set Up: Distribute Sighting Your Partner directions (Appendix Z), graph paper, and centimeter rulers to each student. Put students into pairs.

Implementation: Explain to the students how to correctly hold a ruler to do this project. The student’s arm needs to be straight out in front of them parallel to the ground. In their hand, they need to hold the ruler perpendicular to the ground. You also need to discuss the difference between a heel-to-toe pace and a walking stride. Then go through the Sighting Your Partner directions (Appendix Z) with the students. Provide the students with a wide and long space to measure their partners from various distances.

Processing: Have students present their sighting diagrams to the class. Have students answer and then discuss the following Journal Write: "Does your graph indicate that there is a relationship between the apparent height of a person and that person's distance away from you? Explain."

Lesson #16: People Proportions With Your Pals

In this activity, students will indirectly measure the height of an object by using a proportion with a partner's height.

GOAL: The students will experience a practical application of proportions by using one to determine the height of an object which is too tall for them to measure. They will measure the apparent height of both a "pal" and the object
and use the "pal's" height to set up a proportion. They will use equivalent measures in the proportions and learn how to change from feet and inches to just inches or feet.

TIME REQUIRED: 1 hour.

MATERIALS NEEDED: Metersticks for each pair of student; calculators; copy of the directions worksheet for each student (Appendix AA).

PROCEDURE:

Set Up: Gather up the meter sticks and run off the worksheet (Appendix AA) for each student.

Implementation: Go over the instructions with the class and, before sending them outside, do an example in the class with a tall object in the room. Emphasize that the students need to make sure they record all their data as they go along - either their own data or the data of their pal. Ask the class what kind of a proportion they would use with the measurements they gather. There are many variations and it would be wise to make general statements about how you know when you have set up a good proportion for a problem. They need to have the object on one side of the proportion and the pal on the other (or vice versa), or the object across the top and their pal across the bottom (or vice versa). In addition the apparent heights must be kept together as must the actual heights.

For example:

\[
\frac{\text{height of pal}}{\text{apparent height of pal}} = \frac{\text{height of object}}{\text{apparent height of object}}
\]

Processing: Have the students report back to the class
and share the heights of the object they measured, comparing the differences of the same object. Ask why there was a difference in their calculations. For homework, have students create their own situation where they are to measure indirectly the height of an object. This can be written in the form of a story problem. Have them solve the problem using proportions, and give to a fellow student to solve the following day.

Lesson #17: Investigating Scatter Plots For The Family $XY=K$

In this activity, students will investigate the geometric shape of scatter plots arising from the algebraic relationship $XY=K$, which is called inverse variation.

**GOAL:** Students will create graphs for the $XY=K$ family and formulate connections between $X$, $Y$, and $K$.

**TIME REQUIRED:** 3 hours.

**MATERIALS NEEDED:** Investigating Scatter Plots For The Family $XY=K$ student worksheet (Appendix BB); graph paper; poster board; ruler; markers; and calculators.

**PROCEDURE:**

**Set Up:** Put students into pairs or groups of three. Pass out the worksheet (Appendix BB), ruler, graph paper and poster board to each group.

**Implementation:** Do the following sample problem with the students. Have each group come up with a pair of numbers that multiply to give 36. ($XY=36$) Read through the Investigating Scatter Plots For The Family $XY=K$ directions (Appendix BB) with the students. Give your students ample time to complete the project requirements and the poster.
Processing: Have the students present their poster and give their oral reports to the class about their investigations of $XY=K$. Discuss and come to a consensus about the answers to the report questions. **Homework:** Have students find their K from the Sighting Your Partner activity.

**Lesson #18: How Does Each Pair Vary: Inversely, Directly, Or Neither?**

In this activity, students will apply their knowledge of inverse and direct variation.

**GOAL:** Students will read problems and decide if the relationship is inverse or direct variation.

**TIME REQUIRED:** 30 minutes.

**MATERIALS NEEDED:** How Does Each Pair Vary: Inversely, Directly, or Neither? student worksheet (Appendix CC).

**PROCEDURE:**

**Set Up:** Put students into pairs. Pass out the worksheet (Appendix CC), to each pair.

**Implementation:** Read through the directions with the students. For your information, here are the answers:1. direct, 2. inverse, 3. inverse, 4. inverse, 5. direct, 6. inverse, 7. neither.

**Processing:** After the students finish the worksheet, review the answers to the questions by having the students vote by show of hands to determine if the answer is inverse, direct or neither. Once the students vote have volunteers explain why they chose the answer they chose.
Lesson #19: Converting Pace And Stride Into Meters

In this activity, students will collect data and make a personal conversion graph to convert any number of their own heel-to-toe paces or walking strides into meters. The conversion graph will also serve as another example of direct variation.

GOAL: Students will make a personal graph that converts their own paces and strides into meters.

TIME REQUIRED: 1 hour.

MATERIALS NEEDED: Graph paper; rulers; Pace and Stride homework questions (Appendix DD); and a 40 meter-long walking course.

PROCEDURE:

Set Up: Before class, lay out a straight-line, 40 meter-long walking course either outside or in a long hallway. The course needs to be marked every five meters in chalk or masking tape. Pass out the graph paper to each student.

Implementation: Instruct students to set up a graph with the x axis being the number of heel-to-toes paces / walking strides, the y axis is the distance in meters. Review again with students what heel-to-toe pacing and walking strides mean. Students need to pace off the 40 meter course in heel-to-toe fashion, collect the data they need and plot a graph. Next they need to walk the course again, pace off the 40 meter course in stride fashion, collect the data they need and plot a graph.

Processing: After the graphs are drawn have students compare their graphs with each other and check for gross
errors. **Homework:** Students need to write a formula for each of their scatter plots and answer the Pace and Stride homework questions (Appendix DD).

**Lesson #20: Meter Field Trials**

In this activity, students will work in teams to prepare and participate in a How Far Away is That Meter field trial, in order to investigate inverse variation.

**GOAL:** Students will estimate the distance to a meter stick from eight different locations using indirect measurement techniques.

**TIME REQUIRED:** 2 hours.

**MATERIALS NEEDED:** Graph paper; rulers; meter sticks; student's personal pace/stride graphs; How Far Is That Meter? Field Trial Directions (Appendix EE) and Score Card (Appendix FF) student worksheets; and a large outside area for the meter trial.

**PROCEDURE:**

**Set Up:** Day 1 pretrial day: To prepare the students for the meter trial, group the students into pairs and distribute the graph paper one per person, the rulers and meter sticks one per pair. Day 2 field trial day: Prior to the field trial, somewhere outside put up a meter stick at eye level that is clearly visible from several vantage points. Lay out eight stations that range from 5 to 40 meters away from the meter stick and label them stations 1 to 8. Make eight groups of approximately four students. Students need their rulers, Pace/Stride graph, the Pretrial graph, and Field Trial worksheets (Appendices EE & FF).
Implementation: Day 1 pretrial day: Within the pairs of the students, one student needs to hold a meter stick at arm’s length perpendicular to the ground. The other student needs to measure the meter stick’s apparent height in centimeters and pace off the distance from where they are to the meter stick. Students need to record at least 8 different sightings and distances and graph the results. Repeat the process for the other person in the pair. Day 2 field trial day: Position one group of students at each marker. Give students five minutes to determine their team’s estimated distance from their station to the meter stick and record on the score card (Appendix FF). After five minutes have the groups rotate to the next station and repeat the process until all groups have rotated through the eight stations. Tell students that the smaller the percentage of error is, the higher their grade will be.

Processing: After the meter trial is over, assemble back in class to give the students the actual distances so that they can calculate their error percentages. Assign group grades based on the percent of error. For example, less than 10% error = "A"; between 11% and 20% = "B", 21-30 = "C", etc. Put up and discuss the following Journal Write: “How did your team decide on your team’s measurements for each station? If you could do the meter trial again what would you do to lower your percentage of error?”

Lesson #21: Trigonometric Ratios

In this activity, students will explore the tangent ratio using direct measuring of triangles.
GOAL: Students will learn how to calculate the tangents of various angles of triangles as well as learn the meanings of the ratios of tangent and sine. The student will also learn to find an angle knowing the tangent ratio. They will explore the graphs of the ratios, and connect them to the concept of $\frac{Y}{X}=K$, and also to the concept of indirect measuring - finding a missing part of a triangle when 2 parts are known.

TIME REQUIRED: 2-3 hours.

MATERIALS NEEDED: Scientific calculators for each student; graph paper; rulers; protractors; worksheets for lesson (Appendix GG).

PROCEDURE:

Set Up: Make copies of the worksheets (Appendix GG) for each student. Gather calculators so every student is assured of having one when necessary, and gather all necessary equipment.

Implementation: Pass out rulers, graph paper and lesson instructions to all students. Read through the lesson with the students and have them start drawing the triangles with the various sized angles, and recording the measured and calculated data as they go through it. Discuss the difference between opposite and adjacent sides and where they are located according to their chosen angle. Have the students practice using the tan key on the calculator, and making sure that all calculators are in the degree mode. Discuss how the calculator has a reverse or inverse tan key as well, and practice using these two functions with the
class. Have them graph the indicated data on the reverse side of the graph paper on which they drew the triangles, and answer the questions. They may need to finish the graph and the questions as **homework**.

**Processing:** Ask the following questions either as a classroom discussion or for a **Journal Write:** "What happened to the tan when the angle kept getting larger?" "What happened to the other angle as the focus angle got larger?" "What do you think you could do to your chart and to the ratios you found to find the tan of the other acute angle?"

**Lesson #22: Using The Tan Ratio To Measure Indirectly**

In this activity, the students will make an angle measuring device and use it to measure the height of an object which is either too tall or inaccessible.

**GOAL:** The students will learn to use the tangent function to find the missing side of a right triangle. They will use a meter stick to measure a distance and learn where to place that in the tangent ratio.

**TIME REQUIRED:** 2 hours.

**MATERIALS NEEDED:** Calculators, meter sticks for each group; transparent tape; string; drinking straws for each student; paper clips for each student; index cards for each student; directions for making a hypsometer for each student (Appendix HH); transparency of Tan Ratio to Indirect Height (Appendix II).

**PROCEDURE:**

**Set Up:** Gather up all supplies and reinforce the
concept of the tan function with the class. Run off hypsometer directions (Appendix HH) for all students.

**Implementation:** Go over several problems with the students where they know the angle of a right triangle and are asked to find the measure of one of the legs, knowing the other one. (Appendix II) For example:

\[
\begin{align*}
45^\circ & \quad x \text{ feet} \\
78 \text{ ft.}
\end{align*}
\]

\[
\tan 45^\circ = \frac{x}{78}, \quad \tan 45^\circ = 1, \quad \text{so} \quad \frac{x}{78} = 1, \quad \text{so} \quad x = 78 \text{ ft.}
\]

This is a very easy example, because the angle used yields a very easy tan and equation. Change the angle degree several times and do the problem again, so that all students know what they need to know and how to solve the proportion.

In order to find the angle, you need either a protractor or a hypsometer, which is similar to a sextant that surveyors use. Have the students cut out the copy of the hypsometer and tape or glue it to the index card. Cut out the outline, and tape the straw to it, with the string taped in the middle and the paper clip as a weight to keep the string plumb to the ground. The place where the string is on the hypsometer is the degree and the students hold it up and sight through the straw to the top of the object which they wish to measure. Have the students look through the hypsometer at the top of the object whose height they wish to measure and observe the angle on the hypsometer. Make sure that all data is recorded as the experiment is carried out. Then they measure the distance from where they are standing to the object. Using
that distance as the adjacent side of a triangle, the students use the tangent of the angle and set up the equation: \( \tan(\text{angle}) = \text{height/distance} \). Using their calculators to find the tan of the angle, they then complete the calculation to find the height of the object.

Have students go outside and measure the height using this method for three (3) tall objects, recording all data and returning to class to calculate.

Processing: Have the students share their results with the rest of the class, listing the measurements of the various objects, and comparing the heights each got for the same object. Ask the students the following questions: "What happened when you used 45° on your hypsometer?" "What did you do to account for your height after your calculation to find the object's height?" For homework or a Journal Write, have the students reply to the following questions: "What does a hypsometer do?" "Describe in your own words the process of finding the height of an object using the trigonometry."

Lesson #23: Final Project

In this activity, students will demonstrate their knowledge of ratios, remote measurement, tangents, graphing, determining direct and inverse variation, perspective drawings, people proportions, and shadow proportions in a culminating individual project.

GOAL: Students will apply all of their proportional reasoning skills and demonstrate their full and complete knowledge of the material presented in this unit.
TIME REQUIRED: 5-6 hours.

MATERIALS NEEDED: Graph paper, rulers, calculators, poster board; outside area for students to choose objects to measure; and the Final Project directions (Appendix JJ).

PROCEDURE:

Set Up: Distribute graph paper, rulers, poster board and Final Project Directions (Appendix JJ).

Implementation: Read through the final project directions with the students. Provide them the opportunity to explore outside the classroom for an object to measure and to conduct their final project requirements.

Processing: Students need to come up with a criteria, as a class, to evaluate the final projects. Students then display their final project on poster board and fellow classmates rotate around and evaluate each other's final project.

Piaget Post Test For Cognitive Development Stages

GOAL: To assess students' cognitive developmental stage individually and as a group.

TIME REQUIRED: 20-30 minutes.

MATERIALS NEEDED: Class set of five index cards one of each of the following colors: red, white, blue, yellow, and green and Recording Sheet (Appendix B) for each student.

PROCEDURE:

Set Up: Pass out a set of five color cards to each student. Copy and pass out the Recording Sheet (Appendix B) for each student.

Implementation: Instruct all students as follows: "A
country wants to design a flag with three different colors. I want you to use these cards: red, white, blue, yellow, and green to make as many different flags with three colors as you can. To make a flag, select any three colors, like this: (select the red, white, and blue cards).

When you have a flag, record the colors on the Recording Sheet (Appendix B). See how many other flags you can make. Remember, each flag must have three different colors. It doesn’t matter what order you use the colors-red, white, and blue is the same as blue, white, and red. You can only use a combination of three colors once. Allow the students a maximum of five minutes or until they say they are finished.

Processing: After the student has written each flag’s color entries on their Recording Sheet (Appendix B), have them write a short sentence to explain how they went about generating their list.
**Calendar**

**Late August** (before school starts):
- Inservice all teachers who will be teaching the unit
- Order and gather all materials necessary to teach unit
- Copy all worksheets and create transparencies and other visuals necessary for unit

**September** (at start of school) to **November**:
- Day 1: Administer Piaget Test and evaluate students' cognitive developmental stage.
- Day 2: Administer Multiple Intelligence Test and use to make cooperative learning groups for class.
- Day 3: Administer Mathematical Assessment Test and record and evaluate student needs.
- Day 4: Begin Unit with Lesson #1 - An Introduction to Ratio
- Day 5 & 6: Lesson #2 - The Golden Rectangle and Ratio
- Day 7: Lesson #3 - Scaling to Proportion
- Day 8 & 9: Lesson #4 - Drawing and Graphing on Distorted Grids
- Day 10 & 11: Lesson #5 - Building Similar Figures
- Day 12: Lesson #6 - Sharks In A Box: A Tagging and Recapturing Simulation
- Day 13: Lesson #7 - Exploring Light and Proportion with Pinhole Viewers
- Day 14: Lesson #8 - Color Mixing With Your Pinhole Viewer
- Day 15: Lesson #9 - Ratios of Afterimages of Light
• Day 16: Lesson #10 - Proportions and Projections with Lenses and Lanterns
• Day 17: Lesson #11 - Only the Shadow Knows
• Day 18: Lesson #12 - A Special Ratio for Circles
• Day 19: Lesson #13 - Perspective Drawing and Scatter Plots
• Day 20, 21 & 22: Lesson #14 - Investigating Scatter Plots for the Family $\frac{y}{x} = k$
• Day 23: Lesson #15 - Sighting Your Partner
• Day 24: Lesson #16 - People Proportions with Your Pals
• Day 25, 26 & 27: Lesson #17 - Investigating Scatter Plots for the Family $xy = k$
• Day 28: Lesson #18 - How Does Each Pair Vary: Inversely, Directly, or Neither?
• Day 29: Lesson #19 - Converting Pace and Stride Into Meters
• Day 30 & 31: Lesson #20 - Meter Field Trials
• Day 32, 33, & 34: Lesson #21 - Trigonometric Ratios
• Day 35 & 36: Lesson #22 - Using the Tan Ratio to Measure Indirectly
• Day 37, 38, 39, 40, 41, & 42: Lesson #23 - Final Project
• By Day 42, students should have attained Objective #1 which is Raising Cognitive Levels, Objective #2, which is Making Mathematical Connections; and Objective #3 which is Experiencing Proportionality.
• Day 43 & 44: Test for successful attainment of these goal by means of Piaget Test and Mathematical Post-Assessment Test.
LATE NOVEMBER:
- Student surveys
- Teacher post-inservices to examine student work and to evaluate curriculum for possible improvement

END OF SCHOOL YEAR
- Administer the MDTP Test from California State University, Fullerton
CHAPTER 4

Conclusion

This unit was designed for bilingual immersion students about to enter the seventh grade at Los Alisos Intermediate School, located in Mission Viejo, CA. Our goal in creating this unit was to write curriculum that would prepare this heterogeneous group of students for algebra by the time they entered eighth grade. The present curriculum which is designed for mathematically high achieving students will prepare those students for algebra, in one year. Most of the bilingual immersion students do not fall into that category. On the other hand, the curriculum presently in place for heterogeneous students prepares students for algebra in two years.

Recent research and mathematical reform, encourages teachers and schools to use a more hands-on and interactive approach in order to aid all students in their quest for knowledge. Since a heterogeneous group contains students at varying cognitive development stages, it was the authors’ intentions to address these different levels and thus increase all students’ cognitive levels to the stage of abstract thinking and be prepared to undertake algebra.

Studies also show the need for all students to make connections and see multiple representations of mathematical concepts. This unit on proportional relationships incorporates the concept of proportionality in connection with physics and the practical use of tools to measure objects directly or remotely.
Program Evaluation

The evaluation of this project will take place using both formative and summative components. The formative evaluation will follow the model of evaluability assessment, which we feel is most appropriate for a project designed by teachers for teachers. Teachers who are accustomed to using curriculum of diverse approaches and who have observed students working and their output are the best suited evaluators. This form of evaluation de-emphasizes the role the school administration, but the authors who will implement this project originally, have full support of their administration. The authors are blessed with administrators who possess complete confidence in their ability to instruct students and evaluate how and what students learn.

These procedures will occur throughout the implementation of this project in many ways. The authors will collaborate daily regarding the success or failure of the lessons contained therein. Students’ work will be examined as to the achievement of the project’s goals, which include raising students’ cognitive development stages, creating connections between and among mathematical concepts, and lastly raising students’ levels of proportional reasoning skills. In addition, the author’s plan to observe each other’s classrooms as they look for the successful attainment of these goals.

At the end of the unit, the authors plan to take a day to intensively discuss and review the success of the lessons. The timing, flow, and completeness of the unit as a whole and
broken down into its components will be examined. The need to rewrite, remove, or make additions to the unit will be scrutinized.

The summative evaluation will take place using several forms. The students will be given a survey to ascertain their attitudes regarding the content of this unit and the approaches taken to involve them in their own discovery of proportional relationships. Additionally, the students will be given the algebra readiness exam published and processed by California State University, Fullerton. These results will be compared to other seventh graders who are in a homogeneous group of mathematically gifted students and eighth graders about to enter algebra.

**Recommendations**

The authors recommend that the unit be implemented as a pilot program for the bilingual immersion students. The unit should be expanded upon to complete a year's curriculum. Ultimately, this curriculum should be used with all incoming seventh graders. Inservicing should take place for all teachers.

In the fall of 1998, it is recommended that the unit be piloted in unison with a fellow colleague in order to incorporate daily feedback, confidence, and reliability. This is a vital component to the success of this unit. Teachers need the opportunity to confer with one another regarding the unit's preparation, implementation, or solutions to possible problems that could occur on a daily basis. In addition, it is recommended that teacher's be required to participate in
pre and post inservices in order to be properly trained in teaching this unit.

The reader must realize that this unit is only part of a year's worth of curriculum, therefore the authors recommend creating additional units to complete the year. Each of the units should possess a unifying idea and be created in conjunction with and of the same style and philosophy of this project's unit.

Although this project was created with a particular group of students in mind, namely the bilingual immersion students, it is our recommendation that all incoming seventh graders be exposed to this unit due to the vast approaches to learning that it has to offer.
APPENDICES

APPENDIX A

MULTIPLE INTELLIGENCES SURVEY
This survey will help you identify your strongest intelligences. Read each statement. If it expresses some characteristic of yours and sounds true for the most part, write yes. If it doesn’t, write no. If the statement expresses you sometimes but not other times, put both a yes and a no.

1. I’d rather draw a map than give someone verbal directions.
2. If I am angry or happy, I usually know exactly why.
3. I can play (or used to play) a musical instrument.
4. I like to compose songs or raps.
5. I can add or multiply quickly in my head.
6. I help friends deal with feelings because I deal with my own feelings well.
7. I like to work with calculators and computers.
8. I pick up new dance steps quickly.
9. It’s easy for me to say what I think in an argument or debate.
10. I like word games such as Scrabble, Anagrams, or Password.
11. I enjoy figuring out how to take apart and put back together toys, simple machines and/or puzzles.
12. I like to gather together groups of people for parties or special events.
13. I listen to music on the radio, CDs, or cassettes for much of the day.
15. I like to work puzzles and play games.
16. Learning to ride a bike (or skate) was easy.
17. I am irritated when I hear an argument or statement that sounds illogical.
18. I can convince other people to follow my plans.
19. My sense of balance and coordination is good.
20. My mind searches for patterns, relationships, or logical sequences in things.
21. I need to touch things in order to learn more about them.
22. I like word games such as riddles and tongue twisters.
23. I have vivid and colorful visual dreams.
24. I can identify when there is a key change in a song.
25. I like to work with numbers and figures.
26. I like to sit quietly and reflect on my inner feelings.
27. I can usually find my way around unfamiliar places.
28. I like to hum, whistle and sing in the shower or when I'm alone.
29. I'm good at athletics.
30. I enjoy writing detailed notes to friends.
31. I have lots of hobbies or other activities that I prefer to do on my own.
32. I'm sensitive to the expressions on other people's faces.
33. I stay "in touch" with my moods. I have no trouble identifying them.
34. I am sensitive to the moods of others.
35. I have a good sense of what others think of me.
36. Books are important to me.
37. I enjoy playing around with a chemistry set or other science materials.
38. I prefer reading things that have many illustrations.
39. It is hard for me to sit still for long periods of time.
40. I have a pleasant singing voice.
41. I often have opinions that set me apart from the crowd.
42. I prefer group sports like softball or volleyball to solo sports such as running or swimming.
43. I frequently think about what I want to be when I grow up.
44. I have an easy time remembering stories, poems, or other items.
45. If I hear a piece of music once or twice, I usually can sing it fairly accurately.
46. I frequently ask parents and teachers questions about how and why things work.
47. I have at least three close friends.
48. I enjoy looking at holograms and can see the hidden images in the books and posters.
49. I get a sinking feeling in my stomach when I think I am in trouble.
**Scoring**

Circle each item which you marked as "Yes." Add your totals. A total of four in any of the categories indicates strong ability.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
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<tr>
<td>9</td>
<td>5</td>
<td>1</td>
<td>8</td>
<td>3</td>
<td>2</td>
<td>12</td>
<td></td>
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<td>49</td>
<td>45</td>
<td>43</td>
<td>47</td>
<td></td>
</tr>
</tbody>
</table>

**TOTALS**

Shade in the chart according to how many questions in each category you answered yes to.

**INTELLECTUAL CAPACITY**

<table>
<thead>
<tr>
<th></th>
<th>A=</th>
<th>B=</th>
<th>C=</th>
<th>D=</th>
<th>E=</th>
<th>F=</th>
<th>G=</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>WORD SMART/VERBAL LINGUISTIC</td>
<td>LOGIC SMART/LOGICAL MATHEMATICAL</td>
<td>PICTURE SMART/VISUAL SPATIAL</td>
<td>BODY SMART/BODILY KINESTHETIC</td>
<td>MUSIC SMART/MUSICAL</td>
<td>SELF SMART/INTRAPERSONAL</td>
<td>PEOPLE SMART/INTERPERSONAL</td>
</tr>
</tbody>
</table>
APPENDIX B

PIAGET RECORDING SHEET

STUDENT NAME ________________________________

RECORD YOUR LIST HERE.

When you feel you have a complete list, please discuss the strategies you used to solve the problem.
There are 25 multiple choice questions on this test. Do not write on this test. Answer each question, using scratch paper to do your work. Bubble in the appropriate correct answer on the scantron sheet given to you.

1. Evaluate $20 - 3 \cdot 4 \div 2$
   A 30  B 12  C 4  D 3

2. Evaluate $bc - a$ if $a = 2$, $b = 5$, and $c = 4$
   A 18  B 7.5  C 7  D 1

3. In 1980, the population of Greenville, Indiana, was 40,613 people. By 1990, the population had increased by 4,613 people. What was the population of Greenville in 1990?
   A 46,000 people  B 45,226 people  C 44,226 people  D 36,000 people

4. The quotient of a number and 4 is 20. What is the number?
   A 80  B 16  C 8  D 5

5. The temperature rose 9° in the least hour. If the temperature is 4° now, what was the temperature an hour ago?
   A 13°  B 5°  C -5°  D -13°

   A 36  B 4  C 6  D 2/3

7. What is the greatest common factor of 24 and 45?
   A 2  B 3  C 4  D 5

8. Round 54.763 to the nearest tenth.
   A 55  B 54.8  C 54.76  D 54.7

9. Find $\sqrt{121}$
   A 10  B 11  C 19  D 12
10. Solve: \(- \frac{1}{6} + \frac{3}{4} = g\)
   - A \(\frac{1}{4}\)
   - B \(\frac{1}{3}\)
   - C \(\frac{7}{12}\)
   - D \(\frac{2}{3}\)

11. Find the next term in the arithmetic sequence 5, 2, -1, ...
   - A 2
   - B 1
   - C -3
   - D -4

12. Express 1.66... as a mixed number in simplest form.
   - A \(1 \frac{6}{10}\)
   - B \(1 \frac{3}{5}\)
   - C \(1 \frac{2}{3}\)
   - D \(1 \frac{1}{6}\)

13. If you divide one-fourth of a pizza into 3 equal pieces, what part of the whole pizza is each piece?
   - A \(\frac{3}{4}\)
   - B \(\frac{1}{3}\)
   - C \(\frac{1}{6}\)
   - D \(\frac{1}{12}\)

14. Each day, Shalondo read twice as many pages in a novel as the day before. If she read 3 pages on the first day, how many pages did she read on the fourth day?
   - A 6
   - B 12
   - C 24
   - D 48

15. A recipe calls for 2 cups of flour for 12 muffins. How much flour is needed for 30 muffins?
   - A 3 cups
   - B 4 cups
   - C 5 cups
   - D 6 cups

16. A blouse is on sale for 25% off. If the original price was $34, what is the sale price?
   - A $8.50
   - B $25.50
   - C $26.50
   - D $42.502
17. Find the diameter of a circle whose circumference is 76 meters. Use the formula \( C = \pi d \), where \( \pi = 3.14 \).
   A 238.64 m   B 48.41 m
   C 24.20 m   D 12.1 m

18. Give the value of \( x^3 \), when \( x = 2 \).
   A 6   B 9
   C 5   D 8

19. Find the coordinates of point D.
   A (3, -2)   B (3, 2)
   C (-3, -2)   D (-3, 2)

20. The table below shows the number of students in a class with each eye color. How many students have brown eyes?

<table>
<thead>
<tr>
<th>Eye Color</th>
<th>Tally</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>M</td>
</tr>
<tr>
<td>Brown</td>
<td>M M</td>
</tr>
<tr>
<td>Green</td>
<td>M</td>
</tr>
<tr>
<td>Hazel</td>
<td></td>
</tr>
</tbody>
</table>

   A 22   B 12   C 7   D 4

21. The Vance family is traveling from Jackson, Mississippi, to Dallas, Texas. If they are traveling at an average rate of 50 mph and the distance is 410 miles, how long will it take to make the trip? Use \( d = rt \).
   A 7 hours, 56 minutes   B 8 hours, 2 minutes
   C 8 hours, 12 minutes   D 8 hours, 20 minutes
22. Find the perimeter of the rectangle below if \( x = 3 \).

\[
\text{(x+5) in.} \\
\text{4 in.}
\]


23. Calculate \(-3 + 5 + (-7) - 4\)

A 3  B -3  C -5  D -9

24. Find the area of the following triangle.

\[
\text{15 inches} \\
\text{10 inches}
\]

A 34 sq. in.  B 90 sq. in.  C 45 sq. in.  D 150 sq. in.

25. What is the probability of pulling out a red marble out of a bag which contains 3 white marbles, 8 blue marbles and 5 red marbles?

A 5/11  B 5/16  C 1/5  D 3/16
APPENDIX D

FINDING RATIOS

Use ratios to compare the lengths of the sides of the following figures.

\[
\begin{align*}
A:B &= \quad a:b = \quad a:A = \\
A:C &= \quad b:B = \quad B:A = \\
d:D &= \quad D:d = \quad c:C = \\
\text{Perimeter (abcd)} : \text{Perimeter (ABCD)} &= \\
\text{Area (abcd)} : \text{Area (ABCD)} &= \\
\end{align*}
\]

Reduce all ratios, if possible, and write the decimal equivalent for each.

\[
\begin{align*}
A:B &= \quad a:b = \quad a:A = \\
A:C &= \quad b:B = \quad B:A = \\
d:D &= \quad D:d = \quad c:C = \\
\text{Perimeter (abcd)} : \text{Perimeter (ABCD)} &= \\
\text{Area (abcd)} : \text{Area (ABCD)} &= \\
\end{align*}
\]

\[
\begin{align*}
x/X &= \quad x/z = \quad y/Y = \\
y/Z &= \quad z:Z = \quad X/x = \\
Z/Y &= \quad Z/z = \quad Y/y = \\
\text{Perimeter (xyz)} / \text{Perimeter (XYZ)} &= \\
\text{Area (xyz)} / \text{Area (XYZ)} &= \\
\end{align*}
\]
Measure the following objects and tell the indicated ratios and the decimal equivalent of each.

Flag Pole:Tree = Tree:Flag Pole =
Building/Stop sign = Church/Tree =
Stop sign: Building = Tree:Church =
Flag Pole:Stop Sign =
Stop Sign:Flag Pole =
Oil Well/Tree = Tree: Oil Well =
Silo: Stop Sign = Stop Sign:Silo =
Building/Oil Well =
Oil Well:Building =

Please answer the following questions regarding your results.

Discuss in writing 5 different observations you made regarding your findings of the ratios. Be prepared to share your thoughts with your group and the entire class.
APPENDIX F

INSTRUCTIONS FOR CREATING A VERY SPECIAL RECTANGLE WITH VERY SPECIAL RATIOS. FOLLOW THE INSTRUCTIONS ON THIS SHEET.

1. Draw a square.

2. Find the center of the base.

3. With a compass
   - put metal tip on C
   - Pencil tip on P
   - Swing on arc

4. Complete rectangle with N point

GOLDEN RECTANGLE
APPENDIX G

Classic Beauty: A Look At The Golden Ratios of Ancient Greece

Doryphorus, the Spearbearer
Polykleitos
C. 450-440 B.C.

Aphrodite of Cyrene
APPENDIX H

FIND THE GOLDEN RATIOS IN THE ANIMALS BY MEASURING THEIR DIMENSIONS. SEE IF YOU CAN FIND MORE THAN ONE IN EACH.

DRAGON FLY

SUNFISH

COMMON BLACK HAWK

FLYING SQUIRREL
APPENDIX I

Find All the Golden Ratios That You Can by Measuring. Label the pictures with the measurements you find and compute the ratios.
APPENDIX J

Find as many Golden Ratios that you can and list them using the letters on the picture or your own letters which you place on the picture.
APPENDIX K

Your Own Personal Golden Ratio

HOW DO YOU MEASURE UP TO CLASSICAL STANDARDS?

During the Golden Age of Greece, artists sculpted statues that were not realistic portraits of real people, but were ideal forms, pleasing to the eye. A ratio called the Golden Ratio can be found in many of their sculptures. Fill in the following chart and see if you can find this ratio in your body and those of your family and friends. Using a tape measure start with yourself and find classic beauty!!

1. Use your calculator to find the ratio that the Greeks found in their statues and buildings (for you).
   a. navel to ground ÷ total height = ____________
   b. navel to top of head ÷ navel to ground = ______
   c. top of head to chin ÷ navel to chin = ______

2. What ratio did you find?

3. Can you find the Golden Ratio in your family and friends? Measure four members and complete the chart.

<table>
<thead>
<tr>
<th>Family member</th>
<th>Navel to ground</th>
<th>Navel to Top of head</th>
<th>Total Height</th>
<th>Navel to ground /total height</th>
<th>Navel to top of head / navel to ground</th>
<th>Head / Navel to chin</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

102
EXTENSIONS:
The Fibonacci Sequence begins 1, 1, 2, 3, 5, 8.

4. What is the rule for finding the next number in the sequence?

5. What are the next six numbers in the sequence?

6. Use the numbers in the sequence and see which ratios form the Golden Ratio. Find the decimal equivalences for these.
   \[
   \frac{1}{1} = \quad \frac{1}{2} = \quad \frac{2}{3} = \\
   \frac{3}{5} = \quad \frac{5}{8} = \quad \text{Find the next few ratios and their decimal equivalences.}
   \]

7. What conjecture can you make about the Fibonacci Numbers and the Golden Ratio?
APPENDIX L

Proportional Drawing

PROPORTIONAL DRAWING

The ratio of the segments in the small drawing to the segments in the large drawing is 2 to 1.

The areas have a ratio of 4 to 1.

A line on the small grid should be located on a corresponding position on the large grid.

Make an enlargement of the drawing at the left on the grid below.

104
Examples of Anamorphic Art
Student Work
Graphing Triangles on Distorted Grids

Graph triangle ABC onto each of the following distorted grids. On the lines beneath each grid, describe the distorted triangle.

1. Before you begin, predict which grid will produce the most distorted triangle.

If triangle ABC were translated to a different position on the original grid, it would still have the same shape. Would the resulting image triangle in each of these grids still have the same shape as the first image triangle you graphed?

1. 

2. 

3. 

4. 

5. 

From the Mathematics Teacher, January 1988
You are challenged with graphing a figure that is made of segments and curves. Reproduce the cartoon face on each distorted grid. Be sure that your cartoon lines cross the grid lines at the same place as in the original grid.

Before you begin, predict which graph will produce the most distorted face.
APPENDIX P

Predicting Distortions and Reflections

For each of the following, predict what the reflections of the sets of segments or arcs will look like. Draw your prediction.

1. Place your cylindrical mirror here.

2. Place your cylindrical mirror here.

3. Place your cylindrical mirror here.

4. Place your cylindrical mirror here.

Use what you learned about the reflection of lines to sketch the letters in the boxes so that their reflection in the cylindrical mirror appears with no distortion. Use a pencil so that you can make changes in your sketches.

5. Place your cylindrical mirror here.

6. Place your cylindrical mirror here.

7. Place your cylindrical mirror here.

From the Mathematics Teacher, January 1996
Creating an Amorphic Art Piece

Draw an original sketch in the square grid. Translate your sketch onto the circular grid to form a distorted image. Reflect your distorted image onto a mirrored cylinder to produce a normal image. You may cut along the bold horizontal line to remove the rectangular grid before you display your amorphic art.
Building Similar Figures

Choose a pattern block. Make sure everyone in your group has chosen a different one. Using more of the same block as you have chosen, build three figures that have the same shape as your original pattern block, but are of a different size.

Answer the following questions as you write your individual report:

1. How many blocks did it take to build each figure?

2. What are the scale factors when comparing the different figures you made?

3. How did the scale factors of your figures compare to the other figures made by other group members?

Now work with a partner and have one student choose one of the three figures you have made. Hold the original block directly above the similar figure and look directly down on it. Move the block upwards or downwards so that it appears to be the same size as the figure.

Have your partner measure the distance from your eye to the block and your eye to the figure on the table. Record these measurements and then exchange roles and repeat the
activity until all similar figures have been measured.

Find the perimeter and area of each of the similar figures.

Discuss with your group, and then include in your report the answers to the following questions.

4. What can you say about the relationships between the figures and their distances from your eye?

5. How do the perimeters of the similar figures compare?

6. How do the areas of the similar figures compare?
APPENDIX S

Experiment: Tagging and Recapturing Sharks

1. Remove the cover of the shoe box, look in, and estimate the number of cubes in the box. Record the letter that appears on the cover of the box.

ESTIMATE: ___________ BOX LETTER: ________

2. Write the name of each student in your group in the table below.

3. Have one person in your group take out a handful of cubes. Tag each cube with a circular label. Count the number of cubes tagged and record this amount under "T" in the table below. This amount will be the same for each member in your group. Tagging is to be done only once.

4. Let the sharks take a "swim" (shake up the cubes in the box!). One at a time, each person in your group should remove a handful of cubes. Keep a record of the total of each person's "catch" by writing the number of cubes in that catch in column "n". Count the number of tagged cubes and write this number in column "t." Replace all cubes in the box. Continue this process until everyone has had a chance to capture sharks and record the information in the table.

<table>
<thead>
<tr>
<th>Name</th>
<th>T</th>
<th>t</th>
<th>n</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>2.</td>
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<tr>
<td>3.</td>
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<tr>
<td>4.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MEAN</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

T = number of sharks (cubes) tagged

5. How can you use the information collected by everyone in your group to predict the number of sharks in the lagoon? If you know T, t, and n, how can you find N? Each student should compute N and record its value in the table. Then, find the mean for N.
6. How does your estimate compare with the result determined by finding the mean for N?

7. Count and record the number of cubes in your box.
   ACTUAL COUNT: ______

8. How does your estimate compare with the actual count? How does the mean for N compare with the actual count?
APPENDIX T

Instruction and Recording Sheet
Exploring Light and Proportion Using Pinhole Viewers

1. Build a pinhole viewer using a cardboard tube. Wrap the piece of aluminum foil over one end of the tube and place a rubber band around it to secure it. Do the same with the wax paper on the other end of the tube. Make a pinhole in the center of the aluminum foil with the push pin.

2. Measure the length of your tube (distance from the pinhole to the waxed paper viewing screen) in centimeters, and record that distance in the indicated spot below.

3. After the room has been darkened, observe the projected image through the viewer by holding the tube with the aluminum-wrapped side toward the light, and the waxed paper-wrapped side toward your eye. The waxed paper viewing screen should be about 30 cm away from your eye.

4. How do you need to move the tube to make that image larger? Smaller? (Get up and move and take your pencil, paper and measuring equipment with you!)

5. Position the tube so that the image fits completely on the waxed paper, and measure the height of the image.

6. Carefully set the tube down exactly where you were measuring. (You may want to put a piece of masking tape on the place where you are with your name on it to mark your position.)
7. Write a ratio of the height of the waxed paper image and the distance from the pinhole to the waxed paper (the length of the tube).

\[
\frac{\text{Height of the triangle}}{\text{Distance from pinhole to image on tube}} = \underline{\text{_____}}
\]

8. Measure the height of the projected overhead image on the big screen in front of the class, and record the data below.
9. Set up a proportion with the ratio in #7 above and the ratio of the Projected Height to the Distance from the Pinhole Viewer to the Screen. This last distance is unknown, as yet, so put a variable in the proportion for that value.

\[
\frac{\text{Height of Waxed Paper Image}}{\text{Distance of Tube}} = \frac{\text{Height of Screen Image}}{\text{Distance to Screen}}
\]

10. Use what you learned in the last lesson to solve for the variable, and thus find the distance away from the screen, and record that distance below.
11. Check your algebra by actually measuring the distance with your measuring device and record that below. How accurate were you?

Tube length = \underline{\text{_____}}
Height of triangle on waxed paper = \underline{\text{_____}}
Height of triangle on overhead screen = \underline{\text{_____}}
Distance to screen found using proportion = \underline{\text{_____}}
Actual measured distance to screen = \underline{\text{_____}}

Write a narrative report about this experiment showing all your work and conclusions as if you were going to present it to a classmate who was absent today.
APPENDIX U

Instructions and Recording Sheet for Proportions and Projections With Lenses and Lanterns

FOLLOW THE FOLLOWING INSTRUCTIONS, BEING CAREFUL TO RECORD ALL DATA AS YOU GO ALONG.

1. Set up your light and batteries so that the light works (check connections, wires, light bulbs).

2. Tape the light to the table top or to the side of the battery holder so that it stands upright.

3. Place the foil lantern over the light so that the triangle is evenly lit from behind and light is not "escaping" out the top or sides of the lantern.

4. Describe the shape of the window in the lantern. Sketch it in the space below. Measure the height of this shape.

Shape: ___________________________ Height: ____________

5. IMPORTANT: Place "screen" and light between 50 cm and 1 meter apart from each other.
6. Stretch meter tape flat on the table.

7. Take lens (the top) off of the Bug Box and use the bottom of the box as a "stand" for the lens.

8. Place lens (and stand) somewhere in between the light and the screen.

9. After the room is darkened continue with the following experiment.

10. Experiment with the distances between the light source, the lens, and the screen. (Remember to keep the screen and the light at least 50 cm apart.) Try to get a fairly clear projected image of the window from the lantern on the screen. What do you notice about the projected image? The following proportion exists when a lens focuses an image on a screen:

\[
\text{Height of object (Lantern window)} = \frac{\text{Distance from object to lens}}{\text{Height of projected image}} = \frac{\text{Distance from image to lens}}{\text{Distance from object to lens}}
\]

Using the following abbreviations:

- \( h_{\text{object}} \) = Height of Object
- \( H_{\text{image}} \) = Height of Image
- \( d_{\text{object}} \) = Distance from object to lens
- \( D_{\text{image}} \) = Distance from image to lens

you can now replace the above proportion with the following:

\[
\frac{h_{\text{object}}}{d_{\text{object}}} = \frac{H_{\text{image}}}{D_{\text{image}}}
\]

Using the cross multiplication technique you learned earlier,

\[
H_{\text{image}}d_{\text{object}} = h_{\text{object}}D_{\text{image}}
\]
11. Measure $h_{object}$, $d_{object}$, and $D_{image}$ in centimeters, and record below.

$h_{object} =$ _____; $d_{object} =$ _____; $D_{image} =$ _____

12. Determine $H_{image}$ from your data using the proportion and your knowledge of solving.

$H_{image} =$ _____

13. Check your calculation by measuring the image with your meter stick.

14. How do the calculated and the actual measured distances compare? What could account for the differences?

15. Move the lens only to find a second clear image on your screen. Repeat all the steps above by measuring and setting up the proportion, recording your data here.

$h_{object} =$ _____; $d_{object} =$ _____; $D_{image} =$ _____

16. Do you notice a relationship between your two sets of data?

17. What do you find in everyday life that relates to this experiment. Discuss and tell what you think is happening.
Only The Shadow Knows

DIRECTIONS:

1. Find an object to measure that has cast a shadow on the ground in an area that enables you to measure it.
2. Have a classmate stand near the object and as quickly as you can, measure the shadows of both the object and your friend. Record your data.
3. Draw figures to represent what you have just done, with the correct measurements recorded.
4. Use a proportion to calculate the height of the object.
5. After about 15-25 minutes, repeat the experiment with the same object and use the meterstick instead of your classmate for the creation of the other shadow. Measure the shadows of both the object and the meterstick. Record your data.
6. Draw figures to represent what you have just done, with the correct measurements recorded.
7. Use a proportion to calculate the height of the object.
8. Why were the shadows of the object different?
9. What did you find out about the heights of the object with the two different proportions?
10. What was the difference in the pictures that you drew of the triangles?
11. Write a narrative report of what you did for homework.
Perspective Drawing: Line of Telephone Poles

First the vanishing point $V$ is located on the horizon. Then the first pole $AB$ is drawn vertically and the vanishing envelope $AVB$ is drawn.

The rest of the poles are drawn in.

WHAT IS WRONG WITH THIS DRAWING?
APPENDIX X

**Perspective Drawing Directions**

1. On your construction paper, create a long, vertical line segment. Label one endpoint A and the other endpoint B.
2. Locate the midpoint of segment AB. Label the midpoint M.
3. Put your ruler at M, horizontally, to create a long dotted line for the horizon across the page.
4. Make a vanishing point at the end of the horizon dotted line and label it V.
5. Draw a line connecting point A to point V and draw a line connecting B to V. These are your vanishing lines.
6. Locate the midpoint of MV, on the horizon, and draw the perpendicular bisector, within the vanishing lines. Label the intersection point N and label the endpoints C and D.
7. Locate the midpoint of NV, on the horizon, and draw the perpendicular bisector within the vanishing lines. Label the intersection point O and label the endpoints E and F.
8. Locate the midpoint of OV, on the horizon, and draw the perpendicular bisector, within the vanishing lines. Label the intersection point P and label the endpoints G and H.
9. Locate the midpoint of PV, on the horizon, and draw the perpendicular bisector, within the vanishing lines. Label the intersection point Q and label the endpoints I and J.

Now you need to create a graph. Set up a T chart and measure the height of each line segment (x coordinate) and the distance away the endpoints are, along the vanishing lines, to the vanishing point (y coordinate).
APPENDIX Y

Investigating Scatter Plots For the Family $Y/X=K$ Directions

Each group member must choose two different $K$'s and create two graphs that represent their $K$'s. Each graph must include at least ten number pairs that divide to give $K$. Organize your number pairs in a T chart. Each member's graphs need to be displayed on a group poster.

In addition your group is to prepare an oral report for the class answering the following questions:

1. Describe your scatter plot patterns. What geometric shape was created in your graphs?
2. Suppose you wanted to investigate $Y/X=0.8$. What are some ways of finding pairs of numbers that divide to give 0.8?
3. How many members of the $Y/X=K$ family are there?
4. List other activities that you have done that have a graph like the scatter plots for the $Y/X=K$ family. Is it possible to find a $K$ that works for those graphs? How could you go about find $K$?
5. How are the scatter plots for $Y/X=2$ and $X/Y=2$ alike? How are they different?
6. What happens to your graphs as $K$ gets larger?
7. What can you say for sure about all scatter plots of the form $Y/X=K$?
8. Mathematicians call the $Y/X=K$ relationship direct variation. Why do you think it has this name?
APPENDIX Z

**Sighting Your Partner Directions**

1. In the following chart record at least eight sighting of your partner to determine his/her apparent sizes as measured by a ruler held at arm’s length at various distances.

<table>
<thead>
<tr>
<th>apparent height (cm)</th>
<th>distance apart (paces or strides)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5cm</td>
<td></td>
</tr>
<tr>
<td>10cm</td>
<td></td>
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<tr>
<td>15cm</td>
<td></td>
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<tr>
<td>20cm</td>
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<td>25cm</td>
<td></td>
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<tr>
<td>30cm</td>
<td></td>
</tr>
<tr>
<td>35cm</td>
<td></td>
</tr>
<tr>
<td>40cm</td>
<td></td>
</tr>
</tbody>
</table>

2. On graph paper create a scatter plot of your above data.
3. Describe your scatter plot pattern. What geometric shape was created in your graph?
4. Draw a diagram showing your eye, the ruler at arms length, and your partner. Show the line of sight from your eye through the ruler to the top of your partner’s head and the line of sight from your eye through the ruler to your partner’s feet.
APPENDIX AA

People Proportions With A Pal

DIRECTIONS:
1. Have a pal stand at the base of a tall object, like a tree, a flagpole, a utility pole, etc.
2. With a meterstick in hand, walk about 30 paces away from your pal.
3. Hold the meterstick out vertically at arm’s length and line up the top of the meterstick with the top of your friend’s head in your line of vision. Move your thumb down until it lines up with your classmate’s feet. Measure this distance on your meterstick and record the data.
4. Still standing at the same spot, move the meterstick upward so that the top of the meterstick lines up with the top of the object in your line of vision. Proceed as you did with your “pal” and measure the apparent height of the object and record this measurement.
5. Examine the two distances. How many times larger is the object than your pal?
6. Measure the height of your pal. Then use a proportion to determine the actual height of the object. (You may wait until you get back to the classroom to do this.)
7. Walk another 20 paces away from your pal and repeat the measurements and calculations.
APPENDIX BB

Investigating Scatter Plots for the Family XY=K Directions

Each group member must choose two different K’s and create two graphs that represent their K’s. Each graph must include at least ten number pairs that multiply to give K. Organize your number pairs in a T chart. Each member’s graphs need to be displayed on a group poster.

In addition your group is to prepare an oral report for the class answering the following questions:
1. Describe your scatter plot patterns. What geometric shape was created in your graphs?
2. Suppose you wanted to investigate XY=241.73. What are some ways of finding pairs of numbers that multiply to give 241.73? Is there a limit to the number of pairs you can find? How do you know? Explain.
3. How many members of the XY=K family are there?
4. List other activities that you have done that have a graph like the scatter plots for the XY=K family. Is it possible to find a K that works for those graphs? How could you go about find K?
5. What can you say for sure about all scatter plots of the form XY=K?
6. Mathematicians call the XY=K relationship inverse variation. Why do you think it has this name?
APPENDIX CC

How Does Each Pair Vary: Inversely, Directly, or Neither?

Read the following questions. Sketch a graph for each situation that represents the relationship and list if the relationship is inverse, direct, or neither.

1. The student store sells popcorn before school for 50 cents a bag. How does the money taken in vary with the number of bags of popcorn sold?

2. The basketball coach decides to have each player on the team (including those normally on the bench) play an equal amount of time during the game. How does the length of time each player gets to play vary with the total number of players on the team?

3. How does the number of fingers needed to cover a person’s face vary with the distance away a person is?
4. Mission Viejo is 20 miles away from Huntington Beach. Suppose you could travel from Mission Viejo to Huntington Beach at a constant rate of speed. How does the speed at which you travel vary with the time it takes you to go from Mission Viejo to Huntington Beach?

5. You are driving down Interstate 5 at a speed of 65 mph. How does the distance you travel vary with the time you have been driving?

6. How does the apparent height of a person measured in centimeters with ruler-at-arm’s-length vary with the distance away the person is?
7. Elmer is washing clothes in a washing machine. How does the time it takes to do a load of wash vary with the weight of the clothes he puts into the machine?

8. Make up another situation that represents direct variation.

9. Make up another situation that represents inverse variation.
Pace and Stride Homework Questions

1. For most people, which line do you think is more steeply sloped, the heel-to-toe pacing or the walking stride?

2. Suppose two people have different graphs for their walking stride, who would have the more steeply sloped graph, the person with the shorter stride of the longer stride? Why?

3. If a person takes 50 strides to go 42 meters, how many meters would the same person, walking the same way, go in 5 strides?

4. Suppose you divided all the numbers on the two axes of the graph by 10, would the graph with the new numbers still give valid information?

5. Jared took 27 strides to walk 23 meters, Elsa took 15 strides to walk 12 meters.
   a) Who has the longer stride?
   b) Jared and Elsa both started from the same place and each walked 100 strides in the same direction. How many meters apart are they?
   c) If you started from the same place they did and walked 100 strides in the same direction, would you end up between them, or in back of them, or past them?
APPENDIX EE

"How Far Is That Meter?" Field Trial

Situation
A meter stick will be placed vertically at eye level at a certain location.

Eight markers will be placed at different distances away from the meter stick, up to 50 meters away. On a signal from the teacher, your team will rotate from marker to marker.

Object
At each marker, your team must use remote measuring techniques to estimate the distance from the marker to the meter stick. No direct measurements will be allowed. You may use only a centimeter ruler, a calculator, and your own scatter plots, made up before the field trial.

Scoring
Your team will earn a percentage score depending on the accuracy of your estimates.

For each marker, the error is the difference between your estimate and the actual distance to the meter stick. The total of all your errors will be compared with the total of the actual distances to find the overall percentage of error. The lower your percentage, the better your score.
"How Far Is That Meter?" Field Trial Score Card

Team Members: ____________________________

- During the field trial, find the values for the "Your Estimate" column by sighting apparent lengths with a centimeter ruler and using a scatter plot and/or formula. Teams are encouraged to have several team members sight each distance to check accuracy, but must decide upon only one measure to put in each space.

- Estimate all distances to the nearest tenth of a meter.

- After the field trial, you will be given the values to fill in the "Actual Distance" column. Then you can compute the "Error" and percentage score.

<table>
<thead>
<tr>
<th>Station</th>
<th>Your Estimate</th>
<th>Actual Distance</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
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<td>3</td>
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<td>4</td>
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<td>5</td>
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<td>6</td>
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<td>7</td>
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<tr>
<td>8</td>
<td></td>
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<tr>
<td>TOTALS</td>
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</tr>
</tbody>
</table>

Percentage of Total Error to Total of Actual Distances: ____________________________

Bonus Problem: (write on back)
Smedley thinks that another way to figure a team’s score is to find the percentage of error for each marker and then add up the percentages. Would this give the same result? Is it a fair way of scoring?

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APPENDIX GG

Discovering Tangents:

or "How Good Is Your Tan?"

DIRECTIONS: On your graph paper you are to draw many right triangles with the different sized angles shown in the chart. Choose which angle of the two acute angles you wish to draw with your protractor, as this will become the focus angle. Then measure the opposite and adjacent sides and record their measurements and calculate the ratios in decimal form. This ratio of opposite/adjacent is called the tangent (TAN) of the angle. Every angle has its own unique tangent. Your scientific calculator is programmed to know the relationship between the TAN and the angle’s degree measurement. You can check to see how close your measuring is to the calculators value of TAN for each angle by putting the angle degree in the calculator. Fill in the chart and do the questions following.

<table>
<thead>
<tr>
<th>ANGLE DEGREE</th>
<th>OPP/ADJ (FRACTION)</th>
<th>OPP/ADJ (DECIMAL)</th>
<th>TAN OF ANGLE</th>
<th>DIFFERENCE OF LAST 2 COLUMNS</th>
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1. WHEN IS THE TAN OF THE ANGLE GREATER THAN ONE? LESS THAN ONE? WHY?

2. WHAT HAPPENED TO THE TRIANGLE AT THE POINT WHEN THE TAN WAS EQUAL TO ONE?

3. DESCRIBE THE GRAPHS THAT YOU DREW.

4. WHAT WAS YOUR AVERAGE DIFFERENCE IN THE CALCULATOR AND YOUR MEASURED VALUE OF TAN? WHY WERE THERE ANY DIFFERENCES AT ALL?

5. WHAT HAPPENED AT 90° AND WHY? AT 0° AND WHY?
APPENDIX HH

Instructions for Making a Hypsometer

Place on the edge of the index card.
APPENDIX II

Use the Tangent Ratio to Find the Height

Find the height of each object.

1. 

2. 

1. 

2. 

10 m

2 m
APPENDIX JJ

Final Project Directions

Choose an object to measure, such as a tree, flag pole a building or a light post.

- With your ruler-at-arm’s-length, measure the apparent height and width of your object from at least eight equal distance intervals. Record your information in a T chart to display your data. Graph your results. Write a formula that fits some of your data, (in other words find your K). Show how closely your formula fits your measurement data by including a scatter plot for your formula on the same graph as your data scatter plot. Make at least three different sketches showing your objects apparent height and width (use your actual data) at different distances.

- Use the all three methods: people proportions, shadow proportions, and trigonometric functions to find the actual height of your chosen object. Make a sketch for each method. Show the proportions you used for each method in calculating your object’s actual height. Write about which method you feel most accurately calculates your object’s actual height. Why did you chose that method instead of the other two?

- Neatly display all of your work including graphs and sketches on a poster board.
REFERENCES


