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SYMMETRIC REPRESENTATIONS OF FINITE GROUPS AND RELATED TOPICS

Connie Corona

Symmetric Representations of Finite Groups and Related Topics

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A Thesis

Presented to the

Faculty of

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In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

in

Mathematics

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by

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Abstract

In this thesis, we have presented our discovery of original symmetric presentations of a number of non-abelian simple groups, including several sporatic groups, linear groups, and classical groups.

We have constructed, using our technique of double coset enumeration, J_2, M_{12}, J_1, PSU(3, 3):2, M_{11}, A_{10}, S(4,3), M_{22}:2, PSL(3, 4), S_6, 2:S_5, 2:PSL(3, 4) as homomorphic images of the involutory progenitors 2^{*32}:(2^5:A_5), 2^{*110}: PSL(2, 11), 2^{*5}:A_5, 3^{*4}:D_8, 2^{*110}:PSL(2, 11), 2^{*42}:PSL(2, 7), 2^{*21}:(2×A_7), 2^{*40}:(24:(5:3)), 2^{*15}: A_7, respectively and S_6, S_8, J_2, M_{12}:2, as homomorphic images of the monomial progenitors 2^{*5}:A_5, 2^{*42}:PSL(2, 7), 2^{*18}:(3:A_6), 2^{*8}:(2^3:(4:2)), 2^{*5}:(5:4) respectively.

In addition we have given isomorphism types of all images that were discovered.

Acknowledgements

I would like to take a moment to thank my Advisor/Educator/Mentor, Dr. Hasan. I cannot say thank you enough. Thank you for taking the time, patience, and being the best educator that has made the biggest impact in my life. If I had never taken your calculus course, I'd still be a liberal arts major. Thank you for showing me the joys of mathematics. I would also like to thank my committee members Dr. Scow and Dr. Lim, thank you both for being so kind to me, inspiring me to pursue higher education, and being great educators. I hope to be as great as all three of you one day.

To my family, thank you for the support I needed to get this far in my education. Your love knows no bounds. Biggest thank you to my mom, who would get up with me at 4am to have a cup of coffee with me before catching the bus. Also thank you to my partner Ramses, who would keep me company on those long two-hour car rides from school. Here's to being the first, but not the last.

Table of Contents

List of Figures

Introduction

Magma is extensively used in this thesis. We have used it to compute many of the questions related to the structure of a finite group such as subgroups, normal subgroups, and maximal subgroups, as well coset actions and homomorphic images of presentations.

The infinite semi-direct product, called a progenitor, of the form, m^{*n}:N, where m^{*n} is a free product of n cyclic groups generated by t_1, $1 \leq i \leq n$, of order m and is a group of automorphisms of m^{*n} which permutes the n cyclic subgroups by conjugation. Therefore, for any k ϵ N, $t_i^k = t_i^1$, where gcd(m, 1) = 1. When m=2, N acts as a permutation group on the n-induces of the t_i's; that is, N $\leq$ S_n. The elements of a progenitor can be written, not necessarily uniquely, as $\pi\omega$, where n ϵ N and ω is a word in the t_i's.

Then any relation that a progenitor can be factored by to give a finite image must take the form nω = 1. We will represent this factorization as $\frac{m^{*n}:N}{n\omega}$. We will demonstrate how such factor groups can be identified. WE also note that frequently a progenitor factored by a single relation gives a simple group and a possible sporadic dimple group. Thus, progenitors lay the foundation for the number of topics, related to presentations and representations, that will be discussed throughout the next few chapters.

In **Chapter 1**, we list definitions and theorems pertaining to progenitors. We show how to apply Grindstaff lemma, we will prove the theorem $\frac{2^{*n}:S_n}{t_1t_2t_1=(1,2)} \cong S_{n+1}$ via computing using the computing program MAGMA to verify if we successfully built the progenitor, as well as perform double coset enumeration of the group S_7 over the transitive group S_6. More precisely, we show when a progenitor is factored by given relations it allows us to find the finite homomorphic images of such progenitor. In **Chapter 2** we perform double coset enumeration of the Mathieu sporadic simple group M_{12} over PSL(2,11) and in **Chapter 3** we perform double coset enumeration of the janko sporadic simple group J_2 over (2^5:A_5). In **Chapter 4-5** we continue our discussion of double coset enumeration, we construct the double coset enumeration of a group G over a transitive group N of finite permutation and monomial progenitors. We also show the maximal double coset enumeration for the following groups S_6, S_8, J_2, M_{12}:2. In **Chapter 6** we solve the extension problem in order to classify the isomorphism type of transitive groups as direct product, semi-direct product, or mixed extension.

In **Chapter 7** we discuss a different type of progenitor called the monomial progenitor. We show construction of this progenitor through a process known as the "lifting process", which allows us to build monomial matrices to obtain a new control group on which our monomial progenitor will be constructed from. In **Chapter 8** we show how to construct permutation progenitors of various free product sizes and apply Grindstaff Lemma via computing program MAGMA to verify if we successfully built the progenitor. In **Chapter 9** we either construct or give symmetric presentations of classical groups.. In **Chapter 10** we list the tables of the finite images discovered for each permutation and monomial progenitors, in chapter 7 and 8.

Chapter 1

Preliminary Information

1.1 Preliminary Definitions

We will remind our readers of the following definitions from Group Theory.

Definition 1.1.1 (**Permutation**) A **permutation** of X, where X $\neq \emptyset$, is the bijective mapping α: X $\rightarrow$X. S_x is the set of all permutations of X. [Rot95]

Definition 1.1.2 (**Disjoint**) Two permutations α, $\beta \in S_x$ are **disjoint** if every x moved by one is fixed by the other. In symbols, if α(a) $\neq$ a, then β(a) = a, and if α(b) = b, then β(b) $\neq$ b. [Rot95]

Definition 1.1.3 (**Commute**) Let α and β be permutations. Then α and β are sa1 to **commute** if $\alpha\beta = \beta\alpha$ [Rot95]

Definition 1.1.4. (**Transposition**) A permutation is called a **transposition** whenever it interchanges a pair of elements. [Rot95]

Definition 1.1.5. (**Operation**) For a set G $\neq \emptyset$, a (binary) **operation** is a function μ: G $\rightarrow$ G. [Rot95]

Definition 1.1.6. (**Associative**) The operation * for a set G is **associative** if (a*b)*c = a*(b*c), $\forall$ a,b,c $\in$ G. [Rot95]

Definition 1.1.7. (**Semigroup**) A **semigroup** (G, *) is a nonempty set G equipped with an associative operation *. [Rot95]

Definition 1.1.8. (**Group**). G is a **group** if G is a semi-group with the identity, e $\in$ G, and inverse, b $\in$ G, such that
(i) Identity: e*a = a*e, $\forall$a $\in$ G
(ii) Inverse: $\exists$ b $\in$ G, where a*b = e = b*a, $\forall$a $\in$ G. [Rot95]

Definition 1.1.9. (**Order**) If G is a group, then the **order** of G, denoted |G|, is the number of elements in G. [Rot95]

Definition 1.1.10. (**Free Group**) If X is a subset of a group F, then F is a **free group** with basis X if, for every group G and every function f: X $\rightarrow$ G, there exists a unique homomorphism ϕ: F $\rightarrow$ G extending f. [Rot95]

Definition 1.1.11. (**Presentation**) Let X be a set and let $\triangle$ be a family of words on X. A group G has generators X and relations $\triangle$ if G $\cong$ F/R, where F is the free group with basis X and R is the normal subgroup of F generated by $\triangle$. The ordered pair (X | $\triangle$) is called a **presentation** of G. [Rot95]

Definition 1.1.12. (**Progenitor**) A **progenitor** is a semi-direct product of the following form: P $\cong$ 2^{*n}:N = $\{\pi\omega \mid \pi \in$ N, and ω is a word in the $t_i\}$, where 2^{*n} denotes a free product of n copies of a cyclic group of order 2 generated by involutions t_i for i = 1, 2,...,n; and N is a transitive permutation group of degree n which acts on the free product by permuting the involutory generators. [Curt96]

Definition 1.1.13. (**Symmetric Group**) The **symmetric group**, denoted S_n is the set of all permutations of the nonempty set X = {1, 2, ..., n}. S_n is a group of order n! on n letters.

Definition 1.1.14. (**Fixes, Moves**) Let x $\in$ X, $\alpha \in S_X$. We say α **fixes** x 1 α(x)

= x. We also say α **moves** x if α(x) $\neq$ x. [Rot95]

Definition 1.1.15. (**Center**)Let G be a group. The set of all x $\in$ G who commute with every element of G is called the **center** of G. We denote the center as Z(G). [Rot95]

Definition 1.1.16. (**Abelian Group**) G is an **abelian group** if a*b = b*a, $\forall$ a,b $\in$ G. [Rot95]

Definition 1.1.17. (**Subgroup**) Let G be a group and S be a subset of G, S $\subset$ G, with S $\neq \emptyset$. If s $\in$ G $\rightarrow$ s^{-1} $\in$ G and s,t $\in$ G $\rightarrow$ st $\in$ G, then S is a **subgroup** of G, denoted by S $\leq$ G. [Rot95]

Definition 1.1.18. (**Word**) Let X $\subset$ G with X $\neq \emptyset$. Then an element w $\in$ G of the form w = $x_1^{e_1}x_2^{e_2}...x_n^{e_n}$, where x_i $\in$ X, e_i = $\pm$1, n$\geq$ 1, is called a **word** on X. [Rot95]

Definition 1.1.19. (**Generator**) Let X $\subset$ G. Then the smallest subgroup of G containing X is called the **subgroup generated by X**, denoted as <x>. WE also say that X **generates** <X>. [Rot95]

Definition 1.1.20. (**Proper Subgroup**) Let G be a group and K be a subgroup of G,K $\leq$ G. If gKg^{-1} = K, $\forall$g $\in$ G, then K is a **normal subgroup** of G, denoted by K $\triangle$ G. [Rot95]

Definition 1.1.21. (**Normal Subgroup**) Let G be a group and K be a subgroup of G, K $\leq$ G. If gKg^{-1} = K, $\forall$g $\in$ G, then K is a **normal subgroup** of G, denoted by K $\triangle$ G. [Rot95]

Definition 1.1.22. (**Simple**) Let G be a group with G $\neq$ 1. Then G is **simple** if it has no other normal subgroups than G and 1. [Rot95]

Definition 1.1.23. (**Maximal Normal Subgroup**) Let G be a group and H $\leq$ G. H is a **maximal normal subgroup** of G if $\nexists$ N $\leq$ G with H < N < G. [Rot95]

Definition 1.1.24. (**G-set, Action**) Let X be a set and G be a group. Then X is a **G-set** if there exists a function α: G x X $\rightarrow$ X, called an **action**, such that
(i) Identity: 1x = x, $\forall$ x $\in$ X
and
(ii) Associative: g(hx) = (gh)x, $\forall$g,h,x $\in$ X [Rot95]

Definition 1.1.25. (**Transitive**) A G-set X is **transitive** iff for every x,y $\in$ X, $\exists$ σx. [Rot95]

Definition 1.1.26. (**Homomorphism**) Let G and H be groups with operations * and $\circ$, respectfully. Then there exists a **homomorphism** f: G $\rightarrow$ H if f(a $*$ b) = f(a) $\circ$ f(b), $\forall$a,b $\in$ G. [Rot95]

Definition 1.1.27. (**Isomorphism**) Let G and H be groups. Then G is **isomorphic** to H, denoted by G $\cong$ H, if the homomorphism mapping f: G $\rightarrow$ H is also a bijection. [Rot95]

Definition 1.1.28. (**kernel, Image**) Let G and H be groups with H $\leq$ G and the homomorphic mapping f: G$\rightarrow$ H. Then the **kernel** of f is the set K = {a $\in$ G: f(a) = 1 } and the **image** of f is the set I = {h $\in$ H: h = f(a) for some a $\in$ F }, where K $\leq$ G and I $\leq$ H. [Rot95]

Definition 1.1.29. (**Automorphism**) An **automorphism** of a group G is the isomorphism ϕ: G $\rightarrow$ G. [Rot95]

Definition 1.1.30. (**Representative, Right Coset, Left Coest**) Let S $\leq$ G and t $\in$ G. Then the subset of G, St = {st | s $\in$ S }, is a **right coset** of S in G, where t is a **representative** of St. Also, the subset of G, tS = {ts | s $\in$ S }, is a **left coset** of S in G, where t is a **representative** of tS. [Rot95]

Definition 1.1.31. (**Index**) Let S $\leq$ G. Then the number of right cosets of S in G is the **index** of S in G, denoted [G:S]. [Rot95]

Definition 1.1.32. (**Normal Series**) Let G be a group. The sequence of subgroups G $= G_0 \leq G_1 \leq ... \leq G_n = 1$, where $G_{i+1} \triangle G_i$, $\forall$i, is called a **normal series**. [Rot95]

Definition 1.1.33. (**Factor Groups**) The **factor groups** of a normal series (G = $G_0 \leq G_1 \leq ... \leq G_n = 1$, where $G_{i+1} = 1$) are the groups G_1/G_{i+1}, for i = 0, 1,..., n-1. [Rot95]

Definition 1.1.34. (**Composition Series**) a normal series G $= G_0 \leq G_1 \leq ... \leq G_n = 1$, where $G_{i+1} = 1$ is a **composition series** if G_{i+1} is a maximal normal subgroup of G_i or $G_{i+1} = G_i$, $\forall$i. [Rot95]

Definition 1.1.35. (**Composition Factors**) The factor groups of a composition series are called the **composition factors** of that group. [Rot95].

Definition 1.1.36. (**Direct Product**) Let H and K be groups. The **direct product** of H and K, denoted H $\times$ K, is the group with all elements as ordered pairs having the form (h, k) where h $\in$ H, k $\in$ K, and with operation (h, k)(h$'$,k$'$) = (hh$'$,kk$'$). [Rot95]

Definition 1.1.37. (**Extension**) Let K and Q be groups. Then a group G, with $K_1 \triangle$ G, is an **extension** of K by Q where $K_1 \cong$ K and $G/K_1 \cong$ Q. [Rot95]

Definition 1.1.38. (**complement**) Let K $\leq$ G. Then Q, the **complement** of K in G, exists if K $\cap$ Q = 1 and KQ = G. [Rot95]

Definition 1.1.39. (**Semi-direct Product**) Let G be a group. Then G is a **semi-direct product** of K by Q if K $\triangle$ G and K has a complement of $Q_1 \cong$ Q. [Rot95].

Definition 1.1.40. (**Central Extension**) Let G be a group with H$\leq$ G and N $\leq$ G such that |G| = |N||H|. Then G is a **central extension** by H, denoted G $\cong$ N $\cdot$ H, if N is the center of G. [Rot95]

Definition 1.1.41. (**Mixed Extension**) Let G be a group with H $\leq$ G, N $\leq$ G, and N $\triangle$ G such that |G| = |N||H|. Then G is a **mixed extension** by H, denoted G $\cong$ N$^{\cdot}$:H, if G is formed by both central extension and semi-direct products. [Rot95].

1.2 Preliminary Theorems and Lemmas

Grindstaff's Lemma

States that $\frac{2^{*n}:N}{t_1t_2=t_2t_1, t_1t_n=t_nt_1, \ldots, t_{n-1}t_n=t_nt_{n-1}}$ where we will take the progenitor and make all the t's commute such that we have $\frac{2^{*n}:N}{(t_i,t_j)}$, 1 $\leq$ i $\leq$ n, 1 $\leq$ j $\leq$ n.

We will show the lemma with the following example.

$\frac{2^{*3}:S_3}{t_1t_2=t_2t_1, t_1t_3=t_3t_1, t_2t_3=t_3t_2}$ Grindstaff's lemma says if $\frac{2^{*n}}{t_i,t_j}$ $\cong$ 2^n:N, 1$\leq$ i $\leq$ n, 1 $\leq$ j $\leq$ n. Then $\frac{2^{*3}:S_3}{t_1t_2=t_2t_1}$ $\cong$ 2^3:S_3, then inserting the definition we have $\frac{2^{*3}:S_3}{(t_1,t_2),(t_1,t_3),(t_2,t_3)}$ $\cong$ 2^{*3}:S_3 where x $\sim$ (1, 2, 3) and y $\sim$ (1, 2). We write the progenitor 2^{*3}:S_3 as <x, y, t | x^3, y^2, $(xy)^2$, t^2, (t, N^1) >. N^1 being the stabiliser of 1, therefore N^1 in S_3 = <(2,3)> $\implies$ y^x. Then we test the progenitor G <x, y, t | x^3, y^2, $(xy)^2$, t^2, (t,y^x), (t_1,t_2), (t_1,t_3), (t_2,t_3)> $\implies$ G <x, y, t | x^3, y^2, $(xy)^2$, t^2, (t,y^x), (t,t^x), (t, t^{x^2}), (t^x, t^{x^2}) $\cong$ 2^3:S_3 which is of order 8 * order 6 = order 48. However, there is a short cut, which is of the following. We look at N^1 = <(2,3)> and the orbits of (N^1): {1}, {2, 3} we write as (t_1, t_1), (t_1, t_2). The same job is done by < x,y,t | x^3, y^2, $(xy)^2$, t^2, (t,y^x), (t,t^x)>.

Theorem 1.1:

Every permutation $\alpha \in S_n$ can be written as either a cycle or the product of disjoint cycles. [Rot95]

Theorem 1.3:

Every permutation $\alpha \in S_n$, is either a cycle or a product of disjoint cycles. [Rot95]

Theorem 1.13:

Let f: (G,*)$\rightarrow$ (G,$\circ$) be a homomorphic mapping, then we have the following:

(i) f(e) = e$'$, where e,e$'$ are the identities in G, G$'$, respectfully.

(ii) If x ϵ G, then f(x^{-1}) = f(x)$^{-1}$.

(iii) If x ϵ x ϵ G, n ϵ $\mathbb{Z}$, then f(x^n) = f(x)^n. [Rot95]

Theorem 2.11 (Lagrange's Theorem):
Let G be a finite group with S $\leq$ G. Then |S| | |G| and [G:S] = $\frac{|G|}{|S|}$. [Rot95]

Corollary 2.12:
Let G be a finite group with a ϵ G. Then |a| | |G|. [Rot95]

Theorem 2.24. (First Isomorphism Theorem - FIT):
Let the homomorphism f: G $\rightarrow$ H exisst with kernel K. Then K $\triangle$ G and G/K $\cong$ im(f). [Rot95]

1.3 Proof of Theorem $\frac{2^{*n}:S_n}{t_1t_2t_1=(1,2)} \cong \mathbf{S}_{n+1}$

We want to prove the theorem $\frac{2^{*n}:S_n}{t_1t_2t_1=(1,2)} \cong S_{n+1}$ where n=6. We have S_n, where n=6 so we will be working under S_6 where S_6 is generated by <(1,2,3,4,5,6), (1,2)> where x=(1,2,3,4,5,6) and y=(1,2). By using magma we can find the following

```
S:=Sym(6);
xx:=S!(1,2,3,4,5,6);
yy:=S!(1,2);
N:=sub<S|xx,yy>;
N1:=Stabiliser(N,1);
N1;
/*Permutation group N1 acting on a set of cardinality 6
Order = 120 = 2^3 * 3 * 5
    (2, 3)
    (3, 4)
    (4, 5)
    (5, 6)
*/
xx*yy;
/*(2, 3, 4, 5, 6)*/
N1 eq sub<N|xx*yy>;

/*BEGINNING OF WRITING THE GROUP*/
/*
G<x,y,t>:=Group<x,y,t|x^6,y^2,(x*y)^5,
t^2, */
Orbits(N1);
/*
    GSet{@ 1 @},
```

```
    GSet{@ 2, 3, 4, 5, 6 @}
*/
1^xx;
/*2*/
/*GROUP CHECK USING FPGROUP*/
FPGroup(N);
/*    a^6 ,
    b^2 ,
    (b * a^-1)^5 ,
    (a * b * a^-2 * b * a)^2 ,
    (a^-1 * b * a * b)^3 ,
*/
NN<a,b>:=Group<a,b|    a^6 ,
    b^2 ,
    (b * a^-1)^5 ,
    (a * b * a^-2 * b * a)^2 ,
    (a^-1 * b * a * b)^3>;

word:=function(A)
Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
for i in [2..#NN] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=xx; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=xx^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=yy; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=yy^-1; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
if A eq PP then B:=Sch[i]; end if;
end for;
return B;
end function;

word(N!(2,3));
/*y^x*/
word(N!(3,4));
/*x^-2 * y * x^2*/
word(N!(4,5));
/*x^3 * y * x^3*/
word(N!(5,6));
/*x^2 * y * x^-2*/
```

```
G<x,y,t>:=Group<x,y,t|    x^6 ,
    y^2 ,
    (y * x^-1)^5 ,
    (x * y * x^-2 * y * x)^2 ,
    (x^-1 * y * x * y)^3 ,
t^2,
(t,(y^x)),
(t,(x^-2 * y * x^2)),
(t,(x^3 * y * x^3)),
(t,(x^2 * y * x^-2)),
(t,t^x)>;
#G;
/*46080*/
THE ORDER OF G MUST BE |2^6:S_6|=64*720=46080
THUS OUR PROGENITOR IS GOOD
```

Next we will perform double coset enumeration of G=$\frac{2^{*6}:S_6}{t_1t_2t_1=(1,2)}$ over S_6=N.

First Double Coset [*]

We will start our first double coset labelled [*] with N={1,2,3,4,5,6} where the coset stabiliser is $\frac{|N|}{|N|} = \frac{720}{720} = 1$ and the orbit of [*] on X={1,2,3,4,5} is {1,2,3,4,5,6}. We will then multiply by a orbit representative.

Choose 1 from {1,2,3,4,5,6}

Thus Nt_1 ϵ [1]

Cayley Diagram

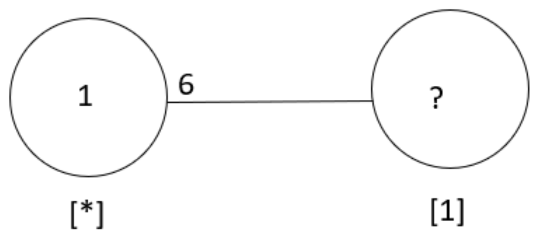


Figure 1.1: Cayley Diagram of [*] for S_6

Second Double Coset [1]

The coset stabiliser Nt_1N=<(2, 3), (2, 4), (2, 5), (2, 6),...> so the number of single right cosets is $\frac{|N|}{|N(1)|}=\frac{720}{120}=6$. The orbit for $N^{(1)}=\{(1)\},\{(2, 3, 4, 5, 6)\}$ we will choose a representative from each orbit and multiply it to Nt_1.

Choose 1 from {1}

Nt_1t_1

$=N(t_1)^2$

$=N \epsilon [*]$

Choose 2 from {2, 3, 4, 5, 6}

Nt_1t_2

Using the relation $t_1t_2t_1=(1,2)$ and multiply on the right by t_1 we have

$\implies t_1t_2t_1t_1=(1,2)t_1$

$\implies t_1t_2=(1,2)t_1$.

Then we have,

$Nt_1t_2 = N(1,2)t_1 = Nt_1 \epsilon [1]$

Cayley Diagram

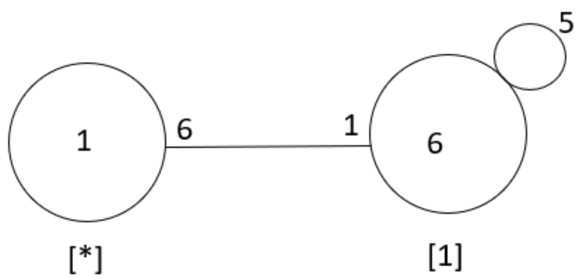


Figure 1.2: Cayley Diagram of [*], [1] for S_6

So G=$\frac{2^{*6}:S_6}{t_1t_2t_1=(1,2)}$ where $|G| \leq 1*|N|+6*|N|=1*720+6*720 = 5040$.

As we proceed we should ask ourselves, what are our cosets? We understand that G acts on x={N, Nt_1,...,Nt_6}. We will begin to preform permutation representation of <x,y,t>.

First x, x=(1,2,3,4,5,6)

which leads us to f(x) ∼ (2,3,4,5,6,7).

1). N	$N^{(1,2,3,4,5,6)}$	N (1)
2). Nt_1	$Nt_1^{(1,2,3,4,5,6)}$	$Nt_1=Nt_2$ (2)
3). Nt_2	$Nt_2^{(1,2,3,4,5,6)}$	$Nt_2=Nt_3$ (3)
4). Nt_3	$Nt_3^{(1,2,3,4,5,6)}$	$Nt_3=Nt_4$ (4)
5). Nt_4	$Nt_4^{(1,2,3,4,5,6)}$	$Nt_4=Nt_5$ (5)
6). Nt_5	$Nt_5^{(1,2,3,4,5,6)}$	$Nt_5=Nt_6$ (6)
7). Nt_6	$Nt_6^{(1,2,3,4,5,6)}$	$Nt_6=Nt_1$ (1)

Figure 1.3: Conjugation of Nt by x

Next y, y=(1,2)

thus f(y) ∼ (2,3)

1). N	$N^{(1,2)}$	N (1)
2). Nt_1	$Nt_1^{(1,2)}$	$Nt_1=Nt_2$ (3)
3). Nt_2	$Nt_2^{(1,2)}$	$Nt_2=Nt_1$ (2)
4). Nt_3	$Nt_3^{(1,2)}$	$Nt_3=Nt_3$ (4)
5). Nt_4	$Nt_4^{(1,2)}$	$Nt_4=Nt_4$ (5)
6). Nt_5	$Nt_5^{(1,2)}$	$Nt_5=Nt_5$ (6)
7). Nt_6	$Nt_6^{(1,2)}$	$Nt_6=Nt_6$ (7)

Figure 1.4: Conjugation of Nt by y

Before the table of t is computed we will conjugate our relation $t_1t_2t_1=(1,2)$ by the following permutations (1,2), (2,3), (2,4), (2,5), (2,6).

Which gives us:

α: (1,2) = $(t_1t_2t_1 = (1,2))^{(1,2)} = t_2t_1t_2$ = (2,1)

β: (2,3) = $(t_1t_2t_1 = (1,2))^{(2,3)} = t_1t_3t_1$ = (1,3)

γ: (2,4) = $(t_1t_2t_1 = (1,2))^{(2,4)}$ = $t_1t_4t_1$ = (1,4)

δ: (2,5) = $(t_1t_2t_1 = (1,2))^{(2,5)}$ = $t_1t_5t_1$ = (1,5)

ζ: (2,6) = $(t_1t_2t_1 = (1,2))^{(2,6)}$ = $t_1t_6t_1$ = (1,6)

Then we have t

Thus f(t) $\sim$ (1,2)

1). N	$*t_1$	N = Nt_1 (2)
2). Nt_1	$*t_1$	$Nt_1 = Nt_1t_1 = Nt_1^2$ = N (1)
3). Nt_2	$*t_1$	$Nt_2 = Nt_2t_1$ by α (3)
4). Nt_3	$*t_1$	$Nt_3 = Nt_3t_1$ by β (4)
5). Nt_4	$*t_1$	$Nt_4 = Nt_4t_1$ by γ (5)
6). Nt_5	$*t_1$	$Nt_5 = Nt_5t_1$ by δ (6)
7). Nt_6	$*t_1$	$Nt_6 = Nt_6t_1$ by ζ (7)

Figure 1.5: Multiplication of Nt by t_1

We have the following permutation representatives that act on G<x,y,t> where the function f, is a homomorphism where we have f(x)=(2,3,4,5,6,7), f(y)=(1,2), and f(z)=(1,2) where G/ker(f) $\cong$ <f(x),f(y),f(t)> = <(2,3,4,5,6,7), (1,2), (1,2)> we will compute the order of the permutation representatives.

$f(t^x) = (f^t)^{f(x)} = (1,2)^{(2,3,4,5,6,7)} = (1,3)$

$f(t^{x^2}) = (f^t)^{f(x)^2} = (1,2)^{(2,3,4,5,6,7)^2} = (1,4)$

$f(t^{x^3}) = (f^t)^{f(x)^3} = (1,2)^{(2,3,4,5,6,7)^3} = (1,5)$

$f(t^{x^4}) = (f^t)^{f(x)^4} = (1,2)^{(2,3,4,5,6,7)^4} = (1,6)$

$f(t^{x^5}) = (f^t)^{f(x)^5} = (1,2)^{(2,3,4,5,6,7)^5} = (1,7)$

$f(t^{x^6}) = (f^t)^{f(x)^6} = (1,2)^{(2,3,4,5,6,7)^6} = (1,2)$

$f(t^{x^7}) = (f^t)^{f(x)^7} = (1,2)^{(2,3,4,5,6,7)^7} = (1,3)$

The order of |f(x),f(y),f(t)|=5040 but $f(t), f(t^x), ..., f(t^{x^7})$ ϵ <f(x),f(y),f(t)>

$\Longrightarrow$ (1,2), (1,3),...,(1,7) $\leq$ <f(x),f(y),f(t)>

So (1,2), (1,3),...,(1,7) ϵ <f(x),f(y),f(t)>

Therefore, $S_7 \leq$ <f(x),f(y),f(t)>

Thus, G $\cong$ <f(x),f(y),f(t)> $\cong S_7$

We use magma to verify our work below.

```
/*IS n=6 ISOMORPHIC TO n+1=6+1=7*/
S:=Sym(7);
fx:=S!(2,3,4,5,6,7);
fy:=S!(2,3);
ft:=S!(1,2);
G1:=sub<S|fx,fy,ft>;
#G1 eq Factorial(7);
/*true*/
sub<G1|(1,2),(1,3),(1,4),(1,5),(1,6),(1,7)> eq G1;
/*true*/
IsIsomorphic(G1,Sym(7));
/*true Isomorphism of GrpPerm: G1,
Degree 7, Order 2^4 * 3^2 * 5 * 7 into GrpPerm: S, Degree 7,
Order 2^4 * 3^2 * 5 * 7 induced by
    (2, 3, 4, 5, 6, 7) |--> (2, 3, 4, 5, 6, 7)
    (2, 3) |--> (2, 3)
    (1, 2) |--> (1, 2)*/
```

Chapter 2

Construction of M_{12} Over PSL(2,11)

We will prove that the progenitor 2^{*110}: PSL(2,11) factored by three relations:
$(xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)^3t_{49}t_{105}t_{25}$,
$(xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)^4t_{102}t_{65}t_{103}t_{29}$,
$(xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)^6t_{45}t_{74}t_{96}t_{46}t_{109}t_{45}$
is isomorphic to M_{12}. where 2^{*110}:PSL(2,11) = < x, y> and x ∼ (1, 2)(3, 8)(4, 11)(5, 13)(6, 17)(7, 20)(9, 26)(10, 23)(12, 34)(14, 40)(15, 42)(16, 45)(18, 48)(19, 50)(21, 52)(22, 54)(24, 58)(25, 61)(27, 57)(28,51)(29, 68)(30, 32)(31, 59)(33, 44)(35, 76)(36, 73)(38, 71)(39, 79)(41, 85)(43, 66)(46, 65)(47, 89)(49, 83)(53, 93)(55, 90)(56, 96)(60, 78)(62, 92)(63, 101)(64, 102)(70, 86)(72, 94)(74, 104)(75, 77)(80, 108)(81, 109)(82, 103)(84, 106)(87, 88)(91, 95)(99, 107)(100, 110) , y ∼ (1, 3, 9)(2, 5, 14)(4, 12, 35)(6, 18, 49)(7, 21, 53)(8, 23, 56)(10, 29,69)(11, 31, 71)(13, 37, 74)(15, 43, 88)(16, 22, 55)(17, 46, 58)(19, 32,48)(20, 51, 41)(24, 59, 98)(25, 45, 65)(26, 64, 92)(27, 67, 36)(30, 70,104)(33, 72, 95)(34, 73, 85)(38, 50, 78)(39, 80, 105)(40, 83, 84)(42, 86, 75)(44, 89, 96)(52, 63, 66)(54, 94, 101)(57, 61, 100)(60, 79,91)(62, 97, 99)(68, 77, 82)(76, 102, 108)(81, 110, 87)(90, 109, 103)(93, 106, 107), factored by three relations is isomorphic to Mathieu sporadic simple group M_{12}. Let G $\cong \frac{2^{*110}:PSL(2,11)}{(xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)^3t_{49}t_{105}t_{25}}$. Thus we show that G ∼ M_{12}

2.1 Expanded Relations

Relation 1 = $(xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)^3 t_{49}t_{105}t_{25}$

Relation 2 = $(xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)^4 t_{102}t_{65}t_{103}t_{29}$

Relation 3 = $(xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)^6 t_{45}t_{74}t_{96}t_{46}t_{109}t_{45}$

2.2 Double Coset Enumeration

First Double Coset

NeN={N(e)^n | n ϵ N } = {N}

The coset Stabilizer of the coset N = Ne is N.

The number of single right cosets in the double coset NeN = [*] is given by

$\frac{|N|}{|N|} = \frac{660}{660} = 1$

X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110} is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110} we will now choose an orbit representative and multiply the representative by N on the right and determine its double coset

Choose 1 from the orbit {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50,51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110}. Then

$Nt_1 \ \epsilon$ [1]

This tells us that one-hundred ten elements move forward towards the double coset [1]

Cayley Diagram

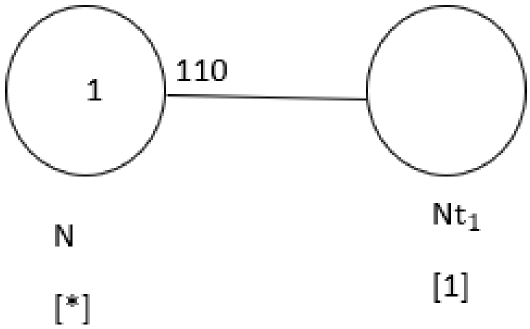


Figure 2.1: Cayley Diagram of [*] for M_{12}

Second Double Coset

$Nt_1N = \{N(t_1)^n \mid n \in N\} = \{Nt_1, Nt_2, ..., Nt_{110}\}$

Point Stabilizer of 1, N^1, is given by $\langle$ $(xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy)$, $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx)$ $\rangle$

Now, $t_1 = t_4$

The coset Stabilizer $N^{(1)}$ is given by $<(y^{-1}xyxy^{-1}xy)>$

The number of single right cosets in the double coset $Nt_1N = [1]$ is given by $\frac{|N|}{|N^{(1)}|} = \frac{660}{6}$ $=110$

The orbits for $N^{(1)}$ on

X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110} are

{1}, {4}, {42, 102, 85}, {45, 64, 56}, {50, 110, 87}, {83, 103, 90}, {2, 105, 93, 7, 22, 68}, {3, 38, 78, 9, 28, 81}, {5, 44, 31, 16, 20, 15}, {6, 94, 108, 19, 18, 73}, {8, 75, 101, 25, 63, 34}, {10, 82, 13, 30, 27, 39}, {11, 97, 91, 33, 43, 57}, {12, 40, 84, 35, 47, 109}, {14, 61, 79, 41, 21, 98}, {17, 99, 100, 29, 36, 59}, {23, 49, 48, 52, 92, 32}, {24, 106, 80, 60, 62, 74}, {26, 76, 54, 66, 65, 86}, {37, 96, 72, 77, 107, 71}, {46, 53, 51, 67, 104, 55}, {58, 89, 88, 70, 69, 95}

Multiply Nt_1 by a representative of each orbit and determine its double coset

<u>Choose 1 from {1}</u>

Nt_1t_1

$= N(t_1)^2$

$= N \epsilon [*]$

<u>Choose 4 from {4}</u>

We have the relation $t_1 = t_4$.

Now $N\underline{t_1},t_4$

$= Nt_4,t_4$

$= N(t_4)^2$

$= N \epsilon [*]$

<u>Choose 42 from {42,102,85}</u>

We have the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)^3)t_{49},t_{105},t_{25} = Id$

conjugated by

$(xyxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})$ to get

$(xyxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{92},t_{44} = t_{42}$.

Now $Nt_1,N\underline{t_{42}}$

$= Nt_1(xyxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{92},t_{44}$

$= Nt_{62},t_{92},t_{44}$.

Then use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx)$

(We will use the inverse since t_{44},t_{50},t_{35}) to get:

$(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx)t_{44} = t_{35},t_{50}$.

Now $Nt_1,(t_{42})$

$= Nt_{62},t_{92},t_{44}$

$= Nt_{62},t_{92},(t_{44})$

$= Nt_{62},t_{92}((yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx)^{-1})t_{35},t_{50}$

$= Nt_{46},t_{69},t_{35},t_{50}$.

Then using the relation $t_1 = t_4$

conjugated by

$(xyxyxyxyxy^{-1}xy^{-1}xyxy^{-1})$ to get $t_{35} = t_9$.

Now $Nt_1,\underline{t_{42}}$

$= Nt_{62},t_{92},t_{44}$

$= N\ t_{62},t_{92},\underline{t_{44}}$

$= Nt_{46},t_{69},t_{35},t_{50}$

$= Nt_{46},t_{69},\underline{t_{35}},t_{50}$

$= Nt_{46},t_{69},t_9,t_{50}$

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(xyxy^{-1}xy)^2$

to get $((xyxy^{-1}xy)^2)t_{69},t_9 = t_4$.

Now $Nt_1,\underline{t_{42}}$

$= Nt_{62},t_{92},t_{44}$

$= Nt_{62},t_{92},\underline{t_{44}}$

$= Nt_{46},t_{69},t_{35},t_{50}$

$= Nt_{46},t_{69},\underline{t_3},t_{50}$

$= Nt_{46},t_{69},t_9,t_{50}$

$= Nt_{46},\underline{t_{69},t_9},t_{50}$

$= Nt_{69},((xyxy^{-1}xy)^2$

to get $((xyxy^{-1}xy)^2)^{-1})t_4,t_{50} = Nt_1,t_4,t_{50}$.

Then using the relation $t_1 = t_4$.

Now $Nt_1,\underline{t_{42}}$

$= Nt_{62},t_{92},t_{44}$

$= Nt_{62},t_{92},\underline{t_{44}}$

$= Nt_{46},t_{69},t_{35},t_{50}$

$= Nt_{46},t_{69},\underline{t_{35}},t_{50}$

$= Nt_{46},t_{69},t_9,t_{50}$

$= Nt_{46},\underline{t_{69},t_9},t_{50}$

$= Nt_1,t_4,t_{50}$

$= N\underline{t_1},t_4,t_{50}$

$= Nt_4,t_4,t_{50}$

$= N(t_4)^2,t_{50}$

$= Nt_{50}$

$= N(t_1)^{(xyxy^{-1}xyxy^{-1}x)} \epsilon$ [1]

<u>Choose 45 from {45,64,56}</u>

Using relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}\ x)^3)t_{49},t_{105},t_{25}$ =Id

conjugated by $(yxy^{-1}x)$ to get $(yxy^{-1}x)t_{101},t_{30} = t_{45}$.

Now

$Nt_1,\underline{t_{45}}$

$= Nt_1(yxy^{-1}x)t_{101},t_{30}$

$= Nt_{96},t_{101},t_{30}$.

Then using the relation t_1 $=t_4$ conjugated by $(y^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)$ to get $t_{101} = t_{63}$.

Now $Nt_1,\underline{t_{45}}$

$= Nt_{96},t_{101},t_{30}$

$= Nt_{96},\underline{t_{101}},t_{30}$

$= Nt_{96},t_{63},t_{30}$.

Then using the relation

$(yxy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{69},t_{18},t_5,t_1$ =Id

conjugated by $(xy^{-1}xy^{-1}xyxy^{-1}xy^{-1})$ to get

$(xy^{-1}xy^{-1}xyxy^{-1}xy^{-1})t_{83},t_{30} = t_{96},t_{63}$.

$Nt_1,\underline{t_{45}}$

$= Nt_{96},t_{101},t_{30}$

$= Nt_{96},\underline{t_{101}},t_{30}$

$= N\underline{t_{96},t_{63},t_{30}}$

$= N(xy^{-1}xy^{-1}xyxy^{-1}xy^{-1})t_{83}$

$= Nt_{83}$

$=N(t_1)^{(xyxy^{-1}xyxyxyx)} \epsilon$ [1]

<u>Choose 50 from {50,110,87}</u>

Using relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25}$ =1

conjugated by

$(y^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}x)t_{57},t_{16} = t_{50}$.

Now $Nt_1,\underline{t_{50}}$

$= Nt_1(y^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}x)t_{57},t_{16}$

$= Nt_{34},t_{57},t_{16}$.

Next, using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25}$ =1

conjugated by $((yxy^{-1}xy^{-1}x)^2)$

to get $((yxy^{-1}xy^{-1}x)^2)t_{34},t_{57},t_6$ =Id.

Now $Nt_1,\underline{t_{50}}$

$= N\underline{t_{34},t_{57}},t_{16}$

$= N(((yxy^{-1}xy^{-1}x)^2)^{-1})t_6,t_{16}$

$= Nt_6,t_{16}$

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25}$ =1

conjugated by $(xyxy^{-1})$ to get $(xyxy^{-1})t_{32},t_{16},t_{42}$ =Id.

Now $Nt_1,\underline{t_{50}}$

$= N\underline{t_{34},t_{57}},t_{16}$

$= N(((yxy^{-1}xy^{-1}x)^2)^{-1})t_6,t_{16}$

$= Nt_6,\underline{t_{16}}$

$= Nt_6,((xyxy^{-1})^{-1})t_{17},t_{42}$

$= Nt_{30},t_{17},t_{42}$.

Using the relation $t_1 = t_4$ conjugated by

$(yxyxy^{-1}xy^{-1}xyxy^{-1}xy^{-1})$ to get $t_{30} = t_{17}$.

Now $Nt_1,\underline{t_{50}}$

$= N\underline{t_{34},t_{57}},t_{16}$

$= N\underline{t_{30}},t_{17},t_{42}$

$= Nt_{17},t_{17},t_{42}$

$= N((t_{17})^2)t_{42}$

$= Nt_{42}$

$= N(t_1)^{(yxy^{-1}xyxyxyxyxy^{-1}x)} \epsilon$ [1]

<u>Choose 83 from {83,103,90}</u>

Using the relation $t_1 = t_4$ conjugated by

$(xy^{-1}xyxy^{-1}xyx)$ to get $t_{83} = t_{50}$.

Now $Nt_1,\underline{t_{83}}$

$= Nt_1,t_{50}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25}$ =Id

conjugated by

$(y^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}x)t_{57},t_{16} = t_{50}$.

Now $Nt_1,\underline{t_{50}}$

$= Nt_1(y^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}x)t_{57},t_{16}$

$= Nt_{34},t_{57},t_{16}$.

Next, using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25}$ =1

conjugated by $((yxy^{-1}xy^{-1}x)^2)$ to get

$((yxy^{-1}xy^{-1}x)^2)t_{34},t_{57},t_6$ =Id.

Now $Nt_1,\underline{t_{50}}$

$= N\underline{t_{34},t_{57}},t_{16}$

$= N(((yxy^{-1}xy^{-1}x)^2)^{-1})t_6,t_{16}$

$= Nt_6,t_{16}$

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25}$ =Id

conjugated by $(xyxy^{-1})$ to get

$(xyxy^{-1})t_{32},t_{16},t_{42}$ =Id.

Now $Nt_1,\underline{t_{50}}$

$= N\underline{t_{34},t_{57}},t_{16}$

$= N(((yxy^{-1}xy^{-1}x)^2)^{-1})t_6,t_{16}$

$= Nt_6,\underline{t_{16}}$

$= Nt_6,((xyxy^{-1})^{-1})t_{17},t_{42}$

$= Nt_{30},t_{17},t_{42}$.

Using the relation $t_1 = t_4$ conjugated by

$(yxyxy^{-1}xy^{-1}xyxy^{-1}xy^{-1})$ to get $t_{30} = t_{17}$.

Now $Nt_1,\underline{t_{50}}$

$= N\underline{t_{34},t_{57}},t_{16}$

$= N\underline{t_{30}},t_{17},t_{42}$

$= Nt_{17},t_{17},t_{42}$

$= N((t_{17})^2)t_{42}$

$= Nt_{42}$

$= N(t_1)^{(yxy^{-1}xyxyxyxyxy^{-1}x)} \epsilon$ [1]

<u>Choose 2 from {2,105,93,7,22,68}</u>

$Nt_1t_2 \ \epsilon$ [1,2]

<u>Choose 3 from {3,38,78,9,28,81}</u>

Using relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by

$(yxyxy^{-1}xy^{-1}xyxy^{-1}xy)$ to get

$(yxyxy^{-1}xy^{-1}xyxy^{-1}xy)t_{58},t_{70} = t_3$.

Now $Nt_1,\underline{t_3} =$

$Nt_1(yxyxy^{-1}xy^{-1}xyxy^{-1}xy)t_{58},t_{70}$

$= Nt_{104},t_{58},t_{70}$.

By using the relation $t_1 = t_4$

conjugated by $(yxyxy^{-1}xy^{-1}xyxy^{-1}xy)$ to get $t_{104} = t_{58}$.

Now $Nt_1,\underline{t_3}$

$= N\underline{t_{104}},t_{58},t_{70}$

$= Nt_{58},t_{58},t_{70}$

$= N((t_{58})^2)t_{70}$

$= Nt_{70}$

$= N(t_1)^{(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})} \epsilon$ [1]

<u>5 from {5,44,31,16,20,15}</u>

$Nt_1t_5 \ \epsilon$ [1,5]

<u>Choose 6 from {6, 94, 108, 19, 18, 73}</u>

Using the relation $t_1 = t_4$ conjugated by $(yxyxyxyxy^{-1}xy^{-1}xyxy^{-1})$ to get $t_6 = t_{32}$.

Now $Nt_1\underline{t_6}$

$= Nt_1,t_{32}$.

Next using relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = 1$

conjugated by

$(xy^{-1}xyxyxy^{-1}xy^{-1}xy^{-1}x)$

which gives

$(xy^{-1}xyxyxy^{-1}xy^{-1}xy^{-1}x)t_{32} = t_{52},t_{107}$.

Now $Nt_1\underline{t_6}$

$= Nt_1,\underline{t_{32}}$

$= Nt_1((xy^{-1}xyxyxy^{-1}xy^{-1}xy^{-1}x)^{-1})t_{52},t_{107}$

$= Nt_{66},t_{52},t_{107}$.

Using the relation $t_1 = t_4$

conjugated by

$(yxyxy^{-1}xyxyxyxyx)$ to get $t_{52} = t_{94}$.

Now $Nt_1\underline{t_6}$

$= Nt_1,\underline{t_{32}}$

$= Nt_{66},\underline{t_{52}},t_{107}$

$= Nt_{66},t_{94},t_{107}$

Next using relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = 1$

conjugated by

$(yxyxyxyxyxy^{-1}xyxy^{-1})$

to get $(yxyxyxyxyxy^{-1}xyxy^{-1})t_{97} = t_{66},t_{94}$.

Now $Nt_1\underline{t_6}$

$= Nt_1,\underline{t_{32}}$

$= Nt_{66},\underline{t_{52}},t_{107}$

$= N\underline{t_{66},t_{94}},t_{107}$

$= N(yxyxyxyxyxy^{-1}xyxy^{-1})t_{97},t_{107}$

$= Nt_{97},t_{107}$

$= N(t_1,t_2)^{(xy^{-1}xy^{-1}xyxyxyxy)} \epsilon [1,2]$

<u>Choose 8 from {8, 75, 101, 25, 63, 34}</u>

Using relation $t_1 = t_4$ conjugated by $(yxyxy^{-1}xyxy^{-1})$ to get $t_8 = t_{34}$.

Now $Nt_1, \underline{t_8}$

$= Nt_1, t_{34}$.

Next using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105} = t_{25}$

conjugated by $(yxyxy^{-1}xyxyxyxyx)$ to get

$(yxyxy^{-1}xyxyxyxyx)t_{34}, t_{65}, t_{74} = Id$.

Now $Nt_1, \underline{t_8}$

$= Nt_1, \underline{t_{34}}$

$= Nt_{33}, t_{74}, t_{65}$

Next using the relation: $(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{92}, t_{107} = t_1, t_2$

conjugated by $(xyxyxy^{-1}xyxyxyx)$ to get $(xyxyxy^{-1}xyxyxyx)t_{33}, t_{74} = t_{31}, t_{78}$.

Now $Nt_1, \underline{t_8}$

$= Nt_1, \underline{t_{34}}$

$= N\underline{t_{33}, t_{74}}, t_{65}$

$= Nt_{31}, t_{78}, t_{65}$.

Then using the relation $t_1 = t_4$ conjugated by

$(y^{-1}xy^{-1}xyxyxy^{-1}xyxy^{-1})$

to get $t_{31} = t_5$ then we have,

$Nt_1, \underline{t_8}$

$= Nt_1, \underline{t_{34}}$

$= N\underline{t_{33}, t_{74}}, t_{65}$

$= N\underline{t_{31}}, t_{78}, t_{65}$

$= Nt_5, t_{78}, t_{65}$.

Then by using the relation again we conjugate $t_1 = t_4$ by

$(xy^{-1}xyxy^{-1}xyxy)$

to get $t_{78} = t_{84}$ thus we have,

$Nt_1, \underline{t_8}$

$= Nt_1, \underline{t_{34}}$

$= N\underline{t_{33}, t_{74}}, t_{65}$

$= N\underline{t_{31}}, t_{78}, t_{65}$

$= Nt_5, \underline{t_{78}}, t_{65}$

$= Nt_5, t_{84}, t_{65}$.

Next, we will use the relation

$(y^{-1}xyxy^{-1}xy^{-1}xyxyxy)t_{109},t_{102} = t_1,t_{26}$

conjugated by $(xyxyxyxy^{-1}xy)$ to get

$(xyxyxyxy^{-1}xy)t_{65},t_5,t_{84},t_{68} = Id$.

Then we have, $((xy)^3)t_{20},t_{68},t_{65}$.

Now, $Nt_1,\underline{t_8}$

$= Nt_1,\underline{t_{34}}$

$= N\underline{t_{33},t_{74}},t_{65}$

$= N\underline{t_{31}},t_{78},t_{65}$

$= Nt_5,\underline{t_{78}},t_{65}$

$= N\underline{t_5,t_{84}},t_{65}$

$= Nt_{20},t_{68},t_{65}$.

Next we will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy* x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by

$(xy^{-1}xyxyxyxyxy^{-1}xy^{-1}xy)$

to get

$(xy^{-1}xyxyxyxyxy^{-1}xy^{-1}xy)t_{68},t_{65} = t_{51}$.

Then we have

$(yxyxyxyxy^{-1}xy^{-1}xyxy^{-1})t_{30},t_{51}$.

Now, $Nt_1,\underline{t_8}$

$= Nt_1,\underline{t_{34}}$

$= N\underline{t_{33},t_{74}},t_{65}$

$= N\underline{t_{31}},t_{78},t_{65}$

$= Nt_5,\underline{t_{78}},t_{65}$

$= N\underline{t_5,t_{84}},t_{65}$

$= Nt_{20},\underline{t_{68},t_{65}}$

$= Nt_{30},t_{51}$.

Then using the relation $t_1 = t_4$ conjugated by

$(yxyxy^{-1}xyxyxy^{-1})$ to get $t_{51} = t_{89}$.

Thus we have $(yxyxyxyxy^{-1}xy^{-1}xyxy^{-1})t_{30},t_{89}$.

Now, $Nt_1,\underline{t_8}$

$= Nt_1,\underline{t_{34}}$

$= N\underline{t_{33},t_{74}},t_{65}$

$= N\underline{t_{31}},t_{78},t_{65}$

$= Nt_5,\underline{t_{78}},t_{65}$

$= N\underline{t_5,t_{84}},t_{65}$

$= Nt_{20},\underline{t_{68},t_{65}}$

$= Nt_{30},t_{51}$

$= Nt_{30},t_{89}$

$= N(t_1,t_2)^{(yxyxy^{-1}xy^{-1}xyxy^{-1}xy^{-1})} \epsilon [1,2]$

Choose 10 from {10, 82, 13, 30, 27, 39}

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id,$

conjugated by $(y^{-1}xyxy^{-1}xyxyxy^{-1}xy^{-1})$

to get

$(y^{-1}xyxy^{-1}xyxyxy^{-1}xy^{-1})t_{46},t_{104} = t_{10}.$

Then we have

$(y^{-1}xyxy^{-1}xyxyxy^{-1}xy^{-1})t_{46},t_{46},t_{104}.$

Now, $Nt_1,\underline{t_{10}}$

$= Nt_{46},t_{46},t_{104}.$

Therefore we have, $Nt_1,\underline{t_{10}}$

$= Nt_{46},t_{46},t_{104}$

$= N(t_{46})^2,t_{104}$

$= Nt_{104}$

$= N(t_1)^{(xyxy^{-1}x)} \epsilon [1]$

Choose 11 from {11, 97, 91, 33, 43, 57}

By using relation $t_1 = t_4$ conjugated by

$(y^{-1}xyxyxy^{-1}xy^{-1}xyxy)$ to get $t_2 = t_{11}$.

Then we have, $Nt_1,\underline{t_{11}}$

$= Nt_1,t_2 \epsilon [1,2]$

Choose 12 from {12, 40, 84, 35, 47, 109}

Using the relation $t_1 = t_4$ conjugated by

$(xy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1}xy^{-1})$ to get $t_3 = t_{12}$

then we have, Nt_1,t_{12}

$= Nt_1,t_3$.

Next using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = 1$

conjugated by $(yxyxy^{-1}xy^{-1}xyxy^{-1}xy)$

to get

$(yxyxy^{-1}xy^{-1}xyxy^{-1}xy)t_{58},t_{70} = t_3$.

Then we have $Nt_1,\underline{t_{12}}$

$= Nt_1,\underline{t_3}$

$= Nt_{104},t_{58},t_{70}$.

Then we will use the relation $t_1 = t_4$ once again conjugated by $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx)$.

to get $t_{58} = t_{104}$.

Then we have $Nt_1,\underline{t_{12}}$

$= Nt_1,\underline{t_3}$

$= N\underline{t_{104}},t_{58},t_{70}$

$= N\underline{t_{58},t_{58}},t_{70}$

$= N(t_{58})^2,t_{70}$

$= Nt_{70}$

$= N(t_1)^{(xyxyxyxy^{-1})} \epsilon [1]$

<u>Choose 14 from {14, 61, 79, 41, 21, 98}</u>

We use the relation $t_1 = t_4$ conjugated by

$(y^{-1}xy^{-1}xyxyxy^{-1}xyx)$ $t_{14} = t_{71}$.

Then we have $Nt_1,\underline{t_{14}}$

$= Nt_1,t_{71}$.

Next we use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = 1$

conjugated by $(y^{-1}xy^{-1}xy^{-1}xyxyx)$

to get

$(y^{-1}xy^{-1}xy^{-1}xyxyx)t_{47},t_{91} = t_{71}$.

Then we have $(y^{-1}xy^{-1}xy^{-1}xyxyx)t_{20},t_{47},t_{91}$.

Next $Nt_1,\underline{t_{14}}$

$= Nt_1,\underline{t_{71}}$

$= Nt_{20},t_{47},t_{91}$.

Then we will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = 1$

conjugated by

$(xyxyxy^{-1}xy^{-1}xy^{-1})$

to get $(xyxyxy^{-1}xy^{-1}xy^{-1})t_{91},t_{22},t_{95} = 1$

then we have

$(xy^{-1}xyxyxyxyxy^{-1}xy)t_{67},t_{40},t_{95},t_{22}$.

Then we have, $Nt_1,\underline{t_{14}}$

$= Nt_1,\underline{t_{71}}$

$= Nt_{20},t_{47},\underline{t_{91}}$

$= Nt_{67},t_{40},t_{95},t_{22}$.

We will use the relation $t_1 = t_4$ conjugated by

$(xy^{-1}xyxy^{-1}xyxy^{-1})$ to get $t_{40} = t_{38}$,

then we have $Nt_1,\underline{t_{14}}$

$= Nt_1,\underline{t_{71}}$

$= Nt_{20},t_{47},\underline{t_{91}}$

$= Nt_{67},\underline{t_{40}},t_{95},t_{22}$

$= Nt_{67},t_{38},t_{95},t_{22}$.

We will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = 1$

conjugated by $(xy^{-1}xy^{-1}xy^{-1}xyxy)$ to get

$(xy^{-1}xy^{-1}xy^{-1}xyxy)t_{89} = t_{38},t_{95}$

then we have

$(xy^{-1}xyxy^{-1}xy^{-1}xyxyxy^{-1})t_{68},t_{89},t_{22}$.

Next $Nt_1,\underline{t_{14}}$

$= Nt_1,\underline{t_{71}}$

$= Nt_{20},t_{47},\underline{t_{91}}$

$= Nt_{67},\underline{t_{40}},t_{95},t_{22}$

$= Nt_{67},\underline{t_{38},t_{95}},t_{22}$

$= Nt_{68},t_{89},t_{22}$.

We will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

again conjugated by $((xyxyxy^{-1})^2)$

to get $((xyxyxy^{-1})^2)t_{24},t_{32} = t_{68}$.

Then we have $(y^{-1}xyxyxyxy^{-1}xy)t_{24},t_{32},t_{89},t_{22}$.

Next $Nt_1,\underline{t_{14}}$

$= Nt_1,\underline{t_{71}}$

$= Nt_{20},t_{47},\underline{t_{91}}$

$= Nt_{67},\underline{t_{40}},t_{95},t_{22}$

$= Nt_{67},\underline{t_{38},t_{95}},t_{22}$

$= N\underline{t_{68}},t_{89},t_{22}$

$= Nt_{24},t_{32},t_{89},t_{22}$.

Then use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by

$(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy)$ to get

$(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy)t_8 = t_{32},t_{89}$

then we have $(y^{-1}xy^{-1}xyxyxy^{-1}xyxy)t_{76},t_8,t_{22}$.

Next $Nt_1,\underline{t_{14}}$

$= Nt_1,\underline{t_{71}}$

$= Nt_{20},t_{47},\underline{t_{91}}$

$= Nt_{67},\underline{t_{40}},t_{95},t_{22}$

$= Nt_{67},\underline{t_{38},t_{95}},t_{22}$

$= N\underline{t_{68}},t_{89},t_{22}$

$= Nt_{24},\underline{t_{32},t_{89}},t_{22}$

$= Nt_{76},t_8,t_{22}$.

We will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by

$((xyxyxy^{-1})^2)$ to get $((xyxyxy^{-1})^2)t_{68} = t_{76},t_8$

then we have $(yxy^{-1}xy^{-1}xy^{-1}x)t_{68},t_{22}$.

Next, $Nt_1,\underline{t_{14}}$

$= Nt_1,\underline{t_{71}}$

$= Nt_{20},t_{47},\underline{t_{91}}$

$= Nt_{67},\underline{t_{40}},t_{95},t_{22}$

$= Nt_{67},\underline{t_{38},t_{95}},t_{22}$

$= N\underline{t_{68}},t_{89},t_{22}$

$= Nt_{24},\underline{t_{32},t_{89}},t_{22}$

$= N\underline{t_{76},t_{8}},t_{22}$

$= Nt_{68},t_{22}$

$= N(t_1,t_2)^{(xyxyxyxy^{-1}xyxy^{-1})} \epsilon [1,2]$

Choose 17 from {17, 99, 100, 29, 36, 59}

We will use the relation $t_1 = t_4$ conjugated by

$(yxyxy^{-1}xy^{-1}xyxy^{-1}xy^{-1})$ to get

$t_{17} = t_{30}$ then we have $Nt_1,(t_{17}) = Nt_1,t_{30}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $((y^{-1}xyxy^{-1}x)^2)$ to get

$((y^{-1}xyxy^{-1}x)^2)t_{46} = t_1,t_{30}$

then we have $((y^{-1}xyxy^{-1}x)^2)t_{46}$.

Next $Nt_1,\underline{t_{17}}$

$= N\underline{t_1,t_{30}}$

$= Nt_{46}$

$= N(t_1)^{(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)} \epsilon [1]$

Choose 23 from {23, 49, 48, 52, 92, 32}

We will use the relation $t_1 = t_4$ to get

$N(t_1),t_{23} = Nt_4,t_{23}$ and use the relation $t_1 = t_4$

again conjugated by $(yxy^{-1}xy^{-1}xy^{-1}xyxy^{-1})$

to get $t_{23} = t_{73}$

then we have $N\underline{t_1},t_{23}$

$= Nt_4,(t_{23})$

$= Nt_4,t_{73}$.

Next we will use the relation

$(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{92},t_{107} = t_1,t_2$

conjugated by

$(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx)$ to get
$(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx)t_{43},t_{96} = t_4,t_{73}$
then we have $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx)t_{43},t_{96}$
$= N(t_1,t_2)^{(xyxy^{-1}xy^{-1}xy^{-1}xy)} \epsilon$ [1,2]

<u>Choose 24 from {24, 106, 80, 60, 62, 74}</u>

Using the relation $t_1 = t_4$ conjugated by $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx)$ to get
$t_{24} = t_{74}$ then we have $Nt_1,\underline{t_{24}} = Nt_1,t_{74}$.
Next using the relation
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = 1$
conjugated by
$(xy^{-1}xy^{-1}xyxyxyxyxy^{-1})$ to get
$(xy^{-1}xy^{-1}xyxyxyxyxy^{-1})t_{20},t_{22} = t_{74}$
then we have
$(xy^{-1}xy^{-1}xyxyxyxyxy^{-1})t_{14},t_{20},t_{22}$.
Next $Nt_1,\underline{t_{24}}$
$= Nt_1,\underline{t_{74}}$
$= Nt_{14},t_{20},t_{22}$.
We will use the relation
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = 1$
again conjugated by $(y^{-1}xyx)$ to get $(y^{-1}xyx)t_{66},t_{48} = t_{22}$
then we have
$(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{37},t_{12},t_{66},t_{48}$.
Next $Nt_1,\underline{t_{24}}$
$= Nt_1,\underline{t_{74}}$
$= Nt_{14},t_{20},\underline{t_{22}}$
$= Nt_{37},t_{12},t_{66},t_{48}$.
Then we will use the relation $t_1 = t_4$ conjugated
by $(xyxyx)$ to get $t_{37} = t_{98}$.
Then we have $Nt_1,\underline{t_{24}}$
$= Nt_1,\underline{t_{74}}$
$= Nt_{14},t_{20},\underline{t_{22}}$
$= N\underline{t_{37}},t_{12},t_{66},t_{48}$

$= Nt_{98},t_{12},t_{66},t_{48}$.

Then we will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^{3})t_{49},t_{105},t_{25} = Id$

conjugated by $(y^{-1}xyxy^{-1}xy^{-1}xyxyxyxy)$

to get $(y^{-1}xyxy^{-1}xy^{-1}xyxyxyxy)t_{38} = t_{98},t_{12}$

then we have $(y^{-1}xy^{-1}xyxyxyxyxy^{-1})t_{38},t_{66},t_{48}$

therefore, $Nt_{1},\underline{t_{24}}$

$= Nt_{1},\underline{t_{74}}$

$= Nt_{14},t_{20},\underline{t_{22}}$

$= N\underline{t_{37}},t_{12},t_{66},t_{48}$

$= N\underline{t_{98},t_{12}},t_{66},t_{48}$

$= Nt_{38},t_{66},t_{48}$.

Next we will use the relation $t_{1} = t_{4}$ conjugated by

$(yxyxy^{-1}xyxyxyxyxy^{-1})$ to get $t_{54} = t_{66}$

then we have $Nt_{1},\underline{t_{24}}$

$= Nt_{1},\underline{t_{74}}$

$= Nt_{14},t_{20},\underline{t_{22}}$

$= N\underline{t_{37}},t_{12},t_{66},t_{48}$

$= N\underline{t_{98},t_{12}},t_{66},t_{48}$

$= Nt_{38},\underline{t_{66}},t_{48}$

$= Nt_{38},t_{54},t_{48}$.

We will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^{3})t_{49},t_{105},t_{25} = Id$

conjugated by $(xy^{-1}xy^{-1}xy^{-1}xyxy)$ to get

$(xy^{-1}xy^{-1}xy^{-1}xyxy)t_{89},t_{95} = t_{38}$

then we have

$(yxyxyxy^{-1}xy^{-1}xyxy^{-1}xy)t_{89},t_{95},t_{54},t_{48}$.

Next $Nt_{1},\underline{t_{24}}$

$= Nt_{1},\underline{t_{74}}$

$= Nt_{14},t_{20},\underline{t_{22}}$

$= N\underline{t_{37}},t_{12},t_{66},t_{48}$

$= N\underline{t_{98},t_{12}},t_{66},t_{48}$

$= Nt_{38},\underline{t_{66}},t_{48}$

$= N\underline{t_{38}},t_{54},t_{48}$

$= Nt_{89},t_{95},t_{54},t_{48}$.

Then we will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(yxyxy^{-1}xy^{-1}xy^{-1}x)$

to get $(yxyxy^{-1}xy^{-1}xy^{-1}x)t_{95},t_{54},t_{91} = Id$

then we have $(y^{-1}xyxyxyx)t_{14},t_{91},t_{48}$.

Next $Nt_1,\underline{t_{24}}$

$= Nt_1,\underline{t_{74}}$

$= Nt_{14},t_{20},\underline{t_{22}}$

$= N\underline{t_{37}},t_{12},t_{66},t_{48}$

$= N\underline{t_{98},t_{12}},t_{66},t_{48}$

$= Nt_{38},\underline{t_{66}},t_{48}$

$= N\underline{t_{38}},t_{54},t_{48}$

$= Nt_{89},\underline{t_{95},t_{54}},t_{48}$

$= Nt_{14},t_{91},t_{48}$.

Then we will use the relation $t_1 = t_4$ conjugated by

$(y^{-1}xy^{-1}xyxyxy^{-1}xyx)$ to get $t_{14} = t_{71}$

then we have $Nt_1,\underline{t_{24}}$

$= Nt_1,\underline{t_{74}}$

$= Nt_{14},t_{20},\underline{t_{22}}$

$= N\underline{t_{37}},t_{12},t_{66},t_{48}$

$= N\underline{t_{98},t_{12}},t_{66},t_{48}$

$= Nt_{38},\underline{t_{66}},t_{48}$

$= N\underline{t_{38}},t_{54},t_{48}$

$= Nt_{89},\underline{t_{95},t_{54}},t_{48}$

$= N\underline{t_{14}},t_{91},t_{48}$

$= Nt_{71},t_{91},t_{48}$.

We will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(y^{-1}xy^{-1}xy^{-1}xyxyx)$ to get $(y^{-1}xy^{-1}xy^{-1}xyxyx)t_{47} = t_{91},t_{71}$ then we have

$(xyxyxy^{-1})t_{47},t_{48}$.

Next $Nt_1,\underline{t_{24}}$

$= Nt_1,\underline{t_{74}}$

$= Nt_{14},t_{20},\underline{t_{22}}$

$= N\underline{t_{37}},t_{12},t_{66},t_{48}$

$= N\underline{t_{98},t_{12}},t_{66},t_{48}$

$= Nt_{38},\underline{t_{66}},t_{48}$

$= N\underline{t_{38}},t_{54},t_{48}$

$= Nt_{89},\underline{t_{95},t_{54}},t_{48}$

$= N\underline{t_{14}},t_{91},t_{48}$

$= N\underline{t_{71},t_{91}},t_{48}$

$= Nt_{47},t_{48}$

$= N(t_1,t_2)^{((yxyxy^{-1}x)^2)} \epsilon [1,2]$

Choose 26 from {26, 76, 54, 66, 65, 86}

$Nt_1t_{26} \epsilon [1,26]$

Choose 37 from {37, 96, 72, 77, 107, 71}

We will use the relation $((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = 1$

conjugated by $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1})$ to get

$(yxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{40},t_{33} = t_{37}$ then we have

$(yxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{44},t_{40},t_{33}$.

Next $Nt_1,\underline{t_{37}} = Nt_{44},t_{40},t_{33}$.

We will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

again and conjugate it by $(y^{-1}xyxyxy^{-1}xy^{-1}x)$

to get $(y^{-1}xyxyxy^{-1}xy^{-1}x)t_{33},t_{105},t_{58} = Id$

then we have

$(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_{51},t_{47},t_{58},t_{105}$.

Next we have $Nt_1,(t_{37})$

$= Nt_{44},t_{40},(t_{33})$

$= Nt_{51},t_{47},t_{58},t_{105}$.

We will use the relation $t_1 = t_4$ conjugated by

$((yxyxy^{-1}x)^2)$ to get $t_{47} = t_{28}$

then we have $Nt_1,\underline{t_{37}}$

$= Nt_{44},t_{40},\underline{t_{33}}$

$= Nt_{51},\underline{t_{47}},t_{58},t_{105}$

$= Nt_{51},t_{28},t_{58},t_{105}$.

We will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(xy^{-1}xyxyxyxyxy^{-1}xyx)$

to get $(xy^{-1}xyxyxyxyxy^{-1}xyx)t_{69} = t_{28},t_{58}$

then we have $(x)t_2,t_{69},t_{105}$.

Next

$Nt_1,\underline{t_{37}}$

$= Nt_{44},t_{40},\underline{t_{33}}$

$= Nt_{51},\underline{t_{47}},t_{58},t_{105}$

$= Nt_{51},\underline{t_{28},t_{58}},t_{105}$

$= Nt_2,t_{69},t_{105}$.

We will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = 1$

conjugated by $(yxyxy^{-1}xyxyxyxyx)$

to get $(yxyxy^{-1}xyxyxyxyx)t_{74},t_{23} = t_2$

then we have

$(y^{-1}xyxy^{-1}xyxyxy^{-1}xy^{-1}x)t_{74},t_{23},t_{69},t_{105}$.

Next $Nt_1,\underline{t_{37}}$

$= Nt_{44},t_{40},\underline{t_{33}}$

$= Nt_{51},\underline{t_{47}},t_{58},t_{105}$

$= Nt_{51},\underline{t_{28},t_{58}},t_{105}$

$= N\underline{t_2},t_{69},t_{105}$

$= Nt_{74},t_{23},t_{69},t_{105}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = Id$

and conjugate it by $(y^{-1}xyxy^{-1}xyxyxyxyxy)$

to get $(y^{-1}xyxy^{-1}xyxyxyxyxy)t_{34} = t_{23},t_{69}$

then we have $f(xyxy^{-1}xy^{-1}xyxy^{-1})t_{65},t_{34},t_{105}$.

$Nt_1, \underline{t_{37}}$

$= Nt_{44}, t_{40}, \underline{t_{33}}$

$= Nt_{51}, \underline{t_{47}}, t_{58}, t_{105}$

$= Nt_{51}, \underline{t_{28}, t_{58}}, t_{105}$

$= N\underline{t_2}, t_{69}, t_{105}$

$= Nt_{74}, \underline{t_{23}, t_{69}}, t_{105}$

$= t_{65}, t_{34}, t_{105}$.

Next we will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(yxyxy^{-1}xyxyxyxyx)$

which gives $(yxyxy^{-1}xyxyxyxyx)t_2 = t_{65}, t_{34}$

then we have $(yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})t_2, t_{105}$.

Next $Nt_1, \underline{t_{37}}$

$= Nt_{44}, t_{40}, \underline{t_{33}}$

$= Nt_{51}, \underline{t_{47}}, t_{58}, t_{105}$

$= Nt_{51}, \underline{t_{28}, t_{58}}, t_{105}$

$= N\underline{t_2}, t_{69}, t_{105}$

$= Nt_{74}, \underline{t_{23}, t_{69}}, t_{105}$

$= N\underline{t_{65}, t_{34}}, t_{105}$

$= Nt_2, t_{105}$

$= N(t_1, t_2)^{(y^{-1}xy^{-1}xyxyxy^{-1}xyxy)} \epsilon [1,2]$

<u>Choose 46 from {46, 53, 51, 67, 104, 55}</u>

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by

$(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)$

to get

$(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)t_{30}, t_{10}, t_{29}$.

Next we have, $Nt_1, \underline{t_{46}} = Nt_{30}, t_{10}, t_{29}$.

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(xy^{-1}xyxyxyxyxy^{-1}xyx)$

which gives

$(xy^{-1}xyxyxyxyxy^{-1}xyx)t_{28},t_4 = t_{29}$

then we have

$(xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}x)t_{36},t_{35},t_{28},t_4$.

Which results in $Nt_1,\underline{t_{46}}$

$= Nt_{30},t_{10},\underline{t_{29}}$

$= Nt_{36},t_{35},t_{28},t_4$.

Using the relation $t_1 = t_4$ conjugated by

$(yxy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)$ to get $t_{36} = t_{10}$.

Then we have $Nt_1,\underline{t_{46}}$

$= Nt_{30},t_{10},\underline{t_{29}}$

$= N\underline{t_{36}},t_{35},t_{28},t_4$

$= Nt_{10},t_{35},t_{28},t_4$.

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by

$(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)$

to get

$(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)t_{28} = t_{10},t_{35}$

then we have

$(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{28},t_{28},t_4$.

Thus we have, $Nt_1,\underline{t_{46}}$

$= Nt_{30},t_{10},\underline{t_{29}}$

$= N\underline{t_{36}},t_{35},t_{28},t_4$

$= N\underline{t_{10},t_{35}},t_{28},t_4$

$= Nt_{28},t_{28},t_4$

$= N(t_{28})^2,t_4$

$= Nt_4$

$= N(t_1)^{(y^{-1}xyxy^{-1}xy)} \in [1]$

$\underline{\text{Choose 58 from } \{58, 89, 88, 70, 69, 95\}}$

We have the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = Id$

and we will conjugate it by $((yxyxy^{-1}x)^2)$ to get

$((yxyxy^{-1}x)^2)t_3,t_{35} = t_{58}$

then we have $((yxyxy^{-1}x)^2)t_{47},t_3,t_{35}$.

Then we have $Nt_1,\underline{t_{58}} = Nt_{47},t_3,t_{35}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

once again conjugated by $((y^{-1}xyxy^{-1}x)^2)t_{30},t_1 = t_{35}$

then we have $(yxy^{-1}xy^{-1}xyxyxyx)t_3,t_{29},t_{30},t_1$.

Next $Nt_1,(t_{58})$

$= Nt_{47},t_3,\underline{t_{35}}$

$= Nt_3,t_{29},t_{30},t_1$.

Next we will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

to get $((yxyxy^{-1}x)^2)t_{30} = t_3,t_{29}$

then we have $(xy^{-1}xyxyxy^{-1}xy^{-1}xyx)t_{30},t_{30},t_1$.

Thus we have $Nt_1,\underline{t_{58}}$

$= Nt_{47},t_3,\underline{t_{35}}$

$= N\underline{t_3,t_{29}},t_{30},t_1$

$= N\underline{t_{30},t_{30}},t_1$

$= N(t_{30})^2,t_1$

$= Nt_1 \epsilon [1]$

<u>Cayley Diagram</u>

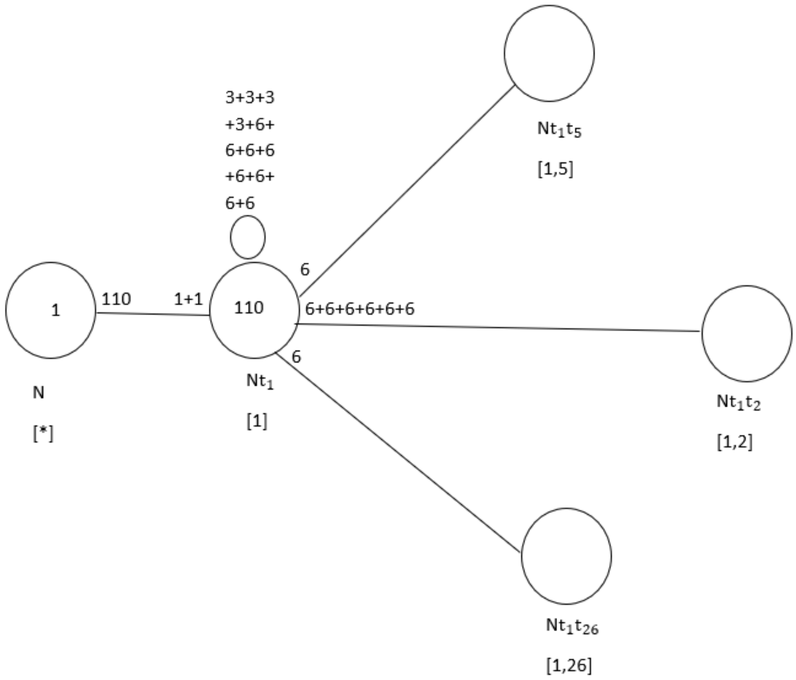


Figure 2.2: Cayley Diagram of [*] and [1] for M_{12}

Third Double Coset

$Nt_1t_2N = \{N(t_1t_2)^n \mid n\epsilon\ N\} = \{Nt_1t_2, Nt_1t_2, ..., Nt_3t_5\}$

The point-stabiliser of 1,2, $N^{1,2}$, is given by $<(xy^{-1}xy^{-1}xyxyx), x^y>$

But $t_1,t_2 = t_{64},t_{83}$

We will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(xy^{-1}xy^{-1}xyxyxy)$

to get $(xy^{-1}xy^{-1}xyxyxy)t_{28},t_{103} = t_2$

then we have $(xy^{-1}xy^{-1}xyxyxy)t_{26},t_{28},t_{103}$.

Next, $Nt_1,\underline{t_2} = Nt_{26},t_{28},t_{103}$.

We will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = 1$

again conjugated by

$(y^{-1}xy^{-1}xyxyxyxyxy^{-1}x)$ to get

$(y^{-1}xy^{-1}xyxyxyxyxy^{-1}x)t_{54},t_{104} = t_{26}$

then we have $(y^{-1}xy^{-1}xy^{-1}xyxy)t_{54},t_{104},t_{28},t_{103}$.

Next $Nt_1,(t_2)$

$= N\underline{t_{26}},t_{28},t_{103}$

$= Nt_{54},t_{104},t_{28},t_{103}$.

We will use the relation $t_1 = t_4$ conjugated by $((yxyxy^{-1}x)^2)$

to get $t_{28} = t_{47}$ then we have $Nt_1,\underline{t_2}$

$= N\underline{t_{26}},t_{28},t_{103}$

$= Nt_{54},t_{104},\underline{t_{28}},t_{103}$

$= Nt_{54},t_{104},t_{47},t_{103}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)$

to get

$(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_1 = t_{104},t_{47}$

then we have

$(yxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{37},t_1,t_{103}$.

Next $Nt_1,\underline{t_2}$

$= N\underline{t_{26}},t_{28},t_{103}$

$= Nt_{54},t_{104},\underline{t_{28}},t_{103}$

$= Nt_{54},\underline{t_{104},t_{47}},t_{103}$

$= Nt_{37},t_1,t_{103}$.

We will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = 1$

once again conjugated by

$(yxy^{-1}xyxyxy^{-1}xy)$

to get $(yxy^{-1}xyxyxy^{-1}xy)t_{98},t_{50} = t_{103}$

then we have $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{110},t_{55},t_{98},t_{50}$.

Next $Nt_1,\underline{t_2}$

$= N\underline{t_{26}},t_{28},t_{103}$

$= Nt_{54},t_{104},\underline{t_{28}},t_{103}$

$= Nt_{54},\underline{t_{104},t_{47}},t_{103}$

$= Nt_{37}, t_1, \underline{t_{103}}$

$= Nt_{110}, t_{55}, t_{98}, t_{50}$.

Using the relation $t_1 = t_4$ conjugated by

$(yxy^{-1}xyxyxyxyxy)$ to get $t_{55} = t_{88}$

then we have $Nt_1, \underline{t_2}$

$= N\underline{t_{26}}, t_{28}, t_{103}$

$= Nt_{54}, t_{104}, \underline{t_{28}}, t_{103}$

$= Nt_{54}, \underline{t_{104}, t_{47}}, t_{103}$

$= Nt_{37}, t_1, \underline{t_{103}}$

$= Nt_{110}, \underline{t_{55}}, t_{98}, t_{50}$

$= Nt_{110}, t_{88}, t_{98}, t_{50}$.

We will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(xyxyxy^{-1}xy^{-1}xy)$ to get

$(xyxyxy^{-1}xy^{-1}xy)t_{97}, t_7 = t_{88}$

then we have

$(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}x)t_{87}, t_{97}, t_7, t_{98}, t_{50}$.

Then we have $Nt_1, (t_2)$

$= N\underline{t_{26}}, t_{28}, t_{103}$

$= Nt_{54}, t_{104}, \underline{t_{28}}, t_{103}$

$= Nt_{54}, \underline{t_{104}, t_{47}}, t_{103}$

$= Nt_{37}, t_1, \underline{t_{103}}$

$= Nt_{110}, \underline{t_{55}}, t_{98}, t_{50}$

$= Nt_{110}, \underline{t_{88}}, t_{98}, t_{50}$

$= Nt_{87}, t_{97}, t_7, t_{98}, t_{50}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(yxyxyxyxy^{-1})$

to get $(yxyxyxyxy^{-1})t_{68} = t_7, t_{98}$

to get $(yxy^{-1}xy^{-1}xyxyxy^{-1}x)t_{29}, t_{28}, t_{68}, t_{50}$.

Next $Nt_1, \underline{t_2}$

$= N\underline{t_{26}}, t_{28}, t_{103}$

$= Nt_{54},t_{104},\underline{t_{28}},t_{103}$

$= Nt_{54},\underline{t_{104},t_{47}},t_{103}$

$= Nt_{37},t_{1},\underline{t_{103}}$

$= Nt_{110},\underline{t_{55}},t_{98},t_{50}$

$= Nt_{110},\underline{t_{88}},t_{98},t_{50}$

$= Nt_{87},t_{97},\underline{t_{7},t_{98}},t_{50}$

$= Nt_{29},t_{28},t_{68},t_{50}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^{3})t_{49},t_{105},t_{25} = Id$

conjugated by $((yxyxy^{-1}x)^{2})$ to get

$((yxyxy^{-1}x)^{2})t_{58},t_{30} = t_{29}$

then we have $(yxyxy^{-1}xyxy^{-1}xy^{-1})t_{58},t_{30},t_{28},t_{68},t_{50}$.

Then we currently have $Nt_{1},\underline{t_{2}}$

$= N\underline{t_{26}},t_{28},t_{103}$

$= Nt_{54},t_{104},\underline{t_{28}},t_{103}$

$= Nt_{54},\underline{t_{104},t_{47}},t_{103}$

$= Nt_{37},t_{1},\underline{t_{103}}$

$= Nt_{110},\underline{t_{55}},t_{98},t_{50}$

$= Nt_{110},\underline{t_{88}},t_{98},t_{50}$

$= Nt_{87},t_{97},\underline{t_{7},t_{98}},t_{50}$

$= N\underline{t_{29}},t_{28},t_{68},t_{50}$

$= Nt_{58},t_{30},t_{28},t_{68},t_{50}$.

By using the relation $t_{1} = t_{4}$ conjugated by $(xy^{-1}xy^{-1}xy^{-1}xyxyx)$ to get $t_{28} = t_{47}$

then we have $Nt_{1},\underline{t_{2}}$

$= N(t_{26}),t_{28},t_{103}$

$= Nt_{54},t_{104},\underline{t_{28}},t_{103}$

$= Nt_{54},\underline{t_{104},t_{47}},t_{103}$

$= Nt_{37},t_{1},\underline{t_{103}}$

$= Nt_{110},\underline{t_{55}},t_{98},t_{50}$

$= Nt_{110},\underline{t_{88}},t_{98},t_{50}$

$= Nt_{87},t_{97},\underline{t_{7},t_{98}},t_{50}$

$= N\underline{t_{29}},t_{28},t_{68},t_{50}$

$= Nt_{58},t_{30},\underline{t_{28}},t_{68},t_{50}$

$= Nt_{58},t_{30},t_{47},t_{68},t_{50}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy^{*}x)^{3})t_{49},t_{105},t_{25} = Id$

conjugated by

$(xy^{-1}xyxyxyxyxy^{-1}xyxy^{-1})$

to get

$(xy^{-1}xyxyxyxyxy^{-1}xyxy^{-1})t_{3} = t_{30},t_{47}$

then we have

$(y^{-1}xyxy^{-1}xy^{-1}xyxy)t_{35},t_{3},t_{68},t_{50}$

which leaves us with $Nt_{1},\underline{t_{2}}$

$= N\underline{t_{26}},t_{28},t_{103}$

$= Nt_{54},t_{104},\underline{t_{28}},t_{103}$

$= Nt_{54},\underline{t_{104},t_{47}},t_{103}$

$= Nt_{37},t_{1},\underline{t_{103}}$

$= Nt_{110},\underline{t_{55}},t_{98},t_{50}$

$= Nt_{110},\underline{t_{88}},t_{98},t_{50}$

$= Nt_{87},t_{97},\underline{t_{7},t_{98}},t_{50}$

$= N\underline{t_{29}},t_{28},t_{68},t_{50}$

$= Nt_{58},t_{30},\underline{t_{28}},t_{68},t_{50}$

$= Nt_{58},\underline{t_{30},t_{47}},t_{68},t_{50}$

$= Nt_{35},t_{3},t_{68},t_{50}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy^{*}x)^{3})t_{49},t_{105},t_{25} = Id$

conjugated by $((yxyxy^{-1}x)^{2})$

to get $((yxyxy^{-1}x)^{2})t_{29} = t_{35},t_{3}$

then we have $(xyxyxy^{-1}xy^{-1}xy^{-1})t_{29},t_{68},t_{50}$.

Then $Nt_{1},(t_{2})$

$= N(t_{26}),t_{28},t_{103}$

$= Nt_{54},t_{104},(t_{28}),t_{103}$

$= Nt_{54},(t_{104},t_{47}),t_{103}$

$= Nt_{37},t_{1},(t_{103})$

$= Nt_{110},(t_{55}),t_{98},t_{50}$

$= Nt_{110},(t_{88}),t_{98},t_{50}$

$= Nt_{87},t_{97},(t_{7},t_{98}),t_{50}$

$= N(t_{29}),t_{28},t_{68},t_{50}$

$= Nt_{58},t_{30},(t_{28}),t_{68},t_{50}$

$= Nt_{58},(t_{30},t_{47}),t_{68},t_{50}$

$= N(t_{35},t_{3}),t_{68},t_{50}$

$= Nt_{29},t_{68},t_{50}$.

Then by using the relation $t_1 = t_4$ conjugated

by $(y^{-1}xyxy^{-1}xy^{-1}xyxy^{-1}xy^{-1}x)$ we have $t_{68} = t_{57}$ then we get

$(xyxyxy^{-1}xy^{-1}xy^{-1})t_{29},t_{57},t_{50}$.

Then we have $Nt_1,t_2)$

$= N(t_{26}),t_{28},t_{103}$

$= Nt_{54},t_{104},(t_{28}),t_{103}$

$= Nt_{54},(t_{104},t_{47}),t_{103}$

$= Nt_{37},t_{1},(t_{103})$

$= Nt_{110},(t_{55}),t_{98},t_{50}$

$= Nt_{110},(t_{88}),t_{98},t_{50}$

$= Nt_{87},t_{97},(t_{7},t_{98}),t_{50}$

$= N(t_{29}),t_{28},t_{68},t_{50}$

$= Nt_{58},t_{30},(t_{28}),t_{68},t_{50}$

$= Nt_{58},(t_{30},t_{47}),t_{68},t_{50}$

$= N(t_{35},t_{3}),t_{68},t_{50}$

$= Nt_{29},(t_{68}),t_{50}$

$= Nt_{29},t_{57},t_{50}$.

Next we will use the relation $t_1 = t_4$ conjugated by

$(xyxy^{-1}xy^{-1}xyxy^{-1}x)$ to get $t_{29} = t_{27}$

and use the same relation $t_1 = t_4$ conjugated by

$(xy^{-1}xyxy^{-1}xyx)$ to get $t_{50} = t_{83}$

to get $(xyxyxy^{-1}xy^{-1}xy^{-1})t_{27},t_{57},t_{83}$

then we have $Nt_1,(t_2)$

$= N(t_{26}),t_{28},t_{103}$

$= Nt_{54},t_{104},(t_{28}),t_{103}$

$= Nt_{54},(t_{104},t_{47}),t_{103}$

$= Nt_{37},t_{1},(t_{103})$

$= Nt_{110},(t_{55}),t_{98},t_{50}$

$= Nt_{110},(t_{88}),t_{98},t_{50}$

$= Nt_{87},t_{97},(t_{7},t_{98}),t_{50}$

$= N(t_{29}),t_{28},t_{68},t_{50}$

$= Nt_{58},t_{30},(t_{28}),t_{68},t_{50}$

$= Nt_{58},(t_{30},t_{47}),t_{68},t_{50}$

$= N(t_{35},t_{3}),t_{68},t_{50}$

$= Nt_{29},(t_{68}),t_{50}$

$= N(t_{29}),t_{57},(t_{50})$

$= Nt_{27},t_{57},t_{83}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^{3})t_{49},t_{105},t_{25} = Id$

conjugated by $((xyxy^{-1}xy^{-1})^{2})$ to get

$((xyxy^{-1}xy^{-1})^{2})t_{9},t_{12} = t_{27}$

then we have $(y^{-1}xyxy^{-1}xy^{-1}xyxy)t_{9},t_{12},t_{57},t_{83}$.

Therefore we have $Nt_{1},(t_{2})$

$= N(t_{26}),t_{28},t_{103}$

$= Nt_{54},t_{104},(t_{28}),t_{103}$

$= Nt_{54},(t_{104},t_{47}),t_{103}$

$= Nt_{37},t_{1},(t_{103})$

$= Nt_{110},(t_{55}),t_{98},t_{50}$

$= Nt_{110},(t_{88}),t_{98},t_{50}$

$= Nt_{87},t_{97},(t_{7},t_{98}),t_{50}$

$= N(t_{29}),t_{28},t_{68},t_{50}$

$= Nt_{58},t_{30},(t_{28}),t_{68},t_{50}$

$= Nt_{58},(t_{30},t_{47}),t_{68},t_{50}$

$= N(t_{35},t_{3}),t_{68},t_{50}$

$= Nt_{29},(t_{68}),t_{50}$

$= N(t_{29}),t_{57},(t_{50})$

$= N(t_{27}),t_{57},t_{83}$

$= Nt_9,t_{12},t_{57},t_{83}$.

By using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})$ to get

$(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{12},t_{57},t_{96} = Id$

we get $(yxy^{-1}xy^{-1}xyxy^{-1}xyx)t_{25},t_{96},t_{83}$.

Next $Nt_1,(t_2)$

$= N(t_{26}),t_{28},t_{103}$

$= Nt_{54},t_{104},(t_{28}),t_{103}$

$= Nt_{54},(t_{104},t_{47}),t_{103}$

$= Nt_{37},t_1,(t_{103})$

$= Nt_{110},(t_{55}),t_{98},t_{50}$

$= Nt_{110},(t_{88}),t_{98},t_{50}$

$= Nt_{87},t_{97},(t_7,t_{98}),t_{50}$

$= N(t_{29}),t_{28},t_{68},t_{50}$

$= Nt_{58},t_{30},(t_{28}),t_{68},t_{50}$

$= Nt_{58},(t_{30},t_{47}),t_{68},t_{50}$

$= N(t_{35},t_3),t_{68},t_{50}$

$= Nt_{29},(t_{68}),t_{50}$

$= N(t_{29}),t_{57},(t_{50})$

$= N(t_{27}),t_{57},t_{83}$

$= Nt_9,(t_{12},t_{57}),t_{83}$

$= Nt_{25},t_{96},t_{83}$.

Then we will use the relation

$t_1 = t_4$ conjugated by

$(yxyxyxy^{-1}xyxyxy^{-1})$ to get $t_{75} = t_{25}$

we get $Nt_1,(t_2)$

$= N(t_{26}),t_{28},t_{103}$

$= Nt_{54},t_{104},(t_{28}),t_{103}$

$= Nt_{54},(t_{104},t_{47}),t_{103}$

$= Nt_{37},t_1,(t_{103})$

$= Nt_{110},(t_{55}),t_{98},t_{50}$

$= Nt_{110},(t_{88}),t_{98},t_{50}$

$= Nt_{87},t_{97},(t_{7},t_{98}),t_{50}$

$= N(t_{29}),t_{28},t_{68},t_{50}$

$= Nt_{58},t_{30},(t_{28}),t_{68},t_{50}$

$= Nt_{58},(t_{30},t_{47}),t_{68},t_{50}$

$= N(t_{35},t_{3}),t_{68},t_{50}$

$= Nt_{29},(t_{68}),t_{50}$

$= N(t_{29}),t_{57},(t_{50})$

$= N(t_{27}),t_{57},t_{83}$

$= Nt_{9},(t_{12},t_{57}),t_{83}$

$= N(t_{25}),t_{96},t_{83}$

$= Nt_{75},t_{96},t_{83}$.

Then we will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^{3})t_{49},t_{105},t_{25} = Id$

conjugated by $(xyxyxyxy^{-1}xy^{-1}xyxyxy)$

to get $(xyxyxyxy^{-1}xy^{-1}xyxyxy)t_{64} = t_{75},t_{96}$

then we have $((xy^{-1}xy)^{2})t_{64},t_{83}$.

Thus, we have $Nt_{1},(t_{2})$

$= N(t_{26}),t_{28},t_{103}$

$= Nt_{54},t_{104},(t_{28}),t_{103}$

$= Nt_{54},(t_{104},t_{47}),t_{103}$

$= Nt_{37},t_{1},(t_{103})$

$= Nt_{110},(t_{55}),t_{98},t_{50}$

$= Nt_{110},(t_{88}),t_{98},t_{50}$

$= Nt_{87},t_{97},(t_{7},t_{98}),t_{50}$

$= N(t_{29}),t_{28},t_{68},t_{50}$

$= Nt_{58},t_{30},(t_{28}),t_{68},t_{50}$

$= Nt_{58},(t_{30},t_{47}),t_{68},t_{50}$

$= N(t_{35},t_{3}),t_{68},t_{50}$

$= Nt_{29},(t_{68}),t_{50}$

$= N(t_{29}),t_{57},(t_{50})$

$= N(t_{27}),t_{57},t_{83}$

$= Nt_9,(t_{12},t_{57}),t_{83}$

$= N(t_{25}),t_{96},t_{83}$

$= N(t_{75},t_{96}),t_{83}$

$= Nt_{64},t_{83}$.

Then $x^y \epsilon N^{(1,2)}$

Also, $t_1,t_2 = t_{92},t_{107}$

Therefore, using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49}, t_{105} ,t_{25} = 1$

conjugated by $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})$ to get

$(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{93} ,t_{79} = t_2$

then we have $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{54} ,t_{93} ,t_{79}$.

Next $Nt_1,(t_2) = Nt_{54} ,t_{93} ,t_{79}$.

Using the relation $t_1 = t_4$ conjugated by

$(xyxy^{-1}xyxyxyxyxy^{-1})$

to get $t_{91} = t_{93}$

then we have $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{54} ,t_{91},t_{79}$.

Next $Nt_1,(t_2)$

$= Nt_{54} ,(t_{93}),t_{79}$

$= Nt_{54} ,t_{91},t_{79}$.

Next we will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49}, t_{105} ,t_{25} = Id$

conjugated by $(yxyxy^{-1}xy^{-1}xy^{-1}x)$ to get

$(yxyxy^{-1}xy^{-1}xy^{-1}x)t_{54} ,t_{91}, t_{108} = Id$

then we have $(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)\ t_{108} ,t_{79}$.

Then we have $Nt_1,(t_2)$

$= Nt_{54} ,(t_{93}),t_{79}$

$= N(t_{54} ,t_{91}),t_{79}$

$= N\ t_{108} ,t_{79}$.

By using the relation $t_1 = t_4$ conjugated by $(y^{-1}xy^{-1}xyxy^{-1}x)$

to get $t_{108} = t_{92}$ we have

$Nt_1,(t_2)$

$= Nt_{54}$,(t_{93}),t_{79}

$= N(t_{54}$,t_{91}),t_{79}

$= N($ t_{108}),t_{79}

$= Nt_{92}$,t_{79}.

We will use the relation $t_1 = t_4$ again conjugated by

($xy^{-1}xyxy^{-1}xyxyxy$) to get $t_{79} = t_{107}$

then we have Nt_1,(t_2)

$= Nt_{54}$,(t_{93}),t_{79}

$= N(t_{54}$,t_{91}),t_{79}

$= N($ t_{108}),t_{79}

$= Nt_{92}$,(t_{79})

$= Nt_{92}$, t_{107}.

Then ($xy^{-1}xy^{-1}xyxyx$) ϵ $N^{(1,2)}$

Therefore the Coset Stabilizer $N^{(12)}$ = <(x^y),($xy^{-1}xy^{-1}xyxyx$)>

The number of single right cosets in the double coset Nt_1t_2N = [1,2] is given by $\frac{|N|}{|N^{(1,2)}|}$ $= \frac{660}{10} = 66$

The orbits for $N^{(12)}$ on

X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110} are:

{6, 84, 22, 65, 99} , {10, 37, 14, 28, 41}, {21, 51, 74, 101, 95}, {24, 72, 89, 63, 53}, {32, 78, 43, 86, 39}, {36, 98, 71, 47, 96}, {1, 64, 92, 57, 108, 87, 4, 90, 102, 68}, {2, 83, 107, 77, 40, 69, 18, 45, 46, 100}, {3, 5, 56, 103, 29, 110, 27, 31, 85, 12}, {7, 106, 25, 60, 58, 33, 19, 75, 49, 104}, {8, 44, 17, 35, 59, 73, 62, 16, 93, 109}, {9, 23, 20, 15, 34, 81, 30, 91, 13, 80}, {11, 50, 79, 61, 38, 67, 48, 42, 70, 82}, {26, 97, 94, 105, 55, 76, 66, 52, 54, 88}

Multiply Nt_1t_2 by a representative of each orbit to determine its double coset.

Multiply by a representative of each orbit:

Choose 6 from {6, 84, 22, 65, 99}

Using the relation $t_1 = t_4$ conjugated by

($y^{-1}xyxyxy^{-1}xy^{-1}xyxy$) to get $t_2 = t_{11}$

we have $Nt_1,(t_2),t_6$

$= Nt_1,t_{11}\ ,t_6.$

Next by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $((yxy^{-1}xyx)^2)$ to get

$((yxy^{-1}xyx)^2)t_{26} = t_6,t_{11}$

then we have $((xy^{-1}xyxy^{-1})^2)t_{42},t_{86}.$

Thus $Nt_1,(t_2),t_6$

$= Nt_1,(t_{11}\ ,t_6)$

$= Nt_{42},t_{86}\ \epsilon\ [1,26]$

Choose 10 from {10, 37, 14, 28, 41}

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by

$((xy^{-1}xyxy)^2)$ to get $((xy^{-1}xyxy)^2)t_{18},t_{75} = t_{10}$

then we have $((xy^{-1}xyxy)^2)t_{71},t_{33},t_{18},t_{75}.$

Next $Nt_1,t_2,(t_{10}\ )$

$= Nt_{71},t_{33}\ ,t_{18},t_{75}.$

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(yxyxyxyxyxy^{-1}xy^{-1}xy)$ then we get

$(yxyxyxyxyxy^{-1}xy^{-1}xy)t_{106}\ ,t_{66} = t_{75}$

then we have $(xyxy^{-1}xyxyx)t_{18},t_{94},t_{44},t_{106}\ ,t_{66}.$

Next $Nt_1,t_2,(t_{10}\ )$

$= Nt_{71},t_{33}\ ,t_{18},(t_{75}\ )$

$= Nt_{18},t_{94},t_{44},t_{106}\ ,t_{66}.$

By using the relation

$(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{92},t_{107} = t_1,t_2$

conjugated by

$(xy^{-1}xy^{-1}xyxyxyxy)$ to get

$(xy^{-1}xy^{-1}xyxyxyxy)t_2,t_{105} = t_{18},t_{94}$

which gives us $(xy^{-1}xyxy)t_2,t_{105},t_{44},t_{106},t_{66}.$

Then we have $Nt_1,t_2,(t_{10})$

$= Nt_{71},t_{33},t_{18},(t_{75})$

$= N(t_{18},t_{94}),t_{44},t_{106},t_{66}$

$= N2,t_{105},t_{44},t_{106},t_{66}$.

By using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(xy^{-1}xyxyxy^{-1}xy^{-1})$

gives

$(xy^{-1}xyxyxy^{-1}xy^{-1})t_{90} = t_{105},t_{44}$

then we have $((yxy^{-1}xy^{-1}x)^2)t_{60},t_{90},t_{106},t_{66}$.

Next $Nt_1,t_2,(t_{10})$

$= Nt_{71},t_{33},t_{18},(t_{75})$

$= N(t_{18},t_{94}),t_{44},t_{106},t_{66}$

$= Nt_2,(t_{105},t_{44}),t_{106},t_{66}$

$= Nt_{60},t_{90},t_{106},t_{66}$.

Using the relation $t_1 = t_4$ conjugated by

$(xyxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})$ to get $t_{90} = t_{87}$

then we have $Nt_1,t_2,(t_{10})$

$= Nt_{71},t_{33},t_{18},(t_{75})$

$= N(t_{18},t_{94}),t_{44},t_{106},t_{66}$

$= Nt_2,(t_{105},t_{44}),t_{106},t_{66}$

$= Nt_{60},(t_{90}),t_{106},t_{66}$

$= Nt_{60},t_{87},t_{106},t_{66}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = 1$

conjugated by $(yxy^{-1}xyxyxyxyxy^{-1})$ gives

$(yxy^{-1}xyxyxyxyxy^{-1})t_{107},t_{91} = t_{87}$

then we have $(yxyxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{47},t_{107},t_{91},t_{106},t_{66}$

which gives us $Nt_1,t_2,(t_{10})$

$= Nt_{71},t_{33},t_{18},(t_{75})$

$= N(t_{18},t_{94}),t_{44},t_{106},t_{66}$

$= Nt_2,(t_{105},t_{44}),t_{106},t_{66}$

$= Nt_{60}$,(t_{90}),t_{106} ,t_{66}

$= Nt_{60}$,(t_{87}),t_{106} ,t_{66}

$= Nt_{47}$,t_{107} ,t_{91},t_{106},t_{66}.

By using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)$ which gives us

$(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_{109} = t_{91},t_{106}$

then we have $((yxy^{-1}x)^2)t_{79}$,t_{11} ,t_{109} ,t_{66}

which then gives us Nt_1,t_2,(t_{10})

$= Nt_{71}$,t_{33} ,t_{18},(t_{75})

$= N(t_{18},t_{94})$,t_{44},t_{106} ,t_{66}

$= Nt_2$,(t_{105} ,t_{44}),t_{106} ,t_{66}

$= Nt_{60}$,(t_{90}),t_{106} ,t_{66}

$= Nt_{60}$,(t_{87}),t_{106} ,t_{66}

$= Nt_{47}$,t_{107} ,(t_{91},t_{106}),t_{66}

$= Nt_{79}$,t_{11} ,t_{109} ,t_{66}.

Using the relation 1=4 conjugated by $(y^{-1}xy^{-1}xyxyxy^{-1}xyxy)$

to get $t_2 = t_{11}$ we then have

Nt_1,t_2,(t_{10})

$= Nt_{71}$,t_{33} ,t_{18},(t_{75})

$= N(t_{18},t_{94})$,t_{44},t_{106} ,t_{66}

$= Nt_2$,(t_{105} ,t_{44}),t_{106} ,t_{66}

$= Nt_{60}$,(t_{90}),t_{106} ,t_{66}

$= Nt_{60}$,(t_{87}),t_{106} ,t_{66}

$= Nt_{47}$,t_{107} ,(t_{91},t_{106}),t_{66}

$= Nt_{79}$,(t_{11}),t_{109} ,t_{66}

$= Nt_{79}$,t_2,t_{109} ,t_{66}.

Then we will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}$,$t_{25} = Id$

conjugated by $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})$ to get

$(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{97},t_2,t_{81} = Id$

then we have $(xyxy^{-1}xy^{-1}xyxyxy^{-1})t_{81}$,t_{109} ,t_{66}

which gives $Nt_1,t_2,(t_{10})$

$= Nt_{71},t_{33},t_{18},(t_{75})$

$= N(t_{18},t_{94}),t_{44},t_{106},t_{66}$

$= Nt_2,(t_{105},t_{44}),t_{106},t_{66}$

$= Nt_{60},(t_{90}),t_{106},t_{66}$

$= Nt_{60},(t_{87}),t_{106},t_{66}$

$= Nt_{47},t_{107},(t_{91},t_{106}),t_{66}$

$= Nt_{79},(t_{11}),t_{109},t_{66}$

$= N(t_{79},2),t_{109},t_{66}$

$= Nt_{81},t_{109},t_{60}$.

Then we will use the relation $t_1 = t_4$ conjugated by $(yxy^{-1}xyxyxy^{-1}xyxyx)$ to get $t_{81} = t_{109}$

then we have $Nt_1,t_2,(t_{10})$

$= Nt_{71},t_{33},t_{18},(t_{75})$

$= N(t_{18},t_{94}),t_{44},t_{106},t_{66}$

$= Nt_2,(t_{105},t_{44}),t_{106},t_{66}$

$= Nt_{60},(t_{90}),t_{106},t_{66}$

$= Nt_{60},(t_{87}),t_{106},t_{66}$

$= Nt_{47},t_{107},(t_{91},t_{106}),t_{66}$

$= Nt_{79},(t_{11}),t_{109},t_{66}$

$= N(t_{79},t_2),t_{109},t_{66}$

$= N(t_{81}),t_{109},t_{60}$

$= N(t_{109},t_{109}),t_{66}$

$= N(t_{109})^2,t_{66} = Nt_{66}$

$= N(t_1)^{(xyxy^{-1}xy^{-1}xy^{-1}xyx)} \epsilon$ [1]

<u>Choose 21 from {21, 51, 74, 101, 95}</u>

By using the relation

$(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{92},t_{107} = t_1,t_2$

then we have $(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{92},t_{107},t_{21}$.

So we have $N(t_1,t_2),t_{21} = Nt_{92},t_{107},t_{21}$.

Using the relation $t_1 = t_4$ conjugated by

$(xy^{-1}xyxy^{-1}xyxyxy)$ to get $t_{107} = t_{79}$

we have N(t_1,t_2),t_{21}

= Nt_{92},(t_{107}),t_{21}

= Nt_{92},t_{79},t_{21}.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25}$ = Id

conjugated by $(xyxyxyxy^{-1}xy^{-1}xyx)$

which gives $(xyxyxyxy^{-1}xy^{-1}xyx)5,t_{94} = t_{79}$

then we have $(xyxyxy^{-1}xy^{-1}xyxy^{-1}xy)t_{87}$,t_{45},t_{94},t_{21}

which leaves us with N(t_1,t_2),t_{21}

= Nt_{92},(t_{107}),t_{21}

= Nt_{92},(t_{79}),t_{21}

= Nt_{87},t_{45},t_{94},t_{21}.

By using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}$,t_{25} = Id

conjugated by $(yxy^{-1}xyxyxyxyxy^{-1}xy)$

gives us $(yxy^{-1}xyxyxyxyxy^{-1}xy)6 = t_{94},t_{21}$

then we have $(yxy^{-1}xyxy^{-1}xy^{-1}xyxyxy)t_{12}$,t_{39} ,t_6

next N(t_1,t_2),t_{21}

= Nt_{92},(t_{107}),t_{21}

= Nt_{92},(t_{79}),t_{21}

= Nt_{87} ,t_{45},(t_{94},t_{21})

= Nt_{12} ,t_{39} ,t_6.

Then by using th relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}$,t_{25} = 1

conjugated by $(xyxyxyxy^{-1}xy^{-1}xyx)$ which gives

$(xyxyxyxy^{-1}xy^{-1}xyx)t_{79} = t_{39},t_6$

then we have $(yxyxyxy^{-1}xyxy^{-1}x)t_{48},t_{79}$.

Next N(t_1,t_2),t_{21}

= Nt_{92},(t_{107}),t_{21}

= Nt_{92},(t_{79}),t_{21}

= Nt_{87} ,t_{45},(t_{94},t_{21})

= Nt_{12} ,(t_{39} ,t_6)

$= Nt_{48},t_{79}$.

Then by using the relation $t_1 = t_4$ conjugated by

$(xy^{-1}xyxy^{-1}xyxyxy)$ gives $t_{79} = t_{107}$

then we have $N(t_1,t_2),t_{21}$

$= Nt_{92},(t_{107}),t_{21}$

$= Nt_{92},(t_{79}),t_{21}$

$= Nt_{87},t_{45},(t_{94},t_{21})$

$= Nt_{12},(t_{39},t_6)$

$= Nt_{48},(t_{79})$

$= Nt_{48},t_{107}$.

By using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(yxy^{-1}xyxyxyxyxy^{-1})$

gives $(yxy^{-1}xyxyxyxyxy^{-1})t_{109},t_{11} = t_{107}$

then we have $(xy^{-1}xy^{-1}xyxyxyxy)t_{100},t_{109},t_{11}$

next $N(t_1,t_2),t_{21}$

$= Nt_{92},(t_{107}),t_{21}$

$= Nt_{92},(t_{79}),t_{21}$

$= Nt_{87},t_{45},(t_{94},t_{21})$

$= Nt_{12},(t_{39},t_6)$

$= Nt_{48},(t_{79})$

$= Nt_{48},(t_{107})$

$= Nt_{100},t_{109},t_{11}$.

Using the relation t_{14} conjugated by $(yxyxyxy^{-1})$ we get $t_{82} = t_{100}$

then we have $N(t_1,t_2),t_{21}$

$= Nt_{92},(t_{107}),t_{21}$

$= Nt_{92},(t_{79}),t_{21}$

$= Nt_{87},t_{45},(t_{94},t_{21})$

$= Nt_{12},(t_{39},t_6)$

$= Nt_{48},(t_{79})$

$= Nt_{48},(t_{107})$

$= N(t_{100}),t_{109},t_{11}$

$= Nt_{82}$,t_{109} ,t_{11}.
Then using the relation
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49}$,$t_{105}$,t_{25} = Id
conjugated by $(y^{-1}xyxyxyxy)$ to get
$(y^{-1}xyxyxyxy)t_{78} = t_{82}$,t_{109}
then we have $(y^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy^{-1})t_{78}$,$t_{11}$
next N(t_1,t_2),t_{21}
$= Nt_{92}$,(t_{107}),t_{21}
$= Nt_{92}$,(t_{79}),t_{21}
$= Nt_{87}$,t_{45},(t_{94},t_{21})
$= Nt_{12}$,(t_{39} ,t_6)
$= Nt_{48}$,(t_{79})
$= Nt_{48}$,(t_{107})
= N(t_{100}),t_{109} ,t_{11}
= N(t_{82} ,t_{109}),t_{11}
$= Nt_{78}$,t_{11}.
Then using the relation
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49}$,$t_{105}$,t_{25} = Id
conjugated by $(y^{-1}xy^{-1}xy^{-1}xyxyx)$
gives $(y^{-1}xy^{-1}xy^{-1}xyxyx)t_{91}$,$t_{71} = t_{11}$
then we have $(xy^{-1}xy^{-1}xyxy^{-1}x)t_{61}$,t_{91},t_{71}
then we have N(t_1,t_2),t_{21}
$= Nt_{92}$,(t_{107}),t_{21}
$= Nt_{92}$,(t_{79}),t_{21}
$= Nt_{87}$,t_{45},(t_{94},t_{21})
$= Nt_{12}$,(t_{39} ,t_6)
$= Nt_{48}$,(t_{79})
$= Nt_{48}$,(t_{107})
= N(t_{100}),t_{109} ,t_{11}
= N(t_{82} ,t_{109}),t_{11}
$= Nt_{78}$,(t_{11})
$= Nt_{61}$,t_{91},t_{71}.

Then by using the relation $t_1 = t_4$ conjugated by
$(xyxy^{-1}xyxyxyxyxy^{-1})$ gives $t_{91} = t_{93}$
then we have $N(t_1,t_2),t_{21}$
$= Nt_{92},(t_{107}),t_{21}$
$= Nt_{92},(t_{79}),t_{21}$
$= Nt_{87},t_{45},(t_{94},t_{21})$
$= Nt_{12},(t_{39},t_6)$
$= Nt_{48},(t_{79})$
$= Nt_{48},(t_{107})$
$= N(t_{100}),t_{109},t_{11}$
$= N(t_{82},t_{109}),t_{11}$
$= Nt_{78},(t_{11})$
$= Nt_{61},(t_{91}),t_{71}$
$= Nt_{61},t_{93},t_{71}$.
Using the same relation again $t_1 = t_4$ conjugated by
$(yxy^{-1}xyxyxy^{-1}xy^{-1}xy^{-1}x)$ gives $t_{61} = t_{77}$
then we have $N(t_1,t_2),t_{21}$
$= Nt_{92},(t_{107}),t_{21}$
$= Nt_{92},(t_{79}),t_{21}$
$= Nt_{87},t_{45},(t_{94},t_{21})$
$= Nt_{12},(t_{39},t_6)$
$= Nt_{48},(t_{79})$
$= Nt_{48},(t_{107})$
$= N(t_{100}),t_{109},t_{11}$
$= N(t_{82},t_{109}),t_{11}$
$= Nt_{78},(t_{11})$
$= Nt_{61},(t_{91}),t_{71}$
$= N(t_{61}),t_{93},t_{71}$
$= Nt_{77},t_{93},t_{71}$.
Then by using the relation
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$
conjugated by $(yxyxyxyxyxy^{-1}xy^{-1})$

gives $(yxyxyxyxyxy^{-1}xy^{-1})t_{41}, t_{31} = t_{77}$

then we have

$(xyxy^{-1}xyxyxyxyxy^{-1}x)t_{41}, t_{31}, t_{93}, t_{71}$.

Next $N(t_1,t_2),t_{21}$

$= Nt_{92},(t_{107}),t_{21}$

$= Nt_{92},(t_{79}),t_{21}$

$= Nt_{87}, t_{45},(t_{94},t_{21})$

$= Nt_{12}, (t_{39}, t_6)$

$= Nt_{48},(t_{79})$

$= Nt_{48},(t_{107})$

$= N(t_{100}),t_{109}, t_{11}$

$= N(t_{82}, t_{109}),t_{11}$

$= Nt_{78},(t_{11})$

$= Nt_{61}, (t_{91}),t_{71}$

$= N(t_{61}),t_{93}, t_{71}$

$= N(t_{77}),t_{93}, t_{71}$

$= Nt_{41}, t_{31}, t_{93}, t_{71}$.

Then the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}, t_{25} = 1$

conjugated by $(xy^{-1}xyxyxyxyx)$

gives $(xy^{-1}xyxyxyxyx)t_{31}, t_{93}, t_{60} = Id$

then we have

$(yxy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{88},t_{60}, t_{71}$.

Next $N(t_1,t_2),t_{21}$

$= Nt_{92},(t_{107}),t_{21}$

$= Nt_{92},(t_{79}),t_{21}$

$= Nt_{87}, t_{45},(t_{94},t_{21})$

$= Nt_{12}, (t_{39}, t_6)$

$= Nt_{48},(t_{79})$

$= Nt_{48},(t_{107})$

$= N(t_{100}),t_{109}, t_{11}$

$= N(t_{82}, t_{109}),t_{11}$

$= Nt_{78},(t_{11})$

$= Nt_{61},(t_{91}),t_{71}$

$= N(t_{61}),t_{93},t_{71}$

$= N(t_{77}),t_{93},t_{71}$

$= Nt_{41},(t_{31},t_{93}),t_{71}$

$= Nt_{88},t_{60},t_{71}$.

Then by using the relation $t_1 = t_4$ conjugated by

$(yxy^{-1}xyxyxyxyxy)$ to get $t_{88} = t_{55}$

we have $N(t_1,t_2),t_{21}$

$= Nt_{92},(t_{107}),t_{21}$

$= Nt_{92},(t_{79}),t_{21}$

$= Nt_{87},t_{45},(t_{94},t_{21})$

$= Nt_{12},(t_{39},t_6)$

$= Nt_{48},(t_{79})$

$= Nt_{48},(t_{107})$

$= N(t_{100}),t_{109},t_{11}$

$= N(t_{82},t_{109}),t_{11}$

$= Nt_{78},(t_{11})$

$= Nt_{61},(t_{91}),t_{71}$

$= N(t_{61}),t_{93},t_{71}$

$= N(t_{77}),t_{93},t_{71}$

$= Nt_{41},(t_{31},t_{93}),t_{71}$

$= N(t_{88}),t_{60},t_{71}$

$= Nt_{55},t_{60},t_{71}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(y^{-1}xyxyxy^{-1}xy^{-1}x)$

we have $(y^{-1}xyxyxy^{-1}xy^{-1}x)t_{58} = t_{55},t_{60}$

which gives $(yxyxy^{-1}xy)t_{58},t_{71}$.

Thus $N(t_1,t_2),t_{21}$

$= Nt_{92},(t_{107}),t_{21}$

$= Nt_{92},(t_{79}),t_{21}$

$= Nt_{87}, t_{45}, (t_{94}, t_{21})$

$= Nt_{12}, (t_{39}, t_{6})$

$= Nt_{48}, (t_{79})$

$= Nt_{48}, (t_{107})$

$= N(t_{100}), t_{109}, t_{11}$

$= N(t_{82}, t_{109}), t_{11}$

$= Nt_{78}, (t_{11})$

$= Nt_{61}, (t_{91}), t_{71}$

$= N(t_{61}), t_{93}, t_{71}$

$= N(t_{77}), t_{93}, t_{71}$

$= Nt_{41}, (t_{31}, t_{93}), t_{71}$

$= N(t_{88}), t_{60}, t_{71}$

$= N(t_{55}, t_{60}), t_{71}$

$= Nt_{58}, t_{71}$

$= N(t_1, t_2)^{(xy-1xyxyxy^{-1}xy^{-1}xyx)} \epsilon$ [1,2]

Choose 24 from {24, 72, 89, 63, 53}

Using the relation $t_1 = t_4$ conjugated by $(y^{-1}xyxy^{-1}xyxyxyxy)$ gives $t_{24} = t_{74}$

then we have $Nt_1, t_2, (t_{24})$

$= Nt_1, t_2, t_{74}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(yxyxy^{-1}xyxyxyxyx)$

gives $(yxyxy^{-1}xyxyxyxyx)t_{34}, t_{65} = t_{74}$.

Then we have $(yxyxy^{-1}xyxyxyxyx)t_{94}, t_{65}, t_{34}, t_{65}$.

Next $Nt_1, t_2, (t_{24})$

$= Nt_1, t_2, (t_{74})$

$= Nt_{94}, t_{65}, t_{34}, t_{65}$.

Next using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by

$(yxyxy^{-1}xyxyxyxyx)$ to get $t_2 = t_{65}, t_{34}$

then we have $(xyxyxy^{-1}xyx)t_{40}, t_2, t_{65}$.

Using the relation
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy^{*}x)^{3})t_{49},t_{105}\ ,t_{25} = Id$
conjugated by $(xy^{-1}xyxyxyxyxy^{-1}xy^{-1}xy)$
gives
$(xy^{-1}xyxyxyxyxy^{-1}xy^{-1}xy)t_{23}\ ,t_{68} = t_{65}$
then we have
$(xyxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{59}\ ,t_{32}\ ,t_{23}\ ,t_{68}$.
Next $Nt_1,t_2,(t_{24}\ )$
$= Nt_1,t_2,(t_{74})$
$= Nt_{94},t_{65}\ ,t_{34},(t_{65}\ )$
$= Nt_{59}\ ,t_{32}\ ,t_{23}\ ,t_{68}$.
Using the relation $t_1 = t_4$ conjugated by $(yxyxyxyxy^{-1}xy^{-1}xyxy^{-1})$ gives $t_6 = t_{32}$
then we have $Nt_1,t_2,(t_{24}\ )$
$= Nt_1,t_2,(t_{74})$
$= Nt_{94},t_{65}\ ,t_{34},(t_{65}\ )$
$= Nt_{59}\ ,(t_{32}\ ),t_{23}\ ,t_{68}$
$= Nt_{59}\ ,t_6,t_{23}\ ,t_{68}$.
Using $((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy^{*}x)^{3})t_{49},t_{105}\ ,t_{25} = Id$
conjugated by
$(y^{-1}xyxy^{-1}xyxyxyxyxy)$ gives
$(y^{-1}xyxy^{-1}xyxyxyxyxy)t_{69} = t_6,t_{23}$
then we have $(xyxyxy^{-1}xy^{-1}x)t_1,t_{69},t_{68}$.
Next $Nt_1,t_2,(t_{24}\ )$
$= Nt_1,t_2,(t_{74})$
$= Nt_{94},t_{65}\ ,t_{34},(t_{65}\ )$
$= Nt_{59}\ ,(t_{32}\ ),t_{23}\ ,t_{68}$
$= Nt_{59}\ ,(t_6,t_{23}\ ),t_{68}$
$= Nt_1,t_{69},t_{68}$.
Then using the relation
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy^{*}x)^{3})t_{49},t_{105}\ ,t_{25} = Id$
conjugated by $(y^{-1}xyxy^{-1}xyxyxy^{-1}xy^{-1})$
which gives $(y^{-1}xyxy^{-1}xyxyxy^{-1}xy^{-1})t_{104}\ ,t_{10} = t_1$

then we have $(xy^{-1}xyxyxy)t_{104}, t_{10}, t_{69}, t_{68}$

which relates to $Nt_1, t_2, (t_{24})$

$= Nt_1, t_2, (t_{74})$

$= Nt_{94}, t_{65}, t_{34}, (t_{65})$

$= Nt_{59}, (t_{32}), t_{23}, t_{68}$

$= Nt_{59}, (t_6, t_{23}), t_{68}$

$= N(t_1), t_{69}, t_{68}$

$= Nt_{104}, t_{10}, t_{69}, t_{68}$.

Then the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by

$(xyxyxyxyxy^{-1}xy^{-1}xyxy^{-1})$

gives

$(xyxyxyxyxy^{-1}xy^{-1}xyxy^{-1})t_{12} = t_{10}, t_{69}$

then we have

$(xyxyxyxy^{-1}xyxy^{-1}x)t_{46}, t_{12}, t_{68}$.

Next $Nt_1, t_2, (t_{24})$

$= Nt_1, t_2, (t_{74})$

$= Nt_{94}, t_{65}, t_{34}, (t_{65})$

$= Nt_{59}, (t_{32}), t_{23}, t_{68}$

$= Nt_{59}, (t_6, t_{23}), t_{68}$

$= N(t_1), t_{69}, t_{68}$

$= Nt_{104}, (t_{10}, t_{69}), t_{68}$

$= Nt_{46}, t_{12}, t_{68}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(y^{-1}xyxy^{-1}xyxyxy^{-1}xy^{-1})$

which gives $(y^{-1}xyxy^{-1}xyxyxy^{-1}xy^{-1})t_1 = t_{46}, t_{12}$

then we have $(yxyxy^{-1})t_1, t_{68}$.

Next $Nt_1, t_2, (t_{24})$

$= Nt_1, t_2, (t_{74})$

$= Nt_{94}, t_{65}, t_{34}, (t_{65})$

$= Nt_{59}, (t_{32}), t_{23}, t_{68}$

$= Nt_{59}, (t_6, t_{23}), t_{68}$

$= N(t_1), t_{69}, t_{68}$

$= Nt_{104}, (t_{10}, t_{69}), t_{68}$

$= N(t_{46}, t_{12}), t_{68}$

$= Nt_1, t_{68}$

$= N(t_1, t_2)^{(xyxyxy^{-1}xy^{-1}x)}. \epsilon [1,2]$

Choose 32 from {32, 78, 43, 86, 39}

Using the relation

$(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{92}, t_{107} = t_1, t_2$

to get $(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{92}, t_{107}, t_{32}$

next $N(t_1, t_2), t_{32}$

$= t_{92}, t_{107}, t_{32}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(xy^{-1}xyxyxy^{-1}xy^{-1}xy^{-1}x)$

which gives $(xy^{-1}xyxyxy^{-1}xy^{-1}xy^{-1}x)t_{99} = t_{107}, t_{32}$

then we have $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{67}, t_{99}$.

Next $N(t_1, t_2), t_{32}$

$= t_{92}, (t_{107}, t_{32})$

$= Nt_{67}, t_{99}$

$= N(t_1, t_{26})^{(yxyxyxy^{-1}xy^{-1}xyxyxy)} \epsilon [1,26]$

Choose 36 from {36, 98, 71, 47, 96}

Using the relation

$(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{92}, t_{107} = t_1, t_2$

to get $(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{92}, t_{107}, t_{36}$

next $N(t_1, t_2), t_{36}$

$= Nt_{92}, t_{107}, t_{36}$.

Using the relation $t_1 = t_4$ conjugated by

$(yxy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)$ to get $t_{10} = t_{36}$

then we have $N(t_1, t_2), t_{36}$

$= Nt_{92}, t_{107}, (t_{36})$

$= Nt_{92},t_{107}\ ,t_{10}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $((xy^{-1}xyxy)^2)$

to get $((xy^{-1}xyxy)^2)t_{18},t_{75} = t_{10}$

then we have $(y^{-1}xy^{-1}xy^{-1}xyxyxy^{-1}xyx)t_{35},t_{54}\ ,t_{18},t_{75}$

which gives us $N(t_1,t_2),t_{36}$

$= Nt_{92},t_{107}\ ,(t_{36})$

$= Nt_{92},t_{107}\ ,(t_{10}\ )$

$= Nt_{35},t_{54}\ ,t_{18},t_{75}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(xy^{-1}xy)$ to get

$(xy^{-1}xy)t_{43} = t_{54}\ ,t_{18}$

which gives $(y^{-1}xy^{-1}xy^{-1}xyxyx)t_{105}\ ,t_{43}\ ,t_{75}$

then we have $N(t_1,t_2),t_{36}$

$= Nt_{92},t_{107}\ ,(t_{36})$

$= Nt_{92},t_{107}\ ,(t_{10}\ )$

$= Nt_{35},(t_{54}\ ,t_{18}),t_{75}$

$= Nt_{105}\ ,t_{43}\ ,t_{75}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(yxyxyxyxyxy^{-1}xy^{-1}xy)$

which gives $(yxyxyxyxyxy^{-1}xy^{-1}xy)t_{106}\ ,t_{66} = t_{75}$

then we have $((yxy^{-1}xy^{-1}x)^2)t_{60}\ ,t_{90}\ ,t_{106}\ ,t_{66}$.

Next $N(t_1,t_2),t_{36}$

$= Nt_{92},t_{107}\ ,(t_{36})$

$= Nt_{92},t_{107}\ ,(t_{10}\ )$

$= Nt_{35},(t_{54}\ ,t_{18}),t_{75}$

$= Nt_{105}\ ,t_{43}\ ,(t_{75}\ )$

$= Nt_{60}\ ,t_{90}\ ,t_{106}\ ,t_{66}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}, t_{25} = Id$

conjugated by $(xy^{-1}xyxyxyxyx)$

which $(xy^{-1}xyxyxyxyx)t_{31}, t_{93} = t_{60}$

then we have $(xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_{31}, t_{93}, t_{90}, t_{106}, t_{66}$.

Next $N(t_1,t_2),t_{36}$

$= Nt_{92},t_{107}, (t_{36})$

$= Nt_{92},t_{107}, (t_{10})$

$= Nt_{35},(t_{54}, t_{18}),t_{75}$

$= Nt_{105}, t_{43}, (t_{75})$

$= N(t_{60}),t_{90}, t_{106}, t_{66}$

$= Nt_{31}, t_{93}, t_{90}, t_{106}, t_{66}$.

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}, t_{25} = Id$

conjugated by $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})$

then we have $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{81} = t_{93}, t_{90}$

which gives $(y^{-1}xyxyxyxyxy)t_{103}, t_{81}, t_{106}, t_{66}$

next $N(t_1,t_2),t_{36}$

$= Nt_{92},t_{107}, (t_{36})$

$= Nt_{92},t_{107}, (t_{10})$

$= Nt_{35},(t_{54}, t_{18}),t_{75}$

$= Nt_{105}, t_{43}, (t_{75})$

$= N(t_{60}),t_{90}, t_{106}, t_{66}$

$= Nt_{31}, (t_{93}, t_{90}),t_{106}, t_{66}$

$= Nt_{103}, t_{81}, t_{106}, t_{66}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}, t_{25} = Id$

conjugated by $(xy^{-1}xyxyxyxyx)$

to give

$(xy^{-1}xyxyxyxyx)\ t_{60} = t_{103}, t_{81}$

then we have $(xyxy^{-1}xy^{-1}xyxyxy^{-1})t_{60}, t_{106}, t_{66}$.

Next $N(t_1,t_2),t_{36}$

$= Nt_{92},t_{107}, (t_{36})$

$= Nt_{92}, t_{107}, (t_{10})$

$= Nt_{35}, (t_{54}, t_{18}), t_{75}$

$= Nt_{105}, t_{43}, (t_{75})$

$= N(t_{60}), t_{90}, t_{106}, t_{66}$

$= Nt_{31}, (t_{93}, t_{90}), t_{106}, t_{66}$

$= N(t_{103}, t_{81}), t_{106}, t_{66}$

$= Nt_{60}, t_{106}, t_{66}.$

By using the relation $t_1 = t_4$ conjugated by $(y^{-1}xyxyxyxyxy^{-1})$

gives $t_{60} = t_{106}$ thus we have $N(t_1,t_2),t_{36}$

$= Nt_{92}, t_{107}, (t_{36})$

$= Nt_{92}, t_{107}, (t_{10})$

$= Nt_{35}, (t_{54}, t_{18}), t_{75}$

$= Nt_{105}, t_{43}, (t_{75})$

$= N(t_{60}), t_{90}, t_{106}, t_{66}$

$= Nt_{31}, (t_{93}, t_{90}), t_{106}, t_{66}$

$= N(t_{103}, t_{81}), t_{106}, t_{66}$

$= N(t_{60}), t_{106}, t_{66}$

$= Nt_{106}, t_{106}, t_{66}$

$= N(t_{106})^2, t_{66}$

$= Nt_{66}$

$= N(t_1)^{(xyxy^{-1}xy^{-1}xy^{-1}xyx)} \epsilon [1]$

<u>Choose 1 from {1, 64, 92, 57, 108, 87, 4, 90, 102, 68}</u>

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $((xy^{-1}xyxy)^2)$ gives

$((xy^{-1}xyxy)^2)t_{18}, t_{75} = t_{10}$

then we have $((xy^{-1}xyxy)^2)t_{71}, t_{33}, t_{18}, t_{75}$.

Next $Nt_1, t_2, (t_1)$

$= Nt_{71}, t_{33}, t_{18}, t_{75}$.

Next we will use the same relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(yxyxyxyxyxy^{-1}xy^{-1}xy)$

which gives

$(yxyxyxyxyxy^{-1}xy^{-1}xy)t_{106}\ ,t_{66} = t_{75}$

then we have $(xyxy^{-1}xyxyx)t_{18},t_{94},t_{44},t_{106}\ ,t_{66}$

next $Nt_1,t_2,(t_1)$

$= Nt_{71},t_{33}\ ,t_{18},(t_{75}\ )$

$= Nt_{18},t_{94},t_{44},t_{106}\ ,t_{66}.$

By using the relation

$(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{92},t_{107} = t_1,t_2$

conjugated by $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx)$

gives $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx)t_{18},t_{94} = t_2,t_{105}$

then we have $(xy^{-1}xyxy)t_2,t_{105}\ ,t_{44},t_{106}\ ,t_{66}.$

Then we have $Nt_1,t_2,(t_1)$

$= Nt_{71},t_{33}\ ,t_{18},(t_{75}\ )$

$= N(t_{18},t_{94}),t_{44},t_{106}\ ,t_{66}$

$= Nt_2,t_{105}\ ,t_{44},t_{106}\ ,t_{66}.$

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(xy^{-1}xyxyxy^{-1}xy^{-1})$

we get $(xy^{-1}xyxyxy^{-1}xy^{-1})t_{90} = t_{105}\ ,t_{44}$

then we have $((yxy^{-1}xy^{-1}x)^2)t_{60}\ ,t_{90}\ ,t_{106}\ ,t_{66}$

which gives $Nt_1,t_2,(t_1)$

$= Nt_{71},t_{33}\ ,t_{18},(t_{75}\ )$

$= N(t_{18},t_{94}),t_{44},t_{106}\ ,t_{66}$

$= Nt_2,(t_{105}\ ,t_{44}),t_{106}\ ,t_{66}$

$= Nt_{60}\ ,t_{90}\ ,t_{106}\ ,t_{66}.$

By using the relation $t_1 = t_4$ conjugated by

$(xyxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})$ to get $t_{90} = t_{87}$

then we have $Nt_1,t_2,(t_1)$

$= Nt_{71},t_{33}\ ,t_{18},(t_{75}\ )$

$= N(t_{18},t_{94}),t_{44},t_{106}\ ,t_{66}$

$= Nt_2,(t_{105}\ ,t_{44}),t_{106}\ ,t_{66}$

$= Nt_{60}\ ,(t_{90}\ ),t_{106}\ ,t_{66}$

$= Nt_{60} ,t_{87} ,t_{106} ,t_{66}$.

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105} ,t_{25} = Id$

conjugated by $(yxy^{-1}xyxyxyxyxy^{-1})$

gives $(yxy^{-1}xyxyxyxyxy^{-1})t_{107} ,t_{91} = t_{87}$

then we have

$(yxyxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{47} ,t_{107} ,t_{91},t_{106} ,t_{66}$

which leds us to $Nt_1,t_2,(t_1)$

$= Nt_{71},t_{33} ,t_{18},(t_{75})$

$= N(t_{18},t_{94}),t_{44},t_{106} ,t_{66}$

$= Nt_2,(t_{105} ,t_{44}),t_{106} ,t_{66}$

$= Nt_{60} ,(t_{90}),t_{106} ,t_{66}$

$= Nt_{60} ,(t_{87}),t_{106} ,t_{66}$

$= Nt_{47} ,t_{107} ,t_{91},t_{106} ,t_{66}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105} ,t_{25} = Id$

conjugated by $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)$

whcih gives us $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_{109} = t_{91},t_{106}$

then we have $((yxy^{-1}x)^2)t_{79} ,t_{11} ,t_{109} ,t_{66}$

which gives us $Nt_1,t_2,(t_1)$

$= Nt_{71},t_{33} ,t_{18},(t_{75})$

$= N(t_{18},t_{94}),t_{44},t_{106} ,t_{66}$

$= Nt_2,(t_{105} ,t_{44}),t_{106} ,t_{66}$

$= Nt_{60} ,(t_{90}),t_{106} ,t_{66}$

$= Nt_{60} ,(t_{87}),t_{106} ,t_{66}$

$= Nt_{47} ,t_{107} ,(t_{91},t_{106}),t_{66}$

$= Nt_{79} ,t_{11} ,t_{109} ,t_{66}$.

Using the relation $t_1 = t_4$ conjugated by

$(xy^{-1}xyxy^{-1}xyxyxy)$ to get $t_{107} = t_{79}$

then we have $Nt_1,t_2,(t_1)$

$= Nt_{71},t_{33} ,t_{18},(t_{75})$

$= N(t_{18},t_{94}),t_{44},t_{106} ,t_{66}$

$= Nt_2,(t_{105}\ ,t_{44}),t_{106}\ ,t_{66}$

$= Nt_{60}\ ,(t_{90}\ ),t_{106}\ ,t_{66}$

$= Nt_{60}\ ,(t_{87}\ ),t_{106}\ ,t_{66}$

$= Nt_{47}\ ,t_{107}\ ,(t_{91},t_{106}\ ),t_{66}$

$= N(t_{79}\ ),t_{11}\ ,t_{109}\ ,t_{66}$

$= Nt_{107}\ ,t_{11}\ ,t_{109}\ ,t_{66}.$

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(yxy^{-1}xyxyxyxyxy^{-1})$

gives $(yxy^{-1}xyxyxyxyxy^{-1})t_{109} = t_{107}\ ,t_{11}$

then we have $(xyxy^{-1}xy^{-1}xyxyxy^{-1})t_{109}\ ,t_{109}\ ,t_{66}$

which gives us $Nt_1,t_2,(t_1)$

$= Nt_{71},t_{33}\ ,t_{18},(t_{75}\ )$

$= N(t_{18},t_{94}),t_{44},t_{106}\ ,t_{66}$

$= Nt_2,(t_{105}\ ,t_{44}),t_{106}\ ,t_{66}$

$= Nt_{60}\ ,(t_{90}\ ),t_{106}\ ,t_{66}$

$= Nt_{60}\ ,(t_{87}\ ),t_{106}\ ,t_{66}$

$= Nt_{47}\ ,t_{107}\ ,(t_{91},t_{106}\ ),t_{66}$

$= N(t_{79}\ ),t_{11}\ ,t_{109}\ ,t_{66}$

$= N(t_{107}\ ,t_{11}\ ),t_{109}\ ,t_{66}$

$= Nt_{109}\ ,t_{109}\ ,t_{66}$

$= N(t_{109}\ )^2,t_{66}$

$= Nt_{66}$

$= N(t_1)^{(xyxy^{-1}xy^{-1}xy^{-1}xyx)}\ \epsilon\ [1]$

Choose 2 from {2, 83, 107, 77, 40, 69, 18, 45, 46, 100}

$Nt_1t_2t_2 = Nt_1(t_2)^2 = Nt_1\ \epsilon\ [1]$

Choose 3 from {3, 5, 56, 103, 29, 110, 27, 31, 85, 12}

Using the relation $t_1 = t_4$

we have $N(t_1),t_2,t_3$

$= Nt_4,t_2,t_3.$

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)^4)t_{102},t_{65} = t_{29}\ ,t_{103}$

conjugated by $((y^{-1}xyx)^2)$

gives $((y^{-1}xyx)^2)t_4,t_2 = t_{64},t_{50}$

then we have $((xy^{-1}xy)^2)t_{64},t_{50},t_3$

then we have $N(t_1),t_2,t_3$

$= N(t_4,t_2),t_3$

$= Nt_{64},t_{50},t_3$.

Then by using the relation $t_1 = t_4$ conjugated by

$(xy^{-1}xyxy^{-1}xyx)$ we have $t_{83} = t_{50}$

then we have $N(t_1),t_2,t_3$

$= N(t_4,t_2),t_3$

$= Nt_{64},(t_{50}),t_3$

$= Nt_{64},t_{83},t_3$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(yxy^{-1}xyxyxyxy)$

we have $(yxy^{-1}xyxyxyxy)t_{68} = t_{83},t_3$

which gives us

$(xyxy^{-1}xyxyxy^{-1})t_{28}\ ,t_{68}$,

next $N(t_1),t_2,t_3 =$

$N(t_4,t_2),t_3 =$

$Nt_{64},(t_{50}),t_3 =$

$Nt_{64},(t_{83},t_3) =$

$Nt_{28}\ ,t_{68}$.

Then using the relation $t_1 = t_4$ conjugated by $(y^{-1}xyxy^{-1}xy^{-1}xyxy^{-1}xy^{-1}x)$ we have t_{68} $= t_{57}$

which gives $N(t_1),t_2,t_3$

$= N(t_4,t_2),t_3$

$= Nt_{64},(t_{50}),t_3$

$= Nt_{64},(t_{83},t_3)$

$= Nt_{28}\ ,(t_{68})$

$= Nt_{28}\ ,t_{57}$.

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1})$

gives $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{33}\ ,t_{37} = t_{57}$

then we have $(xyxyxy^{-1})t_{105}\ ,t_{33}\ ,t_{37}$.

Next $N(t_1),t_2,t_3$

$= N(t_4,t_2),t_3$

$= Nt_{64},(t_{50}),t_3$

$= Nt_{64},(t_{83},t_3)$

$= Nt_{28}\ ,(t_{68})$

$= Nt_{28}\ ,(t_{57})$

$= Nt_{105}\ ,t_{33}\ ,t_{37}$.

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(y^{-1}xyxyxy^{-1}xy^{-1}x)$

gives $(y^{-1}xyxyxy^{-1}xy^{-1}x)t_{55} = t_{105}\ ,t_{33}$

then we have $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)t_{55},t_{37}$.

Then we have $N(t_1),t_2,t_3$

$= N(t_4,t_2),t_3$

$= Nt_{64},(t_{50}),t_3$

$= Nt_{64},(t_{83},t_3)$

$= Nt_{28}\ ,(t_{68})$

$= Nt_{28}\ ,(t_{57})$

$= N(t_{105}\ ,t_{33}\ ),t_{37}$

$= Nt_{55},t_{37}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(y^{-1}xyxy^{-1}xyxyxyxy^{-1})$

gives $(y^{-1}xyxy^{-1}xyxyxyxy^{-1})t_{85}\ ,t_{45} = t_{37}$

then we have $(xy^{-1}xyxy^{-1}xy^{-1}xyxy)1,t_{85}\ ,t_{45}$

then $N(t_1),t_2,t_3$

$= N(t_4,t_2),t_3$

$= Nt_{64},(t_{50}),t_3$

$= Nt_{64},(t_{83},t_3)$

$= Nt_{28} ,(t_{68})$

$= Nt_{28} ,(t_{57})$

$= N(t_{105} ,t_{33}),t_{37}$

$= Nt_{55},(t_{37})$

$= Nt_1,t_{85} ,t_{45}$.

By using the relation $t_1 = t_4$ conjugated by

$(yxyxy^{-1}xyxy)$ we have $t_{56} = t_{85}$

then $N(t_1),t_2,t_3$

$= N(t_4,t_2),t_3$

$= Nt_{64},(t_{50}),t_3$

$= Nt_{64},(t_{83},t_3)$

$= Nt_{28} ,(t_{68})$

$= Nt_{28} ,(t_{57})$

$= N(t_{105} ,t_{33}),t_{37}$

$= Nt_{55},(t_{37})$

$= Nt_1,(t_{85}),t_{45}$

$= Nt_1,t_{56} ,t_{45}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105} ,t_{25} = Id$

conjugated by $(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)$

gives $(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_{67},t_{27} = t_1$

then we have $(xyxy^{-1}xy^{-1}xy^{-1})t_{67},t_{27} ,t_{56} ,t_{45}$.

Then we have $N(t_1),t_2,t_3$

$= N(t_4,t_2),t_3$

$= Nt_{64},(t_{50}),t_3$

$= Nt_{64},(t_{83},t_3)$

$= Nt_{28} ,(t_{68})$

$= Nt_{28} ,(t_{57})$

$= N(t_{105} ,t_{33}),t_{37}$

$= Nt_{55},(t_{37})$

$= Nt_1,(t_{85}),t_{45}$

$= N(t_1), t_{56}, t_{45}$

$= Nt_{67}, t_{27}, t_{56}, t_{45}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

we will conjugated it by $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)$

to get $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)t_{27}, t_{46}, t_{44} = Id$

then we have $(yxyxyxy^{-1}xy^{-1}xyx)t_{97}, t_{44}, t_{45}$

which gives $N(t_1), t_2, t_3$

$= N(t_4, t_2), t_3$

$= Nt_{64}, (t_{50}), t_3$

$= Nt_{64}, (t_{83}, t_3)$

$= Nt_{28}, (t_{68})$

$= Nt_{28}, (t_{57})$

$= N(t_{105}, t_{33}), t_{37}$

$= Nt_{55}, (t_{37})$

$= Nt_1, (t_{85}), t_{45}$

$= N(t_1), t_{56}, t_{45}$

$= Nt_{67}, (t_{27}, t_{56}), t_{45}$

$= Nt_{97}, t_{44}, t_{45}$.

Using the relation $t_1 = t_4$ conjugated by

$(yxy^{-1}xyxy^{-1}xy^{-1}xy^{-1}x)$ we get $t_{97} = t_{105}$

then we have $N(t_1), t_2, t_3$

$= N(t_4, t_2), t_3$

$= Nt_{64}, (t_{50}), t_3$

$= Nt_{64}, (t_{83}, t_3)$

$= Nt_{28}, (t_{68})$

$= Nt_{28}, (t_{57})$

$= N(t_{105}, t_{33}), t_{37}$

$= Nt_{55}, (t_{37})$

$= Nt_1, (t_{85}), t_{45}$

$= N(t_1), t_{56}, t_{45}$

$= Nt_{67}, (t_{27}, t_{56}), t_{45}$

$= N(t_{97}),t_{44},t_{45}$

$= Nt_{105}\ ,t_{44},t_{45}.$

By using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(xy^{-1}xyxyxy^{-1}xy^{-1})$

gives $(xy^{-1}xyxyxy^{-1}xy^{-1})t_{90} = t_{105}\ ,t_{44}$

then we have $N(t_1),t_2,t_3$

$= N(t_4,t_2),t_3$

$= Nt_{64},(t_{50}),t_3$

$= Nt_{64},(t_{83},t_3)$

$= Nt_{28}\ ,(t_{68})$

$= Nt_{28}\ ,(t_{57})$

$= N(t_{105}\ ,t_{33}\ ),t_{37}$

$= Nt_{55},(t_{37})$

$= Nt_1,(t_{85}\ ),t_{45}$

$= N(t_1),t_{56}\ ,t_{45}$

$= Nt_{67},(t_{27}\ ,t_{56}\ ),t_{45}$

$= N(t_{97}),t_{44},t_{45}$

$= N(t_{105}\ ,t_{44}),t_{45}$

$= Nt_{90}\ ,t_{45}$

$= N(t_1,t_2)^{(yxyxyxyxyxy^{-1}xyxy^{-1})}\ \epsilon\ [1,2]$

Choose 7 from {7, 106, 25, 60, 58, 33, 19, 75, 49, 104}

We have the relation

$(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{92},t_{107} = t_1,t_2$

which gives $(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{92},t_{107}\ ,t_7$

then we have $N(t_1,t_2),t_7$

$= Nt_{92},t_{107}\ ,t_7.$

Then by using the relation $t_1 = t_4$ conjugated by $t_7 = t_{33}$

we have $N(t_1,t_2),t_7$

$= Nt_{92},t_{107}\ ,(t_7)$

$= Nt_{92},t_{107}\ ,t_{33}$

. By using the relation $t_1 = t_4$ conjugated by

$(xy^{-1}xyxy^{-1}xyxyxy)$ to get $t_{107} = t_{79}$

we have $N(t_1,t_2),t_7$

$= Nt_{92},t_{107}\ ,(t_7)$

$= Nt_{92},(t_{107}\ ),t_{33}$

$= Nt_{92},t_{79}\ ,t_{33}$.

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = 1$

conjugated by $(xyxyxyxy^{-1}xy^{-1}xyx)$

we have $(xyxyxyxy^{-1}xy^{-1}xyx)t_{45},t_{94} = t_{79}$

then we have $(xyxyxy^{-1}xy^{-1}xyxy^{-1}xy)t_{87}\ ,t_{45},t_{94},t_{33}$

then $N(t_1,t_2),t_7$

$= Nt_{92},t_{107}\ ,(t_7)$

$= Nt_{92},(t_{107}\ ),t_{33}$

$= Nt_{92},(t_{79}\ ),t_{33}$

$= Nt_{87}\ ,t_{45},t_{94},t_{33}$.

Using $((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(xyxy^{-1}xy^{-1}xy^{-1}xy)$ gives

$(xyxy^{-1}xy^{-1}xy^{-1}xy)t_{25} = t_{94},t_{33}$ then we have

$(yxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_{56}\ ,t_{51}\ ,t_{25}$

then $N(t_1,t_2),t_7$

$= Nt_{92},t_{107}\ ,(t_7)$

$= Nt_{92},(t_{107}\ ),t_{33}$

$= Nt_{92},(t_{79}\ ),t_{33}$

$= Nt_{87}\ ,t_{45},(t_{94},t_{33}\ )$

$= Nt_{56}\ ,t_{51}\ ,t_{25}$.

By using $t_1 = t_4$ and conjugating it by

$(yxyxyxy^{-1}xyxyxy^{-1})$ we have $t_{25} = t_{75}$

then we get $N(t_1,t_2),t_7$

$= Nt_{92},t_{107}\ ,(t_7)$

$= Nt_{92},(t_{107}\ ),t_{33}$

$= Nt_{92},(t_{79}\ ),t_{33}$

$= Nt_{87}\ ,t_{45},(t_{94},t_{33}\ )$

$= Nt_{56}\ ,t_{51}\ ,(t_{25})$

$= Nt_{56}\ ,t_{51}\ ,t_{75}$.

Using $((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(xyxyxyxy^{-1}xy^{-1}xyxyxy)$

we have $(xyxyxyxy^{-1}xy^{-1}xyxyxy)t_{75}\ ,t_{61}\ ,t_{85} = Id$

then we have $(xyxy^{-1}xyxyxy^{-1}xy^{-1})t_{25},t_{47}\ ,t_{85}\ ,t_{61}$

then $N(t_1,t_2),t_7$

$= Nt_{92},t_{107}\ ,(t_7)$

$= Nt_{92},(t_{107}\ ),t_{33}$

$= Nt_{92},(t_{79}\ ),t_{33}$

$= Nt_{87}\ ,t_{45},(t_{94},t_{33}\ )$

$= Nt_{56}\ ,t_{51}\ ,(t_{25})$

$= Nt_{56}\ ,t_{51}\ ,(t_{75}\ )$

$= Nt_{25},t_{47}\ ,t_{85}\ ,t_{61}$.

Then using the same relation conjugated by

$(y^{-1}xy^{-1}xy^{-1})$ to get

$(y^{-1}xy^{-1}xy^{-1})t_{94},t_{105} = t_{25}$

then we have

$(yxyxyxy^{-1}xyx)t_{94},t_{104}\ ,t_{47}\ ,t_{85}\ ,t_{61}$

then $N(t_1,t_2),t_7$

$= Nt_{92},t_{107}\ ,(t_7)$

$= Nt_{92},(t_{107}\ ),t_{33}$

$= Nt_{92},(t_{79}\ ),t_{33}$

$= Nt_{87}\ ,t_{45},(t_{94},t_{33}\ )$

$= Nt_{56}\ ,t_{51}\ ,(t_{25})$

$= Nt_{56}\ ,t_{51}\ ,(t_{75}\ )$

$= N(t_{25}),t_{47}\ ,t_{85}\ ,t_{61}$

$= Nt_{94},t_{104}\ ,t_{47}\ ,t_{85}\ ,t_{61}$.

By using the same relation once again conjugated by

$(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)$ we have

$(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)1 = t_{104}\ ,t_{47}$

then we have $(xy^{-1}xyx)t_{108}\ ,t_1,t_{85}\ ,t_{61}$

then $N(t_1,t_2),t_7$

$= Nt_{92},t_{107} ,(t_7)$

$= Nt_{92},(t_{107}),t_{33}$

$= Nt_{92},(t_{79}),t_{33}$

$= Nt_{87} ,t_{45},(t_{94},t_{33})$

$= Nt_{56} ,t_{51} ,(t_{25})$

$= Nt_{56} ,t_{51} ,(t_{75})$

$= N(t_{25}),t_{47} ,t_{85} ,t_{61}$

$= Nt_{94},(t_{104} ,t_{47}),t_{85} ,t_{61}$

$= Nt_{108} ,t_1,t_{85} ,t_{61}$.

By using the relation $t_1 = t_4$ conjugated by

$(yxyxy^{-1}xyxy)$ we have $t_{85} = t_{56}$

then $N(t_1,t_2),t_7$

$= Nt_{92},t_{107} ,(t_7)$

$= Nt_{92},(t_{107}),t_{33}$

$= Nt_{92},(t_{79}),t_{33}$

$= Nt_{87} ,t_{45},(t_{94},t_{33})$

$= Nt_{56} ,t_{51} ,(t_{25})$

$= Nt_{56} ,t_{51} ,(t_{75})$

$= N(t_{25}),t_{47} ,t_{85} ,t_{61}$

$= Nt_{94},(t_{104} ,t_{47}),t_{85} ,t_{61}$

$= Nt_{108} ,t_1,(t_{85}),t_{61}$

$= Nt_{108} ,t_1,t_{56} ,t_{61}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105} ,t_{25} = Id$

conjugated by $(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)$

to get $(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_{67},t_{27} = 1$

then we have $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_{110},t_{67},t_{27} ,t_{56} ,t_{61}$

then $N(t_1,t_2),t_7$

$= Nt_{92},t_{107} ,(t_7)$

$= Nt_{92},(t_{107}),t_{33}$

$= Nt_{92},(t_{79}),t_{33}$

$= Nt_{87}, t_{45}, (t_{94}, t_{33})$

$= Nt_{56}, t_{51}, (t_{25})$

$= Nt_{56}, t_{51}, (t_{75})$

$= N(t_{25}), t_{47}, t_{85}, t_{61}$

$= Nt_{94}, (t_{104}, t_{47}), t_{85}, t_{61}$

$= Nt_{108}, t_{1}, (t_{85}), t_{61}$

$= Nt_{108}, (t_{1}), t_{56}, t_{61}$

$= Nt_{110}, t_{67}, t_{27}, t_{56}, t_{61}$.

Using the same relation conjugated by

$(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)$

gives $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)t_{27}, t_{56}, t_{44} = Id$

then we have $(xy^{-1}xy^{-1}xyxyxy^{-1}xy)t_{101}, t_{97}, t_{44}, t_{61}$

then $N(t_1, t_2), t_7$

$= Nt_{92}, t_{107}, (t_7)$

$= Nt_{92}, (t_{107}), t_{33}$

$= Nt_{92}, (t_{79}), t_{33}$

$= Nt_{87}, t_{45}, (t_{94}, t_{33})$

$= Nt_{56}, t_{51}, (t_{25})$

$= Nt_{56}, t_{51}, (t_{75})$

$= N(t_{25}), t_{47}, t_{85}, t_{61}$

$= Nt_{94}, (t_{104}, t_{47}), t_{85}, t_{61}$

$= Nt_{108}, t_{1}, (t_{85}), t_{61}$

$= Nt_{108}, (t_{1}), t_{56}, t_{61}$

$= Nt_{110}, t_{67}, (t_{27}, t_{56}), t_{61}$

$= Nt_{101}, t_{97}, t_{44}, t_{61}$.

By using the relation $t_1 = t_4$ conjugated

by $(yxy^{-1}xyxy^{-1}xy^{-1}xy^{-1}x)$ gives $t_{97} = t_{105}$

then we have $N(t_1, t_2), t_7$

$= Nt_{92}, t_{107}, (t_7)$

$= Nt_{92}, (t_{107}), t_{33}$

$= Nt_{92}, (t_{79}), t_{33}$

$= Nt_{87}, t_{45}, (t_{94}, t_{33})$

$= Nt_{56}, t_{51}, (t_{25})$

$= Nt_{56}, t_{51}, (t_{75})$

$= N(t_{25}), t_{47}, t_{85}, t_{61}$

$= Nt_{94}, (t_{104}, t_{47}), t_{85}, t_{61}$

$= Nt_{108}, t_{1}, (t_{85}), t_{61}$

$= Nt_{108}, (t_{1}), t_{56}, t_{61}$

$= Nt_{110}, t_{67}, (t_{27}, t_{56}), t_{61}$

$= Nt_{101}, (t_{97}), t_{44}, t_{61}$

$= Nt_{101}, t_{105}, t_{44}, t_{61}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^{3})t_{49}, t_{105}, t_{25} = Id$

conjugated by $(xy^{-1}xyxyxy^{-1}xy^{-1})$

gives $(xy^{-1}xyxyxy^{-1}xy^{-1})t_{90} = t_{105}, t_{44}$

then we have $(xyxyxy^{-1}xy^{-1}xyxy)t_{18}, t_{90}, t_{61}$

then $N(t_1, t_2), t_7$

$= Nt_{92}, t_{107}, (t_{7})$

$= Nt_{92}, (t_{107}), t_{33}$

$= Nt_{92}, (t_{79}), t_{33}$

$= Nt_{87}, t_{45}, (t_{94}, t_{33})$

$= Nt_{56}, t_{51}, (t_{25})$

$= Nt_{56}, t_{51}, (t_{75})$

$= N(t_{25}), t_{47}, t_{85}, t_{61}$

$= Nt_{94}, (t_{104}, t_{47}), t_{85}, t_{61}$

$= Nt_{108}, t_{1}, (t_{85}), t_{61}$

$= Nt_{108}, (t_{1}), t_{56}, t_{61}$

$= Nt_{110}, t_{67}, (t_{27}, t_{56}), t_{61}$

$= Nt_{101}, (t_{97}), t_{44}, t_{61}$

$= Nt_{101}, (t_{105}, t_{44}), t_{61}$

$= Nt_{18}, t_{90}, t_{61}$.

Then using the relation $t_1 = t_4$ conjugated by

$(xyxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})$ to get $t_{90} = t_{87}$

we have $N(t_1, t_2), t_7$

$= Nt_{92}, t_{107}, (t_7)$

$= Nt_{92}, (t_{107}), t_{33}$

$= Nt_{92}, (t_{79}), t_{33}$

$= Nt_{87}, t_{45}, (t_{94}, t_{33})$

$= Nt_{56}, t_{51}, (t_{25})$

$= Nt_{56}, t_{51}, (t_{75})$

$= N(t_{25}), t_{47}, t_{85}, t_{61}$

$= Nt_{94}, (t_{104}, t_{47}), t_{85}, t_{61}$

$= Nt_{108}, t_1, (t_{85}), t_{61}$

$= Nt_{108}, (t_1), t_{56}, t_{61}$

$= Nt_{110}, t_{67}, (t_{27}, t_{56}), t_{61}$

$= Nt_{101}, (t_{97}), t_{44}, t_{61}$

$= Nt_{101}, (t_{105}, t_{44}), t_{61}$

$= Nt_{18}, (t_{90}), t_{61}$

$= Nt_{18}, t_{87}, t_{61}$.

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(xyxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy)$ we have

$(xyxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy)t_{101}, t_{91} = t_{18}$

then we get $(y^{-1}xyxyxyxyxy^{-1}xyxy)t_{101}, t_{91}, t_{87}, t_{61}$

then $N(t_1, t_2), t_7$

$= Nt_{92}, t_{107}, (t_7)$

$= Nt_{92}, (t_{107}), t_{33}$

$= Nt_{92}, (t_{79}), t_{33}$

$= Nt_{87}, t_{45}, (t_{94}, t_{33})$

$= Nt_{56}, t_{51}, (t_{25})$

$= Nt_{56}, t_{51}, (t_{75})$

$= N(t_{25}), t_{47}, t_{85}, t_{61}$

$= Nt_{94}, (t_{104}, t_{47}), t_{85}, t_{61}$

$= Nt_{108}, t_1, (t_{85}), t_{61}$

$= Nt_{108}, (t_1), t_{56}, t_{61}$

$= Nt_{110}, t_{67}, (t_{27}, t_{56}), t_{61}$

$= Nt_{101},(t_{97}),t_{44},t_{61}$

$= Nt_{101},(t_{105},t_{44}),t_{61}$

$= Nt_{18},(t_{90}),t_{61}$

$= N(t_{18}),t_{87},t_{61}$

$= Nt_{101},t_{91},t_{87},t_{61}$.

Then by using the same relation conjugated by

$(yxy^{-1}xyxyxyxyxy^{-1})$ we get

$(yxy^{-1}xyxyxyxyxy^{-1})t_{91},t_{87},t_{109} = Id$

then we have $(xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}x)t_{51},t_{109},t_{61}$

thus $N(t_1,t_2),t_7$

$= Nt_{92},t_{107},(t_7)$

$= Nt_{92},(t_{107}),t_{33}$

$= Nt_{92},(t_{79}),t_{33}$

$= Nt_{87},t_{45},(t_{94},t_{33})$

$= Nt_{56},t_{51},(t_{25})$

$= Nt_{56},t_{51},(t_{75})$

$= N(t_{25}),t_{47},t_{85},t_{61}$

$= Nt_{94},(t_{104},t_{47}),t_{85},t_{61}$

$= Nt_{108},t_1,(t_{85}),t_{61}$

$= Nt_{108},(t_1),t_{56},t_{61}$

$= Nt_{110},t_{67},(t_{27},t_{56}),t_{61}$

$= Nt_{101},(t_{97}),t_{44},t_{61}$

$= Nt_{101},(t_{105},t_{44}),t_{61}$

$= Nt_{18},(t_{90}),t_{61}$

$= N(t_{18}),t_{87},t_{61}$

$= Nt_{101},(t_{91},t_{87}),t_{61}$

$= Nt_{51},t_{109},t_{61}$.

Then by using the relation $t_1 = t_4$ conjugated by

$(yxyxy^{-1}xyxyxy^{-1})$ we have $t_{51} = t_{89}$

then we have $N(t_1,t_2),t_7$

$= Nt_{92},t_{107},(t_7)$

$= Nt_{92},(t_{107}),t_{33}$

$= Nt_{92},(t_{79}),t_{33}$

$= Nt_{87},t_{45},(t_{94},t_{33})$

$= Nt_{56},t_{51},(t_{25})$

$= Nt_{56},t_{51},(t_{75})$

$= N(t_{25}),t_{47},t_{85},t_{61}$

$= Nt_{94},(t_{104},t_{47}),t_{85},t_{61}$

$= Nt_{108},t_{1},(t_{85}),t_{61}$

$= Nt_{108},(t_{1}),t_{56},t_{61}$

$= Nt_{110},t_{67},(t_{27},t_{56}),t_{61}$

$= Nt_{101},(t_{97}),t_{44},t_{61}$

$= Nt_{101},(t_{105},t_{44}),t_{61}$

$= Nt_{18},(t_{90}),t_{61}$

$= N(t_{18}),t_{87},t_{61}$

$= Nt_{101},(t_{91},t_{87}),t_{61}$

$= N(t_{51}),t_{109},t_{61}$

$= Nt_{89},t_{109},t_{61}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^{3})t_{49},t_{105},t_{25} = Id$

conjugated by $(y^{-1}xy^{-1}xyxyxy^{-1}x)$

gives $(y^{-1}xy^{-1}xyxyxy^{-1}x)t_{38} = t_{89},t_{109}$

then we have $(y^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy)t_{38},t_{61}$.

Then $N(t_{1},t_{2}),t_{7}$

$= Nt_{92},t_{107},(t_{7})$

$= Nt_{92},(t_{107}),t_{33}$

$= Nt_{92},(t_{79}),t_{33}$

$= Nt_{87},t_{45},(t_{94},t_{33})$

$= Nt_{56},t_{51},(t_{25})$

$= Nt_{56},t_{51},(t_{75})$

$= N(t_{25}),t_{47},t_{85},t_{61}$

$= Nt_{94},(t_{104},t_{47}),t_{85},t_{61}$

$= Nt_{108},t_{1},(t_{85}),t_{61}$

$= Nt_{108},(t_{1}),t_{56},t_{61}$

$= Nt_{110},t_{67},(t_{27}\ ,t_{56}\ ),t_{61}$

$= Nt_{101},(t_{97}),t_{44},t_{61}$

$= Nt_{101},(t_{105}\ ,t_{44}),t_{61}$

$= Nt_{18},(t_{90}\ ),t_{61}$

$= N(t_{18}),t_{87}\ ,t_{61}$

$= Nt_{101},(t_{91},t_{87}\ ),t_{61}$

$= N(t_{51}\ ),t_{109}\ ,t_{61}$

$= N(t_{89}\ ,t_{109})\ ,t_{61}$

$= Nt_{38},t_{61}$

$= N(1,5)^{(xy^{-1}xyxyxy^{-1}xyxyx)}\ \epsilon\ [1,5]$

Choose 8 from {8, 44, 17, 35, 59, 73, 62, 16, 93, 109}

Using the relation $t_1 = t_4$ conjugated by $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)$ we have $t_2 = t_{11}$

then $Nt_1,(t_2),t_8$

$= Nt_1,t_{11},t_8$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy^{-1})$

gives $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy^{-1})t_{86} = t_{11},t_8$

then we have $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy^{-1})t_{33}\ ,t_{86}$

thus $Nt_1,(t_2),t_8$

$= Nt_1,(t_{11},t_8)$

$= Nt_{33}\ ,t_{86}$.

Then using $t_1 = t_4$ conjugated by

$(yxyxy^{-1}xyxyxyx)$ we have $t_7 = t_{33}$

then $Nt_1,(t_2),t_8$

$= Nt_1,(t_{11},t_8)$

$= N(t_{33}\ ),t_{86}$

$= Nt_7,t_{86}$

$= N(t_1,t_2)^{(xyxy^{-1}xyxyxyxyxy^{-1}xy)}\ \epsilon\ [1,2]$

Choose 9 from {9, 23, 20, 15, 34, 81, 30, 91, 13, 80}

Using the relation $t_1 = t_4$ conjugated by

$(xyxyxyxyxy^{-1}xy^{-1}xyxy^{-1})$ we have $t_9 = t_{35}$

then $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,t_{35}$.

Then the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $((yxyxy^{-1}x)^2)$

we have $((yxyxy^{-1}x)^2)t_{29},t_3 = t_{35}$

then $((yxyxy^{-1}x)^2)t_{47},t_{48},t_{29},t_3$

which leaves us with $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,(t_{35})$

$= Nt_{47},t_{48},t_{29},t_3$.

Then using the same relation conjugated by

$(yxyxy^{-1}xy^{-1}xyxy^{-1}xy)$

then we have $(yxyxy^{-1}xy^{-1}xyxy^{-1}xy)t_{58},t_{70} = t_3$

which gives us

$(xy^{-1}xy^{-1}xyxyxyx)t_9,t_{53},t_{30},t_{58},t_{70}$

then we have $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,(t_{35})$

$= Nt_{47},t_{48},t_{29},(t_3)$

$= Nt_9,t_{53},t_{30},t_{58},t_{70}$.

Using the same relation once again conjugated by $((yxyxy^{-1}x)^2)$

we get $((yxyxy^{-1}x)^2)t_{35} = t_{30},t_{58}$

then we have $(xy^{-1}xyxyxy^{-1}xy^{-1}xyxy)t_{17},t_{11},t_{35},t_{70}$

then $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,(t_{35})$

$= Nt_{47},t_{48},t_{29},(t_3)$

$= Nt_9,t_{53},(t_{30},t_{58}),t_{70}$

$= Nt_{17},t_{11},t_{35},t_{70}$.

By using the relation $t_1 = t_4$ conjugated by

$(yxyxy^{-1}xy^{-1}xyxy^{-1}xy^{-1})$

we get $t_{17} = t_{30}$

then we have $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,(t_{35})$

$= Nt_{47}$,t_{48},t_{29} ,(t_3)

$= Nt_9$,t_{53} ,(t_{30},t_{58}),t_{70}

$= N(t_{17}$),t_{11} ,t_{35},t_{70}

$= Nt_{30}$,t_{11} ,t_{35},t_{70} .

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}$,$t_{105}$,t_{25} = Id

conjugated by $(y^{-1}xyxyxy^{-1}xy^{-1}xy^{-1})$

we get $(y^{-1}xyxyxy^{-1}xy^{-1}xy^{-1})t_{15}$,t_{107} = t_{30}

then we have

$(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)t_{15}$,t_{107} ,t_{11} ,t_{35},t_{70}

then Nt_1,t_2,(t_9)

$= Nt_1$,t_2,(t_{35})

$= Nt_{47}$,t_{48},t_{29} ,(t_3)

$= Nt_9$,t_{53} ,(t_{30},t_{58}),t_{70}

$= N(t_{17}$),t_{11} ,t_{35},t_{70}

$= N(t_{30})$,t_{11} ,t_{35},t_{70}

$= Nt_{15}$,t_{107} ,t_{11} ,t_{35},t_{70} .

Using the same relation conjugated by

$(yxy^{-1}xyxyxyxy^{-1})$

conjugated by $(yxy^{-1}xyxyxyxy^{-1})t_{109}$ = t_{107} ,t_{11}

then we have

$(xyxy^{-1}xyxyxy^{-1}xy^{-1}x)t_{61}$,t_{109} ,t_{35},t_{70}

next Nt_1,t_2,(t_9)

$= Nt_1$,t_2,(t_{35})

$= Nt_{47}$,t_{48},t_{29} ,(t_3)

$= Nt_9$,t_{53} ,(t_{30},t_{58}),t_{70}

$= N(t_{17}$),t_{11} ,t_{35},t_{70}

$= N(t_{30})$,t_{11} ,t_{35},t_{70}

$= Nt_{15}$,(t_{107} ,t_{11}),t_{35},t_{70}

$= Nt_{61}$,t_{109} ,t_{35},t_{70} .

Using the same relation conjugated by

$(yxy^{-1}xyxyxyxy^{-1})$

we have $(yxy^{-1}xyxyxyxyxy^{-1})t_{91},t_{87} = t_{109}$

then we get $(xy^{-1}xyxyxyx)8,t_{91},t_{87}$ $,t_{35},t_{70}$

then $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,(t_{35})$

$= Nt_{47}$ $,t_{48},t_{29}$ $,(t_3)$

$= Nt_9,t_{53}$ $,(t_{30},t_{58}),t_{70}$

$= N(t_{17}$ $),t_{11}$ $,t_{35},t_{70}$

$= N(t_{30}),t_{11}$ $,t_{35},t_{70}$

$= Nt_{15}$ $,(t_{107}$ $,t_{11}$ $),t_{35},t_{70}$

$= Nt_{61}$ $,(t_{109}$ $),t_{35},t_{70}$

$= Nt_8,t_{91},t_{87}$ $,t_{35},t_{70}$.

Using the same relation conjugated by

$(xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy)$

we get

$(xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy)t_{87}$ $,t_{35},t_{97} = Id$

then we have $(xyxyxyxy^{-1}xy)t_{41}$ $,t_{52},t_{97},t_{70}$

then $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,(t_{35})$

$= Nt_{47}$ $,t_{48},t_{29}$ $,(t_3)$

$= Nt_9,t_{53}$ $,(t_{30},t_{58}),t_{70}$

$= N(t_{17}$ $),t_{11}$ $,t_{35},t_{70}$

$= N(t_{30}),t_{11}$ $,t_{35},t_{70}$

$= Nt_{15}$ $,(t_{107}$ $,t_{11}$ $),t_{35},t_{70}$

$= Nt_{61}$ $,(t_{109}$ $),t_{35},t_{70}$

$= Nt_8,t_{91},(t_{87}$ $,t_{35}),t_{70}$

$= Nt_{41}$ $,t_{52},t_{97},t_{70}$.

Then by using the relation $t_1 = t_4$ conjugated by

$(yxyxy^{-1}xyxyxyxyx)$ we have $t_{52} = t_{94}$

then we have $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,(t_{35})$

$= Nt_{47}$ $,t_{48},t_{29}$ $,(t_3)$

$= Nt_9,t_{53}$ $,(t_{30},t_{58}),t_{70}$

$= N(t_{17}),t_{11},t_{35},t_{70}$

$= N(t_{30}),t_{11},t_{35},t_{70}$

$= Nt_{15},(t_{107},t_{11}),t_{35},t_{70}$

$= Nt_{61},(t_{109}),t_{35},t_{70}$

$= Nt_8,t_{91},(t_{87},t_{35}),t_{70}$

$= Nt_{41},(t_{52}),t_{97},t_{70}$

$= Nt_{41},t_{94},t_{97},t_{70}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(yxyxyxyxyxy^{-1}xyxy^{-1})$

gives $(yxyxyxyxyxy^{-1}xyxy^{-1})t_{76} = t_{94},t_{97}$

then we have $(y^{-1}xyxy^{-1}xyxyxyxyx)t_{10},t_{76},t_{70}$

we have $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,(t_{35})$

$= Nt_{47},t_{48},t_{29},(t_3)$

$= Nt_9,t_{53},(t_{30},t_{58}),t_{70}$

$= N(t_{17}),t_{11},t_{35},t_{70}$

$= N(t_{30}),t_{11},t_{35},t_{70}$

$= Nt_{15},(t_{107},t_{11}),t_{35},t_{70}$

$= Nt_{61},(t_{109}),t_{35},t_{70}$

$= Nt_8,t_{91},(t_{87},t_{35}),t_{70}$

$= Nt_{41},(t_{52}),t_{97},t_{70}$

$= Nt_{41},(t_{94},t_{97}),t_{70}$

$= Nt_{10},t_{76},t_{70}$.

Then using the relation $t_1 = t_4$ conjuagated by

$(yxy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy)$ to get $t_{76} = t_{26}$

we have $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,(t_{35})$

$= Nt_{47},t_{48},t_{29},(t_3)$

$= Nt_9,t_{53},(t_{30},t_{58}),t_{70}$

$= N(t_{17}),t_{11},t_{35},t_{70}$

$= N(t_{30}),t_{11},t_{35},t_{70}$

$= Nt_{15}, (t_{107}, t_{11}), t_{35}, t_{70}$

$= Nt_{61}, (t_{109}), t_{35}, t_{70}$

$= Nt_{8}, t_{91}, (t_{87}, t_{35}), t_{70}$

$= Nt_{41}, (t_{52}), t_{97}, t_{70}$

$= Nt_{41}, (t_{94}, t_{97}), t_{70}$

$= Nt_{10}, (t_{76}), t_{70}$

$= Nt_{10}, t_{26}, t_{70}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^{3})t_{49}, t_{105}, t_{25} = Id$

conjugated by

$(y^{-1}xyxy^{-1}xyxyxy^{-1}xy^{-1})$

gives $(y^{-1}xyxy^{-1}xyxyxy^{-1}xy^{-1})t_{46}, t_{104} = t_{10}$

then we have

$(yxy^{-1}xyxy^{-1}xy^{-1}xyxyxy)t_{46}, t_{104}, t_{26}, t_{70}$

next $Nt_{1}, t_{2}, (t_{9})$

$= Nt_{1}, t_{2}, (t_{35})$

$= Nt_{47}, t_{48}, t_{29}, (t_{3})$

$= Nt_{9}, t_{53}, (t_{30}, t_{58}), t_{70}$

$= N(t_{17}), t_{11}, t_{35}, t_{70}$

$= N(t_{30}), t_{11}, t_{35}, t_{70}$

$= Nt_{15}, (t_{107}, t_{11}), t_{35}, t_{70}$

$= Nt_{61}, (t_{109}), t_{35}, t_{70}$

$= Nt_{8}, t_{91}, (t_{87}, t_{35}), t_{70}$

$= Nt_{41}, (t_{52}), t_{97}, t_{70}$

$= Nt_{41}, (t_{94}, t_{97}), t_{70}$

$= Nt_{10}, (t_{76}), t_{70}$

$= N(t_{10}), t_{26}, t_{70}$

$= Nt_{46}, t_{104}, t_{26}, t_{70}$.

Using the same relation conjugated by

$(y^{-1}xy^{-1}xyxyxyxy^{-1}x)$

then $(y^{-1}xy^{-1}xyxyxyxy^{-1}x)t_{104}, t_{26}, t_{49} = Id$

then we have $(xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{98}, t_{49}, t_{70}$

then $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,(t_{35})$

$= Nt_{47}\ ,t_{48},t_{29}\ ,(t_3)$

$= Nt_9,t_{53}\ ,(t_{30},t_{58}),t_{70}$

$= N(t_{17}\ ),t_{11}\ ,t_{35},t_{70}$

$= N(t_{30}),t_{11}\ ,t_{35},t_{70}$

$= Nt_{15}\ ,(t_{107}\ ,t_{11}\ ),t_{35},t_{70}$

$= Nt_{61}\ ,(t_{109}\ ),t_{35},t_{70}$

$= Nt_8,t_{91},(t_{87}\ ,t_{35}),t_{70}$

$= Nt_{41}\ ,(t_{52}),t_{97},t_{70}$

$= Nt_{41}\ ,(t_{94},t_{97}),t_{70}$

$= Nt_{10}\ ,(t_{76}\ ),t_{70}$

$= N(t_{10}\ ),t_{26}\ ,t_{70}$

$= Nt_{46},(t_{104},t_{26}\ ),t_{70}$

$= Nt_{98},t_{49},t_{70}$.

Then by using the relation $t_1 = t_4$ conjugated

by (xyxyx) $t_{98} = t_{37}$

then we have $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,(t_{35})$

$= Nt_{47}\ ,t_{48},t_{29}\ ,(t_3)$

$= Nt_9,t_{53}\ ,(t_{30},t_{58}),t_{70}$

$= N(t_{17}\ ),t_{11}\ ,t_{35},t_{70}$

$= N(t_{30}),t_{11}\ ,t_{35},t_{70}$

$= Nt_{15}\ ,(t_{107}\ ,t_{11}\ ),t_{35},t_{70}$

$= Nt_{61}\ ,(t_{109}\ ),t_{35},t_{70}$

$= Nt_8,t_{91},(t_{87}\ ,t_{35}),t_{70}$

$= Nt_{41}\ ,(t_{52}),t_{97},t_{70}$

$= Nt_{41}\ ,(t_{94},t_{97}),t_{70}$

$= Nt_{10}\ ,(t_{76}\ ),t_{70}$

$= N(t_{10}\ ),t_{26}\ ,t_{70}$

$= Nt_{46},(t_{104},t_{26}\ ),t_{70}$

$= N(t_{98}),t_{49},t_{70}$

$= Nt_{37},t_{49},t_{70}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105} ,t_{25} = Id$

conjugated by $(xy^{-1}xyxy^{-1}xy^{-1}xyxy^{-1}x)$

to get $(xy^{-1}xyxy^{-1}xy^{-1}xyxy^{-1}x)t_{100} = t_{37},t_{49}$

then we have $(yxyxy^{-1}xyx)t_{100} ,t_{70}$

then $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,(t_{35})$

$= Nt_{47} ,t_{48},t_{29} ,(t_3)$

$= Nt_9,t_{53} ,(t_{30},t_{58}),t_{70}$

$= N(t_{17}),t_{11} ,t_{35},t_{70}$

$= N(t_{30}),t_{11} ,t_{35},t_{70}$

$= Nt_{15} ,(t_{107} ,t_{11}),t_{35},t_{70}$

$= Nt_{61} ,(t_{109}),t_{35},t_{70}$

$= Nt_8,t_{91},(t_{87} ,t_{35}),t_{70}$

$= Nt_{41} ,(t_{52}),t_{97},t_{70}$

$= Nt_{41} ,(t_{94},t_{97}),t_{70}$

$= Nt_{10} ,(t_{76}),t_{70}$

$= N(t_{10}),t_{26} ,t_{70}$

$= Nt_{46},(t_{104},t_{26}),t_{70}$

$= N(t_{98}),t_{49},t_{70}$

$= N(t_{37},t_{49}),t_{70}$

$= Nt_{100} ,t_{70}$.

Then by using the relation $t_1 = t_4$ conjugated by

$(yxyxyxy^{-1}xyxyxy^{-1}xy)$ we get $t_{100} = t_{82}$

then we have $Nt_1,t_2,(t_9)$

$= Nt_1,t_2,(t_{35})$

$= Nt_{47} ,t_{48},t_{29} ,(t_3)$

$= Nt_9,t_{53} ,(t_{30},t_{58}),t_{70}$

$= N(t_{17}),t_{11} ,t_{35},t_{70}$

$= N(t_{30}),t_{11} ,t_{35},t_{70}$

$= Nt_{15} ,(t_{107} ,t_{11}),t_{35},t_{70}$

$= Nt_{61}, (t_{109}), t_{35}, t_{70}$

$= Nt_{8}, t_{91}, (t_{87}, t_{35}), t_{70}$

$= Nt_{41}, (t_{52}), t_{97}, t_{70}$

$= Nt_{41}, (t_{94}, t_{97}), t_{70}$

$= Nt_{10}, (t_{76}), t_{70}$

$= N(t_{10}), t_{26}, t_{70}$

$= Nt_{46}, (t_{104}, t_{26}), t_{70}$

$= N(t_{98}), t_{49}, t_{70}$

$= N(t_{37}, t_{49}), t_{70}$

$= N(t_{100}), t_{70}$

$= Nt_{82}, t_{70}$

$= N(t_1, t_2)(yxyxyxy^{-1}) \epsilon [1,2]$

Choose 11 from {11, 50, 79, 61, 38, 67, 48, 42, 70, 82}

Using the relation $t_1 = t_4$ conjugated by

$(y^{-1}xyxyxy^{-1}xy^{-1}xyxy)$ we have $t_2 = t_{11}$

then $Nt_1, (t_2), t_{11}$

$= Nt_1, t_{11}, t_{11}$

$= Nt_1, (t_{11})^2$

$= Nt_1 \epsilon [1]$

Choose 26 from {26, 97, 94, 105, 55, 76, 66, 52, 54, 88}

Using the relation $t_1 = t_4$ conjugated by

$(y^{-1}xyxyxy^{-1}xy^{-1}xyxy)$ we have $t_{11} = t_2$

then $Nt_1, (t_2), t_{26}$

$= Nt_1, t_{11}, t_{26}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $((yxy^{-1}xyx)^2)$

gives $((yxy^{-1}xyx)^2)t_{69} = t_{11}, t_{26}$

then we have $((yxy^{-1}xyx)^2)t_{90}, t_{69}$

then $Nt_1, (t_2), t_{26}$

$= Nt_1, (t_{11}, t_{26})$

$= Nt_{90}, t_{69}$.

Then by using the relation $t_1 = t_4$ conjugated by
$(xyxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})$ we have $t_{90} = t_{87}$
we get $Nt_1,(t_2),t_{26}$
$= Nt_1,(t_{11}, t_{26})$
$= N(t_{90}),t_{69}$
$= Nt_{87}, t_{69}$
$= N(t_1,t_2)^{(xyxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})} \in [1,2]$

Cayley Diagram

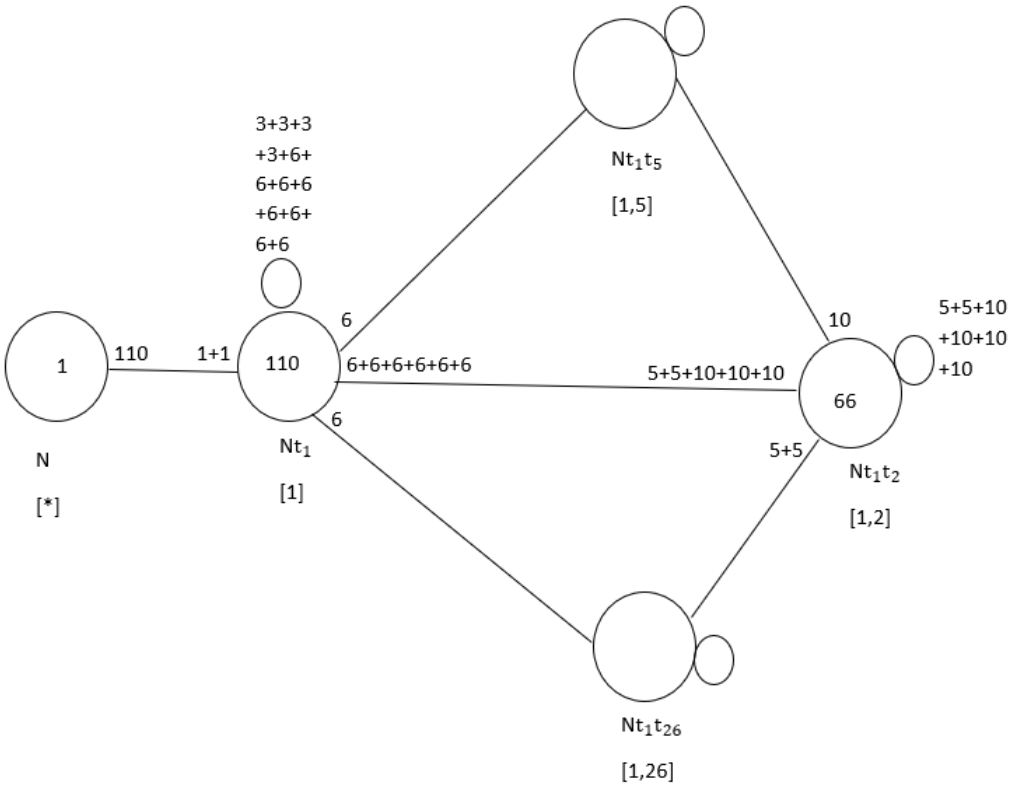


Figure 2.3: Cayley Diagram of [*], [1], [1,2] for M_{12}

Fourth Double Coset

$Nt_1t_{26}N = \{N(t_1t_{26})^n \mid n \epsilon N\} = \{Nt_1t_{26}, Nt_2t_9,..., Nt_3t_{64}\}$

The point-stabiliser 1,26, $N^{1,26}$ is given by <x>

Now $Nt_1,t_{26} = t_2,t_9$

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by conjugated by $((yxy^{-1}xyx)^2)$ which gives

$((yxy^{-1}xyx)^2)t_{86}\ ,t_{69} = t_{26}$

then we have $((yxy^{-1}xyx)^2)t_{90}\ ,t_{86}\ ,t_{69}$

then we have $Nt_1,(t_{26}\ )$

$= Nt_{90}\ ,t_{86}\ ,t_{69}.$

By using the same relation conjugated by

$((xyxy^{-1}xy)^2)$ gives $((xyxy^{-1}xy)^2)t_{69},t_9,t_4 = Id$

then we have $(xy^{-1}xy^{-1}xyxy)t_{62},t_{39}\ ,t_4,t_9$

next $Nt_1,(t_{26}\ )$

$= Nt_{90}\ ,t_{86}\ ,(t_{69})$

$= Nt_{62},t_{39}\ ,t_4,t_9.$

By using the relation $t_1 = t_4$ conjugated by

$(xy^{-1}xyxy^{-1}xyxyxyx)$ we have $t_{99} = t_{39}$

then we have $Nt_1,(t_{26}\ )$

$= Nt_{90}\ ,t_{86}\ ,(t_{69})$

$= Nt_{62},(t_{39}\ ),t_4,t_9$

$= Nt_{62},t_{99}\ ,t_4,t_9.$

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(xyxy^{-1}xyxyxyxy^{-1}x)$

we get $(xyxy^{-1}xyxyxyxy^{-1}x)t_{81} = t_{99}\ ,t_4$

then we have $(yxyxyxy^{-1}xy^{-1}xy^{-1})t_{87}\ ,t_{81}\ ,t_9$

which gives $Nt_1,(t_{26}\ )$

$= Nt_{90}\ ,t_{86}\ ,(t_{69})$

$= Nt_{62},(t_{39}\ ),t_4,t_9$

$= Nt_{62},(t_{99}\ ,t_4),t_9$

$= Nt_{87}, t_{81}, t_9$.

Conjugating the relation $t_1 = t_4$ by

$(xyxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})$ gives $t_{87} = t_{90}$

then we have $Nt_1,(2)$

$= Nt_{90}, t_{86}, (t_{69})$

$= Nt_{62},(t_{39}), t_4, t_9$

$= Nt_{62},(t_{99}, t_4), t_9$

$= N(t_{87}), t_{81}, t_9$

$= Nt_{90}, t_{81}, t_9$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})$

gives $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_2 = t_{90}, t_{81}$

then we have $(y^{-1}xy^{-1}xyxyx)t_2, t_9$

next $Nt_1,(t_{26})$

$= Nt_{90}, t_{86}, (t_{69})$

$= Nt_{62},(t_{39}), t_4, t_9$

$= Nt_{62},(t_{99}, t_4), t_9$

$= N(t_{87}), t_{81}, t_9$

$= N(t_{90}, t_{81}), t_9$

$= Nt_2, t_9$.

Therefore x ϵ $N^{(1,26)}$

Next $Nt_1t_{26} = Nt_{64}t_{97}$

Now using the relation $t_1 = t_4$ conjugated by

$(yxy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy)$ we have $t_{26} = t_{76}$

then we have $Nt_1,(t_{26})$

$= Nt_1, t_{76}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(yxyxyxyxyxy^{-1}xyxy^{-1})$

gives $(yxyxyxyxyxy^{-1}xyxy^{-1})t_{76}, t_{97}, t_{94} = Id$

then we have $(yxy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{102}, t_{94}, t_{97}$

next $Nt_1,(t_{26})$

$= Nt_1,(t_{76})$

$= Nt_{102},t_{94},t_{97}$.

Using the same relation conjugated by

$(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1})$

gives the relation $(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{101},t_{79} = t_{102}$

then we have $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)t_{101},t_{79},t_{94},t_{97}$

next $Nt_1,(t_{26})$

$= Nt_1,(t_{76})$

$= N(t_{102}),t_{94},t_{97}$

$= Nt_{101},t_{79},t_{94},t_{97}$.

Using the same relation conjugated by

$(xyxyxyxy^{-1}xy^{-1}xyx)$

gives $(xyxyxyxy^{-1}xy^{-1}xyx)t_{45} = t_{79},t_{94}$

then we have $(yxy^{-1}xyxyxy^{-1}xy^{-1})t_{72},t_{45},t_{97}$

next $Nt_1,(t_{26})$

$= Nt_1,(t_{76})$

$= N(t_{102}),t_{94},t_{97}$

$= Nt_{101},(t_{79},t_{94}),t_{97}$

$= Nt_{72},t_{45},t_{97}$.

Using the same relation conjugated by

$(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1})$

gives $(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{102} = t_{72},t_{45}$

then we have $(y^{-1}xyxyxy^{-1}xyxy^{-1}x)t_{102},t_{97}$

which leaves $Nt_1,(t_{26})$

$= Nt_1,(t_{76})$

$= N(t_{102}),t_{94},t_{97}$

$= Nt_{101},(t_{79},t_{94}),t_{97}$

$= N(t_{72},t_{45}),t_{97}$

$= Nt_{102},t_{97}$.

Then by using the relation $t_1 = t_4$ conjugated by $(y^{-1}xyxyxyxy^{-1}xy^{-1})$

we have $t_{102} = t_{64}$

thus $Nt_1,(t_{26})$

$= Nt_1,(t_{76})$

$= N(t_{102}),t_{94},t_{97}$

$= Nt_{101},(t_{79},t_{94}),t_{97}$

$= N(t_{72},t_{45}),t_{97}$

$= N(t_{102}),t_{97}$

$= Nt_{64},t_{97}$.

So $x^y \epsilon N^{(1,26)}$

Also $Nt_1t_{26} = Nt_{109}t_{102}$

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(xy^{-1}xy^{-1}xyxyxyxyxy^{-1}xy)$

gives $xy^{-1}xy^{-1}xyxyxyxyxy^{-1}xy)t_{57},t_{73} = t_{56}$

then we have $(xy^{-1}xy^{-1}xyxyxyxyxy^{-1}xy)t_{83},t_{57},t_{73}$

next $Nt_1,(t_{26}) =$

Nt_{83},t_{57},t_{73}.

Then by using the relation $t_1 = t_4$ conjugated by

$(yxy^{-1}xy^{-1}xy^{-1}xyxy^{-1})$ gives $t_{73} = t_{23}$

then we have $Nt_1,(t_{26})$

$= Nt_{83},t_{57},(t_{73})$

$= Nt_{83},t_{57},t_{23}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

Conjugated by $((y^{-1}xyxyx)^2)$

gives $((y^{-1}xyxyx)^2)t_{23},t_{102},t_{59} = Id$

then we have $(yxyxyxyxyxy^{-1}xy)t_{81},t_{91},t_{59},t_{102}$

next $Nt_1,(t_{26})$

$= Nt_{83},t_{57},(t_{73})$

$= Nt_{83},t_{57},(t_{23})$

$= Nt_{81},t_{91},t_{59},t_{102}$.

Using the same relation conjugated by

$(xy^{-1}xy^{-1}xyxyxy^{-1})$ gives

$(xy^{-1}xy^{-1}xyxyxy^{-1})t_{31}$,t_{71} = t_{81}

then we have $(y^{-1}xyxyxyxyx)t_{31}$,t_{71},t_{91},t_{59} ,t_{102}

next Nt_1,(t_{26})

= Nt_{83},t_{57},(t_{73})

= Nt_{83},t_{57},(t_{23})

= N(t_{81}),t_{91},t_{59} ,t_{102}

= Nt_{31} ,t_{71},t_{91},t_{59} ,t_{102}.

Using the same relation conjugated by

$(y^{-1}xy^{-1}xy^{-1}xyxyx)$ gives

$(y^{-1}xy^{-1}xy^{-1}xyxyx)t_{47}$ = t_{71},t_{91}

then we have $(y^{-1}xy^{-1}xyxyxy^{-1})t_{79}$,t_{47} ,t_{59} ,t_{102}

then we have Nt_1,(t_{26})

= Nt_{83},t_{57},(t_{73})

= Nt_{83},t_{57},(t_{23})

= N(t_{81}),t_{91},t_{59} ,t_{102}

= Nt_{31} ,(t_{71},t_{91}),t_{59} ,t_{102}

= Nt_{79} ,t_{47} ,t_{59} ,t_{102}.

Then by using the same relation conjugated by

$(xy^{-1}xy^{-1}xyxyxy^{-1})$ gives

$(xy^{-1}xy^{-1}xyxyxy^{-1})t_{81}$ = t_{79} ,t_{47}

then we have

$(y^{-1}xyxyxyxyxy)t_{81}$,t_{59} ,t_{102}

next Nt_1,(t_{26})

= Nt_{83},t_{57},(t_{73})

= Nt_{83},t_{57},(t_{23})

= N(t_{81}),t_{91},t_{59} ,t_{102}

= Nt_{31} ,(t_{71},t_{91}),t_{59} ,t_{102}

= N(t_{79} ,t_{47}),t_{59} ,t_{102}

= Nt_{81} ,t_{59} ,t_{102}.

By using the same relation conjugated by

$(xyxy^{-1}xyxyxyxyxy^{-1}x)$ gives

$(xyxy^{-1}xyxyxyxyxy^{-1}x)t_{105}$,$t_{88}$ = t_{81}

then we have $(xyxyxyxyxy^{-1}xyxy)t_{105},t_{88},t_{59},t_{102}$

next $Nt_1,(t_{26})$

$= Nt_{83},t_{57},(t_{73})$

$= Nt_{83},t_{57},(t_{23})$

$= N(t_{81}),t_{91},t_{59},t_{102}$

$= Nt_{31},(t_{71},t_{91}),t_{59},t_{102}$

$= N(t_{79},t_{47}),t_{59},t_{102}$

$= N(t_{81}),t_{59},t_{102}$

$= Nt_{95},t_{88},t_{59},t_{102}$.

Using the same relation conjugated by

$(xyxy^{-1}xy^{-1}xyxyxyx)$ gives

$(xyxy^{-1}xy^{-1}xyxyxyx)t_4 = t_{88},t_{59}$

then we have $(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{99},t_4,t_{102}$

which leaves us with $Nt_1,(t_{26})$

$= Nt_{83},t_{57},(t_{73})$

$= Nt_{83},t_{57},(t_{23})$

$= N(t_{81}),t_{91},t_{59},t_{102}$

$= Nt_{31},(t_{71},t_{91}),t_{59},t_{102}$

$= N(t_{79},t_{47}),t_{59},t_{102}$

$= N(t_{81}),t_{59},t_{102}$

$= Nt_{95},(t_{88},t_{59}),t_{102}$

$= Nt_{99},t_4,t_{102}$.

Then once again using the same relation conjugated by

$(xyxy^{-1}xyxyxyxyxy^{-1}x)$ which gives

$(xyxy^{-1}xyxyxyxyxy^{-1}x)t_{81} = t_{99},t_4$

then we have $(y^{-1}xyxy^{-1}xy^{-1}xyxyxy)t_{81},t_{102}$

then we have $Nt_1,(t_{26})$

$= Nt_{83},t_{57},(t_{73})$

$= Nt_{83},t_{57},(t_{23})$

$= N(t_{81}),t_{91},t_{59},t_{102}$

$= Nt_{31},(t_{71},t_{91}),t_{59},t_{102}$

$= N(t_{79},t_{47}),t_{59},t_{102}$

$= N(t_{81}),t_{59},t_{102}$

$= Nt_{95},(t_{88},t_{59}),t_{102}$

$= N(t_{99},t_4),t_{102}$

$= Nt_{109},t_{102}$.

Thus $(yxyxyxy^{-1}xy^{-1}) \epsilon N^{(1,26)}$

Then the coset stabilizer for

$N^{(1,26)} = < x,(x^y), (yxyxyxy^{-1}xy^{-1}) >$

The number of single right cosets in the double coset $Nt_1t_{26}N = [1,26]$ is given by $\frac{|N|}{|N^{(1,26)}|}$ $= \frac{660}{60} = 11$

The orbits for $N^{(1,26)}$ on

X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110} are {18, 48, 46, 70, 65, 86, 22, 43, 54, 66, 62, 80, 91, 93, 63, 101, 92, 108, 53, 95}, {1, 2, 64, 83, 102, 49, 4, 19, 11, 50, 109, 42, 76, 36, 81, 10, 45, 26, 16, 15, 100, 82, 110, 103, 9, 35, 105, 73, 23, 97}, {3, 8, 5, 44, 13, 33, 30, 60, 32, 78, 98, 69, 38, 75, 87, 85,21, 51, 71, 79, 39, 104, 77, 74, 88, 41, 57, 52, 27, 28},{6, 17, 84, 59, 106, 31, 7, 12, 20, 34, 72, 90, 89, 40, 56,37, 25, 67, 94, 55, 47, 96, 68, 29, 24, 99, 14, 107, 58, 61} we will multiply Nt_1t_{26} by a representative of each orbit and determine its double coset.

Choose 18 from {18, 48, 46, 70, 65, 86, 22, 43, 54, 66, 62, 80, 91, 93, 63, 101, 92, 108, 53, 95}

By using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)^4)t_{102},t_{65} = t_{29},t_{103}$

conjugated by $((xyxy^{-1})^2)$

then we have $((xyxy^{-1})^2)t_{26},t_{18} = t_{14},t_{35}$

then we have $((yxy^{-1}x)^2)t_{91},t_{14},t_{35}$

next $Nt_1,(t_{26},t_{18})$

$= Nt_{91},t_{14},t_{35}$.

By using the relation $t_1 = t_4$ conjugated by

$(xyxy^{-1}xyxyxyxyxy^{-1})$ we have $t_{93} = t_{91}$

then we have $Nt_1,(t_{26}\ ,t_{18})$

$= N(t_{91}),t_{14}\ ,t_{35}$

$= Nt_{93}\ ,t_{14}\ ,t_{35}$.

Then by using

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(xy^{-1}xy^{-1}xyxyxy)$

gives $(xy^{-1}xy^{-1}xyxyxy)2 = t_{93}\ ,t_{14}$

then we have $(yxy^{-1}xyxyxy^{-1}xyxyxy)2,t_{35}$

next $Nt_1,(t_{26}\ ,t_{18})$

$= N(t_{91}),t_{14}\ ,t_{35}$

$= N(t_{93}\ ,t_{14}\ ),t_{35}$

$= Nt_2,t_{35}$

$= N(t_1,t_{26}\ )^{(y^{-1}xy^{-1}xyxyxy^{-1}xyxy)}\ \epsilon\ [1,26]$

Choose 26 from {1, 2, 64, 83, 102, 49, 4, 19, 11, 50, 109, 42, 76, 36, 81, 10, 45, 26, 16, 15, 100, 82, 110, 103, 9, 35, 105, 73, 23, 97}

$Nt_1t_{26}t_{26}$

$= Nt_1(t_{26})^2$

$= Nt_1\ \epsilon\ [1]$

Choose 3 from {3, 8, 5, 44, 13, 33, 30, 60, 32, 78, 98, 69, 38, 75, 87, 85, 21, 51, 71, 79, 39, 104, 77, 74, 88, 41, 57, 52, 27, 28}

Using the relation

$(y^{-1}xyxy^{-1}xy^{-1}xyxyxy)t_{109}\ ,t_{102} = t_1,t_{26}$

we have $(y^{-1}xyxy^{-1}xy^{-1}xyxyxy)t_{109}\ ,t_{102},t_3$

then we have $N(t_1,t_{26}\ ),t_3$

$= Nt_{109}\ ,t_{102},t_3$.

Then using the relation

$(yxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{92},t_{107} = t_1,t_2$

conjugated by $(xy^{-1}xyxyxy^{-1}xy^{-1}xy^{-1})$

to get $(xy^{-1}xyxyxy^{-1}xy^{-1}xy^{-1})t_5,t_3,t_{62},t_{97} = Id$

then we have $(yxyxy^{-1})t_{55},t_{30},t_{58},t_{97},t_{62}$

then $N(t_1,t_{26}\ ),t_3$

$= Nt_{109}\ ,t_{102},(t_3)$

$= Nt_{55},t_{30},t_{58},t_{97},t_{62}$.

Then by using

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $((yxyxy^{-1}x)^2)$ gives $((yxyxy^{-1}x)^2)t_{35} = t_{30},t_{58}$

then we have $(yxy^{-1}xy^{-1}xyxyxy^{-1}x)t_{71},t_{35},t_{97},t_{62}$

next $N(t_1,t_{26}\ ),t_3$

$= Nt_{109}\ ,t_{102},(t_3)$

$= Nt_{55},(t_{30},t_{58}),t_{97},t_{62}$

$= Nt_{71},t_{35},t_{97},t_{62}$.

By using the same relation conjugated by

$(xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy)$ gives

$(xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy)t_{35},t_{97},t_{72} = Id$

then we have

$(xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy)t_{15}\ ,t_{72}\ ,t_{62}$

next $N(t_1,t_{26}\ ),t_3$

$= Nt_{109}\ ,t_{102},(t_3)$

$= Nt_{55},(t_{30},t_{58}),t_{97},t_{62}$

$= Nt_{71},(t_{35},t_{97}),t_{62}$

$= Nt_{15}\ ,t_{72}\ ,t_{62}$.

Then by using the relation $t_1 = t_4$ conjugated by

$(yxyxyxy^{-1}xyxy)$ we have $t_{15} = t_{16}$

then we have $N(t_1,t_{26}\ ),t_3$

$= Nt_{109}\ ,t_{102},(t_3)$

$= Nt_{55},(t_{30},t_{58}),t_{97},t_{62}$

$= Nt_{71},(t_{35},t_{97}),t_{62}$

$= N(t_{15}\ ),t_{72}\ ,t_{62}$

$= Nt_{16},t_{72}\ ,t_{62}$.

Then by using

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(yxyxyxy^{-1}xy^{-1}xy)$

gives $t_{16},t_{72}\ ,t_{39} = Id$

then we have $(y^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy^{-1})t_{39}\ ,t_{62}$

next $N(t_1,t_{26}),t_3$

$= Nt_{109},t_{102},(t_3)$

$= Nt_{55},(t_{30},t_{58}),t_{97},t_{62}$

$= Nt_{71},(t_{35},t_{97}),t_{62}$

$= N(t_{15}),t_{72},t_{62}$

$= N(t_{16},t_{72}),t_{62}$

$= Nt_{39},t_{62}$.

Then by using the relation $t_1 = t_4$ conjugated by

$(xy^{-1}xyxy^{-1}xyxyxyx)$ gives $t_{99} = t_{39}$

then we have $N(t_1,t_{26}),t_3$

$= Nt_{109},t_{102},(t_3)$

$= Nt_{55},(t_{30},t_{58}),t_{97},t_{62}$

$= Nt_{71},(t_{35},t_{97}),t_{62}$

$= N(t_{15}),t_{72},t_{62}$

$= N(t_{16},t_{72}),t_{62}$

$= N(t_{39}),t_{62}$

$= Nt_{99},t_{62}$

$= N(t_1,t_2)^{(y^{-1}xy^{-1}xyxy)} \epsilon [1,2]$

<u>Choose 6 from {6, 17, 84, 59, 106, 31, 7, 12, 20, 34, 72, 90, 89, 40, 56,37, 25, 67, 94, 55, 47, 96, 68, 29, 24, 99, 14, 107, 58, 61}</u>

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by

$((yxy^{-1}xy^{-1}x)^2)$

gives $((yxy^{-1}xy^{-1}x)^2)t_{34},t_{57} = t_6$

then we have $((yxy^{-1}xy^{-1}x)^2)t_{91},t_{57},t_{34},t_{57}$

next $Nt_1,t_{26},(t_6)$

$= Nt_{91},t_{57},t_{34},t_{57}$.

Then using the same relation conjugated by $((yxy^{-1}xy^{-1}x)^2)$

gives $((yxy^{-1}xy^{-1}x)^2)t_{26} = t_{57},t_{34}$

then we have $(xyxyxy^{-1})t_{13},t_{26},t_{57}$

next $Nt_1,t_{26},(t_6)$

$= Nt_{91},(t_{57},t_{34}),t_{57}$

$= Nt_{13}\ ,t_{26}\ ,t_{57}.$

Then by using the same relation conjugated by

$(yxy^{-1}xy^{-1}xy^{-1}xy^{-1})$ gives

$(yxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{33}\ ,t_{37} = t_{57}$

then we have $(y^{-1}xy^{-1})t_{92},t_{51}\ ,t_{33}\ ,t_{37}$

next $Nt_1,t_{26}\ ,(t_6)$

$= Nt_{91},(t_{57},t_{34}),t_{57}$

$= Nt_{13}\ ,t_{26}\ ,(t_{57})$

$= Nt_{92},t_{51}\ ,t_{33}\ ,t_{37}.$

Using the same relation conjugated by

$(xyxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})$ conjugated by

$(xyxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{100}\ ,t_{34} = t_{92}$

then we have

$(yxy^{-1}xyxyxyxyxy^{-1}x)t_{100}\ ,t_{34},t_{51}\ ,t_{33}\ ,t_{37}$

next $Nt_1,t_{26}\ ,(t_6)$

$= Nt_{91},(t_{57},t_{34}),t_{57}$

$= Nt_{13}\ ,t_{26}\ ,(t_{57})$

$= N(t_{92}),t_{51}\ ,t_{33}\ ,t_{37}$

$= Nt_{100}\ ,t_{34},t_{51}\ ,t_{33}\ ,t_{37}.$

Then by using the same relation conjugated by

$(y^{-1}xyxy^{-1}xyxyxyxyxy)$ gives

$(y^{-1}xyxy^{-1}xyxyxyxyxy)t_6 = t_{34},t_{51}$

then we have

$(xy^{-1}xy^{-1}xy^{-1}xyxyxy^{-1}xyx)t_{36},t_6,t_{33}\ ,t_{37}$

next $Nt_1,t_{26}\ ,(t_6)$

$= Nt_{91},(t_{57},t_{34}),t_{57}$

$= Nt_{13}\ ,t_{26}\ ,(t_{57})$

$= N(t_{92}),t_{51}\ ,t_{33}\ ,t_{37}$

$= Nt_{100}\ ,(t_{34},t_{51}\ ),t_{33}\ ,t_{37}$

$= Nt_{36},t_6,t_{33}\ ,t_{37}.$

By using the relation $t_1 = t_4$ conjugated by $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xy)$ gives $t_6 = t_{32}$

then we have $Nt_1,t_{26},(t_6)$

$= Nt_{91},(t_{57},t_{34}),t_{57}$

$= Nt_{13},t_{26},(t_{57})$

$= N(t_{92}),t_{51},t_{33},t_{37}$

$= Nt_{100},(t_{34},t_{51}),t_{33},t_{37}$

$= Nt_{36},(t_6),t_{33},t_{37}$

$= Nt_{36},t_{32},t_{33},t_{37}$.

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $((y^{-1}xy^{-1}xyx)^2)$ gives

$((y^{-1}xy^{-1}xyx)^2)t_5,t_{102} = t_{36}$

then we have

$(xyxy^{-1}xyxyxy^{-1}xy^{-1}xy)t_5,t_{102},t_{32},t_{33},t_{37}$

next $Nt_1,t_{26},(t_6)$

$= Nt_{91},(t_{57},t_{34}),t_{57}$

$= Nt_{13},t_{26},(t_{57})$

$= N(t_{92}),t_{51},t_{33},t_{37}$

$= Nt_{100},(t_{34},t_{51}),t_{33},t_{37}$

$= Nt_{36},(t_6),t_{33},t_{37}$

$= N(t_{36}),t_{32},t_{33},t_{37}$

$= Nt_5,t_{102},t_{32},t_{33},t_{37}$.

Then by using the relation $t_1 = t_4$ conjugated by

$(yxyxy^{-1}xyxyxyx)$ gives $t_7 = t_{33}$

then we have $Nt_1,t_{26},(t_6)$

$= Nt_{91},(t_{57},t_{34}),t_{57}$

$= Nt_{13},t_{26},(t_{57})$

$= N(t_{92}),t_{51},t_{33},t_{37}$

$= Nt_{100},(t_{34},t_{51}),t_{33},t_{37}$

$= Nt_{36},(t_6),t_{33},t_{37}$

$= N(t_{36}),t_{32},t_{33},t_{37}$

$= Nt_5,t_{102},t_{32},(t_{33}),t_{37}$

$= Nt_5,t_{102},t_{32},t_7,t_{37}$.

Next using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by

$(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})$ gives

$(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{72} = t_{102},t_{32}$

then we have $((xy^{-1}xy)^2)t_{54}\ ,t_{72}\ ,t_7,t_{37}$

next $Nt_1,t_{26}\ ,(t_6)$

$= Nt_{91},(t_{57},t_{34}),t_{57}$

$= Nt_{13}\ ,t_{26}\ ,(t_{57})$

$= N(t_{92}),t_{51}\ ,t_{33}\ ,t_{37}$

$= Nt_{100}\ ,(t_{34},t_{51}\ ),t_{33}\ ,t_{37}$

$= Nt_{36},(t_6),t_{33}\ ,t_{37}$

$= N(t_{36}),t_{32}\ ,t_{33}\ ,t_{37}$

$= Nt_5,t_{102},t_{32}\ ,(t_{33}\ ),t_{37}$

$= Nt_5,(t_{102},t_{32}\ ),t_7,t_{37}$

$= Nt_{54}\ ,t_{72}\ ,t_7,t_{37}$.

Using the same relation conjugated by

$(yxyxyxy^{-1}xy^{-1}xy)$ gives

$(yxyxyxy^{-1}xy^{-1}xy)t_{79}\ ,t_{16} = t_{72}$ then we have

$(yxy^{-1}xy^{-1}xyxyx)t_{49},t_{79}\ ,t_{16},t_7,t_{37}$

next $Nt_1,t_{26}\ ,(t_6)$

$= Nt_{91},(t_{57},t_{34}),t_{57}$

$= Nt_{13}\ ,t_{26}\ ,(t_{57})$

$= N(t_{92}),t_{51}\ ,t_{33}\ ,t_{37}$

$= Nt_{100}\ ,(t_{34},t_{51}\ ),t_{33}\ ,t_{37}$

$= Nt_{36},(t_6),t_{33}\ ,t_{37}$

$= N(t_{36}),t_{32}\ ,t_{33}\ ,t_{37}$

$= Nt_5,t_{102},t_{32}\ ,(t_{33}\ ),t_{37}$

$= Nt_5,(t_{102},t_{32}\ ),t_7,t_{37}$

$= Nt_{54}\ ,(t_{72}\ ),t_7,t_{37}$

$= Nt_{49},t_{79}\ ,t_{16},t_7,t_{37}$.

Using the same relation again conjugated by

$(xyxy^{-1}xyxyxyxyxy^{-1}xy^{-1})$ gives

$(xyxy^{-1}xyxyxyxyxy^{-1}xy^{-1})t_{16},t_7 = t_{80}$

then we have $(y^{-1}xyxy^{-1}xyxyxyxy)t_{87}$ $,t_{70}$ $,t_{80}$ $,t_{37}$

next Nt_1,t_{26} $,(t_6)$

$= Nt_{91},(t_{57},t_{34}),t_{57}$

$= Nt_{13}$ $,t_{26}$ $,(t_{57})$

$= N(t_{92}),t_{51}$ $,t_{33}$ $,t_{37}$

$= Nt_{100}$ $,(t_{34},t_{51}$ $),t_{33}$ $,t_{37}$

$= Nt_{36},(t_6),t_{33}$ $,t_{37}$

$= N(t_{36}),t_{32}$ $,t_{33}$ $,t_{37}$

$= Nt_5,t_{102},t_{32}$ $,(t_{33}$ $),t_{37}$

$= Nt_5,(t_{102},t_{32}$ $),t_7,t_{37}$

$= Nt_{54}$ $,(t_{72}$ $),t_7,t_{37}$

$= Nt_{49},t_{79}$ $,(t_{16},t_7),$ t_{37}

$= Nt_{87}$ $,t_{70}$ $,t_{80}$ $,t_{37}$.

Then by using the relation $t_1 = t_4$ conjugated by

$((xyxy^{-1}xy)^2)$ gives $t_{70} = t_{46}$

gives Nt_1,t_{26} $,(t_6)$

$= Nt_{91},(t_{57},t_{34}),t_{57}$

$= Nt_{13}$ $,t_{26}$ $,(t_{57})$

$= N(t_{92}),t_{51}$ $,t_{33}$ $,t_{37}$

$= Nt_{100}$ $,(t_{34},t_{51}$ $),t_{33}$ $,t_{37}$

$= Nt_{36},(t_6),t_{33}$ $,t_{37}$

$= N(t_{36}),t_{32}$ $,t_{33}$ $,t_{37}$

$= Nt_5,t_{102},t_{32}$ $,(t_{33}$ $),t_{37}$

$= Nt_5,(t_{102},t_{32}$ $),t_7,t_{37}$

$= Nt_{54}$ $,(t_{72}$ $),t_7,t_{37}$

$= Nt_{49},t_{79}$ $,(t_{16},t_7),$ t_{37}

$= Nt_{87}$ $,(t_{70}$ $),t_{80}$ $,t_{37}$

$= Nt_{87}$ $,t_{46},t_{80}$ $,t_{37}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}$ $,t_{25} = Id$

conjugated by $(xyxyxy^{-1}xy^{-1}xy^{-1})$ gives
$(xyxyxy^{-1}xy^{-1}xy^{-1})t_{95} = t_{46},t_{80}$ then we have
$(yxyxyxyxy^{-1}xyxy^{-1})t_{38}$,t_{95},t_{37}
next Nt_1,t_{26} ,(t_6)
$= Nt_{91},(t_{57},t_{34}),t_{57}$
$= Nt_{13}$,t_{26} ,(t_{57})
$= N(t_{92}),t_{51}$,t_{33} ,t_{37}
$= Nt_{100}$,(t_{34},t_{51}),t_{33} ,t_{37}
$= Nt_{36},(t_6),t_{33}$,t_{37}
$= N(t_{36}),t_{32}$,t_{33} ,t_{37}
$= Nt_5,t_{102},t_{32}$,(t_{33}),t_{37}
$= Nt_5,(t_{102},t_{32}$),t_7,t_{37}
$= Nt_{54}$,(t_{72}),t_7,t_{37}
$= Nt_{49},t_{79}$,(t_{16},t_7), t_{37}
$= Nt_{87}$,(t_{70}),t_{80} ,t_{37}
$= Nt_{87}$,(t_{46},t_{80}),t_{37}
$= Nt_{38}$,t_{95},t_{37}.
Using the same relation conjugated by
$(xy^{-1}xy^{-1}xy^{-1}xyxy)$
gives $(xy^{-1}xy^{-1}xy^{-1}xyxy)t_{89} = t_{38}$,t_{95}
then we have $(yxy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}x)t_{89}$,t_{37}
next Nt_1,t_{26} ,(t_6)
$= Nt_{91},(t_{57},t_{34}),t_{57}$
$= Nt_{13}$,t_{26} ,(t_{57})
$= N(t_{92}),t_{51}$,t_{33} ,t_{37}
$= Nt_{100}$,(t_{34},t_{51}),t_{33} ,t_{37}
$= Nt_{36},(t_6),t_{33}$,t_{37}
$= N(t_{36}),t_{32}$,t_{33} ,t_{37}
$= Nt_5,t_{102},t_{32}$,(t_{33}),t_{37}
$= Nt_5,(t_{102},t_{32}$),t_7,t_{37}
$= Nt_{54}$,(t_{72}),t_7,t_{37}
$= Nt_{49},t_{79}$,(t_{16},t_7), t_{37}

$= Nt_{87}, (t_{70}), t_{80}, t_{37}$

$= Nt_{87}, (t_{46}, t_{80}), t_{37}$

$= N(t_{38}, t_{95}), t_{37}$

$= Nt_{89}, t_{37}$.

Next by using the relation $t_1 = t_4$ conjugated by

$(yxyxy^{-1}xyxyxy^{-1})$ gives $t_{89} = t_{51}$

then we have $Nt_1, t_{26}, (t_6)$

$= Nt_{91}, (t_{57}, t_{34}), t_{57}$

$= Nt_{13}, t_{26}, (t_{57})$

$= N(t_{92}), t_{51}, t_{33}, t_{37}$

$= Nt_{100}, (t_{34}, t_{51}), t_{33}, t_{37}$

$= Nt_{36}, (t_6), t_{33}, t_{37}$

$= N(t_{36}), t_{32}, t_{33}, t_{37}$

$= Nt_5, t_{102}, t_{32}, (t_{33}), t_{37}$

$= Nt_5, (t_{102}, t_{32}), t_7, t_{37}$

$= Nt_{54}, (t_{72}), t_7, t_{37}$

$= Nt_{49}, t_{79}, (t_{16}, t_7), t_{37}$

$= Nt_{87}, (t_{70}), t_{80}, t_{37}$

$= Nt_{87}, (t_{46}, t_{80}), t_{37}$

$= N(t_{38}, t_{95}), t_{37}$

$= N(t_{89}), t_{37}$

$= Nt_{51}, t_{37}$

$= N(t_1, t_2)^{(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1})} \epsilon [1,2]$

Cayley Diagram

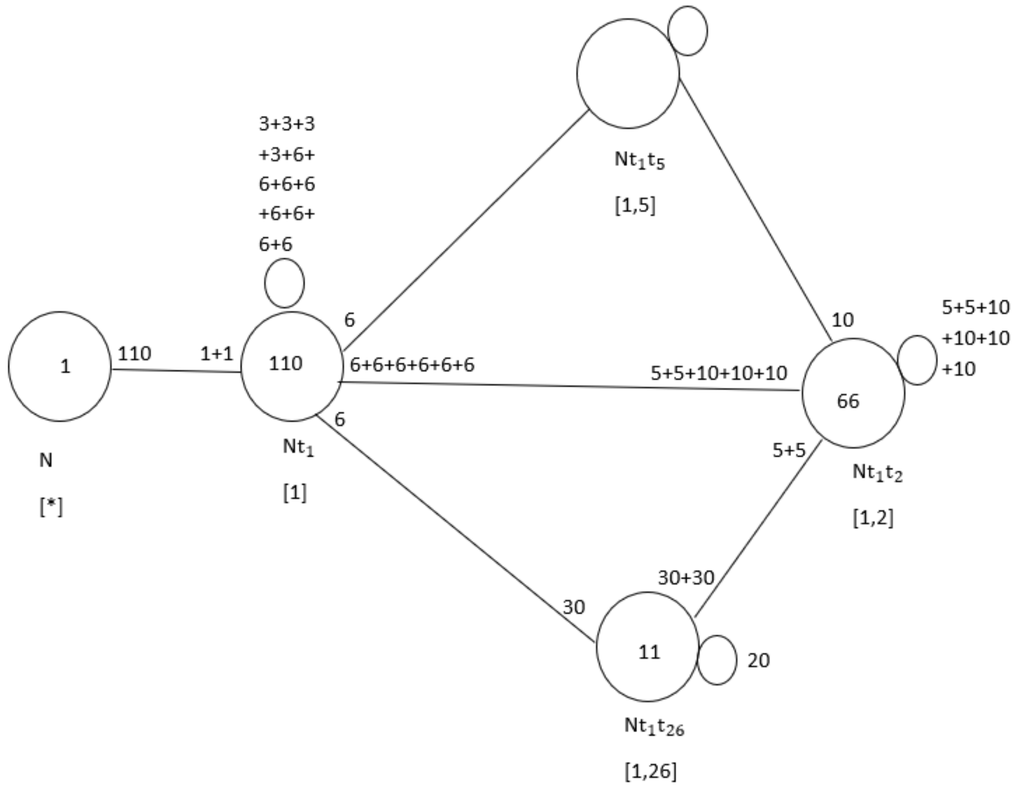


Figure 2.4: Cayley Diagram of [*], [1], [1,2], [1,26] for M_{12}

Fifth Double Coset

$Nt_1t_5N = \{N(t_1t_5)^n \mid n \epsilon N\} = \{Nt_1t_5, Nt_2t_{13}, ..., Nt_3t_{14}\}$

The point-stabiliser 1, 5, N^{15} is given by <(1, 2)(3, 8)(4, 11)(5, 13)(6, 17)(7, 20)(9, 26)(10, 23)(12, 34)(14,40)(15, 42)(16, 45)(18, 48)(19, 50)(21, 52)(22, 54)(24, 58)(25, 61)(27, 57)(28, 51)(29, 68)(30, 32)(31, 59)(33, 44)(35, 76)(36,73)(38, 71)(39, 79)(41, 85)(43, 66)(46, 65)(47, 89)(49, 83)(53,93)(55, 90)(56, 96)(60, 78)(62, 92)(63, 101)(64, 102)(70, 86)(72, 94)(74, 104)(75, 77)(80, 108)(81, 109)(82, 103)(84, 106)(87, 88)(91, 95)(99, 107)(100, 110)>

Now $Nt_1t_5 = Nt_2t_{13}$

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $((y^{-1}xy^{-1}xyx)^2)$

gives $((y^{-1}xy^{-1}xyx)^2)t_{25},t_{48} = t_5$

then we have $((y^{-1}xy^{-1}xyx)^2)t_{80}$ $,t_{25},t_{48}$

next $Nt_1,(t_5)$

$= Nt_{80}$ $,t_{25},t_{48}$.

Next we will use the same relation conjugated by

$((y^{-1}xyxyx)^2)$ gives

$((y^{-1}xyxyx)^2)t_{102},t_{59} = t_{48}$ then we have

$(yxy^{-1}xy^{-1}xyxyxyxyx)t_{26}$ $,t_{36},t_{102},t_{59}$

therefore we have $Nt_1,(t_5)$

$= Nt_{80}$ $,t_{25},(t_{48})$

$= Nt_{26}$ $,t_{36},t_{102},t_{59}$.

By using the same relation we will conjugate it by

$((y^{-1}xy^{-1}xyx)^2)$ then we have

$((y^{-1}xy^{-1}xyx)^2)t_5 = t_{36},t_{102}$ thus we have

$(yxyxyxy^{-1}xy^{-1}xy^{-1})t_{87}$ $,t_5,t_{59}$

next we have $Nt_1,(t_5)$

$= Nt_{80}$ $,t_{25},(t_{48})$

$= Nt_{26}$ $,(t_{36},t_{102}),t_{59}$

$= Nt_{87}$ $,t_5,t_{59}$.

Next by using the relation $t_1 = t_4$ conjugated by

$(xyxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})$

we have $t_{87} = t_{90}$

then we have $Nt_1,(t_5)$

$= Nt_{80}$ $,t_{25},(t_{48})$

$= Nt_{26}$ $,(t_{36},t_{102}),t_{59}$

$= N(t_{87}$ $),t_5,t_{59}$

$= Nt_{90}$ $,t_5,t_{59}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}$ $,t_{25} = Id$

conjugated by $(y^{-1}xy^{-1}xy^{-1}xyxyxy^{-1})$

then we have

$(y^{-1}xy^{-1}xy^{-1}xyxyxy^{-1})t_{90}$ $,t_5,t_2 = Id$

thus we have $(yxy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy)t_2,t_{59}$

next we have $Nt_1,(t_5)$

$= Nt_{80}\ ,t_{25},(t_{48})$

$= Nt_{26}\ ,(t_{36},t_{102}),t_{59}$

$= N(t_{87}\ ),t_5,t_{59}$

$= N(t_{90}\ ,t_5),t_{59}$

$= Nt_2,t_{59}$.

Next by using the relation $t_1 = t_4$ conjugated by

$(yxyxyxyxy^{-1}xyxy)$ we have $t_{59} = t_{13}$

thus we have $Nt_1,(t_5)$

$= Nt_{80}\ ,t_{25},(t_{48})$

$= Nt_{26}\ ,(t_{36},t_{102}),t_{59}$

$= N(t_{87}\ ),t_5,t_{59}$

$= N(t_{90}\ ,t_5),t_{59}$

$= Nt_2,(t_{59}\ )$

$= Nt_2,t_{13}$.

Then x ϵ $N^{(1,5)}$ since $N(t_1t_5)^x = Nt_2t_{13}$

Now $Nt_1t_5 = Nt_{69}t_{18}$

We have t_1,t_5 we are going to multiply t_{18},t_{18} to the right side since $(t_{18})^2 = Id$

then we have Nt_1,t_5

$= Nt_1,t_5,t_{18},t_{18}$.

Next by using the relation $t_1 = t_4$ conjugated by

$(y^{-1}xy^{-1}xyxyxy^{-1}xyxy^{-1})$

we have $t_5 = t_{31}$

then we have Nt_1,t_5

$= Nt_1,(t_5),t_{18},t_{18}$

$= Nt_1,t_{31}\ ,t_{18},t_{18}$.

Next by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $((xy^{-1}xyxy)^2)$

we have $((xy^{-1}xyxy)^2)t_{31}\ ,t_{18},t_{75} = Id$

then we have $((y^{-1}xy^{-1}xyx)^2)t_{80}\ ,t_{75}\ ,t_{18}$

next we have Nt_1,t_5

$= Nt_1,(t_5),t_{18},t_{18}$

$= Nt_1,(t_{31}\ ,t_{18}),t_{18}$

$= Nt_{80}\ ,t_{75}\ ,t_{18}$.

By using the same relation conjugated by

$(xyxy^{-1}xyxyxyxyxy^{-1}xy^{-1})$ gives

$(xyxy^{-1}xyxyxyxyxy^{-1}xy^{-1})t_{16},t_7 = t_{80}$

then we have $(xyxyxyxy^{-1}xyxyx)t_{16},7,t_{75}$

then we have Nt_1,t_5

$= Nt_1,(t_5),t_{18},t_{18}$

$= Nt_1,(t_{31}\ ,t_{18}),t_{18}$

$= N(t_{80}\ ),t_{75}\ ,t_{18}$

$= Nt_{16},7,t_{75}\ ,t_{18}$.

By using the same relation conjugated by

$(y^{-1}xyxyxyxy^{-1}x)$ we have

$(y^{-1}xyxyxyxy^{-1}x)t_7,t_{75}\ ,t_{26} = Id$

then we have $(xy^{-1}xy^{-1}xyxyxyxyxy)t_2,t_{26}\ ,t_{18}$

next Nt_1,t_5

$= Nt_1,(t_5),t_{18},t_{18}$

$= Nt_1,(t_{31}\ ,t_{18}),t_{18}$

$= N(t_{80}\ ),t_{75}\ ,t_{18}$

$= Nt_{16},(t_7,t_{75}\ ),t_{18}$

$= Nt_2,t_{26}\ ,t_{18}$.

By using the relation $t_1 = t_4$ conjugated by

$(yxy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy)$ we have $t_{26} = t_{76}$

then we have Nt_1,t_5

$= Nt_1,(t_5),t_{18},t_{18}$

$= Nt_1,(t_{31}\ ,t_{18}),t_{18}$

$= N(t_{80}\ ),t_{75}\ ,t_{18}$

$= Nt_{16},(t_7,t_{75}\ ),t_{18}$

$= Nt_2,(t_{26}\ ),t_{18}$

$= Nt_2,t_{76}\ ,t_{18}$.

Next using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $((xy^{-1}xyxy^{-1})^2)$ gives

$((xy^{-1}xyxy^{-1})^2)t_2,t_{76}\ ,t_{67} = Id$

then we have $(yxy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{67},t_{18}$

then we have Nt_1,t_5

$= Nt_1,(t_5),t_{18},t_{18}$

$= Nt_1,(t_{31}\ ,t_{18}),t_{18}$

$= N(t_{80}\ ),t_{75}\ ,t_{18}$

$= Nt_{16},(t_7,t_{75}\ ),t_{18}$

$= Nt_2,(t_{26}\ ),t_{18}$

$= N(t_2,\ t_{76}\ ),t_{18}$

$= Nt_{67},t_{18}$.

Then using the relation $t_1 = t_4$ conjugated by $(xy^{-1}xyxyxyxyxy^{-1}xyx)$ we have $t_{67} = t_{69}$

then we have Nt_1,t_5

$= Nt_1,(t_5),t_{18},t_{18}$

$= Nt_1,(t_{31}\ ,t_{18}),t_{18}$

$= N(t_{80}\ ),t_{75}\ ,t_{18}$

$= Nt_{16},(t_7,t_{75}\ ),t_{18}$

$= Nt_2,(t_{26}\ ),t_{18}$

$= N(t_2,\ t_{76}\ ),t_{18}$

$= N(t_{67}),t_{18}$

$= Nt_{69},t_{18}$.

So $(yxyxy^{-1})\ \epsilon\ N^{(1,5)}$ since $N(t_1t_5)^{(yxyxy^{-1})} = Nt_{69}t_{18}$

Thus the coset stabilizer for $N^{(1,5)} = <x, (yxyxy^{-1})>$

The number of single right cosets in the double coset $Nt_1t_5N = [1,5]$ is given by $\frac{|N|}{|N^{(15)}|} = \frac{660}{60} = 11$

The orbits for $N^{(15)}$ on

X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110} are {7, 20, 94, 72, 15, 50, 42, 19, 105, 37}, {16, 45, 33, 98, 44, 52, 49, 21, 83, 97}, {1, 2, 69, 13, 89, 5, 70, 47, 18, 86, 23, 48, 93, 109, 10, 95,91, 53, 81, 31, 28, 36, 59, 51, 73, 65, 46, 4, 11, 67}, {3, 8, 76, 82, 35, 99, 103, 29, 24, 107, 25, 68, 58, 78, 41,61, 101, 88, 32, 60, 85, 66, 43, 63, 87, 30, 40, 102, 108, 90, 110, 77, 14, 64, 80, 71, 55, 100, 75, 22, 56, 62, 38, 17, 27, 54, 96, 92,106, 104, 6, 57, 34, 79, 84, 74, 12, 39, 9, 26} we will multiply Nt_1t_5 by a representative of each orbit and determine its double coset

Choose 7 from {7, 20, 94, 72, 15, 50, 42, 19, 105, 37}

By using the relation $t_1 = t_4$ conjugated by

$(y^{-1}xy^{-1}xyxyxy^{-1}xyxy^{-1})$ we have $t_5 = t_{31}$

then we have $Nt_1,(t_5)$

$= Nt_1,t_{31}\ ,t_7$.

Next by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}\ ,t_{25}, = Id$

conjugated by $(yxyxyxyxyxy^{-1}xy^{-1})$

gives $(yxyxyxyxyxy^{-1}xy^{-1})t_{59}\ ,t_{41} = t_{31}$

then we have $(yxyxyxyxyxy^{-1}xy^{-1})t_{110},t_{59}\ ,t_{41}\ ,t_7$

next we have $Nt_1,(t_5)$

$= Nt_1,(t_{31}\ ),t_7$

$= Nt_{110},t_{59}\ ,t_{41}\ ,t_7$.

Next by using the same relation conjugated by

$(yxy^{-1}xyxyxyxy)$ then

$(yxy^{-1}xyxyxyxy)t_{83} = t_{41}\ ,t_7$

then we have $(xy^{-1}x)t_{40}\ ,t_{70}\ ,t_{83}$

next we have $Nt_1,(t_5)$

$= Nt_1,(t_{31}\ ),t_7$

$= Nt_{110},t_{59}\ ,(t_{41}\ ,t_7)$

$= Nt_{40}\ ,t_{70}\ ,t_{83}$.

Using the same relation conjugated by

$(y^{-1}xyxy^{-1}xy^{-1}xyxy^{-1})$ then we have

$(y^{-1}xyxy^{-1}xy^{-1}xyxy^{-1})t_{41}\ ,t_{110} = t_{83}$

next we have

$(xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy^{-1})t_{31},t_{11},t_{41},t_{110}$

then $Nt_1,(t_5)$

$= Nt_1,(t_{31}),t_7$

$= Nt_{110},t_{59},(t_{41},t_7)$

$= Nt_{40},t_{70},(t_{83})$

$= Nt_{31},t_{11},t_{41},t_{110}$.

Using the same relation conjugated by

$(y^{-1}xy^{-1}xy^{-1}xyxyx)$ then we have

$(y^{-1}xy^{-1}xy^{-1}xyxyx)t_{71},t_{11} = t_{110}$

then we have

$(y^{-1}xy^{-1}xyxy^{-1}x)t_{92},t_{47},t_{101},t_{71},t_{11}$

next $Nt_1,(t_5)$

$= Nt_1,(t_{31}),t_7$

$= Nt_{110},t_{59},(t_{41},t_7)$

$= Nt_{40},t_{70},(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},t_{47},t_{101},t_{71},t_{11}$.

Then by using the relation $t_1 = t_4$ conjugated by

$((yxyxy^{-1}x)^2)$ we have $t_{47} = t_{28}$

and by using the same relation conjugated by $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)$ we have $t_{101} = t_{63}$

then we have $Nt_1,(t_5)$

$= Nt_1,(t_{31}),t_7$

$= Nt_{110},t_{59},(t_{41},t_7)$

$= Nt_{40},t_{70},(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}$

$= Nt_{92},t_{28},t_{63},t_{71},t_{11}$.

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25}, = Id$

conjugated by

$(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)$

then we have

$(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)t_{29}$,$t_{46} = t_{28}$

then we have

$(xyxy^{-1}xy^{-1}xyxyxy)t_{90}$,t_{29} ,t_{46},t_{63},t_{71},t_{11}

next Nt_1,(t_5)

= Nt_1,(t_{31}),t_7

= Nt_{110},t_{59} ,(t_{41} ,t_7)

= Nt_{40} ,t_{70} ,(t_{83})

= Nt_{31},t_{11},t_{41},(t_{110})

= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}

= Nt_{92},(t_{28}),t_{63},t_{71},t_{11}

= Nt_{90} ,t_{29} ,t_{46},t_{63},t_{71},t_{11} .

Next by using the same relation conjugated by

$(yxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)$

we have $(yxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_{46}$,$t_{63}$,$t_{95}$ = Id

then we have $(xy^{-1}xyxyxy^{-1}xy^{-1}xy)t_{24}$,t_{59} ,t_{95},t_{71},t_{11}

next Nt_1,(t_5)

= Nt_1,(t_{31}),t_7

= Nt_{110},t_{59} ,(t_{41} ,t_7)

= Nt_{40} ,t_{70} ,(t_{83})

= Nt_{31},t_{11},t_{41},(t_{110})

= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}

= Nt_{92},(t_{28}),t_{63},t_{71},t_{11}

= Nt_{90} ,t_{29} ,(t_{46},t_{63}),t_{71},t_{11}

= Nt_{24} ,t_{59} ,t_{95},t_{71},t_{11} .

Next by using the relation $t_1 = t_4$ conjugated by

$(yxyxyxyxy^{-1}xyxy)$ we have $t_{59} = t_{13}$

then we have Nt_1,(t_5)

= Nt_1,(t_{31}),t_7

= Nt_{110},t_{59} ,(t_{41} ,t_7)

= Nt_{40} ,t_{70} ,(t_{83})

= Nt_{31},t_{11},t_{41},(t_{110})

= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}

$= Nt_{92},(t_{28}),t_{63},t_{71},t_{11}$

$= Nt_{90},t_{29},(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24},(t_{59}),t_{95},t_{71},t_{11}$

$= Nt_{24},t_{13},t_{95},t_{71},t_{11}$.

Next by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25}, = Id$

conjugated by $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy)$

then we have $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy)t_{84} = t_{13},t_{95}$

then $(y^{-1}xyxyxyxy^{-1}xy^{-1})t_{102},t_{84},t_{71},t_{11}$

next $Nt_1,(t_5)$

$= Nt_1,(t_{31}),t_7$

$= Nt_{110},t_{59},(t_{41},t_7)$

$= Nt_{40},t_{70},(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}),t_{63},t_{71},t_{11}$

$= Nt_{90},t_{29},(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24},(t_{59}),t_{95},t_{71},t_{11}$

$= Nt_{24},(t_{13},t_{95}),t_{71},t_{11}$

$= Nt_{102},t_{84},t_{71},t_{11}$.

By using the same relation conjugated by

$(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy)$ we have $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy)t_{100},t_{81} = t_{84}$

then we have $(xy^{-1}xy^{-1}xy^{-1}xyxyxy)t_{28},t_{100},t_{81},t_{71},t_{11}$

next $Nt_1,(t_5)$

$= Nt_1,(t_{31}),t_7$

$= Nt_{110},t_{59},(t_{41},t_7)$

$= Nt_{40},t_{70},(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}),t_{63},t_{71},t_{11}$

$= Nt_{90},t_{29},(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24},(t_{59}),t_{95},t_{71},t_{11}$

$= Nt_{24}, (t_{13}, t_{95}), t_{71}, t_{11}$

$= Nt_{102}, (t_{84}), t_{71}, t_{11}$

$= Nt_{28}, t_{100}, t_{81}, t_{71}, t_{11}$.

By using the same relation conjugated by

$(xy^{-1}xy^{-1}xyxyxy^{-1})$ gives

$(xy^{-1}xy^{-1}xyxyxy^{-1})t_{31} = t_{81}, t_{71}$

then we have $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_5, t_{17}, t_{31}, t_{11}$

next $Nt_1, (t_5)$

$= Nt_1, (t_{31}), t_7$

$= Nt_{110}, t_{59}, (t_{41}, t_7)$

$= Nt_{40}, t_{70}, (t_{83})$

$= Nt_{31}, t_{11}, t_{41}, (t_{110})$

$= Nt_{92}, (t_{47}), (t_{101}), t_{71}, t_{11}$

$= Nt_{92}, (t_{28}), t_{63}, t_{71}, t_{11}$

$= Nt_{90}, t_{29}, (t_{46}, t_{63}), t_{71}, t_{11}$

$= Nt_{24}, (t_{59}), t_{95}, t_{71}, t_{11}$

$= Nt_{24}, (t_{13}, t_{95}), t_{71}, t_{11}$

$= Nt_{102}, (t_{84}), t_{71}, t_{11}$

$= Nt_{28}, t_{100}, (t_{81}, t_{71}), t_{11}$

$= Nt_5, t_{17}, t_{31}, t_{11}$.

Next by using the relation $t_1 = t_4$ conjugated by

$(y^{-1}xy^{-1}xyxyxy^{-1}xyxy^{-1})$ gives $t_5 = t_{31}$

then we have $Nt_1, (t_5)$

$= Nt_1, (t_{31}), t_7$

$= Nt_{110}, t_{59}, (t_{41}, t_7)$

$= Nt_{40}, t_{70}, (t_{83})$

$= Nt_{31}, t_{11}, t_{41}, (t_{110})$

$= Nt_{92}, (t_{47}), (t_{101}), t_{71}, t_{11}$

$= Nt_{92}, (t_{28}), t_{63}, t_{71}, t_{11}$

$= Nt_{90}, t_{29}, (t_{46}, t_{63}), t_{71}, t_{11}$

$= Nt_{24}, (t_{59}), t_{95}, t_{71}, t_{11}$

$= Nt_{24}, (t_{13}, t_{95}), t_{71}, t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28}\ ,t_{100}\ ,(t_{81}\ ,t_{71}),t_{11}$

$= N(t_5),t_{17}\ ,t_{31}\ ,t_{11}$

$= Nt_{31}\ ,t_{17}\ ,t_{31}\ ,t_{11}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}\ ,t_{25}, = Id$

conjugated by $(xy^{-1}xy^{-1}xyxyxy^{-1})$

we have $(xy^{-1}xy^{-1}xyxyxy^{-1})t_{47}\ ,t_{79} = t_{31}$

then we have

$(yxyxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{47}\ ,t_{79}\ ,t_{17}\ ,t_{31}\ ,t_{11}$

next $Nt_1,(t_5)$

$= Nt_1,(t_{31}\ ),t_7$

$= Nt_{110},t_{59}\ ,(t_{41}\ ,t_7)$

$= Nt_{40}\ ,t_{70}\ ,(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}\ ),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}\ ),t_{63},t_{71},t_{11}$

$= Nt_{90}\ ,t_{29}\ ,(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24}\ ,(t_{59}\ ),t_{95},t_{71},t_{11}$

$= Nt_{24}\ ,(t_{13}\ ,t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28}\ ,t_{100}\ ,(t_{81}\ ,t_{71}),t_{11}$

$= N(t_5),t_{17}\ ,t_{31}\ ,t_{11}$

$= N(t_{31}\ ),t_{17}\ ,t_{31}\ ,t_{11}$

$= Nt_{47}\ ,t_{79}\ ,t_{17}\ ,t_{31}\ ,t_{11}$.

By using the same relation conjugated by

$(yxyxyxy^{-1}xy^{-1}xy)$ then that gives

$(yxyxyxy^{-1}xy^{-1}xy)t_{39} = t_{79}\ ,t_{17}$

then we have $(yxyxyxy^{-1}xy)t_{83},t_{39}\ ,t_{31}\ ,t_{11}$

next $Nt_1,(t_5)$

$= Nt_1,(t_{31}\ ),t_7$

$= Nt_{110},t_{59}\ ,(t_{41}\ ,t_7)$

$= Nt_{40}, t_{70}, (t_{83})$

$= Nt_{31}, t_{11}, t_{41}, (t_{110})$

$= Nt_{92}, (t_{47}), (t_{101}), t_{71}, t_{11}$

$= Nt_{92}, (t_{28}), t_{63}, t_{71}, t_{11}$

$= Nt_{90}, t_{29}, (t_{46}, t_{63}), t_{71}, t_{11}$

$= Nt_{24}, (t_{59}), t_{95}, t_{71}, t_{11}$

$= Nt_{24}, (t_{13}, t_{95}), t_{71}, t_{11}$

$= Nt_{102}, (t_{84}), t_{71}, t_{11}$

$= Nt_{28}, t_{100}, (t_{81}, t_{71}), t_{11}$

$= N(t_5), t_{17}, t_{31}, t_{11}$

$= N(t_{31}), t_{17}, t_{31}, t_{11}$

$= Nt_{47}, (t_{79}, t_{17}), t_{31}, t_{11}$

$= Nt_{83}, t_{39}, t_{31}, t_{11}$.

Using the relation $t_1 = t_4$ conjugated by $(xy^{-1}xyxy^{-1}xyxyxyx)$ we get $t_{39} = t_{99}$ then we have $Nt_1, (t_5)$

$= Nt_1, (t_{31}), t_7$

$= Nt_{110}, t_{59}, (t_{41}, t_7)$

$= Nt_{40}, t_{70}, (t_{83})$

$= Nt_{31}, t_{11}, t_{41}, (t_{110})$

$= Nt_{92}, (t_{47}), (t_{101}), t_{71}, t_{11}$

$= Nt_{92}, (t_{28}), t_{63}, t_{71}, t_{11}$

$= Nt_{90}, t_{29}, (t_{46}, t_{63}), t_{71}, t_{11}$

$= Nt_{24}, (t_{59}), t_{95}, t_{71}, t_{11}$

$= Nt_{24}, (t_{13}, t_{95}), t_{71}, t_{11}$

$= Nt_{102}, (t_{84}), t_{71}, t_{11}$

$= Nt_{28}, t_{100}, (t_{81}, t_{71}), t_{11}$

$= N(t_5), t_{17}, t_{31}, t_{11}$

$= N(t_{31}), t_{17}, t_{31}, t_{11}$

$= Nt_{47}, (t_{79}, t_{17}), t_{31}, t_{11}$

$= Nt_{83}, (t_{39}), t_{31}, t_{11}$

$= Nt_{83}, t_{99}, t_{31}, t_{11}$.

Next by using the relation
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25}, = Id$
conjugated by $(yxyxyxyxyxy^{-1}xyx)$ gives
$(yxyxyxyxyxy^{-1}xyx)t_{88},t_{82},t_{99}$
then we have
$(yxy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy)t_{49},t_{88},t_{82},t_{31},t_{11}$
then we have $Nt_1,(t_5)$
$= Nt_1,(t_{31}),t_7$
$= Nt_{110},t_{59},(t_{41},t_7)$
$= Nt_{40},t_{70},(t_{83})$
$= Nt_{31},t_{11},t_{41},(t_{110})$
$= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}$
$= Nt_{92},(t_{28}),t_{63},t_{71},t_{11}$
$= Nt_{90},t_{29},(t_{46},t_{63}),t_{71},t_{11}$
$= Nt_{24},(t_{59}),t_{95},t_{71},t_{11}$
$= Nt_{24},(t_{13},t_{95}),t_{71},t_{11}$
$= Nt_{102},(t_{84}),t_{71},t_{11}$
$= Nt_{28},t_{100},(t_{81},t_{71}),t_{11}$
$= N(t_5),t_{17},t_{31},t_{11} = N(t_{31}),t_{17},t_{31},t_{11}$
$= Nt_{47},(t_{79},t_{17}),t_{31},t_{11}$
$= Nt_{83},(t_{39}),t_{31},t_{11}$
$= Nt_{83},(t_{99}),t_{31},t_{11}$
$= Nt_{49},t_{88},t_{82},t_{31},t_{11}$.
By using the same relation conjugated by
$(y^{-1}xy^{-1}xy^{-1})$ gives $(y^{-1}xy^{-1}xy^{-1})t_{25}, = t_{49},t_{88}$
then we have
$(xy^{-1}xyxy^{-1}xyxyx)t_{25},,t_{82},t_{31},t_{11}$
next $Nt_1,(t_5)$
$= Nt_1,(t_{31}),t_7$
$= Nt_{110},t_{59},(t_{41},t_7)$
$= Nt_{40},t_{70},(t_{83})$
$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}),t_{63},t_{71},t_{11}$

$= Nt_{90},t_{29},(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24},(t_{59}),t_{95},t_{71},t_{11}$

$= Nt_{24},(t_{13},t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28},t_{100},(t_{81},t_{71}),t_{11}$

$= N(t_{5}),t_{17},t_{31},t_{11}$

$= N(t_{31}),t_{17},t_{31},t_{11}$

$= Nt_{47},(t_{79},t_{17}),t_{31},t_{11}$

$= Nt_{83},(t_{39}),t_{31},t_{11}$

$= Nt_{83},(t_{99}),t_{31},t_{11}$

$= N(t_{49},t_{88}),t_{82},t_{31},t_{11}$

$= Nt_{25},,t_{82},t_{31},t_{11}$.

Using the relation $t_1 = t_4$ conjugated by $(xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx)$ gives $t_{82} = t_{100}$ then we have $Nt_1,(t_5)$

$= Nt_1,(t_{31}),t_7 = Nt_{110},t_{59},(t_{41},t_7)$

$= Nt_{40},t_{70},(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}),t_{63},t_{71},t_{11}$

$= Nt_{90},t_{29},(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24},(t_{59}),t_{95},t_{71},t_{11}$

$= Nt_{24},(t_{13},t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28},t_{100},(t_{81},t_{71}),t_{11}$

$= N(t_{5}),t_{17},t_{31},t_{11}$

$= N(t_{31}),t_{17},t_{31},t_{11}$

$= Nt_{47},(t_{79},t_{17}),t_{31},t_{11}$

$= Nt_{83},(t_{39}),t_{31},t_{11}$

$= Nt_{83},(t_{99}),t_{31},t_{11}$

$= N(t_{49},t_{88}),t_{82},t_{31},t_{11}$

$= Nt_{25},,(t_{82}),t_{31},t_{11}$

$= Nt_{25},,t_{100},t_{31},t_{11}$.

Then by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^{3})t_{49},t_{105},t_{25}, = Id$

conjugated by $(xy^{-1}xyxy^{-1}xy^{-1}xyxy^{-1}x)$

gives $(xy^{-1}xyxy^{-1}xy^{-1}xyxy^{-1}x)t_{92},t_{85} = t_{100}$

then we have

$(yxy^{-1}xyxyxyxy^{-1}xy)t_{66},,t_{92},t_{85},t_{31},t_{11}$

next $Nt_{1},(t_{5})$

$= Nt_{1},(t_{31}),t_{7}$

$= Nt_{110},t_{59},(t_{41},t_{7})$

$= Nt_{40},t_{70},(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}),t_{63},t_{71},t_{11}$

$= Nt_{90},t_{29},(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24},(t_{59}),t_{95},t_{71},t_{11}$

$= Nt_{24},(t_{13},t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28},t_{100},(t_{81},t_{71}),t_{11}$

$= N(t_{5}),t_{17},t_{31},t_{11}$

$= N(t_{31}),t_{17},t_{31},t_{11}$

$= Nt_{47},(t_{79},t_{17}),t_{31},t_{11}$

$= Nt_{83},(t_{39}),t_{31},t_{11}$

$= Nt_{83},(t_{99}),t_{31},t_{11}$

$= N(t_{49},t_{88}),t_{82},t_{31},t_{11}$

$= Nt_{25},,(t_{82}),t_{31},t_{11}$

$= Nt_{25},,(t_{100}),t_{31},t_{11}$

$= Nt_{66},,t_{92},t_{85},t_{31},t_{11}$.

By using the same relation conjugated by

$(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)$ we have

$(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)t_{23} = t_{85},t_{31}$

then we have $(y^{-1}xyxy^{-1}xyxyxy)t_{103},,t_7,t_{23}\ ,t_{11}$

next $Nt_1,(t_5)$

$= Nt_1,(t_{31}\ ),t_7$

$= Nt_{110},t_{59}\ ,(t_{41}\ ,t_7)$

$= Nt_{40}\ ,t_{70}\ ,(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}\ ),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}\ ),t_{63},t_{71},t_{11}$

$= Nt_{90}\ ,t_{29}\ ,(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24}\ ,(t_{59}\ ),t_{95},t_{71},t_{11}$

$= Nt_{24}\ ,(t_{13}\ ,t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28}\ ,t_{100}\ ,(t_{81}\ ,t_{71}),t_{11}$

$= N(t_5),t_{17}\ ,t_{31}\ ,t_{11}$

$= N(t_{31}\ ),t_{17}\ ,t_{31}\ ,t_{11}$

$= Nt_{47}\ ,(t_{79}\ ,t_{17}\ ),t_{31}\ ,t_{11}$

$= Nt_{83},(t_{39}\ ),t_{31}\ ,t_{11}$

$= Nt_{83},(t_{99}\ ),t_{31}\ ,t_{11}$

$= N(t_{49},t_{88}),t_{82}\ ,t_{31}\ ,t_{11}$

$= Nt_{25},,(t_{82}\ ),t_{31}\ ,t_{11}$

$= Nt_{25},,(t_{100}\ ),t_{31}\ ,t_{11}$

$= Nt_{66},,t_{92},(t_{85}\ ,t_{31}\ ),t_{11}$

$= Nt_{103},,t_7,t_{23}\ ,t_{11}$.

Then by using the relation $t_1 = t_4$ conjugated by $(xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)$

we have $t_{103}, = t_{110}$

then we have $Nt_1,(t_5)$

$= Nt_1,(t_{31}\ ),t_7$

$= Nt_{110},t_{59}\ ,(t_{41}\ ,t_7)$

$= Nt_{40}\ ,t_{70}\ ,(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}\ ),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}\ ),t_{63},t_{71},t_{11}$

$= Nt_{90}$ $,t_{29}$ $,(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24}$ $,(t_{59}$ $),t_{95},t_{71},t_{11}$

$= Nt_{24}$ $,(t_{13}$ $,t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28}$ $,t_{100}$ $,(t_{81}$ $,t_{71}),t_{11}$

$= N(t_{5}),t_{17}$ $,t_{31}$ $,t_{11}$

$= N(t_{31}$ $),t_{17}$ $,t_{31}$ $,t_{11}$

$= Nt_{47}$ $,(t_{79}$ $,t_{17}$ $),t_{31}$ $,t_{11}$

$= Nt_{83},(t_{39}$ $),t_{31}$ $,t_{11}$

$= Nt_{83},(t_{99}$ $),t_{31}$ $,t_{11}$

$= N(t_{49},t_{88}),t_{82}$ $,t_{31}$ $,t_{11}$

$= Nt_{25},,(t_{82}$ $),t_{31}$ $,t_{11}$

$= Nt_{25},,(t_{100}$ $),t_{31}$ $,t_{11}$

$= Nt_{66},,t_{92},(t_{85}$ $,t_{31}$ $),t_{11}$

$= N(t_{103},),t_{7},t_{23}$ $,t_{11}$

$= Nt_{110},t_{7},t_{23}$ $,t_{11}$.

Next using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy^{*}x)^{3})t_{49},t_{105}$ $,t_{25},= Id$

conjugated by $(y^{-1}xyxy^{-1}xy^{-1}xyxy^{-1})$ we have

$(y^{-1}xyxy^{-1}xy^{-1}xyxy^{-1})t_{62},t_{41} = t_{110}$

then we have $(yxyxy^{-1}xyxyx)t_{62},t_{41}$ $,t_{7},t_{23}$ $,t_{11}$

next $Nt_{1},(t_{5})$

$= Nt_{1},(t_{31}$ $),t_{7}$

$= Nt_{110},t_{59}$ $,(t_{41}$ $,t_{7})$

$= Nt_{40}$ $,t_{70}$ $,(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}$ $),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}$ $),t_{63},t_{71},t_{11}$

$= Nt_{90}$ $,t_{29}$ $,(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24}$ $,(t_{59}$ $),t_{95},t_{71},t_{11}$

$= Nt_{24}$ $,(t_{13}$ $,t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28}\ ,t_{100}\ ,(t_{81}\ ,t_{71}),t_{11}$

$= N(t_5),t_{17}\ ,t_{31}\ ,t_{11}$

$= N(t_{31}\ ),t_{17}\ ,t_{31}\ ,t_{11}$

$= Nt_{47}\ ,(t_{79}\ ,t_{17}\ ),t_{31}\ ,t_{11}$

$= Nt_{83},(t_{39}\ ),t_{31}\ ,t_{11}$

$= Nt_{83},(t_{99}\ ),t_{31}\ ,t_{11}$

$= N(t_{49},t_{88}),t_{82}\ ,t_{31}\ ,t_{11}$

$= Nt_{25},,(t_{82}\ ),t_{31}\ ,t_{11}$

$= Nt_{25},,(t_{100}\ ),t_{31}\ ,t_{11}$

$= Nt_{66},,t_{92},(t_{85}\ ,t_{31}\ ),t_{11}$

$= N(t_{103},),t_7,t_{23}\ ,t_{11}$

$= N(t_{110}),t_7,t_{23}\ ,t_{11}$

$= Nt_{62},t_{41}\ ,t_7,t_{23}\ ,t_{11}$.

Using the same relation conjugated by

$(yxy^{-1}xyxyxyxy)$

gives $(yxy^{-1}xyxyxyxy)t_{83} = t_{41}\ ,t_7$

then we have $(y^{-1}xyxy^{-1}xy^{-1})t_{37},t_{83},t_{23}\ ,t_{11}$

next $Nt_1,(t_5)$

$= Nt_1,(t_{31}\ ),t_7$

$= Nt_{110},t_{59}\ ,(t_{41}\ ,t_7)$

$= Nt_{40}\ ,t_{70}\ ,(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}\ ),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}\ ),t_{63},t_{71},t_{11}$

$= Nt_{90}\ ,t_{29}\ ,(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24}\ ,(t_{59}\ ),t_{95},t_{71},t_{11}$

$= Nt_{24}\ ,(t_{13}\ ,t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28}\ ,t_{100}\ ,(t_{81}\ ,t_{71}),t_{11}$

$= N(t_5),t_{17}\ ,t_{31}\ ,t_{11}$

$= N(t_{31}\ ),t_{17}\ ,t_{31}\ ,t_{11}$

$= Nt_{47}\ ,(t_{79}\ ,t_{17}\ ),t_{31}\ ,t_{11}$

$= Nt_{83},(t_{39}),t_{31},t_{11}$

$= Nt_{83},(t_{99}),t_{31},t_{11}$

$= N(t_{49},t_{88}),t_{82},t_{31},t_{11}$

$= Nt_{25},,(t_{82}),t_{31},t_{11}$

$= Nt_{25},,(t_{100}),t_{31},t_{11}$

$= Nt_{66},,t_{92},(t_{85},t_{31}),t_{11}$

$= N(t_{103},),t_7,t_{23},t_{11}$

$= N(t_{110}),t_7,t_{23},t_{11}$

$= Nt_{62},(t_{41},t_7),t_{23},t_{11}$

$= Nt_{37},t_{83},t_{23},t_{11}$.

By using the same relation conjugated by

$(y^{-1}xyxy^{-1}xy^{-1}xyxy^{-1})$ gives

$(y^{-1}xyxy^{-1}xy^{-1}xyxy^{-1})t_{110} = t_{37},t_{83}$

then we have $(yxyxyxy^{-1}xy^{-1}xyxy)t_{110},t_{23},t_{11}$

next $Nt_1,(t_5)$

$= Nt_1,(t_{31}),t_7$

$= Nt_{110},t_{59},(t_{41},t_7)$

$= Nt_{40},t_{70},(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}),t_{63},t_{71},t_{11}$

$= Nt_{90},t_{29},(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24},(t_{59}),t_{95},t_{71},t_{11}$

$= Nt_{24},(t_{13},t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28},t_{100},(t_{81},t_{71}),t_{11}$

$= N(t_5),t_{17},t_{31},t_{11}$

$= N(t_{31}),t_{17},t_{31},t_{11}$

$= Nt_{47},(t_{79},t_{17}),t_{31},t_{11}$

$= Nt_{83},(t_{39}),t_{31},t_{11}$

$= Nt_{83},(t_{99}),t_{31},t_{11}$

$= N(t_{49},t_{88}),t_{82},t_{31},t_{11}$

$= Nt_{25},,(t_{82}\ ),t_{31}\ ,t_{11}$

$= Nt_{25},,(t_{100}\ ),t_{31}\ ,t_{11}$

$= Nt_{66},,t_{92},(t_{85}\ ,t_{31}\ ),t_{11}$

$= N(t_{103},),t_{7},t_{23}\ ,t_{11}$

$= N(t_{110}),t_{7},t_{23}\ ,t_{11}$

$= Nt_{62},(t_{41}\ ,t_{7}),t_{23}\ ,t_{11}$

$= N(t_{37},t_{83}),t_{23}\ ,t_{11}$

$= Nt_{110},t_{23}\ ,t_{11}$.

Next using the same relation conjugated by

$(y^{-1}xyxy^{-1}xy^{-1}xyxy^{-1})$ gives

$(y^{-1}xyxy^{-1}xy^{-1}xyxy^{-1})t_{62},t_{41} = t_{110}$

then we have

$(xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy)t_{62},t_{41}\ ,t_{23}\ ,t_{11}$

next $Nt_{1},(t_{5})$

$= Nt_{1},(t_{31}\ ),t_{7}$

$= Nt_{110},t_{59}\ ,(t_{41}\ ,t_{7})$

$= Nt_{40}\ ,t_{70}\ ,(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}\ ),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}\ ),t_{63},t_{71},t_{11}$

$= Nt_{90}\ ,t_{29}\ ,(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24}\ ,(t_{59}\ ),t_{95},t_{71},t_{11}$

$= Nt_{24}\ ,(t_{13}\ ,t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28}\ ,t_{100}\ ,(t_{81}\ ,t_{71}),t_{11}$

$= N(t_{5}),t_{17}\ ,t_{31}\ ,t_{11}$

$= N(t_{31}\ ),t_{17}\ ,t_{31}\ ,t_{11}$

$= Nt_{47}\ ,(t_{79}\ ,t_{17}\ ),t_{31}\ ,t_{11}$

$= Nt_{83},(t_{39}\ ),t_{31}\ ,t_{11}$

$= Nt_{83},(t_{99}\ ),t_{31}\ ,t_{11}$

$= N(t_{49},t_{88}),t_{82}\ ,t_{31}\ ,t_{11}$

$= Nt_{25},,(t_{82}\ ),t_{31}\ ,t_{11}$

$= Nt_{25},,(t_{100}),t_{31},t_{11}$

$= Nt_{66},,t_{92},(t_{85},t_{31}),t_{11}$

$= N(t_{103},),t_7,t_{23},t_{11}$

$= N(t_{110}),t_7,t_{23},t_{11}$

$= Nt_{62},(t_{41},t_7),t_{23},t_{11}$

$= N(t_{37},t_{83}),t_{23},t_{11}$

$= N(t_{110}),t_{23},t_{11}$

$= Nt_{62},t_{41},t_{23},t_{11}$.

Using the relation $t_1 = t_4$ conjugated by

$(yxy^{-1}xy^{-1}xy^{-1}xyxy^{-1})$ gives $t_{23} = t_{73}$

then we have $Nt_1,(t_5)$

$= Nt_1,(t_{31}),t_7$

$= Nt_{110},t_{59},(t_{41},t_7)$

$= Nt_{40},t_{70},(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}),t_{63},t_{71},t_{11}$

$= Nt_{90},t_{29},(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24},(t_{59}),t_{95},t_{71},t_{11}$

$= Nt_{24},(t_{13},t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28},t_{100},(t_{81},t_{71}),t_{11}$

$= N(t_5),t_{17},t_{31},t_{11}$

$= N(t_{31}),t_{17},t_{31},t_{11}$

$= Nt_{47},(t_{79},t_{17}),t_{31},t_{11}$

$= Nt_{83},(t_{39}),t_{31},t_{11}$

$= Nt_{83},(t_{99}),t_{31},t_{11}$

$= N(t_{49},t_{88}),t_{82},t_{31},t_{11}$

$= Nt_{25},,(t_{82}),t_{31},t_{11}$

$= Nt_{25},,(t_{100}),t_{31},t_{11}$

$= Nt_{66},,t_{92},(t_{85},t_{31}),t_{11}$

$= N(t_{103},),t_7,t_{23},t_{11}$

$= N(t_{110}),t_7,t_{23} ,t_{11}$

$= Nt_{62},(t_{41} ,t_7),t_{23} ,t_{11}$

$= N(t_{37},t_{83}),t_{23} ,t_{11}$

$= N(t_{110}),t_{23} ,t_{11}$

$= Nt_{62},t_{41} ,(t_{23}),t_{11}$

$= Nt_{62},t_{41} ,t_{73},t_{11}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105} ,t_{25}, = Id$

conjugated by $(yxyxy^{-1}xyxyxyxyxy^{-1})$ gives

$(yxyxy^{-1}xyxyxyxyxy^{-1})t_{41} ,t_{73},t_{10} = Id$

then we have

$(xyxy^{-1}xyxy^{-1}xy^{-1}x)t_{27} ,t_{10} ,t_{11}$

next $Nt_1,(t_5)$

$= Nt_1,(t_{31}),t_7$

$= Nt_{110},t_{59} ,(t_{41} ,t_7)$

$= Nt_{40} ,t_{70} ,(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}),t_{63},t_{71},t_{11}$

$= Nt_{90} ,t_{29} ,(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24} ,(t_{59}),t_{95},t_{71},t_{11}$

$= Nt_{24} ,(t_{13} ,t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28} ,t_{100} ,(t_{81} ,t_{71}),t_{11}$

$= N(t_5),t_{17} ,t_{31} ,t_{11}$

$= N(t_{31}),t_{17} ,t_{31} ,t_{11}$

$= Nt_{47} ,(t_{79} ,t_{17}),t_{31} ,t_{11}$

$= Nt_{83},(t_{39}),t_{31} ,t_{11}$

$= Nt_{83},(t_{99}),t_{31} ,t_{11}$

$= N(t_{49},t_{88}),t_{82} ,t_{31} ,t_{11}$

$= Nt_{25},,(t_{82}),t_{31} ,t_{11}$

$= Nt_{25},(t_{100}),t_{31} ,t_{11}$

$= Nt_{66},,t_{92},(t_{85} ,t_{31}),t_{11}$

$= N(t_{103},),t_7,t_{23} ,t_{11}$

$= N(t_{110}),t_7,t_{23} ,t_{11}$

$= Nt_{62},(t_{41} ,t_7),t_{23} ,t_{11}$

$= N(t_{37},t_{83}),t_{23} ,t_{11}$

$= N(t_{110}),t_{23} ,t_{11}$

$= Nt_{62},t_{41} ,(t_{23}),t_{11}$

$= Nt_{62},(t_{41} ,t_{73}),t_{11}$

$= Nt_{27} ,t_{10} ,t_{11}$.

Using the relation $t_1 = t_4$ conjugated by

$(yxy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)$ gives $t_{10} = t_{36}$

then we have $Nt_1,(t_5)$

$= Nt_1,(t_{31}),t_7$

$= Nt_{110},t_{59} ,(t_{41} ,t_7)$

$= Nt_{40} ,t_{70} ,(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}),t_{63},t_{71},t_{11}$

$= Nt_{90} ,t_{29} ,(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24} ,(t_{59}),t_{95},t_{71},t_{11}$

$= Nt_{24} ,(t_{13} ,t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28} ,t_{100} ,(t_{81} ,t_{71}),t_{11}$

$= N(t_5),t_{17} ,t_{31} ,t_{11}$

$= N(t_{31}),t_{17} ,t_{31} ,t_{11}$

$= Nt_{47} ,(t_{79} ,t_{17}),t_{31} ,t_{11}$

$= Nt_{83},(t_{39}),t_{31} ,t_{11}$

$= Nt_{83},(t_{99}),t_{31} ,t_{11}$

$= N(t_{49},t_{88}),t_{82} ,t_{31} ,t_{11}$

$= Nt_{25},,(t_{82}),t_{31} ,t_{11}$

$= Nt_{25},,(t_{100}),t_{31} ,t_{11}$

$= Nt_{66},,t_{92},(t_{85} ,t_{31}),t_{11}$

$= N(t_{103},),t_7,t_{23}\ ,t_{11}$

$= N(t_{110}),t_7,t_{23}\ ,t_{11}$

$= Nt_{62},(t_{41}\ ,t_7),t_{23}\ ,t_{11}$

$= N(t_{37},t_{83}),t_{23}\ ,t_{11}$

$= N(t_{110}),t_{23}\ ,t_{11}$

$= Nt_{62},t_{41}\ ,(t_{23}\ ),t_{11}$

$= Nt_{62},(t_{41}\ ,t_{73}),t_{11}$

$= Nt_{27}\ ,(t_{10}\ ),t_{11}$

$= Nt_{27}\ ,t_{36},t_{11}$.

Next using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25}, = Id$

conjugated by $(xyxyxyxyxy^{-1}xy^{-1}xyxy)$ gives

$(xyxyxyxyxy^{-1}xy^{-1}xyxy)t_{27}\ ,t_{36},t_9 = Id$

next we have $(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_9,t_{11}$

next we have $Nt_1,(t_5)$

$= Nt_1,(t_{31}\ ),t_7$

$= Nt_{110},t_{59}\ ,(t_{41}\ ,t_7)$

$= Nt_{40}\ ,t_{70}\ ,(t_{83})$

$= Nt_{31},t_{11},t_{41},(t_{110})$

$= Nt_{92},(t_{47}\ ),(t_{101}),t_{71},t_{11}$

$= Nt_{92},(t_{28}\ ),t_{63},t_{71},t_{11}$

$= Nt_{90}\ ,t_{29}\ ,(t_{46},t_{63}),t_{71},t_{11}$

$= Nt_{24}\ ,(t_{59}\ ),t_{95},t_{71},t_{11}$

$= Nt_{24}\ ,(t_{13}\ ,t_{95}),t_{71},t_{11}$

$= Nt_{102},(t_{84}),t_{71},t_{11}$

$= Nt_{28}\ ,t_{100}\ ,(t_{81}\ ,t_{71}),t_{11}$

$= N(t_5),t_{17}\ ,t_{31}\ ,t_{11}$

$= N(t_{31}\ ),t_{17}\ ,t_{31}\ ,t_{11}$

$= Nt_{47}\ ,(t_{79}\ ,t_{17}\ ),t_{31}\ ,t_{11}$

$= Nt_{83},(t_{39}\ ),t_{31}\ ,t_{11}$

$= Nt_{83},(t_{99}\ ),t_{31}\ ,t_{11}$

$= N(t_{49},t_{88}),t_{82}\ ,t_{31}\ ,t_{11}$

$= Nt_{25},,(t_{82}),t_{31},t_{11}$

$= Nt_{25},,(t_{100}),t_{31},t_{11}$

$= Nt_{66},,t_{92},(t_{85},t_{31}),t_{11}$

$= N(t_{103},),t_7,t_{23},t_{11}$

$= N(t_{110}),t_7,t_{23},t_{11}$

$= Nt_{62},(t_{41},t_7),t_{23},t_{11}$

$= N(t_{37},t_{83}),t_{23},t_{11}$

$= N(t_{110}),t_{23},t_{11}$

$= Nt_{62},t_{41},(t_{23}),t_{11}$

$= Nt_{62},(t_{41},t_{73}),t_{11}$

$= Nt_{27},(t_{10}),t_{11}$

$= N(t_{27},t_{36}),t_{11}$

$= Nt_9,t_{11}$

$= N(t_1,t_5)^{(y^{-1}xy^{-1}xyxy^{-1}xyx)} \epsilon [1,5]$

Choose 16 from {16, 45, 33, 98, 44, 52, 49, 21, 83, 97}

Using the relation $t_1 = t_4$ conjugated by

$(y^{-1}xy^{-1}xyxyxy^{-1}xyxy^{-1})$ gives $t_5 = t_{31}$

then we have $Nt_1,(t_5),t_{16}$

$= Nt_1,t_{31},t_{16}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(xy^{-1}xyxyxyxyx)$ gives

$(xy^{-1}xyxyxyxyx)t_{81},t_{103} = t_{31}$

then we have $(xy^{-1}xyxyxyxyx)t_{61},t_{81},t_{103},t_{16}$

next $Nt_1,(t_5),t_{16}$

$= Nt_1,(t_{31}),t_{16}$

$= Nt_{61},t_{81},t_{103},t_{16}$.

Using the same relation conjugated by

$(xyxy^{-1}xyxyxyxyxy^{-1}xy^{-1})$ gives

$(xyxy^{-1}xyxyxyxyxy^{-1}xy^{-1})t_{103},t_{16},t_7 = Id$

then we have $(yxy^{-1}xy^{-1})t_{89},t_{18},t_7$

next $Nt_1,(t_5),t_{16}$

$= Nt_1,(t_{31}),t_{16}$

$= Nt_{61},t_{81},(t_{103},t_{16})$

$= Nt_{89},t_{18},t_7$.

Using the same relation conjugated by

$(y^{-1}xy^{-1}xyxyxyxyxy^{-1}x)$ gives

$(y^{-1}xy^{-1}xyxyxyxyxy^{-1}x)t_{26},t_{49} = t_7$

then we have $(xy^{-1}xy^{-1}xy^{-1}xyxyxy^{-1}xy)t_{41},t_{74},t_{26},t_{49}$

next $Nt_1,(t_5),t_{16}$

$= Nt_1,(t_{31}),t_{16}$

$= Nt_{61},t_{81},(t_{103},t_{16})$

$= Nt_{89},t_{18},(t_7)$

$= Nt_{41},t_{74},t_{26},t_{49}$.

Again using the same relation conjugated by

$((yxy^{-1}xy^{-1}x)^2)$ gives

$((yxy^{-1}xy^{-1}x)^2)t_{74},t_{26},t_{34} = Id$

then we have $(xyxyxyxyxy^{-1}xyx)t_{60},t_{34},t_{49}$

next $Nt_1,(t_5),t_{16}$

$= Nt_1,(t_{31}),t_{16}$

$= Nt_{61},t_{81},(t_{103},t_{16})$

$= Nt_{89},t_{18},(t_7)$

$= Nt_{41},(t_{74},t_{26}),t_{49}$

$= Nt_{60},t_{34},t_{49}$.

Using the same relation conjugated by

$(xyxy^{-1}xyxyxyxyx)$ gives

$(xyxy^{-1}xyxyxyxyx)t_{49},t_{29},t_{85} = Id$

which gives us $(yxy^{-1}xyxy^{-1}xy^{-1}xyxyxy)t_4,t_{56},t_{85},t_{20}$

next $Nt_1,(t_5),t_{16}$

$= Nt_1,(t_{31}),t_{16}$

$= Nt_{61},t_{81},(t_{103},t_{16})$

$= Nt_{89},t_{18},(t_7)$

$= Nt_{41},(t_{74},t_{26}),t_{49}$

$= Nt_{60},t_{34},(t_{49})$

$= Nt_4, t_{56}, t_{85}, t_{20}$.

Using the relation $t_1 = t_4$ conjugated by

$(yxyxy^{-1}xyxy)$ we get $t_{56} = t_{85}$

next we have $Nt_1,(t_5),t_{16}$

$= Nt_1,(t_{31}),t_{16}$

$= Nt_{61}, t_{81}, (t_{103}, t_{16})$

$= Nt_{89}, t_{18}, (t_7)$

$= Nt_{41}, (t_{74}, t_{26}), t_{49}$

$= Nt_{60}, t_{34}, (t_{49})$

$= Nt_4, (t_{56}), t_{85}, t_{20}$

$= Nt_4, t_{85}, t_{85}, t_{20}$

$= Nt_4, (t_{85})^2, t_{20}$

$= Nt_4, t_{20}$

$= N(t_1,t_5)^{(xyxy^{-1}xyxy^{-1}xy^{-1}xyx)} \epsilon [1,5]$

Choose 5 from {1, 2, 69, 13, 89, 5, 70, 47, 18, 86, 23, 48, 93, 109,
10, 95,91, 53, 81, 31, 28, 36, 59, 51, 73, 65, 46, 4, 11, 67}

$Nt_1t_5t_5$

$= Nt_1(t_5)^2$

$= Nt_1 \epsilon [1]$

Choose 3 from {3, 8, 76, 82, 35, 99, 103, 29, 24, 107, 25, 68, 58, 78, 41,61, 101,
88, 32, 60, 85, 66, 43, 63, 87, 30, 40, 102, 108, 90, 110, 77, 14, 64, 80, 71,
55, 100, 75, 22, 56, 62, 38, 17, 27, 54, 96, 92,106, 104, 6, 57, 34, 79, 84, 74, 12,
39, 9, 26}

Using the relation $t_1 = t_4$ conjugated by

$(y^{-1}xy^{-1}xyxyxy^{-1}xyxy^{-1})$ we have $t_5 = t_{31}$

next we have $Nt_1,(t_5),t_3$

$= Nt_1, t_{31}, t_3$.

Next we will use the relation (

$(xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by

$(xy^{-1}xyxyxyxyx)$ gives

$(xy^{-1}xyxyxyxyx)t_{81}, t_{103} = t_{31}$

we have $(xy^{-1}xyxyxyxyx)t_{61}$,t_{81} ,t_{103},t_3

next Nt_1,(t_5),t_3

= Nt_1,(t_{31}),t_3

= Nt_{61} ,t_{81} ,t_{103},t_3.

By using the same relation we have

$(xyxy^{-1}xyxyxyxyxy^{-1}xy^{-1})$

which gives

$(xyxy^{-1}xyxyxyxyxy^{-1}xy^{-1})t_{80} = t_{103},t_3$

then we have $(yxyxyxy^{-1}xy^{-1}xyx)t_{100}$,t_{97},t_{80}

next Nt_1,(t_5),t_3

= Nt_1,(t_{31}),t_3

= Nt_{61} ,t_{81} ,(t_{103},t_3)

= Nt_{100} ,t_{97},t_{80} .

By using the same relation conjugated by

$(y^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}x)$

gives $(y^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}x)t_{50},t_{40}$,t_{80} = Id

then we have

$(y^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy^{-1})t_{76}$,t_{18},t_{50},t_{40}

next Nt_1,(t_5),t_3

= Nt_1,(t_{31}),t_3

= Nt_{61} ,t_{81} ,(t_{103},t_3)

= Nt_{100} ,t_{97},(t_{80})

= Nt_{76} ,t_{18},t_{50},t_{40} .

Using the same relation conjugated by

$(yxyxyxyxyxy^{-1}xyx)$

gives $(yxyxyxyxyxy^{-1}xyx)t_{99}$,$t_{78} = t_{40}$

then we have $(y^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)t_{101}$, t_{10}, t_{19}, t_{99}, t_{78}

next Nt_1,(t_5),t_3

= Nt_1,(t_{31}),t_3

= Nt_{61} ,t_{81} ,(t_{103},t_3)

= Nt_{100} ,t_{97},(t_{80})

= Nt_{76} ,t_{18},t_{50},(t_{40})

$= Nt_{101}, t_{10}, t_{19}, t_{99}, t_{78}$.

Using the relation $t_1 = t_4$ conjugated by $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)$ we have $t_{101} = t_{63}$

then $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}),t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{63}, t_{10}, t_{19}, t_{99}, t_{78}$.

Next using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(xyxy^{-1})$ gives $(xyxy^{-1})t_{42}, t_{17} = t_{63}$

then we have

$(xy^{-1}xyxy^{-1}xy^{-1}xyxy^{-1}x)t_{42}, t_{17}, t_{10}, t_{19}, t_{99}, t_{78}$

next we have $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}),t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, t_{17}, t_{10}, t_{19}, t_{99}, t_{78}$.

Using the same relation conjugated by

$(xyxyxyxyxy^{-1}xy^{-1}xyxy^{-1})$ gives

$(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)t_{38}, t_{69}, t_{19}, t_{99}, t_{78}$

then $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}),t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42},(t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, t_{69}, t_{10}, t_{99}, t_{78}$.

By using the same relation conjugated by

$(y^{-1}xyxy^{-1}xyxyxyxyxy)$ gives

$(y^{-1}xyxy^{-1}xyxyxyxyxy)t_{51}, t_{34} = t_{69}$

then we have

$(yxyxyxy^{-1}xy^{-1})t_{47}, t_{51}, t_{34}, t_{19}, t_{99}, t_{78}$

next $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}),t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42},(t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, t_{34}, t_{19}, t_{99}, t_{78}$.

Next by using the same relation conjugated by

$(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)$ gives

$(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)t_{44} = t_{34}, t_{19}$

then we have

$(xy^{-1}xyxy^{-1}xyxyxyx)t_{78}, t_{102}, t_{44}, t_{99}, t_{78}$

then $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}),t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42},(t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78},t_{102},t_{44},t_{99}\ ,t_{78}$.

By using the relation $t_1 = t_4$ conjugated by $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1})$ gives $t_{44} = t_{20}$

then we have $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}\ ),t_3$

$= Nt_{61}\ ,t_{81}\ ,(t_{103},t_3)$

$= Nt_{100}\ ,t_{97},(t_{80}\ )$

$= Nt_{76}\ ,t_{18},t_{50},(t_{40}\ )$

$= N(t_{101}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= N(t_{63}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{42},(t_{17}\ ,t_{10}\ ),t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{38}\ ,(t_{69}),t_{10}\ ,t_{99}\ ,t_{78}$

$= Nt_{47}\ ,t_{51}\ ,(t_{34},t_{19}\ ),t_{99}\ ,t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99}\ ,t_{78}$

$= Nt_{78},t_{102},t_{20}\ ,t_{99}\ ,t_{78}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by

$(xy^{-1}xyxyxyxy^{-1})$ gives

$(xy^{-1}xyxyxyxy^{-1})t_{84},t_{21} = t_{20}$

then we have

$(y^{-1}xyxy^{-1}xy^{-1}xy^{-1})t_{44},t_{13}\ ,t_{84},t_{21}\ ,t_{99}\ ,t_{78}$

next $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}\ ),t_3$

$= Nt_{61}\ ,t_{81}\ ,(t_{103},t_3)$

$= Nt_{100}\ ,t_{97},(t_{80}\ )$

$= Nt_{76}\ ,t_{18},t_{50},(t_{40}\ )$

$= N(t_{101}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= N(t_{63}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{42},(t_{17}\ ,t_{10}\ ),t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{38}\ ,(t_{69}),t_{10}\ ,t_{99}\ ,t_{78}$

$= Nt_{47}\ ,t_{51}\ ,(t_{34},t_{19}\ ),t_{99}\ ,t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99}\ ,t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, t_{21}, t_{99}, t_{78}$.

Using the same relation conjugated by

$(y^{-1}xyxyxy^{-1}xy^{-1}xy^{-1})$ gives

$(y^{-1}xyxyxy^{-1}xy^{-1}xy^{-1})t_{30} = t_{21}, t_{99}$

next we have $((y^{-1}x)^5)t_{106}, t_{98}, t_{103}, t_{30}, t_{78}$

then $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= Nt_{106}, t_{98}, t_{103}, t_{30}, t_{78}$.

Then by using same relation conjugated by

$(yxyxyxyxyxy^{-1}xy^{-1}xy)$ gives

$(yxyxyxyxyxy^{-1}xy^{-1}xy)t_{97}, t_{19} = t_{106}$

then we have

$(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{97}, t_{19}, t_{98}, t_{103}, t_{30}, t_{78}$

then $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42},(t_{17}\ ,t_{10}\ ),t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{38}\ ,(t_{69}),t_{10}\ ,t_{99}\ ,t_{78}$

$= Nt_{47}\ ,t_{51}\ ,(t_{34},t_{19}\ ),t_{99}\ ,t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99}\ ,t_{78}$

$= Nt_{78},t_{102},(t_{20}\ ),t_{99}\ ,t_{78}$

$= Nt_{44},t_{13}\ ,t_{84},(t_{21}\ ,t_{99}\ ),t_{78}$

$= N(t_{106}\ ),t_{98},t_{103},t_{30},t_{78}$

$= Nt_{97},t_{19}\ ,t_{98},t_{103},t_{30},t_{78}.$

Using the same relation conjugated by

$(xyxy^{-1}xyxyxy^{-1}xyx)$ gives

$(xyxy^{-1}xyxyxy^{-1}xyx)t_{108} = t_{19}\ ,t_{98}$

then we have

$(yxy^{-1}xyxyxyxy^{-1}xy^{-1})t_{102},t_{108}\ ,t_{103},t_{30},t_{78}$

next $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}\ ),t_3$

$= Nt_{61}\ ,t_{81}\ ,(t_{103},t_3)$

$= Nt_{100}\ ,t_{97},(t_{80}\ )$

$= Nt_{76}\ ,t_{18},t_{50},(t_{40}\ )$

$= N(t_{101}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= N(t_{63}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{42},(t_{17}\ ,t_{10}\ ),t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{38}\ ,(t_{69}),t_{10}\ ,t_{99}\ ,t_{78}$

$= Nt_{47}\ ,t_{51}\ ,(t_{34},t_{19}\ ),t_{99}\ ,t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99}\ ,t_{78}$

$= Nt_{78},t_{102},(t_{20}\ ),t_{99}\ ,t_{78}$

$= Nt_{44},t_{13}\ ,t_{84},(t_{21}\ ,t_{99}\ ),t_{78}$

$= N(t_{106}\ ),t_{98},t_{103},t_{30},t_{78}$

$= Nt_{97},(t_{19}\ ,t_{98}),t_{103},t_{30},t_{78}$

$= Nt_{102},t_{108}\ ,t_{103},t_{30},t_{78}.$

Using the relation $t_1 = t_4$ conjugated by $(yxyxyxy^{-1}xy^{-1}xy)$ we have $t_{110} = t_{103}$

then $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}\ ),t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97}, (t_{19}, t_{98}), t_{103}, t_{30}, t_{78}$

$= Nt_{102}, t_{108}, (t_{103}), t_{30}, t_{78}$

$= Nt_{102}, t_{108}, t_{110}, t_{30}, t_{78}$.

Using the same relation conjugated by $(xyxy^{-1}xy)$ we have $t_{30} = t_{17}$

then $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97}, (t_{19}, t_{98}), t_{103}, t_{30}, t_{78}$

$= Nt_{102}, t_{108}, (t_{103}), t_{30}, t_{78}$

$= Nt_{102}, t_{108}, t_{110}, (t_{30}), t_{78}$

$= Nt_{102}, t_{108}, t_{110}, t_{17}, t_{78}$.

Next we will use the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(y^{-1}xy^{-1}xyxyxyxyxy)$ gives

$(y^{-1}xy^{-1}xyxyxyxyxy)t_{106}, t_{79} = t_{110}$

then $(y^{-1}xy^{-1}xyxy^{-1})t_{22}, t_{30}, t_{106}, t_{79}, t_{17}, t_{78}$

then $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97}, (t_{19}, t_{98}), t_{103}, t_{30}, t_{78}$

$= Nt_{102}, t_{108}, (t_{103}), t_{30}, t_{78}$

$= Nt_{102}, t_{108}, t_{110}, (t_{30}), t_{78}$

$= Nt_{102}, t_{108}, (t_{110}), t_{17}, t_{78}$

$= Nt_{22}, t_{30}, t_{106}, t_{79}, t_{17}, t_{78}$.

By using the same relation conjugated by

$(yxyxyxy^{-1}xy^{-1}xy)$ gives

$(yxyxyxy^{-1}xy^{-1}xy)t_{39} = t_{79}, t_{17}$

next we have $(y^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy)t_{58}, t_{15}, t_{44}, t_{39}, t_{78}$.

Then $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97}, (t_{19}, t_{98}), t_{103}, t_{30}, t_{78}$

$= Nt_{102}, t_{108}, (t_{103}), t_{30}, t_{78}$

$= Nt_{102}, t_{108}, t_{110}, (t_{30}), t_{78}$

$= Nt_{102}, t_{108}, (t_{110}), t_{17}, t_{78}$

$= Nt_{22}, t_{30}, t_{106}, (t_{79}, t_{17}), t_{78}$

$= Nt_{58}, t_{15}, t_{44}, t_{39}, t_{78}$.

By using the relation $t_1 = t_4$ conjugated by $(yxy^{-1}xy^{-1}xy^{-1}xy^{-1})$ we have $t_{44} = t_{20}$ therefore we have $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}),t_{98},t_{103},t_{30},t_{78}$

$= Nt_{97},(t_{19},t_{98}),t_{103},t_{30},t_{78}$

$= Nt_{102},t_{108},(t_{103}),t_{30},t_{78}$

$= Nt_{102},t_{108},t_{110},(t_{30}),t_{78}$

$= Nt_{102},t_{108},(t_{110}),t_{17},t_{78}$

$= Nt_{22},t_{30},t_{106},(t_{79},t_{17}),t_{78}$

$= Nt_{58},t_{15},(t_{44}),t_{39},t_{78}$

$= Nt_{58},t_{15},t_{20},t_{39},t_{78}$.

Next using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(y^{-1}xyxyxy^{-1}xy^{-1}xy^{-1})$ gives

$(y^{-1}xyxyxy^{-1}xy^{-1}xy^{-1})t_{99},t_{21} = t_{15}$

then we have $(y^{-1}xy^{-1}xyxy^{-1})t_{22},t_{99},t_{21},t_{20},t_{39},t_{78}$

then we have $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}),t_3$

$= Nt_{61},t_{81},(t_{103},t_3)$

$= Nt_{100},t_{97},(t_{80})$

$= Nt_{76},t_{18},t_{50},(t_{40})$

$= N(t_{101}),t_{10},t_{19},t_{99},t_{78}$

$= N(t_{63}),t_{10},t_{19},t_{99},t_{78}$

$= Nt_{42},(t_{17},t_{10}),t_{19},t_{99},t_{78}$

$= Nt_{38},(t_{69}),t_{10},t_{99},t_{78}$

$= Nt_{47},t_{51},(t_{34},t_{19}),t_{99},t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99},t_{78}$

$= Nt_{78},t_{102},(t_{20}),t_{99},t_{78}$

$= Nt_{44},t_{13},t_{84},(t_{21},t_{99}),t_{78}$

$= N(t_{106}),t_{98},t_{103},t_{30},t_{78}$

$= Nt_{97},(t_{19},t_{98}),t_{103},t_{30},t_{78}$

$= Nt_{102},t_{108},(t_{103}),t_{30},t_{78}$

$= Nt_{102},t_{108},t_{110},(t_{30}),t_{78}$

$= Nt_{102},t_{108},(t_{110}),t_{17},t_{78}$

$= Nt_{22},t_{30},t_{106},(t_{79},t_{17}),t_{78}$

$= Nt_{58}, t_{15}, (t_{44}), t_{39}, t_{78}$

$= Nt_{58}, (t_{15}), t_{20}, t_{39}, t_{78}$

$= Nt_{22}, t_{99}, t_{21}, t_{20}, t_{39}, t_{78}$.

Next by using the same relation conjugated by

$(xy^{-1}xyxyxyxy^{-1})$

gives $(xy^{-1}xyxyxyxy^{-1})t_{21}, t_{20}, t_{70} = 1$ then we have

$((y^{-1}xyx)^2)t_{74}, t_{36}, t_{77}, t_{39}, t_{78}$

next $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97}, (t_{19}, t_{98}), t_{103}, t_{30}, t_{78}$

$= Nt_{102}, t_{108}, (t_{103}), t_{30}, t_{78}$

$= Nt_{102}, t_{108}, t_{110}, (t_{30}), t_{78}$

$= Nt_{102}, t_{108}, (t_{110}), t_{17}, t_{78}$

$= Nt_{22}, t_{30}, t_{106}, (t_{79}, t_{17}), t_{78}$

$= Nt_{58}, t_{15}, (t_{44}), t_{39}, t_{78}$

$= Nt_{58}, (t_{15}), t_{20}, t_{39}, t_{78}$

$= Nt_{22}, t_{99}, (t_{21}, t_{20}), t_{39}, t_{78}$

$= Nt_{74}, t_{36}, t_{77}, t_{39}, t_{78}$.

Then by using the relation $t_1 = t_4$ conjugated by

$(yxy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)$ we have $t_{36} = t_{10}$

then $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}),t_3$

$= Nt_{61},t_{81},(t_{103},t_3)$

$= Nt_{100},t_{97},(t_{80})$

$= Nt_{76},t_{18},t_{50},(t_{40})$

$= N(t_{101}),t_{10},t_{19},t_{99},t_{78}$

$= N(t_{63}),t_{10},t_{19},t_{99},t_{78}$

$= Nt_{42},(t_{17},t_{10}),t_{19},t_{99},t_{78}$

$= Nt_{38},(t_{69}),t_{10},t_{99},t_{78}$

$= Nt_{47},t_{51},(t_{34},t_{19}),t_{99},t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99},t_{78}$

$= Nt_{78},t_{102},(t_{20}),t_{99},t_{78}$

$= Nt_{44},t_{13},t_{84},(t_{21},t_{99}),t_{78}$

$= N(t_{106}),t_{98},t_{103},t_{30},t_{78}$

$= Nt_{97},(t_{19},t_{98}),t_{103},t_{30},t_{78}$

$= Nt_{102},t_{108},(t_{103}),t_{30},t_{78}$

$= Nt_{102},t_{108},t_{110},(t_{30}),t_{78}$

$= Nt_{102},t_{108},(t_{110}),t_{17},t_{78}$

$= Nt_{22},t_{30},t_{106},(t_{79},t_{17}),t_{78}$

$= Nt_{58},t_{15},(t_{44}),t_{39},t_{78}$

$= Nt_{58},(t_{15}),t_{20},t_{39},t_{78}$

$= Nt_{22},t_{99},(t_{21},t_{20}),t_{39},t_{78}$

$= Nt_{74},(t_{36}),t_{77},t_{39},t_{78}$

$= Nt_{74},t_{10},t_{77},t_{39},t_{78}$.

By using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105},t_{25} = Id$

conjugated by $(yxyxyxyxyxy^{-1}xy^{-1})$ gives

$(yxyxyxyxyxy^{-1}xy^{-1})t_{31} = t_{10},t_{77}$

then we have $(yxyxyxyxy^{-1}xy)t_{55},t_{31},t_{39},t_{78}$

then $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}),t_3$

$= Nt_{61},t_{81},(t_{103},t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97}, (t_{19}, t_{98}), t_{103}, t_{30}, t_{78}$

$= Nt_{102}, t_{108}, (t_{103}), t_{30}, t_{78}$

$= Nt_{102}, t_{108}, t_{110}, (t_{30}), t_{78}$

$= Nt_{102}, t_{108}, (t_{110}), t_{17}, t_{78}$

$= Nt_{22}, t_{30}, t_{106}, (t_{79}, t_{17}), t_{78}$

$= Nt_{58}, t_{15}, (t_{44}), t_{39}, t_{78}$

$= Nt_{58}, (t_{15}), t_{20}, t_{39}, t_{78}$

$= Nt_{22}, t_{99}, (t_{21}, t_{20}), t_{39}, t_{78}$

$= Nt_{74}, (t_{36}), t_{77}, t_{39}, t_{78}$

$= Nt_{74}, (t_{10}, t_{77}), t_{39}, t_{78}$

$= Nt_{55}, t_{31}, t_{39}, t_{78}$.

By using the same relation conjugated by

$((xy^{-1}xyxy)^2)$ gives $((xy^{-1}xyxy)^2)t_{10}, t_{64} = t_{31}$

then we have

$(yxy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy^{-1})t_{42}, t_{10}, t_{64}, t_{39}, t_{78}$

next $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{42},(t_{17}\ ,t_{10}\ ),t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{38}\ ,(t_{69}),t_{10}\ ,t_{99}\ ,t_{78}$

$= Nt_{47}\ ,t_{51}\ ,(t_{34},t_{19}\ ),t_{99}\ ,t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99}\ ,t_{78}$

$= Nt_{78},t_{102},(t_{20}\ ),t_{99}\ ,t_{78}$

$= Nt_{44},t_{13}\ ,t_{84},(t_{21}\ ,t_{99}\ ),t_{78}$

$= N(t_{106}\ ),t_{98},t_{103},t_{30},t_{78}$

$= Nt_{97},(t_{19}\ ,t_{98}),t_{103},t_{30},t_{78}$

$= Nt_{102},t_{108}\ ,(t_{103}),t_{30},t_{78}$

$= Nt_{102},t_{108}\ ,t_{110},(t_{30}),t_{78}$

$= Nt_{102},t_{108}\ ,(t_{110}),t_{17}\ ,t_{78}$

$= Nt_{22}\ ,t_{30},t_{106}\ ,(t_{79}\ ,t_{17}\ ),t_{78}$

$= Nt_{58},t_{15}\ ,(t_{44}),t_{39}\ ,t_{78}$

$= Nt_{58},(t_{15}\ ),t_{20}\ ,t_{39}\ ,t_{78}$

$= Nt_{22}\ ,t_{99}\ ,(t_{21}\ ,t_{20}\ ),t_{39}\ ,t_{78}$

$= Nt_{74},(t_{36}),t_{77}\ ,t_{39}\ ,t_{78}$

$= Nt_{74},(t_{10}\ ,t_{77}\ ),t_{39}\ ,t_{78}$

$= Nt_{55},(t_{31}\ ),t_{39}\ ,t_{78}$

$= Nt_{42},t_{10}\ ,t_{64},t_{39}\ ,t_{78}$.

By using the same relation conjugated by

$(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}x)$ gives

$(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}x)t_{63} = t_{64},t_{39}$

then we have

$(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_{21},t_{49},t_{63},t_{78}$

then $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}\ ),t_3$

$= Nt_{61}\ ,t_{81}\ ,(t_{103},t_3)$

$= Nt_{100}\ ,t_{97},(t_{80}\ )$

$= Nt_{76}\ ,t_{18},t_{50},(t_{40}\ )$

$= N(t_{101}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= N(t_{63}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{42},(t_{17},t_{10}),t_{19},t_{99},t_{78}$

$= Nt_{38},(t_{69}),t_{10},t_{99},t_{78}$

$= Nt_{47},t_{51},(t_{34},t_{19}),t_{99},t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99},t_{78}$

$= Nt_{78},t_{102},(t_{20}),t_{99},t_{78}$

$= Nt_{44},t_{13},t_{84},(t_{21},t_{99}),t_{78}$

$= N(t_{106}),t_{98},t_{103},t_{30},t_{78}$

$= Nt_{97},(t_{19},t_{98}),t_{103},t_{30},t_{78}$

$= Nt_{102},t_{108},(t_{103}),t_{30},t_{78}$

$= Nt_{102},t_{108},t_{110},(t_{30}),t_{78}$

$= Nt_{102},t_{108},(t_{110}),t_{17},t_{78}$

$= Nt_{22},t_{30},t_{106},(t_{79},t_{17}),t_{78}$

$= Nt_{58},t_{15},(t_{44}),t_{39},t_{78}$

$= Nt_{58},(t_{15}),t_{20},t_{39},t_{78}$

$= Nt_{22},t_{99},(t_{21},t_{20}),t_{39},t_{78}$

$= Nt_{74},(t_{36}),t_{77},t_{39},t_{78}$

$= Nt_{74},(t_{10},t_{77}),t_{39},t_{78}$

$= Nt_{55},(t_{31}),t_{39},t_{78}$

$= Nt_{42},t_{10},(t_{64},t_{39}),t_{78}$

$= Nt_{21},t_{49},t_{63},t_{78}$.

By using the relation $t_1 = t_4$ conjugated by $((xy^{-1})^4)$ we have $t_{21} = t_{72}$

next $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}),t_3$

$= Nt_{61},t_{81},(t_{103},t_3)$

$= Nt_{100},t_{97},(t_{80})$

$= Nt_{76},t_{18},t_{50},(t_{40})$

$= N(t_{101}),t_{10},t_{19},t_{99},t_{78}$

$= N(t_{63}),t_{10},t_{19},t_{99},t_{78}$

$= Nt_{42},(t_{17},t_{10}),t_{19},t_{99},t_{78}$

$= Nt_{38},(t_{69}),t_{10},t_{99},t_{78}$

$= Nt_{47},t_{51},(t_{34},t_{19}),t_{99},t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99},t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97}, (t_{19}, t_{98}), t_{103}, t_{30}, t_{78}$

$= Nt_{102}, t_{108}, (t_{103}), t_{30}, t_{78}$

$= Nt_{102}, t_{108}, t_{110}, (t_{30}), t_{78}$

$= Nt_{102}, t_{108}, (t_{110}), t_{17}, t_{78}$

$= Nt_{22}, t_{30}, t_{106}, (t_{79}, t_{17}), t_{78}$

$= Nt_{58}, t_{15}, (t_{44}), t_{39}, t_{78}$

$= Nt_{58}, (t_{15}), t_{20}, t_{39}, t_{78}$

$= Nt_{22}, t_{99}, (t_{21}, t_{20}), t_{39}, t_{78}$

$= Nt_{74}, (t_{36}), t_{77}, t_{39}, t_{78}$

$= Nt_{74}, (t_{10}, t_{77}), t_{39}, t_{78}$

$= Nt_{55}, (t_{31}), t_{39}, t_{78}$

$= Nt_{42}, t_{10}, (t_{64}, t_{39}), t_{78}$

$= N(t_{21}), t_{49}, t_{63}, t_{78}$

$= Nt_{72}, t_{49}, t_{63}, t_{78}$.

Next by using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})$

gives

$(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{17}, t_{52} = t_{72}$

then we have

$(yxy^{-1}xyxy)t_{17}, t_{52}, t_{49}, t_{63}, t_{78}$.

By using the relation $t_1 = t_4$ conjugated by

$(y^{-1}xy^{-1}xyxyxy^{-1}xy^{-1}x)$ we have $t_{49} = t_{19}$

next $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{42},(t_{17}\ ,t_{10}\ ),t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{38}\ ,(t_{69}),t_{10}\ ,t_{99}\ ,t_{78}$

$= Nt_{47}\ ,t_{51}\ ,(t_{34},t_{19}\ ),t_{99}\ ,t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99}\ ,t_{78}$

$= Nt_{78},t_{102},(t_{20}\ ),t_{99}\ ,t_{78}$

$= Nt_{44},t_{13}\ ,t_{84},(t_{21}\ ,t_{99}\ ),t_{78}$

$= N(t_{106}\ ),t_{98},t_{103},t_{30},t_{78}$

$= Nt_{97},(t_{19}\ ,t_{98}),t_{103},t_{30},t_{78}$

$= Nt_{102},t_{108}\ ,(t_{103}),t_{30},t_{78}$

$= Nt_{102},t_{108}\ ,t_{110},(t_{30}),t_{78}$

$= Nt_{102},t_{108}\ ,(t_{110}),t_{17}\ ,t_{78}$

$= Nt_{22}\ ,t_{30},t_{106}\ ,(t_{79}\ ,t_{17}\ ),t_{78}$

$= Nt_{58},t_{15}\ ,(t_{44}),t_{39}\ ,t_{78}$

$= Nt_{58},(t_{15}\ ),t_{20}\ ,t_{39}\ ,t_{78}$

$= Nt_{22}\ ,t_{99}\ ,(t_{21}\ ,t_{20}\ ),t_{39}\ ,t_{78}$

$= Nt_{74},(t_{36}),t_{77}\ ,t_{39}\ ,t_{78}$

$= Nt_{74},(t_{10}\ ,t_{77}\ ),t_{39}\ ,t_{78}$

$= Nt_{55},(t_{31}\ ),t_{39}\ ,t_{78}$

$= Nt_{42},t_{10}\ ,(t_{64},t_{39}\ ),t_{78}$

$= Nt_{21},t_{49},t_{63},t_{78}$.

By using the relation $t_1 = t_4$ conjugated by $((xy^{-1})^4)$ we have $t_{21} = t_{72}$

next $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}\ ),t_3$

$= Nt_{61}\ ,t_{81}\ ,(t_{103},t_3)$

$= Nt_{100}\ ,t_{97},(t_{80}\ )$

$= Nt_{76}\ ,t_{18},t_{50},(t_{40}\ )$

$= N(t_{101}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= N(t_{63}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{42},(t_{17}\ ,t_{10}\ ),t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{38}\ ,(t_{69}),t_{10}\ ,t_{99}\ ,t_{78}$

$= Nt_{47}\ ,t_{51}\ ,(t_{34},t_{19}\ ),t_{99}\ ,t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97}, (t_{19}, t_{98}), t_{103}, t_{30}, t_{78}$

$= Nt_{102}, t_{108}, (t_{103}), t_{30}, t_{78}$

$= Nt_{102}, t_{108}, t_{110}, (t_{30}), t_{78}$

$= Nt_{102}, t_{108}, (t_{110}), t_{17}, t_{78}$

$= Nt_{22}, t_{30}, t_{106}, (t_{79}, t_{17}), t_{78}$

$= Nt_{58}, t_{15}, (t_{44}), t_{39}, t_{78}$

$= Nt_{58}, (t_{15}), t_{20}, t_{39}, t_{78}$

$= Nt_{22}, t_{99}, (t_{21}, t_{20}), t_{39}, t_{78}$

$= Nt_{74}, (t_{36}), t_{77}, t_{39}, t_{78}$

$= Nt_{74}, (t_{10}, t_{77}), t_{39}, t_{78}$

$= Nt_{55}, (t_{31}), t_{39}, t_{78}$

$= Nt_{42}, t_{10}, (t_{64}, t_{39}), t_{78}$

$= N(t_{21}), t_{49}, t_{63}, t_{78}$

$= Nt_{72}, (t_{49}), t_{63}, t_{78}$

$= Nt_{72}, t_{19}, t_{63}, t_{78}$.

Using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49}, t_{105}, t_{25} = Id$

conjugated by $(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})$

gives $(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{17}, t_{52} = t_{72}$

then $(yxy^{-1}xyxy)t_{17}, t_{52}, t_{19}, t_{63}, t_{78}$

then $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}$,(t_{69}),t_{10} ,t_{99} ,t_{78}

$= Nt_{47}$,t_{51} ,$(t_{34}$,t_{19}),t_{99} ,t_{78}

$= Nt_{78}$,t_{102},(t_{44}),t_{99} ,t_{78}

$= Nt_{78}$,t_{102},$(t_{20}$),t_{99} ,t_{78}

$= Nt_{44}$,t_{13} ,t_{84},$(t_{21}$,t_{99}),t_{78}

$= N(t_{106}$),t_{98},t_{103},t_{30},t_{78}

$= Nt_{97}$,$(t_{19}$,$t_{98})$,t_{103},t_{30},t_{78}

$= Nt_{102}$,t_{108} ,(t_{103}),t_{30},t_{78}

$= Nt_{102}$,t_{108} ,t_{110},(t_{30}),t_{78}

$= Nt_{102}$,t_{108} ,(t_{110}),t_{17} ,t_{78}

$= Nt_{22}$,t_{30},t_{106} ,$(t_{79}$,t_{17}),t_{78}

$= Nt_{58}$,t_{15} ,(t_{44}),t_{39} ,t_{78}

$= Nt_{58}$,$(t_{15}$),t_{20} ,t_{39} ,t_{78}

$= Nt_{22}$,t_{99} ,$(t_{21}$,t_{20}),t_{39} ,t_{78}

$= Nt_{74}$,(t_{36}),t_{77} ,t_{39} ,t_{78}

$= Nt_{74}$,$(t_{10}$,t_{77}),t_{39} ,t_{78}

$= Nt_{55}$,$(t_{31}$),t_{39} ,t_{78}

$= Nt_{42}$,t_{10} ,$(t_{64}$,t_{39}),t_{78}

$= Nt_{21}$,t_{49},t_{63},t_{78}.

By using the relation $t_1 = t_4$ conjugated by $((xy^{-1})^4)$ we have $t_{21} = t_{72}$

next Nt_1,(t_5),t_3

$= Nt_1$,$(t_{31}$),t_3

$= Nt_{61}$,t_{81} ,$(t_{103}$,$t_3)$

$= Nt_{100}$,t_{97},$(t_{80}$)

$= Nt_{76}$,t_{18},t_{50},$(t_{40}$)

$= N(t_{101})$,t_{10} ,t_{19} ,t_{99} ,t_{78}

$= N(t_{63})$,t_{10} ,t_{19} ,t_{99} ,t_{78}

$= Nt_{42}$,$(t_{17}$,t_{10}),t_{19} ,t_{99} ,t_{78}

$= Nt_{38}$,(t_{69}),t_{10} ,t_{99} ,t_{78}

$= Nt_{47}$,t_{51} ,$(t_{34}$,t_{19}),t_{99} ,t_{78}

$= Nt_{78}$,t_{102},(t_{44}),t_{99} ,t_{78}

$= Nt_{78}$,t_{102},$(t_{20}$),t_{99} ,t_{78}

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97}, (t_{19}, t_{98}), t_{103}, t_{30}, t_{78}$

$= Nt_{102}, t_{108}, (t_{103}), t_{30}, t_{78}$

$= Nt_{102}, t_{108}, t_{110}, (t_{30}), t_{78}$

$= Nt_{102}, t_{108}, (t_{110}), t_{17}, t_{78}$

$= Nt_{22}, t_{30}, t_{106}, (t_{79}, t_{17}), t_{78}$

$= Nt_{58}, t_{15}, (t_{44}), t_{39}, t_{78}$

$= Nt_{58}, (t_{15}), t_{20}, t_{39}, t_{78}$

$= Nt_{22}, t_{99}, (t_{21}, t_{20}), t_{39}, t_{78}$

$= Nt_{74}, (t_{36}), t_{77}, t_{39}, t_{78}$

$= Nt_{74}, (t_{10}, t_{77}), t_{39}, t_{78}$

$= Nt_{55}, (t_{31}), t_{39}, t_{78}$

$= Nt_{42}, t_{10}, (t_{64}, t_{39}), t_{78}$

$= N(t_{21}), t_{49}, t_{63}, t_{78}$

$= Nt_{72}, (t_{49}), t_{63}, t_{78}$

$= N(t_{72}), t_{19}, t_{63}, t_{78}$

$= Nt_{17}, t_{52}, t_{19}, t_{63}, t_{78}$.

Using the same relation conjugated by (yxyxy) gives

$(yxyxy)t_{52}, t_{19}, t_{55} = Id$

then we have $(yxy)t_{59}, t_{55}, t_{63}, t_{78}$

then $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97}, (t_{19}, t_{98}), t_{103}, t_{30}, t_{78}$

$= Nt_{102}, t_{108}, (t_{103}), t_{30}, t_{78}$

$= Nt_{102}, t_{108}, t_{110}, (t_{30}), t_{78}$

$= Nt_{102}, t_{108}, (t_{110}), t_{17}, t_{78}$

$= Nt_{22}, t_{30}, t_{106}, (t_{79}, t_{17}), t_{78}$

$= Nt_{58}, t_{15}, (t_{44}), t_{39}, t_{78}$

$= Nt_{58}, (t_{15}), t_{20}, t_{39}, t_{78}$

$= Nt_{22}, t_{99}, (t_{21}, t_{20}), t_{39}, t_{78}$

$= Nt_{74}, (t_{36}), t_{77}, t_{39}, t_{78}$

$= Nt_{74}, (t_{10}, t_{77}), t_{39}, t_{78}$

$= Nt_{55}, (t_{31}), t_{39}, t_{78}$

$= Nt_{42}, t_{10}, (t_{64}, t_{39}), t_{78}$

$= Nt_{21}, t_{49}, t_{63}, t_{78}.$

By using the relation $t_1 = t_4$ conjugated by $((xy^{-1})^4)$ we have $t_{21} = t_{72}$

next $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97}, (t_{19}, t_{98}), t_{103}, t_{30}, t_{78}$

$= Nt_{102}, t_{108}, (t_{103}), t_{30}, t_{78}$

$= Nt_{102}, t_{108}, t_{110}, (t_{30}), t_{78}$

$= Nt_{102}, t_{108}, (t_{110}), t_{17}, t_{78}$

$= Nt_{22}, t_{30}, t_{106}, (t_{79}, t_{17}), t_{78}$

$= Nt_{58}, t_{15}, (t_{44}), t_{39}, t_{78}$

$= Nt_{58}, (t_{15}), t_{20}, t_{39}, t_{78}$

$= Nt_{22}, t_{99}, (t_{21}, t_{20}), t_{39}, t_{78}$

$= Nt_{74}, (t_{36}), t_{77}, t_{39}, t_{78}$

$= Nt_{74}, (t_{10}, t_{77}), t_{39}, t_{78}$

$= Nt_{55}, (t_{31}), t_{39}, t_{78}$

$= Nt_{42}, t_{10}, (t_{64}, t_{39}), t_{78}$

$= N(t_{21}), t_{49}, t_{63}, t_{78}$

$= Nt_{72}, (t_{49}), t_{63}, t_{78}$

$= N(t_{72}), t_{19}, t_{63}, t_{78}$

$= Nt_{17}, (t_{52}, t_{19}), t_{63}, t_{78}$

$= Nt_{59}, t_{55}, t_{63}, t_{78}$.

Then using the relation $t_1 = t_4$ conjugated by

$(yxyxyxyxy^{-1}xyxy)$ gives $t_{59} = t_{13}$

then we have $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97},(t_{19}\ ,t_{98}),t_{103},t_{30},t_{78}$

$= Nt_{102},t_{108}\ ,(t_{103}),t_{30},t_{78}$

$= Nt_{102},t_{108}\ ,t_{110},(t_{30}),t_{78}$

$= Nt_{102},t_{108}\ ,(t_{110}),t_{17}\ ,t_{78}$

$= Nt_{22}\ ,t_{30},t_{106}\ ,(t_{79}\ ,t_{17}\ ),t_{78}$

$= Nt_{58},t_{15}\ ,(t_{44}),t_{39}\ ,t_{78}$

$= Nt_{58},(t_{15}\ ),t_{20}\ ,t_{39}\ ,t_{78}$

$= Nt_{22}\ ,t_{99}\ ,(t_{21}\ ,t_{20}\ ),t_{39}\ ,t_{78}$

$= Nt_{74},(t_{36}),t_{77}\ ,t_{39}\ ,t_{78}$

$= Nt_{74},(t_{10}\ ,t_{77}\ ),t_{39}\ ,t_{78}$

$= Nt_{55},(t_{31}\ ),t_{39}\ ,t_{78}$

$= Nt_{42},t_{10}\ ,(t_{64},t_{39}\ ),t_{78}$

$= N(t_{21}\ ),t_{49},t_{63},t_{78}$

$= Nt_{72}\ ,(t_{49}),t_{63},t_{78}$

$= N(t_{72}\ ),t_{19}\ ,t_{63},t_{78}$

$= Nt_{17}\ ,(t_{52},t_{19}\ ),t_{63},t_{78}$

$= N(t_{59}\ ),t_{55},t_{63},t_{78}$

$= Nt_{13}\ ,t_{55},t_{63},t_{78}.$

By using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by

$(xy^{-1}xy^{-1}xy^{-1}xyxyxy^{-1}x)$ gives

$(xy^{-1}xy^{-1}xy^{-1}xyxyxy^{-1}x)t_{51} = t_{13}\ ,t_{55}$

then we have

$(xy^{-1}xyxyxyxyx)t_{51}\ ,t_{63},t_{78}$

then we have $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}\ ),t_3$

$= Nt_{61}\ ,t_{81}\ ,(t_{103},t_3)$

$= Nt_{100}\ ,t_{97},(t_{80}\ )$

$= Nt_{76}\ ,t_{18},t_{50},(t_{40}\ )$

$= N(t_{101}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= N(t_{63}),t_{10}\ ,t_{19}\ ,t_{99}\ ,t_{78}$

$= Nt_{42},(t_{17},t_{10}),t_{19},t_{99},t_{78}$

$= Nt_{38},(t_{69}),t_{10},t_{99},t_{78}$

$= Nt_{47},t_{51},(t_{34},t_{19}),t_{99},t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99},t_{78}$

$= Nt_{78},t_{102},(t_{20}),t_{99},t_{78}$

$= Nt_{44},t_{13},t_{84},(t_{21},t_{99}),t_{78}$

$= N(t_{106}),t_{98},t_{103},t_{30},t_{78}$

$= Nt_{97},(t_{19},t_{98}),t_{103},t_{30},t_{78}$

$= Nt_{102},t_{108},(t_{103}),t_{30},t_{78}$

$= Nt_{102},t_{108},t_{110},(t_{30}),t_{78}$

$= Nt_{102},t_{108},(t_{110}),t_{17},t_{78}$

$= Nt_{22},t_{30},t_{106},(t_{79},t_{17}),t_{78}$

$= Nt_{58},t_{15},(t_{44}),t_{39},t_{78}$

$= Nt_{58},(t_{15}),t_{20},t_{39},t_{78}$

$= Nt_{22},t_{99},(t_{21},t_{20}),t_{39},t_{78}$

$= Nt_{74},(t_{36}),t_{77},t_{39},t_{78}$

$= Nt_{74},(t_{10},t_{77}),t_{39},t_{78}$

$= Nt_{55},(t_{31}),t_{39},t_{78}$

$= Nt_{42},t_{10},(t_{64},t_{39}),t_{78}$

$= N(t_{21}),t_{49},t_{63},t_{78}$

$= Nt_{72},(t_{49}),t_{63},t_{78}$

$= N(t_{72}),t_{19},t_{63},t_{78}$

$= Nt_{17},(t_{52},t_{19}),t_{63},t_{78}$

$= N(t_{59}),t_{55},t_{63},t_{78}$

$= N(t_{13},t_{55}),t_{63},t_{78}$

$= Nt_{51},t_{63},t_{78}$.

Then by using the relation $t_1 = t_4$ conjugated by $(y^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)$ we have $t_{101} = t_{63}$

then $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}),t_3$

$= Nt_{61},t_{81},(t_{103},t_3)$

$= Nt_{100},t_{97},(t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97}, (t_{19}, t_{98}), t_{103}, t_{30}, t_{78}$

$= Nt_{102}, t_{108}, (t_{103}), t_{30}, t_{78}$

$= Nt_{102}, t_{108}, t_{110}, (t_{30}), t_{78}$

$= Nt_{102}, t_{108}, (t_{110}), t_{17}, t_{78}$

$= Nt_{22}, t_{30}, t_{106}, (t_{79}, t_{17}), t_{78}$

$= Nt_{58}, t_{15}, (t_{44}), t_{39}, t_{78}$

$= Nt_{58}, (t_{15}), t_{20}, t_{39}, t_{78}$

$= Nt_{22}, t_{99}, (t_{21}, t_{20}), t_{39}, t_{78}$

$= Nt_{74}, (t_{36}), t_{77}, t_{39}, t_{78}$

$= Nt_{74}, (t_{10}, t_{77}), t_{39}, t_{78}$

$= Nt_{55}, (t_{31}), t_{39}, t_{78}$

$= Nt_{42}, t_{10}, (t_{64}, t_{39}), t_{78}$

$= N(t_{21}), t_{49}, t_{63}, t_{78}$

$= Nt_{72}, (t_{49}), t_{63}, t_{78}$

$= N(t_{72}), t_{19}, t_{63}, t_{78}$

$= Nt_{17}, (t_{52}, t_{19}), t_{63}, t_{78}$

$= N(t_{59}), t_{55}, t_{63}, t_{78}$

$= N(t_{13}, t_{55}), t_{63}, t_{78}$

$= Nt_{51}, (t_{63}), t_{78}$

$= Nt_{51}, t_{101}, t_{78}.$

By using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy{*}x)^{3})t_{49}, t_{105}, t_{25} = Id$

conjugated by $(xy^{-1}xyxyxyxyxy^{-1}xy^{-1}xy)$ gives

$(xy^{-1}xyxyxyxyxy^{-1}xy^{-1}xy)t_{68},t_{65},t_{51} = Id$

then we have

$(y^{-1}xyxy^{-1}xyxyxy^{-1}xy^{-1})t_{68},t_{65},t_{101},t_{78}$

then $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}),t_3$

$= Nt_{61},t_{81},(t_{103},t_3)$

$= Nt_{100},t_{97},(t_{80})$

$= Nt_{76},t_{18},t_{50},(t_{40})$

$= N(t_{101}),t_{10},t_{19},t_{99},t_{78}$

$= N(t_{63}),t_{10},t_{19},t_{99},t_{78}$

$= Nt_{42},(t_{17},t_{10}),t_{19},t_{99},t_{78}$

$= Nt_{38},(t_{69}),t_{10},t_{99},t_{78}$

$= Nt_{47},t_{51},(t_{34},t_{19}),t_{99},t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99},t_{78}$

$= Nt_{78},t_{102},(t_{20}),t_{99},t_{78}$

$= Nt_{44},t_{13},t_{84},(t_{21},t_{99}),t_{78}$

$= N(t_{106}),t_{98},t_{103},t_{30},t_{78}$

$= Nt_{97},(t_{19},t_{98}),t_{103},t_{30},t_{78}$

$= Nt_{102},t_{108},(t_{103}),t_{30},t_{78}$

$= Nt_{102},t_{108},t_{110},(t_{30}),t_{78}$

$= Nt_{102},t_{108},(t_{110}),t_{17},t_{78}$

$= Nt_{22},t_{30},t_{106},(t_{79},t_{17}),t_{78}$

$= Nt_{58},t_{15},(t_{44}),t_{39},t_{78}$

$= Nt_{58},(t_{15}),t_{20},t_{39},t_{78}$

$= Nt_{22},t_{99},(t_{21},t_{20}),t_{39},t_{78}$

$= Nt_{74},(t_{36}),t_{77},t_{39},t_{78}$

$= Nt_{74},(t_{10},t_{77}),t_{39},t_{78}$

$= Nt_{55},(t_{31}),t_{39},t_{78}$

$= Nt_{42},t_{10},(t_{64},t_{39}),t_{78}$

$= N(t_{21}),t_{49},t_{63},t_{78}$

$= Nt_{72},(t_{49}),t_{63},t_{78}$

$= N(t_{72}), t_{19}, t_{63}, t_{78}$

$= Nt_{17}, (t_{52}, t_{19}), t_{63}, t_{78}$

$= N(t_{59}), t_{55}, t_{63}, t_{78}$

$= N(t_{13}, t_{55}), t_{63}, t_{78}$

$= Nt_{51}, (t_{63}), t_{78}$

$= N(t_{51}), t_{101}, t_{78}$

$= Nt_{68}, t_{65}, t_{101}, t_{78}$.

By using the same relation conjugated by

$(xyxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy)$ then it gives

$(xyxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy)t_{65}, t_{101}, t_{91} = Id$

then we have

$(xy^{-1}xy^{-1}xyxyxy^{-1}xy)t_{31}, t_{91}, t_{78}$

then $Nt_1, (t_5), t_3$

$= Nt_1, (t_{31}), t_3$

$= Nt_{61}, t_{81}, (t_{103}, t_3)$

$= Nt_{100}, t_{97}, (t_{80})$

$= Nt_{76}, t_{18}, t_{50}, (t_{40})$

$= N(t_{101}), t_{10}, t_{19}, t_{99}, t_{78}$

$= N(t_{63}), t_{10}, t_{19}, t_{99}, t_{78}$

$= Nt_{42}, (t_{17}, t_{10}), t_{19}, t_{99}, t_{78}$

$= Nt_{38}, (t_{69}), t_{10}, t_{99}, t_{78}$

$= Nt_{47}, t_{51}, (t_{34}, t_{19}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{44}), t_{99}, t_{78}$

$= Nt_{78}, t_{102}, (t_{20}), t_{99}, t_{78}$

$= Nt_{44}, t_{13}, t_{84}, (t_{21}, t_{99}), t_{78}$

$= N(t_{106}), t_{98}, t_{103}, t_{30}, t_{78}$

$= Nt_{97}, (t_{19}, t_{98}), t_{103}, t_{30}, t_{78}$

$= Nt_{102}, t_{108}, (t_{103}), t_{30}, t_{78}$

$= Nt_{102}, t_{108}, t_{110}, (t_{30}), t_{78}$

$= Nt_{102}, t_{108}, (t_{110}), t_{17}, t_{78}$

$= Nt_{22}, t_{30}, t_{106}, (t_{79}, t_{17}), t_{78}$

$= Nt_{58}, t_{15}, (t_{44}), t_{39}, t_{78}$

$= Nt_{58},(t_{15}),t_{20},t_{39},t_{78}$

$= Nt_{22},t_{99},(t_{21},t_{20}),t_{39},t_{78}$

$= Nt_{74},(t_{36}),t_{77},t_{39},t_{78}$

$= Nt_{74},(t_{10},t_{77}),t_{39},t_{78}$

$= Nt_{55},(t_{31}),t_{39},t_{78}$

$= Nt_{42},t_{10},(t_{64},t_{39}),t_{78}$

$= N(t_{21}),t_{49},t_{63},t_{78}$

$= Nt_{72},(t_{49}),t_{63},t_{78}$

$= N(t_{72}),t_{19},t_{63},t_{78}$

$= Nt_{17},(t_{52},t_{19}),t_{63},t_{78}$

$= N(t_{59}),t_{55},t_{63},t_{78}$

$= N(t_{13},t_{55}),t_{63},t_{78}$

$= Nt_{51},(t_{63}),t_{78}$

$= N(t_{51}),t_{101},t_{78}$

$= Nt_{68},(t_{65},t_{101}),t_{78}$

$= Nt_{31},t_{91},t_{78}$.

Next using the relation $t_1 = t_4$ conjugated by

$(y^{-1}xy^{-1}xyxyxy^{-1}xyxy^{-1})$ gives $t_{31} = t_5$

then we have $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}),t_3$

$= Nt_{61},t_{81},(t_{103},t_3)$

$= Nt_{100},t_{97},(t_{80})$

$= Nt_{76},t_{18},t_{50},(t_{40})$

$= N(t_{101}),t_{10},t_{19},t_{99},t_{78}$

$= N(t_{63}),t_{10},t_{19},t_{99},t_{78}$

$= Nt_{42},(t_{17},t_{10}),t_{19},t_{99},t_{78}$

$= Nt_{38},(t_{69}),t_{10},t_{99},t_{78}$

$= Nt_{47},t_{51},(t_{34},t_{19}),t_{99},t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99},t_{78}$

$= Nt_{78},t_{102},(t_{20}),t_{99},t_{78}$

$= Nt_{44},t_{13},t_{84},(t_{21},t_{99}),t_{78}$

$= N(t_{106}),t_{98},t_{103},t_{30},t_{78}$

$= Nt_{97},(t_{19}\ ,t_{98}),t_{103},t_{30},t_{78}$

$= Nt_{102},t_{108}\ ,(t_{103}),t_{30},t_{78}$

$= Nt_{102},t_{108}\ ,t_{110},(t_{30}),t_{78}$

$= Nt_{102},t_{108}\ ,(t_{110}),t_{17}\ ,t_{78}$

$= Nt_{22}\ ,t_{30},t_{106}\ ,(t_{79}\ ,t_{17}\ ),t_{78}$

$= Nt_{58},t_{15}\ ,(t_{44}),t_{39}\ ,t_{78}$

$= Nt_{58},(t_{15}\ ),t_{20}\ ,t_{39}\ ,t_{78}$

$= Nt_{22}\ ,t_{99}\ ,(t_{21}\ ,t_{20}\ ),t_{39}\ ,t_{78}$

$= Nt_{74},(t_{36}),t_{77}\ ,t_{39}\ ,t_{78}$

$= Nt_{74},(t_{10}\ ,t_{77}\ ),t_{39}\ ,t_{78}$

$= Nt_{55},(t_{31}\ ),t_{39}\ ,t_{78}$

$= Nt_{42},t_{10}\ ,(t_{64},t_{39}\ ),t_{78}$

$= N(t_{21}\ ),t_{49},t_{63},t_{78}$

$= Nt_{72}\ ,(t_{49}),t_{63},t_{78}$

$= N(t_{72}\ ),t_{19}\ ,t_{63},t_{78}$

$= Nt_{17}\ ,(t_{52},t_{19}\ ),t_{63},t_{78}$

$= N(t_{59}\ ),t_{55},t_{63},t_{78}$

$= N(t_{13}\ ,t_{55}),t_{63},t_{78}$

$= Nt_{51}\ ,(t_{63}),t_{78}$

$= N(t_{51}\ ),t_{101},t_{78}$

$= Nt_{68},(t_{65}\ ,t_{101}),t_{78}$

$= N(t_{31}\ ),t_{91},t_{78}$

$= Nt_{5},t_{91},t_{78}.$

Then using the relation

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy*x)^3)t_{49},t_{105}\ ,t_{25} = Id$

conjugated by $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)$ gives

$(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_{106} = t_5,t_{91}$

then we have $(xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_{106}\ ,t_{78}$

next $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}\ ),t_3$

$= Nt_{61}\ ,t_{81}\ ,(t_{103},t_3)$

$= Nt_{100}\ ,t_{97},(t_{80}\ )$

$= Nt_{76} ,t_{18},t_{50},(t_{40})$

$= N(t_{101}),t_{10} ,t_{19} ,t_{99} ,t_{78}$

$= N(t_{63}),t_{10} ,t_{19} ,t_{99} ,t_{78}$

$= Nt_{42},(t_{17} ,t_{10}),t_{19} ,t_{99} ,t_{78}$

$= Nt_{38} ,(t_{69}),t_{10} ,t_{99} ,t_{78}$

$= Nt_{47} ,t_{51} ,(t_{34},t_{19}),t_{99} ,t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99} ,t_{78}$

$= Nt_{78},t_{102},(t_{20}),t_{99} ,t_{78}$

$= Nt_{44},t_{13} ,t_{84},(t_{21} ,t_{99}),t_{78}$

$= N(t_{106}),t_{98},t_{103},t_{30},t_{78}$

$= Nt_{97},(t_{19} ,t_{98}),t_{103},t_{30},t_{78}$

$= Nt_{102},t_{108} ,(t_{103}),t_{30},t_{78}$

$= Nt_{102},t_{108} ,t_{110},(t_{30}),t_{78}$

$= Nt_{102},t_{108} ,(t_{110}),t_{17} ,t_{78}$

$= Nt_{22} ,t_{30},t_{106} ,(t_{79} ,t_{17}),t_{78}$

$= Nt_{58},t_{15} ,(t_{44}),t_{39} ,t_{78}$

$= Nt_{58},(t_{15}),t_{20} ,t_{39} ,t_{78}$

$= Nt_{22} ,t_{99} ,(t_{21} ,t_{20}),t_{39} ,t_{78}$

$= Nt_{74},(t_{36}),t_{77} ,t_{39} ,t_{78}$

$= Nt_{74},(t_{10} ,t_{77}),t_{39} ,t_{78}$

$= Nt_{55},(t_{31}),t_{39} ,t_{78}$

$= Nt_{42},t_{10} ,(t_{64},t_{39}),t_{78}$

$= N(t_{21}),t_{49},t_{63},t_{78}$

$= Nt_{72} ,(t_{49}),t_{63},t_{78}$

$= N(t_{72}),t_{19} ,t_{63},t_{78}$

$= Nt_{17} ,(t_{52},t_{19}),t_{63},t_{78}$

$= N(t_{59}),t_{55},t_{63},t_{78}$

$= N(t_{13} ,t_{55}),t_{63},t_{78}$

$= Nt_{51} ,(t_{63}),t_{78}$

$= N(t_{51}),t_{101},t_{78}$

$= Nt_{68},(t_{65} ,t_{101}),t_{78}$

$= N(t_{31}),t_{91},t_{78}$

$= N(t_5,t_{91}),t_{78}$

$= Nt_{106} ,t_{78}$.

Next using the relation $t_1 = t_4$ conjugated by $(xy^{-1}xy^{-1}x)$ gives $t_{106} = t_{60}$

then we have $Nt_1,(t_5),t_3$

$= Nt_1,(t_{31}),t_3$

$= Nt_{61} ,t_{81} ,(t_{103},t_3)$

$= Nt_{100} ,t_{97},(t_{80})$

$= Nt_{76} ,t_{18},t_{50},(t_{40})$

$= N(t_{101}),t_{10} ,t_{19} ,t_{99} ,t_{78}$

$= N(t_{63}),t_{10} ,t_{19} ,t_{99} ,t_{78}$

$= Nt_{42},(t_{17} ,t_{10}),t_{19} ,t_{99} ,t_{78}$

$= Nt_{38} ,(t_{69}),t_{10} ,t_{99} ,t_{78}$

$= Nt_{47} ,t_{51} ,(t_{34},t_{19}),t_{99} ,t_{78}$

$= Nt_{78},t_{102},(t_{44}),t_{99} ,t_{78}$

$= Nt_{78},t_{102},(t_{20}),t_{99} ,t_{78}$

$= Nt_{44},t_{13} ,t_{84},(t_{21} ,t_{99}),t_{78}$

$= N(t_{106}),t_{98},t_{103},t_{30},t_{78}$

$= Nt_{97},(t_{19} ,t_{98}),t_{103},t_{30},t_{78}$

$= Nt_{102},t_{108} ,(t_{103}),t_{30},t_{78}$

$= Nt_{102},t_{108} ,t_{110},(t_{30}),t_{78}$

$= Nt_{102},t_{108} ,(t_{110}),t_{17} ,t_{78}$

$= Nt_{22} ,t_{30},t_{106} ,(t_{79} ,t_{17}),t_{78}$

$= Nt_{58},t_{15} ,(t_{44}),t_{39} ,t_{78}$

$= Nt_{58},(t_{15}),t_{20} ,t_{39} ,t_{78}$

$= Nt_{22} ,t_{99} ,(t_{21} ,t_{20}),t_{39} ,t_{78}$

$= Nt_{74},(t_{36}),t_{77} ,t_{39} ,t_{78}$

$= Nt_{74},(t_{10} ,t_{77}),t_{39} ,t_{78}$

$= Nt_{55},(t_{31}),t_{39} ,t_{78}$

$= Nt_{42},t_{10} ,(t_{64},t_{39}),t_{78}$

$= N(t_{21}),t_{49},t_{63},t_{78}$

$= Nt_{72} ,(t_{49}),t_{63},t_{78}$

$= N(t_{72}),t_{19} ,t_{63},t_{78}$

$= Nt_{17}$,(t_{52},t_{19}),t_{63},t_{78}

$= N(t_{59}$),t_{55},t_{63},t_{78}

$= N(t_{13}$,t_{55}),t_{63},t_{78}

$= Nt_{51}$,(t_{63}),t_{78}

$= N(t_{51}$),t_{101},t_{78}

$= Nt_{68}$,(t_{65} ,t_{101}),t_{78}

$= N(t_{31}$),t_{91},t_{78}

$= N(t_5$,t_{91}),t_{78}

$= N(t_{106}$),t_{78}

$= Nt_{60}$,t_{78}

$= N(t_1,t_2)^{(yxy^{-1}xyxy^{-1}xy)}$ ϵ [1,2]

Cayley Diagram

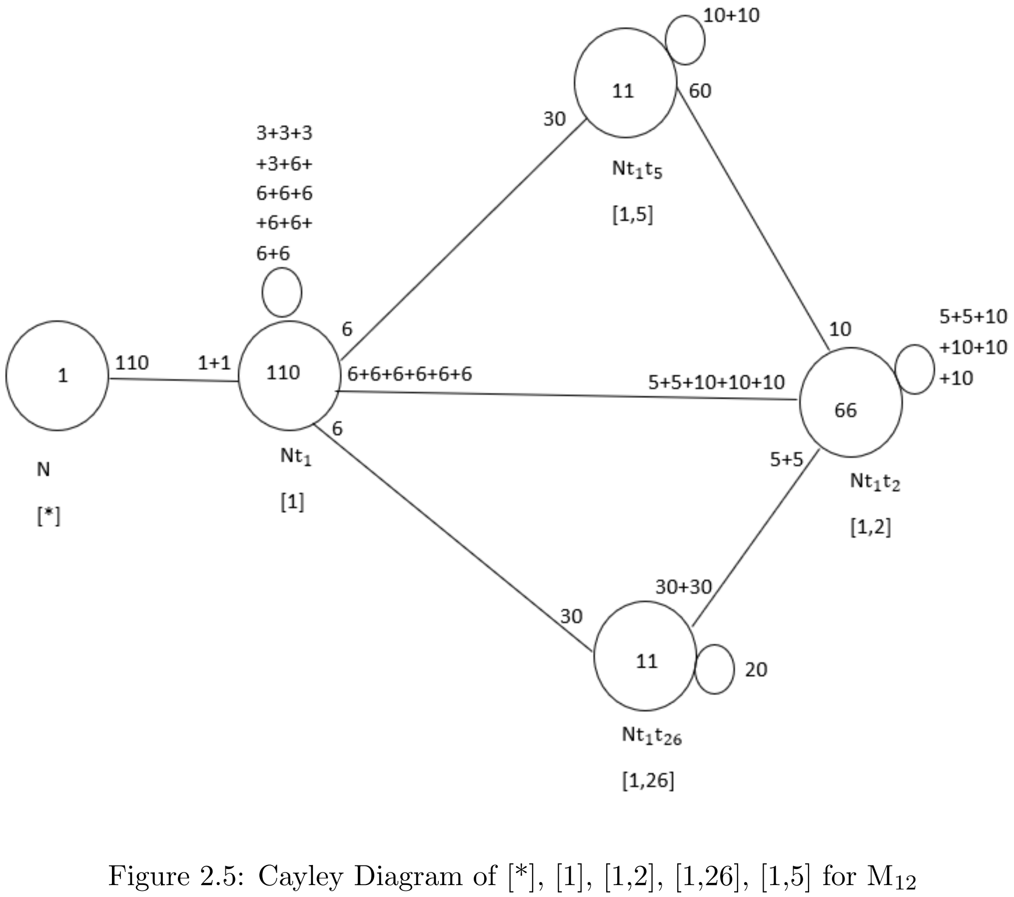


Figure 2.5: Cayley Diagram of [*], [1], [1,2], [1,26], [1,5] for M_{12}

Chapter 3

$$\frac{2^{*32}{:}(2^5{:}A_5)}{(xy^{-2}xy)^6t_2t_9t_{13}t_{15}t_4t_2,(x^5{*}y^3)t_1t_4t_1} \cong \mathbf{J}_2$$

We will prove that the progenitor 2^{*32}:(2^5:A_5), where 2^5:A_5 = < x, y> and x ∼ (1, 2)(3, 5, 7, 11, 17, 4, 6, 9, 14, 22)(8, 13, 20, 29, 23, 10, 16, 25, 30,18)(12, 19, 28, 32, 26, 15, 24, 27, 31, 21) , y ∼ (1, 3)(2, 4)(5, 8)(6, 10)(7, 12, 17, 27, 14, 23) (9, 15, 22, 28, 11, 18)(13, 21, 24, 30, 32, 20)(16, 26, 19, 29, 31, 25), factored by two relations is isomorphic to the janko sporadic simple group J_2. Let G $\cong \frac{2^{*32}:(2^5:A_5)}{(xy^{-2}xy)^6t_2t_9t_{13}t_{15}t_4t_2,(x^5*y^3)t_1t_4t_1}$ = J_2. Thus we show that G ∼ J_2.

3.1 Expanding Relations

<u>Relation Given:</u>

$$((xy^{-2}xy)t^x)^6$$

$((\mathrm{xy}^{-2}\mathrm{xy})\mathrm{t}^x)^6$

$= (\mathrm{xy}^{-2}\mathrm{xy})\mathrm{t}_2(\mathrm{xy}^{-2}\mathrm{xy})\mathrm{t}_2(\mathrm{xy}^{-2}\mathrm{xy})\mathrm{t}_2(\mathrm{xy}^{-2}\mathrm{xy})\mathrm{t}_2(\mathrm{xy}^{-2}\mathrm{xy})\mathrm{t}_2(\mathrm{xy}^{-2}\mathrm{xy})\mathrm{t}_2$

$= (\mathrm{xy}^{-2}\mathrm{xy})^6\mathrm{t}_2^{(xy^{-2}xy)^5}\mathrm{t}_2^{(xy^{-2}xy)^4}\mathrm{t}_2^{(xy^{-2}xy)^3}\mathrm{t}_2^{(xy^{-2}xy)^2}\mathrm{t}_2^{(xy^{-2}xy)}\mathrm{t}_2$

$= (\mathrm{xy}^{-2}\mathrm{xy})^6\mathrm{t}_2\mathrm{t}_9\mathrm{t}_{13}\mathrm{t}_1\mathrm{t}_5\mathrm{t}_4\mathrm{t}_2$

$$(\tau t)^3 \text{ Relation (1)}$$

$= (\tau\ \mathrm{t})^3$

$= \tau\mathrm{t}\ \tau\mathrm{t}\tau\mathrm{t}$

$= \tau^3\mathrm{t}^{\tau^2}\mathrm{t}^{\tau}\mathrm{t}$

$= \tau^3\mathrm{t}_1\mathrm{t}_4\mathrm{t}_1$

$\tau^3 = t_1 t_4 t_1$

$$(\pi t^x)^6 \text{ Relation (2)}$$

$= (\pi t^x)^6$

$= (\pi t_2)^6$

$= \pi t_2\ \pi t_2\ \pi t_2\ \pi t_2\ \pi t_2\ \pi t_2$

$= \pi^6 t_2^{\pi^5} t_2^{\pi^4} t_2^{\pi^3} t_2^{\pi^2} t_2^{\pi} t_2$

$= \pi^6 t_2 t_9 t_{13} t_{15} t_4 t_2\ \pi^6 = t_2 t_9 t_{13} t_{15} t_4 t_2$

We now prove using lemmas 1-4, that $t_1, t_2, t_1, t_2, t_1 = x^5$

Lemma 1:

$\underline{t_1,\ t_2,\ t_1,\ t_2 = ((yx^{-1}yxy^{-1})^3)\ t_1,\ t_2,\ t_1,\ t_2,\ t_5}$

First

$t_1,\ t_2,\ t_1,\ \underline{t_2},\ t_1$

$= ((yx^{-1}yxy^{-1})^3) t_5,\ t_6,\ t_5,\ t_2,\ t_6,\ t_1,\ t_5,\ t_1,\ t_1,\ t_5$ (Relation (1) $t_2 \sim t_2 t_6,\ t_5 t_1 t_1 t_5 = e$)

$= t_1,\ t_2,\ t_1,\ t_6,\ t_2,\ t_1,\ t_5$

$\implies (t_1,\ t_2,\ t_1,\ t_2,\ t_1)^{-1} = t_5,\ t_1,\ t_2,\ t_6,\ t_1,\ t_2,\ t_1$

$= t_5,\ t_1,\ \underline{t_2,\ t_6},\ t_1,\ t_2,\ t_1$ (By relation (1) $t_2 t_6 \sim t_2$)

$= ((yx^{-1}yxy^{-1})^3) t_1,\ t_5,\ t_2,\ t_1,\ t_2,\ t_1$

$\implies (t_1,\ t_2,\ t_1,\ t_2,\ t_1)^{-1} = ((yx^{-1}yxy^{-1})^3) t_1,\ t_2,\ t_1,\ t_2,\ t_5,\ t_1$

$\implies t_1,\ t_2,\ t_1,\ t_2,\ t_1 = ((yx^{-1}yxy^{-1})^3) t_1,\ t_2,\ t_1,\ t_2,\ t_5,\ t_1$

$\implies t_1,\ t_2,\ t_1,\ t_2 = ((yx^{-1}yxy^{-1})^3) t_1,\ t_2,\ t_1,\ t_2,\ t_5$

Lemma 2:

$\underline{t_1,\ t_2,\ t_1,\ t_2 = ((y^{-1}xyx^{-1}y)^3)\ t_9,\ t_7,\ t_9,\ t_2,\ t_7}$

Next

$t_1,\ t_2,\ t_1,\ t_2$

$= t_1,\ t_2,\ t_1, \underline{t_2,\ t_7},\ t_7$ (Relation (1))

$= ((y^{-1}xyx^{-1}y)^3)\ t_9,\ t_7,\ t_9,\ t_2,\ t_7$

Lemma 3:

$t_1, t_2, t_1, t_2 = (x^{-1}y^{-1}x^{-1}y^3x^{-1})\ t_{22}, t_{23}, t_{10}, t_7$

Proof:

$t_1,t_2,t_1,t_2 = ((y^{-1}xyx^{-1}y)^3)t_9,t_7,\underline{t_9,t_2},t_7$ (lemma 2)

$= t_2t_1t_9\underline{t_{27}t_{19}t_5}t_7$ (Relation (2))

$= \underline{t_{27}t_{29}t_{27}t_3}t_{10}t_7$ (Relation (2))

$= t_{22}t_{23}t_{10}t_7$ (Relation (2))

Lemma 4:

$Nt_2,t_5,t_{10} = Nt_2,t_5,t_{31} = Nt_2,t_{22},t_{23}$

Proof:

$t_2,t_5,t_{10},t_{31},t_5,t_2$

$= t_2,t_5,t_{10},t_{31},t_5,\underline{t_2,t_7},t_7$

$=((y^{-1}xyx^{-1}y)^3)\ \underline{t_7,t_{16},t_{29}},t_{18},t_{16},t_2,t_7$ (Relation (1))

$=(xyx^{-1}yx^2y^{-1})\ t_7,t_1,t_{14},t_{18},t_{16},t_2,t_7$ (Relation (2))

$=(x^3y^3x)\ t_2,t_9,t_{20},\underline{t_{31},t_5,t_2}$ (Relation (1))

$= ((xy^{-2})^2)t_4,t_{18},t_{20},t_{31},t_{28},t_4$ (Relation (2))

$= (x^2y^2)\ t_4,t_8,t_{26},t_{31},t_{28},t_4$ (Relation (2))

$= (x^2yxyxy^{-1})$ (Relation (2)

This gives $Nt_2,t_5,t_{10} = Nt_2,t_5,t_{31}$

Now $t_2,t_5,t_{31},t_{23},t_{22},t_2 = (xyxy^2)$. (Relation (2))

So $Nt_2,t_5,t_{31} = Nt_2,t_{22},t_{23}$

Thus $Nt_2,t_5,t_{10},t_{23},t_{22},t_2 = Nt_2,t_5,t_{10},t_{31},t_5,t_2$

Lemma 5

$t_1,t_2,t_1,t_2,t_1 = x^5$

Proof:

$t_1t_2t_1t_2t_1$

$= (yx^{-1}yxy^{-1})\ \underline{t_1t_2t_1t_2}t_5t_1$ (lemma 1)

$= (yx^{-1}yxy^{-1}\ )(x^{-1}y^{-1}x^{-1}y^3x^{-1})t_{22}t_{23}t_{10}t_7t_5t_1$ (lemma 2)

$=(y^{-1}xy^{-1}x^{-1}y^{-1}x\ )t_{22}t_{23}t_{10}t_7t_5t_1$

$=((y^{-1}xy^{-1}x^{-1}y^{-1}x\ )^{-1})\underline{t_7,t_{27}}t_{25}t_{32}t_{22}t_2$ ($t_1t_2t_1t_2t_1$ is an involution)

$=(yx^2yx^{-1}yx)\underline{t_7,t_{25}},t_{32},t_{22},t_2$ (Relation (1) $t_7,t_{27} \sim t_7$)

$=(y^{-1}xy^{-2}xy^{-2})t_7,t_{21},t_{22},\underline{t_{23},t_{32}},t_{22},t_2$ (Relation (2) $t_7,t_{25} \sim t_7,t_{21},t_{22},t_{23}$)

$=(yx^{-1}y^{-1}xy^{-2}x)\underline{t_2,t_{22},t_{21}},t_{23},t_{22},t_2$ (Relation (1))

$=(xy^3xy^{-1}x)t_2,t_5,t_{10},\underline{t_{23},t_{22},t_2}$ (Relation (2))

$=(y^{-2}xyx^{-2}y)t_2,t_5,t_{10},t_{31},t_5,t_2$ (Lemma 4)

$=(y^{-2}xyx^{-2}y)t_2,t_5,t_{10},t_{31},t_5,\underline{t_2,t_7},t_7$

$=((xyx^{-1}yx^2y^{-1})t_7,t_1,t_{14},t_{18},t_{16},\underline{t_2,t_7}$ (Relation (1))

$=(x^{-1}yxy^{-1}x^{-1}y^{-2})t_2,t_9,t_{20},\underline{t_{31},t_5,t_2}$ (Relation (1))

$=(yx^{-1}yx^{-1}y^{-1}x^{-1})\underline{t_4,t_{18},t_{20}},t_{31},t_{28},t_4$ (Relation (2))

$=(x^{-2}yx^{-1}y^{-1}x)t_4,t_8,t_{26},t_4,t_{31},t_{28},t_4$ (Relation (2))

$= x^5$

Lemma 6

$t_1,t_8,t_{23},t_{29} = x^4yxyx^{-1}t_{18}t_{12}t_{20}t_{30}$

Proof:

$t_1,t_8,\underline{t_{23}},t_{29}$

$= (yx^2y^{-2}\ x^{-2})t_6t_9t_{23}t_{13}t_6\underline{t_1t_3}t_{29}$ (Relation (2))

$= (yxy^{-1})^4t_{12}t_2t_{30}t_{11}t_{12}t_1t_7t_{21}\underline{t_{10}t_{29}}$ (Relation (2))

$= (x^2yx^2y^{-1}x^{-1}y^{-1})t_{19}t_7t_8t_{25}t_{19}\underline{t_9t_2t_{22}}t_{10}$ (Relation (1))

$= (xyx^4y^{-1})t_{15}t_{30}t_{32}t_{21}t_{15}t_9t_{27}\underline{t_{29}t_{10}}$ (Relation (2))

$= (yxyx^{-1}y^{-1}xyx^{-1})t_{24}t_8\underline{t_{23}t_{22}t_{24}}t_1t_3t_{29}$ (Relation (1))

$= (x^4yx^{-1}y)t_{28}t_8\underline{t_{23}t_{31}}t_{28}t_1t_3t_{29}$ (Relation (2))

$= (yxy^{-2}x)t_7t_{22}t_{23}t_{25}\underline{t_7t_1t_3}t_{29}$ (Relation (2))

$= (yxyx^{-1}y^{-1})^2t_1t_{28}t_{30}t_{17}t_7t_{21}\underline{t_{10}t_{29}}$ (Relation (2))

$= (y^2x^{-1}y^{-1}x^2)t_9\underline{t_4t_8t_{26}}t_2t_{22}t_{10}$ (Relation (1))

$= (xy^2x^{-1}y^{-2}x)t_2\underline{t_4t_2t_9}t_2t_{22}t_{10}$ (Relation (2))

$= (y^2xy^3x^{-1}y)t_{32}t_4t_2\underline{t_{11}t_2t_{22}}t_{10}$ (Relation (2))

$= (yx^{-1}y^2\ x)t_{17}t_7t_{16}t_{11}\underline{t_2t_5t_{10}}$ (Relation (2))

$= (yxy^{-2})^2t_{32}t_{24}t_{20}t_2t_2t_{11}t_{30}$ (Relation (2))

$= (yxy^{-2})^2t_{32}t_{24}\underline{t_{20}t_{11}}t_{30}$

Lemma 7:

<u>**$t_1, t_8, t_{23}, t_{29} = y^{-1}x^{-3}$**</u>

We have

$t_1, t_8, t_{23}, t_{29} = (x^4yxyx^{-1})\ t_{18}, t_{12}, t_{20}, t_{30}$ (Lemma 5)

Then,

$\Longrightarrow$ $(t_1, t_8, t_{23}, t_{29} = (x^4yxyx^{-1})\ t_{18}, t_{12}, t_{20}, t_{30})^{-1}$

$\Longrightarrow$ $t_{29}, t_{23}, t_8, t_1 = t_{30}, t_{20}, t_{12}, t_{18}\ (x^4yxyx^{-1})^{-1}$

$\Longrightarrow$ $t_{29}, t_{23}, t_8, t_1 = (xyxy^2xyx)\ t_8, t_{14}, t_{23}, t_1$

$\Longrightarrow$ $t_{29}, t_{23}, t_8 = (xyxy^2xyx)\ t_8, t_{14}, t_{23}$

$\Longrightarrow$ $(x^4yxyx^{-1})\ t_{29}, t_{23}, t_8 = t_8, t_{14}, t_{23}$

Conjugate by $(y^2xy^{-1}xy^{-1}x)$

$\Longrightarrow$ $(x^{-3}y^{-1}x^{-1}yx^{-1})\ t_4, t_{14}, t_{28} = t_{28}, t_{18}, t_{14}$

$\Longrightarrow$ $(x^{-3}y^{-1}x^{-1}yx^{-1})\ t_4,\underline{t_{14}, t_{28}, t_{14}}, t_{18}, t_{28} = e$

$\Longrightarrow$ $(x^{-2}y^{-1}x^{-1}y^{-1}x^2)\ t_{25}, t_{32}, t_{19},\underline{t_{24}, t_{18}, t_{28}} = e$ (Relation (2))

$\Longrightarrow$ $(yxy^{-1}x^{-2}y^{-1})\ t_{30}, t_{20}, t_{28}, t_{28}, t_{12}, t_{18} = e$ (Relation (2)

$\Longrightarrow$ $(yxy^{-1}x^{-2}y^{-1})\ t_{30}, t_{20}, t_{12}, t_{18} = e$

$\Longrightarrow$ $(yxy^{-1}x^{-2}y^{-1})\ t_{30}, t_{20}, t_{12}, t_{18} = e$

Thus

$t_{18}, t_{12}, t_{20}, t_{30} = (yxy^{-1}x^{-2}y^{-1})$.

Now $t_1, t_8, t_{23}, t_{29} = (x^4yxyx^{-1})(yxy^{-1}x^{-2}y^{-1})$ (Lemma 5)

$=y^{-1}x^{-3}$

3.2 Double Coset Enumeration

<u>**First Double Coset**</u>

NeN = { $N(e)^n$ | n ϵ N } = { N }

The coset stabiliser the coset of N = Ne is N.

The number of single right cosets the double coset NeN = [*] is given by $\frac{|N|}{|N|} = \frac{1920}{1920} = 1$

The orbits of N on

X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32} is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32} we will now choose an orbit representative and multiply it by N on the right and determine its double coset.

Choose 1 from {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32}

Then

$Nt_1 \epsilon$ [1]

This tells us that thirty two elements move forward to the double coset [1]

Cayley Diagram

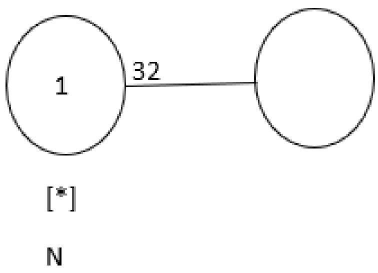


Figure 3.1: Cayley Diagram of [*] for J_2

Second Double Coset

$Nt_1N = \{ N(t_1)^n \mid n \epsilon N \} = \{Nt_1, Nt_2, ..., Nt_{32}\}$

The point-stabiliser of 1, N^1 is given by $< x^2, y^2 >$

The Coset Stabilizer of $N^{(1)} = < x^2, y^2 >$

The number of single right cosets in the double coset Nt_1N = [1] is given by $\frac{|N|}{|N^{(1)}|} = \frac{1920}{60}$ = 32

The orbits for $N^{(1)}$ on

X = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31,32 } are {1}, {2}, {3, 7, 17, 6, 14} , {4,9,22,5,11}, {8, 20, 23, 21, 16, 12, 19, 30, 28, 27, 32, 31, 26, 18, 15, 13, 24, 29, 10, 25 }

Multiply Nt_1 by a representative of each orbit and determine its double coset

Choose 1 from {1}

Nt_1t_1

$= N(t_1)^2$

$= N \epsilon$ [*]

Choose 2 from {2}

$Nt_1t_2 \epsilon$ [1,2]

Choose 3 from {3,7,17,6,14}

$Nt_1t_3 \epsilon$ [1,3]

Choose 4 from {4, 9, 22, 5, 11}

Nt_1t_4

$= (x^5y^3)t_1$ (By relation (1))

$= (x^5y^3)(t_1) \epsilon$ [1]

Choose 8 from {8, 20, 23, 21, 16, 12, 19, 30, 28, 27, 32, 31, 26, 18, 15, 13, 24, 29, 10, 25}

$Nt_1t_8 \epsilon$ [1,8]

Cayley Diagram

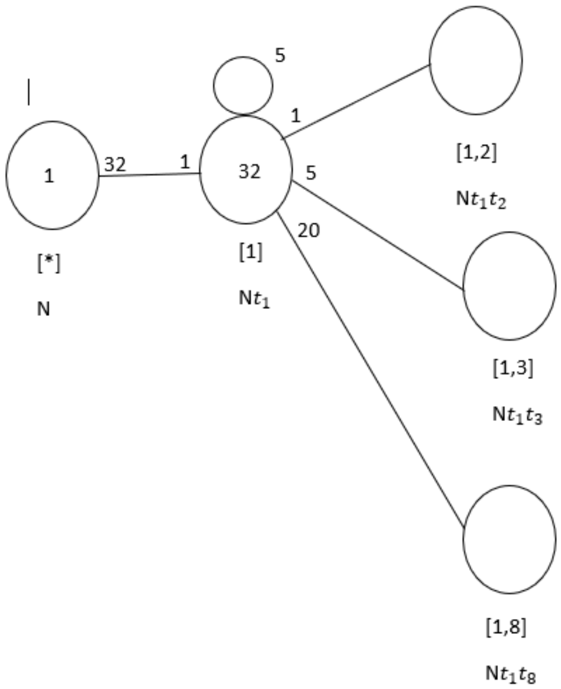


Figure 3.2: Cayley Diagram of [*], [1] for J_2

Third Double Coset

$Nt_1t_2N = \{ N(t_1t_2)^n \mid n \epsilon N \} = \{Nt_1t_2, t_2t_1, ..., Nt_3t_4\}$

The point-stabiliser of 1,2, N^{12} is given by $< x^2, y^2 >$

The coset Stabilizer of $N^{(12)} = < x^2, y^2 >$

The number of single right cosets in the double coset $Nt_1t_2N = [1,2]$ is given by $\frac{|N|}{|N^{(12)}|} = \frac{1920}{60} = 32$

The orbits for $N^{(12)}$ on

X={ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32 } are {1}, {2}, {3, 7, 17, 6, 14}, {4, 9, 22, 5, 11}, {8, 20, 23, 21, 16, 12, 19, 30, 28, 27, 32, 31, 26, 18, 15, 13, 24, 29, 10, 25}

Multiply Nt_1t_2 by a representative of each orbit and determine its double coset.

Choose 1 from {1}

$t_1t_2t_1$

Then

$(x^5)t_1t_2t_1t_2t_1 = e$ (lemma 1)

So

$(x^5)t_1t_2 = t_1t_2t_1$

Thus

$Nt_1t_2 = Nt_1t_2t_1$.

Then $Nt_1t_2t_1 = Nt_1t_2 \ \epsilon \ [1,2]$.

Choose 2 from {2}

$Nt_1t_2t_2$

$= t_1(t_2)^2$

$= t_1 \ \epsilon \ [1]$

Choose 3 from {3, 7, 17, 6, 14}

t_1,t_2,t_3

$= t_1,t_2,t_1,\underline{t_1},t_3$

$= (x^5)t_2,t_1,t_2,t_1,t_2,t_1,t_2,t_3$ (Lemma (1))

$= \underline{t_2,t_1,t_2,t_1,t_1,t_2,t_1,t_2},t_1,t_2,t_3$

$= t_1,\underline{t_2,t_3}$

$= (x^5y^3)t_4,t_2$ (Relation (1))

$= N(t_1,t_3)^{(x^{-2}y^{-2}xy)} \ \epsilon \ [1,3]$

Choose 4 from {4, 9, 22, 5, 11}

$Nt_1t_2t_4$ ϵ [1,2,4]

Choose 8 from {8,20,23,21,16,12,19,30,28,27,32,31,26,18,15,13,24,29,10,25}

$t_1,\underline{t_2,t_8}$

$= (xy^2xy^{-1}x^{-1})\underline{t_{22},t_{25}},t_{16}$ (Lemma 7)

$= (xyx^2yxy)t_7,\underline{t_6,t_{16}}$ (Lemma 7)

$(x^{-1}y^3xy)t_1,t_6$ (Relation (1))

$= N(t_1,t_3)^{(x^{-4}y^2)}$ ϵ [1,3]

Cayley Diagram

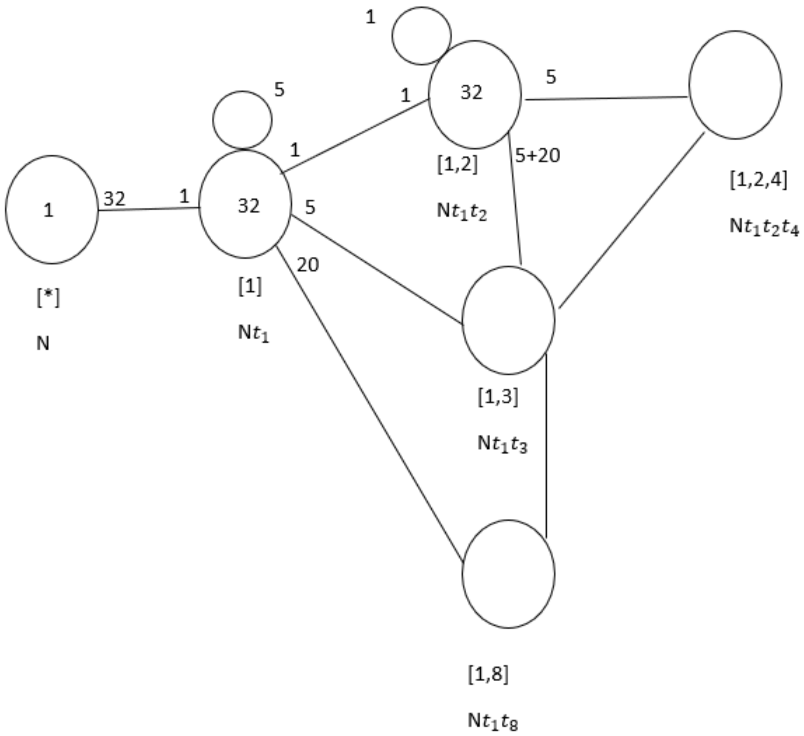


Figure 3.3: Cayley Diagram of [*], [1], [1,2] for J_2

<u>Fourth Double Coset</u>

$Nt_1t_3N = \{ N(t_1t_3)^n \mid n \epsilon N \} = \{Nt_1t_3, Nt_2t_5, ..., Nt_3t_4\}$

The point-stabiliser of 1, 3, N^{13} is given by $< y^2, (xy^{-2}x^{-1}y^2) >$

The coset Stabilizer of

$N^{(13)} = < y^2, (xy^{-2}x^{-1}y^2) >$

The number of single right cosets in the double coset $Nt_1t_3N = [1,3]$ is given by $\frac{|N|}{|N^{(13)}|} = \frac{1920}{12} = 160$

The orbits for $N^{(13)}$ on

X = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32}

are {1}, {2}, {3}, {4}, {5, 22, 11, 9}, {6, 17, 14, 7}, {8, 18, 15, 28}, {10, 23, 12, 27}, {13, 24, 26, 32, 30, 29, 20, 19, 31, 25, 21, 16}

We will multiply Nt_1t_3 by a representative of each orbit and determine its double coset.

<u>Choose 1 from {1}</u>

t_1,t_3,t_1

$=t_5,\underline{t_5,t_1},t_3,t_1$

$= ((yx^{-1}yxy^{-1})^3)t_1,\underline{t_5},t_3,t_1$ (Relation (1))

$= (xy^2xy^{-1}xy^{-1})t_3,t_5,\underline{t_8,t_3},t_1$ (Relation (1))

$= (y^3)t_8,t_2,\underline{t_8,t_1}$ (Relation (1))

$(xyx^{-1}y^{-2}x^{-1}y)t_{21},\underline{t_{30},t_{14}},t_{18}$ (Lemma 7)

$= (yx^{-1}yx^{-1}y^{-2}x)\underline{t_9,t_{30}},t_{18}$ (Relation (1))

$= (xy^{-1}xy^2xy^{-1})t_{17},\underline{t_3,t_{18}}$ (Lemma 7)

$= (y^3)t_1,t_3$ (Relation (1))

$= Nt_1t_3 \epsilon [1,3]$

<u>Choose 2 from {2}</u>

$t_1,\underline{t_3,t_2}$

$= (x^5y^3)t_4,t_3$ (Relation (1))

$= N(t_1,t_2)^{(x^3y^{-1})}$.

<u>Choose 3 from {3}</u>

$Nt_1\ t_3\ t_3$

$= t_1(t3)^2$

$= t_1$

$= Nt_1 \epsilon [1]$

$\underline{\text{Choose 4 from \{4\}}}$

$t_1,t_3,\underline{t_4}$

$= (xyx^3y^{-1}x)t_{31},\underline{t_{28},t_{16}},t_{26},t_5$ (Lemma 7)

$= (x^{-1}yx^{-1}yx^{-1}y^{-1})\underline{t_{21},t_{12}},t_{10},t_{26},t_5$ (Lemma 7)

$= (xy^{-1}xyx^{-2}y^{-1}x^{-1})t_{21},\underline{t_{10},t_{26}},t_5$ (Relation (1))

$= (y)t_8,t_{10},t_5$ (Relation (1))

$= N(t_1\ ,t_8)^{(x^{-1}yx^3yx)} \epsilon [1,8]$

$\underline{\text{Choose 5 from \{5, 22, 11, 9\}}}$

$t_1,\underline{t_3,t_5}$

$= (x^{-3}y^{-1})\underline{t_4,t_{19}},t_{14}$ (Lemma 7)

$= (x^5y^{-2})\underline{t_{25},t_{31}},t_{14}$ (Lemma 7)

$= (x^5y^{-2})t_{26},\underline{t_2,t_{14}}$ (Lemma 7)

$= (y^2)t_{10},t_2$ (Relation (1))

$\underline{\text{Choose 6 from \{6, 17, 14, 7\}}}$

t_1,t_3,t_6

$= t_1,t_3,\underline{t_6,t_2},t_2$

$= ((yx^{-1}yxy^{-1})^3)t_5,\underline{t_{10},t_6},t_2$ (Relation (1))

$(xy^2xy^{-1}xy^{-1})t_8,\underline{t_{10},t_2}$ (Relation (1))

$= (y^3xy^{-1}x^{-2})t_{30},t_9,\underline{t_{12}}$ (Lemma 7)

$= (yx^{-3}y^2)\underline{t_{22},t_{25},t_{12}},t_4$ (Relation (1))

$= (y^{-2})t_8,t_4$ Lemma 7)

$= N(1,3)^{(x^{-4})}$

$\underline{\text{Choose 8 from \{8, 18, 15, 28\}}}$

$t_1,\underline{t_3,t_8}$

$= ((x^{-1}yx)^3)t_6,t_3$ (Relation (1))

$= N(t_1,t_8)^{(y^2x^{-1}y^3x)} \epsilon [1,8]$

$\underline{\text{Choose 10 from \{10, 23, 12, 27\}}}$

Proof of $Nt_1t_3t_{10} \epsilon Nt_1,t_2N$

$t_1,t_3,\underline{t_{10}}$

$= (y^3)t_3,t_1,t_{10},\underline{t_6}$ (Relation (1))

$= (yx^{-1}yx^{-1}y^{-2}x^{-1})t_{10},\underline{t_5,t_3},t_6,t_2$ (Lemma 7)

$= (x^{-1}yx^2y^{-2})t_{11},t_{26},\underline{t_{19},t_6},t_2$ (Lemma 7)

$= (yx^{-2}y^{-1}xy^{-1})\underline{t_{29},t_7,t_{19}},t_2$ (Relation (1))

$= (x^{-1}y^3xy)t_1,t_2$ (Lemma 7)

$= t_1,t_2.$

Thus, $t_1t_3t_{10} = (x^{-1}y^3xy)t_1,t_2$.

Then $Nt_1t_3t_{10} = Nt_1,t_2$ in $Nt_1,t_2N = [1,2]$.

Choose 13 from {13,24,26,32,30,29,20,19,31,25,21,16}

$t_1,\underline{t_3,t_{13}}$

$= (xy^{-2}x^{-1}yx^{-1})\underline{t_{22},t_{30}},t_{24}$ (Lemma 7)

$= (y^{-1}xy^{-1}x^{-2})t_{22},t_{24}$ (Relation (1))

$= N(t_1,t_3)^{(x^{-2}y^{-1}x^{-1})} \epsilon [1,3]$

Cayley Diagram

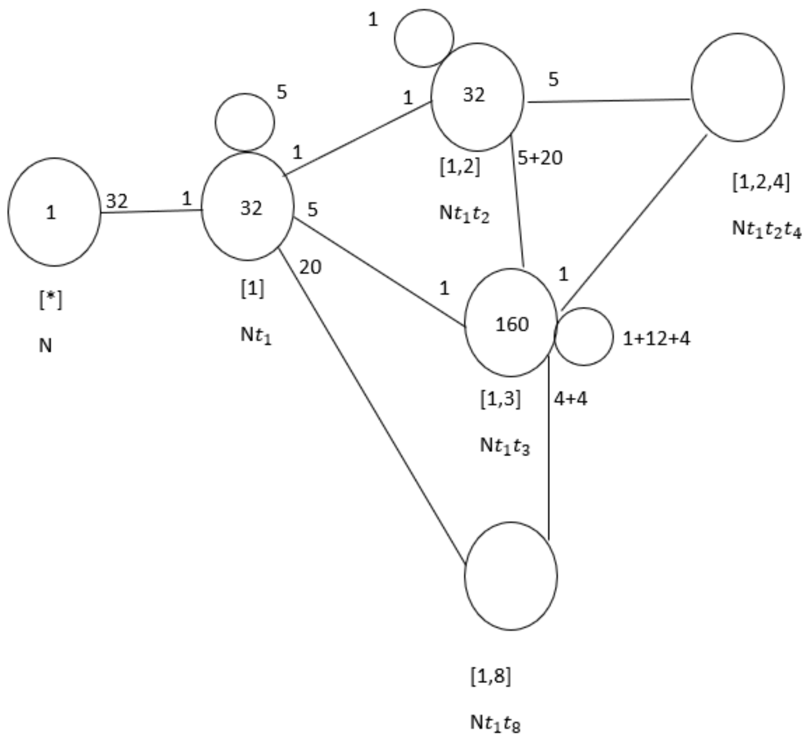


Figure 3.4: Cayley Diagram of [*], [1], [1,2], [1,3] for J_2

Fifth Double Coset

$Nt_1t_8N = \{ N(t_1t_8)^n \mid n \epsilon N \} = \{Nt_1t_8, Nt_2t_{13}, ..., Nt_3t_5 \}$

The point-stabiliser of 1, 8, N^{18} is given by $< y^2 >$

Now $Nt_1t_8 = Nt_3t_5$

Then

Now $N^{1,8} \cong 3$ (3 denote $\mathbb{Z}_3$ or C_3)

We have $Nt_1t_8 = Nt_{29}t_{23}$ (Lemma 7)

So (1, 29, 3, 16)(2, 30, 4, 13)(5, 17, 8, 23)(6, 22, 10, 18)(7, 14, 27, 12)(9, 11, 28, 15)(19, 26, 25, 31)(20, 32, 24, 21) ϵ $N^{(1,8)}$ and $Nt_{29}t_{23} = Nt_3Nt_5$.

Also, $N^{(1,8)} \geq < N^{1,8}$, (1, 29, 3, 16)(2, 30, 4, 13)(5, 17, 8, 23)(6, 22, 10, 18)(7, 14, 27, 12)(9, 11, 28, 15)(19, 26, 25, 31)(20, 32, 24, 21)> $\cong 2^{\cdot}A_4$ (central extension of 2 by A_4)

So

$(x^{-1}yx^{-1}y^3x^{-1})$ ϵ $N^{(18)}$

Thus The coset stabiliser $N^{(18)} \geq < N^{18}, y, (x^{-1}yx^{-1}y^3x^{-1}) > =$

$<y^2, y, (x^{-1}yx^{-1}y^3x^{-1})>$

The number of single right cosets in the double coset Nt_1t_8 = [1,8] is given by $\frac{|N|}{|N^{(18)}|} = \frac{1920}{24} = 80$

The orbits for $N^{(18)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32} are {1, 3, 29, 16, 25, 31, 19, 26}, {2, 4, 30, 13, 20, 32, 24, 21}, {5, 8, 17, 23, 14, 27, 12, 7}, {6, 10, 22, 18, 11, 28, 15, 9}

Multiply Nt_1t_8 by a representative of each orbit and determine its double coset.

Choose 1 from {1, 3, 29, 16, 25, 31, 19, 26}

$\underline{t_1},t_8,t_1$

$= ((yx^{-1}yxy^{-1})^3)t_1,\underline{t_5,t_8},t_1$ (Relation (1))

$= (xy^2xy^{-1}xy^{-1})t_3,\underline{t_5,t_1}$ (Relation (1))

$= (x^5y^3)t_{10},t_5$ (Relation (1))

$= N(t_1,t_8)^{(xyxy^{-1}xy^{-1}x)}$ ϵ [1,8]

Choose 8 from {5, 8, 17, 23, 14, 27, 12, 7}

$Nt_1t_8t_8$

$= t_1(t_8)^2$

$= t_1$ ϵ [1]

Choose 2 from {2, 4, 30, 13, 20, 32, 24, 21}

1,8,2

= $(y^{-1}xy^{-1}x^{-2})t_{30},t_{22},t_{32}$ (Lemma 7)

= $(xy^{-2}x^{-1}yx^{-1})t_{30},t_{32}$(Relation (1))

Choose 6 from {6, 10, 22, 18, 11, 28, 15, 9}

t_1,t_8,t_6

= $(y^3x^3)t_{26},t_{27},t_6$ (Lemma 7)

= $(x^{-1}yxyx^{-1}yx)t_{10},t_{26},t_3$ (Lemma 7)

= $(y^2)t_{10},t_3$(Relation (1))

= $Nt_{10},t_3 = N(t_1,t_3)^{(x^{-1}yx^3yx)}$ ϵ [1,3]

Cayley Diagram

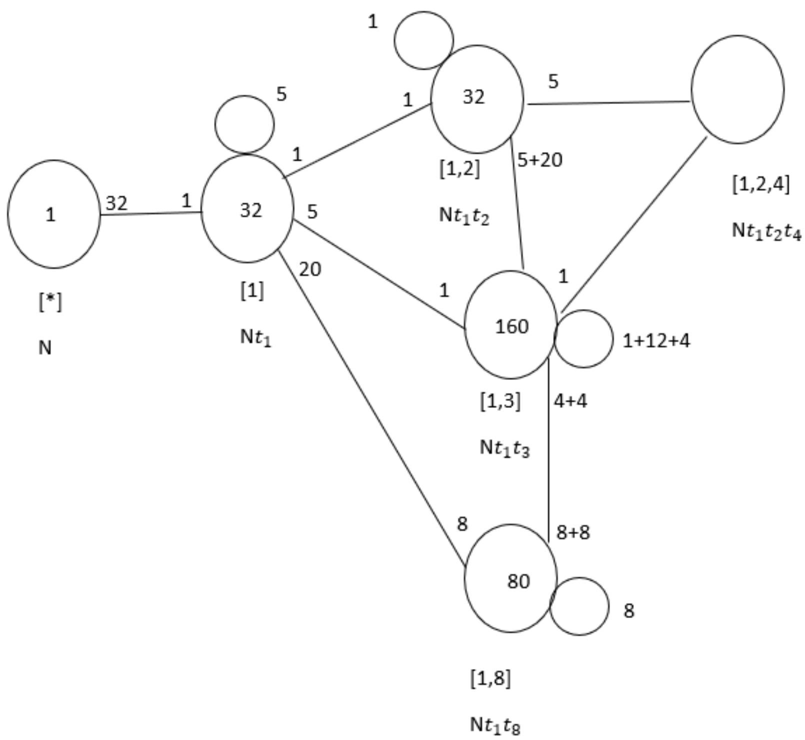


Figure 3.5: Cayley Diagram of [*], [1], [1,2], [1,3], [1,8] for J_2

Sixth Double Coset

N$t_1t_2t_4$N ={N$(t_1t_2t_4)^n$ | nϵ N} = {N$t_1t_2t_4$, N$t_2t_1t_6$,..., N$t_3t_4t_2$}

The point-stabiliser of 1, 2, 4, $N^{1,2,4}$ is given by < $(xyx^{-1})^2$, $(xy^{-2}x^{-1}y^2, y^{-2})$ >

$N^{1,2,4} \cong 2^2{:}3$ We have N$t_1t_2t_4$ = N$t_{24}t_{19}t_{25}$ (See the proof below)

$t_1,t_2,\underline{t_4}$

$= (y^3xy^{-2})t_5,\underline{t_6,t_{19}},t_{32},t_{25}$ (Lemma 7)

$= (y^{-1}xy^{-1}x^{-1}y^2x)t_{24},t_6,\underline{t_{32}},t_{25}$ (Relation (1))

$= (y^{-1}xyx^{-2}yx)t_{26},\underline{t_{31},t_{15}},t_1,t_{19},t_{25}$ (Lemma 7)

$= (yx^{-3}y^{-1}x^{-1}y)t_{32},t_{19},\underline{t_9,t_1},t_{19},t_{25}$ (Lemma 7)

$= (yx^{-1}y^{-2}xy)\underline{t_{23},t_{12}},t_9,t_{19},t_{25}$ (Relation (1))

$= (y^{-1}x^3yx)t_{28},\underline{t_{25},t_9},t_{19},t_{25}$ (Lemma 7)

$= (xyxyx^2y^2)\underline{t_{24},t_{25}},t_{19},t_{25}$ (Relation (1))

$= (xyxyx^{-3}y^{-1})t_{24},t_{19},t_{25}$ (Relation (1))

Thus N$t_1,t_2,\underline{t_4}$ = Nt_{24},t_{19},t_{25}

N$t_1t_2t_4$ = N$t_{24}t_{19}t_{25}$ $\implies$ (1, 24, 3, 20)(2, 19, 4, 25)(5, 15, 8, 11)(6, 12, 10, 14)(7, 22, 27, 18)(9, 17, 28, 23)(13, 26, 30, 31)(16, 21, 29, 32) ϵ $N^{(1,2,4)}$ and N$t_{24}t_{19}t_{25}$ = N$t_3t_4t_2$.

Now $N^{(124)} \geq$ < N124,(1, 24, 3, 20)(2, 19, 4, 25)(5, 15, 8, 11)(6, 12, 10, 14)(7, 22, 27, 18)(9, 17, 28, 23)(13, 26, 30, 31)(16, 21, 29, 32) > $\cong 2^4{:}(A_4)$

Proof of N$t_1t_3t_4$ = N$t_8t_{10}t_5$ ϵ [1,2,4]

We have $t_1,t_3,\underline{t_4}$

$=(xyx^3y^{-1}x)t_{31},\underline{t_{28},t_{16}},t_{26},t_5$ (Lemma 7)

$=(x^{-1}yx^{-1}yx^{-1}y^{-1})\underline{t_{21},t_{12}},t_{10},t_{26},t_5$ (Relation (1))

$=(xy^{-1}xyx^{-2}y^{-1}x^{-1})t_{21},\underline{t_{10},t_{26}},t_5$ (Relation (1))

$=(y)t_8,t_{10},t_5$ (Relation (1)) Note Nt_8,t_{10},t_5 ϵ N$t_1t_2t_4$ N since N$t_1t_2t_4$ conjugate by (1, 8, 19, 14, 26, 27)(2, 10, 24, 11, 21, 28)(3, 6, 25, 15, 31, 9)(4, 5, 20, 12, 32, 7)(13, 23)(16, 18)(17, 29)(22, 30) is Nt_8,t_{10},t_5.

Thus The coset stabiliser $N^{(124)} \geq$ < N^{124}, $(xy^{-2}x^{-1}y)$, $(x^2yx^{-2}y^{-1}x^{-1})$ >

= < $(x^2yx^{-2}y^{-1}x^{-1})$, $(xy^{-2}x^{-1}y)$, $(x^2yx^{-2}y^{-1}x^{-1})$ >

The number of single right cosets in the double coset N$t_1t_2t_4$N = [1,2,4] is given by $\frac{|N|}{|N^{(124)}|}$ $= \frac{1920}{192} = 10$

The orbits for $N^{(124)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32} are {1, 3, 2, 24, 4, 20, 19, 13, 29, 30, 21, 32, 16, 25, 26, 31}, {5, 11, 22, 8, 28, 18, 15, 6, 17, 14, 7, 27, 9, 10, 23, 12}

Multiply $Nt_1t_2t_4$ by a representative of each orbit and determine its double coset.

Choose 4 from {1, 3, 2, 24, 4, 20, 19, 13, 29, 30, 21, 32, 16, 25, 26, 31}

$Nt_1t_2t_4t_4$

$= t_1t_2(t_4)^2$

$= Nt_1t_2 \epsilon$ [1,2]

Choose 5 from {5, 11, 22, 8,2 8, 18, 15, 6, 17, 14, 7, 27, 9, 10, 23, 12}

$\underline{t_1,t_2},t_4,t_5$

$= (x^5)t_1,t_2,\underline{t_1,t_4},t_5$ (Lemma 1)

$= (y^3)t_4,t_3,\underline{t_1,t_5}$ (Relation (1))

$= (yx^{-1}yx^{-1}y^{-2}x^{-1})t_8,\underline{t_{10},t_1}$ (Relation (1))

$= (x^{-1}yx^{-3}y^{-1}x^{-1})\underline{t_{30},t_7},t_{19}$ (Lemma 7)

$= (y^{-1}x^{-1}y^2)\underline{t_{21},t_{23}},t_{19}$ (Lemma 7)

$= (y^2xy^3x^{-1})t_{25},\underline{t_6,t_{19}}$ (Lemma 7)

$= (y^2)t_8,t_6$(Relation (1))

$N(t_1,t_3)^{(y^{-1}x^3yx)} \epsilon [t_1,t_3]$

Cayley Diagram

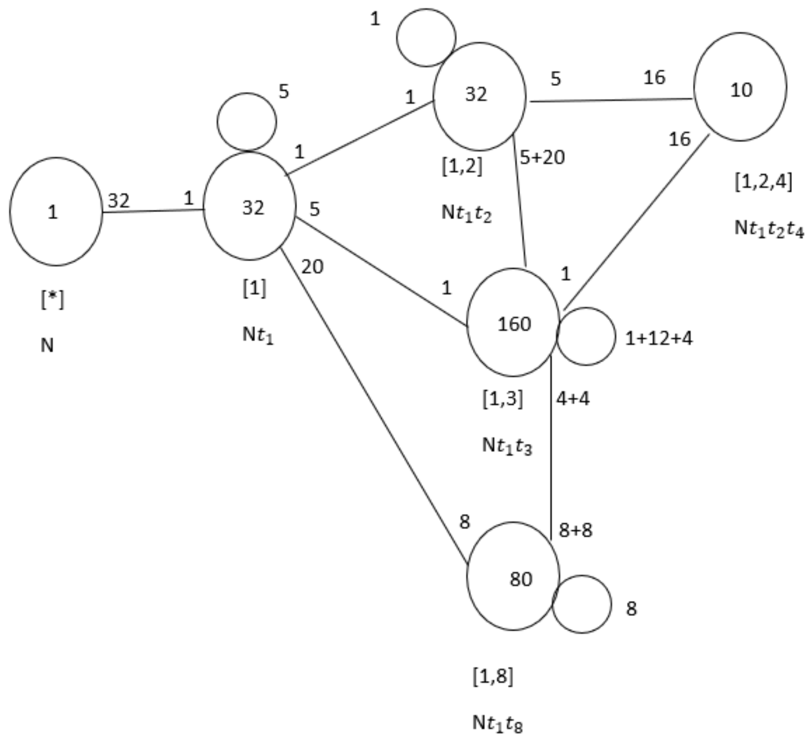


Figure 3.6: Cayley Diagram of [*], [1], [1,2], [1,3], [1,8], [1,2,4] for J_2

Chapter 4

Double Coset Enumeration

4.1 $\frac{2^{*5}:A_5}{t_1t_2t_3t_1=(1,2,3)} \cong \mathbf{S}_6$

We will prove that the progenitor 2^{*5}:A_5, where 2^{*5}:A_5 = <x, y> and x ∼ (3, 4, 5), y ∼ (1, 2, 3), factored by one relation is isomorphic to S_6. Let G ≅ $\frac{2^{*5}:A5}{t_1t_2t_3t_1=(1,2,3)}$. Thus we will show that G ∼ S_6

Expanded Relation

Relation 1 = $t_1t_2t_3t_1$ = (1,2,3)

The elements of our N are { (1, 5, 4, 2, 3), (1, 3, 4), (1, 3)(2, 4), (1, 2, 3, 4, 5), (1, 5)(3, 4), (1, 3)(2, 5), (3, 5, 4), Id(N), (1, 4)(2, 5), (1, 4, 2, 5, 3), (1, 4, 3, 5, 2), (1, 4, 2), (1, 5)(2, 3), (1, 5, 2, 3, 4), (1, 5, 2), (2, 5)(3, 4), (2, 5, 3), (1, 3)(4, 5), (1, 4)(3, 5), (1, 2, 5, 3, 4), (1, 2, 3, 5, 4), (1, 4, 3), (1, 2, 4, 3, 5), (1, 4, 5, 2, 3), (1, 4, 5, 3, 2), (1, 2, 4, 5, 3), (1, 3, 5, 2, 4), (1, 5, 3, 4, 2), (1, 5, 4, 3, 2), (2, 5, 4), (2, 3)(4, 5), (2, 3, 4), (1, 2, 5), (1, 2, 5, 4, 3), (1, 2)(3, 5), (1, 2)(4, 5), (1, 5)(2, 4), (1, 5, 2, 4, 3), (1, 3, 5, 4, 2), (2, 4, 3), (1, 4)(2, 3), (1, 3, 2, 4, 5), (1, 3, 5), (2, 3, 5), (1, 4, 5), (1, 2)(3, 4), (1, 2, 3), (1, 3, 4, 2, 5), (1, 5, 3, 2, 4), (1, 5, 3), (2, 4, 5), (1, 3, 2, 5, 4), (1, 3, 4, 5, 2), (1, 5, 4), (2, 4)(3, 5), (3, 4, 5), (1, 3, 2), (1, 4, 3, 2, 5), (1, 4, 2, 3, 5), (1, 2, 4)}

First Double Coset

NeN = { $N(e)^n$ | n ϵ N} = {N}

The coset stabiliser the coset N = Ne is N.

The number of single right cosets of the double coset NeN = [*] is given by $\frac{|N|}{|N|} = \frac{60}{60} = 1$

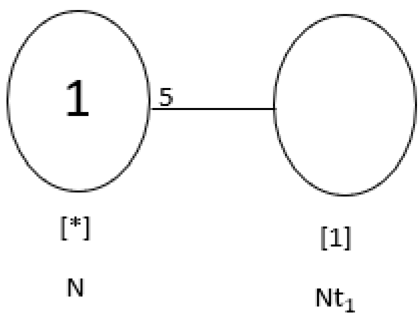
The orbit of N on X = {1, 2, 3, 4, 5} is {1, 2, 3, 4, 5} we will choose a representative of the orbit and multiply N on the right and determine its double coset.

Choose 1 from {1, 2, 3, 4, 5}

$Nt_1 \epsilon$ [1].

This tells us that five elements move forward to the double coset [1].

Cayley Diagram



Second double coset

$Nt_1N = \{ N(t_1)^n \mid n \epsilon N \} = \{ Nt_1, Nt_2, Nt_3, Nt_4, Nt_5\}$

The point-stabiliser 1, N^1 is given by < (2, 3, 4), (3, 4, 5)>

The coset stabiliser $N^{(1)}$ = < (2, 3, 4), (2, 4, 5), (2, 4, 3)>

The number of single right cosets of the double coset Nt_1N = [1] is given by $\frac{|N|}{|N^{(1)}|} = \frac{60}{12}$ = 5

The orbits of $N^{(1)}$ on X = { 1, 2, 3, 4, 5} are {1}, {2, 3, 4, 5}

We will multiply Nt_1 by a representative of each orbit and determine its double coset.

Choose 1 from {1}

Nt_1t_1

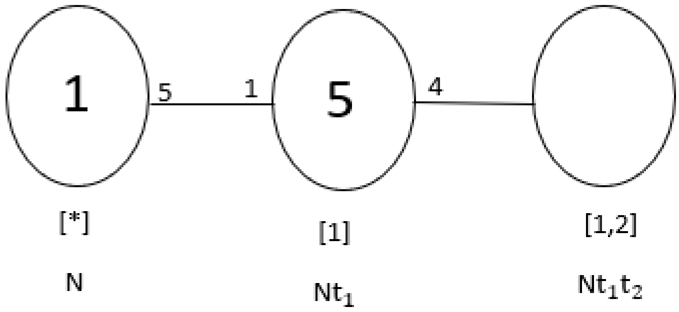
$= N(t_1)^2$

$= N \epsilon$ [*]

Choose 2 from {2, 3, 4, 5}

$Nt_1t_2 \epsilon$ [1,2]

Cayley Diagram



Third double coset

$Nt_1t_2N = \{ N(t_1t_2)^n \mid n \in N \} = \{Nt_1t_2, Nt_1t_3, ..., Nt_2t_1\}$

The The point-stabiliser 1,2, $N^{1,2}$ is given by $< (3, 4, 5) >$

But $Nt_1t_2 = Nt_1t_3$

We will use our relation

$t_1t_2t_3t_1 = (1,2,3)$ we will multiply the right by t_1 and t_3 we get $t_1t_2t_3t_1t_1t_3 = (1, 2, 3)t_1t_3$ thus we have $t_1t_2 = (1, 2, 3)t_1t_3$ since $t_1^2 = e$ and $t_3^2 = e$, we will label the relation $t_1t_2 = (1, 2, 3)t_1t_3$ as (α). Thus we have

Nt_1t_2

$= (1, 2, 3)t_1t_3$

$= Nt_1t_3$

$= N(t_1t_2)^{(2,3,4)}$

$= Nt_1t_3$

So $(2, 3, 4) \in N^{(1,2)}$

But we also have $Nt_1t_2 = Nt_1t_4$

Using our relation $t_1t_2t_3t_1 = (1,2,3)$ conjugated by (3,4) gives us $t_1t_2t_4t_1 = (1,2,4)$ then we will multiply on the right by t_1 and t_4 which gives us $t_1t_2 = (1,2,4)t_1t_4$ we will label this relation (β).

Therefore we have

Nt_1t_2

$= Nt_1t_4$

$=N(t_1t_2)^{(2,4,5)}$

$= Nt_1t_4$

$= Nt_1t_2$

So $(2, 4, 5) \epsilon N^{(1,2)}$

Next $Nt_1t_2 = Nt_1t_5$, we will use our relation $t_1t_2t_3t_1 = (1,2,3)$ conjugated by (3,5) to get $t_1t_2t_5t_1 = (1,2,5)$ we will multiply on the right by t_1 and t_5 which gives us $t_1t_2 = (1,2,5)t_1t_5$ we will label this relation (γ). Thus we have

Nt_1t_2

$= Nt_1t_5$

$=N(t_1t_5)^{(2,5,4)}$

$= Nt_1t_5$

$= Nt_1t_2$

So $(2, 5, 4) \epsilon N^{(1,2)}$

Therefore the coset stabiliser $N^{(1,2)} \geq < N^{1,2}, (2, 3, 4), (2, 4, 5), (2, 5, 4)> = <(3, 4, 5), (2, 3, 4), (2, 4, 5), (2, 5, 4)>$ therefore $N^{(1,2)} \geq <(3, 4, 5), (2, 3, 4), (2, 4, 5), (2, 5, 4))>$

The number of single right cosets of the double coset $Nt_1t_2N = [1,2]$ is given by $\frac{|N|}{|N^{(1,2)}|} = \frac{60}{12} = 5$

The orbits of $N^{(1,2)}$ on $X = \{1, 2, 3, 4, 5\}$ are $\{1\}$,] $\{2, 3, 4, 5\}$

Multiply Nt_1t_2 by a representative of each orbit and determine its double coset

Choose 1 from {1}

$Nt_1t_2t_1 \epsilon [1]$

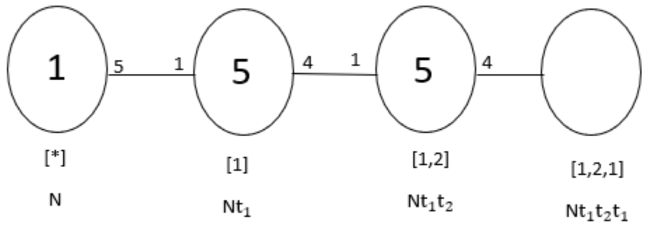
Choose 2 from {2, 3, 4, 5}

$Nt_1t_2t_2$

$= Nt_1(t_2)^2$

$= Nt_1 \epsilon [1]$

Cayley Diagram



Fourth double coset

$Nt_1t_2t_1N = \{ N(t_1t_2t_1)^n \mid n \epsilon N \} = \{Nt_1t_2t_1, Nt_1t_3t_1,..., Nt_2t_1t_2\}$

The The point-stabiliser 1, 2, 1, $N^{1,2,1}$ is given by $< (3, 4, 5) >$

But $Nt_1t_2t_1 = Nt_1t_3t_1 = Nt_1t_4t_1 = Nt_1t_5t_1$

Using our relation (α) we have $t_1t_2 = (1, 2, 3)t_1t_3$ if we multiply on the right by t_1 we have $t_1t_2t_1 = (1, 2, 3)t_1t_3t_1$ Therefore we have

$Nt_1t_2t_1$

$= (1, 2, 3)t_1t_3t_1$

$= Nt_1t_3t_1$

$= N(t_1t_2t_1)^{(2,3,4)}$

$= Nt_1t_3t_1$

$= Nt_1t_2t_1$

So $(2, 3, 4) \epsilon N^{(1,2,1)}$

We also have $Nt_1t_2t_1 = Nt_1t_4t_1$

We will use our relation (β) $t_1t_2 = (1,2,4)t_1t_4$ we will multiply on the right by t_1 which gives us $t_1t_2t_1 = (1,2,4)t_1t_4t_1$, therefore we have

$Nt_1t_2t_1$

$= (1, 2, 4)t_1t_4t_1$

$= Nt_1t_4t_1$

$= N(t_1t_2t_1)^{(2,4,5)}$

$= Nt_1t_4t_1$

$= Nt_1t_2t_1$

So (2, 4, 5) ϵ $N^{(1,2,1)}$

Next we have $Nt_1t_2t_1 = Nt_1t_5t_1$ we will use our relation (γ) $t_1t_2 = (1,2,5)t_1t_5$ we will then multiply on the right by t_1 which gives us $t_1t_2t_1 = (1,2,5)t_1t_5t_1$ therefore we have

$Nt_1t_2t_1$

$= (1, 2, 5)t_1t_5t_1$

$= Nt_1t_5t_1$

$= N(t_1t_2t_1)^{(2,5,3)}$

$= Nt_1t_5t_1$

$= Nt_1t_2t_1$

So (2, 5, 3) ϵ $N^{(1,2,1)}$

But $Nt_1t_2t_1$

$= t_1\underline{t_2t_1}$

Conjugate our relation $t_1t_2 = (1,2,3)\ t_1t_3$ by (2,1) gives $t_2t_1 = (213)t_2t_3$

Therefore,

$t_1\underline{t_2t_1}$

$=t_1(2,1,3)t_2t_3$

$=(2,1,3)\underline{t_3t_2}t_3$

Conjugate our relation $t_1t_2 = (1,2,3)t_1t_3$ by (1,3) which gives $t_3t_2 = (321)t_3t_1$

Thus,

$Nt_1\underline{t_2t_1}$

$= N\underline{t_3t_2}t_3$

$=Nt_3(1,3)t_3t_1$

$= Nt_1t_3t_1$

Therefore our coset stabiliser $N^{(1,2,1)} = < N^{1,2,1}$, (2,5,3), (2,3,4), (2, 4, 5)$> = <$ (1, 2, 3, 4, 5)$>$

The number of single right cosets of the double coset $Nt_1t_2t_1N$ = [1,2,1] is given by $\frac{|N|}{|N^{(1,2,1)}|} = \frac{60}{60} = 1$

The orbits of $N^{(1,2,1)}$ on X = {1, 2, 3, 4, 5} are {1, 2, 3, 4, 5}

We will multiply $Nt_1t_2t_1$ by a representative of each orbit and determine its double coset.

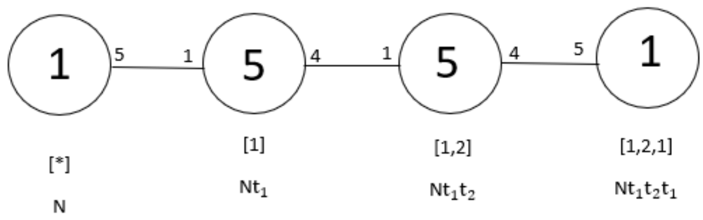
Choose 1 from {1, 2, 3, 4, 5}

$Nt_1t_2t_1t_1$

$= Nt_1t_2(t_1)^2$

$= Nt_1t_2 \epsilon [1,2]$

Cayley Diagram



We will write G as a union of the right cosets. This us a cayley diagram of G over N

$$|G| \leq \frac{|N|}{|N|} + \frac{|N|}{|N^{(1)}|} + \frac{|N|}{|N^{(1,2)}|} + \frac{|N|}{|N^{(1,2,1)}|} * N$$
$$= (1+5+5+1) * |N|$$
$$= (12) * 60$$
$$= 720$$

This implies that our G's upper-bound limit is of order 720.

4.2 $\frac{3^{*4}:D_8}{(13)(24)t_2t_1t_4=t_3t_4t_1} \cong$ 2:S$_5$

We will prove that the progenitor 3^4:D_8 where 3^4:D_8 = <x, y> and x ∼ (1, 2, 3, 4), y ∼ (2, 4), factored by one relation is isomorphic to 2:S_5. Let G ≅ $\frac{3^{*4}:N}{(13)(24)t_2t_1t_4=t_3t_4t_1}$. Thus we show that G ∼ 2:S_5.

Relation Given:

$(xt)^6$

Expanding the relation given

where, x = (1, 2, 3, 4) and y = (2, 4)

$(xt)^6$ = xtxtxtxtxtxt (Expand)

$= xtxtxtxtxxx^{-1}txt$

$= xtxxtxx^2x^{-2}tx^2t^xt$

$= xtxtxtxx^2x^{-2}tx^2t^xt$

$= xtxx^4x^{-4}tx^4t^{x^3}t^{x^2}t^xt$

$= xx^5x^{-5}tx^5t^{x^4}t^{x^3}t^{x^2}t^xt$

$= x^6t^{x^5}t^{x^4}t^{x^3}t^{x^2}t$ (simplify)

$(1, 2, 3, 4)^6\ t^{(1,2,3,4)^5}t^{(1,2,3,4)^4}t^{(1,2,3,4)^3}t^{(1,2,3,4)^2}t^{(1,2,3,4)}$ (Plug in x)

$(1, 3)(2, 4)\ t_2t_1t_4t_3t_2t_1$ (Simplify)

$(1, 3)(2, 4)t_2t_1t_4 = t_3t_4t_1$

We label our t_i's as follows:

Labeling			
1	2	3	4
1	2	1^2	2^2

First Double Coset

NeN = { $N(e)^n$ | n ϵ N } = { N }

The coset Stabilizer of N = Ne is N

The number of single right cosets of the double coset NeN = [*] is given by $\frac{|N|}{|N|} = \frac{8}{8} = 1$

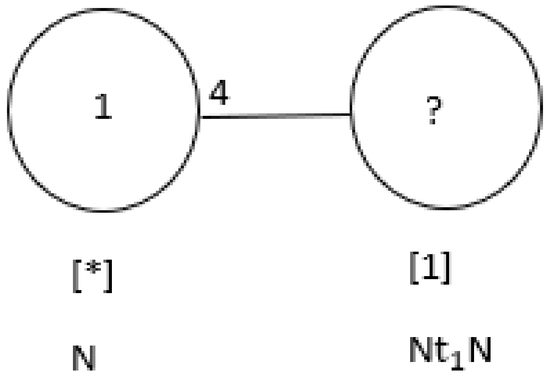
The orbits of N on X = {1, 2, 3, 4} are {1, 2, 3, 4} we will now choose an orbit representative and multiply it by N on the right and determine its double coset.

Choose 1 from {1, 2, 3, 4}

$Nt_1\ \epsilon$ [1]

This tells us that four elements move forward to the double coset [1]

Cayley Diagram



Second Double Coset

Nt_1N = { $N(t_1)^n$ | n ϵ N } = {Nt_1, Nt_2, Nt_3, Nt_4}

The point-stabiliser of 1, N^1 is given by { (2, 4)}

The coset Stabilizer $N^{(1)}$ = { (2, 4) }

The number of single right cosets in the double coset Nt_1N = [1] $\frac{|N|}{|N^{(1)}|} = \frac{8}{2} = 4$

The orbits for $N^{(1)}$ on X = {1,2,3,4} are: {1},{2,4}, {3}

Multiply Nt_1 by a representative of each orbit and determine its double coset.

<u>Choose 1 from {1}</u>

Nt_1t_1

= Nt_1^2 (Simplify)

= Nt_3 ϵ [1] ($t_3 = t_1^2$)

This means that one element loops back the double coset [1]

<u>Choose 2 from {2,4}</u>

Nt_1t_2 ϵ [12]

This means two elements move forward to the double coset [1, 2]

<u>Choose 3 from {3}</u>

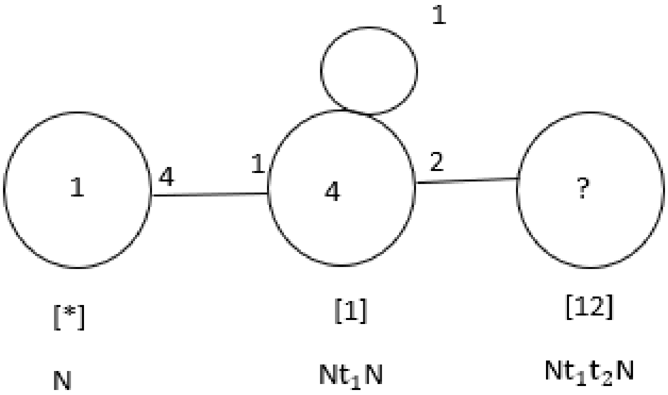
Nt_1t_3 ($t_3 = t_1^2$)

= $Nt_1t_1^2$

= N ϵ [*] ($t_1^3 = e$)

This means one element moves back to the double coset [*]

Cayley Diagram



Third Double Coset

$Nt_1t_2N = \{ N(t_1t_2)^n \mid n \epsilon N \} = \{Nt_1t_2, Nt_4t_1, Nt_2t_3, Nt_3t_4, Nt_2t_1, Nt_4t_3, Nt_3t_2, Nt_1t_4 \}$

The point Stabilizers of 1, 2, N^{12} is given by { 1 }

The coset Stabilizers $N^{(12)} = \{ 1 \}$

The number of single right cosets in the double coset $Nt_1t_2N = [1,2]$ is given by $\frac{|N|}{|N^{(12)}|} = \frac{8}{1} = 8$

The orbits of $N^{(12)}$ on X = {1,2,3,4} are {1}, {2},{3},{4}

Multiply Nt_1t_2 by a representative of each orbit and determine its double coset.

Choose 1 from {1}

$Nt_1t_2t_1 \ \epsilon \ [1, 2, 1]$

This tells us that one element moves to the double coset [1, 2, 1]

Choose 3 from {3} $Nt_1t_2t_2$

$Nt_1t_2^2$ (but $t_2^2 = 4$)

$= Nt_1t_4$

$= N(t_1t_2)^{(24)} \ \epsilon \ [12]$

This means one element loops back to the double coset [1, 2]

Choose 4 from {4}

$Nt_1t_2t_4$

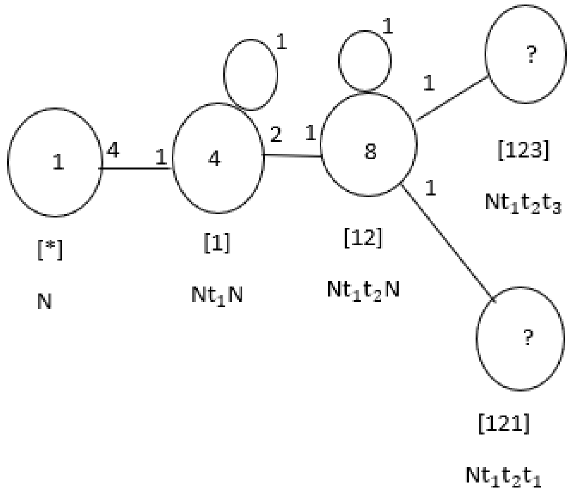
$= Nt_1t_2t_2^2$ (Since $t_4 = t_2^2$)

$= Nt_1t_2$ ($t_2^3 = e$)

$Nt_1 \epsilon$ [1]

This means one element moves back to the double coset [1]

Cayley Graph



Fourth Double Coset

$Nt_1t_2t_1N = \{ N(t_1t_2t_1)^n \mid n \,\epsilon\, N\} = \{ Nt_1t_2t_1, Nt_4t_1t_4, Nt_2t_3t_2, Nt_3t_4t_3, Nt_2t_1t_2, Nt_4t_3t_4, Nt_3t_2t_3, Nt_1t_4t_1\}$

The point-stabiliser of 1, 2, 1, N^{121} is given by {e}

The coset Stabilizer $N^{(121)}$ = {e}

The number of single right cosets in the double coset $Nt_1t_2t_1N$ = [1,2,1] is given by $\frac{|N|}{|N^{(121)}|}$ $= \frac{8}{1} = 8$

The orbits of N^{121} on X= {1,2,3,4} are {1}, {2},{3},{4}

Multiply $Nt_1t_2t_1$ by a representative of each orbit to determine its double coset.

Choose 1 from {1}

$Nt_1t_2t_1t_1$

$= Nt_1t_2t_1^2$ (Simplify)

$= Nt_1t_2t_3$ ($t_1^2 = t_3$)

$= Nt_1t_2t_3 \,\epsilon$ [123]

This means one element moves to the double coset [1, 2, 3]

Choose 2 from {2}

$Nt_1t_2t_1t_2 \epsilon [1212]$

One element moves to the double coset [1, 2, 1, 2]

Choose 3 from {3}

$Nt_1t_2t_1t_3$

$= Nt_1t_2t_1t_1^2 \; (t_3 = t_1^2)$

$= Nt_1t_2t_1^3 \; (t_1^3 = e)$

$= Nt_1t_2 \epsilon [12]$

One element moves back to the double coset [1, 2]

Choose 4 from {4}

$Nt_1t_2t_1t_4$

$= t_1\underline{t_2t_1t_4}$

Using relation $(13)(24)t_2t_1t_4 = t_3t_4t_1$ conjugated by (14)(23)

$(13)(24)t_2t_1t_4^{14)(23)} = t_3t_4t_1$

Which gives $(42)(31)t_3t_4t_1 = t_2t_1t_4$

Then

$Nt_1t_2t_1t_4$

$= t_1\underline{t_2t_1t_4}$

$= t_1(42)(31)t_3t_4t_1$

$= (42)(31)t_3t_3t_4t_1$

$= (42)(31)t_3^2t_4t_1$

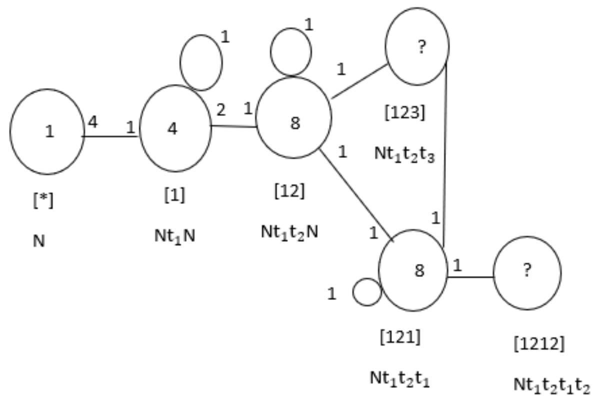
$= (42)(31)t_1t_4t_1 \; (t_3^2 = t_1)$

$= Nt_1t_4t_1$

$= N(t_1t_2t_1)^{(2,4)} \epsilon [1, 2, 1]$

This means one element loops back to the double coset [1, 2, 1]

Cayley Diagram



Fifth Double Coset

$Nt_1t_2t_3N = \{ N(t_1t_2t_3)^n \mid n \epsilon N\} = \{Nt_1t_2t_3, Nt_4t_1t_2, Nt_2t_3t_4, Nt_3t_4t_1, Nt_2t_1t_4, Nt_4t_3t_2, Nt_3t_2t_1, Nt_1t_4t_3 \}$

The point-stabilisers of 1, 2, 3, N^{123} is given by {e}

We have $N\underline{t_1t_2t_3}$

Using the relation $(13)(24)t_2t_1t_4 = t_3t_4t_1$

Conjugate the relation by (12)(34) which gives, $(13)(24)t_1t_2t_3 = t_4t_3t_2$

Then we have

$N\underline{t_1t_2t_3}$

$= (24)(13)t_4t_3t_2$

$= Nt_4t_3t_2$

$= N(t_1t_2t_3)^{(14)(23)} \epsilon N^{(123)}$

Therefore, the coset stabiliser $N^{123} \leq N^{(123)} = \{ e, (1, 3)(2, 4) \}$

The number of single right cosets in the double coset $Nt_1t_2t_3N = [1,2,3]$ is given by $\frac{|N|}{|N^{(123)}}$ $= \frac{8}{2} = 4$

The orbits of N^{123} on X = {1, 2, 3, 4} are {1, 4},{2, 3}

We will multiply $Nt_1t_2t_3$ by a representative of each orbit and determine its double coset

<u>Choose 1 from {1,4}</u>

$Nt_1t_2t_3t_1$

$= Nt_1t_2t_1^2t_1$ (Since $t_1^2 = t_3$)

$= Nt_1t_2t_1^3$

$= Nt_1t_2$

$= Nt_1t_2 \epsilon [1, 2]$

Two elements move back to the double coset [1, 2]

<u>Choose 2 from {2,3}</u>

$Nt_1t_2t_3t_2$

$= N\underline{t_1t_2t_3}t_2$

Since Conjugating our relation $(13)(24)t_2t_1t_4 = t_3t_4t_1$ by $(12)(34)$

Then, $(13)(24)t_2t_1t_4^{(12)(34)} = t_3t_4t_1^{(12)(34)}$

This gives us, $(24)(13)t_1t_2t_3 = t_4t_3t_2$

Then

$Nt_1t_2t_3t_2$

$= N\underline{t_1t_2t_3}t_2$

$= (31)(42)t_4t_3t_2t_2$

$= (31)(42)t_4t_3t_2^2$ (Simplify)

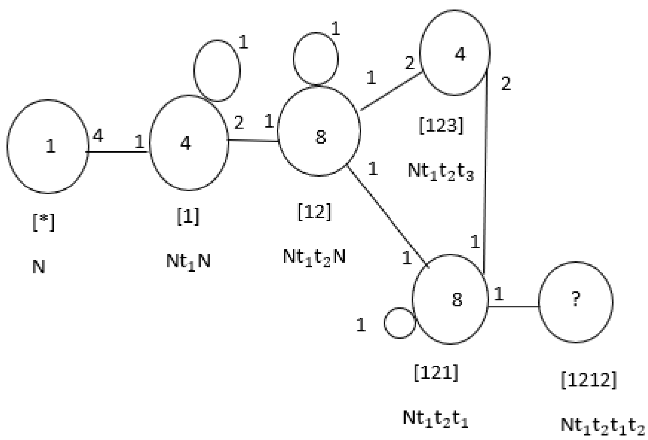
$= (31)(42)t_4t_3t_4$

$= Nt_4t_3t_4$

$= N(t_1t_2t_1)^{(14)(23)} \epsilon [1, 2, 1]$

This tells us that two elements move to the double coset [1, 2, 1]

<u>Cayley Diagram</u>



Sixth Double Coset

$Nt_1t_2t_1t_2N$ = { $N(t_1t_2t_1t_2)^n$ | n ϵ N} = {$Nt_4t_1t_4t_1$, $Nt_2t_3t_2t_3$, $Nt_1t_2t_1t_2$, $Nt_3t_4t_3t_4$, $Nt_2t_1t_2t_1$, $Nt_4t_3t_4t_3$, $Nt_3t_2t_3t_2$, $Nt_1t_4t_1t_4$ }

The point-stabiliser of 1, 2, 1, 2, N^{1212} is given by { 1 }

We have, $Nt_1t_2t_1t_2$

= $Nt_1t_2t_1t_2$

= $Nt_1\underline{t_2t_1t_4}t_4$

Since conjugating the relation $(13)(24)t_2t_1t_4 = t_3t_4t_1$ by (14)(23)

Then, $(13)(24)t_2t_1t_4^{(14)(23)} = t_3t_4t_1^{(14)(23)}$

Which gives us, $(42)(31)t_3t_4t_1 = t_2t_1t_4$

Then

$Nt_1t_2t_1t_2$

= $Nt_1t_2t_1t_2$

= $Nt_1\underline{t_2t_1t_4}t_4$

= $t_1(42)(31)t_3t_4t_1t_4$

= $(42)(31)t_3t_3t_4t_1t_4$

= $Nt_3t_3t_4t_1t_4$

= $N(t_3)^2t_4t_1t_4$

= $Nt_1t_4t_1t_4$

Then, $N(t_1t_2t_1t_2)^{(24)}$ ϵ $N^{(1212)}$

Thae coset stbiliser $N^{1212} \geq N^{(1212)}$ = {e, (2,4)}

The number of single right cosets in the double coset $Nt_1t_2t_1t_2N$ = [1,2,1,2] is given by $\frac{|N|}{|N^{(1212)}|} = \frac{8}{2} = 4$

The orbits of $N^{(1212)}$ on X = {1,2,3,4} are {1}, {3}, {2,4}

We will multiply $Nt_1t_2t_1t_2$ by a representative of each orbit to determine its double coset.

Choose 1 from {1}

$Nt_1t_2t_1t_2t_1$ ϵ [12121]

This means one element moves forward to the double coset [1, 2, 1, 2, 1]

Choose 2 from {2,4}

$Nt_1t_2t_1t_2t_2$

= $Nt_1t_2t_1t_2^2$

= N $t_1\underline{t_2t_1t_4}$

Since conjugating the relation $(13)(24)t_2t_1t_4 = t_3t_4t_1$ by $(14)(23)$

Then $(13)(24)t_2t_1t_4^{(14)(23)} = t_3t_4t_1^{(14)(23)}$

Which gives: $(42)(31)t_3t_4t_1 = t_2t_1t_4$

Then

$Nt_1t_2t_1t_2t_2$

$= Nt_1t_2t_1t_2^2$

$= N\ t_1\underline{t_2t_1t_4}$

$= t_1(42)(31)t_3t_4t_1$

$= (42)(31)t_3t_3t_4t_1$

$= (42)(31)t_3^2t_4t_1$

$= (42)(31)t_1t_4t_1$

$= Nt_1t_4t_1$

$= N(t_1t_2t_1)^{(2,4)}\ \epsilon\ [1, 2, 1]$

This means two elements move back to the double coset [1, 2, 1]

<u>Choose 3 from {3}</u>

$Nt_1t_2\underline{t_1t_2t_3}$

$= t_1t_2(31)(42)t_4t_3t_2$

Since Conjugating the relation $(13)(24)t_2t_1t_4 = t_3t_4t_1$ by $(13)(24)$

Then, $(13)(24)t_2t_1t_4^{(13)(24)} = t_3t_4t_1^{(13)(24)}$

Which gives, $(31)(42)t_4t_3t_2 = t_1t_2t_3$

$Nt_1t_2t_1t_2t_3$

$= (31)(42)t_3t_4t_4t_3t_2$

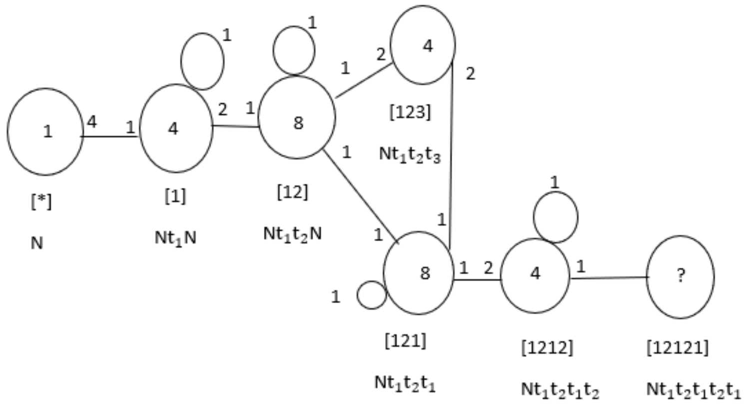
$= (31)(42)t_3t_2t_3t_2$

$= Nt_3t_2t_3t_2\ \epsilon\ [1212]$

since $Nt_3t_2t_3t_2$ is an element of [1212]

This tells us one element loops back to the double coset [1, 2, 1, 2]

<u>Cayley Diagram</u>



Seventh Double Coset

$Nt_1t_2t_1t_2t_1N = \{ N(t_1t_2t_1t_2t_1)^n \mid n \epsilon N\} = \{Nt_1t_2t_1t_2t_1, Nt_4t_1t_4t_1t_4, Nt_2t_3t_2t_3t_2,$
$Nt_3t_4t_3t_4t_3, Nt_2t_1t_2t_1t_2, Nt_4t_3t_4t_3t_4\ Nt_3t_2t_3t_2t_3, Nt_1t_4t_1t_4t_1 \}$

The point-stabiliser of 1, 2, 1, 2, 1, N^{12121} is given by { 1 }

Next

$Nt_1t_2t_1t_2$

$= Nt_1t_2t_1t_4t_4\ (t_2 = t_4^2 = t_4t_4)$

$= Nt_1\underline{t_2}t_1t_2t_1$

$= Nt_1t_4\underline{t_4t_1t_2}t_1$

Since conjugating the relation $(13)(24)t_2t_1t_4 = t_3t_4t_1$ by (1234)

Then, $(13)(24)t_2t_1t_4^{(1234)} = t_3t_4t_1^{(1234)}$

Which gives, $(24)(31)t_3t_2t_1 = Nt_4t_1t_2$

Then we have

$Nt_1t_2t_1t_2$

$= Nt_1t_2t_1t_4t_4$

$= Nt_1\underline{t_2}t_1t_2t_1$

$= Nt_1t_4\underline{t_4t_1t_2}t_1$

$= t_1t_4(24)(31)t_3t_2t_1t_1$

$= (24)(31)t_3t_2\underline{t_3t_2t_1}t_1$

Since Conjugating the relation $(13)(24)t_2t_1t_4 = t_3t_4t_1$ by (24)

Then, $(13)(24)t_2t_1t_4^{(24)} = t_3t_4t_1^{(24)}$

Which gives $(24)(31)t_3t_2t_1 = t_4t_1t_2$

Then

$Nt_1t_2t_1t_2$

$= Nt_1t_2t_1t_4t_4$

$= Nt_1\underline{t_2}t_1t_2t_1$

$= Nt_1t_4\underline{t_4t_1t_2}t_1$

$= t_1t_4(24)(31)t_3t_2t_1t_1$

$= (24)(31)t_3t_2t_3t_2\underline{t_1t_1}$

$=(24)(31)t_3t_2t_3t_2t_3\underline{t_3t_3t_3}$

$= (24)(31)t_3t_2\underline{t_3}t_2t_3$

$= (24)(31)\underline{t_3t_2t_1}t_1t_2t_3$

$= (24)(31)(13)(42)t_4t_1t_2\underline{t_1t_2t_3}$

Since conjugating the relation $(13)(24)t_2t_1t_4 = t_3t_4t_1$ by $(13)(24)$

Then, $(13)(24)t_2t_1t_4^{(13)(24)} = t_3t_4t_1^{(13)(24)}$

Which gives, $(31)(42)t_1t_3t_2 = t_1t_2t_3$

Then

$Nt_1t_2t_1t_2$

$= Nt_1t_2t_1t_4t_4$

$= Nt_1\underline{t_2}t_1t_2t_1$

$= Nt_1t_4\underline{t_4t_1t_2}t_1$

$= t_1t_4(24)(31)t_3t_2t_1t_1$

$= (24)(31)t_3t_2t_3t_2\underline{t_1t_1}$

$=(24)(31)t_3t_2t_3t_2t_3\underline{t_3t_3t_3}$

$= (24)(31)t_3t_2\underline{t_3}t_2t_3$

$= (24)(31)\underline{t_3t_2t_1}t_1t_2t_3$

$= (24)(31)(13)(42)t_4t_1t_2\underline{t_1t_2t_3}$

$= (24)(31)(13)(42)t_4t_1t_2(31)(42)t_4t_3t_2$

$= (24)(31)(13)(42)(31)(42)t_2t_3t_4t_4t_3t_2$

$= (21)(31)(13)(42)(31)(42)t_2t_3\underline{t_4t_4}t_3t_2$

$= (24)(31)(13)(42)(31)(42)t_2t_3t_2t_3t_2$

$= Nt_2t_3t_2t_3t_2$

$\Longrightarrow\ N(t_1t_2t_1t_2t_1)^{(1234)}\ \epsilon\ N^{(12121)}$

Next

$Nt_1t_2t_1t_2$

$= Nt_1\underline{t_2t_1t_4}t_4$ ($t_2 = t_4^2 = t_4t_4$)

Reasoning: Conjugate the relation

$(13)(24)t_2t_1t_4 = t_3t_4t_1$ by $(14)(23)$

Then $(13)(24)t_2t_1t_4^{(14)(23)} = t_3t_4t_1^{(14)(23)}$

This gives us, $(42)(31)t_3t_4t_1 = t_2t_1t_4$

Then

$Nt_1t_2t_1t_2$

$= Nt_1\underline{t_2t_1t_4}t_4$

$= t_1(42)(31)t_3t_4t_1t_4$

$= (42)(31)\underline{t_3t_3}t_4t_1t_4t_1t_2t_1t_2$

$= (42)(31)t_1t_4t_1t_4$

So, we have $Nt_1t_2t_1t_2t_1 = Nt_1t_4t_1t_4t_1$

$\implies N(t_1t_2t_1t_2t_1)^{(2,4)} \epsilon N^{(1,2,1,2,1)}$

$N^{(1,2,1,2,1)} \geq <(1, 2, 3, 4), (2, 4)> = \{N\}$

The number of single right cosets in the double coset $Nt_1t_2t_1t_2t_1 = [1,2,1,2,1]$ is given by $\frac{|N|}{|N^{(12121)}|} = \frac{8}{8} = 1$

The orbits of $N^{(12121)}$ on X = {1,2,3,4} is {1,2,3,4}. We will multiply $Nt_1t_2t_1t_2t_1$ on the right by an orbit representative and determine its double coset.

<u>Choose 1 from {1234}</u>

$Nt_1t_2t_1t_2t_1t_1$

$t_1t_2t_1t_2\underline{t_1t_1}$

$= t_1t_2\underline{t_1t_2t_3}$

Since conjugating the relation $(13)(24)t_2t_1t_4 = t_3t_4t_1$ by $(13)(24)$

Then, $(13)(24)t_2t_1t_4^{(13)(24)} = t_3t_4t_1^{(13)(24)}$

Therefore, $(31)(42)t_4t_3t_2 = t_1t_2t_3$

Then

$Nt_1t_2t_1t_2t_1t_1$

$t_1t_2t_1t_2\underline{t_1t_1}$

$= t_1t_2\underline{t_1t_2t_3}$

$= t_1t_2(31)(42)t_4t_3t_2$

$= (31)(42)t_3\underline{t_4t_4}t_3t_2$

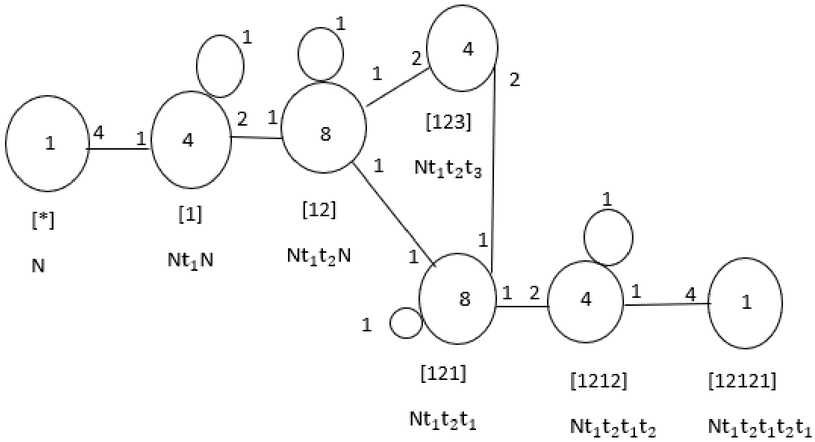
$= (31)(42)t_3t_2t_3t_2$

$= Nt_3t_2t_3t_2$

$= N(t_1t_2t_1t_2)^{(13)} \epsilon$ [1212]

This tells us four elements move back to the double coset [1, 2, 1, 2]

Cayley Diagram



4.3 $\frac{2^{*110}:PSL(2,11)}{(xyxy^{-1}xyxyxy^{-1}xy^{-1}x)^3t_{106}t_{92}t_{24}} \cong \mathbf{J}_1$

We will prove that the progenitor 2^{*110}:PSL(2,11), where 2^{*110}:PSL(2,11) = < x, y> and x ∼ (1, 2)(3, 8)(4, 11)(5, 13)(6, 17)(7, 20)(9, 26)(10, 23)(12, 34)(14, 40)(15, 42)(16, 45)(18, 48)(19, 50)(21, 52)(22, 54)(24, 58)(25, 61)(27, 57)(28, 51)(29, 68)(30, 32)(31, 59)(33, 44)(35, 76)(36, 73)(38, 71)(39, 79)(41, 85)(43, 66)(46, 65)(47, 89)(49, 83)(53, 93)(55, 90)(56, 96)(60, 78)(62, 92)(63, 101)(64, 102)(70, 86)(72, 94)(74, 104)(75, 77)(80, 108)(81, 109)(82, 103)(84, 106)(87, 88)(91, 95)(99, 107)(100, 110), y ∼ (1, 3, 9)(2, 5, 14)(4, 12, 35)(6, 18, 49)(7, 21, 53)(8, 23, 56)(10, 29, 69)(11, 31, 71)(13, 37, 74)(15, 43, 88)(16, 22, 55)(17, 46, 58)(19, 32, 48)(20, 51, 41)(24, 59, 98)(25, 45, 65)(26, 64, 92)(27, 67, 36)(30, 70, 104)(33, 72, 95)(34, 73, 85)(38, 50, 78)(39, 80, 105)(40, 83, 84)(42, 86, 75)(44, 89, 96)(52, 63, 66)(54, 94, 101)(57, 61, 100)(60, 79, 91)(62, 97, 99)(68, 77, 82)(76, 102, 108)(81, 110, 87)(90, 109, 103)(93, 106, 107), factored by one relation is isomorphic to the janko sporadic simple group J_1. Let G ≅ $\frac{2^{*110}:PSL(2,11)}{(xyxy^{-1}xyxyxy^{-1}xy^{-1}x)^3t_{106}t_{92}t_{24}}$. Thus we show that G ∼ J_1

<u>Expanded Relation</u>

Relation 1: $(xyxy^{-1}xyxyxy^{-1}xy^{-1}x)^3t_{106}t_{92}t_{24}$

<u>First Double Coset</u>

NeN = {N(e)^n | n ϵ N } = {N}

The coset Stabilizer of the coset N = Ne is N.

The number of single right cosets the double coset NeN = [*] is given by $\frac{|N|}{|N|} = \frac{660}{660} = 1$

The orbits of N on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110} is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110}

We will now choose an orbit representative and multiply it by N on the right and determine its double coset.

Choose 1 from {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110}

$Nt_1 \epsilon$ [1]

This tells us that one-hundred ten elements move forward to the double coset [1]

Cayley Diagram

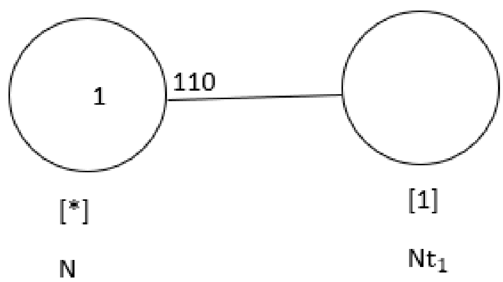


Figure 4.1: Cayley Diagram of [*] for J_1

Second Double Coset

$Nt_1N = \{ N(t_1)^n \mid n \epsilon N\} = \{Nt_1, Nt_2,..., Nt_{110}\}$

The point-stabiliser of 1, N^1 is given by $<(xyxyxy^{-1}xy^{-1}x),$
$(xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy)>$

The coset Stabilizer of $N^{(1)} = <(xyxyxy^{-1}xy^{-1}x), (xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy)>$

The number of single right cosets in the double coset Nt_1N = [1] is given by $\frac{|N|}{|N^{(1)}|} = \frac{660}{6}$ = 110

The orbits for $N^{(1)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110} are {1}, {4}, {42, 102, 85}, {45, 64, 56}, {50, 110, 87}, {83, 103, 90}, {2, 105, 93, 7, 22, 68}, {3, 38, 78, 9, 28, 81}, {5, 44, 31, 16, 20, 15}, {6, 94, 108, 19, 18, 73}, {8, 75, 101, 25, 63, 34}, {10, 82, 13, 30, 27, 39}, {11, 97, 91, 33, 43, 57}, {12, 40, 84, 35, 47, 109}, {14, 61, 79, 41, 21, 98}, {17, 99, 100, 29, 36, 59}, {23, 49, 48, 52, 92, 32}, {24, 106, 80, 60, 62, 74}, {26, 76, 54, 66, 65, 86}, {37, 96, 72, 77, 107, 71}, {46, 53, 51, 67, 104, 55}, {58, 89, 88, 70, 69, 95}

Multiply Nt_1 by a representative of each orbit and determine its double coset.

Choose 1 from {1}

Nt_1t_1

$= N(t_1)^2$

$= N \epsilon [*]$

Choose 4 from {4}

$Nt_1t_4 \epsilon [1,4]$

Choose 42 from {42, 102, 85}

Nt_1t_{42}

$= (yxyxy^{-1}xyxy^{-1}xy^{-1})t_{83}$

$= Nt_{83}$

$= N(t_1)^{xy^{-1}xy} \epsilon [1]$

Choose 45 from {45, 64, 56}

Nt_1t_{45}

$= (yxyxyxy^{-1})t_{110}t_{103}$

$= Nt_{110}t_{103}$

$= N(t_1t_4)^{xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}} \epsilon [1,4]$

Choose 50 from {50, 110, 87}

Nt_1t_{50}

$= (xy^{-1}xyxy^{-1}xyx)t_1$

$= Nt_1$

$Nt_1 \epsilon [1]$

Choose 83 from {83, 103, 90}

Nt_1t_{83}

$= (yxyxy^{-1}xyxy^{-1}xy^{-1})t_{42}$

$=Nt_{42}$

$= N(t_1)^{(xy^{-1}xyxy^{-1})^2}$ ϵ [1]

Choose 2 from {2, 105, 93, 7, 22, 68}

Nt_1t_2 ϵ [1,2]

Choose 3 from {3, 38, 78, 9, 28, 81}

Nt_1t_3

$= (yxyxyxyxy^{-1}x)t_{110}$

$= Nt_{110}$

$= N(t_1)^{xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}}$ ϵ [1]

Choose 5 from {5, 44, 31, 16, 20, 15}

Nt_1t_5

$= (y^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}x)t_4$

$= Nt_4$

$= N(t_1)^{xy^{-1}xyxyxy^{-1}xyxy^{-1}x}$ ϵ [1]

Choose 6 from {6, 94, 108, 19, 18, 73}

Nt_1t_6

$= (y^{-1}xy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})t_{80}t_{17}$

$= Nt_{80}t_{17}$

$= N(t_1t_2)^{y^{-1}xyxyx}$ ϵ [1,2]

Choose 8 from {8, 75, 101, 25, 63, 34}

t_1t_8

$= (x^y)t_{93}t_{26}$

$= N(t_1t_2)^{y^{-1}xy^{-1}xyxyxy}$ ϵ [1,2]

Choose 10 from {10, 82, 13, 30, 27, 39}

Nt_1t_{10}

$= (xy^{-1}xy^{-1}x)t_{91}t_{50}$

$= Nt_{91}t_{50}$

$= N(t_1t_2)^{(yxy^{-1}x)^2}$ ϵ [1,2]

Choose 11 from {11, 97, 91, 33, 43, 57}

Nt_1t_{11}

$= (xy^{-1}xy^{-1}xy^{-1}xyxyxy^{-1}xy)t_{96}t_{82}$

$= Nt_{96}t_{82}$

$= N(t_1t_2)^{yxy^{-1}xy^{-1}xyxy} \epsilon [1,2]$

Choose 12 from {12, 40, 84, 35, 47, 109}

Nt_1t_{12}

$= (y^{-1}xy^{-1})t_{54}t_4$

$= Nt_{54}t_4$

$= N(t_1t_2)^{yxy^{-1}xyxyxy^{-1}} \epsilon [1,2]$

Choose 14 from {14, 61, 79, 41, 21, 98}

Nt_1t_{14}

$= (yxy^{-1}xyxy^{-1})t_{62}t_{64}$

$= Nt_{62}t_{64}$

$= N(t_1t_2)^{xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}} \epsilon [1,2]$

Choose 17 from {17, 99, 100, 29, 36, 59}

Nt_1t_{17}

$= (xy^{-1}xyxy^{-1}xy^{-1}x)t_{14}$

$= Nt_{14}$

$= N(t_1)^{xyxyxy^{-1}xy} \epsilon [1]$

Choose 23 from {23, 49, 48, 52, 92, 32}

Nt_1t_{23}

$= (yxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)t_{92}t_{97}$

$= Nt_{92}t_{97}$

$= N(t_1t_2)^{y^{-1}xy^{-1}xyxy^{-1}x} \epsilon [1,2]$

Choose 24 from {24, 106, 80, 60, 62, 74}

Nt_1t_{24}

$= (xyxyxy^{-1}xyxy^{-1}x)t_{92}$

$= Nt_{92}$

$= N(t_1)^{xy^{-1}xy^{-1}xyxyx} \epsilon [1]$

Choose 26 from {26, 76, 54, 66, 65, 86}

Nt_1t_{26}

$= (xyxyxyxyxy^{-1}xy)t_{25}$

$= Nt_{25}$

$= N(t_1)^{xyxy^{-1}xy^{-1}xyxyxyxyxy^{-1}}$ ϵ [1]

Choose 37 from {37, 96, 72, 77, 107, 71}

Nt_1t_{37}

$= (xyxy^{-1}xy^{-1}xyx)t_{32}$

$= Nt_{32}$

$= N(t_1)^{xyxyxy-1xyxyxy^{-1}xy^{-1}}$ ϵ [1]

Choose 46 from {46, 53, 51, 67, 104, 55}

Nt_1t_{46} ϵ [1,46]

Choose 58 from {58, 89, 88, 70, 69, 95}

Nt_1t_{58}

$= (xyxy^{-1}xyxyx)t_{75}t_{90}$

$= Nt_{75}t_{90}$

$= N(t_1t_2)^{xyxyxyxy^{-1}xy}$ ϵ [1,2]

Cayley Diagram

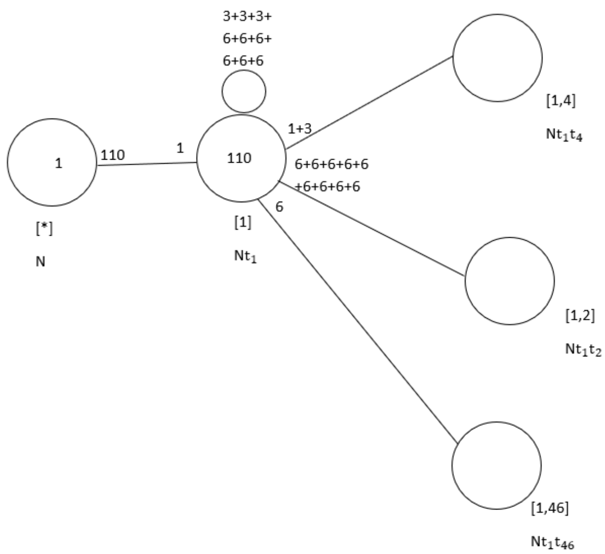


Figure 4.2: Cayley Diagram of [*], [1] for J_1

Third Double Coset

$Nt_1t_2N = \{N(t_1t_2)^n \mid n \epsilon N\} = \{Nt_1t_2, Nt_2t_1,..., Nt_3t_5\}$

The point-stabiliser 1,2, $N^{1,2}$ is given by $< 1 >$

Now $Nt_1t_2 = Nt_{38}t_{67}$

Now $N(t_1t_2)^{(y^{-1}xyxyxyxyxy^{-1}xy)} = Nt_{38}t_{67}$.

Thus $(y^{-1}xyxyxyxyxy^{-1}xy) \epsilon N^{(12)}$

Thus the coset stabiliser $N^{(1,2)} \geq <N^{1,2}, (y^{-1}xyxyxyxyxy^{-1}xy)> =$
$< (y^{-1}xyxyxyxyxy^{-1}xy) >$

The number of single right cosets in the double coset $Nt_1t_2N = [1,2]$ is given by $\frac{|N|}{|N^{(1,2)}|}$
$= \frac{660}{5} = 132$

The orbits for $N^{(1,2)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110} are {1, 38, 50, 87, 28}, {2, 67, 89, 57, 74}, {3, 78, 110, 9, 81}, {4, 40, 83, 90, 47}, {5, 104, 100, 80, 72}, {6, 23, 76, 8, 86}, {7, 44, 91, 17, 13}, {10, 55, 98, 43, 107}, {11, 69, 51, 68, 24}, {12, 84, 103, 35, 109}, {14, 46, 39, 96, 97}, {15, 77, 60, 29, 53}, {16, 61, 106, 27, 95}, {18, 52, 56, 54, 42}, {19, 108, 63, 64, 75}, {20, 93, 30, 59, 33}, {21, 31, 58, 82, 62}, {22, 79, 36, 88, 37}, {25, 49, 92, 101, 102}, {26, 34, 65, 32, 73}, {41, 105, 71, 70, 99}, {45, 48, 94, 85, 66}. We will multiply Nt_1t_2 on the right by an orbit representative and determine its double coset.

Choose 1 from{1, 38, 50, 87, 28}

$Nt_1t_2t_1$

$= (x^y)t_3t_5$

$= N(t_1t_2)^y \epsilon [1,2]$

Choose 2 from {2, 67, 89, 57, 74}

$Nt_1t_2t_2$

$= Nt_1(t_2)^2$

$= Nt_1 \epsilon [1]$

Choose 3 from {3, 78, 110, 9, 81}

$Nt_1t_2t_3$

$= (xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy)t_3t_{39}$

$= Nt_3t_{39}$

$= N(t_1t_2)^{xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy^{-1}} \epsilon [1,2]$

Choose 4 from {4, 40, 83, 90, 47}

$Nt_1t_2t_4$

$= (xy^{-1}xy^{-1}xy^{-1}xyxyxy^{-1}xy)t_{68}t_{22}$

$= Nt_{68}t_{22}$

$= N(t_1t_2)^{xyxyxyxy^{-1}xyxy^{-1}} \epsilon [1,2]$

Choose 5 from {5, 104, 100, 80, 72}

$Nt_1t_2t_5$

$= (xyxy^{-1}xy^{-1}xy^{-1}x)t_{91}$

$= Nt_{91}$

$= N(t_1)^{xyxyxyxyxy^{-1}xyxy^{-1}} \epsilon [1]$

Choose 6 from {6, 23, 76, 8, 86}

$Nt_1t_2t_6$

$= (xyxyxyxyxy^{-1}xy^{-1})t_{91}$

$= Nt_{91}$

$= N(t_1)^{xyxyxyxyxy^{-1}xyxy^{-1}} \epsilon [1]$

Choose 7 from {7, 44, 91, 17, 13}

$Nt_1t_2t_7$

$= (y^{-1}xyxy^{-1}xy)t_{75}$

$= Nt_{75}$

$= N(t_1)^{yxy^{-1}xyxyxyxyxy^{-1}xy^{-1}} \epsilon [1]$

Choose 10 from {10, 55, 98, 43, 107}

$Nt_1t_2t_{10}$

$= (yxyxyxy^{-1}xy^{-1}xyxy^{-1})t_{56}t_{60}$

$= Nt_{56}t_{60}$

$= N(t_1t_2)^{xyxy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy} \epsilon [1,2]$

Choose 11 from {11, 69, 51, 68, 24}

$Nt_1t_2t_{11}$

$= (y^{-1}xy^{-1}xyxy^{-1}xy)t_{10}$

$= Nt_{10}$

$= N(t_1)^{(yx)^2} \epsilon$ [1]

Choose 12 from {12, 84, 103, 35, 109}

$t_1t_2t_{12}$

$= (xyxyxy^{-1}xy^{-1}xyxyxyx)t_{41}$

$= Nt_{41}$

$= N(t_1)^{y^{-1}xyxy^{-1}xy^{-1}xy^{-1}x} \epsilon$ [1]

Choose 14 from {14, 46, 39, 96, 97}

$Nt_1t_2t_{14}$

$= (yxyxy\ )t_{71}t_{33}$

$= Nt_{71}t_{33}$

$= N(t_1t_2)^{(xy^{-1}xyxy)^2} \epsilon$ [1,2]

Choose 15 from {15, 77, 60, 29, 53}

$Nt_1t_2t_{15}$

$= (yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy^{-1})t_{75}t_{47}$

$= Nt_{75}t_{47}$

$= N(t_1t_2)^{xyxy^{-1}xy^{-1}xy} \epsilon$ [1,2]

Choose 16 from {16, 61, 106, 27, 95}

$Nt_1t_2t_{16}$

$= (yxyxy^{-1}xy^{-1}xyxyxy^{-1})t_{48}$

$= Nt_{48}$

$= N(t_1)^{y^{-1}xy^{-1}xyxyxy^{-1}xy^{-1}xy^{-1}x} \epsilon$ [1]

Choose 18 from{18, 52, 56, 54, 42}

$Nt_1t_2t_{18}$

$= (y^{-1}xyxyx)t_2t_{29}$

$= Nt_2t_{29}$

$= N(t_1t_2)^{xyxyxy^{-1}xy^{-1}} \epsilon$ [1,2]

Choose 19 from {19, 108, 63, 64, 75}

$Nt_1t_2t_{19}$

$= (y^{-1}xyxy^{-1}xy^{-1}xyxy^{-1})t_{105}t_{31}$

$= Nt_{105}t_{31}$

$= N(t_1t_2)^{yxy^{-1}xyxy^{-1}xy^{-1}xy^{-1}} \epsilon$ [1,2]

Choose 20 from {20, 93, 30, 59, 33}

$Nt_1t_2t_{20}$

$= (y^{-1}xy^{-1}xy^{-1}xy^{-1}xyx)t_{50}t_{89}$

$= Nt_{50}t_{89}$

$= N(t_1t_2)^{xy^{-1}xyxyxy^{-1}xyxyxy} \epsilon$ [1,2]

Choose 21 from {21, 31, 58, 82, 62}

$Nt_1t_2t_{21}$

$= (xy^{-1}xyxyxy^{-1})t_{57}t_{68}$

$= Nt_{57}t_{68}$

$= N(t_1t_4)^{y^{-1}xyxy^{-1}xy^{-1}xyxyx} \epsilon$ [1,4]

Choose 22 from {22, 79, 36, 88, 37}

$Nt_1t_2t_{22}$

$= (yxyxyxyxyxy^{-1}x)t_2t_{53}$

$= Nt_2t_{53}$

$=N(t_1t_2)^{yxyxy^{-1}xyxy^{-1}xy^{-1}x} \epsilon$ [1,2]

Choose 25 from {25, 49, 92, 101, 102}

$Nt_1t_2t_{25}$

$= (xyxy^{-1}xyxyxy^{-1}xy^{-1})t_{27}$

$= Nt_{27}$

$= N(t_1)^{y^{-1}xyxy^{-1}xy^{-1}xyxy} \epsilon$ [1]

Choose 26 from {26, 34, 65, 32, 73}

$Nt_1t_2t_{26}$

$= (y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})t_{46}t_{55}$

$= Nt_{46}t_{55}$

$= N(t_1t_2)^{(xyxy^{-1}xy)^2} \epsilon$ [1,2]

Choose 41 from {41, 105, 71, 70, 99}

$Nt_1t_2t_{41}$

$= (xyxyxy^{-1}xyxy^{-1})t_{100}t_{84}$

$= Nt_{100}t_{84}$

$= N(t_1t_{46})^{(y^{-1}xyxy^{-1}xy^{-1}xyxyxy^{-1})} \epsilon$ [1,46]

Choose 45 from {45, 48, 94, 85, 66}

$Nt_1t_2t_{45}$

$= (xyxyxy^{-1}xy^{-1}xy^{-1}xy^{-1})t_{46}$

$= Nt_{46}$
$= N(t_1)^{xy^{-1}xyxyxy^{-1}xy^{-}1xyxy^{-1}} \epsilon$ [1]
Cayley Diagram

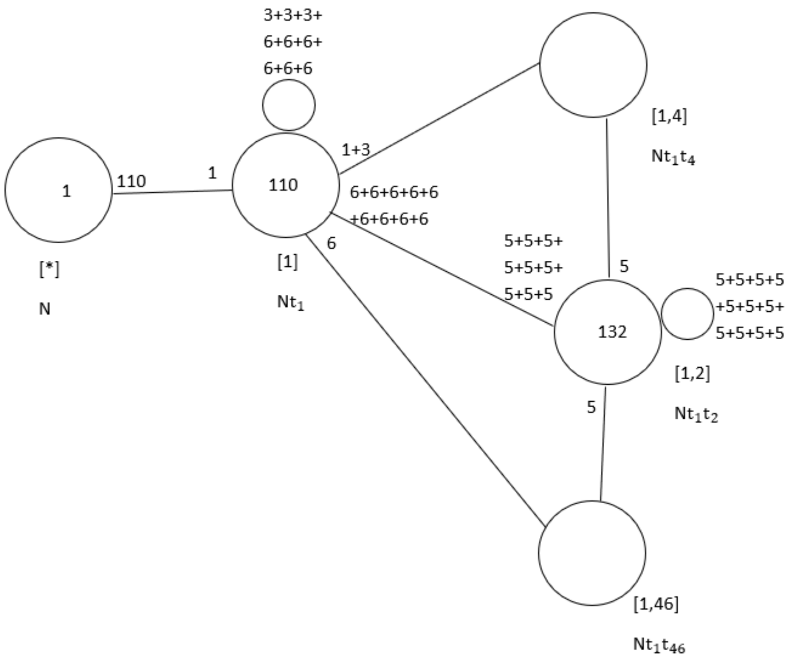


Figure 4.3: Cayley Diagram of [*], [1], [1,2] for J_1

Fourth Double Coset

$Nt_1t_4N = \{ N(t_1t_4)^n \mid n \epsilon N\} = \{Nt_1t_4, Nt_2t_{11},..., Nt_3t_{12}\}$
The point-stabiliser 1,4, $N^{1,4}$ is given by
$<(xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy), (yxyxy^{-1}xyxy^{-1}xy^{-1}) >$
But $t_1t_4^{(xy^{-1}xy^{-1}xyxyxyxy^{-1})} = t_3t_{12}$ therefore $(xy^{-1}xy^{-1}xyxyxyxy^{-1}) \epsilon N^{(1,4)}$
Therefore the coset stabiliser $N^{(1,4)} = < (xy^{-1}xy^{-1}xyxyxyxy^{-1}),$
$(xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy), (yxyxy^{-1}xyxy^{-1}xy^{-1}), (yxyxy^{-1}xyxy^{-1}x),$
$(yxy^{-1}xy^{-1}xyxyxyx), y, (xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy^{-1}), (xyxyxy^{-1}xy^{-1}xy) >$
The number of single right cosets in the double coset $Nt_1t_4N = [1,4]$ is given by $\frac{|N|}{|N^{(1,4)}|}$
$= \frac{660}{60} = 11$

The orbits on $N^{(1,4)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110} are {1, 3, 78, 38, 9, 50, 28, 110, 81, 87}, {4, 12, 84, 40, 35, 83, 47, 103, 109, 90}, {6, 108, 94, 76, 18, 32, 101, 49, 85, 19, 66, 26, 52, 102, 64, 75, 25, 73, 92, 48, 63, 56, 8, 23, 54, 42, 45, 34, 86, 65}, {2, 93, 105, 106, 5, 21, 39, 55, 77, 22, 7, 60, 24, 79, 107, 59, 13, 97, 31, 44, 71, 14, 89, 51, 43, 98, 53, 91, 20, 100, 27, 30, 67, 80, 68, 70, 37, 29, 104, 16, 36, 41, 58, 96, 82, 11, 74, 95, 72, 62, 69, 57, 46, 10, 33, 99, 61, 88, 15, 17}

We will multiply Nt_1t_4 on the right by an orbit representative and determine its double coset.

Choose 1 from {1, 3, 78, 38, 9, 50, 28, 110, 81, 87}

$Nt_1t_4t_1$

$= (yxy^{-1})t_{87}t_{90}$

$= N(t_1t_4)^{yxyxyxyxyxy^{-1}x} \epsilon$ [1,4]

Choose 4 from {4, 12, 84, 40, 35, 83, 47, 103, 109, 90}

$Nt_1t_4t_4$

$= Nt_1(t_4)^2$

$= Nt_1 \epsilon$ [1]

Choose 6 from {6, 108, 94, 76, 18, 32, 101, 49, 85, 19, 66, 26, 52, 102, 64, 75, 25, 73, 92, 48, 63, 56, 8, 23, 54, 42, 45, 34, 86, 65}

$Nt_1t_4t_6$

$= N(y^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)t_{110}$

$= Nt_{110}$

$= N(t_1)^{(xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1})} \epsilon$ [1]

Choose 2 from {2, 93, 105, 106, 5, 21, 39, 55, 77, 22, 7, 60, 24, 79, 107, 59, 13, 97, 31, 44, 71, 14, 89, 51, 43, 98, 53, 91, 20, 100, 27, 30, 67, 80, 68, 70, 37, 29, 104, 16, 36, 41, 58, 96, 82, 11, 74, 95, 72, 62, 69, 57, 46, 10, 33, 99, 61, 88, 15, 17}

$Nt_1t_4t_2$

$= N(yxyxyxy^{-1}xy^{-1}xyx)t_{24}t_{38}$

$= Nt_{24}t_{38}$

$= N(t_1t_2)^{(xy^{-1}xyxyxy^{-1}xy^{-1}xy)} \epsilon$ [1,2]

Cayley Diagram

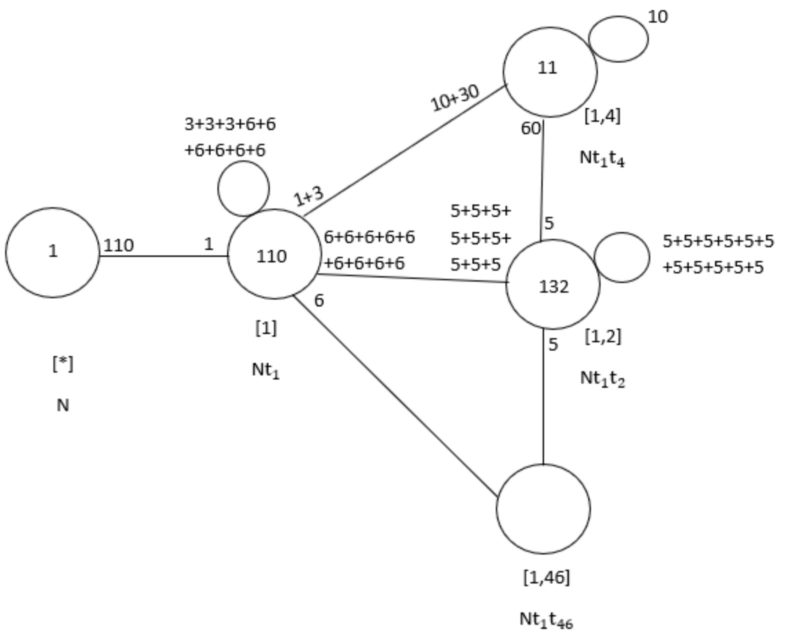


Figure 4.4: Cayley Diagram of [*], [1], [1,2], [1,4] for J_1

Fifth Double Coset

$Nt_1t_{46}N = \{ N(t_1t_{46})^n \mid n \epsilon N\} = \{Nt_1t_{46}, Nt_2t_{65},..., Nt_3t_{58}\}$

The point-stabiliser 1,46, $N^{1,46}$ is given by { 1 }

But $t_1t_{46} = t_{80}t_{50}$

Therefore the coset stabiliser $N^{(1,46)}$ are < $(xyxy^{-1})$, $(y^{-1}xyxyx)$>

The number of single right cosets in the double coset $Nt_1t_{46}N$ = [1,46] is given by $\frac{|N|}{|N^{(1,46)}|}$ $= \frac{660}{55} = 12$

Then the orbits of $N^{(1,46)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110} are {1, 74, 80, 19, 32, 12, 44, 91, 106, 42, 39, 67, 98, 54, 26, 96, 35, 5, 28, 63, 55, 72, 95, 34, 16, 46, 103, 87, 14, 107, 56, 57, 13, 10, 73, 84, 52, 89, 50, 43, 108, 17, 100, 27, 75, 18, 104, 64, 97, 65, 109, 7, 38, 61, 2}, {3, 69, 37, 8, 15, 70, 49, 92, 94, 30, 82, 68, 83, 93, 60, 62, 105, 79, 81, 6, 40, 22, 66, 29, 45, 58, 41, 9, 31, 47, 20, 11, 102, 4, 77, 99, 33, 24, 110, 90, 86, 101, 88, 85, 76, 59, 36, 23, 21, 53, 71, 25, 78, 48, 51 } We will multiply Nt_1t_{46} by a orbit representative and determine its double coset.

We will choose 46 and 3 to be our orbit representatives from the orbits above.

Choose 46 from {1, 74, 80, 19, 32, 12, 44, 91, 106, 42, 39, 67, 98, 54, 26, 96, 35, 5, 28, 63, 55, 72, 95, 34, 16, 46, 103, 87, 14, 107, 56, 57, 13, 10, 73, 84, 52, 89, 50, 43, 108, 17, 100, 27, 75, 18, 104, 64, 97, 65, 109, 7, 38, 61, 2}

$Nt_1t_{46}t_{46}$

$= Nt_1(t_{46})^2$

$= Nt_1 \ \epsilon \ [1]$

Choose 3 from {3, 69, 37, 8, 15, 70, 49, 92, 94, 30, 82, 68, 83, 93, 60, 62, 105, 79, 81, 6, 40, 22, 66, 29, 45, 58, 41, 9, 31, 47, 20, 11, 102, 4, 77, 99, 33, 24, 110, 90, 86, 101, 88, 85, 76, 59, 36, 23, 21, 53, 71, 25, 78, 48, 51 }

$Nt_1t_{46}t_3$

$= N(y^{-1}xyxyxy)t_{21}t_{106}$

$= Nt_{21}t_{106}$

$= N(t_1t_2)^{(xy^{-1})^4} \ \epsilon \ [1,2]$

Cayley Diagram

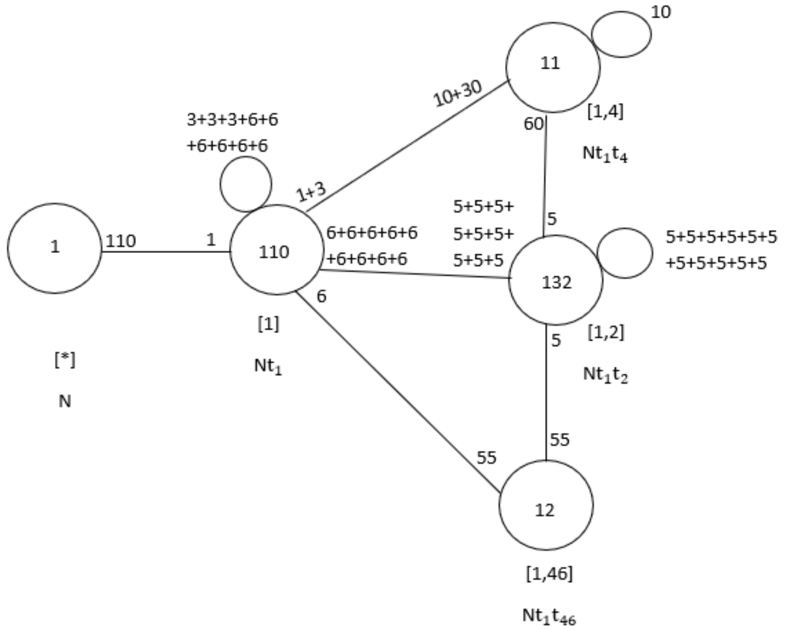


Figure 4.5: Cayley Diagram of [*], [1], [1,2], [1,4], [1,46] for J_1

4.4 $\frac{2^{*42}:PSL(2,7)}{(yx)^5t_{32}t_{36}t_{30}t_{20}t_{12},(yx)^4t_{35}t_{24}t_{21}t_{31},(y)t_{20}t_{26}t_{31}t_{25}t_{20}}$ ≅ 2:PSL(3,4)

we will prove that the progenitor 2^{*42}:(PSL(2,7), where 2∗42:(PSL(2,7) = < x, y> and x ∼ (1, 2)(5, 7)(6, 9)(8, 12)(10, 14)(11, 15)(13, 17)(16, 20)(18,22)(19, 23)(21, 26)(24, 27)(25, 30)(28, 33)(29, 35)(32, 37)(39, 41)(40, 42), y ∼ (1, 3, 5, 8)(2, 4, 6, 10)(7, 11, 9, 13)(12, 16)(14, 18)(15, 19, 24, 29)(17, 21, 27, 32)(20, 25, 31, 26)(22, 28, 34, 23)(30, 36, 37, 40)(33, 38, 35,39)(41, 42)(106, 107), factored by three relations is isomorphic to PSL(3,4):2. Let

G ≅ $\frac{2*42:(PSL(2,7)}{(yx)^5t_{32}t_{36}t_{30}t_{20}t_{12},(yx)^4t_{35}t_{24}t_{21}t_{31},(y)t_{20}t_{26}t_{31}t_{25}t_{20}}$. Thus we show that G ∼ 2:PSL(3,4).

Expanded Relations

Relation 1 = (yx)*$t^{(y^{-1}x)^5}$ = $(y*x)^5$*t_{32}*t_{36}*t_{30}*t_{20}*t_{12}

Relation 2 = (yx)*$t^{(y^{-1}xy^{-1}xy^2))^4}$= $(y*x)^4$*t_{35}*t_{24}*t_{21}*t_{31}

Relation 3 = (y)*$t^{(y^{-1}x)^2)^5}$ = (y)*t_{20}*t_{26}*t_{31}*t_{25}*t_{20}

First Double Coset

NeN = { N(e)^n | n ϵ N } = {N}

The coset stabiliser of the coset N = Ne is N.

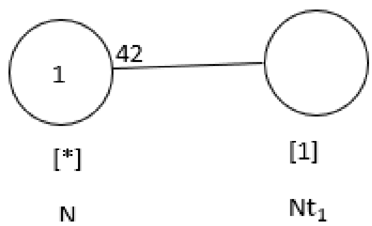
The number of single right cosets in the double coset NeN = [*] is given by $\frac{|N|}{|N|} = \frac{42}{42} = 1$

The orbits of N on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42}. We will multiply N on the right by an orbit representative and determine its double coset.

Choosing 1 from {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42}

Nt_1 ϵ [1]

Cayley Diagram



Second Double Coset

Nt_1 = { N$(t_1)^n$ | n ϵ N} = {Nt_1, Nt_2, Nt_3,..., Nt_{42}}

The point-stabiliser of 1, N^1 is given by <x, y>

But Nt_1 = Nt_2

Nt_1

= Nt_2

= N$(t_1)^x$ ϵ [1]

The coset Stabilizer $N^{(1)} = <(xyxy^{-1})^2, xyxy^{-1}x, xyxy^{-1}, x, yxy^{-1}x, yxy^{-1}xyxy^{-1}>$
The number of single right cosets in the double coset Nt_1N = [1] is given by $\frac{|N|}{|N^{(1)}|} = \frac{168}{8}$ = 21
The orbits for $N^{(1)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are { 1, 2 }, { 25, 30 }, { 28, 33 }, { 3, 17, 31, 13 }, { 4, 15, 34, 11 }, { 24, 36, 27, 38 }, { 5, 35, 42, 20, 40, 7, 29, 16 }, { 6, 37, 41, 22, 39, 9, 32, 18 }, { 8, 21, 19, 14, 23, 12, 26, 10 }. We will multiply Nt_1 on the right by an orbit representative and determine its double coset.

Choosing 1 from { 1, 2 }

Nt_1t_1

$= N(t_1)^2$

$= N \epsilon$ [*]

Choosing 25 from { 25, 30 }

Nt_1t_{25}

$= Nt_{30}$

$= N(t_1)^{(y^{-1}xy^{-1}xyxyxy^{-1})} \epsilon$ [1]

Choosing 28 from { 28, 33 }

Nt_1t_{28}

$= Nt_{30}$

$= N(t_1)^{(y^{-1}xy^{-1}xyxyxy^{-1})} \epsilon$ [1]

Choosing 3 from { 3, 17, 31, 13 }

$Nt_1t_3 \epsilon$ [1,3]

Choosing 4 from { 4, 15, 34, 11 }

Nt_1t_4

$= Nt_2t_3$

$= N(t_1t_3)^x \epsilon$ [1,3]

Choosing 5 from { 5, 35, 42, 20, 40, 7, 29, 16 }

$t_1t_5 \epsilon$ [1,5]

Choosing 6 from { 6, 37, 41, 22, 39, 9, 32, 18 }

Nt_1t_6

$= Nt_{11}t_{34}$

$= N(t_1t_5)^{(xyxyxy^{-1})} \epsilon$ [1,5]

Choosing 8 from { 8, 21, 19, 14, 23, 12, 26, 10 }

Nt_1t_8

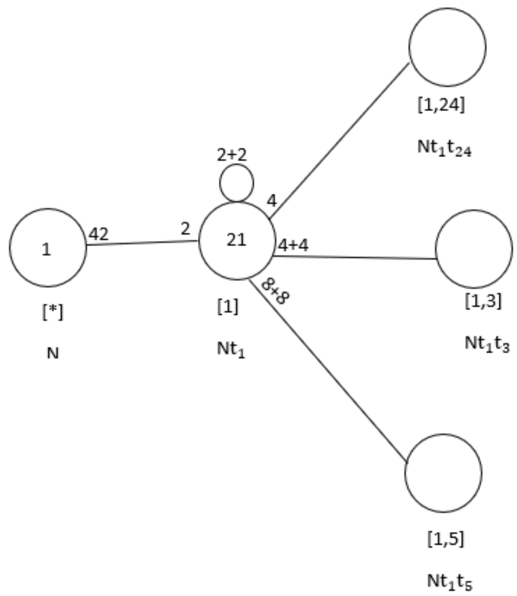
$= Nt_{21}t_1$

$= N(t_1t_3)^{(y^{-1}xyxy^{-1}x)} \epsilon$ [1,3]

Choosing 24 { 24, 36, 27, 38 }

$Nt_1t_{24} \epsilon$ [1,24]

Cayley Diagram



Third Double Coset

Nt_1t_3N = { $N(t_1t_3)^n$ | n ϵ N} = {Nt_1t_3, Nt_2t_3,..., Nt_3t_5}

The point-stabilizer of 1, 3, N^{13} is given by < 1>

But $Nt_1t_3 = Nt_2t_3$

Nt_1t_3

$= Nt_2t_3$

$= N(t_1t_3)^x \ \epsilon$ [1,3]

The coset Stabilizer of $N^{(13)}$ = <x>

The number of single right cosets in the double coset $N^{(1,3)}$ = [1,3] is given by $\frac{|N|}{|N^{(12)}|} = \frac{168}{2} = 84$

The orbits for $N^{(13)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are:

{3}, {4}, {31}, {34}, {36}, {1,2}, {5, 7}, {6, 9}, {8, 12}, {10, 14}, {11, 15}, {13, 17}, {16, 20}, {18, 22}, {19, 23}, {21, 26}, {24, 27}, {25, 30}, {28, 33}, {29, 35}, {32, 37}, {39, 41}, {40, 42}

We will multiply Nt_1t_3 by an orbit representative and determine its double coset.

<u>Choosing 3 from {3}</u>

$Nt_1t_3t_3$

$= Nt_1(t_3)^2$

$= Nt_1 \ \epsilon$ [1]

<u>Choosing 4 from {4}</u>

$Nt_1t_3t_4$

$= Nt_1$

$= N(t_1)^{(xyxy^{-1})^2} \ \epsilon$ [1]

<u>Choosing 31 from {31}</u>

$Nt_1t_3t_{31}$

$= Nt_{38}t_1$

$= N(t_1t_{24})^{(xy^{-1}xyxyxy)} \ \epsilon$ [1,24]

<u>Choosing 34 from {34}</u>

$Nt_1t_3t_{34}$

$= Nt_{38}t_1$

$= (t_1t_{24})^{(xy^{-1}xyxyxy)}$ ϵ [1,24]

Choosing 36 from {36}

$Nt_1t_3t_{36}$

$= Nt_1t_{31}$

$= N(t_1t_3)^{(xyxy^{-1})^2}$ ϵ [1,3]

Choosing 1 from {1,2}

$Nt_1t_3t_1$

$= Nt_{22}t_{18}$

$= N(t_1t_{24})^{(xyxy^{-1})^2}$ ϵ [1,24]

Choosing 5 from {5,7}

$Nt_1t_3t_5$

$= Nt_8t_{25}$

$= N(t_1t_3)^{(xyxy^{-1}xyxy^2)}$ ϵ [1,3]

Choosing 6 from {6,9}

$Nt_1t_3t_6$

$= Nt_8t_{25}$

$= N(t_1t_3)^{(xyxy^{-1}xyxy^2)}$ ϵ [1,3]

Choosing 8 from {8,12}

$Nt_1t_3t_8$

$= Nt_{41}t_{24}$

$= N(t_1t_3)^{(y^2xyxy^{-1}xyx)}$ ϵ [1,3]

Choosing 10 from {10,14}

$Nt_1t_3t_{10}$

$= Nt_{41}t_{24}$

$= N(t_1t_3)^{(y^2xyxy^{-1}xyx)}$ ϵ [1,3]

Choosing 11 from {11,15}

$Nt_1t_3t_{11}$

$= Nt_{23}t_{19}$

$= N(t_1t_5)^{(xyxyxy^{-1}xyx)}$ ϵ [1,5]

Choosing 13 from {13,17}

$Nt_1t_3t_{13}$

$= Nt_{23}t_{19}$

$= N(t_1t_5)^{(xyxyxy^{-1}xyx)} \epsilon [1,5]$

Choosing 16 from {16,20}

$Nt_1t_3t_{16}$

$= Nt_4t_{25}$

$= N(t_1t_5)^{(yxy^{-1}xy)} \epsilon [1,5]$

Choosing 18 from {18,22}

$Nt_1t_3t_{18}$

$= Nt_4t_{25}$

$= N(t_1t_5)^{(yxy^{-1}xy)} \epsilon [1,5]$

Choosing 19 from {19,23}

$Nt_1t_3t_{19}$

$= Nt_{34}t_{42}$

$= N(t_1t_3)^{(y^2xyxyxy^{-1}x)} \epsilon [1,3]$

Choosing 21 from {21,26}

$Nt_1t_3t_{21}$

$= Nt_{34}t_{42}$

$= N(t_1t_3)^{(y^2xyxyxy^{-1}x)} \epsilon [1,3]$

Choosing 24 from {24,27}

$Nt_1t_3t_{24}$

$= Nt_3$

$= N(t_1)^{(xyxy^{-1}xyx)} \epsilon [1]$

Choosing 25 from {25,30}

$Nt_1t_3t_{25} \epsilon [1,3,25]$

Choosing 28 from {28,33}

$Nt_1t_3t_{28}$

$= Nt_{16}t_{10}t_{12}$

$= N(t_1t_3t_{25})^{(y^{-1}xy^{-1})} \epsilon [1,3,25]$

Choosing 29 from {29,35}

$Nt_1t_3t_{29}$

$= Nt_{24}t_3t_{15}$

$= N(t_1t_3t_{25}^{(xy^{-1}xy)^2} \epsilon [1,3,25]$

Choosing 32 from {32,37}

$Nt_1t_3t_{32}$

$= Nt_{24}t_3t_{15}$

$= N(t_1t_3t_{25})^{(xy^{-1}xy)^2} \epsilon$ [1,3,25]

Choosing 39 from {39,41}

$Nt_1t_3t_{39}$

$= Nt_{35}t_7$

$= N(t_1t_3)^{(xy^{-1}xyxyxy^2)} \epsilon$ [1,3]

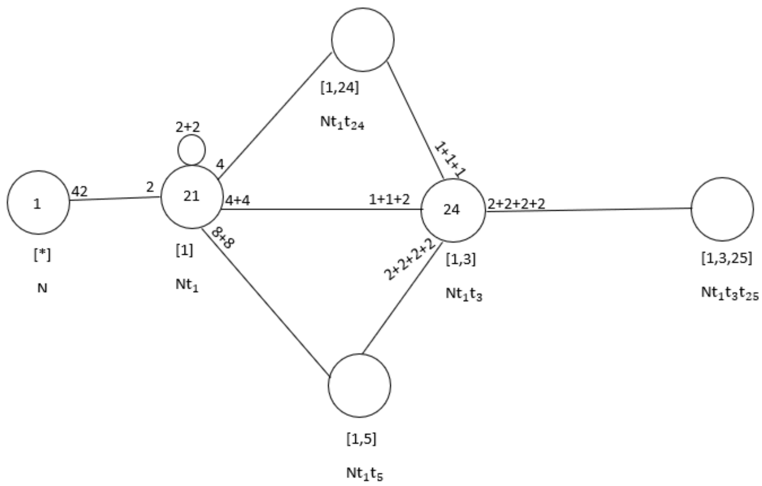
Choosing 40 from {40,42}

$Nt_1t_3t_{40}$

$= Nt_{35}t_7$

$= N(t_1t_3)^{(xy^{-1}xyxyxy^2)} \epsilon$ [1,3]

Cayley Diagram



Fourth Double Coset

Nt_1t_5N = { $N(t_1t_5)^n$ | n ϵ N} = {Nt_1t_5, Nt_2t_7, ..., Nt_3t8}

The point-stabilizer of 1, 5, $N^{1,5}$ is given by $< 1 >$

Now $Nt_1t_5 = Nt_{11}t_{34}$

Nt_1t_5

$= Nt_{11}t_{34}$

$= N(t_1t_5)^{(xyxyxy^{-1})}$ ϵ [1,5]

Thus ($xyxyxy^{-1}$) ϵ [1,5]

Thus $N^{(15)} \geq <N^{15}, (xyxyxy^{-1})> = <(xyxyxy^{-1}))>$

The number of single right cosets in the double coset Nt_1t_5 = [1,5] is given by $\frac{|N|}{|N^{(15)}|} = \frac{168}{3} = 56$

The orbits for $N^{(15)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are {1, 11, 14}, {2, 13, 12}, {3, 29, 37}, {4, 32, 35}, {5, 34, 21}, {6, 31, 19}, {7, 26, 38}, {8, 40, 22}, {9, 23, 36}, {10, 39, 20}, {15, 18, 33}, {16, 30, 17}, {24, 41, 28}, {25, 27, 42}

We will multiply Nt_1t_5 on the right by an orbit representative and determine its double coset.

<u>Choosing 1 from {1,11,14}</u>

$Nt_1t_5t_1$

$= Nt_{11}t_{34}$

$= N(t_1t_5)^{(xyxyxy^{-1})}$ ϵ [1,5]

<u>Choosing 2 from {2,13,12}</u>

$Nt_1t_5t_2$

$= Nt_{11}t_{34}$

$= N(t_1t_5)^{(xyxyxy^{-1})}$ ϵ [1,5]

<u>Choosing 3 from {3,29,37}</u>

$Nt_1t_5t_3$

$= Nt_1t_{13}$

$= N(t_1t_3)^{(yxy^{-1})}$ ϵ [1,3]

<u>Choosing 4 from {4,32,35}</u>

$Nt_1t_5t_4$

$= Nt_1t_{13}$

$= N(t_1t_3)^{(yxy^{-1})} \epsilon$ [1,3]

Choosing 5 from {5,34,21}

$Nt_1t_5t_5$

$= Nt_1(t_5)^2 \epsilon$ [1]

Choosing 6 from {6,31,19}

$Nt_1t_5t_6$

$= Nt_1$

$= N(t_1)^{(xyxy^{-1})^2} \epsilon$ [1]

Choosing 7 from {7,26,38}

$Nt_1t_5t_7$

$= Nt_{14}t_{32}$

$= N(t_1t_5)^{(yxy^{-1}xy^{-1}xy^2)} \epsilon$ [1,5]

Choosing 8 from {8,40,22}

$Nt_1t_5t_8$

$= Nt_{23}t_2$

$= N(t_1t_5)^{(xy^{-1}xyxy^{-1})} \epsilon$ [1,3]

Choosing 9 from {9,23,36}

$Nt_1t_5t_9$

$= Nt_{14}t_{32}$

$= N(t_1t_5)^{(yxy^{-1}xy^{-1}xy^2)} \epsilon$ [1,5]

Choosing 10 from {10,39,20}

$Nt_1t_5t_{10}$

$= Nt_{23}t_2$

$= N(t_1t_3)^{(xy^{-1}xyxy^{-1})} \epsilon$ [1,3]

Choosing 15 from {15,18,33}

$Nt_1t_5t_{15}$

$= Nt_{41}t_{29}t_{18}$

$= N(t_1t_3t_{25})^{(y^{-1}xyxyxy^{-1}xy^{-1})} \epsilon$ [1,3,25]

Choosing 16 from {16,30,17}

$Nt_1t_5t_{16}$

$= Nt_{40}t_{27}t_{10}$

$= N(t_1t_3t_{25})^{(y^2xy^{-1}xy^{-1}xy)} \epsilon$ [1,3,25]

Choosing 24 from {24,41,28}

$Nt_1t_5t_{24}$

$= Nt_{41}t_{29}t_{18}$

$= N(t_1t_3t_{25})^{(y^{-1}xyxyxy^{-1}xy^{-1})} \epsilon$ [1,3,25]

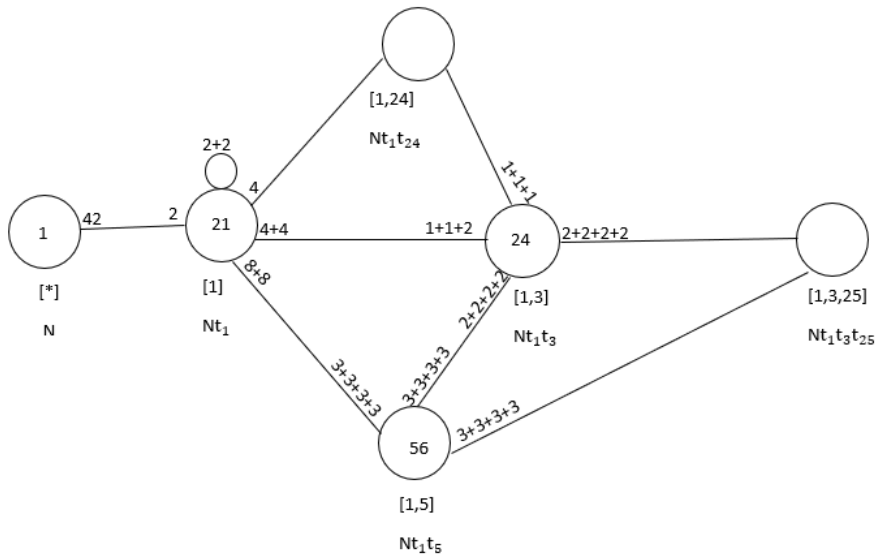
Choosing 25 from {25,27,42}

$Nt_1t_5t_{25}$

$= Nt_{26}t_{30}t_5$

$= N(t_1t_3t_{25})^{(y^2xy^{-1}xyx)} \epsilon$ [1,3,25]

Cayley Diagram



Fifth Double Coset

$Nt_1t_{24}N = \{ N(t_1t_{24})^n \mid n \epsilon N\} = \{Nt_1t_{26}, Nt_2t_{21},..., Nt_3t_{20}\}$

The point-stabilizer of 1, 24, $N^{1,24}$ is given by $< 1 >$

Now $Nt_1t_{24} = Nt_2t_{27}$

Nt_1t_{26}

$= Nt_2t_{27}$

$= N(t_1t_{27})^x \epsilon$ [1,24]

Also $Nt_1t_{24} = Nt_2t_{24}$

Nt_1t_{24}

$= Nt_2t_{24}$

$= N(t_1t_{24})^{(yxy^{-1}xyxy^{-1})}$

So $(yxy^{-1}xyxy^{-1}) \epsilon N^{(124)}$

Thus $N^{(124)} \geq < N^{124}, x, (yxy^{-1}xyxy^{-1})> = <x, (yxy^{-1}xyxy^{-1})>$

The number of single right cosets in the double coset Nt_1t_{24} = [1,24] is given by $\frac{|N|}{|N^{(1,24)}|}$ $= \frac{168}{8} = 21$

The orbits for $N^{(1,24)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are {3, 31}, {4, 34}, {36, 38}, {1, 2, 24, 27}, {11, 15, 25, 30}, {13, 17, 28, 33}, {5, 7, 10, 14, 40, 42, 26, 21}, {6, 9, 8, 12, 39, 41, 23, 19}, {16, 20, 37, 32, 29, 35, 22, 18}

We will multiply Nt_1t_{26} on the right by an orbit representative and determine its double coset.

Choosing 3 from {3, 31}

$Nt_1t_{24}t_3$

$= Nt_{22}t_{18}$

$= N(t_1t_{24})^{(yxy^{-1}xy^{-1}xy^{-1}x)} \epsilon$ [1,24]

Choosing 4 from {4, 34}

$Nt_1t_{24}t_4$

$= Nt_{22}t_{18}$

$= N(t_1t_{24})^{(yxy^{-1}xy^{-1}xy^{-1}x)} \epsilon$ [1,24]

Choosing 36 from {36, 38}

$Nt_1t_{24}t_{36}$

$= Nt_{30}t_{21}t_{10}$

$= N(t_1t_{24}t_{36})^{(y^{-1}xy^{-1}xyxyxy^{-1})}$ ϵ [1,24,36]

Choosing 24 from {1, 2, 24, 27}

$Nt_1t_2t_{24}t_{24}$

$= Nt_1t_2(t_{24}^2$

$= Nt_1t_2$ ϵ [1,2]

Choosing 11 from {11, 15, 25, 30}

$Nt_1t_{24}t_{11}$

$= Nt_2t_{17}$

$= N(t_1t_3)^{(yxy^{-1}x)}$ ϵ [1,3]

Choosing 13 from {13, 17, 28, 33}

$Nt_1t_{24}t_{13}$

$= Nt_2t_{17}$

$= N(t_1t_3)^{(yxy^{-1}x)}$ ϵ [1,3]

Choosing 5 from {5, 7, 10, 14, 40, 42, 26, 21}

$Nt_1t_{24}t_5$

$= Nt_{35}t_{34}t_{23}$

$= N(t_1t_3t_{25})^{(yxyxyxy^{-1}x)}$ ϵ [1,3,25]

Choosing 6 from {6, 9, 8, 12, 39, 41, 23, 19}

$Nt_1t_{24}t_6$

$= Nt_{35}t_{34}t_{23}$

$= N(t_1t_3t_{25})^{(yxyxyxy^{-1}x)}$ ϵ [1,3,25]

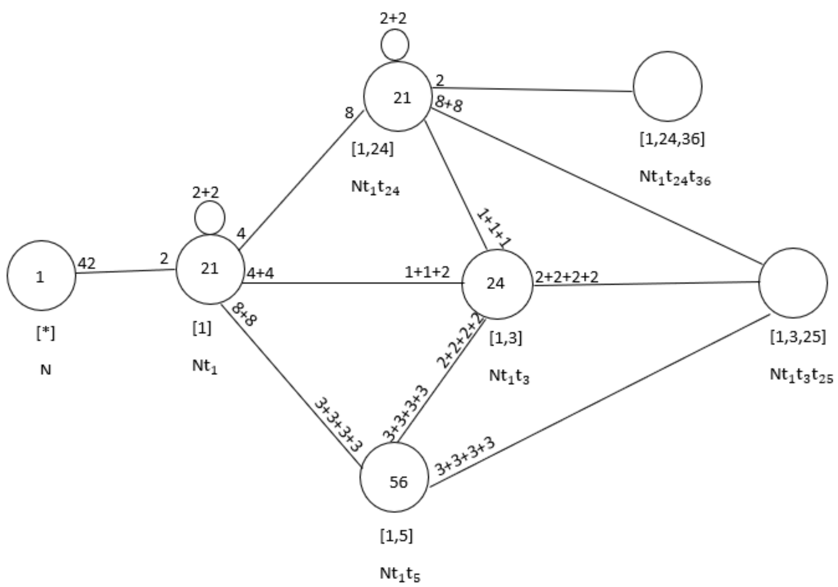
Choosing 16 from {16, 20, 37, 32, 29, 35, 22, 18}

$Nt_1t_{24}t_{16}$

$= Nt_{16}t_4$

$= N(t_1t_3)^{(x^y)}$ ϵ [1,3]

Cayley Diagram



<u>**Sixth Double Coset**</u>

$Nt_1t_3t_{25}N = \{ N(t_1t_3t_{25})^n \mid n \epsilon N\} = \{Nt_1t_3t_{25}, Nt_2t_3t_{30}, ..., Nt_3t_5t_{31}\}$

The point-stabilizer of 1, 2, 25, $N^{1,3,25}$ is given by $< 1 >$

Now $Nt_1t_3t_{25} = Nt_{16}t_{10}t_{12}$

$Nt_1t_3t_{25}$

$= Nt_{16}t_{10}t_{12}$

Now $N(t_1t_3t_{25})^{(y^{-1}xy^{-1})}$ ϵ [1,3,25]

Thus the coset stabiliser $N^{(1,3,25)} \geq < N^{1,3,25}, (y^{-1}xy^{-1}) > = < (y^{-1}xy^{-1}) >$

The number of single right cosets in the double coset $Nt_1t_3t_{25}$ = [1,3,25] is given by $\frac{|N|}{|N^{(1,3,25)}|} = \frac{168}{3} = 56$

The orbits for $N^{(1,3,25)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are {1, 16, 5}, {2, 18, 6}, {3, 10, 11}, {4, 8, 13}, {7, 32, 19}, {9, 29, 21}, {12, 26, 25}, {14, 23, 28}, {15, 38, 22}, {17, 36, 20}, {24, 34, 39}, {27, 31, 40}, {30, 41, 37}, {33, 42, 35}

We will multiply $Nt_1t_3t_{25}$ on the right by an orbit representative and determine its double coset.

<u>Choosing 1 from {1,16,5}</u>

$Nt_1t_3t_{25}t_1$

$= Nt_7t_{15}t_{21}$

$= N(t_1t_3t_{25})^{(yx)^2} \epsilon [1,3,25]$

Choosing 2 from {2,18,6}

$Nt_1t_3t_{25}t_2$

$= Nt_7t_{15}t_{21}$

$= N(t_1t_3t_{25})^{(yx)^2} \epsilon [1,3,25]$

Choosing 3 from {3,10,11}

$Nt_1t_3t_{25}t_3$

$= Nt_{27}t_{32}$

$= N(t_1t_5)^{(yxyxy^{-1}xy^2)} \epsilon [1,5]$

Choosing 4 from {4,8,13}

$Nt_1t_3t_{25}t_4$

$= Nt_{27}t_{32}$

$= N(t_1t_5)^{(yxyxy^{-1}xy^2)} \epsilon [1,5]$

Choosing 7 from {7,32,19}

$Nt_1t_3t_{25}t_7$

$= Nt_{40}t_{11}$

$= N(t_1t_3)^{(y^{-1}xyxyxy^{-1})} \epsilon [1,3]$

Choosing 9 from {9,29,21}

$Nt_1t_3t_{25}t_9$

$= Nt_{39}t_{11}$

$= N(t_1t_3)^{(xy^{-1}xyxy*xy^{-1})} \epsilon [1,3]$

Choosing 25 from {12,26,25}

$Nt_1t_3t_{25}t_{25}$

$= Nt_1t_3(t_{25})^2$

$= Nt_1t_3 \epsilon [1,3]$

Choosing 14 from {14,23,28}

$Nt_1t_3t_{25}t_{14}$

$= Nt_{18}t_{10}$

$= N(t_1t_3)^{(xy^{-1})^2} \epsilon [1,3]$

Choosing 15 from {15,38,22}

$Nt_1t_3t_{25}t_{15}$

$= Nt_{30}t_{34}$

$= N(t_1t_5)^{(y^2xy^{-1}xy^{-1}xy^2)}$ ϵ [1,5]

Choosing 17 from {17,36,20}

$Nt_1t_3t_{25}t_{17}$

$= Nt_{30}t_{34}$

$= N(t_1t_5)^{(y^2xy^{-1}xy^{-1}xy^2)}$ ϵ [1,5]

Choosing 24 from {24,34,39}

$Nt_1t_3t_{25}t_{24}$

$= Nt_1t_3t_{25}$

$= Nt_1t_3t_{25}$ ϵ [1,3,25]

Choosing 27 from {27,31,40}

$Nt_1t_3t_{25}t_{27}$

$= Nt_1t_3t_{25}$

$= Nt_1t_3t_{25}$ ϵ [1,3,25]

Choosing 30 from {30,41,37}

$Nt_1t_3t_{25}t_{30}$

$= Nt_{22}t_{18}$

$= N(t_1t_{24})^{(yxy^{-1}xy^{-1}xy^{-1}x)}$ ϵ [1,24]

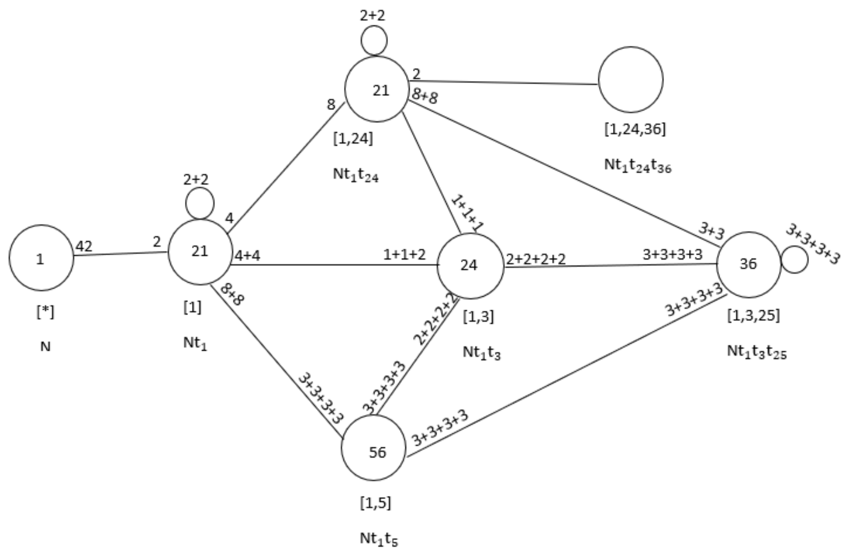
Choosing 33 from {33,42,35}

$Nt_1t_3t_{25}t_{33}$

$= Nt_{22}t_{18}$

$= N(t_1t_{24})^{(yxy^{-1}xy^{-1}xy^{-1}x)}$ ϵ [1,24]

Cayley Diagram



Seventh Double Coset

$Nt_1t_{24}t_{36}N = \{ N(t_1t_{24}t_{36})^n \mid n \epsilon N\} = \{Nt_1t_{24}t_{36}, Nt_2t_{27}t_{36},..., Nt_3t_{29}t_{37}\}$

The point-stabilizer of 1, 24, 36, $N^{1,24,36}$ is given by $< 1>$

Now $Nt_1t_{24}t_{36} = Nt_2t_{27}t_{36}\ Nt_1t_{24}t_{36}$

$= Nt_2t_{27}t_{36}$

$= N(t_1t_{24}t_{36})^x \epsilon [1,24,36]$

Also $Nt_1t_{24}t_{36} = Nt_3t_{29}t_{37}$

$Nt_1t_{24}t_{36}$

$= Nt_3t_{29}t_{37}$

$=N(t_1t_{24}t_{36})^y \epsilon [1,24,36]$

Thus $N^{(1,24,36)} \geq < N^{1,24,36}, x, y > = < x, y >$

The number of single right cosets in the double coset $Nt_1t_{24}t_{36} = [1,24,36]$ is given by $\frac{|N|}{|N^{(12436)}|} = \frac{168}{168} = 1$

The orbits for $N^{(1,24,36)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42}

We will multiply $Nt_1t_{24}t_{36}$ on the right by an orbit representative and determine its double coset.

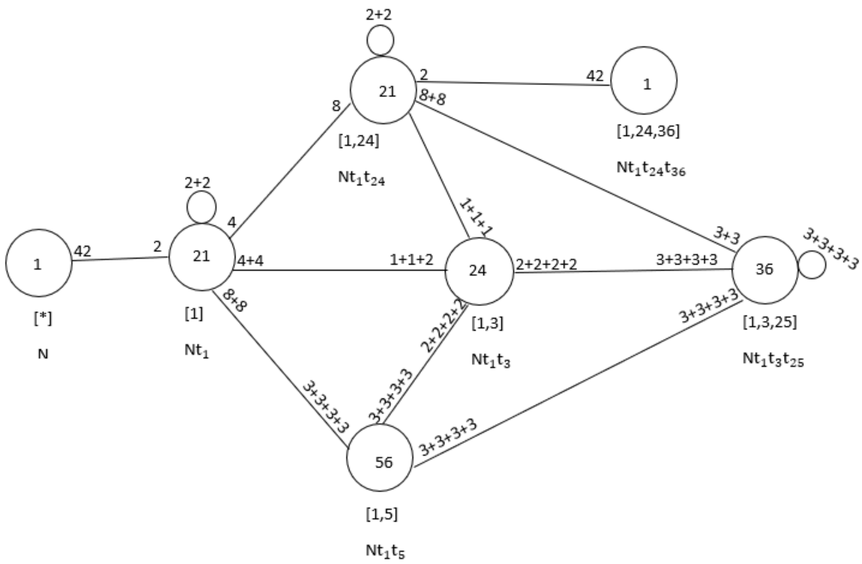
Choosing 36 from

$Nt_1t_{24}t_{36}t_{36}$

$= Nt_1t_{24}(t_{36})^2$

$= Nt_1t_{24} \epsilon [1,24]$

Cayley Diagram



4.5 $\frac{2^{*110}:PSL(2,11)}{(yxyxyxyxyxy^{-1}xy^{-1}xy)^5 t_{38}t_{89}t_{67}t_{30}t_{15},(yxyxyxyxyxy^{-1}xy^{-1}xy)^3 t_{69}t_{17}t_{16}} \cong \mathbf{M}_{11}$

We will prove that the progenitor 2^{*110}: PSL(2,11) where 2^{*110}: PSL(2,11) = < x, y> and x ∼ (1, 2)(3, 8)(4, 11)(5, 13)(6, 17)(7, 20)(9, 26)(10, 23)(12, 34)(14, 40)(15, 42)(16, 45)(18, 48)(19, 50)(21, 52)(22, 54)(24, 58)(25, 61)(27, 57)(28, 51)(29, 68)(30, 32)(31, 59)(33, 44)(35, 76)(36, 73)(38, 71)(39, 79)(41, 85)(43, 66)(46, 65)(47, 89)(49, 83)(53, 93)(55, 90)(56, 96)(60, 78)(62, 92)(63, 101)(64, 102)(70, 86)(72, 94)(74, 104)(75, 77)(80, 108)(81, 109)(82, 103)(84, 106)(87, 88)(91, 95)(99, 107)(100, 110), y ∼ (1, 3, 9)(2, 5, 14)(4, 12, 35)(6, 18, 49)(7, 21, 53)(8, 23, 56)(10, 29, 69)(11, 31, 71)(13, 37, 74)(15, 43, 88)(16, 22, 55)(17, 46, 58)(19, 32, 48)(20, 51, 41)(24, 59, 98)(25, 45, 65)(26, 64, 92)(27, 67, 36)(30, 70, 104)(33, 72, 95)(34, 73, 85)(38, 50, 78)(39, 80, 105)(40, 83, 84)(42, 86, 75)(44, 89, 96)(52, 63, 66)(54, 94, 101)(57, 61, 100)(60, 79, 91)(62, 97, 99)(68, 77, 82)(76, 102, 108)(81, 110, 87)(90, 109, 103)(93, 106, 107), factored by two relations is isomorphic to Mathieu sporadic simple group M_{11}.

Let G ≅ $\frac{2^{*110}:PSL(2,11)}{(yxyxyxyxyxy^{-1}xy^{-1}xy)^5 t_{38}t_{89}t_{67}t_{30}t_{15},(yxyxyxyxyxy^{-1}xy^{-1}xy)^3 t_{69}t_{17}t_{16}}$. Thus we will show that G ∼ M_{11}.

We will use the following relations

Relation 1 = $((yxyxyxyxyxy^{-1}xy^{-1}xy)^5 t_{38}t_{89}t_{67} = t_{15}t_{30}$

Relation 2 = $((yxyxyxyxyxy^{-1}xy^{-1}xy)^3 t_{69}t_{17} = t_{16}$

First Double Coset

NeN = { N(e)^n | n ϵ N }

The coset Stabilizer of the coset N = Ne is N.

The number of single right cosets in the double coset NeN = [*] is given by $\frac{|N|}{|N|} = \frac{660}{660} = 1$

The orbit of N on X = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110 } is

{ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110 }

Multiply by N on the right by a orbit representative and determine its double coset.

Choose 1 from { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110 }

$Nt_1 \epsilon [1]$

This tells us that one-hundred ten elements move to the double coset [1]

Cayley Diagram

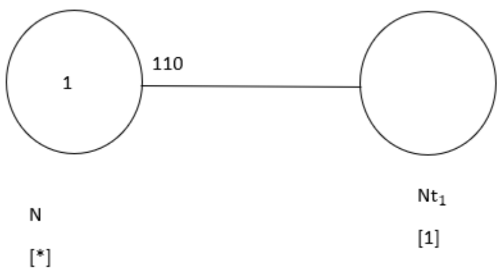


Figure 4.6: Cayley Diagram of [*] for M_{11}

Second Double Coset

$Nt_1N = \{ N(t_1)^n \mid n \epsilon N\} = \{Nt_1, Nt_2, ..., Nt_{110}\}$

The point-stabiliser 1, N^1 is given by $< xyxyxy^{-1}xy^{-1}x, yxyxy^{-1}xyxy^{-1}xy^{-1} >$

Now $t_1 = t_3$

Then $N(t_1)^y = t_3$

Therefore y ϵ $N^{(1)}$

Thus the coset stabiliser $N^{(1)} \geq < N^1, y > =$

$< xy^{-1}xy^{-1}xyxyxyxy^{-1}$, $xyxyxy^{-1}xy^{-1}x$, $y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx$, $yxy^{-1}xy^{-1}xyxyxyx$, $yxyxy^{-1}xyxy^{-1}x$, y, $xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy^{-1}$, $xyxyxy^{-1}xy^{-1}xy>$

The number of single right cosets in the double coset $Nt_1N = [1]$ is given by $\frac{|N|}{|N^{(1)}|} = \frac{660}{60}$ = 11

The orbits for $N^{(1)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110} are {1}, {4}, {42, 85, 102}, {45, 56, 64}, {50, 87, 110}, {83, 90, 103}, {2, 93, 68, 7, 22, 105}, {3, 78, 81, 9, 28, 38}, {5, 31, 15, 16, 20, 44}, {6, 108, 73, 19, 18, 94}, {8, 101, 34, 25, 63, 75}, {10, 13, 39, 30, 27, 82}, {11, 91, 57, 33, 43, 97}, {12, 84, 109, 35, 47, 40}, {14, 79, 98, 41, 21, 61}, {17, 100, 59, 29, 36, 99}, {23, 48, 32, 52, 92, 49}, {24, 80, 74, 60, 62, 106}, {26, 54, 86, 66, 65, 76}, {37, 72, 71, 77, 107, 96}, {46, 51, 55, 67, 104, 53}, {58, 88, 95, 70, 69, 89}

Multiply Nt_1 by a representative of each orbit and determine its double coset.

<u>Choose 1 from {1}</u>

Nt_1t_1

$= N(t_1)^2$

$= N \ \epsilon$ [*]

<u>Choose 4 from {4}</u>

Nt_1t_4

$= (yxyxyxyxyxy^{-1}xyx)t_{28}t_{47}$

$= N(t_1t_4)^{(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1})}$ ϵ [1]

<u>Choose 42 from {42,85,102}</u>

t_1t_{42}

$= (yxy^{-1}xy^{-1}x)t_{81}$

$=(t_1)^{(yxyxyxy^{-1}xy^{-1}x)}$ ϵ [1]

<u>Choose 45 from {45,56,64}</u>

t_1t_{45}

$= (yxy^{-1}xy^{-1}xyxyxyxyxy)t_{87}$

$= (t_1)^{(xyxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})}$ ϵ [1]

Choose 50 from {50,87,110}

Nt_1t_{50}

$= Nt_1t_1$

$= N(t_1)^2$

$= N$ ϵ [*]

Choose 83 from {83,90,103}

t_1t_{83}

$= (yxyxy^{-1}xy^{-1}xyxy^{-1})t_9$

$= (t_1)^{(yxy^{-1}xy^{-1}xyxyxyxy)}$ ϵ [1]

Choose 2 from {2,93,68,7,22,105}

t_1t_2

$= (x)t_{89}$

$= (t_1)^{(yxy^{-1}xy^{-1})}$ ϵ [1]

Choose 3 from {3, 78, 81, 9, 28, 38}

Nt_1t_3

$= Nt_1t_1$

$= N(t_1)^2$

$= N$ ϵ [*]

Choose 5 from {5, 31, 15, 16, 20, 44}

t_1t_5

$= (yxyxy^{-1})t_7$

$= (t_1)^{(y^{-1}xy^{-1}xy^{-1}xyxy)}$ ϵ [1]

Choose 6 from {6, 108, 73, 19, 18, 94}

t_1t_6

$= (xyxyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)t_{28}$

$= (t_1)^{(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1})}$ ϵ [1]

Choose 8 from {8, 101, 34, 25, 63, 75}

t_1t_8

$= (yxy^{-1}xy^{-1}xyx)t_{87}$

$= (t_1)^{(xyxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})} \epsilon [1]$

Choose 10 from {10, 13, 39, 30, 27, 82}

t_1t_{10}

$= (xyxyxy^{-1})t_{13}$

$= (t_1)^{(y^{-1}xy^{-1}xy^{-1}xyxy)} \epsilon [1]$

Choose 11 from {11, 91, 57, 33, 43, 97}

t_1t_{11}

$= (xyxyxy^{-1}x)t_{70}$

$= (t_1)^{(yxyxyxyxy^{-1}x)} \epsilon [1]$

Choose 12 from {12, 84, 109, 35, 47, 40}

t_1t_{12}

$= ((xy)^2)t_{81}$

$= (t_1)^{(yxyxyxy^{-1}xy^{-1}x)} \epsilon [1]$

Choose 14 from {14, 79, 98, 41, 21, 61}

t_1t_{14}

$= (xyxyxy^{-1}xy^{-1}x)t_{98}$

$= (t_1)^{(xyxy^{-1}xyxyxy^{-1}xy^{-1}xy^{-1})} \epsilon [1]$

Choose 17 from {17, 100, 59, 29, 36, 99}

t_1t_{17}

$= (xyxyxy^{-1}xy^{-1}xy^{-1}x)t_{32}$

$= (t_1)^{(yxyxyxyxy^{-1}xy^{-1}x)} \epsilon [1]$

Choose 23 from {23, 48, 32, 52, 92, 49}

t_1t_{23}

$= (xyxyxy^{-1}xy^{-1}xy^{-1}xy)t_9$

$= (t_9)^{(y^{-1})} \epsilon [1]$

Choose 24 from {24, 80, 74, 60, 62, 106}

t_1t_{24}

$= (y^{-1}xy^{-1}xy^{-1})t_{60}$

$= (t_1)^{(y^{-1}xyxyxy^{-1}xy^{-1})} \epsilon [1]$

Choose 26 from {26, 54, 86, 66, 65, 76}

t_1t_{26}

$= (yxyxyxy^{-1})t_{28}$

$= (t_1)^{(xy^{-1}xy^{-1}xy^{-1}xyxyxy)} \epsilon$ [1]

Choose 37 from {37, 72, 71, 77, 107, 96}

t_1t_{37}

$= (yxy^{-1}xy^{-1}x)t_{42}$

$= (t_1)^{(yxyxyxyxy)} \epsilon$ [1]

Choose 46 from {46, 51, 55, 67, 104, 53}

t_1t_{46}

$= (y^{-1}xyxyxy^{-1}xyx)t_{31}$

$= (t_1)^{(y^{-1}xyxyxy^{-1}xy^{-1}xyxy^{-1})} \epsilon$ [1]

Choose 58 from {58, 88, 95, 70, 69, 89}

t_1t_{58}

$= t_{88}$

$= (t_1)^{(yxyxyxyxyxy^{-1})} \epsilon$ [1]

Cayley Diagram

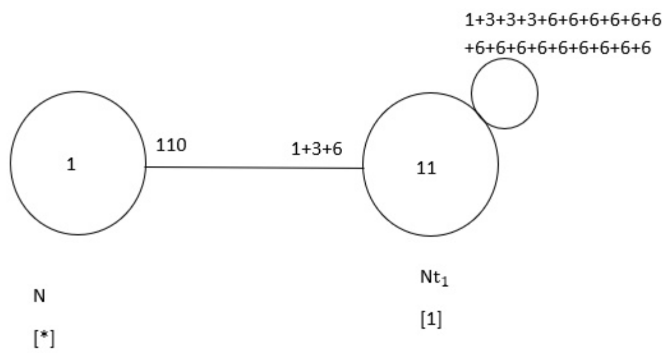


Figure 4.7: Cayley Diagram of [*], [1] for M_{11}

4.6 $\frac{2^{*21}:(2\times A_7)}{(y^{-2}y^{-1}x^4)^5t_5t_1t_7t_5t_1,(xy^{-1}xyx^2y)^7t_6t_2t_{15}t_{10}t_5t_1} \cong \mathbf{A}_{10}$

We will prove that the progenitor 2^{*21}:(2×A_7), where 2^{*21}:(2×A_7) = <x, y> and x ∼ (1, 11, 17, 21, 7, 5, 13, 2, 12, 18, 19, 8, 6, 14, 3, 10, 16, 20, 9, 4, 15), y ∼ (1, 16, 15, 10, 2, 17, 13, 11, 3, 18, 14, 12)(4, 8, 6, 7, 5, 9)(19, 20, 21), factored by two relations isomorphic to the Alternating simple group A_{10}. Let G ≅ $\frac{2^{*21}:(2\times A_7)}{(y^{-2}y^{-1}x^4)^5t_5t_1t_7t_5t_1,(xy^{-1}xyx^2y)^7t_6t_2t_{15}t_{10}t_5t_1}$.
Thus we will show that G ∼ A_{10}.

We have the following relations

Relation 1 = $((yx^{-2}y^{-1}x^4)*(t))^5 = (yx^{-2}y^{-1}x^4)^5t_5t_1t_7t_5t_1$

Relation 2 = $((xy^{-1}xyx^2y)*t)^7 = (xy^{-1}xyx^2y)^7t_6t_2t_{15}t_{10}t_5t_1$

<u>First Double Coset</u>

NeN = { $N(e)^n$ | n ϵ N } = {N}

The coset Stabilizer of the coset N = Ne is N.

The number of single right cosets in the double coset NeN = [*] is given by $\frac{|N|}{|N|} = \frac{7560}{7560} = 1$

The orbit of N on X={ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21} is {1, 11, 16, 17, 3, 20, 15, 21, 13, 10, 18, 9, 7, 19, 2, 14, 4, 5, 8, 12, 6}

We will multiply N on the right by an orbit representative and determine its double coset.

<u>Choose 1 from {1, 11, 16, 17, 3, 20, 15, 21, 13, 10, 18, 9, 7, 19, 2, 14, 4, 5, 8, 12, 6}</u>

Nt_1 ϵ [1]

This tells us that twenty-one elements move forward to the double coset [1]

<u>Cayley Diagram</u>

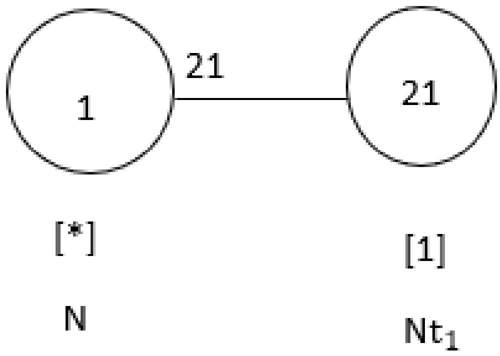


Figure 4.8: Cayley Diagram of [*] for A_{10}

Second Double Coset

$Nt_1N = \{N(t_1)^n \mid n \epsilon N\} = Nt_1, Nt_2, ..., Nt_{21}\}$

The point-stabiliser of 1, N^1 is given by $< (x, y^{-1}), y^2x>$

The coset stabiliser $N^{(1)} = < (x, y^{-1}), y^2x>$

The number of single right cosets in the double coset Nt_1N = [1] is given by $\frac{|N|}{|N^{(1)}|} = \frac{7560}{360}$ = 21

The orbits for $N^{(1)}$ on

X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21} are {1}, {2}, {3}, {4, 14, 16, 11, 7, 19}, {5, 15, 17, 12, 8, 20}, {6, 13, 18, 10, 9, 21}

Multiply Nt_1 by a representative of each orbit and determine its double coset.

Choose 1 from {1}

$Nt_1\ t_1$

$= N(t_1)^2$

$= N \epsilon$ [*]

Choose 2 from {2}

Nt_1t_2

$= Nt_3$

$= N(t_1)^{x^{-2}y^2xy^{-2}} \epsilon$ [1]

Choose 3 from {3}

Nt_1t_3

$= Nt_2$

$= N(t_1)^{x^{-1}y^{-1}x^{-2}y^{-2}x} \epsilon$ [1]

Choose 4 from {4, 14, 16, 11, 7, 19}

$Nt_1t_4 \ \epsilon$ [1,4]

Choose 5 from {5, 15, 17, 12, 8, 20}

$Nt_1t_5 \ \epsilon$ [1,4]

Choose 6 from {6, 13, 18, 10, 9, 21}

$Nt_1t_6 \ \epsilon$ [1,6]

Cayley Diagram

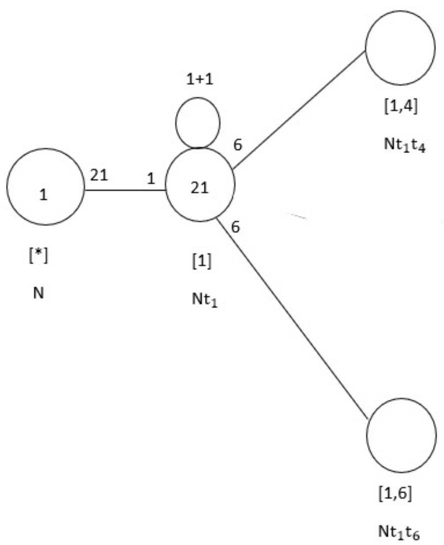


Figure 4.9: Cayley Diagram of [*], [1] for A_{10}

Third Double Coset

Nt_1t_4N = { $N(t_1t_4)^n$ | n ϵ N } = {Nt_1t_4, $Nt_{11}t_{15}$,...,$Nt_{16}t_8$}

The point-stabiliser of 1, 4, $N^{1,4}$ is given by < (x, y^{-1}), y^2x>

The coset stabiliser $N^{(1,4)}$ = < (x, y^{-1}), y^2x>

The number of single right cosets in the double coset Nt_1t_4N = [1,4] is given by $\frac{|N|}{|N^{(1,4)}|}$ = $\frac{7560}{60}$ = 126

The orbits for $N^{(1,4)}$ on

X={ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21 } are:

{1}, {2}, {3}, {4}, {5}, {6}, {7, 16, 11, 14, 19}, {8, 17, 12, 15, 20}, {9, 18, 10, 13, 21}

Multiply Nt_1t_4 by a representative of each orbit and determine its double coset.

Choose 1 from {1}

$Nt_1t_4t_1$

$= Nt_6t_2$

$= N(t_1t_4)^{xyxy^{-1}x^{-1}y^2}$ ϵ [1,4]

Choose 2 from {2}

Nt_1 t_4 t_2

$= Nt_1t_{10}$

$= N(t_1t_6)^{x^{-4}y^{-1}x^2}$ ϵ [1,6]

Choose 3 from {3}

$Nt_1t_4t_3$

$= Nt_6$

$= N(t_1)^{x^{-1}y^{-1}x^{-1}yx^{-2}y^{-2}}$ ϵ [1]

Choose 4 from {4}

$Nt_1t_4t_4$

$= Nt_1(t_4)^2$

$= Nt_1$ ϵ [1]

Choose 5 from {5}

$Nt_1t_4t_5$

$= N_1t_6$ ϵ [1,6]

Choose 6 from {6}

$Nt_1t_4t_6$

$= Nt_6t_2$

$= \mathrm{N}(t_1t_4)^{xyxy^{-1}x^{-1}y^2}$ ϵ [1,4]

Choose 7 from {7, 16, 11, 14, 19}

$\mathrm{Nt_1t_4t_7}$

$= \mathrm{Nt_3t_6}$

$= \mathrm{N}(t_1t_4)^{x^3yx^{-2}y^{-1}x}$ ϵ [1,4]

Choose 8 from {8, 17, 12, 15, 20}

$\mathrm{Nt_1t_4t_8}$

$= \mathrm{Nt_2t_5}$

$= \mathrm{N}(t_1t_4)^{y^2xyx^2y^{-1}x^2}$ ϵ [1,4]

Choose 9 from {9, 18, 10, 13, 21}

$\mathrm{Nt_1t_4t_9}$ ϵ [1,4,9]

Cayley Diagram

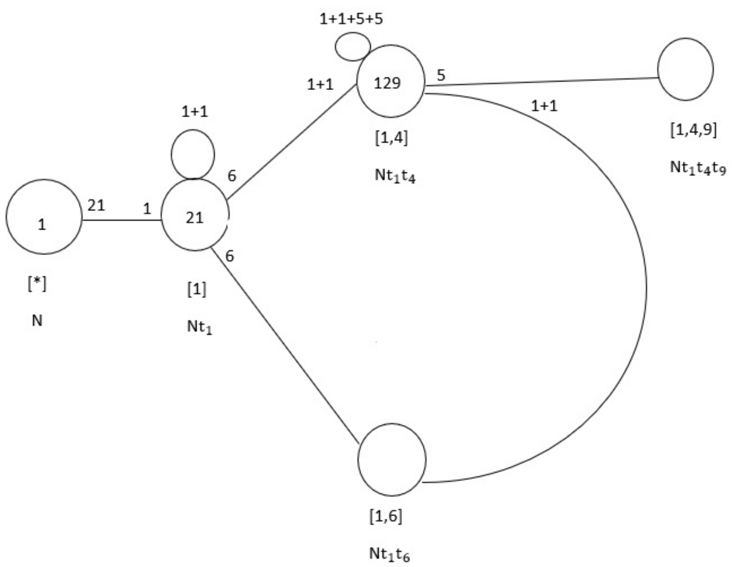


Figure 4.10: Cayley Diagram of [*], [1], [1,4] for A_{10}

Fourth Double Coset

$Nt_1t_6N = \{ N(t_1t_6)^n \mid n \in N \} = \{Nt_1t_6, Nt_{11}t_{14},..., Nt_{16}t_7\}$

The point-stabiliser 1, 6, $N^{1,6}$ is given by

$< yx^{-1}y^{-1}x^2yxy^{-1}x, x^3y^{-1}x^{-1}yx, yx^{-1}y^{-1}x^{-1}y^{-1}x^{-2}yx, yx^3yx^{-2}y^{-2}x^{-1} >$

Now $t_1t_6 = t_1t_{13}$

Then $N(t_1t_6)^{y^2x} = t_1t_{13}$

Thus the coset stabiliser $N^{(1,6)} \geq < N^{1,6}, x^{-1}yx^{-2}y^{-1}x^{-1}y, x^{-3}y^{-1}xyx^{-1}, x^2y^2xy^{-1}xy,$
$(y^{-1}xyxy)^2,$
$y^{-1}x^{-1}y^{-1}xyx, x^2yxyx^{-1}y^{-2}x^{-2}, x^2y^{-1}x^{-1}y^{-1}x^{-2}, yxy^{-1}x^3y^2,$
$y^{-1}x^2yxy^{-1}xyx^{-1}, y^{-1}x^{-2}y^{-2}xy, yx^{-1}y^{-1}x^2yxy^{-1}x, xy^{-1}x^2y^{-1}x^3y^{-1},$
$x^5yx^3, xy^{-1}x^{-3}y^{-1}x^{-1}y^{-1}, x^2y^2xy^{-1}x^{-1}y^{-1}x^{-2}, (x^{-2}y^{-1})^2,$
$yx^2y^{-2}x^{-1}y^{-1}x^{-1}y^{-1}, y^{-1}x^2y^{-1}x^{-3}, (x, y)^2, yx^{-3}y^{-3}x^{-1},$
$xyxy^2x^{-2}, xyxy^{-3}, y^2x^{-1}y^{-1}x^2y^{-2}, xy^2x^{-2}y^{-2}x,$
$x^{-3}y^{-1}x^{-2}yx^{-2}y, xy^{-1}x^{-1}y^{-1}x^2yx^{-1}, x^4yx^3y, xy^2xy^{-2}x^{-1}y^{-1},$
$x^2y^{-1}x^{-1}yx^{-2}y^{-2}, xy^3x^2y^{-1}x, yxy^{-2}x^3yx^{-1}, y^{-2}x^3y^{-1},$
$yx^{-1}y^{-1}x^{-1}y^{-1}x^{-2}yx, x^{-1}y^{-2}x^{-1}yx^{-1}y^{-1}x^{-1}, y^{-1}xy^2xyxy^{-1}x^{-1}, y^{-1}x^{-1}yx^{-1}y^{-1}x^2y,$
$x^3y^{-1}x^{-2}y^{-1}x, yx^3yx^{-2}y^{-2}x^{-1}, x^{-2}y^{-1}x^3, yxy^{-1}x^{-2}y,$
$xy^2x^{-1}y^{-1}x^2y^{-1}x, y^{-1}x^{-2}yxy^{-1}xy, x^3y^2x^{-1}y^{-1}, x^2yxy^{-1}xyx,$
$yxyx^{-2}y^{-1}x^2y, x^{-2}yxy^{-2}x^2, y^2x, x^3y^{-1}x^{-1}yx,$
$x^{-1}y^3xy^{-1}x^{-1}y^{-1}, yx^{-2}yxyx^{-1}y^{-1}, x^2yx^2y^3x, xy^{-1}x^{-2}y^{-1}x^3y^{-1}x,$
$yx^{-2}y^{-1}x^{-1}, yxyxy^3x^2, x^2y^{-1}xy, x^{-1}yx^2yx^{-3},$
$(x^2y^{-1}x^2)^2, x^{-1}y^{-1}x^{-1}y^{-1}x, x^{-1}y^2x^3yx^{-1}y^{-1}, yx^2y^{-1}x^{-1}yxyx^{-1},$
$xyx^{-3}y^{-1}x^2, x^3y^2x^4, xyx^2y^2x^{-1}y^{-1}x^{-1}, x^{-1}yx^{-1}y^{-1}xy^2x^{-1} >$
$= < x^{-1}yx^{-2}y^{-1}x^{-1}y, x^{-3}y^{-1}xyx^{-1}, x^2y^2xy^{-1}xy, (y^{-1}xyxy)^2,$
$y^{-1}x^{-1}y^{-1}xyx, x^2yxyx^{-1}y^{-2}x^{-2}, x^2y^{-1}x^{-1}y^{-1}x^{-2}, yxy^{-1}x^3y^2,$
$y^{-1}x^2yxy^{-1}xyx^{-1}, y^{-1}x^{-2}y^{-2}xy, yx^{-1}y^{-1}x^2yxy^{-1}x, xy^{-1}x^2y^{-1}x^3y^{-1},$
$x^5yx^3, xy^{-1}x^{-3}y^{-1}x^{-1}y^{-1}, x^2y^2xy^{-1}x^{-1}y^{-1}x^{-2}, (x^{-2}y^{-1})^2,$
$yx^2y^{-2}x^{-1}y^{-1}x^{-1}y^{-1}, y^{-1}x^2y^{-1}x^{-3}, (x, y)^2, yx^{-3}y^{-3}x^{-1},$
$xyxy^2x^{-2}, xyxy^{-3}, y^2x^{-1}y^{-1}x^2y^{-2}, xy^2x^{-2}y^{-2}x,$
$x^{-3}y^{-1}x^{-2}yx^{-2}y, xy^{-1}x^{-1}y^{-1}x^2yx^{-1}, x^4yx^3y, xy^2xy^{-2}x^{-1}y^{-1},$
$x^2y^{-1}x^{-1}yx^{-2}y^{-2}, xy^3x^2y^{-1}x, yxy^{-2}x^3yx^{-1}, y^{-2}x^3y^{-1},$
$yx^{-1}y^{-1}x^{-1}y^{-1}x^{-2}yx, x^{-1}y^{-2}x^{-1}yx^{-1}y^{-1}x^{-1}, y^{-1}xy^2xyxy^{-1}x^{-1}, y^{-1}x^{-1}yx^{-1}y^{-1}x^2y,$

$x^3y^{-1}x^{-2}y^{-1}x$, $yx^3yx^{-2}y^{-2}x^{-1}$, $x^{-2}y^{-1}x^3$, $yxy^{-1}x^{-2}y$,
$xy^2x^{-1}y^{-1}x^2y^{-1}x$, $y^{-1}x^{-2}yxy^{-1}xy$, $x^3y^2x^{-1}y^{-1}$, $x^2yxy^{-1}xyx$,
$yxyx^{-2}y^{-1}x^2y$, $x^{-2}yxy^{-2}x^2$, y^2x, $x^3y^{-1}x^{-1}yx$,
$x^{-1}y^3xy^{-1}x^{-1}y^{-1}$, $yx^{-2}yxyx^{-1}y^{-1}$, $x^2yx^2y^3x$, $xy^{-1}x^{-2}y^{-1}x^3y^{-1}x$,
$yx^{-2}y^{-1}x^{-1}$, $yxyxy^3x^2$, $x^2y^{-1}xy$, $x^{-1}yx^2yx^{-3}$,
$(x^2y^{-1}x^2)^2$, $x^{-1}y^{-1}x^{-1}y^{-1}x$, $x^{-1}y^2x^3yx^{-1}y^{-1}$, $yx^2y^{-1}x^{-1}yxyx^{-1}$,
$xyx^{-3}y^{-1}x^2$, $x^3y^2x^4$, $xyx^2y^2x^{-1}y^{-1}x^{-1}$, $x^{-1}yx^{-1}y^{-1}xy^2x^{-1}$ >

The number of single right cosets in the double coset Nt_1t_6N = [1,6] is given by $\frac{|N|}{|N^{(1,4)}|}$ = $\frac{7560}{360}$ = 21

The orbits for $N^{(1,6)}$ on

X={ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21 } are:

{1}, {2}, {3}, {4, 14, 16, 11, 19, 7}, {5, 15, 17, 12, 20, 8}, {6, 13, 18, 10, 21, 9}

Multiply Nt_1t_6 by a representative of each orbit and determine its double coset

Choose 1 from {1}

$Nt_1t_6t_1$ ϵ [1,6,1]

Choose 2 from {2}

$Nt_1t_6t_2$

$= Nt_3t_{17}$

$= N(t_1t_6)^{y^2xyx^2y^{-1}x^2}$ ϵ [1,6]

Choose 3 from {3}

$Nt_1t_6t_3$

$= Nt_2t_{19}$

$= N(t_1t_6)^{yxy^{-2}xyx^2}$ ϵ [1,6]

Choose 4 from {4, 14, 16, 11, 19, 7}

$Nt_1t_6t_4$

$= Nt_6t_2$

$= N(t_1t_4)^{xyxy^{-1}x^-1y^2}$ ϵ [1,4]

Choose 5 from {5, 15, 17, 12, 20, 8 }

$Nt_1t_6t_5$

$= Nt_1t_4$ ϵ [1,4]

Choose 6 from {6, 13, 18, 10, 21, 9}

$Nt_1t_6t_6$

$= Nt_1(t_6)^2$

$= Nt_1 \ \epsilon \ [1]$

<u>Cayley Diagram</u>

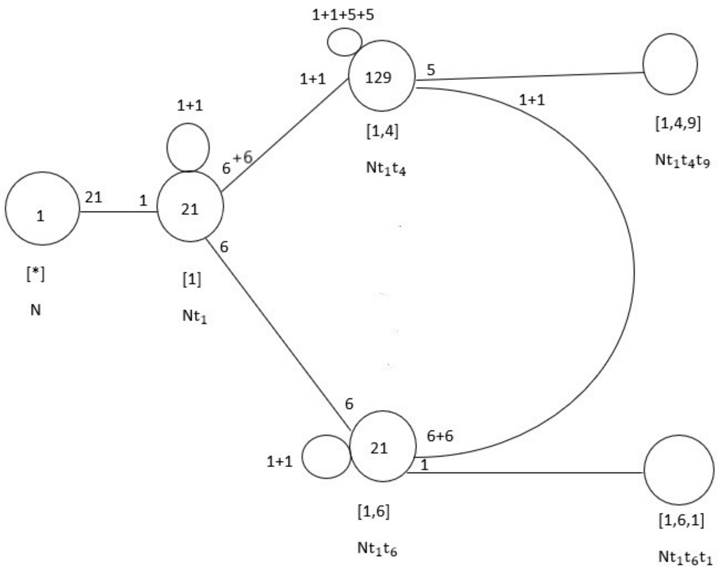


Figure 4.11: Cayley Diagram of [*], [1], [1,4], [1,6], for A_{10}

Fifth Double Coset

$Nt_1t_6t_1N = \{ N(t_1t_6t_1)^n \mid n \epsilon N \} = \{Nt_1t_6t_1, Nt_{11}t_{14}t_{11},..., Nt_{16}t_7t_{16}\}$

The point-stabiliser 1, 6, 1, $N^{1,6,1}$ is given by $< yxy^{-1}xyxy^2, x^2yxyxy^{-1}x^{-1}y^{-1}, y^3xyx, yx^3yx^{-2}y^{-2}x^{-1} >$

Now $t_1t_6t_1 = t_{11}t_{14}t_{11}$

Then $N(t_1t_6t_1)^{xyx^{-2}y^{-1}x^{-2}y^{-2}} = t_1t_{14}t_{11}$

Thus the coset stabiliser $N^{(1,6,1)} \geq < N^{1,6,1}, xyx^{-2}y^{-1}x^{-2}y^{-2}, (xy^{-1}x^{-1})^2, yxyxy^{-1}xy^2x,$
$xyx^{-1}y^{-1}x^{-2}y^{-1}x^{-1}y^{-1}, y^2x^{-2}yx^3, y^2x^2yx^2y^{-1}x, y^3xy^{-1}x^{-1}y^{-1}x^{-2}y^{-1},$
$xy^{-1}x^2yx, y^{-1}x^{-2}yxy^2, xy^2xy^{-1}x^3y, x^{-2}y^{-1}x^{-1}y^{-2}x^{-2},$
$y^3x^2yx^{-2}, x^{-1}y^{-1}xyx^{-2}, yxy^{-1}xyxy^2x, x^{-1}yxy^{-2}x^{-1}y^{-1}x^{-1}y^{-1},$
$y^{-1}x^{-2}y^{-1}x^{-1}y^{-2}x^{-1}, x^2y^{-1}x^{-1}yxy^{-1}xyx, yxyx^{-2}y^{-1}xyx^{-1}, yxyx^{-1}y^{-1}x^{-1}y^{-1}x^{-1},$
$yx^3yx^{-2}y^{-2}, x^2yx^2yx, xy^{-2}x^{-2}y^{-1}x^{-2}y, yxy^{-1}x^4y^{-1},$
$xy^{-2}x^{-1}y^{-2}x^{-1}, y^2xyxy^{-2}x^{-2}, yx^{-1}y^{-2}x^{-2}y^{-1}, x^{-2}yx^2y^2x,$
$x^{-1}y^2x^3y^{-1}x^{-2}, x^4yx^2yx^{-1}, x, x^2y^{-1}x^{-1}yxy^{-1}x^{-1}y,$
$y^2xy^3x^{-1}y^{-1}, xy^{-1}x^{-3}yx^{-1}y^{-2}, x^2y^{-1}x^{-2}y^{-1}x^{-3}, y^{-1}x^{-1}yx^{-1}y^{-1}xyx^{-1},$
$yx^{-1}y^{-1}x^{-1}y^{-2}xy, x^{-2}yx^{-2}yxy^{-1}, y^2xy^{-1}x^{-1}yx^{-1}, y^3xyx^2,$
$yx^{-4}yx^{-3}, yxyxy^{-1}x^{-1}y^{-1}x^2, x^{-1}y^{-1}x^{-1}y^{-3}x, yxy^{-1}x^{-3}yx^{-1},$
$x^{-2}y^2xy^{-1}x^{-1}yx, x^{-1}y^{-1}x^{-1}yx^2y^{-2}, x^{-1}y^{-1}x^2y^{-1}xy, (y^{-1}x^{-1}y)^2,$
$yx^2y^{-1}x^{-1}y^{-1}x^2y^{-1}, x^3yxy^{-1}xyx^{-1}y^{-1}, yx^2y^{-1}x^2y^{-2}x, x^2y^2xy^2,$
$yxyx^{-1}y^{-1}x^{-1}y, yx^{-2}yxy^{-1}x^{-2}, y^{-1}xyx^2y^2x^{-1}, x^2y^2,$
$x^2yx^{-3}yx^{-1}y^{-2}, yx^3y^2x^2y^{-1}, x^{-3}y^{-1}x^{-1}yxy, xy^2x^2yx^2y^{-1},$
$x^2yx^{-2}y^2x^{-2}y^{-1}, x^{-2}y^{-2}x^{-4}, y^{-1}x^{-2}y^{-1}x^{-1}, x^{-2}yxy^{-1}x^3y^{-1},$
$x^2yxyxy^{-1}x^{-1}y^{-1} >$

$= < xyx^{-2}y^{-1}x^{-2}y^{-2}, (xy^{-1}x^{-1})^2, yxyxy^{-1}xy^2x, xyx^{-1}y^{-1}x^{-2}y^{-1}x^{-1}y^{-1},$
$y^2x^{-2}yx^3, y^2x^2yx^2y^{-1}x, y^3xy^{-1}x^{-1}y^{-1}x^{-2}y^{-1}, xy^{-1}x^2yx,$
$y^{-1}x^{-2}yxy^2, xy^2xy^{-1}x^3y, x^{-2}y^{-1}x^{-1}y^{-2}x^{-2}, y^3x^2yx^{-2},$
$x^{-1}y^{-1}xyx^{-2}, yxy^{-1}xyxy^2x, x^{-1}yxy^{-2}x^{-1}y^{-1}x^{-1}y^{-1}, y^{-1}x^{-2}y^{-1}x^{-1}y^{-2}x^{-1},$
$x^2y^{-1}x^{-1}yxy^{-1}xyx, yxyx^{-2}y^{-1}xyx^{-1}, yxyx^{-1}y^{-1}x^{-1}y^{-1}x^{-1}, yx^3yx^{-2}y^{-2},$
$x^2yx^2yx, xy^{-2}x^{-2}y^{-1}x^{-2}y, yxy^{-1}x^4y^{-1}, xy^{-2}x^{-1}y^{-2}x^{-1},$
$y^2xyxy^{-2}x^{-2}, yx^{-1}y^{-2}x^{-2}y^{-1}, x^{-2}yx^2y^2x, x^{-1}y^2x^3y^{-1}x^{-2},$
$x^4yx^2yx^{-1}, x, x^2y^{-1}x^{-1}yxy^{-1}x^{-1}y, y^2xy^3x^{-1}y^{-1},$
$xy^{-1}x^{-3}yx^{-1}y^{-2}, x^2y^{-1}x^{-2}y^{-1}x^{-3}, y^{-1}x^{-1}yx^{-1}y^{-1}xyx^{-1}, yx^{-1}y^{-1}x^{-1}y^{-2}xy,$

$x^{-2}yx^{-2}yxy^{-1}$, $y^2xy^{-1}x^{-1}yx^{-1}$, y^3xyx^2, $yx^{-4}yx^{-3}$,
$yxyxy^{-1}x^{-1}y^{-1}x^2$, $x^{-1}y^{-1}x^{-1}y^{-3}x$, $yxy^{-1}x^{-3}yx^{-1}$, $x^{-2}y^2xy^{-1}x^{-1}yx$,
$x^{-1}y^{-1}x^{-1}yx^2y^{-2}$, $x^{-1}y^{-1}x^2y^{-1}xy$, $(y^{-1}x^{-1}y)^2$, $yx^2y^{-1}x^{-1}y^{-1}x^2y^{-1}$,
$x^3yxy^{-1}xyx^{-1}y^{-1}$, $yx^2y^{-1}x^2y^{-2}x$, $x^2y^2xy^2$, $yxyx^{-1}y^{-1}x^{-1}y$,
$yx^{-2}yxy^{-1}x^{-2}$, $y^{-1}xyx^2y^2x^{-1}$, x^2y^2, $x^2yx^{-3}yx^{-1}y^{-2}$,
$yx^3y^2x^2y^{-1}$, $x^{-3}y^{-1}x^{-1}yxy$, $xy^2x^2yx^2y^{-1}$, $x^2yx^{-2}y^2x^{-2}y^{-1}$,
$x^{-2}y^{-2}x^{-4}$, $y^{-1}x^{-2}y^{-1}x^{-1}$, $x^{-2}yxy^{-1}x^3y^{-1}$, $x^2yxyxy^{-1}x^{-1}y^{-1}$ >

The number of single right cosets in the double coset $Nt_1t_6t_1N$ = [1,6,1] is given by $\frac{|N|}{|N^{(1,6,1)}|} = \frac{7560}{7560} = 1$

The orbits for $N^{(1,6,1)}$ on

X={ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21 } are:

{1, 11, 14, 19, 17, 8, 3, 5, 20, 7, 16, 12, 15, 21, 18, 9, 6, 10, 13, 2, 4}

Multiply $Nt_1t_6t_1$ by a representative of each orbit and determine its double coset.

Choose 1 from {1, 11, 14, 19, 17, 8, 3, 5, 20, 7, 16, 12, 15, 21, 18, 9, 6, 10, 13, 2, 4}

$Nt_1t_6t_1t_1$

$= Nt_1t_6(t_1)^2$

$= Nt_1t_6 \epsilon$ [1,6]

Cayley Diagram

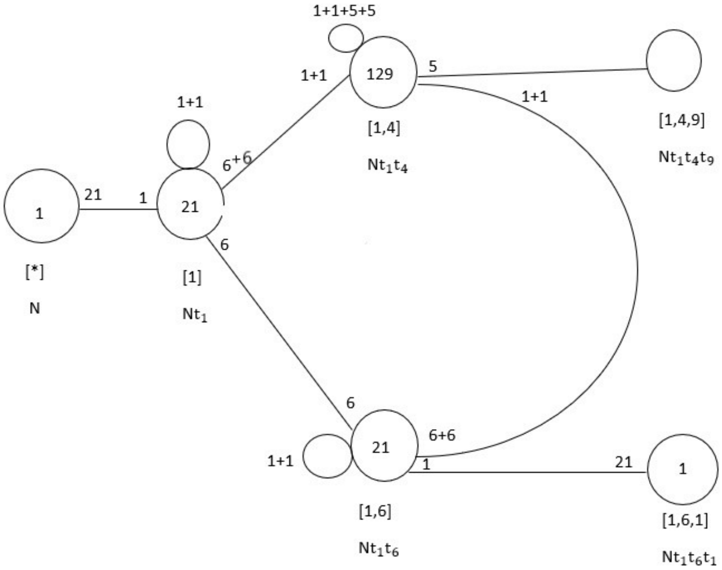


Figure 4.12: Cayley Diagram of [*], [1], [1,4], [1,6], [1,6,1] for A_{10}

Sixth Double Coset

$Nt_1t_4t_9N = \{N(t_1t_4t_9)^n \mid n \in N\} = \{Nt_1t_4t_9, Nt_{11}t_{15}t_4, ..., Nt_{16}t_8t_4\}$

The point-stabiliser 1, 4, 9, $N^{1,4,9}$ is given by $< yxyxy^{-1}x^{-1}y^{-1}x^2, yx^3yx^{-2}y^{-2}x^{-1} >$

Now $t_1t_4t_9 = t_4t_8t_2$

Then $N(t_1t_4t_9)^{yxy^{-1}x^{-2}yx^{-2}} = t_4t_8t_2$

Thus the coset stabiliser $N^{(1,4,9)} \geq < N^{1,4,9}, yxy^{-1}x^{-2}yx^{-2}, yx^{-2}y^{-1}x^{-3}, y^2x^3y^{-1}x^{-2}y^2,$
$yxyxy^3,$
$y^{-2}x^2y^{-1}x^3y^{-1}, x^3y^{-1}x^{-1}yx^{-1}y^{-1}x, x^{-1}y^{-1}x^{-2}yx^2y^{-1}, y^3xyxy^2x^{-1},$
$y^2xyx^3, yxy^{-2}x^{-2}yxy, y^4, xyx^2yx^2y^{-1}x,$
$y^2x^{-1}y^{-1}x^{-1}yx^{-1}y^{-1}, xy^2x^{-1}y^{-1}x^2y^{-1}x^{-1}, yx^2y^{-1}x^2y^2x, y^{-1}x^{-1}y^{-1}xyx^{-1},$
$x^5yx, y^2xyx^2y^{-1}x^2, yx^3yx^{-2}y^{-2}x^{-1}, x^2y^{-2}x^{-1}y^{-1}xyx,$
$x^2y^{-1}x^{-2}y^{-1}x^3, y^{-1}x^{-1}yxy^2xy, x^2yxyx^{-2}y^{-1}x^{-1}, yxyxy^{-1}x^{-1}y^{-1}x^2,$
$yx^{-3}yx^{-2}y^{-2}, y^{-1}x^{-3}y^{-1}x^{-2}y^{-1} >$
$= < yxy^{-1}x^{-2}yx^{-2}, yx^{-2}y^{-1}x^{-3}, y^2x^3y^{-1}x^{-2}y^2, yxyxy^3,$
$y^{-2}x^2y^{-1}x^3y^{-1}, x^3y^{-1}x^{-1}yx^{-1}y^{-1}x, x^{-1}y^{-1}x^{-2}yx^2y^{-1}, y^3xyxy^2x^{-1},$

y^2xyx^3, $yxy^{-2}x^{-2}yxy$, y^4, $xyx^2yx^2y^{-1}x$,
$y^2x^{-1}y^{-1}x^{-1}yx^{-1}y^{-1}$, $xy^2x^{-1}y^{-1}x^2y^{-1}x^{-1}$, $yx^2y^{-1}x^2y^2x$, $y^{-1}x^{-1}y^{-1}xyx^{-1}$,
x^5yx, $y^2xyx^2y^{-1}x^2$, $yx^3yx^{-2}y^{-2}x^{-1}$, $x^2y^{-2}x^{-1}y^{-1}xyx$,
$x^2y^{-1}x^{-2}y^{-1}x^3$, $y^{-1}x^{-1}yxy^2xy$, $x^2yxyx^{-2}y^{-1}x^{-1}$, $yxyxy^{-1}x^{-1}y^{-1}x^2$,
$yx^{-3}yx^{-2}y^{-2}$, $y^{-1}x^{-3}y^{-1}x^{-2}y^{-1}$ >

The number of single right cosets in the double coset $Nt_1t_4t_9N$ = [1,4,9] is given by $\frac{|N|}{|N^{(1,4,9)}|} = \frac{7560}{108} = 70$

The orbits for $N^{(1,4,9)}$ on

X={ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21 } are:

{1, 2, 4, 3, 5, 8, 6, 9, 7}, {10, 13, 16, 11, 14, 19, 18, 21, 20, 15, 12, 17}

Multiply $Nt_1t_4t_9$ by a representative of each orbit and determine its double coset

Choose 9 from {1, 2, 4, 3, 5, 8, 6, 9, 7}

$Nt_1t_4t_9t_9$

$= Nt_1t_4(t_9)^2$

$= Nt_1t_4 \epsilon$ [1,4]

Choose 10 from {10, 13, 16, 11, 14, 19, 18, 21, 20, 15, 12, 17}

$Nt_1t_4t_{10}$

$= Nt_2t_5t_7$

$= N(t_1t_4t_9)^{y^2xyx^2y^{-1}x^2} \epsilon$ [1,4,9]

Cayley Diagram

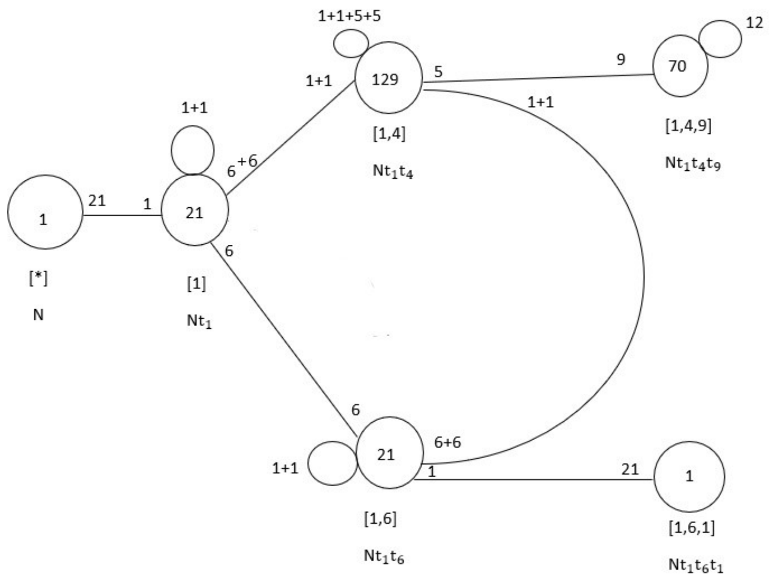


Figure 4.13: Cayley Diagram of [*], [1], [1,4], [1,6], [1,6,1], [1,4,9] for A_{10}

4.7 $\frac{2^{*15}:A_7}{((y^{-1}x^4y^{-1})t^{(x^3y^{-1}xyx^2yx)})^4, x^2ytx^{-2}tyxty^{-1}tx^{-1}ty^{-1}t} \cong \mathbf{M_{22}:2}$

We will prove that the progenitor 2^{*15}: (A_7), where 2^{*15}: (A_7), = <x, y> and x ∼ (1, 2, 5, 10, 13, 7, 3)(4, 8, 14,15, 11, 6, 9), y ∼ (1, 2, 5, 4)(3, 6)(7, 12, 9,14)(8, 10,13, 11), factored by two relations is isomorphic to the mathieu sporadic group M_{22}:2. Let G ≅ $\frac{2^{*15}:A_7}{((y^{-1}x^4y^{-1})t^{(x^3y^{-1}xyx^2yx)})^4, x^2ytx^{-2}tyxty^{-1}tx^{-1}ty^{-1}t}$. Thus we show that G ∼ M_{22}:2.

We have the following relations

Relation 1 = $((y^{-1}x^4y^{-1})* \, t^{(x^3y^{-1}xyx^2yx)})^4 = (y^{-1}x^4y^{-1})^4 t_5t_{15}t_{14}t_5$

Relation 2 = $x^2ytx^{-2}tyxty^{-1}tx^{-1}ty^{-1}t = (x^2yx^{-2} * yx * y^{-1}x^{-1}y^{-1})t_{10}t_4t_{12}t_6t_4t_1$

<u>First Double Coset</u>

NeN = { $N(e)^n$ | n ϵ N } = {N}

The coset stabiliser of N = Ne is N

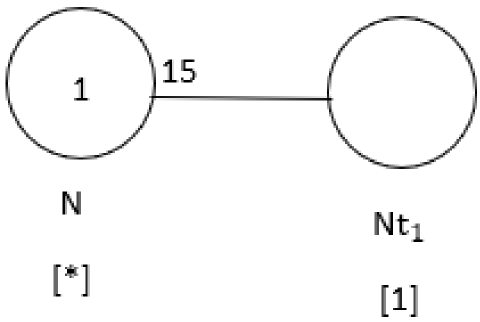
The number of single right cosets in the double coset NeN = [*] is given by $\frac{|N|}{|N|} = \frac{2520}{2520} = 1$

The orbit of N on X= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}. We multiply N on the right by an orbit representative and determine its double coset.

Choose 1 from {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

$Nt_1 \epsilon [1]$

Cayley Diagram



Second Double Coset

$Nt_1N = \{N(t_1)^n \mid n\epsilon\ N\} = \{Nt_1, Nt_2, ..., Nt_{15}\}$

The point-stabilizer of 1, N^1, is given by $<(x^{-1}yx)^2, (x^{-1}, y^{-1}), xyx^{-2}yxy^2, x^{-1}yxy^{-1}xyxy, x^{-2}y^2x^{-1}y^{-1}x, x^2yxyx^{-2}yx^{-2}>$

The coset Stabilizer $N^{(1)} = < yxyx^{-2}yx, y^2xyx^3yx^{-1}, xy^{-1}x^{-1}y^{-1}x^2y>$

The number of single right cosets in the double right coset $Nt_1N = [1]$ is given by $N^{(1)} = \frac{|N|}{|N^{(1)}|} = \frac{2520}{160} = 15$

The orbits for $N^{(1)}$ on

X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} are:

{1} and {2, 10, 12, 3, 5, 15, 8, 7, 11, 4, 9, 13, 14, 6}. We will multiply Nt_1 on the right by an orbit representative and determine its double coset.

Choose 1 from {1}

Nt_1t_1

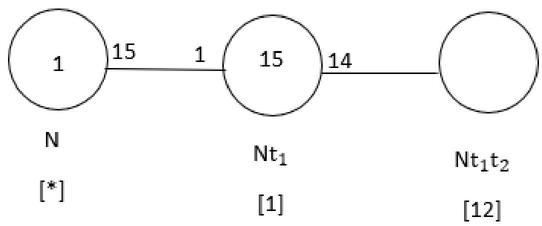
$= N(t_1)^2$

$= N \epsilon [*]$

Choose 2 from {2, 10, 12, 3, 5, 15, 8, 7, 11, 4, 9, 13, 14, 6}

Nt_1t_2 ϵ [1,2]

Cayley Diagram



Third Double Coset

$Nt_1t_2N = \{N(t_1t_2)^n \mid n\epsilon\ N\} = \{Nt_1t_2,...,Nt_2t_5\}$

The point-stabilizer of 1, 2, $N^{1,2}$ is given by $< xyxy^{-1}xyx^2, xy^{-1} >$

Now $t_1t_2 = t_1t_{10}$

Nt_1t_2

$= Nt_1t_{10}$

$= N(t_1t_2)^{y^2x^3y^{-1}x^{-2}y}$ ϵ [1,2]

Thus the coset stabiliser $N^{(1,2)} \geq < N^{1,2}, y^2x^3y^{-1}x^{-2}y, xyx^{-1}y^{-1}x^{-1}yx^2y, xyxy^{-1}xyxy^2x,$
$x^{-1}yxy^{-1}xyxy,$
$y^2x^{-1}y^{-1}x^{-1}yx^2y, xy^{-1}x^{-1}y^2x, yxyxy^2x^{-2}y^2, xy^{-1},$
$x^2y^{-1}xyx^2yx^2, xyx^3y^{-1}x^{-2}y, yxyxy^2x^{-2}y^{-1}x^{-1}, xyxy^{-1}xyx^2,$
$(x^{-1}yx)^2, yx^{-2}y^2x>$

The number of single right cosets in the double coset Nt_1t_2N = [1,2] is given by $N^{(1,2)} = \frac{|N|}{|N^{(1,2)}|} = \frac{2520}{24} = 105$

The orbits for $N^{(1,2)}$ on

X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} are: {1}, {2, 10}, {3, 14, 4, 6, 12, 8, 9, 5, 7, 11, 13, 15}. We will multiply Nt_1t_2on the right by an orbit representative and determine its double coset.

Choose 1 from $\{1\}$

$Nt_1t_2t_1 \epsilon [1,2,1]$

Choose 2 from $\{2, 10\}$

$Nt_1t_2t_2$

$= Nt_1t_2t_2$

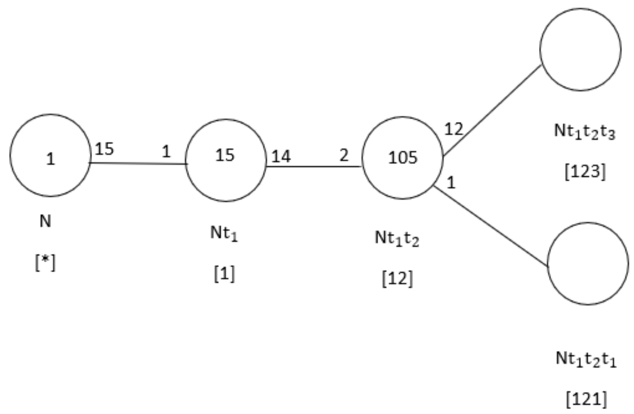
$= Nt_1(t_2)^2$

$= Nt_1 \epsilon [1]$

Choose 3 from $\{3, 14, 4, 6, 12, 8, 9, 5, 7, 11, 13, 15\}$

$Nt_1t_2t_3 \epsilon [1,2,3]$

Cayley Diagram



Fourth Double Coset

$Nt_1t_2t_3N = \{N(t_1t_2t_3)^n \mid n\epsilon N\} = \{Nt_1t_2t_3, Nt_2t_5t_1, ..., Nt_2t_5t_6\}$

The point-stabilizer of 1, 2, 3, $N^{1,2,3}$ is given by $< 1 >$

Now $t_1t_2t_3 = t_3t_{12}t_1$

$Nt_1t_2t_3$

$= Nt_3t_{12}t_1$

$= N(t_1t_2t_3)^{(x^{-2}yxy^{-1}x)} \epsilon [1,2,3]$

Thus the coset stabiliser $N^{(1,2,3)} \geq < N^{1,2,3}, x^{-2}yxy^{-1}x, x^2yx^3y^2, (x^{-2}yxy^{-1}x)>$

The number of single right cosets in the double coset $Nt_1t_2t_3N = [1,2,3]$ is given by

$N^{(1,2,3)} = \frac{|N|}{|N^{(1,2,3)}|} = \frac{2520}{20} = 126$

The orbits for $N^{(1,2,3)}$ on

X={1,2,3,4,5,6,7,8,9,10,11,12,13,14,15} are: {2, 5, 12, 10, 14}, {1, 15, 3, 13, 9, 6, 4, 11, 7, 8}. We will multiply $Nt_1t_2t_3$ on the right by an orbit representative and determine its double coset.

Choose 2 from {2, 5, 12, 10, 14}

$Nt_1t_2t_1 \epsilon [1,2,1]$

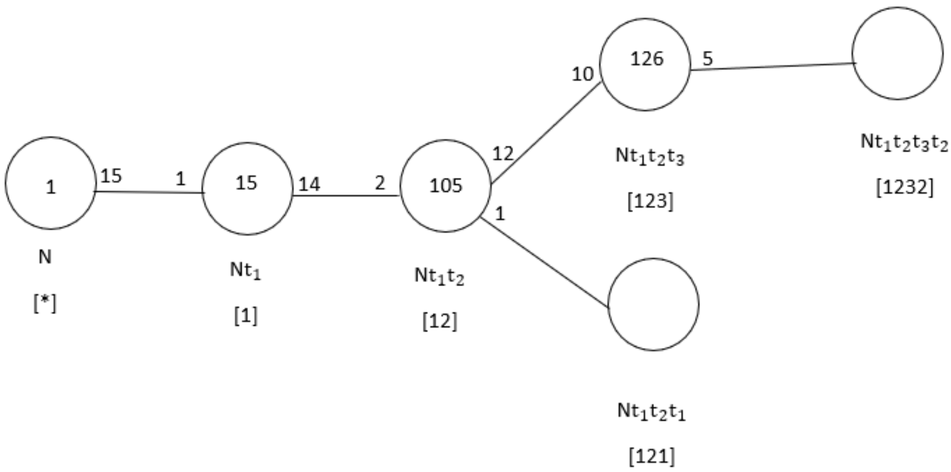
Choose 3 from {1, 15, 3, 13, 9, 6, 4, 11, 7, 8}

$Nt_1t_2t_3t_3$

$= Nt_1t_2(t_3)^2$

$= Nt_1t_2 \epsilon [1,2]$

Cayley Diagram



Fifth Double Coset

$Nt_1t_2t_1N = \{N(t_1t_2t_1)^n \mid n\epsilon N\} = \{Nt_1t_2t_1, ..., Nt_2t_5t_2\}$

The point-stabilizer of 1, 2, 1, $N^{1,2,1}$ is given by $< xyxyx^{-1}y^{-1}x^{-1}y, yx^{-1} >$

Now $t_1t_2t_1 = t_1 t_{10} t_1$

$Nt_1t_2t_1$

$= Nt_1t_{10}t_1$

$=N(t_1t_2t_1)^{((y^{-1}xy)^2)}$ ϵ $N^{(1,2,1)}$

Thus the coset stabiliser $N^{(1,2,1)} \geq <N^{1,2,1}$, yxy^2xyx, $yxy^2xyx^2y^{-1}$, $y^{-1}x^{-1}y^{-1}x^2y^{-1}x^{-2}$, $xyxyx^{-1}y^{-1}x^{-1}y$,

$xy^{-1}x^2y^2xyx$, $xyxy^{-1}xyxy^2x$, x^2y^2xyx, $y^2x^3y^{-1}x^{-2}y$,

yx^{-1}, $xy^{-1}x^{-1}y^2x$, $x^{-1}yxy^{-1}xyxy$, $xyx^3y^{-1}x^{-2}y$,

$(x^{-1}yx)^2$, $y^2x^{-1}y^{-1}x^{-1}yx^2y$, $x^{-1}yxyx^{-2}y^{-1}x$, $x^{-3}y^2x^{-1}y^{-1}x$,

$x^{-2}yx^2y^{-1}x^{-2}$, $yxyxy^2x^{-2}y^2$, $xyx^{-1}y^{-1}x^2y^{-1}x^2$, $xyx^{-1}y^{-1}x^{-1}yx^2y$,

$x^2y^{-1}xyx^2yx^2$, $(x^{-1}yx^3)^2$, $yxyxy^2x^{-2}y^{-1}x^{-1}$, $yx^{-2}y^2x$,

$xyx^{-1}y^{-1}x^2yxy$, $y^2x^{-1}y^{-1}x^2y^{-1}x^2$, $(y^{-1}xy)^2) >$

The number of single right cosets in the double coset $Nt_1t_2t_1N$ = [1,2,1] is given by $N^{(1,2,1)} = \frac{|N|}{|N^{(1,2,1)}|} = \frac{2520}{72} = 35$

The orbits for $N^{(1,2,1)}$ on

X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} are: {1, 10, 2}, {3, 9, 11, 13, 14, 7, 15, 12, 6, 5, 8, 4}. We will multiply $Nt_1t_2t_1$ by an orbit representative and determine its double coset.

Choose 1 from {1, 10, 2}

$Nt_1t_2t_1t_1$

$= Nt_1t_2(t_1)^2$

$= Nt_1t_2$ ϵ [1,2]

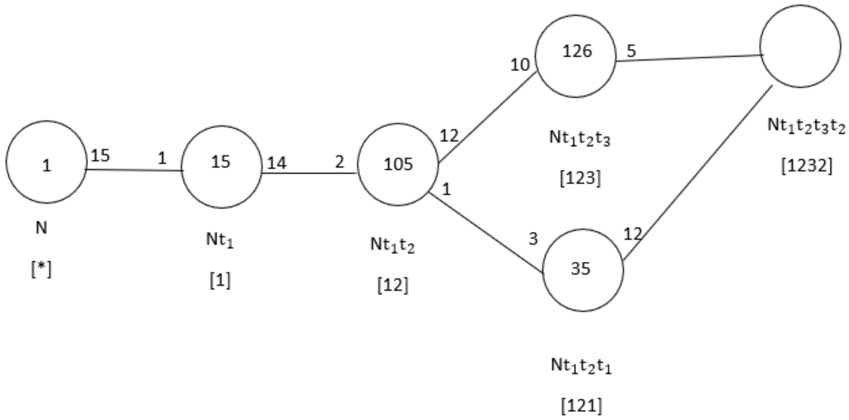
Choose 3 from {1, 15, 3, 13, 9, 6, 4, 11, 7, 8}

$Nt_1t_2t_1t_3$

$= (x^{-1}y^{-1}xyx^3)t_1t_9t_8t_9$

$= Nt_1t_2$ ϵ [1,2,3,2]

Cayley Diagram



Sixth Double Coset

$Nt_1t_2t_3t_2N = \{N(t_1t_2t_3t_2)^n \mid n\epsilon\ N\} = \{Nt_1t_2t_3t_2, Nt_2t_5t_1t_5, ..., Nt_2t_5t_6t_5\}$

The point-stabilizer of 1, 2, 3, 2, $N^{1,2,3,2}$ is given by $< 1 >$

Now $t_1t_2t_3t_2 = t_{13}t_2t_4t_2$

$Nt_1t_2t_3t_2$

$= Nt_{13}t_2t_4t_2$

$= N(t_1t_2t_3t_2)^{(y^2x^2y^{-1}x)}\ \epsilon\ N^{(1,2,3,2)}$

Thus the coset stabiliser $N^{(1,2,3,2)} \geq < N^{1,2,3,2}, yxy^2xyx^2, x^2yx, (y^2x^2y^{-1}x) >$

The number of single right cosets in the double coset $Nt_1t_2t_3t_2N = [1,2,3,2]$ is given by $N^{(1,2,3,2)} = \frac{|N|}{|N^{(1,2,3,2)}|} = \frac{2520}{36} = 70$

The orbits for $N^{(1,2,3,2)}$ on

X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} are {1, 8, 13, 11, 9, 15}, {2, 7, 5, 6, 12, 3, 4, 14, 10}. We will multiply $Nt_1t_2t_3t_2$ on the right by an orbit representative and determine its double coset.

Choose 1 from {1, 8, 13, 11, 9, 15}

$Nt_1t_2t_3t_2t_1$

$= (x^{-1}yxy^{-1}x^{-1}y^{-1}x^{-1}y)t_{15}t_{13}t_{15}\ \epsilon\ [1,2,1]$

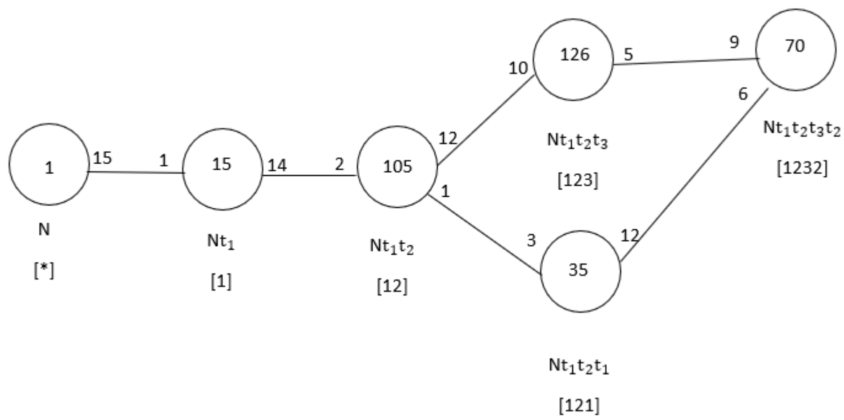
Choose 2 from {2, 7, 5, 6, 12, 3, 4, 14, 10}

$Nt_1t_2t_3t_2t_2$

$= Nt_1t_2t_3(t_2)^2$

$= Nt_1t_2t_3\ \epsilon\ [1,2,3]$

Cayley Diagram



4.8 $\frac{2^{*42}:PSL(2,7)}{((yx)*t^{(y^{-1}x)})^5,((yx)*t^{(y^{-1}xy^{-1}xy^2)})^4,(y*t^{(y^{-1}x)^2})^5,(y*t)^5} \cong$ PSL(3,4)

We will prove that the progenitor 2^{*42}:PSL(2,7) where 2^{*42}:PSL(2,7) = < x, y > and x ~ (1, 2)(5, 7)(6, 9)(8, 12)(10, 14)(11, 15)(13, 17)(16, 20)(18,22)(19, 23)(21, 26)(24, 27)(25, 30)(28, 33)(29, 35)(32, 37)(39, 41)(40, 42), y ~ (1, 3, 5, 8)(2, 4, 6, 10)(7, 11, 9, 13)(12, 16)(14, 18)(15, 19, 24, 29)(17, 21, 27, 32)(20, 25, 31, 26)(22, 28, 34, 23)(30, 36, 37, 40)(33, 38, 35,39)(41, 42)(106, 107), factored by four relations isomorphic to PSL(3,4). Let G ≅ $\frac{2^{*42}:PSL(2,7)}{((yx)*t^{(y^{-1}x)})^5,((yx)*t^{(y^{-1}xy^{-1}xy^2)})^4,(y*t^{(y^{-1}x)^2})^5,(y*t)^5}$. Thus we show that G ~ PSL(3,4).

We have the following relations

Relation 1 = $((yx)*t^{(y^{-1}x)})^5 = (y*x)^5 t_{32}t_{36}t_{30}t_{20}t_{12}$

Relation 2 = $((yx)*t^{(y^{-1}xy^{-1}xy^2)})^4 = (y*x)^4 t_{35}t_{24}t_{21}t_{31}$

Relation 3 = $(y*t^{(y^{-1}x)^2})^5 = y^5 t_{20}t_{26}t_{31}t_{25}t_{20}$

Relation 4 = $(yt)^5 = y^5 t_3t_1t_8t_5t_3$

First Double Coset

NeN = { N(e)^n | n ϵ N } = {N}

The coset stabilizer of the coset N = Ne is N.

The number of single right cosets in the double coset NeN = [*] is given by $\frac{|N|}{|N|} = \frac{42}{42} = 1$

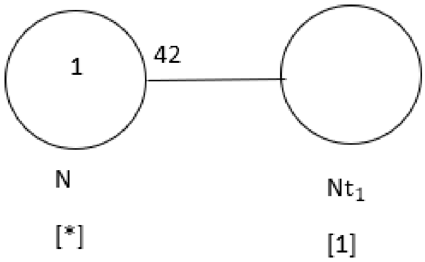
The orbits of N on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42}

We will multiply N on the right by a representative of each orbit and determine its double coset.

Choose 1 from {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42}

Nt_1 ϵ [1]

Cayley Diagram



<u>Second Double Coset</u>

$Nt_1N = \{ N(t_1)^n \mid n \epsilon N\} = \{Nt_1, Nt_2,...,Nt_{42}\}$

The point-stabiliser of 1, N^1 is given by $<yxy^{-1}, (xyxy^{-1})^2>$

But $Nt_1 = Nt_2$

Nt_1

$= Nt_2$

$= N(t_1)^{((xyxy^{-1})^2)} \epsilon [1]$

Then the coset Stabilizer $N^{(1)} = <yxy^{-1}, (xyxy^{-1})^2>$

The number of single right cosets in the double coset $Nt_1N = [1]$ is given by $\frac{|N|}{|N^{(1)}|} = \frac{168}{4}$ $= 42$

The orbits for $N^{(1)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are {1}, {2}, {25}, {28}, {30}, {33}, {3, 13, 31, 17}, {4, 11, 34, 15}, {5, 16, 42, 35}, {6, 18, 41, 37}, {7, 29, 40, 20}, {8, 10, 19, 21}, {9, 32, 39, 22}, {12, 26, 23, 14}, {24, 38, 27, 36}

Multiply Nt_1 by a representative of each orbit and determine its double coset.

<u>Choose 1 from {1}</u>

Nt_1t_1

$= N(t_1)^2$

$= Nt_1 \epsilon [*]$

<u>Choose 2 from {2}</u>

Nt_1t_2

$= Nt_1t_1$

$= N(t_1)^2$

$= N \epsilon$ [*]

Choose 25 from {25}

Nt_1t_{25}

$= Nt_{30}$

$= N(t_1)^{(y^2xy^{-1}xy^{-1}xy^2)} \epsilon$ [1]

Choose 28 from {28}

Nt_1t_{28}

$= Nt_{30}$

$= N(t_1)^{(y^2xy^{-1}xy^-1xy^2)} \epsilon$ [1]

Choose 30 from {30}

Nt_1t_{30}

$= Nt_{25}$

$= N(t_1)^{(y^2xy^{-1}xyxy^2)} \epsilon$ [1]

Choose 33 from {33}

Nt_1t_{33}

$= Nt_{25}$

$= N(t_1)^{(y^2xy^{-1}xyxy^2)} \epsilon$ [1]

Choose 3 from {3, 13, 31, 17}

$Nt_1t_3 \epsilon$ [1,3]

Choose 4 from {4, 11, 34, 15}

Nt_1t_4

$= Nt_{27}t_{31}$

$= N(t_1t_3)^{(y^2xy^{-1}xy^2)} \epsilon$ [1,3]

Choose 5 from {5,16,42,35}

$Nt_1t_5 \epsilon$ [1,5]

Choose 6 from {6, 18, 41, 37}

Nt_1t_6

$= N(yxy^2)t_{11}t_{34}$

$= N(t_1t_5)^{(xyxyxy^{-1})} \epsilon$ [1,5]

Choose 7 from {7, 29, 40, 20}

Nt_1t_7

$= Nt_2t_7$

$= N(t_1t_5)^{(x)} \epsilon [1,5]$

Choose 8 from {8, 10, 19, 21}

Nt_1t_8

$= N(yxy^{-1})t_{18}t_1$

$= N(t_1t_3)^{(xy^{-1}xyxy^{-1}x)} \epsilon [1,3]$

Choose 9 from {9, 32, 39, 22}

Nt_1t_9

$= Nt_2t_7$

$= N(t_1t_5)^{(x)} \epsilon [1,5]$

Choose 12 from {12, 26, 23, 14}

Nt_1t_{12}

$= N(xyxy^{-1}x)t_{12}t_{30}$

$= N(t_1t_3)^{(xyxy^{-1}xy^{-1}xy^2)} \epsilon [1,3]$

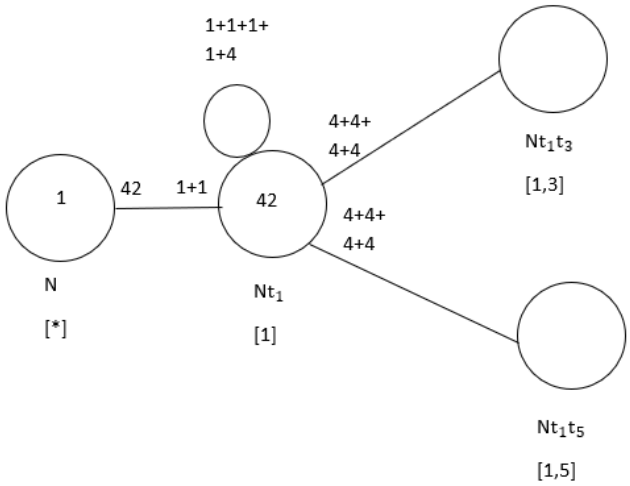
Choose 24 from {24, 38, 27, 36}

Nt_1t_{24}

$= Nt_{36}$

$= N(t_1)^{(y^2xy^{-1}xy^-1xy^{-1})} \epsilon [1]$

Cayley Diagram



<u>**Third Double Coset**</u>

$Nt_1t_3N = \{ N(t_1t_3)^n \mid n \epsilon N\} = \{Nt_1t_3, Nt_2t_3,..., Nt_3t_5\}$

The point-stabiliser 1, 3, N^{13} is given by $< 1 >$

Now $Nt_1t_3 = Nt_{24}t_{31}$

$= Nt_1t_3$

$= Nt_{24}t_{31}$

$= N(t_1t_3)^{(y^2xyxy^2)} \epsilon$ [1,3]

Thus, $(y^2xyxy^2) \epsilon$ [1,3]

Thus the coset stabiliser $N^{13)} \geq < N^{13}, x, y^2xyxy^2 > =$

$<x, y^2xyxy^2 >$

The number of single right cosets in the double coset Nt_1t_3N = [1,3] is given by $\frac{|N|}{|N^{(13)}|} = \frac{168}{4} = 42$

The orbits for $N^{(13)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are: {3, 31}, {4, 34}, {36, 38}, {1, 2, 24, 27}, {5, 7, 10, 14}, {6, 9, 8, 12}, {11, 15, 25, 30}, {13, 17, 28, 33}, {16, 20, 37, 32}, {18, 22, 35, 29}, {19, 23, 41, 39}, {21, 26, 42, 40}

Multiply Nt_1t_3 by a representative of each orbit and determine its double coset.

<u>Choose 3 from {3, 31}</u>

$Nt_1t_3t_3$

$= Nt_1(t_3)^2$

$= Nt_1 \; \epsilon \; [1]$

Choose 4 from {4, 34}

$Nt_1t_3t_4$

$= Nt_1$

$= N(t_1)^{(x*yxy^{-1})^2} \; \epsilon \; [1]$

Choose 36 from {36, 38}

$Nt_1t_3t_{36}$

$= Nt_2t_{31}$

$= N(t_1t_3)^{(yxy^{-1}xyxxy^{-1})} \; \epsilon \; [1,3]$

Choose 1 from {1, 2, 24, 27}

$Nt_1t_3t_1$

$= N(x)t_3t_1$

$= N(t_1)^{(y^2xy^{-1}xyxy^{-1})} \; \epsilon \; [1]$

Choose 5 from {5, 7, 10, 14}

$Nt_1t_3t_5$

$= N(y^2xy^{-1})t_{18}t_1$

$= N(t_1t_3)^{(xy^{-1}xyxy^{-1}x)} \; \epsilon \; [1,3]$

Choose 6 from {6, 9, 8, 12}

$Nt_1t_3t_6$

$= N(y^2xy^{-1})t_{18}t_1$

$= N(t_1t_3)^{(xy^{-1}xyxy^{-1}x)} \; \epsilon \; [1,3]$

Choose 11 from {11, 15, 25, 30}

$Nt_1t_3t_{11}$

$= N((y^{-1}x)^3)t_{23}t_{18}$

$= N(t_1t_5)^{(xyxyxy^{-1}xyx)} \; \epsilon \; [1,5]$

Choose 13 from {13, 17, 28, 33}

$Nt_1t_3t_{13}$

$= N((y^{-1}x)^3)t_{23}t_{18}$

$= N(t_1t_5)^{(xyxyxy^{-1}xyx)} \; \epsilon \; [1,5]$

Choose 16 from {16, 20, 37, 32}

$Nt_1t_3t_{16}$

$= N(xyxyxy^2)t_4t_{25}$

$= N(t_1t_5)^{yxy^{-1}xy)}$ ϵ [1,5]

Choose 18 from {18, 22, 35, 29}

$Nt_1t_3t_{18}$

$= Nt_{24}t_{22}$

$= N(t_1t_5)^{(y^{-1}xy^{-1}xy^{-1}xyx)}$ ϵ [1,5]

Choose 19 from {19, 23, 41, 39}

$Nt_1t_3t_{19}$

$= N(y^2xy^{-1}xy)t_{34}t_{42}$

$= N(t_1t_3)^{(y^2xyxyxy^{-1}x)}$ ϵ [1,3]

Choose 21 from {21, 26, 42, 40}

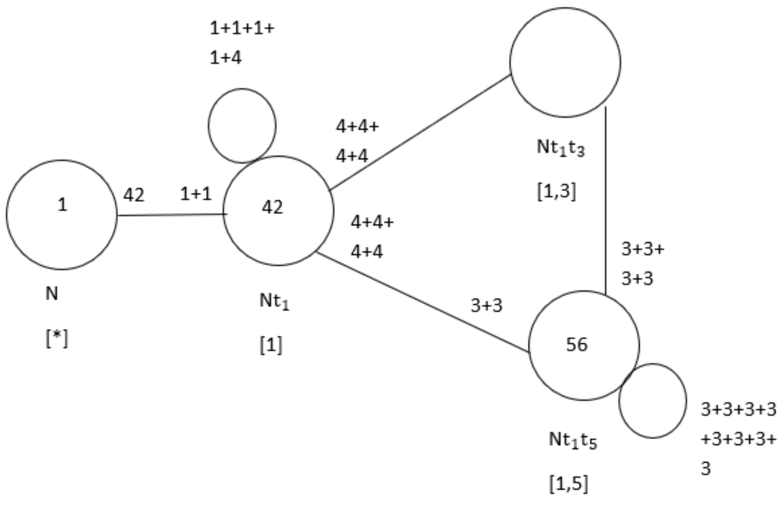
$Nt_1t_3t_{21}$

$= N(\ y^2xy^{-1}xy)t_{34}t_{42}$

$= Nt_{34}t_{42}$

$= N(t_1t_3)^{(y^2xyxyxy^{-1}x)}$ ϵ [1,3]

Cayley Diagram



<u>Fourth Double Coset</u>

$Nt_1t_5N = \{ N(t_1t_5)^n \mid n \epsilon N\} = \{Nt_1t_5, Nt_2t_7, ..., Nt_3t_8\}$

The point-stabiliser 1, 5, $N^{1,5}$ is given by $< 1 >$

Now $Nt_1t_{24} = Nt_{11}t_{34}$

Nt_1t_{24}

$= Nt_{11}t_{34}$

$= N(t_1t_{24})^{(xyxyxy^{-1})} \epsilon [1,24]$

Thus $xyxyxy^{-1} \epsilon N^{(15)}$

Thus the coset stabiliser $N^{(15)} \geq < N^{15}, xyxyxy^{-1} > =$

$<xyxyxy^{-1}>$

The number of single right cosets in the double coset Nt_1t_5N = [1,5] is given by $\frac{|N|}{|N^{(15)}|} = \frac{168}{3} = 56$

The orbits for $N^{(15)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are {1, 11, 14}, {2, 13, 12}, {3, 29, 37}, {4, 32, 35}, {5, 34, 21}, {6, 31, 19}, {7, 26, 38}, {8, 40, 22}, {9, 23, 3}, {10, 39, 30}, {15, 18, 33}, {16, 30, 17}, {24, 41, 28}, {25, 27, 42}

Multiply Nt_1t_5 by a representative of each orbit and determine its double coset.

<u>Choose 1 from {1, 11, 14}</u>

$Nt_1t_5t_1$

$= N(y^2)^*t_1t_5$

$= Nt_1t_5 \epsilon [1,5]$

<u>Choose 2 from {2, 13, 12}</u>

$Nt_1t_5t_2$

$= N(y^2)t_1t_5$

$= Nt_1t_5 \epsilon [1,5]$

<u>Choose 3 from {3, 29, 37}</u>

$Nt_1t_5t_3$

$= N(xyxyxy^{-1}xyx)t_{36}t_{17}$

$= N(t_1t_3)^{(y^2xy^{-1}xy^{-1}xy^{-1})} \epsilon [1,3]$

<u>Choose 4 from {4, 32, 35}</u>

$Nt_1t_5t_4$

$= N(xyxyxy^{-1}xyx)t_{36}t_{17}$

$= N(t_1t_3)^{(y^2xy^{-1}xy^{-1}xy^{-1})} \epsilon$ [1,3]

Choose 5 from {5, 34, 21}

$Nt_1t_5t_5$

$= Nt_1(t_5)^2$

$= Nt_1 \epsilon$ [1]

Choose 6 from {6, 31, 19}

$Nt_1t_5t_6$

$= Nt_1$

$= N(t_1)^{((xyxy^{-1})^2)} \epsilon$ [1]

Choose 7 from {7, 26, 38}

$Nt_1t_5t_7$

$= N(y^2xy^{-1}x)t_{13}t_{22}$

$= N(t_1t_5)^{(xy)^3} \epsilon$ [1,5]

Choose 8 from {8, 40, 22}

$Nt_1t_5t_8$

$= N(xy)t_{12}t_{30}$

$= N(t_1t_3)^{(xyxy^{-1}xy^{-1}xy^2)} \epsilon$ [1,3]

Choose 9 from {9, 23, 3}

$Nt_1t_5t_9$

$= N(y^2xy^{-1}x)t_{13}t_{22}$

$= N(t_1t_5)^{(xy)^3}$

Choose 10 from {10, 39, 30}

$Nt_1t_5t_{10}$

$= N(\ xy)t_{12}t_{30}$

$= N(t_1t_3)^{xyxy^{-1}xy^{-1}xy^2)} \epsilon$ [1,3]

Choose 15 from {15, 18, 33}

$Nt_1t_5t_{15}$

$= N(y^{-1}xy^{-1}xy^{-1})t_{11}t_4$

$= N(t_1t_3)^{(y^2xy)} \epsilon$ [1,3]

Choose 16 from {16, 30, 17}

$Nt_1t_5t_{16}$

$= N(y^{-1}xy^{-1})t_5t_1$

$= N(t_1t_5)^{(y^{-1}xy^{-1})}$ ϵ [1,5]

Choose 24 from {24, 41, 28}

$Nt_1t_5t_{24}$

$= N(y^{-1}xy^{-1}xyxy)t_2t_{40}$

$= N(t_1t_5)^{(yxy^{-1}xyxy^{-1})}$ ϵ [1,5]

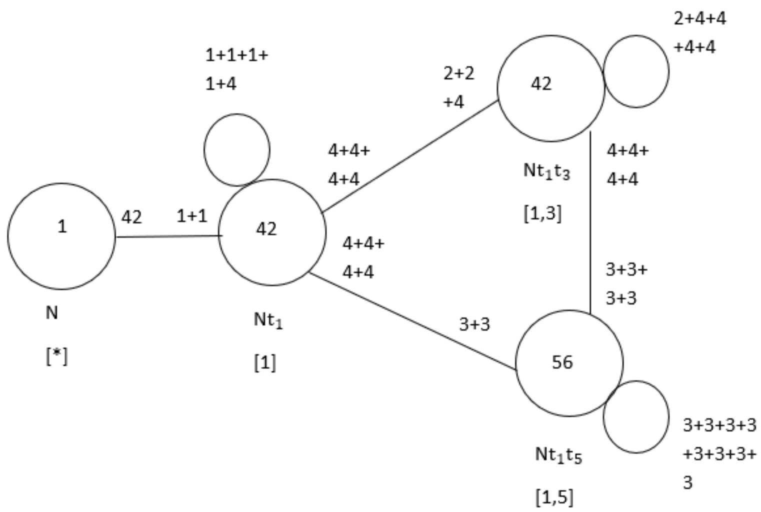
Choose 25 from {25, 27, 42}

$Nt_1t_5t_{25}$

$= N(y^2xyxy^{-1}xy^2)t_1t_{35}$

$= N(t_1t_5)^{(xyxy^{-1}x)}$ ϵ [1,5]

Cayley Diagram



Chapter 5

Double Coset Enumeration Involving Maximal Subgroup

5.1 S_6 over S_5 and A_5

We perform double coset enumeration of G over H = S_5 = < x, y, $t_1t_2t_1$ > and A_5 = N. It should be noted that the order of H is 160 and $A_5 \leq$ H. We will prove that the progenitor 2^{*5}: A_5, where 2^{*5}:A_5 = < x, y> and x ∼ (3, 4, 5), y ∼ (1, 2, 3), factored by one relations is isomorphic to S_6. Let G $\cong \frac{2^{*5}:A_5}{t_1t_2t_1}$. Thus we show that G ∼ S_6.

<u>**First Double Coset**</u>

HeN = { $H(e)^n$ | n ϵ N } = {H | n ϵ N }

{H} since N ⊆ H

The coset stabiliser H = He is H.

The number of single right cosets in the double coset HeN = [*] is given by $\frac{|H|}{|H|} = \frac{120}{120} =$ 1

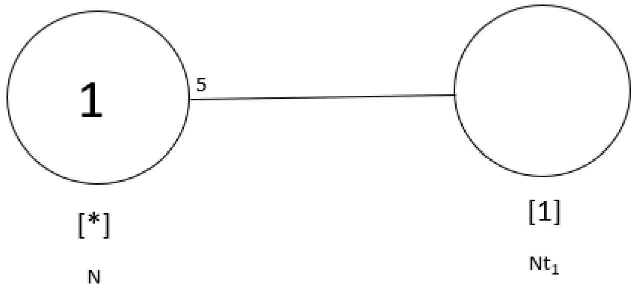
The orbits of H on X = {1, 2, 3, 4, 5} is {1, 2, 3, 4, 5}

We will now choose an orbit representative and multiply and multiply the representative by H on the right and determine its double coset.

<u>Choose 1 from {1, 2, 3, 4, 5}</u>

Ht_1 ϵ [1]

<u>Cayley Diagram</u>



Second Double Coset

$Ht_1N = \{ H(t_1)^n \mid n \epsilon N\} = \{Ht_1, Ht_2, Ht_3, Ht_4, Ht_5\}$

The The point-stabiliser 1, N^1 is given by $< (2, 3, 4), (3, 4, 5) >$

The coset stabiliser $N^{(1)} = < (2, 3, 4), (3, 4, 5) >$

THe number of single right cosets in the double coset $Ht_1N = [1]$ is given by $\frac{|N|}{|N^{(1)}|} = \frac{120}{24}$ $= 5$.

The orbits of $N^{(1)}$ on $X = \{1, 2, 3, 4, 5\}$ are $\{1\}$, $\{2, 3, 4, 5\}$, we will now choose a representative of each orbit and multiply Ht_1 and determine its double coset.

Choose 1 from {1} Ht_1t_1

$= H(t_1)^2$

$= H \epsilon [*]$

Choose 2 from {2, 3, 4, 5}

We will investigate using our relation. Where $H = < N, t_1t_2t_1 >$ since $t_1t_2t_1 \epsilon H$.

Then by a theorem,

$\implies Ht_1t_2t_1 = H$

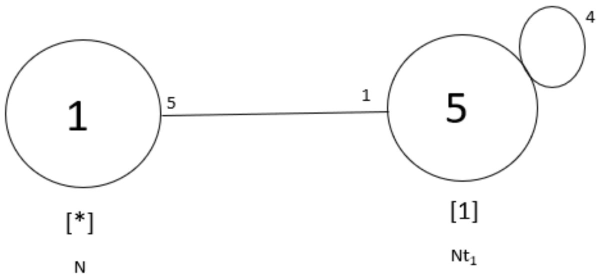
We will multiply on the right by t_1 since $t_1^2 = e$

$\implies t_1t_2t_1t_1 = Ht_1$

$\implies Ht_1t_2$

$= Ht_1 \epsilon [1]$

Cayley Diagram



Thus, $G = H \cup Ht_1 \cup Ht_2 \cup Ht_3 \cup Ht_4 \cup Ht_5$

Now $H = < N, t_1t_2t_1 >$, which is double the order of N, then $H \cup Nt_1t_2t_1$ since the order of H is 120 and the order of N is 60. So,

$G = N \cup Nt_1t_2t_1$ which gives

$G = N \cup t_1t_2t_1 \cup (N \cup Nt_1t_2t_1)t_1 \cup (N \cup Nt_1t_2t_1)t_2 \cup (N \cup Nt_1t_2t_1)t_3 \cup (N \cup Nt_1t_2t_1)t_4 \cup (N \cup Nt_1t_2t_1)t_5$

$\implies G = N \cup t_1t_2t_1 \cup Nt_1 \cup Nt_1t_2t_1t_1 \cup Nt_2 \cup Nt_1t_2t_1t_2 \cup Nt_3 \cup Nt_1t_2t_1t_3 \cup Nt_4 \cup Nt_1t_2t_1t_4 \cup Nt_5 \cup Nt_1t_2t_1t_5$

Our goal is to get a G similar to our DCE of 2^{*5}: A_5 which is $G = N \cup Nt_1 \cup Nt_2 \cup Nt_3 \cup Nt_4 \cup Nt_5 \cup Nt_1t_2 \cup Nt_1t_3 \cup Nt_1t_4 \cup Nt_1t_5 \cup Nt_2t_3 \cup Nt_2t_4 \cup Nt_2t_5 \cup Nt_3t_4 \cup Nt_3t_5 \cup Nt_4t_5 \cup Nt_1t_2t_1$

We need to investigate further, we are going to take the following elements of our G, and use our relation to see where they belong: $Nt_1t_2t_1t_1$, $Nt_1t_2t_1t_2$, $Nt_1t_2t_1t_3$, $Nt_1t_2t_1t_4$, $Nt_1t_2t_1t_5$.

First $Nt_1t_2t_1t_1$

$Nt_1t_2t_1t_1$

$= Nt_1t_2(t_1)^2$

$= Nt_1t_2$

Next $Nt_1t_2t_1t_2$

$= \underline{t_1t_2t_1}t_2$

Conjugate our H relation $t_1t_2t_1$ by (1,2,3) to get $t_2t_3t_2$

Then we have $\underline{t_1t_2t_1}t_2$

$= t_2t_3t_2t_2$

$= t_2t_3(t_2)^2$

$= t_2t_3$

Similarly $Nt_1t_2t_1t_3$

$= \underline{t_1t_2t_1}t_3$

Conjugate our H relation $t_1t_2t_1$ by (1,3,2) to get $t_3t_2t_3$

Then we have $\underline{t_1t_2t_1}t_3$

$= t_3t_2t_3t_2$

$= t_3t_2(t_3)^2$

$= t_3t_2$

Next $Nt_1t_2t_1t_4$

$= \underline{t_1t_2t_1}t_4$

Conjugate our H relation $t_1t_2t_1$ by (1,4,2) to get $t_4t_1t_4$

Then we have $\underline{t_1t_2t_1}t_4$

$= t_4t_1t_4t_4$

$= t_4t_1(t_4)^2$

$= t_4t_1$

Lastly $Nt_1t_2t_1t_5$

$= \underline{t_1t_2t_1}t_5$

Conjugate our H relation $t_1t_2t_1$ by (1,5,2) to get $t_5t_1t_5$

Then we have $\underline{t_1t_2t_1}t_5$

$= t_5t_1t_5t_5$

$= t_5t_1(t_5)^2$

$= t_5t_1$

Therefore we have

$H = N \cup Nt_1 \cup t_2 \cup Nt_3 \cup Nt_4 \cup Nt_5\ Nt_1t_2 \cup Nt_2t_1 \cup Nt_3t_2 \cup Nt_4t_1 \cup Nt_5t_1 \cup t_1t_2t_1$

which is the same as

$G = N \cup Nt_1 \cup Nt_2 \cup Nt_3 \cup Nt_4 \cup Nt_5 \cup Nt_1t_2 \cup Nt_1t_3 \cup Nt_1t_4 \cup Nt_1t_5 \cup Nt_2t_3 \cup Nt_2t_4 \cup Nt_2t_5 \cup Nt_3t_4 \cup Nt_3t_5 \cup Nt_4t_5 \cup Nt_1t_2t_1$

5.2 S_8 over S_7 and PSL(2,7)

We perform double coset enumeration of G over H = S_7 = $< x, y, tyty^{-1}xty^{-1}ty^{-1}xt >$ and PSL(2, 7) = N. It should be noted that the order of H is 5040. We will prove that the progenitor 2^{*42}: PSL(2, 7), where 2^{*42}:PSL(2, 7) = < x, y> and x ~ (1, 2)(5, 7)(6, 9)(8, 12)(10, 14)(11, 15)(13, 17)(16, 20)(18, 22)(19, 23)(21, 26)(24, 27)(25, 30)(28, 33)(29, 35)(32, 37)(39, 41)(40, 42), y ~ (1, 3, 5, 8)(2, 4, 6, 10)(7, 11, 9, 13)(12, 16)(14, 18)(15, 19, 24, 29)(17,21, 27, 32)(20, 25, 31, 26)(22, 28, 34, 23)(30, 36, 37, 40)(33, 38, 35, 39)(41, 42), factored by three relations is isomorphic to 2^2:4. Let G $\cong$ $\frac{2^{*42}:PSL(2,7)}{(yxy)^8t_8t_{13}t_4t_8t_{13}t_4t_8t_{13}, y^2t_{27}t_{21}, t_1t_8t_7t_3t_1}$. Thus we show that G ~ 2^2:4.

We have the following relations

Relation 1 = $(yxy)t^((xy)^3)^8$ = $(yxy)^8t_8t_{13}t_4t_8t_{13}t_4t_8t_{13}$ = e

Relation 2 = $((y)t^{(y^{-1}xyxy^{-1}xy^{-1})^2}$ = $y^2t_{27}t_{21}$ = e

Relation 3 = $tyty^{-1}xty^{-1}ty^{-1}xt$ = $t_1t_8t_7t_3t_1$ = $Ht_1t_8t_7$ = Ht_1t_3

<u>First Double Coset</u>

HeN= { $H(e)^n$ | n ϵ N } = H since N $\subseteq$ H

The coset stabiliser H = He is H.

The number of single right cosets in the double coset HeN = [*] is given by $\frac{|N|}{|N|} = \frac{168}{168} =$ 1

The orbit of N on X={ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} We will multiply H on the right by an orbit representative and determine its double coset.

Ht_1 ϵ [1]

This tells us that forty-two elements move forward to the double coset [1]

<u>Cayley Diagram</u>

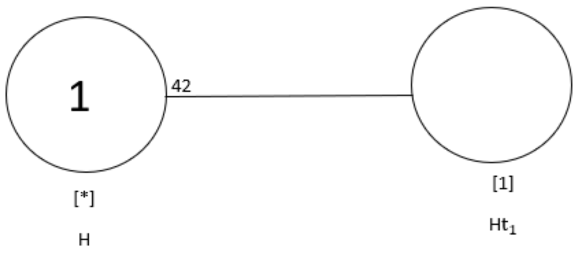


Figure 5.1: Cayley Diagram of [*] for S_8 over S_7 and PSL(2,7)

Second Double Coset

$Ht_1N = \{ H(t_1)^n \mid n \in N \} = \{Ht_1, Ht_2, ..., Ht_{42}\}$

Point Stabilizer 1, $N^1 = <(xyxy^{-1})^2, yxy^{-1}>$

Now $Ht_1 = Ht_5$

Ht_1

$= (y^2t)^2t_{33}$

$= H(t_1)^{xy^{-1}xyxyx}$

Then $(xy^{-1}xyxyx) \in N^{(1)}$

Thus $N^{(1)} \geq < N^1, yxy^{-1}, y^2, (xyxy^{-1})^2, yxy, xyxy^{-1}xy^2, xyxy^{-1}xyxy > = <(xyxy^{-1})^2, yxy^{-1}, yxy^{-1}, y^2, (xyxy^{-1})^2, yxy, xyxy^{-1}xy^2, xyxy^{-1}xyxy>$

The number of single right cosets in the double coset $Ht_1N = [1]$ is given by $N^{(1)} = \frac{|N|}{|N^{(1)}|} = \frac{168}{24} = 7$

The orbits for $N^{(1)}$ on

X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} is

{1, 5, 42, 16, 33, 35}, {2, 6, 41, 18, 30, 37}, {12, 23, 26, 25, 14, 28}, {3, 31, 13, 8, 11, 27, 20, 17, 19, 10, 4, 32, 29, 15, 22, 24, 36, 40, 39, 9, 21, 34, 7, 38}

Multiply H by a representative of each orbit and determine its orbit.

Choose 1 from {1, 5, 42, 16, 33, 35}

Ht_1t_1

$= H(t_1)^2$

$= H \epsilon$ [*]

Choose 2 from {2, 6, 41, 18, 30, 37}

Ht_1t_2

$= (txy^{-1}xy^{-1}txyt)t_1^{(xy^{-1}xyxyxy^{-1}xy^{-1})}$

$(txy^{-1}xy^{-1}txyt)t_{42}$

$= (xy^{-1}xy^{-1}xy)t_{28}t_4t_1t_{42}$

So

$= (xy^{-1}xy^{-1}xy)t_{28}t_4t_1t_{42}$

Then $Ht_1t_2 = (xy^{-1}xy^{-1}xy)t_{28}t_4t_1t_{42}$

$\implies Ht_1t_2 = Ht_{28}t_4t_1t_{42} \epsilon$ [1]

Choose 12 from {12, 23, 26, 25, 14, 28}

Nt_1t_{12}

$= (xty^{-1}txyxy^2tx)t_1^{xy^{-1}xyxyxy^{-1}xy^{-1}}$

$= (xty^{-1}txyxy^2tx)t_{42}$

$= (xy^{-1}xyxy^2x)t_{31}t_{14}t_2t_{42}$

Then

$= (xy^{-1}xyxy^2x)t_{31}t_{14}t_2t_{42}$

Next,

$Ht_1t12 = (xy^{-1}xyxy^2x)t_{31}t_{14}t_2t_{42}$

$\implies Ht_1t12 = Ht_{31}t_{14}t_2t_{42} \epsilon$ [1]

Choose 3 from {3, 31, 13, 8, 11, 27, 20, 17, 19, 10, 4, 32, 29, 15, 22, 24, 36, 40, 39, 9, 21, 34, 7, 38}

Ht_1t_3

$= (y^{-1}xtxy^{-1}xtytx)t_1^{yxyxy^{-1}xy^{-1}}$

$= (y^{-1}xtxy^{-1}xtytx)t_{32}$

$= (y^{-1}xxy^{-1}xyx)t_{22}t_3t_2t_{32}$

So $= (y^{-1}xxy^{-1}xyx)t_{22}t_3t_2t_{32}$

Then

$Ht_1t_3 = (y^{-1}xxy^{-1}xyx)t_{22}t_3t_2t_{32}$

$\implies Ht_1t_3 = Ht_{22}t_3t_2t_{32} \epsilon$ [1]

Cayley Diagram

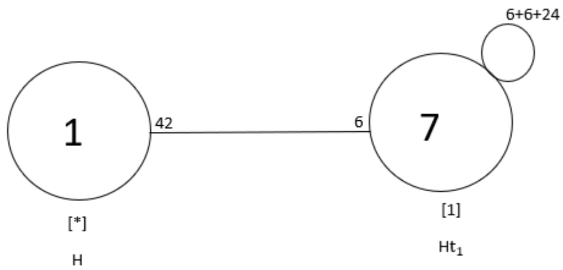


Figure 5.2: Cayley Diagram of [*], [1] for S_8 over S_7 and PSL(2,7)

5.3 J_2 over M_{10} and (3:A_6)

We perform double coset enumeration of G over H = M_{10} = < x, y, (txytxty^{-1}tx^{-1}t)> and (3:A_6) = N. We will prove that the progenitor 2^{*18}:(3:A_6), where (3:A_6) = < x, y> and x ∼ (1, 18, 4, 17, 3, 15)(2, 5, 14, 11, 6, 8, 16, 13, 7, 10, 12, 9), y ∼ (1, 16, 10, 15, 9)(3, 12, 5, 17, 11)(4, 14, 8, 18, 13), factored by two relations isomorphic to the janko sporadic simple group J_2. Let G ≅ $\frac{2^{*18}:(3\times A_6)}{(xy)t_1t_{15}t_{13}t_{12}t_5}$ = J_2. Thus we show that G ∼ J_2.
We have the following relations
Relation 1 = (txytxty^{-1}tx^{-1}t) = (xy)$t_1t_{15}t_{13}t_{12}t_5$
Relation 2 = ((yxy^{-1}x^{-2})$t^{(xyx^{-1}y^{-1}xy^{-2}x)}$)^9 = (yxy^{-1}x^{-2})^{9}*$t_9t_{13}t_{11}t_9t_{13}t_{11}t_9t_{13}t_{11}$

<u>**First Double Coset**</u>

HeN = { H(e)^n | n ϵ N } = {H}
The coset stabiliser of the the coset H = He is H.
The number of single right cosets in the double coset HeN= [*] is given by $\frac{|N|}{|N|} = \frac{1080}{1080} =$ 1
The orbit of N on X= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18} is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18} which is transitive. We will multiply H on the right by a representative of the orbit and determine its double coset.
<u>Choose 1 from {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18}</u>
Ht_1 ϵ [1]
This tells us eighteen elements move forward to the double coset [1].

Cayley Diagram

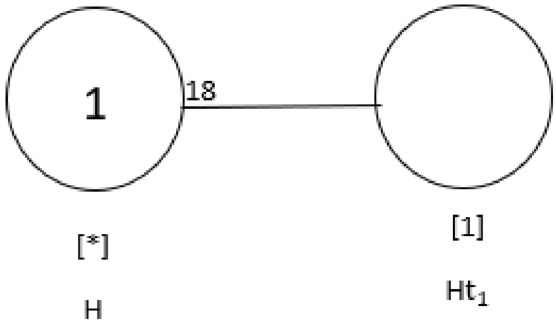


Figure 5.3: Cayley Diagram of [*] for J_2 over M_{10} and (3:A_6)

Second Double Coset

$Ht_1N = \{ H(t_1)^n \mid n \epsilon N \} = \{Ht_1, Ht_2, ..., Ht_{18}\}$

The point-stabiliser 1, N^1 is given by $< (xyx)^2, y^2x^2y, y^2xy^{-1}x^2 >$

The coset stabiliser $N^{(1)} = < (xyx)^2, y^2x^2y, y^2xy^{-1}x^2 >$

The number of single right cosets in the double coset Ht_1N = [1] is given by $\frac{|N|}{|N|} = \frac{1080}{60}$ = 18

The orbit of $N^{(1)}$ on X= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18} are {1}, {3}, {4}, {2, 17, 16, 9, 5, 11, 10, 13, 18, 8, 14, 15, 12, 7, 6}. We will multiply Ht_1 on the right by an orbit representative and determine its double coset.

Choose 1 from {1}

Ht_1t_1

$= H(t_1)^2$

$= H \epsilon$ [*]

Choose 3 from {3}

Ht_1t_3

$= Ht_4$

$H(t_1)^{(y^{-1}x^2yx^{-1})} \epsilon$ [1]

Choose 4 from {4}

Ht_1t_4

$= Ht_3$

$= H(t_1)^{(y^{-1}x^2y^{-1}x^{-1}yx^{-1})} \epsilon$ [1]

Choose 2 from {2, 17, 16, 9, 5, 11, 10, 13, 18, 8, 14, 15, 12, 7, 6}

Ht_1t_2 ϵ [1,2]

Cayley Diagram

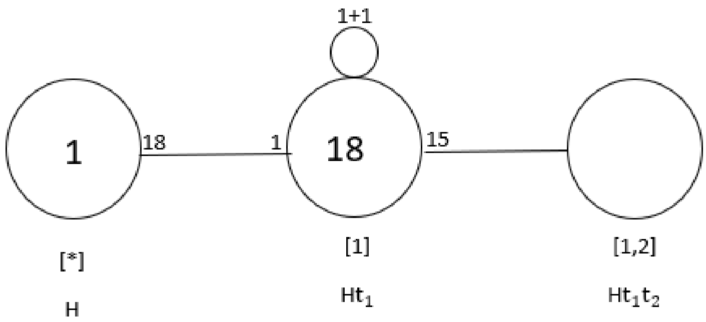


Figure 5.4: Cayley Diagram of [*], [1] for J_2 over M_{10} and (3:A_6)

Third Double Coset

$Ht_1t_2N = \{ H(t_1t_2)^n \mid n \epsilon N \} = \{Ht_1t_2, Ht_{18}t_5,...,Ht_{16}t_2\}$

The point-stabiliser 1, 2, $N^{1,2}$ is given by $<(xyx)^2, x^2y^2x^2y^{-1}x^{-1}>$

But $t_1t_2 = t_2t_1$

Therefore The coset stabiliser $N^{(1,2)} = < x^2yxyx^2y^{-1}x, (xyx)^2, (y^{-1}x^{-1}y^2)^2,$
$x^2y^2x^2y^{-1}x^{-1}, x^2yx^{-1}y^{-1}x^2y^{-1}, yxyx^{-2}y^{-1}xy^{-1} >$

The number of single right cosets in the double coset Ht_1t_2N = [1,2] is given by $\frac{|N|}{|N^{(1,2)}|}$
$= \frac{1080}{8} = 135$

The orbits of $N^{(1,2)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18} are {1, 2}, {3, 6}, {4, 7}, {5, 17, 9, 16}, {8, 18, 11, 12}, {10, 15, 13, 14}. We will multiply Nt_1t_2 on the right by an orbit representative and determine its doouble coset.

Choose 2 from {1, 2}

$Ht_1t_2t_2$

$= Ht_1(t_2)^2$

$= Ht_1 \ \epsilon$ [1]

Choose 3 from {3, 6}

$Ht_1t_2t_3$

$= (x^2y^2x^2y^{-1}x^{-1})(t_1t_2)^{(x^{-1}y^2x^{-1}yx)}$

$= (x^2y^2x^2y^{-1}x^{-1})t_2t_4$

$= Ht_2t_4$

$= H(t_1t_2)^{(x^{-1}y^2x^{-1}yx)} \ \epsilon$ [1,2]

Choose 4 from {4, 7}

$Ht_1t_2t_4$

$= (x^2y^2x^2y^{-1}x^{-1})(t_1t_2)^{(y^{-1}xy^{-2})}$

$= (x^2y^2x^2y^{-1}x^{-1})t_2t_3$

$= Ht_2t_3$

$= H(t_1t_2)^{(y^{-1}xy^{-2})} \ \epsilon$ [1,2]

Choose 5 from {5, 17, 9, 16}

$Ht_1t_2t_5$

$= ((x^{-2}y^{-1}x)^{-1}*(txytxty^{-1}tx^{-1}t))(t_1t_2)^{(x^2yx^2y^{-1}x^2)}$

$= ((x^{-2}y^{-1}x)^{-1}*(txytxty^{-1}tx^{-1}t))t_{16}t_{17}$

$= ((x^{-2}y^{-1}x)^{-1}*(xy))t_1*t_{15}*t_{13}*t_{12}*t_5*t_{16}t_{17}$

So

$= ((x^{-2}y^{-1}x)^{-1}*(xy))t_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_{16}t_{17}$

Then

$Ht_1t_2t_5 = ((x^{-2}y^{-1}x)^{-1}*(xy))t_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_{16}t_{17}$

$\implies Ht_1t_2t_5 = Ht_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_{16}t_{17} \epsilon [1,2]$

Choose 8 from {8, 18, 11, 12}

$Ht_1t_2t_8 \epsilon [1, 2, 8]$

Choose 10 from {10, 15 ,13, 14}

$Ht_1t_2t_{10}$

$= ((x^{-2}y^{-1}x)^{-1}*(txytxty^{-1}tx^{-1}t))(t_1t_2t_8)^{(x^2yx^2y^{-1}x^2)}$

$= ((x^{-2}y^{-1}x)^{-1}*(txytxty^{-1}tx^{-1}t))t_{16}t_{17}t_8$

$= ((x^{-2}y^{-1}x)^{-1}*(xy))t_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_{16}t_{17}t_8$

So

$= ((x^{-2}y^{-1}x)^{-1}*(xy))t_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_{16}t_{17}t_8$

Then

$Ht_1t_2t_{10} = ((x^{-2}y^{-1}x)^{-1}*(xy))t_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_{16}t_{17}t_8$

$\implies Ht_1t_2t_{10} = Ht_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_{16}t_{17}t_8 \epsilon [1,2,8]$

Cayley Diagram

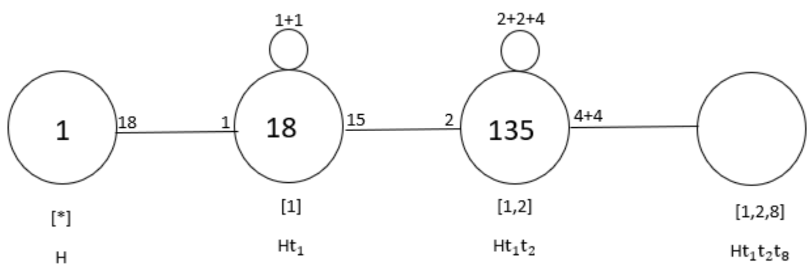


Figure 5.5: Cayley Diagram of [*], [1], [1,2] for J_2 over M_{10} and $(3:A_6)$

Fourth Double Coset

$Ht_1t_2t_8N$ = { $H(t_1t_2t_8)^n$ | n ϵ N } = {$Ht_1t_2t_8$, $Ht_{18}t_5t_{16}$,..., $Ht_{16}t_2t_{18}$}

The point-stabiliser 1, 2, 8, $N^{1,2,8}$ is given by $< 1 >$

But $Ht_1t_2t_8 = Ht_{14}t_8t_2$

$= Ht_{14}t_8t_2$

$= H(t_1t_2t_8)^{(xy^2xy^{-1}x)}$

Also $Ht_1t_2t_8 = Ht_2t_1t_8$

$Ht_1t_2t_8$

$= Ht_2t_1t_8$

$= H(t_1t_2t_8)^{(y^{-1}x^{-1}y^2)^2}$

Therefore the coset stabiliser $N^{(1,2,8)} = < xy^2xy^{-1}x, (y^{-1}x^{-1}y^2)^2 >$

The number of single right cosets in the double coset $Ht_1t_2t_8N$ = [1,2,8] is given by $\frac{|N|}{|N|}$ $= \frac{1080}{10} = 108$

The orbits of $N^{(1,2,8)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18} are {9}, {11}, {13}, {1, 14, 2, 15, 8 }, {3, 16, 6, 17, 10}, {4, 12, 7, 18, 5}. We will multiply $Ht_1t_2t_8$ on the right by a orbit representative and determine its double coset.

Choose 9 from {9}

$Ht_1t_2t_8t_9$

$= (y^{-1}x^{-1}y^{-1}x^{-2})^{-1}(txytxty^{-1}tx^{-1}t)(t_1t_2t_8)^{(xy^{-1}xyx^2y)}$

$= (y^{-1}x^{-1}y^{-1}x^{-2})^{-1}(txytxty^{-1}tx^{-1}t)t_1{*}t_{14}{*}t_{15}$

$= ((y^{-1}x^{-1}y^{-1}x^{-2})^{-1}(x{*}y)){*}t_1{*}t_{15}{*}t_{13}{*}t_{12}{*}t_5{*}t_1{*}t_{14}{*}t_{15}$

So $= ((y^{-1}x^{-1}y^{-1}x^-2)^{-1}(x{*}y)){*}t_1{*}t_{15}{*}t_{13}{*}t_{12}{*}t_5{*}t_1{*}t_{14}{*}t_{15}$

Then

$Ht_1t_2t_8t_9 = ((y^{-1}x^{-1}y^{-1}x^-2)^{-1}(x{*}y)){*}t_1{*}t_{15}{*}t_{13}{*}t_{12}{*}t_5{*}t_1{*}t_{14}{*}t_{15}$

$\Longrightarrow Ht_1t_2t_8t_9 = Ht_1{*}t_{15}{*}t_{13}{*}t_{12}{*}t_5{*}t_1{*}t_{14}{*}t_{15}$ ϵ [1,2,8]

Choose 11 from {11}

$Ht_1t_2t_8t_{11}$ ϵ [1,2,8,11]

Choose 13 from {13}

$Ht_1t_2t_8t_{13}$

$= (x^{-1}yx^{-1}y^{-1})^{-1}{*}(txytxty^{-1}tx^{-1}t)(t_1t_2t_8t_{13})^{(xyxy^{-1}xyx)}$

$= (x^{-1}yx^{-1}y^{-1})^{-1}{*}(txytxty^{-1}tx^{-1}t)t_1{*}t_8{*}t_2{*}t_{15}$

$= ((x^{-1}yx^{-1}y^{-1})^{-1}(x{*}y))t_1{*}t_{15}{*}t_{13}{*}t_{12}{*}t_5{*}t_1{*}t_8{*}t_2{*}t_{15}$

So

$= ((x^{-1}yx^{-1}y^{-1})^{-1}(x^*y))t_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_1{}^*t_8{}^*t_2{}^*t_{15}$

Then

$Ht_1t_2t_8t_{13} = ((x^{-1}yx^{-1}y^{-1})^{-1}(x^*y))t_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_1{}^*t_8{}^*t_2{}^*t_{15}$

$\Longrightarrow Ht_1t_2t_8t_{13} = Ht_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_1{}^*t_8{}^*t_2{}^*t_{15}\ \epsilon\ [1,2,8,11]$

<u>Choose 8 from {1, 14, 2, 15, 8}</u>

$Ht_1t_2t_8t_8$

$= Ht_1t_2(t_8)^2$

$= Ht_1t_2\ \epsilon\ [1,2]$

<u>Choose 3 from {3, 16, 6, 17, 10}</u>

$Ht_1t_2t_8t_3$

$= ((x^2y^{-1}x^{-1}yxy^{-1})^{-1}*(txytxty^{-1}tx^{-1}t))(t_1t_2)^{(xy^{-1}x^{-1}y^{-1})}$

$= ((x^2y^{-1}x^{-1}yxy^{-1})^{-1}*(txytxty^{-1}tx^{-1}t))t_6t_{16}$

$= ((x^2y^{-1}x^{-1}yxy^{-1})^{-1}(x^*y))t_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_6{}^*t_{16}$

So $= ((x^2y^{-1}x^{-1}yxy^{-1})^{-1}(x^*y))t_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_6{}^*t_{16}$

Then $Ht_1t_2t_8t_3 = ((x^2y^{-1}x^{-1}yxy^{-1})^{-1}(x^*y))t_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_6{}^*t_{16}$

$\Longrightarrow Ht_1t_2t_8t_3 = Ht_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_6{}^*t_{16}\ \epsilon\ [1,2]$

<u>Choose 4 from {4, 12, 7, 18, 5}</u>

$Ht_1t_2t_8t_4$

$= ((y^{-1}x^{-1}yx^2)^{-1}(txytxty^{-1}tx^{-1}t))(t_1t_2t_8)^{(yx^{-1}y^{-1}x^2)}$

$= ((y^{-1}x^{-1}yx^2)^{-1}(txytxty^{-1}tx^{-1}t))t_6t_{18}t_5$

$= ((y^{-1}x^{-1}yx^2)^{-1}*(x^*y))t_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_6{}^*t_{18}{}^*t_5$

So

$= ((y^{-1}x^{-1}yx^2)^{-1}*(x^*y))t_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_6{}^*t_{18}{}^*t_5$

Then

$Ht_1t_2t_8t_4 = ((y^{-1}x^{-1}yx^2)^{-1}*(x^*y))t_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_6{}^*t_{18}{}^*t_5$

$\Longrightarrow Ht_1t_2t_8t_4 = Ht_1{}^*t_{15}{}^*t_{13}{}^*t_{12}{}^*t_5{}^*t_6{}^*t_{18}{}^*t_5\ \epsilon\ [1,2,8]$

<u>Cayley Diagram</u>

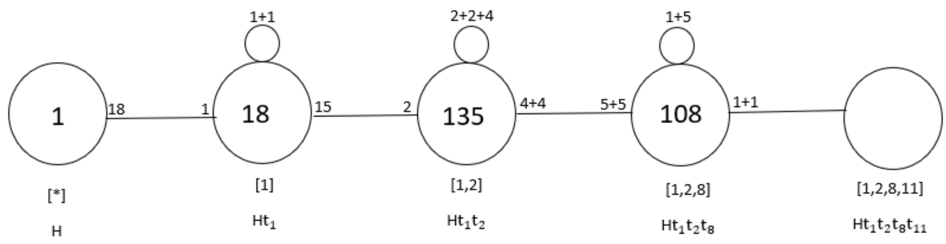


Figure 5.6: Cayley Diagram of [*], [1], [1,2], [1,2,8] for J_2 over M_{10} and (3:A_6)

Fifth Double Coset

$Ht_1t_2t_8t_{11}N = \{ H(t_1t_2t_8t_{11})^n \mid n \epsilon N \} = \{Ht_1t_2t_8t_{11}, Ht_{18}t_5t_{16}t_6, ..., Ht_{16}t_2t_{18}t_3\}$

The point-stabiliser 1, 2, 8, 11, $N^{1,2,8,11}$ is given by $< 1 >$

But $Ht_1t_2t_8t_{11} = Ht_{14}t_{11}t_8t_1$

$Ht_1t_2t_8t_{11}$

$= Ht_{14}t_{11}t_8t_1$

$= H(t_1t_2t_8t_{11})^{(xy^{-1}x^{-1}yx)}$

Also $Ht_1t_2t_8t_{11} = Ht_1t_8t_{15}t_2$

$Ht_1t_2t_8t_{11}$

$= Ht_1t_8t_{15}t_2$

$= H(t_1t_2t_8t_{11})^{(x^3y^{-1}x^2yx)}$

Therefore the coset stabiliser $N^{(1,2,8,11)} = < yxy^2, y^2x^2y >$

The number of single right cosets in the double coset $Ht_1t_2t_8t_{11}$ = [1,2,8,11] is given by $\frac{|N|}{|N^{(1,2,8,11)}|} = \frac{1080}{60} = 18$

The orbits of $N^{(1,2,8,11)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18} are {1, 14, 15, 11, 2, 8}, {3, 16, 17, 13, 6, 10}, {4, 12, 18, 9, 7, 5}

We will multiply $Ht_1t_2t_8t_{11}$ by an orbit representative and determine its double coset.

<u>Choose 11 from {1, 14, 15, 11, 2, 8}</u>

$Ht_1t_2t_8t_{11}t_{11}$

$= Ht_1t_2t_8(t_{11})^2$

$= Ht_1t_2t_8 \epsilon$ [1,2,8]

<u>Choose 3 from {3, 16, 17, 13, 6, 10}</u>

$Ht_1t_2t_8t_{11}t_3$

$= (txytxty^{-1}tx^{-1}t)(t_1t_2t_8)^{(yxy^2)}$

$= (txytxty^{-1}tx^{-1}t)t_{14}t_{11}t_8$

$= ((xy^{-1}x^2y^{-1}x^{-1})^{-1}*(x*y))*t_1*t_{15}*t_{13}*t_{12}*t_5*t_{14}*t_{11}*t_8$

So

$= ((xy^{-1}x^2y^{-1}x^{-1})^{-1}*(x*y))*t_1*t_{15}*t_{13}*t_{12}*t_5*t_{14}*t_{11}*t_8$

Then

$Ht_1t_2t_8t_{11}t_{11} = ((xy^{-1}x^2y^{-1}x^{-1})^{-1}*(x*y))*t_1*t_{15}*t_{13}*t_{12}*t_5*t_{14}*t_{11}*t_8$

$Ht_1t_2t_8t_{11}t_{11} = Ht_1*t_{15}*t_{13}*t_{12}*t_5*t_{14}*t_{11}*t_8 \epsilon$ [1,2,8]

<u>Choose 4 from {4, 12, 18, 9, 7, 5}</u>

$Ht_1t_2t_8t_{11}t_4$

$= (txytxty^{-1}tx^{-1}t)(t_1*t_2*t_8*t_{11})^{(y^{-1}xy^{-1}x^{-1}y)}$

$= (txytxty^{-1}tx^{-1}t)t_1*t_{15}*t_{14}*t_8$

$= ((x^3y^{-1}xyx)^{-1}*(x*y))*t_1*t_{15}*t_{13}*t_{12}*t_5*t_1*t_{15}*t_{14}*t_8$

So

$= ((x^3y^{-1}xyx)^{-1}*(x*y))*t_1*t_{15}*t_{13}*t_{12}*t_5*t_1*t_{15}*t_{14}*t_8$

Then

$Ht_1t_2t_8t_{11}t_4 = ((x^3y^{-1}xyx)^{-1}*(x*y))*t_1*t_{15}*t_{13}*t_{12}*t_5*t_1*t_{15}*t_{14}*t_8$

$Ht_1t_2t_8t_{11}t_4 = Ht_1*t_{15}*t_{13}*t_{12}*t_5*t_1*t_{15}*t_{14}*t_8 \epsilon$ [1,2,8,11]

<u>Cayley Diagram</u>

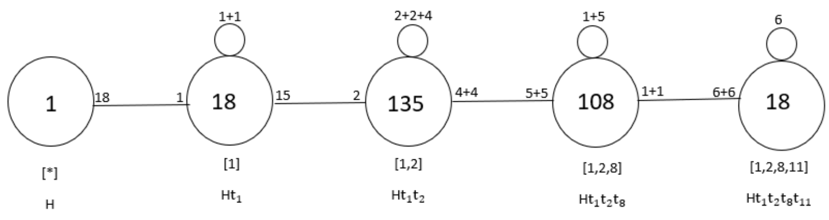


Figure 5.7: Cayley Diagram of [*], [1], [1,2], [1,2,8], [1,2,8,11] for J_2 over M_{10} and $(3{:}A_6)$

5.4 S_8 over S_7 and PSL(2,7)

We perform double coset enumeration of G over H = S_7 = <x, y, ($tyty^{-1}xty^{-1}ty^{-1}xt$)> and PSL(2, 7) = N. We will prove that the progenitor 2^{*42}: PSL(2, 7), where PSL(2,7) = <x, y> and x ∼ (1, 2)(5, 7)(6, 9)(8, 12)(10, 14)(11, 15)(13, 17)(16, 20)(18, 22)(19,23)(21, 26)(24, 27)(25, 30)(28, 33)(29, 35)(32, 37)(39, 41)(40, 42), y ∼ (1, 3, 5, 8)(2, 4, 6, 10)(7, 11, 9, 13)(12, 16)(14, 18)(15, 19, 24, 29)(17,21, 27, 32)(20, 25, 31, 26)(22, 28, 34, 23)(30, 36, 37, 40)(33, 38, 35,39)(41, 42), factored by three relations isomorphic to S_8. Let G ≅ $\frac{2^{*42}:PSL(2,7)}{y^2t_{21}t_{17},(yxy)^8t_8t_{13}t_4t_8t_{13}t_4t_8t_{13},t_1t_2t_3t_5t_{12}}$. Thus we will show G ∼ S_8.

We have the following relations

Relation 1: $((yxy)t^{((xy)^3)})^8 = (yxy)^8t_8t_{13}t_4t_8t_{13}t_4t_8t_{13}$

Relation 2: $((y)t^{(y^{-1}xyxy^{-1}xy^{-1})})^2 = (y)^2t_{21}t_{17}$

Relation 3: $(tyty^{-1}xty^{-1}ty^{-1}xt) = t_1t_2t_3t_5t_{12}$

<u>First Double Coset</u>

HeN = {$H(e)^n$ | n ϵ N }= {H}

The coset stabiliser of H = He = H.

The number of single right cosets in the double coset HeN = [*] is given by $\frac{|N|}{|N|} = \frac{168}{168} =$ 1

The orbit of N on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} is transitive therefore we have {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23,24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42 }

Next we will multiply H by an orbit representative and determine its double coset.

<u>Choose 1 from the orbit</u>

Ht_1 ϵ [1]

Therefore all 42 elements proceed forward to the double coset Ht_1N.

<u>Cayley Diagram</u>

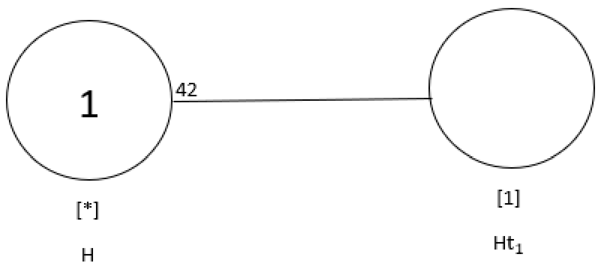


Figure 5.8: Cayley Diagram of [*] for S_8 over S_7 and PSL(2,7)

Second Double Coset

$Ht_1N = \{ H(t_1)^n \mid n \epsilon N \} = \{Ht_1, Ht_2, ..., Ht_3\}$

The point-stabiliser N^1 is given by $\{yxy^{-1}, xyxy^{-1}x\}$

But $Ht_1 = Ht_5$

Then

Ht_1

$= ((y^2t)^2)(t_1)^{(xy^{-1}xyxyx)}$

$= ((y^2t)^2)t_{33}$

$= H(t_1t_1t_1t_5)t_{33}$

$= H(t_1t_5)t_{33}$

The coset stabiliser $N^{(1)} = \{y^2, (xyxy^{-1})^2, xyxy^{-1}x, yxy, xyxy^{-1}xy^2, xyxy^{-1}xyxy\}$

The number of single right cosets in the double coset Ht_1N = [1] is given by $\frac{|N|}{|N^{(1)}|} = \frac{168}{24}$ $= 7$

The orbits of $N^{(1)}$ on X={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are {1}, {2}, {25}, {28}, {30}, {33}, {3, 17, 31, 13}, {4, 15, 34, 11}, {5, 35, 42, 16}, {6, 37, 41, 18}, {7, 20, 40, 29}, {8, 21, 19, 10}, {9, 22, 39, 32}, {12, 14, 23, 26}, {24, 36, 27, 38}.

We multiply Ht_1 on the right by an orbit representative and determine its double coset.

Choose 1 from {1}

Ht_1t_1

$= H(t_1)^2$

$= H \epsilon [*]$

Choose 2 from {2}

Ht_1t_2

$= (txy^{-1}xy^{-1}txyt)(t_1)^{(xy^{-1}xyxyxy^{-1}xy^{-1})}$

$= (txy^{-1}xy^{-1}txyt)t_{42}$

$= ((x)t_1t_7t_7t_4)t_{42}$

So

$Ht_1t_2 = ((x)t_1t_7t_7t_4)t_{42}$

Then

$Ht_1t_2 = H(t_1t_7t_7t_4)t_{42} \epsilon [1]$

Choose 25 from {25}

Ht_1t_{25}

$= (xyxty^{-1}ty^{-1}xyxty^{-1})(t_1)^{(xy^{-1}xyxyx)}$

$= (xyxty^{-1}ty^{-1}xyxty^{-1})t_{33}$

$= (t_5t_2t_5)t_{33}$

Then

$Ht_1t_{25} = (t_5t_2t_5)t_{33}$

Thus

$Ht_1t_{25} = H(t_5t_2t_5)t_{33} \epsilon [1]$

Choose 28 from {28}

Ht_1t_{28}

$= (txy^{-1}xy^{-1}txyt)(t_1)^{(xy^{-1}xyxyxy^{-1}xy^{-1})}$

$= (txy^{-1}xy^{-1}txyt)t_{42}$

$= ((x)t_1t_7t_4)t_{42}$

We have

$Ht_1t_{28} = ((x)t_1t_7t_4)t_{42}$

Then

$Ht_1t_{28} = H(t_1t_7t_4)t_{42} \epsilon [1]$

Choose 30 from {30}

Ht_1t_{30}

$= (xyxty^{-1}ty^{-1}xyxty^{-1})(t_1)^{xy^{-1}xyxyx}$

$= (xyxty^{-1}ty^{-1}xyxty^{-1})t_{33}$

$= (t_5 t_2 t_5) t_{33}$

So

$Ht_1 t_{30} = (t_5 t_2 t_5) t_{33}$

Then

$Ht_1 t_{30} = H(t_5 t_2 t_5) t_{33} \epsilon [1]$

<u>Choose 33 from {33}</u>

$Ht_1 t_{33}$

$= Ht_1 t_1$

$= H(t_1)^2$

$= H \epsilon [*]$

<u>Choose 3 from {3, 17, 31, 13}</u>

$Ht_1 t_3$

$= (y^{-1}xtxy^{-1}xtytx)(t_1)^{(yxyxy^{-1}xy^{-1})}$

$= (y^{-1}xtxy^{-1}xtytx)t_{32}$

$= ((y^2x)t_6 t_3 t_6)t_{32}$ So

$= ((y^2x)t_6 t_3 t_6)t_{32}$

Then

$Ht_1 t_3 = ((y^2x)t_6 t_3 t_6)t_{32}$

Thus

$Ht_1 t_3 = H(t_6 t_3 t_6)t_{32} \epsilon [1]$

<u>Choose 4 from {4, 15, 34, 11}</u>

$Ht_1 t_4$

$= (y^{-1}xtyxyxtxt)(t_1)^{(xy^{-1}xyxy^{-1}xy^2)}$

$= (y^{-1}xtyxyxtxt)t_{29}$

$= (t_1 t_4 t_7)t_{29}$

So

$Ht_1 t_4 = (t_1 t_4 t_7)t_{29}$

$Ht_1 t_4 = H(t_1 t_4 t_7)t_{29} \epsilon [1]$

<u>Choose 5 from {5, 35, 42, 16}</u>

$Ht_1 t_5$

$= Ht_1 t_1$

$H(t_1)^2$

H ϵ [*]

<u>Choose 6 from {6, 37, 41, 18}</u>

Ht_1t_6

$= (ty^2xtxy^2t)(t_1)^{(xy^{-1}xyxy^{-1}xy^2)}$

$= (ty^2xtxy^2t)t_{33}$

$= (t_1t_6t_1)t_{33}$

So

$Ht_1t_6 = (t_1t_6t_1)t_{33}$

Then

$Ht_1t_6 = H(t_1t_6t_1)t_{33}$ ϵ [1]

<u>Choose 7 from {7, 20, 40, 29}</u>

Ht_1t_7

$= (txy^{-1}xytytxy)(t_1)^{(y^{-1}xy^{-1}xyxyxy^{-1})}$

$= (txy^{-1}xytytxy)t_{30}$

$= ((y^{-1}xy^{-1}xy)t_7t_1t_7)t_{30}$

So

$Ht_1t_7 = ((y^{-1}xy^{-1}xy)t_7t_1t_7)t_{30}$

Then

$Ht_1t_7 = H(t_7t_1t_7)t_{30}$ ϵ [1]

<u>Choose 8 from {8, 21, 19, 10}</u>

Ht_1t_8

$= (xyxtxy^{-1}xty^{-1}ty^{-1})(t_1)^{(xy^{-1}xy)}$

$= (xyxtxy^{-1}xty^{-1}ty^{-1})t_{18}$

$= ((y^2xyxy)t_2t_5t_4)t_{18}$

So

$Ht_1t_8 = ((y^2xyxy)t_2t_5t_4)t_{18}$

Then

$Ht_1t_8 = H(t_2t_5t_4)t_{18}$ ϵ [1]

<u>Choose 9 from {9, 22, 39, 32}</u>

Ht_1t_9

$= (y^{-1}txtxy^2tx)(t_1)^{((xy^{-1}xy)^2)}$

$= (y^{-1}txtxy^2tx)t_{24}$

$=(((y^{-1}x)^3)t_6t_8t_4)t_{24}$

So

$Ht_1t_8 =(((y^{-1}x)^3)t_6t_8t_4)t_{24}$

Then

$Ht_1t_8 = H(t_6t_8t_4)t_{24} \epsilon [1]$

Choose 12 from {12, 14, 23, 26}

Ht_1t_{12}

$= (xty^{-1}txyxy^2tx)(t_1)^{(xy^{-1}xyxyxy^{-1}xy^{-1})}$

$= (xty^{-1}txyxy^2tx)t_{42}$

$= (t_1t_{12}t_{16})t_{42}$

So

$Ht_1t_{12} = (t_1t_{12}t_{16})t_{42}$

Then

$Ht_1t_{12} = H(t_1t_{12}t_{16})t_{42} \epsilon [1]$

Choose 24 from {24, 36, 27, 38}

Ht_1t_{24}

$= (yxyxtxyty^{-1}xty)(t_1)^{(xy^{-1}xyxy^{-1})}$

$= (yxyxtxyty^{-1}xty)t_{23}$

$= ((xyxyxyxy^{-1})t_7t_1t_7)t_{23}$

So

$Ht_1t_{24} = ((xyxyxyxy^{-1})t_7t_1t_7)t_{23}$

Then

$Ht_1t_{24} = H(t_7t_1t_7)t_{23} \epsilon [1]$

Cayley Diagram

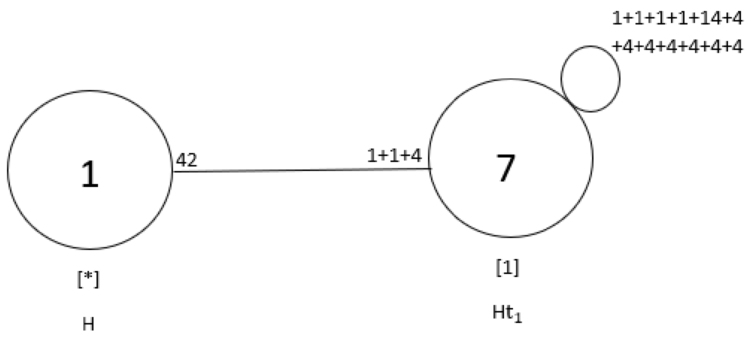


Figure 5.9: Cayley Diagram of [*], [1] for S_8 over S_7 and PSL(2,7)

5.5 M_{12}:2 over $((4:2^2):S_4)$ and $(2^3:(4:2))$

We will perform double coset enumeration of G over H = $((4:2^2):S_4)$ = <x, y, (tytxtxtytytyty)> and $((2^3):(4:2))$ = N, where the order of H is 384 and the order of N is 192. We will prove that the progenitor $2^{*8}:((2^3):(4:2))$, where $((2^3):(4:2))$ = <x, y> and x $\sim$ (1, 2, 5, 4)(3, 6, 8, 7), y $\sim$ (1, 3, 6, 4)(5, 7), factored by seven relations is isomorphic to the mathieu sporadic simple group M_{12}:2. Let G $\cong \frac{2^{*8}:((2^3):(4:2))}{(x^2)t_1t_4t_6t_7t_2t_5}$. Thus we show G $\sim M_{12}$:2

We have the following relations

Relation 1: $(((xyx^{-1})^2)*t^{(x^2y^2x^{-1}y^{-1})})^6 = ((xyx^{-1})^2)t_2t_2t_2t_2t_2t_2 = ((xyx^{-1})^2)$

Relation 2: $(((xy)^3)*t^{(x^2y^{-1}x^{-1}yx)})^3 = ((xy)^3)^3t_7t_5t_7$

Relation 3: $(x^2*t)^5 = (x^2)^5t_1t_5t_1t_5t_1$

Relation 4: $(x^2txty^{-1}tx^{-1}tx^2txtx^2tyty) = t_1t_1t_1t_3t_8t_8t_3t_1 = 1$

Relation 5: $(x^2yxtxtxytytxtxy^{-1}tytxty^{-1}t) = t_1t_1t_1t_2t_1t_6t_2t_6t_8 = t_1t_2t_1t_6t_2t_6t_8$

Relation 6: $(xytytxt)^3 = ((xy)^3)t_1t_6t_1$

Relation 7: $(tytxtxtytyty) = (x^2)t_1t_4t_6t_7t_2t_5$

First Double Coset

HeN = { H(e)^n | n ϵ N } = {H}

The coset stabiliser H is He = H

The number of single right cosets in the double coset HeN = [*] is given by $\frac{|N|}{|N|} = \frac{192}{192} =$ 1

Next we will find the orbits of N on X = {1, 2, 3, 4, 5, 6, 7, 8} which are {1, 2, 3, 4, 5, 6, 7, 8}

We will multiply H on the right by an orbit representative and determine its double coset.

Choose 1 from {1, 2, 3, 4, 5, 6, 7, 8}

Ht_1 ϵ [1]

Thus all eight elements proceed to the double coset Ht_1N which is labelled as [1].

Cayley Diagram

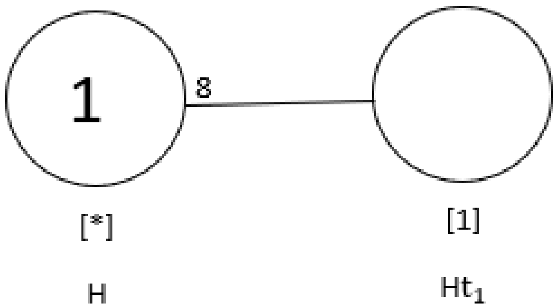


Figure 5.10: Cayley Diagram of [*] for M_{12}:2 over ((4:2^2):S_4) and (2^3:(4:2))

Second Double Coset

Ht_1N = { H(t_1)^n | n ϵ N } = {Ht_1, Ht_2,..., Ht_8}

The point-stabiliser 1, N^1 is given by <$(x^{-1}y^{-1})^2$, $(xyx^{-1})^2$, $x^2y^2x^{-1}y^{-1}x^{-1}$ >

The coset stabiliser $N^{(1)}$ = <$(x^{-1}y^{-1})^2$, $(xyx^{-1})^2$, $x^2y^2x^{-1}y^{-1}x^{-1}$ >

The number of single right cosets in the double coset Ht_1N = [1] is given by $\frac{|N|}{|N^{(1)}|} = \frac{192}{24}$ = 8

The orbits of $N^{(1)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8} are {1}, {6}, {2, 7, 4, 8, 5, 3}. We will multiply Ht_1 on the right by an orbit representative and determine its double coset.

Choose 1 as the representative or the orbit {1}

Ht_1t_1

$= Ht_1^2$

$= H \epsilon [*]$

Choose 6 from the orbit {6}

Ht_1t_6

$= (xy)^3t_1$

$= Ht_1 \epsilon [1]$

Choose 2 from {2,7,4,8,5,3}

$Ht_1t_2 \epsilon [1,2]$

Cayley Diagram

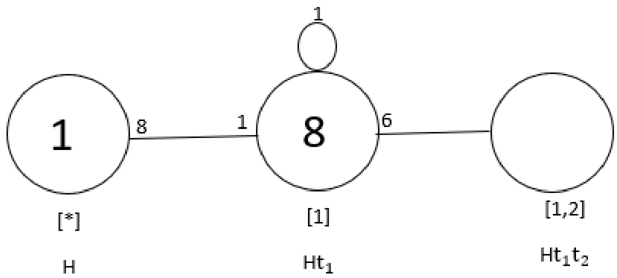


Figure 5.11: Cayley Diagram of [*], [1] for M_{12}:2 over ((4:2^2):S_4) and (2^3:(4:2))

Third Double Coset

$Ht_1t_2N = \{ H(t_1t_2)^n \mid n \epsilon N \} = \{Ht_1t_2, Ht_2t_1,.., Ht_3t_2\}$

The point-stabiliser 1, 2, $N^{1,2}$ is given by $< x^2yxy^{-1}, x^2y^2x^{-1}y^{-1}x^{-1} >$

The coset stabiliser $N^{(1,2)} = < x^2yxy^{-1}, x^2y^2x^{-1}y^{-1}x^{-1} >$

The number of single right cosets in the double coset Ht_1t_2N = [1,2] is given by $\frac{|N|}{|N^{(1,2)}|}$

$= \frac{192}{4} = 48.$

The orbits of $N^{(1,2)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8} are {1}, {2}, {6}, {8}, {3, 7, 4, 5} we will multiply Ht_1t_2 by a orbit representative and determine its double coset.

Choose 1 from {1}

$Ht_1t_2t_1$

$= (y^{-1}xy)^2t_1t_2$

$= Ht_1t_2 \epsilon [1,2]$

Choose 2 from {2}

$Ht_1t_2t_2$

$= Ht_1(t_2)^2$

$= Ht_1 \epsilon [1]$

Choose 6 from {6}

$Ht_1t_2t_6 \epsilon [1,2,6]$

Choose 8 from {8}

$Ht_1t_2t_8$

$= ((xy)^3)t_6t_2$

$= H(t_1t_2)^{(y^{-1}x^{-1}y^{-1}x^2)} \epsilon [1,2]$

Choose 3 from {3, 7, 4, 5}

$Ht_1t_2t_3 \epsilon [1,2,3]$

Cayley Diagram

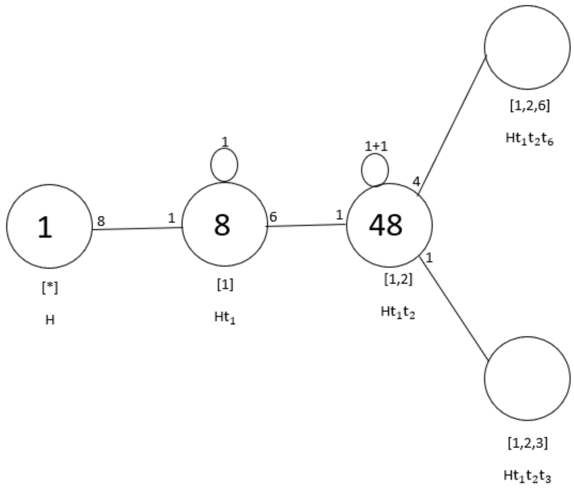


Figure 5.12: Cayley Diagram of [*], [1], [1,2] for M_{12}:2 over ((4:2^2):S_4) and (2^3:(4:2))

Fourth Double Coset

$Ht_1t_2t_6N = \{ H(t_1t_2t_6)^n \mid n \epsilon N \} = \{Ht_1t_2t_6, Ht_2t_1t_8, ..., Ht_3t_2t_4\}$

The point-stabiliser 1, 2, 6, $N^{1,2,6}$ is given by $< x^2yxy^{-1}, (xyx^{-1})^2 >$

Now $Ht_1t_2t_6$

$= (x^2tx^{-1}ty^{-1}tx^2tytxt)t_5t_4t_7$

$= (t_1t_2t_6t_7t_4t_5)t_5t_4t_7$

Then

$Ht_1t_2t_6 = (t_1t_2t_6t_7t_4t_5)t_5t_4t_7$

Thus

$Ht_1t_2t_6 = H(t_1t_2t_6t_7t_4t_5)t_5t_4t_7$

The coset stabiliser $N^{(1,2,6)} = < x^2yxy^{-1}, (xyx^{-1})^2, xy^2x, x^2, xyxy^2, yx^{-1}y^{-1} >$

The number of single right cosets in the double coset $Ht_1t_2t_6N$ = [1,2,6] is given by $\frac{|N|}{|N^{(1,2,6)}|} = \frac{192}{32} = 6$

The orbit of $N^{(1,2,6)}$ on X={1, 2, 3, 4, 5, 6, 7, 8} is transitive therefore the orbit is {1, 5, 3, 7, 8, 2, 6, 4}

We will multiply $Ht_1t_2t_6$ by an orbit representative and determine its double coset.

Choose 6 from {1, 2, 3, 4, 5, 6, 7, 8}

$Ht_1t_2t_6t_6$

$= Ht_1t_2(t_6)^2$

$= Ht_1t_2 \epsilon [1,2]$

<u>Cayley Diagram</u>

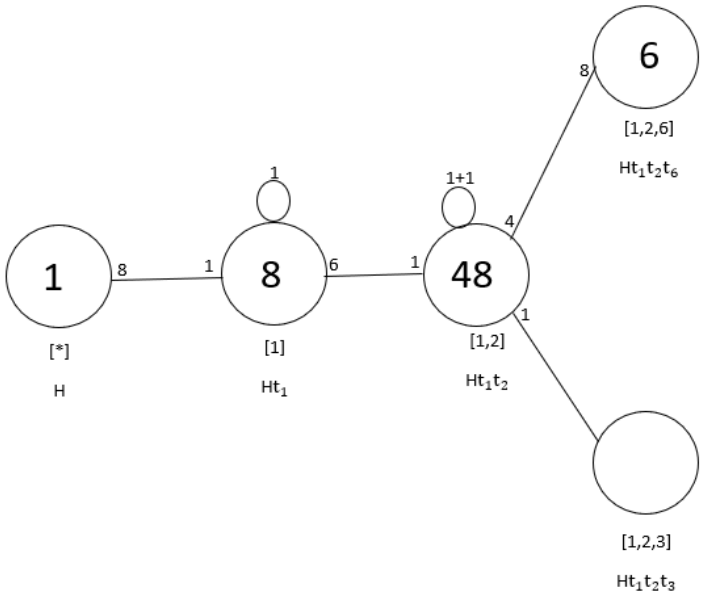


Figure 5.13: Cayley Diagram of [*], [1], [1,2], [1,2,6] for M_{12}:2 over ((4:2^2):S_4) and (2^3:(4:2))

Fifth Double Coset

$Ht_1t_2t_3N = \{H(t_1t_2t_3)^n \mid n \epsilon N\} = \{Ht_1t_2t_3, Ht_2t_1t_6, ..., Ht_3t_2t_6\}$

The point-stabiliser 1, 2, 3, $N^{1,2,3}$ is given by <e>

The coset stabiliser $N^{(1,2,3)} = < 1 >$

The number of single right cosets in the double coset $Ht_1t_2t_3N$ = [1,2,3] is given by $\frac{|N|}{|N^{(1,2,3)}|} = \frac{192}{1} = 192$

The orbits of $N^{(1,2,3)}$ on X={1,2,3,4,5,6,7,8} are {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}.

Multiplying $Ht_1t_2t_3$ by an orbit representative and determine its double coset.

<u>Choose 1 from {1}</u>

$Ht_1t_2t_3t_1 \epsilon$ [1,2,3,1]

<u>Choose 2 from {2}</u>

$Ht_1t_2t_3t_2$

$= (xy^{-1}x^{-1}y^{-1}x^{-1}y^{-1})t_7t_2t_3$

$= H(t_1t_2t_3)^{(xyxy^{-1}x^{-1}yx)} \epsilon$ [1,2,3]

<u>Choose 3 from{3}</u>

$Ht_1t_2t_3t_3$

$= Ht_1t_2(t_3)^2$

$= Ht_1t_2 \epsilon$ [1,2]

<u>Choose 4 from {4}</u>

$Ht_1t_2t_3t_4$

$= ((xy)^3)t_6t_8t_3$

$= H(t_1t_2t_3)^{(xyxy^{-1}x^{-1}yx)} \epsilon$ [1,2,3]

<u>Choose 5 from {5}</u>

$Ht_1t_2t_3t_5 \epsilon$ [1,2,3,5]

<u>Choose 6 from {6}</u>

$Ht_1t_2t_3t_6$

$= (tx^{-1}ty^{-1}tx^2tytxt)t_1t_2t_3$

$= (t_1t_2t_6t_7t_4t_5)t_1t_2t_3$

So

$Ht_1t_2t_3t_6 = (t_1t_2t_6t_7t_4t_5)t_1t_2t_3$

Then

$Ht_1t_2t_3t_6 = H(t_1t_2t_6t_7t_4t_5)t_1t_2t_3 \epsilon$ [1,2,3]

<u>Choose 7 from {7}</u>

$Ht_1t_2t_3t_7 \epsilon$ [1,2,3,7]

<u>Choose 8 from {8}</u>

$Ht_1t_2t_3t_8 \epsilon$ [1,2,3,8]

<u>Cayley Diagram</u>

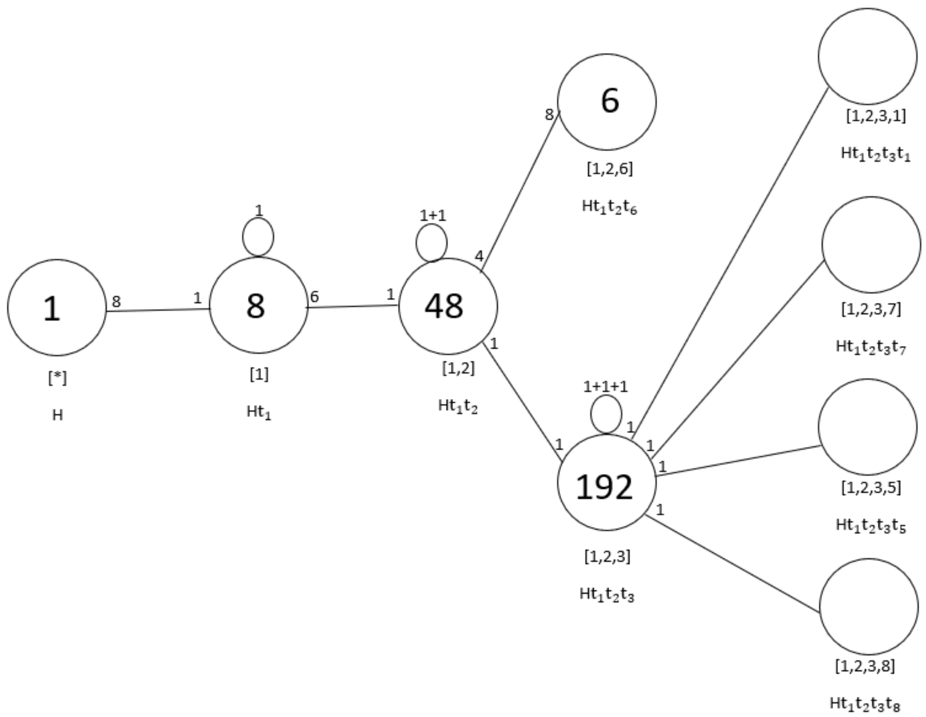


Figure 5.14: Cayley Diagram of [*],[1], [1,2], [1,2,6], [1,2,3] for M_{12}:2 over ((4:2^2):S_4) and (2^3:(4:2))

Sixth Double Coset

$Ht_1t_2t_3t_7N = \{ H(t_1t_2t_3t_7)^n \mid n \epsilon N \} = \{Ht_1t_2t_3t_7, Ht_2t_5t_6t_3,..., Ht_3t_2t_6t_5\}$

The point-stabiliser 1, 2, 3, 7, $N^{1,2,3,7}$ is given by $< 1 >$

Now $Ht_1t_2t_3t_7$

$= t_2t_7t_3t_1$

$= ((x^{-1}y^{-1})^2)t_2t_7t_3t_1$

So

$Ht_1t_2t_3t_7 = ((x^{-1}y^{-1})^2)t_2t_7t_3t_1$

Then

$Ht_1t_2t_3t_7 = Ht_2t_7t_3t_1$

Thus The coset stabiliser $N^{(1,2,3,7)}$ is $< xy^{-1} >$

The number of single right cosets in the double coset $Ht_1t_2t_3t_7N = [1,2,3,7]$ is given by $\frac{|N|}{|N^{(1,2,3,7)}|} = \frac{192}{3} = 64$

The orbits of $N^{(1,2,3,7)}$ on X={1, 2, 3, 4, 5, 6, 7, 8} are {3}, {4}, {1,2,7}, {5,6,8}

Multiplying $Ht_1t_2t_3t_7$ by a representative of each orbit and determine its double coset.

Choose 3 from {3}

$Ht_1t_2t_3t_7t_3$

$= (xy^{-1})t_1t_7t_3t_2$

$= H(t_1t_2t_3t_7)^{(xyx^{-1}y^{-1}x)} \epsilon [1,2,3,7]$

Choose 4 from {4}

$Ht_1t_2t_3t_7t_4 \epsilon [1,2,3,7]$

Choose 7 from {1,2,7}

$Ht_1t_2t_3t_7t_7$

$= Ht_1t_2t_3t(_7)^2$

$= Ht_1t_2t_3 \epsilon [1,2,3]$

Choose 5 from {5,6,8}

$Ht_1t_2t_3t_7t_5$

$= ((xy)^3)t_6t_8t_4t_7$

$= H(t_1t_2t_3t_5)^{((xy)^3)} \epsilon [1,2,3,5]$

Cayley Diagram

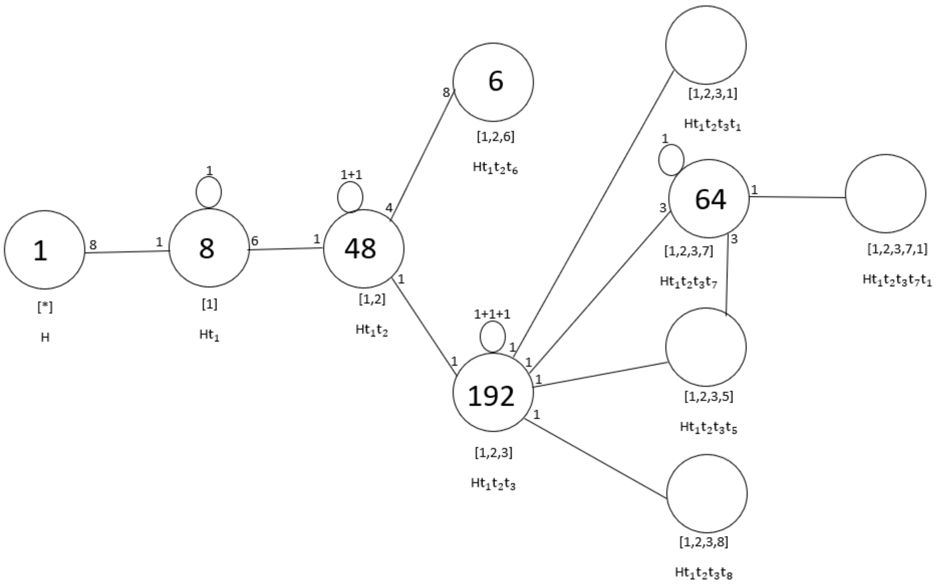


Figure 5.15: Cayley Diagram of [*], [1], [1,2], [1,2,6], [1,2,3], [1,2,3,7] for M_{12}:2 over $((4:2^2):S_4)$ and $(2^3:(4:2))$

Seventh Double Coset

$Ht_1t_2t_3t_5N = \{ H(t_1t_2t_3t_5)^n \mid n \epsilon N \} = \{Ht_1t_2t_3t_5, Ht_2t_5t_6t_4, ..., Ht_3t_2t_5t_7\}$

The point-stabiliser 1, 2, 3, 5, $N^{1,2,3,5}$ is given by $<e>$

But $Ht_1t_2t_3t_5$

$= t_4t_5t_6t_2$

$= (ytytxtxtytytyt)t_4t_5t_6t_2$

$= ((x^2)t_1t_3t_6t_5t_2t_7)t_4t_5t_6t_2$

So

$Ht_1t_2t_3t_5 = ((x^2)t_1t_3t_6t_5t_2t_7)t_4t_5t_6t_2$

Then

$Ht_1t_2t_3t_5 = H(t_1t_3t_6t_5t_2t_7)t_4t_5t_6t_2$

Therefore the coset stabiliser $N^{(1,2,3,5)} = < x^2y^{-1}x^2y >$

The number of single right cosets in the double coset $Ht_1t_2t_3t_5N$ = [1,2,3,5] is given by $\frac{|N|}{|N^{(1,2,3,5)}|} = \frac{192}{2} = 96$

The orbits of $N^{(1,2,3,5)}$ on X = {1,2,3,4,5,6,7,8} are {1, 4}, {2, 5}, {3, 6}, {7, 8}

We will multiply $Ht_1t_2t_3t_5$ by an orbit representative and determine its double coset.

Choose 1 from {1, 4}

$Ht_1t_2t_3t_5t_1$

$(xyx^{-1}tytxtxtytxt)(t_1t_2t_3t_5)^{(yxy^{-1}x)}$

$= (xyx^{-1}tytxtxtytxt)t_6t_3t_7t_4$

$= ((x^2)t_1t_3t_6t_5t_2t_7)t_4t_5t_6t_2 =$

So

$Ht_1t_2t_3t_5t_1 = ((x^2)t_1t_3t_6t_5t_2t_7)t_4t_5t_6t_2$

Then

$Ht_1t_2t_3t_5t_1 = H(t_1t_3t_6t_5t_2t_7)t_4t_5t_6t_2 \ \epsilon$ [1,2,3,8]

Choose 5 from {2, 5}

$Ht_1t_2t_3t_5t_5$

$= Ht_1t_2t_3(t_5)^2$

$= Ht_1t_2t_3 \ \epsilon$ [1,2,3]

Choose {3, 6}

$Ht_1t_2t_3t_5t_3$

$= (x^2ytytxtxtytxt)(t_1t_2t_3t_5)^{(x^{-1}y^{-1}x^{-1}y^2)}$

$= (x^2ytytxtxtytxt)t_4t_5t_2t_6$

$= (t_1t_5t_6t_8t_4t_2)t_2t_5t_2t_6$

So

$Ht_1t_2t_3t_5t_3 = (t_1t_5t_6t_8t_4t_2)t_2t_5t_2t_6$

Then

$Ht_1t_2t_3t_5t_3 = H(t_1t_5t_6t_8t_4t_2)t_2t_5t_2t_6 \ \epsilon$ [1,2,3,5]

Choose 7 from {7, 8}

$Ht_1t_2t_3t_5t_7$

$= (xy^{-1}x^2)t_5t_6t_4t_8$

$= H(t_1t_2t_3t_7)^{(y^2xyxy)} \ \epsilon$ [1,2,3,7]

Cayley Diagram

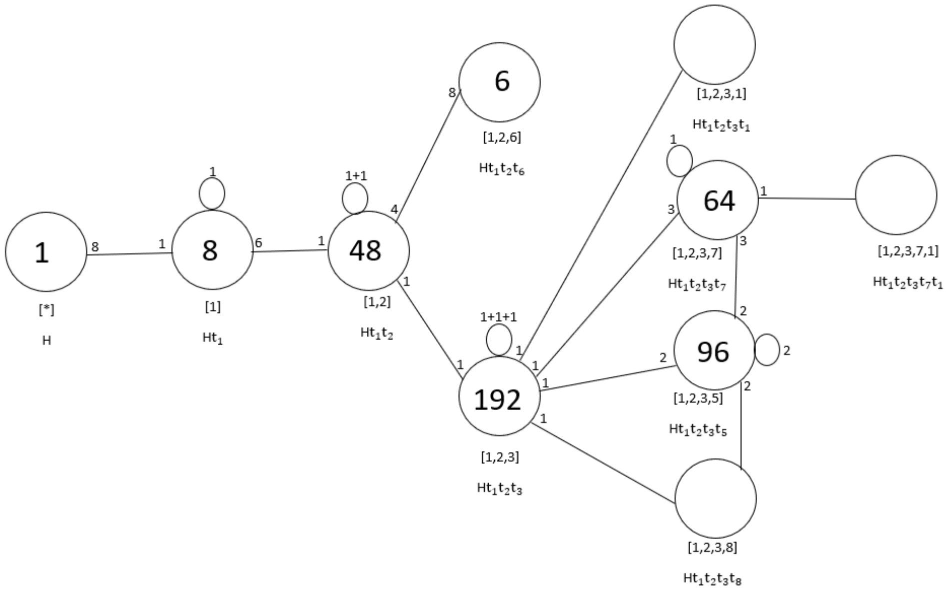


Figure 5.16: Cayley Diagram of [*],[1],[1,2],[1,2,6],[1,2,3],[1,2,3,7],[1,2,3,5] for M_{12}:2 over ((4:2^2):S_4) and (2^3:(4:2))

Eighth Double Coset

$Ht_1t_2t_3t_8N = \{H(t_1t_2t_3t_8)^n \mid n \epsilon N\} = \{Ht_1t_2t_3t_8, Ht_2t_5t_6t_7, ..., Ht_3t_2t_6t_8\}$

The point-stabiliser 1, 2, 3, 8, $N^{1,2,3,8}$ is given by $< 1 >$

Now $Ht_1t_2t_3t_8$

$= Ht_7t_4t_8t_3$

$= ((xy)^3)t_6t_3t_2t_4$

So

$Ht_1t_2t_3t_8 = ((xy)^3)t_6t_3t_2t_4$

Then

$Ht_1t_2t_3t_8 = Ht_6t_3t_2t_4$

and $Ht_1t_2t_3t_8$

$= Ht_6t_3t_2t_4$

$= ((xy)^3)t_6t_3t_2t_4$

Then

$Ht_1t_2t_3t_8 = ((xy)^3)t_6t_3t_2t_4$

Thus

$Ht_1t_2t_3t_8 = Ht_6t_3t_2t_4$

Therefore The coset stabiliser $N^{(1,2,3,8)} = < xyx^2y^{-1}, xyx^{-1}y^{-1}x^{-1}y >$

The number of single right cosets in the double coset $Ht_1t_2t_3t_8N$ = [1,2,3,8] is given by $\frac{|N|}{|N^{1,2,3,8)}|} = \frac{192}{4} = 48$

The orbits of $N^{(1,2,3,8)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8} are {1,7,6,5} and {2,4,3,8}.

We will multiply $Ht_1t_2t_3t_8$ by a representative of each orbit and determine its double coset.

Choose 1 from {1,7,6,5}

$Ht_1t_2t_3t_8t_1$

$= (yxy^2xy^{-1})t_4t_8t_6t_7$

$= H(t_1t_2t_3t_8)^{(x^{-1}y^2xy)}$ ϵ [1,2,3,5]

Choose 8 from {2,4,3,8}

$Ht_1t_2t_3t_8t_8$

$= Ht_1t_2t_3(t_8)^2$

$= Ht_1t_2t_3$ ϵ [1,2,3]

Cayley Diagram

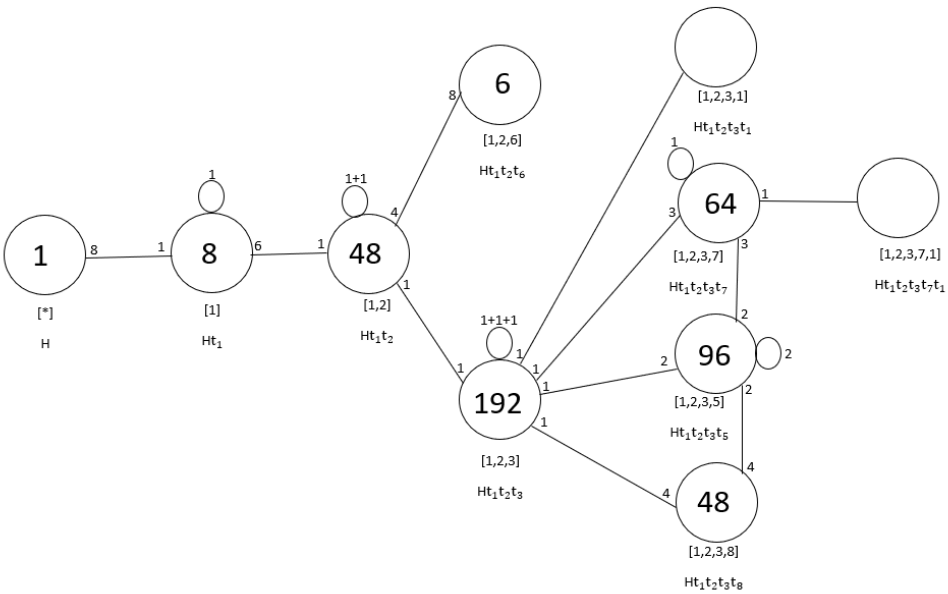


Figure 5.17: Cayley Diagram of [*],[1],[1,2],[1,2,6],[1,2,3],[1,2,3,7],[1,2,3,5], [1,2,3,5,8] for M_{12}:2 over $((4:2^2):S_4)$ and $(2^3:(4:2))$

Ninth Double Coset

$Ht_1t_2t_3t_1N = \{ H(t_1t_2t_3t_1)^n \mid n \epsilon N \} = \{Ht_1t_2t_3t_1, Ht_2t_5t_6t_2,..., Ht_3t_2t_6t_3\}$

The point-stabiliser 1, 2, 3, 1, $N^{1,2,3,1}$ is given by $< 1 >$

But $Nt_1t_2t_3t_1$

$= Nt_7t_1t_2t_7$

$=(ty^2xty^{-1}x^{-1}tytx^{-1}ty^{-1}t)t_5t_6t_8t_5$

$=((yx)t_1t_7t_6t_8t_3t_2)t_5t_6t_8t_5$

Also $Ht_1t_2t_3t_1$

$= Ht_6t_8t_4t_6$

$= (x^2y^{-1}tx^{-1}ty^{-1}tx^{-1}tx^{-1}ty^{-1}t)t_6t_8t_4t_6$

$=((x^2)t_1t_2t_6t_3t_5t_4)t_6t_8t_4t_6$

So

$Ht_1t_2t_3t_1 = ((x^2)t_1t_2t_6t_3t_5t_4)t_6t_8t_4t_6$

Then

$Ht_1t_2t_3t_1 = H(t_1t_2t_6t_3t_5t_4)t_6t_8t_4t_6$

Therefore The coset stabiliser $N^{(1,2,3,1)} = < (xy)^3, xy^2xy >$

The number of single right cosets in the double coset $Ht_1t_2t_3t_1N = [1,2,3,1]$ is given by $\frac{|N|}{|N^{(1,2,3,1)}|} = \frac{192}{8} = 24$

The orbit of $N^{(1,2,3,1)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8} is transitive therefore the orbit is {1,7,6,3,5,2,4,8}. We will multiply $Ht_1t_2t_3t_1$ on the right by an orbit representative and determine its double coset.

Choose 1 from the orbit {1,2,3,4,5,6,7,8}

$Ht_1t_2t_3t_1t_1$

$= Ht_1t_2t_3(t_1)^2$

$= Ht_1t_2t_3 \epsilon [1,2,3]$

Cayley Diagram

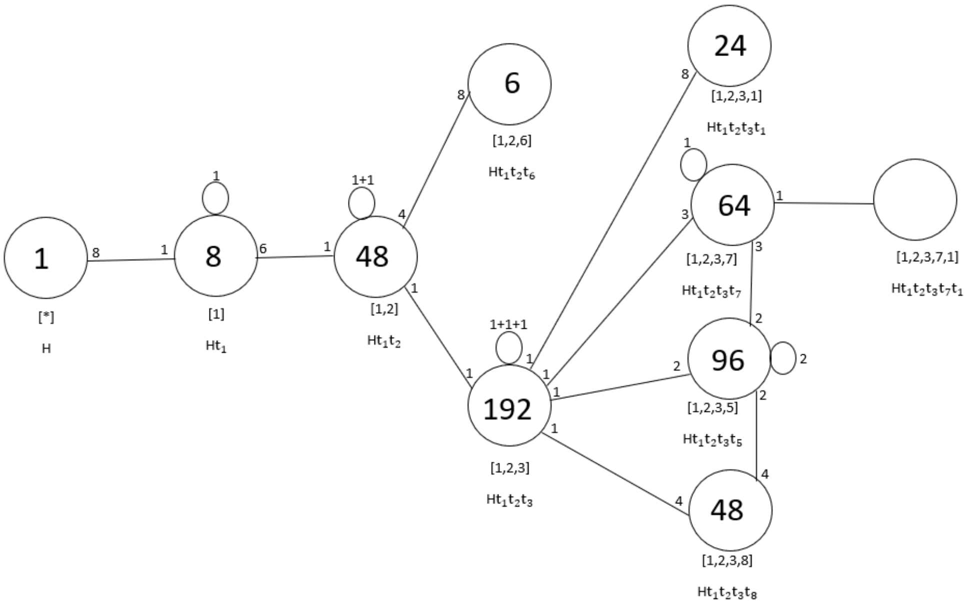


Figure 5.18: Cayley Diagram of [*], [1], [1,2], [1,2,6], [1,2,3], [1,2,3,7], [1,2,3,5], [1,2,3,5,8], [1,2,3,1] for M_{12}:2 over $((4:2^2):S_4)$ and $(2^3:(4:2))$

Tenth Double Coset

$Ht_1t_2t_3t_7t_4N = \{ H(t_1t_2t_3t_7t_4)^n \mid n \epsilon N \} = \{Ht_1t_2t_3t_7t_4, Ht_2t_5t_6t_3t_1, ..., Ht_3t_2t_6t_5t_1\}$

The point-stabiliser 1, 2, 3, 7, 4, $N^{1,2,3,7,4}$ is given by $< 1 >$

However $Ht_1t_2t_3t_7t_4$

$= Ht_5t_3t_1t_8t_6$

$=(y^2x^{-1}y^{-1})t_2t_4t_6t_7t_1$

Then $Ht_1t_2t_3t_7t_4 = (y^2x^{-1}y^{-1})t_2t_4t_6t_7t_1$

Therefore

$Ht_1t_2t_3t_7t_4 = Ht_2t_4t_6t_7t_1$

Also $Ht_1t_2t_3t_7t_4$

$= Ht_8t_5t_1t_3t_6$

$= (yx)t_8t_5t_1t_3t_6$

Then

$Ht_1t_2t_3t_7t_4 = (yx)t_8t_5t_1t_3t_6$

Thus

$Ht_1t_2t_3t_7t_4 = Ht_8t_5t_1t_3t_6$

Therefore The coset stabiliser $N^{(1,2,3,7,4)} = < xy^2xy >$

The number of single right cosets in the double coset $Ht_1t_2t_3t_7t_4N$ = [1,2,3,7,4] is given by $\frac{|N|}{|N^{(1,2,3,7,4)}|} = \frac{192}{24} = 8$

The orbit of $N^{(1,2,3,7,4)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8} is transitive so we have the orbit {1, 2, 3, 4, 5, 6, 7, 8}.

We will multiply $Ht_1t_2t_3t_7t_4$ by an orbit representative and determine its double coset.

Choose 4 from {1,2,3,4,5,6,7,8}

$Ht_1t_2t_3t_7t_4t_4$

$= Ht_1t_2t_3t_7(t_4)^2$

$= Ht_1t_2t_3t_7 \in [1,2,3,7]$

Cayley Diagram

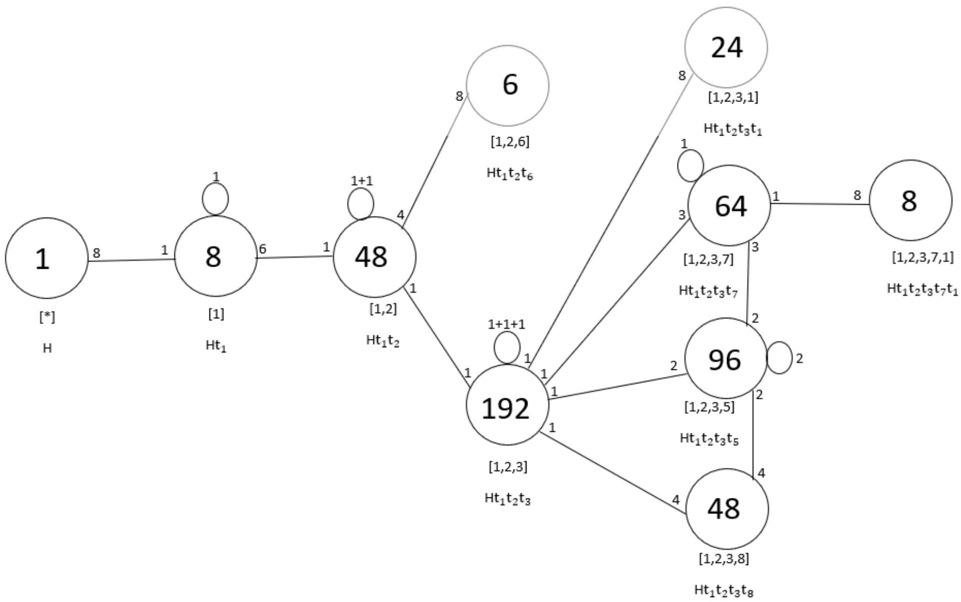


Figure 5.19: Cayley Diagram of [*], [1], [1,2], [1,2,6], [1,2,3], [1,2,3,7], [1,2,3,5], [1,2,3,5,8], [1,2,3,1], [1,2,3,7,4] for M_{12}:2 over $((4:2^2):S_4)$ and $(2^3:(4:2))$

5.6 S_6 over S_5 and (5:4)

We perform double coset enumeration of G over H = < (5:4), $t_2t_5t_2$ > and . We note that the order of H is 120 and (5:4) ≤ H. We will prove that the progenitor 2^{*5}:(5:4), where 2^{*5}:(5:4) = <x, y> and x ∼ (1, 2, 3, 4, 5), y ∼ (1, 2, 4, 3), factored by one relation is isomorphic to S_6. Let G $\cong \frac{2^{*5}:(5:4)}{t_2t_5t_2}$. Thus we show G ∼ S_6

First double Coset

HeN = $\{H(e)^n \mid n \epsilon N\}$ = {H}

The coset stabiliser H = He = H.

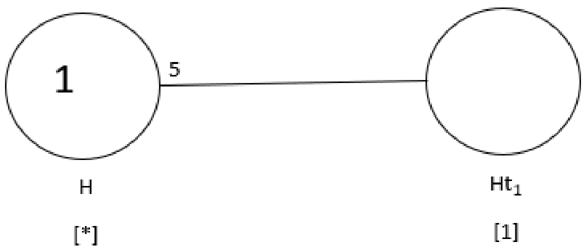
The number of single right cosets in the double coset HeN = [*] is given by $\frac{|N|}{|N|} = \frac{20}{20} = 1$

The orbits of H on X = {1, 2, 3, 4, 5} is {1, 2, 3, 4, 5}. We will multiply H on the right by an orbit representative and determine its double coset.

Choose 1 from {1, 2, 3, 4, 5}

$Ht_1 \epsilon$ [1]

Cayley Diagram



Second double coset

$Ht_1N = \{H(t_1)^n \mid n \epsilon N\} = Ht_1, Ht_2, Ht_3, Ht_4, Ht_5$

The point-stabiliser 1, N^1 given by < (2, 4, 5, 3) >

The coset stabiliser $N^{(1)}$ = < (2, 4, 5, 3) >.

The number of single right cosets in the double coset Ht_1N = [1] is given by $\frac{|N|}{|N^{(1)}|} = \frac{20}{4}$ = 5

The orbits of $N^{(1)}$ on X = {1, 2, 3, 4, 5} are {1}, {2, 3, 4, 5}. We will multiply Ht_1 on the right by a orbit representative and determine its double coset.

Choose 1 from {1}

Ht_1t_1

$= H(t_1)^2$

$= H \; \epsilon$ [*]

Choose 2 from {2, 3, 4, 5}

Ht_1t_2

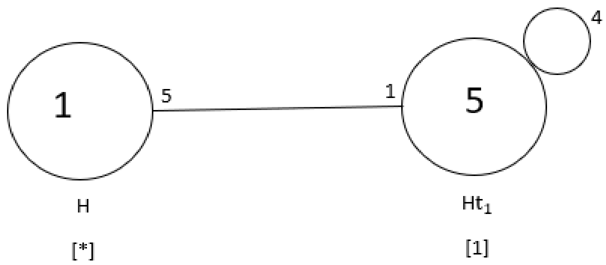
We have the relation $Ht_2t_5t_2 = H$ conjugated by the permutation (1, 4, 5, 2) gives H(1, 4, 5, 2)$t_1t_2t_1 = H$.

= H(1, 4, 5, 2)$t_1t_2t_1$

$\implies Ht_1t_2 = Ht_1 \; \epsilon$ [1]

Then $Ht_1t_2 = Ht_1 \; \epsilon$ [1]

Cayley Diagram



For DCE over N see CSUSB Thesis, February 2020, by Mayra McGrath.

Chapter 6

Isomorphism Types

6.1 (6:2^2):(7×3)

Largest Normal Abelian Subgroup

Given N, a transitive group on 42 letters, where N is generated by

xx = (1, 26, 41)(2, 25, 42)(3, 27, 38)(4, 28, 37)(5, 30, 40)(6, 29, 39)(7, 12, 10)(8, 11, 9)(13, 36, 23)(14, 35, 24)(15, 31, 19)(16, 32, 20)(17, 33, 22)(18, 34, 21), and

yy = (1, 7, 20, 2, 8, 19)(3, 9, 21, 4, 10, 22)(5, 11, 24, 6, 12, 23)(13, 32, 29)(14, 31, 30)(15, 34, 25)(16, 33, 26)(17, 36, 28)(18, 35, 27)(37, 38)(39, 40)(41, 42)

We can see the normal lattice of N by using the magma code below:

```
S:=Sym(42);
xx:=S!(1, 26, 41)(2, 25, 42)(3, 27, 38)(4, 28, 37)(5, 30, 40)(6, 29, 39)(7, 12, 10)(8, 11, 9)(13, 36, 23)(14, 35, 24)(15, 31, 19)(16, 32, 20)(17, 33, 22)(18, 34, 21);
yy:=S!(1, 7, 20, 2, 8, 19)(3, 9, 21, 4, 10, 22)(5, 11, 24, 6, 12, 23)(13, 32, 29)(14, 31, 30)(15, 34, 25)(16, 33, 26)(17, 36, 28)(18, 35, 27)(37, 38)(39, 40)(41, 42);
N:=sub<S|xx,yy>;
#N;
504
CompositionFactors(N);
NL:= NormalLattice(N);
NL;
```

```
    Normal subgroup lattice
-----------------------
```

```
[10]  Order 504  Length 1  Maximal Subgroups: 6 7 8 9
---
[ 9]  Order 168  Length 1  Maximal Subgroups: 5
[ 8]  Order 168  Length 1  Maximal Subgroups: 4 5
[ 7]  Order 168  Length 1  Maximal Subgroups: 5
[ 6]  Order 168  Length 1  Maximal Subgroups: 5
---
[ 5]  Order 56   Length 1  Maximal Subgroups: 3
[ 4]  Order 24   Length 1  Maximal Subgroups: 2 3
---
[ 3]  Order 8    Length 1  Maximal Subgroups: 1
---
[ 2]  Order 3    Length 1  Maximal Subgroups: 1
---
[ 1]  Order 1    Length 1  Maximal Subgroups:
```

Next, we find the largest normal abelian subgroup of N, which can be found using the magma code:

for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if; end for;

4

From the above magma code we get NL[4] is the largest normal abelian subgroup. The order of NL[4] is 24 and the order of N is 504. N has no normal subgroup of order 24. Therefore, N is not a direct-product. Thus, N is an extension of NL[4] by a group, we'll call q, where q = N/NL[4]. Next, we will check if N is a semi-direct product of q or a mixed extension of q. We will use the factors of the order of NL[4] to find the generators of a group isomorphic NL[4]. We can find the generators on magma by using the following code:

Order (NL4);

24

Order (N);

504

IsIsomorphic(NL[4],AbelianGroup(GrpPerm,[6,2,2]));

True

Then NL[4] $\cong 6:2:2$, where NL[4] is generated by

A $\sim$ (1, 3, 6, 2, 4, 5)(7, 9, 12, 8, 10, 11)(13, 15, 17, 14, 16, 18)(19, 21, 24)(20, 22, 23)(25, 28, 30, 26, 27, 29)(31, 34, 35)(32, 33, 36)(37, 39, 41)(38, 40, 42)

B ∼ (13, 14)(15, 16)(17, 18)(19, 20)(21, 22)(23, 24)(25, 26)(27, 28)(29, 30)(37, 38)(39, 40)(41, 42)

C ∼ (7, 8)(9, 10)(11, 12)(25, 26)(27, 28)(29, 30)(31, 32)(33, 34)(35, 36)(37, 38)(39, 40)(41,42)

Therefore we will consider A,B,C to be generators of NL[4]. Next we consider our q of order 21 = 73, where q is a group in two generator relations <d,e| d^3 , e^3 , e^{-1} d^{-1} e^{-1} d e d^{-1} > Thus, the presentation of NL[4] is Group<a,b,c,d,e|a^6, b^2, c^2, (a,b), (a,c), (b,c), d^3, e^3, $e^{-1}d^{-1}e^{-1}ded^{-1}$, a^d=?, a^e=?, b^d=?,b^e=?, c^d=?, c^e=?>

Semi-Direct Product of NL[4] by N/NL[4]

Next, we raise A, B, and C by the d and e from q to find what combination of a, b, and c we have. So we have a^d=?, b^d=?,c^d=?, a^e=?, b^e=?, and c^e=?. However, by using magma we can confirm that $a^d = a^4b^2c$, $a^e = a^4bc$, $b^d = a^3b^2c$, $b^e = a^3b^2c$, $c^d = a^3bc^2$, $c^e = a^6bc^2$. We will include these in our presentation of G and verify it's isomorphic to N to find the presentation of N.

The following magma code:

Group<a, b, c, d, e | a^6, b^2, c^2, (a, b), (a, c), (b, c), d^3, e^3, $e^{-1}d^{-1}e^{-1}ded^{-1}$, $a^d = a^4b^2c$, $a^e = a^4bc$, $b^d = a^3b^2c$, $b^e = a^3b^2c$, $c^d = a^3bc^2$, $c^e = a^6bc^2$ >

#G;

504

f,G1,k:=CosetAction(G,sub<G|Id(G)>);

IsIsomorphic(G1,N);

True

Thus we have a true isomorphism that tells us that we have a semi-direct product. Now we need to consider how do we write our isomorphism. We do so, by looking at the order of NL[4] which was confirmed to us using Isomorphism of NL[4] in respects to group permutation to be $6{:}2^2$. Next we look at the order of our q, which is of order 21 = 7×3 thus we have the following isomorphism $(6{:}2^2){:}(7\times3)$.

6.2 (21×3):12

Largest Normal Abelian Subgroup

Given N, a transitive group on 42 letters, where N is generated by

xx = (1, 17, 20, 28, 15, 10)(2, 18, 19, 30, 14, 11, 3, 16, 21, 29, 13, 12)(4, 26, 6, 27)(5, 25)(7, 34, 32, 24, 37, 42, 8, 36, 31, 23, 38, 40)(9, 35, 33, 22, 39, 41), and
yy = (1, 22, 3, 23)(2, 24)(4, 33, 28, 19, 34, 39, 5, 32, 29, 20, 35, 38)(6, 31, 30, 21, 36, 37)(7, 40, 14, 16, 27, 12)(8, 42, 15, 17, 26, 11, 9, 41, 13, 18, 25, 10).
We can see the normal lattice of N by using the magma code below:

```
S:=Sym(42);
xx:=S!(1, 17, 20, 28, 15, 10)(2, 18, 19, 30, 14, 11, 3, 16, 21, 29, 13, 12)(4, 26, 6, 27)(5, 25)(7, 34, 32, 24, 37, 42, 8, 36, 31, 23, 38, 40)(9, 35, 33, 22, 39, 41);
yy:=S!(1, 22, 3, 23)(2, 24)(4, 33, 28, 19, 34, 39, 5, 32, 29, 20, 35, 38)(6, 31, 30, 21, 36, 37)(7, 40, 14, 16, 27, 12) (8, 42, 15, 17, 26, 11, 9, 41, 13, 18, 25, 10);
N:=sub<S|xx,yy>;
#N;
756
CompositionFactors(N);
NL:= NormalLattice(N);
NL;
```

```
Normal subgroup lattice
-----------------------

[11]  Order 756  Length 1  Maximal Subgroups: 9 10
---
[10]  Order 378  Length 1  Maximal Subgroups: 7 8
[ 9]  Order 252  Length 1  Maximal Subgroups: 7
---
[ 8]  Order 189  Length 1  Maximal Subgroups: 4 6
[ 7]  Order 126  Length 1  Maximal Subgroups: 5 6
---
[ 6]  Order 63   Length 1  Maximal Subgroups: 2 3
[ 5]  Order 18   Length 1  Maximal Subgroups: 3
---
[ 4]  Order 21   Length 1  Maximal Subgroups: 2
[ 3]  Order 9    Length 1  Maximal Subgroups: 1
---
[ 2]  Order 7    Length 1  Maximal Subgroups: 1
---
[ 1]  Order 1    Length 1  Maximal Subgroups:
```

Next, we find the largest normal abelian subgroup of N, which can be found using the magma code:

```
for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if; end for;
6
```

From the above magma code we get NL[6] is the largest normal abelian subgroup. The order of NL[6] is 63 and the order of N is 756. N has no normal subgroup of order 12. Therefore, N is not a direct-product. Thus, N is an extension of NL[6] by a group, we'll call q, where q = N/NL[6] and q is of order 12. Next, we will check if N is a semi-direct product of q or a mixed extension of q. We will use the factors of the order of NL[6] to find the generators of a group isomorphic NL[6]. We can find the generators on magma by using the following code:

```
Order (NL6);
63
Order (N);
756
X:=[21,3]; IsIsomorphic(NL6,AbelianGroup(GrpPerm,X));
true
for g,h in NL6 do if Order(g) eq 21 and Order(h) eq 21 and NL6 eq sub<NL6|g,h> then A:=g; B:=h; end if; end for;
```

Then NL[6] $\cong 12 \times 12$, where NL[6] is generated by A ∼(1, 37, 32, 25, 21, 13, 9, 2, 38, 33, 27, 19, 15, 7, 3, 39, 31, 26, 20, 14, 8)(4, 42, 34, 29, 23, 18, 11)(5, 41, 35, 28, 22, 17, 10)(6, 40, 36, 30, 24, 16, 12) and B∼ (1, 13, 27, 39, 8, 21, 33, 3, 14, 25, 38, 7, 20, 32, 2, 15, 26, 37, 9, 19, 31)(4, 17, 30, 42, 10, 24, 34, 5, 16, 29, 41, 12, 23, 35, 6, 18, 28, 40, 11, 22, 36). We also have q to have the following two generators c ∼ (1, 2, 4, 6, 8, 10, 12, 3, 5, 7, 9, 11) and d ∼ (1, 3, 4, 7, 8, 11, 12, 2, 5, 6, 9, 10). We will raise A and B to powers of c and d such that we have a^c=?, b^c=?, a^d=?, b^d=?. Thus, the presentation of NL[6] is G<a,b,c>:=Group<a,b,c|a^{21},b^3,$a^b = a$, c^{12}, $a^c = ?$, $b^c = ?$>;

Semi-Direct Product of NL[6] by N/NL[6]

Next, we have to conjugate A and B by C. We find that c^{12}, $a^c = a^{17}b$, $b^c = a^7b$. We will include these in our presentation of G and verify it is isomorphic to N to find the presentation of N.

The following magma code:

G<a,b,c>:=Group<a,b,c|a^{21}, b^3, $a^b = a$, c^{12}, $a^c = a^{17}b$, $b^c = a^7b$>

#G;

756

f,G1,k:=CosetAction(G,sub<G|Id(G)>); IsIsomorphic(G1,N);

true

Thus N is a semi direct product of (21×3):12 and the presentation of N is:

G <a, b, c>:=Group<a, b, c | a^{21}, b^3, $a^b = a$, c^{12}, $a^c = a^{17}b$, $b^c = a^7b$>

6.3 7×S_5

Largest Normal Abelian Subgroup

Given N, a transitive group on 42 letters, where N is generated by xx = (1, 20, 39, 15, 35, 12, 26, 2, 22, 38, 16, 36, 10, 25, 6, 19, 37, 18, 32, 8, 27, 3, 21, 42, 14, 34, 7, 29, 4, 24, 40, 13, 33, 9, 30, 5, 23, 41, 17, 31, 11, 28) and yy = (1, 19, 41, 18, 33, 11, 26, 3, 20, 42, 17, 35, 10, 29, 2, 24, 39, 16, 32, 9, 25, 5, 22, 37, 14, 31, 8, 28, 6, 21, 40, 15, 34, 12, 27, 4, 23, 38, 13, 36, 7, 30)

We can see the normal lattice of N by using the magma code below:

S:=Sym(42);

xx:=S!(1, 20, 39, 15, 35, 12, 26, 2, 22, 38, 16, 36, 10, 25, 6, 19, 37, 18, 32, 8, 27, 3, 21, 42, 14, 34, 7, 29, 4, 24, 40, 13, 33, 9, 30, 5, 23, 41, 17, 31, 11, 28);

yy:=S!(1, 19, 41, 18, 33, 11, 26, 3, 20, 42, 17, 35, 10, 29, 2, 24, 39, 16, 32, 9, 25, 5, 22, 37, 14, 31, 8, 28, 6, 21, 40, 15, 34, 12, 27, 4, 23, 38, 13, 36, 7, 30);

N:=sub<S|xx,yy>;

#N;

840

NL:= NormalLattice(N);

NL;

```
Normal subgroup lattice
-----------------------

[6]  Order 840  Length 1  Maximal Subgroups: 4 5
---
[5]  Order 420  Length 1  Maximal Subgroups: 2 3
[4]  Order 120  Length 1  Maximal Subgroups: 3
```

```
---
[3]  Order 60   Length 1  Maximal Subgroups: 1
---
[2]  Order 7    Length 1  Maximal Subgroups: 1
---
[1]  Order 1    Length 1  Maximal Subgroups:
```

Next, we find the largest normal abelian subgroup of N, which can be found using the magma code:

for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if; end for;

2

From the above magma code we get NL[2] is the largest normal abelian subgroup. The order of NL[2] is 7 and the order of N is 840. N has no normal subgroups of order 120. Therefore, N is not a direct-product. Thus, N is an extension of NL[2] by a group, we'll call q, where q = N/NL[2]. Next, we will check if N is a semi-direct product of q or a mixed extension of q. We will use the factors of the order of NL[2] to find the generators of a group isomorphic NL[2]. We can find the generators on magma by using the following code:

Order (NL2);

7

Order (N);

X:=[7]; IsIsomorphic(NL2,AbelianGroup(GrpPerm,X));

True

Then NL[2] is generated by A $\sim$ (1, 26, 10, 32, 14, 40, 23)(2, 25, 8, 34, 13, 41, 20)(3, 29, 9, 31, 15, 38, 19)(4, 30, 11, 35, 16, 37, 21)(5, 28, 12, 36, 18, 42, 24)(6, 27, 7, 33, 17, 39, 22). We also have q which is of order 120 and is isomorphic to S_5 and has the following generators d $\sim$ (2, 3, 5, 4, 6, 8)(7, 10, 9) and e $\sim$ (1, 2, 4)(3, 5, 7, 8, 6, 9). Next we will raise A with d and e. Thus, the presentation of NL[2] is:

G<a,d,e>:= Group $<$ a,d,e $|a^7, d^6$, e^6 , $(e^{-1}d^{-1})^3$, $(e^{-1}d)^3$, e^{-3} d^{-1} e^3 d^{-1} , d^{-1} e^{-1} d^3 e^{-1} d^{-2} , a^d = ?, a^e = ? $>$

Semi-Direct Product of NL[2] by N/NL[2]

We now have the following presentation of G and verify it's isomorphic to N to find the presentation of N. The following magma code:

G<a,d,e>:= Group $<$ a,d,e $|a^7, d^6$, e^6 , $(e^{-1}d^{-1})^3$, $(e^{-1}d)^3$, e^{-3} d^{-1} e^3 d^{-1} , d^{-1} e^{-1}

d^3 e^{-1} d^{-2} , a^d = a,a^e = a>

#G;

840

f,G1,k:=CosetAction(G,sub<G|Id(G)>); IsIsomorphic(G1,N);

Thus, N is a semi direct product of 7×S_5 and the presentation of N is G<a,d,e>:= Group < a,d,e | a^7,d^6 , e^6 , $(e^{-1}d^{-1})^3$, $(e^{-1}d)^3$, e^{-3} d^{-1} e^3 d^{-1} , d^{-1} e^{-1} d^3 e^{-1} d^{-2} , a^d = a, a^e = a >

6.4 (2^5:(3×7))

Largest Normal Abelian Subgroup

Given N, a transitive group on 42 letters, where N is generated by xx= (1, 3, 6, 2, 4, 5)(7, 10, 11)(8, 9, 12) (13, 16, 17, 14, 15, 18)(19, 22, 24)(20, 21, 23) (25, 27, 30)(26, 28, 29)(31, 34, 35, 32, 33, 36) (37, 40, 42, 38, 39, 41) and yy = (1, 18, 27, 40, 8, 19, 34, 2, 17, 28, 39, 7, 20, 33) (3, 14, 29, 41, 10, 22, 35)(4, 13, 30, 42, 9, 21, 36) (5, 15, 26, 37, 11, 23, 31, 6, 16, 25, 38, 12, 24, 32)

We can see the normal lattice of N by using the magma code below:

S:=Sym(42);

xx= (1, 3, 6, 2, 4, 5)(7, 10, 11)(8, 9, 12) (13, 16, 17, 14, 15, 18)(19, 22, 24)(20, 21, 23)(25, 27, 30) (26, 28, 29)(31, 34, 35, 32, 33, 36)(37, 40, 42, 38, 39, 41)

yy = (1, 18, 27, 40, 8, 19, 34, 2, 17, 28, 39, 7, 20, 33) (3, 14, 29, 41, 10, 22, 35)(4, 13, 30, 42, 9, 21, 36) (5, 15, 26, 37, 11, 23, 31, 6, 16, 25, 38, 12, 24, 32)

N:=sub<S|xx,yy>;

#N;

672

NL:= NormalLattice(N);

NL;

```
Normal subgroup lattice
-----------------------

[9]  Order 672  Length 1  Maximal Subgroups: 7 8
---
[8]  Order 224  Length 1  Maximal Subgroups: 5 6
[7]  Order 96   Length 1  Maximal Subgroups: 4 6
```

```
---
[6]  Order 32   Length 1  Maximal Subgroups: 2 3
---
[5]  Order 56   Length 1  Maximal Subgroups: 3
---
[4]  Order 12   Length 1  Maximal Subgroups: 2
[3]  Order 8    Length 1  Maximal Subgroups: 1
---
[2]  Order 4    Length 1  Maximal Subgroups: 1
---
[1]  Order 1    Length 1  Maximal Subgroups:
```

Next, we find the largest normal Abelian subgroup of N, which can be found using the magma code:

for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if; end for;

6

From the above magma code we get NL[6] is the largest normal abelian subgroup. The order of NL[6] is 32 and the order of N is 672. N has no normal subgroup of order 21. Therefore, N is not a direct-product. Thus, N is an extension of NL[6] by a group, we'll call q, where q = N/NL[6]. Next, we will check if N is a semi-direct product of q or a mixed extension of q. We will use the factors of the order of NL[6] to find the generators of a group isomorphic NL[6]. We can find the generators on magma by using the following code:

Order (NL6);

32

Order (N);

X:=[2,2,2,2,2]; IsIsomorphic(NL6,AbelianGroup(GrpPerm,X));

True

for g, h, i, j, k in NL6 do if Order(g) eq 10 and Order(h) eq 10 and NL6 eq sub<NL6|g, h, i, j, k> then A:=g; B:=h; C:=i; D:=j; E:=k; end if; end for;

Then NL[6] is generated by:

A ∼ (3, 4)(5, 6)(9, 10)(11, 12)(13, 14)(15, 16)(21, 22)(23, 24)(25, 26)(29, 30)(31, 32)(35, 36)(37, 38)(41, 42)

B ∼ (3, 4)(9, 10)(15, 16)(17, 18)(21, 22)(25, 26)(27, 28)(31, 32)(33, 34)(41, 42)

C ∼ (5, 6)(11, 12)(15, 16)(19, 20)(21, 22)(25, 26)(33, 34)(35, 36)(39, 40)(41, 42)

D ∼ (3, 4)(5, 6)(7, 8)(13, 14)(15, 16)(21, 22)(23, 24)(27, 28)(33, 34)(39, 40)

E ∼ (1, 2)(5, 6)(7, 8)(11, 12)(15, 16)(17, 18)(19, 20)(23, 24)(25, 26)(27, 28)(31, 32)(33, 34)(37, 38)(39, 40)

Next we consider our q, where our q is of order 21 = 7×3, and is generated by f ∼ (1, 2, 4)(3, 5, 7)(6, 8, 10)(9, 11, 13)(12, 14, 16)(15, 17, 19)(18, 20, 21) and g ∼ (1, 3, 6, 9, 12, 15, 18)(2, 5, 8, 11, 14, 17, 20)(4, 7, 10, 13, 16, 19, 21). Thus the presentation of NL[6] is:

G<a,b> = Group <a^3 , (a, b) , b^{-7} >

Semi-Direct Product of NL[6] by N/NL[6]

Next, we have to conjugate A, B, C, D, E by f and g. We find that

$a^f = a^2b^2c^2d^2e$,

$a^g = ab^2c^2d^2e^2$,

$b^f = abc^2d^2e^2$,

$b^g = a^2bcde$,

$c^f = a^2b^2cd^2e$,

$c^g = a^2bcd^2e$,

$d^f = ab^2c^2de$,

$d^g = abc^2d^2e$,

$e^f = ab^2c^2d^2e$,

$e^g = a^2b^2c^2d^2e$

We will include these in our presentation of G and verify it is isomorphic to N to find the presentation of N. The following magma code:

G<a, b, c, d, e, f, g>:=Group < a, b, c, d, e, f, g | a^2, b^2,

c^2, d^2, e^2,

(a,b), (a,c), (a,d), (a,e), (b,c), (b,d), (d,e), (c,d),

(c,e), f^3 , (f, g),

g^{-7},

$a^f = a^2b^2c^2d^2e$,

$a^g = ab^2c^2d^2e^2$,

$b^f = abc^2d^2e^2$,

$b^g = a^2bcde$,

$c^f = a^2b^2cd^2e$,

$c^g = a^2bcd^2e$,

$d^f = ab^2c^2de$,

$d^g = abc^2d^2e$,

$e^f = ab^2c^2d^2e$,

$e^g = a^2b^2c^2d^2e$ >;

#G;

672

f,G1,k:=CosetAction(G,sub<G|Id(G)>);

IsIsomorphic(G1,N);

True

Thus, N is a semi direct product of N$\sim$ (2^5:(3×7)) and the presentation of N is:

G<a, b, c, d, e, f, g>:=Group< a, b, c, d, e, f, g | a^2, b^2,

c^2,d^2,e^2,

(a,b), (a,c), (a,d), (a,e), (b,c), (b,d), (d,e), (c,d),

(c,e),f^3 , (f, g),

g^{-7},

$a^f = a^2b^2c^2d^2e$,

$a^g = ab^2c^2d^2e^2$,

$b^f = abc^2d^2e^2$,

$b^g = a^2bcde$,

$c^f = a^2b^2cd^2e$,

$c^g = a^2bcd^2e$,

$d^f = ab^2c^2de$,

$d^g = abc^2d^2e$,

$e^f = ab^2c^2d^2e$,

$e^g = a^2b^2c^2d^2e$ >

6.5 14^2:3

Largest Normal Abelian Subgroup

Given N, a transitive group on 42 letters, where N is generated by:

xx= (1, 19, 42)(2, 20, 41)(3, 21, 39)(4, 22, 40)(5, 24, 38)(6, 23, 37)(7, 26, 36)(8, 25, 35)(9, 27, 34)(10, 28, 33)(11, 16, 31)(12, 15, 32)(13, 18, 30)(14, 17, 29),

yy = (1, 32, 23)(2, 31, 24)(3, 29, 26)(4, 30, 25)(5, 42, 27)(6, 41, 28)(7, 40, 15)(8, 39, 16)(9, 37, 17)(10, 38, 18)(11, 36, 19)(12, 35, 20)(13, 34, 21)(14, 33, 22)

We can see the normal lattice of N by using the magma code below:
S:=Sym(42);
xx:=S!(1, 19, 42)(2, 20, 41)(3, 21, 39)(4, 22, 40)(5, 24, 38)(6, 23, 37)(7, 26, 36)(8, 25, 35)(9, 27, 34)(10, 28, 33)(11, 16, 31)(12, 15, 32)(13, 18, 30)(14, 17, 29);
yy:=S!(1, 32, 23)(2, 31, 24)(3, 29, 26)(4, 30, 25)(5, 42, 27)(6, 41, 28)(7, 40, 15)(8, 39, 16)(9, 37, 17)(10, 38, 18)(11, 36, 19)(12, 35, 20)(13, 34, 21)(14, 33, 22);
N:=sub<S|xx,yy>;
#N;
588
NL:= NormalLattice(N);
NL;

```
Normal subgroup lattice
-----------------------

[9]  Order 588  Length 1  Maximal Subgroups: 8
---
[8]  Order 196  Length 1  Maximal Subgroups: 5 6 7
---
[7]  Order 28   Length 1  Maximal Subgroups: 3 4
[6]  Order 28   Length 1  Maximal Subgroups: 2 4
---
[5]  Order 49   Length 1  Maximal Subgroups: 2 3
[4]  Order 4    Length 1  Maximal Subgroups: 1
---
[3]  Order 7    Length 1  Maximal Subgroups: 1
[2]  Order 7    Length 1  Maximal Subgroups: 1
---
[1]  Order 1    Length 1  Maximal Subgroups:
```

Next, we find the largest normal Abelian subgroup of N, which can be found using the magma code:
for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if; end for;
8
From the above magma code we get NL[8] is the largest normal abelian subgroup. The order of NL[8] is 196 and the order of N is 588. N has no normal subgroup of order 3. Therefore, N is not a direct-product. Thus, N is an extension of NL[8] by a group, we'll call q, where q = N/NL[8]. Next, we will check if N is a semi-direct product of q or a

mixed extension of q. We will use the factors of the order of NL[8] to find the generators of a group isomorphic NL[8]. We can find the generators in magma using the following code:

Order (NL[8]);

196

Order (N);

X:=[14,14]; IsIsomorphic(NL8,AbelianGroup(GrpPerm,X));

True

for g,h in NL8 do if Order(g) eq 14 and Order(h) eq 14 and NL8 eq sub<NL8|g,h> then A:=g; B:=h; end if; end for;

Then NL[8] is generated by:

A ∼ (15, 25, 21, 17, 28, 24, 19)(16, 26, 22, 18, 27, 23, 20)(29, 39, 35, 32, 42, 38, 33)(30, 40, 36, 31, 41, 37, 34)

B ∼ (1, 10, 3, 12, 5, 14, 8)(2, 9, 4, 11, 6, 13, 7)(15, 24, 17, 25, 19, 28, 21)(16, 23, 18, 26, 20, 27, 22)(29, 32, 33, 35, 38, 39, 42)(30, 31, 34, 36, 37, 40, 41)

C ∼ (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 28, 25, 24, 21, 19, 17)(16, 27, 26, 23, 22, 20, 18)(29, 41, 39, 37, 35, 34, 32, 30, 42, 40, 38, 36, 33, 31)

D ∼ (15, 18, 19, 22, 24, 26, 28, 16, 17, 20, 21, 23, 25, 27)(29, 31, 33, 36, 38, 40, 42, 30, 32, 34, 35, 37, 39, 41)

We also consider our q, with is of order 3 and has the following generators d ∼ (1, 2, 3) and e ∼ (1, 3, 2).

Thus the presentation of NL[8] is:

G<a,b> = Group $< b^{-1}\ a^{-1}\ ,\ b^{-3} >$

Semi-Direct Product of NL[8] by N/NL[8]

Next,we will conjugate A and B by d

$a^{d} = a^{2}b^{7}$

$b^{d} = ab^{11}$

We will include these in our presentation of G and verify it is isomorphic to N to find the presentation of N. The following magma code:

G<a, b, d, e>:=Group<a, b, d, e | a^{14}, b^{14}, (a, b), $e^{-1}\ d^{-1}$, d^{-3}, $a^{d} = a^{2}b^{7}$, $b^{d} = ab^{11}$ >;

6.6 $\frac{2^{*12}:(2^3:(3:2))}{(tt^(x))^2=(xyxy^-1xy),(xyt^x)^3} \cong \mathbf{2^3} \times \mathbf{S}_4$

We will work with the following group

Group<x, y, t| x^2 , y^2 , $(xy)^3$, y^{-1} x y^{-1} x y^{-1} x y x y x y x , t^2,(t,(x y x y^{-1} x y)), $(tt^x)^2$ = (x y x y^{-1} x y), $(xyt^x)^3$ >

where G is of order 192, and has x = (1, 8)(2, 7)(3, 6)(4, 5)(9, 12)(10, 11) and y = (1, 6, 7, 12)(2, 11)(3, 10, 9, 4)(5, 8) with the goal to find the isomorphism type of this group. We will begin by looking at the normal lattice of our G,

```
     Normal subgroup lattice
-----------------------

[8]  Order 192  Length 1  Maximal Subgroups: 7
---
[7]  Order 96   Length 1  Maximal Subgroups: 6
---
[6]  Order 32   Length 1  Maximal Subgroups: 3 4 5
---
[5]  Order 8    Length 1  Maximal Subgroups: 2
[4]  Order 8    Length 1  Maximal Subgroups: 2
[3]  Order 8    Length 1  Maximal Subgroups: 2
---
[2]  Order 2    Length 1  Maximal Subgroups: 1
---
[1]  Order 1    Length 1  Maximal Subgroups:
```

Our objective after analyzing our normal lattice of G to is find the largest normal abelian subgroup of G,

for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if;end for;

5

We find 5 to be the largest normal abelian subgroup, which we will denote as NL[5]. Next NL[5] is of order 8 and is generated by < (1, 26)(2, 15)(3, 22)(4, 29)(5, 19)(6, 14)(7, 8)(9, 17)(10, 31)(11, 21)(12, 24)(13, 18)(16, 30)(20, 25)(23, 27)(28, 32), (1, 32)(2, 30)(3, 25)(4, 21)(5, 18)(6, 31)(7, 23)(8, 27)(9, 12)(10, 14)(11, 29)(13, 19)(15, 16)(17, 24)(20, 22)(26, 28), (1, 31)(2, 25)(3, 30)(4, 19)(5, 29)(6, 32)(7, 24)(8, 12)(9, 27)(10, 26)(11, 18)(13, 21)(14, 28)(15, 20)(16, 22)(17, 23) > we should also record that the order of our G is so far 2^3 due to the permutations of NL[5] are each of order 2. We will use the following code to confirm with magma that NL[5] is isomorphic to the group permutation 2^3

IsIsomorphic(NL[5],AbelianGroup(GrpPerm,[2,2,2]));
True
Next we will investigate our q = G/NL[5], where q is of order 24 and is generated by < (3, 4), (2, 3), (1, 2)(3, 4)>. We will further investigate q, and look at the normal lattice of q

```
Normal subgroup lattice
-----------------------

[4]  Order 24  Length 1  Maximal Subgroups: 3
---
[3]  Order 12  Length 1  Maximal Subgroups: 2
---
[2]  Order 4   Length 1  Maximal Subgroups: 1
---
[1]  Order 1   Length 1  Maximal Subgroups:
```

We then will look for the largest normal abelian subgroup which we find to be nl[2] and nl[2] is of order 4 and generated by <(1, 2)(3, 4), (1, 4)(2, 3)>. Then we apply the following magma code to confirm that nl[2] is isomorphic to the group permutation 2.
IsIsomorphic(nl[2],AbelianGroup(GrpPerm,[4]));
True
However, we know that q is of order 24, therefore we are still missing an order of 6. Thus we will continue our search with qq = q/nl[2], where qq is the quotient group of q with respects to nl[2]. We find that qq is of order 6 and has the following composition factors

```
G
|  Cyclic(2)

|  Cyclic(3)
1
```

Thus we have the following isomorphism for G $\sim$ (2^3:(2^2:(3:2))).
Then since G $\sim$ (2^3):((2^2):(3:2)) then we write (2^3):((2^2):(3:2)) = (2^3):([(2^2):3]:2) = (2^3):([(2^2):3]:2) = 2^3:(A_4:2) = $2^3 \times S_4$
Next we will investigate our N where N is of order 48 and is generated by x $\sim$ (1, 8)(2, 7)(3, 6)(4, 5)(9, 12)(10, 11) and y $\sim$ (1, 6, 7, 12)(2, 11)(3, 10, 9, 4)(5, 8). We will now

look at the normal lattice of N

```
Normal subgroup lattice
-----------------------

[9]  Order 48  Length 1  Maximal Subgroups: 6 7 8
---
[8]  Order 24  Length 1  Maximal Subgroups: 5
[7]  Order 24  Length 1  Maximal Subgroups: 5
[6]  Order 24  Length 1  Maximal Subgroups: 4 5
---
[5]  Order 12  Length 1  Maximal Subgroups: 3
[4]  Order 8   Length 1  Maximal Subgroups: 2 3
---
[3]  Order 4   Length 1  Maximal Subgroups: 1
---
[2]  Order 2   Length 1  Maximal Subgroups: 1
---
[1]  Order 1   Length 1  Maximal Subgroups:
```

Next we will find the largest normal abelian subgroup of the normal lattice of N

for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if;end for;

4

we find to be NL[4] to be the largest normal abelian subgroup of N, where NL[4] is of order 8 and is generated by $<$ (3, 9)(6, 12), (2, 8)(3, 9)(5, 11)(6, 12), (1, 7)(3, 9)(4, 10)(6, 12)$>$. By looking at the permutations NL[4] produces we can see there are three permutations of order two. We will use the following magma code to confirm if NL[4] is isomorphic to 2^3.

IsIsomorphic(NL[4],AbelianGroup(GrpPerm,[2,2,2]));

True

Thus NL[4] $\sim 2^3$. Next we will investigate q = N/NL[4], where q is of order 6, generated by $<$ (2, 3), (1, 2)$>$ and has the following composition factor.

```
G
|  Cyclic(2)

|  Cyclic(3)
1
```

We will continue our investigation by looking at the normal lattice of q, which we will denote as nl.

```
Normal subgroup lattice
-----------------------

[3]  Order 6  Length 1  Maximal Subgroups: 2
---
[2]  Order 3  Length 1  Maximal Subgroups: 1
---
[1]  Order 1  Length 1  Maximal Subgroups:
```

Next we will use the following code to find the largest normal abelian subgroup of q.
for i in [1..#nl] do if IsAbelian(nl[i]) then i; end if;end for;
2
We find that the largest normal abelian subgroup of q to be nl[2]. Next we find nl[2] is of order 3 and is generated by <(1, 3, 2)> then this indicates to use that nl[2] is isomorphic to the group permutation 3, we will confirm this using magma.
IsIsomorphic(nl[2],AbelianGroup(GrpPerm,[3]));
True
However, by the composition factor of q we should notice that we are missing a cyclic group of 2. Therefore we investigate even further by looking at qq = q/nl[2] which is the quotient group in respects to q and nl[2] which we find qq to be of order 2 and generated by < (1, 2)>. We check with magma to confirm if qq is isomorphic to cyclic group(2)
IsIsomorphic(qq,CyclicGroup(2));
True
Thus we have the isomorphism type of N, where N $\sim$ (2^3):(3:2).

6.7 $\frac{2^{*18}:(3:A_6)}{((xy)^2)t^{(xyx^{-1}y^{-1}xy^{-2}x)})^{10},((x^{-3}y)t^{(x^{-1}y^{-1}x^{-1}y^{-1}x^{-1})})^{10}} \cong \mathbf{2^5{:}A_6}$

We will investigate the following group G = Group<x,y,t| y^5 , y^{-2} x^3 y^{-2} x^{-1} , $(x\ y^{-1})^4$, y^{-1} x y^{-1} x^{-1} y^{-1} x y x y^{-1} x^{-1} , t^2, (t,y^{-1} x^{-1} y x y^{-1} x^{-2}), (t,$(x^{-1}\ y^2)^2$), (t,$(x\ y\ x)^2$), $((x\ y\ x\ y^{-2}\ x)t^{(xyxyx^2yx^{-1})}t^{((xyx^{-1})^2)})^3$, $(((x\ y)^2)t^{(xyx^{-1}y^{-1}xy^{-2}x)})^{10}$,
$((x^{-3}\ y)t^{(x^{-1}y^{-1}x^{-1}y^{-1}x^{-1})})^{10}$ > where G is of order 11520. To find what G is isomorphic to, we will begin by looking at the normal lattice of G with the goal of finding the largest normal abelian subgroup of G.

```
    Normal subgroup lattice
-----------------------

[5]  Order 11520  Length 1  Maximal Subgroups: 3 4
---
[4]  Order 5760   Length 1  Maximal Subgroups: 2
---
[3]  Order 32     Length 1  Maximal Subgroups: 2
---
[2]  Order 16     Length 1  Maximal Subgroups: 1
---
[1]  Order 1      Length 1  Maximal Subgroups:
```

From the normal lattice of G we find then use the following code to find the largest normal abelian subgroup.

for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if;end for;

1, 2, 3

We have following abelian subgroups of G, 1, 2, and 3. However, since 3 is the largest number we will use 3 as our largest normal abelian subgroup which we will denote as NL[3]. We find the generators of NL[3] is of order 32 and NL[3] = < (1, 6)(2, 3)(4, 29)(5, 24)(7, 27)(8, 19)(9, 28)(10, 14)(11, 26)(12, 22)(13, 16)(15, 20)(17, 18)(21, 32)(23, 31)(25, 30), (1, 15)(2, 25)(3, 30)(4, 7)(5, 28)(6, 20)(8, 13)(9, 24)(10, 18)(11, 12)(14, 17)(16, 19)(21, 23)(22, 26)(27, 29)(31, 32), (1, 10)(2, 5)(3, 24)(4, 21)(6, 14)(7, 23)(8, 11)(9, 30)(12, 13)(15, 18)(16, 22)(17, 20)(19, 26)(25, 28)(27, 31)(29, 32), (1, 2)(3, 6)(4, 8)(5, 10)(7, 13)(9, 17)(11, 21)(12, 23)(14, 24)(15, 25)(16, 27)(18, 28)(19, 29)(20, 30)(22, 31)(26, 32), (1, 26)(2, 32)(3, 21)(4, 24)(5, 29)(6, 11)(7, 9)(8, 14)(10, 19)(12, 20)(13, 17)(15, 22)(16, 18)(23, 30)(25, 31)(27, 28) >. Since NL[3] has five permutations of order 2 we can write NL[3] $\sim 2^5$. We will use magma to confirm that NL[3] is isomorphic to the group permutation 2^5.

IsIsomorphic(NL[3],AbelianGroup(GrpPerm,[2,2,2,2,2]));

True

Next we will investigate q = G/NL[3] and the transversal of (NL[3],G). First q, we find that q is of order 360 is generated by < (1, 2, 3, 5)(4, 6), (2, 4, 5, 3, 6)>. We then will investigate the composition factors of q.

```
    G
    |  Alternating(6)
    1
```

Therefore since the composition factors of q is A_6 we check with magma to confirm that q is isomorphic to A_6.

IsIsomorphic(q,Alt(6));

True

Therefore we find the isomorphism of G to be G $\sim$ (2^5:A_6).

Next we will investigate our N, where our N is of order 1080 and is generated by x $\sim$ (1, 13, 2, 11)(3, 9, 10, 7)(4, 6, 8, 5)(12, 16, 14, 15) and y $\sim$ (1, 4, 15, 10)(2, 5, 9, 16)(6, 11, 7, 14)(8, 12). We will begin by looking at the normal lattice of N

```
Normal subgroup lattice
-----------------------

[3]  Order 1080  Length 1  Maximal Subgroups: 2
---
[2]  Order 3     Length 1  Maximal Subgroups: 1
---
[1]  Order 1     Length 1  Maximal Subgroups:
```

Next will now look for the largest normal abelian subgroup in our normal lattice of N

for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if;end for;

2

We are given that NL[2] is the largest normal abelian subgroup of N. Then NL[2] is generated by the following < (1, 3, 4)(2, 6, 7)(5, 8, 10)(9, 11, 13)(12, 14, 16)(15, 17, 18)> and is of order 3. By looking at the only permutation of NL[2] to be of order 3 we can write NL[2] isomorphic to the group permutation 3. Therefore, we will use the following code in magma to confirm,

IsIsomorphic(NL[2],AbelianGroup(GrpPerm,[3]));

True

Since magma gave us true we can write NL[2] $\sim$ 3. Next we will investigate q = N/NL[2], where q is of order 360 and generated by <(1, 2, 3, 5)(4, 6), (2, 4, 5, 3, 6)>. We will now look at the composition factors of q.

```
G
|  Alternating(6)
1
```

We can see by the composition factor of q that the image q produces is A_6 therefore, we will use magma to confirm if q is isomorphic to A_6.

IsIsomorphic(q,Alt(6));

True

Thus our isomorphism for N is N $\sim$ 3:A_6.

6.8 $\frac{2^{*32}:(2^5:A_5)}{(xy^{-2}xy)^6t_2t_9t_{13}t_{15}t_4t_2} \cong \mathbf{J}_2$

We will find the isomorphism type of our group

Group<x,y,t|x^{10} , y^6 , $(x\ y^{-2}\ x)^2$, $(x\ y^2\ x^2)^2$, $(y^{-1}\ x^{-1})^5$, $(x\ y^2\ x^{-1}\ y^{-1})^2$, $x^{-1}\ y^{-1}\ x^5$ y x^{-4} , y x^{-2} y^{-1} x^3 y x y^3 x^{-1} , t^2, (t,x^2), (t,y^2), $((x\ y^{-2}\ x\ y)t^{(x))^6}$, $(x\ t)^5$ >

where our N is of order 1920. We will proceed by analyzing our normal lattice of our N, where we will look for the largest normal abelian subgroup.

```
Normal subgroup lattice
-----------------------

[4]  Order 1920  Length 1  Maximal Subgroups: 3
---
[3]  Order 32    Length 1  Maximal Subgroups: 2
---
[2]  Order 2     Length 1  Maximal Subgroups: 1
---
[1]  Order 1     Length 1  Maximal Subgroups:
```

We would like to know what is the largest normal abelian subgroup of N. To do so we will use the following code in magma.

for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if;end for;

2

In the normal lattice of N, we have 2 to be the largest normal abelian subgroup, which we will label NL[2]. Our NL[2] is of order 2 and is generated by < Identity, (1, 2)(3, 4)(5, 6)(7, 9)(8, 10)(11, 14)(12, 15)(13, 16)(17, 22)(18, 23)(19, 24)(20,25)(21, 26)(27, 28)(29, 30)(31, 32)>. By the generators of NL[2] we see that there is only one permutation that is of order 2, therefore we can write NL[2] isomorphic to the group permutation 2. However, we need to use magma to confirm that we are correct.

IsIsomorphic(NL[2],AbelianGroup(GrpPerm,[2]));

True

Therefore we can write NL[2] $\sim$ 2. Next will look at our q = N/NL[2], where q is of order 960 and is generated by < (2, 3, 4, 6, 9)(5, 8, 12, 15, 10)(7, 11, 14, 16, 13), (1, 2)(3, 5)(4, 7, 9, 14, 6, 10)(8, 13, 11, 15, 16, 12) >. However, before we continue let us look at the composition factors of q. We have

```
G
|  Alternating(5)

|  Cyclic(2)

|  Cyclic(2)

|  Cyclic(2)

|  Cyclic(2)
1
```

Since there is an A_5 contained in the composition factors of q as well as a string of cyclic(2) we need to investigate q further by looking at the normal lattice of q.

```
Normal subgroup lattice
-----------------------

[3]  Order 960  Length 1  Maximal Subgroups: 2
---
[2]  Order 16   Length 1  Maximal Subgroups: 1
---
[1]  Order 1    Length 1  Maximal Subgroups:
```

Next we need to find the largest normal abelian subgroup of q.

for i in [1..#nl] do if IsAbelian(nl[i]) then i; end if;end for;

2

Since we find that 2 is the highest abelian group of q, we will denote 2 as nl[2]. Now nl[2] is generated by < (1, 2)(3, 5)(4, 14)(6, 7)(8, 15)(9, 10)(11, 12)(13, 16), (1, 3)(2, 5)(4, 8)(6, 16)(7, 13)(9, 11)(10, 12)(14, 15), (1, 4)(2, 14)(3, 8)(5, 15)(6, 12)(7, 11)(9, 13)(10, 16), (1, 6)(2, 7)(3, 16)(4, 12)(5, 13)(8, 10)(9, 15)(11, 14)> since there are four permutations of order 2 in nl[2] we can write nl[2] is isomorphic to the group permutation 2^4. But we need to confirm with magma if this is correct.

IsIsomorphic(nl[2],AbelianGroup(GrpPerm,[2,2,2,2]));

True

Therefore we can write nl[2] $\sim 2^4$. Now notice how A_5 is not apparent in the isomorphism of nl[2] this means we must investigate even further with qq that is the quotient group of q with respects to nl[2]. We find that qq = q/nl[2] is of order 60 and has the following composition factor

```
G
|  Alternating(5)
1
```

To confirm that q is isomorphic to A_5 we use the following code in magma
IsIsomorphic(q,Alt(5));
True
Thus we have the following semi direct product 2^{32}:(2^5:A_5).

6.9 $\frac{2^{*110}:PSL(2,11)}{(xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)^3 t_{49} t_{105} t_{25}} \cong \mathbf{M}_{12}$

We will determine the isomorphism type of the following group
Group<x,y,t | x^2, y^3, $(y^{-1}xyx)^5$, $(xy^{-1})^{11}$,
$(yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2$,
t^2, $(t, xyxyxy^{-1}xy^{-1}x)$,
$(t, xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy)$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)t^{(y^{-1}xyxy^{-1}xy^{-1}xyxyxyx)})^3$ >
where our N is of order 660. We will proceed by analyzing out normal lattice of N, where we will look for the largest possible abelian group.

```
Normal subgroup lattice
-----------------------

[2]  Order 660  Length 1  Maximal Subgroups: 1
---
[1]  Order 1    Length 1  Maximal Subgroups:
```

In our normal lattice we only have two possible choices for our largest normal abelian subgroup, either NL[2] which is 660, our whole N or NL[1] which is our identity. Therefore, our largest normal abelian subgroup in our N, is NL[1]. However, NL[1] is of order 1, thus our N is simple. Therefore, our N is PSL(2, 11).
Next we will do the same process however we want to bring our attention to finding the isomorphism type of our G. Where we have the following normal lattice

```
Normal subgroup lattice
-----------------------

[2]  Order 95040  Length 1  Maximal Subgroups: 1
---
[1]  Order 1      Length 1  Maximal Subgroups:
```

Similar to the situation with our N, we only have two possible choices for the largest normal abelian subgroup. However, the largest possible abelian group for the normal lattice of G is NL[1]. This gives us a good sense that our G is simple. Therefore, our G is M_{12}.

Thus our isomorphism type is PSL(2, 11):M_{12}.

6.10 $\frac{2^{*6}:S_3}{(tt^{(xyxy^{-2}xy)})^2=(xyxy^{-1}xy),(xyt^x)^3} \cong \mathbf{2^3}$:$\mathbf{S_4}$

We factor the progenitor 2^{12}: (S_4:2) by the relations $(tt^{(xyxy^{-2}xy)})^2 = (x\ y\ x\ y^{-1}\ x\ y)$, and $(xyt^x)^3$ and obtain a finite group G. However, the action of G on the cosets of S_4:2=<x,y> is not faithful. Thus, we modify the progenitor to 2^6: S_3 and factor by the two relations given above and obtain the same group G. We will work in magma to produce a faithful kernel of order 1, find the order of x and y, and have the proper index order of 48. Therefore, we start by working with the following progenitor.

Group<x, y, t | x^2 , y^4 , y^{-1} x y^{-2} x y^{-1} x y^2 x , y^{-1} x y^{-1} x y^{-1} x y x y x y x , t^2,(t,(x y x y^{-1} x y)),(t,(x y^2 x)), $(tt^{(xyxy^{-2}xy)})^2 = (x\ y\ x\ y^{-1}\ x\ y)$, $(xyt^x)^3$ >

That has the following information given from magma

Index(G,sub<G|x,y>);

32

f,G1,k:=CosetAction(G,sub<G|x,y>);

CompositionFactors(G1);

```
G
|  Cyclic(2)
*
|  Cyclic(3)
*
|  Cyclic(2)
*
|  Cyclic(2)
```

```
|  Cyclic(2)

|  Cyclic(2)

|  Cyclic(2)
1
```

#k;

8

Order(f(x));

2

Order(f(y));

2

Order(f(xy));

3

#G;

384

#sub<G|x,y>;

48

This progenitor stems from the Transitive Group (12,21) which has an N of order 48, which is exactly 48 = #sub<G|x,y> from our progenitor. We will begin to modify our progenitor by first changing the order of y from y^4 to y^2 and adding $(xy)^3$. We will then check the if our order of G, N, and the kernel have changed in magma.

Group<x, y, t | x^2 , y^2, $(xy)^3$ y^{-1} x y^{-2} x y^{-1} x y^2 x , y^{-1} x y^{-1} x y^{-1} x y x y x y x , t^2,(t,(x y x y^{-1} x y)),(t,(x y^2 x)), $(tt^{(xyxy^{-2}xy)})^2$ = (x y x y^{-1} x y), $(xyt^x)^3$ >;

f,G1,k:=CosetAction(G,sub<G|x,y>);

#k;

1

#G1;

192

#sub<G|x,y>;

6

By changing the order of y and adding the relation $(xy)^3$ into the progenitor our information of order of G, kernel, and N have changed. In the case of our order of G we now have 192 where as previously it was 384, the order of N is now 6 when it was an order of 48, and now our kernel is of order 1 which means that we have a faithful presentation where as we previously had a kernel of order 8.

We will proceed by making a new group, by finding our new N generators which we will label as NNN, in NNN we will input the stabilisers of our original progenitor.

NNN<x,y>:=Group<x, y | x^2, y^2, $(xy)^3$, y^{-1} x y^{-1} x y^{-1} x y x y x y x >;

ff, NNN1, k:=CosetAction(NNN, sub<NNN | (x y x y^{-1} x y)>);

NNN1;

```
Permutation group NNN1 acting on a set of cardinality 6
Order = 6 = 2  3
    (1, 2)(3, 5)(4, 6)
    (1, 3)(2, 4)(5, 6)
```

By using the stabilisers of our original progenitor magma helped us find two new generators which we will use to make a new group. Since the highest number in the generators is 6 we will be working in S_6, we will label the first generator as xx, and the second generator as yy. We will then continue in magma by confirming the order of our N is 6.

S:=Sym(6);

xx:=S!(1, 2)(3, 5)(4, 6);

yy:=S!(1, 3)(2, 4)(5, 6);

N:=sub<S|xx, yy>;

#N

6

We will now check what is our N isomorphic to as well check if the following N,1 stabiliser (x y x y^{-1} x y) applies to our progenitor.

IsIsomorphic(N,Sym(3));

```
true Mapping from: GrpPerm: N to GrpPerm: $, Degree 3, Order 2  3
Composition of Mapping from: GrpPerm: N to GrpPC and
Mapping from: GrpPC to GrpPC and
Mapping from: GrpPC to GrpPerm: $, Degree 3, Order 2  3
```

Stabiliser(N,1) eq sub<N|(xx yy xx yy^{-1} xx yy)>;

true

Next we will find the isomorphism type of our G1 by looking at the normal lattice G1 produces, then we will proceed by looking at its largest normal Abelian subgroup, look at the generators of that abelian group, and then look at our quotient group which we will denote as q = G1/NL[5].

NL:=NormalLattice(G1);

NL;

```
Normal subgroup lattice
-----------------------

[8]  Order 192  Length 1  Maximal Subgroups: 7
---
[7]  Order 96   Length 1  Maximal Subgroups: 6
---
[6]  Order 32   Length 1  Maximal Subgroups: 3 4 5
---
[5]  Order 8    Length 1  Maximal Subgroups: 2
[4]  Order 8    Length 1  Maximal Subgroups: 2
[3]  Order 8    Length 1  Maximal Subgroups: 2
---
[2]  Order 2    Length 1  Maximal Subgroups: 1
---
[1]  Order 1    Length 1  Maximal Subgroups:
```

for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if; end for;

1, 2, 5

NL[5];

```
Permutation group acting on a set of cardinality 32
Order = 8 = 2^3
(1, 26)(2, 15)(3, 22)(4, 29)(5, 19)(6, 14)(7, 8)(9, 17)(10, 31)
 (11, 21)(12, 24)(13, 18)(16, 30)(20, 25)(23, 27)(28, 32)
(1, 32)(2, 30)(3, 25)(4, 21)(5, 18)(6, 31)(7, 23)(8, 27)(9, 12)
 (10, 14)(11, 29)(13, 19)(15, 16)(17, 24)(20, 22)(26, 28)
(1, 31)(2, 25)(3, 30)(4, 19)(5, 29)(6, 32)(7, 24)(8, 12)(9, 27)(10, 26)
 (11, 18)(13, 21)(14, 28)(15, 20)(16, 22)(17, 23)
```

IsIsomorphic(NL[5],AbelianGroup(GrpPerm,[2,2,2]));

True

q,ff:=quo<G1|NL[5]>;

q;

```
Permutation group q acting on a set of cardinality 4
Order = 24 = 2^3  3
    (3, 4)
```

```
    (2, 3)
    (1, 2)(3, 4)
```

IsIsomorphic(q,Sym(4));

```
true Isomorphism of GrpPerm: q, Degree 4, Order 2^3  3
into GrpPerm: $, Degree 4, Order 2^3
 3 induced by
    (3, 4) |--> (3, 4)
    (2, 3) |--> (2, 3)
    (1, 2)(3, 4) |--> (1, 2)(3, 4)
```

Thus our N is isomorphic to S_3 and our G1 is isomorphic to 2^3:S_4. Therefore we are left with the following progenitor.

S:=Sym(6);

xx:=S!(1, 2)(3, 5)(4, 6);

yy:=S!(1, 3)(2, 4)(5, 6);

N:=sub<S|xx, yy>;

#N

6

G<x, y, t>:=Group<x, y, t | x^2 , y^2 , $(xy)^3$, t^2, (t,(x y x y^{-1} x y)), $(tt^{(x)})^2$ = (x y x y^{-1} x y), $(xyt^x)^3$ >;

f,G1,k:=CosetAction(G,sub<G|x,y>);

#k;

1

#sub<G|x,y>;

6

CompositionFactors(G1);

```
    G
    |  Cyclic(2)
    
    |  Cyclic(3)
    
    |  Cyclic(2)
    
    |  Cyclic(2)
    
    |  Cyclic(2)
```

```
|  Cyclic(2)

|  Cyclic(2)
1
```

Now the action of G on the cosets of S_3 = <x,y> is faithful.

6.11 $\frac{2^{*6}:S_3}{(tt^{(xyxy^{-2}xy)})^2=(xyxy^{-1}xy),(xt)^2,(xyt^x)^5} \cong \mathbf{2 \times A_5}$

We factor the progenitor 2^{12}: (S_4:2) by the relations $(tt^{(xyxy^{-2}xy)})^2$ = (x y x y^{-1} x y), $(xt)^2$, and $(xyt^x)^5$ and obtain a finite group G. However, the action of G on the cosets of S_4:2 = <x, y> is not faithful. Thus, we modify the progenitor to 2^6: 2^3 and factor by two relations given above and obtain the same group G. We will work with a progenitor that stemmed from transitive group(12,21) that has an N of order 48. We will use magma to find what is the order of sub<G|x,y> = |N|, the index of G, order of the kernel, the order of f(x), f(y), and f(xy), and lastly the composition factors of G.

G<x,y,t>:=Group<x,y,t | x^2 , y^4 , y-1 x y^{-2} x y^{-1} x y^2 x , y^{-1} x y^{-1} x y^{-1} x y x y x y x , t^2, (t,(x y x y^{-1} x y)), (t,(x y^2 x)), $(tt^{(xyxy^{-2}xy)})^2$ = (x y x y^{-1} x y), $(xt)^2$, $(xyt^x)^5$ >;

Index(G,sub<G|x,y>);

20

f,G1,k:=CosetAction(G,sub<G|x,y>);

#k;

1

#sub<G|x,y>;

6

Order(f(x));

2

Order(f(y));

2

Order(f(xy));

3

CompositionFactors(G1);

```
G
|  Alternating(5)
```

```
    |  Cyclic(2)
    1
```

Next we will apply the information we found from the order of f(x), f(y), and f(xy) into our progenitor, then we will recheck the order of the kernel, N, and G1 to check if we have a faithful group of k = 1 and if so we will continue with looking for NNN1 which gives the generators of our new group.

G<x,y,t>:=Group<x,y,t | x^2 , y^2 , $(xy)^3$, y-1 x y^{-2} x y^{-1} x y^2 x , y^{-1} x y^{-1} x y^{-1} x y x y x y x , t^2, (t,(x y x y^{-1} x y)), (t,(x y^2 x)), $(tt^{(xyxy^{-2}xy)})^2$ = (x y x y^{-1} x y), $(xt)^2$, $(xyt^x)^5$ >;

f,G1,k:=CosetAction(G,sub<G|x,y>);

#k;

1

#G;

120

Index(G,sub<G|x,y>);

20

Now the action of G on the cosets of N = <x,y> is faithful.

Next looking for NNN

NNN<x,y>:=Group<x,y|x^2,y^2,$(xy)^3$,y^{-1} x y^{-2} x y^{-1} x y^2 x,y^{-1} x y^{-1} x y^{-1} x y x y x y x >;

ff,NNN1,k:=CosetAction(NNN,sub<NNN|(x y x y^{-1} x y),(x y^2 x)>);

NNN1;

```
Permutation group NNN1 acting on a set of cardinality 6
Order = 6 = 2  3
    (1, 2)(3, 5)(4, 6)
    (1, 3)(2, 4)(5, 6)
```

Thus we begin to write our new group. NN1 gives us two generators, which we will denote as xx and yy respectively, next we will be working under Sym(6) due to the highest number in the generators is 6. Thus we have the following information.

S:=Sym(6);

xx:=S!(1, 2)(3, 5)(4, 6);

yy:=S!(1, 3)(2, 4)(5, 6);

N:=sub<S|xx,yy>;

```
CompositionFactors(N);
/ G
    |  Cyclic(2)

    |  Cyclic(3)
    1
```

Next we will look for the stabiliser of N in respect to 1, then we will find the isomorphism type of N.

Stabiliser(N,1);

```
Permutation group acting on a set of cardinality 6
Order = 1
```

IsIsomorphic(N,Sym(3));

```
true Mapping from: GrpPerm: N to GrpPerm: $, Degree 3, Order 2  3
Composition of Mapping from: GrpPerm: N to GrpPC and
Mapping from: GrpPC to GrpPC and
Mapping from: GrpPC to GrpPerm: $, Degree 3, Order 2  3
```

Next we want to find the isomorphism type of G1, to do so we will look at the normal lattice of G1, find the largest normal abelian subgroup, and then look at our quotient group which we will denote as q = G1/NL[2].

NL:=NormalLattice(G1);

NL;

```
Normal subgroup lattice
-----------------------

[4]  Order 120  Length 1  Maximal Subgroups: 2 3
---
[3]  Order 60   Length 1  Maximal Subgroups: 1
---
[2]  Order 2    Length 1  Maximal Subgroups: 1
---
[1]  Order 1    Length 1  Maximal Subgroups:
```

for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if; end for;

1, 2

NL[2];

```
Permutation group acting on a set of cardinality 20
Order = 2
Id($)
(1, 20)(2, 19)(3, 16)(4, 18)(5, 13)(6, 15)(7, 10)(8, 17)(9, 12)(11, 14)
```

T:=Transversal(G1,NL[2]);

q,ff:=quo<G1|NL[2]>;

q;

```
Permutation group q acting on a set of cardinality 6
Order = 60 = 2^2  3  5
    (1, 2)(5, 6)
    (2, 3)(4, 5)
    (3, 4)(5, 6)
```

CompositionFactors(q);

```
    G
    |  Alternating(5)
    1
```

IsIsomorphic(q,Alt(5));

True

Thus we have our N isomorphic to S_3 and our G isomorphic to $2\times A_5$.

6.12 $\frac{2^{*6}:A_6}{(((xy)^2)t^{(xyx^{-1}y^{-1}xy^{-2}x)})^{10},((x^{-3}y)t^{(x^{-1}y^{-1}x^{-1}y^{-1}x^{-1})})^{10}} \cong \mathbf{2^5{:}A_6}$

We will work with a progenitor from $3{:}A_6$ where $3{:}A_6$ has an N of order 1080. with the progenitor given below we will find the order of sub<G|x,y> also known as N, the order of k, the order of G, composition factors of G, and find how many double cosets G has in respects to N.

G<x,y,t>:=Group<x,y,t| y^5 ,x^4, y^{-2} x^3 y^{-2} x^{-1} , (x $y^{-1})^4$, y^{-1} x y^{-1} x^{-1} y^{-1} x y x y^{-1} x^{-1}, t^2, (t,y^{-1} x^{-1} y x y^{-1} x^{-2}), (t,(x^{-1} $y^2)^2$), (t,(x y $x)^2$), ((x y x y^{-2} x)$t^{(xyxyx^2yx^{-1})}t^{((xyx^{-1})\ 2)})^3$, (((x $y)^2$)$t^{(xyx^{-1}y^{-1}xy^{-2}x)})^{10}$, (($x^{-3}$ y)$t^{(x^{-1}y^{-1}x^{-1}y^{-1}x^{-1})})^{10}$ >;

f,G1,k:=CosetAction(G,sub<G|x,y>);

#k;

1

CompositionFactors(G1);

```
G
|  Cyclic(2)

|  Alternating(6)

|  Cyclic(2)

|  Cyclic(2)

|  Cyclic(2)

|  Cyclic(2)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

4

#G;

11520

#sub<G|x,y>;

360

Order(f(x));

4

Order(f(y));

5

Order(f(xy));

5

We will then incorporate the orders of f(x), f(y), and f(xy) into our progenitor and then proceed to check the Index of G in respects to N, the order of sub<G|x,y>, if we get the order of sub<G|x,y> to equal 360 we will proceed to find NNN1, which will give us generators of our new group.

G<x,y,t>:=Group<x,y,t| y^5 ,x^4, $(x,y)^5$, $y^{-2}\ x^3\ y^{-2}\ x^{-1}$, $(x\ y^{-1})^4$, $y^{-1}\ x\ y^{-1}\ x^{-1}\ y^{-1}\ x\ y\ x\ y^{-1}\ x^{-1}$, t^2, $(t,y^{-1}\ x^{-1}\ y\ x\ y^{-1}\ x^{-2})$, $(t,(x^{-1}\ y^2)^2)$, $(t,(x\ y\ x)^2)$, $((x\ y\ x\ y^{-2}\ x)t^{(xyxyx^2yx^{-1})}t^{((xyx^{-1})\ 2)})^3$, $(((x\ y)^2)t^{(xyx^{-1}y^{-1}xy^{-2}x)})^{10}$, $((x^{-3}\ y)t^{(x^{-1}y^{-1}x^{-1}y^{-1}x^{-1})})^{10}$ >;

f,G1,k:=CosetAction(G,sub<G|x,y>);

#k;

1

Index(G,sub<G|x,y>);

32

#sub<G|x,y>;

360

Now the action of G on the cosets of A_6 = <x, y> is faithful since #k = 1.

Next we proceed in finding NNN

NNN<x,y>:=Group<x,y| y^5 ,x^4, $(x,y)^5$, y^{-2} x^3 y^{-2} x^{-1} , $(x\ y^{-1})^4$, y^{-1} x y^{-1} x^{-1} y^{-1} x y x y^{-1} x^{-1} >;

#NNN;

360

ff,NNN1,k:=CosetAction(NNN,sub<NNN| (y^{-1} x^{-1} y x y^{-1} x^{-2}), $(x^{-1}\ y^2)^2$,$(x\ y\ x)^2$ >);

NNN1;

```
Permutation group NNN1 acting on a set of cardinality 6
Order = 360 = 2^3  3^2  5
    (1, 2)(3, 4, 6, 5)
    (1, 3, 5, 2, 4)
```

Since NNN1 gave us two generators we will label them as xx and yy, respectively. Also since the highest number in the generators is 6, we will work with Sym(6) in our new group. Then we will proceed to find the isomorphism type of our N by looking at the normal lattice of N then largest normal abelian subgroup of N.

S:=Sym(6);

xx:=S!(1, 2)(3, 4, 6, 5);

yy:=S!(1, 3, 5, 2, 4);

N:=sub<S|xx,yy>;

CompositionFactors(N);

```
    G
    |  Alternating(6)
    1
```

NL:=NormalLattice(N);

NL;

```
Normal subgroup lattice
-----------------------
```

```
[2]  Order 360  Length 1  Maximal Subgroups: 1
---
[1]  Order 1    Length 1  Maximal Subgroups:
```

for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if;end for;

1

NL[1];

```
Permutation group acting on a set of cardinality 6
Order = 1
```

IsSimple(N);

True

Since N is simple that means we will consider its isomorphism type to be its composition factor image which is A_6. Next we will look for the isomorphism type of G to do so we will look at the normal lattice of G.

NL:=NormalLattice(G1);

NL;

```
Normal subgroup lattice
-----------------------

[5]  Order 11520  Length 1  Maximal Subgroups: 3 4
---
[4]  Order 5760   Length 1  Maximal Subgroups: 2
---
[3]  Order 32     Length 1  Maximal Subgroups: 2
---
[2]  Order 16     Length 1  Maximal Subgroups: 1
---
[1]  Order 1      Length 1  Maximal Subgroups:
```

Then we will look for the largest normal abelian subgroup of G by doing the following.

for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if;end for;

1, 2, 3

We find that 3 is the largest normal abelian subgroup of G, therefore we will denote 3 as NL[3] and look for the generators it produces.

NL[3];

```
Permutation group acting on a set of cardinality 32
Order = 32 = 2^5
Id($)
(1, 6)(2, 3)(4, 29)(5, 24)(7, 27)(8, 19)(9, 28)(10, 14)
(11, 26)(12, 22)(13, 16)(15, 20)(17, 18)(21, 32)(23, 31)(25, 30)
(1, 15)(2, 25)(3, 30)(4, 7)(5, 28)(6, 20)(8, 13)(9, 24)(10, 18)(11, 12)
(14, 17)(16, 19)(21, 23)(22, 26)(27, 29)(31, 32)
(1, 10)(2, 5)(3, 24)(4, 21)(6, 14)(7, 23)(8, 11)(9, 30)(12, 13)
(15, 18)(16, 22)(17, 20)(19, 26)(25, 28)(27, 31)(29, 32)
(1, 2)(3, 6)(4, 8)(5, 10)(7, 13)(9, 17)(11, 21)(12, 23)(14, 24)
(15, 25)(16, 27)(18, 28)(19, 29)(20, 30)(22, 31)(26, 32)
(1, 26)(2, 32)(3, 21)(4, 24)(5, 29)(6, 11)(7, 9)(8, 14)(10, 19)
(12, 20)(13, 17)(15, 22)(16, 18)(23, 30)(25, 31)(27, 28)
```

Next we can see that NL[3] has five permutations each of order 2, therefore we can write it NL[3] $\sim 2^5$. However, we need to use the following code in magma to confirm if NL[3] $\sim 2^5$.

IsIsomorphic(NL[3],AbelianGroup(GrpPerm,[2,2,2,2,2]));

True

Since we get true to NL[3] being isomorphic to the group permutation we can proceed by analyzing q = G1/NL[3].

q,ff:=quo<G1|NL[3]>;

T:=Transversal(G1,NL[3]);

q;

```
Permutation group q acting on a set of cardinality 6
Order = 360 = 2^3  3^2  5
    (1, 2, 3, 5)(4, 6)
    (2, 4, 5, 3, 6)
    Id(q)
```

Since q has two generators and is of order 360 we will investigate its composition factors.

CompositionFactors(q);

```
    G
    |  Alternating(6)
    1
```

We get the composition factor of q to be A_6 we will confirm with magma that q is isomorphic to a_6.

IsIsomorphic(q,Alt(6));

True

Thus we have our N which is isomorphic to A_6 and our G which is isomorphic to $2^5{:}A_6$.

Chapter 7

Monomial Representations

7.1 $\mathbf{7^{*5}{:}_m(A_5)}$

Let H = $((2^2){:}(3))$ = <(1, 7)(2, 10)(3, 9)(5, 6), (1, 6, 5)(2, 3, 8)(4, 10, 9)> where the order of $((2^2){:}(3))$ is 12. Then H′ = (2^2) = < 1, (1, 6)(2, 10)(4, 8)(5, 7), (1, 7)(2, 10)(3, 9)(5, 6) > and the order of (2^2) is 4. Then $((2^2){:}(3))/(2^2)$ = H/H′ = { e, H′(1, 2, 3) }. The classes of $((2^2){:}(3))$ are:

C_1 = {e}

C_2 = {(1, 6)(2, 10)(4, 8)(5, 7), (1, 5)(3, 9)(4, 8)(6, 7), (1, 7)(2, 10)(3, 9)(5, 6)}

C_3 = {(1, 7, 6)(2, 9, 8)(3, 4, 10), (1, 5, 7)(2, 9, 4)(3, 8, 10), (2, 3, 4)(5, 6, 7)(8, 10, 9), (1, 6, 5)(2, 3, 8)(4, 10, 9)}

C_4 = {(1, 6, 7)(2, 8, 9)(3, 10, 4), (2, 4, 3)(5, 7, 6)(8, 9, 10), (1, 7, 5)(2, 4, 9)(3, 10, 8), (1, 5, 6)(2, 8, 3)(4, 9, 10)}

We know that the character table of H/H′ is given by:

Character Table			
	e	a=T[2]=H′(ba^{-1}bab)	a^2=b=T[3]=H′a
X.1	1	1	1
X.2	1	w	w^2
X.3	1	w^2	w

We lift the characters of H/H′ to H. To calculate the lift of X of character X of H/H′, we note that
X(C[1][3]) = X(1)=1,
X(C[2][3]) = X(1)=1,
X(C[3][3]) = XT[2],
X(C[4][3]) = XT[3]

Lifted-Character Table				
Class Representative	H′	H′=(ba^{-1}bab)	H′(a)	H′(a^{-1})
X.1	1	1	1	1
X.2	1	1	w	w^2
X.3	1	1	w^2	w

Thus X.1,X.2,X.3 are irreducible characters of H since X.1,X.2,X.3 are irreducible characters in H/H′.
We consider G = A_5 = <(1, 6, 5)(2, 3, 8)(4, 10, 9),(1, 9)(2, 7)(3, 10)(6, 8)>. We want to find the monomial representative of A_5, if possible. In order to see this we first have to look at the character table of A_5.

```
 Character Table of Group G
---------------------------
Class |   1  2  3    4    5
Size  |   1 15 20   12   12
Order |   1  2  3    5    5
---------------------------
p  =  2   1  1  3    5    4
p  =  3   1  2  1    5    4
p  =  5   1  2  3    1    1
---------------------------
X.1   +   1  1  1    1    1
X.2   +   3 -1  0   Z1 Z1#2
X.3   +   3 -1  0 Z1#2   Z1
X.4   +   4  0  1   -1   -1
X.5   +   5  1 -1    0    0
```

Now the character table of A_5 has characters whose degree is greater than one. It should be noted that all characters of degree larger than one have degree three, four, and five. Since there are two characters of degree three, once character of degree four, and one character of degree five, it is possible for A_5 to have four different monomial representations. All of these will be irreducible monomial representations. So we need to determine which of the five irreducible characters of A_5 are faithful. We note that there is one faithful characters namely X.5, where X.5 = (5, 1, -1, 0, 0). In order for A_5 to have an irreducible faithful monomial representation, A_5 is required to have a subgroup of H to the index five in A_5 and that such H must have a linear character that induces up to the character X.5 of A_5.

We find H = <(1, 2, 5)(3, 8, 6)(4, 9, 7), (1, 8)(2, 6)(4, 10)(7, 9), (1, 8)(3, 5)(4, 9)(7, 10)> has an index five in A_5 and that of H induces up to the character X.5 of A_5.

Therefore, A_5 has a faithful irreducible monomial representations of degree five. The field entries of the representation is determined by the character values ϕ of H that is being induced, which brings our focus to (1, 1, -$\mathbb{Z}_{33}$ - 1, $\mathbb{Z}_{33}$) has $\mathbb{Z}_3$ since the character values are three roots of unity, the field of entries is the cycloatomic field of 3rd roots of unity. Thus, explicitly this representation is

$\rho : G \longrightarrow GL(C)$, where C=CyclotomicField(3),

ρ (xx) =

$T[1]xxT[1]^{-1}\ T[1]xxT[2]^{-1}\ T[1]xxT[3]^{-1}\ T[1]xxT[4]^{-1}\ T[1]xxT[5]^{-1}\ T[1]xxT[6]^{-1}T[1]xxT[7]^{-1}$
$T[2]xxT[1]^{-1}\ T[2]xxT[2]^{-1}\ T[2]xxT[3]^{-1}\ T[2]xxT[4]^{-1}\ T[2]xxT[5]^{-1}\ T[2]xxT[6]^{-1}T[2]xxT[7]^{-1}$
$T[3]xxT[1]^{-1}\ T[3]xxT[2]^{-1}\ T[3]xxT[3]^{-1}\ T[3]xxT[4]^{-1}\ T[3]xxT[5]^{-1}\ T[3]xxT[6]^{-1}T[3]xxT[7]^{-1}$
$T[4]xxT[1]^{-1}\ T[4]xxT[2]^{-1}\ T[4]xxT[3]^{-1}\ T[4]xxT[4]^{-1}\ T[4]xxT[5]^{-1}\ T[4]xxT[6]^{-1}T[4]xxT[7]^{-1}$
$T[5]xxT[1]^{-1}\ T[5]xxT[2]^{-1}\ T[5]xxT[3]^{-1}\ T[5]xxT[4]^{-1}\ T[5]xxT[5]^{-1}\ T[5]xxT[6]^{-1}T[5]xxT[7]^{-1}$

ρ (yy) =

$T[1]yyT[1]^{-1}\ T[1]yyT[2]^{-1}\ T[1]yyT[3]^{-1}\ T[1]yyT[4]^{-1}\ T[1]yyT[5]^{-1}\ T[1]yyT[6]^{-1}T[1]yyT[7]^{-1}$
$T[2]yyT[1]^{-1}\ T[2]yyT[2]^{-1}\ T[2]yyT[3]^{-1}\ T[2]yyT[4]^{-1}\ T[2]yyT[5]^{-1}\ T[2]yyT[6]^{-1}T[2]yyT[7]^{-1}$
$T[3]yyT[1]^{-1}\ T[3]yyT[2]^{-1}\ T[3]yyT[3]^{-1}\ T[3]yyT[4]^{-1}\ T[3]yyT[5]^{-1}\ T[3]yyT[6]^{-1}T[3]yyT[7]^{-1}$
$T[4]yyT[1]^{-1}\ T[4]yyT[2]^{-1}\ T[4]yyT[3]^{-1}\ T[4]yyT[4]^{-1}\ T[4]yyT[5]^{-1}\ T[4]yyT[6]^{-1}T[4]yyT[7]^{-1}$
$T[5]yyT[1]^{-1}\ T[5]yyT[2]^{-1}\ T[5]yyT[3]^{-1}\ T[5]yyT[4]^{-1}\ T[5]yyT[5]^{-1}\ T[5]yyT[6]^{-1}T[5]yyT[7]^{-1}$

where G=<xx,yy> and G= HT[1] $\cup$ HT[2] $\cup$ HT[3] $\cup$ HT[4] $\cup$ HT[5]

$$A = \begin{bmatrix} \mathbb{Z} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\mathbb{Z}_3 - 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbb{Z}_3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\mathbb{Z}_3 - 1 & 0 & 0 \end{bmatrix}$$

where A= ρ(xx) and B=ρ(yy).

Therefore the generators of the faithful irreducible monomial representation are A and B. Now the smallest finite field that contains the third root of unity (elements of order 7) is $\mathbb{Z}_7$. The elements of order three in $\mathbb{Z}_7$ are generators of $\mathbb{Z}_7$-{0}.

These elements of order 3 are 2 and 4.

$$2^1 = 2 \qquad\qquad 2^2 = 4 \tag{7.1}$$

We now find a permutation representation of our monomial representation. we denote the permutation representative of A, B and Axx, Byy respectively. Our progenitor is: 7^{*5}:A_5. We have five t_i's: t_1, t_2, t_3, t_4, t_5. We now will interpret the automorphisms given by the two matrices A and B by using the formula $a_{ij} \Leftrightarrow t_i \to t_j$.

First we will consider the A matrix,

$$A = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

So,

$a_{11} = 4$

$a_{23} = 1$

$a_{34} = 1$

$a_{42} = 1$

$a_{55} = 2$

Then we have,

$t_1 \to t_1^4$

$t_2 \to t_3$

$t_3 \to t_4$

$t_4 \to t_2$

$t_5 \to t_5^2$

We have three distinct powers of each of the five t_i's and to simplify we will use modulo 7. We label t_1, t_2, t_3, t_4, t_5 by 1, 2, 3, 4, 5 respectively. Apply $t_1 \to t_1^4$ and $t_2 \to t_3$ to form the permutation.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
t_1	t_2	t_3	t_4	t_5	t_1^2	t_2^2	t_3^2	t_4^2	t_5^2	t_1^3	t_2^3	t_3^3	t_4^3	t_5^3	t_1^4	t_2^4	t_3^4	t_4^4	t_5^4	t_1^5	t_2^5	t_3^5	t_4^5	t_5^6	t_1^6	t_2^6	t_3^6	t_4^6	t_5^6
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
16	3	4	2	10	1	8	9	7	20	21	13	14	12	30	6	18	19	17	5	26	23	24	22	15	11	28	29	27	25

Therefore, Axx = (1, 16, 6)(2, 3, 4)(5, 10, 20) (7, 8, 9)(11, 21, 26)(12, 13, 14)(15, 30, 25) (17, 18, 19)(22, 23, 24)(27, 28, 29)

Next we consider

$$B=\begin{bmatrix}0&1&0&0&0\\1&0&0&0&0\\0&0&0&0&4\\0&0&0&1&0\\0&0&2&0&0\end{bmatrix}$$

a_{11} =1

a_{21} = 1

a_{35} = 4

a_{44} = 1

a_{53} = 2

Then we have,

$t_1 \rightarrow t_2$

$t_2 \rightarrow t_1$

$t_3 \rightarrow t_5^4$

$t_4 \rightarrow t_4$

$t_5 \rightarrow t_3^2$

We label t_1, t_2, t_3, t_4, t_5 by 1, 2, 3, 4, 5 respectively. Apply $t_1 \rightarrow t_2$, $t_2 \rightarrow t_1$ to form the permutation. Thus the permutation for

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
t_1	t_2	t_3	t_4	t_5	t_1^2	t_2^2	t_3^2	t_4^2	t_5^2	t_1^3	t_2^3	t_3^3	t_4^3	t_5^3	t_1^4	t_2^4	t_3^4	t_4^4	t_5^4	t_1^5	t_2^5	t_3^5	t_4^5	t_5^6	t_1^6	t_2^6	t_3^6	t_4^6	t_5^6
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
2	1	20	4	8	7	6	5	9	18	12	11	25	14	28	17	16	10	19	3	22	21	30	24	13	27	26	15	29	23

Byy = (1, 2)(3, 20)(5, 8)(6, 7)(10, 18)(11, 12)(13, 25)(15, 28)(16, 17)(21, 22)(23, 30)(26, 27).

Now we want to find a symmetric presentation for our progenitor. The stabilizer (N, { 1, 6, 11, 16, 21, 26 } where, $<t_1> = \{t_1, t_1^2, ..., t_1^6\}$ and $\{g \in N \mid <t>^g = <t>\}$ is called the normalizer of <t> in G. This tells us the number of different conjugates of $<t_1>$ is five. Thus, our presentation of the progenitor 7^{*5}:A_5 is

$<x,y,t \mid x^3$, y^2 ,

$(yx^{-1})^5$,

t^7,

$(t,(yx^{-1}yxy))$,

(t,($xyx^{-1}yxy^{-1}x^{-1}$)),

t$^{(xyx^{-1}yxy^{-1}x^{-2})}$=t^4 >

where we have $7^{*5}:_m:(A_5) \cong 7^{*5}:(A_5)$.

We check using Grindstaff's lemma:

<x,y,t | x^3 ,

y^2 ,

(yx^{-1})^5 ,

t^7,

(t,(yx^{-1}yxy)),

(t,($xyx^{-1}yxy^{-1}x^{-1}$)),

t$^{(xyx^{-1}yxy^{-1}x^{-2})}$=t^4,

(t,t$^{(x^{-1}yxyx^{-1}yx^{-1})}$),

(t,t$^{(yx)}$),

(t,t$^{(x^-1yxyx^-1yx)}$),

(t,t$^{(x^{-1}yxyx)}$),

(t$^{(x^{-1}yxyx^{-1}yx^{-1})}$,t$^{(yx)}$),

(t$^{(x^{-1}yxyx^{-1}yx^{-1})}$,t$^{(x^{-1}yxyx^{-1}yx)}$),

(t$^{(x^{-1}yxyx^{-1}yx^{-1})}$,t$^{(x^{-1}yxyx)}$),

(t$^{(yx)}$,t$^{(x^{-1}yxyx^{-1}yx)}$),

(t$^{(yx)}$,t$^{(x^{-1}yxyx)}$),

(t$^{(x^{-1}yxyx^{-1}yx)}$,t$^{(x^{-1}yxyx)}$);

Index(G,sub<G|x,y>);

16807

♯N;

60

Next we want to find our first order relations and to do so we must investigate the five conjugacy classes of PSL(5,7) . In the table below we list the conjugacy classes and their representatives.

Conjugacy Classes and First Order Relations		
Class	Class Representative	Elements of the form πt_i
2	(xyx^{-1}) = (1, 9)(2, 5)(4, 16)(6, 19)(7, 10)(11, 29)(12, 15)(14, 21)(17, 20)(22, 25)(24, 26)(27, 30)	$(xyx^{-1})t_5$, $(xyx^{-1})t_6$, $(xyx^{-1})t_7$, $(xyx^{-1})t$
3	x = (1, 6, 16)(2, 3, 4)(5, 20, 10)(7, 8, 9)(11, 26, 21)(12, 13, 14)(15, 25, 30)(17, 18, 19)(22, 23, 24)(27, 28, 29)	xt_7, xt, xt_2, xt_4
4	(xy) = (1, 7, 5, 3, 4)(2, 20, 18, 19, 16)(6, 17, 10, 8, 9)(11, 27, 15, 13, 14)(12, 25, 23, 24, 21)(22, 30, 28, 29, 26)	(xy)t, $(xy)t_3$
5	$((xy)^2)$ = (1, 5, 4, 7, 3)(2, 18, 16, 20, 19)(6, 10, 9, 17, 8)(11, 15, 14, 27, 13)(12, 23, 21, 25, 24)(22, 28, 26, 30, 29)	$((xy)^2)t$, $((xy)^2)t_3$

7.2 $5^{*6}:_m(S_5)$

Let H = (5:4) = <(1, 4, 3, 2)(5, 7, 10, 8)(6, 9), (1, 3)(2, 4)(5, 10)(7, 8), (1, 5, 9, 10, 3)(2, 7, 8, 4, 6)> where the order of H′ (5:4) is 20. Then consider (5:5) = < 1, (1, 3, 10, 9, 5)(2, 6, 4, 8, 7)> where the order of (5:5) is 5. Then (5:4)/(5:5) = {(1, 2)(3, 4), (1, 3, 2, 4), (1, 2)(3, 4)}. The classes of (5:4) are:

C_1 = {e}

C_2 = {(1, 9)(2, 7)(3, 10)(6, 8), (2, 6)(3, 5)(4, 7)(9, 10), (1, 10)(2, 8)(4, 6)(5, 9), (1, 3)(2, 4)(5, 10)(7, 8), (1, 5)(3, 9)(4, 8)(6, 7)}

C_3 ={(1, 6, 5, 7)(2, 10)(3, 8, 9, 4), (1, 2, 3, 4)(5, 8, 10, 7)(6, 9), (1, 4, 10, 6)(2, 9, 8, 5)(3, 7), (1, 8)(2, 5, 6, 3)(4, 9, 7, 10), (1, 7, 9, 2)(3, 6, 10, 8)(4, 5)}

C_4 = {(1, 2, 9, 7)(3, 8, 10, 6)(4, 5), (1, 6, 10, 4)(2, 5, 8, 9)(3, 7), (1, 8)(2, 3, 6, 5)(4, 10, 7, 9), (1, 7, 5, 6)(2, 10)(3, 4, 9, 8), (1, 4, 3, 2)(5, 7, 10, 8)(6, 9)}

C_5= {(1, 5, 9, 10, 3)(2, 7, 8, 4, 6), (1, 10, 5, 3, 9)(2, 4, 7, 6, 8), (1, 3, 10, 9, 5)(2, 6, 4, 8, 7), (1, 9, 3, 5, 10)(2, 8, 6, 7, 4)}

Character Table				
	e=H′a^0	T[2]=H′a	T[3]=H′a^2	T[4]=H′a^3
X.1(i^0)	1	1	1	1
X.2(i^1)	1	i	(i^2)=-1	i^3=-i
X.3(i^2)	1	i^2=-1	(i^2)^2=i^4=1	(i^2)^3=i^6=-1
X.4	1	i^3=-i	(i^3)^2=i^6=-1	(i^3)^3=i^9=i

Then we are able to consider the character table of (5:4)/(5:5) to be given by:
We then lift the characters of (5:4)/(5:5) to (5:4). To calculate the lift of X of character X of (5:4)/(5:5), we note that
X(C[1][3]) = X(1)=1,
X(C[2][3]) = XT[2],
X(C[3][3]) = XT[2],
X(C[4][3]) = XT[4],
X(C[5][3]) = X(1)=1,

Lifted-Character Table					
Class Rep	H′e=a^0	H′T[2]=(bab^2)	H′T[3]=(a^{-2}b^{-1})	H′T[4]=(ba^2)	H′T[5]=(ab^{2}a)
X.1(i^0)	1	1	1	1	1
X.2(i^1)	1	i	-1	-i	1
X.3(i^2)	1	-1	1	-1	1
X.4(i^3)	1	-i	-1	i	1

Therefore X.1, X.2, X.3, X.4 are irreducible characters of H since X.1, X.2, X.3, X.4 are irreducible characters in H/H′.
We will consider G = S_5 = <(2, 6, 4, 5, 3, 7)(8, 10, 9), (1, 7, 4)(2, 6, 9, 3, 5, 10)> we will find a possible monomial representative of G = S_5. To do so we must first investigate the character table of S_5.

```
    Character Table of Group G
--------------------------
```

Class		1	2	3	4	5	6	7
Size		1	10	15	20	30	24	20
Order		1	2	2	3	4	5	6
p = 2		1	1	1	4	3	6	4
p = 3		1	2	3	1	5	6	2
p = 5		1	2	3	4	5	1	7
X.1	+	1	1	1	1	1	1	1
X.2	+	1	-1	1	1	-1	1	-1
X.3	+	4	-2	0	1	0	-1	1
X.4	+	4	2	0	1	0	-1	-1
X.5	+	5	1	1	-1	-1	0	1
X.6	+	5	-1	1	-1	1	0	-1
X.7	+	6	0	-2	0	0	1	0

The character table of S_5 has characters whose degree is larger than one. All characters greater than one are X.3 and X.4 with degree four, X.5 and X.6 with degree five, X.7 with degree six. This tells us that is is possible for S_5 to have three different monomial representations. Therefore, we need to determine which of the five irreducible characters of S_5 are faithful. We have one faithful character X.7, where X.7= (6, 0, -2, 0, 0, 1, 0). In order for S_5 to have an irreducible faithful monomial representation, S_5 is required to have a subgroup of H to the index six in S_5 that such H must have a linear character that induces up to character X.7 of S_5.

We find H = <(1, 9)(2, 7)(3, 10)(6, 8), (1, 4, 10, 6)(2, 9, 8, 5)(3, 7), (1, 9)(2, 7)(3, 10)(6, 8)> has an index six in S_5 and that such character is $\phi(3)$ of H and induces up to character X.7 of S_5.

Then S_5 has a faithful irreducible monomial representation of degree six. The field entries of the representation is determined by the character ϕ of H that is being induced, which then brings our focus towards (1, -1, $-\mathbb{Z}(4)_4$, $\mathbb{Z}(4)_4$, 1) since the values are four roots od unity, the field entries is the cycloatomic field of 4th root of unity. Thus the representation is

$\rho : S_5 \longrightarrow GL(C)$, where C=CyclotomicField(4),

where S_5 =<xx, yy> and $S_5 = HT[1] \cup HT[2] \cup HT[3] \cup HT[4] \cup HT[5]$

$$A = \begin{bmatrix} \mathbb{Z}_4 & 0 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathbb{Z}_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbb{Z}_4 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \mathbb{Z} & 0 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -\mathbb{Z}_4 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & \mathbb{Z}_4 & 0 & 0 \end{bmatrix}$$

where A= ρ(xx) and B=ρ(yy).

Therefore the generators of the faithful irreducible monomial representation are A and B. Now the smallest finite field that contains the fourth root of unity (elements of order 5) is $\mathbb{Z}_5$.

We will now find a permutation representation of our monomial representation. we denote the permutation representative of A, B and Axx, Byy respectively. Our progenitor is: 5^{*6}:S_5. We have five t_i's: t_1, t_2, t_3, t_4, t_5, t_6. We now will interpret the automorphisms given by the two matrices A and B by using the formula $a_{ij} \Leftrightarrow t_i \rightarrow t_j$.

First we will consider the A matrix,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & & 0 & -i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a_{12} = 1

a_{23} = -i

a_{36} = i

a_{45} = -1

a_{53} = -1

$a_{6,1}$ = 1

Then we have,

$t_1 \rightarrow t_2$

$t_2 \rightarrow t_3^{-i}$

$t_3 \rightarrow t_6^{i}$

$t_4 \rightarrow t_5^{-1}$

$t_5 \rightarrow t_3^{-1}$

$t_6 \rightarrow t_1$

We have four distinct of each of the six t_i's and to simplify we will use modulo five. We label t_1, t_2, t_3, t_4, t_5, t_6 by 1, 2, 3, 4, 5, 6 respectively. Apply $t_1 \rightarrow t_2$, $t_2 \rightarrow t_5$ to form the permutation.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
t_1	t_2	t_3	t_4	t_5	t_6	t_1^2	t_2^2	t_3^2	t_4^2	t_5^2	t_6^2	t_1^3	t_2^3	t_3^3	t_4^3	t_5^3	t_6^3	t_1^4	t_2^4	t_3^4	t_4^4	t_5^4	t_6^4
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_2	t_4^3	t_6^2	t_5^4	t_3^4	t_1	t_2^2	t_4	t_6^4	t_5^3	t_3^3	t_1^2	t_2^3	t_4^4	t_6	t_5^2	t_3^2	t_1^3	t_2^4	t_4^2	t_6^3	t_5	t_3	t_1^4
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
2	16	12	23	21	1	8	4	24	17	15	7	14	22	6	11	9	13	20	10	18	5	3	19

Then the permutation for

Axx = (1, 2, 16, 11, 15, 6)(3, 12, 7, 8, 4, 23)(5, 21, 18, 13, 14, 22)(9, 24, 19, 20, 10, 17)

Next we will consider our next matrix B,

where B= $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -i & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & i & 0 & 0 \end{bmatrix}$

a_{13} =1

a_{25} = 1

a_{32} = -i

a_{46} = 1

a_{54} = i

$a_{64} = i$

Then we have,

$t_1 \rightarrow t_3$

$t_2 \rightarrow t_5$

$t_3 \rightarrow t_2^{-i}$

$t_4 \rightarrow t_6$

$t_5 \rightarrow t_4^{i}$

$t_6 \rightarrow t_4^{i}$

Then we have the labeling t_1, t_2, t_3, t_4, t_5, t_6 by 1, 2, 3, 4, 5, 6 respectively. Apply $t_1 \rightarrow t_3$, $t_2 \rightarrow t_5$ to form the permutation.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
t_1	t_2	t_3	t_4	t_5	t_6	t_1^2	t_2^2	t_3^2	t_4^2	t_5^2	t_6^2	t_1^3	t_2^3	t_3^3	t_4^3	t_5^3	t_6^3	t_1^4	t_2^4	t_3^4	t_4^4	t_5^4	t_6^4
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_3	t_5	t_2^3	t_1	t_6	t_4^2	t_3^2	t_5^2	t_2	t_1^2	t_6^2	t_4^4	t_3^3	t_5^3	t_2^4	t_1^3	t_6^3	t_4	t_3^4	t_5^4	t_2^2	t_1^4	t_6^4	t_4^3
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
3	5	14	1	6	10	9	11	2	7	12	22	15	17	20	13	18	4	21	23	8	19	24	16

Thus the permutation for Byy is

Byy = (1, 3, 14, 17, 18, 4)(2, 5, 6, 10, 7, 9)(8, 11, 12, 22, 19, 21)(13, 15, 20, 23, 24, 16)

Next we will find a symmetric presentation for our progenitor (N, { 1, 7, 13, 19 } where $<t_1> = \{ t_1, t_1^2, ..., t_1^6 \}$ and $\{g \epsilon N \mid <t>^g = <t>\}$ is called the normalizer of $<t>$ in G.This tells us the number of different conjugates of $<t_1>$ is six. Thus our progenitor of the presentation 5^{*6}:S_5 is

<x,y,t>:=Group<x,y,t| x^6 ,

y^6 ,

$(y^{-1}x^{-1})^3$,

$(y^{-1}x)^3$,

$y^{-3}x^{-1}y^3x^{-1}$,

$x^{-1}y^{-1}x^3y^{-1}x^{-2}$,

t^5,

$(t,(y^{-1}x^2y^{-1}))$,

$(t,(y^{-1}x^2y^{-2}x^2y^{-2}x^2y^{-2}x^2y^{-1}xy^{-2})=t^{(xy^2x^2)})$

where we have $5^{*6}:_m(S_5) \cong 5^{*6}:(S_5)$

We check using Grindstaff's lemma

G<x,y,t>:=Group<x,y,t|

x^6 ,

y^6 ,

$(y^{-1}x^{-1})^3$,

$(y^{-1}x)^3$,

$y^{-3}x^{-1}y^3x^{-1}$,

$x^{-1}y^{-1}x^3y^{-1}x^{-2}$,

t^5,

$(t,(y^{-1}x^2y^{-1}))$,

$(t,(y^{-1}x^2y^{-2}x^2y^{-2}x^2y^{-2}x^2y^{-1}xy^{-2})=t^{(}xy^2x^2))$

$(t^{(x^{-1}y^{-2}x^{-1})},t^{(xy^2x^2)})$,

$(t^{(x^{-1}y^{-2}x^{-1})},t^{(xy^{-2}x^{-2})})$,

$(t^{(x^{-1}y^{-2}x^{-1})},t^{(y^{-1}})$,

$(t^{(x^{-1}y^{-2}x^{-1})},t^{(xy)})$,

$(t^{(x^{-1}y^{-2}x^{-1})},t^{(yxy^{-1}x^2)})$,

$(t^{(xy^2x^2)},t^{(xy^{-2}x^{-2})})$,

$(t^{(xy^2x^2)},t^{(y^{-1})})$,

$(t^{(xy^2x^2)},t^{(xy)})$,

$(t^{(xy^2x^2)},t^{(yxy^{-1}x^2)})$,

$(t^{(xy^{-2}x^{-2})},t^{(y^{-1})})$,

$(t^{(xy^{-2}x^{-2})},t^{(xy)})$,

$(t^{(xy^{-2}x^{-2})},t^{(yxy^{-1}x^2)})$,

$(t^{(y^{-1})},t^{(xy)})$,

$(t^{(y^{-1})},t^{(yxy^{-1}x^2)})$,

$(t^{(xy)},t^{(yxy^{-1}x^2)})$>

Index(G,sub<G|x,y>)

15625

Next we obtain a list of elements of the form πt_i such that $\pi \epsilon S_5$, up to conjugacy. One element from each of the 7 conjugacy classes of PSL(6,5) will be considered. In the table below we list the conjugacy classes and their representatives.

Conjugacy Classes		
Class	Class Representative	Elements of the form πt_i
2	(x^3) = (1, 11)(2, 15)(3, 8)(4, 12)(5, 13)(6, 16)(7, 23)(9, 20)(10, 24)(14, 21)(17, 19)(18, 22)	$(x^3)t$, $(x^3)t_3$
3	(yxy^2) = (1, 19)(2, 11)(3, 21)(4, 6)(5, 14)(7, 13)(8, 23)(9, 15)(10, 12)(16, 18)(17, 20)(22, 24)	$(yxy^2)t$, $(yxy^2)t_2$, $(yxy^2)t_4$
4	(x^2) = (1, 16, 15)(2, 11, 6)(3, 7, 4)(5, 18, 14)(8, 23, 12)(9, 19, 10)(13, 22, 21)(17, 24, 20)	$(x^2)t$, $(x^2)t_3$, $(x^2)t_5$,$(x^2)t_9$
5	(x^2y) = (1, 13, 19, 7)(2, 12, 11, 10)(3, 9, 21, 15)(4, 14, 6, 5)(8, 24, 23, 22)(16, 20, 18, 17)	$(x^2y)t$, $(x^2y)t_2$, $(x^2y)t_3$, $(x^2y)t_4$, $(x^2y)t_8$, $(x^2y)t_{16}$
6	$(y^{-1}x^2y^{-1})$ = (2, 22, 21, 6, 17)(3, 24, 11, 20, 4)(5, 8, 16, 15, 12)(9, 18, 23, 14, 10)	$(y^{-1}x^2y^{-1})t$, $(y^{-1}x^2y^{-1})t_7$, $(y^{-1}x^2y^{-1})t_{13}$, $(y^{-1}x^2y^{-1})t_{19}$, $(y^{-1}x^2y^{-1})t_2$, $(y^{-1}x^2y^{-1})t_3$, $(y^{-1}x^2y^{-1})t_5$, $(y^{-1}x^2y^{-1})t_9$
7	(x)= (1, 2, 16, 11, 15, 6)(3, 12, 7, 8, 4, 23)(5, 21, 18, 13, 14, 22)(9, 24, 19, 20, 10, 17)	$(x)t$, $(x)t_3$, $(x)t_5$, $(x)t_9$

7.3 $37^{*2}:_m(12:2)$

We consider G = (12:2) = <(1, 5, 9)(2, 6, 10)(3, 7, 11)(4, 8, 12), (1, 4, 7, 10)(2, 5, 8, 11)(3, 6, 9, 12), (1, 11)(2, 10)(3, 9)(4, 8)(5, 7)>. We want to find the monomial representatives of (12:2), if possible. In order to see this, we first look at the character table of (12:2).

```
    Character Table of Group G
-------------------------------------
Class |   1  2  3  4  5  6  7   8   9
Size  |   1  1  6  6  2  2  2   2   2
Order |   1  2  2  2  3  4  6  12  12
-------------------------------------
p  =  2   1  1  1  1  5  2  5   7   7
p  =  3   1  2  3  4  1  6  2   6   6
-------------------------------------
```

X.1	+	1	1	1	1	1	1	1	1	1
X.2	+	1	1	-1	-1	1	1	1	1	1
X.3	+	1	1	1	-1	1	-1	1	-1	-1
X.4	+	1	1	-1	1	1	-1	1	-1	-1
X.5	+	2	2	0	0	-1	2	-1	-1	-1
X.6	+	2	2	0	0	-1	-2	-1	1	1
X.7	+	2	-2	0	0	2	0	-2	0	0
X.8	+	2	-2	0	0	-1	0	1	Z1	-Z1
X.9	+	2	-2	0	0	-1	0	1	-Z1	Z1

Now the character table of (12:2) has characters whose degree is greater than one. We note that all characters of degree larger than one have degree two. Thus, it is only possible to find monomial representation of (12:2) of degree two. Since there are five characters of degree two, it is possible for G to have five different monomial representations. All of these will all be irreducible monomial representations. We are looking for faithful monomial representation. So we need to determine which of the five irreducible characters of (12:2) are faithful. We note that there are only two faithful characters namely X.8, X.9, where X.8 = (2, -2, 0, 0, -1, 0, 1, w, -w) and similarly X.9= (2, -2, 0, 0, -1, 0, 1, -w, w). We will consider the character X.9= (2, -2, 0, 0, -1, 0, 1, -2, 2) for this paper. In order for (12:2) to have an irreducible faithful monomial representation, (12:2) is required to have a subgroup of H to the index two in (12:2) and that such H must have a linear character that induces up to the character X.9 of (12:2).

We find (4:3) = <(1, 4, 7, 10)(2, 5, 8, 11)(3, 6, 9, 12), (1, 7)(2, 8)(3, 9)(4, 10)(5, 11)(6, 12), (1, 9, 5)(2, 10, 6)(3, 11, 7)(4, 12, 8)> has an index two in (12:2) and that of (4:3) induces up to the character X.9 of (12:2).

Thus, G has a faithful irreducible monomial representations of degree 2. The field of entries of the representation is determined by the character values of the character of H that is being induced; namely, $\phi(6)$, given by (1, -1, $\mathbb{Z}_{(12)_3}$, $-\mathbb{Z}_{(12)_3}$ - 1, $-\mathbb{Z}_{(12)_4}$, $\mathbb{Z}_{(12)_4}$, $\mathbb{Z}_{(12)_3} + 1$, $-\mathbb{Z}_{(12)_3}$, $-\mathbb{Z}_{(12)_4}\mathbb{Z}_{(12)_3}$, $\mathbb{Z}_{(12)_4}\mathbb{Z}_{(12)_3} + \mathbb{Z}_{(12)_4}$,$\mathbb{Z}_{(12)_4}\mathbb{Z}_{(12)_3}$, $-\mathbb{Z}_{(12)_4}\mathbb{Z}_{(12)_3}$ - $\mathbb{Z}_{(12)_4}$) has $\mathbb{Z}_{12}$. Since $\phi(6)$ values are twelve roots of unit, the field of entries is the cyclotomic field of the 12th roots of unity. Thus, explicitly this representation is

$\rho : G \longrightarrow GL(C)$, where C=CyclotomicField(12),

ρ (xx) $\begin{bmatrix} T[1]xxT[1]^{-1} & T[1]xxT[2]^{-1} \\ T[2]xxT[1]^{-1} & T[2]xxT[2]^{-1} \end{bmatrix}$

$\rho(yy)\begin{bmatrix} T[1]yyT[1]^{-1} & T[1]yyT[2]^{-1} \\ T[2]yyT[1]^{-1} & T[2]yyT[2]^{-1} \end{bmatrix}$, and

$\rho(zz)\begin{bmatrix} T[1]zzT[1]^{-1} & T[1]zzT[2]^{-1} \\ T[2]zzT[1]^{-1} & T[2]zzT[2]^{-1} \end{bmatrix}$,

where $G =< xx, yy, zz >$ and $G = HT[1] \cup HT[2]$

Then

A = $\begin{bmatrix} -\mathbb{Z}^2_{(12)} & 0 \\ 0 & \mathbb{Z}_{(12)} - 1 \end{bmatrix}$, B = $\begin{bmatrix} -\mathbb{Z}^3_{(12)} & 0 \\ 0 & \mathbb{Z}^3_{(12)} \end{bmatrix}$, D = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

where $A = \rho(xx)$, $B = \rho(yy)$, and $D = \rho(zz)$. Thus, the generators of the faithful irreducible monomial representation are A, B, D.

Now the smallest finite field that contains the twelfth root of unity (elements of order 37) is $\mathbb{Z}_{37}$. The elements of order twelve in $\mathbb{Z}_{37}$ are generators (every element of $\mathbb{Z}_{37}$-{0} is a power of each of these elements of $\mathbb{Z}_{37}$ - {0}) of $\mathbb{Z}_{37}$-{0} .

These elements of order 12 are 8, 10, 26, 31, 29 have a primitive root of two.

We now find a permutation representation of our monomial representation. We denote the permutation representations of A, B, and by Axx, Byy, and Dzz, respectively. Our progenitor is: 37^{*2}:(12:2). We have two t_i's: t_1, t_2. We now interpret the automorphisms given by the three matrices A, B, and D by using the formula $a_{ij} \Leftrightarrow t_i \rightarrow t_j$.

First consider A=$\begin{bmatrix} 10 & 0 \\ 0 & 26 \end{bmatrix}$

So $a_{11} = 10$ and $a_{22} = 26$

Therefore,

$t_1 \rightarrow t_1^{10}$

$t_2 \rightarrow t_2^{26}$

Since $|t_1| = 12 = |t_2|$ we have twelve distinct powers of each of the two t_i's and to simplify we use modulo 37.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
t_1	t_2	t_1^2	t_2^2	t_1^3	t_2^3	t_1^4	t_2^4	t_1^5	t_2^5	t_1^6	t_2^6	t_1^7	t_2^7	t_1^8	t_2^8	t_1^9	t_2^9
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_1^{10}	t_2^{26}	t_1^{20}	t_2^{15}	t_1^{30}	t_2^4	t_1^3	t_2^{30}	t_1^{13}	t_2^{19}	t_1^{23}	t_2^8	t_1^{33}	t_2^{34}	t_1^6	t_2^{23}	t_1^{16}	t_2^{12}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
19	52	39	30	59	8	5	60	25	38	45	16	65	68	11	46	31	24

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
t_1^{10}	t_2^{10}	t_1^{11}	t_2^{11}	t_1^{12}	t_2^{12}	t_1^{13}	t_2^{13}	t_1^{14}	t_2^{14}	t_1^{15}	t_2^{15}	t_1^{16}	t_2^{16}	t_1^{17}	t_2^{17}	t_1^{18}	t_2^{18}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_1^{26}	t_2	t_1^{36}	t_2^{27}	t_1^9	t_2^{16}	t_1^{19}	t_2^5	t_1^{29}	t_2^{31}	t_1^2	t_2^{20}	t_1^{12}	t_2^9	t_1^{22}	t_2^{35}	t_1^{32}	t_2^{24}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
51	2	71	54	17	32	37	10	57	62	3	40	23	18	43	70	63	48

37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
t_1^{19}	t_2^{19}	t_1^{20}	t_2^{20}	t_1^{21}	t_2^{21}	t_1^{22}	t_2^{22}	t_1^{23}	t_2^{23}	t_1^{24}	t_2^{24}	t_1^{25}	t_2^{25}	t_1^{26}	t_2^{26}	t_1^{27}	t_2^{27}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_1^5	t_2^{13}	t_1^{15}	t_2^2	t_1^{25}	t_2^{28}	t_1^{35}	t_2^{17}	t_1^8	t_2^6	t_1^{18}	t_2^{32}	t_1^{28}	t_2^{21}	t_1	t_2^{10}	t_1^{11}	t_2^{36}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
9	26	29	4	49	56	69	34	15	12	35	64	55	42	1	20	21	72

55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
t_1^{28}	t_2^{28}	t_1^{29}	t_2^{29}	t_1^{30}	t_2^{30}	t_1^{31}	t_2^{31}	t_1^{32}	t_2^{32}	t_1^{33}	t_2^{33}	t_1^{34}	t_2^{34}	t_1^{35}	t_2^{35}	t_1^{36}	t_2^{36}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_1^{21}	t_2^{25}	t_1^{31}	t_2^{14}	t_1^4	t_2^3	t_1^{14}	t_2^{29}	t_1^{24}	t_2^{18}	t_1^{34}	t_2^7	t_1^7	t_2^{33}	t_1^{17}	t_2^{22}	t_1^{27}	t_2^{11}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
41	50	61	28	7	6	27	58	47	36	67	14	13	66	33	44	53	22

Which gives us the permutation

Axx = (1, 19, 51)(2, 52, 20)(3, 39, 29)(4, 30, 40)(5, 59, 7)(6, 8, 60)(9, 25, 37)(10, 38, 26)(11, 45, 15)(12, 16, 46)(13, 65, 67)(14, 68, 66)(17, 31, 23)(18, 24, 32)(21, 71, 53)(22, 54, 72)(27, 57, 61)(28, 62, 58)(33, 43, 69)(34, 70, 44)(35, 63, 47)(36, 48, 64)(41, 49, 55)(42, 56, 50)

Next we will investigate the B matrix. So we have

$$B=\begin{bmatrix} 6 & 0 \\ 0 & 31 \end{bmatrix}$$

So $b_{11} = 6$ and $b_{22} = 31$

Therefore,

$t_1 \to t_1^6$

$t_2 \to t_2^{31}$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
t_1	t_2	t_1^2	t_2^2	t_1^3	t_2^3	t_1^4	t_2^4	t_1^5	t_2^5	t_1^6	t_2^6	t_1^7	t_2^7	t_1^8	t_2^8	t_1^9	t_2^9
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_1^6	t_2^{31}	t_1^{12}	t_2^{25}	t_1^{18}	t_2^{19}	t_1^{24}	t_2^{13}	t_1^{30}	t_2^7	t_1^{36}	t_2	t_1^5	t_2^{32}	t_1^{11}	t_2^{26}	t_1^{17}	t_2^{20}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
11	62	23	50	35	38	47	26	59	14	71	2	9	64	21	52	33	40

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
t_1^{10}	t_2^{10}	t_1^{11}	t_2^{11}	t_1^{12}	t_2^{12}	t_1^{13}	t_2^{13}	t_1^{14}	t_2^{14}	t_1^{15}	t_2^{15}	t_1^{16}	t_2^{16}	t_1^{17}	t_2^{17}	t_1^{18}	t_2^{18}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_1^{23}	t_2^{14}	t_1^{29}	t_2^8	t_1^{35}	t_2^2	t_1^4	t_2^{33}	t_1^{10}	t_2^{27}	t_1^{16}	t_2^{21}	t_1^{22}	t_2^{15}	t_1^{28}	t_2^9	t_1^{34}	t_2^3
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
45	28	57	16	69	4	7	66	19	54	31	42	43	30	55	18	67	6

37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
t_1^{19}	t_2^{19}	t_1^{20}	t_2^{20}	t_1^{21}	t_2^{21}	t_1^{22}	t_2^{22}	t_1^{23}	t_2^{23}	t_1^{24}	t_2^{24}	t_1^{25}	t_2^{25}	t_1^{26}	t_2^{26}	t_1^{27}	t_2^{27}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_1^3	t_2^{34}	t_1^9	t_2^{28}	t_1^{15}	t_2^{22}	t_1^{21}	t_2^{16}	t_1^{27}	t_2^{10}	t_1^{33}	t_2^4	t_1^2	t_2^{35}	t_1^8	t_2^{29}	t_1^{14}	t_2^{23}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
5	68	17	56	29	44	41	32	53	20	65	8	3	70	15	58	27	46

55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
t_1^{28}	t_2^{28}	t_1^{29}	t_2^{29}	t_1^{30}	t_2^{30}	t_1^{31}	t_2^{31}	t_1^{32}	t_2^{32}	t_1^{33}	t_2^{33}	t_1^{34}	t_2^{34}	t_1^{35}	t_2^{35}	t_1^{36}	t_2^{36}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_1^{20}	t_2^{17}	t_1^{26}	t_2^{11}	t_1^{32}	t_2^{5}	t_1	t_2^{36}	t_1^{7}	t_2^{30}	t_1^{13}	t_2^{24}	t_1^{19}	t_2^{18}	t_1^{25}	t_2^{12}	t_1^{31}	t_2^{6}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
39	34	51	22	63	10	1	72	13	60	25	48	37	36	49	24	61	12

Therefore our permutation for B is

Byy = (1, 11, 71, 61)(2, 62, 72, 12)(3, 23, 69, 49)(4, 50, 70, 24)(5, 35, 67, 37)(6, 38, 68, 36)(7, 47, 65, 25)(8, 26, 66, 48)(9, 59, 63, 13)(10, 14, 64, 60)(15, 21, 57, 51)(16, 52, 58, 22)(17, 33, 55, 39)(18, 40, 56, 34)(19, 45, 53, 27)(20, 28, 54, 46)(29, 31, 43, 41)(30, 42, 44, 32)

where D= $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

So $d_{12} = 1$ and $d_{21} = 1$

Therefore,

$t_1 \rightarrow t_2$

$t_2 \rightarrow t_1$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
t_1	t_2	t_1^2	t_2^2	t_1^3	t_2^3	t_1^4	t_2^4	t_1^5	t_2^5	t_1^6	t_2^6	t_1^7	t_2^7	t_1^8	t_2^8	t_1^9	t_2^9
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_2	t_1	t_2^2	t_1^2	t_2^3	t_1^3	t_2^4	t_1^4	t_2^5	t_1^5	t_2^6	t_1^6	t_2^7	t_1^7	t_2^8	t_1^8	t_2^9	t_1^9
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15	18	17

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
t_1^{10}	t_2^{10}	t_1^{11}	t_2^{11}	t_1^{12}	t_2^{12}	t_1^{13}	t_2^{13}	t_1^{14}	t_2^{14}	t_1^{15}	t_2^{15}	t_1^{16}	t_2^{16}	t_1^{17}	t_2^{17}	t_1^{18}	t_2^{18}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_2^{10}	t_1^{10}	t_2^{11}	t_1^{11}	t_2^{12}	t_1^{12}	t_2^{13}	t_1^{13}	t_2^{14}	t_1^{14}	t_2^{15}	t_1^{15}	t_2^{16}	t_1^{16}	t_2^{17}	t_1^{17}	t_2^{18}	t_1^{18}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
20	19	22	21	24	23	26	25	28	27	30	29	32	31	34	33	36	35

37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
t_1^{19}	t_2^{19}	t_1^{20}	t_2^{20}	t_1^{21}	t_2^{21}	t_1^{22}	t_2^{22}	t_1^{23}	t_2^{23}	t_1^{24}	t_2^{24}	t_1^{25}	t_2^{25}	t_1^{26}	t_2^{26}	t_1^{27}	t_2^{27}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_2^{19}	t_1^{19}	t_2^{20}	t_1^{20}	t_2^{21}	t_1^{21}	t_2^{22}	t_1^{22}	t_2^{23}	t_1^{23}	t_2^{24}	t_1^{24}	t_2^{25}	t_1^{25}	t_2^{26}	t_1^{26}	t_2^{27}	t_1^{27}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
38	37	40	39	42	41	44	43	46	45	48	47	50	49	52	51	54	53

55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
t_1^{28}	t_2^{28}	t_1^{29}	t_2^{29}	t_1^{30}	t_2^{30}	t_1^{31}	t_2^{31}	t_1^{32}	t_2^{32}	t_1^{33}	t_2^{33}	t_1^{34}	t_2^{34}	t_1^{35}	t_2^{35}	t_1^{36}	t_2^{36}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_2^{28}	t_1^{28}	t_2^{29}	t_1^{29}	t_2^{30}	t_1^{30}	t_2^{31}	t_1^{31}	t_2^{32}	t_1^{32}	t_2^{33}	t_1^{33}	t_2^{34}	t_1^{34}	t_2^{35}	t_1^{35}	t_2^{36}	t_1^{36}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
56	55	58	57	60	59	62	61	64	63	66	65	68	67	70	69	72	71

Therefore we get the permutation

Dzz = (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16)(17, 18)(19, 20)(21, 22)(23, 24)(25, 26)(27, 28)(29, 30)(31, 32)(33, 34)(35, 36)(37, 38)(39, 40)(41, 42)(43, 44)(45, 46)(47, 48)(49, 50)(51, 52)(53, 54)(55, 56)(57, 58)(59, 60)(61, 62)(63, 64)(65, 66)(67, 68)(69, 70)(71, 72)

Now we want to find a symmetric presentation for our progenitor. The stabiliser (N,{ 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71 } where

$<t_1> = \{t_1, t_1^2, ..., t_1^{37}\}$ and $\{g \epsilon N \mid <t>^g = <t>\}$ is called the normalizer of <t> in G. This tells us the number of different conjugates of $<t_1>$ is two. Thus, our presentation of the progenitor 37^{*2}:(12:2) is

$<x,y,z,t \mid x^3, y^4, z^2, (x, y), (x^{-1}z)^2, (y^{-1}z)^2, t^{37}, (t^{(x^{-1}y^{-1})}=t^{29})>$ where we have

$37^{*2}:_m(12:2) \cong 37^{*2}:(12:2)$.

We know $(t_1,t_2)=1 \rightarrow (t_1^1, t_2^1)=1$. Thus $t_1, t^z)=1$ gives $(t_1^i, t_2^j=1\ \forall$ i,j = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71}

Then we will use Grindstaff's lemma where we check to our work to make sure we are indeed using the correct progenitor.

```
G<x,y,z,t>:=Group<x,y,z,t|
x^3 ,
y^4 ,
z^2 ,
(x, y) ,
(x^-1z)^2 ,
(y^-1z)^2 ,
t^37,
(t^(x^-1y^-1)=t^29),(t,t^z)>;
#G;
32856
Index(G,sub<G|x,y,z>);
1369
#N;
24
```

Since we have two by two matrices and we are working with the relatively prime number p = 37. We can follow that we need to have 37^2 = 1,369 and we multiply that by our order N, where our N=24. Therefore, we have $37^2(24)$ = 32856, which is the order of G. This informs us that our progenitor is correct.

Next we obtain a list of elements of the form πt_i such that $\pi \epsilon$ N, up to conjugacy. One element from each of the 9 conjugacy classes of PSL(2,37) will be considered. For example (y^2) is in class 2 so $(y^2 t_{17})$ is one element of the required form. Since the centralizer of (y^2) is transitive the other elements of $(y^2)t_i$ are all conjugate to $(y^2 t_{17})$. Thus by looking at the centralizer of each class representative a list of distinct (up to conjugacy) elements of the form πt_i is found. In the table below we list the conjugacy classes and their representatives.

Conjugacy Classes		
Class	Class Representative	Elements of the form πt_i
2	(y^2) = (1, 23)(2, 24)(3, 21)(4, 22)(5, 19)(6, 20)(7, 17)(8, 18)(9, 15)(10, 16)(11, 13)(12, 14)	$(y^2 t_{17})$
3	z = (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16)(17, 18)(19, 20)(21, 22)(23, 24)	zt_2, zt_3, zt_5,zt_7, zt_9, zt_{11}
4	(yz) = (1, 10)(2, 15)(3, 20)(4, 5)(6, 21)(7, 14)(8, 11)(9, 24)(12, 17)(13, 18)(16, 23)(19, 22)	$(yz)t_{10}$, $(yz)t_2$, $(yz)t_3$, $(yz)t_4$, $(yz)t_7$, $(yz)t_8$
5	x = (1, 17, 5)(2, 6, 18)(3, 9, 11)(4, 12, 10)(7, 19, 23)(8, 24, 20)(13, 21, 15)(14, 16, 22)	xt_{17}, xt_2
6	y = (1, 9, 23, 15)(2, 16, 24, 10)(3, 19, 21, 5)(4, 6, 22, 20)(7, 13, 17, 11)(8, 12, 18, 14)	yt_9, yt_2
7	(y^2x^{-1}) = (1, 19, 17, 23, 5, 7)(2, 8, 6, 24, 18, 20)(3, 13, 9, 21, 11, 15)(4, 16, 12, 22, 10, 14)	$(y^2x^{-1})t_{19}$, $(y^2x^{-1})t_2$
8	(xy) = (1, 11, 19, 15, 17, 3, 23, 13, 5, 9, 7, 21)(2, 22, 8, 10, 6, 14, 24, 4, 18, 16, 20, 12)	$(xy)t_{11}$, $(xy)t_2$
9	(yx^-1) = (1, 3, 7, 15, 5, 11, 23, 21, 17, 9, 19, 13)(2, 14, 20, 10, 18, 22, 24, 12, 6, 16, 8, 4)	$(yx^-1)t_3$, $(yx^-1)t_2$

7.4 $7^{*8}:_m(2^3:6)$

Let H = ((4:2):2) = < (2, 3)(4, 5), (1, 5)(2, 6)(3, 7)(4, 8), (1, 3)(2, 8)(4, 6)(5, 7), (1, 2)(3, 8)(4, 7)(5, 6) > , The order of ((4:2):2) is = 16. Then 2 = H' = < 1, (1, 8)(2, 3)(4, 5)(6, 7)> and the order of H' = 2. Then ((4:2):2)/2 = H/H' = { H',$H'(xy^2x)$, $H'(xyxy^{-1}xy^{-1})$, $H'((xy)^3)$, $H'(xyxyxy^{-1})$, $H'((xy^2)^2)$, $H'(xy^{-1}xy^2)$, $H'(x^y)$ }. The classes of H are :

C_1 = {e }

C_2 = { (1, 8)(2, 3)(4, 5)(6, 7) }

C_3 = { (1, 7)(2, 4)(3, 5)(6, 8) }

C_4 = { (1, 6)(2, 5)(3, 4)(7, 8) }

C_5 = { (1, 5)(2, 6)(3, 7)(4, 8), (1, 4)(2, 7)(3, 6)(5, 8) }

$C_6 = \{ (1, 3)(2, 8)(4, 6)(5, 7), (1, 2)(3, 8)(4, 7)(5, 6) \}$

$C_7 = \{ (2, 3)(4, 5), (1, 8)(6, 7) \}$

$C_8 = \{ (1, 7)(2, 5)(3, 4)(6, 8), (1, 6)(2, 4)(3, 5)(7, 8) \}$

$C_9 = \{ (1, 4, 8, 5)(2, 6, 3, 7), (1, 5, 8, 4)(2, 7, 3, 6) \}$

$C_{10} = \{ (1, 3, 8, 2)(4, 7, 5, 6), (1, 2, 8, 3)(4, 6, 5, 7) \}$

We know that a Character Table of H/H′ is given by:

Character Table								
	e	H′T[2]	H′T[3]	H′T[4]	H′T[5]	H′T[6]	H′T[7]	H′T[8]
$\hat{X}.1$	1	1	1	1	1	1	1	1
$\hat{X}.2$	1	-1	1	1	-1	-1	1	-1
$\hat{X}.3$	1	1	-1	1	-1	1	-1	-1
$\hat{X}.4$	1	1	1	-1	1	-1	-1	-1
$\hat{X}.5$	1	-1	-1	1	1	-1	-1	1
$\hat{X}.6$	1	1	-1	-1	-1	-1	1	1
$\hat{X}.7$	1	-1	1	-1	-1	1	-1	1
$\hat{X}.8$	1	-1	-1	-1	1	1	1	-1

We lift the characters of H/H$'$ to ((4:2):2). To calculate the lift of X of character $\hat{X}$ of H/H$'$, we note that

X(C[1][3]) = $\hat{X}$(1)=1,

X(C[2][3]) = $\hat{X}$(1)=1,

X(C[3][3]) = $\hat{X}$T[7],

X(C[4][3]) = $\hat{X}$T[7],

X(C[5][3]) = $\hat{X}$T[3],

X(C[6][3]) = $\hat{X}$T[4],

X(C[7][3]) = $\hat{X}$T[2],

X(C[8][3]) = $\hat{X}$T[8],

X(C[9][3]) = $\hat{X}$T[5],

X(C[10][3]) = $\hat{X}$T[6]

Lifted-Character Table										
Classes	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Rep	H$'$	H$'$(xy^{2}x)	H$'$(xyxy^{-1}xy^{-1})	H$'$((xy)^3)	H$'$(xyxyxy^{-1})	H$'$((xy^2)^2)	H$'$(xy^{-1}xy^2)	H$'$(x^y)	H$'$(y^{-1}xy^{-1})	H$'$(xyx)
X.1	1	1	1	1	1	1	1	1	1	1
X.2	1	1	1	1	1	1	-1	-1	-1	-1
X.3	1	1	-1	-1	-1	1	1	-1	-1	1
X.4	1	1	-1	-1	1	-1	1	-1	1	-1
X.5	1	1	-1	-1	-1	1	-1	1	1	-1
X.6	1	1	1	1	-1	-1	1	1	-1	-1
X.7	1	1	-1	-1	1	-1	-1	1	-1	1
X.8	1	1	1	1	-1	-1	-1	-1	1	1

Then X.1, X.2, X.3, X.4, X.5, X.6, X.7, X.8 are irreducible characters of ((4:2):2), since $\hat{X}$.1, $\hat{X}$.2, $\hat{X}$.3, $\hat{X}$.4, $\hat{X}$.5, $\hat{X}$.6, $\hat{X}$.7, $\hat{X}$.8 are irreducible characters in H/H$'$.

We have G = (2^3:6) generated by x = (1, 5, 9, 3)(2, 4, 7, 6) and y = (1, 2, 4, 8)(3, 6, 9, 5) that has an order of 72 and let there be an ((4:2):2), where ((4:2):2) is generated by <(1, 8, 9)(2, 4, 3)(5, 6, 7), (1, 3, 6)(2, 7, 8)(4, 5, 9)> and has an order of 9.

We will begin to induce the $\phi(2)$ up to X.6 where X is the character table of (2^3:6)

```
Character Table of Group G
--------------------------



--------------------------
Class |   1  2  3  4  5  6
Size  |   1  9  8 18 18 18
```

```
Order |   1  2  3  4  4  4
--------------------------
p  =  2   1  1  3  2  2  2
p  =  3   1  2  1  4  5  6
--------------------------
X.1   +   1  1  1  1  1  1
X.2   +   1  1  1 -1  1 -1
X.3   +   1  1  1  1 -1 -1
X.4   +   1  1  1 -1 -1  1
X.5   -   2 -2  2  0  0  0
X.6   +   8  0 -1  0  0  0
```

and ϕ the character table of ((4:2):2)

```
Character Table of Group H
-------------------------------------------
Class |   1   2   3   4   5   6   7   8   9
Size  |   1   1   1   1   1   1   1   1   1
Order |   1   3   3   3   3   3   3   3   3
-------------------------------------------
p  =  3   1   1   1   1   1   1   1   1   1
-------------------------------------------
X.1   +   1   1   1   1   1   1   1   1   1
X.2   0   1   J-1-J   1   1   J-1-J   J-1-J
X.3   0   1-1-J   J   1   1-1-J   J-1-J   J
X.4   0   1   1   1   J-1-J   J-1-J-1-J   J
X.5   0   1   J-1-J   J-1-J-1-J   J   1   1
X.6   0   1-1-J   J   J-1-J   1   1   J-1-J
X.7   0   1   1   1-1-J   J-1-J   J   J-1-J
X.8   0   1   J-1-J-1-J   J   1   1-1-J   J
X.9   0   1-1-J   J-1-J   J   J-1-J   1   1
```

Where $\phi(2)$ = (1, $\mathbb{Z}(3)_3$, -$\mathbb{Z}(3)_3$ - 1, 1, 1, $\mathbb{Z}(3)_3$, -$\mathbb{Z}(3)_3$ - 1, $\mathbb{Z}(3)_3$, -$\mathbb{Z}(3)_3$ - 1). We want to focus on the degrees of the character table of (2^3:6), it should be noted that we want degrees greater than one and in the character table of (2^3:6), the only degrees greater than one are X.5= (2, -2, 2, 0, 0, 0) with degree 2 and X.6= (8, 0, -1, 0, 0, 0) with degree 8. Therefore it is only possible to find monomial representations of (2^3:6) with either a degree of 2 or a degree of 8, and it is only possible for (2^3:6) to have two different possible monomial representations. We need to determine which of the two irreducible characters of (2^3:6) are faithful, we note that there is one faithful character namely X.6. For (2^3:6) to have a faithful irreducible monomial representation, (2^3:6) is required to have a

subgroup of H to the index eight in (2^3:6), and that such H must be a linear character that induces up to X.6 of (2^3:6). From previously stated we understand that H=<(1, 8, 9)(2, 4, 3)(5, 6, 7), (1, 3, 6)(2, 7, 8)(4, 5, 9)> has an index of eight in (2^3:6) and CH[2] induces up to character X.6 of (2^3:6). Thus (2^3:6) has a faithful irreducible monomial representation of degree 8. The field of entries of the representation is determined by the character values ϕ of H that is being induced namely CH[2] given by (1, $\mathbb{Z}(3)_3$, -$\mathbb{Z}(3)_3$ - 1, 1, 1, $\mathbb{Z}(3)_3$, -$\mathbb{Z}(3)_3$ - 1, $\mathbb{Z}(3)_3$, -$\mathbb{Z}(3)_3$ - 1) has $\mathbb{Z}_3$ where the values of CH[2] values are third root of unity, the field entries is of the cycloatomic field 3rd root unity. Explicitly the representation is ρ: (2^3:6) $\rightarrow$ GL(C) where C= Cycloatomic Field (3),
Where (2^3:6) = <xx, yy> and G = HT[1] $\cup$ HT[2] $\cup$ HT[3] $\cup$ HT[4] $\cup$ HT[5] $\cup$ HT[6] $\cup$ HT[7] $\cup$ HT[8]
Then

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbb{Z}_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \mathbb{Z}_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbb{Z}_3 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbb{Z}_3 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mathbb{Z}_3 - 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where A=ρxx and B=ρyy. Thus the generators of the faithful irreducible monomial representation are A and B. Now the smallest finite field that contains the third root of unity (elements of order 7) is $\mathbb{Z}_7$. We now find the permutation representation of our monomial representation.We denote permutation representation of A and B by Axx and Byy respectively. Our progenitor is 7^{*8}:(2^3:6). We have eight t_i's: t_1, t_2, t_3, t_4, t_5, t_6, t_7,

t_8. We will use the formula $a_{ij} \Leftrightarrow t_i \to t_j$. We have the labeling t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8 by 1, 2, 3, 4, 5, 6, 7, 8 respectively. Apply $t_1 \to t_2$, $t_2 \to t_4$ to form the permutation.

Lets first consider $A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \end{bmatrix}$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_1^2	t_2^2	t_3^2	t_4^2	t_5^2	t_6^2	t_7^2	t_8^2	t_1^3	t_2^3	t_3^3	t_4^3	t_5^3	t_6^3	t_7^3	t_8^3
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_2	t_4	t_6^2	t_7	t_3^2	t_8	t_1	t_5^2	t_2^2	t_4^2	t_6^4	t_7^2	t_3^4	t_8^2	t_1^2	t_5^4	t_2^3	t_4^3	t_6^6	t_7^3	t_3^6	t_8^3	t_1^3	t_5^6
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
2	4	14	7	11	8	1	13	10	12	30	15	27	16	9	29	18	20	46	23	43	24	17	45

25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
t_1^4	t_2^4	t_3^4	t_4^4	t_5^4	t_6^4	t_7^4	t_8^4	t_1^5	t_2^5	t_3^5	t_4^5	t_5^5	t_6^5	t_7^5	t_8^5	t_1^6	t_2^6	t_3^6	t_4^6	t_5^6	t_6^6	t_7^6	t_8^6
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_2^4	t_4^4	t_6	t_7^4	t_3	t_8^4	t_1^4	t_5	t_2^5	t_4^5	t_6^3	t_7^5	t_3^3	t_8^5	t_1^5	t_5^3	t_2^6	t_4^6	t_6^5	t_7^6	t_3^5	t_8^6	t_1^6	t_5^5
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
26	28	6	31	3	32	25	5	34	36	22	39	19	40	33	21	42	44	38	47	35	48	41	37

Therefore Axx= (1, 2, 4, 7)(3, 14, 16, 29)(5, 11, 30, 32)(6, 8, 13, 27)(9, 10, 12, 15)(17, 18, 20, 23)(19, 46, 48, 37)(21, 43, 38, 40)(22, 24, 45, 35)(25, 26, 28, 31)(33, 34, 36, 39)(41, 42, 44, 47)

Next we will consider B, with the following labeling t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8 by 1, 2, 3, 4, 5, 6, 7, 8 respectively. Apply $t_1 \to t_5$, $t_2 \to t_8$ to form the permutation.

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix}$$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_1^2	t_2^2	t_3^2	t_4^2	t_5^2	t_6^2	t_7^2	t_8^2	t_1^3	t_2^3	t_3^3	t_4^3	t_5^3	t_6^3	t_7^3	t_8^3
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_5	t_8	t_2^2	t_6^4	t_4^2	t_1	t_3	t_7^4	t_5^2	t_8^2	t_2^4	t_6	t_4^4	t_1^2	t_3^2	t_7	t_5^3	t_8^3	t_2^6	t_6^5	t_4^6	t_1^3	t_3^3	t_7^5
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
5	8	10	30	12	1	3	31	13	16	26	6	28	9	11	7	21	24	42	38	44	17	19	39

25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
t_1^4	t_2^4	t_3^4	t_4^4	t_5^4	t_6^4	t_7^4	t_8^4	t_1^5	t_2^5	t_3^5	t_4^5	t_5^5	t_6^5	t_7^5	t_8^5	t_1^6	t_2^6	t_3^6	t_4^6	t_5^6	t_6^6	t_7^6	t_8^6
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_5^4	t_8^4	t_2	t_6^2	t_4	t_1^4	t_3^4	t_7^2	t_5^5	t_8^5	t_2^3	t_6^6	t_4^6	t_1^5	t_3^5	t_7^6	t_5^6	t_8^6	t_2^5	t_6^3	t_4^5	t_1^6	t_3^6	t_7^3
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
32	29	2	14	4	25	27	15	37	40	18	46	44	33	35	47	45	48	34	22	36	41	43	23

Therefore Byy = (1, 5, 12, 6)(2, 8, 31, 27)(3, 10, 16, 7)(4, 30, 25, 29)(9, 13, 28, 14)(11, 26, 32, 15)(17, 21, 44, 22)(18, 24, 39, 35)(19, 42, 48, 23)(20, 38, 33, 37)(34, 40, 47, 43)(36, 46, 41, 45). Next we want to find our symmetric presentation for our progenitor. The stabiliser(N,{1, 9, 17, 25, 33, 41}) where $<t_1> = \{t_1, t_1^2, ..., t_1^7\}$ and the normilizer of <t> of G. This tells us the number of different conjugates of $<t_1>$ is eight. Thus our progenitor 7^{*8}:$(2^3$:6) is $<x,y,t| \; x^4 , y^4 , y^{-1}x^{-2}yxy^2x^{-1} , x^{-1}y^{-2}x^{-2}yx^{-1}y^{-1}, t^7,(t,((y^{-1}xy^{-1}x^{-1})^2)),((t^{(y^{-1}x^2y^{-1})})=t^2)>$ where we have $7^{*8}:_m(2^3:6) \cong 7^{*8}:(2^3:6)$

7.5 $17^{*2}:_m((2:8):2)$

We have G = ((2:8):2) generated by x=(1, 14, 6, 9, 2, 13, 5, 10)(3, 15, 8, 12, 4, 16, 7, 11), y=(1, 6, 2, 5)(3, 8, 4, 7)(9, 14, 10, 13)(11, 16, 12, 15) and z=(1, 16)(2, 15)(3, 10)(4, 9)(5, 12)(6, 11)(7, 13)(8, 14) that has an order of 32 and let there be H = (8:2), where

(8:2) is generated by <(1, 16, 2, 15)(3, 10, 4, 9)(5, 12, 6, 11)(7, 13, 8, 14), (1, 14, 6, 9, 2, 13, 5, 10)(3, 15, 8, 12, 4, 16, 7, 11), (1, 6, 2, 5)(3, 8, 4, 7)(9, 13, 10, 14)(11, 15, 12, 16), (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16))> and has an order of 16.

We will begin to induce from ((2:8):2) to H

Where $\phi(7)$ = (1, -1, 1, -1, $\mathbb{Z}(8)_8^2$, -$\mathbb{Z}(8)_8^2$, -$\mathbb{Z}(8)_8^2$, $\mathbb{Z}(8)_8^2$, $\mathbb{Z}(8)_8^3$, $\mathbb{Z}(8)_8$, -$\mathbb{Z}(8)_8^3$, -$\mathbb{Z}(8)_8$, -$\mathbb{Z}(8)_8$, -$\mathbb{Z}(8)_8^3$, $\mathbb{Z}(8)_8$, $\mathbb{Z}(8)_8^3$). We want to focus on the degrees of ((2:8):2), it should be noted that we want degrees greater than one and in the character table of ((2:8):2), the only degrees greater than one are X.17, X.18, X.19, X.20 who all have a degree of 2. Therefore it is only possible to find monomial representations of ((2:8):2) with either a degree of 2 and it is only possible for ((2:8):2) to have four different possible monomial representations. We need to determine which of the four irreducible characters of ((2:8):2) are faithful, we note that there is one faithful character namely X.18. For ((2:8):2) to have a faithful irreducible monomial representation, ((2:8):2) is required to have a subgroup of (8x2) to the index eight in ((2:8):2), and that such (8:2) must be a linear character that induces up to X.18 of ((2:8):2). From previously stated we understand that (8:2) = <(1, 16, 2, 15)(3, 10, 4, 9)(5, 12, 6, 11)(7, 13, 8, 14), (1, 14, 6, 9, 2, 13, 5, 10)(3, 15, 8, 12, 4, 16, 7, 11), (1, 6, 2, 5)(3, 8, 4, 7)(9, 13, 10, 14)(11, 15, 12, 16), (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16))> has an index of two in ((2:8):2) and $\phi(7)$ induces up to character X.18 of ((2:8):2). Thus ((2:8):2) has a faithful irreducible monomial representation of degree 2. The field of entries of the representation is determined by the character values ϕ of (8x2) that is being induced namely $\phi(7)$ given by (1, -1, 1, -1, $\mathbb{Z}(8)_8^2$, -$\mathbb{Z}(8)_8^2$, -$\mathbb{Z}(8)_8^2$, $\mathbb{Z}(8)_8^2$, $\mathbb{Z}(8)_8^3$, $\mathbb{Z}(8)_8$, -$\mathbb{Z}(8)_8^3$, -$\mathbb{Z}(8)_8$, -$\mathbb{Z}(8)_8$, -$\mathbb{Z}(8)_8^3$, $\mathbb{Z}(8)_8$, $\mathbb{Z}(8)_8^3$) where the values of $\phi(7)$ values are eighth root of unity, the field entries is of the cycloatomic field 8th root unity. Explicitly the representation is ρ: G $\rightarrow$ GL(C) where C= Cycloatomic Field (8),

ρ (xx)= $\begin{bmatrix} T[1]xxT[1]^{-1} & T[1]xxT[2]^{-1} \\ T[2]xxT[1]^{-1} & T[2]xxT[2]^{-1} \end{bmatrix}$

ρ (yy)= $\begin{bmatrix} T[1]xxT[1]^{-1} & T[1]xxT[2]^{-1} \\ T[2]xxT[1]^{-1} & T[2]xxT[2]^{-1} \end{bmatrix}$

ρ (zz) = $\begin{bmatrix} T[1]xxT[1]^{-1} & T[1]xxT[2]^{-1} \\ T[2]xxT[1]^{-1} & T[2]xxT[2]^{-1} \end{bmatrix}$

Where G=<xx,yy,zz> and G= HT[1] $\cup$ HT[2]

Then

A = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, B = $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, C= $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$
where A=ρxx, B=ρyy and C= ρzz. Thus the generators of the faithful irreducible monomial representation are A, B, and C. Now the smallest finite field that contains the eighth root of unity (elements of order 17) is $\mathbb{Z}_{17}$. We now find the permutation representation of our monomial representation. We denote permutation representation of A, B, C by Axx, Byy, Czz respectively. Our progenitor is 17^{*2}:((2:8):2).
We have two t_i's: t_1, t_2. We will use the formula $a_{ij} \Leftrightarrow t_i \rightarrow t_j$.
We have the following labeling t_1, t_2 by 1, 2 respectively. Apply $t_1 \rightarrow t_1^8$, $t_2 \rightarrow t_2^8$ to form the permutation.
Lets first consider A = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
t_1	t_2	t_1^2	t_2^2	t_1^3	t_2^3	t_1^4	t_2^4	t_1^5	t_2^5	t_1^6	t_2^6	t_1^7	t_2^7	t_1^8	t_2^8	t_1^9	t_2^9	t_1^{10}	t_2^{10}	t_1^{11}	t_2^{11}	t_1^{12}	t_2^{12}	t_1^{13}	t_2^{13}	t_1^{14}	t_2^{14}	t_1^{15}	t_2^{15}	t_1^{16}	t_2^{16}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_1^8	t_2^8	t_1^{16}	t_2^{16}	t_1^7	t_2^7	t_1^{15}	t_2^{15}	t_1^6	t_2^6	t_1^{14}	t_2^{14}	t_1^5	t_2^5	t_1^{13}	t_2^{13}	t_1^4	t_2^4	t_1^{12}	t_2^{12}	t_1^3	t_2^3	t_1^{11}	t_2^{11}	t_1^2	t_2^2	t_1^{10}	t_2^{10}	t_1	t_2	t_1^9	t_2^9
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
15	16	31	32	13	14	29	30	11	12	27	28	9	10	25	26	7	8	23	24	5	6	21	22	3	4	19	20	1	2	17	18

Therefore Axx = (1, 15, 25, 3, 31, 17, 7, 29)(2, 16, 26, 4, 32, 18, 8, 30)(5, 13, 9, 11, 27, 19, 23, 21)(6, 14, 10, 12, 28, 20, 24, 22)
Next we will consider the matrix B, with the following labeling. t_1, t_2 by 1, 2 respectively. Apply $t_1 \rightarrow t_2$, $t_2 \rightarrow t_1^{16}$ t form the permutation.
B = $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
t_1	t_2	t_1^2	t_2^2	t_1^3	t_2^3	t_1^4	t_2^4	t_1^5	t_2^5	t_1^6	t_2^6	t_1^7	t_2^7	t_1^8	t_2^8	t_1^9	t_2^9	t_1^{10}	t_2^{10}	t_1^{11}	t_2^{11}	t_1^{12}	t_2^{12}	t_1^{13}	t_2^{13}	t_1^{14}	t_2^{14}	t_1^{15}	t_2^{15}	t_1^{16}	t_2^{16}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_2	t_1^{16}	t_2^2	t_1^{15}	t_2^3	t_1^{14}	t_2^4	t_1^{13}	t_2^5	t_1^{12}	t_2^6	t_1^{11}	t_2^7	t_1^{10}	t_2^8	t_1^9	t_2^9	t_1^8	t_2^{10}	t_1^7	t_2^{11}	t_1^6	t_2^{12}	t_1^5	t_2^{13}	t_1^4	t_2^{14}	t_1^3	t_2^{15}	t_1^2	t_2^{16}	t_1
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
2	31	4	29	6	27	8	25	10	23	27	12	21	14	19	16	17	18	15	20	13	22	11	24	9	26	7	28	5	30	3	1

Therefore Byy = (1, 2, 31, 32)(3, 4, 29, 30)(5, 6, 27, 28)(7, 8, 25, 26)(9, 10, 23, 24)(11, 12, 21, 22)(13, 14, 19, 20)(15, 16, 17, 18).
Then we consider C= $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ We label t_1, t_2 by 1, 2 respectively. Apply $t_1 \rightarrow t_2^{16}$, $t_2 \rightarrow t_1^{16}$ to form the permutation.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
t_1	t_2	t_1^2	t_2^2	t_1^3	t_2^3	t_1^4	t_2^4	t_1^5	t_2^5	t_1^6	t_2^6	t_1^7	t_2^7	t_1^8	t_2^8
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_2^{16}	t_1^{16}	t_2^{15}	t_1^{15}	t_2^{14}	t_1^{14}	t_2^{13}	t_1^{13}	t_2^{12}	t_1^{12}	t_2^{11}	t_1^{11}	t_2^{10}	t_1^{10}	t_2^9	t_1^9
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17

17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
t_1^9	t_2^9	t_1^{10}	t_2^{10}	t_1^{11}	t_2^{11}	t_1^{12}	t_2^{12}	t_1^{13}	t_2^{13}	t_1^{14}	t_2^{14}	t_1^{15}	t_2^{15}	t_1^{16}	t_2^{16}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_2^8	t_1^8	t_2^7	t_1^7	t_2^6	t_1^6	t_2^5	t_1^5	t_2^4	t_1^4	t_2^3	t_1^3	t_2^2	t_1^2	t_2	t_1
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Therefore Czz = (1, 32)(2, 31)(3, 30)(4, 29)(5, 28)(6, 27)(7, 26)(8, 25)(9, 24)(10, 23)(11, 22)(12, 21)(13, 20)(14, 19)(15, 18)(16, 17).

Next we want to fine our symmetric presentation for our progenitor. The stabiliser(N,{ 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31 }) where $<t_1>=\{t_1, t_1^2,..., t_1^{17}\}$ and the normilizer of <t> of G. This tells us the number of different conjugates of $<t_1>$ is two. Thus our progenitor 17^{*2}:((2:8):2) is $<x, y, z, t| y^4, z^2, (x, y), x^{-1}zxz, (y^{-1}z)^2, x^{-3}y^2x^{-1}, t^{17}, (t^{(zy)}=t),(t^{(y^5x^5z)}=t^8)>$ where we have $17^{*2}:_m((2:8):2) \cong 17^{*2}:((2:8):2)$

7.6 $13^{*2}:_m(12:2)$

We consider G = (12:2) = < (1, 5, 9)(2, 6, 10)(3, 7, 11)(4, 8, 12), (1, 4, 7, 10)(2, 5, 8, 11)(3, 6, 9, 12), (1, 11)(2, 10)(3, 9)(4, 8)(5, 7) >. We want to find the monomial representatives of (12:2), if possible. In order to see this, we first look at the character table of (12:2).

```
    Character Table of Group G
-------------------------------------
Class |   1  2  3  4  5  6  7   8   9
Size  |   1  1  6  6  2  2  2   2   2
Order |   1  2  2  2  3  4  6  12  12
-------------------------------------
p  =  2   1  1  1  1  5  2  5   7   7
p  =  3   1  2  3  4  1  6  2   6   6
-------------------------------------
X.1   +   1  1  1  1  1  1  1   1   1
X.2   +   1  1 -1 -1  1  1  1   1   1
X.3   +   1  1  1 -1  1 -1  1  -1  -1
X.4   +   1  1 -1  1  1 -1  1  -1  -1
X.5   +   2  2  0  0 -1  2 -1  -1  -1
X.6   +   2  2  0  0 -1 -2 -1   1   1
X.7   +   2 -2  0  0  2  0 -2   0   0
X.8   +   2 -2  0  0 -1  0  1  Z1 -Z1
X.9   +   2 -2  0  0 -1  0  1 -Z1  Z1
```

Now the character table of (12:2) has characters whose degree is greater than one. We note that all characters of degree larger one have degree two. Thus, it is only possible to find monomial representation of (12:2) of degree two. Since there are five characters of degree two, it is possible for (12:2) to have five different monomial representations. All of these will all be irreducible monomial representations. We are looking for faithful monomial representation. So we need to determine which of the five irreducible characters of (12:2) are faithful. We note that there are only two faithful characters namely X.8, X.9, where X.8 = (2, -2, 0, 0, -1, 0, 1, w, -w) and similarly X.9= (2, -2, 0, 0, -1, 0, 1, -w, w). We will consider the character X.9= (2, -2, 0, 0, -1, 0, 1, -2, 2) for this paper. In order for (12:2) to have an irreducible faithful monomial representation, (12:2) is required to have a subgroup of H = (4:3) to the index two in (12:2) and that such (4:3) must have a linear character that induces up to the character X.9 of (12:2).
We find H = (4:3) = <(1, 4, 7, 10)(2, 5, 8, 11)(3, 6, 9, 12), (1, 7)(2, 8)(3, 9)(4, 10)(5, 11)(6, 12), (1, 9, 5)(2, 10, 6)(3, 11, 7)(4, 12, 8)> has an index two in (12:2) and that of (4:3) induces up to the character X.9 of (12:2).
Thus, (12:2) has a faithful irreducible monomial representations of degree 2. The field of entries of the representation is determined by the character values of the character ϕ of (4:3) that is being induced; namely, given by (1, -1, $\mathbb{Z}_{(12)_3}$, -$\mathbb{Z}_{(12)_3}$ - 1, -$\mathbb{Z}_{(12)_4}$, $\mathbb{Z}_{(12)_4}$, $\mathbb{Z}_{(12)_3}$ + 1, -$\mathbb{Z}_{(12)_3}$, -$\mathbb{Z}_{(12)_4}\mathbb{Z}_{(12)_3}$, $\mathbb{Z}_{(12)_4}\mathbb{Z}_{(12)_3}$ + $\mathbb{Z}_{(12)_4}$, $\mathbb{Z}_{(12)_4}\mathbb{Z}_{(12)_3}$, -$\mathbb{Z}_{(12)_4}\mathbb{Z}_{(12)_3}$ - $\mathbb{Z}_{(12)_4}$) has $\mathbb{Z}_{12}$. Since $\phi(6)$ values are twelve roots of unit, the field of entries is the cyclotomic field of the 12th roots of unity. Thus, explicitly this representation is
$\rho : G \longrightarrow GL(C)$, where C=CyclotomicField(12),

ρ (xx) $\begin{bmatrix} T[1]xxT[1]^{-1} & T[1]xxT[2]^{-1} \\ T[2]xxT[1]^{-1} & T[2]xxT[2]^{-1} \end{bmatrix}$

$\rho(yy)$ $\begin{bmatrix} T[1]yyT[1]^{-1} & T[1]yyT[2]^{-1} \\ T[2]yyT[1]^{-1} & T[2]yyT[2]^{-1} \end{bmatrix}$, and

$\rho(zz)$ $\begin{bmatrix} T[1]zzT[1]^{-1} & T[1]zzT[2]^{-1} \\ T[2]zzT[1]^{-1} & T[2]zzT[2]^{-1} \end{bmatrix}$,

where $G =< xx, yy, zz >$ and $G = HT[1] \cup HT[2]$

Then

A = $\begin{bmatrix} -\mathbb{Z}_{(12)}^2 & 0 \\ 0 & \mathbb{Z}_{(12)} - 1 \end{bmatrix}$, B = $\begin{bmatrix} -\mathbb{Z}_{(12)}^3 & 0 \\ 0 & \mathbb{Z}_{(12)}^3 \end{bmatrix}$, D = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

where $A = \rho(xx)$, $B = \rho(yy)$, and $D = \rho(zz)$. Thus, the generators of the faithful irreducible monomial representation are A, B, D. Now the smallest finite field that contains the twelfth root of unity (elements of order 13) is $\mathbb{Z}_{13}$. The elements of order twelve in $\mathbb{Z}_{13}$ are generators (every element of $\mathbb{Z}_{13}$-$\{0\}$ is a power of each of these elements of $\mathbb{Z}_{13}$ - $\{0\}$) of $\mathbb{Z}_{13}$-$\{0\}$.
These elements of order 12 are 2, 6, 10, and 11.

```
Primitiveroot(12);
 2
```

Choose 2:

$$2^1 = 2 \qquad 2^5 = 6 \qquad 2^7 = 11 \qquad 2^{11} = 10$$

We now find a permutation representation of our monomial representation. We denote the permutation representations of A, B, and by Axx, Byy, and Dzz, respectively. Our progenitor is: 13^{*2}:(12:2). We have two t_i's: t_1, t_2. We now interpret the automorphisms given by the three matrices A, B, and D by using the formula $a_{ij} \Leftrightarrow t_i \rightarrow t_j$. First consider A=$\begin{bmatrix} 9 & 0 \\ 0 & 3 \end{bmatrix}$
So $a_{11} = 9$ and $a_{22} = 3$
Then,
$t_1 \rightarrow t_1^9$ and $t_2 \rightarrow t_2^3$.
Since $|t_1| = 12 = |t_2|$ we have twelve distinct powers of each of the two t_i's and to simplify we use modulo 13. We label t_1, t_2 by 1, 2 respectively. Apply $t_1 \rightarrow t_1^9$ and $t_2 \rightarrow t_2^3$ to form the permutation.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
t_1	t_2	t_1^2	t_2^2	t_1^3	t_2^3	t_1^4	t_2^4	t_1^5	t_2^5	t_1^6	t_2^6	t_1^7	t_2^7	t_1^8	t_2^8	t_1^9	t_2^9	t_1^{10}	t_2^{10}	t_1^{11}	t_2^{11}	t_1^{12}	t_2^{12}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_1^9	t_2^3	t_1^{18}	t_2^6	t_1^{27}	t_2^9	t_1^{36}	t_2^{12}	t_1^{45}	t_2^{15}	t_1^{54}	t_2^{18}	t_1^{63}	t_2^{21}	t_1^{72}	t_2^{24}	t_1^{81}	t_2^{27}	t_1^{90}	t_2^{30}	t_1^{99}	t_2^{33}	t_1^{108}	t_2^{36}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
17	6	9	12	1	18	19	24	11	4	3	10	21	16	13	22	5	2	23	8	15	14	7	20

Therefore Axx=(1, 17, 5)(2, 6, 18)(3, 9, 11)(4, 12, 10)(7, 19, 23)(8, 24, 20)(13, 21, 15)(14, 16, 22)
Next,

we consider B = $\begin{bmatrix} 5 & 0 \\ 0 & 8 \end{bmatrix}$ where $a_{11} = 5$ and $a_{22} = 8$

Therefore,

$t_1 \rightarrow t_1^5$

$t_2 \rightarrow t_2^8$.

We label t_1, t_2 by 1, 2 respectively. Apply $t_1 \rightarrow t_1^5$ and $t_2 \rightarrow t_2^8$ to form the permutation.

Therefore the permutation for

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
t_1	t_2	t_1^2	t_2^2	t_1^3	t_2^3	t_1^4	t_2^4	t_1^5	t_2^5	t_1^6	t_2^6	t_1^7	t_2^7	t_1^8	t_2^8	t_1^9	t_2^9	t_1^{10}	t_2^{10}	t_1^{11}	t_2^{11}	t_1^{12}	t_2^{12}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_1^5	t_2^8	t_1^{10}	t_2^{16}	t_1^{15}	t_2^{24}	t_1^{20}	t_2^{32}	t_1^{25}	t_2^{40}	t_1^{30}	t_2^{48}	t_1^{35}	t_2^{56}	t_1^{40}	t_2^{64}	t_1^{45}	t_2^{72}	t_1^{50}	t_2^{80}	t_1^{55}	t_2^{88}	t_1^{60}	t_2^{96}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
9	16	19	6	3	22	13	12	23	2	7	18	17	8	1	24	11	14	21	4	5	20	15	10

Byy=(1, 9, 23, 15)(2, 16, 24, 10)(3, 19, 21, 5)(4, 6, 22, 20)(7, 13, 17, 11)(8, 12, 18, 14)

Then,

the matrix D = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ has a_{12}=1 and a_{21}=1. Therefore,

$t_1 = t_2$

$t_2 = t_1$

We label t_1, t_2 by 1, 2 respectively. Apply $t_1 \rightarrow t_2$ and $t_2 \rightarrow t_1$ to form the permutation.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
t_1	t_2	t_1^2	t_2^2	t_1^3	t_2^3	t_1^4	t_2^4	t_1^5	t_2^5	t_1^6	t_2^6	t_1^7	t_2^7	t_1^8	t_2^8	t_1^9	t_2^9	t_1^{10}	t_2^{10}	t_1^{11}	t_2^{11}	t_1^{12}	t_2^{12}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_2	t_1	t_2^2	t_1^2	t_2^3	t_1^3	t_2^4	t_1^4	t_2^5	t_1^5	t_2^6	t_1^6	t_2^7	t_1^7	t_2^8	t_1^8	t_2^9	t_1^9	t_2^{10}	t_1^{10}	t_2^{11}	t_1^{11}	t_2^{12}	t_1^{12}
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15	18	17	20	19	22	21	24	23

Thus the permutation for

Dzz=(1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16)(17, 18)(19, 20)(21, 22)(23, 24)

Now we want to find a symmetric presentation for our progenitor. The stabiliser(N,{ 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23 } where

$<t_1> = \{t_1, t_1^2, ..., t_1^{13}\}$ and $\{g \epsilon N \mid <t>^g = <t>\}$ is called the normalizer of <t> in G. This tells us the number of different conjugates of $<t_1>$ is two. Thus, our presentation of the progenitor 13^{*2}:(12:2) is $<x,y,z,t \mid x^3, y^4, z^2, (x, y), (x^{-1}z)^2, (y^{-1}z)^2, t^{13}, (t^{(x^2)}=t^3),(t^{(y^3)}=t^8)>$ where we have $13^{*2}:_m(12:2) \cong 13^{*2}:(12:2)$.

We check using Grindstaff's Lemma:

We know $(t_1, t_2)=1 \rightarrow (t_1^1, t_2^1)=1$. Thus $(t_1, t^z)=1$ gives $(t_1^i, t_2^j = 1)$ $\forall$ i,j = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 }

```
G<x,y,z,t>:=Group<x,y,z,t|
 x^3 ,
y^4 ,
z^2 ,
(x, y) ,
(x^-1z)^2 ,
(y^-1z)^2 ,
t^13,
(t^(x^2)=t^3),(t^(y^3)=t^8),
(t,t^z)>;
#G;
4056
Index(G,sub<G|x,y,z>);
24
#N;
24
13^224;
4056
```

Next we obtain a list of elements of the form πt_i such that $\pi \epsilon$ N, up to conjugacy. One element from each of the 9 conjugacy classes of PSL(2,13) will be considered. For example (y^2) is in class 2 so (y^2t_{17}) is one element of the required form. Since the centralizer of (y^2) is transitive the other elements of $(y^2)t_i$ are all conjugate to (y^2t_{17}). Thus by looking at the centralizer of each class representative a list of distinct (up to conjugacy) elements of the form πt_i is found. In the table below we list the conjugacy classes and their representatives.

Conjugacy Classes		
Class	Class Representative	Elements of the form πt_i
2	(y^2) = (1, 23)(2, 24)(3, 21)(4, 22)(5, 19)(6, 20)(7, 17)(8, 18)(9, 15)(10, 16)(11, 13)(12, 14)	$(y^2 t_{17})$
3	z = (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16)(17, 18)(19, 20)(21, 22)(23, 24)	zt_2, zt_3, zt_5,zt_7, zt_9, zt_{11}
4	(yz) = (1, 10)(2, 15)(3, 20)(4, 5)(6, 21)(7, 14)(8, 11)(9, 24)(12, 17)(13, 18)(16, 23)(19, 22)	$(yz)t_{10}$, $(yz)t_2$, $(yz)t_3$, $(yz)t_4$, $(yz)t_7$, $(yz)t_8$
5	x = (1, 17, 5)(2, 6, 18)(3, 9, 11)(4, 12, 10)(7, 19, 23)(8, 24, 20)(13, 21, 15)(14, 16, 22)	xt_{17}, xt_2
6	y = (1, 9, 23, 15)(2, 16, 24, 10)(3, 19, 21, 5)(4, 6, 22, 20)(7, 13, 17, 11)(8, 12, 18, 14)	yt_9, yt_2
7	(y^2x^{-1}) = (1, 19, 17, 23, 5, 7)(2, 8, 6, 24, 18, 20)(3, 13, 9, 21, 11, 15)(4, 16, 12, 22, 10, 14)	$(y^2x^{-1})t_{19}$, $(y^2x^{-1})t_2$
8	(xy) = (1, 11, 19, 15, 17, 3, 23, 13, 5, 9, 7, 21)(2, 22, 8, 10, 6, 14, 24, 4, 18, 16, 20, 12)	$(xy)t_{11}$, $(xy)t_2$
9	(yx^-1) = (1, 3, 7, 15, 5, 11, 23, 21, 17, 9, 19, 13)(2, 14, 20, 10, 18, 22, 24, 12, 6, 16, 8, 4)	$(yx^-1)t_3$, $(yx^-1)t_2$

7.7 $3^{*3}:_m((2^3):(3:2))$

Let H = ((4:2):(2)) such that H = ((4:2):(2)) = $<$ (2, 3)(4, 5), (1, 5)(2, 6)(3, 7)(4, 8), (1, 3)(2, 8)(4, 6)(5, 7), (1, 2)(3, 8)(4, 7)(5, 6) $>$ with the following character table for H.

```
Character Table of Group H
--------------------------------------
Class |   1  2  3  4  5  6  7  8  9 10
Size  |   1  1  1  1  2  2  2  2  2  2
Order |   1  2  2  2  2  2  2  2  4  4
--------------------------------------
p  =  2   1  1  1  1  1  1  1  1  2  2
--------------------------------------
X.1   +   1  1  1  1  1  1  1  1  1  1
X.2   +   1  1 -1 -1  1 -1  1 -1  1 -1
X.3   +   1  1 -1 -1 -1  1 -1  1  1 -1
X.4   +   1  1  1  1 -1 -1 -1 -1  1  1
X.5   +   1  1 -1 -1 -1  1  1 -1 -1  1
```

```
X.6    +   1  1 -1 -1  1 -1 -1  1 -1  1
X.7    +   1  1  1  1 -1 -1  1  1 -1 -1
X.8    +   1  1  1  1  1  1 -1 -1 -1 -1
X.9    +   2 -2 -2  2  0  0  0  0  0  0
X.10   +   2 -2  2 -2  0  0  0  0  0  0
```

We consider G = ((2^3):(3:2)) = < (1, 4)(2, 5)(3, 6)(7, 8), (1, 8, 2, 3)(4, 6, 7, 5)>. We want to find the monomial representative of ((2^3):(3:2)), if possible. In order to see if this is possible we consider the character table of ((2^3):(3:2)).

```
Character Table of Group G
----------------------------
Class |   1  2  3  4  5  6  7  8  9 10
Size  |   1  1  3  3  6  6  8  6  6  8
Order |   1  2  2  2  2  2  3  4  4  6
--------------------------------------
p  =  2   1  1  1  1  1  1  7  3  3  7
p  =  3   1  2  3  4  5  6  1  8  9  2
--------------------------------------
X.1    +   1  1  1  1  1  1  1  1  1  1
X.2    +   1 -1  1 -1  1 -1  1 -1  1 -1
X.3    +   1  1  1  1 -1 -1  1 -1 -1  1
X.4    +   1 -1  1 -1 -1  1  1  1 -1 -1
X.5    +   2 -2  2 -2  0  0 -1  0  0  1
X.6    +   2  2  2  2  0  0 -1  0  0 -1
X.7    +   3  3 -1 -1  1  1  0 -1 -1  0
X.8    +   3 -3 -1  1 -1  1  0 -1  1  0
X.9    +   3 -3 -1  1  1 -1  0  1 -1  0
X.10   +   3  3 -1 -1 -1 -1  0  1  1  0
```

Now the character table of ((2^3):(3:2)) has characters whose degree is greater than one. It should be noted that all characters larger than one have degree two and three. Since there are two characters of degree two and four characters of degree three, it is possible for ((2^3):(3:2)) to have six different monomial representations. All of these will be irreducible monomial representations. So we need to determine which of the six irreducible characters of ((2^3):(3:2)) are faithful. We note that there is one faithful character namely X.8, where X.8 = (3, -3, -1, 1, 1, -1, 1, 0, -1, 1, 0). In order for ((2^3):(3:2)) to have an irreducible faithful monomial representation, ((2^3):(3:2)) is required to have a subgroup of H to the index three in ((2^3):(3:2)) and that such H must be a linear character that induces up to the character X.8 of ((2^3):(3:2)).

We find H = ((4:2):(2)) = < (2, 3)(4, 5), (1, 5)(2, 6)(3, 7)(4, 8), (1, 3)(2, 8)(4, 6)(5, 7), (1, 2)(3, 8)(4, 7)(5, 6) > has an index of three in ((2^3):(3:2)) and that H induces up to the character X.8 of ((2^3):(3:2)).

Therefore, ((2^3):(3:2)) has a faithful irreducible monomial representations of degree two. The field entries of the representation is determined by the character values ϕ of H that is being induced, which brings our attention to $\mathbb{Z}_2$ since the character values are 2nd roots of unity. Therefore, explicit this representation is $\rho : G \longrightarrow GL(C)$, where C=CyclotomicField(2).

ρ (xx) =

$T[1]xxT[1]^{-1}$ $T[1]xxT[2]^{-1}$ $T[1]xxT[3]^{-1}$
$T[2]xxT[1]^{-1}$ $T[2]xxT[2]^{-1}$ $T[2]xxT[3]^{-1}$
$T[3]xxT[1]^{-1}$ $T[3]xxT[2]^{-1}$ $T[3]xxT[3]^{-1}$

ρ (yy) =

$T[1]yyT[1]^{-1}$ $T[1]yyT[2]^{-1}$ $T[1]yyT[3]^{-1}$
$T[2]yyT[1]^{-1}$ $T[2]yyT[2]^{-1}$ $T[2]yyT[3]^{-1}$
$T[3]yyT[1]^{-1}$ $T[3]yyT[2]^{-1}$ $T[3]yyT[3]^{-1}$

where G= <xx, yy> and G = HT[1] ∪ HT[2] ∪ HT[3]

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

where A = ρ(xx) and B = ρ(yy).

Therefore the generators of the faithful irreducible monomial representation are A and B. Now the smallest finite field that contains the second root of unity (elements of order 3) is $\mathbb{Z}_3$. The elements of order two in $\mathbb{Z}_3$ are generators of $\mathbb{Z}_3$-{0}.

Now we find a permutation representation of our monomial representation. We denote the permutation representative A, B, And Axx, Byy, respectively. Our progenitor is 3^{*3}:((2^3):(3:2)). We have three t_i's: t_1, t_2, t_3. We will now interpret the automorphisms given by the two matrices A and B by using the formula $a_{ij} \Leftrightarrow t_i \rightarrow t_j$.

First we consider the A matrix,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

So,

$a_{1,2}$ = 1

$a_{2,1}$ = 1

$a_{3,3}$ = -1

Then we have,

$a_{1,2}$, $t_1 \to t_2$

$a_{1,1}$, $t_2 \to t_1$

$a_{3,3}$, $t_3 \to t_3^{-1}$

We have two distinct powers of each of three t_i's and to simplify we will use modulo 7. We label t_1, t_2, t_3, t_1^2, t_2^2 and t_3^2 are 1, 2, 3, 4, 5, 6 respectively. Apply $t_1 \to t_2$, $t_2 \to t_1$, and $t_3 \to t_3^{-1}$ to form the permutation.

1	2	3	4	5	6
t_1	t_2	t_3	t_1^2	t_2^2	t_3^2
↓	↓	↓	↓	↓	↓
t_2	t_1	t_3^{-1}	$(t_2)^2$	$(t_1)^2$	$(t_3^{-1})^2$
↓	↓	↓	↓	↓	↓
2	1	5	5	4	3

Therefore our permutation for Axx = (1, 2)(3, 6)(4, 5).

Next we consider

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$b_{1,3} = 1$

$b_{22} = -1$

$b_{3,1} = -1$

Then we have

$t_1 \to t_3$

$t_2 \to t_2^{-1}$

$t_3 \to t_1^{-1}$

We label t_1, t_2,t_3, t_3 by 1, 2, 3 respectively. Apply $t_1 \to t_3$, $t_2 \to t_2^{-1}$, and $t_3 \to t_1^{-1}$.

Thus our permutation for Byy = (1, 3, 4, 6)(2, 5).

Now we want to find a symmetric presentation for our progenitor. The stabiliser (N, {1,4}) where, $<t_1> = \{t_1, t_1^3\}$ and $\{g \in N \mid <t>^g = <t>\}$ is called the normalizer of $< t >$ in G. Thus tells us the number of different conjugates of $< t_1 >$ is two. Thus our

1	2	3	4	5	6
t_1	t_2	t_3	t_1^2	t_2^2	t_3^2
↓	↓	↓	↓	↓	↓
t_3	t_2^{-1}	t_1^{-1}	t_3^2	$(t_2^{-1})^2$	$(t_1^{-1})^2$
↓	↓	↓	↓	↓	↓
3	2	4	6	5	1

presentation of the progenitor 3^{*3}:((2^3):(3:2)) is

G<x,y,t>:=Group<x,y,t| x^2 , y^4 ,

$y^{-1}xy^{-2}xy^{-1}xy^2x$, $y^{-1}xy^{-1}xy^{-1}xyxyxyx$,

t^9,

(t,yxy),

(t,xyx $y^{-3}x^{-1}y^{-2}x^{-1}y^{-2}x^{-1}y^{-1}$),

$t^{(y^2)}=t^2$ >

where we have $3^{*3}:_m((2^3):(3:2)) \cong 3^{*3}:((2^3):(3:2))$.

We check using Grindstaff's lemma, if we add (t_1,t_2), (t_1, t_3), (t_2, t_3) to the presentation of ((2^3):(3:2)) then the |((2^3):(3:2))| = | 3^3:((2^3):(3:2))| = 2748 = 1296 or index of G in respects to x and y is 3^3. Now $t_1 = t$, $t_2 = t^x$, $t_3 = t^y$ then we have

G<x,y,t>:=Group<x,y,t| x^2 , y^4 , $y^{-1}xy^{-2}xy^{-1}xy^2x$,

$y^{-1}xy^{-1}xy^{-1}xyxyxyx$,

t^9, (t,yxy),

(t,xyx $y^{-3}x^{-1}y^{-2}x^{-1}y^{-2}x^{-1}y^{-1}$), $t^{(y^2)}=t^2$,

(t,t^x),(t,t^y),(t^x,t^y) >

#G;

1296

Index(G,sub<G|x,y>);

27

Which gives the order of G to be 1296 and the index of G in respects to x and y is 27 = 3^3 thus our progenitor is correct.

7.8 $7^{*3}:_m(2:(2:4))$

Let H = ((4:2):(2)) = < (2, 3)(4, 5), (1, 5)(2, 6)(3, 7)(4, 8), (1, 3)(2, 8)(4, 6)(5, 7), (1, 2)(3, 8)(4, 7)(5, 6) > where the order of ((4:2):(2)) is 16.

Next we will investigate the classes of H where we have

Class 2 Representative: xy^2x

Class 3 Representative: $xyxy^{-1}xy^{-1}$

Class 4 Representative: $(xy)^3$

Class 5 Representative: $xyxyxy^{-1}$

Class 6 Representative: $(xy^2)^2$

Class 7 Representative: $xy^{-1}xy^2$

Class 8 Representative: x^y

Class 9 Representative: $y^{-1}xy^{-1}$

Class 10 Representative: xyx

We also have the character table of H, which is as follows.

Character Table of Group H

Class		1	2	3	4	5	6	7	8	9	10
Size		1	1	1	1	2	2	2	2	2	2
Order		1	2	2	2	2	2	2	2	4	4
p =	2	1	1	1	1	1	1	1	1	2	2
X.1	+	1	1	1	1	1	1	1	1	1	1
X.2	+	1	1	-1	-1	1	-1	1	-1	1	-1
X.3	+	1	1	-1	-1	-1	1	-1	1	1	-1
X.4	+	1	1	1	1	-1	-1	-1	-1	1	1
X.5	+	1	1	-1	-1	-1	1	1	-1	-1	1
X.6	+	1	1	-1	-1	1	-1	-1	1	-1	1
X.7	+	1	1	1	1	-1	-1	1	1	-1	-1
X.8	+	1	1	1	1	1	1	-1	-1	-1	-1
X.9	+	2	-2	-2	2	0	0	0	0	0	0
X.10	+	2	-2	2	-2	0	0	0	0	0	0

We consider G = (2:(2:4)) = < (1, 4)(2, 5)(3, 6)(7, 8), (1, 8, 2, 3)(4, 6, 7, 5)>. We want to find the monomial representative of (2:(2:4)), if possible. In order to see if this is possible we have to consider the character table of (2:(2:4)).

Character Table of Group G

```
----------------------------
Class |    1  2  3  4  5  6  7  8  9 10
Size  |    1  1  3  3  6  6  8  6  6  8
Order |    1  2  2  2  2  2  3  4  4  6
--------------------------------------
p  =  2    1  1  1  1  1  1  7  3  3  7
p  =  3    1  2  3  4  5  6  1  8  9  2
--------------------------------------
X.1    +   1  1  1  1  1  1  1  1  1  1
X.2    +   1 -1  1 -1  1 -1  1 -1  1 -1
X.3    +   1  1  1  1 -1 -1  1 -1 -1  1
X.4    +   1 -1  1 -1 -1  1  1  1 -1 -1
X.5    +   2 -2  2 -2  0  0 -1  0  0  1
X.6    +   2  2  2  2  0  0 -1  0  0 -1
X.7    +   3  3 -1 -1  1  1  0 -1 -1  0
X.8    +   3 -3 -1  1 -1  1  0 -1  1  0
X.9    +   3 -3 -1  1  1 -1  0  1 -1  0
X.10   +   3  3 -1 -1 -1 -1  0  1  1  0
```

Now the character table of (2:(2:4)) has characters whose degree is greater than one. It should be noted that all characters of degree larger than one have degree two and three. Since there are two characters of degree two and four characters of degree three, it is possible for (2:(2:4)) to have six different monomial representations. All of these will be irreducible monomial representations. So we need to determine which of the six irreducible characters of (2:(2:4)) are faithful. We note that there is one faithful character namely X.8, where X.8 = (3, -3, -1, 1, -1, 1, 0, -1, 1, 0). In order for (2:(2:4)) to have an irreducible faithful monomial representation, (2:(2:4)) is required to have a subgroup of H to the index three in (2:(2:4)) and that such H must have a linear character that induces up to that character X.8 of (2:(2:4)).

We find ((4:2):(2)) = H = ((4:2):(2)) = $<$ (2, 3)(4, 5), (1, 5)(2, 6)(3, 7)(4, 8), (1, 3)(2, 8)(4, 6)(5, 7), (1, 2)(3, 8)(4, 7)(5, 6) $>$ has an index of three in (2:(2:4)) and that H induces up to the character X.8 of (2:(2:4)).

Therefore, (2:(2:4)) has a faithful irreducible monomial representations of degree three. The field entries of the representation is determined by the character values ϕ of H that is being induced, which brings our attention to $\mathbb{Z}_3$ since the character values are 3rd roots of unity. Therefore, explicitly this representation is $\rho : G \longrightarrow GL(C)$, where C=CyclotomicField(3).

ρ (xx) =

$T[1]xxT[1]^{-1}$ $T[1]xxT[2]^{-1}$ $T[1]xxT[3]^{-1}$
$T[2]xxT[1]^{-1}$ $T[2]xxT[2]^{-1}$ $T[2]xxT[3]^{-1}$
$T[3]xxT[1]^{-1}$ $T[3]xxT[2]^{-1}$ $T[3]xxT[3]^{-1}$

ρ (yy) =
$T[1]yyT[1]^{-1}$ $T[1]yyT[2]^{-1}$ $T[1]yyT[3]^{-1}$
$T[2]yyT[1]^{-1}$ $T[2]yyT[2]^{-1}$ $T[2]yyT[3]^{-1}$
$T[3]yyT[1]^{-1}$ $T[3]yyT[2]^{-1}$ $T[3]yyT[3]^{-1}$
where G= <xx, yy> and G= HT[1] ∪ HT[2] ∪ HT[3]

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

where A = ρ(xx) and B = ρ(yy).

Therefore the generators of the faithful irreducible monomial representation are A and B. Now the smallest finite field that contais the third root of unity (elements of order 7) is $\mathbb{Z}_7$. The elements of order three in $\mathbb{Z}_7$ are generators of $\mathbb{Z}_7$-{0}.

We now find a permutation representation of our monomial representation. We denote the permutation representative of A, B and Axx, Byy respectively. Our progenitor is 7^{*3}:(2:(2:4)). We have three t_i's: t_1, t_2, t_3. We will now interpret the automorphisms given by the two matrices A and B by using the formula $a_{ij} \Leftrightarrow t_i \to t_j$.

First we will consider the A matrix,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

So,

$a_{1,2}$ = 1

$a_{2,1}$ = 1

$a_{3,3}$ = -1

Then we have,

$a_{1,2}$, $t_1 \to t_2$

$a_{1,1}$, $t_2 \to t_1$

$a_{3,3}$, $t_3 \to t_3^{-1}$

We have two distinct powers of each of three t_i's and to simplify we will use modulo 7. We label t_1, t_2, t_3, t_1^2, t_2^2 and t_3^2 are 1, 2, 3, 4, 5, 6 respectively. Apply $t_1 \to t_2$, $t_2 \to t_1$, and $t_3 \to t_3^{-1}$ to form the permutation.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
t_1	t_2	t_3	t_1^2	t_2^2	t_3^2	t_1^3	t_2^3	t_3^3	t_1^4	t_2^4	t_3^4	t_1^5	t_2^5	t_3^5	t_1^6	t_2^6	t_3^6
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_2	t_1	t_3^6	t_2^2	t_1^2	t_3^6	t_2^3	t_1^3	t_3^4	t_2^4	t_1^4	t_3^3	t_2^5	t_1^5	t_3^2	t_2^6	t_1^6	t_3
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
2	1	18	5	4	15	8	7	12	11	10	9	14	13	6	17	16	3

Thus,

Axx = (1, 2)(3, 18)(4, 5)(6, 15)(7, 8)(9, 12)(10, 11)(13, 14)(16, 17)

Next we consider

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$b_{1,3} = 1$

$b_{22} = -1$

$b_{3,1} = -1$

Then we have

$t_1 \rightarrow t_3$

$t_2 \rightarrow t_2^{-1}$

$t_3 \rightarrow t_1^{-1}$

We label t_1, t_2,t_3, t_3 by 1, 2, 3 respectively. Apply $t_1 \rightarrow t_3$, $t_2 \rightarrow t_2^{-1}$, and $t_3 \rightarrow t_1^{-1}$.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
t_1	t_2	t_3	t_1^2	t_2^2	t_3^2	t_1^3	t_2^3	t_3^3	t_1^4	t_2^4	t_3^4	t_1^5	t_2^5	t_3^5	t_1^6	t_2^6	t_3^6
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
t_3	t_2^6	t_1^6	t_3^2	t_2^5	t_1^5	t_3^3	t_2^4	t_1^4	t_3^4	t_2^3	t_1^3	t_3^5	t_2^2	t_1^2	t_3^6	t_2	t_1
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
3	17	16	6	14	13	9	11	10	12	8	7	15	5	4	18	2	1

Therefore we have the permutation B = (1, 3, 16, 18)(2, 17)(4, 6, 13, 15)(5, 14)(7, 9, 10, 12)(8,11)

Now we want to find a symmetric presentation for our progenitor. The stabiliser (N, {1,4,7} where, $<t_1> = \{t_1, t_1^2, ..., t_1^6\}$ and $\{g \epsilon N \mid <t>^g = <t>\}$ is called the normilzer of $< t >$ in G. Thus tells us the numbrt of different conjugates of $< t_1 >$ is two. Thus our presentation of the progenitor 7^{*3}:(2:(2:4)) is

G<x,y,t>:=Group<x,y,t| x^2 , y^4 , $y^{-1}xy^{-2}xy^{-1}xy^2x$, $y^{-1}xy^{-1}xy^{-1}xyxyxyx$, t^7,
(t, $(y^{-1}xy^{-1})$)>

where we have $7^{*3}:_m(2:(2:4)) \cong 7^{*3}:(2:(2:4))$.

We check using Grindstaff's lemma

G<x,y,t>:=Group<x,y,t| x^2 , y^4 ,
$y^{-1}xy^{-2}xy^{-1}xy^2x$,
$y^{-1}xy^{-1}xy^{-1}xyxyxyx$,
t^3,
(t,$(y^{-1}xy^{-1})$),
($t^{(xy^2x)}$,$t^{(y^2xy)}$),
($t^{(xy^2x)}$,t^y),
($t^{(y^2xy)}$,t^y)>

Using the following magma code

print Index(G,sub<G|x,y>: CosetLimit:=9^{10}, Hard:=true, Print:=2);

We get true for our index

N;

48

Chapter 8

Symmetric Presentations

8.1 2^{*12}:(2^3:2^2)

Let there be an N = (2^3:2^2) where N is of order 48 and is generated by <(1, 8)(2, 7)(3, 6)(4, 5)(9, 12)(10, 11), (1, 6, 7, 12)(2, 11)(3, 10, 9, 4)(5, 8) > where x= (1, 8)(2, 7)(3, 6)(4, 5)(9, 12)(10, 11) and y=(1, 6, 7, 12)(2, 11)(3, 10, 9, 4)(5, 8). We are able to build a progenitor for our N. We wish to write symmetric presentation for the progenitor 2^{*12}:(2^3:2^2). A presentation for N is

(G<x,y>=<x,y,t| x^2 , y^4 , $y^{-1}xy^{-2}xy^{-1}xy^2x$, $y^{-1}xy^{-1}xy^{-1}xyxyxyx$), t^2, Stabilisers of (N,1) >

We need to find the stabilisers of (N,1) meaning we need to find permutations contained in N, that fix 1. We find the following permutations stabilise 1.

(2, 8)(3, 9)(5, 11)(6, 12)=(xy^2x)

(3, 9)(6, 12)=($xyxy^{-1}xy$)

We will add these additional words into our progenitor to complete it.

G<x,y>=<x,y,t| x^2 , y^4 ,

$y^{-1}xy^{-2}xy^{-1}xy^2x$, $y^{-1}xy^{-1}xy^{-1}xyxyxyx$,

t^2, (xy^2x), ($xyxy^{-1}xy$) >

However, the question is raised, how do we know our progenitor is in fact correct? To understand if we do indeed build the correct progenitor we apply Grindstaff's Lemma, where we look at the orbits of N1, and find what word (permutation) takes one to an orbit representative and we continue this process until all orbits are exhausted. Therefore, the orbits of N1 include {1}, {4}, {7}, {10}, {2, 8}, {3, 9}, {5, 11}, {6,12} we will choose the following orbit representatives 3, 4, 5, 6, 7, 8, 10.

$1^{(xyxy)}$=3

$1^{(xyx)}$=4

$1^{(xy)}$=5

$1^{(y)}$=6

$1^{(y^2)}$=7

$1^{(x)}$=8

$1^{(xyxy^2)}$=10

Next we will put the following permutations into a t-cycle and then add them into our progenitor.

(t,$t^{(xyxy)}$),

(t,$t^{(xyx)}$),

(t,$t^{(xy)}$),

(t,$t^{(y)}$),

(t,$t^{(y^2)}$),

(t,$t^{(x)}$),

(t,$t^{(xyxy^2)}$)

G<x,y>=<x,y,t| x^2 , y^4 , $y^{-1}xy^{-2}xy^{-1}xy^2x$, $y^{-1}xy^{-1}xy^{-1}xyxyxyx$, t^2, (xy^2x), ($xyxy^{-1}xy$), (t, $t^{(xyxy)}$), (t, $t^{(xyx)}$), (t, $t^{(xy)}$), (t, $t^{(y)}$), (t, $t^{(y^2)}$), (t, $t^{(x)}$), (t, $t^{(xyxy^2)}$)> what we want is $2^{12}(48)$ = 196608 if we get this we know our progenitor is correct. Using magma to compute the order of our G we find that

#G;

196608

Thus our progenitor is correct. Next we will use Lemma 3.3 to find additional relations to add to our progenitor. Lemma 3.3 is only applicable to t_1t_2 where permutations need to be of order two, and need to stabilise t_1t_2. We will look at the set of permutations of the centraliser in N of the point-stabiliser in N of t_1 and t_2. We find the following permutations:

(1, 8, 7, 2)(3, 12)(4, 5, 10, 11)(6, 9),

(1, 7)(2, 8)(4, 10)(5, 11),

(1, 8)(2, 7)(3, 12)(4, 5)(6, 9)(10, 11),

(3, 9)(6, 12),
(1, 2)(3, 6)(4, 11)(5, 10)(7, 8)(9, 12),
(1, 7)(3, 9)(4, 10)(6, 12),
(2, 8)(3, 9)(5, 11)(6, 12),
(1, 2, 7, 8)(3, 6)(4, 11, 10, 5)(9, 12),
(1, 7)(2, 8)(3, 9)(4, 10)(5, 11)(6, 12),
(1, 2)(3, 12)(4, 11)(5, 10)(6, 9)(7, 8),
Identity,
(1, 2, 7, 8)(3, 12)(4, 11, 10, 5)(6, 9),
(2, 8)(5, 11),
(1, 8, 7, 2)(3, 6)(4, 5, 10, 11)(9, 12),
(1, 7)(4, 10),
(1, 8)(2, 7)(3, 6)(4, 5)(9, 12)(10, 11)
However, the only applicable permutations are the following
(3, 9)(6, 12) = ($xyxy^{-1}xy$)
(1, 2)(3, 6)(4, 11)(5, 10)(7, 8)(9, 12) = (y^2xy^2)
(1, 2)(3, 12)(4, 11)(5, 10)(6, 9)(7, 8) = (yxyxy)
It should be noted that selected permutations that do not contain (1,2), when added to the permutation are of even order, and should be added into the permutation as follows ($(tt^{(xyxy^{-2}xy)})^k$=($xyxy^{-1}xy$) note that ($tt^{(xyxy^{-2}xy)}$) is simply t_1t_2 it is necessary we add this equal to the permutation since there is no (1,2) contained in its permutation, and we put it to the power of k, for when we find first order relations they will go from different powers ranging from [0..10], but for the relations found using lemma 3.3 they are of even or odd power. If they are of even power it means that the permutation(s) found do not have (1,2) contained in their permutation hence why it is necessary to input t_1t_2 to be equal to that sa1 permutation(s) and when it comes to adding the permutation to first order relations the even relation found from lemma 3.3 their power ranges from [2..10 by 2]. In the case of a permutation that contains (1,2) to be added into the progenitor it is the word of that progenitor multiplied by t. So for example using (1, 2)(3, 6)(4, 11)(5, 10)(7, 8)(9, 12) = (y^2xy^2) to add this permutation into our progenitor we will have ((y^2xy^2)t) and we will raise this to an odd power ranges such as [3..9 by 2].

8.2 2^{*24}:(2^3:(2:3))

Let there by an N= (2^3:(2:3)) where N is of order 48 and is generated by <(1, 6, 12, 23)(2, 5, 11, 24)(3, 8, 14, 19)(4, 7, 13, 20)(9, 15, 22, 18)(10, 16, 21, 17), (1, 24, 4, 18, 21, 14)(2, 23, 3, 17, 22, 13)(5, 7, 11, 10, 19, 16)(6, 8, 12, 9, 20, 15) > where x= (1, 6, 12, 23)(2, 5, 11, 24)(3, 8, 14, 19)(4, 7, 13, 20)(9, 15, 22, 18)(10, 16, 21, 17) and y=(1, 24, 4, 18, 21, 14)(2, 23, 3, 17, 22, 13)(5, 7, 11, 10, 19, 16)(6, 8, 12, 9, 20, 15). We wish to write a symmetric presentation for the progenitor 2^{*24}:N. A presentation for N is (G<x,y> = <x,y,t| x^4 , $(x^{-1}y^{-1})^2$, $(xy^{-2})^2$, y^6,) t^2, Stabilisers of (N,1) > We need to find the stabilisers of (N,1) meaning we need to find words contained in N, that fix t_1. We find the following words stabilise t_1.

(3, 22)(4, 21)(5, 8)(6, 7)(9, 19)(10, 20)(11, 15)(12, 16)(13, 23)(14, 24)=($x^2y^{-1}xy$)

We will add these additional permutations into our progenitor to complete it.

G<x,y>=<x,y,t| x^4 , $(x^{-1}y^{-1})^2$, $(xy^{-2})^2$, y^6, t^2, ($x^2y^{-1}xy$)>

To understand if we do indeed build the correct progenitor we apply Grindstaff's Lemma, where we look at the orbits of N1, and find what word (permutation) takes one to an orbit representative and we continue this process until all orbits are exhausted. Therefore, the orbits of N1 include {1}, {2}, {17}, {18}, {3,22}, {4,21}, {5,8}, {6,7}, {9,19}, {10,20}, {11,15}, {12,16}, {13,23}, {14,24} we will choose the following orbit representatives 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 17, 18, 24.

$1^{(x^3y^{-1})}$=2

$1^{(x^3y)}$=3

$1^{(y^2)}$=4

$1^{(xyx^2y^{-4})}$=5

$1^{(x)}$=6

$1^{(xy^5x^{-1})}$=9

$1^{(xyx^2y^{-1})}$=10

$1^{(x^3y^{-1}x^2)}$=11

$1^{(xy^2)}$=12

$1^{(x^3y^4)}$=13

$1^{(y^4x)}$=17

$1^{(y^3)}$=18

$1^{(y)}$=24

Next we will put the following permutations into a t-cycle and then add them into our progenitor.

$(t,t^{(x^3y^{-1})})$

$(t,t^{(x^3y)})$

$(t,t^{(y^2)})$

$(t,t^{(xyx^2y^{-4})})$

$(t,t^{(x)})$

$(t,t^{(xy^5x^{-1})})$

$(t,t^{(xyx^2y^{-1})})$

$(t,t^{(x^3y^{-1}x^2)})$

$(t,t^{(xy^2)})$

$(t,t^{(x^3y^4)})$

$(t,t^{(y^4x)})$

$(t,t^{(y^3)})$

$(t,t^{(y)})$

G<x,y>=<x,y,t| x^4 , $(x^{-1}y^{-1})^2$, $(xy^{-2})^2$, y^6, t^2, $(x^2y^{-1}xy)$, $(t, t^{(x^3y^{-1})})$, $(t, t^{(x^3y)})$, $(t, t^{(y^2)})$, $(t, t^{(xyx^2y^{-4})})$, $(t, t^{(x)})$, $(t, t^{(xy^5x^{-1})})$, $(t, t^{(xyx^2y^{-1})})$, $(t, t^{(x^3y^{-1}x^2)})$, $(t, t^{(xy^2)})$, $(t, t^{(x^3y^4)})$, $(t, t^{(y^4x)})$, $(t, t^{(y^3)})$, $(t, t^{(y)})$> what we want is $2^{24}(48) = 805306368$ if we get this we know our progenitor is correct. Using magma to compute the order of our G we find that

#G;

0

Our number must be very large if the order of G is given to be zero. Therefore, we must use

print Index(G,sub<G|x,y>: CosetLimit:=9^{10}, Hard:=true, Print:=2);

INDEX = 16777216 (a=16777216 r=16911 h=16778532 n=16778532; l=33560 c=149.39; m=16777216 t=16778531) 16777216

Thus our progenitor is correct. Next we will use Lemma 3.3 to find additional relations to add to our progenitor. Lemma 3.3 is only applicale to t_1t_2 where permutations need to be of order two, and need to stabilise t_1t_2. We will look at the set of permutations of the centraliser in N of the point-stabiliser in N of t_1 and t_2. We find the following permutations:

(1, 18)(2, 17)(3, 13)(4, 14)(5, 10)(6, 9)(7, 19)(8, 20)(11, 16)(12, 15)(21, 24)(22, 23)
(1, 17)(2, 18)(3, 8)(4, 7)(5, 22)(6, 21)(9, 24)(10, 23)(11, 15)(12, 16)(13, 20)(14, 19)
Identity
(1, 17)(2, 18)(3, 5)(4, 6)(7, 21)(8, 22)(9, 14)(10, 13)(19, 24)(20, 23)
(1, 18)(2, 17)(3, 23)(4, 24)(5, 20)(6, 19)(7, 9)(8, 10)(11, 12)(13, 22)(14, 21)(15, 16)
(3, 22)(4, 21)(5, 8)(6, 7)(9, 19)(10, 20)(11, 15)(12, 16)(13, 23)(14, 24)
(1, 2)(3, 20)(4, 19)(5, 23)(6, 24)(7, 14)(8, 13)(9, 21)(10, 22)(11, 12)(15, 16)(17, 18)
(1, 2)(3, 10)(4, 9)(5, 13)(6, 14)(7, 24)(8, 23)(11, 16)(12, 15)(17, 18)(19, 21)(20, 22)
However, the only applicable permutations are the following
(1, 2)(3, 20)(4, 19)(5, 23)(6, 24)(7, 14)(8, 13)(9, 21)(10, 22)(11, 12)(15, 16)(17, 18) = (xy^2xy^{-1})
(3, 22)(4, 21)(5, 8)(6, 7)(9, 19)(10, 20)(11, 15)(12, 16)(13, 23)(14, 24) = $(x^2y^{-1}xy)$
(1, 2)(3, 10)(4, 9)(5, 13)(6, 14)(7, 24)(8, 23)(11, 16)(12, 15)(17, 18)(19, 21)(20, 22) = (yx)
Therefore by lemma 3.3 we have the following progenitors that contain the permutations above as different cases.

Case: (xy^2xy^{-1}) ODD

for k in [2..10 by 2] do G<x,y>=<x,y,t| x^4 , $(x^{-1}y^{-1})^2$, $(xy^{-2})^2$, y^6, t^2, $(x^2y^{-1}xy)$, $((xy^2xy^{-1})t)^k$ > k, #G; end for;

Case: $(x^2y^{-1}xy)$ EVEN

G<x,y>=<x,y,t| x^4 , $(x^{-1}y^{-1})^2$, $(xy^{-2})^2$, y^6, t^2, $(x^2y^{-1}xy)$, $(tt^{(x^3y^{-1})})^k$= $(x^2y^{-1}xy)$> k, #G; end for;

Case: (yx) ODD

for k in [2..10 by 2] do G<x,y>=<x,y,t| x^4 , $(x^{-1}y^{-1})^2$, $(xy^{-2})^2$, y^6, t^2, $(x^2y^{-1}xy)$, $((yx)t)^k$ >; k, #G; end for;

8.3 2^{*24}:((4:3):2^2)

Let there by an N = ((4:3):2^2) where N is of order 48 and is generated by $<$(1, 13)(2, 14)(5, 18)(6, 17)(9, 22)(10, 21), (1, 16, 18, 7, 9, 23)(2, 15, 17, 8, 10, 24)(3, 5, 20, 22, 11, 13)(4, 6, 19, 21, 12, 14), (1, 17)(2, 18)(3, 15)(4, 16)(5, 14)(6, 13)(7, 12)(8, 11)(9, 10)(19, 23)(20, 24)(21, 22) $>$ where x= (1, 13)(2, 14)(5, 18)(6, 17)(9, 22)(10, 21), y=(1, 16, 18, 7, 9, 23)(2, 15, 17, 8, 10, 24)(3, 5, 20, 22, 11, 13)(4, 6, 19, 21, 12, 14) and z=(1, 17)(2, 18)(3, 15)(4, 16)(5, 14)(6, 13)(7, 12)(8, 11)(9, 10)(19, 23)(20, 24)(21, 22). We wish to write a symmetric presentation for the progenitor 2^{*24}:((4:3):2^2). A presentation for N is (G$<$x,y$>$=$<$x,y,t| x^2 , y^6 , $y^{-1}xy^2xy^{-1}$, $y^{-1}xy^{-1}xyxyx$,) t^2, Stabilisers of (N,1) $>$ We still need to find the stabilisers of (N,1) meaning we need to find permutations contained in N, fix t_1. We find the following permutations stabilise t_1.

(3, 16)(4, 15)(7, 20)(8, 19)(11, 23)(12, 24)=(yxy^{-1})

We will add these additional words into our progenitor to complete it.

G$<$x,y$>$=$<$x,y,t| x^2 , y^6 , $y^{-1}xy^2xy^{-1}$, $y^{-1}xy^{-1}xyxyx$, t^2, (yxy^{-1})$>$

However, the question is raised, how do we know our progenitor is in fact correct? To understand if we did indeed build the correct progenitor we apply Grindstaff's Lemma, where we look at the orbits of N1, and find what word (permutation) takes one to an orbit representative and we continue this process until all orbits. Therefore, the orbits of N1 include {1}, {2}, {5}, {6}, {9}, {10}, {13}, {14}, {17}, {18}, {21}, {22}, {3, 16}, {4, 15}, {7, 20}, {8, 19}, {11, 23}, {12, 24} we will choose the following orbit representatives 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 17, 18, 21, 22, 24.

$1^{(zy^{-2})}=2$

$1^{(y^4xy^{-3})}=3$

$1^{(yz)}=4$

$1^{(y^2x)}=5$

$1^{(xz)}=6$

$1^{(y^3)}=7$

$1^{(zy)}=8$

$1^{(y^4)}=9$

$1^{(zy^2)}=10$

$1^{(xy^{-1})}=11$

$1^{(x)}=13$

$1^{(xy^2z)}$=14

$1^{(z)}$=17

$1^{(y^2)}$=18

$1^{(zy^2x)}$=21

$1^{(y^4x)}$=22

$1^{(zy^3)}$=24

Next we will put the following permutations into a t-cycle and then add them into our progenitor.

$(t,t^{(zy^{-2})})$

$(t,t^{(y^4xy^{-3})})$

$(t,t^{(yz)})$

$(t,t^{(y^2x)})$

$(t,t^{(xz)})$

$(t,t^{(y^3)})$

$(t,t^{(zy)})$

$(t,t^{(y^4)})$

$(t,t^{(zy^2)})$

$(t,t^{(xy^{-1})})$

$(t,t^{(x)})$

$(t,t^{(xy^2z)})$

$(t,t^{(z)})$

$(t,t^{(y^2)})$

$(t,t^{(zy^2x)})$

$(t,t^{(y^4x)})$

$(t,t^{(zy^3)})$

$G<x,y>=<x,y,t|\ x^2,\ y^6,\ y^{-1}xy^2xy^{-1},\ y^{-1}xy^{-1}xyxyx,\ t^2,\ (yxy^{-1}),$
$(t,t^{(zy^{-2})}),\ (t,t^{(y^4xy^{-3})}),\ (t,t^{(yz)}),\ (t,t^{(y^2x)}),$
$(t,t^{(xz)}),\ (t,t^{(y^3)}),\ (t,t^{(zy)}),\ (t,t^{(y^4)}),$
$(t,t^{(zy^2)}),\ (t,t^{(xy^{-1})}),\ (t,t^{(x)}),\ (t,t^{(xy^2z)}),$
$(t,t^{(z)}),\ (t,t^{(y^2)}),\ (t,t^{(zy^2x)}),\ (t,t^{(y^4x)}),\ (t,t^{(zy^3)})>$

what we want is $2^{(24)}(48) = 805306368$ if we get this we know our progenitor is correct. Using magma to compute the order of our G we find that

#G;

0

Our number must be very large if the order of G is given to be zero. Therefore, we must use

print Index(G,sub<G|x,y>: CosetLimit:=9^{10}, Hard:=true, Print:=2);

INDEX = 16777216 (a=16777216 r=16911 h=16778532 n=16778532; l=33560 c=149.39; m=16777216 t=16778531) 16777216

Thus our progenitor is correct. Next we will use Lemma 3.3 to find additional relations to add to our progenitor. Lemma 3.3 is only applicable to t_1t_2 where permutations need to be of order two, and need to stabilise t_1t_2. We will look at the set of permutations of the centraliser in N of the point-stabiliser in N of t_1 and t_2. We find the following permutations:

Identity

(1, 9, 18)(2, 10, 17)(3, 23, 20, 16, 11, 7)(4, 24, 19, 15, 12, 8)(5, 13, 22)(6, 14, 21)

(1, 22, 18, 13, 9, 5)(2, 21, 17, 14, 10, 6)(3, 23, 20, 16, 11, 7)(4, 24, 19, 15, 12, 8)

(1, 13)(2, 14)(5, 18)(6, 17)(9, 22)(10, 21)

(1, 9, 18)(2, 10, 17)(3, 11, 20)(4, 12, 19)(5, 13, 22)(6, 14, 21)(7, 16, 23)(8, 15, 24)

(1, 5, 9, 13, 18, 22)(2, 6, 10, 14, 17, 21)(3, 7, 11, 16, 20, 23)(4, 8, 12, 15, 19, 24)

(1, 22, 18, 13, 9, 5)(2, 21, 17, 14, 10, 6)(3, 11, 20)(4, 12, 19)(7, 16, 23)(8, 15, 24)

(1, 5, 9, 13, 18, 22)(2, 6, 10, 14, 17, 21)(3, 20, 11)(4, 19, 12)(7, 23, 16)(8, 24, 15)

(1, 18, 9)(2, 17, 10)(3, 7, 11, 16, 20, 23)(4, 8, 12, 15, 19, 24)(5, 22, 13)(6, 21, 14)

(3, 16)(4, 15)(7, 20)(8, 19)(11, 23)(12, 24)

(1, 13)(2, 14)(3, 16)(4, 15)(5, 18)(6, 17)(7, 20)(8, 19)(9, 22)(10, 21)(11, 23)(12, 24)

(1, 18, 9)(2, 17, 10)(3, 20, 11)(4, 19, 12)(5, 22, 13)(6, 21, 14)(7, 23, 16)(8, 24, 15)

However, the only applicable permutation is the following

(3, 16)(4, 15)(7, 20)(8, 19)(11, 23)(12, 24) = (yxy^{-1})

Therefore by lemma 3.3 we have the following progenitor that contain the permutation above is:

Case: (xy^2xy^{-1}) EVEN

for k in [2..10 by 2] do G<x,y> = <x,y,t| x^2 , y^6 , $y^{-1}xy^2xy^{-1}$, $y^{-1}xy^{-1}xyxyx$, t^2, (yxy^{-1}), $(tt^{(zy^{-2})})^k$ = (yxy^{-1})>; k, #G; end for;

8.4 2^{*24}:((8:2^2):3)

Let there by an N = ((8:2^2):3) where N is of order 96 and is generated by <(1, 9, 20, 6, 14, 24, 2, 10, 19, 5, 13, 23)(3, 12, 21, 7, 16, 18, 4, 11, 22, 8, 15, 17), (1, 16, 23, 8, 13, 22, 6, 11, 20, 3, 9, 17, 2, 15, 24, 7, 14, 21, 5, 12, 19, 4, 10, 18) > where x= (1, 9, 20, 6, 14, 24, 2, 10, 19, 5, 13, 23)(3, 12, 21, 7, 16, 18, 4, 11, 22, 8, 15, 17) and y=(1, 16, 23, 8, 13, 22, 6, 11, 20, 3, 9, 17, 2, 15, 24, 7, 14, 21, 5, 12, 19, 4, 10, 18). We wish to write a symmetric presentation for the progenitor 2^{*24}:((8:2^2):3). A presentation for N is (G<x, y> = <x, y, t| $(x^{-1}y^{-2})^2$, $x^{-4}yx^{-1}y$, $y^{-1}x^{-1}y^{-1}x^2y^2x^{-1}$,) t^2, Stabilisers of (N,1) >

We need to find the stabilisers of (N,1) meaning we need to find permutations contained in N,fix t_1. We find the following permutations stabilise t_1.

(17, 18)(19, 20)(21, 22)(23, 24)=(yxy)

(9, 10)(11, 12)(13, 14)(15, 16)=(y^2x)

We will add these additional words into our progenitor to complete it.

G<x,y>=<x,y,t| $(x^{-1}y^{-2})^2$, $x^{-4}yx^{-1}y$, $y^{-1}x^{-1}y^{-1}x^2y^2x^{-1}$, t^2, (yxy), (y^2x)>

However, the question is raised, how do we know our progenitor is in fact correct? To understand if we did build the correct progenitor we apply Grindstaff's Lemma, where we look at the orbits of N1, and find what word (permutation) takes one to an orbit representative and we continue this process until all orbits are exhausted. Therefore, the orbits of N1 include {1}, {2}, {5}, {6}, {7}, {8}, {9, 10}, {11, 12}, {13, 14}, {15, 16}, {17, 18}, {19, 20}, {21, 22}, {23, 24} we will choose the following orbit representatives 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 15, 17, 19, 22, 23.

$1^{(x^6)}$=2

$1^{(y^9)}$=3

$1^{(y^{21})}$=4

$1^{(x^9)}$=5

$1^{(x^3)}$=6

$1^{(y^{15})}$=7

$1^{(y^3)}$=8

$1^{(x)}$=9

$1^{(y^7)}$=11

$1^{(y^4)}$=13

$1^{(y^{13})}$=15

$1^{(y^{11})}$=17

$1^{(x^{8})}$=19

$1^{(y^{5})}$=22

$1^{(x^{11})}$=23

Next we will put the following permutations into a t-cycle and then add them into our progenitor.

(t,$t^{(x^{6})}$)

(t,$t^{(y^{9})}$)

(t,$t^{(y^{21})}$)

(t,$t^{(x^{9})}$)

(t,$t^{(x^{3})}$)

(t,$t^{(y^{15})}$)

(t,$t^{(y^{3})}$)

(t,$t^{(x)}$)

(t,$t^{(y^{7})}$)

(t,$t^{(y^{4})}$)

(t,$t^{(y^{13})}$)

(t,$t^{(y^{11})}$)

(t,$t^{(x^{8})}$)

(t,$t^{(y^{5})}$)

(t,$t^{(x^{11})}$)

G<x,y>=<x,y,t| $(x^{-1}y^{-2})^2$, $x^{-4}yx^{-1}y$, $y^{-1}x^{-1}y^{-1}x^2y^2x^{-1}$, t^2, (yxy), (y^2x), (t,$t^{(x^{6})}$), (t,$t^{(y^{9})}$), (t,$t^{(y^{21})}$), (t,$t^{(x^{9})}$), (t,$t^{(x^{3})}$), (t,$t^{(y^{15})}$), (t,$t^{(y^{3})}$), (t,$t^{(x)}$), (t,$t^{(y^{7})}$), (t,$t^{(y^{4})}$), (t,$t^{(y^{13})}$), (t,$t^{(y^{11})}$), (t,$t^{(x^{8})}$), (t,$t^{(y^{5})}$), (t,$t^{(x^{11})}$)>

what we want is $2^($24)96 = 1610612736 if we get this we know our progenitor is correct. Next we will use Lemma 3.3 to find additional relations to add to our progenitor. Lemma 3.3 is only applicable to t_1t_2 where permutations need to be of order two, and need to stabilise t_1t_2. We will look at the set of permutations of the centraliser in N of the point-stabiliser in N of t_1 and t_2. We find the following permutations:

(1, 7, 6, 4, 2, 8, 5, 3)(9, 15, 14, 12, 10, 16, 13, 11)(17, 23, 21, 20, 18, 24, 22, 19),

Identity,

(1, 8, 6, 3, 2, 7, 5, 4)(9, 16, 14, 11, 10, 15, 13, 12)(17, 24, 21, 19, 18, 23, 22, 20),
(1, 4, 5, 7, 2, 3, 6, 8)(9, 11, 13, 16, 10, 12, 14, 15)(17, 20, 22, 23, 18, 19, 21, 24),
(1, 3, 5, 8, 2, 4, 6, 7)(9, 11, 13, 16, 10, 12, 14, 15)(17, 20, 22, 23, 18, 19, 21, 24),
(1, 2)(3, 4)(5, 6)(7, 8)(17, 18)(19, 20)(21, 22)(23, 24),
(1, 4, 5, 7, 2, 3, 6, 8)(9, 12, 13, 15, 10, 11, 14, 16)(17, 20, 22, 23, 18, 19, 21, 24),
(1, 4, 5, 7, 2, 3, 6, 8)(9, 11, 13, 16, 10, 12, 14, 15)(17, 19, 22, 24, 18, 20, 21, 23),
(1, 3, 5, 8, 2, 4, 6, 7)(9, 12, 13, 15, 10, 11, 14, 16)(17, 20, 22, 23, 18, 19, 21, 24),
(9, 10)(11, 12)(13, 14)(15, 16),
(1, 8, 6, 3, 2, 7, 5, 4)(9, 15, 14, 12, 10, 16, 13, 11)(17, 24, 21, 19, 18, 23, 22, 20),
(1, 3, 5, 8, 2, 4, 6, 7)(9, 11, 13, 16, 10, 12, 14, 15)(17, 19, 22, 24, 18, 20, 21, 23),
(1, 6, 2, 5)(3, 7, 4, 8)(9, 13, 10, 14)(11, 16, 12, 15)(17, 22, 18, 21)(19, 24, 20, 23),
(1, 6, 2, 5)(3, 7, 4, 8)(9, 13, 10, 14)(11, 16, 12, 15)(17, 21, 18, 22)(19, 23, 20, 24),
(1, 4, 5, 7, 2, 3, 6, 8)(9, 12, 13, 15, 10, 11, 14, 16)(17, 19, 22, 24, 18, 20, 21, 23),
(1, 7, 6, 4, 2, 8, 5, 3)(9, 16, 14, 11, 10, 15, 13, 12)(17, 24, 21, 19, 18, 23, 22, 20),
(1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16)(17, 18)(19, 20)(21, 22)(23, 24),
(1, 8, 6, 3, 2, 7, 5, 4)(9, 16, 14, 11, 10, 15, 13, 12)(17, 23, 21, 20, 18, 24, 22, 19),
(1, 3, 5, 8, 2, 4, 6, 7)(9, 12, 13, 15, 10, 11, 14, 16)(17, 19, 22, 24, 18, 20, 21, 23),
(1, 6, 2, 5)(3, 7, 4, 8)(9, 14, 10, 13)(11, 15, 12, 16)(17, 21, 18, 22)(19, 23, 20, 24),
(1, 6, 2, 5)(3, 7, 4, 8)(9, 14, 10, 13)(11, 15, 12, 16)(17, 22, 18, 21)(19, 24, 20, 23),
(1, 2)(3, 4)(5, 6)(7, 8),
(17, 18)(19, 20)(21, 22)(23, 24),
(1, 5, 2, 6)(3, 8, 4, 7)(9, 13, 10, 14)(11, 16, 12, 15)(17, 21, 18, 22)(19, 23, 20, 24),
(1, 5, 2, 6)(3, 8, 4, 7)(9, 13, 10, 14)(11, 16, 12, 15)(17, 22, 18, 21)(19, 24, 20, 23),
(1, 7, 6, 4, 2, 8, 5, 3)(9, 15, 14, 12, 10, 16, 13, 11)(17, 24, 21, 19, 18, 23, 22, 20),
(1, 8, 6, 3, 2, 7, 5, 4)(9, 15, 14, 12, 10, 16, 13, 11)(17, 23, 21, 20, 18, 24, 22, 19),
(1, 7, 6, 4, 2, 8, 5, 3)(9, 16, 14, 11, 10, 15, 13, 12)(17, 23, 21, 20, 18, 24, 22, 19),
(1, 5, 2, 6)(3, 8, 4, 7)(9, 14, 10, 13)(11, 15, 12, 16)(17, 21, 18, 22)(19, 23, 20, 24),
(1, 5, 2, 6)(3, 8, 4, 7)(9, 14, 10, 13)(11, 15, 12, 16)(17, 22, 18, 21)(19, 24, 20, 23),
(1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16),
(9, 10)(11, 12)(13, 14)(15, 16)(17, 18)(19, 20)(21, 22)(23, 24)

However, the only applicaible permutations are the following

(1, 2)(3, 4)(5, 6)(7, 8)(17, 18)(19, 20)(21, 22)(23, 24) = (x^{-1}, y^{-1})

(1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16)(17, 18)(19, 20)(21, 22)(23, 24) = ($x^2yx^{-1}y$)

(9, 10)(11, 12)(13, 14)(15, 16)(17, 18)(19, 20)(21, 22)(23, 24) = (x,y)

Therefore by lemma 3.3 we have the following permutations can be added into the progenitor as extra relations.

8.5 Linear Maps of PSL(2,29)

We will write linear maps that generate PSL(2,29) namely α, β, γ assist us in doing so. We should note that α: x $\rightarrow$ x+1, β: x $\rightarrow$ xK, $\gamma \rightarrow -\frac{1}{x}$.

We will begin with α: x $\rightarrow$ x+1

$\{\infty\} \cup \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$ thus our α permutation is α = (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29)

In order to find β we need non-zero squares of F_{29} and we will use modulo 29 to simplify.

β: x $\rightarrow$ xK

$0^2 = 0$

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16$

$5^2 = 25$

$6^2 = 36 \equiv_{29} 7$

$7^2 = 49 \equiv_{29} 20$

$8^2 = 64 \equiv_{29} 6$

$9^2 = 81 \equiv_{29} 23$

$10^2 = 100 \equiv_{29} 13$

$11^2 = 121 \equiv_{29} 5$

$12^2 = 144 \equiv_{29} 28$

$13^2 = 169 \equiv_{29} 24$

$14^2 = 196 \equiv_{29} 22$

$15^2 = 225 \equiv_{29} 22$

$16^2 = 256 \equiv_{29} 24$

$17^2 = 289 \equiv_{29} 28$

$18^2 = 324 \equiv_{29} 5$

$19^2 = 361 \equiv_{29} 13$

$20^2 = 400 \equiv_{29} 23$

$21^2 = 441 \equiv_{29} 6$

$22^2 = 484 \equiv_{29} 20$

$23^2 = 529 \equiv_{29} 7$

$24^2 = 576 \equiv_{29} 25$

$25^2 = 625 \equiv_{29} 16$

$26^2 = 676 \equiv_{29} 9$

$27^2 = 729 \equiv_{29} 4$

$28^2 = 784 \equiv_{29} 1$

$29^2 = 841 \equiv_{29} 0$

Therefore we find the following squares 0, 1, 4, 5, 6, 7, 9, 13, 16, 20, 22, 23, 24, 25, 28. Next we will find the smallest non-zero square k, whose powers gives all of non-zero squares.

We will choose 4 to start with

$4^1 = 4$

$4^2 = 16$

$4^3 = 64 \equiv_{29} 6$

$4^4 = 256 \equiv_{29} 24$

$4^5 = 1024 \equiv_{29} 9$

$4^6 = 4096 \equiv_{29} 7$

$4^7 = 16384 \equiv_{29} 28$

$4^8 = 65536 \equiv_{29} 25$

$4^9 = 262144 \equiv_{29} 13$

$4^{10} = 1048576 \equiv_{29} 23$

$4^{11} = 4194304 \equiv_{29} 5$

$4^{12} = 16777216 \equiv_{29} 20$

$4^{13} = 67108864 \equiv_{29} 22$

$4^{14} = 268435456 \equiv_{29} 1$

Notice that four worked and four being one of the smallest non-zero square we will use K=4.

Now β: x $\to$ 4x

$4(1) = 4$

$4(2) = 8$

$4(3) = 12$

$4(4) = 16$

$4(5) = 20$

$4(6) = 24$

$4(7) = 28$

$4(8) = 32 \equiv_{29} 3$

$4(9) = 36 \equiv_{29} 7$

$4(10) = 40 \equiv_{29} 11$

$4(11) = 44 \equiv_{29} 15$

$4(12) = 48 \equiv_{29} 19$

$4(13) = 52 \equiv_{29} 23$

$4(14) = 56 \equiv_{29} 27$

$4(15) = 60 \equiv_{29} 2$

$4(16) = 64 \equiv_{29} 6$

$4(17) = 68 \equiv_{29} 10$

$4(18) = 72 \equiv_{29} 14$

$4(19) = 76 \equiv_{29} 18$

$4(20) = 80 \equiv_{29} 22$

$4(21) = 84 \equiv_{29} 26$

$4(22) = 88 \equiv_{29} 21$

$4(23) = 92 \equiv_{29} 5$

$4(24) = 96 \equiv_{29} 9$

$4(25) = 100 \equiv_{29} 13$

$4(26) = 104 \equiv_{29} 17$

$4(27) = 108 \equiv_{29} 21$

$4(28) = 112 \equiv_{29} 25$

$4(29) = 116 \equiv_{29} 0$

Thus our permutation for β = (1, 4, 16, 6, 24, 9, 7, 28, 25, 13, 23, 5, 20, 22)(2, 8, 3, 12, 19, 18, 14, 27, 21, 26, 17, 10, 11, 15)

Next we will find γ where γ: x $\rightarrow$ -$\frac{1}{x}$ = -1x^{-1}

-1(1)$^{-1}$ = -1(1) = -1 $\equiv_{29}$ 28

-1(2)$^{-1}$ = -1(15) = -15 $\equiv_{29}$ 14

-1(3)$^{-1}$ = -1(10) = -10 $\equiv_{29}$ 19

-1(4)$^{-1}$ = -1(22) = -22 $\equiv_{29}$ 7

-1(5)$^{-1}$ = -1(6) = -6 $\equiv_{29}$ 23

-1(6)$^{-1}$ = -1(5) = -5 $\equiv_{29}$ 24

-1(7)$^{-1}$ = -1(25) = -25 $\equiv_{29}$ 4

-1(8)$^{-1}$ = -1(11) = -11 $\equiv_{29}$ 18

-1(9)$^{-1}$ = -1(13) = -13 $\equiv_{29}$ 16

-1(10)$^{-1}$ = -1(3) = -3 $\equiv_{29}$ 26

-1(11)$^{-1}$ = -1(8) = -8 $\equiv_{29}$ 21

-1(12)$^{-1}$ = -1(17) = -17 $\equiv_{29}$ 12

-1(13)$^{-1}$ = -1(9) = -9 $\equiv_{29}$ 20

-1(14)$^{-1}$ = -1(27) = -27 $\equiv_{29}$ 2

-1(15)$^{-1}$ = -1(2) = -2 $\equiv_{29}$ 27

-1(16)$^{-1}$ = -1(20) = -20 $\equiv_{29}$ 9

-1(17)$^{-1}$ = -1(12) = -12 $\equiv_{29}$ 17

-1(18)$^{-1}$ = -1(21) = -21 $\equiv_{29}$ 8

-1(19)$^{-1}$ = -1(26) = -26 $\equiv_{29}$ 3

-1(20)$^{-1}$ = -1(16) = -16 $\equiv_{29}$ 13

-1(21)$^{-1}$ = -1(18) = -18 $\equiv_{29}$ 11

-1(22)$^{-1}$ = -1(4) = -4 $\equiv_{29}$ 25

-1(23)$^{-1}$ = -1(24) = -24 $\equiv_{29}$ 5

-1(24)$^{-1}$ = -1(23) = -23 $\equiv_{29}$ 6

-1(25)$^{-1}$ = -1(7) = -7 $\equiv_{29}$ 22

-1(26)$^{-1}$ = -1(19) = -19 $\equiv_{29}$ 10

-1(27)$^{-1}$ = -1(14) = -14 $\equiv_{29}$ 15

-1(28)$^{-1}$ = -1(28) = -28 = $\equiv_{29}$ 1

Thus the permutation for γ = (0, ∞)(1, 28)(2, 14)(3, 19)(4, 7)(5, 23)(6, 24)(8, 18)(9, 16)(10, 26)(11, 21)(13, 20)(15, 27)(22, 25).

We will now use magma to confirm if our work above is correct.

```
S:=Sym(30);
a:=S!(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,
18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29);
b:=S!(1, 4, 16, 6, 24, 9, 7, 28, 25, 13, 23, 5, 20, 22)
(2, 8, 3, 12, 19, 18, 14, 27, 21, 26, 17, 10, 11, 15);
g:=S!(29, 30)(1, 28)(2, 14)(3, 19)(4, 7)(5, 23)(6, 24)(8, 18)
(9, 16)(10, 26)(11, 21)(13, 20)(15, 27)(22, 25);
psl229:=sub<S|a,b,g>;
#psl229;
/12180/
IsIsomorphic(PSL(2,29),psl229);
/
true Homomorphism of GrpPerm: $,
Degree 30, Order 2^235729 into GrpPerm: psl229,
Degree 30, Order 2^235729 induced by
(3, 23, 21, 6, 19, 20, 28, 16, 15, 22, 9, 25, 4, 7)
(5, 13, 14, 27, 12, 10, 30, 26, 29, 8, 24, 11, 18, 17)
|--> (1, 10, 26, 4, 14, 19, 16, 21, 2, 9, 25, 5, 8, 27)
(3, 23, 29, 13, 24, 20, 28, 30, 7, 15, 11, 22, 6, 12)
(1, 8, 2)(3, 12, 20)(4, 23, 24)(5, 16, 21)(6, 7, 26)
(9, 14, 25)(10, 18, 27)(11, 15, 19)(13, 30, 17)(22, 29, 28)
|--> (1, 19, 21)(2, 10, 4)(3, 13, 8)(5, 18, 17)(6, 14, 12)
(7, 22, 25)(9, 20, 23)(11, 28, 27)(15, 24, 26)(16, 30, 29)
/
```

Magma confirms that our work is correct, thus we found PSL(2,29)'s map where PSL(2,29) $= <\alpha, \beta, \gamma>$

We next need to find PGL(2,29), where we will find map $(\frac{ax+b}{cx+d})$ such that ad-bc $\neq 1$, ad-bc $\neq$ non-zero square and ad-bc $\neq 0$.

Now we will use the following permutations given to use from magma to find linear fractional maps (1, 10, 26, 4, 14, 19, 16, 21, 2, 9, 25, 5, 8, 27)(3, 23, 29, 13, 24, 20, 28, 30, 7, 15, 11, 22, 6, 12) and (1, 19, 21)(2, 10, 4)(3, 13, 8)(5, 18, 17)(6, 14, 12)(7, 22, 25)(9, 20, 23)(11, 28, 27)(15, 24, 26)(16, 30, 29) it should be noted that $30 = \infty$ and $29 = 0$.

We will first use the permutation (1, 10, 26, 4, 14, 19, 16, 21, 2, 9, 25, 5, 8, 27)(3, 23, 29, 13, 24, 20, 28, 30, 7, 15, 11, 22, 6, 12)

Choose 29, where 29 maps to 13

$\frac{a(29)+b}{c(29)+d} = 13$

$\implies \frac{b}{d} = 13$

$\implies$ b = 13d

Choose 30, where 30 maps to 7

$\frac{a(30)+b}{c(30)+d} = 7$

$\implies \frac{a+\frac{b}{30}}{c+\frac{d}{30}} = 7$

$\implies \frac{a}{c} = 7$

$\implies$ a = 7c

Choose 1, where 1 maps to 10

$\frac{a(1)+b}{c(1)+d} = 10$

$\implies \frac{a+b}{c+d} = 10$

$\implies$ a+b = 10c+10d

Using a+b=10c+10d we will be plugging in b=1d and a=7c

$\implies$ 7c + 13d = 10c + 10d

$\implies$ 3d = 3c

$\implies$ d=c

Thus we have d=c, b=13d, and a=7c.

We will pick a=7, b=13, c=1, d=1 and plug them into the $\frac{ax+b}{cx+d}$ to find a linear map. Thus we have x $\to \frac{7x+13}{x+1}$.

We will now check to see if this is the correct linear map by picking x=1 and x=4.

Choose 1, where 1 maps to 10

$\frac{7(1)+13}{1+1}$

$= \frac{7+13}{2}$

$= \frac{20}{2}$

= 10, which is true.

Choose 4, where 4 maps to 14

$\frac{7(4)+13}{4+1}$

$= \frac{28+13}{5}$

$= \frac{41}{5}$

$= 415^{-1}$

= 416

$= 246 \equiv_{29} 14$, which is true.

Thus our linear map is correct.

Next we will find a linear map using our second permutation (1, 19, 21)(2, 10, 4)(3, 13, 8)(5, 18, 17)(6, 14, 12)(7, 22, 25)(9, 20, 23)(11, 28, 27)(15, 24, 26)(16, 30, 29).

Choose 29, which maps to 16

$\frac{a(29)+b}{c(29)+d} = 16$

$\implies \frac{b}{d} = 16$

$\implies$ b = 16d

Choose 30, which maps to 29

$\frac{a(30)+b}{c(30)+d} = 29$

$\implies \frac{a}{c} = 0$

$\implies$ a = 0

Choose 2, which maps to 10

$\frac{a(2)+b}{c(2)+d} = 10$

$\frac{2a+b}{2c+d} = 10$

$\implies$ 2a+b = 20c+10d

We will plug in b=16d and a=0 into 2a+b=20c+10d

$\implies$ 2(0)+16d=20c+10d

$\implies$ 16d=20c+10d

$\implies$ 6d=20c

$\implies$ 3d=10c

So we have a=0, b=16d and 3d=10c

We will pick a=0, b=160, c=0, and d=10 we will plug in these variables into $\frac{ax+b}{cx+d}$ to find the linear map.

Thus we have x $\rightarrow \frac{160}{3x+10}$

We will check if this is correct by Choose x=1 and x=2.

Choose 1, which maps to 10

$\frac{160}{3(1)+10}$

$= \frac{160}{13}$

$= 160(3^{-1})$

$= 160(9)$

$= 1440 \equiv_{29} 19$, which is true

Choose 2, which maps to 10

$\frac{160}{3(2)+10}$

$= \frac{160}{16}$
$= 160(16^{-1})$
$= 160(20)$
$= 3200 \equiv_{29} 10$, which is true.
Thus our linear map is correct.

8.6 2^{*9}:(3^2:(2^2))

We will work with an N = (3^2:(2^2)) who has an order of 36 and is generated by <(1, 3, 2)(4, 7, 9)(5, 8, 6), (1, 2, 5, 6, 4, 9)(3, 7, 8), (1, 5, 4)(2, 6, 9)(3, 8, 7), (1, 4)(2, 6)(7, 8)> where x = (1, 6, 7)(2, 3, 4, 9, 8, 5) and y = (1, 4)(2, 7)(3, 9)(6, 8). We wish to write a symmetric presentation for the progenitor 2^{*9}:(3^2:(2^2)). A presentation for N is
(G< x, y > = < x, y, t | y^2 , x^6 , $(xyx)^2$, $x^{-1}yx^{-1}yx^{-1}yxyxyx^{-1}y$,) t^2, Stabilisers of (N,1) >.
We now need to analyse the stabilisers of N fix t_1.. The permutations in N that stabilise t_1 are as follows.
(2, 9)(3, 8)(4, 5) = x^3
(2, 8)(3, 9)(6, 7) = $xyx^{-1}yxy$
We will now add the additional words above into our progenitor.
G< x, y > = < x, y, t — y^2 , x^6 , $(xyx)^2$, $x^{-1}yx^{-1}yx^{-1}yxyxyx^{-1}y$, t^2, x^3, $xyx^{-1}yxy$ >.
Next we will apply Grindstaff's Lemma, we will look at the orbits of N in respects to one and find what word takes one to an orbit representative, we will continue with this process until the orbits representatives are exhausted. Therefore, the orbits of N in respects to one are {1}, {4, 5}, {6, 7}, {2, 9, 8, 3} we will choose the following orbit representatives: 2, 4, 6.
$1^{(yx^{-2})} = 2$
$1^y = 4$
$1^x = 6$
We will now put the permutations above into a t-cycle and include them into our progenitor.
G< x, y > = < x, y, t | y^2 , x^6 , $(xyx)^2$, $x^{-1}yx^{-1}yx^{-1}yxyxyx^{-1}y$, t^2, x^3, $xyx^{-1}yxy$, (t,$t^{(yx^{-2})}$), (t,t^y), (t,t^y)>.

We need to have $2^{*9}26 = 18432$ to know if our progenitor is correct. We will use magma to compute the order of our G we find that

#G;

18432

Thus our progenitor is correct. Next we will apply lemma 3.3 to find additional relations to add to our progenitor. It should be noted that Lemma 3.3 is only applicable to t_1t_2 where permutations need to be of order two, and need to stabilise t_1t_2. We will look at the set of words of the centraliser in N of the point-stabiliser in N of t_1t_2.

(1, 2)(4, 6)(5, 9)(7, 8) = x^2y

(x^2y) is the only permutation given and applicable besides the identity. Therefore since (x^2y) contains (1, 2) in its permutation, that makes it odd thus we will have to put it to the power of k and raise it to an odd power range such as [3..9 by 2].

8.7 2^{*18}:(3:A_6)

Let there be an N = (3:A_6) where N is generated by < (1, 2, 4, 7, 3, 6)(5, 8, 10)(9, 12, 13, 16, 11, 14)(15, 17, 18), (1, 2, 5, 9)(3, 6, 8, 11)(4, 7, 10, 13)(12, 15)(14, 17)(16, 18), (1, 3, 4)(2, 6, 7)(5, 8, 10)(9, 11, 13)(12, 14, 16)(15, 17, 18) > where x = (1, 18, 4, 17, 3, 15)(2, 5, 14, 11, 6, 8, 16, 13, 7, 10, 12, 9) and y = (1, 16, 10, 15, 9)(3, 12, 5, 17, 11)(4, 14, 8, 18, 13).

We wish to write symmetric presentation for the progenitor 2^{*18}:(3:A_6). A presentation for N is (G< x, y > = < x, y, t | y^5 , $y^{-2}x^3y^{-2}x^{-1}$, $(xy^{-1})^4$, $y^{-1}xy^{-1}x^{-1}y^{-1}xyxy^{-1}x^{-1}$,) t^2, Stabilisers of (N,1) >.

We need to find the stabiliser of (N, 1) meaning we need to dins permutations contained in N, that fix t_1. The following permutations stabilise t_1.

(2, 11, 18)(5, 10, 8)(6, 13, 15)(7, 9, 17)(12, 14, 16) = $y^{-1}x^{-1}yxy^{-1}x^{-2}$

(2, 13, 5, 16, 15)(6, 9, 8, 12, 17)(7, 11, 10, 14, 18) = $(x^{-1}y^2)^2$

(5, 17)(8, 18)(9, 16)(10, 15)(11, 12)(13, 14) = $(xyx)^2$

(5, 16)(8, 12)(9, 17)(10, 14)(11, 18)(13, 15) = $y^2xy^{-1}x^2$

We will add the additional words into our progenitor to complete it.

G< x, y > = < x, y, t | y^5 , $y^{-2}x^3y^{-2}x^{-1}$, $(xy^{-1})^4$, $y^{-1}xy^{-1}x^{-1}y^{-1}xyxy^{-1}x^{-1}$, t^2, $y^{-1}x^{-1}yxy^{-1}x^{-2}$, $(x^{-1}y^2)^2$, $(xyx)^2$, $y^2xy^{-1}x^2$ >

we will look at the orbits of N in repsects to t_1 and and find the word of each of the orbit representatives. We continue this process until all the orbits are exhausted. The orbits are as follows { 1 }, { 3 }, { 4 }, { 2, 11, 13, 18, 10, 12, 15, 5, 14, 7, 8, 17, 6, 16, 9 } we will choose 3, 4, 18 as the representatives.

$1^{(x^4)} = 3$

$1^{(x)^2} = 4$

$1^x = 18$

Next we will put the permutations above into t-cycles and then proceed to add them to our progenitor.

G< x, y > = < x, y, t | y^5 , $y^{-2}x^3y^{-2}x^{-1}$, $(xy^{-1})^4$, $y^{-1}xy^{-1}x^{-1}y^{-1}xyxy^{-1}x^{-1}$, t^2, $y^{-1}x^{-1}yxy^{-1}x^{-2}$, $(x^{-1}y^2)^2$, $(xyx)^2$, $y^2xy^{-1}x^2$

(t, $t^{(x^4)}$), (t, $t^{(x)^2}$), (t,t^x) >

We want is to have $2^{*18}1080 = 283115520$ if we get this number we know our progenitor is correct. we will use magma to confirm if our progenitor is correct by computing the order of G.

#G;

283115520

Thus our progenitor is correct. Next we will apply Lemma 3.3 to find any additional relations to add to our progenitor. Lemma 3.3 is only applicable to t_1t_2 where the permutations need to be of the order two, and need to stabilise t_1t_2. We will look at the set of permutations of the centraliser in N of the point-stabiliser in N of t_1 and t_2.

(1, 2)(3, 6)(4, 7)(12, 18)(14, 15)(16, 17) = $x^2yxyx^2y^{-1}x$

(1, 2)(3, 6)(4, 7)(5, 9)(8, 11)(10, 13) = $(y^{-1}x^{-1}y^2)^2$

(5, 9)(8, 11)(10, 13)(12, 18)(14, 15)(16, 17) = $x^2yxyx^2y^{-1}x$

(1, 2)(3, 6)(4, 7)(12, 18)(14, 15)(16, 17) = $y^2xy^{-1}x^2$

(5, 9)(8, 11)(10, 13)(12, 18)(14, 15)(16, 17) = $(xyx)^2$

(1, 2)(3, 6)(4, 7)(5, 9)(8, 11)(10, 13) = $x^2y^2x^2y^{-1}x^{-1}$

However, even though Lemma 3.3 gives us multiple permutations that stabilise [1, 2] it also states that there is only one unique permutation that can be added to the progenitor. In addition to stabilising [1, 2] the permutation must also equal itself when raised to the other permutations and the permutation that meets that requirement is $(xyx)^2$. It should be noted that this permutation does not contain (1, 2) therefore it is of even order and should be added to the permutation as follows $(tt^{(x^4y^2x^{-1})})^j = (xyx)^2$ and we put it to the power of j for when we find our first order relations its power range will be from [0..10].

Chapter 9

Classical Groups

9.1 Unitary Groups

In this section we give symmetric presentations of groups involving PSU(3,3), U(3,7), PSU(3,9), and PSU(3,11).

9.1.1 Construction of the Unitary Group PSU(3,3):2

We will prove that the progenitor 2^{*42}:PSL(2,7) where 2^{*42}:PSL(2,7) = $<$ x, y $>$ and x $\sim$ (1, 2)(5, 7)(6, 9)(8, 12)(10, 14)(11, 15)(13, 17)(16, 20)(18,22)(19, 23)(21, 26)(24, 27)(25, 30)(28, 33)(29, 35)(32, 37)(39, 41)(40, 42), y $\sim$ (1, 3, 5, 8)(2, 4, 6, 10)(7, 11, 9, 13)(12, 16)(14, 18)(15, 19, 24, 29)(17, 21, 27, 32)(20, 25, 31, 26)(22, 28, 34, 23)(30, 36, 37, 40)(33, 38, 35,39)(41, 42)(106, 107), factored by three relations isomorphic to the Unitary classical simple group PSU(3, 3):2.

Let G $\cong \frac{2^{*42}:PSL(2,7)}{y^2 t_{21} t_{17}, (yx)^8 t_{21} t_{31} t_{25} t_{40} t_{41} t_{35} t_{24}, ((yx)^3)^8 t_{21} t_{40} t_{24} t_{25} t_{35} t_{31}}$ $\sim$ PSU(3, 3):2.

We have the following relations

Relation 1 = $((y)t^{(y^{-1}xyxy^{-1}xy^{-1})})^2 = y^2 t_{21} t_{17}$

Relation 2 = $((yx)t^{(y^{-1}xy^{-1}xy^{-1}xyx))})^8 = (yx)^8 t_{21} t_{31} t_{25} t_{40} t_{41} t_{35} t_{24}$

Relation 3 = $(((yx)^3)t^{(y^{-1}xy^{-1}xy^2))})^6 = ((yx)^3)^8 t_{21} t_{40} t_{24} t_{25} t_{35} t_{31}$

First Double Coset

NeN = { $N(e)^n$ | n ϵ N } = {N}

The coset Stabilizer of the coset N = Ne is N.

The number of single right cosets in the double coset NeN = [] is given by $\frac{|N|}{|N|} = \frac{42}{42} = 1$

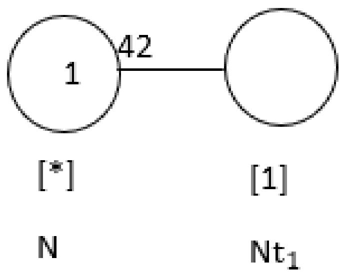
X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} is{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} we will multiply N on the right by an orbit representative and determine its double coset.

Choose 1 from {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42}.

Then

Nt_1 ϵ [1]

Cayley Diagram



Second Double Coset

Nt_1N = { $N(t_1)^n$ | n ϵ N} = {Nt_1, Nt_2,..., Nt_{42}}

The point-stabiliser 1, N^1 is given by <yxy^{-1}, $xyxy^{-1}x$>

But $Nt_1 = Nt_5$

Nt_1

$= Nt_5$

$= N(t_1^{y^2}$ ϵ [1]

But $Nt_1 = Nt_5$

Nt_1

$= Nt_5$

$= N(t_1)^{yxy}$

The coset stabilizer $N^{(1)} = <yxy^{-1}, xyxy^{-1}x>$

The number of single right cosets in the double coset $Nt_1N = [1]$ is given by $\frac{|N|}{|N^{(1)}|} = \frac{168}{4}$ $= 42$

The orbits for $N^{(1)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are {1}, {2}, {25}, {28}, {30}, {33}, {3, 13, 31, 17}, {4, 11, 34, 15}, {5, 16, 42, 35}, {6, 18, 41, 37}, {7, 29, 40, 20}, {8, 10, 19, 21}, {9, 32, 39, 22}, {12, 26, 23, 14}, {24, 38, 27, 36}

Multiply Nt_1 by a representative of each orbit and determine its double coset.

<u>Choose 1 from {1}</u>

Nt_1t_1

$= N(t_1)^2$

$= N \epsilon [*]$

<u>Choose 2 from {2}</u>

Nt_1t_2

$= ((xyxy^{-1})^2)t_2t_1$

$= N(t_1t_2)^{(yxy^{-1}xyxy^{-1})} \epsilon [1,2]$

<u>Choose 25 from {25}</u>

Nt_1t_{25}

$= (yxy^{-1})t_{30}t_{33}$

$= N(t_1t_2)^{(y^{-1}xy^{-1}xyx)} \epsilon [1,2]$

<u>Choose 28 from {28}</u>

Nt_1t_{28}

$= ((xyxy^{-1})^2)t_2t_1$

$= N(t_1t_2)^{(yxy^{-1}xyxy^{-1})} \epsilon [1,2]$

<u>Choose 30 from {30}</u>

Nt_1t_{30}

$= (yxy^{-1})t_{30}t_{33}$

$= N(t_1t_2)^{(y^{-1}xy^{-1}xyx)} \epsilon [1,2]$

Choose 33 from {33}

Nt_1t_{33}

$= Nt_1t_1$

$= N(t_1)^2$

$= N \epsilon$ [*]

Choose 3 from {3, 13, 31, 17}

Nt_1t_3

$= (yxyxy^{-1}xy^2)t_{12}t_{14}$

$= N(t_1t_2)^{(xyxy^{-1}xy^{-1}x)} \epsilon$ [1,2]

Choose 4 from {4, 11, 34, 15}

Nt_1t_4

$= (yxy^{-1}xyxy^{-1})t_{28}t_{26}$

$= N(t_1t_2)^{(yxy^{-1}xy^{-1}xy^{-1}xy)} \epsilon$ [1,2]

Choose 5 from {5, 16, 42, 35}

Nt_1t_5

$= Nt_1t_1$

$= N(t_1)^2$

$= N \epsilon$ [*]

Choose 6 from {6, 18, 41, 37}

Nt_1t_6

$= (xyxy^{-1}xy^{-1}xy)t_6t_5$

$= N(t_1t_2)^{(yxy^{-1}xyxy)} \epsilon$ [1,2]

Choose 7 from {7, 29, 40, 20}

Nt_1t_7

$= Nt_{28}t_{25}$

$= N(t_1t_2)^{(yxy^{-1}xy^{-1}xy^{-1}xy)} \epsilon$ [1,2]

Choose 8 from {8, 10, 19, 21}

Nt_1t_8

$= (y^{-1}xyxy^{-1}xy^2)t_{12}t_{14}$

$= N(t_1t_2)^{(xyxy^{-1}xy^{-1}x)} \epsilon$ [1,2]

Choose 9 from {9, 32, 39, 22}

Nt_1t_9

= $(y^{-1}xyxy^{-1}xy^{-1}x)t_{26}t_{23}$

= $N(t_1t_2)^{(yxyxy^{-1}xyx)}$ ϵ [1,2]

Choose 12 from {12, 26, 23, 14}

Nt_1t_{12}

= $(y^2xyxy^{-1}xy^2\ )t_{42}t_{41}$

= $N(t_1t_2)^{(y^2xy^{-1}xy^{-1}xyx)}$ ϵ [1,2]

Choose 24 from {24, 38, 27, 36}

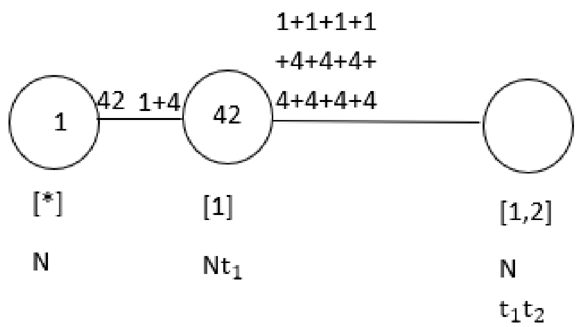
Nt_1t_{24}

= $(y^2xy^{-1}xy^{-1}xy^{-1})t_{36}t_1$

= $Nt_{36}t_1$

= $N(t_1t_2)^{(y^2xy^{-1}xy^{-1}xy^{-1})}$ ϵ [1,2]

Cayley Diagram



Third Double Coset

Nt_1t_3N = { $N(t_1t_3)^n$ | n ϵ N} = {Nt_1t_3, Nt_2t_3,..., Nt_3t_5}

The point-stabiliser 1, 2, N^{12} is given by <yxy^{-1}, $(xyxy^{-1})^2$ >

Now $Nt_1t_2 = Nt_2t_1$

Now $(t_1t_2)^{xyxy^{-1}}$

Thus, $(xyxy^{-1})$ ϵ [1,2]

Thus the coset stabiliser $N^{12)} \geq < N^{12}, yxy^{-1}, (xyxy^{-1})^2, xyxy^{-1}, x,$ $yxy^{-1}x, yxy^{-1}xyxy^{-1} >$

The number of single right cosets in the double coset Nt_1t_2N = [1,2] is given by $\frac{|N|}{|N^{(12)}|} = \frac{168}{8} = 21$

The orbits for $N^{(12)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are {1, 2}, {25, 30}, {28, 33}, {3, 13, 31, 17}, {4, 11, 34, 15}, {24, 38, 27, 36}, {5, 16, 42, 20, 40, 7, 29, 35}, {6, 18, 41, 22, 39, 9, 32, 37}, {8, 10, 19, 14, 23, 12, 26, 21}

Multiply Nt_1t_2 by a representative of each orbit and determine its double coset.

Choose 2 from {1, 2}

$Nt_1t_2t_2$

$= Nt_1(t_2)^2$

$= Nt_1 \epsilon [1]$

Choose 25 from {25, 30}

$Nt_1t_2t_{25}$

$= (yxy^{-1}xyxy^{-1})t_2t_1t_{34}$

$= N(t_1t_2t_4)^{(yxy^{-1}xyxy^{-1})} \epsilon [1,2,4]$

Choose 28 from {28, 33}

$Nt_1t_2t_{28}$

$= (y^2xyxy^{-1}xy^2)t_{16}$

$= N(t_1)^{(xyxy^{-1}xy^{-1}xy^{-1})} \epsilon [1]$

Choose 3 from {3, 13, 31, 17}

$Nt_1t_2t_3$

$= (y^2xyxyxy^{-1}x)t_{18}t_{16}t_{19}$

$= N(t_1t_2t_6)^{(xy^{-1}xy)} \epsilon [1,2,6]$

Choose 4 from {4, 11, 34, 15}

$Nt_1t_2t_4$

$= (xy^{-1}xy^{-1}xy)t_{33}t_{30}t_{29}$

$= N(t_1t_2t_4)^{(y^2xyxy^{-1}xy^2)} \epsilon [1,2,4]$

Choose 24 from {24, 38, 27, 36}

$Nt_1t_2t_{24}$

$= (xyxyxyx\ )t_{39}t_{40}t_{14}$

$= N(t_1t_2t_6)^{(xy^{-1}xyxyxy^{-1})} \epsilon [1,2,6$

Choose 5 from {5, 16, 42, 20, 40, 7, 29, 35}

$Nt_1t_2t_5$

$= (yxy^{-1}xy^{-1}xy^{-1}xy)t_4$

$= N(t_1)^{(yxy^{-1}xy)} \epsilon$ [1]

Choose 6 from {6, 18, 41, 22, 39, 9, 32, 37}

$Nt_1t_2t_6$

$= (y^2xyxy^2)t_{23}t_{26}t_{21}$

$= N(t_1t_2t_6)^{(xyxyxy^{-1}xyx)} \epsilon$ [1,2,6]

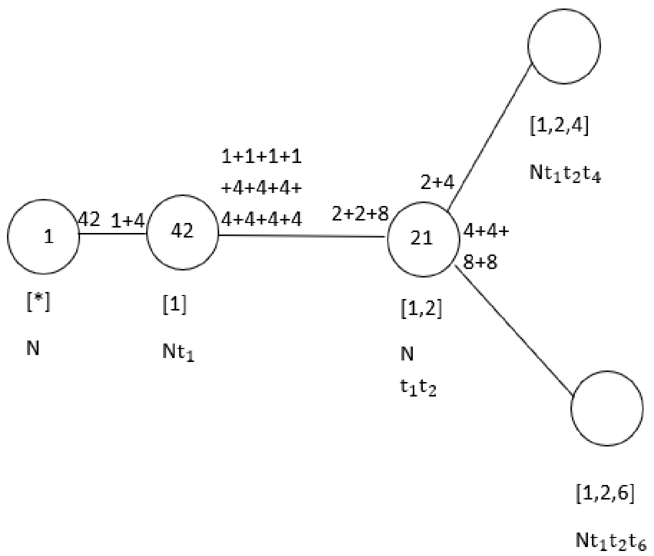
Choose 8 from {8, 10, 19, 14, 23, 12, 26, 21}

$Nt_1t_2t_8$

$= ((y^2xy^{-1})^3)t_{20}t_{22}t_{25}$

$= N(t_1t_2t_6)^{(y^2xy^{-1}xyxy)} \epsilon$ [1,2,6]

Cayley Diagram



Fourth Double Coset

$Nt_1t_2t_4N = \{ N(t_1t_2t_4)^n \mid n \epsilon N\} = \{Nt_1t_2t_4, Nt_2t_1t_4, ..., Nt_3t_4t_6\}$

The point-stabiliser 1, 2, 4, $N^{1,2,4}$ is given by $< 1 >$

Now $Nt_1t_2t_4 = Nt_1t_2t_{11}$

Now $N(t_1t_2t_4)^{(yxy^{-1})}$

Thus $(yxy^{-1}) \epsilon N^{(124)}$

Also $Nt_1t_2t_4 = Nt_{25}t_{28}t_5$

Now $N(t_1t_2t_4)^{(y^{-1}xyxy)}$

Thus $(y^{-1}xyxy) \epsilon N^{(124)}$

Thus the coset stabiliser $N^{(1,2,4)} \geq < N^{124}, yxy^{-1}, y^{-1}xyxy, x > =$

$<yxy^{-1}, y^{-1}xyxy, x>$

The number of single right cosets in the double coset $Nt_1t_2t_4N = [1,2,4]$ is given by $\frac{|N|}{|N^{(1,2,4)}|} = \frac{168}{24} = 7$

The orbits for $N^{(1,2,4)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are {1, 2, 25, 28, 30, 33}, {3, 13, 6, 17, 18, 9, 31, 41, 22, 32, 37, 39}, {4, 11, 5, 15, 16, 7, 34, 42, 20, 29, 35, 40}, {8, 12, 10, 19, 26, 36, 14, 21, 23, 27, 38, 24}

Multiply $Nt_1t_2t_4$ by a representative of each orbit and determine its double coset.

Choose 1 from {1, 2, 25, 28, 30, 33}

$Nt_1t_2t_4t_1$

$= (x)t_{25}t_{28}$

$= N(t_1t_2)^{(y^{-1}xy^{-1}xy)} \epsilon [1,2]$

Choose 3 from {3, 13, 6, 17, 18, 9, 31, 41, 22, 32, 37, 39}

$t_1t_2t_4t_3 \epsilon [1,2,4,3]$

Choose 4 from {4, 11, 5, 15, 16, 7, 34, 42, 20, 29, 35, 40}

$Nt_1t_2t_4t_4$

$= Nt_1t_2(t_4)^2$

$= Nt_1t_2 \epsilon [1,2]$

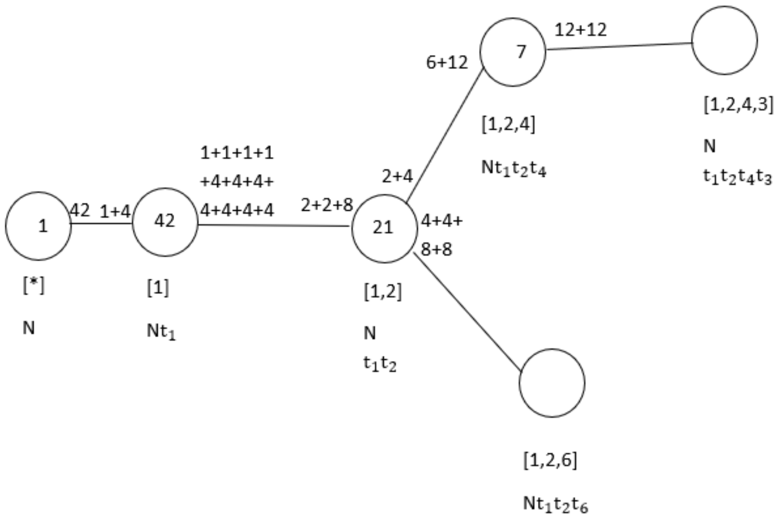
Choose 8 from {8, 12, 10, 19, 26, 36, 14, 21, 23, 27, 38, 24}

$Nt_1t_2t_4t_8$

$= (xyxy^{-1}xy^{-1}x)t_{11}t_{13}t_{18}t_{16}$

$= N(t_1t_2t_4t_3)^{(y^2xy)} \epsilon [1,2,4,3]$

Cayley Diagram



Fifth Double Coset

$Nt_1t_2t_6N = \{ N(t_1t_2t_6)^n \mid n \epsilon N\} = \{Nt_1t_2t_6, Nt_2t_1t_9, ..., Nt_3t_4t_{10}\}$

The point-stabiliser 1, 2, 6, $N^{1,2,6}$ is given by $< 1 >$

Now $Nt_1t_2t_6 = Nt_{23}t_{24}t_{19}$

Now $N(t_1t_2t_6)^{(xyxyxy^{-1}xyx)} = Nt_{23}t_{24}t_{19}$.

Thus $(xyxyxy^{-1}xyx) \epsilon N^{(126)}$

Also, $Nt_1t_2t_6 = Nt_{27}t_{24}t_{19}$

Now $N(t_1t_2t_6)^{(y^{-1}xyxy^{-1}xy)} = Nt_{27}t_{24}t_{19}$.

Thus the coset stabiliser $N^{(126)} \geq < N^{126}, xy^2x, xyxyxy^{-1}xyx, y^{-1}xyxy^{-1}xy,$ $yxyxy^{-1}xy^{-1} > = <xy^2x, xyxyxy^{-1}xyx, y^{-1}xyxy^{-1}xy, yxyxy^{-1}xy^{-1} >$

The number of single right cosets in the double coset $Nt_1t_2t_6N$ = [1,2,6] is given by $\frac{|N|}{|N^{(126)}|} = \frac{168}{8} = 21$

The orbits for $N^{(126)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are {8, 39}, {10, 40}, {20, 22}, {3, 12, 38, 41}, {4, 14, 36, 42}, {16, 31, 18, 34}, {1, 9, 32, 23, 27, 25, 11, 35}, {2, 7, 29, 26, 24, 28, 13, 37}, {5, 6, 17, 19, 21, 15, 30, 33}

Multiply $Nt_1t_2t_6$ by a representative of each orbit and determine its double coset.

Choose 8 from {8, 39}

$Nt_1t_2t_6t_8$

$= (xyxy^{-1}xy^{-1}xy^2)t_{33}t_{30}t_7t_9$

$= N(t_1t_2t_4t_3)^{(xy^{-1}xyxyx)}$ ϵ [1243]

Choose 10 from {10, 40}

$Nt_1t_2t_6t_{10}$

$= (xyxy^{-1}xy^{-1}xy^{-1})t_{37}t_{35}t_{27}t_{35}$

$= N(t_1t_2t_6t_2)^{(y^{-1}xy^2)^3)}$ ϵ [1,2,6,2]

Choose 20 from {20, 22}

$Nt_1t_2t_6t_{20}$

$= (xyxy^{-1}xy^{-1})t_{31}t_{34}t_7t_{34}$

$= N(t_1t_2t_6t_2)^{(y^{-1}xy^2)^2}$ ϵ [1262]

Choose 3 from {3, 12, 38, 41}

$Nt_1t_2t_6t_3$

$= ((yx)^3)t_{34}t_{31}t_{41}t_{42}$

$= N(t_1t_2t_4t_3)^{(y^2xyxyxy^{-1}x)}$ ϵ [1243]

Choose 4 from {4, 14, 36, 42}

$Nt_1t_2t_6t_4$

$= ((y^2xy^{-1})^3)t_{22}t_{20}$

$= N(t_1t_2)^{(yxy^{-1}xy^{-1}xyx)}$ ϵ [1,2]

Choose 16 from {16, 31, 18, 34}

$Nt_1t_2t_6t_{16}$

$= (y^{-1}xyxy^{-1}xy^2)t_{20}t_{22}$

$= N(t_1t_2)^{(xyxy^{-1}xy^{-1}xyx)}$ ϵ [1,2]

Choose 1 from {1, 9, 32, 23, 27, 25, 11, 35}

$Nt_1t_2t_6t_1$

$= (xy^{-1}xy^{-1}xyxyxy^{-1})t_{13}t_{11}$

$= N(t_1t_2)^{(xy)^3}$ ϵ [1,2]

Choose 2 from {2, 7, 29, 26, 24, 28, 13, 37}

$Nt_1t_2t_6t_2$ ϵ [1,2,6,2]

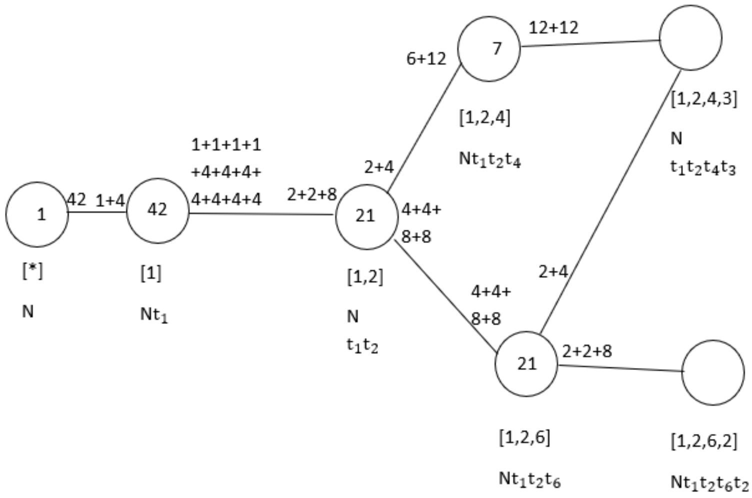
Choose 6 from {5, 6, 17, 19, 21, 15, 30, 33}

$Nt_1t_2t_6t_6$

$= Nt_1t_2(t_6)^2$

$= Nt_1t_2 \epsilon [1,2]$

Cayley Diagram



Sixth Double Coset

$Nt_1t_2t_4t_3N = \{ N(t_1t_2t_4t_3)^n \mid n \epsilon N \} = \{Nt_1t_2t_4t_3, Nt_2t_1t_4t_3,..., Nt_3t_4t_6t_5\}$

The point-stabiliser 1, 2, 4, 3, $N^{1,2,4,3}$ is given by $< 1 >$

Now $Nt_1t_2t_4t_3 = Nt_5t_6t_{10}t_8$

Now $N(t_1t_2t_4t_3)^{(y^2)} = Nt_5t_6t_{10}t_8$.

Thus $(y^2) \epsilon N^{(1243)}$

Also $Nt_1t_2t_4t_3 = Nt_6t_5t_{22}t_{20}$

therefore, $(yxy^{-1}xyxy) \epsilon N^{(1,2,4,3)}$

Thus the coset stabiliser $N^{(1,2,4,3)} \geq < N^{1,2,4,3}, y^2, x, yxy^{-1}xyxy > =$

$<y^2, x, yxy^{-1}xyxy>$

The number of single right cosets in the double coset $Nt_1t_2t_4t_3N = [1,2,4,3]$ is given by $\frac{|N|}{|N^{(1243)}|} = \frac{168}{24} = 7$

The orbits for $N^{(1243)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are {11, 15, 13, 24, 17, 27}, {1, 2, 5, 6, 7, 36, 9, 38, 42, 40, 39, 41}, {3, 8, 20, 12, 28, 16, 31, 33, 29, 23, 35, 19}, {4, 10, 22, 14, 25, 18, 34, 30, 32, 26, 37, 21}

Multiply $Nt_1t_2t_4t_3$ by a representative of each orbit and determine its double coset.

Choose 11 from {11, 15, 13, 24, 17, 27}

$Nt_1t_2t_4t_3t_{11}$

$= (yxy^{-1}xy)t_{12}t_{14}t_{40}$

$= N(t_1t_2t_6)^{(xyxy^{-1}xy^{-1}xy^2)} \epsilon$ [1,2,6]

Choose 1 from {1, 2, 5, 6, 7, 36, 9, 38, 42, 40, 39, 41}

$Nt_1t_2t_4t_3t_1$

$= (yxy^{-1}xy^{-1}xy^2)t_{34}t_{31}t_{39}$

$= N(t_1t_2t_4)^{(xyxy^{-1}xy^{-1}xy^2)} \epsilon$ [1,2,4]

Choose 3 from {3, 8, 20, 12, 28, 16, 31, 33, 29, 23, 35, 19}

$Nt_1t_2t_4t_3t_3$

$= Nt_1t_2t_4(t_3)^2$

$= Nt_1t_2t_4 \epsilon$ [1,2,4]

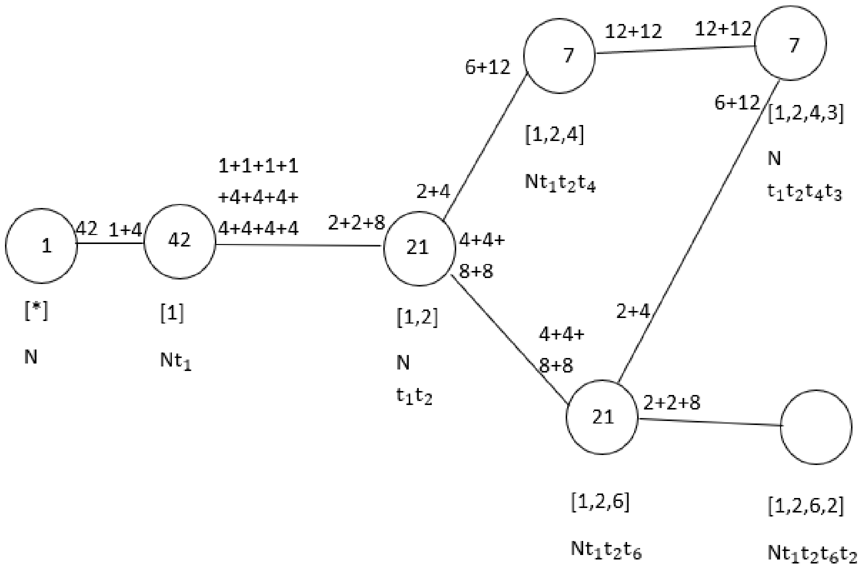
Choose 4 from {4, 10, 22, 14, 25, 18, 34, 30, 32, 26, 37, 21}

$Nt_1t_2t_4t_3t_4$

$= (y^{-1}xy^{-1}xyxy^2)t_{35}t_{37}t_{41}$

$= N(t_1t_2t_6)^{(yxyxyxy^{-1}x)} \epsilon$ [1,2,6]

Cayley Diagram



Seventh Double Coset

$Nt_1t_2t_6t_2N = \{ N(t_1t_2t_6t_2)^n \mid n \epsilon N\} = \{Nt_1t_2t_6t_2, Nt_2t_1t_9t_1, ..., Nt_3t_4t_{10}t_4\}$

The point-stabiliser 1, 2, 5, 2, $N^{1,2,5,2}$ is given by $< 1 >$

Now $Nt_1t_2t_6t_2 = Nt_5t_6t_2t_6$

Now $N(t_1t_2t_6t_2)^{y^2} = Nt_5t_6t_2t_6$.

Thus $y^2 \epsilon N^{(1262)}$

Next $Nt_1t_2t_6t_2 = Nt_{21}t_{19}t_{24}t_{19}$

Then $N(t_1t_2t_6t_2)^{(yxyxy^{-1}xy)} = Nt_{21}t_{19}t_{24}t_{19}$. Therefore $(yxyxy^{-1}xy) \epsilon N^{(1262)}$

Thus the coset stabiliser $N^{(1,2,6,2)} \geq < N^{1262}, y^2, x, yxyxy^{-1}xy > =$

$<y^2, x, yxyxy^{-1}xy>$

The number of single right cosets in the double coset $Nt_1t_2t_6t_2 = [1,2,6,2]$ is given by $\frac{|N|}{|N^{(1,2,6,2)}|} = \frac{168}{24} = 7$

The orbits for $N^{(1,2,6,2)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} are {3, 8, 39, 12, 41, 38}, {4, 10, 40, 14, 42, 36}, {16, 20, 18, 31, 22, 34}, {1, 2, 5, 21, 6, 19, 7, 27, 26, 32, 35, 9, 24, 23, 29, 37, 17, 33, 25, 11, 15, 30, 28, 13}

Multiply $Nt_1t_2t_6t_2$ by a representative of each orbit and determine its double coset.

Choose 3 from {3, 8, 39, 12, 41, 38}

$Nt_1t_2t_6t_2t_3$

$= (xyxy^{-1}xy^{-1})t_5t_6t_{30}t_6t_{27}$

$= N(t_1t_2t_6t_2t_3)^{(xyxy^{-1}xy^2)} \epsilon [1,2,6,2,3]$

Choose 4 from {4, 10, 40, 14, 42, 36}

$Nt_1t_2t_6t_2t_4$

$= (yxy^{-1}xy^{-1}xy^{-1}xy)t_{14}t_{12}t_{24}$

$= N(t_1t_2t_6)^{(xy^{-1}xy^2)} \epsilon [1,2,6]$

Choose 16 from {16, 20, 18, 31, 22, 34}

$Nt_1t_2t_6t_2t_{16}$

$= (xyxy^{-1}xy^{-1}xyx)t_{10}t_8t_{23}$

$= N(t_1t_2t_6)^{(yxy^{-1}xy^{-1})} \epsilon [1,2,6]$

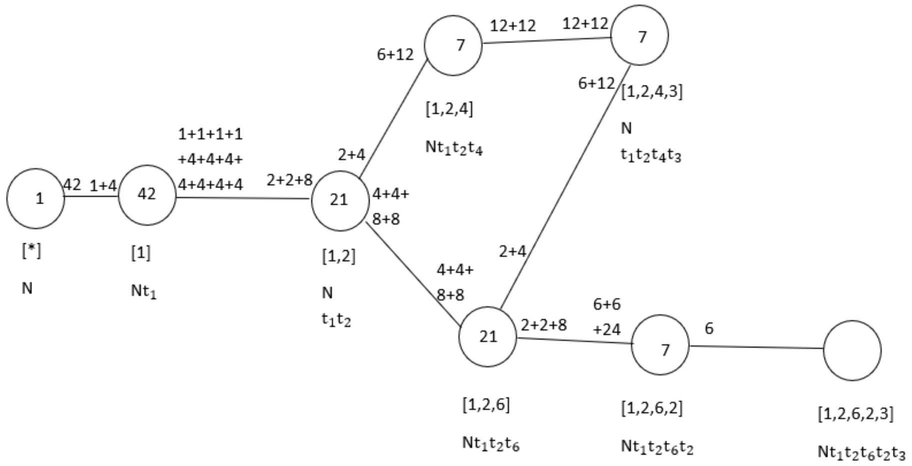
Choose 2 from {1, 2, 5, 21, 6, 19, 7, 27, 26, 32, 35, 9, 24, 23, 29, 37, 17, 33, 25, 11, 15, 30, 28, 13}

$Nt_1t_2t_6t_2t_2$

$= Nt_1t_2t_6(t_2)^2$

$= Nt_1t_2t_6 \epsilon [1,2,6]$

Cayley Diagram



Eighth Double Coset

$Nt_1t_2t_6t_2t_3N = \{ N(t_1t_2t_6t_2t_3)^n \mid n \epsilon N\} = \{Nt_1t_2t_6t_2t_3, Nt_2t_1t_9t_1t_3,..., Nt_3t_4t_{10}t_4t_5\}$

The point-stabiliser 1, 2, 6, 2, 3, $N^{1,2,6,2,3}$ is given by $< 1 >$

Now $Nt_1t_2t_6t_2t_3 = Nt_3t_4t_{10}t_4t_5$

Now $N(t_1t_2t_6t_2t_3)^y = Nt_3t_4t_{10}t_4t_5$.

Thus y ϵ $N^{(1,2,6,2,3)}$

Thus the coset stabiliser $N^{(1,2,6,2,3)} \geq < N^{1,2,6,2,3}, x, y > =$

$<x, y>$

The number of single right cosets in the double coset $Nt_1t_2t_6t_2t_3N = [1,2,6,2,3]$ is given by $\frac{|N|}{|N^{(1,2,6,2,3)}|} = \frac{168}{168} = 1$

The orbits for $N^{(1,2,6,2,3)}$ on

X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42} is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20,21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42}

Multiply $Nt_1t_2t_6t_2t_3$ by a representative of the orbit and determine its double coset.

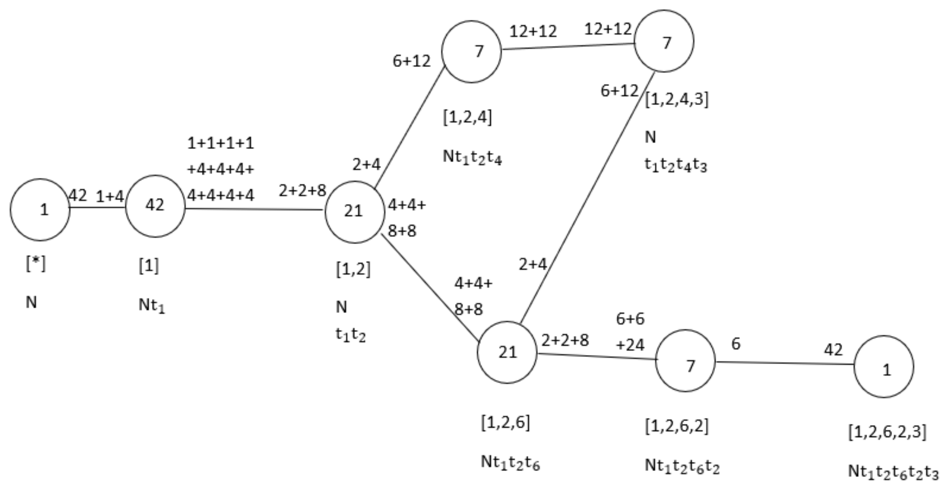
Choose 3 from {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42}

$Nt_1t_2t_6t_2t_3t_3$

$= Nt_1t_2t_6t_2(t_3)^2$

$= Nt_1t_2t_6t_2 \epsilon$ [1,2,6,2]

Cayley Diagram



9.1.2 Composition Factors

2^{*42}:PSL(2,7)

S:=Sym(42);

xx:=S!(1, 2)(5, 7)(6, 9)(8, 12)(10, 14)(11, 15)(13, 17)(16, 20)(18, 22)(19, 23)(21, 26)(24, 27)(25, 30)(28, 33)(29, 35)(32, 37)(39, 41)(40, 42);

yy:=S!(1, 3, 5, 8)(2, 4, 6, 10)(7, 11, 9, 13)(12, 16)(14, 18)(15, 19, 24, 29)(17, 21, 27, 32)(20, 25, 31, 26)(22, 28, 34, 23)(30, 36, 37, 40)(33, 38, 35, 39)(41, 42);

N:=sub<S|xx,yy>;

#N;

168

COMPOSITION FACTOR

a:=0; b:=0; c:=0; d:=0; e:=0; f:=0; g:=0; h:=6; i:=2;

G<x,y,t>:=Group<x,y,t|x^2 ,y^4 ,$y^{-2}xy^2xy^2x$,

$(xy^{-1})^7$, t^2, (t,yxy^{-1}), $(t,xyxy^{-1}x)$,

$((y)*t^{(yxyxyxy^{-1}x)})^a$,

$((yx\ )*t^{(y)})^b$,

$(((yx)^3\ )*t^{((y^2xy^{-1})^3)})^c$,

$(((yx)^3\ )*t^{((xy)^3)})^d$,

$((y^2)*t^{(y^2xy)})^e$,

$((y^2)*t^{(y^{-1}x)})^f$,

$((y^2)*t^{((xy)^3)})^g$,

$((yxy)*t^{(y^{-1}xyxy^{-1}xy^{-1})})^h$,

$((yxy)*t^{((xy^{-1})^2)})^i$ >;

f,G1,k:=CosetAction(G,sub<G|x,y>);

#G1;

12096

#k;

1 CompositionFactors(G1);

```
G
|  Cyclic(2)
*
|  2A(2, 3)                   = PSU(3, 3)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

8

#sub<G|x,y>;

168

2^{*42}:((6:2^2):(7×3))

S:=Sym(42);

xx:=S!(1, 26, 41)(2, 25, 42)(3, 27, 38)(4, 28, 37)(5, 30, 40)(6, 29, 39)(7, 12, 10)(8, 11, 9)(13, 36, 23)(14, 35, 24)(15, 31, 19)(16, 32, 20)(17, 33, 22)(18, 34, 21);

yy:=S!(1, 7, 20, 2, 8, 19)(3, 9, 21, 4, 10, 22)(5, 11, 24, 6, 12, 23)(13, 32, 29)(14, 31, 30)(15, 34, 25)(16, 33, 26)(17, 36, 28)(18, 35, 27)(37, 38)(39, 40)(41, 42);

N:=sub<S|xx,yy>;

#N;

504

COMPOSITION FACTOR

a:=0; b:=8; c:=0; d:=2; e:=6; f:=0; g:=0; h:=0; i:=0; j:=0;

G<x,y,t>:=Group<x,y,t|x^2 ,y^4 ,$y^{-2}xy^2xy^2x$,

$(xy^{-1})^7$, t^2, (t,yxy^{-1}), $(t,xyxy^{-1}x)$,

$((y)*t^{(xy^{-1}xy^{-1}xyxyx)})^a$,

$((y)*t^{(xy^{-1}xyxyxy^{-1})})^b$,

$((y)*t^{(y^{-1}xyxyxy^{-1})})^c$,

$((yxy)*t^{(yxy^{-1}xy^{-1}xyxy)})^d$,

$((yxy)*t^{(xyxyxy^{-1}xy^{-1})})^e$,

$((yxy)*t^{(y^2xy^{-1}xy^{-1}xy^2)})^f$,

$((yxy)*t^{(y^{-1}xy^{-1}xy^2)})^g$,

$((y^2)*t^{(xy^{-1}xy^{-1}xyxyx)})^h$,

$((y^2)*t^{(xy^{-1}xyxyxy^{-1})})^i$,

$((y^2)*t^{(y^{-1}xyxyxy^{-1})})^j$ >;

f,G1,k:=CosetAction(G,sub<G|x,y>);

#G1;

12096

#k;

1 CompositionFactors(G1);

```
G
|  Cyclic(2)
*
|  2A(2, 3)                    = PSU(3, 3)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

8

#sub<G|x,y>;

168

2^{*24}:$(2^3$:$(2\times3))$

S:=Sym(24);

xx:=S!(1, 6, 12, 23)(2, 5, 11, 24)(3, 8, 14, 19)(4, 7, 13, 20)(9, 15, 22, 18)(10, 16, 21, 17);

yy:=S!(1, 24, 4, 18, 21, 14)(2, 23, 3, 17, 22, 13)(5, 7, 11, 10, 19, 16)(6, 8, 12, 9, 20, 15);

#N;

48

COMPOSITION FACTOR

a:=0; b:=0; c:=0; d:=0; e:=0; f:=0; g:=0; h:=7; i:=3; j:=9; k:=3;

G<x,y,t>:=Group<x,y,t| x^4 , $(x^{-1}y^{-1})^2$, $(xy^{-2})^2$, y^6,

t^2, (t,$(x^2y^{-1}xy)$),

$((x^2y^3)*t^{(y)})^a$,

$((y^3)*t^{(xy^{-1}x)})^b$,

$((xy)*t^{(x^y)})^c$,

$((x^2)*t^{(x^{-1}yx^{-1})})^d$,

$((xy^{-1})*t^{(y^3)})^e$,

$(x*t^{(xyx^{-1}y)})^f$,

$(y*t^{(xy^{-1})})^g$,

$((y^2)*t^{(x^2y^{-1}x)})^h$,

$((xy)*t^{(x^2y^2)})^i$,

$((xy^{-2})*t^{(x^2y^{-1}x)})^j$,

$((yx)*t)^k$ >;

f,G1,k:=CosetAction(G,sub<G|x,y>);

#G1;

235984

#k; 0

CompositionFactors(G1);

```
G
|  2A(2, 7)                    = PSU(3, 7)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

10220

#sub<G|x,y>;

0

2^{*12}:(2^3(2×3))

N:=TransitiveGroup(12,37);

S:=Sym(12);

xx:=S!(1, 8)(2, 7)(3, 6)(4, 5)(9, 12)(10, 11);

yy:=S!(1, 6, 7, 12)(2, 11)(3, 10, 9, 4)(5, 8);

N:=sub<S|xx,yy>;

#N;

48

COMPOSITION FACTOR

a:=0; b:=2; c:=8; d:=0; e:=0; f:=0; g:=0; h:=0; i:=0; j:=0; k:=5;

G<x,y,t>:=Group<x,y,t| x^2 , y^4 ,

$y^{-1}xy^{-2}xy^{-1}xy^2x$,

$y^{-1}xy^{-1}xy^{-1}xyxyxyx$,

t^2, (t,(xy^2x)),(t,($xyxy^{-1}xy$)),

$(((xy)^2)*t^{(y^2xy)})^a$,

$((yxy)*t^{(x^y)})^b$,

$((y)*t^{(yxy)})^c$,

$((xy^2xy)*t^{(yx)})^d$,

$((xy)*t^{(yxy^2)})^e$,

$((y^2)*t^{(yxy^{-1}xy^{-1})})^f$,

$(((xy)^3)*t^{((xy)^3)})^g$,

$((xy)*t^{(xy^2xy)})^h$,

$((xy^2xy)*t^{(y^{-1}xyx)})^i$,

$((xyxy^{-1}xy)*t^{(yxyxy^{-1})})^j$,

((yxyxy)*t)^k >;

f,G1,k:=CosetAction(G,sub<G|x,y>);

#G1;

3547800

#k;

0

CompositionFactors(G1);

```
G
|  Cyclic(2)
*
|  2A(2, 9)                    = PSU(3, 9)
*
|  Cyclic(2)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

74600

#sub<G|x,y>;

0

$\underline{2^{*9}{:}(3^2{:}2^2)}$

S:=Sym(9);

xx:=S!(1, 6, 7)(2, 3, 4, 9, 8, 5);

yy:=S!(1, 4)(2, 7)(3, 9)(6, 8);

N:=sub:=<S|xx,yy>;

#N;

36

COMPOSITION FACTOR

a:=0; b:=0; c:=0; d:=0; e:=0; f:=0; g:=3; h:=0; i:=7; j:=8;

G<x,y,t>:=Group<x,y,t|y^2 , x^6 ,

(xyx)^2 ,

x^{-1}yx^{-1}yx^{-1}yxyxyx^{-1}y ,

t^2, (t,x^3), (t,(xyx^{-1}yxy)),

((yx^{-1}yx)*t$^{(xyx^{-1}yx)}$)^a,

((xy)*t$^{(xyx^{-1}yx)}$)^b,

(((xy)^2)*t$^{(xyx)}$)^c,

((xyx)*t$^{(yxy)}$)^{d},

(((xy)^{3})*t$^{(yx^{-1}yx)}$)^{e},

((x^{3})*t$^{(yx^{-1}y)}$)^{f},

((x^{2})*t$^{(xyxyx^{-1}y)}$)^{g},

((x)*t$^{(xyx^{-1}yx)}$)^{h},

((xy)*t$^{((xy)^{3})}$)^{i},

((yx^{-1}yx)*t$^{(xyxyx^{-1}y)}$)^{j} >;

f,G1,k:=CosetAction(G,sub<G|x,y>);

#G1;

43008

#k;

1

CompositionFactors(G1);

```
G
|  Cyclic(2)
*
|  2A(2, 3)                    = PSU(3, 3)
*
|  Cyclic(2)
*
|  Cyclic(2)
*
|  Cyclic(2)
*
|  Cyclic(2)
*
|  Cyclic(2)
*
|  Cyclic(2)
*
|  Cyclic(2)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

1330

#sub<G|x,y>;

36

2^{*18}:(2:A_6)

S:=Sym(18);

xx:=S!(1, 18, 4, 17, 3, 15)(2, 5, 14, 11, 6, 8, 16, 13, 7, 10, 12, 9);

yy:=S!(1, 16, 10, 15, 9)(3, 12, 5, 17, 11)(4, 14, 8, 18, 13);

N:=sub<S|xx,yy>;

#N;

1080

COMPOSTION FACTOR

a:=0; b:=0; c:=0; d:=0; e:=0; f:=0; g:=0; h:=6; i:=0; j:=8;

G<x,y,t>:=Group<x,y,t| y^5 ,

$y^{-2}x^3y^{-2}x^{-1}$,

$(xy^{-1})^4$,

$y^{-1}xy^{-1}x^{-1}y^{-1}xyxy^{-1}x^{-1}$,

t^2, $(t,y^{-1}x^{-1}yxy^{-1}x^{-2})$, $(t,(x^{-1}y^2)^2)$, $(t,(xyx)^2)$,

$(((xy)^2)*t^{((xyx^{-1})^2)})^a$,

$((x^5)*t^{(x^{-1}yx^{-1}y^{-1}x^2y^{-1})})^b$,

$((x^2)*t^{(yx^2yxy^{-1}x^2)})^c$,

$((xy^{-1}x^{-1}y^{-1})*t^{(x^{-1}yx^{-1}y^{-1}x^2y^{-1})})^d$,

$((x^5y)*t^{(yx^{-2}y^{-2})})^e$,

$(((xy)^3)*t^{(xy^{-1}x^{-1}yx^{-3})})^f$,

$((xyxy^{-2}x)*t^{(yxyx^{-2}yx^{-1}y^{-1})})^g$,

$((yxy^{-1}x^{-2})*t^{(xy^{-1}x^{-1}yx^2yx)})^h$,

$((x^6)*t^{(yxyx^{-2}yx^{-1}y^{-1})})^i$,

$((x^{-4})*t^{(yxyx^{-2}yx^{-1}y^{-1})})^j$ >;

f,G1,k:=CosetAction(G,sub<G|x,y>);

Index(G,sub<G|x,y>);

393976

#k;

0

CompositionFactors(G1);

```
G
|  Cyclic(2)
*
```

```
|  2A(2, 11)                  = PSU(3, 11)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

1286

#sub<G|x,y>;

0

9.2 Symplectic

In this section we give symmetric presentations of groups involving S(4,3), S(4,5), S(4,7), S(6,2) and exceptional groups G(2,3) and G(2,4).

9.2.1 Construction of the Symplectic Group S(4,3)

We will prove that the progenitor 2^{*40}:(24:(5:3)), where 2^{*40}:(24:(5:3)) = <x, y> and x $\sim$ (1, 2, 6, 13, 4)(3, 9, 15, 5, 11)(7, 19, 35, 38, 21)(8, 14, 29, 28, 22) (10, 20, 34, 17, 24)(12, 25, 40, 39, 27)(16, 32, 37, 36, 33)(18, 31, 23, 30,26), y $\sim$ (1, 3, 10, 11, 16, 5)(2, 7, 20, 35, 23, 8)(4, 12, 26, 22, 30, 14)(6, 17, 18)(13, 21, 37, 39, 24, 28)(15, 31)(19, 34, 27, 33, 25, 36)(29, 38, 40), factored by two relations is isomorphic to the symplectic classical simple group S(4,3). Let G $\cong \frac{2^{40}:(2^4:(5:3))}{((x^4)t^(x^{-1}y^{-2}x^{-1}y))^4, x^2yx^3y^{-1}xtxy^{-1}txy^{-1}t}$. Thus we show that G $\sim$ S(4,3).
We expand the following relations
Relation 1 = $((x^4)t^(x^{-1}y^{-2}x^{-1}y))^4$ = $(x^4)^4t_{29}t_{28}t_{22}t_8$
Relation 2 = $x^2yx^3y^{-1}xtxy^{-1}txy^{-1}t$ = $(x^2yx^3y^{-1}x(xy^{-1})^2)t_{30}t_8t_1$

<u>First Double Coset</u>

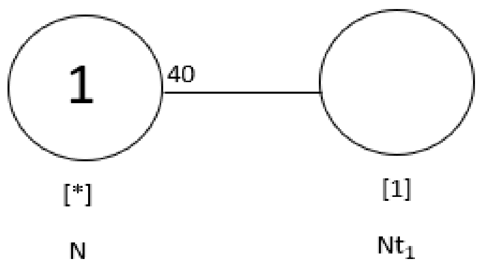
NeN = { N(e)^n | n ϵ N } = {N}
The coset stabiliser N = Ne is N
The number of single right cosets in the double coset NeN = [] is given by $\frac{|N|}{|N|} = \frac{960}{960} = 1$
The orbit of N on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40} is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40}. we multiply N on the right by an orbit representative

and determine its double coset.

Choose 1 from {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40}

$Nt_1 \in [1]$

Cayley Diagram



Second Double Coset

$Nt_1N = \{N(t_1)^n \mid n \in N\} = \{Nt_1, Nt_2,.., Nt_{40}\}$

The point-stabiliser 1, N^1 is given by $< xy^2x^{-1}y^{-2}, y^{-1}x^{-2}yx^2y, yxyx^{-1}y^{-1} >$

But $Nt_1 = Nt_{20}$

Nt_1

$= (x^2y^{-1}xyxy)t_1$

$= N(t_1)^{(y^{-1}x^{-2}y^{-1}x^2y)}$

Thus $(y^{-1}x^{-2}y^{-1}x^2y) \in N^{(1)}$

The coset stabiliser $N^{(1)} = < y^{-1}x^{-1}y^{-1}xy^{-2}, y^2x^2y^{-1}x^{-1}y, xy^{-1}x^{-2}y^{-1}x^2, y^2x, y^3xyx, xyx^2y^{-1}, (y^{-1}, x), xy^2xy^{-1}xy^{-1}, xy^2xy^{-1}x^{-1}y, xyxyx^{-1}, yxy^{-1}x^{-1}yx, x^{-2}yx^{-2}y^{-1}, (y^{-1}x)^2, x^{-1}yx^{-1}y^{-1}x^{-1}, x^{-2}y^{-1}x^{-1}yx^{-1}y^{-1}, yxy^2x^{-1}yx, y^{-1}x^2yx, xy^2, yxyx^{-1}yx, x^{-1}y^{-1}x^{-1}yx^{-1}, x^{-1}y^{-1}x^{-1}y^3, x^{-2}yxy, y^2x^2y^{-1}xy^{-1}, y^2xyx^{-1}yx^{-1}, (yxyx^{-1}y^{-1})^2, x^2yx^{-2} >$

The number of single right cosets in the double coset $Nt_1N = [1]$ is given by $\frac{|N|}{|N^{(1)}|} = \frac{960}{96}$ $= 10$

The orbits if $N^{(1)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40} are {1, 20, 33, 30}, {8, 12, 19, 11}, {2, 18, 37, 3, 24, 4, 34, 7, 39, 35, 16, 31, 14, 17, 9, 6, 38, 36, 29, 5, 21, 10, 26, 27, 28, 32, 15, 23, 25, 13, 22, 40}. We multiply Nt_1 on the right by an orbit representative and determine its double coset.

Choose 1 from {1, 20, 33, 30} Nt_1t_1

$= Nt_1^2$

$= N \epsilon$ [*]

Choose 8 from {8, 12, 19, 11}

Nt_1t_8

$= (x^{-1}yx^{-2}yxy)t_{20}$

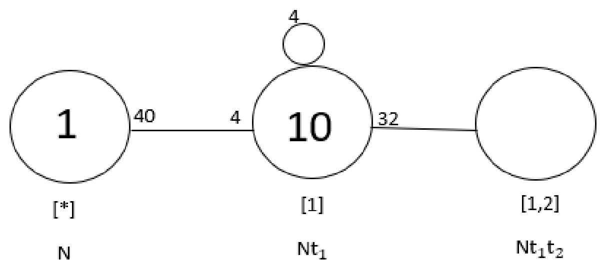
$= Nt_{20}$

$= (Nt_1)^{(y^2x)} \epsilon$ [1]

Choose 2 from{2, 18, 37, 3, 24, 4, 34, 7, 39, 35, 16, 31, 14, 17, 9, 6, 38, 36, 29, 5, 21, 10, 26, 27, 28, 32, 15, 23, 25, 13, 22, 40}

$Nt_1t_2 \epsilon$ [1,2]

Cayley Diagram



Third Double Coset

$Nt_1t_2N = \{N(t_1t_2)^n \mid n \epsilon N\} = \{Nt_1t_2, Nt_2t_6, ..., Nt{-}3t_7\}$

The point-stabiliser 1, 2, $N^{1,2}$ is given by $< 1 >$

But $Nt_1t_2 = Nt_9t_{34}$

Nt_1t_2

$= Nt_9t_{34}$

$= N(t_1t_2)^{(yxy)}$

Thus (yxy) ϵ $N^{(1,2)}$

Also $Nt_1t_2 = t_{35}t_{10}$

Nt_1t_2

$= (y^2xy^{-1}x^{-1}y^{-1})t_1t_{15}$

$= Nt_1t_{15}$

$= N(t_1t_2)^{(x^{-1}yxy^{-1}xy^2)}$

Therefore $(x^{-1}yxy^{-1}xy^2)$ ϵ $N^{(1,2)}$

Coset Stabilisers of $N^{(1,2)} = < xyx^2, yxy, (yxy), (x^{-1}yxy^{-1}xy^2) >$

The number if single right cosets in the double coset Nt_1t_2N is given by $\frac{|N|}{|N^{(1,2)}|} = \frac{960}{60} =$ 16

The orbits of $N^{(1,2)}$ on X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40} are {13, 20, 40, 25, 22}, {1, 9, 35, 31, 30, 14, 27, 29, 38, 3, 7, 5, 37, 24, 33}, {2, 34, 10, 26, 23, 21, 4, 28, 19, 17, 36, 15, 12, 18, 8, 39, 6, 16, 11, 32}. We multiply Nt_1t_2 on the right by an orbit representative and determine its double coset.

Choose 13 from {13, 20, 40, 25, 22}

$Nt_1t_2t_{13}$

$= (yx^{-2})t_{23}t_{35}$

$= Nt_{23}t_{35}$

$= (Nt_1t_2)^{(x^{-2}y^{-1}xyx^{-1})}$ ϵ [1,2]

Choose 1 from {1, 9, 35, 31, 30, 14, 27, 29, 38, 3, 7, 5, 37, 24, 33}

$Nt_1t_2t_1$

$= (yx^{-2}y)t_5t_{21}$

$= Nt_5t_{21}$

$= N(t_1t_2)^{(yxy^2x^{-1}y^{-2})}$ ϵ [1,2]

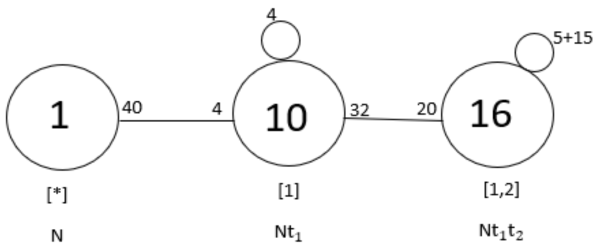
Choose 2 from {2, 34, 10, 26, 23, 21, 4, 28, 19, 17, 36, 15, 12, 18, 8, 39, 6, 16, 11, 32}

$Nt_1t_2t_2$

$= Nt_1(t_2)^2$

$= Nt_1$ ϵ [1]

Cayley Diagram



9.2.2 Composition Factors

2^{*12}:(2^3:2^2)

S:=Sym(12);

xx:=S!(1, 8)(2, 7)(3, 6)(4, 5)(9, 12)(10, 11);

yy:=S!(1, 6, 7, 12)(2, 11)(3, 10, 9, 4)(5, 8);

N:=sub<S|xx,yy>;

#N;

48

COMPOSITION FACTORS

a:=0; b:=0; c:=0; d:=0; e:=0; f:=0; g:=0; h:=6; i:=3; j:=4; k:=5;

G<x,y,t>:=Group<x,y,t| x^2 , y^4 ,

$y^{-1}xy^{-2}xy^{-1}xy^2x$,

$y^{-1}xy^{-1}xy^{-1}xyxyxyx$,

t^2, (t,(xy^2x)),(t,($xyxy^{-1}xy$)),

$((xy^2xy)*t^{(yx)})^a$,

$((xyxy^{-1}xy)*t^{(x^y)})^b$,

$(((xy)^3)*t^{(y^{-1}xyx)})^c$,

$((yxy)*t^{(xy^2xy)})^d$,

$((y)*t^{((xy)^3)})^e$,

$(((xy)^2)*t^{(yxy^{-1}xy^{-1})})^f$,

$((xy^2xy)*t^{(y^2xy)})^g$,

$((xy)*t^{(xyxy^{-1}x)})^h$,

$((y^2)*t^{(yxy^2)})^i$,

$((yxy)*t^{(yxy)})^j$,

$((yxyxy)*t)^k$ >;

Index(G,sub<G|x,y>);

1080

f,G1,k:=CosetAction(G,sub<G|x,y>);

#k;

1

CompositionFactors(G1);

```
G
```

```
    |  Cyclic(2)
    *
    |  C(2, 3)                     = S(4, 3)
    1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

41

#sub<G|x,y>;

48

2^{*80}:(2:˙(5:6))

S:=Sym(80);

xx:=S!(1, 2, 4, 8, 15, 27, 40, 59, 73, 80)(3, 6, 12, 22, 14, 25, 13, 24, 37, 56)(5, 10, 19, 32, 47, 60, 61, 71, 75, 16)(7, 11, 21, 35, 52, 38, 57, 20, 9, 17)(18, 31, 46, 63, 42, 53, 23, 33, 49, 66)(26, 39, 58, 72, 55, 70, 79, 29, 43, 51)(28, 41, 48, 64, 36, 54, 68, 78, 44, 34)(30, 45, 62, 74, 65, 69, 76, 50, 67, 77) ;

yy:=S!(1, 3, 7, 14, 26, 37)(2, 5, 11, 19, 33, 50)(4, 9, 18)(6, 13)(8, 16, 29, 44, 17, 30)(10, 20, 34, 51, 68, 72)(12, 23)(15, 28, 42, 45, 49, 67)(21, 36, 55, 41, 43, 61)(22, 27, 25, 38, 56, 70)(24, 31)(32, 48, 65, 75, 78, 77)(35, 53, 59)(40, 60, 57, 71, 46, 62)(47, 58, 64, 52, 69, 73)(54, 66, 76, 63, 74, 80);

N:=sub<S|xx,yy>;

#N;

1920

COMPOSITION FACTORS

a:=2; b:=0; c:=0; d:=4; e:=0; f:=0; g:=0; h:=0; i:=0; j:=0;

G<x,y,t>:=Group<x,y,t|x^{10} , y^6 ,

$(xy^{-2}x)^2$,

$(xy^2x^2)^2$,

$(y^{-1}x^{-1})^5$,

$(xy^2x^{-1}y^{-1})^2$,

$x^{-1}y^{-1}x^5yx^{-4}$,

$yx^{-2}y^{-1}x^3yxy^3x^{-1}$,

t^2, (t,$(yx^{-2}y)$), (t,$(y^{-1}x^{-2}yx^{-1}y^{-1})$),

$((x^5)*t^{(x^{-1}y^{-1}xy^{-1}xy^{-1})})^a$,

$((yx^4yx^{-1})*t^{(xyxyx^{-2}y)})^b$,

$((x^2y^{-1}x^{-1}y)*t^{(y^2xyx)})^c$,

$((x^4)*t^{(x^{-1}y^{-2}x^{-1}y)})^d$,

$(((xy^{-1})^2)*t^{(yx^{-1}y^{-2})})^e$,

$((xyx^{-1})*t^{(yx^{-2}yx^2)})^f$,

$((xy^{-1}x^{-1})*t^{(yx^2y^{-1}xyx)})^g$,

$((x^2y)*t^{(y^{-1}x^{-1}y^{-1}x^{-1}yx^{-1})})^h$,

$((x)*t^{((yxy^{-1})^4)})^i$,

$((x^3)*t^{((xy^2)^2)})^j$ >;

Index(G,sub<G|x,y>);

54

f,G1,k:=CosetAction(G,sub<G|x,y>);

#k;

0

CompositionFactors(G1);

```
G
|  C(2, 3)                    = S(4, 3)
*
|  Cyclic(2)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

6

#sub<G|x,y>;

0

2^{*40}:(2^4:(5:3))

S:=Sym(40);

xx:=S!(1, 2, 6, 13, 4)(3, 9, 15, 5, 11)(7, 19, 35, 38, 21)(8, 14, 29, 28, 22) (10, 20, 34, 17, 24)(12, 25, 40, 39, 27)(16, 32, 37, 36, 33)(18, 31, 23, 30,26);

yy:=S!(1, 3, 10, 11, 16, 5)(2, 7, 20, 35, 23, 8)(4, 12, 26, 22, 30, 14)(6, 17, 18)(13, 21, 37, 39, 24, 28)(15, 31)(19, 34, 27, 33, 25, 36)(29, 38, 40);

N:=sub:=<S|xx,yy>;

#N;

960

COMPOSITION FACTORS

a:=0; b:=0; c:=0; d:=0; e:=0; f:=0; g:=0; h:=0; i:=3; j:=4;

G<x,y,t>:=Group<x,y,t| x^5 , y^6 ,

$(xy^{-2}x)^2$,

$(y^{-1}x^{-1})^5$,

$(xy^2x^{-1}y^{-1})^2$,

$x^{-1}y^{-3}xy^{-1}x^{-1}y^3xy$,

t^2,

$(t,(y^{-1}x^{-2}y^{-1}x^{-1}y^{-1}))$, $(t,((yxyx^{-1}y^{-1})^2))$,

$((xy^{-1}x^{-1})*t^{(xyx^{-1}yx^{-1}yx)})^a$,

$((x^2)*t^{(x^2yx^{-2})})^b$,

$((x)*t^{(x^{-1}yx^2y^3)})^c$,

$((xyx)*t^{(xy^{-2}xy^{-1}x^{-1})})^d$,

$((x^2y^{-1}x^{-1}y)*t^{(y^{-1}xy^3xy)})^e$,

$(((x^{-1}y)^2)*t^{(x^{-1}y^2xy^{-1}x^{-1}y^{-1})})^f$,

$((xyx^{-1}y^{-1}x^{-1}y^{-1})*t^{(x^{-1}y^2xy^{-1}x^{-1}y^{-1})})^g$,

$(((x^{-1}y)^3)*t^{(xy^2xy^{-1}x^{-1}y^{-1})})^h$,

$(((xyx^{-1})^3)*t^{(y^{-1}xy^{-1}x^{-1}y^{-1}x)})^i$,

$((x)*t^{(y^{-1}xy^3xy)})^j$ >;

#G1;

27

f,G1,k:=CosetAction(G,sub<G|x,y>);

#k;

1

CompositionFactors(G1);

```
G
|  C(2, 3)                    = S(4, 3)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

3

#sub<G|x,y>;

960

2^{*32}(2:·:(5:6)

S:=Sym(32);

xx:=S!(1, 2)(3, 5, 7, 11, 17, 4, 6, 9, 14, 22)(8, 13, 20, 29, 23, 10, 16, 25, 30,18) (12, 19, 28, 32, 26, 15, 24, 27, 31, 21) ;

yy:=S!(1, 3)(2, 4)(5, 8)(6, 10)(7, 12, 17, 27, 14, 23) (9, 15, 22, 28, 11, 18) (13, 21, 24, 30, 32, 20)(16, 26, 19, 29, 31, 25);

N:=sub<S|xx,yy>;

#N;

1920

a:=6; b:=6; c:=0; d:=0; e:=0; f:=0; g:=0; h:=0; i:=0; j:=0; k:=6;

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 ,

$(xy^{-2}x)^2$,

$(xy^2x^2)^2$,

$(y^{-1}x^{-1})^5$,

$(xy^2x^{-1}y^{-1})^2$,

$x^{-1}y^{-1}x^5yx^{-4}$,

$yx^{-2}y^{-1}x^3yxy^3x^{-1}$,

t^2,

$(t,(x^2))$,

$(t,(y^2))$,

$((x^2)*(t^{(x^{-2}yx^2)}))^a$,

$((y^2)*(t^{(x^2y^{-1}x^{-2}y^{-1}x)})*(t^{(y^{-1}xy^{-1}xy^{-2})}))^b$,

$((y^3)*(t^{(y^{-1}x^{-1}yx^{-3})}))^c$,

$((x^5)*t*(t^{(x^{-2}yx^2)}))^d$,

$((x)*(t^{(xy^{-2}x^{-2}y^{-1})}))^e$,

$((yx^{-1})*(t^{(x^2y^{-1}xyx^{-1})})*(t^{(y^3xy^{-2})}))^f$,

$((x)*(t^{(xy^{-1}x^{-1}yx^{-1}yx^{-1})})*(t^{(y^2x^{-1}y^2x^2)}))^g$,

$((y^{-1})*(t^{(x^{-2}y^{-1}x^2yx)}))^h$,

$(((yx^{-1})^2)*(t^{(x^{-2}y^{-1}x^2yx)})*(t^{(y^{-1}x^{-1}yx^{-3})}))^i$,

$(((xy)^2)*(t^{(xyx^{-2}yx)})*(t^{(x^{-2}yx^2)}))^j$,

$(x^5*t)^k$ >;

f,G1,k:=CosetAction(G,sub<G|x,y>);

#G1;

54

#k;

0

CompositionFactors(G1);

```
G
|  Cyclic(2)
*
|  C(2, 3)                    = S(4, 3)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

6

#sub<G|x,y>;

0

2^{*9}:(3^2:(2^2))

S:=Sym(9);

xx:=S!(1, 6, 7)(2, 3, 4, 9, 8, 5);

yy:=S!(1, 4)(2, 7)(3, 9)(6, 8);

N:=sub:=<S|xx,yy>;

#N;

36

COMPOSITION FACTORS

a:=0; b:=0; c:=0; d:=0; e:=0; f:=0; g:=4; h:=0; i:=5; j:=6;

G<x,y,t>:=Group<x,y,t|y^2 , x^6 ,

$(xyx)^2$,

$x^{-1}yx^{-1}yx^{-1}yxyxyx^{-1}y$,

t^2, (t,x^3), $(t,(xyx^{-1}yxy))$,

$((yx^{-1}yx)*t^{(xyx^{-1}yx)})^a$,

$((x^2)*t^{(yxyx^{-1})})^b$,

$((x)*t^{(yxy)})^c$,

$((xy)*t^{(xyx)})^d$,

$(((xy)^2)*t^{(xyxyx^{-1}y)})^e$,

$((x^2)*t^{((xy)^3)})^f$,

$((xyx)*t^{(yx^{-1}y)})^g$,

$(((xy)^3)*t^{(yx^{-1}yx)})^h$,

((x^3)*t$^{(yxy)}$)^i,

((xyx)*t$^{(yxyx^{-1})}$)^j >;

f,G1,k:=CosetAction(G,sub<G|x,y>);

#G1;

1560000

#k;

0

CompositionFactors(G1);

```
G
|  Cyclic(2)
*
|  C(2, 5)                    = S(4, 5)
*
|  Cyclic(2)
*
|  Cyclic(3)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

45268

#sub<G|x,y>;

0

2^{32}:(2:·:(5:6))

S:=Sym(32);

xx:=S!(1, 2)(3, 5, 7, 11, 17, 4, 6, 9, 14, 22)(8, 13, 20, 29, 23, 10, 16, 25, 30,18) (12, 19, 28, 32, 26, 15, 24, 27, 31, 21) ;

yy:=S!(1, 3)(2, 4)(5, 8)(6, 10)(7, 12, 17, 27, 14, 23) (9, 15, 22, 28, 11, 18) (13, 21, 24, 30, 32, 20)(16, 26, 19, 29, 31, 25);

N:=sub<S|xx,yy>;

#N;

1920

COMPOSITION FACTORS

a:=3; b:=0; c:=8; d:=0; e:=0; f:=0; g:=0; h:=0; i:=0; j:=0;

G<x,y,t>:=Group<x,y,t|x^{10} , y^6 ,

(xy^{-2}x)^2 ,

$(xy^2x^2)^2$,

$(y^{-1}x^{-1})^5$,

$(xy^2x^{-1}y^{-1})^2$,

$x^{-1}y^{-1}x^5yx^{-4}$,

$yx^{-2}y^{-1}x^3yxy^3x^{-1}$,

t^2, (t,($yx^{-2}y$)), (t,($y^{-1}x^{-2}yx^{-1}y^{-1}$)),

((x^2y)*$t^{(xy^{-1}xy)}$)^a,

((x^2)*$t^{((xy^2)^2)}$)^b,

((yx^4yx^{-1})*$t^{(x^{-1}y^{-1}x^{-1}y^{-1}xyx)}$)^c,

(($xy^{-2}x$)*$t^{(x^2yxyx^3)}$)^d,

((x^5)*$t^{(y^{-2})}$)^e,

(($x^2y^{-1}x^{-1}y$)*$t^{(xyx^{-1}yxy^{-1})}$)^f,

((x^4)*$t^{(y^{-1}x^3y^{-1}x)}$)^g,

((xyx^{-1})*$t^{(yx^{-2}yx^2)}$)^h,

((xy^{-1})*$t^{(xyx^2y^2x^{-1})}$)^i,

((x^2y)*$t^{(yx^{-1}yx^2y^{-1})}$)^j >;

Index(G,sub<G|x,y>);

144060

f,G1,k:=CosetAction(G,sub<G|x,y>);

#k;

0

CompositionFactors(G1);

```
G
|  C(2, 7)                    = S(4, 7)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

186

#sub<G|x,y>;

0

2^{*160}:(2:˙:(5:6))

S:=Sym(160);

xx:=S!(1, 2, 6, 19, 5, 16, 47, 119, 52, 50)(3, 10, 33, 35, 77, 48, 93, 98, 137, 140)(4, 13, 41, 100, 97, 49, 106, 150, 139, 89)(7, 22, 65, 38, 64, 120, 53, 129, 141, 108)(8, 25, 71, 107, 80, 121, 157, 61, 30, 86)(9, 29, 82, 148, 123, 122, 158, 136, 135, 11)(12, 37, 63, 81, 152, 87, 28, 78, 102, 20)(14, 26, 75, 73, 56, 124, 159, 105, 94, 111)(15, 45, 113, 149, 145, 125, 115, 39, 103, 118)(17, 51, 66, 142, 155, 126, 96, 130, 43, 34)(18, 55, 91, 32, 85, 67, 60, 59, 133, 127)(21, 44, 114, 110, 154, 83, 92, 58, 72, 146)(23, 46, 31, 88, 95, 128, 151, 143, 70, 57)(24, 69, 117, 132, 42, 109, 138, 116, 160, 156)(27, 68, 144, 101, 36, 99, 134, 104, 79, 62)(40, 76, 74, 147, 131, 112, 54, 84, 153, 90);

yy:=S!(1, 3, 11, 33, 91, 93)(2, 7)(4, 14)(5, 17, 52, 95, 85, 142)(6, 20, 60, 90, 136, 71)(8, 26, 76, 24, 70, 72)(9, 30, 32, 23, 68, 103)(10, 16, 48, 123, 98, 59)(12, 38, 78, 129, 80, 146)(13, 42, 100, 114, 150, 64)(15, 46, 102, 153, 157, 96)(18, 56, 29, 83, 36, 69)(19, 57, 127, 43, 50, 126)(21, 62, 138, 67, 111, 158)(22, 66, 44, 115, 116, 31)(25, 51, 125, 151, 81, 147)(27, 77)(28, 79, 113, 144, 74, 148)(34, 94, 112, 75, 118, 132)(35, 97)(37, 101, 39, 104, 84, 135)(40, 105, 145, 160, 155, 73)(41, 108, 106, 156, 139, 58)(45, 117, 143, 53, 130, 92)(47, 120)(49, 124)(54, 109, 88, 110, 121, 159)(55, 131, 82, 61, 119, 152)(63, 65, 86, 154, 87, 141)(89, 137)(99, 140)(107, 133, 128, 134, 149, 122);

N:=sub<S|xx,yy>;

#N;

1920

COMPOSITION FACTORS

a:=3; b:=0; c:=0; d:=0; e:=0; f:=0; g:=0; h:=0; i:=0; j:=0;

G<x,y,t>:=Group<x,y,t|x^{10} , y^{6} ,

$(xy^{-2}x)^{2}$,

$(xy^{2}x^{2})^{2}$,

$(y^{-1}x^{-1})^{5}$,

$(xy^{2}x^{-1}y^{-1})^{2}$,

$x^{-1}y^{-1}x^{5}yx^{-4}$,

$yx^{-2}y^{-1}x^{3}yxy^{3}x^{-1}$,

t^{2},

$(t,yx^{-4}y)$,

$(t,y^{-1}xy^{-2}xy)$,

$((x^{3}y^{-1}x^{-1}y^{-1}x^{-1}y)*t^{(yx)})^{a}$,

$((x^{3}y^{-1}x^{-1}y^{-1}x^{-1}y)*t^{(x^{-3}y^{-1}x^{-1}y^{2})})^{b}$,

$((x^{3}y^{-1}x^{-1}y^{-1}x^{-1}y)*t^{(y^{-1}x^{-1}y^{-1}x^{-2})})^{c}$,

((y^2)*t$^{(x)}$)^d,

((y^2)*t$^{(yxy^{-2}xyx)}$)^e,

((y^2)*t$^{(y^2)}$)^f,

((y^2)*t$^{(x^{-2}y^{-2})}$)^g,

(((x^{-1}y)^3)*t$^{(x^2yx^{-1}y^{-2}x^{-1})}$)^h,

(((x^{-1}y)^3)*t$^{(x^{-1}yxyx^{-1}y^{-1}x^{-1}y^{-1})}$)^i,

(((x^{-1}y)^3)*t$^{(x)}$)^j >;

Index(G,sub<G|x,y>);

1451520

f,G1,k:=CosetAction(G,sub<G|x,y>);

#k;

1

CompositionFactors(G1);

```
G
|  C(3, 2)                    = S(6, 2)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

36

#sub<G|x,y>;

960

2^{*2}:(4^2):(2:6))

S:=Sym(16);

xx:=S!(1, 3, 7, 8)(2, 15, 12, 4)(5, 13, 10, 14)(6, 9, 16, 11);

yy:=S!(2, 5, 11, 15, 13, 9)(3, 4, 10, 8, 12, 14)(6, 7, 16);

N:=sub<S|xx,yy>;

#N;

192

COMPOSITION FACTORS

a:=0; b:=0; c:=0; d:=0; e:=0; f:=0; g:=0; h:=9; i:=0; j:=4; k:=4;

G<x,y,t>:=Group<x,y,t| x^4 , y^6 ,

$y^{-3}x^{-1}y^3x^{-1}$,

$yx^{-2}y^{-1}x^2y^{-1}x^2y$,

$x^{-1}y^{-1}x^{-1}y^{-1}xy^{-1}xy^{-1}$,

t^2, (t,y), (t,$(xyx^{-1}y^2x^{-1})$),

$((y^2x^2)*t^{(y^{-1}xyx)})^a$,

$(((xy)^2)*t^{(y^2x)})^b$,

$(((yxy)^2)*t^{((y,x))})^c$,

$((x)*t^{(x^2y^2x^{-1}y^{-1})})^d$,

$((xy^2x^2y)*t^{(yx^2y^2)})^e$,

$((x^2y)*t^{(y^2x^2)})^f$,

$((xy)*t^{(y^{-2}x^{-1})})^g$,

$((yxy)*t^{(xy^{-1}x^2y)})^h$,

$((x^2)*t^{(x^{-1}y^2x^{-1})})^i$,

$((y^3)*t^{(x^{-1}yx^{-1}y^{-1})})^j$,

$(x^2y^{-1}xy*t)^k$ >;

Index(G,sub<G|x,y>);

30240

f,G1,k:=CosetAction(G,sub<G|x,y>);

#k;

1

CompositionFactors(G1);

```
G
|  C(3, 2)                    = S(6, 2)
*
|  Cyclic(2)
*
|  Cyclic(2)
1
```

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

296

#sub<G|x,y>;

192

9.2.3 Exceptional Groups

$2^{*9}:(3^2:(2^2))$

S:=Sym(9);

xx:=S!(1, 6, 7)(2, 3, 4, 9, 8, 5);

yy:=S!(1, 4)(2, 7)(3, 9)(6, 8);

N:=sub:=<S—xx,yy>;

#N;

36

COMPOSITION FACTORS

a:=0; b:=0; c:=0; d:=0; e:=0; f:=0; g:=3; h:=7; i:=0; j:=7;

G<x,y,t>:=Group<x,y,t—y^{2} , x^{6} ,

(xyx)^{2} ,

x^{-1}yx^{-1}yx^{-1}yxyxyx^{-1}y ,

t^{2}, (t,x^{3}), (t,(xyx^{-1}yxy)),

((yx^{-1}yx)*t$^{(xyx^{-1}yx)}$)^{a},

((xy)*t$^{(xyx^{-1}yx)}$)^{b},

(((xy)^{2})*t$^{(xyx)}$)^{c},

((xyx)*t$^{(yxy)}$)^{d},

(((xy)^{3})*t$^{(yx^{-1}yx)}$)^{e},

((x^{3})*t$^{(yx^{-1}y)}$)^{f},

((x^{2})*t$^{(xyxyx^{-1}y)}$)^{g},

((x)*t$^{(xyx^{-1}yx)}$)^{h},

((xy)*t$^{((xy)^{3})}$)^{i},

((yx^{-1}yx)*t$^{(xyxyx^{-1}y)}$)^{j} >;

f,G1,k:=CosetAction(G,sub<G—x,y>);

#G1;

235872

#k;

1

CompositionFactors(G1);

```
G
|  Cyclic(2)
*
|  G(2, 3)
1
```

#DoubleCosets(G,sub<G—x,y>,sub<G—x,y>);

6912

#sub<G—x,y>;

36

<u>5^{*32}: (2:˙(5:6))</u>

COMPOSITION FACTORS

a:=3; b:=8; c:=0; d:=0; e:=0; f:=0; g:=0; h:=0; i:=0; j:=0; k:=5;

G<x,y,t>:=Group<x,y,t| x^{10} , y^{6} ,

$(xy^{-2}x)^2$,

$(xy^2x^2)^2$,

$(y^{-1}x^{-1})^5$,

$(xy^2x^{-1}y^{-1})^2$,

$x^{-1}y^{-1}x^5yx^{-4}$,

$yx^{-2}y^{-1}x^3yxy^3x^{-1}$,

t^5,

$(t,(x^2))$,

$(t,(y^2))$,

$((y^3)*(t^{(xy^{-1}x^{-1}y^3x)}))^a$,

$(((yx^{-1})^2)*(t^{(y^2x^{-1}y^2x^2)})*(t^{(y^{-1}xy^{-1}xy^{-1})}))^b$,

$((yx^{-1}y^{-1}x^{-1}y^{-1}x)*(t^{(xyx^{-1}yxy^{-1})}))^c$,

$((yx^2y^{-1}x^{-1})*(t^{(xyx^{-1}yxy^{-1})})*(t^{(xy^{-1}x^{-1}y^3x)}))^d$,

$((x^2)*(t^{(yxy^{-1}x^{-1}yx)})*(t^{(y^{-1}x^{-1}y^{-1})}))^e$,

$((y^{-1})*(t^{(yx^{-1}yx^{-1}yx^{-1}y)}))^f$,

$((x)*(t^{(xy^{-2}x^{-2}y^{-1})})*(t^{(yx^{-1}y^{-2})}))^g$,

$((y)*(t^{(y^2x^{-1}yx^{-1}y)}))^h$,

$((yx^{-1}y^{-1}x^{-1}y^{-1}x)*(t^{(x^{-2}y^{-1}x^2yx)})*(t^{(y^2xy^{-1}xy^{-1}x^{-1})}))^i$,

$((x^5)*(t^{(xy^{-1}x^{-1}yx^{-1}yx^{-1})}))^j$,

$(x^5*t)^k$ >;

f,G1,k:=CosetAction(G,sub<G—x,y>);

#G1;

131040

#k;

0

CompositionFactors(G1);

```
G
|  G(2, 4)
1
```

#DoubleCosets(G,sub<G—x,y>,sub<G—x,y>);

214

#sub<G—x,y>;

0

#G;

0

$\underline{2^{*18}:(2:A_6)}$

S:=Sym(18);

xx:=S!(1, 18, 4, 17, 3, 15)(2, 5, 14, 11, 6, 8, 16, 13, 7, 10, 12, 9);

yy:=S!(1, 16, 10, 15, 9)(3, 12, 5, 17, 11)(4, 14, 8, 18, 13);

N:=sub<S—xx,yy>;

#N;

1080

COMPOSTION FACTORS

a:=0; b:=0; c:=0; d:=0; e:=0; f:=3; g:=0; h:=6; i:=0; j:=0;

G<x,y,t>:=Group<x,y,t| y^5 ,

$y^{-2}x^3y^{-2}x^{-1}$,

$(xy^{-1})^4$,

$y^{-1}xy^{-1}x^{-1}y^{-1}xyxy^{-1}x^{-1}$,

t^2, $(t,y^{-1}x^{-1}yxy^{-1}x^{-2})$, $(t,(x^{-1}y^2)^2)$, $(t,(xyx)^2)$,

$((x^5y)*t^{(xyx^{-1}y^2x^{-2})})^a$,

$((x^{-2})*t^{(xy^{-1}x^{-2}yx^{-2})})^b$,

$((x)*t^{(xyx^{-1}y^{-1}xy^{-2}x)})^c$,

$((x^3)*t^{(yxyx^{-2}yx^{-1}y^{-1})})^d$,

$((yxy^{-1}x^{-2})*t^{(xy^{-1}x^{-1}yx^2yx)})^e$,

$((x^4)*t^{((xyx^{-1})^2)})^f$,

$((x^6)*t^{(xyx^{-1})})^g$,

$(((xy)^3)*t^{(xyxyx^2yx^{-1})})^h$,

((x^{-2})*t$^{(xyx^{-1})}$)^{i},

((x)*t$^{(x^{-1}y^{-1}x^{-1}y^{-1}x^{-1})}$)^{j} >;

Index(G,sub<G|x,y>);

232960

f,G1,k:=CosetAction(G,sub<G—x,y>);

#k;

0

CompositionFactors(G1);

```
G
|  G(2, 4)
1
```

#DoubleCosets(G,sub<G—x,y>,sub<G—x,y>);

342

#sub<G—x,y>;

0

Chapter 10

Composition Charts

10.1 2^{*110}:PSL(2,11)

We have the following information

S:=Sym(110), we are working with 110 letters.

x ∼ (1, 2)(3, 8)(4, 11)(5, 13)(6, 17)(7, 20)(9, 26)(10, 23)(12, 34)(14, 40)(15, 42)(16, 45)(18, 48)(19, 50)(21, 52)(22, 54)(24, 58)(25, 61)(27, 57)(28, 51)(29, 68)(30, 32)(31, 59)(33, 44)(35, 76)(36, 73)(38, 71)(39, 79)(41, 85)(43, 66)(46, 65)(47, 89)(49, 83)(53, 93)(55, 90)(56, 96)(60, 78)(62, 92)(63, 101)(64, 102)(70, 86)(72, 94)(74, 104)(75, 77)(80, 108)(81, 109)(82, 103)(84, 106)(87, 88)(91, 95)(99, 107)(100, 110),

y ∼ (1, 3, 9)(2, 5, 14)(4, 12, 35)(6, 18, 49)(7, 21, 53)(8, 23, 56)(10, 29, 69)(11, 31, 71)(13, 37, 74)(15, 43, 88)(16, 22, 55)(17, 46, 58)(19, 32, 48)(20, 51, 41)(24, 59, 98)(25, 45, 65)(26, 64, 92)(27, 67, 36)(30, 70, 104)(33, 72, 95)(34, 73, 85)(38, 50, 78)(39, 80, 105)(40, 83, 84)(42, 86, 75)(44, 89, 96)(52, 63, 66)(54, 94, 101)(57, 61, 100)(60, 79, 91)(62, 97, 99)(68, 77, 82)(76, 102, 108)(81, 110, 87)(90, 109, 103)(93, 106, 107),

The order of N = 660.

J1 progenitor 2										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	3	0	J_1
2	2	0	0	0	0	0	0	0	0	2×PSL(2, 11)
4	2	0	0	0	0	0	0	0	0	2^{11}:PSL(2, 11)
6	2	6	0	0	0	0	0	3	0	3^{10}:(2:PSL(2, 11))

Progenitor of J1 Progenitor 2:

G<x, y, t>:=Group<x, y, t|x^2, y^3, $(y^{-1}xyx)^5$, $(xy^{-1})^{11}$, $(yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2$, t^2, (t, $xyxyxy^{-1}xy^{-1}x$), (t, $xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy$),
$(x*t^{y^{-1}xyxy^{-1}xy^{-1}xyxyxyx)})^a$,
$(x*t^{y^{-1}xyxy^{-1}x)})^b$,
$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{xyxyxy^{-1}xy^{-1}xyxyxy^{-1})})^c$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{y^{-1})})^d$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(yx)^2)})^e$,
$((yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})*t^{(yx)^2)})^f$,
$((yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})*t^{y^{-1}xyxy^{-1}xy^{-1})})^g$,
$((y^{-1}xyxyxyxy^{-1}xyx)*t^{(yx)^2)})^h$,
$((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t^{xy^{-1}xyxyxy^{-1}xy^{-1}xy)})^i$,
$((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t^{(yx)^2)})^j$ > Progenitor of J1 progenitor 3:

J1 progenitor 3										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	3	M_{12}
0	0	0	0	0	0	0	0	0	0	2×PSL(2, 11)

G<x, y, t>:=Group<x, y, t|x^2, y^3, $(y^{-1}xyx)^5$, $(xy^{-1})^{11}$, $(yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2$, t^2, (t, $xyxyxy^{-1}xy^{-1}x$), (t, $xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy$),
$((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t^{(xyxyxyxyx)})^a$,
$((y^{-1}xyxyxyxy^{-1}xyx)*t^{(xyxyxy^{-1}xyxyxyx)})^b$,
$((yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})*t^{(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)})^c$,
$((yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})*t^{(y^{-1}xy^{-1}xyxy^{-1}xy)})^d$,
$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{(y^{-1}xyxyxyxyxy^{-1}xyxy^{-1})})^e$,
$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{(yxyxyxy^{-1}xyxyx)})^f$,
$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{(xy^{-1}xy^{-1}xyxyxyxy^{-1})})^g$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(yxyxyxy^{-1}xyxyx)})^h$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(xyxy^{-1}xy^{-1}xyxy^{-1}x)})^i$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(y^{-1}xyxy^{-1}xy^{-1}xyxyxyx)})^j$ >

J1 progenitor 4										
a	b	c	d	e	f	g	h	i	j	G
0	2	0	0	0	0	0	0	3	0	2×PSL(2, 11)
3	4	0	0	0	0	0	0	0	0	2^{11}:PSL(2, 11)

Progenitor of J1 Progenitor 4:

G<x, y, t>:=Group<x, y, t|x^2, y^3, $(y^{-1}xyx)^5$, $(xy^{-1})^{11}$, $(yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2$, t^2, (t, $xyxyxy^{-1}xy^{-1}x$), (t, $xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy$),
$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(yxy)})^a$,
$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(y^{-1}xy^{-1}xyxy^{-1}xy)})^b$,
$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)})^c$,
$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1})})^d$,
$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(yxyxy^{-1}xyxyxy^{-1}xy)})^e$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(xyxyxyxyxyx)})^f$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(y^{-1}xy^{-1}xyxyxyxy^{-1})})^g$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(xy^{-1}xyxyxy^{-1}xy^{-1}xy)})^h$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(y^{-1}xy^{-1}xyxy^{-1}xy)})^i$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(y^{-1}xyxyxyxyxy^{-1}xyxy^{-1})})^j$ >

J1 progenitor 5						
a	b	c	d	e	f	G
0	0	0	0	0	3	J_1
0	0	2	0	0	6	2×J_1
5	3	0	0	0	0	M_{11}

Progenitor J1 progenitor 5:

G<x, y, t>:=Group<x, y, t|x^2, y^3, $(y^{-1}xyx)^5$, $(xy^{-1})^{11}$, $(yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2$, t^2, (t, $xyxyxy^{-1}xy^{-1}x$), (t, $xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy$),
$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{((yx)^5)})^a$,
$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{(yxy^{-1}xyxyxy^{-1}xy^{-1})})^b$,
$(x*t^{(y^{-1}xy^{-1}xyxyxyxy)})^c$,
$((y^{-1}xyxyxyxy^{-1}xyx)*t^{(y^{-1}xy^{-1}xyxyxyxy)})^d$,
$((y^{-1}xyxyxyxy^{-1}xyx)*t^{((yx)^2)})^e$,

((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t$^{(xy^{-1}xyxyxy^{-1}xy^{-1}xy)}$))^{f} >

J1 progenitor 6							
a	b	c	d	e	f	g	G
0	0	0	0	0	2	6	2×M_{11}
0	2	0	0	0	0	0	2×PSL(2, 11)

Progenitor of J1 progenitor 6:

G<x, y, t>:=Group<x, y, t|x^2, y^3, (y^{-1}xyx)^5, (xy^{-1})^{11}, (yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2, t^2, (t, xyxyxy^{-1}xy^{-1}x), (t, xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy),
((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t$^{((yx)^2)} * t^{(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy))^a}$,
((yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})*t$^{(y^{-1}xy^{-1}xyxy^{-1}xy))^b}$,
((yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})*t$^{(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}x))^c}$,
((yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})*t$^{(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy))^d}$,
((yxyxyxyxyxy^{-1}xy^{-1}xy)*t$^{(y^{-1})} * t^{(y^{-1}xyxy^{-1}xy^{-1}xyxyx))^e}$,
((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t$^{(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}))^f}$,
((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t$^{(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xy))^g}$ >

J1 progenitor 7					
a	b	c	d	e	G
0	0	0	3	0	2^{10}:PSL(2, 11)

Progenitor J1 Progenitor 7:

G<x, y, t>:=Group<x, y, t|x^2, y^3, (y^{-1}xyx)^5, (xy^{-1})^{1}1, (yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2, t^2, (t, xyxyxy^{-1}xy^{-1}x), (t, xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy),
(x*t$^{(xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}))^a}$,
((y^{-1}xyxyxyxy^{-1}xyx)*t$^{((yx)^2))^b}$,
((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t$^{(yxyxyxy^{-1}xyxyx)} * t^{(y^{-1}xyxy^{-1}xy^{-1}))^c}$,
((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t$^{((xy)^5))^d}$,
(x*t$^{(yxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxyx)} * t^{(yxyxyxyxy^{-1}xy^{-1}xyxy))^e}$ > Progenitor J1 Progenitor 8:

J1 progenitor 8					
a	b	c	d	e	G
2	0	0	0	0	2×PSL(2, 11)

G<x, y, t>:=Group<x, y, t|x^2, y^3, $(y^{-1}xyx)^5$, $(xy^{-1})^{1}1$, $(yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2$, t^2, (t, $xyxyxy^{-1}xy^{-1}x$), (t, $xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy$),
$(x*t^{(xyxy^{-1}xyxyxyxy^{-1}x)})^a$,
$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(yxy)}*t^{(xy^{-1}xy^{-1}xyxyxyxy^{-1})})^b$,
$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{(xyxy^{-1}xy^{-1}xyxy^{-1}x)})^c$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(yxyxy^{-1}xyxy^{-1})})^d$,
$((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t^{(xyxyxyxyx)})^e$ >

J1 progenitor 10						
a	b	c	d	e	f	G
3	0	0	0	0	0	J_1

progenitor of J1 Progenitor 10:
G<x, y, t>:=Group<x, y, t|x^2, y^3, $(y^{-1}xyx)^5$, $(xy^{-1})^{1}1$, $(yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2$, t^2, (t, $xyxyxy^{-1}xy^{-1}x$), (t, $xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy$),
$((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t^{(xy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1}xy)})^a$,
$((yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})*t^{(yxyxy^{-1}xyxy^{-1})})^b$,
$((y^{-1}xyxyxyxy^{-1}xyx)*t^{(yx)^2}*t^{(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)})^c$,
$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{(xyxy^{-1}xy^{-1}xyxy^{-1}x)})^d$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(yxy)})^e$,
$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)}*t^{(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)})^f$ > pro-

J1 progenitor 11							
a	b	c	d	e	f	g	G
0	0	0	0	0	0	2	2×PSL(2, 11)
5	2	0	0	0	0	0	M_{11}
4	0	2	0	0	0	0	2^{11}:PSL(2, 11)

genitor of J1 progenitor 11:
G<x, y, t>:=Group<x, y, t|x^2, y^3, $(y^{-1}xyx)^5$, $(xy^{-1})^{1}1$, $(yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2$, t^2, (t, $xyxyxy^{-1}xy^{-1}x$), (t, $xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy$),
$(x*t^{(xy^{-1}xy^{-1}xyx)})^a$,
$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1})})^b$,
$(x*t^{((yx)^5)}*t^{(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)})^c$,

$((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t^{(y^{-1}xy^{-1}xyxyxyxy))^{d}}$,
$((yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})*t^{(y^{-1}xy^{-1}xyxy^{-1}xy)} * t^{(xy^{-1}xy^{-1}xyxyxyxy^{-1}))^{e}}$,
$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{(yxy))^{f}}$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(y^{-1}xyxy^{-1}xy^{-1})} * t^{(xy^{-1}xyxyxyxyxy^{-1}x))^{g}} >$

J1 progenitor 12							
a	b	c	d	e	f	g	G
0	0	0	0	0	0	2	2×PSL(2, 11)
0	2	0	0	3	0	0	J_1
3	0	0	0	0	0	0	M_{12}
5	0	3	0	0	0	0	2^{10}:PSL(2, 11)
8	2	6	0	0	0	0	2×M_{11}

progenitor of J1 progenitor 12:

G<x, y, t>:=Group<x, y, t|x^2, y^3, $(y^{-1}xyx)^5$, $(xy^{-1})^11$, $(yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2$, t^2, (t, $xyxyxy^{-1}xy^{-1}x$), (t, $xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy$),
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(y^{-1}xyxy^{-1}xy^{-1}xyxyxyx))^{a}}$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})} * t^{((yx)^5))^{b}}$,
$((yxyxyxyxyxy^{-1}xy^{-1}xy)* t^{(y^{-1}xy^{-1}xyxyxyxy))^{c}}$,
$((yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})* t^{(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy))^{d}}$,
$((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t^{(xy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1}xy))^{e}}$,
$((y^{-1}xyxyxyxy^{-1}xyx)*t^{((yx)^2)} * t^{(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy))^{f}}$,
$(x*t^{(yxy^{-1}))^{g}} >$

J1 progenitor 13					
a	b	c	d	e	G
0	3	0	0	0	J_1
0	6	2	0	0	2×M_{11}

progenitor of J1 progenitor 13:

G<x, y, t>:=Group<x, y, t|x^2, y^3, $(y^{-1}xyx)^5$, $(xy^{-1})^11$, $(yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2$, t^2, (t, $xyxyxy^{-1}xy^{-1}x$), (t, $xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy$),
$(x*t^{(yxy^{-1})} * t^{(((yx)^2)))^{a}}$,
$((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t^{(xy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1}xy))^{b}}$,

$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(xy^{-1}xyxyxy^{-1}xy^{-1}xy))^{c}}$,
$((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t^{(xy^{-1}xyxyxy^{-1}xy^{-1}xy)} * t^{((yx)^{2}))^{d}}$,
$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{(y^{-1}xyxy^{-1}xy^{-1}xyxyxyx))^{e}}$ >

J1 progenitor 14						
a	b	c	d	e	f	G
0	0	3	0	0	0	J_1

progenitor of J1 Prgogenitor 14:
G<x, y, t>:=Group<x, y, t|x^2, y^3, $(y^{-1}xyx)^5$, $(xy^{-1})^11$, $(yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2$, t^2, (t, $xyxyxy^{-1}xy^{-1}x$), (t, $xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy$),
$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(y^{-1}xy^{-1}xyxy^{-1}xy)} * t^{(y^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1}))^{a}}$,
$((yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})*t^{(y^{-1}xy^{-1}xy^{-1}xy^{-1}xy^{-1}xy))^{b}}$,
$((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t^{(xy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1}xy))^{c}}$,
$(x*t^{(xyxy^{-1}xy^{-1}xyxy)} * t^{(y^{-1}xyxy^{-1}x))^{d}}$,
$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(yxy^{-1}xyxyxyxy))^{e}}$,
$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{(yxyxyxy^{-1}xyxyx)} * t^{(xyxyxyxyxyx))^{f}}$ >

J1 progenitor 15									
a	b	c	d	e	f	g	h	i	G
0	2	0	0	0	0	0	0	3	$2\times$PSL(2, 11)
3	4	0	0	0	0	0	0	0	2^{11}:PSL(2, 11)

Progenitor of J1 progenitor 15:
G<x, y, t>:=Group<x, y, t|x^2, y^3, $(y^{-1}xyx)^5$, $(xy^{-1})^11$, $(yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2$, t^2, (t, $xyxyxy^{-1}xy^{-1}x$), (t, $xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy$),
$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{(xy^{-1}xyxyxyxyxy^{-1}x)} * t^{(y^{-1}xy^{-1}xyxyxyxy))^{a}}$,
$(x*t^{(yxy^{-1}xyxyxyxyx))^{b}}$,
$(x*t^{((yx)^{5}))^{c}}$,
$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(yxy)} * t^{(y^{-1}))^{d}}$,
$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(yxy))^{f}}$,
$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{(yxy^{-1}xyxyxy^{-1}xy^{-1})} * t^{(yxyxy^{-1}xyxy^{-1}))^{g}}$,
$((y^{-1}xyxyxyxy^{-1}xyx)*t^{((yx)^{2}))^{h}}$,
$((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t^{(xy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1}xy)} * t^{(xyxyxy^{-1}xyxyxyx))^{i}}$ >

J1 progenitor 16								
a	b	c	d	e	f	g	h	G
0	2	0	0	0	0	2	0	$2\times$PSL(2, 11)
4	3	0	0	0	0	0	0	2^{10}:PSL(2, 11)

Progenitor of J1 Progenitor 16:

$G<x, y, t>:=Group<x, y, t|x^2, y^3, (y^{-1}xyx)^5, (xy^{-1})^{1}1, (yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2, t^2,$ $(t, xyxyxy^{-1}xy^{-1}x), (t, xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy),$

$((yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})*t^{(y^{-1}xy^{-1}xyxyxyxy^{-1})} * t^{(xy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1}xy)})^a,$

$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{(y^{-1}xy^{-1}xyxyxyxy)})^b,$

$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(y^{-1}xyxy^{-1}xy^{-1})})^c,$

$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}x)})^d,$

$(x*t^{(xyxy^{-1}xy^{-1}xyxy)} * t^{(y^{-1}xy^{-1}xyxyxyxyxy^{-1})})^e,$

$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(y^{-1})})^f,$

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(y^{-1}xyxy^{-1}xy^{-1})})^g >$

J1 progenitor SOR										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	3	6	0	0	J_1

Progenitor of J1 Progenitor SOR:

$G<x, y, t>:=Group<x, y, t|x^2, y^3, (y^{-1}xyx)^5, (xy^{-1})^{1}1, (yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2, t^2,$ $(t, xyxyxy^{-1}xy^{-1}x), (t, xy^{-1}xy^{-1}xyxy^{-1}xy^{-1}xyxy),$

$(x*t^{(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)} * t^{(yxyxyxyxyxy^{-1}xy)})^a,$

$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{((yx)^5)})^b,$

$((xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)*t^{(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy^{-1})})^c,$

$((yxyxyxyxyxy^{-1}xy^{-1}xy)*t^{(y^{-1}xyxy^{-1}xy^{-1}xyxyx)})^d,$

$((yxy^{-1}xy^{-1}xyxyxy^{-1}xy^{-1})*t^{(xy^{-1}xyxyxy^{-1}xy^{-1}xy)})^e,$

$((y^{-1}xy^{-1}xyxyxy^{-1}xyxy)*t^{(y^{-1}xy^{-1}xy^{-1}xy^{-1}xyxy)} * t^{(yxy^{-1}xyxyxyxy)})^f,$

$((y^{-1}xyxyxyxy^{-1}xyx)*t^{(xyxyxyxyx)})^g,$

$(x*t^{(y^{-1}xyxy^{-1}xy^{-1})} * t^{(xy^{-1}xyxy^{-1}xy^{-1}xy^{-1}xy^{-1})})^h,$

$((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t^{(xyxyxy^{-1}xyxyxyx)})^i,$

$((xyxy^{-1}xyxyxy^{-1}xy^{-1}x)*t^{(xyxyxyxyx)})^j >$

10.2 2^{*160}:PSL(2,11)

We have the following information

S:=Sym(160)

x ∼ (1, 2, 6, 19, 5, 16, 47, 119, 52, 50)(3, 10, 33, 35, 77, 48, 93, 98, 137, 140)(4, 13, 41, 100, 97, 49, 106, 150, 139, 89)(7, 22, 65, 38, 64, 120, 53, 129, 141, 108)(8, 25, 71, 107, 80, 121, 157, 61, 30, 86)(9, 29, 82, 148, 123, 122, 158, 136, 135, 11)(12, 37, 63, 81, 152, 87, 28, 78, 102, 20)(14, 26, 75, 73, 56, 124, 159, 105, 94, 111)(15, 45, 113, 149, 145, 125, 115, 39, 103, 118)(17, 51, 66, 142, 155, 126, 96, 130, 43, 34)(18, 55, 91, 32, 85, 67, 60, 59, 133, 127)(21, 44, 114, 110, 154, 83, 92, 58, 72, 146)(23, 46, 31, 88, 95, 128, 151, 143, 70, 57)(24, 69, 117, 132, 42, 109, 138, 116, 160, 156)(27, 68, 144, 101, 36, 99, 134, 104, 79, 62)(40, 76, 74, 147, 131, 112, 54, 84, 153, 90)

y ∼ (1, 3, 11, 33, 91, 93)(2, 7)(4, 14)(5, 17, 52, 95, 85, 142)(6, 20, 60, 90, 136, 71)(8, 26, 76, 24, 70, 72)(9, 30, 32, 23, 68, 103)(10, 16, 48, 123, 98, 59)(12, 38, 78, 129, 80, 146)(13, 42, 100, 114, 150, 64)(15, 46, 102, 153, 157, 96)(18, 56, 29, 83, 36, 69)(19, 57, 127, 43, 50, 126)(21, 62, 138, 67, 111, 158)(22, 66, 44, 115, 116, 31)(25, 51, 125, 151, 81, 147)(27, 77)(28, 79, 113, 144, 74, 148)(34, 94, 112, 75, 118, 132)(35, 97)(37, 101, 39, 104, 84, 135)(40, 105, 145, 160, 155, 73)(41, 108, 106, 156, 139, 58)(45, 117, 143, 53, 130, 92)(47, 120)(49, 124)(54, 109, 88, 110, 121, 159)(55, 131, 82, 61, 119, 152)(63, 65, 86, 154, 87, 141)(89, 137)(99, 140)(107, 133, 128, 134, 149, 122)

#N = 1920

J1 progenitor SOR										
a	b	c	d	e	f	g	h	i	j	G
3	0	0	0	0	0	0	0	0	0	J_2

Progenitor of J1 Progenitor SOR:

G<x,y,t>:=Group<x,y,t|x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$, t^2, (t,y * x^{-4} * y), (t,$y^{-1} * x * y^{-2} * x * y$),

$((x * y^{-1} * x * y^{-2} * x * y)*t^{(x^4*y^{-1}*x)})^a$,

$((x * y^{-1} * x * y^{-2} * x * y)*t^{(x)})^b$,

$((x * y^{-1} * x * y^{-2} * x * y)*t^{(x^2*y^2*x*y*x^{-1})})^c$,

$((x^2)*t^{(y^{-1}*x^{-1}*y^{-1}*x^{-3})})^d$,

$((x^2)*t^{(x^2*y^2*x*y*x^{-1})})^e$,
$((x^2)*t^{(y*x*y^{-2}*x*y*x)})^f$,
$((x * y^{-2} * x * y)*t^{(x^2*y^{-1}*x*y*x^{-2}*y^{-1})})^g$,
$((x * y^{-2} * x * y)*t^{(x)})^h$,
$((x * y^{-2} * x * y)*t^{(x^2*y^2*x*y*x^{-1})})^i$,
$(((y * x^{-1})^2)*t^{(y*x^{-2}*y*x*y^{-1}*x)})^j$ >

10.3 2^{*330}: PSL(2,11)

We have the following information

S:=Sym(330)

x ∼ (1, 31, 61)(2, 32, 62)(3, 33, 63)(4, 34, 64)(5, 35, 65)(6, 36, 66)(7, 37, 67)(8, 38, 68)(9, 39, 69)(10, 40, 70)(11, 41, 71)(12, 42, 72)(13, 43, 73)(14, 44, 74)(15, 45, 75)(16, 46, 76)(17, 47, 77)(18, 48, 78)(19, 49, 79)(20, 50, 80)(21, 51, 81)(22, 52, 82)(23, 53, 83)(24, 54, 84)(25, 55, 85)(26, 56, 86)(27, 57, 87)(28, 58, 88)(29, 59, 89)(30, 60, 90)(91, 121, 151)(92, 122, 152)(93, 123, 153)(94, 124, 154)(95, 125, 155)(96, 126, 156)(97, 127, 157)(98, 128, 158)(99, 129, 159)(100, 130, 160)(101, 131, 161)(102, 132, 162)(103, 133, 163)(104, 134, 164)(105, 135, 165)(106, 136, 166)(107, 137, 167)(108, 138, 168)(109, 139, 169)(110, 140, 170)(111, 141, 171)(112, 142, 172)(113, 143, 173)(114, 144, 174)(115, 145, 175)(116, 146, 176)(117, 147, 177)(118, 148, 178)(119, 149, 179)(120, 150, 180)(181, 196, 187)(182, 198, 189)(183, 197, 188)(184, 190, 193)(185, 192, 195)(186, 191, 194)(199, 209, 204)(200, 210, 203)(201, 208, 202)(205, 206, 207)(211, 241, 271)(212, 242, 272)(213, 243, 273)(214, 244, 274)(215, 245, 275)(216, 246, 276)(217, 247, 277)(218, 248, 278)(219, 249, 279)(220, 250, 280)(221, 251, 281)(222, 252, 282)(223, 253, 283)(224, 254, 284)(225, 255, 285)(226, 256, 286)(227, 257, 287)(228, 258, 288)(229, 259, 289)(230, 260, 290)(231, 261, 291)(232, 262, 292)(233, 263, 293)(234, 264, 294)(235, 265, 295)(236, 266, 296)(237, 267, 297)(238, 268, 298)(239, 269, 299)(240, 270, 300)(301, 309, 328)(302, 307, 329)(303, 308, 330)(304, 326, 314)(305, 327, 313)(306, 325, 315)(310, 311, 312)(316, 321, 324)(317, 320, 323)(318, 319, 322)

y ∼ (1, 4)(2, 5)(3, 6)(7, 13)(8, 15)(9, 14)(11, 12)(17, 18)(19, 23)(20, 22)(21, 24)(25, 30)(26, 28)(27, 29)(31, 55)(32, 57)(33, 56)(34, 48)(35, 47)(36, 46)(37, 49)(38, 51)(39, 50)(40, 52)(41, 53)(42, 54)(43, 45)(59, 60)(61, 91)(62, 92)(63, 93)(64, 94)(65, 95)(66, 96)(67, 97)(68, 98)(69, 99)(70, 100)(71, 101)(72, 102)(73, 103)(74, 104)(75, 105)(76, 106)(77,

107)(78, 108)(79, 109)(80, 110)(81, 111)(82, 112)(83, 113)(84, 114)(85, 115)(86, 116)(87, 117)(88, 118)(89, 119)(90, 120)(121, 181)(122, 182)(123, 183)(124, 184)(125, 185)(126, 186)(127, 187)(128, 188)(129, 189)(130, 190)(131, 191)(132, 192)(133, 193)(134, 194)(135, 195)(136, 196)(137, 197)(138, 198)(139, 199)(140, 200)(141, 201)(142, 202)(143, 203)(144, 204)(145, 205)(146, 206)(147, 207)(148, 208)(149, 209)(150, 210)(151, 211)(152, 212)(153, 213)(154, 214)(155, 215)(156, 216)(157, 217)(158, 218)(159, 219)(160, 220)(161, 221)(162, 222)(163, 223)(164, 224)(165, 225)(166, 226)(167, 227)(168, 228)(169, 229)(170, 230)(171, 231)(172, 232)(173, 233)(174, 234)(175, 235)(176, 236)(177, 237)(178, 238)(179, 239)(180, 240)(241, 304)(242, 305)(243, 306)(244, 301)(245, 302)(246,303)(247, 313) (248, 315)(249, 314)(250, 310)(251, 312)(252, 311)(253, 307)(254, 309)(255, 308)(256, 316)(257, 318)(258, 317)(259, 323)(260, 322)(261, 324)(262, 320)(263, 319)(264, 321)(265, 330)(266, 328)(267, 329)(268, 326)(269, 327)(270, 325)(272, 273)(275, 276)(277, 283)(278, 284)(279, 285)(280, 286)(281, 288)(282, 287)(289, 296)(290, 297)(291, 295)(292, 299)(293, 298)(294, 300)
#N = 660

PSL 330 letters										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	3	0	3	3	M_{12}

Progenitor of PSL330letters:

G<x,y,t>:=Group<x,y,t| x^3 , y^2 , $(x^{-1} * y * x * y)^5$, $(y * x^{-1})^{11}$, $(x * y * x * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1} * y)^2$, t^2, $(t, x^{-1} * y * x^{-1} * y * x^{-1} * y * x * y * x * y * x)$,
$((x * y * x * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1} * y) * t^{(y*x^{-1}*y*x^{-1}*y*x*y*x^{-1})})^a$,
$(x * t^{((y*x^{-1}*y*x*y*x)^2)})^b$,
$(x * t^{(x^{-1}*y*x*y*x*y*x*y*x^{-1}*y*x*y)})^c$,
$((x^{-1} * y * x * y * x * y * x * y * x * y * x^{-1} * y * x^{-1}) * t^{(x*y*x*y*x^{-1}*y*x^{-1}*y*x*y*x*y*x*y)})^d$,
$((x^{-1} * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1} * y) * t^{(y*x)})^e$,
$((x * y) * t^{((x*y)^4)})^f$,
$(((x * y)^2) * t^{(x*y*x^{-1}*y*x*y)})^g$,
$(((x * y)^2) * t^{(x^{-1}*y*x^{-1}*y*x^{-1}*y*x^{-1}*y*x)})^h$,
$((x * y) * t^{(x*y*x^{-1}*y*x^{-1}*y*x*y*x^{-1}*y)})^i$,

$((x * y)*t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y*x*y*x*y*x^{-1}*y*x)})^j >$

PSL330letters2					
a	b	c	d	e	G
0	0	0	0	2	2

Progenitor of PSL330letters2:

$G<x,y,t>:=Group<x,y,t|\ x^3 , y^2 , (x^{-1} * y * x * y)^5 , (y * x^{-1})^{11} ,$

$(x * y * x * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1} * y)^2 , t^2,$

$(t,x^{-1} * y * x^{-1} * y * x^{-1} * y * x * y * x * y * x),$

$(((x * y)^2)*t^{(x*y*x^{-1}*y*x*y*x^{-1}*y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})^a,$

$(((x * y)^2)*t^{(x*y*x^{-1}*y*x*y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})^b,$

$((x^{-1} * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1} * y)*t^{((x^{-1}*y)^2)})^c,$

$((x^{-1} * y * x * y * x * y * x * y * x * y * x^{-1} * y * x^{-1})*t^{(x^{-1}*y*x^{-1}*y*x*y*x^{-1}*y)})^d,$

$((y * x * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1})*t^{(x^{-1}*y*x^{-1}*y*x*y*x^{-1}*y*x^{-1}*y*x^{-1}*y*x^{-1})})^e >$

PSL330letters3						
a	b	c	d	e	f	G
0	0	0	0	2	0	2
0	0	0	2	6	0	$2\times J_1$

progenitor of PSL330letters3:

$G<x,y,t>:=Group<x,y,t|\ x^3 ,$

$y^2 ,$

$(x^{-1} * y * x * y)^5 ,$

$(y * x^{-1})^{11} ,$

$(x * y * x * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1} * y)^2 ,$

$t^2,$

$(t,x^{-1} * y * x^{-1} * y * x^{-1} * y * x * y * x * y * x),$

$y * x * y * x * y * x^{-1} * y * x * y)*t^{(x^{-1})})^a,$

$(x*t^{(y*x*y*x*y*x*y*x^{-1}*y*x^{-1}*y)})^b,$

$((y * x * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1})$

$t^{(x^{-1}*y*x^{-1})} * t^{(x^{-1}*y*x*y*x^{-1}*y*x^{-1}*y*x*y*x^{-1}*y*x^{-1})})^c,$

$((x^{-1} * y * x * y * x * y * x * y * x * y * x^{-1} * y * x^{-1})*t^{(x^{-1}*y*x^{-1}*y*x*y*x^{-1}*y*x^{-1}*y*x)})^{d}$,
$((x^{-1} * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1} * y)*t^{(x*y*x*y*x^{-1}*y*x*y*x*y*x*y*x*y*x)})^{e}$,
$((x * y)*t^{(y)}*t^{(y*x*y*x*y*x*y*x^{-1}*y*x^{-1}*y*x)})^{f}$ >

PSL330letters4							
a	b	c	d	e	f	g	G
0	0	0	0	0	5	2	$2\times J_1$

Progenitor of PSL330letters4:
G<x,y,t> = Group<x,y,t| x^3 ,
y^2 ,
$(x^{-1} * y * x * y)^5$,
$(y * x^{-1})^{11}$,
$(x * y * x * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1} * y)^2$,
t^2,
$(t,x^{-1} * y * x^{-1} * y * x^{-1} * y * x * y * x * y * x)$,
$((x * y)*t^{(x*y*x^{-1})})^{a}$,
$(((x * y)^2)*t^{(x^{-1}*y*x*y*x^{-1}*y*x^{-1}*y*x*y)}*t^{(x*y*x)})^{b}$,
$((x^{-1} * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1} * y)*t^{(x^{-1}*y*x*y*x^{-1})})^{c}$,
$((x^{-1} * y * x * y * x * y * x * y * x * y * x^{-1} * y * x^{-1})*t^{(x*y*x*y*x*y*x^{-1}*y*x*y*x*y)})^{d}$,
$((x^{-1} * y * x * y * x * y * x * y * x * y * x^{-1} * y * x^{-1})*t^{(x^{-1}*y*x^{-1}*y*x^{-1}*y*x^{-1}*y*x^{-1}*y*x*y*x^{-1}*y*x)}$
$* t^{(x^{-1}*y*x^{-1}*y*x*y*x^{-1}*y*x*y)})^{e}$,
$((y * x * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1})*t^{(x^{-1}*y*x^{-1}*y*x^{-1}*y*x^{-1}*y*x^{-1}*y*x*y*x^{-1}*y)})^{f}$,
$(x*t^{(x*y*x*y*x*y*x^{-1}*y*x*y*x*y*x)})^{g}$ >

PSL330letters5					
a	b	c	d	e	G
0	0	2	0	6	$2\times J_1$

Progenitor of PSL330letters5:
G<x,y,t> = Group<x,y,t| x^3 ,
y^2 ,
$(x^{-1} * y * x * y)^5$,
$(y * x^{-1})^{11}$,

$(x * y * x * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1} * y)^2$,

t^2,

$(t,x^{-1} * y * x^{-1} * y * x^{-1} * y * x * y * x * y * x)$,

$((y * x * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1})*t^{(x)}*t^{(x^{-1}*y*x*y*x^{-1}*y)})^a$,

$((y * x * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1})*t^{(y*x*y*x^{-1})})^b$,

$((x^{-1} * y * x * y * x * y * x * y * x * y * x^{-1} * y * x^{-1})*t^{((y*x^{-1}*y*x*y*x)^2)})^c$,

$((x * y)*t^{((y*x*y*x^{-1}*y*x^{-1})^2)}*t^{(x^{-1}*y)})^d$,

$(((x * y)^2)*t^{(y)})^e$ >

PSL330letters6						
a	b	c	d	e	f	G
0	0	0	0	0	2	2

Progenitor of PSL330letters6:

G<x,y,t> = Group<x,y,t| x^3 ,

y^2 ,

$(x^{-1} * y * x * y)^5$,

$(y * x^{-1})^{11}$,

$(x * y * x * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1} * y)^2$,

t^2,

$(t,x^{-1} * y * x^{-1} * y * x^{-1} * y * x * y * x * y * x)$,

$((x * y)*t^{(y*x*y*x*y*x^{-1}*y*x*y*x*y*x*y)})^a$,

$((x * y)*t^{(x*y*x^{-1}*y*x^{-1}*y*x^{-1}*y*x^{-1})})^b$,

$(((x * y)^2)*t^{(y*x*y*x*y)}*t^{((x*y)^2)})^c$,

$((x * y)*t^{(x*y*x)})^d$,

$((x^{-1} * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1} * y) * t^{(yx^{-1}yx^{-1}yxyxy)} * t^{(xyxyxyxyxyx^{-1}y)})^e$,

$((y * x * y * x * y * x^{-1} * y * x^{-1} * y * x^{-1})*t^{(y*x*y*x*y*x^{-1}*y*x^{-1})})^f$ >

10.4 2^{*49}: $(7^2{:}3)$

We have the following information

S:=Sym(49);

x ∼ (1, 31, 11, 3, 39, 21, 7, 46, 29, 14, 2, 37, 24, 6, 44, 32, 13, 49, 40, 23, 15)(4, 38, 26,

10, 45, 27, 19, 8, 35, 20, 5, 42, 28, 12, 48, 36, 22, 9, 43, 30, 18)(16, 41, 33, 17, 47, 34, 25)
y ∼ (1, 2, 5, 11, 20, 27, 34)(3, 6, 12, 21, 28, 35, 41)(4, 9, 17, 24, 31, 38, 44)(7,13, 22, 29, 36, 42, 47)(8, 15, 19, 26, 33, 40, 46)(10, 18, 25, 32, 39, 45, 49)(14, 23, 30, 37, 43, 48, 16)
#N = 147

Sym49progenitor										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	6	3	PSL(2,13)
0	0	0	0	0	0	0	0	8	3	2^6:PSL(2,7)
0	0	0	0	0	0	2	0	0	4	2^6:(7:6)
2	0	6	0	3	0	0	0	0	0	PSL(2,13)
2	0	8	0	3	0	0	0	0	0	PSL(2,13)

Progenitor of Sym49progenitor:
G<x,y,t> = Group<x,y,t|y^7 ,y^{-2} * x^2 * y * x ,y * x^4 * y^{-2} * x^{-1},
t^2, (t,y * x^2 * y * x^{-1}),
((y * x^{-1} * y * x^{-1} * y)*$t^{(y^x)}$)^a,
((y * x^{-1} * y * x^{-1} * y)*$t^{(x*y^{-1}*x^{-1}*y*x^{-1})}$)^b,
((x^6)*$t^{(x*y*x)}$)^c,
((x^6)*$t^{(x^2*y^2)}$)^d,
((x * y^{-1} * x^{-1} * y^{-3})*$t^{(y^3*x)}$)^e,
(((x * y^{-1})^3)*$t^{(x*y^3)}$)^f,
(((x * y^{-1})^3)*$t^{(x^2*y^2)}$)^g,
((y)*$t^{(x*y*x)}$)^h,
((y)*$t^{(x^2*y^2)}$)^i,
((y^2)*$t^{(y^3*x)}$)^j >

Sym49progenitor2										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	6	3	PSL(2,13)
0	0	0	0	0	0	0	0	8	3	2^6:PSL(2,7)

Progenitor of Sym49progenitor2:

G<x,y,t> = Group<x,y,t|y^7 ,y^{-2} * x^2 * y * x ,y * x^4 * y^{-2} * x^{-1},
t^2, (t,y * x^2 * y * x^{-1}),
((y^3)*$t^{(x^2*y^{-2})}$)^a,
((y^3)*$t^{(y^3*x)}$)^b,
((y^{-3})*$t^{(x^2*y^{-2})}$)^c,
((y^{-3})*$t^{(y^3*x)}$)^d,
((y^{-2})*$t^{(x^{-1}*y^{-2}*x^{-1})}$)^e,
((y^{-2})*$t^{(y^x)}$)^f,
((y^{-1})*$t^{((y^{-1}*x^{-1})^2)}$)^g,
((y^{-1})*$t^{(y^3*x*y)}$)^h,
((x^3 * y^{-1})*$t^{(y^3*x)}$)^i,
((x^3 * y^{-1})*$t^{(x*y^3)}$)^j >

Sym49progenitor3											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	7	3	PSL(2,13)
0	0	0	0	0	0	0	0	0	8	3	PSL(2,13)
0	0	6	3	0	0	0	0	0	0	0	PSL(2,13)
0	0	8	3	0	0	0	0	0	0	0	2^6:PSL(2,7)

Progenitor of Sym49progenitor3:
G<x,y,t> = Group<x,y,t|y^7 ,y^{-2} * x^2 * y * x ,y * x^4 * y^{-2} * x^{-1},
t^2, (t,y * x^2 * y * x^{-1}),
(((x^{-1}, y))*$t^{(y^3*x)}$)^a,
(((x^{-1}, y))*$t^{(x*y^3)}$)^b,
((x * y * x^{-1} * y)*$t^{(x^2*y^{-2})}$)^c,
((x^{-1} * y^{-1} * x * y^{-2})*$t^{(x^2*y^{-2})}$)^d,
((x * y^{-1} * x * y^{-1} * x)*$t^{(x^2*y^{-2})}$)^e,
((x^2 * y^{-1} * x * y)*$t^{(x^2*y^{-2})}$)^f,
((x^2 * y^2 * x)*$t^{(x^2*y^{-2})}$)^g,
((x * y^{-1} * x^{-1} * y^{-1})*$t^{(x^2*y^{-2})}$)^h,
((x)*$t^{(x^2*y^{-2})}$)^i,
((x^2)*$t^{(x^2*y^{-2})}$)^j,

$((\mathrm{x}^4)^*\mathrm{t}^{(x^2*y^{-2})})^k >$

Sym49progenitor4										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	3	0	6	PSL(2,13)
0	0	0	0	0	0	0	3	0	7	PSL(2,13)
0	0	0	0	0	0	0	3	0	8	PSL(2,13)
0	0	0	0	0	0	3	9	6	0	PSL(2,13)
3	7	0	0	0	0	0	0	0	0	PSL(2,13)
3	8	0	0	0	0	0	0	0	0	PSL(2,13)

Progenitor of Sym49progenitor4:

G<x,y,t> = Group<x,y,t|y^7 ,$y^{-2} * x^2 * y * x$,$y * x^4 * y^{-2} * x^{-1}$,

t^2, $(t, y * x^2 * y * x^{-1})$,

$((x^5) * t^{(x^2 * y^{-2})})^a$,

$((y^{-1} * x^{-1} * y^{-3}) * t^{(x^2 * y^{-2})})^b$,

$((y^{-1} * x^{-1} * y^{-1} * x^{-1} * y^{-1}) * t^{(x^2 * y^{-2})})^c$,

$((y^3 * x * y) * t^{(x^2 * y^{-2})})^d$,

$((y * x * y * x * y) * t^{(x^2 * y^{-2})})^e$,

$((x^{-4}) * t^{(x^2 * y^{-2})})^f$,

$((y * x^{-2} * y^{-2}) * t^{(x^2 * y^{-2})})^g$,

$((x^{-2}) * t^{(y * x^{-1} * y^{-2} * x^{-1})})^h$,

$((x^{-1}) * t^{(x * y * x)})^i$,

$((x^{-1}) * t^{(x * y^{-1} * x * y * x)})^j$ >

Sym49progenitor5										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	3	6	PSL(2,13)
0	0	0	0	0	0	0	0	3	8	2^6:PSL(2,7)
0	0	0	0	0	0	0	6	8	2	2×PSL(2,7)
0	0	0	0	0	0	0	9	0	2	PSL(2,8)
0	0	0	0	0	0	7	0	6	2	PSL(2,13)
0	0	0	0	0	0	8	8	3	0	PSL(2,13)
7	0	6	3	0	0	0	0	0	0	PSL(2,13)
0	0	8	3	0	0	0	0	0	0	PSL(2,13)
3	9	0	6	0	0	0	0	0	0	PSL(2,13)

Progenitor of Sym49progenitor5:

G<x,y,t> = Group<x,y,t|y^7 ,$y^{-2} * x^2 * y * x$,$y * x^4 * y^{-2} * x^{-1}$,

t^2, (t,$y * x^2 * y * x^{-1}$),

$((x^{-1})*t^{(x*y*x^{-1}*y^{-2})})^a$,

$((x^{-1})*t^{(x^4)})^b$,

$((y * x * y * x * y)*t^{(x^2*y^2)}*t^{(x^4*y)})^c$,

$((y^{-1} * x^{-1} * y^{-1} * x^{-1} * y^{-1})*t^{(x^2)})^d$,

$((x^5)*t^{(y^x)})^e$,

$((x^2)*t^{(y^{-1})}*t^{(y*x^{-1}*y*x^{-1}*y)})^f$,

$((x)*t^{(y*x^{-2})})^g$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(y^2*x*y)})^h$,

$((x^2 * y^2 * x)*t^{(x^{-1}*y^{-3}*x^{-1})})^i$,

$((x^2 * y^{-1} * x * y)*t^{(x^{-1}*y^{-1}*x*y^{-1})}*t^{(x*y*x^2*y)})^j$ >

Sym49progenitor6										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	3	6	PSL(2,13)
0	0	0	0	0	0	0	0	3	8	2^6:PSL(2,7)
0	0	0	0	0	0	0	2	0	6	2×PSL(2,13)
0	0	0	0	0	0	0	2	0	7	PSL(2,8)
0	0	0	0	0	0	0	2	0	8	2×PSL(2,7)
0	0	0	0	0	0	0	2	0	9	PSL(2,71)
0	0	0	0	0	0	0	2	0	10	2×PSL(2,29)
6	2	0	0	0	0	0	0	0	0	2×PSL(2,7)
6	0	3	0	0	0	0	2	0	6	PSL(2,13)
8	0	3	0	0	0	0	0	0	0	2^6:PSL(2,7)
10	3	4	0	2	0	0	0	0	0	3×S_7
0	2	9	0	2	0	0	0	0	0	PSL(2,71)
0	2	10	0	2	0	0	0	0	0	2×PSL(2,29)
0	0	4	2	2	0	0	0	0	0	2×PSL(2,13)
0	5	6	2	0	0	0	0	0	0	2×A_5
7	5	6	2	2	0	0	0	0	0	A_7
10	7	6	2	2	0	0	0	0	0	2^6:S_7
10	5	8	2	2	0	0	0	0	0	4:(2:PSL(3,4))
5	4	0	3	2	0	0	0	0	0	4:(2:PSL(3,4))
5	4	7	3	2	0	0	0	0	0	4:PSL(3,4)

Progenitor of Sym49progenitor6:

G<x,y,t> = Group<x,y,t|y^7 ,y^{-2} * x^2 * y * x ,y * x^4 * y^{-2} * x^{-1},

t^2, (t,y * x^2 * y * x^{-1}),

((x * y^{-1} * x * y^{-1} * x)*$t^{((y*x^2))}$)^a,

((x^{-1} * y^{-1} * x * y^{-2})*$t^{(y^{-1})}$*$t^{(x^2*y^{-1}*x^{-1}*y^{-1}*x^{-1})}$)^b,

((x * y * x^{-1} * y)*$t^{(x^2*y^2)}$)^c,

(((x^{-1}, y))*$t^{(x^{-1}*y^{-3}*x^{-1})}$*$t^{(y*x^{-1}*y*x^{-1}*y)}$)^d,

((x^3 * y^{-1})*$t^{((y*x^{-1}*y^{-1})^2)}$)^e,

((y^{-1})*$t^{(x^{-1}*y^{-3}*x^{-1})}$)^f,

$((y^{-2})*t^{(x^{-1}*y^{-3}*x^{-1})})^{g}$,
$((y^{-3})*t^{(x^{4}*y)}*t^{(x*y*x)})^{h}$,
$((y^{3})*t^{(y^{-3})})^{i}$,
$((y^{2})*t^{(y*x^{2})})^{j}$ >

Sym49progenitor8										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	3	3	8	0	0	2^6:PSL(2,7)

Progenitor of Sym49progenitor8:
G<x,y,t> = Group<x,y,t|y^7 ,$y^{-2} * x^2 * y * x$,$y * x^4 * y^{-2} * x^{-1}$,
t^2, (t,$y * x^2 * y * x^{-1}$),
$((y^{-3})*t^{(y*x^{-1}*y*x^{-1}*y)})^{a}$,
$((y^{-3})*t^{(y^{2}*x*y)})^{b}$,
$(((x^{-1}, y))*t^{(x*y^{2}*x^{2})})^{c}$,
$(((x^{-1}, y))*t^{(y*x^{-1})})^{d}$,
$(((x^{-1}, y))*t^{(y*x^{-1}*y*x)})^{e}$,
$((x^{2} * y^{-1} * x * y)*t^{(x*y*x)})^{f}$,
$((x^{2} * y^{-1} * x * y)*t^{(x^{2}*y^{2})})^{g}$,
$((x^{2} * y^{2} * x)*t^{(x*y*x^{-1}*y^{-2})})^{h}$,
$((x^{2} * y^{2} * x)*t^{(x^{4})})^{i}$,
$((x^{2} * y^{2} * x)*t^{(x^{-3})})^{j}$ >

Sym49progenitor9										
a	b	c	d	e	f	g	h	i	j	G
3	6	2	0	0	0	0	0	0	0	7^2:(7:3)
3	0	4	1	0	0	0	0	0	0	7^2:(7:6)
0	2	3	6	0	0	0	0	0	0	PSL(2,13)
0	2	3	8	0	0	0	0	0	0	PSL(2,13)

Progenitor of Sym49progenitor9:
G<x,y,t> = Group<x,y,t|y^7 ,$y^{-2} * x^2 * y * x$,$y * x^4 * y^{-2} * x^{-1}$,
t^2, (t,$y * x^2 * y * x^{-1}$),
$((y * x^{-1} * y * x^{-1} * y)*t^{(y^{-1}*x^{-1}*y*x^{-1})}*t^{(y^{x})})^{a}$,

$((y^{-1} * x * y^{-1} * x * y^{-1})*t^{((y^{-1}*x^{-1})^2)})^b$,

$((x^3)*t^{(x^2*y^{-2})})^c$,

$((x^3)*t^{(x^4*y)}*t^{(y^x)})^d$,

$((x^6)*t^{(y^x)})^e$,

$((x^6)*t^{(x^{-2}*y^{-2}*x^{-1})})^f$,

$((x * y^{-1} * x^{-1} * y^{-3})*t^{(y^3*x*y)}*t^{(x*y^2*x^2)})^g$,

$((x * y^{-1}* x^{-1} * y^{-3})*t^{(y^2*x*y)})^h$,

$(((x * y)^3)*t^{(x^{-3})}*t^{(x^4)})^i$,

$((x^3)*t^{(x^{-1}*y^{-2}*x^{-1})})^j >$

Sym49progenitor10										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	2	0	4	2^6:(7:6)
0	0	0	0	0	2	2	2	7	7	2^6:(7:3)

Progenitor of Sym49progenitor10:

$G<x,y,t> = Group<x,y,t|y^7 ,y^{-2} * x^2 * y * x ,y * x^4 * y^{-2} * x^{-1}$,

t^2, $(t,y * x^2 * y * x^{-1})$,

$((x^6)*t^{(x*y^{-1}*x^{-1}*y*x^{-1})})^a$,

$((x * y^{-1} * x^{-1} * y^{-3})*t^{(y*x^{-1}*y^{-2}*x^{-1})})^b$,

$((x * y^{-1} * x^{-1} * y^{-3})*t^{(y*x*y*x*y)}*t^{(x^{-1}*y)})^c$,

$(((x * y)^3)*t^{(y*x^{-1})})^d$,

$(((x * y)^3)*t^{(x^4*y)})^e$,

$(((x * y^{-1})^3)*t^{(x^2)}*t^{(x*y*x^2*y)})^f$,

$((x^{-3})*t^{(x*y^{-2})})^g$,

$((x^{-3})*t^{(x*y*x^2*y)})^h$,

$((y)*t^{(y^3*x)}*t^{(x^2*y^{-2})})^i$,

$((y)*t^{(y^2*x*y)})^j >$

Sym49progenitor11										
a	b	c	d	e	f	g	h	i	j	G
0	3	6	0	0	0	0	0	0	0	PSL(2,13)
3	9	8	0	0	0	0	0	0	0	2^6:PSL(2,7)

Progenitor of Sym49progenitor11:

G<x,y,t> = Group<x,y,t|y^7 ,y^{-2} * x^2 * y * x ,y * x^4 * y^{-2} * x^{-1},

t^2, (t,y * x^2 * y * x^{-1}),

$((y^2)*t^{(x^{-1}*y^{-2}*x^{-1})})^a$,

$((y^2)*t^{(y^x)})^b$,

$((y^3)*t^{(y^x)}*t^{(x*y^{-1}*x^{-1}*y*x^{-1})})^c$,

$((y^{-3})*t^{(y*x^{-1})})^d$,

$((y^{-3})*t^{(x*y^{-1}*x*y*x)}*t^{(x*y*x^{-1}*y^{-2})})^e$,

$((y^{-2})*t^{(x*y*x)})^f$,

$((y^{-1})*t^{(y*x^{-1}*y*x)}*t^{(x^{-2}*y^{-2}*x^{-1})})^g$,

$((x^3 * y^{-1})*t^{(y^x)})^h$,

$((x^3 * y^{-1})*t^{(y*x^{-2})})^i$,

$((x * y * x^{-1} * y)*t^{(y*x^{-1}*y*x)}*t^{((y^{-1}*x^{-1})^2)})^j$ >

Sym49progenitor12										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	9	2	PSL(2,71)
0	0	0	0	0	0	0	2	3	6	PSL(2,13)
0	0	0	0	0	0	0	2	3	8	PSL(2,13)
0	0	0	0	0	0	3	5	7	0	PSL(2,29)
0	0	0	0	0	0	6	4	3	0	PSL(2,13)
0	0	0	0	0	0	8	0	3	8	PSL(2,13)
6	7	0	3	0	0	0	0	9	2	PSL(2,13)
8	7	0	3	0	0	0	0	9	2	2^6:PSL(2,7)

Progenitor of Sym49progenitor12:

G<x,y,t> = Group<x,y,t|y^7 ,y^{-2} * x^2 * y * x ,y * x^4 * y^{-2} * x^{-1},

t^2, (t,y * x^2 * y * x^{-1}),

$((x^{-1} * y^{-1} * x * y^{-2})*t^{(y*x^{-1})})^a$,

$((x * y * x^{-1} * y)*t^{((x^{-1},y))}*t^{(x^2*y*x^{-1}*y)})^b$,

$((x * y^{-1} * x * y^{-1} * x)*t^{(x^{-1}*y^{-2}*x^{-1})}*t^{(y^{-3})})^c$,

$((x^2 * y^{-1} * x * y)*t^{(y*x^{-1}*y*x)})^d$,

$((x^2 * y^2 * x)*t^{(x^2*y^{-2})}*t^{(x^2*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^e$,

$((x^2 * y^2 * x) * t^{(y^{-1}*x^{-1}*y^{-1})})^f$,
$((x * y^{-1} * x^{-1} * y^{-1}) * t^{(x^{-1}*y)})^g$,
$((x) * t^{(x*y^2*x^2)} * t^{(x^2*y*x^{-1})})^h$,
$((x^2) * t^{(x^2*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^i$,
$((x^4) * t^{(x^2*y^2)} * t^{((y*x^{-1}*y^{-1})^2)})^j$ >

Sym49progenitor15										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	2	0	4	2×PSL(2,13)
0	0	0	0	0	0	3	5	3	0	PSL(2,29)
0	0	0	0	0	0	4	2	0	0	PSL(2,13)×2
0	0	0	0	0	0	0	2	0	4	2×PSL(2,13)
3	4	0	2	0	0	0	0	0	0	PSL(2,13)×2
3	5	0	2	0	0	0	0	0	0	PSL(2,29)
2	10	0	2	0	0	0	0	0	0	2×PSL(2,29)
2	0	7	2	0	0	0	0	0	0	PSL(2,13)

Progenitor of Sym49progenitor15:
G<x,y,t> = Group<x,y,t| y^7 , $y^{-2} * x^2 * y * x$, $y * x^4 * y^{-2} * x^{-1}$,
t^2, $(t, y * x^2 * y * x^{-1})$,
$((x * y^{-1} * x^{-1} * y^{-3}) * t^{(x^2*y^{-1}*x^{-1}*y^{-1}*x^{-1})} * t^{(x^2*y^{-2})})^a$,
$((x^4) * t^{(y*x^{-2})})^b$,
$((y * x * y * x * y) * t^{(y*x^2)})^c$,
$((y * x^{-1} * y * x^{-1} * y) * t^{(x^{-1},y)})^d$,
$((x^2) * t^{(x^{-2}*y^{-2}*x^{-1})})^e$,
$((x^{-4}) * t^{(x^2*y^2)} * t^{(y^3*x)})^f$,
$((x^{-1}) * t^{(x^4)})^g$,
$((x^3 * y^{-1}) * t^{(x^2*y^{-2})} * t^{(x^2*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^h$,
$(((x * y^{-1})^3) * t^{(y*x^{-1}*y)})^i$,
$(((x * y^{-1})^3) * t^{(y*x^2)})^j$ >

Sym49progenitor16								
a	b	c	d	e	f	g	h	G
0	0	0	0	0	0	7	3	PSL(2,13)
0	0	0	0	0	6	6	3	PSL(2,13)
0	0	0	0	0	8	0	3	2^6:PSL(2,7)
0	0	0	0	0	8	8	3	PSL(2,13)
0	0	0	0	2	2	0	6	2×PSL(2,13)
0	0	0	0	2	2	0	9	PSL(2,71)
0	0	0	0	2	2	0	10	2:PSL(2,29)
0	0	0	0	2	3	3	4	PSL(2,13)×2
0	0	0	0	2	3	3	5	A_7
6	7	3	0	0	0	0	0	PSL(2,13)
8	7	3	0	0	0	0	0	2^6:PSL(2,7)

Progenitor ofSym49progenitor16:

G<x,y,t> = Group<x,y,t|y^7 ,y^{-2} * x^2 * y * x ,y * x^4 * y^{-2} * x^{-1},

t^2, (t,y * x^2 * y * x^{-1}),

((x^2 * y^2 * x)*$t^{(x*y^-1*x^{-1}*y*x^{-1})}$$)^a$,

((x^2 * y^{-1} * x * y)*$t^{(x^2*y*x^{-1}*y)}$*$t^{(x^{-3})}$$)^b$,

((x^{-4})*$t^{(y*x^{-1}*y*x)}$$)^c$,

((x^{-1})*$t^{(x^2*y^{-1}*x*y^{-1})}$*$t^{(y*x^{-1}*y*x)}$$)^d$,

((x^{-1}, y)*$t^{(y*x^{-1}*y^{-1})}$$)^e$,

((y^3)*$t^{(y*x^2)}$*$t^{(x^2*y*x^{-1}*y)}$$)^f$,

((y * x^{-1} * y * x^{-1} * y)*$t^{(x*y^{-1}*x^{-1}*y*x^{-1})}$*$t^{(x*y^2)}$$)^g$,

((y^3 * x * y)*$t^{(x*y^2*x^2)}$$)^h$ >

10.5 2^{*32}: (2:˙(5:6))

We have the following information

S:=Sym(32)

x ∼ (1, 2)(3, 5, 7, 11, 17, 4, 6, 9, 14, 22)(8, 13, 20, 29, 23, 10, 16, 25, 30,18)(12, 19, 28, 32, 26, 15, 24, 27, 31, 21)

y ∼ (1, 3)(2, 4)(5, 8)(6, 10)(7, 12, 17, 27, 14, 23) (9, 15, 22, 28, 11, 18)(13, 21, 24, 30, 32, 20)(16, 26, 19, 29, 31, 25)

#N = 1920

Progenitor of Corncob

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x^5)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^a$,

$((y^3)*(t^{(y^2*x^{-1}*y^2*x^2)}))^b$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^c$,

$((y^2)*(t^{(y^3*x*y^{-2})}))^d$,

$(((y * x^{-1})^3)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^e$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-2}*y*x^2)}))^f$,

$((x^2)*(t^{(x*y*x^{-2}*y*x)}))^g$,

$(((x * y)^2)*(t^{(y*x^{-1}*y^{-2})}))^h$,

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^i$,

$((y)*(t^{(x^3*y*x^2*y^{-2})}))^j$,

$((y^{-1})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^k$,

$((x * y * x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^l$,

$((x)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^m$,

$((x^3)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^n$,

$((y * x^{-1})*(t^{(x^{-1}*y*x*y*x)}))^o$,

$(((y * x^{-1})^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^p$,

$(((x * y)^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^q$,

$((x^2)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^r$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^s$,

$(((y * x^{-1})^3)*(t^{(x*y^2*x*y)}))^u$ >

Corncob																				
a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	u	G
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	6	3^5:A_5
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	8	2^{10}:2^5
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2

Progenitor of Corncob1

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^a$,

$((y^{-1})*(t^{(x*y^2*x*y)}))^b$,

$(((x * y)^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^c$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-1}*y*x*y*x)}))^d$,

$(((y * x^{-1})^3)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^e$,

$((y^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^f$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^g$,

$((y^3)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^h$,

$((x^5)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^i$,

$((x^3)*(t^{(x*y^2*x*y)}))^j$ >

Corncob1										
a	b	c	d	e	f	g	h	i	j	G
8	3	0	0	0	0	0	0	0	0	J_2
6	0	0	4	0	0	0	0	0	0	2
0	6	0	4	0	0	0	0	0	0	3^5:(2^5:A_5)

Progenitor of Corncob2

$G<x,y,t>:=Group<x,y,t| x^{10}, y^{6}, (x * y^{-2} * x)^{2}, (x * y^{2} * x^{2})^{2}, (y^{-1} * x^{-1})^{5}, (x * y^{2} * x^{-1} * y^{-1})^{2}, x^{-1} * y^{-1} * x^{5} * y * x^{-4}, y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1},$

$t^{2}, (t,(x^{2})), (t,(y^{2})),$

$((x^{3})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^{a},$

$((y)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^{b},$

$(((y * x^{-1})^{2})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^{c},$

$(((x * y)^{2})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^{d},$

$((y * x^{2} * y^{-1} * x^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^{e},$

$(((y * x^{-1})^{3})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^{f},$

$((y^{2})*(t^{(x^{4}*y*x^{-1}*y)}))^{g},$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{2}*x*y^{-1}*x*y^{-1}*x^{-1})}))^{h},$

$((x^{5})*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)}))^{i},$

$((x^{3})*(t^{(x^{-2}*y*x^{2})}))^{j} >$

Corncob2										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	1	0	2
0	0	0	0	0	0	0	0	2	6	S(4,3)×2
0	0	0	0	0	0	0	2	0	0	2
5	0	0	0	0	0	0	0	0	0	2
0	0	4	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob3

$G<x,y,t>:=Group<x,y,t| x^{10}, y^{6}, (x * y^{-2} * x)^{2}, (x * y^{2} * x^{2})^{2}, (y^{-1} * x^{-1})^{5}, (x * y^{2} * x^{-1} * y^{-1})^{2}, x^{-1} * y^{-1} * x^{5} * y * x^{-4}, y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1},$

t^2, (t,(x^2)), (t,(y^2)),
((x^3)*($t^{(y^{-1}*x^{-1}*y^{-1})}$))^a,
((y^{-1})*($t^{(x*y^2*x*y)}$))^b,
((($y * x^{-1})^2$)*($t^{(x^2*y^{-1}*x*y*x^{-1})}$))^c,
((y^2)*($t^{(x^{-2}*y^{-1}*x^2*y*x)}$))^d,
((($y * x^{-1})^3$)*($t^{(y*x*y^{-1}*x^{-1}*y*x)}$))^e,
((y^3)*($t^{(y^{-1}*x*y^{-1}*x*y^{-1})}$))^f,
((x^2)*($t^{(y^{-1}*x*y^{-1}*x*y^{-1})}$))^g,
((($x * y)^2$)*($t^{(x^{-2}*y^{-1}*x^2*y*x)}$))^h,
((y^3)*($t^{(x*y^{-2}*x^{-2}*y^{-1})}$))^i,
((y^{-1})*($t^{(x^{-1}*y*x*y*x)}$))^j >

Corncob3										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	2	0	2
0	0	0	0	0	0	0	0	3	8	J_2
0	0	0	0	0	0	0	5	0	0	2
5	0	0	0	0	0	0	0	0	0	2
6	3	0	0	0	0	0	0	0	0	J_2
0	0	6	4	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob4

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, (t,(x^2)), (t,(y^2)),
((x^5)*($t^{(y^2*x^{-1}*y^2*x^2)}$))^a,
((y^3)*($t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}$))^b,
(($y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x$)*($t^{(y^3*x*y^{-2})}$))^c,
((y^2)*($t^{(y^{-1}*x^{-1}*y^{-1})}$))^d,
((($y * x^{-1})^3$)*($t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}$))^e,
(($y * x^2 * y^{-1} * x^{-1}$)*($t^{(x^{-2}*y^{-1}*x^2*y*x)}$))^f,
((x^2)*($t^{(x*y*x^{-2}*y*x)}$))^g,

$(((x * y)^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^h$,
$(((y * x^{-1})^2)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^i$,
$((y)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^j >$

Corncob4										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	5	0	0	2
1	0	0	0	0	0	0	0	0	0	2
0	2	0	0	0	0	0	0	0	0	2

Progenitor of Corncob5

$G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x^5)*(t^{(x*y*x^{-2}*y*x)}))^a$,
$((y^2)*(t^{(x^{-1}*y*x*y*x)}))^b$,
$((y^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^c$,
$((y)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^d$,
$(((y * x^{-1})^2)*(t^{(x^4*y*x^{-1}*y)}))^e$,
$((y^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^f$,
$((x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^g$,
$((x * y * x)*(t^{(x^4*y*x^{-1}*y)}))^h$,
$((y * x^{-1})*(t^{(x*y^2*x*y)}))^i$,
$((x^3)*(t^{(x^3*y*x^2*y^{-2})}))^j >$

Corncob5										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	6	0	3^5:A_5
0	0	0	0	0	0	0	4	0	0	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	4	0	10	2^8:(2:A_5)
2	4	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob6

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^a$,

$((x)*(t^{(x^4*y*x^{-1}*y)}))^b$,

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^c$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^3*y*x^2*y^{-2})}))^d$,

$((y^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^e$,

$((x^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^f$,

$(((y * x^{-1})^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^g$,

$((y^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^h$,

$((y * x^{-1})*(t^{(x^{-2}*y*x^2)}))^i$,

$((x * y * x)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^j$ >

Corncob6										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	4	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	6	0	4	2^8:(2:A_5)
0	0	0	0	0	0	0	6	6	0	3^5:A_5
0	0	0	0	0	0	4	0	0	0	S(4,3)×2
6	0	0	0	0	0	0	0	0	0	3^5:A_5
0	0	4	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob7

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y^{-1})*(t^{(x^{-2}*y*x^2)}))^a$,

$(((y * x^{-1})^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^b$,

$(((y * x^{-1})^3)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^c$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^d$,

$((y^{-1})*(t^{(y*x^{-1}*y^{-2})}))^e$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^f$,

$((x^5)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^g$,

$((y * x^{-1})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^h$,

$((x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^i$,

$((x * y * x)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^j$ >

Corncob7										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	4	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	6	10	0	3^5:A_5
0	0	0	0	0	0	2	0	0	4	2^8:(2:A_5)
0	4	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob8

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x)*(t^{(y*x^{-1}*y^{-2})}))^a$,

$((y^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^b$,

$(((x * y)^2)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^c$,

$((x^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^d$,

$(((y * x^{-1})^3)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^e$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^f$,

$((x^5)*(t^{(x*y*x^{-2}*y*x)}))^g$,

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^h$,

$((y)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^i$,

$((x * y * x)*(t^{(x^3*y*x^2*y^{-2})}))^j$ >

Corncob8										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	4	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	4	0	0	S(4,3)×2
0	0	0	0	0	0	2	0	0	4	2^8:(2:A_5)
0	0	0	0	0	0	2	0	6	0	2
6	3	0	0	0	0	0	0	0	0	J_2

Progenitor of Corncob9

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x)*(t^{(x^4*y*x^{-1}*y)}))^a$,

$((x * y * x)*(t^{(x^{-1}*y*x*y*x)}))^b$,

$((y)*(t^{(y^3*x*y^{-2})}))^c$,

$(((y * x^{-1})^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^d$,

$((x^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^e$,

$((y * x^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^f$,

$((x^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^g$,
$((x * y * x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^h$,
$((y^{-1})*(t^{(x^4*y*x^{-1}*y)}))^i$,
$((y)*(t^{(x^{-2}*y*x^2)}))^j$ >

Corncob9										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	4	0	0	2^6:(2:PSL(2,7))
0	4	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
7	0	3	0	0	0	0	0	0	0	J_2
0	10	0	4	0	0	0	0	0	0	S(4,3)×2
0	4	0	6	0	0	0	0	0	0	2^6:(2:PSL(2,7))

Progenitor of Corncob10

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x * y * x)*(t^{(x^{-2}*y*x^2)}))^a$,
$((y^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^b$,
$((y)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^c$,
$(((x * y)^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^d$,
$((x^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^e$,
$(((y * x^{-1})^3)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^f$,
$((y^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^g$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^4*y*x^{-1}*y)}))^h$,
$((x^5)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^i$,
$(((y * x^{-1})^2)*(t^{(x*y*x^{-2}*y*x)}))^j$ >

Corncob10										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	4	S(4,3)×2
4	0	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))

Progenitor of Corncob11

$G<x,y,t>:=Group<x,y,t|\ x^{10}\ ,\ y^6\ ,\ (x * y^{-2} * x)^2\ ,\ (x * y^2 * x^2)^2\ ,\ (y^{-1} * x^{-1})^5\ ,\ (x * y^2$
$* x^{-1} * y^{-1})^2\ ,\ x^{-1} * y^{-1} * x^5 * y * x^{-4}\ ,\ y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}\ ,$
$t^2,\ (t,(x^2)),\ (t,(y^2)),$
$(((x * y)^2)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^a,$
$((y)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^b,$
$((y^{-1})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^c,$
$((x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^d,$
$((x^3)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^e,$
$((y^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^f,$
$(((x * y)^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^g,$
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^4*y*x^{-1}*y)}))^h,$
$((y^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^i,$
$((y^2)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^j >$

Corncob11										
a	b	c	d	e	f	g	h	i	j	G
6	3	0	0	0	0	0	0	0	0	J_2

Progenitor of Corncob12

$G<x,y,t>:=Group<x,y,t|\ x^{10}\ ,\ y^6\ ,\ (x * y^{-2} * x)^2\ ,\ (x * y^2 * x^2)^2\ ,\ (y^{-1} * x^{-1})^5\ ,\ (x * y^2$
$* x^{-1} * y^{-1})^2\ ,\ x^{-1} * y^{-1} * x^5 * y * x^{-4}\ ,\ y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}\ ,$
$t^2,\ (t,(x^2)),\ (t,(y^2)),$
$((y^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^a,$
$(((y * x^{-1})^3)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^b,$
$((x^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^c,$
$(((y * x^{-1})^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^d,$

$((y)*(t^{(y*x^{-1}*y^{-2})}))^{e}$,
$((y^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^{f}$,
$((x*y*x)*(t^{(y^{2}*x^{-1}*y*x^{-1}*y)}))^{g}$,
$((x^{3})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^{h}$,
$((y*x^{-1})*(t^{(y*x^{-1}*y^{-2})}))^{i}$,
$((x*y*x)*(t^{(x^{2}*y^{-1}*x^{-2}*y^{-1}*x)}))^{j} >$

Corncob12										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	4	2^{6}:(2:PSL(2,7))
4	6	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob13

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^{6}, (x*y^{-2}*x)^{2}, (x*y^{2}*x^{2})^{2}, (y^{-1}*x^{-1})^{5}, (x*y^{2}*x^{-1}*y^{-1})^{2}, x^{-1}*y^{-1}*x^{5}*y*x^{-4}, y*x^{-2}*y^{-1}*x^{3}*y*x*y^{3}*x^{-1}$,
t^{2}, $(t,(x^{2}))$, $(t,(y^{2}))$,
$((x*y*x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^{a}$,
$((y^{-1})*(t^{(x^{2}*y^{-1}*x^{-2}*y^{-1}*x)}))^{b}$,
$((y)*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{c}$,
$(((y*x^{-1})^{2})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^{d}$,
$(((x*y)^{2})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^{e}$,
$((x^{2})*(t^{(y^{3}*x*y^{-2})}))^{f}$,
$((y*x^{2}*y^{-1}*x^{-1})*(t^{(y^{2}*x*y*x^{-1}*y*x^{-1})}))^{g}$,
$(((y*x^{-1})^{3})*(t^{(x^{-1}*y*x*y*x)}))^{h}$,
$((y^{2})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^{i}$,
$((y*x^{-1}*y^{-1}*x^{-1}*y^{-1}*x)*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{j} >$

Corncon13										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	4	4	S(4,3)×2
4	0	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
0	0	0	4	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob14

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x^5)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^a$,

$((y^3)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^b$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^c$,

$((y^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^d$,

$(((y * x^{-1})^3)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^e$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^f$,

$((x^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^g$,

$(((x * y)^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^h$,

$(((y * x^{-1})^2)*(t^{(x^4*y*x^{-1}*y)}))^i$,

$((y)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^j$ >

Corncob14										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	4	0	S(4,3)×2

Progenitor of Corncob15

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^a$,

$((x^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^b$,

$((x)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^c$,

$((x * y * x)*(t^{(x^{-2}*y*x^{2})}))^{d}$,
$((y^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^{e}$,
$((y)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^{f}$,
$(((y * x^{-1})^{2})*(t^{(x^{-2}*y*x^{2})}))^{g}$,
$(((x * y)^{2})*(t^{(y^{2}*x^{-1}*y^{2}*x^{2})}))^{h}$,
$((x^{2})*(t^{(y*x^{-1}*y^{-2})}))^{i}$,
$((y * x^{2} * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^{j}$ >

Corncob15										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	4	0	6	0	S(4,3)×2
0	10	0	4	0	0	0	0	0	0	2^{6}:(2:PSL(2,7))
0	0	10	4	0	0	0	0	0	0	2^{8}:(2:A_{5})
0	0	0	4	0	0	0	0	0	0	2^{6}:(2:PSL(2,7))

Progenitor of Corncob17

G<x,y,t>:=Group<x,y,t| x^{10} , y^{6} , $(x * y^{-2} * x)^{2}$, $(x * y^{2} * x^{2})^{2}$, $(y^{-1} * x^{-1})^{5}$, $(x * y^{2} * x^{-1} * y^{-1})^{2}$, $x^{-1} * y^{-1} * x^{5} * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1}$,
t^{2}, $(t,(x^{2}))$, $(t,(y^{2}))$,
$((y * x^{-1})*(t^{(x^{-2}*y*x^{2})}))^{a}$,
$((x^{3})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^{b}$,
$((x)*(t^{(y^{2}*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^{c}$,
$((x * y * x)*(t^{(y^{2}*x*y^{-1}*x*y^{-1}*x^{-1})}))^{d}$,
$((y^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^{e}$,
$((y)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^{f}$,
$(((y * x^{-1})^{2})*(t^{(x^{-2}*y*x^{2})}))^{g}$,
$(((x * y)^{2})*(t^{(y^{2}*x*y^{-1}*x*y^{-1}*x^{-1})}))^{h}$,
$((x^{2})*(t^{(x^{-1}*y*x*y*x)})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^{i}$,
$((y * x^{2} * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{j}$ >

Corncob17										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	5	4	2^8:(2^5:A_5)

Progenitor of Corncob18

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^a$,
$(((y * x^{-1})^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^b$,
$((y^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^c$,
$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^d$,
$((y^{-1})*(t^{(y^3*x*y^{-2})}))^e$,
$((x)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x^3*y*x^2*y^{-2})}))^f$,
$((y * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^g$,
$((y^2)*(t^{(x*y^2*x*y)}))^h$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^3*y*x^2*y^{-2})}))^i$,
$((x^5)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^j$ >

Corncob18										
a	b	c	d	e	f	g	h	i	j	G
4	4	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
4	6	0	0	0	0	0	0	0	0	3^5:A_5
4	8	0	0	0	0	0	0	0	0	2^{10}:2^5
4	10	0	0	0	0	0	0	0	0	5^5:(2^5:A_5)
0	6	4	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob20

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x)*(t^{(x*y^2*x*y)}))^a$,

$((x * y * x)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^b$,
$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^c$,
$(((x * y)^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^3*x*y^{-2})}))^d$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^e$,
$(((y * x^{-1})^3)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^f$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(x^3*y*x^2*y^{-2})}))^g$,
$((y^3)*(t^{(y^2*x^{-1}*y^2*x^2)}))^h$,
$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^i$,
$((y^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^j$ >

Corncob20										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	3	2
0	4	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
10	4	0	0	0	0	0	0	0	0	2^8:(2:A_5)
0	0	4	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob21

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^a$,
$((y^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^b$,
$((x^5)*(t^{(x^3*y*x^2*y^{-2})}))^c$,
$((y^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^d$,
$(((y * x^{-1})^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(x*y*x^{-2}*y*x)}))^e$,
$((y^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^f$,
$((x^3)*(t^{(x*y*x^{-2}*y*x)}))^g$,
$((y * x^{-1})*(t^{(x^3*y*x^2*y^{-2})}))^h$,
$((x)*(t^{(x*y^2*x*y)}))^i$,
$((x * y * x)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^4*y*x^{-1}*y)}))^j$ >

Corncob21										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	3	S(4,3)×2

Progenitor of Corncob22

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^a$,

$((x * y * x)*(t^{(x^4*y*x^{-1}*y)}))^b$,

$((y^{-1})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^c$,

$((y)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^d$,

$(((y * x^{-1})^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^e$,

$(((x * y)^2)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^{-2}*y*x^2)}))^f$,

$((x^2)*(t^{(x^{-2}*y*x^2)}))^g$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^h$,

$(((y * x^{-1})^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^i$,

$((y^2)*(t^{(x^4*y*x^{-1}*y)})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^j$ >

Corncob22										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	6	3	3^5:A_5
0	0	0	0	0	0	0	4	0	3	3^5:(2^5:A_5)
0	0	0	0	0	0	0	4	4	0	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	4	8	0	2^{10}:2^5
0	0	0	0	0	0	0	4	10	0	5^5:(2^5:A_5)
0	4	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))

Progenitor of Corncob23

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^a$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^b$,
$((y^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^c$,
$((x^5)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^d$,
$((x^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^e$,
$(((x * y)^2)*(t^{(x^3*y*x^2*y^{-2})}))^f$,
$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^g$,
$((y)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^h$,
$((x * y * x)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^i$,
$((x^3)*(t^{(y^3*x*y^{-2})}))^j$ >

Corncob23										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	4	0	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	0	4	10	2^8:(2:A_5)
4	0	4	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob24

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(x*y^2*x*y)}))^a$,
$((x^3)*(t^{(x^{-2}*y*x^2)}))^b$,
$((x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^c$,
$((y^{-1})*(t^{(x^3*y*x^2*y^{-2})}))^d$,
$(((y * x^{-1})^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x^{-1}*y*x*y*x)}))^e$,
$(((x * y)^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^f$,
$((x^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^g$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^h$,
$(((y * x^{-1})^3)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^3*y*x^2*y^{-2})}))^i$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y*x^{-2}*y*x)}))^j$ >

Corncob24										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	2	0	2^8:(2:A_5)
0	0	0	0	0	0	0	4	0	4	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	4	0	6	3^5:(2^5:A_5)
0	0	0	0	0	0	0	4	0	8	2^{10}:2^5

Progenitor of Corncob25

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^a$,

$((y^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^b$,

$((x^5)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^c$,

$((y^2)*(t^{(x^4*y*x^{-1}*y)})*(t^{(y^3*x*y^{-2})}))^d$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^3*y*x^2*y^{-2})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^e$,

$((x^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^f$,

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^g$,

$((y)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^h$,

$((x * y * x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^i$ >

Corncob25									
a	b	c	d	e	f	g	h	i	G
0	0	0	0	0	0	0	0	4	2^6:(2:PSL(2,7))

Progenitor of Corncob28

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x^5)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^a$,

$((y^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^b$,

$((y * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^c$,

$((x)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^{d}$,
$((y^{-1})*(t^{(x^{-1}*y*x*y*x)})*(t^{(y*x^{-1}*y^{-2})}))^{e}$,
$((y)*(t^{(x^{-2}*y*x^{2})}))^{f}$,
$(((y * x^{-1})^{2})*(t^{(y*x^{-1}*y^{-2})}))^{g}$,
$(((x * y)^{2})*(t^{(x^{3}*y*x^{2}*y^{-2})})*(t^{(x^{-2}*y*x^{2})}))^{h}$,
$((x^{2})*(t^{(x^{4}*y*x^{-1}*y)}))^{i}$,
$((y * x^{2} * y^{-1} * x^{-1})*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)}))^{j} >$

Corncob28										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	5	0	4	2^{8}:(2^{5}:A_5)
2	4	0	0	0	0	0	0	0	0	S(4,3)×2
2	0	6	0	0	0	0	0	0	0	3^{5}:A_5

Progenitor of Corncob29

$G<x,y,t>:=Group<x,y,t|\ x^{10}\ ,\ y^{6}\ ,\ (x * y^{-2} * x)^{2}\ ,\ (x * y^{2} * x^{2})^{2}\ ,\ (y^{-1} * x^{-1})^{5}\ ,\ (x * y^{2} * x^{-1} * y^{-1})^{2}\ ,\ x^{-1} * y^{-1} * x^{5} * y * x^{-4}\ ,\ y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1}$,
t^{2}, $(t,(x^{2}))$, $(t,(y^{2}))$,
$(((y * x^{-1})^{3})*(t^{(x^{3}*y*x^{2}*y^{-2})}))^{a}$,
$((y^{2})*(t^{(x^{4}*y*x^{-1}*y)}))^{b}$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^{c}$,
$((y^{3})*(t^{(y*x^{-1}*y^{-2})})*(t^{(y^{-1}*x^{-1}*y*x^{2})}))^{d}$,
$((x^{5})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^{e}$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^{-2}*y*x^{2})}))^{f}$,
$(((y * x^{-1})^{3})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(y^{3}*x*y^{-2})}))^{g}$,
$((x^{2})*(t^{(x^{3}*y*x^{2}*y^{-2})}))^{h}$,
$(((x * y)^{2})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{3}*x*y^{-2})}))^{i}$,
$((x * y * x)*(t^{(y^{2}*x^{-1}*y^{2}*x^{2})}))^{j} >$

Corncob29										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	4	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	0	5	4	2^8:(2:A_5)
6	4	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob30

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^6, (x*y^{-2}*x)^2, (x*y^2*x^2)^2, (y^{-1}*x^{-1})^5, (x*y^2*x^{-1}*y^{-1})^2, x^{-1}*y^{-1}*x^5*y*x^{-4}, y*x^{-2}*y^{-1}*x^3*y*x*y^3*x^{-1},$

$t^2, (t,(x^2)), (t,(y^2)),$

$((x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^a,$

$((y*x^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^b,$

$((x^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^c,$

$((x*y*x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^d,$

$((y^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^e,$

$((y)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^f,$

$(((y*x^{-1})^2)*(t^{(x*y*x^{-2}*y*x)}))^g,$

$(((x*y)^2)*(t^{(x^3*y*x^2*y^{-2})}))^h,$

$((y*x^2*y^{-1}*x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^i,$

$((y*x^{-1}*y^{-1}*x^{-1}*y^{-1}*x)*(t^{(x^{-2}*y*x^2)})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^j >$

Corncob30										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	4	4	2^6:(2:PSL(2,7))
0	6	0	0	0	0	0	0	0	0	3^5:A_5

Progenitor of Corncob32

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^{-1})*(t^{(y*x^{-1}*y^{-2})}))^a$,
$((y^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^b$,
$((y)*(t^{(x^{-2}*y*x^2)}))^c$,
$(((y * x^{-1})^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^d$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^e$,
$(((y * x^{-1})^3)*(t^{(x^3*y*x^2*y^{-2})})*(t^{(y*x^{-1}*y^{-2})}))^f$,
$((y^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^g$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^{-1}*y*x*y*x)})*(t^{(x^3*y*x^2*y^{-2})}))^h$,
$((y^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^i$,
$(((y * x^{-1})^2)*(t^{(y*x^{-1}*y^{-2})}))^j$ >

Corncob32										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	4	S(4,3)×2

Progenitor of Corncob33

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x^3)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^a$,
$((x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^b$,
$((x * y * x)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(x^{-1}*y*x*y*x)}))^c$,
$(((x * y)^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^d$,

$((x^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^e$,
$((x^5)*(t^{(y*x*y^{-1}*x^{-1}*y*x)})*(t^{(y^3*x*y^{-2})}))^f$,
$((x^2)*(t^{(x*y*x^{-2}*y*x)}))^g$,
$((x * y * x)*(t^{(x^{-1}*y*x*y*x)}))^h$,
$((x^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^i$,
$((x)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^j >$

Corncob33										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	4	0	0	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	4	0	5	2^8:(2:A_5)

Progenitor of Corncob34

$G<x,y,t>:=Group<x,y,t|\ x^{10}\ ,\ y^6\ ,\ (x * y^{-2} * x)^2\ ,\ (x * y^2 * x^2)^2\ ,\ (y^{-1} * x^{-1})^5\ ,\ (x * y^2 * x^{-1} * y^{-1})^2\ ,\ x^{-1} * y^{-1} * x^5 * y * x^{-4}\ ,\ y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}\ ,$
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x)*(t^{(x*y^2*x*y)}))^a$,
$((x * y * x)*(t^{(x^{-1}*y*x*y*x)}))^b$,
$((y^{-1})*(t^{(x^{-1}*y*x*y*x)}))^c$,
$((y)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^d$,
$(((y * x^{-1})^2)*(t^{(x^3*y*x^2*y^{-2})})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^e$,
$((x^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^{-2}*y*x^2)}))^f$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^g$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^h$,
$((y^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^i$,
$((x^5)*(t^{(x^3*y*x^2*y^{-2})})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^j >$

Corncob34										
a	b	c	d	e	f	g	h	i	j	G
0	4	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
10	4	0	0	0	0	0	0	0	0	2^8:(2:A_5)

Progenitor of Corncob35

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^{6}, (x * y^{-2} * x)^{2}, (x * y^{2} * x^{2})^{2}, (y^{-1} * x^{-1})^{5}, (x * y^{2} * x^{-1} * y^{-1})^{2}, x^{-1} * y^{-1} * x^{5} * y * x^{-4}, y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1},$

$t^{2}, (t,(x^{2})), (t,(y^{2})),$

$((x^{5})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^{a},$

$((y^{2})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^{b},$

$(((y * x^{-1})^{3})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{c},$

$(((x * y)^{2})*(t^{(x^{4}*y*x^{-1}*y)})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^{d},$

$((y)*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)}))^{e},$

$((y^{-1})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{f},$

$((x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^{g},$

$((x * y * x)*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^{h},$

$((y^{-1})*(t^{(x^{-1}*y*x*y*x)}))^{i},$

$(((y * x^{-1})^{2})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^{j} >$

Corncob35										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	4	S(4,3)×2
2	4	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob36

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^{6}, (x * y^{-2} * x)^{2}, (x * y^{2} * x^{2})^{2}, (y^{-1} * x^{-1})^{5}, (x * y^{2} * x^{-1} * y^{-1})^{2}, x^{-1} * y^{-1} * x^{5} * y * x^{-4}, y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1},$

$t^{2}, (t,(x^{2})), (t,(y^{2})),$

$((x^{5})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^{a},$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{3}*x*y^{-2})})*(t^{(x^{-2}*y*x^{2})}))^{b},$

$(((y * x^{-1})^{3})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^{c},$

$((x^{2})*(t^{(y^{2}*x^{-1}*y^{2}*x^{2})})*(t^{(x*y^{2}*x*y)}))^{d},$

$((y * x^{2} * y^{-1} * x^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^{e},$

$(((y * x^{-1})^{2})*(t^{(y^{3}*x*y^{-2})})*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)}))^{f},$

$((y * x^{-1})*(t^{(y^{3}*x*y^{-2})})*t)^{g},$

$((x^{3})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^{h},$

$((x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(x*y*x^{-2}*y*x)}))^{i}$,
$((x * y * x)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^{j}$ >

Corncob36										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	4	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	0	5	4	2^8:(2:A_5)

Progenitor of Corncob37
G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x * y * x)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(x^3*y*x^2*y^{-2})}))^{a}$,
$((x)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^{b}$,
$((x^3)*(t^{(x^{-1}*y*x*y*x)}))^{c}$,
$((y * x^{-1})*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^{d}$,
$((x)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^{e}$,
$((y * x^{-1})*(t^{(y^3*x*y^{-2})}))^{f}$,
$((y^{-1})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^{g}$,
$((y)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^{h}$,
$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^{i}$,
$(((x * y)^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*t)^{j}$ >

Corncob37										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	4	0	S(4,3)×2
3	0	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob38
G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,

$(((x * y)^2)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^a$,
$((x^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^b$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^c$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^d$,
$((y^3)*(t^{(x^3*y*x^2*y^{-2})}))^e$,
$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^f$,
$((y)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^g$,
$((x * y * x)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^h$,
$((x^3)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^i$,
$((y * x^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^j >$

Corncob38										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	4	0	0	2^6:(2:PSL(2,7))

Progenitor of Corncob39

$G<x,y,t>:=Group<x,y,t|\ x^{10}\ ,\ y^6\ ,\ (x * y^{-2} * x)^2\ ,\ (x * y^2 * x^2)^2\ ,\ (y^{-1} * x^{-1})^5\ ,\ (x * y^2 * x^{-1} * y^{-1})^2\ ,\ x^{-1} * y^{-1} * x^5 * y * x^{-4}\ ,\ y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^a$,
$((y * x^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(x*y^2*x*y)}))^b$,
$((x)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^c$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^2*x*y)})*t)^d$,
$(((y * x^{-1})^3)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^e$,
$((y^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^f$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^3*x*y^{-2})})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^g$,
$((y^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^h$,
$((x^5)*(t^{(x^{-2}*y*x^2)}))^i$,
$((y)*(t^{(y^3*x*y^{-2})}))^j >$

Corncob39										
a	b	c	d	e	f	g	h	i	j	G
6	0	0	0	0	0	0	0	0	0	3^5:A_5

Progenitor of Corncob40

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^6, (x * y^{-2} * x)^2, (x * y^2 * x^2)^2, (y^{-1} * x^{-1})^5, (x * y^2 * x^{-1} * y^{-1})^2, x^{-1} * y^{-1} * x^5 * y * x^{-4}, y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1},$

$t^2, (t,(x^2)), (t,(y^2)),$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^3*y*x^2*y^{-2})})*(t^{(y*x^{-1}*y^{-2})}))^a,$

$(((y * x^{-1})^3)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^b,$

$((x^2)*(t^{(y^2*x^{-1}*y^2*x^2)})*(t^{(x*y^2*x*y)}))^c,$

$((y^{-1})*(t^{(x*y^2*x*y)})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^d,$

$((x * y * x)*(t^{(y*x^{-1}*y^{-2})}))^e,$

$((x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^f,$

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^g,$

$((y^2)*(t^{(y^3*x*y^{-2})})*t)^h,$

$((y^3)*(t^{(y^3*x*y^{-2})}))^i,$

$((x^5)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^j >$

Corncob40										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	2	10	2

Progenitor of Corncob41

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^6, (x * y^{-2} * x)^2, (x * y^2 * x^2)^2, (y^{-1} * x^{-1})^5, (x * y^2 * x^{-1} * y^{-1})^2, x^{-1} * y^{-1} * x^5 * y * x^{-4}, y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1},$

$t^2, (t,(x^2)), (t,(y^2)),$

$((y^3)*(t^{(y*x^{-1}*y^{-2})}))^a,$

$((y^2)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^b,$

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^4*y*x^{-1}*y)}))^c,$

$(((y * x^{-1})^2)*(t^{(x*y^2*x*y)}))^d >$

Corncob41				
a	b	c	d	G
4	4	0	0	S(4,3)×2
2	6	0	0	2
4	0	4	0	2^6:(2:PSL(2,7))
6	0	4	0	3^5:(2^5:A_5)
8	0	4	0	2^{10}:2^5
3	0	7	0	J_2

Progenitor of Corncob46

$G<x,y,t>:=Group<x,y,t| x^{10}, y^6, (x*y^{-2}*x)^2, (x*y^2*x^2)^2, (y^{-1}*x^{-1})^5, (x*y^2*x^{-1}*y^{-1})^2, x^{-1}*y^{-1}*x^5*y*x^{-4}, y*x^{-2}*y^{-1}*x^3*y*x*y^3*x^{-1},$

$t^2, (t,(x^2)), (t,(y^2)),$

$((x)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^a,$

$((x*y*x)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^b,$

$((y^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^c,$

$((y)*(t^{(x^4*y*x^{-1}*y)}))^d,$

$(((y*x^{-1})^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^e,$

$(((x*y)^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^f,$

$((x^2)*(t^{(x*y*x^{-2}*y*x)})*(t^{(y*x^{-1}*y^{-2})}))^g,$

$((y*x^2*y^{-1}*x^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^h,$

$(((y*x^{-1})^3)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^i,$

$((y^2)*(t^{(y^{-1}*x^{-1}*y*x^2)})*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^j >$

Corncob46										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	4	0	6	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	4	0	9	3^5:(2^5:A_5)
0	0	0	0	0	0	0	4	6	0	3^5:A_5
0	0	0	0	0	0	0	4	8	0	2^{10}:2^5

Progenitor of Corncob48

$G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,$

$t^2, (t,(x^2)), (t,(y^2)),$

$((x^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^a,$

$((y)*(t^{(y*x*y^{-1}*x^{-1}*y*x)})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^b,$

$((x * y * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^c,$

$((y * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^d,$

$((y * x^{-1})*(t^{(x^4*y*x^{-1}*y)}))^e,$

$((y^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^f,$

$((y)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(x^4*y*x^{-1}*y)}))^g,$

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^h,$

$(((x * y)^2)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^i,$

$((x^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^j >$

Corncob48										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	4	0	0	S(4,3)×2
0	0	4	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
10	0	4	0	0	0	0	0	0	0	2^8:(2:A_5)

Progenitor of Corncob49

$G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,$

$t^2, (t,(x^2)), (t,(y^2)),$

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^a,$

$((x^2)*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x^{-1}*y*x*y*x)}))^b,$

$(((y * x^{-1})^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^c,$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^d,$

$((x^5)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^e,$

$(((y * x^{-1})^2)*(t^{(x^{-2}*y*x^2)}))^f,$

$((y)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^g,$

$((x * y * x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(y^3*x*y^{-2})}))^h,$

$((x)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^{i}$,
$((y*x^{-1})*(t^{(y*x^{-1}*y^{-2})})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{j}$ >

Corncob49										
a	b	c	d	e	f	g	h	i	j	G
4	0	4	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
4	0	6	0	0	0	0	0	0	0	3^5:A_5

Progenitor of Corncob50

G<x,y,t>:=Group<x,y,t| x^{10} , y^{6} , $(x*y^{-2}*x)^{2}$, $(x*y^{2}*x^{2})^{2}$, $(y^{-1}*x^{-1})^{5}$, $(x*y^{2}*x^{-1}*y^{-1})^{2}$, $x^{-1}*y^{-1}*x^{5}*y*x^{-4}$, $y*x^{-2}*y^{-1}*x^{3}*y*x*y^{3}*x^{-1}$,
t^{2}, (t,(x^2)), (t,(y^2)),
$((y*x^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x^{-1}*y*x*y*x)}))^{a}$,
$((x)*(t^{(x^{-1}*y*x*y*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^{b}$,
$((x*y*x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^{c}$,
$((y^{-1})*(t^{(y^{2}*x*y*x^{-1}*y*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^{d}$,
$(((y*x^{-1})^{2})*(t^{(y^{2}*x^{-1}*y*x^{-1}*y)}))^{e}$,
$(((x*y)^{2})*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)}))^{f}$,
$(((y*x^{-1})^{3})*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^{g}$,
$((y^{2})*(t^{(y^{2}*x^{-1}*y^{2}*x^{2})}))^{h}$,
$((y^{3})*(t^{(y^{2}*x*y*x^{-1}*y*x^{-1})}))^{i}$,
$((y*x^{-1}*y^{-1}*x^{-1}*y^{-1}*x)*(t^{(x^{2}*y^{-1}*x*y*x^{-1})})*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)}))^{j}$ >

Corncob50										
a	b	c	d	e	f	g	h	i	j	G
0	0	4	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
6	0	4	0	0	0	0	0	0	0	2^8:(2:A_5)
0	5	4	0	0	0	0	0	0	0	2^8:(2:A_5)
0	10	0	0	3	0	0	0	0	0	2
0	0	0	0	4	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob51

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^a$,

$((y^2)*(t^{(x*y^2*x*y)}))^b$,

$((y^3)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^c$,

$((y^2)*(t^{(x^{-1}*y*x*y*x)}))^d$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^e$,

$(((x * y)^2)*(t^{(y*x^{-1}*y^{-2})}))^f$,

$((x^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^g$,

$((y)*(t^{(x*y*x^{-2}*y*x)}))^h$,

$((y^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^i$,

$((y)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^j$ >

Corncob51										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	5	3	8	0	J_2
0	0	0	0	0	5	5	0	6	0	2
2	0	0	0	0	0	0	0	0	0	2
0	3	0	0	0	0	0	0	0	0	2
4	6	0	4	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob52

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^a$,

$((x * y * x)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^b$,

$((y^{-1})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^c$,

$((x^3)*(t^{(y^3*x*y^{-2})}))^d$,

$((y * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^e$,

$((x^3)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^f$,

$((y^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^g$,

$((y)*(t^{(x^{-1}*y*x*y*x)}))^h$,

$((x^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^i$,

$((y^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^j$ >

Corncob52										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	3	2
0	0	0	0	0	0	0	6	0	6	2
0	4	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
0	4	6	0	0	0	0	0	0	0	2

Progenitor of Corncob53

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$((x^5)*(t^{(x^{-2}*y*x^2)}))^a$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^b$,

$((y^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^c$,

$((y^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^d$,

$((x^2)*(t^{(x*y^2*x*y)}))^e$,

$(((x * y)^2)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^f$,

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^g$,

$((y)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^h$,

$((x * y * x)*(t^{(x*y*x^{-2}*y*x)}))^i$,

$((x)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^j$ >

Corncob53										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	5	2
0	0	0	0	0	0	0	0	4	0	2^6:(2:PSL(2,7))
0	0	0	0	0	0	4	0	0	0	S(4,3)×2
1	0	0	0	0	0	0	0	0	0	2
0	2	0	0	0	0	0	0	0	0	2
2	0	4	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob54

$G<x,y,t>:=Group<x,y,t| x^{10}, y^6, (x * y^{-2} * x)^2, (x * y^2 * x^2)^2, (y^{-1} * x^{-1})^5, (x * y^2 * x^{-1} * y^{-1})^2, x^{-1} * y^{-1} * x^5 * y * x^{-4}, y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1},$

$t^2, (t,(x^2)), (t,(y^2)),$

$((y * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^a,$

$((y * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^b,$

$((y * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^c,$

$((x)*(t^{(x*y*x^{-2}*y*x)}))^d,$

$((y^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^e,$

$((y^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^f,$

$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^g,$

$(((x * y)^2)*(t^{(x^{-2}*y*x^2)}))^h,$

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^i,$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^j >$

Corncob54										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	0	0	4	4	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	0	4	6	3^5:(2^5:A_5)
0	0	0	0	0	0	0	0	4	8	2^{10}:2^5
0	0	0	0	0	0	3	0	0	0	2
0	0	0	0	0	0	4	0	0	6	S(4,3)×2
6	0	0	0	0	0	0	0	0	0	3^5:A_5

Progenitor of Corncob55

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x^5)*(t^{(x^{-2}*y*x^2)}))^a$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^b$,

$(((y * x^{-1})^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^c$,

$(((y * x^{-1})^3)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^d$,

$((x^2)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^e$,

$(((x * y)^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^f$,

$((y)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^g$,

$((x * y * x)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^h$,

$((x)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^i$,

$((y * x^{-1})*(t^{(x^{-2}*y*x^2)}))^j$ >

Corncob55										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	5	0	2
0	0	0	0	0	0	0	4	0	0	2^6:(2:PSL(2,7))
0	0	0	0	0	0	3	0	7	10	J_2
1	0	0	0	0	0	0	0	0	0	2
0	2	0	0	0	0	0	0	0	0	2
0	0	0	0	0	0	3	0	7	0	J_2

Progenitor of Corncob56

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^a$,

$((y)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^b$,

$((y^{-1})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^c$,

$(((x * y)^2)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^d$,

$((x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^e$,

$((y * x^{-1})*(t^{(x^3*y*x^2*y^{-2})}))^f$,

$((x * y * x)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^g$,

$(((y * x^{-1})^2)*(t^{(x^3*y*x^2*y^{-2})}))^h$,

$((x^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^i$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^2*x*y)}))^j$ >

Corncob56										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	5	0	2
0	0	0	0	0	0	0	4	0	0	S(4,3)×2
0	0	0	0	0	0	4	0	10	0	2^8:(2:A_5)
0	0	0	0	0	0	4	6	0	0	2^6:(2:PSL(2,7))

Progenitor of Corncob57

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^a$,
$((y^2)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^b$,
$((x^5)*(t^{(x^{-1}*y*x*y*x)}))^c$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^d$,
$(((y * x^{-1})^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^e$,
$((x^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^f$,
$((y)*(t^{(x*y^2*x*y)}))^g$,
$((y^{-1})*(t^{(y*x^{-1}*y^{-2})}))^h$,
$((y * x^{-1})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^i$,
$((y)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^j$ >

Corncob57										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	3	0	0	8	J_2
0	3	0	0	0	0	0	0	0	0	2
0	4	2	0	0	0	0	0	0	0	S(4,3)×2
0	0	6	2	0	0	0	0	0	0	2
4	0	2	4	0	0	0	0	0	0	2^6:(2:PSL(2,7))
4	0	2	6	0	0	0	0	0	0	3^5:(2^5:A_5)
4	0	2	8	0	0	0	0	0	0	2^{10}:2^5

Progenitor of Corncob58

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y)*(t^{(x^{-1}*y*x*y*x)}))^a$,
$(((x * y)^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^b$,
$((y^2)*(t^{(x^4*y*x^{-1}*y)}))^c$,
$((y^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^d$,

$((x^5)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^e$,
$((y^2)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^f$,
$(((x * y)^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^g$,
$((y^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^h$,
$((y * x^{-1})*(t^{(x^3*y*x^2*y^{-2})}))^i$,
$(((y * x^{-1})^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^j$ >

Corncob58										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	0	0	6	6	3^5:A_5
0	5	0	0	0	0	0	0	0	0	2

Progenitor of Corncob59

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$(((y * x^{-1})^3)*(t^{(x^3*y*x^2*y^{-2})}))^a$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-1}*y*x*y*x)}))^b$,
$(((x * y)^2)*(t^{(x^3*y*x^2*y^{-2})}))^c$,
$(((y * x^{-1})^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^d$,
$((y)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^e$,
$((x * y * x)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^f$,
$((x)*(t^{(y*x^{-1}*y^{-2})}))^g$,
$((y * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^h$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^i$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^j$ >

Corncob59										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	0	6	0	0	3^5:A_5
2	0	0	0	0	0	0	0	0	0	2
4	4	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
6	4	0	0	0	0	0	0	0	0	3^5:A_5
8	4	0	0	0	0	0	0	0	0	2^{10}:2^5
10	4	0	0	0	0	0	0	0	0	5^5:(2^5:A_5)

Progenitor of Corncob60

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^a$,

$((y^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^b$,

$((x^5)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^c$,

$(((x * y)^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^d$,

$(((y * x^{-1})^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^e$,

$((y)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^f$,

$((x * y * x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^g$,

$((x^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^h$,

$((y^2)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^i$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^3*x*y^{-2})}))^j$ >

Corncob60										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	0	5	0	0	2
0	0	0	0	0	0	0	0	4	4	S(4,3)×2
0	0	0	0	0	0	4	0	0	0	2^6:(2:PSL(2,7))
0	0	0	0	0	0	4	0	6	0	2^8:(2:A_5)
2	0	0	0	0	0	0	0	0	0	2
0	3	0	0	0	0	0	0	0	0	2
0	4	2	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob61

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^a$,

$((x^5)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^b$,

$((y^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^c$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^d$,

$(((x * y)^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^e$,

$(((y * x^{-1})^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^f$,

$((y)*(t^{(x^{-1}*y*x*y*x)}))^g$,

$((x * y * x)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^h$,

$((x^3)*(t^{(y*x^{-1}*y^{-2})}))^i$,

$((y * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^j$ >

Corncob61										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	4	0	0	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	4	0	6	2
0	0	0	0	0	0	0	4	10	0	2^8:(2:A_5)
2	0	0	0	0	0	0	0	0	0	2
0	1	0	0	0	0	0	0	0	0	2

Progenitor of Corncob62

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^a$,
$((x * y * x)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^b$,
$((x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^c$,
$(((y * x^{-1})^2)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^d$,
$((x^2)*(t^{(x^4*y*x^{-1}*y)}))^e$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^f$,
$(((y * x^{-1})^3)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^g$,
$((y^2)*(t^{(x^3*y*x^2*y^{-2})}))^h$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^i$,
$((y^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^j$ >

Corncob62										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	0	3	0	0	2
0	0	0	0	0	0	0	4	0	4	S(4,3)×2
6	0	0	0	0	0	0	0	0	0	3^5:A_5
0	4	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
6	4	0	0	0	0	0	0	0	0	2

Progenitor of Corncob63

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^a$,

$((y^3)*(t^{(x^4*y*x^{-1}*y)}))^b$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^c$,

$((x^5)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^d$,

$((y^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^e$,

$(((x * y)^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^f$,

$((y^{-1})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^g$,

$((x)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^h$,

$((y * x^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^i$,

$((y^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^j$ >

Corncob63										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	3	2
0	0	0	0	0	0	0	0	6	4	S(4,3)×2
0	0	0	0	0	0	3	0	5	0	J_2
2	0	0	0	0	0	0	0	0	0	2

Progenitor of Corncob64

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^a$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y*x^{-1}*y^{-2})}))^b$,

$((y^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^c$,

$((x^5)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^d$,

$(((y * x^{-1})^3)*(t^{(y^3*x*y^{-2})}))^e$,

$((x^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^f$,

$((y^{-1})*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^g$,

$((x^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^h$,

$((x)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^i$,

$((y * x^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^j$ >

Corncob64										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	6	3^5:A_5
0	0	0	0	0	0	0	5	5	0	2
3	0	0	0	0	0	0	0	0	0	2
0	0	2	0	0	0	0	0	0	0	2

Progenitor of Corncob65

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,

t^2, (t,(x^2)), (t,(y^2)),

((x^3)*($t^{(x^{-2}*y*x^2)}$))^a,

((y^{-1})*($t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}$))^b,

(((x * y)^2)*($t^{(y^{-1}*x^{-1}*y*x^{-3})}$))^c,

((x^2)*($t^{(x*y^2*x*y)}$))^d,

((y * x^2 * y^{-1} * x^{-1})*($t^{(y^{-1}*x*y^{-1}*x*y^{-1})}$))^e,

((y^2)*($t^{(y^2*x*y*x^{-1}*y*x^{-1})}$))^f,

((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*($t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}$))^g,

((x^5)*($t^{(y^{-1}*x*y^{-1}*x*y^{-1})}$))^h,

((x * y * x)*($t^{(x^{-1}*y*x*y*x)}$))^i,

((x^3)*($t^{(x^{-1}*y*x*y*x)}$))^j >

Corncob65										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	5	2
0	0	0	0	0	0	0	0	4	0	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	2	4	0	2^8:(2:A_5)
0	0	0	0	0	0	2	0	8	0	2
5	6	0	0	0	0	0	0	0	0	2

Progenitor of Corncob66

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2

$* x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,$
$t^2, (t,(x^2)), (t,(y^2)),$
$((x^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^a,$
$((x * y * x)*(t^{(x*y*x^{-2}*y*x)}))^b,$
$((y^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^c,$
$((y)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^d,$
$(((y * x^{-1})^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^e,$
$((x^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^f,$
$(((y * x^{-1})^3)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^g,$
$((y^2)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^h,$
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^i,$
$((x^3)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^j >$

Corncob66										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	2	0	2
0	0	0	0	0	0	0	0	2	5	2
0	0	0	0	0	0	0	4	0	6	S(4,3)×2
5	0	0	0	0	0	0	0	0	0	2
0	4	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))

Progenitor of Corncob67

$G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2$
$* x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,$
$t^2, (t,(x^2)), (t,(y^2)),$
$((x^3)*(t^{(x^4*y*x^{-1}*y)}))^a,$
$(((y * x^{-1})^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^b,$
$((x^2)*(t^{(y*x^{-1}*y^{-2})}))^c,$
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^d,$
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^e,$
$((x^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^f,$
$(((x * y)^2)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^g,$

$((y^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^i$,
$((x^2)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^j >$

Corncob67										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	6	5	2
5	0	0	0	0	0	0	0	0	0	2
0	4	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob68

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^a$,
$((y^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^b$,
$((x * y * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^c$,
$((y * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^d$,
$((x^3)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^e$,
$((y^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^f$,
$((y)*(t^{(x^4*y*x^{-1}*y)}))^g$,
$(((y * x^{-1})^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^h$,
$(((x * y)^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^i$,
$(((y * x^{-1})^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^j >$

Corncob68										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	0	4	0	0	S(4,3)×2
0	3	0	0	0	0	0	0	0	0	2
0	0	4	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
4	0	4	0	0	0	0	0	0	0	2^8:(2:A_5)

Progenitor of Corncob69

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y^2)*(t^{(x^3*y*x^2*y^{-2})}))^a$,

$((y^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^b$,

$(((y * x^{-1})^3)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^c$,

$((x^2)*(t^{(x*y*x^{-2}*y*x)}))^d$,

$(((y * x^{-1})^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^e$,

$((x * y * x)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^f$,

$((x * y * x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^g$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^h$,

$(((y * x^{-1})^3)*(t^{(x*y^2*x*y)}))^i$,

$((y^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^j$ >

Corncob69										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	3	2
0	0	0	0	0	0	0	0	6	4	S(4,3)×2
0	0	0	0	0	0	0	4	4	0	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	4	6	0	3^5:A_5
0	0	0	0	0	0	0	4	8	0	2^{10}:2^5
0	0	0	0	0	0	0	4	10	0	5^5:(2^5:A_5)
3	0	0	0	0	0	0	0	0	0	2
0	2	0	0	0	0	0	0	0	0	2
4	4	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob70

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$(((y * x^{-1})^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^a$,

$(((x * y)^2)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^b$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^c$,

$((x^5)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^d$,

$((x^2)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^e$,

$(((y * x^{-1})^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^f$,

$((x^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^g$,

$((y * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^h$,

$(((y * x^{-1})^3)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^i$,

$(((y * x^{-1})^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^j$ >

Corncob70										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	0	6	0	0	3^5:A_5
2	0	0	0	0	0	0	0	0	0	2
0	0	2	0	0	0	0	0	0	0	2

Progenitor of Corncob71

$G<x,y,t>:=Group<x,y,t|\ x^{10}\ ,\ y^6\ ,\ (x * y^{-2} * x)^2\ ,\ (x * y^2 * x^2)^2\ ,\ (y^{-1} * x^{-1})^5\ ,\ (x * y^2 * x^{-1} * y^{-1})^2\ ,\ x^{-1} * y^{-1} * x^5 * y * x^{-4}\ ,\ y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}\ ,$

$t^2,\ (t,(x^2)),\ (t,(y^2)),$

$((y^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^a,$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^b,$

$((y^3)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^c,$

$((x^5)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^d,$

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^e,$

$(((y * x^{-1})^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^f,$

$((y^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^g,$

$((y * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^h,$

$((x^3)*(t^{(y^3*x*y^{-2})}))^i,$

$((x * y * x)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^j >$

Corncob71										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	4	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	0	10	4	2^8:(2:A_5)
0	0	0	0	0	0	0	6	10	4	2
3	0	0	0	0	0	0	0	0	0	2
0	2	0	0	0	0	0	0	0	0	2
4	0	4	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob72

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x * y * x)*(t^{(x*y*x^{-2}*y*x)}))^a$,
$((y)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^b$,
$(((y * x^{-1})^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^c$,
$(((x * y)^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^d$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^e$,
$((x^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^f$,
$((y^3)*(t^{(y^2*x^{-1}*y^2*x^2)}))^g$,
$(((y * x^{-1})^3)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^h$,
$((y^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^i$,
$((x)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^j$ >

Corncob72										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	5	2
0	0	0	0	0	0	0	6	4	0	S(4,3)×2
4	0	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
0	0	3	0	0	0	0	0	0	0	2
10	0	4	0	0	0	0	0	0	0	S(4,3)×2
0	6	6	0	0	0	0	0	0	0	2

Progenitor of Corncob73

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y^{-1})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^a$,
$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^b$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^c$,
$((x^2)*(t^{(x^3*y*x^2*y^{-2})}))^d$,
$(((y * x^{-1})^3)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^e$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^3*y*x^2*y^{-2})}))^f$,
$((y^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^g$,
$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^h$,
$(((y * x^{-1})^3)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^i$,
$((y)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^j$ >

Corncob73										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	2	6	2
0	0	0	0	0	0	0	0	4	6	2
0	3	0	0	0	0	0	0	0	0	2
0	4	0	0	0	0	0	0	0	0	S(4,3)×2
6	0	4	0	0	0	0	0	0	0	2

Progenitor of Corncob74

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$(((y * x^{-1})^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^a$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^b$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^c$,
$((x^5)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^d$,
$(((x * y)^2)*(t^{(x^3*y*x^2*y^{-2})}))^e$,
$((x * y * x)*(t^{(x*y^2*x*y)}))^f$,
$((y * x^{-1})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^g$,
$((x^2)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^h$,
$(((y * x^{-1})^3)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^i$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^j$ >

Corncob74										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	0	0	2	0	2
3	0	0	0	0	0	0	0	0	0	2
4	0	0	0	0	0	0	0	0	0	S(4,3)×2
0	0	2	0	0	0	0	0	0	0	2
0	4	6	0	0	0	0	0	0	0	3^5:(2^5:A_5)
0	4	8	0	0	0	0	0	0	0	2^{10}:2^5

Progenitor of Corncob75

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^{-1}*y*x*y*x)}))^a$,
$((y^3)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^b$,
$((x^5)*(t^{(x*y*x^{-2}*y*x)}))^c$,
$(((x * y)^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^d$,
$((y)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^e$,
$((x * y * x)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^f$,
$((x)*(t^{(x^4*y*x^{-1}*y)}))^g$,
$((y * x^{-1})*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^h$,
$((y)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^i$,
$((y^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^j$ >

Corncob75										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	3	2
0	0	0	0	0	0	0	6	0	0	3^5:A_5
2	0	0	0	0	0	0	0	0	0	2
0	0	1	0	0	0	0	0	0	0	2

Progenitor of Corncob76

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y^2)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^a$,

$((y^3)*(t^{(x^{-2}*y*x^2)}))^b$,

$((x^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^c$,

$((x * y * x)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^d$,

$((x)*(t^{(x^{-2}*y*x^2)}))^e$,

$((x^3)*(t^{(x^{-1}*y*x*y*x)}))^f$,

$((y * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^g$,

$((x * y * x)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^h$,

$(((x * y)^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^i$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^j$ >

Corncob76										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	5	0	2
0	0	0	0	0	0	0	4	0	0	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	4	0	4	2^8:(2:A_5)
0	0	0	0	0	0	6	0	0	0	3^5:A_5
3	0	0	0	0	0	0	0	0	0	2
0	2	0	0	0	0	0	0	0	0	2
4	4	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob77

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y*x^{-2}*y*x)}))^a$,

$(((y * x^{-1})^3)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^b$,

$((x^5)*(t^{(x*y^2*x*y)}))^c$,

$((y * x^{-1})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^d$,

$((x)*(t^{(x^4*y*x^{-1}*y)}))^e$,

$((y)*(t^{(x^{-2}*y*x^2)}))^f$,

$(((y * x^{-1})^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^g$,

$(((x * y)^2)*(t^{(y*x^{-1}*y^{-2})}))^h$,

$((x^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^i$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^j$ >

Corncob77										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	5	0	2
0	2	0	0	0	0	0	0	0	0	2
4	4	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
4	6	0	0	0	0	0	0	0	0	3^5:A_5
4	8	0	0	0	0	0	0	0	0	2^{10}:2^5
4	10	0	0	0	0	0	0	0	0	5^5:(2^5:A_5)

Progenitor of Corncob78

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$(((y * x^{-1})^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^a$,
$((y^3)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^b$,
$((y^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^c$,
$((x^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^d$,
$((y)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^e$,
$(((y * x^{-1})^3)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^f$,
$((y^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^g$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^2*x^{-1}*y^2*x^2)}))^h$,
$((y^3)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^i$,
$((x^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^j$ >

Corncob78										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	2	0	2
0	0	0	0	0	0	0	0	3	7	J_2
2	0	0	0	0	0	0	0	0	0	2
0	2	0	0	0	0	0	0	0	0	2

Progenitor of Corncob79

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^a$,
$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^b$,
$((x^3)*(t^{(y^2*x^{-1}*y^2*x^2)}))^c$,
$((y^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^d$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^e$,
$((y^3)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^f$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^g$,
$((y^{-1})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^h$,
$((x)*(t^{(y^3*x*y^{-2})}))^i$,
$(((x * y)^2)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^j$ >

Corncob79										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	4	0	10	5	2
0	3	0	0	0	0	0	0	0	0	2
0	4	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob80

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^a$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^b$,
$((y^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^c$,
$((x)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^d$,
$((x^3)*(t^{(y^3*x*y^{-2})}))^e$,
$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^f$,
$((x^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^g$,
$(((y * x^{-1})^3)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^h$,

$((y^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^i$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^j >$

Corncob80										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	0	0	3	0	2
0	0	0	0	0	0	0	6	4	0	S(4,3)×2
0	0	2	0	0	0	0	0	0	0	2
4	0	4	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))

Progenitor of Corncob82

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$(((y * x^{-1})^3)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^a$,
$((y)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^b$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^c$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^d$,
$((x^3)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^e$,
$(((y * x^{-1})^3)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^f$,
$(((x * y)^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^g$,
$((y)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^h$,
$((x)*(t^{(x^3*y*x^2*y^{-2})}))^i$,
$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^j >$

Corncob82										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	3	2
0	0	0	0	0	0	0	0	0	4	S(4,3)×2
2	0	0	0	0	0	0	0	0	0	2
4	0	4	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
6	0	4	0	0	0	0	0	0	0	3^5:A_5
8	0	4	0	0	0	0	0	0	0	2^{10}:2^5
0	6	4	0	0	0	0	0	0	0	3^5:(2^5:A_5)
4	6	4	0	0	0	0	0	0	0	2
0	3	7	0	0	0	0	0	0	0	J_2

Progenitor of Corncob83

$G<x,y,t>:=Group<x,y,t|\ x^{10},\ y^6,\ (x * y^{-2} * x)^2,\ (x * y^2 * x^2)^2,\ (y^{-1} * x^{-1})^5,\ (x * y^2 * x^{-1} * y^{-1})^2,\ x^{-1} * y^{-1} * x^5 * y * x^{-4},\ y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1},$

$t^2, (t,(x^2)), (t,(y^2)),$

$((y^2)*(t^{(x^{-1}*y*x*y*x)}))^a,$

$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^b,$

$(((y * x^{-1})^3)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^c,$

$((x^5)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^d,$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^{-2}*y*x^2)}))^e,$

$((y^2)*(t^{(x^{-1}*y*x*y*x)}))^f,$

$(((y * x^{-1})^3)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^g,$

$((x^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^h,$

$((y^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^i,$

$((x)*(t^{(y^3*x*y^{-2})}))^j >$

Corncob83										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	3	0	2
0	0	0	0	0	0	0	0	4	6	S(4,3)×2
3	0	0	0	0	0	0	0	0	0	2
0	4	0	0	0	0	0	0	0	0	S(4,3)×2
0	0	2	0	0	0	0	0	0	0	2^8:(2:A_5)

Progenitor of Corncob84

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^a$,

$((x^5)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^b$,

$(((x * y)^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^c$,

$((y^2)*(t^{(x*y^2*x*y)}))^d$,

$(((x * y)^2)*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^e$,

$((y^3)*(t^{(x^{-2}*y*x^2)}))^f$,

$((y^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^g$,

$((y)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^h$,

$(((y * x^{-1})^3)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(x*y*x^{-2}*y*x)}))^i$,

$((y * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^j$ >

Corncob84										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	6	3^5:A_5
0	0	0	0	0	0	0	0	4	6	2
0	0	0	0	0	2	0	0	8	0	2
0	1	0	0	0	0	0	0	0	0	2

Progenitor of Corncob85

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2$

$* x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^a$,
$((x * y * x)*(t^{(x*y*x^{-2}*y*x)})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^b$,
$(((x * y)^2)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^c$,
$((x^3)*(t^{(y*x^{-1}*y^{-2})})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^d$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^4*y*x^{-1}*y)}))^e$,
$((x)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^f$,
$((x^5)*(t^{(y*x^{-1}*y^{-2})}))^g$,
$(((y * x^{-1})^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^h$,
$((y^{-1})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^i$,
$((y)*(t^{(y*x^{-1}*y^{-2})}))^j >$

Corncob85										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	4	0	0	S(4,3)×2
0	0	0	0	0	0	1	0	0	0	2
0	0	0	0	0	0	2	0	0	6	2

Progenitor of Corncob86

$G<x,y,t>:=Group<x,y,t|\ x^{10}$, y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2$
$* x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y^{-1})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^a$,
$((y)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^b$,
$(((y * x^{-1})^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^c$,
$(((x * y)^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^d$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-1}*y*x*y*x)}))^e$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})})*(t^{(y*x^{-1}*y^{-2})}))^f$,
$((y * x^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^g$,
$((x)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^h$,
$(((y * x^{-1})^2)*(t^{(x*y*x^{-2}*y*x)}))^i$,

$((x^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^j$ >

Corncob86										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	5	2
0	0	0	0	0	2	0	10	6	0	2
0	0	4	0	0	0	0	0	0	0	S(4,3)×2
0	0	6	5	0	0	0	0	0	0	2
6	0	0	0	4	0	0	0	0	0	2

Progenitor of Corncob87

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^a$,

$((x^5)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^b$,

$((y)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^c$,

$((y^{-1})*(t^{(y^3*x*y^{-2})}))^d$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^e$,

$((x * y * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^f$,

$((y * x^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^g$,

$(((y * x^{-1})^3)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^i$,

$((y^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^j$ >

Corncob87										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	3	2
0	0	0	0	0	0	0	0	6	4	S(4,3)×2
0	0	0	0	0	4	0	2	8	0	2^6:(2:PSL(2,7))
0	0	0	0	0	4	0	2	8	6	2^8:(2:A_5)
2	0	0	0	0	0	0	0	0	0	2
0	1	0	0	0	0	0	0	0	0	2

Progenitor of Corncob88

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y^2)*(t^{(y^3*x*y^{-2})}))^a$,

$(((y * x^{-1})^2)*(t^{(y^2*x^{-1}*y^2*x^2)}))^b$,

$((x)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^c$,

$((x^3)*(t^{(x^{-1}*y*x*y*x)}))^d$,

$((y^{-1})*(t^{(x^{-2}*y*x^2)}))^e$,

$((y)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^f$,

$(((y * x^{-1})^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^g$,

$((x^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^h$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y*x^{-2}*y*x)}))^i$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^j$ >

Corncob88										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	0	0	4	4	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	0	4	6	3^5:(2^5:A_5)
0	0	0	0	0	0	0	0	4	8	2^{10}:2^5
0	0	0	0	0	0	4	0	0	4	S(4,3)×2
3	0	0	0	0	0	0	0	0	0	2
0	4	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob89

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^a$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^3*y*x^2*y^{-2})}))^b$,
$((x^5)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^c$,
$((y^2)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^d$,
$((x * y * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^e$,
$((x)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^f$,
$((y^{-1})*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^g$,
$((x^2)*(t^{(x^3*y*x^2*y^{-2})}))^h$,
$(((y * x^{-1})^3)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^i$,
$((x^5)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^j$ >

Corncob89										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	1	2
0	0	0	0	0	0	0	0	0	2	S(4,3)×2
0	0	0	0	0	0	6	0	4	10	2
3	0	0	0	0	0	0	0	0	0	2
4	2	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob90

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^a$,
$((y^2)*(t^{(x^3*y*x^2*y^{-2})})*(t^{(x^4*y*x^{-1}*y)}))^b$,
$((x^3)*(t^{(x*y^2*x*y)}))^c$,
$((y * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^d$,
$((y)*(t^{(x^{-1}*y*x*y*x)})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^e$,
$(((y * x^{-1})^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^f$,
$((x^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^g$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-2}*y*x^2)}))^h$,
$(((y * x^{-1})^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^i$,
$((y^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^j$ >

Corncob90										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	2	0	2
0	0	0	0	0	0	0	0	4	3	2
0	0	0	0	0	0	0	4	0	6	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	4	0	9	3^5:(2^5:A_5)
0	0	0	0	0	0	0	4	6	0	3^5:A_5
0	0	0	0	0	0	0	4	8	0	2^{10}:2^5
0	0	0	0	0	0	0	4	10	0	5^5:(2^5:A_5)
4	3	0	0	0	0	0	0	0	0	2
4	6	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
4	0	10	6	0	0	0	0	0	0	3^5:A_5

Progenitor of Corncob91

$G<x,y,t>:=Group<x,y,t|\ x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,$

$t^2, (t,(x^2)), (t,(y^2)),$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-1}*y*x^2)})*(t^{(x*y*x^{-2}*y*x)}))^a,$

$((x^5)*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^b,$

$((x * y * x)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})})*(t^{(x*y^2*x*y)}))^c,$

$((y^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^d,$

$((y)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^e,$

$((x^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^f,$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(x^4*y*x^{-1}*y)}))^g,$

$((y^3)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(x^3*y*x^2*y^{-2})}))^h,$

$((y * x^{-1})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^i,$

$((x)*(t^{(x*y*x^{-2}*y*x)})*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^j >$

Corncob91										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	2	6	0	2^8:(2:A_5)
0	0	0	0	0	0	0	2	6	5	2^8:(2:A_5)
2	0	0	0	0	0	0	0	0	0	2
0	1	0	0	0	0	0	0	0	0	2

Progenitor of Corncob92

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x * y * x)*(t^{(x^4*y*x^{-1}*y)}))^a$,

$((y)*(t^{(y^{-1}*x^{-1}*y*x^2)})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^b$,

$(((y * x^{-1})^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^c$,

$((x^2)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^d$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^4*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^e$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^f$,

$((y^3)*(t^{(y*x^{-1}*y^{-2})}))^g$,

$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^h$,

$(((x * y)^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^i$,

$(((y * x^{-1})^3)*(t^{(x^4*y*x^{-1}*y)}))^j$ >

Corncob92										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	0	0	5	4	2^8:(2:A_5)
4	0	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
4	6	0	0	0	0	0	0	0	0	2^8:(2:A_5)
0	0	3	0	0	0	0	0	0	0	2
0	0	4	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob93

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(y*x^{-1}*y^{-2})}))^a$,
$((x^5)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^b$,
$(((y * x^{-1})^3)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^c$,
$(((x * y)^2)*(t^{(x^2*y^{-1}*x*y*x^{-1})})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^d$,
$((y * x^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^e$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^f$,
$(((y * x^{-1})^3)*(t^{(y*x^{-1}*y^{-2})})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^g$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^h$,
$((y^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^i$,
$((x^5)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^j$ >

Corncob93										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	1	2
0	0	0	0	0	0	0	0	2	0	2^8:(2:A_5)
0	0	0	0	0	0	2	0	2	0	2^8:(2:A_5)
0	0	0	0	0	0	4	4	0	0	2^{10}:2^5
2	0	0	0	0	0	0	0	0	0	2
0	1	0	0	0	0	0	0	0	0	2

Progenitor of Corncob94

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y*x^{-2}*y*x)}))^a$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^b$,
$(((x * y)^2)*(t^{(x^{-2}*y*x^2)}))^c$,
$((x * y * x)*(t^{(x^{-2}*y*x^2)})*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^d$,
$((x^3)*(t^{(x^3*y*x^2*y^{-2})}))^e$,

$((y * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-1}*y*x^{2})}))^{f}$,
$((x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^{g}$,
$((x * y * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(x^{4}*y*x^{-1}*y)}))^{h}$,
$(((y * x^{-1})^{2})*(t^{(y^{2}*x*y^{-1}*x*y^{-1}*x^{-1})}))^{i}$,
$((y * x^{2} * y^{-1} * x^{-1})*(t^{(y^{2}*x^{-1}*y*x^{-1}*y)})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{j}$ >

Corncob94										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	5	0	9	0	2
2	0	0	0	0	0	0	0	0	0	2
2	0	10	4	0	0	0	0	0	0	2

Progenitor of Corncob95

G<x,y,t>:=Group<x,y,t| x^{10} , y^{6} , $(x * y^{-2} * x)^{2}$, $(x * y^{2} * x^{2})^{2}$, $(y^{-1} * x^{-1})^{5}$, $(x * y^{2} * x^{-1} * y^{-1})^{2}$, $x^{-1} * y^{-1} * x^{5} * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1}$,
t^{5}, $(t,(x^{2}))$, $(t,(y^{2}))$,
$((y * x^{2} * y^{-1} * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^{a}$,
$((y^{2})*(t^{(x^{3}*y*x^{2}*y^{-2})})*(t^{(x^{4}*y*x^{-1}*y)}))^{b}$,
$((x^{3})*(t^{(x*y^{2}*x*y)}))^{c}$,
$((y * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^{d}$,
$((y)*(t^{(x^{-1}*y*x*y*x)})*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)}))^{e}$,
$(((y * x^{-1})^{2})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^{f}$,
$((x^{2})*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)})*(t^{(y^{2}*x^{-1}*y^{2}*x^{2})}))^{g}$,
$((y * x^{2} * y^{-1} * x^{-1})*(t^{(x^{-2}*y*x^{2})}))^{h}$,
$(((y * x^{-1})^{3})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^{i}$,
$((y^{2})*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(y^{2}*x*y^{-1}*x*y^{-1}*x^{-1})}))^{j}$ >

Corncob95										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	4	4	0	2
4	0	0	0	0	0	0	0	0	0	2

Progenitor of Corncob96

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^{6}, (x * y^{-2} * x)^{2}, (x * y^{2} * x^{2})^{2}, (y^{-1} * x^{-1})^{5}, (x * y^{2} * x^{-1} * y^{-1})^{2}, x^{-1} * y^{-1} * x^{5} * y * x^{-4}, y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1},$

$t^{5}, (t,(x^{2})), (t,(y^{2})),$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-1}*y*x^{2})})*(t^{(x*y*x^{-2}*y*x)}))^{a},$

$((x^{5})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)})*(t^{(x^{2}*y^{-1}*x*y*x^{-1})}))^{b},$

$((x * y * x)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})})*(t^{(x*y^{2}*x*y)}))^{c},$

$((y^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{d},$

$((y)*(t^{(y^{2}*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^{e},$

$((x^{2})*(t^{(y*x*y^{-1}*x^{-1}*y*x)})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^{f},$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(x^{4}*y*x^{-1}*y)}))^{g},$

$((y^{3})*(t^{(y^{3}*x^{-1}*y^{-2}*x^{-1})})*(t^{(x^{3}*y*x^{2}*y^{-2})}))^{h},$

$((y * x^{-1})*(t^{(y^{3}*x^{-1}*y^{-2}*x^{-1})})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^{i},$

$((x)*(t^{(x*y*x^{-2}*y*x)})*(t^{(y^{2}*x*y*x^{-1}*y*x^{-1})}))^{j} >$

Corncon96										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	2	0	0	2
2	0	0	0	0	0	0	0	0	0	2

Progenitor of Corncob97

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^{6}, (x * y^{-2} * x)^{2}, (x * y^{2} * x^{2})^{2}, (y^{-1} * x^{-1})^{5}, (x * y^{2} * x^{-1} * y^{-1})^{2}, x^{-1} * y^{-1} * x^{5} * y * x^{-4}, y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1},$

$t^{5}, (t,(x^{2})), (t,(y^{2})),$

$((x * y * x)*(t^{(x^{4}*y*x^{-1}*y)}))^{a},$

$((y)*(t^{(y^{-1}*x^{-1}*y*x^{2})})*(t^{(x^{2}*y^{-1}*x*y*x^{-1})}))^{b},$

$(((y * x^{-1})^{2})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^{c},$

$((x^2)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^d$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^4*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^e$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^f$,
$((y^3)*(t^{(y*x^{-1}*y^{-2})}))^g$,
$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^h$,
$(((x * y)^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^i$,
$(((y * x^{-1})^3)*(t^{(x^4*y*x^{-1}*y)}))^j$ >

Corncob97										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	2	5	4	2

Progenitor of Corncob98

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^5, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(y*x^{-1}*y^{-2})}))^a$,
$((x^5)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^b$,
$(((y * x^{-1})^3)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^c$,
$(((x * y)^2)*(t^{(x^2*y^{-1}*x*y*x^{-1})})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^d$,
$((y * x^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^e$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^f$,
$(((y * x^{-1})^3)*(t^{(y*x^{-1}*y^{-2})})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^g$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^h$,
$((y^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^i$,
$((x^5)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^j$ >

Corncob98										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	2	0	5^5:(2^5:A_5)
0	0	0	0	0	0	0	4	0	0	2
2	0	0	0	0	0	0	0	0	0	2

Progenitor of Corncob99

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^5, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y*x^{-2}*y*x)}))^a$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^b$,
$(((x * y)^2)*(t^{(x^{-2}*y*x^2)}))^c$,
$((x * y * x)*(t^{(x^{-2}*y*x^2)})*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^d$,
$((x^3)*(t^{(x^3*y*x^2*y^{-2})}))^e$,
$((y * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^f$,
$((x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^g$,
$((x * y * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^h$,
$(((y * x^{-1})^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^i$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^j$ >

Corncob99										
a	b	c	d	e	f	g	h	i	j	G
2	0	0	0	0	0	0	0	0	0	2

Progenitor of Corncob100

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y^2)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^a$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^2*x*y)}))^b$,
$((y * x^{-1})*(t))^c$,

$(((y * x^{-1})^2)*(t^{(x^4*y*x^{-1}*y)}))^d$,
$((x^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*t)^e$,
$((y^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^f$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^g$,
$((x^5)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^h$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*t*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^i$,
$((x^3)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^j$ >

Corncob100										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	5	2
0	0	0	0	0	0	0	0	2	0	2
3	0	0	0	0	0	0	0	0	0	2
0	0	6	0	0	0	0	0	0	0	3^5:A_5

Progenitor of Corncob101

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y^2)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^a$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y*x^{-1}*y^{-2})}))^b$,
$((y * x^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})})*(t^{(x^3*y*x^2*y^{-2})}))^c$,
$((x^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^d$,
$((x)*(t^{(y*x^{-1}*y^{-2})}))^e$,
$((x * y * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*t)^f$,
$((y^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^g$,
$((y)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^h$,
$(((y * x^{-1})^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^i$,
$((x^2)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^j$ >

Corncob101										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	5	2
0	0	0	0	0	0	3	0	0	7	J_2
3	0	0	0	0	0	0	0	0	0	2

Progenitor of Corncob102

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^a$,
$(((y * x^{-1})^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^b$,
$((y^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^c$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)})*(t^{(x^4*y*x^{-1}*y)}))^d$,
$((y^3)*(t^{(y*x^{-1}*y^{-2})}))^e$,
$((x^5)*(t^{(x^{-2}*y*x^2)}))^f$,
$((x^2)*t*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^g$,
$((y * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^h$,
$((x^3)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^i$,
$((x)*(t^{(x*y^2*x*y)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^j$,
$((x^3)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*t)^k$ >

Corncob102											
a	b	c	d	e	f	g	h	i	j	k	G
4	4	0	0	0	0	0	0	0	0	0	2^6:(2:PSL(2,7))
4	6	0	0	0	0	0	0	0	0	0	3^5:A_5
4	8	0	0	0	0	0	0	0	0	0	2^{10}:2^5
4	10	0	0	0	0	0	0	0	0	0	5^5:(2^5:A_5)
0	6	4	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob103

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2$

$* x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^a$,
$((x^3)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^b$,
$((x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^c$,
$((x * y * x)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^d$,
$((y^{-1})*(t^{(x^4*y*x^{-1}*y)}))^e$,
$((y)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^f$,
$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^g$,
$(((x * y)^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^h$,
$((x^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^i$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^j$,
$(((y * x^{-1})^3)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^k$ >

Corncob103											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	0	2	2
0	0	0	0	0	0	0	0	0	4	4	2^6:(2:PSL(2,7))
0	0	0	0	0	0	0	0	0	4	6	3^5:A_5
0	0	0	0	0	0	0	0	0	4	8	2^{10}:2^5
0	0	0	0	0	0	0	0	0	4	10	5^5:(2^5:A_5)
0	0	0	0	0	0	0	0	0	4	10	2

Progenitor of Corncob104
G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y^2)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^a$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^b$,
$((y^3)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^c$,
$((x^5)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^d$,
$((y * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^e$,

$((x^3)*(t^{(x^{-1}*y*x*y*x)}))^f$,
$((x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^g$,
$((x*y*x)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^h$,
$((y^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^i$,
$((y)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^j$,
$(((y*x^{-1})^2)*(t^{(x^3*y*x^2*y^{-2})}))^k$ >

Corncob104											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	0	3	2
0	0	0	0	0	0	0	0	0	0	4	S(4,3)×2
3	0	0	0	0	0	0	0	0	0	0	2
0	8	2	0	0	0	0	0	0	0	0	2
4	0	4	0	0	0	0	0	0	0	0	S(4,3)×2

Progenitor of Corncob105

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x*y^{-2}*x)^2$, $(x*y^2*x^2)^2$, $(y^{-1}*x^{-1})^5$, $(x*y^2*x^{-1}*y^{-1})^2$, $x^{-1}*y^{-1}*x^5*y*x^{-4}$, $y*x^{-2}*y^{-1}*x^3*y*x*y^3*x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$(((x*y)^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^a$,
$((x^2)*(t^{(y*x^{-1}*y^{-2})}))^b$,
$((y*x^2*y^{-1}*x^{-1})*(t^{(x*y*x^{-2}*y*x)}))^c$,
$(((y*x^{-1})^3)*(t^{(x^{-2}*y*x^2)}))^d$,
$((y^2)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^e$,
$((y*x^{-1}*y^{-1}*x^{-1}*y^{-1}*x)*(t^{(y^3*x*y^{-2})}))^f$,
$((y^3)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^g$,
$((x^5)*(t^{(x*y^2*x*y)}))^h$,
$((y*x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)})*t)^i$ >

Corncob105									
a	b	c	d	e	f	g	h	i	G
0	0	0	0	0	0	0	1	0	2
0	0	0	0	0	0	2	0	0	2
5	0	0	0	0	0	0	0	0	2

Progenitor of Corncob106

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(y^2*x^{-1}*y^2*x^2)}))^a$,

$((x^5)*(t^{(x*y^2*x*y)}))^b$,

$((x)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^c$,

$((y^3)*(t^{(y^3*x*y^{-2})}))^d$,

$((x * y * x)*(t^{(x^4*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^e$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^f$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^{-2}*y*x^2)}))^g$,

$((y^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^h$,

$((y^2)*(t^{(x*y*x^{-2}*y*x)}))^i$,

$((y)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^j$,

$(((y * x^{-1})^3)*(t^{(y*x^{-1}*y^{-2})}))^k$ >

Corncob106											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	0	2	2
0	1	0	0	0	0	0	0	0	0	0	2

Progenitor of Grapejelly2

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^a$,

$((y^{-1})*(t^{(x*y^2*x*y)}))^b,$

$(((x*y)^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^c,$

$((y*x^2*y^{-1}*x^{-1})*(t^{(x^{-1}*y*x*y*x)}))^d,$

$(((y*x^{-1})^3)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^e,$

$((y^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^f,$

$((y*x^{-1}*y^{-1}*x^{-1}*y^{-1}*x)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^g,$

$((y^3)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^h,$

$((x^5)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^i,$

$((x^3)*(t^{(x*y^2*x*y)}))^j,$

$(x^5*t)^k >$

Grapejelly2											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	5	5	$2{:}(A_5{:}J_2)$
0	0	0	0	0	0	0	0	2	6	2	$S(4,3)\times 2$
0	3	0	0	0	0	0	0	0	0	5	$A_5 \times J_2$
0	3	0	0	0	0	0	0	0	0	10	$2{:}(A_5{:}J_2)$

Progenitor of Grapejelly3

$G<x,y,t>:=Group<x,y,t|\ x^{10},\ y^6,\ (x*y^{-2}*x)^2,\ (x*y^2*x^2)^2,\ (y^{-1}*x^{-1})^5,\ (x*y^2*x^{-1}*y^{-1})^2,\ x^{-1}*y^{-1}*x^5*y*x^{-4},\ y*x^{-2}*y^{-1}*x^3*y*x*y^3*x^{-1},$

$t^2,\ (t,(x^2)),\ (t,(y^2)),$

$((x^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^a,$

$((y)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^b,$

$(((y*x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^c,$

$(((x*y)^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^d,$

$((y*x^2*y^{-1}*x^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^e,$

$(((y*x^{-1})^3)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^f,$

$((y^2)*(t^{(x^4*y*x^{-1}*y)}))^g,$

$((y*x^{-1}*y^{-1}*x^{-1}*y^{-1}*x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^h,$

$((x^5)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^i,$

$((x^3)*(t^{(x^{-2}*y*x^2)}))^j,$

$(x^5*t)^k$ >

Grapejelly3											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	0	5	2:(A_5:J_2)

Progenitor of Grapejelly4

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x^3)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^a$,

$((y^{-1})*(t^{(x*y^2*x*y)}))^b$,

$(((y * x^{-1})^2)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^c$,

$((y^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^d$,

$(((y * x^{-1})^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^e$,

$((y^3)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^f$,

$((x^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^g$,

$(((x * y)^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^h$,

$((y^3)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^i$,

$((y^{-1})*(t^{(x^{-1}*y*x*y*x)}))^j$,

$(x^5*t)^k$ >

Grapejelly4											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	3	0	5	$A_5 \times J_2$
0	0	0	0	0	0	0	0	3	0	10	2:(A_5:J_2)
0	0	0	0	0	0	0	0	3	8	5	J_2
0	3	0	0	0	0	0	0	0	0	5	$A_5 \times J_2$
0	3	0	0	0	0	0	0	0	0	10	2:(A_5:J_2)
6	3	0	0	0	0	0	0	0	0	5	J_2

Progenitor of Grapejelly5

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x^5)*(t^{(y^2*x^{-1}*y^2*x^2)}))^a$,

$((y^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^b$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^3*x*y^{-2})}))^c$,

$((y^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^d$,

$(((y * x^{-1})^3)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^e$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^f$,

$((x^2)*(t^{(x*y*x^{-2}*y*x)}))^g$,

$(((x * y)^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^h$,

$(((y * x^{-1})^2)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^i$,

$((y)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^j$,

$(x^5*t)^k$ >

Grapejelly5											
a	b	c	d	e	f	g	h	i	j	k	G
0	3	0	0	0	0	0	0	0	0	5	$A_5 \times J_2$
0	3	0	0	0	0	0	0	0	0	10	2:(A_5:J_2)

Progenitor of Grapejelly6

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2$

$* x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x^5)*(t^{(x*y*x^{-2}*y*x)}))^a$,

$((y^2)*(t^{(x^{-1}*y*x*y*x)}))^b$,

$((y^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^c$,

$((y)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^d$,

$(((y * x^{-1})^2)*(t^{(x^4*y*x^{-1}*y)}))^e$,

$((y^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^f$,

$((x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^g$,

$((x * y * x)*(t^{(x^4*y*x^{-1}*y)}))^h$,

$((y * x^{-1})*(t^{(x*y^2*x*y)}))^i$,

$((x^3)*(t^{(x^3*y*x^2*y^{-2})}))^j$,

$(x^5*t)^k$ >

Grapejelly6											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	5	5	$2{:}(A_5{:}J_2)$
0	0	0	0	0	0	0	0	6	0	2	$3^5{:}(2^5{:}A_5)$
0	0	0	0	0	0	0	0	0	0	2	S(4,3)×2
0	0	0	0	0	0	0	0	0	0	4	$2^6{:}(S(4,3){:}2)$
0	0	0	0	0	0	0	0	0	0	8	$2{:}(2^6{:}(S(4,3){:}2))$

Progenitor of Grapejelly7

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2$
$* x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^a$,

$((x)*(t^{(x^4*y*x^{-1}*y)}))^b$,

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^c$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^3*y*x^2*y^{-2})}))^d$,

$((y^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^e$,

$((x^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^f$,

$(((y * x^{-1})^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^g$,
$((y^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^h$,
$((y * x^{-1})*(t^{(x^{-2}*y*x^2)}))^i$,
$((x * y * x)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^j$,
$(x^5*t)^k >$

Grapejelly7											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	6	0	2	3^5:(2^5:A_5)
0	0	0	0	0	0	0	4	0	6	4	2^6:(S(4,3):2)
0	0	0	0	0	0	0	4	0	6	8	2:(2^6:(S(4,3):2))

Progenitor of Grapejelly8

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y^{-1})*(t^{(x^{-2}*y*x^2)}))^a$,
$(((y * x^{-1})^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^b$,
$(((y * x^{-1})^3)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^c$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^d$,
$((y^{-1})*(t^{(y*x^{-1}*y^{-2})}))^e$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^f$,
$((x^5)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^g$,
$((y * x^{-1})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^h$,
$((x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^i$,
$((x * y * x)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^j$,
$(x^5*t)^k >$

Grapejelly8											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	7	0	5	J_2

Progenitor of Grapejelly9

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$((x)*(t^{(y*x^{-1}*y^{-2})}))^a$,

$((y^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^b$,

$(((x * y)^2)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^c$,

$((x^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^d$,

$(((y * x^{-1})^3)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^e$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^f$,

$((x^5)*(t^{(x*y*x^{-2}*y*x)}))^g$,

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^h$,

$((y)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^i$,

$((x * y * x)*(t^{(x^3*y*x^2*y^{-2})}))^j$,

$(x^5*t)^k$ >

Grapejelly9											
a	b	c	d	e	f	g	h	i	j	k	G
5	0	0	0	0	0	0	0	0	0	5	2:(A_5:J_2)
0	3	0	0	0	0	0	0	0	0	5	$A_5 \times J_2$
0	3	0	0	0	0	0	0	0	0	10	2:(A_5:J_2)
6	3	0	0	0	0	0	0	0	0	5	J_2

Progenitor of Grapejelly10

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$((x)*(t^{(x^4*y*x^{-1}*y)}))^a$,

$((x * y * x)*(t^{(x^{-1}*y*x*y*x)}))^b$,

$((y)*(t^{(y^3*x*y^{-2})}))^c$,

$(((y * x^{-1})^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^d$,

$((x^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^e$,

$((y * x^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^f$,

$((x^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^g$,

$((x * y * x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^h$,

$((y^{-1})*(t^{(x^4*y*x^{-1}*y)}))^i$,

$((y)*(t^{(x^{-2}*y*x^2)}))^j$,

$(x^5*t)^k$ >

Grapejelly10											
a	b	c	d	e	f	g	h	i	j	k	G
7	0	0	0	0	0	0	0	0	0	5	J_2

Progenitor of Grapejelly11

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x * y * x)*(t^{(x^{-2}*y*x^2)}))^a$,

$((y^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^b$,

$((y)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^c$,

$(((x * y)^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^d$,

$((x^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^e$,

$(((y * x^{-1})^3)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^f$,

$((y^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^g$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^4*y*x^{-1}*y)}))^h$,

$((x^5)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^i$,

$(((y * x^{-1})^2)*(t^{(x*y*x^{-2}*y*x)}))^j$,

$(x^5*t)^k$ >

Grapejelly11											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	4	2	2^8:(2:A_5)
4	0	0	0	0	0	0	0	0	0	4	2^5:(2^4:A_5)
0	0	3	0	0	0	0	0	0	0	5	$A_5 \times J_2$
0	0	3	0	0	0	0	0	0	0	10	2:(A_5:J_2)
10	8	3	0	0	0	0	0	0	0	5	J_2

Progenitor of Grapejelly12

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$(((x * y)^2)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^a$,

$((y)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^b$,

$((y^{-1})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^c$,

$((x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^d$,

$((x^3)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^e$,

$((y^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^f$,

$(((x * y)^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^g$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^4*y*x^{-1}*y)}))^h$,

$((y^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^i$,

$((y^2)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^j$,

$(x^5*t)^k$ >

Grapejelly12											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	4	4	4	2^6:(S(4,3):2)
0	0	0	0	0	0	0	0	4	4	6	S(4,3)×2
0	0	0	0	0	0	0	0	4	4	8	2:(2^6:(S(4,3):2))
6	3	0	0	0	0	0	0	0	0	5	J_2
6	6	0	0	0	0	0	0	0	0	10	2×J_2

Progenitor of Grapejelly13

$G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,$

$t^2, (t,(x^2)), (t,(y^2)),$

$((y^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^a,$

$(((y * x^{-1})^3)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^b,$

$((x^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^c,$

$(((y * x^{-1})^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^d,$

$((y)*(t^{(y*x^{-1}*y^{-2})}))^e,$

$((y^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^f,$

$((x * y * x)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^g,$

$((x^3)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^h,$

$((y * x^{-1})*(t^{(y*x^{-1}*y^{-2})}))^i,$

$((x * y * x)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^j,$

$(x^5*t)^k >$

Grapejelly13											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	4	2	2^8:(2:A_5)
0	0	0	0	0	0	0	0	0	4	4	2^5:(2^4:A_5)

Progenitor of Grapejelly14

$G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,$

$t^2, (t,(x^2)), (t,(y^2)),$

$((x * y * x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^a,$

$((y^{-1})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^b,$

$((y)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^c,$

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^d,$

$(((x * y)^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^e,$

$((x^2)*(t^{(y^3*x*y^{-2})}))^f,$

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^g,$

$(((y * x^{-1})^3)*(t^{(x^{-1}*y*x*y*x)}))^h$,

$((y^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^i$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^j$,

$(x^5*t)^k >$

Grapejelly14											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	4	0	4	2^6:(S(4,3):2)
0	0	0	0	0	0	0	0	4	0	6	S(4,3)×2
0	0	0	0	0	0	0	0	4	0	8	2:(2^6:(S(4,3):2))
0	0	0	0	0	0	0	4	0	4	2	2^{16}:(2^4:A_5)
4	0	0	0	0	0	0	0	0	0	2	2^8:(2:A_5)
4	0	0	0	0	0	0	0	0	0	4	2^5:(2^4:A_5)

Progenitor of Grapejelly15

$G<x,y,t>:=Group<x,y,t|\ x^{10}\ ,\ y^6\ ,\ (x * y^{-2} * x)^2\ ,\ (x * y^2 * x^2)^2\ ,\ (y^{-1} * x^{-1})^5\ ,\ (x * y^2 * x^{-1} * y^{-1})^2\ ,\ x^{-1} * y^{-1} * x^5 * y * x^{-4}\ ,\ y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}\ ,$

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x^5)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^a$,

$((y^3)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^b$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^c$,

$((y^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^d$,

$(((y * x^{-1})^3)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^e$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^f$,

$((x^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^g$,

$(((x * y)^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^h$,

$(((y * x^{-1})^2)*(t^{(x^4*y*x^{-1}*y)}))^i$,

$((y)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^j$,

$(x^5*t)^k >$

Grapejelly15											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	4	0	2	S(4,3)×2
0	3	0	0	0	0	0	0	0	0	5	$A_5 \times J_2$
0	3	0	0	0	0	0	0	0	0	10	$2{:}(A_5{:}J_2)$

Progenitor of Grapejelly16

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^a$,

$((x^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^b$,

$((x)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^c$,

$((x * y * x)*(t^{(x^{-2}*y*x^2)}))^d$,

$((y^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^e$,

$((y)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^f$,

$(((y * x^{-1})^2)*(t^{(x^{-2}*y*x^2)}))^g$,

$(((x * y)^2)*(t^{(y^2*x^{-1}*y^2*x^2)}))^h$,

$((x^2)*(t^{(y*x^{-1}*y^{-2})}))^i$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^j$,

$(x^5*t)^k$ >

Grapejelly16											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	5	7	5	J_2
0	0	0	0	0	0	0	0	6	0	2	S(4,3)×2
0	0	0	0	0	0	0	0	6	0	4	$2^6{:}(S(4,3){:}2)$
0	0	0	0	0	0	0	0	6	0	8	$2{:}(2^6{:}(S(4,3){:}2))$
6	0	0	0	0	0	0	0	0	0	2	$3^5{:}(2^5{:}A_5)$

Progenitor of Grapejelly17

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2$

$* x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^a$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(y^2*x^{-1}*y^2*x^2)}))^b$,
$(((y * x^{-1})^3)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^c$,
$((x^2)*(t^{(x*y^2*x*y)}))^d$,
$(((x * y)^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^e$,
$(((y * x^{-1})^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^f$,
$((y)*(t^{(x^4*y*x^{-1}*y)}))^g$,
$((y^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^h$,
$((x)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^i$,
$((y * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^j$,
$(x^5*t)^k >$

Grapejelly17											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	5	0	5	2:(A_5:J_2)
3	0	0	0	0	0	0	0	0	0	5	$A_5 \times J_2$
3	0	0	0	0	0	0	0	0	0	10	2:(A_5:J_2)

Progenitor of Grapejelly18

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2$
$* x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^{-1})*(t^{(x^{-2}*y*x^2)}))^a$,
$((x^3)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^b$,
$((x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^c$,
$((x * y * x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^d$,
$((y^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^e$,
$((y)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^f$,
$(((y * x^{-1})^2)*(t^{(x^{-2}*y*x^2)}))^g$,
$(((x * y)^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^h$,

$((x^2)*(t^{(x^{-1}*y*x*y*x)})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^i$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^j$,
$(x^5*t)^k$ >

Grapejelly18											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	10	4	2	2^{13}:(2^5:A_5)
6	0	0	0	0	0	0	0	0	0	2	3^5:(2^5:A_5)

Progenitor of Grapejelly20

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x^5)*(t^{(y*x^{-1}*y^{-2})})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^a$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^b$,
$(((y * x^{-1})^3)*(t^{(x^{-2}*y*x^2)}))^c$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^4*y*x^{-1}*y)}))^d$,
$((x^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(x*y^2*x*y)}))^e$,
$(((x * y)^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^f$,
$(((y * x^{-1})^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^g$,
$((y)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^h$,
$((x * y * x)*(t^{(y^3*x*y^{-2})})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^i$,
$((x)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^j$,
$(x^5*t)^k$ >

Grapejelly20											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	2	0	5	2:(A_5:J_2)
0	0	0	0	0	0	0	0	2	0	10	2^2:(A_5:J_2)
2	4	0	0	0	0	0	0	0	0	2	2^{16}:(2^4:A_5)

Progenitor of Grapejelly21

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^6, (x*y^{-2}*x)^2, (x*y^2*x^2)^2, (y^{-1}*x^{-1})^5, (x*y^2*x^{-1}*y^{-1})^2, x^{-1}*y^{-1}*x^5*y*x^{-4}, y*x^{-2}*y^{-1}*x^3*y*x*y^3*x^{-1},$

$t^2, (t,(x^2)), (t,(y^2)),$

$((x)*(t^{(x*y^2*x*y)}))^a,$

$((x*y*x)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^b,$

$(((y*x^{-1})^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^c,$

$(((x*y)^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^3*x*y^{-2})}))^d,$

$((y*x^2*y^{-1}*x^{-1})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^e,$

$(((y*x^{-1})^3)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^f,$

$((y*x^{-1}*y^{-1}*x^{-1}*y^{-1}*x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(x^3*y*x^2*y^{-2})}))^g,$

$((y^3)*(t^{(y^2*x^{-1}*y^2*x^2)}))^h,$

$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^i,$

$((y^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^j,$

$(x^5*t)^k >$

Grapejelly21											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	3	0	0	5	$A_5 \times J_2$
0	0	0	0	0	0	0	3	0	0	10	$2{:}(A_5{:}J_2)$
5	0	0	0	0	0	0	0	0	0	5	$2{:}(A_5{:}J_2)$

Progenitor of Grapejelly22

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^6, (x*y^{-2}*x)^2, (x*y^2*x^2)^2, (y^{-1}*x^{-1})^5, (x*y^2*x^{-1}*y^{-1})^2, x^{-1}*y^{-1}*x^5*y*x^{-4}, y*x^{-2}*y^{-1}*x^3*y*x*y^3*x^{-1},$

$t^2, (t,(x^2)), (t,(y^2)),$

$((y*x^{-1}*y^{-1}*x^{-1}*y^{-1}*x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^a,$

$((y^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^b,$

$((x^5)*(t^{(x^3*y*x^2*y^{-2})}))^c,$

$((y^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^d,$

$(((y*x^{-1})^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(x*y*x^{-2}*y*x)}))^e,$

$((y^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^f,$

$((x^3)*(t^{(x*y*x^{-2}*y*x)}))^g,$

$((y * x^{-1})*(t^{(x^3*y*x^2*y^{-2})}))^h$,
$((x)*(t^{(x*y^2*x*y)}))^i$,
$((x * y * x)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^4*y*x^{-1}*y)}))^j$,
$(x^5*t)^k$ >

Grapejelly22											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	5	0	5	2:(A_5:J_2)
8	3	0	0	0	0	0	0	0	0	5	$A_5 \times J_2$
8	3	0	0	0	0	0	0	0	0	10	2:(A_5:J_2)

Progenitor of Grapejelly23

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^a$,
$((x * y * x)*(t^{(x^4*y*x^{-1}*y)}))^b$,
$((y^{-1})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^c$,
$((y)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^d$,
$(((y * x^{-1})^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^e$,
$(((x * y)^2)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^{-2}*y*x^2)}))^f$,
$((x^2)*(t^{(x^{-2}*y*x^2)}))^g$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^h$,
$(((y * x^{-1})^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^i$,
$((y^2)*(t^{(x^4*y*x^{-1}*y)})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^j$,
$(x^5*t)^k$ >

Grapejelly23											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	3	2	3^5:(2^5:A_5)
0	0	0	0	0	0	0	0	0	4	2	PSL(4,3)×2
7	0	0	0	0	0	0	0	0	0	5	J_2

Progenitor of Grapejelly24

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$((y^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^a$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^b$,

$((y^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^c$,

$((x^5)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^d$,

$((x^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^e$,

$(((x * y)^2)*(t^{(x^3*y*x^2*y^{-2})}))^f$,

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^g$,

$((y)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^h$,

$((x * y * x)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^i$,

$((x^3)*(t^{(y^3*x*y^{-2})}))^j$,

$(x^5*t)^k$ >

Grapejelly24											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	5	5	2:(A_5:J_2)
0	0	3	0	0	0	0	0	0	0	5	$A_5 \times J_2$
0	0	3	0	0	0	0	0	0	0	10	2:(A_5:J_2)

Progenitor of Grapejelly25

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$((y * x^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(x*y^2*x*y)}))^a$,
$((x^3)*(t^{(x^{-2}*y*x^2)}))^b$,
$((x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^c$,
$((y^{-1})*(t^{(x^3*y*x^2*y^{-2})}))^d$,
$(((y * x^{-1})^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x^{-1}*y*x*y*x)}))^e$,
$(((x * y)^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^f$,
$((x^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^g$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^h$,
$(((y * x^{-1})^3)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^3*y*x^2*y^{-2})}))^i$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y*x^{-2}*y*x)}))^j$,
$(x^5*t)^k$ >

Grapejelly25											
a	b	c	d	e	f	g	h	i	j	k	G
0	5	0	0	0	0	0	0	0	0	5	2:(A_5:J_2)

Progenitor of Grapejelly26

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^a$,
$((y^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^b$,
$((x^5)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^c$,
$((y^2)*(t^{(x^4*y*x^{-1}*y)})*(t^{(y^3*x*y^{-2})}))^d$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^3*y*x^2*y^{-2})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^e$,
$((x^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^f$,
$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^g$,
$((y)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^h$,
$((x * y * x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^i$,
$(x^5*t)^j$ >

Grapejelly26										
a	b	c	d	e	f	g	h	i	j	G
0	3	0	0	0	0	0	0	0	5	$A_5 \times J_2$
0	3	0	0	0	0	0	0	0	10	2:(A_5:J_2)
4	4	0	0	0	0	0	0	0	2	2^{16}:(2^4:A_5)

Progenitor of Grapejelly27

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^a$,

$((y^2)*(t^{(x^3*y*x^2*y^{-2})})*(t^{(x^4*y*x^{-1}*y)}))^b$,

$((x^3)*(t^{(x*y^2*x*y)}))^c$,

$((y * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^d$,

$((y)*(t^{(x^{-1}*y*x*y*x)})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^e$,

$(((y * x^{-1})^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^f$,

$((x^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^g$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-2}*y*x^2)}))^h$,

$(((y * x^{-1})^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^i$,

$((y^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^j$,

$(x^5*t)^k$ >

Grapejelly27											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	4	6	2	2^{16}:(2^4:A_5)
0	0	0	0	0	0	0	4	0	9	2	3^5:(2^5:A_5)
0	0	0	0	0	0	0	4	8	0	2	2^5:(2^5:A_5)
0	0	0	0	0	0	0	4	10	0	2	5^5:(2^5:A_5)
4	2	0	0	0	0	0	0	0	0	2	2^{16}:(2^4:A_5)
5	2	0	0	0	0	0	0	0	0	2	PSL(4,3)×2
0	2	4	0	0	0	0	0	0	0	2	2^5:(2^5:A_5)

Progenitor of Grapejelly28

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x * y * x)*(t^{(x^4*y*x^{-1}*y)}))^a$,

$((y)*(t^{(y^{-1}*x^{-1}*y*x^2)})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^b$,

$(((y * x^{-1})^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^c$,

$((x^2)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^d$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^4*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^e$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^f$,

$((y^3)*(t^{(y*x^{-1}*y^{-2})}))^g$,

$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^h$,

$(((x * y)^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^i$,

$(((y * x^{-1})^3)*(t^{(x^4*y*x^{-1}*y)}))^j$,

$(x^5*t)^k$ >

Grapejelly28											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	3	0	0	0	5	$A_5 \times J_2$
0	0	0	0	0	0	3	0	0	0	10	$2{:}(A_5{:}J_2)$
0	0	0	0	0	0	4	0	0	4	2	$2^{16}{:}(2^4{:}A_5)$

Progenitor of Grapejelly31

$G<x,y,t>:=Group<x,y,t| \; x^{10}, y^6, (x * y^{-2} * x)^2, (x * y^2 * x^2)^2, (y^{-1} * x^{-1})^5, (x * y^2 * x^{-1} * y^{-1})^2, x^{-1} * y^{-1} * x^5 * y * x^{-4}, y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1},$

$t^5, (t,(x^2)), (t,(y^2)),$

$((x * y * x)*(t^{(x^4*y*x^{-1}*y)}))^a,$

$((y)*(t^{(y^{-1}*x^{-1}*y*x^2)})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^b,$

$(((y * x^{-1})^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^c,$

$((x^2)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^d,$

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^4*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^e,$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^f,$

$((y^3)*(t^{(y*x^{-1}*y^{-2})}))^g,$

$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^h,$

$(((x * y)^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^i,$

$(((y * x^{-1})^3)*(t^{(x^4*y*x^{-1}*y)}))^j,$

$(x^5*t)^k >$

Grapejelly31											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	3	10	0	0	2	$A_5 \times J_2$
0	0	0	5	0	0	0	0	0	0	2	2:(A_5:J_2)

Progenitor of Grapejelly32

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, (t,(x^2)), (t,(y^2)),

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(y*x^{-1}*y^{-2})}))^a$,

$((x^5)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^b$,

$(((y * x^{-1})^3)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^c$,

$(((x * y)^2)*(t^{(x^2*y^{-1}*x*y*x^{-1})})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^d$,

$((y * x^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^e$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^f$,

$(((y * x^{-1})^3)*(t^{(y*x^{-1}*y^{-2})})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^g$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^h$,

$((y^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^i$,

$((x^5)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^j$,

$(x^5*t)^k$ >

Grapejelly32											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	2	0	5	5^5:(2^4:A_5)

Progenitor of Grapejelly33

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, (t,(x^2)), (t,(y^2)),

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y*x^{-2}*y*x)}))^a$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^b$,

$(((x * y)^2)*(t^{(x^{-2}*y*x^2)}))^c$,

$((x * y * x)*(t^{(x^{-2}*y*x^{2})})*(t^{(y^{2}*x*y*x^{-1}*y*x^{-1})}))^{d}$,
$((x^{3})*(t^{(x^{3}*y*x^{2}*y^{-2})}))^{e}$,
$((y * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-1}*y*x^{2})}))^{f}$,
$((x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^{g}$,
$((x * y * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(x^{4}*y*x^{-1}*y)}))^{h}$,
$(((y * x^{-1})^{2})*(t^{(y^{2}*x*y^{-1}*x*y^{-1}*x^{-1})}))^{i}$,
$((y * x^{2} * y^{-1} * x^{-1})*(t^{(y^{2}*x^{-1}*y*x^{-1}*y)})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{j}$,
$(x^{5}*t)^{k}$ >

Grapejelly33											
a	b	c	d	e	f	g	h	i	j	k	G
10	0	5	0	0	0	0	0	0	0	2	2:(A_5:J_2)

Progenitor of Grapejelly34

G<x,y,t>:=Group<x,y,t| x^{10} , y^{6} , $(x * y^{-2} * x)^{2}$, $(x * y^{2} * x^{2})^{2}$, $(y^{-1} * x^{-1})^{5}$, $(x * y^{2} * x^{-1} * y^{-1})^{2}$, $x^{-1} * y^{-1} * x^{5} * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1}$,
t^{2}, $(t,(x^{2}))$, $(t,(y^{2}))$,
$((y^{2})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^{a}$,
$((y * x^{2} * y^{-1} * x^{-1})*(t^{(x*y^{2}*x*y)}))^{b}$,
$((y * x^{-1})*(t))^{c}$,
$(((y * x^{-1})^{2})*(t^{(x^{4}*y*x^{-1}*y)}))^{d}$,
$((x^{2})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*t)^{e}$,
$((y^{2})*(t^{(y^{2}*x^{-1}*y*x^{-1}*y)}))^{f}$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{2}*x*y*x^{-1}*y*x^{-1})}))^{g}$,
$((x^{5})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^{h}$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*t*(t^{(y^{2}*x^{-1}*y*x^{-1}*y)}))^{i}$,
$((x^{3})*(t^{(y^{-1}*x^{-1}*y*x^{2})}))^{j}$,
$(x^{5}*t)^{k}$ >

Grapejelly34											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	6	0	0	0	0	0	0	0	2	3^5:(2^5:A_5)

Progenitor of Grapejelly36

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,

t^2, (t,(x^2)), (t,(y^2)),

((y * x^2 * y^{-1} * x^{-1})*($t^{(y^{-1}*x*y^{-1}*x*y^{-1})}$))^a,

(((y * x^{-1})^3)*($t^{(x*y^{-1}*x^{-1}*y^3*x)}$))^b,

((y^2)*($t^{(y^2*x^{-1}*y*x^{-1}*y)}$))^c,

((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*($t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}$)*($t^{(x^4*y*x^{-1}*y)}$))^d,

((y^3)*($t^{(y*x^{-1}*y^{-2})}$))^e,

((x^5)*($t^{(x^{-2}*y*x^2)}$))^f,

((x^2)*t*($t^{(y^3*x^{-1}*y^{-2}*x^{-1})}$))^g,

((y * x^{-1})*($t^{(y^2*x^{-1}*y^2*x^2)}$))^h,

((x^3)*($t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}$))^i,

((x)*($t^{(x*y^2*x*y)}$)*($t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}$))^j,

((x^3)*($t^{(y^2*x^{-1}*y*x^{-1}*y)}$)*t)^k,

(x^5*t)^l >

Grapejelly36												
a	b	c	d	e	f	g	h	i	j	k	l	G
4	8	0	0	0	0	0	0	0	0	0	2	2^5:(2^5:A_5)
4	10	0	0	0	0	0	0	0	0	0	2	5^5:(2^5:A_5)

Progenitor of Grapejelly37

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,

t^2, (t,(x^2)), (t,(y^2)),

((y * x^{-1})*($t^{(y^{-1}*x*y^{-1}*x*y^{-2})}$))^a,

((x^3)*($t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}$))^b,

((x)*(t$^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}$))^c,
((x * y * x)*(t$^{(x*y*x^{-1}*y*x*y^{-1})}$))^d,
((y^{-1})*(t$^{(x^4*y*x^{-1}*y)}$))^e,
((y)*(t$^{(y^2*x^{-1}*y*x^{-1}*y)}$))^f,
(((y * x^{-1})^2)*(t$^{(y^{-1}*x^{-1}*y*x^2)}$))^g,
(((x * y)^2)*(t$^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}$))^h,
((x^2)*(t$^{(y^2*x*y*x^{-1}*y*x^{-1})}$))^i,
((y * x^2 * y^{-1} * x^{-1})*(t$^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}$))^j,
(((y * x^{-1})^3)*(t$^{(y^{-1}*x^{-1}*y*x^{-3})}$))^k,
(x^5*t)^l >

Grapejelly37												
a	b	c	d	e	f	g	h	i	j	k	l	G
0	0	0	0	0	0	0	0	0	4	8	2	2^5:2^5:A_5
0	0	0	0	0	0	0	0	0	4	10	2	5^5:(2^5:A_5)
5	6	0	0	0	0	0	0	0	0	0	5	J_2

Progenitor of Grapejelly38

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,
t^2, (t,(x^2)), (t,(y^2)),
((y^2)*(t$^{(y^{-1}*x^{-1}*y*x^{-3})}$))^a,
((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t$^{(y^{-1}*x*y^{-1}*x*y^{-1})}$))^b,
((y^3)*(t$^{(x^{-2}*y^{-1}*x^2*y*x)}$))^c,
((x^5)*(t$^{(x^2*y^{-1}*x*y*x^{-1})}$))^d,
((y * x^{-1})*(t$^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}$))^e,
((x^3)*(t$^{(x^{-1}*y*x*y*x)}$))^f,
((x)*(t$^{(y^{-1}*x^{-1}*y^{-1})}$))^g,
((x * y * x)*(t$^{(y*x*y^{-1}*x^{-1}*y*x)}$))^h,
((y^{-1})*(t$^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}$))^i,
((y)*(t$^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}$))^j,
(((y * x^{-1})^2)*(t$^{(x^3*y*x^2*y^{-2})}$))^k,

$(x^5*t)^l >$

Grapejelly38												
a	b	c	d	e	f	g	h	i	j	k	l	G
4	0	0	0	0	0	0	0	0	0	0	4	2^6:(S(4,3):2)
4	0	0	0	0	0	0	0	0	0	0	8	2:(2^6:(S(4,3):2))
6	4	4	4	0	0	0	0	0	0	0	2	2^{16}:(2^4:A_5)
0	8	3	5	0	0	0	0	0	0	0	5	$A_5 \times J_2$

Progenitor of Grapejelly39

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$(((x * y)^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^a$,

$((x^2)*(t^{(y*x^{-1}*y^{-2})}))^b$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y*x^{-2}*y*x)}))^c$,

$(((y * x^{-1})^3)*(t^{(x^{-2}*y*x^2)}))^d$,

$((y^2)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^e$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^3*x*y^{-2})}))^f$,

$((y^3)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^g$,

$((x^5)*(t^{(x*y^2*x*y)}))^h$,

$((y * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)})*t)^i$,

$(x^5*t)^j >$

Grapejelly39										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	3	0	10	5	J_2

Progenitor of Grapejelly41

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y^{-1})*(t^{(x^{-2}*y*x^{2})}))^{a}$,

$((x)*(t^{(y^{-1}*x^{-1}*y*x^{2})})*(t^{(y^{2}*x^{-1}*y*x^{-1}*y)}))^{b}$,

$((y * x^{-1})*(t^{(y^{2}*x*y*x^{-1}*y*x^{-1})}))^{c}$,

$(((y * x^{-1})^{2})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^{d}$,

$((x^{2})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^{e}$,

$(x^{5}*t)^{f}$ >

Grapejelly41						
a	b	c	d	e	f	G
0	0	0	0	6	5	J_2
0	0	0	0	6	10	$2\times J_2$
0	0	6	0	10	2	3^5:(2^5:A_5)

Progenitor of Grapejelly42

G<x,y,t>:=Group<x,y,t| x^{10} , y^{6} , $(x * y^{-2} * x)^{2}$, $(x * y^{2} * x^{2})^{2}$, $(y^{-1} * x^{-1})^{5}$, $(x * y^{2} * x^{-1} * y^{-1})^{2}$, $x^{-1} * y^{-1} * x^{5} * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1}$,

t^{2}, $(t,(x^{2}))$, $(t,(y^{2}))$,

$(((y * x^{-1})^{3})*(t^{(y^{-1}*x^{-1}*y*x^{2})}))^{a}$,

$((y * x^{2} * y^{-1} * x^{-1})*(t^{(x*y*x^{-2}*y*x)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^{b}$,

$((x^{2})*(t^{(x^{2}*y^{-1}*x^{-2}*y^{-1}*x)}))^{c}$,

$(((x * y)^{2})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x*y^{2}*x*y)}))^{d}$,

$((y)*(t^{(y^{2}*x^{-1}*y*x^{-1}*y)}))^{e}$,

$((x * y * x)*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)}))^{f}$,

$((x)*(t^{(x^{4}*y*x^{-1}*y)})*(t^{(y^{2}*x*y*x^{-1}*y*x^{-1})}))^{g}$,

$((x^{3})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{h}$,

$((y * x^{-1})*(t^{(x^{4}*y*x^{-1}*y)})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{i}$,

$((x^{5})*(t^{(y^{2}*x^{-1}*y^{2}*x^{2})}))^{j}$,

$(x^{5}*t)^{k}$ >

Grapejelly42											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	6	7	0	0	5	J_2
8	4	0	0	0	0	0	0	0	0	2	2^9:(2^5:A_5)
8	4	0	0	0	0	0	0	0	0	4	2^5:(2^5:A_5)

Progenitor of Grapejelly43

$G<x,y,t>:=Group<x,y,t|\ x^{10}\ ,\ y^6\ ,\ (x * y^{-2} * x)^2\ ,\ (x * y^2 * x^2)^2\ ,\ (y^{-1} * x^{-1})^5\ ,\ (x * y^2 * x^{-1} * y^{-1})^2\ ,\ x^{-1} * y^{-1} * x^5 * y * x^{-4}\ ,\ y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}\ ,$

$t^2,\ (t,(x^2)),\ (t,(y^2)),$

$((y^3)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^a,$

$((x^5)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^b,$

$((x^3)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^c,$

$((x^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^d,$

$((x)*(t^{(x^{-2}*y*x^2)})*(t^{(y*x^{-1}*y^{-2})}))^e,$

$((x * y * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^f,$

$((y^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^g,$

$((y)*(t^{(x^{-1}*y*x*y*x)}))^h,$

$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(y^3*x*y^{-2})}))^i,$

$(((x * y)^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(x^4*y*x^{-1}*y)}))^j,$

$(x^5*t)^k >$

Grapejelly43											
a	b	c	d	e	f	g	h	i	j	k	G
3	5	0	0	0	0	0	0	0	0	5	$A_5 \times J_2$
3	10	0	0	0	0	0	0	0	0	10	2:(A_5:J_2)

Progenitor of Grapejelly44

$G<x,y,t>:=Group<x,y,t|\ x^{10}\ ,\ y^6\ ,\ (x * y^{-2} * x)^2\ ,\ (x * y^2 * x^2)^2\ ,\ (y^{-1} * x^{-1})^5\ ,\ (x * y^2 * x^{-1} * y^{-1})^2\ ,\ x^{-1} * y^{-1} * x^5 * y * x^{-4}\ ,\ y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}\ ,$

$t^2,\ (t,(x^2)),\ (t,(y^2)),$

$((x^2)*(t^{(x^4*y*x^{-1}*y)})*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^a,$

$(((y * x^{-1})^3)*(t^{(y^{-1}*x^{-1}*y*x^2)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^b$,
$((y^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^c$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^d$,
$((x^5)*(t^{(x*y*x^{-2}*y*x)}))^e$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^f$,
$(((y * x^{-1})^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(y^2*x^{-1}*y^2*x^2)}))^g$,
$((y^{-1})*(t^{(y*x^{-1}*y^{-2})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^h$,
$((y)*(t^{(y^3*x*y^{-2})})*t)^i$,
$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^3*x*y^{-2})}))^j$,
$(x^5*t)^k$ >

Grapejelly44											
a	b	c	d	e	f	g	h	i	j	k	G
3	0	0	0	0	0	0	0	0	0	5	J_2
3	0	0	0	0	0	0	0	0	0	10	$2\times J_2$
6	3	4	0	0	0	0	0	0	0	4	2^6:(S(4,3):2)

Progenitor of Grapejelly45

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$(((y * x^{-1})^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^a$,
$((x * y * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(x*y^2*x*y)}))^b$,
$((x)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^c$,
$((y * x^{-1})*(t^{(x^4*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^d$,
$((x^3)*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^e$,
$((y^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^f$,
$((y)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^g$,
$(((y * x^{-1})^2)*(t^{(y^3*x*y^{-2})})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^h$,
$((x^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})})*t)^i$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^4*y*x^{-1}*y)})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^j$,
$(x^5*t)^k$ >

Grapejelly45											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	4	2	$2^9{:}(2^5{:}A_5)$
0	0	0	0	0	0	0	0	0	4	4	$2^5{:}(2^5{:}A_5)$

Progenitor of Grapejelly46

$G<x,y,t>:=Group<x,y,t|\ x^{10}\ ,\ y^6\ ,\ (x * y^{-2} * x)^2\ ,\ (x * y^2 * x^2)^2\ ,\ (y^{-1} * x^{-1})^5\ ,\ (x * y^2 * x^{-1} * y^{-1})^2\ ,\ x^{-1} * y^{-1} * x^5 * y * x^{-4}\ ,\ y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}\ ,$

$t^2,\ (t,(x^2)),\ (t,(y^2)),$

$((x^2)*(t^{(x^{-2}*y*x^2)}))^a,$

$((y^2)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^b,$

$((y^3)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^c,$

$((x^5)*t*(t^{(x^{-2}*y*x^2)}))^d,$

$((x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^e,$

$((y * x^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})})*(t^{(y^3*x*y^{-2})}))^f,$

$((x)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(y^2*x^{-1}*y^2*x^2)}))^g,$

$((y^{-1})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^h,$

$(((y * x^{-1})^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^i,$

$(((x * y)^2)*(t^{(x*y*x^{-2}*y*x)})*(t^{(x^{-2}*y*x^2)}))^j,$

$(x^5*t)^k >$

Grapejelly46											
a	b	c	d	e	f	g	h	i	j	k	G
6	6	0	0	0	0	0	0	0	0	4	2^6:(S(4,3):2)
6	6	0	0	0	0	0	0	0	0	6	S(4,3)×2
6	6	0	0	0	0	0	0	0	0	8	2:(2^6:(S(4,3):2))
0	0	3	0	0	0	0	0	0	0	5	$A_5 \times J_2$
0	0	3	0	0	0	0	0	0	0	10	2:(A_5:J_2)
0	10	3	0	0	0	0	0	0	0	5	J_2

Progenitor of Grapejelly47

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$(((x * y)^2)*(t^{(x*y^2*x*y)})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^a$,

$((x^2)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^b$,

$((y * x^2 * y^{-1} * x^{-1})*t*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^c$,

$((y * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^d$,

$((x^3)*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x^3*y*x^2*y^{-2})}))^e$,

$((x)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^f$,

$((x * y * x)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})})*(t^{(y*x^{-1}*y^{-2})}))^g$,

$((y^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^h$,

$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^i$,

$((y^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*t)^j$,

$(x^5*t)^k$ >

Grapejelly47											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	8	3	2	2^8:(2:A_5)
0	6	0	0	0	0	0	0	0	0	4	2^6:(S(4,3):2)
0	6	0	0	0	0	0	0	0	0	8	2:(2^6:(S(4,3):2))
0	6	0	0	0	0	0	0	0	0	6	S(4,3)×2
5	0	4	0	0	0	0	0	0	0	2	2^8:(2^5:A_5)
10	0	4	0	0	0	0	0	0	0	2	2^9:(2^5:A_5)

Progenitor of Grapejelly48

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y^3)*(t^{(y^2*x^{-1}*y^2*x^2)}))^a$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y*x^{-2}*y*x)}))^b$,

$(((y * x^{-1})^2)*(t^{(y*x^{-1}*y^{-2})})*(t^{(x^3*y*x^2*y^{-2})}))^c$,

$((x^5)*(t^{(x*y^2*x*y)})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^d$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^e$,

$((y^2)*(t^{(x^4*y*x^{-1}*y)}))^f$,

$(((y * x^{-1})^3)*(t^{(x^3*y*x^2*y^{-2})})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^g$,

$((x * y * x)*(t^{(x^4*y*x^{-1}*y)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^h$,

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^i$,

$((y^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^j$,

$(x^5*t)^k$ >

Grapejelly48											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	8	4	2	S(4,3)×2
0	0	0	0	0	0	0	0	8	4	4	2^6:(S(4,3):2)
0	0	0	0	0	0	0	0	8	4	8	2:(2^6:(S(4,3):2))
9	6	0	0	0	0	0	0	0	0	5	$A_5 \times J_2$
9	6	0	0	0	0	0	0	0	0	10	2:(A_5:J_2)
6	0	3	0	0	0	0	0	0	0	0	3^5:(2^5:A_5)
6	6	4	0	0	0	0	0	0	0	2	PSL(4,3)×2

Progenitor of Grapejelly49

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$((y^2)*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^a$,

$((y^{-1})*(t^{(x^{-1}*y*x*y*x)})*(t^{(x*y^2*x*y)}))^b$,

$(((y * x^{-1})^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^c$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y*x^{-2}*y*x)}))^d$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^e$,

$((y^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*t)^f$,

$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^g$,

$((x * y * x)*(t^{(x^4*y*x^{-1}*y)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^h$,

$((x^3)*(t^{(x^{-2}*y*x^2)}))^i$,

$(((y * x^{-1})^3)*(t^{(x*y^2*x*y)})*(t^{(x^3*y*x^2*y^{-2})}))^j$,

$(x^5*t)^k$ >

Grapejelly49											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	5	0	5	2:(A_5:J_2)
0	0	0	0	0	0	0	5	0	4	2	PSL(4,3)×2
2	10	0	0	0	0	0	0	0	0	2	S(4,3)×2
2	10	0	0	0	0	0	0	0	0	4	2^6:(S(4,3):2)
2	10	0	0	0	0	0	0	0	0	8	2:(2^6:(S(4,3):2))
3	6	0	4	0	0	0	0	0	0	2	2^5:(2^5:A_5)

Progenitor of Grapejelly50

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^6, (x * y^{-2} * x)^2, (x * y^2 * x^2)^2, (y^{-1} * x^{-1})^5, (x * y^2 * x^{-1} * y^{-1})^2, x^{-1} * y^{-1} * x^5 * y * x^{-4}, y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1},$

$t^5, (t,(x^2)), (t,(y^2)),$

$((x^2)*(t^{(x^{-2}*y*x^2)}))^a,$

$((y^2)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^b,$

$((y^3)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^c,$

$((x^5)*t*(t^{(x^{-2}*y*x^2)}))^d,$

$((x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^e,$

$((y * x^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})})*(t^{(y^3*x*y^{-2})}))^f,$

$((x)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(y^2*x^{-1}*y^2*x^2)}))^g,$

$((y^{-1})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^h,$

$(((y * x^{-1})^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^i,$

$(((x * y)^2)*(t^{(x*y*x^{-2}*y*x)})*(t^{(x^{-2}*y*x^2)}))^j,$

$(x^5*t)^k >$

Grapejelly50											
a	b	c	d	e	f	g	h	i	j	k	G
5	0	0	0	0	0	0	0	0	0	2	2:(A_5:J_2)
0	0	3	0	0	0	0	0	0	0	2	$A_5 \times J_2$
6	0	3	0	0	0	0	0	0	0	2	J_2

Progenitor of Grapejelly51

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^{6}, (x * y^{-2} * x)^{2}, (x * y^{2} * x^{2})^{2}, (y^{-1} * x^{-1})^{5}, (x * y^{2} * x^{-1} * y^{-1})^{2}, x^{-1} * y^{-1} * x^{5} * y * x^{-4}, y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1},$

$t^{5}, (t,(x^{2})), (t,(y^{2})),$

$(((x * y)^{2})*(t^{(x*y^{2}*x*y)})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^{a},$

$((x^{2})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^{b},$

$((y * x^{2} * y^{-1} * x^{-1})*t*(t^{(x^{2}*y^{-1}*x^{-2}*y^{-1}*x)}))^{c},$

$((y * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^{d},$

$((x^{3})*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x^{3}*y*x^{2}*y^{-2})}))^{e},$

$((x)*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)}))^{f},$

$((x * y * x)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})})*(t^{(y*x^{-1}*y^{-2})}))^{g},$

$((y^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(y^{3}*x^{-1}*y^{-2}*x^{-1})}))^{h},$

$(((y * x^{-1})^{2})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^{i},$

$((y^{3})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)})*t)^{j},$

$(x^{5}*t)^{k} >$

Grapejelly51											
a	b	c	d	e	f	g	h	i	j	k	G
0	5	0	0	0	0	0	0	0	0	2	2:(A_5:J_2)

Progenitor of Grapejelly54

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^{6}, (x * y^{-2} * x)^{2}, (x * y^{2} * x^{2})^{2}, (y^{-1} * x^{-1})^{5}, (x * y^{2} * x^{-1} * y^{-1})^{2}, x^{-1} * y^{-1} * x^{5} * y * x^{-4}, y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1},$

$t^{5}, (t,(x^{2})), (t,(y^{2})),$

$(((y * x^{-1})^{2})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^{a},$

$((y^{3})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^{3}*y*x^{2}*y^{-2})}))^{b},$

$((y)*(t^{(x^{2}*y^{-1}*x^{-2}*y^{-1}*x)}))^{c},$

$(((x * y)^{2})*(t^{(x*y*x^{-2}*y*x)})*(t^{(x*y^{2}*x*y)}))^{d},$

$((x^{2})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{e},$

$((y * x^{2} * y^{-1} * x^{-1})*(t^{(x^{-1}*y*x*y*x)})*(t^{(x^{-1}*y*x*y*x)}))^{f},$

$((y^{2})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^{g},$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{2}*x*y^{-1}*x*y^{-1}*x^{-1})})*t)^{h},$

$((y^{3})*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)}))^{i},$

$((x^5)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(x^4*y*x^{-1}*y)}))^j$,

$(x^5*t)^k >$

Grapejelly54											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	3	0	2	$A_5 \times J_2$

Progenitor of Grapejelly55

$G<x,y,t>:=Group<x,y,t|\ x^{10}\ ,\ y^6\ ,\ (x * y^{-2} * x)^2\ ,\ (x * y^2 * x^2)^2\ ,\ (y^{-1} * x^{-1})^5\ ,\ (x * y^2 * x^{-1} * y^{-1})^2\ ,\ x^{-1} * y^{-1} * x^5 * y * x^{-4}\ ,\ y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, $(t,(x^2))$, $(t,(y^2))$,

$((x^5)*(t^{(x*y^2*x*y)})*(t^{(x^4*y*x^{-1}*y)}))^a$,

$((x * y * x)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^b$,

$((x^3)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^c$,

$((y * x^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^d$,

$((y^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^e$,

$((y)*(t^{(x^{-2}*y*x^2)}))^f$,

$(((y * x^{-1})^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^g$,

$((x^2)*(t^{(x^{-1}*y*x*y*x)})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^h$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^i$,

$((y^2)*(t^{(y^{-1}*x^{-1}*y*x^2)})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^j$,

$(x^5*t)^k >$

Grapejelly55											
a	b	c	d	e	f	g	h	i	j	k	G
5	4	0	0	0	0	0	0	0	0	5	5^5:(2^4:A_5)

Progenitor of Grapejelly56

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^a$,

$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^b$,

$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^2)})*(t^{(x^3*y*x^2*y^{-2})}))^c$,

$((y * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^d$,

$((y^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^e$,

$((y)*(t^{(y^2*x^{-1}*y^2*x^2)}))^f$,

$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^g$,

$(((x * y)^2)*(t^{(x^{-2}*y*x^2)}))^h$,

$((x^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^i$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^j$,

$(x^5*t)^k$ >

Grapejelly56											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	5	0	8	2	2:(A_5:J_2)

Progenitor of Grapejelly57

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^a$,

$(((y * x^{-1})^2)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^b$,

$((y^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^c$,

$((x * y * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^d$,

$((x^3)*(t^{(y^{-1}*x^{-1}*y^{-1})})*t)^e$,
$((y * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^f$,
$((x^2)*(t^{(x^3*y*x^2*y^{-2})})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^g$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^h$,
$(((y * x^{-1})^3)*t*(t^{(y*x^{-1}*y^{-2})}))^i$,
$((y^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^j$,
$(x^5*t)^k >$

Grapejelly57											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	2	0	5	5^5:(2^4:A_5)
8	0	0	4	0	0	0	0	0	0	5	5^5:(2^4:A_5)

Progenitor of Grapejelly58

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$(((y * x^{-1})^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^a$,
$((y^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^3*y*x^2*y^{-2})}))^b$,
$((y)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^c$,
$(((x * y)^2)*(t^{(x*y*x^{-2}*y*x)})*(t^{(x*y^2*x*y)}))^d$,
$((x^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^e$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-1}*y*x*y*x)})*(t^{(x^{-1}*y*x*y*x)}))^f$,
$((y^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^g$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*t)^h$,
$((y^3)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^i$,
$((x^5)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(x^4*y*x^{-1}*y)}))^j$,
$(x^5*t)^k >$

Grapejelly58											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	3	0	5	$A_5 \times J_2$
0	0	0	0	0	0	0	0	3	0	10	2:(A_5:J_2)
0	0	0	0	0	0	0	4	4	0	2	2^{16}:(2^4:A_5)
0	0	0	0	0	0	4	0	0	0	4	2^6:(S(4,3):2)
0	0	0	0	0	0	4	0	0	0	6	S(4,3)×2
0	0	0	0	0	0	4	0	0	0	8	2:(2^6:(S(4,3):2))
10	0	8	3	0	0	0	0	0	0	5	J_2
10	0	8	3	0	0	0	0	0	0	10	2×J_2
6	0	0	4	0	0	0	0	0	0	2	PSL(4,3)×2

Progenitor of Grapejelly59

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x^5)*(t^{(x*y^2*x*y)})*(t^{(x^4*y*x^{-1}*y)}))^a$,

$((x * y * x)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^b$,

$((x^3)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^c$,

$((y * x^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^d$,

$((y^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^e$,

$((y)*(t^{(x^{-2}*y*x^2)}))^f$,

$(((y * x^{-1})^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^g$,

$((x^2)*(t^{(x^{-1}*y*x*y*x)})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^h$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^i$,

$((y^2)*(t^{(y^{-1}*x^{-1}*y*x^2)})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^j$,

$(x^5*t)^k$ >

Grapejelly59											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	3	7	0	5	J_2
0	4	0	0	0	0	0	0	0	0	2	2^8:(2:A_5)
0	4	0	0	0	0	0	0	0	0	4	2^5:(2^4:A_5)

Progenitor of Grapejelly60

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^a$,

$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^b$,

$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^2)})*(t^{(x^3*y*x^2*y^{-2})}))^c$,

$((y * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^d$,

$((y^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^e$,

$((y)*(t^{(y^2*x^{-1}*y^2*x^2)}))^f$,

$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^g$,

$(((x * y)^2)*(t^{(x^{-2}*y*x^2)}))^h$,

$((x^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^i$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^j$,

$(x^5*t)^k$ >

Grapejelly60											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	6	0	5	J_2
0	0	0	0	0	0	0	0	6	0	10	$2\times J_2$
0	0	0	0	0	0	0	6	0	8	4	2^6:(S(4,3):2)
0	0	0	0	0	0	0	6	0	8	6	S(4,3)$\times$2
0	0	0	0	0	0	0	6	0	8	8	2:(2^6:(S(4,3):2))
0	0	3	6	0	0	0	0	0	0	2	3^5:(2^5:A_5)

Progenitor of Grapejelly61

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^a$,

$(((y * x^{-1})^2)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^b$,

$((y^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^c$,

$((x * y * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^d$,

$((x^3)*(t^{(y^{-1}*x^{-1}*y^{-1})})*t)^e$,

$((y * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^f$,

$((x^2)*(t^{(x^3*y*x^2*y^{-2})})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^g$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^h$,

$(((y * x^{-1})^3)*t*(t^{(y*x^{-1}*y^{-2})}))^i$,

$((y^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^j$,

$(x^5*t)^k$ >

Grapejelly61											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	4	6	6	2	2^4:(3^5:2^5)
4	0	0	0	0	0	0	0	0	0	2	2^9:(2^5:A_5)
4	0	0	0	0	0	0	0	0	0	4	2^5:(2^5:A_5)

Progenitor of Grapejelly62

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^6, (x * y^{-2} * x)^2, (x * y^2 * x^2)^2, (y^{-1} * x^{-1})^5, (x * y^2 * x^{-1} * y^{-1})^2, x^{-1} * y^{-1} * x^5 * y * x^{-4}, y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1},$

$t^2, (t,(x^2)), (t,(y^2)),$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^a,$

$((x^5)*(t^{(x*y*x^{-2}*y*x)}))^b,$

$((y^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^c,$

$(((y * x^{-1})^3)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^d,$

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-1}*y*x*y*x)})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^e,$

$(((y * x^{-1})^3)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^f,$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^4*y*x^{-1}*y)}))^g,$

$((y^3)*(t^{(x^4*y*x^{-1}*y)})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^h,$

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)})*(t^{(x*y^2*x*y)}))^i,$

$((x^3)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^j,$

$(x^5*t)^k >$

Grapejelly62											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	4	0	2	2^9:(2^5:A_5)
0	0	0	0	0	0	0	0	4	0	4	2^5:(2^5:A_5)

Progenitor of Grapejelly63

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x^3)*(t^{(y^{-1}*x^{-1}*y*x^2)})*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^a$,

$((y * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^b$,

$((x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(y^2*x^{-1}*y^2*x^2)}))^c$,

$((x * y * x)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^d$,

$((y^{-1})*(t^{(y^3*x*y^{-2})})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^e$,

$((y)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^f$,

$(((y * x^{-1})^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(x*y^2*x*y)}))^g$,

$(((x * y)^2)*(t^{(y*x^{-1}*y^{-2})}))^h$,

$((x^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})})*(t^{(y*x^{-1}*y^{-2})}))^i$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^4*y*x^{-1}*y)})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^j$,

$(x^5*t)^k$ >

Grapejelly63											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	6	0	0	2	S(4,3)×2
0	0	0	0	0	0	0	6	0	0	4	2^6:(S(4,3):2)
0	0	0	0	0	0	0	6	0	0	8	2:(2^6:(S(4,3):2))
0	6	0	0	0	0	0	0	0	0	2	3^5:(2^5:A_5)

Progenitor of Grapejelly64

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{2} * y^{-1} * x^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^{a},$

$(((y * x^{-1})^{3})*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)})*(t^{(x^{3}*y*x^{2}*y^{-2})}))^{b},$

$((y^{2})*(t^{(y^{3}*x*y^{-2})})*(t^{(y^{2}*x^{-1}*y^{2}*x^{2})}))^{c},$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^{d},$

$((y^{3})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-1}*y*x^{2})}))^{e},$

$((x^{5})*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)})*(t^{(y^{2}*x*y*x^{-1}*y*x^{-1})}))^{f},$

$((x^{3})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(x^{4}*y*x^{-1}*y)}))^{g},$

$((x)*(t^{(x*y^{2}*x*y)})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^{h},$

$((x * y * x)*(t^{(y^{2}*x^{-1}*y^{2}*x^{2})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^{i},$

$((y^{-1})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})})*(t^{(x^{4}*y*x^{-1}*y)}))^{j},$

$(x^{5}*t)^{k} >$

Grapejelly64											
a	b	c	d	e	f	g	h	i	j	k	G
4	2	0	0	0	0	0	0	0	0	2	2^8:(2:A_5)
4	4	0	0	0	0	0	0	0	0	2	2^9:(2^5:A_5)
4	6	0	0	0	0	0	0	0	0	2	2^4:(3^5:2^5)
4	0	3	0	0	0	0	0	0	0	2	3^5:(2^5:A_5)
4	0	6	0	0	0	0	0	0	0	2	2^5:(3^5:(2^5:A_5))
4	4	6	0	0	0	0	0	0	0	2	2^5:(2^4:A_5)

Progenitor of Grapejelly66

$G<x,y,t>:=Group<x,y,t|\ x^{10} , y^{6} , (x * y^{-2} * x)^{2} , (x * y^{2} * x^{2})^{2} , (y^{-1} * x^{-1})^{5} , (x * y^{2} * x^{-1} * y^{-1})^{2} , x^{-1} * y^{-1} * x^{5} * y * x^{-4} , y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1} ,$

$t^{5}, (t,(x^{2})), (t,(y^{2})),$

$((x^{3})*(t^{(y^{-1}*x^{-1}*y*x^{2})})*(t^{(y^{2}*x*y*x^{-1}*y*x^{-1})}))^{a},$

$((y * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^{b},$

$((x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(y^{2}*x^{-1}*y^{2}*x^{2})}))^{c},$

$((x * y * x)*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{d},$

$((y^{-1})*(t^{(y^{3}*x*y^{-2})})*(t^{(y^{3}*x^{-1}*y^{-2}*x^{-1})}))^{e},$

$((y)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^{f},$

$(((y * x^{-1})^{2})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(x*y^{2}*x*y)}))^{g},$

$(((x * y)^2)*(t^{(y*x^{-1}*y^{-2})}))^h$,
$((x^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})})*(t^{(y*x^{-1}*y^{-2})}))^i$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^4*y*x^{-1}*y)})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^j$,
$(x^5*t)^k$ >

Grapejelly66											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	5	0	0	2	2:(A_5:J_2)

Progenitor of Grapejelly68

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^5, $(t,(x^2))$, $(t,(y^2))$,
$((y^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^a$,
$(((y * x^{-1})^2)*(t^{(y^2*x^{-1}*y^2*x^2)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^b$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^c$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^d$,
$((x^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^e$,
$((y^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^f$,
$((x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(y*x^{-1}*y^{-2})}))^g$,
$((y)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^h$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^i$,
$((x^5)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^j$,
$(x^5*t)^k$ >

Grapejelly68											
a	b	c	d	e	f	g	h	i	j	k	G
3	8	0	0	0	0	0	0	0	0	5	G(2,4)
3	0	8	0	0	0	0	0	0	0	2	$A_5 \times J_2$

Progenitor of Grapejelly70

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2$

$* x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^5, $(t,(x^2))$, $(t,(y^2))$,
$(((y * x^{-1})^2)*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^a$,
$(((x * y)^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^b$,
$(((y * x^{-1})^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^c$,
$((y^2)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^d$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^e$,
$((y^3)*(t^{(x^{-1}*y*x*y*x)})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^f$,
$((x^5)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^g$,
$((y^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^h$,
$((y * x^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^i$,
$((x^3)*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^j$,
$(x^5*t)^k >$

Grapejelly70											
a	b	c	d	e	f	g	h	i	j	k	G
0	5	0	0	0	0	0	0	0	0	2	2:(A_5:J_2)

Progenitor of Grapejelly71

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2$
$* x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^a$,
$(((y * x^{-1})^2)*(t^{(y^2*x^{-1}*y^2*x^2)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^b$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^c$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^d$,
$((x^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^e$,
$((y^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^f$,
$((x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(y*x^{-1}*y^{-2})}))^g$,
$((y)*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^h$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^i$,
$((x^5)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^j$,

$(x^5*t)^k >$

Grapejelly71											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	3	0	5	0	2	S(4,3)×2
3	0	0	0	0	0	0	0	0	0	5	$A_5 \times J_2$
3	0	0	0	0	0	0	0	0	0	10	$2:(A_5:J_2)$
0	3	0	0	0	0	0	0	0	0	2	$3^5:(2^5:A_5)$
0	4	0	0	0	0	0	0	0	0	2	PSL(4,3)×2
3	10	0	0	0	0	0	0	0	0	5	J_2

Progenitor of Grapejelly72

$G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,$

$t^2, (t,(x^2)), (t,(y^2)),$

$((x^5)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^a,$

$(((y * x^{-1})^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^3*y*x^2*y^{-2})}))^b,$

$((x^2)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^c,$

$((y)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^d,$

$((x * y * x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^e,$

$((x)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^f,$

$((x^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^g,$

$((y * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^h,$

$((y^{-1})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^i,$

$(((y * x^{-1})^2)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^j,$

$(x^5*t)^k >$

Grapejelly72											
a	b	c	d	e	f	g	h	i	j	k	G
0	2	0	0	0	0	0	0	0	0	2	2^8:(2:A_5)
0	2	0	0	0	0	0	0	0	0	4	2^5:(2^4:A_5)

Progenitor of Grapejelly73

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$(((y * x^{-1})^2)*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^a$,
$(((x * y)^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^b$,
$(((y * x^{-1})^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^c$,
$((y^2)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^d$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^e$,
$((y^3)*(t^{(x^{-1}*y*x*y*x)})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^f$,
$((x^5)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^g$,
$((y^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^h$,
$((y * x^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^i$,
$((x^3)*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^j$,
$(x^5*t)^k$ >

Grapejelly73											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	4	0	0	2	S(4,3)×2
0	0	0	0	0	0	0	4	0	0	4	2^6:(S(4,3):2)
0	0	0	0	0	0	0	4	0	0	8	2:(2^6:(S(4,3):2))
0	6	0	0	0	0	0	0	0	0	2	S(4,3)×2
0	6	0	0	0	0	0	0	0	0	4	2^6:(S(4,3):2)
0	6	0	0	0	0	0	0	0	0	8	2:(2^6:(S(4,3):2))

Progenitor of Grapejelly74

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2$

$* x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^a$,
$((y * x^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})})*(t^{(x^{-2}*y*x^2)}))^b$,
$((y^{-1})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^c$,
$((x)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^d$,
$((x^2)*(t^{(y^3*x*y^{-2})}))^e$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^f$,
$((y^2)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^g$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^h$,
$((x^5)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^i$,
$((y^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^j$,
$(x^5*t)^k$ >

Grapejelly74											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	4	2	0	2	$2^{16}:(2^4:A_5)$

Progenitor of Grapejelly75

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y^3)*(t^{(y^3*x*y^{-2})})*t)^a$,
$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^b$,
$((x * y * x)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^c$,
$((x^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^d$,
$(((y * x^{-1})^3)*t*(t^{(y*x^{-1}*y^{-2})}))^e$,
$((y * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^f$,
$((x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^g$,
$((y^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^h$,
$(((y * x^{-1})^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^i$,
$((x^2)*(t^{(x*y^2*x*y)}))^j$,

$(x^5*t)^k$ >

Grapejelly75											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	6	4	2^6:(S(4,3):2)
0	0	0	0	0	0	0	0	0	6	8	2:(2^6:(S(4,3):2))
3	8	0	0	0	0	0	0	0	0	2	2^8:(2:A_5)

Progenitor of Grapejelly76

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^a$,

$(((x * y)^2)*t*(t^{(y^3*x*y^{-2})}))^b$,

$((x * y * x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^c$,

$((x^3)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^d$,

$((y * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^e$,

$(((y * x^{-1})^2)*(t^{(x^{-2}*y*x^2)}))^f$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*t)^g$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^h$,

$((y^3)*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^i$,

$((x^5)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^j$,

$(x^5*t)^k$ >

Grapejelly76											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	2	2	2	2^5:(2^4:A_5)
0	0	0	0	0	0	0	0	2	3	2	3^5:(2^5:A_5)
0	0	0	0	0	0	0	0	2	4	2	2^5:(2^5:A_5)
0	0	0	0	0	0	0	0	2	5	2	5^5:(2^5:A_5)
0	0	0	0	0	0	0	0	2	6	2	2^5:(3^5:(2^5:A_5))
0	0	0	0	0	0	4	0	0	0	2	2^9:(2^5:A_5)

Progenitor of Grapejelly77

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, (t,(x^2)), (t,(y^2)),

$((y^{-1})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^a$,

$((y * x^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})})*(t^{(x^{-2}*y*x^2)}))^b$,

$((y^{-1})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^c$,

$((x)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^d$,

$((x^2)*(t^{(y^3*x*y^{-2})}))^e$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^f$,

$((y^2)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^g$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^h$,

$((x^5)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^i$,

$((y^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^j$,

$(x^5*t)^k$ >

Grapejelly77											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	2	5	5^5:(2^4:A_5)

Progenitor of Grapejelly78

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, (t,(x^2)), (t,(y^2)),

$((y^3)*(t^{(y^3*x*y^{-2})})*t)^a$,

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^b$,

$((x * y * x)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^c$,

$((x^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^d$,

$(((y * x^{-1})^3)*t*(t^{(y*x^{-1}*y^{-2})}))^e$,

$((y * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^f$,

$((x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^g$,

$((y^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^h$,

$(((y * x^{-1})^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^i$,

$((x^2)*(t^{(x*y^2*x*y)}))^j$,

$(x^5*t)^k$ >

Grapejelly78											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	5	2	2:(A_5:J_2)
2	0	0	0	0	0	0	0	0	0	5	5^5:(2^4:A_5)

Progenitor of Grapejelly82

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, (t,(x^2)), (t,(y^2)),

$((x^5)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^a$,

$((y^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^b$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x^{-1}*y*x^2)})*t)^c$,

$(((y * x^{-1})^2)*(t^{(x*y*x^{-2}*y*x)}))^d$,
$((y)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^4*y*x^{-1}*y)}))^e$,
$((x * y * x)*(t^{(y^2*x^{-1}*y^2*x^2)})*t)^f$,
$((x^3)*(t^{(x^{-1}*y*x*y*x)}))^g$,
$((y * x^{-1})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^h$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^i$,
$((x^5)*t*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^j$,
$(x^5*t)^k$ >

Grapejelly82											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	6	0	8	0	2	J_2

Progenitor of Grapejelly83

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x^3)*(t^{(x*y^2*x*y)}))^a$,
$((x^5)*(t^{(y*x*y^{-1}*x^{-1}*y*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^b$,
$((y^{-1})*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^c$,
$(((x * y)^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^d$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-2}*y*x^2)})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^e$,
$((y^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^f$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^g$,
$((x^5)*(t^{(y^2*x^{-1}*y^2*x^2)}))^h$,
$((y * x^{-1})*(t^{(x^4*y*x^{-1}*y)})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^i$,
$((x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(x^{-2}*y*x^2)}))^j$,
$(x^5*t)^k$ >

Grapejelly83											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	4	0	0	8	2	PSL(4,3)×2

Progenitor of Grapejelly84

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^{6}, (x*y^{-2}*x)^{2}, (x*y^{2}*x^{2})^{2}, (y^{-1}*x^{-1})^{5}, (x*y^{2}*x^{-1}*y^{-1})^{2}, x^{-1}*y^{-1}*x^{5}*y*x^{-4}, y*x^{-2}*y^{-1}*x^{3}*y*x*y^{3}*x^{-1},$

$t^{2}, (t,(x^{2})), (t,(y^{2})),$

$((x)*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x^{3}*y*x^{2}*y^{-2})}))^{a},$

$((y*x^{-1})*(t^{(x^{4}*y*x^{-1}*y)})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^{b},$

$((x*y*x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^{c},$

$(((y*x^{-1})^{2})*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^{d},$

$(((x*y)^{2})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^{e},$

$((y*x^{2}*y^{-1}*x^{-1})*(t^{(y^{3}*x*y^{-2})})*(t^{(x*y^{2}*x*y)}))^{f},$

$(((y*x^{-1})^{3})*(t^{(y*x^{-1}*y^{-2})})*(t^{(x^{4}*y*x^{-1}*y)}))^{g},$

$((y*x^{-1}*y^{-1}*x^{-1}*y^{-1}*x)*t*(t^{(x^{-2}*y*x^{2})}))^{h},$

$((y^{3})*(t^{(y^{2}*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^{3}*x^{-1}*y^{-2}*x^{-1})}))^{i},$

$((x^{5})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*t)^{j},$

$(x^{5}*t)^{k} >$

Grapejelly84											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	2	2	2	2^5:(2^4:A_5)
0	0	0	0	0	0	0	0	2	3	2	3^5:(2^5:A_5)
0	0	0	0	0	0	0	0	2	4	2	2^5:(2^5:A_5)
0	0	0	0	0	0	0	0	2	5	2	5^5:(2^5:A_5)
0	0	0	0	0	0	0	0	2	6	2	2^5:(3^5:(2^5:A_5))
0	0	0	0	0	0	0	0	2	8	2	2^{16}:(2^4:A_5)

Progenitor of Grapejelly85

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^{6}, (x*y^{-2}*x)^{2}, (x*y^{2}*x^{2})^{2}, (y^{-1}*x^{-1})^{5}, (x*y^{2}*x^{-1}*y^{-1})^{2}, x^{-1}*y^{-1}*x^{5}*y*x^{-4}, y*x^{-2}*y^{-1}*x^{3}*y*x*y^{3}*x^{-1},$

t^2, (t,(x^2)), (t,(y^2)),
((x^5)*($t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}$))^a,
((y^2)*($t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}$)*($t^{(y^{-1}*x*y^{-1}*x*y^{-2})}$))^b,
((y * x^2 * y^{-1} * x^{-1})*($t^{(y^{-1}*x^{-1}*y*x^2)}$)*t)^c,
(((y * x^{-1})^2)*($t^{(x*y*x^{-2}*y*x)}$))^d,
((y)*($t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}$)*($t^{(x^4*y*x^{-1}*y)}$))^e,
((x * y * x)*($t^{(y^2*x^{-1}*y^2*x^2)}$)*t)^f,
((x^3)*($t^{(x^{-1}*y*x*y*x)}$))^g,
((y * x^{-1})*($t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}$)*($t^{(x^4*y*x^{-1}*y)}$))^h,
((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*($t^{(y^{-1}*x^{-1}*y*x^{-3})}$))^i,
((x^5)*t*($t^{(y^{-1}*x*y^{-1}*x*y^{-1})}$))^j,
(x^5*t)^k >

Grapejelly85											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	4	0	2	2	PSL(4,3)×2

Progenitor of Grapejelly86

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,
t^2, (t,(x^2)), (t,(y^2)),
(((y * x^{-1})^3)*($t^{(x*y*x^{-1}*y*x*y^{-1})}$)*($t^{(x^{-1}*y*x*y*x)}$))^a,
((x^2)*($t^{(x*y^2*x*y)}$))^b,
(((y * x^{-1})^2)*($t^{(y^2*x^{-1}*y^2*x^2)}$)*($t^{(x*y^{-1}*x^{-1}*y^3*x)}$))^c,
((y^{-1})*($t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}$)*($t^{(y^{-1}*x^{-1}*y*x^{-3})}$))^d,
((x * y * x)*($t^{(x^{-2}*y^{-1}*x^2*y*x)}$)*($t^{(x^{-2}*y*x^2)}$))^e,
((x)*($t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}$)*($t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}$))^f,
((x^3)*($t^{(x*y^{-1}*x^{-1}*y^3*x)}$)*($t^{(y*x*y^{-1}*x^{-1}*y*x)}$))^g,
((y * x^{-1})*($t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}$)*($t^{(y^{-1}*x*y^{-1}*x*y^{-2})}$))^h,
((x^5)*($t^{(y^2*x*y*x^{-1}*y*x^{-1})}$)*($t^{(y^{-1}*x^{-1}*y*x^2)}$))^i,
((y^3)*($t^{(x^{-2}*y^{-1}*x^2*y*x)}$)*($t^{(x^{-1}*y*x*y*x)}$))^j,
(x^5*t)^k >

Grapejelly86											
a	b	c	d	e	f	g	h	i	j	k	G
0	6	0	0	0	0	0	0	0	0	4	2^6:(S(4,3):2)
0	6	0	0	0	0	0	0	0	0	8	2:(2^6:(S(4,3):2))

Progenitor of Grapejelly87

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((x^5)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})((x^5)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^a$,

$((x * y * x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^b$,

$((y)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^c$,

$((y * x^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^d$,

$((x^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(y*x^{-1}*y^{-2})}))^e$,

$(((x * y)^2)*(t^{(x^4*y*x^{-1}*y)})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^f$,

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^g$,

$((y)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})})*(t^{(y^3*x*y^{-2})}))^h$,

$((y^3)*(t^{(x^4*y*x^{-1}*y)}))^i$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^j$,

$(x^5*t)^k$ >

Grapejelly87											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	3	0	5	$A_5 \times J_2$
0	0	0	0	0	0	0	0	3	0	10	2:(A_5:J_2)
0	4	0	0	0	0	0	0	0	0	2	2^8:(2:A_5)
0	4	0	0	0	0	0	0	0	0	4	2^5:(2^4:A_5)

Progenitor of Grapejelly90

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, (t,(x^2)), (t,(y^2)),
$(((y * x^{-1})^3)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(x^{-1}*y*x*y*x)}))^a$,
$((x^2)*(t^{(x*y^2*x*y)}))^b$,
$(((y * x^{-1})^2)*(t^{(y^2*x^{-1}*y^2*x^2)})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^c$,
$((y^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^d$,
$((x * y * x)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(x^{-2}*y*x^2)}))^e$,
$((x)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^f$,
$((x^3)*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^g$,
$((y * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^h$,
$((x^5)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^i$,
$((y^3)*(t^{(x^{-2}*y^{-1}*x^2*y*x)})*(t^{(x^{-1}*y*x*y*x)}))^j$,
$(x^5*t)^k$ >

Grapejelly90											
a	b	c	d	e	f	g	h	i	j	k	G
0	5	0	0	0	0	0	0	0	0	2	2:(A_5:J_2)

Progenitor of Grapejelly91

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^5, (t,(x^2)), (t,(y^2)),
$((x^5)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^a$,
$((x * y * x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^b$,
$((y)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^c$,
$((y * x^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^d$,
$((x^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(y*x^{-1}*y^{-2})}))^e$,
$(((x * y)^2)*(t^{(x^4*y*x^{-1}*y)})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^f$,
$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^g$,
$((y)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})})*(t^{(y^3*x*y^{-2})}))^h$,
$((y^3)*(t^{(x^4*y*x^{-1}*y)}))^i$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^j$,
$(x^5*t)^k$ >

Grapejelly91											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	3	0	2	$A_5 \times J_2$

Progenitor of Grapejelly94

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, $(t,(x^2))$, $(t,(y^2))$,

$((x^3)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^a$,

$((y * x^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^b$,

$((y)*(t^{(y^2*x^{-1}*y^2*x^2)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^c$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^d$,

$(((y * x^{-1})^3)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^e$,

$((y^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^f$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^g$,

$((y^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^h$,

$((x^5)*(t^{(x*y*x^{-2}*y*x)})*(t^{(x^3*y*x^2*y^{-2})}))^i$,

$((y * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^j$,

$(x^5*t)^k$ >

Grapejelly94											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	3	0	6	2	$A_5 \times J_2$

Progenitor of Grapejelly95

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^4*y*x^{-1}*y)})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^a$,

$((y^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^b$,

$(((y * x^{-1})^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^c$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^d$,
$(((x * y)^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^e$,
$((x^5)*(t^{(y^2*x^{-1}*y^2*x^2)}))^f$,
$((y^3)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(x^{-2}*y*x^2)}))^g$,
$((x^3)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^h$,
$((y * x^{-1})*(t^{(y*x^{-1}*y^{-2})}))^j$,
$(x^5*t)^k$ >

Grapejelly95											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	6	0	0	2	J_2

Progenitor of Grapejelly96

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^5, $(t,(x^2))$, $(t,(y^2))$,
$((x * y * x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^a$,
$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^b$,
$((x * y * x)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^c$,
$((x^3)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^d$,
$((y * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^e$,
$((x)*(t^{(x*y*x^{-2}*y*x)})*(t^{(y*x^{-1}*y^{-2})}))^f$,
$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y*x^{-1}*y^{-2})}))^g$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^h$,
$((y^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^i$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^j$,
$(x^5*t)^k$ >

Grapejelly96											
a	b	c	d	e	f	g	h	i	j	k	G
3	0	0	0	0	0	0	0	0	0	2	J_2
0	10	0	0	0	0	0	0	0	0	10	5^5:(2^4:A_5)

Progenitor of Grapejelly97

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, (t,(x^2)), (t,(y^2)),

$((y^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^a$,

$((x * y * x)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^b$,

$((y^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^c$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^d$,

$(((y * x^{-1})^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^e$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^f$,

$((x^5)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^g$,

$(((y * x^{-1})^2)*(t^{(x^3*y*x^2*y^{-2})}))^i$,

$((x^5)*(t^{(x^{-2}*y*x^2)})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^j$,

$(x^5*t)^k$ >

Grapejelly97											
a	b	c	d	e	f	g	h	i	j	k	G
3	10	0	0	0	0	0	0	0	0	2	$A_5 \times J_2$
3	10	10	0	0	0	0	0	0	0	2	J_2

Progenitor of Grapejelly98

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$((x^3)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^a$,

$((y * x^{-1})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^b$,

$((y)*(t^{(y^2*x^{-1}*y^2*x^2)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^c$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^d$,

$(((y * x^{-1})^3)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^e$,

$((y^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(y^2*x^{-1}*y*x^{-1}*y)}))^f$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^g$,

$((y^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^h$,

$((x^5)*(t^{(x*y*x^{-2}*y*x)})*(t^{(x^3*y*x^2*y^{-2})}))^i$,

$((y * x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x)})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^j$,

$(x^5*t)^k$ >

Grapejelly98											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	3	0	0	5	$A_5 \times J_2$
0	0	0	0	0	0	0	3	0	0	10	$2{:}(A_5{:}J_2)$
0	0	0	0	0	0	0	4	2	0	2	$2^{16}{:}(2^4{:}A_5)$
0	0	0	0	0	0	4	0	2	6	2	$2^8{:}(2{:}A_5)$
0	6	0	0	0	0	0	0	0	0	2	$3^5{:}(2^5{:}A_5)$
4	10	0	4	0	0	0	0	0	0	2	PSL(4,3)×2

Progenitor of Grapejelly99

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^4*y*x^{-1}*y)})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^a$,

$((y^2)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^b$,

$(((y * x^{-1})^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^c$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^d$,

$(((x * y)^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^e$,

$((x^5)*(t^{(y^2*x^{-1}*y^2*x^2)}))^f$,

$((y^3)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(x^{-2}*y*x^2)}))^g$,

$((x^3)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^h$,

$((y * x^{-1})*(t^{(y*x^{-1}*y^{-2})}))^j$,

$(x^5*t)^k$ >

Grapejelly99											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	6	2	3^5:(2^5:A_5)
0	0	0	0	0	0	0	7	0	10	4	J_2
0	0	0	0	0	0	0	8	2	10	8	PSL(4,3)×2
0	4	0	0	0	0	0	0	0	0	2	S(4,3)×2
0	4	0	0	0	0	0	0	0	0	4	2^6:(S(4,3):2)
0	4	0	0	0	0	0	0	0	0	2	2:(2^6:(S(4,3):2))
4	6	0	4	0	0	0	0	0	0	4	2^5:(2^5:A_5)
4	6	0	4	0	0	0	0	0	0	2	2^{16}:(2^4:A_5)
6	6	0	4	0	0	0	0	0	0	2	3^5:(2^5:A_5)
10	6	10	4	0	0	0	0	0	0	4	5^5:(2^5:A_5)

Progenitor of Grapejelly100

$G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,$

$t^2, (t,(x^2)), (t,(y^2)),$

$((x * y * x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^a,$

$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^b,$

$((x * y * x)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^c,$

$((x^3)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^d,$

$((y * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^e,$

$((x)*(t^{(x*y*x^{-2}*y*x)})*(t^{(y*x^{-1}*y^{-2})}))^f,$

$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y*x^{-1}*y^{-2})}))^g,$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^h,$

$((y^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^i,$

$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^j,$

$(x^5*t)^k >$

Grapejelly100											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	4	6	0	2	2^{16}:(2^4:A_5)
4	0	0	0	0	0	0	0	0	0	2	2^5:(2^4:A_5)

Progenitor of Grapejelly101

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^a$,

$((x * y * x)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^b$,

$((y^2)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^c$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^d$,

$(((y * x^{-1})^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^e$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^f$,

$((x^5)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^g$,

$(((y * x^{-1})^2)*(t^{(x^3*y*x^2*y^{-2})}))^i$,

$((x^5)*(t^{(x^{-2}*y*x^2)})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^j$,

$(x^5*t)^k$ >

Grapejelly101											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	4	6	2	S(4,3)×2
3	0	0	0	0	0	0	0	0	0	5	$A_5 \times J_2$
3	0	0	0	0	0	0	0	0	0	10	$2{:}(A_5{:}J_2)$
0	4	0	0	0	0	0	0	0	0	2	$2^8{:}(2{:}A_5)$
0	4	0	0	0	0	0	0	0	0	4	$2^5{:}(2^4{:}A_5)$
0	0	3	0	0	0	0	0	0	0	2	$3^5{:}(2^5{:}A_5)$
6	0	4	0	0	0	0	0	0	0	2	PSL(4,3)×2
3	0	10	0	0	0	0	0	0	0	5	J_2
8	0	0	4	0	0	0	0	0	0	2	$2^5{:}(2^5{:}A_5)$
10	0	0	4	0	0	0	0	0	0	2	$5^5{:}(2^5{:}A_5)$
0	8	0	4	0	0	0	0	0	0	2	$2^9{:}(2^5{:}A_5)$
0	0	6	4	0	0	0	0	0	0	2	$2^5{:}(3^5{:}(2^5{:}A_5))$

Progenitor of Grapejelly102

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^a$,

$((x * y * x)*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*t)^b$,

$((x^3)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^c$,

$((y^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^d$,

$((y^3)*(t^{(x^{-2}*y*x^2)})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^e$,

$((x^5)*(t^{(y^2*x^{-1}*y^2*x^2)})*t)^f$,

$((y)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^g$,

$((x * y * x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^3*y*x^2*y^{-2})}))^h$,

$((x)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^i$,

$((y * x^{-1})*t*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^j$,

$(x^5*t)^k$ >

Grapejelly102											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	3	0	0	5	J_2
0	0	0	0	0	0	0	3	0	0	10	$2\times J_2$
0	0	0	0	0	0	0	8	5	0	5	$2{:}(A_5{:}J_2)$

Progenitor of Grapejelly103

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^6, (x*y^{-2}*x)^2, (x*y^2*x^2)^2, (y^{-1}*x^{-1})^5, (x*y^2*x^{-1}*y^{-1})^2, x^{-1}*y^{-1}*x^5*y*x^{-4}, y*x^{-2}*y^{-1}*x^3*y*x*y^3*x^{-1},$

$t^2, (t,(x^2)), (t,(y^2)),$

$((y*x^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^a,$

$((x)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^b,$

$((y^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^c,$

$(((y*x^{-1})^2)*(t^{(y^2*x^{-1}*y^2*x^2)}))^c,$

$(((x*y)^2)*t*(t^{(y^{-1}*x^{-1}*y^{-1})}))^d,$

$((x^2)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^e,$

$((y*x^2*y^{-1}*x^{-1})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^f,$

$(((y*x^{-1})^3)*(t^{(x^{-2}*y*x^2)}))^g,$

$((y^2)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*t)^h,$

$((y*x^{-1}*y^{-1}*x^{-1}*y^{-1}*x)*(t^{(x^4*y*x^{-1}*y)}))^i,$

$((y^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y*x^{-1}*y^{-2})}))^j,$

$(x^5*t)^k >$

Grapejelly103											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	6	0	2	2	$2^5{:}(2^4{:}A_5)$
0	0	0	0	0	0	0	9	2	2	2	$3^5{:}(2^5{:}A_5)$
0	5	0	0	0	0	0	0	0	0	5	$2{:}(A_5{:}J_2)$

Progenitor of Grapejelly104

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^6, (x*y^{-2}*x)^2, (x*y^2*x^2)^2, (y^{-1}*x^{-1})^5, (x*y^2*x^{-1}*y^{-1})^2, x^{-1}*y^{-1}*x^5*y*x^{-4}, y*x^{-2}*y^{-1}*x^3*y*x*y^3*x^{-1},$

t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x^5)*(t^{(x^4*y*x^{-1}*y)}))^a$,
$((x*y*x)*(t^{(y^3*x*y^{-2})}))^b$,
$((y^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})})*t)^c$,
$((y*x^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^d$,
$((x^3)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^e$,
$((x*y*x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^f$,
$((y)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^g$,
$(((x*y)^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^h$,
$((y*x^2*y^{-1}*x^{-1})*(t^{(y^3*x*y^{-2})}))^i$,
$(((y*x^{-1})^3)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^j$,
$(x^5*t)^k$ >

Grapejelly104											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	2	2	2^8:(2:A_5)
0	0	0	0	0	0	0	0	0	2	4	2^5:(2^4:A_5)
0	0	0	0	0	0	0	0	4	4	2	2^9:(2^5:A_5)
0	0	0	0	0	0	0	0	4	6	2	2^4:(3^5:2^5)
0	0	0	0	0	0	0	0	4	8	2	2^8:2:$A_5$786
6	0	0	6	0	0	0	0	0	0	2	3^5:(2^5:A_5)

Progenitor of Grapejelly105

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x*y^{-2}*x)^2$, $(x*y^2*x^2)^2$, $(y^{-1}*x^{-1})^5$, $(x*y^2*x^{-1}*y^{-1})^2$, $x^{-1}*y^{-1}*x^5*y*x^{-4}$, $y*x^{-2}*y^{-1}*x^3*y*x*y^3*x^{-1}$,
t^5, $(t,(x^2))$, $(t,(y^2))$,
$((y^{-1})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^a$,
$((x*y*x)*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*t)^b$,
$((x^3)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^c$,
$((y^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^d$,
$((y^3)*(t^{(x^{-2}*y*x^2)})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^e$,
$((x^5)*(t^{(y^2*x^{-1}*y^2*x^2)})*t)^f$,

$((y)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^{g}$,
$((x * y * x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^{3}*y*x^{2}*y^{-2})}))^{h}$,
$((x)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^{i}$,
$((y * x^{-1})*t*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^{j}$,
$(x^{5}*t)^{k}$ >

Grapejelly105											
a	b	c	d	e	f	g	h	i	j	k	G
8	0	6	0	0	0	0	0	0	0	2	J_2

Progenitor of Grapejelly109

G<x,y,t>:=Group<x,y,t| x^{10} , y^{6} , $(x * y^{-2} * x)^{2}$, $(x * y^{2} * x^{2})^{2}$, $(y^{-1} * x^{-1})^{5}$, $(x * y^{2} * x^{-1} * y^{-1})^{2}$, $x^{-1} * y^{-1} * x^{5} * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1}$,
t^{5}, $(t,(x^{2}))$, $(t,(y^{2}))$,
$((x^{2})*(t^{(x^{2}*y^{-1}*x^{-2}*y^{-1}*x)})*(t^{(y^{3}*x^{-1}*y^{-2}*x^{-1})}))^{a}$,
$((y^{-1})*(t^{(y*x^{-1}*y^{-2})}))^{b}$,
$((x * y * x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^{4}*y*x^{-1}*y)}))^{c}$,
$((x * y * x)*(t^{(y^{2}*x^{-1}*y*x^{-1}*y)})*(t^{(y^{2}*x^{-1}*y^{2}*x^{2})}))^{d}$,
$(((y * x^{-1})^{2})*(t^{(y^{2}*x*y*x^{-1}*y*x^{-1})})*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)}))^{e}$,
$((y * x^{2} * y^{-1} * x^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(y*x^{-1}*y^{-2})}))^{f}$,
$(((y * x^{-1})^{3})*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^{g}$,
$((x^{5})*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(x^{4}*y*x^{-1}*y)}))^{h}$,
$((y^{3})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^{i}$,
$((x^{5})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(x^{2}*y^{-1}*x^{-2}*y^{-1}*x)}))^{j}$,
$(x^{5}*t)^{k}$ >

Grapejelly109											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	3	0	2	$A_5 \times J_2$

Progenitor of Grapejelly111

G<x,y,t>:=Group<x,y,t| x^{10} , y^{6} , $(x * y^{-2} * x)^{2}$, $(x * y^{2} * x^{2})^{2}$, $(y^{-1} * x^{-1})^{5}$, $(x * y^{2}$

$* x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^5, $(t,(x^2))$, $(t,(y^2))$,
$(((x * y)^2)*(t^{(x^4*y*x^{-1}*y)}))^a$,
$((y)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^b$,
$(((y * x^{-1})^2)*(t^{(x*y^2*x*y)}))^c$,
$((x * y * x)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^d$,
$((y * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^e$,
$((y^3)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^f$,
$((y * x^{-1})*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^g$,
$(x^5*t)^k$ >

Grapejelly111							
a	b	c	d	e	f	g	G
0	0	0	0	3	0	2	A_5:J_2
0	0	0	4	0	0	5	5^5:(2^4:A_5)

Progenitor of Grapejelly112

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^a$,
$((y)*(t^{(y*x^{-1}*y^{-2})}))^b$,
$((x^5)*(t^{(y^2*x^{-1}*y^2*x^2)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^c$,
$((x^3)*(t^{(x^3*y*x^2*y^{-2})}))^d$,
$((x * y * x)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^e$,
$(((y * x^{-1})^2)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^f$,
$((x^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})})*(t^{(x^3*y*x^2*y^{-2})}))^g$,
$(((y * x^{-1})^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^i$,
$((y^2)*(t^{(y*x^{-1}*y^{-2})}))^j$,
$(x^5*t)^k$ >

Grapejelly112											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	4	2	S(4,3)×2
0	0	0	0	0	0	0	0	0	4	4	2^6:(S(4,3):2)
0	0	0	0	0	0	0	0	0	4	8	2:(2^6:(S(4,3):2))
0	0	0	0	0	0	0	0	2	0	2	2^5:(2^4:A_5)
8	8	2	0	0	0	0	0	0	0	2	2^2:(2^8:(S(4,3)×2))

Progenitor of Grapejelly113

$G<x,y,t>:=Group<x,y,t|\ x^{10}\ ,\ y^6\ ,\ (x * y^{-2} * x)^2\ ,\ (x * y^2 * x^2)^2\ ,\ (y^{-1} * x^{-1})^5\ ,\ (x * y^2 * x^{-1} * y^{-1})^2\ ,\ x^{-1} * y^{-1} * x^5 * y * x^{-4}\ ,\ y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}\ ,$

$t^2,\ (t,(x^2)),\ (t,(y^2)),$

$((x^2)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^a,$

$((y^{-1})*(t^{(y*x^{-1}*y^{-2})}))^b,$

$((x * y * x)*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^4*y*x^{-1}*y)}))^c,$

$((x * y * x)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(y^2*x^{-1}*y^2*x^2)}))^d,$

$(((y * x^{-1})^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^e,$

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(y*x^{-1}*y^{-2})}))^f,$

$(((y * x^{-1})^3)*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^g,$

$((x^5)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(x^4*y*x^{-1}*y)}))^h,$

$((y^3)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^i,$

$((x^5)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^j,$

$(x^5*t)^k >$

Grapejelly113											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	3	0	5	$A_5 \times J_2$
0	0	0	0	0	0	0	0	3	0	10	2:(A_5:J_2)

Progenitor of Grapejelly114

$G<x,y,t>:=Group<x,y,t|\ x^{10}\ ,\ y^6\ ,\ (x * y^{-2} * x)^2\ ,\ (x * y^2 * x^2)^2\ ,\ (y^{-1} * x^{-1})^5\ ,\ (x * y^2 * x^{-1} * y^{-1})^2\ ,\ x^{-1} * y^{-1} * x^5 * y * x^{-4}\ ,\ y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}\ ,$

t^2, (t,(x^2)), (t,(y^2)),
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x^{-1}*y*x*y*x)})*(t^{(x^4*y*x^{-1}*y)}))^a$,
$((y^3)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^b$,
$((y * x^{-1})*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})}))^c$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^d$,
$((y)*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*(t^{(x*y*x^{-2}*y*x)}))^e$,
$((x^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})})*(t^{(x^3*y*x^2*y^{-2})}))^f$,
$((x^5)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^g$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^3*x*y^{-2})}))^h$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^i$,
$((y * x^2 * y^{-1} * x^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^j$,
$(x^5*t)^k$ >

Grapejelly114											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	4	4	0	2	2^{16}:(2^4:A_5)
0	0	0	0	0	0	0	4	5	4	2	2^2:(2^8:(S(4,3)×2))
4	2	0	0	0	0	0	0	0	0	2	2^9:(2^5:A_5)
0	2	6	0	0	0	0	0	0	0	2	3^5:(2^5:A_5)

Progenitor of Grapejelly116

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, (t,(x^2)), (t,(y^2)),
$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^a$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*t)^b$,
$((y^3)*(t^{(y^3*x*y^{-2})}))^c$,
$((x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(x^{-2}*y*x^2)}))^d$,
$((x^3)*(t^{(y*x^{-1}*y^{-2})}))^e$,
$((y * x^{-1})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^f$,
$((x)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^g$,
$((y^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*t^x)^h$,

$(((y * x^{-1})^2)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^i$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^4*y*x^{-1}*y)}))^j$,
$(x^5*t)^k$ >

Grapejelly116											
a	b	c	d	e	f	g	h	i	j	k	G
6	5	0	0	0	0	0	0	0	0	2	PSL(4,3)×2
0	0	3	0	0	0	0	0	0	0	5	$A_5 \times J_2$
0	0	3	0	0	0	0	0	0	0	10	2:(A_5:J_2)
0	4	4	0	0	0	0	0	0	0	2	2^{16}:(2^4:A_5)

Progenitor of Grapejelly120

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^5, $(t,(x^2))$, $(t,(y^2))$,
$((x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^a$,
$((y^{-1})*(t^{(x^{-1}*y*x*y*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^b$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(x^{-1}*y*x*y*x)}))^c$,
$((x^5)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})})*(t^{(x^{-2}*y*x^2)}))^d$,
$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^e$,
$((y * x^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^f$,
$((x^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^g$,
$((x)*(t^{(x*y*x^{-2}*y*x)})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^h$,
$((x * y * x)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})})*(t^{(y^2*x^{-1}*y^2*x^2)}))^i$,
$((y^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^4*y*x^{-1}*y)}))^j$,
$(x^5*t)^k$ >

Grapejelly120											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	3	2	2:(A_5:J_2)
0	0	0	0	0	0	0	0	2	10	2	J_2

Progenitor of Grapejelly121

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, $(t,(x^2))$, $(t,(y^2))$,

$((y^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^a$,

$((y * x^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})})*(t^{(x^{-2}*y*x^2)}))^b$,

$((y^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^c$,

$((y)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^d$,

$(((x * y)^2)*(t^{(y^{-1}*x^{-1}*y*x^2)})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^e$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(x*y^2*x*y)}))^f$,

$((y^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^g$,

$((x^5)*(t^{(x^{-1}*y*x*y*x)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^h$,

$((x^2)*(t^{(x^{-2}*y*x^2)})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^i$,

$(((y * x^{-1})^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^j$,

$(x^5*t)^k$ >

Grapejelly121											
a	b	c	d	e	f	g	h	i	j	k	G
0	3	0	0	0	0	0	0	0	0	2	2:(A_5:J_2)

Progenitor of Grapejelly124

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^5, $(t,(x^2))$, $(t,(y^2))$,

$((y^2)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^a$,

$((y)*(t^{(x^3*y*x^2*y^{-2})})*(t^{(y^2*x^{-1}*y^2*x^2)}))^b$,

$((x * y * x)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^c$,

$((y * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(y^3*x*y^{-2})}))^d$,

$((x^5)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^e$,

$((y^2)*(t^{(y^2*x^{-1}*y*x^{-1}*y)})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^f$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^g$,

$(((y * x^{-1})^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^h$,

$((x^5)*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^i$,
$((y^3)*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^j$,
$(x^5*t)^k$ >

Grapejelly124											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	2	5	5^5:(2^4:A_5)

Progenitor of Grapejelly125

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^a$,
$((y^{-1})*(t^{(x^{-1}*y*x*y*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x)}))^b$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(x^{-1}*y*x*y*x)}))^c$,
$((x^5)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})})*(t^{(x^{-2}*y*x^2)}))^d$,
$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^e$,
$((y * x^{-1})*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^f$,
$((x^3)*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^g$,
$((x)*(t^{(x*y*x^{-2}*y*x)})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^h$,
$((x * y * x)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})})*(t^{(y^2*x^{-1}*y^2*x^2)}))^i$,
$((y^{-1})*(t^{(y^{-1}*x^{-1}*y*x^{-3})})*(t^{(x^4*y*x^{-1}*y)}))^j$,
$(x^5*t)^k$ >

Grapejelly125											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	3	5	2:(A_5:J_2)
0	0	0	0	0	0	0	0	0	3	10	2^2:(A_5:J_2)
5	0	0	2	0	0	0	0	0	0	2	2^8:(2:A_5)
8	0	0	2	0	0	0	0	0	0	2	PSL(4,3)×2

Progenitor of Grapejelly126

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$((y^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^a$,

$((y * x^{-1})*(t^{(x^2*y^{-1}*x*y*x^{-1})})*(t^{(x^{-2}*y*x^2)}))^b$,

$((y^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^c$,

$((y)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^d$,

$(((x * y)^2)*(t^{(y^{-1}*x^{-1}*y*x^2)})*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^e$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})})*(t^{(x*y^2*x*y)}))^f$,

$((y^2)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^g$,

$((x^5)*(t^{(x^{-1}*y*x*y*x)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^h$,

$((x^2)*(t^{(x^{-2}*y*x^2)})*(t^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}))^i$,

$(((y * x^{-1})^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})})*(t^{(y^{-1}*x^{-1}*y*x^2)}))^j$,

$(x^5*t)^k$ >

Grapejelly126											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	2	0	0	0	2	0	6	2	2^{16}:(2^4:A_5)
0	0	0	2	0	0	0	2	5	6	2	2^8:(2:A_5)

Progenitor of Grapejelly127

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,

$(((y * x^{-1})^2)$*(t$^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}$)*(t$^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}$))^a,
((y * x^2 * y^{-1} * x^{-1})*(t$^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}$)*(t$^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}$))^b,
$(((y * x^{-1})^3)$*(t$^{(y^2*x^{-1}*y*x^{-1}*y)}$)*(t$^{(y^{-1}*x^{-1}*y*x^{-3})}$))^c,
((y^2)*(t$^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}$)*(t$^{(x*y^{-2}*x^{-2}*y^{-1})}$))^d,
((x^5)*(t$^{(y^{-1}*x^{-1}*y*x^2)}$)*(t$^{(y*x^{-1}*y^{-2})}$))^e,
((y^3)*(t$^{(x*y^{-1}*x^{-1}*y^3*x)}$)*(t$^{(y^2*x*y^{-1}*x*y^{-1}*x^{-1})}$))^f,
((x^5)*(t$^{(y^3*x*y^{-2})}$)*(t$^{(y^2*x^{-1}*y^2*x^2)}$))^g,
((y^{-1})*(t$^{(x^2*y^{-1}*x*y*x^{-1})}$)*(t$^{(x^4*y*x^{-1}*y)}$))^h,
((x * y * x)*(t$^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}$)*(t$^{(y^{-1}*x*y^{-1}*x*y^{-2})}$))^i,
((x)*(t$^{(y^3*x^{-1}*y^{-2}*x^{-1})}$)*(t$^{(y^{-1}*x^{-1}*y*x^{-3})}$))^j,
(x^5*t)^k >

Grapejelly127											
a	b	c	d	e	f	g	h	i	j	k	G
6	0	2	0	0	0	0	0	0	0	2	2^5:(2^4:A_5)

Progenitor of Grapejelly129

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, (t,(x^2)), (t,(y^2)),
((y^2)*(t$^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}$)*(t$^{(x*y^{-1}*x^{-1}*y^3*x)}$))^a,
((y)*(t$^{(x^3*y*x^2*y^{-2})}$)*(t$^{(y^2*x^{-1}*y^2*x^2)}$))^b,
((x * y * x)*(t$^{(y^2*x^{-1}*y*x^{-1}*y)}$)*(t$^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}$))^c,
((y * x^{-1})*(t$^{(y^{-1}*x*y^{-1}*x*y^{-1})}$)*(t$^{(y^3*x*y^{-2})}$))^d,
((x^5)*(t$^{(y^3*x^{-1}*y^{-2}*x^{-1})}$)*(t$^{(x*y^{-1}*x^{-1}*y^3*x)}$))^e,
((y^2)*(t$^{(y^2*x^{-1}*y*x^{-1}*y)}$)*(t$^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}$))^f,
((y * x^2 * y^{-1} * x^{-1})*(t$^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}$)*(t$^{(y^{-1}*x^{-1}*y*x^2)}$))^g,
$(((y * x^{-1})^2)$*(t$^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}$)*(t$^{(y^{-1}*x^{-1}*y*x^2)}$))^h,
((x^5)*(t$^{(y^{-1}*x*y^{-1}*x*y^{-1})}$)*(t$^{(y^{-1}*x^{-1}*y*x^{-3})}$))^i,
((y^3)*(t$^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}$)*(t$^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}$))^j,
(x^5*t)^k >

Grapejelly129											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	3	6	6	2	3^5:(2^5:A_5)

Progenitor of Grapejelly130

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$((y^2)*(t^{(x*y*x^{-1}*y*x*y^{-1})})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^a$,

$(((x * y)^2)*(t^{(y^2*x^{-1}*y^2*x^2)}))^b$,

$((y)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^c$,

$((y^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^d$,

$((x)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^e$,

$((y * x^{-1})*(t^{(y^3*x*y^{-2})}))^f$,

$((y^3)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)})*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^g$,

$((x^5)*(t^{(y^2*x*y*x^{-1}*y*x^{-1})}))^h$,

$(((x * y)^2)*(t^{(y^3*x*y^{-2})}))^i$,

$((y)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^j$,

$(x^5*t)^k$ >

Grapejelly130											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	6	0	2	S(4,3)×2
0	0	0	0	0	0	0	0	6	0	4	2^6:(S(4,3):2)
0	0	0	0	0	0	0	0	6	0	8	2:(2^6:(S(4,3):2))

Progenitor of Grapejelly131

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$((y)*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^a$,

$((y^{-1})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^b$,

$((x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^{c}$,
$((y * x^{2} * y^{-1} * x^{-1})*(t^{(x^{2}*y^{-1}*x*y*x^{-1})})*(t^{(x*y*x^{-2}*y*x)}))^{d}$,
$(((y * x^{-1})^{3})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^{e}$,
$((y^{2})*(t^{(y^{2}*x*y^{-1}*x*y^{-1}*x^{-1})})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^{f}$,
$((y^{3})*(t^{(y^{2}*x^{-1}*y^{2}*x^{2})}))^{g}$,
$((x^{5})*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^{h}$,
$(((y * x^{-1})^{2})*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^{i}$,
$((y * x^{-1})*(t^{(x*y*x^{-2}*y*x)})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^{j}$,
$(x^{5}*t)^{k}$ >

Grapejelly131											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	4	6	2	S(4,3)×2
0	0	0	0	0	0	3	0	0	0	5	$A_5 \times J_2$
0	0	0	0	0	0	3	0	0	0	10	2:(A_5:J_2)
8	0	8	0	0	0	0	0	0	0	2	2^2:(2^8:(S(4,3)×2))

Progenitor of Grapejelly132

G<x,y,t>:=Group<x,y,t| x^{10} , y^{6} , $(x * y^{-2} * x)^{2}$, $(x * y^{2} * x^{2})^{2}$, $(y^{-1} * x^{-1})^{5}$, $(x * y^{2} * x^{-1} * y^{-1})^{2}$, $x^{-1} * y^{-1} * x^{5} * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1}$,
t^{2}, $(t,(x^{2}))$, $(t,(y^{2}))$,
$((y * x^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^{a}$,
$((x^{3})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(y^{2}*x*y*x^{-1}*y*x^{-1})}))^{b}$,
$((x)*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^{c}$,
$((x * y * x)*(t^{(x^{4}*y*x^{-1}*y)}))^{d}$,
$((y^{-1})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})})*(t^{(y^{2}*x*y*x^{-1}*y*x^{-1})}))^{e}$,
$((y)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^{f}$,
$(((y * x^{-1})^{2})*(t^{(x^{4}*y*x^{-1}*y)}))^{g}$,
$(((x * y)^{2})*(t^{(y^{3}*x*y^{-2})}))^{h}$,
$((x^{2})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)})*(t^{(x^{-2}*y*x^{2})}))^{i}$,
$((y * x^{2} * y^{-1} * x^{-1})*(t^{(x*y^{2}*x*y)}))^{j}$,
$(x^{5}*t)^{k}$ >

Grapejelly132											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	10	4	2	2^{13}:(2^5:A_5)
0	0	0	0	0	0	0	6	0	0	2	S(4,3)×2
0	0	0	0	0	0	0	6	0	0	4	2^6:(S(4,3):2)
0	0	0	0	0	0	0	6	0	0	8	2:(2^6:(S(4,3):2))
0	0	0	0	0	0	0	10	5	4	2	2^8:(2^5:A_5)

Progenitor of Grapejelly133

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(x^2))$, $(t,(y^2))$,
$((x^5)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^a$,
$((y^3)*(t^{(x*y^2*x*y)})*(t^{(x^3*y*x^2*y^{-2})}))^b$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^3*x^{-1}*y^{-2}*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^c$,
$((y^2)*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(x^{-2}*y*x^2)}))^d$,
$(((y * x^{-1})^3)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^e$,
$((y * x^2 * y^{-1} * x^{-1})*t*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^f$,
$((x^2)*(t^{(x*y*x^{-2}*y*x)})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^g$,
$(((x * y)^2)*(t^{(x^{-1}*y*x*y*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^h$,
$(((y * x^{-1})^2)*(t^{(y^3*x*y^{-2})})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^i$,
$((y)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^j$,
$(x^5*t)^k$ >

Grapejelly133											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	3	5	$A_5 \times J_2$
0	0	0	0	0	0	0	0	0	3	10	2:(A_5:J_2)
0	2	0	0	0	0	0	0	0	0	2	2^8:(2:A_5)

Progenitor of Grapejelly134

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^{6}, (x * y^{-2} * x)^{2}, (x * y^{2} * x^{2})^{2}, (y^{-1} * x^{-1})^{5}, (x * y^{2} * x^{-1} * y^{-1})^{2}, x^{-1} * y^{-1} * x^{5} * y * x^{-4}, y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1},$

$t^{2}, (t,(x^{2})), (t,(y^{2})),$

$((y)*(t^{(x*y^{2}*x*y)})*(t^{(y^{3}*x*y^{-2})}))^{a},$

$((y^{-1})*(t^{(x*y*x^{-2}*y*x)})*(t^{(x^{2}*y^{-1}*x*y*x^{-1})}))^{b},$

$((x * y * x)*(t^{(y^{2}*x^{-1}*y^{2}*x^{2})})*(t^{(x^{2}*y^{-1}*x^{-2}*y^{-1}*x)}))^{c},$

$((x)*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)})*t)^{d},$

$((x^{3})*(t^{(x*y*x^{-2}*y*x)})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^{e},$

$((y * x^{-1})*(t^{(y^{3}*x*y^{-2})})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^{f},$

$((x^{5})*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^{g},$

$((y^{3})*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(y^{2}*x^{-1}*y^{2}*x^{2})}))^{h},$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y*x^{-1}*y^{-2})})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^{i},$

$(((y * x^{-1})^{3})*(t^{(x^{-2}*y^{-1}*x^{2}*y*x)}))^{j},$

$(x^{5}*t)^{k} >$

Grapejelly134											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	4	4	6	2^{16}:(2^{4}:A_5)
3	7	4	0	0	0	0	0	0	0	2	2^{8}:(2:A_5)
0	5	5	0	0	0	0	0	0	0	2	2^{8}:S(4,3)×2

Progenitor of Grapejelly135

$G<x,y,t>:=Group<x,y,t|\ x^{10}, y^{6}, (x * y^{-2} * x)^{2}, (x * y^{2} * x^{2})^{2}, (y^{-1} * x^{-1})^{5}, (x * y^{2} * x^{-1} * y^{-1})^{2}, x^{-1} * y^{-1} * x^{5} * y * x^{-4}, y * x^{-2} * y^{-1} * x^{3} * y * x * y^{3} * x^{-1},$

$t^{5}, (t,(x^{2})), (t,(y^{2})),$

$((x^{5})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^{a},$

$((y^{3})*(t^{(x*y^{2}*x*y)})*(t^{(x^{3}*y*x^{2}*y^{-2})}))^{b},$

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{3}*x^{-1}*y^{-2}*x^{-1})})*(t^{(x*y^{-1}*x^{-1}*y*x^{-1}*y*x^{-1})}))^{c},$

$((y^{2})*(t^{(x*y^{-2}*x^{-2}*y^{-1})})*(t^{(x^{-2}*y*x^{2})}))^{d},$

$(((y * x^{-1})^{3})*(t^{(y^{-1}*x^{-1}*y^{-1})}))^{e},$

$((y * x^{2} * y^{-1} * x^{-1})*t*(t^{(x*y^{-1}*x^{-1}*y^{3}*x)}))^{f},$

$((x^{2})*(t^{(x*y*x^{-2}*y*x)})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^{g},$

$(((x * y)^2)*(t^{(x^{-1}*y*x*y*x)})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^h$,
$(((y * x^{-1})^2)*(t^{(y^3*x*y^{-2})})*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^i$,
$((y)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^j$,
$(x^5*t)^k$ >

Grapejelly135											
a	b	c	d	e	f	g	h	i	j	k	G
0	2	0	0	0	0	0	0	0	0	5	5^5:(2^4:A_5)

Progenitor of Grapejelly136

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^5, $(t,(x^2))$, $(t,(y^2))$,
$((y)*(t^{(x*y^2*x*y)})*(t^{(y^3*x*y^{-2})}))^a$,
$((y^{-1})*(t^{(x*y*x^{-2}*y*x)})*(t^{(x^2*y^{-1}*x*y*x^{-1})}))^b$,
$((x * y * x)*(t^{(y^2*x^{-1}*y^2*x^2)})*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^c$,
$((x)*(t^{(x*y^{-1}*x^{-1}*y^3*x)})*t)^d$,
$((x^3)*(t^{(x*y*x^{-2}*y*x)})*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^e$,
$((y * x^{-1})*(t^{(y^3*x*y^{-2})})*(t^{(y^{-1}*x*y^{-1}*x*y^{-2})}))^f$,
$((x^5)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^g$,
$((y^3)*(t^{(y^{-1}*x^{-1}*y^{-1})})*(t^{(y^2*x^{-1}*y^2*x^2)}))^h$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y*x^{-1}*y^{-2})})*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^i$,
$(((y * x^{-1})^3)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^j$,
$(x^5*t)^k$ >

Grapejelly136											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	2	10	0	5	5^5:(2^4:A_5)

Progenitor of Grapejelly137

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, $(t,(x^2))$, $(t,(y^2))$,
$((y^3)*(t^{(x*y^{-2}*x^{-2}*y^{-1})}))^a$,
$((x^5)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^b$,
$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^{-2}*y^{-1}*x^2*y*x)}))^c$,
$(((y * x^{-1})^3)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^d$,
$(((x * y)^2)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^e$,
$((y)*(t^{(y*x*y^{-1}*x^{-1}*y*x^{-3})}))^f$,
$((x * y * x)*(t^{(x*y^{-2}*x^{-1}*y^{-1}*x^{-1})}))^g$,
$((x^3)*(t^{(y^{-1}*x^{-1}*y*x^{-3})}))^h$,
$((y * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^i$,
$((y^3)*(t^{(x*y^2*x*y)}))^j$,
$((x^5)*t)^k >$

Grapejelly137											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	0	3	0	2:(A_5:J_2)
0	0	0	0	0	0	0	0	6	6	0	3^5:(2^5:A_5)
0	0	0	0	0	0	0	0	10	3	5	J_2
0	0	0	0	0	0	6	0	0	4	2	2^2:(2^8:(S(4,3)×2))
3	0	0	0	0	0	0	0	0	0	0	2:(A_5:J_2)
3	5	0	0	0	0	0	0	0	0	0	$A_5 \times J_2$
4	2	4	0	0	0	0	0	0	0	0	2^{16}:(2^4:A_5)

Progenitor of Grapejelly138

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^a$,

$(((y * x^{-1})^3)*(t^{(y^2*x^{-1}*y^2*x^2)}))^b$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(x^2*y^{-1}*x^{-2}*y^{-1}*x)}))^c$,

$((y^3)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^d$,

$((x^5)*(t^{(y*x^{-1}*y^{-2})}))^e$,

$((y * x^{-1})*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^f$,

$((x)*(t^{(x*y*x^{-1}*y*x*y^{-1})}))^g$,

$((x * y * x)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^h$,

$((y^{-1})*(t^{(y^{-1}*x*y^{-1}*x*y^{-1})}))^i$,

$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^j$,

$((x^5)*t)^k$ >

Grapejelly138											
a	b	c	d	e	f	g	h	i	j	k	G
4	4	0	0	0	0	0	0	0	0	0	2^5:(2^4:A_5)
4	10	0	0	0	0	0	0	0	0	0	5^5:(2^5:A_5)
4	6	6	0	0	0	0	0	0	0	0	3^5:(2^5:A_5)
4	0	8	0	0	0	0	0	0	0	0	2^5:(2^5:A_5)
0	0	0	3	0	0	0	0	0	0	0	2:(A_5:J_2)

Progenitor of Grapejelly139

G<x,y,t>:=Group<x,y,t| x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(x^2)), (t,(y^2)),

$(((y * x^{-1})^2)*(t^{(y^{-1}*x^{-2}*y^{-1}*x^{-1}*y)}))^a$,

$((x^2)*(t^{(x*y^{-1}*x*y*x^{-2}*y^{-1}*x^{-1})}))^b$,

$((y * x^2 * y^{-1} * x^{-1})*(t^{(y^2*x^{-1}*y^2*x^2)}))^c$,

$(((y * x^{-1})^3)*(t^{(x^4*y*x^{-1}*y)}))^d$,

$((y^2)*(t^{(x*y*x^{-2}*y*x)}))^e$,

$((y * x^{-1} * y^{-1} * x^{-1} * y^{-1} * x)*(t^{(y^{-1}*x^{-1}*y*x^2)}))^f$,

$((y^3)*(t^{(y^{-1}*x^{-1}*y^{-1})}))^g$,

$((x^5)*(t^{(x*y^{-1}*x^{-1}*y^3*x)}))^h$,

$((x * y * x)*(t^{(y*x^{-1}*y*x^{-1}*y*x^{-1}*y)}))^i$,

$((x^3)*(t^{(x^4*y*x^{-1}*y)}))^j$,

$((x^5)*t)^k >$

Grapejelly139											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	4	4	10	0	2^5:(2^4:A_5)
0	0	0	0	0	0	0	4	4	10	2	2^8:(2:A_5)
0	0	0	0	0	0	3	0	0	0	0	2:(A_5:J_2)
0	0	0	0	0	0	3	0	0	0	5	$A_5 \times J_2$
4	0	0	0	0	0	0	0	0	0	0	S(4,3)×2
0	4	6	0	0	0	0	0	0	0	0	3^5:(2^5:A_5)
0	4	8	0	0	0	0	0	0	0	0	2^5:(2^5:A_5)
0	10	4	10	0	0	0	0	0	0	0	5^5:(2^5:A_5)

10.6 2^{*16}:(2^4:(4:2))

We have the following information

S:=Sym(16)

x ∼ (1, 13, 2, 11)(3, 9, 10, 7)(4, 6, 8, 5)(12, 16, 14, 15)

y ∼ (1, 4, 15, 10)(2, 5, 9, 16)(6, 11, 7, 14)(8, 12)

#N = 384

Progenitor of Garlicbread

$G<x,y,t>:=Group<x,y,t|x^4 , y^4 , x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1} , x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,

t^2, $(t, (x * y^2 * x * y * x * y^{-1}))$, $(t,(x^2 * y * x^2 * y * x))$,

$((x^2 * y)*t^{(y*x*y^{-1}*x*y^{-1})})^a$,

$((x * y * x^{-1})*t^{(x*y*x*y*x^2*y^{-1})})^b$,
$(((x^2 * y)^2)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^c$,
$(((x * y)^2)*t^{(x^2)})^d$,
$((x^{-1} * y^{-1} * x^2 * y)*t^{((x^2*y)^2)})^e$,
$((x^2)*t^{(x^2*y^{-1}*x^{-1}*y*x)})^f$,
$(((x * y^{-1})^3)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^g$,
$((x * y)*t^{(x^2*y^2*x^{-1}*y^{-1})})^h$,
$((y^{-1} * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^i$,
$((x^2 * y)*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^j$,
$(x^2 * y * x^{-1} * y * x * y*t)^k$ >

Garlicbread											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	4	4	2^6:(2^3:(2:3))
0	0	0	0	0	0	0	0	0	4	8	2^6:(2^4:D_6)
0	0	0	0	0	0	0	0	0	5	3	2^4:S_5
0	0	0	0	0	0	0	0	0	6	8	2^3:(2^6:3)
0	0	0	0	0	0	0	0	6	0	6	3^3:(2^3:(2:3))
0	0	0	0	0	0	0	0	6	0	8	2^8:(2^6:2):(3:2^2)
0	0	0	0	0	0	0	0	6	0	10	5^3:(2^3:(2:3))
0	0	0	0	0	0	0	0	8	6	8	2^7:(PSL(2,7):2)
0	0	0	0	0	0	0	6	0	8	8	2^8:(2^4:(2^3:3))

Garlicbread											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	0	3	$2^4{:}S_5$
4	0	0	0	0	0	0	0	0	0	4	$2^6{:}(2^3{:}(2{:}3))$
4	0	0	0	0	0	0	0	0	0	8	$2^6{:}(2^4{:}D_6)$
6	0	0	0	0	0	0	0	0	0	8	$2^3{:}(2^6{:}3)$
0	4	0	0	0	0	0	0	0	0	6	$2{:}(3^4{:}2^3){:}(2^3{:}3)$
0	4	0	0	0	0	0	0	0	0	8	$2^8{:}(2^6{:}2){:}(3{:}2^2)$
0	4	0	0	0	0	0	0	0	0	10	$2{:}(5^4{:}2^3){:}(2^3{:}3)$
0	5	10	0	0	0	0	0	0	0	5	$2{:}(A_5{:}(A_5{:}2))$
0	5	10	0	0	0	0	0	0	0	10	$2^2{:}(2^8{:}A_5){:}(A_5{:}2)$
0	4	0	6	0	0	0	0	0	0	8	$2^8{:}(2^4{:}(2^3{:}3))$
0	6	0	6	0	0	0	0	0	0	6	$2{:}(2^8{:}(2^7{:}3^4))$
0	6	0	6	0	0	0	0	0	0	8	$2{:}(2^8{:}2){:}(3^4{:}2^5){:}(3{:}2)$
0	6	0	6	0	0	0	0	0	0	10	$2^6{:}(3{:}2^5)$

Progenitor of Garlicbread2

$G<x,y,t>:=Group<x,y,t|x^4 , y^4 , x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1} , x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y ,$

$t^2, (t, (x * y^2 * x * y * x * y^{-1})), (t,(x^2 * y * x^2 * y * x)),$

$((x^2 * y)*t^{(x^2)})^a,$

$((x * y^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^b,$

$((y^{-1} * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^c,$

$((x * y)*t^{(x*y*x*y*x^2*y^{-1})})^d,$

$((x * y * x^{-1})*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^e,$

$((x)*t^{(x*y*x^2*y^{-1}*x*y^{-1})})^f,$

$((y^2 * x * y * x^{-1})*t^{((x^2*y)^2)})^g,$

$((y * x^2 * y)*t^{((x^2*y^{-1}*x^{-1})^2)})^h,$

$(((x^2 * y)^2)*t^{(y*x^2*y*x*y^{-1})})^i,$

$(((x * y)^2)*t^{(y^2*x*y^{-1})})^j,$

$(x^2 * y * x^{-1} * y * x * y*t)^k >$

Garlicbread2											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	4	0	0	4	2^6:(3:2^5)
0	0	0	0	0	0	0	4	0	0	6	2:(3^4:2^3):(2^3:3)
0	0	0	0	0	0	0	4	0	0	8	2^8:(2^6:2):(3:2^2)
0	0	0	0	0	0	0	4	0	0	10	2:(5^4:2^3):(2^3:3)
0	0	0	0	0	0	0	6	5	0	10	2^3:(2:(M_{12}:2))
0	0	0	0	0	0	0	6	6	8	6	2^2:(2^5:(A_5:2))
0	0	0	0	0	0	0	8	6	6	6	2:(2^8:2):(3^4:2^5):(3:2)
0	0	0	0	0	0	5	0	10	10	5	2:(A_5:(A_5:2))
0	0	0	0	0	0	0	0	10	10	10	2^2:(2^8:A_5):(A_5:2)

Progenitor of Garlicbread3

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} *$
$x^{-2} * y^{-1} * x * y * x * y$,
t^2, $(t, (x * y^2 * x * y * x * y^{-1}))$, $(t,(x^2 * y * x^2 * y * x))$,
$((x^{-1} * y^{-1} * x^2 * y)*t^{(x^2)})^a$,
$(((x * y * x^{-1})^2)*t^{(x^2*y^2*x^{-1}*y^{-1})})^b$,
$((x^2)*t^{(y*x*y^{-1}*x*y^{-1})})^c$,
$(((x * y)^3)*t^{(x^2*y^{-1}*x^{-1}*y*x)})^d$,
$(((x * y^{-1})^3)*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^e$,
$(((x^2 * y)^2)*t^{(y^{-1}*x^2*y^{-1}*x^{-1})})^f$,
$((y * x^2 * y)*t^{(y^2*x*y^{-1})})^g$,
$((y^2 * x * y * x^{-1})*t^{(y*x^2*y*x*y^{-1})})^h$,
$((x)*t^{((x^2*y^{-1}*x^{-1})^2)})^i$,
$((x * y * x^{-1})*t^{((x^2*y)^2)})^j$,
$(x^2 * y * x^{-1} * y * x * y*t)^k$ >

Garlicbread3											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	0	3	$2^4{:}S_5$
0	0	0	0	0	0	0	0	0	4	4	$2^6{:}(2^4{:}D_6)$
0	0	0	0	0	0	0	0	0	4	6	$2{:}(3^4{:}2^3){:}(2^3{:}3)$
0	0	0	0	0	0	0	0	0	4	8	$2^8{:}(2^6{:}2){:}(3{:}2^2)$
0	0	0	0	0	0	0	0	0	4	10	$2{:}(5^4{:}2^3){:}(2^3{:}3)$
0	0	0	0	0	0	0	0	0	5	5	$2{:}(A_5{:}(A_5{:}2))$
0	0	0	0	0	0	0	0	0	5	10	$2^2{:}(2^8{:}A_5){:}(A_5{:}2)$
0	0	0	0	0	0	0	0	4	8	4	$2^8{:}(2^4{:}(2^2{:}3))$
0	0	0	0	0	0	0	0	8	4	8	$2^8{:}(2^4{:}(2^3{:}3))$
10	0	0	2	0	0	0	0	0	0	5	$2{:}(A_5{:}(A_5{:}2))$
0	4	0	4	0	0	0	0	0	0	4	$(2^2{:}3^6){:}(2^3{:}2){:}(3{:}2^4){:}(2{:}2^2)$

Progenitor of Garlicbread4

$G<x,y,t>:=Group<x,y,t|x^4 , y^4 , x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1} , x^{-1} * y^{-2} *$
$x^{-2} * y^{-1} * x * y * x * y ,$
$t^2, (t, (x * y^2 * x * y * x * y^{-1})), (t,(x^2 * y * x^2 * y * x)),$
$((x^2 * y)*t^{(y*x*y^{-1}*x*y^{-1})})^a,$
$((x * y * x^{-1})*t^{(x*y*x*y*x^2*y^{-1})})^b,$
$(((x^2 * y)^2)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^c,$
$(((x * y)^2)*t^{(x^2)})^d,$
$((x^{-1} * y^{-1} * x^2 * y)*t^{((x^2*y)^2)})^e,$
$((x^2)*t^{(x^2*y^{-1}*x^{-1}*y*x)})^f,$
$(((x * y^{-1})^3)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^g,$
$((x * y)*t^{(x^2*y^2*x^{-1}*y^{-1})})^h,$
$((y^{-1} * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^i,$
$((x^2 * y)*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^j,$
$(x^2*t)^k >$

Progenitor of Garlicbread5

$G<x,y,t>:=Group<x,y,t|x^4 , y^4 , x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1} , x^{-1} * y^{-2} *$

Garlicbread4											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	6	0	4	2^6:(2^3:(2:3))
0	0	0	0	0	0	0	0	6	0	6	3^3:(2^3:(2:3))
0	0	0	0	0	0	0	0	6	0	8	2^8:(2^6:2):(3:2^2)
0	0	0	0	0	0	0	0	6	0	10	5^3:(2^3:(2:3))
6	0	0	0	0	0	0	0	0	0	8	2^3:(2^6:3)
8	4	0	0	0	0	0	0	0	0	4	2^6:(2^4:D_6)
8	4	0	0	0	0	0	0	0	0	8	2^8:(2^4:(2^3:3))
0	5	0	0	0	0	0	0	0	0	10	2^2:(2^8:A_5):(A_5:2)
0	10	0	0	0	0	0	0	0	0	4	2:(5^4:2^3):(2^3:3)

$x^{-2} * y^{-1} * x * y * x * y$,

t^2, (t, $(x * y^2 * x * y * x * y^{-1})$), (t,$(x^2 * y * x^2 * y * x)$),

$((x^2 * y)*t^{(x^2)})^a$,

$((x * y^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^b$,

$((y^{-1} * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^c$,

$((x * y)*t^{(x*y*x*y*x^2*y^{-1})})^d$,

$((x * y * x^{-1})*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^e$,

$((x)*t^{(x*y*x^2*y^{-1}*x*y^{-1})})^f$,

$((y^2 * x * y * x^{-1})*t^{((x^2*y)^2)})^g$,

$((y * x^2 * y)*t^{((x^2*y^{-1}*x^{-1})^2)})^h$,

$(((x^2 * y)^2)*t^{(y*x^2*y*x*y^{-1})})^i$,

$(((x * y)^2)*t^{(y^2*x*y^{-1})})^j$,

$(x^2*t)^k$ >

Garlicbread5											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	4	0	6	4	2^6:(3:2^5)
0	0	0	0	0	0	0	4	0	6	6	2:(3^4:2^3):(2^3:3)
0	0	0	0	0	0	0	4	0	6	8	2^8:(2^6:2):(3:2^2)
0	0	0	0	0	0	0	4	0	6	10	2:(5^4:2^3):(2^3:3)
0	0	0	0	0	0	0	6	6	6	6	3^3:(2^3:(2:3))
0	0	0	0	0	0	0	6	6	8	6	2^2:(2^5:(A_5:2))

Progenitor of Garlicbread6

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,

t^2, (t, ($x * y^2 * x * y * x * y^{-1}$)), (t,($x^2 * y * x^2 * y * x$)),

$((x^{-1} * y^{-1} * x^2 * y) * t^{(x^2)})^a$,

$(((x * y * x^{-1})^2) * t^{(x^2*y^2*x^{-1}*y^{-1})})^b$,

$((x^2) * t^{(y*x*y^{-1}*x*y^{-1})})^c$,

$(((x * y)^3) * t^{(x^2*y^{-1}*x^{-1}*y*x)})^d$,

$(((x * y^{-1})^3) * t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^e$,

$(((x^2 * y)^2) * t^{(y^{-1}*x^2*y^{-1}*x^{-1})})^f$,

$((y * x^2 * y) * t^{(y^2*x*y^{-1})})^g$,

$((y^2 * x * y * x^{-1}) * t^{(y*x^2*y*x*y^{-1})})^h$,

$((x) * t^{((x^2*y^{-1}*x^{-1})^2)})^i$,

$((x * y * x^{-1}) * t^{((x^2*y)^2)})^j$,

$(x^2 * t)^k$ >

Garlicbread6											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	4	4	2^6:(2^4:D_6)
0	0	0	0	0	0	0	0	0	4	6	2:(3^4:2^3):(2^3:3)
0	0	0	0	0	0	0	0	0	4	8	2^8:(2^6:2):(3:2^2)
0	0	0	0	0	0	0	0	0	4	10	2:(5^4:2^3):(2^3:3)
0	0	0	0	0	0	0	0	0	5	10	2^2:(2^8:A_5):(A_5:2)
0	0	0	0	0	0	0	0	8	10	4	2:(5^4:2^3):(2^3:3)
0	4	5	2	0	0	0	0	0	0	5	3:(PSL(2,11):2)
0	6	0	3	0	0	0	0	0	0	5	2^2:(3:(M_{12}:2))
8	4	4	4	0	0	0	0	0	0	4	(2^2:3^6):(2^3:2):(3:2^4):(2:2^2)

Progenitor of Garlicbread7

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,

t^2, (t, ($x * y^2 * x * y * x * y^{-1}$)), (t,($x^2 * y * x^2 * y * x$)),

$((x^2 * y)*t^{(y*x*y^{-1}*x*y^{-1})})^a$,

$((x * y * x^{-1})*t^{(x*y*x*y*x^2*y^{-1})})^b$,

$(((x^2 * y)^2)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^c$,

$(((x * y)^2)*t^{(x^2)})^d$,

$((x^{-1} * y^{-1} * x^2 * y)*t^{((x^2*y)^2)})^e$,

$((x^2)*t^{(x^2*y^{-1}*x^{-1}*y*x)})^f$,

$(((x * y^{-1})^3)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^g$,

$((x * y)*t^{(x^2*y^2*x^{-1}*y^{-1})})^h$,

$((y^{-1} * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^i$,

$((x^2 * y)*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^j$,

$(t*t^{(x^2)})^k$= ($y * x^{-1} * y * x * y$)>

Garlicbread7											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	0	2	2^3:(2^6:(PSL(2,7):2))

Progenitor of Garlicbread8

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,

t^2, (t, $(x * y^2 * x * y * x * y^{-1})$), (t,$(x^2 * y * x^2 * y * x)$),

$((x^2 * y)*t^{(x^2)})^a$,

$((x * y^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^b$,

$((y^{-1} * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^c$,

$((x * y)*t^{(x*y*x*y*x^2*y^{-1})})^d$,

$((x * y * x^{-1})*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^e$,

$((x)*t^{(x*y*x^2*y^{-1}*x*y^{-1})})^f$,

$((y^2 * x * y * x^{-1})*t^{((x^2*y)^2)})^g$,

$((y * x^2 * y)*t^{((x^2*y^{-1}*x^{-1})^2)})^h$,

$(((x^2 * y)^2)*t^{(y*x^2*y*x*y^{-1})})^i$,

$(((x * y)^2)*t^{(y^2*x*y^{-1})})^j$,

$(t*t^{(x^2)})^k$= $(y * x^{-1} * y * x * y)^k$ >

Garlicbread8											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	10	6	10	$2:(5^4:2^3):(2^3:3)$
0	0	0	0	0	0	0	6	6	8	6	$2^2:(2^5:(A_5:2))$
0	0	0	0	0	0	0	8	4	6	2	$2^8:(2^6:2):(3:2^2)$
0	0	0	0	0	0	0	8	4	6	6	$2:(PSL(2,7):2^4)$
0	0	0	0	0	0	0	8	6	6	6	$2:(2^8:2):(3^4:2^5):(3:2)$
4	0	0	0	0	0	0	0	0	0	2	$2^6:(2^3:(2:3))$
4	0	0	0	0	0	0	0	0	0	4	$2^6:(2^4:D_6)$
6	0	0	0	0	0	0	0	0	0	4	$2^3:(2^6:3)$
0	6	6	0	0	0	0	0	0	0	4	$2^8:(2^4:(2^2:3))$
0	6	6	0	0	0	0	0	0	0	6	$2^3:(3^3:2^3):(3:2)$
0	6	6	0	0	0	0	0	0	0	8	$2^8:(2^6:2):(3:2^2)$
0	6	6	0	0	0	0	0	0	0	10	$2^3:(5^3:2^3):(3:2)$

Progenitor of Garlicbread9

$G<x,y,t>:=Group<x,y,t|x^4 , y^4 , x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1} , x^{-1} * y^{-2} *$
$x^{-2} * y^{-1} * x * y * x * y ,$

$t^2, (t, (x * y^2 * x * y * x * y^{-1})), (t,(x^2 * y * x^2 * y * x)),$

$((x^{-1} * y^{-1} * x^2 * y)*t^{(x^2)})^a,$

$(((x * y * x^{-1})^2)*t^{(x^2*y^2*x^{-1}*y^{-1})})^b,$

$((x^2)*t^{(y*x*y^{-1}*x*y^{-1})})^c,$

$(((x * y)^3)*t^{(x^2*y^{-1}*x^{-1}*y*x)})^d,$

$(((x * y^{-1})^3)*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^e,$

$(((x^2 * y)^2)*t^{(y^{-1}*x^2*y^{-1}*x^{-1})})^f,$

$((y * x^2 * y)*t^{(y^2*x*y^{-1})})^g,$

$((y^2 * x * y * x^{-1})*t^{(y*x^2*y*x*y^{-1})})^h,$

$((x)*t^{((x^2*y^{-1}*x^{-1})^2)})^i,$

$((x * y * x^{-1})*t^{((x^2*y)^2)})^j,$

$(t*t^{(x^2)})^k= (y * x^{-1} * y * x * y)^k >$

Garlicbread9											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	4	2	2^6:(2^4:D_6)
0	0	0	0	0	0	0	0	0	4	4	2^8:(2^6:2):(3:2^2)
0	0	0	0	0	0	0	0	0	4	6	(2^5:3^4):(2^3:2^2):(3:2)
0	0	0	0	0	0	0	0	4	8	2	2^8:(2^4:(2^2:3))
0	0	0	0	0	0	0	5	0	5	10	2^2:(2^8:A_5):(A_5:2)
0	0	0	0	0	0	0	0	0	10	2	2:(5^4:2^3):(2^3:3)
4	0	0	2	0	0	0	0	0	0	2	2^6:(2^3:(2:3))
5	0	0	2	0	0	0	0	0	0	10	2:(A_5:(A_5:2))

Progenitor of Garlicbread14

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,

t^2, (t, ($x * y^2 * x * y * x * y^{-1}$)), (t,($x^2 * y * x^2 * y * x$)),

$(((x * y)^2)*t^{(y*x*y^{-1}*x*y^{-1})})^a$,

$((x)*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^b$,

$((y^2 * x * y * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^c$,

$((x * y)*t^{((x^2*y)^2)})^d$,

$((x^2 * y)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^e$,

$(((x * y)^3)*t^{(x^2)})^f$,

$(((x * y * x^{-1})^2)*t^{(y*x*y^{-1}*x*y^{-1})})^g$,

$(((x^2 * y)^2)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^h$,

$((y^2 * x * y * x^{-1})*t^{(y*x*y^{-1}*x*y^{-1})})^i$,

$((x)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^j$,

$(x^2 * y * x^{-1} * y * x * y*t)^k$ >

Garlicbread14											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	5	0	5	2:(A_5:(A_5:2))
0	0	0	0	0	0	0	0	8	4	4	2^8:(2^4:(2^2:3))
0	0	0	0	0	0	2	0	8	0	4	2^6:(2^3:(2:3))
0	0	0	0	0	0	2	0	10	0	5	2:(A_5:(A_5:2))
0	0	0	0	0	0	2	5	0	0	10	3:(PSL(2,11):2)
0	0	0	0	0	0	4	4	0	8	4	2^8:(2^4:(2^3:3))
0	0	0	0	0	0	4	4	0	8	8	2^9:(2^6:(2^2:3))
0	0	2	0	0	0	0	0	0	0	5	2:(A_5:(A_5:2))
6	0	6	0	0	0	0	0	0	0	4	(2^3:3^4):(2^4:2^6):(3:2)
6	0	10	0	0	0	0	0	0	0	4	(2^3:5^4):(2^4:2^2):(3:2)

Progenitor of Garlicbread15

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,

t^2, (t, ($x * y^2 * x * y * x * y^{-1}$)), (t,($x^2 * y * x^2 * y * x$)),

$((y * x^2 * y)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^a$,

$((y^2 * x * y * x^{-1})*t^{(x*y*x^2*y^{-1}*x*y^{-1})})^b$,

$(((x^2 * y)^2)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^c$,

$((x^{-1} * y^{-1} * x^2 * y)*t^{(x^2)})^d$,

$(((x * y * x^{-1})^2)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^e$,

$((x^2)*t^{(x^2*y^2*x^{-1}*y^{-1})})^f$,

$(((x * y)^3)*t^{(y*x^2*y*x*y^{-1})})^g$,

$(((x * y^{-1})^3)*t^{(y^2*x*y^{-1})})^h$,

$((y^{-1} * x^{-1})*t^{((x^2*y)^2)})^i$,

$((x * y)*t^{(y*x*y^{-1}*x*y^{-1})})^j$,

$(x^2 * y * x^{-1} * y * x * y*t)^k$ >

Garlicbread15											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	2	0	6	4	$(2^3{:}3^4){:}(2^4{:}2^6){:}(3{:}2)$
0	0	0	0	0	0	2	0	6	0	4	$2^6{:}(2^3{:}(2{:}3))$
0	0	0	0	0	0	2	0	6	0	5	$2{:}(A_5{:}(A_5{:}2))$
4	0	0	0	0	0	0	0	0	0	4	$2^6{:}(3{:}2^5)$
4	0	0	0	0	0	0	0	0	0	6	$2{:}(3^4{:}2^3){:}(2^3{:}3)$
4	0	0	0	0	0	0	0	0	0	8	$2^8{:}(2^6{:}2){:}(3{:}2^2)$
4	0	0	0	0	0	0	0	0	0	10	$2{:}(5^4{:}2^3){:}(2^3{:}3)$
4	4	0	0	0	0	0	0	0	0	8	$2^8{:}(2^4{:}(2^3{:}3))$
0	10	0	0	0	0	0	0	0	0	4	$(2^3{:}5^4){:}(2^4{:}2^2){:}(3{:}2)$
0	4	4	0	0	0	0	0	0	0	6	$(2{:}3^8{:})2^4{:}2^2){:}(3{:}2)$
6	0	5	0	0	0	0	0	0	0	10	$2^3{:}(2{:}(M_{12}{:}2))$

Progenitor of Garlicbread16

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,

t^2, $(t, (x * y^2 * x * y * x * y^{-1}))$, $(t,(x^2 * y * x^2 * y * x))$,

$((x * y)*t^{(x^2*y^2*x^{-1}*y^{-1})})^a$,

$((x^2 * y)*t^{(x^2)})^b$,

$((x * y * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^c$,

$((x)*t^{(x*y*x^2*y^{-1}*x*y^{-1})})^d$,

$((y^2 * x * y * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^e$,

$((y * x^2 * y)*t^{(y^2*x*y^{-1})})^f$,

$(((x * y)^2)*t^{(y*x*y^{-1}*x*y^{-1})})^g$,

$(((x * y * x^{-1})^2)*t^{((x^2*y)^2)})^h$,

$((x^2)*t^{((x^2*y^{-1}*x^{-1})^2)})^i$,

$(((x * y)^3)*t^{(y^{-1}*x^2*y^{-1}*x^{-1})})^j$,

$(x^2 * y * x^{-1} * y * x * y*t)^k$ >

Garlicbread16											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	4	2	4	2^6:(2^3:(2:3))
0	0	0	0	0	0	0	0	10	2	5	2:(A_5:(A_5:2))
0	0	0	0	0	0	0	2	5	0	10	3:(PSL(2,11):2)
6	0	0	0	0	0	0	0	0	0	8	2^8:(2^6:2):(3:2^2)
6	0	0	0	0	0	0	0	0	0	10	5^3:(2^3:(2:3))

Progenitor of Garlicbread17

$G<x,y,t>:=Group<x,y,t|x^4 , y^4 , x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1} , x^{-1} * y^{-2} *$
$x^{-2} * y^{-1} * x * y * x * y$,
t^2, $(t, (x * y^2 * x * y * x * y^{-1}))$, $(t,(x^2 * y * x^2 * y * x))$,
$((x)*t^{(y^2*x*y^{-1})})^a$,
$((y^2 * x * y * x^{-1})*t^{(x*y*x*y*x^2*y^{-1})})^b$,
$(((x * y^{-1})^3)*t^{(x^2*y^{-1}*x^{-1}*y*x)})^c$,
$(((x * y)^3)*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^d$,
$((x * y * x^{-1})*t^{(x*y*x^2*y^{-1}*x*y^{-1})})^e$,
$((x * y)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^f$,
$((x * y^{-1})*t^{((x^2*y)^2)})^g$,
$((y * x^2 * y)*t^{(x^2*y^{-1}*x^{-1}*y*x)})^h$,
$(((x * y)^3)*t^{(y*x*y^{-1}*x*y^{-1})})^i$,
$((x^2)*t^{(x^2)})^j$,
$(x^2 * y * x^{-1} * y * x * y*t)^k >$

Garlicbread17											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	2	5	10	3:(PSL(2,11):2)
0	0	0	0	0	0	0	4	0	6	6	2:(3^4:2^3):(2^3:3)
0	0	0	0	0	0	0	4	0	8	4	2^6:(3:2^5)
0	0	0	0	0	0	0	4	0	8	8	2^8:(2^6:2):(3:2^2)
0	0	0	0	0	0	0	4	4	0	8	2^8:(2^4:(2^3:3))
0	0	0	0	0	0	0	4	5	0	10	5^4:(2^3:2^2):(3:2)
0	0	0	0	0	0	0	4	10	0	10	2:(5^4:2^3):(2^3:3)
0	0	0	0	0	0	0	10	4	0	5	2^2:(2^8:A_5):(A_5:2)
0	0	0	0	0	0	6	6	9	5	10	2^2:(3:(M_{12}:2))
0	0	0	0	0	0	6	6	6	5	10	2^3:(2:(M_{12}:2))
0	5	0	0	0	0	0	0	0	0	5	2:(A_5:(A_5:2))
0	5	10	0	0	0	0	0	0	0	5	2:(A_5:(A_5:2))

Progenitor of Garlicbread18

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,

t^2, (t, $(x * y^2 * x * y * x * y^{-1})$), (t,$(x^2 * y * x^2 * y * x)$),

$(((x * y)^2)*t^{(y*x*y^{-1}*x*y^{-1})})^a$,

$((x)*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^b$,

$((y^2 * x * y * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^c$,

$((x * y)*t^{((x^2*y)^2)})^d$,

$((x^2 * y)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^e$,

$(((x * y)^3)*t^{(x^2)})^f$,

$(((x * y * x^{-1})^2)*t^{(y*x*y^{-1}*x*y^{-1})})^g$,

$(((x^2 * y)^2)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^h$,

$((y^2 * x * y * x^{-1})*t^{(y*x*y^{-1}*x*y^{-1})})^i$,

$((x)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^j$,

$(x^2*t)^k$ >

Garlicbread18											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	2	0	0	6	8	3:(2:(PSL(2,7):2))
0	0	0	0	0	0	4	4	0	8	4	2^8:(2^4:(2^3:3))
0	0	0	0	0	0	4	4	0	8	8	2^9:(2^6:(2^2:3))
0	0	0	0	0	0	6	0	4	0	4	(2^3:3^4):(2^4:2^6):(3:2)

Progenitor of Garlicbread19

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1} , x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y ,

t^2, (t, (x * y^2 * x * y * x * y^{-1})), (t,(x^2 * y * x^2 * y * x)),

$((y * x^2 * y)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^a$,

$((y^2 * x * y * x^{-1})*t^{(x*y*x^2*y^{-1}*x*y^{-1})})^b$,

$(((x^2 * y)^2)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^c$,

$((x^{-1} * y^{-1} * x^2 * y)*t^{(x^2)})^d$,

$(((x * y * x^{-1})^2)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^e$,

$((x^2)*t^{(x^2*y^2*x^{-1}*y^{-1})})^f$,

$(((x * y)^3)*t^{(y*x^2*y*x*y^{-1})})^g$,

$(((x * y^{-1})^3)*t^{(y^2*x*y^{-1})})^h$,

$((y^{-1} * x^{-1})*t^{((x^2*y)^2)})^i$,

$((x * y)*t^{(y*x*y^{-1}*x*y^{-1})})^j$,

$(x^2*t)^k$ >

Garlicbread19											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	2	0	3	5	2^2:(3:(M_{12}:2))
4	0	0	0	0	0	0	0	0	0	4	2^6:(3:2^5)
4	0	0	0	0	0	0	0	0	0	6	2:(3^4:2^3):(2^3:3)
4	0	0	0	0	0	0	0	0	0	8	2^8:(2^6:2):(3:2^2)
4	0	0	0	0	0	0	0	0	0	10	2:(5^4:2^3):(2^3:3)
0	6	0	0	0	0	0	0	0	0	4	(2^3:3^4):(2^4:2^6):(3:2)
5	6	0	0	0	0	0	0	0	0	5	3:(PSL(2,11):2)
0	10	0	0	0	0	0	0	0	0	4	(2^3:5^4):(2^4:2^2):(3:2)
0	4	4	0	0	0	0	0	0	0	6	(2:3^8:)2^4:2^2):(3:2)
6	0	5	0	0	0	0	0	0	0	10	2^3:(2:(M_{12}:2))

Progenitor of Garlicbread20

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,

t^2, (t, $(x * y^2 * x * y * x * y^{-1})$), (t,$(x^2 * y * x^2 * y * x)$),

$((x * y)*t^{(x^2*y^2*x^{-1}*y^{-1})})^a$,

$((x^2 * y)*t^{(x^2)})^b$,

$((x * y * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^c$,

$((x)*t^{(x*y*x^2*y^{-1}*x*y^{-1})})^d$,

$((y^2 * x * y * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^e$,

$((y * x^2 * y)*t^{(y^2*x*y^{-1})})^f$,

$(((x * y)^2)*t^{(y*x*y^{-1}*x*y^{-1})})^g$,

$(((x * y * x^{-1})^2)*t^{((x^2*y)^2)})^h$,

$((x^2)*t^{((x^2*y^{-1}*x^{-1})^2)})^i$,

$(((x * y)^3)*t^{(y^{-1}*x^2*y^{-1}*x^{-1})})^j$,

$(x^2*t)^k$ >

Garlicbread20											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	10	2	5	3:(PSL(2,11):2)
10	10	2	0	0	0	0	0	0	0	8	2^2:(3:(PSL(3,4):2))

Progenitor of Garlicbread21

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,

t^2, (t, $(x * y^2 * x * y * x * y^{-1})$), (t,$(x^2 * y * x^2 * y * x)$),

$((x)*t^{(y^2*x*y^{-1})})^a$,

$((y^2 * x * y * x^{-1})*t^{(x*y*x*y*x^2*y^{-1})})^b$,

$(((x * y^{-1})^3)*t^{(x^2*y^{-1}*x^{-1}*y*x)})^c$,

$(((x * y)^3)*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^d$,

$((x * y * x^{-1})*t^{(x*y*x^2*y^{-1}*x*y^{-1})})^e$,

$((x * y)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^f$,

$((x * y^{-1})*t^{((x^2*y)^2)})^g$,

$((y * x^2 * y)*t^{(x^2*y^{-1}*x^{-1}*y*x)})^h$,

$(((x * y)^3)*t^{(y*x*y^{-1}*x*y^{-1})})^i$,

$((x^2)*t^{(x^2)})^j$,

$(x^2*t)^k$ >

Garlicbread21											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	2	0	5	3:(PSL(2,11):2)
0	0	0	0	0	0	0	4	0	8	8	2^8:(2^6:2):(3:2^2)
0	0	0	0	0	0	0	4	10	10	10	2:(5^4:2^3):(2^3:3)
6	8	2	0	0	0	0	0	0	0	8	2^3:(2^6:3)

Progenitor of Garlicbread22

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,
t^2, (t, $(x * y^2 * x * y * x * y^{-1})$), (t,$(x^2 * y * x^2 * y * x)$),
$(((x * y)^2)*t^{(y*x*y^{-1}*x*y^{-1})})^a$,
$((x)*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^b$,
$((y^2 * x * y * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^c$,
$((x * y)*t^{((x^2*y)^2)})^d$,
$((x^2 * y)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^e$,
$(((x * y)^3)*t^{(x^2)})^f$,
$(((x * y * x^{-1})^2)*t^{(y*x*y^{-1}*x*y^{-1})})^g$,
$(((x^2 * y)^2)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^h$,
$((y^2 * x * y * x^{-1})*t^{(y*x*y^{-1}*x*y^{-1})})^i$,
$((x)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^j$,
$(t*t^{(x^2)})^k$= $(y * x^{-1} * y * x * y)$>

Garlicbread22											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	0	2	2^3:(2^6:(PSL(2,7):2))
0	0	0	0	0	0	4	8	8	0	2	2^7:(PSL(2,7):2)
0	0	0	0	0	0	0	0	0	0	2	2^3:(2^6:(PSL(2,7):2))

Progenitor of Garlicbread23

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,
t^2, (t, $(x * y^2 * x * y * x * y^{-1})$), (t,$(x^2 * y * x^2 * y * x)$),

$((y * x^2 * y)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^a$,
$((y^2 * x * y * x^{-1})*t^{(x*y*x^2*y^{-1}*x*y^{-1})})^b$,
$(((x^2 * y)^2)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^c$,
$((x^{-1} * y^{-1} * x^2 * y)*t^{(x^2)})^d$,
$(((x * y * x^{-1})^2)*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^e$,
$((x^2)*t^{(x^2*y^2*x^{-1}*y^{-1})})^f$,
$(((x * y)^3)*t^{(y*x^2*y*x*y^{-1})})^g$,
$(((x * y^{-1})^3)*t^{(y^2*x*y^{-1})})^h$,
$((y^{-1} * x^{-1})*t^{((x^2*y)^2)})^i$,
$((x * y)*t^{(y*x*y^{-1}*x*y^{-1})})^j$,
$(t*t^{(x^2)})^k = (y * x^{-1} * y * x * y)>$

Garlicbread23											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	0	2	2^3:(2^6:(PSL(2,7):2))
0	0	0	0	0	0	0	10	0	0	2	2^7:(PSL(2,7):2)
0	0	0	0	0	0	0	0	0	0	2	2^3:(2^6:(PSL(2,7):2))

Progenitor of Garlicbread24

$G<x,y,t>:=Group<x,y,t|x^4 , y^4 , x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1} , x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,
t^2, $(t, (x * y^2 * x * y * x * y^{-1}))$, $(t,(x^2 * y * x^2 * y * x))$,
$((x * y)*t^{(x^2*y^2*x^{-1}*y^{-1})})^a$,
$((x^2 * y)*t^{(x^2)})^b$,
$((x * y * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x^{-1})})^c$,
$((x)*t^{(x*y*x^2*y^{-1}*x*y^{-1})})^d$,
$((y^2 * x * y * x^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^e$,
$((y * x^2 * y)*t^{(y^2*x*y^{-1})})^f$,
$(((x * y)^2)*t^{(y*x*y^{-1}*x*y^{-1})})^g$,
$(((x * y * x^{-1})^2)*t^{((x^2*y)^2)})^h$,
$((x^2)*t^{((x^2*y^{-1}*x^{-1})^2)})^i$,
$(((x * y)^3)*t^{(y^{-1}*x^2*y^{-1}*x^{-1})})^j$,
$(t*t^{(x^2)})^k = (y * x^{-1} * y * x * y)>$

Garlicbread24											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	0	2	2^3:(2^6:(PSL(2,7):2))
0	0	0	0	0	0	0	0	0	4	2	2^7:(PSL(2,7):2)
0	0	0	0	0	0	0	0	0	0	2	2^3:(2^6:(PSL(2,7):2))

Progenitor of Garlicbread25

$G<x,y,t>:=Group<x,y,t|x^4 , y^4 , x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1} , x^{-1} * y^{-2} *$
$x^{-2} * y^{-1} * x * y * x * y ,$
$t^2, (t, (x * y^2 * x * y * x * y^{-1})), (t,(x^2 * y * x^2 * y * x)),$
$((x)*t^{(y^2*x*y^{-1})})^a,$
$((y^2 * x * y * x^{-1})*t^{(x*y*x*y*x^2*y^{-1})})^b,$
$(((x * y^{-1})^3)*t^{(x^2*y^{-1}*x^{-1}*y*x)})^c,$
$(((x * y)^3)*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^d,$
$((x * y * x^{-1})*t^{(x*y*x^2*y^{-1}*x*y^{-1})})^e,$
$((x * y)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^f,$
$((x * y^{-1})*t^{((x^2*y)^2)})^g,$
$((y * x^2 * y)*t^{(x^2*y^{-1}*x^{-1}*y*x)})^h,$
$(((x * y)^3)*t^{(y*x*y^{-1}*x*y^{-1})})^i,$
$((x^2)*t^{(x^2)})^j,$
$(t*t^{(x^2)})^k= (y * x^{-1} * y * x * y)>$

Garlicbread25											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	0	2	2^3:(2^6:(PSL(2,7):2))
0	0	0	0	0	0	0	0	4	8	2	2^3:(2^6:(PSL(2,7):2))
0	0	0	0	0	0	0	0	0	0	2	2^7:(PSL(2,7):2)

Progenitor of Garlicbread26

$G<x,y,t>:=Group<x,y,t|x^4 , y^4 , x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1} , x^{-1} * y^{-2} *$
$x^{-2} * y^{-1} * x * y * x * y ,$
$t^2, (t, (x * y^2 * x * y * x * y^{-1})), (t,(x^2 * y * x^2 * y * x)),$
$((y^{-1} * x^{-1})*t^{(x*y*x*y*x^2*y^{-1})}*t^{(x^2*y^{-1}*x^{-1}*y*x)})^a,$

$((x * y^{-1})*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^b$,
$((x^2 * y)*t^{(x^2*y^2*x^{-1}*y^{-1})})^c$,
$(((x * y^{-1})^3)*t^{(x*y*x*y*x^2*y^{-1})}*t^{((x^2*y)^2)})^d$,
$(((x * y)^3)*t^{(x^2*y^{-1}*x^{-1}*y*x)})^e$,
$((x^2)*t^{((x^2*y^{-1}*x^{-1})^2)})^f$,
$(((x * y * x^{-1})^2)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})}*t^{(y^{-1}*x^2*y^{-1}*x^{-1})})^g$,
$((x^{-1} * y^{-1} * x^2 * y)*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^h$,
$(((x * y)^2)*t^{((x^2*y^{-1}*x^{-1})^2)}*t^{(y*x^2*y*x*y^{-1})})^i$,
$(((x^2 * y)^2)*t^{(x^2)})^j$,
$(t*t^{(x^2)})^k$= (y * x^{-1} * y * x * y)>

Garlicbread26											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	0	2	2^3:(2^6:(PSL(2,7):2))
0	0	0	0	0	0	0	0	0	0	2	2^3:(2^6:(PSL(2,7):2))
7	0	0	0	0	0	0	0	0	0	2	2^7:(PSL(2,7):2)

Progenitor of Garlicbread27

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,
t^2, (t, $(x * y^2 * x * y * x * y^{-1})$), (t,$(x^2 * y * x^2 * y * x)$),
$((y * x^2 * y)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^a$,
$((y^2 * x * y * x^{-1})*t^{((x^2*y^{-1}*x^{-1})^2)})^b$,
$((x)*t^{(x^2)}*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^c$,
$((x * y * x^{-1})*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^d$,
$((x * y)*t^{(x*y*x*y*x^2*y^{-1})}*t)^e$,
$((y^{-1} * x^{-1})*t^{((x^2*y)^2)}*t^{((x^2*y^{-1}*x^{-1})^2)})^f$,
$((x * y^{-1})*t^{(y^{-1}*x^2*y^{-1}*x^{-1})})^g$,
$((x^2 * y)*t^{(x^2*y^{-1}*x^{-1}*y*x)}*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^h$,
$(((x * y^{-1})^3)*t^{(x^2)})^i$,
$((x^2)*t^{(x^2*y^2*x^{-1}*y^{-1})}*t^{(x^2*y^{-1}*x^{-1}*y*x)})^j$,
$(t*t^{(x^2)})^k$= (y * x^{-1} * y * x * y)>

Garlicbread27											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	0	2	2^3:(2^6:(PSL(2,7):2))
0	0	0	0	0	0	0	0	0	3	2	2^7:(PSL(2,7):2)
0	0	0	0	0	0	0	0	0	0	2	2^3:(2^6:(PSL(2,7):2))
0	8	4	0	0	0	0	0	0	0	2	2^7:(PSL(2,7):2)

Progenitor of Garlicbread28

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,

t^2, (t, $(x * y^2 * x * y * x * y^{-1})$), (t,$(x^2 * y * x^2 * y * x)$),

$((y * x^2 * y)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^a$,

$((y^2 * x * y * x^{-1})*t^{((x^2*y^{-1}*x^{-1})^2)})^b$,

$((x)*t^{(x^2)}*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^c$,

$((x * y * x^{-1})*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^d$,

$((x * y)*t^{(x*y*x*y*x^2*y^{-1})}*t)^e$,

$((y^{-1} * x^{-1})*t^{((x^2*y)^2)}*t^{((x^2*y^{-1}*x^{-1})^2)})^f$,

$((x * y^{-1})*t^{(y^{-1}*x^2*y^{-1}*x^{-1})})^g$,

$((x^2 * y)*t^{(x^2*y^{-1}*x^{-1}*y*x)}*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^h$,

$(((x * y^{-1})^3)*t^{(x^2)})^i$,

$((x^2)*t^{(x^2*y^2*x^{-1}*y^{-1})}*t^{(x^2*y^{-1}*x^{-1}*y*x)})^j$,

$(x^2*t)^k$ >

Garlicbread28											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	4	10	4	2^6:(2^4:D_6)
0	0	0	0	0	0	0	0	6	3	8	2^3:(2^6:3)
4	0	0	0	0	0	0	0	0	0	4	2^6:(3:2^5)
4	0	0	0	0	0	0	0	0	0	6	2:(3^4:2^3):(2^3:3)
4	0	0	0	0	0	0	0	0	0	8	2^8:(2^6:2):(3:2^2)
4	0	0	0	0	0	0	0	0	0	10	2:(5^4:2^3):(2^3:3)
10	10	4	0	0	0	0	0	0	0	5	2^{12}:(3:(PSL(2,11):2))
0	5	0	2	0	0	0	0	0	0	10	2:(A_5:(A_5:2))
5	10	0	2	0	0	0	0	0	0	5	3:(PSL(2,11):2)

Progenitor of Garlicbread29

$G<x,y,t>:=Group<x,y,t|x^4 , y^4 , x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1} , x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y ,$

$t^2, (t, (x * y^2 * x * y * x * y^{-1})), (t,(x^2 * y * x^2 * y * x)),$

$((y * x^2 * y)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^a,$

$((y^2 * x * y * x^{-1})*t^{((x^2*y^{-1}*x^{-1})^2)})^b,$

$((x)*t^{(x^2)}*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^c,$

$((x * y * x^{-1})*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^d,$

$((x * y)*t^{(x*y*x*y*x^2*y^{-1})}*t)^e,$

$((y^{-1} * x^{-1})*t^{((x^2*y)^2)}*t^{((x^2*y^{-1}*x^{-1})^2)})^f,$

$((x * y^{-1})*t^{(y^{-1}*x^2*y^{-1}*x^{-1})})^g,$

$((x^2 * y)*t^{(x^2*y^{-1}*x^{-1}*y*x)}*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^h,$

$(((x * y^{-1})^3)*t^{(x^2)})^i,$

$((x^2)*t^{(x^2*y^2*x^{-1}*y^{-1})}*t^{(x^2*y^{-1}*x^{-1}*y*x)})^j,$

$(x^2*t)^k >$

Garlicbread29											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	2	4	2^6:(2^4:D_6)
0	0	0	0	0	0	0	0	0	3	8	2^3:(2^6:3)
4	0	0	0	0	0	0	0	0	0	4	2^6:(3:2^5)
4	0	0	0	0	0	0	0	0	0	6	2:(3^4:2^3):(2^3:3)
4	0	0	0	0	0	0	0	0	0	8	2^8:(2^6:2):(3:2^2)
4	0	0	0	0	0	0	0	0	0	10	2:(5^4:2^3):(2^3:3)
10	0	4	0	0	0	0	0	0	0	5	2^{12}:(3:(PSL(2,11):2))
10	10	4	2	0	0	0	0	0	0	5	3:(PSL(2,11):2)

Progenitor of Garlicbread30

G<x,y,t>:=Group<x,y,t|x^4 , y^4 , $x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1}$, $x^{-1} * y^{-2} * x^{-2} * y^{-1} * x * y * x * y$,

t^2, (t, ($x * y^2 * x * y * x * y^{-1}$)), (t,($x^2 * y * x^2 * y * x$)),

$((y * x^2 * y)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^a$,

$((y^2 * x * y * x^{-1})*t^{((x^2*y^{-1}*x^{-1})^2)})^b$,

$((x)*t^{(x^2)}*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^c$,

$((x * y * x^{-1})*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^d$,

$((x * y)*t^{(x*y*x*y*x^2*y^{-1})}*t)^e$,

$((y^{-1} * x^{-1})*t^{((x^2*y)^2)}*t^{((x^2*y^{-1}*x^{-1})^2)})^f$,

$((x * y^{-1})*t^{(y^{-1}*x^2*y^{-1}*x^{-1})})^g$,

$((x^2 * y)*t^{(x^2*y^{-1}*x^{-1}*y*x)}*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^h$,

$(((x * y^{-1})^3)*t^{(x^2)})^i$,

$((x^2)*t^{(x^2*y^2*x^{-1}*y^{-1})}*t^{(x^2*y^{-1}*x^{-1}*y*x)})^j$,

$(x^2 * y * x^{-1} * y * x * y*t)^k$ >

Garlicbread30											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	2	3	8	$2^3:(2^6:3)$
0	0	0	0	0	0	0	3	0	0	8	$2^7:(PSL(2,7):2)$
4	0	0	0	0	0	0	0	0	0	4	$2^6:(3:2^5)$
4	0	0	0	0	0	0	0	0	0	6	$2:(3^4:2^3):(2^3:3)$
4	0	0	0	0	0	0	0	0	0	8	$2^8:(2^6:2):(3:2^2)$
4	0	0	0	0	0	0	0	0	0	10	$2:(5^4:2^3):(2^3:3)$
0	5	4	0	0	0	0	0	0	0	5	$2:(A_5:(A_5:2))$
0	4	8	0	0	0	0	0	0	0	4	$2^9:(2^6:(2^2:3))$
0	5	8	0	0	0	0	0	0	0	5	$2:(A_5:(A_5:2))$
5	10	4	2	0	0	0	0	0	0	10	$3:(PSL(2,11):2)$
10	5	0	4	0	0	0	0	0	0	5	$2:(A_5:(A_5:2))$
0	4	8	4	0	0	0	0	0	0	4	$2^9:(2^6:(2^2:3))$

Progenitor of Garlicbread31

$G<x,y,t>:=Group<x,y,t|x^4 , y^4 , x * y^{-2} * x^{-1} * y^{-1} * x * y^2 * x^{-1} * y^{-1} , x^{-1} * y^{-2} *$
$x^{-2} * y^{-1} * x * y * x * y ,$
$t^2, (t, (x * y^2 * x * y * x * y^{-1})), (t,(x^2 * y * x^2 * y * x)),$
$((y * x^2 * y)*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^a,$
$((y^2 * x * y * x^{-1})*t^{((x^2*y^{-1}*x^{-1})^2)})^b,$
$((x)*t^{(x^2)}*t^{(x^{-1}*y^{-1}*x*y^{-1}*x)})^c,$
$((x * y * x^{-1})*t^{(y*x^2*y^{-1}*x^{-1}*y^{-1}*x)})^d,$
$((x * y)*t^{(x*y*x*y*x^2*y^{-1})}*t)^e,$
$((y^{-1} * x^{-1})*t^{((x^2*y)^2)}*t^{((x^2*y^{-1}*x^{-1})^2)})^f,$
$((x * y^{-1})*t^{(y^{-1}*x^2*y^{-1}*x^{-1})})^g,$
$((x^2 * y)*t^{(x^2*y^{-1}*x^{-1}*y*x)}*t^{(x^{-1}*y*x^{-1}*y*x*y^{-1})})^h,$
$(((x * y^{-1})^3)*t^{(x^2)})^i,$
$((x^2)*t^{(x^2*y^2*x^{-1}*y^{-1})}*t^{(x^2*y^{-1}*x^{-1}*y*x)})^j,$
$(x^2 * y * x^{-1} * y * x * y*t)^k >$

Garlicbread31											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	0	3	2^4:S_5
0	0	0	0	0	0	0	0	0	10	4	2^6:(2^4:D_6)
0	0	0	0	0	0	0	0	6	3	8	2^3:(2^6:3)
0	0	0	0	0	0	0	0	0	0	3	2^4:S_5
4	0	0	0	0	0	0	0	0	0	4	2^6:(3:2^5)
4	0	0	0	0	0	0	0	0	0	6	2:(3^4:2^3):(2^3:3)
4	0	0	0	0	0	0	0	0	0	8	2^8:(2^6:2):(3:2^2)
4	0	0	0	0	0	0	0	0	0	10	2:(5^4:2^3):(2^3:3)
10	5	0	0	0	0	0	0	0	0	5	2:(A_5:(A_5:2))
10	5	4	0	0	0	0	0	0	0	10	2:(A_5:(A_5:2))
0	4	8	4	0	0	0	0	0	0	4	2^9:(2^6:(2^2:3))
0	5	8	4	0	0	0	0	0	0	5	2:(A_5:(A_5:2))

10.7 2^{*18}:(3:A_6)

We have the following information

S:=Sym(18)

x ∼ (1, 18, 4, 17, 3, 15)(2, 5, 14, 11, 6, 8, 16, 13, 7, 10, 12, 9)

y ∼ (1, 16, 10, 15, 9)(3, 12, 5, 17, 11)(4, 14, 8, 18, 13)

#N = 1080

Progenitor of Cucumber

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x^5 * y)*t^{(x*y*x*y*x^2*y*x^{-1})})^a$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x^{-1}*y^2*x^{-2})})^b$,

$(((x * y)*2)*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^c$,

$((x * y)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^d$,

$((x^5)*t^{(y*x^{-2}*y^{-2})})^e$,

$((x)*t^{(y*x^2*y*x*y^{-1}*x^2)})^f$,

$((x^{-2})*t^{(x^{-3}*y*x^{-1}*y)})^g$,

$((x^2)*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^h$,

$((x^{-3} * y)*t^{(x*y*x^2)})^i$,

$(((x * y)^{*3})*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^j >$

Cucumber										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	4	0	0	2^6:A_6
0	0	0	0	0	0	0	0	0	6	S_7
6	0	0	0	0	0	0	0	0	0	S_7

Progenitor of Cucumber2

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x^3)*t^{((x*y*x^{-1})^2)})^a$,

$((x * y * x * y^{-2} * x)*t^{(x*y*x*y*x^2*y*x^{-1})})^b$,

$((y * x * y^{-1} * x^{-2})*t^{(x^{-1}*y^2*x^{-2})})^c$,

$((x^{-4})*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^d$,

$((x^4)*t^{(y*x^{-2}*y^{-2})})^e$,

$((x^6)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^f$,

$((x^{-2})*t^{(x^{-3}*y*x^{-1}*y)})^g$,

$((x * y)*t^{(y*x^2*y*x*y^{-1}*x^2)})^h$,

$((x^5 * y)*t^{(y^{-1}*x^2*y*x^2)})^i$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x*y*x^2)})^j >$

Cucumber2										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	6	0	S_7
0	0	0	0	0	0	4	10	10	0	2^6:A_6
0	0	0	0	0	0	6	0	10	10	3^5:(2:A_6)
8	0	6	2	0	0	0	0	0	0	2^{10}:(2:A_6)
0	6	0	3	0	0	0	0	0	0	2^{12}:(3:A_6)

Progenitor of Cucumber3

$G<x,y,t>:=Group<x,y,t|\ y^{5}\ ,\ y^{-2} * x^{3} * y^{-2} * x^{-1}\ ,\ (x * y^{-1})^{4}\ ,\ y^{-1} * x * y^{-1} * x^{-1} *$
$y^{-1} * x * y * x * y^{-1} * x^{-1}\ ,$
$t^{2},\ (t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2}),\ (t,(x^{-1} * y^{2})^{2}),\ (t,(x * y * x)^{2}),$
$(((x * y)^{*2})*t^{(x*y*x*y*x^{2}*y*x^{-1})})^{a},$
$((x^{-2})*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^{b},$
$(((x * y)^{*3})*t^{(y*x^{2}*y*x*y^{-1}*x^{2})})^{c},$
$((x^{-3} * y)*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^{2}*y^{-1})})^{d},$
$((y * x * y^{-1} * x^{-2})*t^{(x*y*x^{2})})^{e},$
$((x^{4})*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^{f},$
$((x^{5} * y)*t^{(x*y*x^{-1})})^{g},$
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^{h},$
$((x * y)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^{i},$
$((x^{5})*t^{((x*y*x^{-1})^{2})})^{j}\ >$

Cucumber3										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	4	2^{6}:A_{6}
6	0	0	0	0	0	0	0	0	0	S_{7}

Progenitor of Cucumber4

$G<x,y,t>:=Group<x,y,t|\ y^{5}\ ,\ y^{-2} * x^{3} * y^{-2} * x^{-1}\ ,\ (x * y^{-1})^{4}\ ,\ y^{-1} * x * y^{-1} * x^{-1} *$
$y^{-1} * x * y * x * y^{-1} * x^{-1}\ ,$
$t^{2},\ (t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2}),\ (t,(x^{-1} * y^{2})^{2}),\ (t,(x * y * x)^{2}),$
$((x^{-2})*t^{((x*y*x^{-1})^{2})})^{a},$
$((x^{2})*t^{(x*y*x^{-1}*y^{2}*x^{-2})})^{b},$
$(((x * y)^{*3})*t^{(x*y*x^{-1})})^{c},$
$(((x * y)^{*2})*t^{(y^{-1}*x^{2}*y*x^{2})})^{d},$
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(y^{-1}*x^{2}*y*x^{2})})^{e},$
$((x^{5} * y)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^{f},$
$((x^{3})*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^{g},$
$((x * y * x * y^{-2} * x)*t^{(x*y^{-1}*x^{-1}*y*x^{2}*y*x)})^{h},$
$((y * x * y^{-1} * x^{-2})*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^{i},$

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^{j} >$

Cucumber4										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	6	0	9	0	J_2
0	0	0	0	0	0	8	0	6	10	2^2:(2^5:A_6)
0	0	0	0	0	0	8	6	0	10	2^6:A_6
0	0	0	0	0	0	8	6	3	0	2^{12}:(3:A_6)
2	6	6	0	0	0	0	0	0	0	S_7

Progenitor of Cucumber5

$G<x,y,t>:=Group<x,y,t|\ y^5\ ,\ y^{-2} * x^3 * y^{-2} * x^{-1}\ ,\ (x * y^{-1})^4\ ,\ y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}\ ,$

$t^2,\ (t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2}),\ (t,(x^{-1} * y^2)^2),\ (t,(x * y * x)^2),$

$((x^{-3} * y)*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^{a},$

$((x^{-2})*t^{(x*y*x^2)})^{b},$

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^{c},$

$((x^5 * y)*t^{(x*y*x*y*x^2*y*x^{-1})})^{d},$

$(((x * y)^2)*t^{(x^{-1}*y^2*x^{-2})})^{e},$

$((x * y)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^{f},$

$((x * y * x * y^{-2} * x)*t^{((x*y*x^{-1})^2)})^{g},$

$((x^{-4})*t^{(x*y*x^2)})^{h},$

$((y * x * y^{-1} * x^{-2})*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^{i},$

$((x^6)*t^{(y^{-1}*x^2*y*x^2)})^{j} >$

Cucumber5										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	4	0	S_7
0	0	0	0	0	0	6	3	0	0	2^{12}:(3:A_6)
0	4	0	0	0	0	0	0	0	0	2^6:A_6

Progenitor of Cucumber6

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x^6)*t^{(x*y*x^{-1})})^a$,

$((x^{-2})*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^b$,

$((x^{-3} * y)*t^{(y*x^2*y*x*y^{-1}*x^2)})^c$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^d$,

$((x^5 * y)*t^{(x^{-1}*y^2*x^{-2})})^e$,

$(((x * y)^2)*t^{(x*y*x^2)})^f$,

$((x^{-3} * y)*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^g$,

$((x^3)*t^{(x^{-3}*y*x^{-1}*y)})^h$,

$((x^5 * y)*t^{(x*y*x^{-1}*y^2*x^{-2})})^i$,

$((x * y)*t^{((x*y*x^{-1})^2)})^j$ >

Cucumber6										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	6	S_7
0	0	0	0	0	0	0	6	10	0	2×J_2
0	2	6	0	0	0	0	0	0	0	S_7
4	2	10	0	0	0	0	0	0	0	2^6:A_6
6	2	10	0	0	0	0	0	0	0	3^5:(2:A_6)

Progenitor of Cucumber7

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x^5 * y)*t^{(x*y*x^{-1}*y^2*x^{-2})})^a$,
$((x^{-2})*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^b$,
$((x)*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^c$,
$((x^3)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^d$,
$((y * x * y^{-1} * x^{-2})*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^e$,
$((x^4)*t^{((x*y*x^{-1})^2)})^f$,
$((x^6)*t^{(x*y*x^{-1})})^g$,
$(((x * y)^3)*t^{(x*y*x*y*x^2*y*x^{-1})})^h$,
$((x^{-2})*t^{(x*y*x^{-1})})^i$,
$((x)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^j$ >

Cucumber7										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	7	J_2
0	0	0	0	0	0	4	0	4	8	$2^6{:}A_6$
0	0	0	0	0	0	4	0	6	0	$2^{13}{:}(2{:}A_6)$
0	0	0	0	0	2	0	10	6	0	$3^5{:}(2{:}A_6)$
0	0	0	0	0	3	0	6	0	0	G(2,4)
0	0	0	0	0	3	0	8	0	7	J_2
0	0	0	0	0	3	4	0	6	0	$2^{12}{:}(3{:}A_6)$
0	0	0	0	0	4	6	6	0	10	S_7
6	0	0	0	0	0	0	0	0	0	S_7
0	6	0	4	0	0	0	0	0	0	$2^2{:}(2^5{:}A_6)$
0	6	0	6	0	0	0	0	0	0	$2\times J_2$
10	6	7	6	0	0	0	0	0	0	J_2
0	10	8	8	6	0	0	0	0	0	$2^6{:}A_6$

Progenitor of Cucumber8

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,
$(((x * y)^2)*t^{((x*y*x^{-1})^2)})^a$,
$((x^5)*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^b$,

$((x^2)*t^{(y*x^2*y*x*y^{-1}*x^2)})^c$,
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^d$,
$((x^5 * y)*t^{(y*x^{-2}*y^{-2})})^e$,
$(((x * y)^3)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^f$,
$((x * y * x * y^{-2} * x)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^g$,
$((y * x * y^{-1} * x^{-2})*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^h$,
$((x^6)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^i$,
$((x^{-4})*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^j$ >

Cucumber8										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	4	2	2^6:A_6
0	0	0	0	0	0	0	0	4	3	2^{12}:(3:A_6)
0	0	0	0	0	0	0	4	6	0	S_7
0	0	0	0	0	0	0	6	6	2	3^5:(2:A_6)
0	0	0	0	0	0	0	6	8	2	2^{10}:(2:A_6)
0	0	0	0	0	0	0	6	10	2	5^5:(2:A_6)
6	0	0	0	0	0	0	0	0	0	S_7
0	8	4	0	0	0	0	0	0	0	2^6:A_6

Progenitor of Cucumber9

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,
$((x * y^{-1} * x^{-1} * y^{-1})*t^{((x*y*x^{-1})^2)})^a$,
$(((x * y)^3)*t^{(x*y*x^{-1}*y^2*x^{-2})})^b$,
$((x^6)*t^{(x*y*x^{-1})})^c$,
$((x^5)*t^{(y^{-1}*x^2*y*x^2)})^d$,
$((x^5)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^e$,
$((x * y * x * y^{-2} * x)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^f$,
$((x^4)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^g$,
$(((x * y)^2)*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^h$,
$((x^3)*t^{(x*y*x^2)})^i$,

$((x)*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^j$ >

Cucumber9										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	4	0	$2^2{:}(2^5{:}A_6)$
0	0	0	0	0	0	0	0	6	0	$2^2{:}J_2$
0	0	0	0	0	0	0	0	6	7	J_2
0	0	0	0	0	0	0	6	10	10	S_7
0	0	0	0	0	0	0	10	6	0	$2\times J_2$
6	0	0	0	0	0	0	0	0	0	S_7
10	10	4	0	0	0	0	0	0	0	$2^6{:}A_6$

Progenitor of Cucumber10

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x * y)*t^{(x^{-3}*y*x^{-1}*y)})^a$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(y*x^2*y*x*y^{-1}*x^2)})^b$,

$((y * x * y^{-1} * x^{-2})*t^{(y*x^{-2}*y^{-2})})^c$,

$(((x * y)^2)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^d$,

$((x^6)*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^e$,

$((x^5)*t^{(x^{-1}*y^2*x^{-2})})^f$,

$((x)*t^{(x*y*x*y*x^2*y*x^{-1})})^g$,

$((x^2)*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^h$,

$(((x * y)^3)*t^{(x*y*x^2)})^i$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^j$ >

Cucumber10										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	6	S_7
0	0	0	0	0	0	0	4	10	0	2^6:A_6

Progenitor of Cucumber11

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x^6)*t^{(x*y*x^2)})^a$,

$((x^{-3} * y)*t^{(x*y*x*y*x^2*y*x^{-1})})^b$,

$((x)*t^{(y*x^{-2}*y^{-2})})^c$,

$((x * y * x * y^{-2} * x)*t^{(x^{-3}*y*x^{-1}*y)})^d$,

$((y * x * y^{-1} * x^{-2})*t^{(x*y*x^2)})^e$,

$((y * x * y^{-1} * x^{-2})*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^f$,

$((x^{-2})*t^{(y^{-1}*x^2*y*x^2)})^g$,

$((x)*t^{(x*y*x^{-1})})^h$,

$((x^3)*t^{(x*y*x^{-1}*y^2*x^{-2})})^i$,

$(((x * y)^2)*t^{((x*y*x^{-1})^2)})^j$ >

Cucumber11										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	6	S_7
0	0	0	0	0	0	0	0	4	10	2^6:A_6
0	0	0	0	0	0	2	8	8	0	2^2:(2^5:A_6)
0	0	7	0	0	0	0	0	0	0	J_2
4	10	8	0	0	0	0	0	0	0	2^6:A_6

Progenitor of Cucumber12

$G<x,y,t>:=Group<x,y,t|\ y^5\ ,\ y^{-2} * x^3 * y^{-2} * x^{-1}\ ,\ (x * y^{-1})^4\ ,\ y^{-1} * x * y^{-1} * x^{-1} *$
$y^{-1} * x * y * x * y^{-1} * x^{-1}\ ,$
$t^2,\ (t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2}),\ (t,(x^{-1} * y^2)^2),\ (t,(x * y * x)^2),$
$((x^4)*t^{((x*y*x^{-1})^2)})^a,$
$((x^{-3} * y)*t^{(y^{-1}*x^2*y*x^2)})^b,$
$((x^5 * y)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^c,$
$(((x * y)^3)*t^{(x*y*x^2)})^d,$
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^e,$
$((x^{-3} * y)*t^{(y*x^2*y*x*y^{-1}*x^2)})^f,$
$((x * y * x * y^{-2} * x)*t^{(y*x^{-2}*y^{-2})})^g,$
$(((x * y)^2)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^h,$
$((x^5)*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^i,$
$((y * x * y^{-1} * x^{-2})*t^{(x^{-1}*y^2*x^{-2})})^j >$

Cucumber12										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	4	S_7
0	0	0	0	0	0	0	10	4	0	$2^6{:}A_6$
2	6	0	0	0	0	0	0	0	0	S_7
3	6	0	0	0	0	0	0	0	0	G(2,4)

Progenitor of Cucumber13

$G<x,y,t>:=Group<x,y,t|\ y^5\ ,\ y^{-2} * x^3 * y^{-2} * x^{-1}\ ,\ (x * y^{-1})^4\ ,\ y^{-1} * x * y^{-1} * x^{-1} *$
$y^{-1} * x * y * x * y^{-1} * x^{-1}\ ,$
$t^2,\ (t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2}),\ (t,(x^{-1} * y^2)^2),\ (t,(x * y * x)^2),$
$((x)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^a,$
$((x^6)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})}*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^b,$
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(y^{-1}*x^2*y*x^2)})^c,$
$((x^5 * y)*t^{(x*y*x^{-1})})^d,$
$((x^6)*t^{((x*y*x^{-1})^2)})^e,$
$((x * y)*t^{(x*y*x^2)}*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^f,$
$((x * y * x * y^{-2} * x)*t^{(x*y*x^2)})^g,$

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^h$,
$((x^2)*t^{(x^{-3}*y*x^{-1}*y)})^i$,
$(((x * y)^2)*t^{(y*x^2*y*x*y^{-1}*x^2)})^j$ >

Cucumber13										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	6	S_7
0	0	0	0	0	0	0	0	4	10	$2^6{:}A_6$
0	5	6	0	0	0	0	0	0	0	S_7

Progenitor of Cucumber14
G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,
$((x^6)*t*t^{(x*y*x*y*x^2*y*x^{-1})})^a$,
$((x^4)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^b$,
$((x^{-4})*t^{(x*y*x^{-1}*y^2*x^{-2})})^c$,
$((y * x * y^{-1} * x^{-2})*t^{(y*x^2*y*x*y^{-1}*x^2)}*t^{(x*y*x^2)})^d$,
$((x * y * x * y^{-2} * x)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^e$,
$((x^3)*t^{(y^{-1}*x^2*y*x^2)})^f$,
$(((x * y)^3)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})}*t)^g$,
$((x^{-3} * y)*t^{(x^{-3}*y*x^{-1}*y)})^h$,
$((x^2)*t^{((x*y*x^{-1})^2)})^i$,
$((x^{-2})*t^{(x^{-1}*y^2*x^{-2})})^j$ >

Cucumber14										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	5	0	8	2	$2^4{:}(3^4{:}(2{:}A_6))$
4	0	2	3	0	0	0	0	0	0	$A_6{:}(A_7{:}2)$

Progenitor of Cucumber15
G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x^{-2})*t^{(x*y*x^{-1}*y^{2}*x^{-2})})^{a}$,
$((x)*t^{(x*y^{-1}*x^{-1}*y*x^{2}*y*x)})^{b}$,
$((x * y)*t^{(x*y^{-1}*x^{-1}*y*x^{2}*y*x)})^{c}$,
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x*y*x*y*x^{2}*y*x^{-1})})^{d}$,
$((x^{5} * y)*t^{((x*y*x^{-1})^{2})})^{e}$,
$((x^{3})*t^{(y*x^{-2}*y^{-2})})^{f}$,
$((x * y * x * y^{-2} * x)*t^{(y^{-1}*x^{2}*y*x^{2})})^{g}$,
$((y * x * y^{-1} * x^{-2})*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^{h}$,
$((x^{-4})*t^{(x*y^{-1}*x^{-1}*y*x^{2}*y*x)})^{i}$,
$((x^{4})*t^{(x*y*x^{2})})^{j}$ >

Cucumber15										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	6	0	6	3	2^{12}:(3:A_6)
0	0	0	0	0	0	10	0	0	0	2^2:J_2
0	0	0	0	0	6	10	0	0	3	J_2
0	0	0	0	0	6	10	0	0	6	2×J_2
2	8	0	0	0	0	0	0	0	0	2^2:(2^5:A_6)
0	10	6	0	0	0	0	0	0	0	S_7

Progenitor of Cucumber16

G<x,y,t>:=Group<x,y,t| y^{5} , $y^{-2} * x^{3} * y^{-2} * x^{-1}$, $(x * y^{-1})^{4}$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^{2}, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^{2})^{2})$, $(t,(x * y * x)^{2})$,
$((x^{5} * y)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})}*t^{(y^{-1}*x^{2}*y*x^{2})})^{a}$,
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^{b}$,
$(((x * y)^{2})*t^{(x*y^{-1}*x^{-1}*y*x^{2}*y*x)})^{c}$,
$((x * y)*t^{(x*y^{-1}*x^{-1}*y*x^{2}*y*x)})^{d}$,
$((x^{5})*t^{(x*y*x^{-1})}*t^{(x*y*x^{-1}*y^{2}*x^{-2})})^{e}$,
$((x)*t^{(x*y*x*y*x^{2}*y*x^{-1})})^{f}$,
$((x^{-2})*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^{g}$,
$((x^{2})*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^{h}$,
$((x^{-3} * y)*t^{(x*y^{-1}*x^{-1}*y*x^{2}*y*x)}*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^{i}$,

$((x * y * x * y^{-2} * x)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^j$ >

Cucumber16										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	6	0	8	PSL(3,11)×2
0	0	0	0	0	0	0	6	5	6	2^6:(3^5:(2:3))
0	0	0	0	0	0	2	0	5	0	2^4:(3^4:(2:A_6))
0	0	0	0	0	0	2	0	10	6	2^9:(3^4:(A_6:2))
0	0	0	0	0	0	2	4	10	0	2^6:A_6
0	0	0	0	0	0	2	6	0	6	3^5:(2:A_6)
0	0	0	0	0	0	2	6	10	6	3^4:(2:A_6)
0	0	0	0	0	0	2	8	0	6	2^{10}:(2:A_6)
0	0	0	0	0	0	2	8	10	6	2^9:(2:A_6)

Progenitor of Cucumber17

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^a$,

$((x^5 * y)*t^{(y*x^{-2}*y^{-2})})^b$,

$((x * y)*t^{(x*y*x^{-1})})^c$,

$((x)*t^{(y^{-1}*x^2*y*x^2)})^d$,

$((x^2)*t^{(x*y*x*y*x^2*y*x^{-1})})^e$,

$((x^{-3} * y)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^f$,

$(((x * y)^3)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^g$,

$((x * y * x * y^{-2} * x)*t^{(y*x^{-2}*y^{-2})})^h$,

$((y * x * y^{-1} * x^{-2})*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^i$,

$(t*t^{(x^4*y^2*x^{-1})})^j=(x^2 * y^2 * x^2 * y^{-1} * x^{-1})$>

Cucumber17										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	2	2^2:J_2

Progenitor of Cucumber18

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,
$((y * x * y^{-1} * x^{-2})*t^{(x*y*x*y*x^2*y*x^{-1})})^a$,
$(((x * y)^3)*t^{(x*y*x^{-1})}*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^b$,
$((x^{-2})*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^c$,
$((x)*t^{(x*y*x^{-1}*y^2*x^{-2})})^d$,
$((x * y)*t^{(x*y*x^2)})^e$,
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^f$,
$((x^5 * y)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^g$,
$((x^{-3} * y)*t^{((x*y*x^{-1})^2)*t^{(x*y*x*y*x^2*y*x^{-1})}})^h$,
$((x^4)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^i$,
$(t*t^{(x^4*y^2*x^{-1})})^j$=$(x^2 * y^2 * x^2 * y^{-1} * x^{-1})$>

Cucumber18										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	2	2^2:J_2
0	0	0	0	0	0	0	0	3	2	J_2
0	0	0	0	0	0	0	0	6	2	2×J_2

Progenitor of Cucumber19

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,
$((x^6)*t^{(y*x^2*y*x*y^{-1}*x^2)})^a$,
$(((x * y)^2)*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^b$,
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})}*t^{(x^{-3}*y*x^{-1}*y)})^c$,
$((x^5 * y)*t^{(x*y*x^{-1}*y^2*x^{-2})})^d$,

$((x^5)*t^{(y^{-1}*x^2*y*x^2)}*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^e$,

$((x^{-2})*t^{(y*x^2*y*x*y^{-1}*x^2)})^f$,

$((x^{-3} * y)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^g$,

$((y * x * y^{-1} * x^{-2})*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})}*t^{((x*y*x^{-1})^2)})^h$,

$((x^4)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^i$,

$(t*t^{(x^4*y^2*x^{-1})})^j=(x^2 * y^2 * x^2 * y^{-1} * x^{-1})$>

Cucumber19										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	2	2^2:J_2
0	0	0	0	0	0	0	0	3	2	J_2
0	0	0	0	0	0	8	0	6	2	$2\times J_2$

Progenitor of Cucumber20

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$(((x * y)^3)*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^a$,

$((x^6)*t*t^{(y*x^2*y*x*y^{-1}*x^2)})^b$,

$((x)*t^{(x*y*x^{-1}*y^2*x^{-2})})^c$,

$((x^5 * y)*t^{(x*y*x^{-1}*y^2*x^{-2})})^d$,

$((y * x * y^{-1} * x^{-2})*t^{((x*y*x^{-1})^2)}*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^e$,

$((x^2)*t^{(y*x^{-2}*y^{-2})})^f$,

$((x * y * x * y^{-2} * x)*t^{(x*y*x*y*x^2*y*x^{-1})}*t^{((x*y*x^{-1})^2)})^g$,

$(((x * y)^2)*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^h$,

$((x^{-3} * y)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^i$,

$(((x * y)^3)*t^{(x*y*x^2)}*t^{(x^{-3}*y*x^{-1}*y)})^j$ >

Cucumber20										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	3	10	10	0	2^5:A_6
0	0	0	0	0	4	0	10	0	0	2^6:A_6
0	0	0	0	0	6	7	0	6	0	S_7
6	4	0	0	0	0	0	0	0	0	3^6:S_7
0	0	4	0	0	0	0	0	0	0	2^6:A_6

Progenitor of Cucumber21

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} *$
$y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,
$((x^2)*t^{(y^{-1}*x^2*y*x^2)}*t^{(x*y*x^{-1}*y^2*x^{-2})})^a$,
$((x)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^b$,
$((x^5 * y)*t^{(x*y*x^2)})^c$,
$((x^{-2})*t^{(x^{-1}*y^2*x^{-2})}*t^{(x*y*x*y*x^2*y*x^{-1})})^d$,
$((x^4)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^e$,
$((x^6)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^f$,
$((x)*t^{((x*y*x^{-1})^2)}*t^{(x*y*x^{-1})})^g$,
$((x^5 * y)*t^{(y*x^2*y*x*y^{-1}*x^2)})^h$,
$((x)*t^{(y*x^2*y*x*y^{-1}*x^2)})^i$,
$((x * y * x * y^{-2} * x)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^j$ >

Cucumber21										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	4	10	0	6	3^4:(2:A_6)
0	0	0	0	0	0	8	10	0	6	2^5:(3^4:(2:A_6))
0	0	0	0	0	4	0	10	8	0	2^6:A_6
0	0	0	0	0	8	0	10	0	6	2^{10}:(2:A_6)
2	10	0	4	0	0	0	0	0	0	S_7
2	8	10	4	0	0	0	0	0	0	2^6:A_6
2	8	0	8	0	0	0	0	0	0	2^2:(2^5:A_6)

Progenitor of Cucumber22

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x^5)*t^{(x^{-1}*y^2*x^{-2})}*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^a$,

$((x^{-4})*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^b$,

$((x * y)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})}*t)^c$,

$((x^4)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^d$,

$(((x * y)^2)*t^{(x^{-3}*y*x^{-1}*y)}*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^e$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x*y*x^2)}*t^{(y*x^2*y*x*y^{-1}*x^2)})^f$,

$(((x * y)^3)*t^{(y*x^2*y*x*y^{-1}*x^2)})^g$,

$((x)*t^{(x*y*x*y*x^2*y*x^{-1})})^h$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^i$,

$((x^5 * y)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^j$ >

Cucumber22										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	6	S_7
0	0	0	0	0	0	8	7	10	0	J_2
4	0	0	2	0	0	0	0	0	0	3^4:(2:A_6)

Progenitor of Cucumber23

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} *$

$y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t, y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t, (x^{-1} * y^2)^2)$, $(t, (x * y * x)^2)$,

$((x^5 * y) * t^{(x*y^{-1}*x^{-1}*y*x^{-3})} * t)^a$,

$((x^5 * y) * t^{(x^{-1}*y^2*x^{-2})})^b$,

$((x * y) * t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^c$,

$((x^{-3} * y) * t^{(x*y^{-1}*x^{-1}*y*x^{-3})} * t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^d$,

$(((x * y)^3) * t^{(x*y*x^{-1})})^e$,

$((y * x * y^{-1} * x^{-2}) * t^{(x*y*x^{-1}*y^2*x^{-2})})^f$,

$((x^2) * t^{(x*y*x^2)})^g$,

$((x^6) * t^{(x*y*x^{-1}*y^2*x^{-2})} * t^{(x*y*x*y*x^2*y*x^{-1})})^h$,

$((x^{-4}) * t^{(x*y*x*y*x^2*y*x^{-1})})^i$,

$((y * x * y^{-1} * x^{-2}) * t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})} * t^{(x^{-3}*y*x^{-1}*y)})^j$ >

Cucumber23										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	4	0	3	$3^2{:}(A_6{:}(A_7{:}2))$
0	0	0	0	0	0	0	4	2	3	$A_6{:}(A_7{:}2)$
0	0	0	0	0	0	8	0	2	6	$2^6{:}A_6$
0	6	0	0	0	0	0	0	0	0	S_7

Progenitor of Cucumber24

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} *$

$y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t, y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t, (x^{-1} * y^2)^2)$, $(t, (x * y * x)^2)$,

$((x * y^{-1} * x^{-1} * y^{-1}) * t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^a$,

$(((x * y)^2) * t^{(x*y*x^{-1})})^b$,

$((x^{-2}) * t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^c$,

$((x^3) * t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^d$,

$((x^6) * t^{(x*y*x*y*x^2*y*x^{-1})} * t)^e$,

$((x^{-4}) * t^{(y^{-1}*x^2*y*x^2)})^f$,

$((x * y * x * y^{-2} * x) * t * t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^g$,

$((x^{-3} * y) * t^{(x*y*x^{-1})})^h$,

$((x * y^{-1} * x^{-1} * y^{-1}) * t^{(x*y*x*y*x^2*y*x^{-1})})^i$,

$(t*t^{(x^4*y^2*x^{-1})})^j=(x^2 * y^2 * x^2 * y^{-1} * x^{-1})$>

Cucumber24										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	2	2^2:J_2
0	0	0	0	0	0	0	0	10	2	2×J_2
0	0	0	0	0	3	0	8	0	2	J_2

Progenitor of Cucumber25

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x*y*x^{-1})}*t^{((x*y*x^{-1})^2)})^a$,

$((x^4)*t^{(y^{-1}*x^2*y*x^2)})^b$,

$((x^6)*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^c$,

$((x * y * x * y^{-2} * x)*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^d$,

$((x^3)*t^{(x*y*x^2)})^e$,

$((x^{-3} * y)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^f$,

$((x^{-2})*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^g$,

$((x^5)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})}*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^h$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x*y*x*y*x^2*y*x^{-1})})^i$,

$(t*t^{(x^4*y^2*x^{-1})})^j=(x^2 * y^2 * x^2 * y^{-1} * x^{-1})$>

Cucumber25										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	2	2^2:J_2
0	0	0	0	0	0	0	0	10	2	2×J_2

Progenitor of Cucumber26

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$(((x * y)^2)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^a$,

$((x * y)*t^{(x*y*x^{-1})})^b$,

$((x^3)*t^{(x*y*x*y*x^2*y*x^{-1})})^c$,
$((x * y * x * y^{-2} * x)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^d$,
$((x^6)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^e$,
$((x^3)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^f$,
$((x^2)*t^{(y*x^{-2}*y^{-2})})^g$,
$((x)*t^{(x*y*x^2)})^h$,
$((x^5)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^i$,
$((x * y)*t^{(x^{-3}*y*x^{-1}*y)})^j$ >

Cucumber26										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	7	7	0	J_2
0	0	0	0	0	6	0	0	0	0	2^2:J_2
0	0	0	0	0	8	4	0	8	0	2^6:A_6
6	0	0	0	0	0	0	0	0	0	S_7
0	0	6	0	0	0	0	0	0	0	2^2:J_2
10	0	6	0	0	0	0	0	0	0	$2\times J_2$
0	0	4	6	0	0	0	0	0	0	2^2:(2^5:A_6)
0	10	8	6	0	0	0	0	0	0	2^6:A_6

Progenitor of Cucumber27

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,
$(((x * y)^2)*t^{(x*y*x*y*x^2*y*x^{-1})})^a$,
$((x^5 * y)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^b$,
$((x * y^{-1} * x^{-1} * y^{-1})*t^{((x*y*x^{-1})^2)})^c$,
$((x^2)*t^{(x*y*x^2)}*t^{(x^{-1}*y^2*x^{-2})})^d$,
$((x^{-3} * y)*t^{(x^{-1}*y^2*x^{-2})})^e$,
$(((x * y)^3)*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^f$,
$((y * x * y^{-1} * x^{-2})*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^g$,
$((x^4)*t^{(y*x^2*y*x*y^{-1}*x^2)}*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^h$,
$((x^6)*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^i$,

$((x^5 * y)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^j >$

Cucumber27										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	6	S_7
0	0	0	0	0	0	0	2	0	0	2^6:A_6
0	0	0	0	0	0	6	3	0	10	3^5:(2:A_6)
6	0	0	0	0	0	0	0	0	0	S_7

Progenitor of Cucumber28

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x^5 * y)*t^{(x*y*x^{-1}*y^2*x^{-2})}*t^{(x^{-1}*y^2*x^{-2})})^a$,

$((x^5)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^b$,

$((x^{-3} * y)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^c$,

$((y * x * y^{-1} * x^{-2})*t^{((x*y*x^{-1})^2)})^d$,

$((x * y * x * y^{-2} * x)*t^{(x^{-1}*y^2*x^{-2})})^e$,

$((x^6)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})}*t)^f$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^g$,

$((x^5 * y)*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^h$,

$((x^5)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})}*t^{(x*y*x^2)})^i$,

$((x^2)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^j >$

Cucumber28										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	4	2^6:A_6
0	0	0	0	0	0	0	10	8	6	3^5:(2:A_6)
0	0	0	0	0	0	10	10	8	8	2^{11}:2^5:A_6
0	0	0	0	0	0	10	10	8	10	5^5:(2:A_6)
3	0	0	0	0	0	0	0	0	0	S_7
10	8	0	6	0	0	0	0	0	0	2^6:A_6
0	7	8	8	0	0	0	0	0	0	J_2

Progenitor of Cucumber29

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x^{-3} * y)*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^a$,

$((x * y * x * y^{-2} * x)*t^{(y^{-1}*x^2*y*x^2)})^b$,

$((y * x * y^{-1} * x^{-2})*t^{((x*y*x^{-1})^2)})^c$,

$((x^{-4})*t^{(y*x^{-2}*y^{-2})})^d$,

$((x^4)*t^{(x*y*x^{-1}*y^2*x^{-2})})^e$,

$((x^6)*t^{(x^{-3}*y*x^{-1}*y)})^f$,

$((x^3)*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^g$,

$((x^{-3} * y)*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^h$,

$((x)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^i$,

$((x^5)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^j$ >

Cucumber29										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	7	0	J_2
0	0	0	0	0	8	4	0	0	0	2^2:(2^5:A_6)
0	6	6	0	3	0	0	0	0	0	2^{12}:(3:A_6)
6	0	10	0	3	0	0	0	0	0	G(2,4)

Progenitor of Cucumber30

$G<x,y,t>:=Group<x,y,t|\ y^5\ ,\ y^{-2} * x^3 * y^{-2} * x^{-1}\ ,\ (x * y^{-1})^4\ ,\ y^{-1} * x * y^{-1} * x^{-1} *$
$y^{-1} * x * y * x * y^{-1} * x^{-1}\ ,$

$t^2,\ (t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2}),\ (t,(x^{-1} * y^2)^2),\ (t,(x * y * x)^2),$

$((x^5)*t^{(x*y*x^{-1})})^a,$

$(((x * y)^2)*t^{(x^{-1}*y^2*x^{-2})})^b,$

$((x^5 * y)*t^{((x*y*x^{-1})^2)})^c,$

$((x^{-2})*t^{(x*y*x^2)})^d,$

$((y * x * y^{-1} * x^{-2})*t^{(y^{-1}*x^2*y*x^2)})^e,$

$((x^{-4})*t^{(x^{-3}*y*x^{-1}*y)})^f,$

$((x^4)*t^{(x*y*x^{-1})})^g,$

$((x^6)*t^{(x*y*x^2)})^h,$

$((y * x * y^{-1} * x^{-2})*t^{(x^{-3}*y*x^{-1}*y)})^i,$

$((x^5)*t^{(y*x^{-2}*y^{-2})})^j >$

Cucumber30										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	7	J_2
0	0	0	0	0	0	3	4	6	0	2^{12}:(3:A_6)
0	0	0	0	0	2	4	0	0	8	2^2:(2^5:A_6)
0	0	0	0	0	2	6	4	6	8	2^6:A_6
0	0	0	0	0	3	0	4	6	0	2^{12}:(3:A_6)
7	0	0	0	0	0	0	0	0	0	J_2
10	6	0	0	0	0	0	0	0	0	S_7

Progenitor of Cucumber31

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t, y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x * y)*t^{(y*x^2*y*x*y^{-1}*x^2)}*t^{(x*y*x^{-1})})^a$,

$((x^5 * y)*t^{(y^{-1}*x^2*y*x^2)})^b$,

$((x^5)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^c$,

$((x^{-3} * y)*t^{(x*y*x^{-1}*y^2*x^{-2})}*t^{(x*y*x^{-1})})^d$,

$((x * y * x * y^{-2} * x)*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^e$,

$((y * x * y^{-1} * x^{-2})*t^{(y*x^{-2}*y^{-2})})^f$,

$((x^{-4})*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)}*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^g$,

$((x^4)*t^{(y*x^{-2}*y^{-2})}*t^{(x^{-1}*y^2*x^{-2})})^h$,

$((x^6)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^i$,

$((x)*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^j$ >

Cucumber31										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	4	4	0	2^6:A_6
0	0	0	0	0	0	3	6	4	0	2^{13}:(2:A_6)
3	0	0	0	0	0	0	0	0	0	S_7
0	10	0	3	0	0	0	0	0	0	2×J_2
0	10	7	3	0	0	0	0	0	0	J_2

Progenitor of Cucumber32

$G<x,y,t>:=Group<x,y,t|\ y^5\ ,\ y^{-2} * x^3 * y^{-2} * x^{-1}\ ,\ (x * y^{-1})^4\ ,\ y^{-1} * x * y^{-1} * x^{-1} *$
$y^{-1} * x * y * x * y^{-1} * x^{-1}\ ,$
$t^2,\ (t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2}),\ (t,(x^{-1} * y^2)^2),\ (t,(x * y * x)^2),$
$((x^5 * y)*t^{((x*y*x^{-1})^2)})^a,$
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x*y*x^{-1}*y^2*x^{-2})})^b,$
$(((x * y)^2)*t^{(x*y*x^{-1})})^c,$
$((x * y)*t^{(y^{-1}*x^2*y*x^2)})^d,$
$((x^5)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^e,$
$((x)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^f,$
$((x^{-2})*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^g,$
$((x^2)*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^h,$
$((x^{-3} * y)*t^{(x*y*x^2)})^i,$
$(((x * y)^3)*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^j >$

Cucumber32										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	4	0	0	$2^6{:}A_6$
6	0	0	0	0	0	0	0	0	0	S_7

Progenitor of Cucumber33

$G<x,y,t>:=Group<x,y,t|\ y^5\ ,\ y^{-2} * x^3 * y^{-2} * x^{-1}\ ,\ (x * y^{-1})^4\ ,\ y^{-1} * x * y^{-1} * x^{-1} *$
$y^{-1} * x * y * x * y^{-1} * x^{-1}\ ,$
$t^2,\ (t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2}),\ (t,(x^{-1} * y^2)^2),\ (t,(x * y * x)^2),$
$((x^3)*t^{(x^{-3}*y*x^{-1}*y)})^a,$
$((x * y * x * y^{-2} * x)*t^{(y*x^2*y*x*y^{-1}*x^2)})^b,$
$((y * x * y^{-1} * x^{-2})*t^{(y*x^{-2}*y^{-2})})^c,$
$((x^{-4})*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^d,$
$((x^4)*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^e,$
$((x^6)*t^{(x^{-1}*y^2*x^{-2})})^f,$
$((x^5 * y)*t^{(x*y*x*y*x^2*y*x^{-1})})^g,$
$((x * y^{-1} * x^{-1} * y^{-1})*t^{((x*y*x^{-1})^2)}*t^{(x*y*x^{-1}*y^2*x^{-2})})^h,$
$(((x * y)^2)*t^{(x*y*x^{-1})})^i,$

$((x * y)*t^{(y^{-1}*x^2*y*x^2)}*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^j$ >

Cucumber33										
a	b	c	d	e	f	g	h	i	j	Index
4	0	0	0	0	0	0	0	0	0	$2^2{:}(2^5{:}A_6)$
6	0	0	0	0	0	0	0	0	0	$2^2{:}J_2$
0	6	3	0	0	0	0	0	0	0	$2^{12}{:}(3{:}A_6)$
4	6	3	0	0	0	0	0	0	0	$2^6{:}A_6$
6	10	3	0	0	0	0	0	0	0	J_2
6	10	6	6	0	0	0	0	0	0	$2\times J_2$

Progenitor of Cucumber34

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,
$((x^{-4})*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^a$,
$((y * x * y^{-1} * x^{-2})*t^{(x*y^{-1}*x^{-1}*y*x^{-3})}*t^{(y*x^2*y*x*y^{-1}*x^2)})^b$,
$(((x * y)^3)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^c$,
$((x^2)*t^{(x^{-1}*y^2*x^{-2})}*t^{(y*x^{-2}*y^{-2})})^d$,
$((x)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})}*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^e$,
$((x^{-3} * y)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^f$,
$((x^3)*t^{(x*y*x^{-1})})^g$,
$((x * y * x * y^{-2} * x)*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^h$,
$((y * x * y^{-1} * x^{-2})*t^{(x*y*x*y*x^2*y*x^{-1})})^i$,
$((x^{-4})*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^j$ >

Cucumber34										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	6	3	0	2^{12}:(3:A_6)
0	0	0	0	0	0	4	0	6	0	2^2:(2^5:A_6)
0	0	0	0	0	0	4	0	6	0	2^6:A_6
0	0	0	0	0	0	6	0	0	0	2^2:J_2
0	0	0	0	0	0	6	0	0	0	J_2
0	0	0	0	0	0	6	0	0	0	2×J_2
2	6	6	0	0	0	0	0	0	0	S_7
3	6	6	0	0	0	0	0	0	0	G(2,4)
2	3	10	0	0	0	0	0	0	0	3^5:(2:A_6)
0	3	0	4	0	0	0	0	0	0	A_6:(A_7:2)

Progenitor of Cucumber35

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,
$((x^4)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})}*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^a$,
$((x^6)*t^{(x*y*x^{-1}*y^2*x^{-2})})^b$,
$((x^5 * y)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})}*t^{(x*y*x^{-1})})^c$,
$((x^3)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^d$,
$((x * y * x * y^{-2} * x)*t^{(x*y*x*y*x^2*y*x^{-1})})^e$,
$((x^4)*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^f$,
$((x^6)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)}*t^{(y*x^{-2}*y^{-2})})^g$,
$((x * y * x * y^{-2} * x)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^h$,
$(((x * y)^3)*t^{(x^{-1}*y^2*x^{-2})})^i$,
$((x^2)*t^{(x*y*x^{-1})})^j$ >

Cucumber35										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	4	2^6:A_6
0	0	0	0	0	0	0	0	6	6	S_7
0	0	0	0	0	0	0	6	10	8	2^{10}:(2:A_6)
0	0	0	0	0	0	0	6	10	10	5^5:(2:A_6)
0	0	0	0	0	0	0	8	0	6	PSL(3,11)×2
0	0	0	0	0	2	0	0	10	6	3^5:(2:A_6)
2	0	0	0	0	0	0	0	0	0	2^6:A_6
0	0	0	4	0	0	0	0	0	0	2^2:(2^5:A_6)
0	4	0	6	0	0	0	0	0	0	2^2:J_2

Progenitor of Cucumber36

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t, y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})}*t^{(x^{-1}*y^2*x^{-2})})^a$,

$((x^5)*t^{((x*y*x^{-1})^2)})^b$,

$((x^5 * y)*t^{(y*x^{-2}*y^{-2})}*t^{(y*x^2*y*x*y^{-1}*x^2)})^c$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^d$,

$((x^5)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^e$,

$(((x * y)^3)*t^{(x*y*x^{-1})})^f$,

$((y * x * y^{-1} * x^{-2})*t^{(y*x^2*y*x*y^{-1}*x^2)}*t^{(y*x^{-2}*y^{-2})})^g$,

$((x^{-4})*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^h$,

$((x^4)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)}*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^i$,

$((x^6)*t^{(x*y*x^2)})^j$ >

Cucumber36										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	2	0	$2^6{:}A_6$
0	0	0	0	0	0	0	0	3	4	$2^{13}{:}(2{:}A_6)$
0	0	0	0	0	0	0	3	0	4	$2^{12}{:}(3{:}A_6)$
0	4	0	0	0	0	0	0	0	0	$2^6{:}A_6$

Progenitor of Cucumber37

$G<x,y,t>:=Group<x,y,t|\ y^5\ ,\ y^{-2} * x^3 * y^{-2} * x^{-1}\ ,\ (x * y^{-1})^4\ ,\ y^{-1} * x * y^{-1} * x^{-1} *$
$y^{-1} * x * y * x * y^{-1} * x^{-1}\ ,$
$t^2,\ (t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2}),\ (t,(x^{-1} * y^2)^2),\ (t,(x * y * x)^2),$
$((x^6)*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^a,$
$((x^4)*t)^b,$
$((y * x * y^{-1} * x^{-2})*t^{(x*y*x^{-1}*y^2*x^{-2})})^c,$
$(((x * y)^3)*t^{(x*y*x^{-1})})^d,$
$((x^2)*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^e,$
$((x)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^f,$
$((x^5)*t^{(y^{-1}*x^2*y*x^2)})^g,$
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x^{-3}*y*x^{-1}*y)})^h,$
$((x^5 * y)*t^{(x*y*x*y*x^2*y*x^{-1})})^i,$
$((x * y * x * y^{-2} * x)*t^{(x*y*x^{-1})})^j >$

Cucumber37										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	4	10	10	0	2^6:A_6
0	0	0	0	0	0	8	0	10	0	2^{10}:(2:A_6)
0	0	0	0	0	7	6	0	0	0	J_2
0	10	4	0	0	0	0	0	0	0	S_7
4	2	6	0	0	0	0	0	0	0	2^6:A_6
6	2	6	0	0	0	0	0	0	0	3^5:(2:A_6)
8	2	6	0	0	0	0	0	0	0	2^{10}:(2:A_6)
10	2	6	0	0	0	0	0	0	0	5^5:(2:A_6)
4	3	6	0	0	0	0	0	0	0	2^{12}:(3:A_6)
0	3	0	6	0	0	0	0	0	0	G(2,4)

Progenitor of Cucumber38

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,

t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((y * x * y^{-1} * x^{-2})*t^{(x*y*x^2)})^a$,

$((x^{-4})*t^{(x*y*x^2)})^b$,

$((x^4)*t^{(y*x^{-2}*y^{-2})}*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^c$,

$((x^6)*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^d$,

$((x^5 * y)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^e$,

$((x * y^{-1} * x^{-1} * y^{-1})*t^{(y^{-1}*x^2*y*x^2)})^f$,

$(((x * y)^2)*t^{(y*x^2*y*x*y^{-1}*x^2)})^g$,

$((x)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^h$,

$((x^{-2})*t^{(x*y^{-1}*x^{-2}*y*x^{-2})})^i$,

$((x^2)*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)}*t)^j$ >

Cucumber38										
a	b	c	d	e	f	g	h	i	j	Index
0	8	2	0	0	0	0	0	0	0	2^6:A_6
0	0	3	4	0	0	0	0	0	0	2^{13}:(2:A_6)
0	9	3	4	0	0	0	0	0	0	2^{12}:(3:A_6)

Progenitor of Cucumber39

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} *$
$y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,
$(((x * y)^3)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^a$,
$((x^3)*t^{((x*y*x^{-1})^2)})^b$,
$((x^{-4})*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})})^c$,
$((x^4)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^d$,
$((x^6)*t^{(x^{-1}*y^2*x^{-2})})^e$,
$((x^5 * y)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^f$,
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^g$,
$(((x * y)^2)*t^{(y*x^{-2}*y^{-2})}*t^{(x*y*x*y*x^2*y*x^{-1})})^h$,
$((x)*t^{(y*x^2*y*x*y^{-1}*x^2)})^i$,
$((x^{-2})*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^j$ >

Cucumber39										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	4	2^6:A_6
0	0	0	0	0	0	6	5	10	0	S_7
0	4	2	0	0	0	0	0	0	0	2^6:A_6
0	4	3	0	0	0	0	0	0	0	2^{12}:(3:A_6)
6	0	0	3	0	0	0	0	0	0	G(2,4)

Progenitor of Cucumber40

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} *$
$y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,

$((x^{-3} * y)*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^a$,
$((x^3)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^b$,
$((y * x * y^{-1} * x^{-2})*t^{((x*y*x^{-1})^2)})^c$,
$((x^4)*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)}*t^{(x*y^{-1}*x^{-1}*y*x^{-3})})^d$,
$((x^6)*t^{(y*x^{-2}*y^{-2})})^e$,
$((x^5 * y)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^f$,
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x*y*x^2)})^g$,
$(((x * y)^2)*t^{(x^{-1}*y^2*x^{-2})})^h$,
$((x * y)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^i$,
$((x)*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)}*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^j$ >

Cucumber40										
a	b	c	d	e	f	g	h	i	j	Index
0	4	0	0	0	0	0	0	0	0	2^2:(2^5:A_6)
0	6	0	0	0	0	0	0	0	0	2^2:J_2
10	0	0	2	0	0	0	0	0	0	2^6:A_6

Progenitor of Cucumber41

G<x,y,t>:=Group<x,y,t| y^5 , $y^{-2} * x^3 * y^{-2} * x^{-1}$, $(x * y^{-1})^4$, $y^{-1} * x * y^{-1} * x^{-1} * y^{-1} * x * y * x * y^{-1} * x^{-1}$,
t^2, $(t,y^{-1} * x^{-1} * y * x * y^{-1} * x^{-2})$, $(t,(x^{-1} * y^2)^2)$, $(t,(x * y * x)^2)$,
$((x^2)*t^{(x*y*x^{-1}*y^2*x^{-2})})^a$,
$(((x * y)^3)*t^{(y^{-1}*x^2*y*x^2)})^b$,
$((x^3)*t^{(x^{-1}*y*x^{-1}*y^{-1}*x^2*y^{-1})}*t^{(y*x^{-2}*y^{-2})})^c$,
$((x * y * x * y^{-2} * x)*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x^{-1})})^d$,
$((y * x * y^{-1} * x^{-2})*t^{(x*y*x^{-1}*y^{-1}*x*y^{-2}*x)})^e$,
$((x^4)*t^{(x^{-1}*y^2*x^{-2})}*t^{(y*x*y*x^{-2}*y*x^{-1}*y^{-1})})^f$,
$((x^6)*t^{(y*x^{-2}*y^{-2})})^g$,
$((x^5 * y)*t^{(x*y^{-1}*x^{-2}*y*x^{-2})}*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^h$,
$((x * y^{-1} * x^{-1} * y^{-1})*t^{(x*y^{-1}*x^{-1}*y*x^2*y*x)})^i$,
$((x * y)*t^{((x*y*x^{-1})^2)})^j$ >

Cucumber41										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	6	S_7
0	0	0	0	0	0	4	5	0	0	2^6:A_6
0	0	0	0	0	3	6	5	0	0	3^5:(2:A_6)
2	6	0	0	0	0	0	0	0	0	S_7

10.8 2^{*9}:(3^2:(2^2))

We have the following information

S:=Sym(9)

x ∼ (1, 6, 7)(2, 3, 4, 9, 8, 5)

y ∼ (1, 4)(2, 7)(3, 9)(6, 8)

#N = 36

Progenitor of Zesty

G<x,y,t>:=Group<x,y,t|y^2 , x^6 , (x * y * x)^2 , x^{-1} * y * x^{-1} * y * x^{-1} * y * x * y * x * y * x^{-1} * y ,

t^2, (t,x^3), (t,(x * y * x^{-1} * y * x * y)),

((y * x^{-1} * y * x)*$t^{(x*y*x^{-1}*y*x)}$)^a,

((x * y)*$t^{(x*y*x^{-1}*y*x)}$)^b,

(((x * y)^2)*$t^{(x*y*x)}$)^c,

((x * y * x)*$t^{(y*x*y)}$)^d,

(((x * y)^3)*$t^{(y*x^{-1}*y*x)}$)^e,

((x^3)*$t^{(y*x^{-1}*y)}$)^f,

((x^2)*$t^{(x*y*x*y*x^{-1}*y)}$)^g,

((x)*$t^{(x*y*x^{-1}*y*x)}$)^h,

((x * y)*$t^{((x*y)^3)}$)^i,

((y * x^{-1} * y * x)*$t^{(x*y*x*y*x^{-1}*y)}$)^j >

Zesty										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	4	5	$2\times S_6$
0	0	0	0	0	0	0	0	4	6	$2^4{:}(2^5{:}(3^2{:}2^2))$
0	0	0	0	0	0	0	0	5	5	$3^4{:}(2{:}(A_5{:}A_5))$
0	0	0	0	0	0	0	5	5	4	$2\times A_5$
0	0	0	0	0	0	2	9	7	0	PSL(2,8)
0	0	0	0	0	0	2	9	9	0	PSL(2,19)
0	0	0	0	0	0	2	9	10	0	(3:2):PSL(2,19)
0	0	0	0	0	0	2	10	0	8	$(2{:}3){:}(A_6{:}2)$
0	0	0	0	0	0	3	0	0	5	$2^5{:}S_6$
0	0	0	0	0	0	3	0	8	7	$PSL(3,4){:}2^2$
0	0	0	0	0	0	3	0	9	7	$2\times A_9$
0	0	0	0	0	0	3	4	0	0	$(3{:}2){:}S_6$
0	0	0	0	0	0	3	6	0	6	$2^6{:}(2^4{:}(2^2{:}3^2))$
0	0	0	0	0	0	3	6	8	9	$2^6{:}(3^3{:}2^3)$

Zesty										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	4	0	4	0	2^3:(3:2):(A_6:2)
0	0	0	0	0	0	4	0	4	10	(2:3):(2:S_6)
0	0	0	0	0	0	4	0	5	6	2^6:S_6
0	0	0	0	0	0	4	0	6	4	2^7:(3^2:2^2)
0	0	0	0	0	0	4	8	6	6	2^9:(2^2:3):(2^2:3)
0	0	0	0	0	0	5	0	4	0	2:(PSL(2,81):2)
3	0	0	0	0	0	0	0	0	0	2^4:(3^2:2^2)

Zesty										
a	b	c	d	e	f	g	h	i	j	G
5	4	0	0	0	0	0	0	0	0	2×S_6
6	4	0	0	0	0	0	0	0	0	2^4:(2^5:(3^2:2^2))
5	5	0	0	0	0	0	0	0	0	3^4:(2:(A_5:A_5))
10	5	2	0	0	0	0	0	0	0	2×A_5
8	6	2	0	0	0	0	0	0	0	2^4:(2:(3:2))
10	6	2	0	0	0	0	0	0	0	5^2:(2:(3:2))
8	7	2	0	0	0	0	0	0	0	PSL(2,7)×2
9	7	2	0	0	0	0	0	0	0	PSL(2,8)
8	8	2	0	0	0	0	0	0	0	2:(PSL(2,7):2)
10	8	2	0	0	0	0	0	0	0	(2:3):(A_6:2)
9	9	2	0	0	0	0	0	0	0	PSL(2,19)
10	9	2	0	0	0	0	0	0	0	(3:2):PSL(2,19)
5	0	3	0	0	0	0	0	0	0	2^5:S_6
6	6	3	0	0	0	0	0	0	0	2^6:(2^4:(2^2:3^2))
7	6	3	0	0	0	0	0	0	0	PSL(3,4):2^2
5	5	6	0	0	0	0	0	0	0	3^4:(2:A_5)
5	5	10	0	0	0	0	0	0	0	2:(A_5:A_5)
4	6	4	6	0	0	0	0	0	0	2^7:(3^2:2^2)
8	6	4	6	0	0	0	0	0	0	2^9:(2^2:3):(2^2:3)

Progenitor of Zesty2

G<x,y,t>:=Group<x,y,t|y^2 , x^6 , $(x * y * x)^2$, $x^{-1} * y * x^{-1} * y * x^{-1} * y * x * y * x *$
$y * x^{-1} * y$,
t^2, (t,x^3), $(t,(x * y * x^{-1} * y * x * y))$,
$((y * x^{-1} * y * x)*t^{(x*y*x^{-1}*y*x)})^a$,
$((x^2)*t^{(y*x*y*x^{-1})})^b$,
$((x)*t^{(y*x*y)})^c$,
$((x * y)*t^{(x*y*x)})^d$,
$(((x * y)^2)*t^{(x*y*x*y*x^{-1}*y)})^e$,
$((x^2)*t^{((x*y)^3)})^f$,
$((x * y * x)*t^{(y*x^{-1}*y)})^g$,
$(((x * y)^3)*t^{(y*x^{-1}*y*x)})^h$,
$((x^3)*t^{(y*x*y)})^i$,
$((x * y * x)*t^{(y*x*y*x^{-1})})^j$ >

Zesty2										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	5	6	5	$2{\times}A_5$
0	0	0	0	0	0	0	6	4	3	2^2:(3:2^5):(3^2:5)
0	0	0	0	0	0	4	0	3	0	(3:2):S_6
0	0	0	0	0	0	4	0	4	4	2^4:(2^5:(3^2:2^2))
0	0	0	0	0	0	4	0	5	6	3:(2:(S(4,5):2))
0	0	0	0	0	0	5	6	0	3	2^5:S_6
0	0	0	0	0	0	6	0	6	3	2^6:(2^4:(2^2:3^2))
5	2	0	0	0	0	0	0	0	0	$2{\times}A_5$
5	3	0	0	0	0	0	0	0	0	2^5:S_6
4	0	5	0	0	0	0	0	0	0	$2{\times}S_6$
5	0	5	0	0	0	0	0	0	0	3^4:(2:(A_5:A_5))
6	4	5	0	0	0	0	0	0	0	2^6:S_6
5	6	5	0	0	0	0	0	0	0	3^4:(2:A_5)
10	2	6	0	0	0	0	0	0	0	5^2:(2:(3:2))
6	3	6	0	0	0	0	0	0	0	2^6:(2^4:(2^2:3^2))
7	3	6	0	0	0	0	0	0	0	PSL(3,4):2^2

Zesty2										
8	2	10	0	0	0	0	0	0	0	(2:3):(A_6:2)
9	2	10	0	0	0	0	0	0	0	(3:2):PSL(2,19)
0	4	0	4	0	0	0	0	0	0	2^3:(3:2):(A_6:2)
10	4	0	4	0	0	0	0	0	0	(2:3):(2:S_6)
0	5	0	4	0	0	0	0	0	0	2:(PSL(2,81):2)
6	6	0	4	0	0	0	0	0	0	2^4:(2^5:(3^2:2^2))
5	5	8	8	0	0	0	0	0	0	2^3:PSL(3,4):2
7	3	0	9	0	0	0	0	0	0	$2{\times}A_9$
4	4	0	9	0	0	0	0	0	0	2^6:(3^3:2^3)
10	2	9	10	0	0	0	0	0	0	(3:2):PSL(2,19)

Progenitor of Zesty3

G<x,y,t>:=Group<x,y,t|y^2 , x^6 , $(x * y * x)^2$, $x^{-1} * y * x^{-1} * y * x^{-1} * y * x * y * x * y * x^{-1} * y$,

t^2, (t,x^3), $(t,(x * y * x^{-1} * y * x * y))$,

$((x^2)^{*}t^{(y*x^{-1}*y)})^a$,

$((x * y * x)^{*}t^{(y*x^{-1}*y*x)})^b$,

$((x^3)^{*}t^{((x*y)^3)})^c$,

$((x * y)^{*}t^{(x*y*x*y*x^{-1}*y)})^d$,

$((y * x^{-1} * y * x)^{*}t^{(x*y*x)})^e$,

$((x)^{*}t^{(x*y*x^{-1}*y*x)})^f$,

$(((x * y)^2)^{*}t^{(y*x*y)})^g$,

$((x^2)^{*}t^{(y*x*y*x^{-1})})^h$,

$((x * y * x)^{*}t^{(x*y*x*y*x^{-1}*y)})^i$,

$(((x * y)^3)^{*}t^{(y*x^{-1}*y)})^j$ >

Zesty3										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	2	5	0	$2{\times}A_5$
0	0	0	0	0	0	0	3	4	0	$2^6{:}3^2{:}2^2$
0	0	0	0	0	0	0	4	4	6	$2^3{:}(3{:}2){:}(A_6{:}2)$
0	0	0	0	0	0	2	7	8	0	PSL(2,7)×2
0	0	0	0	0	0	2	7	9	0	PSL(2,8)
0	0	0	0	0	0	2	8	6	0	$2^4{:}(2{:}(3{:}2))$
0	0	0	0	0	0	2	8	8	0	2:(PSL(2,7):2)
0	0	0	0	0	0	2	8	10	0	$(2{:}3){:}(A_6{:}2)$
0	0	0	0	0	0	2	9	9	0	PSL(2,19)
0	0	0	0	0	0	2	9	10	0	(3:2):PSL(2,19)
0	0	0	0	0	0	2	10	6	0	$5^2{:}(2{:}(3{:}2))$
0	0	0	0	0	0	3	3	4	0	$2^4{:}(3^2{:}2^2)$
0	0	0	0	0	0	4	4	4	0	$2^2{:}S_6$
6	8	0	2	0	0	0	0	0	0	$2^4{:}(2{:}(3{:}2))$
7	8	0	2	0	0	0	0	0	0	PSL(2,7)×2
8	8	0	2	0	0	0	0	0	0	2:(PSL(2,7):2)
10	8	0	2	0	0	0	0	0	0	$(2{:}3){:}(A_6{:}2)$
7	9	0	2	0	0	0	0	0	0	PSL(2,8)
9	9	0	2	0	0	0	0	0	0	PSL(2,19)
10	9	0	2	0	0	0	0	0	0	(3:2):PSL(2,19)
5	10	0	2	0	0	0	0	0	0	$2{\times}A_5$
6	10	0	2	0	0	0	0	0	0	$5^2{:}(2{:}(3{:}2))$
0	4	8	3	0	0	0	0	0	0	$2^2{:}(3{:}2^5){:}(3^2{:}5)$

Progenitor of Zesty4

G<x,y,t>:=Group<x,y,t|y^2 , x^6 , $(x * y * x)^2$, $x^{-1} * y * x^{-1} * y * x^{-1} * y * x * y * x * y * x^{-1} * y$,

t^2, (t,x^3), $(t,(x * y * x^{-1} * y * x * y))$,

$(((x * y)^3)*t^{(y*x*y*x^{-1})})^a$,

$((y * x^{-1} * y * x)*t^{(y*x*y)})^b$,

$(((x * y)^3)*t^{(x*y*x^{-1}*y*x)})^c$,
$((x^3)*t^{(x*y*x)})^d$,
$((y * x^{-1} * y * x)*t^{(x*y*x*y*x^{-1}*y)})^e$,
$(((x * y)^2)*t^{((x*y)^3)})^f$,
$((x * y)*t^{(y*x^{-1}*y*x)})^g$,
$((x)*t^{(y*x^{-1}*y)})^h$,
$((x * y * x)*t^{(y*x*y*x^{-1})})^i$,
$(((x * y)^3)*t^{(y*x*y)})^j$ >

Zesty4										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	4	3	0	3^3:2^3:
0	0	0	0	0	0	0	4	4	0	2^3:(3:2):(A_6:2)
0	0	0	0	0	0	0	4	5	0	2:(PSL(2,81):2)
0	0	0	0	0	0	0	5	2	0	2×A_5
0	0	0	0	0	0	6	8	6	2	2^4:(2:(3:2))
0	0	0	0	0	0	7	8	4	2	PSL(2,7)×2
0	0	0	0	0	0	8	10	8	2	(2:3):(A_6:2)
0	0	0	0	0	0	9	10	2	0	(3:2):PSL(2,19)
6	10	9	2	0	0	0	0	0	0	(3:2):PSL(2,19)
0	8	10	2	0	0	0	0	0	0	(2:3):(A_6:2)
0	6	4	3	0	0	0	0	0	0	2^4:(2:(3:2))
0	8	5	3	0	0	0	0	0	0	2:(3:A_6):(A_6:2)
0	4	0	5	0	0	0	0	0	0	2:(PSL(2,81):2)

10.9 2^{*12}:(2^3:2^2)

We have the following information

N:=TransitiveGroup(12,37)

S:=Sym(12)

x $\sim$ (1, 8)(2, 7)(3, 6)(4, 5)(9, 12)(10, 11)

y $\sim$ (1, 6, 7, 12)(2, 11)(3, 10, 9, 4)(5, 8)

#N = 48

Progenitor of juniper

G<x,y,t>:=Group<x,y,t| x^2 , y^4 , $y^{-1} * x * y^{-2} * x * y^{-1} * x * y^2 * x$, $y^{-1} * x * y^{-1} * x * y^{-1} * x * y * x * y * x * y * x$,

t^2, (t,(x * y^2 * x)), (t,(x * y * x * y^{-1} * x * y)),

$((x * y * x * y^{-1} * x * y)*t^{(y*x*y*x*y^{-1})})^a$,

$(x*t^{(y*x*y)})^b$,

$(((x * y)^2)*t^{(y*x)})^c$,

$(((x * y)^3)*t^{(y^2*x*y)})^d$,

$(y^2*t^{(y*x)})^e$,

$((y * x * y)*t^{((x*y)^3)})^f$,

$((x * y)*t^{(y^2*x*y)})^g$,

$(y*t^{(y*x*y^2)})^h$,

$(x*t^{(x*y^2*x*y)})^i$,

$((x * y * x * y^{-1} * x * y)*t^{(y*x)})^j$,

$(t*t^{(y^2*x)})^k = (x * y * x * y^{-1} * x * y)$>;

Juniper											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	5	6	1	$2\times A_5$
0	0	2	0	0	0	0	0	0	0	5	2:(5:2)
0	0	2	0	0	0	0	0	0	0	6	2^2:(3:2)
0	0	2	0	0	0	0	0	0	0	7	2:7:2
0	0	2	0	0	0	0	0	0	0	9	2:(3^2:2)
0	0	2	0	0	0	0	0	0	0	10	$2\times A_5$
6	6	3	0	0	0	0	0	0	0	3	3:(PSL(2,7):2)
6	6	3	0	0	0	0	0	0	0	4	PSL(2,7):(2^2:(3:2))
2	7	3	0	0	0	0	0	0	0	3	PSL(2,13)
2	7	3	0	0	0	0	0	0	0	4	PSL(2,127)

Juniper											
a	b	c	d	e	f	g	h	i	j	k	Index
0	9	3	0	0	0	0	0	0	0	3	PSL(2,37)
0	4	4	0	0	0	0	0	0	0	2	2^2:(A_5:2)
2	5	4	0	0	0	0	0	0	0	2	$2\times A_6$
6	7	4	0	0	0	0	0	0	0	2	2×PSL(2,41)
6	3	5	0	0	0	0	0	0	0	2	A_6
6	3	5	0	0	0	0	0	0	0	3	PSL(2,11)
4	4	5	0	0	0	0	0	0	0	2	PSL(2,19)×2
10	2	8	0	0	0	0	0	0	0	4	2:(PSL(2,7):2)
10	2	8	0	0	0	0	0	0	0	5	3:(2:(A_6:2))
4	2	9	0	0	0	0	0	0	0	5	3:(2:PSL(2,19))
0	3	9	0	0	0	0	0	0	0	2	PSL(2,73)
8	7	9	0	0	0	0	0	0	0	1	PSL(2,8)
2	9	9	0	0	0	0	0	0	0	1	PSL(2,19)
8	2	7	2	0	0	0	0	0	0	6	PSL(2,13):2

Progenitor of juniper2

G<x,y,t>:=Group<x,y,t| x^2 , y^4 , y^{-1} * x * y^{-2} * x * y^{-1} * x * y^2 * x , y^{-1} * x * y^{-1} *

x * y^{-1} * x * y * x * y * x * y * x ,

t^2, (t,(x * y^2 * x)), (t,(x * y * x * y^{-1} * x * y)),

$(y*t^{(y*x*y*x*y^{-1})})^a$,

$(((x*y)^3)*t^{(x^y)})^b$,

$(x*t^{(y^{-1}*x*y*x)})^c$,

$((x*y)*t^{((x*y)^3)})^d$,

$((y*x*y)*t^{(x*y^2*x*y)})^e$,

$(((x*y)^2)*t^{(y^2*x*y)})^f$,

$((x*y^2*x*y)*t^{(x*y*x*y^{-1}*x)})^g$,

$((x*y*x*y^{-1}*x*y)*t^{(x*y*x*y^{-1}*x)})^h$,

$((y^2)*t^{(y^2*x*y)})^i$,

$(((x*y)^3)*t^{((x*y)^3)})^j$,

$(t*t^{(y^2*x)})^k$ = (x * y * x * y^{-1} * x * y)>;

Juniper2											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	10	6	1	2×A_5
5	8	2	0	0	0	0	0	0	0	5	2×A_5
4	0	3	0	0	0	0	0	0	0	2	2×PSL(2,7)
5	4	3	0	0	0	0	0	0	0	2	3:(2:A_6)

Juniper2											
a	b	c	d	e	f	g	h	i	j	k	Index
6	0	0	3	0	0	0	0	0	0	3	3:(PSL(2,7):2)
6	0	0	3	0	0	0	0	0	0	4	PSL(2,7):(2^2:(3:2))
7	0	0	3	0	0	0	0	0	0	3	PSL(2,13)
7	0	0	3	0	0	0	0	0	0	4	PSL(2,127)
9	0	0	3	0	0	0	0	0	0	3	PSL(2,37)
4	2	0	4	0	0	0	0	0	0	2	2^2:(A_5:2)
7	2	0	4	0	0	0	0	0	0	2	2×PSL(2,41)
5	2	4	4	0	0	0	0	0	0	5	2×A_6
8	2	4	4	0	0	0	0	0	0	4	2:(A_5:(A_5:2^3))
8	2	4	4	0	0	0	0	0	0	5	2:(PSL(2,81):2)
3	6	0	5	0	0	0	0	0	0	3	PSL(2,11)
4	10	0	5	0	0	0	0	0	0	2	PSL(2,19)×2
9	0	3	5	0	0	0	0	0	0	5	PSL(2,19)
5	0	4	5	0	0	0	0	0	0	3	2^5:A_6

Progenitor of juniper3

G<x,y,t>:=Group<x,y,t| x^2 , y^4 , $y^{-1} * x * y^{-2} * x * y^{-1} * x * y^2 * x$, $y^{-1} * x * y^{-1} * x * y^{-1} * x * y * x * y * x * y * x$,

t^2, (t,(x * y^2 * x)), (t,(x * y * x * y^{-1} * x * y)),

$(((x * y)^2)*t^{(y^2*x*y)})^a$,

$((y * x * y)*t^{(x^y)})^b$,

$((y)*t^{(y*x*y)})^c$,

$((x * y^2 * x * y)*t^{(y*x)})^d$,

$((x * y)*t^{(y*x*y^2)})^e$,

$((y^2)*t^{(y*x*y^{-1}*x*y^{-1})})^f$,

$(((x * y)^3)*t^{((x*y)^3)})^g$,

$((x * y)*t^{(x*y^2*x*y)})^h$,

$((x * y^2 * x * y)*t^{(y^{-1}*x*y*x)})^i$,

$((x * y * x * y^{-1} * x * y)*t^{(y*x*y*x*y^{-1})})^j$,

$(t*t^{(y^2*x)})^k = (x * y * x * y^{-1} * x * y)$>;

Juniper3											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	2	0	2	5	2:(5:2)
0	0	0	0	0	0	0	2	0	2	6	2^2:(3:2)
0	0	0	0	0	0	0	2	0	2	7	2:(7:2)
0	0	0	0	0	0	0	2	0	2	9	2:(3^2:2)
0	0	0	0	0	0	0	2	0	2	10	$2\times A_5$
0	0	0	0	0	0	0	3	6	2	3	3:(PSL(2,7):2)
0	0	0	0	0	0	0	3	6	2	4	PSL(2,7):(2^2:(3:2))
0	0	0	0	0	0	0	3	7	0	3	PSL(2,13)
0	0	0	0	0	0	0	3	7	0	4	PSL(2,127)
0	0	0	0	0	0	0	3	9	0	3	PSL(2,37)
0	0	0	0	0	0	0	4	4	4	2	2^2:(A_5:2)
0	0	0	0	0	0	0	4	5	2	2	$2\times A_6$
0	0	0	0	0	0	0	4	7	2	2	2×PSL(2,41)

Juniper3											
a	b	c	d	e	f	g	h	i	j	k	Index
5	3	0	0	0	0	0	0	0	0	2	A_6
5	3	0	0	0	0	0	0	0	0	3	PSL(2,11)
6	3	0	0	0	0	0	0	0	0	2	3:(PSL(2,7):2)
4	4	0	0	0	0	0	0	0	0	2	2^2:(A_5:2)
3	6	0	0	0	0	0	0	0	0	4	PSL(2,7):(2^2:(3:2))
3	7	0	0	0	0	0	0	0	0	3	PSL(2,13)
3	7	0	0	0	0	0	0	0	0	4	PSL(2,127)
4	7	0	0	0	0	0	0	0	0	2	2×PSL(2,41)
8	10	0	0	0	0	0	0	0	0	1	3:(2:(A_6:2))
9	10	0	0	0	0	0	0	0	0	1	3:(2:PSL(2,19))
7	8	2	0	0	0	0	0	0	0	1	2×PSL(2,7)
8	8	2	0	0	0	0	0	0	0	1	2:(PSL(2,7):2)
7	9	2	0	0	0	0	0	0	0	1	PSL(2,8)
9	9	2	0	0	0	0	0	0	0	1	PSL(2,19)
9	4	3	0	0	0	0	0	0	0	3	PSL(2,73)

Progenitor of juniper4

G<x,y,t>:=Group<x,y,t| x^2 , y^4 , y^{-1} * x * y^{-2} * x * y^{-1} * x * y^2 * x , y^{-1} * x * y^{-1} * x * y^{-1} * x * y * x * y * x * y * x ,

t^2, (t,(x * y^2 * x)), (t,(x * y * x * y^{-1} * x * y)),

((x * y^2 * x * y)*$t^{(y*x)}$)^a,

((x * y * x * y^{-1} * x * y)*$t^{(x^y)}$)^b,

(((x * y)^3)*$t^{(y^{-1}*x*y*x)}$)^c,

((y * x * y)*$t^{(x*y^2*x*y)}$)^d,

((y)*$t^{((x*y)^3)}$)^e,

(((x * y)^2)*$t^{(y*x*y^{-1}*x*y^{-1})}$)^f,

((x * y^2 * x * y)*$t^{(y^2*x*y)}$)^g,

((x * y)*$t^{(x*y*x*y^{-1}*x)}$)^h,

((y^2)*$t^{(y*x*y^2)}$)^i,

((y * x * y)*$t^{(y*x*y)}$)^j,

$(t{*}t^{(y^2*x)})^k = (x * y * x * y^{-1} * x * y)$>;

Juniper4											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	3	0	9	3	PSL(2,37)
0	0	0	0	0	0	0	3	2	6	3	3:(PSL(2,7):2)
0	0	0	0	0	0	0	3	2	6	4	PSL(2,7):(2^2:(3:2))
0	0	0	0	0	0	0	3	2	7	3	PSL(2,13)
0	0	0	0	0	0	0	3	2	7	4	PSL(2,127)
0	0	0	0	0	0	0	4	0	7	2	2×PSL(2,41)
0	0	0	0	0	0	0	4	2	4	2	2^2:(A_5:2)
0	0	0	0	0	0	0	4	2	5	2	2×A_6
0	0	0	0	0	0	0	5	4	3	2	A_6
0	0	0	0	0	0	0	5	4	3	3	PSL(2,11)

Progenitor of juniper5

G<x,y,t>:=Group<x,y,t| x^2 , y^4 , $y^{-1} * x * y^{-2} * x * y^{-1} * x * y^2 * x$, $y^{-1} * x * y^{-1} * x * y^{-1} * x * y * x * y * x * y * x$,

t^2, (t,(x * y^2 * x)), (t,(x * y * x * y^{-1} * x * y)),

$(((x * y)^2)*t^{(y^2*x*y)})^a$,

$((y * x * y)*t^{(x^y)})^b$,

$((y)*t^{(y*x*y)})^c$,

$((x * y^2 * x * y)*t^{(y*x)})^d$,

$((x * y)*t^{(y*x*y^2)})^e$,

$((y^2)*t^{(y*x*y^{-1}*x*y^{-1})})^f$,

$(((x * y)^3)*t^{((x*y)^3)})^g$,

$((x * y)*t^{(x*y^2*x*y)})^h$,

$((x * y^2 * x * y)*t^{(y^{-1}*x*y*x)})^i$,

$((x * y * x * y^{-1} * x * y)*t^{(y*x*y*x*y^{-1})})^j$,

$((y * x * y * x * y)*t)^k$ >

Juniper5											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	3	0	0	3	2:(5:2)
0	0	0	0	0	0	0	3	0	0	5	2:(A_5:A_5)
0	0	0	0	0	0	0	3	6	0	6	3:(PSL(2,7):2)
0	0	0	0	0	0	0	3	6	0	7	PSL(2,13)
0	0	0	0	0	0	0	3	6	0	8	PSL(2,7):(2^2:(3:2))
0	0	0	0	0	0	0	3	6	0	9	PSL(2,37)
0	0	0	0	0	0	0	3	7	0	8	PSL(2,127)
0	0	0	0	0	0	0	4	7	0	4	2×PSL(2,41)
0	0	0	0	0	0	0	5	2	10	10	2^2:(A_5:2)
0	0	0	0	0	0	0	5	3	0	5	A_6

Juniper5											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	5	3	0	6	PSL(2,11)
0	0	0	0	0	0	0	5	3	0	9	PSL(2,19)
5	2	0	0	0	0	0	0	0	0	10	2^2:(A_5:2)
7	2	0	0	0	0	0	0	0	0	5	A_7 ×2
7	2	0	0	0	0	0	0	0	0	6	2:(A_7 ×2)
5	3	0	0	0	0	0	0	0	0	5	A_6
5	3	0	0	0	0	0	0	0	0	6	PSL(2,11)
5	3	0	0	0	0	0	0	0	0	9	PSL(2,19)
9	3	0	0	0	0	0	0	0	0	4	PSL(2,73)

Juniper5											
a	b	c	d	e	f	g	h	i	j	k	Index
4	4	0	0	0	0	0	0	0	0	7	2×PSL(2,41)
5	4	0	0	0	0	0	0	0	0	4	PSL(2,19)×2
3	7	0	0	0	0	0	0	0	0	8	PSL(2,127)
3	7	0	0	0	0	0	0	0	0	6	PSL(2,13)
3	9	0	0	0	0	0	0	0	0	6	PSL(2,37)
5	9	0	0	0	0	0	0	0	0	3	PSL(2,19)
10	9	0	0	0	0	0	0	0	0	2	3:(2:PSL(2,19))
6	3	3	0	0	0	0	0	0	0	3	3:(PSL(2,7):2)
9	3	4	0	0	0	0	0	0	0	4	PSL(2,73)
8	2	7	0	0	0	0	0	0	0	10	2:(A_9:2)
8	2	7	0	0	0	0	0	0	0	7	2×PSL(2,7)
3	6	7	0	0	0	0	0	0	0	7	PSL(2,13)
0	2	8	0	0	0	0	0	0	0	5	2:(PSU(3,9):2)
9	3	8	0	0	0	0	0	0	0	4	PSL(2,73)
3	6	8	0	0	0	0	0	0	0	8	PSL(2,7):(2^2:(3:2))

Progenitor of juniper6

G<x,y,t>:=Group<x,y,t| x^2 , y^4 , y^{-1} * x * y^{-2} * x * y^{-1} * x * y^2 * x , y^{-1} * x * y^{-1} * x * y^{-1} * x * y * x * y * x * y * x ,

t^2, (t,(x * y^2 * x)), (t,(x * y * x * y^{-1} * x * y)),

((x * y^2 * x * y)*$t^{(y*x)}$)^a,

((x * y * x * y^{-1} * x * y)*$t^{(x^y)}$)^b,

(((x * y)^3)*$t^{(y^{-1}*x*y*x)}$)^c,

((y * x * y)*$t^{(x*y^2*x*y)}$)^d,

((y)*$t^{((x*y)^3)}$)^e,

(((x * y)^2)*$t^{(y*x*y^{-1}*x*y^{-1})}$)^f,

((x * y^2 * x * y)*$t^{(y^2*x*y)}$)^g,

((x * y)*$t^{(x*y*x*y^{-1}*x)}$)^h,

((y^2)*$t^{(y*x*y^2)}$)^i,

((y * x * y)*$t^{(y*x*y)}$)^j,

((y * x * y * x * y)*t)^k >

Juniper6											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	5	2	5	PSL(3,11):2
0	0	0	0	0	0	0	3	0	0	5	2:(A_5:A_5)
0	0	0	0	0	0	0	3	0	6	6	3:(PSL(2,7):2)
0	0	0	0	0	0	0	3	0	6	7	PSL(2,13)
0	0	0	0	0	0	0	3	0	6	8	PSL(2,7):(2^2:(3:2))
0	0	0	0	0	0	0	3	0	6	9	PSL(2,37)
0	0	0	0	0	0	0	3	0	7	8	PSL(2,127)
0	0	0	0	0	0	0	4	0	4	7	2×PSL(2,41)
0	0	0	0	0	0	0	6	3	4	5	S(4,3)×2
0	0	0	0	0	0	0	8	0	10	2	3:(2:(A_6:2))
0	0	0	0	0	0	0	8	3	2	8	2:(PSL(2,49):2)
0	0	0	0	0	0	0	8	8	8	2	2:(PSL(2,7):2)
0	0	0	0	0	0	0	9	0	10	2	3:(2:PSL(2,19))
0	0	0	0	0	0	0	9	2	4	3	PSL(2,73)
0	0	0	0	0	0	0	9	2	7	2	PSL(2,8)

Progenitor of juniper7

G<x,y,t>:=Group<x,y,t| x^2 , y^4 , y^{-1} * x * y^{-2} * x * y^{-1} * x * y^2 * x , y^{-1} * x * y^{-1} * x * y^{-1} * x * y * x * y * x * y * x ,

t^2, (t,(x * y^2 * x)), (t,(x * y * x * y^{-1} * x * y)),

$(((x * y)^2)*t^{(y^2*x*y)})^a$,

$((y * x * y)*t^{(x^y)})^b$,

$((y)*t^{(y*x*y)})^c$,

$((x * y^2 * x * y)*t^{(y*x)})^d$,

$((x * y)*t^{(y*x*y^2)})^e$,

$((y^2)*t^{(y*x*y^{-1}*x*y^{-1})})^f$,

$(((x * y)^3)*t^{((x*y)^3)})^g$,

$((x * y)*t^{(x*y^2*x*y)})^h$,

$((x * y^2 * x * y)*t^{(y^{-1}*x*y*x)})^i$,

$((x * y * x * y^{-1} * x * y)*t^{(y*x*y*x*y^{-1})})^j$,

$((y^2 * x * y^2)*t)^k$ >

Juniper7											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	3	0	8	5	2:(A_5:A_5)
0	0	0	0	0	0	0	3	6	2	6	3:(PSL(2,7):2)
0	0	0	0	0	0	0	3	6	2	7	PSL(2,13)
0	0	0	0	0	0	0	3	6	2	8	PSL(2,7):(2^2:(3:2))
0	0	0	0	0	0	0	3	6	2	9	PSL(2,37)
0	0	0	0	0	0	0	3	7	0	8	PSL(2,127)
0	0	0	0	0	0	0	4	4	2	4	2^2:(A_5:2)
0	0	0	0	0	0	0	4	4	2	7	2×PSL(2,41)
0	0	0	0	0	0	0	5	9	4	3	PSL(2,19)
2	0	0	0	0	0	0	0	0	0	10	2:(5:2)
3	0	0	0	0	0	0	0	0	0	5	2:(A_5:A_5)
5	2	0	0	0	0	0	0	0	0	10	2^2:(A_5:2)

Juniper7											
a	b	c	d	e	f	g	h	i	j	k	Index
7	2	0	0	0	0	0	0	0	0	6	2:(A_7:2)
5	3	0	0	0	0	0	0	0	0	4	A_6
5	3	0	0	0	0	0	0	0	0	6	PSL(2,11)
5	3	0	0	0	0	0	0	0	0	9	PSL(2,19)
6	3	0	0	0	0	0	0	0	0	4	3:(PSL(2,7):2)
9	3	0	0	0	0	0	0	0	0	4	PSL(2,73)
4	4	0	0	0	0	0	0	0	0	5	2×A_6
4	4	0	0	0	0	0	0	0	0	7	2×PSL(2,41)
5	4	0	0	0	0	0	0	0	0	4	PSL(2,19)×2
3	6	0	0	0	0	0	0	0	0	7	PSL(2,13)
3	6	0	0	0	0	0	0	0	0	8	PSL(2,7):(2^2:(3:2))
3	6	0	0	0	0	0	0	0	0	9	PSL(2,37)
3	7	0	0	0	0	0	0	0	0	8	PSL(2,127)
9	10	0	0	0	0	0	0	0	0	2	3:(2:PSL(2,19))
0	2	5	0	0	0	0	0	0	0	8	2:(PSU(3,9):2)
7	2	5	0	0	0	0	0	0	0	6	A_7 ×2

Progenitor of juniper8

G<x,y,t>:=Group<x,y,t| x^2 , y^4 , $y^{-1} * x * y^{-2} * x * y^{-1} * x * y^2 * x$, $y^{-1} * x * y^{-1} * x * y^{-1} * x * y * x * y * x * y * x$,

t^2, (t,(x * y^2 * x)), (t,(x * y * x * y^{-1} * x * y)),

$((x * y^2 * x * y)*t^{(y*x)})^a$,

$((x * y * x * y^{-1} * x * y)*t^{(x^y)})^b$,

$(((x * y)^3)*t^{(y^{-1}*x*y*x)})^c$,

$((y * x * y)*t^{(x*y^2*x*y)})^d$,

$((y)*t^{((x*y)^3)})^e$,

$(((x * y)^2)*t^{(y*x*y^{-1}*x*y^{-1})})^f$,

$((x * y^2 * x * y)*t^{(y^2*x*y)})^g$,

$((x * y)*t^{(x*y*x*y^{-1}*x)})^h$,

$((y^2)*t^{(y*x*y^2)})^i$,

$((y * x * y)*t^{(y*x*y)})^j$,

$((y^2 * x * y^2)*t)^k$ >

Juniper8											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	3	2	0	5	2:(A_5:A_5)
0	0	0	0	0	0	0	3	2	6	6	3:(PSL(2,7):2)
0	0	0	0	0	0	0	3	2	6	7	PSL(2,13)
0	0	0	0	0	0	0	3	2	6	8	PSL(2,7):(2^2:(3:2))
0	0	0	0	0	0	0	3	2	6	9	PSL(2,37)
0	0	0	0	0	0	0	3	2	7	8	PSL(2,127)
0	0	0	0	0	0	0	4	0	4	4	2^2:(A_5:2)
0	0	0	0	0	0	0	4	0	4	7	2×PSL(2,41)
0	0	0	0	0	0	0	8	2	2	10	3:(2:(A_6:2))
0	0	0	0	0	0	0	8	3	2	8	2:(PSL(2,49):2)
0	0	0	0	0	0	0	9	0	9	2	PSL(2,19)
0	0	0	0	0	0	0	9	0	10	2	3:(2:PSL(2,19))

10.10 2^{*16}:((4^2):(2:6))

We have the following information

S:=Sym(16)

x ∼ (1, 3, 7, 8)(2, 15, 12, 4)(5, 13, 10, 14)(6, 9, 16, 11)

y ∼ (2, 5, 11, 15, 13, 9)(3, 4, 10, 8, 12, 14)(6, 7, 16)

#N = 192

Progenitor of piggy

G<x,y,t>:=Group<x,y,t| x^4 , y^6 , y^{-3} * x^{-1} * y^3 * x^{-1} , y * x^{-2} * y^{-1} * x^2 * y^{-1} * x^2 * y , x^{-1} * y^{-1} * x^{-1} * y^{-1} * x * y^{-1} * x * y^{-1} , t^2, (t,y), (t,(x * y * x^{-1} * y^2 * x^{-1})),

((y^2 * x * y^{-1} * x)*$t^{(x^2*y^{-1}*x^{-1}*y)}$)^a,

(((x * y)^2)*$t^{(x*y^{-1}*x^2*y)}$)^b,

((x * y^2 * x^2 * y)*$t^{(y^{-1}*x*y*x)}$)^c,

((x * y)*$t^{(y^2*x^2)}$)^d,

((y * x * y)*$t^{(y^2*x)}$)^e,

((x^2)*$t^{(x*y^{-1})}$)^f,

((y^3)*$t^{((y,x))}$)^g,

$((x * y * x^2 * y^{-1})*t^{(x^2*y^2*x^{-1}*y^{-1})})^h$,
$((y^2 * x * y)*t^{(x*y^{-2})})^i$,
$((y^2 * x * y^{-1} * x)*t^{(x^2*y^{-1}*x^{-1}*y)})^j$,
$(x^2 * y^{-1} * x * y*t)^k$ >

Piggy											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	2	0	10	0	5	2:(A_5:(A_5:2))
0	0	0	0	0	0	2	0	10	5	10	3:(PSL(2,11):2)
10	0	2	0	0	0	0	0	0	0	5	2:(A_5:(A_5:2))
0	5	2	0	0	0	0	0	0	0	10	3:(PSL(2,11):2)
4	0	5	0	0	0	0	0	0	0	8	2^2:(PSL(3,4):2^2)
6	0	5	0	0	0	0	0	0	0	5	2×M_{12}
6	6	5	8	0	0	0	0	0	0	10	2^2:M_{12}

Progenitor of piggy2

G<x,y,t>:=Group<x,y,t| x^4 , y^6 , $y^{-3} * x^{-1} * y^3 * x^{-1}$, $y * x^{-2} * y^{-1} * x^2 * y^{-1} * x^2 * y$, $x^{-1} * y^{-1} * x^{-1} * y^{-1} * x * y^{-1} * x * y^{-1}$, t^2, (t,y), $(t,(x * y * x^{-1} * y^2 * x^{-1}))$,
$((y^2 * x^2)*t^{(y^{-1}*x*y*x)})^a$,
$(((x * y)^2)*t^{(y^2*x)})^b$,
$(((y * x * y)^2)*t^{((y,x))})^c$,
$((x)*t^{(x^2*y^2*x^{-1}*y^{-1})})^d$,
$((x * y^2 * x^2 * y)*t^{(y*x^2*y^2)})^e$,
$((x^2 * y)*t^{(y^2*x^2)})^f$,
$((x * y)*t^{(y^{-2}*x^{-1})})^g$,
$((y * x * y)*t^{(x*y^{-1}*x^2*y)})^h$,
$((x^2)*t^{(x^{-1}*y^2*x^{-1})})^i$,
$((y^3)*t^{(x^{-1}*y*x^{-1}*y^{-1})})^j$,
$(x^2 * y^{-1} * x * y*t)^k$ >

Piggy2											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	2	4	5	2:(A_5:(A_5:2))
0	0	0	0	0	0	0	9	0	4	4	2^2:S(6,2)
8	0	0	2	0	0	0	0	0	0	7	PSL(2,49)×2
10	0	0	2	0	0	0	0	0	0	5	2:(A_5:(A_5:2))
0	5	0	2	0	0	0	0	0	0	10	3:(PSL(2,11):2)
6	6	0	5	0	0	0	0	0	0	4	2^2:(PSL(3,4):2^2)

Progenitor of piggy4

G<x,y,t>:=Group<x,y,t| x^4 , y^6 , $y^{-3} * x^{-1} * y^3 * x^{-1}$, $y * x^{-2} * y^{-1} * x^2 * y^{-1} * x^2 * y$, $x^{-1} * y^{-1} * x^{-1} * y^{-1} * x * y^{-1} * x * y^{-1}$, t^2, (t,y), (t,($x * y * x^{-1} * y^2 * x^{-1}$)),
$((y^2 * x^2)*t^{(y^{-1}*x*y*x)})^a$,
$(((x * y)^2)*t^{(y^2*x)})^b$,
$(((y * x * y)^2)*t^{((y,x))})^c$,
$((x)*t^{(x^2*y^2*x^{-1}*y^{-1})})^d$,
$((x * y^2 * x^2 * y)*t^{(y*x^2*y^2)})^e$,
$((x^2 * y)*t^{(y^2*x^2)})^f$,
$((x * y)*t^{(y^{-2}*x^{-1})})^g$,
$((y * x * y)*t^{(x*y^{-1}*x^2*y)})^h$,
$((x^2)*t^{(x^{-1}*y^2*x^{-1})})^i$,
$((y^3)*t^{(x^{-1}*y*x^{-1}*y^{-1})})^j$,
$(t*t^{(x*y^4*x^{-2})})^k=(x * y * x^{-1} * y^2 * x^{-1})$>

Piggy4											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	2	2	PSL(2,7)×2
0	0	0	0	0	0	0	8	4	2	3	PSL(3,3)×2

Progenitor of piggy5

G<x,y,t>:=Group<x,y,t| x^4 , y^6 , $y^{-3} * x^{-1} * y^3 * x^{-1}$, $y * x^{-2} * y^{-1} * x^2 * y^{-1} * x^2 * y$, $x^{-1} * y^{-1} * x^{-1} * y^{-1} * x * y^{-1} * x * y^{-1}$, t^2, (t,y), (t,($x * y * x^{-1} * y^2 * x^{-1}$)),
$((y^2 * x * y^{-1} * x)*t^{(x^2*y^{-1}*x^{-1}*y)})^a$,

$(((x * y)^2)*t^{(x*y^{-1}*x^2*y)})^b$,
$((x * y^2 * x^2 * y)*t^{(y^{-1}*x*y*x)})^c$,
$((x * y)*t^{(y^2*x^2)})^d$,
$((y * x * y)*t^{(y^2*x)})^e$,
$((x^2)*t^{(x*y^{-1})})^f$,
$((y^3)*t^{((y,x))})^g$,
$((x * y * x^2 * y^{-1})*t^{(x^2*y^2*x^{-1}*y^{-1})})^h$,
$((y^2 * x * y)*t^{(x*y^{-2})})^i$,
$((y^2 * x * y^{-1} * x)*t^{(x^2*y^{-1}*x^{-1}*y)})^j$,
$(y * x^{-1} * y^{-2} * x*t)^k$ >

Piggy5											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	2	0	5	10	10	2:(A_5:(A_5:2))
0	0	0	0	0	0	2	0	10	10	5	3:(PSL(2,11):2)
5	5	0	0	0	0	0	0	0	0	5	3:(PSL(2,11):2)
4	0	5	0	0	0	0	0	0	0	8	2^2:(PSL(3,4):2^2)
8	0	10	5	0	0	0	0	0	0	8	2:(PSL(3,4):(2^2))
6	6	5	8	0	0	0	0	0	0	10	2^2:M_{12}

Progenitor of piggy6

G<x,y,t>:=Group<x,y,t| x^4 , y^6 , $y^{-3} * x^{-1} * y^3 * x^{-1}$, $y * x^{-2} * y^{-1} * x^2 * y^{-1} * x^2 * y$, $x^{-1} * y^{-1} * x^{-1} * y^{-1} * x * y^{-1} * x * y^{-1}$, t^2, (t,y), (t,$(x * y * x^{-1} * y^2 * x^{-1})$),
$((y^2 * x^2)*t^{(y^{-1}*x*y*x)})^a$,
$(((x * y)^2)*t^{(y^2*x)})^b$,
$(((y * x * y)^2)*t^{((y,x))})^c$,
$((x)*t^{(x^2*y^2*x^{-1}*y^{-1})})^d$,
$((x * y^2 * x^2 * y)*t^{(y*x^2*y^2)})^e$,
$((x^2 * y)*t^{(y^2*x^2)})^f$,
$((x * y)*t^{(y^{-2}*x^{-1})})^g$,
$((y * x * y)*t^{(x*y^{-1}*x^2*y)})^h$,
$((x^2)*t^{(x^{-1}*y^2*x^{-1})})^i$,
$((y^3)*t^{(x^{-1}*y*x^{-1}*y^{-1})})^j$,

(y * x^{-1} * y^{-2} * x*t)^k >

Piggy6											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	2	0	5	3:(PSL(2,11):2)
0	0	0	0	0	0	0	9	0	4	4	2^2:S(6,2)
0	0	0	0	0	0	6	0	2	8	8	3:(2:(PSL(2,7):2))
0	0	10	2	0	0	0	0	0	0	5	3:(PSL(2,11):2)

10.11 2^{*24}:((8:2^2):3)

We have the following information

S:=Sym(24)

x ∼ (1, 9, 20, 6, 14, 24, 2, 10, 19, 5, 13, 23)(3, 12, 21, 7, 16, 18, 4, 11, 22, 8, 15, 17)

y ∼ (1, 16, 23, 8, 13, 22, 6, 11, 20, 3, 9, 17, 2, 15, 24, 7, 14, 21, 5, 12, 19, 4, 10, 18)

#N = 96

Progenitor of Fishcakes

G<x,y,t>:=Group<x,y,t| (x^{-1} * y^{-2})^2 , x^{-4} * y * x^{-1} * y , y^{-1} * x^{-1} * y^{-1} * x^2 * y^2 * x^{-1},

t^2, (t,(y * x * y)), (t,(y^2 * x)),

((y^{-1} * x * y^{-1})*t$^{(x*y^2)}$)^a,

((x^{-3})*t$^{(x^2*y)}$)^b,

((x * y^{-1})*t$^{(x^{-3})}$)^c,

((y^{-3})*t$^{(x*y^2*x^{-2}*y^{-1})}$)^d,

((y * x^2)*t$^{(x^{-1}*y)}$)^e,

((y * x^{-1} * y)*t$^{(y*x^{-1}*y^2)}$)^f,

((x^3 * y^3)*t$^($$x^2 * y^{-1} * x^{-1}$))^g,

((y * x^{-2} * y)*t$^{(x^{-2}*y^2)}$)^h,

((x^2)*t$^{(y*x*y^{-1})}$)^i,

((x * y^{-2} * x)*t$^{(y*x)}$)^j,

(x^{-1}, y^{-1})^k >

Fishcakes											
a	b	c	d	e	f	g	h	i	j	k	Index
5	6	2	0	0	0	0	0	0	0	1	A_5
4	10	3	0	0	0	0	0	0	0	1	3:(A_5:2)
2	5	4	0	0	0	0	0	0	0	1	5^2:2^2
3	5	4	0	0	0	0	0	0	0	1	2:(3:PSL(2,11)):(PSL(2,11):2)
2	7	4	0	0	0	0	0	0	0	1	7^2:2^2

Progenitor of Fishcakes2

$G<x,y,t>:=Group<x,y,t| (x^{-1} * y^{-2})^2 , x^{-4} * y * x^{-1} * y , y^{-1} * x^{-1} * y^{-1} * x^2 * y^2 * x^{-1},$

$t^2, (t,(y * x * y)), (t,(y^2 * x)),$

$((x^{-3} * y^{-1})*t^{(y^{-2}*x)})^a,$

$((x^2 * y * x^{-1})*t^{(x*y*x^{-1})})^b,$

$((y * x^2 * y)*t^{(y*x^{-1}*y)})^c,$

$((y * x^{-1})*t^{(x^2*y^{-1})})^d,$

$((x^{-1} * y * x^{-1})*t^{(x^2*y^2)})^e,$

$((y * x^2)*t^{(x^{-1})})^f,$

$((x * y * x^{-1} * y)*t^{(y*x^{-1}*y^{-1})})^g,$

$((y^{-1})*t^{(}x^2 * y^{-1} * x^{-1}))^h,$

$((x^3 * y)*t^{(x^2*y)})^i,$

$((x * y^{-1} * x)*t^{(x*y^2)})^j,$

$(x^{-1}, y^{-1})^k >$

Fishcakes2											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	2	10	10	0	1	2:(5:2)
0	0	0	0	0	0	3	4	0	6	1	PSL(2,121)×2
10	10	2	0	0	0	0	0	0	0	1	2:(5:2)
4	6	3	0	0	0	0	0	0	0	1	PSL(2,121)×2

Progenitor of Fishcakes3

$G<x,y,t>:=Group<x,y,t|\ (x^{-1} * y^{-2})^2\ ,\ x^{-4} * y * x^{-1} * y\ ,\ y^{-1} * x^{-1} * y^{-1} * x^2 * y^2 * x^{-1},$

$t^2,\ (t,(y * x * y)),\ (t,(y^2 * x)),$

$((y^{-1} * x * y^{-1})*t^{(x*y^2)})^a,$

$((x^{-3})*t^{(x^2*y)})^b,$

$((x * y^{-1})*t^{(x^{-3})})^c,$

$((y^{-3})*t^{(x*y^2*x^{-2}*y^{-1})})^d,$

$((y * x^2)*t^{(x^{-1}*y)})^e,$

$((y * x^{-1} * y)*t^{(y*x^{-1}*y^2)})^f,$

$((x^3 * y^3)*t^(x^2 * y^{-1} * x^{-1}))^g,$

$((y * x^{-2} * y)*t^{(x^{-2}*y^2)})^h,$

$((x^2)*t^{(y*x*y^{-1})})^i,$

$((x * y^{-2} * x)*t^{(y*x)})^j,$

$(x^2 * y * x^{-1} * y)^k >$

Fishcakes3											
a	b	c	d	e	f	g	h	i	j	k	Index
5	4	2	0	0	0	0	0	0	0	1	A_5
4	10	3	0	0	0	0	0	0	0	1	3:(A_5:2)
3	5	4	0	0	0	0	0	0	0	1	2:(3:PSL(2,11)):(PSL(2,11):2)
3	2	5	0	0	0	0	0	0	0	1	3:PSL(2,11)
2	5	5	0	0	0	0	0	0	0	1	A_6

Progenitor of Fishcakes4

G<x,y,t>:=Group<x,y,t| $(x^{-1} * y^{-2})^2$, $x^{-4} * y * x^{-1} * y$, $y^{-1} * x^{-1} * y^{-1} * x^2 * y^2 * x^{-1}$,

t^2, (t,(y * x * y)), (t,(y^2 * x)),

$((x^{-3} * y^{-1})*t^{(y^{-2}*x)})^a$,

$((x^2 * y * x^{-1})*t^{(x*y*x^{-1})})^b$,

$((y * x^2 * y)*t^{(y*x^{-1}*y)})^c$,

$((y * x^{-1})*t^{(x^2*y^{-1})})^d$,

$((x^{-1} * y * x^{-1})*t^{(x^2*y^2)})^e$,

$((y * x^2)*t^{(x^{-1})})^f$,

$((x * y * x^{-1} * y)*t^{(y*x^{-1}*y^{-1})})^g$,

$((y^{-1})*t^(x^2 * y^{-1} * x^{-1}))^h$,

$((x^3 * y)*t^{(x^2*y)})^i$,

$((x * y^{-1} * x)*t^{(x*y^2)})^j$,

$(x^2 * y * x^{-1} * y)^k$ >

Fishcakes4											
a	b	c	d	e	f	g	h	i	j	k	Index
0	10	2	0	0	0	0	0	0	0	1	2:(5:2)
6	4	3	0	0	0	0	0	0	0	1	PSL(2,121)×2

Progenitor of Fishcakes5

G<x,y,t>:=Group<x,y,t| $(x^{-1} * y^{-2})^2$, $x^{-4} * y * x^{-1} * y$, $y^{-1} * x^{-1} * y^{-1} * x^2 * y^2 * x^{-1}$,

t^2, (t,(y * x * y)), (t,(y^2 * x)),

$((y^{-1} * x * y^{-1})*t^{(x*y^2)})^a$,

$((x^{-3})*t^{(x^2*y)})^b$,

$((x * y^{-1})*t^{(x^{-3})})^c$,

$((y^{-3})*t^{(x*y^2*x^{-2}*y^{-1})})^d$,

$((y * x^2)*t^{(x^{-1}*y)})^e$,

$((y * x^{-1} * y)*t^{(y*x^{-1}*y^2)})^f$,

$((x^3 * y^3)*t(x^2 * y^{-1} * x^{-1}))^g$,

$((y * x^{-2} * y)*t^{(x^{-2}*y^2)})^h$,

$((x^2)*t^{(y*x*y^{-1})})^i$,

$((x * y^{-2} * x)*t^{(y*x)})^j$,

$(t*t^{(x^6)})^k$ = (x,y)>

Fishcakes5											
a	b	c	d	e	f	g	h	i	j	k	Index
5	6	2	0	0	0	0	0	0	0	1	A_5
4	10	3	0	0	0	0	0	0	0	1	3:(A_5:2)

Progenitor of Fishcakes6

G<x,y,t>:=Group<x,y,t| $(x^{-1} * y^{-2})^2$, $x^{-4} * y * x^{-1} * y$, $y^{-1} * x^{-1} * y^{-1} * x^2 * y^2 * x^{-1}$,

t^2, (t,(y * x * y)), (t,(y^2 * x)),

$((x^{-3} * y^{-1})*t^{(y^{-2}*x)})^a$,

$((x^2 * y * x^{-1})*t^{(x*y*x^{-1})})^b$,

$((y * x^2 * y)*t^{(y*x^{-1}*y)})^c$,

$((y * x^{-1})*t^{(x^2*y^{-1})})^d$,

$((x^{-1} * y * x^{-1})*t^{(x^2*y^2)})^e$,

$((y * x^2)*t^{(x^{-1})})^f$,

$((x * y * x^{-1} * y)*t^{(y*x^{-1}*y^{-1})})^g$,

$((y^{-1})*t^(x^2 * y^{-1} * x^{-1}))^h$,

$((x^3 * y)*t^{(x^2*y)})^i$,

$((x * y^{-1} * x)*t^{(x*y^2)})^j$,

$(t*t^{(x^6)})^k$ = (x,y)>

Fishcakes6											
a	b	c	d	e	f	g	h	i	j	k	Index
10	10	2	0	0	0	0	0	0	0	1	2:(5:2)
6	4	3	0	0	0	0	0	0	0	1	PSL(2,121)×2

10.12 2^{*24}:(2^3:(2:3))

We have the following information

S:=Sym(24)

x ∼ (1, 6, 12, 23)(2, 5, 11, 24)(3, 8, 14, 19)(4, 7, 13, 20)(9, 15, 22, 18)(10, 16, 21, 17)

y ∼ (1, 24, 4, 18, 21, 14)(2, 23, 3, 17, 22, 13)(5, 7, 11, 10, 19, 16)(6, 8, 12, 9, 20, 15)

#N = 48

Progenitor of apple

G<x,y,t>:=Group<x,y,t| x^4 , $(x^{-1} * y^{-1})^2$, $(x * y^{-2})^2$, y^6, t^2, (t,($x^2 * y^{-1} * x * y$)),
$((x * y^{-1})*t^{(x^2*y^2*x)})^a$,
$(y*t^{(y*x^2)})^b$,
$(x*t^{(x)})^c$,
$((x * y^{-2})*t^{(y*x^2*y)})^d$,
$((x * y)*t^{(x^2*y^{-1})})^e$,
$((x^2 * y^3)*t^{(x*y^3)})^f$,
$((x^2)*t^{(y*x^{-1}*y)})^g$,
$((y^3)*t^{(y*x^2*y^2)})^h$,
$((x^2 * y^3)*t^{(x*y^2)})^i$,
$((x * y)*t^{(x*y^{-1}*x*y)})^j$,
$((y * x)*t)^k$ >

Apple											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	5	2	10	3:(PSL(2,11):2)
0	0	0	0	0	0	0	3	2	5	6	3×A_7
0	0	0	0	0	0	2	0	10	3	5	5^2:(2:(3:2))
0	4	4	0	0	0	0	0	0	0	2	2^2:(A_5:(A_5:2^2))
3	0	0	0	3	0	0	0	0	0	5	5^2:(2:(3:2))
3	0	0	0	3	0	0	0	0	0	7	7^2:(2:(3:2))
4	0	0	0	3	0	0	0	0	0	8	2×PSL(2,17)

Progenitor of apple2

G<x,y,t>:=Group<x,y,t| x^4 , $(x^{-1} * y^{-1})^2$, $(x * y^{-2})^2$, y^6, t^2, (t,($x^2 * y^{-1} * x * y$)),
$((x^2 * y^3)*t^{(y)})^a$,
$((y^3)*t^{(x*y^{-1}*x)})^b$,
$((x * y)*t^{(x^y)})^c$,
$((x^2)*t^{(x^{-1}*y*x^{-1})})^d$,
$((x * y^{-1})*t^{(y^3)})^e$,
$(x*t^{(x*y*x^{-1}*y)})^f$,

$(y*t^{(x*y^{-1})})^g$,
$((y^2)*t^{(x^2*y^{-1}*x)})^h$,
$((x * y)*t^{(x^2*y^2)})^i$,
$((x * y^{-2})*t^{(x^2*y^{-1}*x)})^j$,
$((y * x)*t)^k$ >

Apple2											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	2	0	2	10	2:(5:2)
0	0	0	0	0	0	0	2	0	4	4	2^3:(A_6:2)
0	0	0	0	0	0	0	2	0	4	5	5:(2:(S(4,3):2))
0	0	0	0	0	0	0	2	6	4	10	2^2:(A_7:2)
0	0	0	0	0	0	0	2	6	9	7	PSL(2,8):(PSL(2,8):(3:2))
0	0	0	0	0	0	0	2	7	4	6	2:(A_8:2)
0	0	0	0	0	0	0	4	4	2	5	2:(A_7:2)
0	0	0	0	0	0	0	7	3	9	3	PSU(3,7)
0	0	0	0	0	0	0	8	2	8	7	PSL(2,49)×2
5	2	2	0	0	0	0	0	0	0	10	3:(PSL(2,11):2)
7	2	3	0	0	0	0	0	0	0	3	PSU(3,7)
3	10	3	0	0	0	0	0	0	0	4	2^3:PSL(2,7)
5	2	3	3	0	0	0	0	0	0	6	3×A_7
6	2	3	3	0	0	0	0	0	0	6	PSL(3,3)
6	4	3	3	0	0	0	0	0	0	3	2:(J_2:2)

10.13 2^{*12}:(2^3:2^2)

We have the following information

S:=Sym(12)

x ∼ (1, 8)(2, 7)(3, 6)(4, 5)(9, 12)(10, 11)

y ∼ (1, 6, 7, 12)(2, 11)(3, 10, 9, 4)(5, 8)

#N = 48

Progenitor for peaches

G<x, y, t>:=Group< x, y, t | x^2 , y^4 , $y^{-1} * x * y^{-2} * x * y^{-1} * x * y^2 * x$, $y^{-1} * x *$
$y^{-1} * x * y^{-1} * x * y * x * y * x * y * x$,
t^2, $(t,(x * y * x * y^{-1} * x * y))$, $(t,(x * y^2 * x))$,
$((x * y)^*(t^{(y*x)}))^a$,
$((y * x * y)^*(t^{(x^y)}))^b$,
$((x)^*(t^{(y^{-1}*x*y*x)}))^c$,
$((x * y * x * y^{-1} * x * y)^*(t^{(x*y*x*y^{-1}*x)}))^d$,
$((x * y)^{3*}(t^{(y*x)}))^e$,
$((x * y * x * y^{-1} * x * y)^*(t^{(x*y^2*x*y)}))^f$,
$((x * y^2 * x * y)^*(t^{(y*x*y)}))^g$,
$(((x * y)^2)^*(t^{(y*x*y)}))^h$,
$((y)^*(t^{(y*x)}))^i$,
$((x * y)^*(t^{((x*y)^3)}))^j$,
$((y^2 * x * y^2)^*t)^k$ >

Peaches											
a	b	c	d	e	f	g	h	i	j	k	Index
2	0	0	0	0	0	0	0	0	0	2	2
4	0	2	0	0	0	0	0	0	0	2	2
5	0	2	0	0	0	0	0	0	0	5	A_5
4	2	3	0	0	0	0	0	0	0	2	PSL(2,7)×2
5	2	3	0	0	0	0	0	0	0	2	3:(2:A_6)
0	0	0	0	0	0	0	3	3	0	3	A_5
0	0	0	0	0	0	0	3	5	0	5	2:(A_5:A_5)
0	0	0	0	0	0	0	4	0	10	2	2
0	0	0	0	0	0	0	4	0	10	3	2:3
0	0	0	0	0	0	0	4	0	10	5	2:5
0	0	0	0	0	0	0	4	0	10	6	2:(3:2)
0	0	0	0	0	0	0	4	0	10	7	7:2
0	0	0	0	0	0	0	4	0	10	9	3^2:2
2	0	0	0	0	0	0	0	0	0	2	2
2	0	0	0	0	0	0	0	0	0	3	2:3
2	0	0	0	0	0	0	0	0	0	5	2:5
2	0	0	0	0	0	0	0	0	0	6	2:(3:2)

Peaches											
a	b	c	d	e	f	g	h	i	j	k	Index
2	0	0	0	0	0	0	0	0	0	7	7:2
2	0	0	0	0	0	0	0	0	0	8	2
2	0	0	0	0	0	0	0	0	0	9	3^2:2
2	0	0	0	0	0	0	0	0	0	10	A_5
3	0	0	0	0	0	0	0	0	0	4	2
3	0	0	0	0	0	0	0	0	0	5	2:(A_5:A_5)
5	3	0	0	0	0	0	0	0	0	9	PSL(2,19)
0	5	0	0	0	0	0	0	0	0	2	A_5
3	6	0	0	0	0	0	0	0	0	9	PSL(2,37)
8	6	0	0	0	0	0	0	0	0	2	2
7	8	0	0	0	0	0	0	0	0	2	PSL(2,7)×2
8	8	0	0	0	0	0	0	0	0	2	2:(PSL(2,7):2)
3	9	0	0	0	0	0	0	0	0	6	PSL(2,37)
10	9	0	0	0	0	0	0	0	0	2	3:(2:PSL(2,19))
9	0	2	0	0	0	0	0	0	0	9	PSL(2,19)
0	3	4	0	0	0	0	0	0	0	5	3:(2:A_6)
7	4	4	0	0	0	0	0	0	0	4	3:(A_8:2)
4	8	4	0	0	0	0	0	0	0	10	2:(PSL(2,81):2)
10	2	5	0	0	0	0	0	0	0	6	2:(PSL(2,25):2)
5	5	5	0	0	0	0	0	0	0	5	2^2:PSL(3,4)

Progenitor for peaches2

G<x, y, t>:=Group< x, y, t | x^2 , y^4 , y^{-1} * x * y^{-2} * x * y^{-1} * x * y^2 * x , y^{-1} * x * y^{-1} * x * y^{-1} * x * y * x * y * x * y * x ,

t^2, (t,(x * y * x * y^{-1} * x * y)), (t,(x * y^2 * x)),

((x * y)*($t^{(y*x*y^2)}$))^a,

((y * x * y)*($t^{(y*x*y^{-1}*x*y^{-1})}$))^b,

((x * y * x * y^{-1} * x * y)*($t^{(y^2*x*y)}$))^c,

(y^2*($t^{(y*x)}$))^d,

((x * y)^3*($t^{(y*x*y)}$))^e,

((x * y * x * y^{-1} * x * y)*($t^{(y^2*x*y)}$))^f,

(y^2*($t^{(x^y)}$))^g,

(((x * y)^2)*($t^{(x*y^2*x*y)}$))^h,

((x * y)*($t^{(y*x*y*x*y^{-1})}$))^i,

((x * y)*($t^{(y^2*x*y)}$))^j,

((y^2 * x * y^2)*t)^k >

Peaches2											
a	b	c	d	e	f	g	h	i	j	k	Index
0	0	0	0	0	0	0	0	0	3	5	2:(A_5:A_5)
0	0	0	0	0	0	0	0	0	5	2	A_5
2	0	0	0	0	0	0	0	0	0	10	A_5
3	0	0	0	0	0	0	0	0	0	5	2:(A_5:A_5)
5	0	0	0	0	0	0	0	0	0	2	A_5
7	2	0	0	0	0	0	0	0	0	8	PSL(2,7)×2
8	2	0	0	0	0	0	0	0	0	8	2:(PSL(2,7):2)
9	2	0	0	0	0	0	0	0	0	9	PSL(2,19)
3	6	0	0	0	0	0	0	0	0	9	PSL(2,37)
3	9	0	0	0	0	0	0	0	0	6	PSL(2,37)
10	9	0	0	0	0	0	0	0	0	2	3:(2:PSL(2,19))

10.14 2^{*31}:(31:5)

We have the following information

S:=Sym(31)

x ∼ (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31)

y ∼ (1, 16, 8, 4, 2)(3, 17, 24, 12, 6)(5, 18, 9, 20, 10)(7, 19, 25, 28, 14)(11, 21, 26, 13, 22)(15, 23, 27, 29, 30)

#N = 155

Progenitor of fruitsalad

$G<x,y,t>:=Group<x,y,t|\ y^5\ ,\ y^{-1}*x^{-2}*y*x\ ,$

$t^2,\ (t,(x*y)^2),$

$(((x^{-1}*y^{-1}*x)^2)*t^{((y*x^{-1}*y^{-1})^2)})^a,$

$((x^5)*t^{(x*y^{-2}*x*y)})^b,$

$((x*y)*t^{(x*y^{-1}*x)})^c,$

$((y^{-1}*x^{-1}*y)*t^{((y*x^{-1}*y)^2)})^d,$

$((y^2*x^{-1}*y^2*x^{-1}*y)*t^{(x^{-1}*y^{-1})})^e,$

$(x*t^{((y*x^{-1}*y^{-1})^2)})^f,$

$((y^{-1}*x^{-1})*t^{(y^2*x^2)})^g,$

$(((x*y)^2)*t^{((y*x*y^{-1})^3)})^h,$

$((x^3)*t^{(x)})^i,$

$((x^5)*t^{(x*y^{-1}*x)}*t^{(x*y^{-1}*x)})^j >$

Fruitsalad										
a	b	c	d	e	f	g	h	i	j	G
10	0	0	2	0	0	0	0	0	0	2

Progenitor of fruitsalad2

$G<x,y,t>:=Group<x,y,t|\ y^5\ ,\ y^{-1}*x^{-2}*y*x\ ,$

$t^2,\ (t,(x*y)^2),$

$((x*y)*t^{(y)})^a,$

$(((x*y)^2)*t^{(x*y^2*x^3)})^b,$

$(((x^{-1}*y^{-1}*x)^2)*t^{(y^{-1}*x*y^{-1})})^c,$

$((y^{-1}*x^{-1})*t^{(x*y*x^3)})^d,$

$((x)*t^{(y*x*y)})^{e}$,

$((x^{3})*(t^{(x^{-3})}))^{f}$,

$((x^{5})*t^{(y^{-1}*x*y^{2}*x)})^{g}$,

$((x*y*x^{3}*y^{-1})*t^{(x*y^{-1}*x*y)})^{h}$,

$((y^{2}*x^{-1}*y^{2}*x^{-1}*y)*t^{((x*y*x^{-1})^{2})})^{i}$,

$((y^{-1}*x^{-1}*y)*t^{((y*x^{-1}*y)^{2})})^{j} >$

Fruitsalad2										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2

Progenitor of fruitsalad3

$G<x,y,t>:=Group<x,y,t|\ y^{5}\ ,\ y^{-1}*x^{-2}*y*x\ ,$

$t^{2},\ (t,(x*y)^{2})$,

$((y^{2}*x^{-1}*y^{2}*x^{-1}*y)*t^{(x^{5}*y^{-1})})^{a}$,

$((y^{-1}*x^{-1})*t^{((y*x^{-1}*y^{-1})^{2})})^{b}$,

$((x^{5})*t^{(y*x*y)})^{c}$,

$((x*y)*t^{(x*y^{-1}*x)})^{d}$,

$((x^{3})*t^{(y^{-1}*x^{-1}*y^{2})})^{e}$,

$((y^{-1}*x^{-1}*y)*t^{(y)})^{f}$,

$((x*y*x^{3}*y^{-1})*t^{(x^{2}*y^{-2}*x^{-1})})^{g}$,

$((x)*t^{(x^{5}*y^{-1})})^{h}$,

$((x^{3})*t^{(x*y^{-2}*x*y)})^{i}$,

$((x*y)*t^{(y^{2}*x^{-2}*y^{-1}*x^{-1}*y^{-1})})^{j}$,

$(((x*y)^{2})*t^{(x^{-3}*y^{-1}*x^{-1})})^{k} >$

Fruitsalad3											
a	b	c	d	e	f	g	h	i	j	k	G
0	0	0	0	0	0	0	0	2	0	0	2
2	0	0	0	0	0	0	0	0	0	0	2

Progenitor of fruitsalad4

$G<x,y,t>:=Group<x,y,t|\ y^{5}\ ,\ y^{-1}*x^{-2}*y*x\ ,$

$t^{2},\ (t,(x*y)^{2})$,

$((x * y * x^3 * y^{-1}) * t^{(x^5 * y^{-1})})^a$,

$((y^{-1} * x^{-1} * y) * t^{(y^{-1} * x * y^{-1})})^b$,

$((x * y) * t^{(x^3 * y^{-2} * x)})^c$,

$((x) * t^{(x * y * x^3)})^d$,

$((x^3) * t^{(y^{-1} * x * y^2 * x)})^e$,

$((x^5) * t^{(y * x^{-2} * y^{-1} * x^{-1} * y^{-1})})^f$,

$(((x * y)^2) * t^{(x)})^g$,

$(((x^{-1} * y^{-1} * x)^2) * t^{(x * y^{-1} * x * y^2)})^h$,

$((y^2 * x^{-1} * y^2 * x^{-1} * y) * t^{(y * x * y)})^i$,

$((y^{-1} * x^{-1} * y) * t^{(y * x^3 * y^{-2})})^j >$

Fruitsalad4										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	2	2
2	0	0	0	0	0	0	0	0	0	2

10.15 2^{*72}:((4:3):2)

We have the following information

S:=Sym(72)

x ∼ (1, 19, 51)(2, 52, 20)(3, 39, 29)(4, 30, 40)(5, 59, 7)(6, 8, 60)(9, 25, 37)(10, 38, 26)(11, 45, 15)(12, 16, 46)(13, 65, 67)(14, 68, 66)(17, 31, 23)(18, 24, 32)(21, 71, 53)(22, 54, 72)(27, 57, 61)(28, 62, 58)(33, 43, 69)(34, 70, 44)(35, 63, 47)(36, 48, 64)(41, 49, 55)(42, 56, 50)

y ∼ (1, 11, 71, 61)(2, 62, 72, 12)(3, 23, 69, 49)(4, 50, 70, 24)(5, 35, 67, 37)(6, 38, 68, 36)(7, 47, 65, 25)(8, 26, 66, 48)(9, 59, 63, 13)(10, 14, 64, 60)(15, 21, 57, 51)(16, 52, 58, 22)(17, 33, 55, 39)(18, 40, 56, 34)(19, 45, 53, 27)(20, 28, 54, 46)(29, 31, 43, 41)(30, 42, 44, 32)

z ∼ (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16)(17, 18)(19, 20)(21, 22)(23, 24)(25, 26)(27, 28)(29, 30)(31, 32)(33, 34)(35, 36)(37, 38)(39, 40)(41, 42)(43, 44)(45, 46)(47, 48)(49, 50)(51, 52)(53, 54)(55, 56)(57, 58)(59, 60)(61, 62)(63, 64)(65, 66)(67, 68)(69, 70)(71, 72)

#N = 24

Progenitor of hotcheetos

G<x,y,z,t>:=Group<x,y,z,t| x^3 , y^4 , z^2 , (x, y) , $(x^{-1} * z)^2$, $(y^{-1} * z)^2$, t^{37}, $(t^{(x^{-1}*y^{-1})}=t^{29})$,

$(y*t^{(z)})^a$,

$((y^2 * x^{-1})*t^{(x*y^2*z)})^b$,

$((y * z)*t^{(x*z*y)})^c$,

$((y^2)*t^{(x*z)})^d$,

$((x * y)*t^{(z*y)})^e$,

$((y * x^{-1})*t^{(z*x*y)})^f$,

$(x*t^{(z*x)})^g$,

$((y * z)*t^{(y*z*x)})^h$,

$(z*t^{(y*x^{-1})})^i$,

$(y*t^{(y^2)})^j$ >

Hotcheetos										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	3	0	PSL(2,37)
0	0	3	0	0	0	0	0	0	0	PSL(2,37)

Progenitor of hotcheetos1

G<x,y,z,t>:=Group<x,y,z,t| x^3 , y^4 , z^2 , (x, y) , $(x^{-1} * z)^2$, $(y^{-1} * z)^2$, t^{37},

$(t^{(x^{-1}*y^{-1})}=t^{29})$, $((y^2 * x^{-1})*t^{(y^{-1})})^a$,

$((x * y)*t^{(z*y)})^b$,

$((y * x^{-1})*t^{(y^2)})^c$,

$(x*t^{(y^2*z)})^d$,

$(y*t^{(z)})^e$,

$((y * z)*t^{(y)})^f$,

$(z*t^{(y*z)})^g$,

$((y^2)*t^{(y*x^{-1})})^h$,

$((x * y)*t^{(y*z*x)})^i$,

$((y * x^{-1})*t^{(x)})^j$ >

Hotcheetos1										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	3	0	0	0	PSL(2,37)

Progenitor of hotcheetos2

G<x,y,z,t>:=Group<x,y,z,t| x^3 , y^4 , z^2 , (x, y) , $(x^{-1} * z)^2$, $(y^{-1} * z)^2$, t^{37}, $(t^{(x^{-1}*y^{-1})}=t^{29})$,

$((y * x^{-1})*(t^{(x*y^2*z)}))^a$,

$((x * y)*(t^{(x)})*(t^{(x*z)}))^b$,

$((x)*(t^{(y^{-1})}))^c$,

$((z)*(t^{(y^2*z)}))^d$,

$((y * z)*(t^{(z*x)}))^e$,

$((y^2)*(t^{(y)}))^f$,

$((y * z)*(t^{(x*y^2*z)}))^g$,

$((x)*(t^{(y*z)}))^h$,

$((y)*(t^{(x)})*(t^{(x*z)}))^i$,

$((x * y)*(t^{(y*z*x)}))^j$ >

Hotcheetos2										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	2	0	3:(2:(PSL(2,37):2))
0	0	0	0	0	0	3	0	0	0	PSL(2,37)
0	2	0	0	0	0	0	0	0	0	2:(PSL(2,37):2)
0	0	6	3	0	0	0	0	0	0	PSL(2,37)

Progenitor of hotcheetos3

G<x,y,z,t>:=Group<x,y,z,t| x^3 , y^4 , z^2 , (x, y) , $(x^{-1} * z)^2$, $(y^{-1} * z)^2$, t^{37}, $(t^{(x^{-1}*y^{-1})}=t^{29})$,

$((y * x^{-1})*(t^{(y*z)}))^a$,

$((x * y)*(t^{(y^2*z*x)}))^b$,

$((y)*(t^{(y^{-1})})*(t^{(z*y)}))^c$,

$((x)*(t^{(y*z)}))^d$,

$((z)*(t^{(y^2*x^{-1})}))^e$,

$((y^2)*(t^{(x^{-1})}))^f$,

$((y * z)*(t^{(x*y^2*z)}))^g$,

$((y)*(t^{(y*z*x)}))^h$,
$((x)*(t^{(x*z)}))^i$,
$((x * y)*(t^{(x*y^2*z)}))^j$ >

Hotcheetos3										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	3	4	0	0	PSL(2,37)
0	0	2	0	0	0	0	0	0	0	3:(2:(PSL(2,37):2))

Progenitor of hotcheetos4

G<x,y,z,t>:=Group<x,y,z,t| x^3 , y^4 , z^2 , (x, y) , $(x^{-1} * z)^2$, $(y^{-1} * z)^2$, t^{37}, $(t^{(x^{-1}*y^{-1})}=t^{29})$,
$((y * x^{-1})*(t^{(y*x^{-1})})*(t^{(x*z)}))^a$,
$((x * y)*(t^{(x*y^2)}))^b$,
$((y)*(t^{(x^{-1}*y^{-1})}))^c$,
$((y * z)*(t^{(y^2)})*(t^{(y^2*z)}))^d$,
$((z)*(t^{(y)}))^e$,
$((y^2)*(t^{(y^2*z*x)}))^f$,
$((x)*(t^{(y^2*x^{-1})}))^g$,
$((y * z)*(t^{(y^2*z*x)}))^h$,
$((z)*(t^{(y^{-1})}))^i$,
$((y)*(t^{(x*y)}))^j$ >

Hotcheetos4										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	3	0	0	PSL(2,37)
2	0	0	0	0	0	0	0	0	0	2:(PSL(2,37):2)
0	0	8	3	0	0	0	0	0	0	PSL(2,37)

Progenitor of hotcheetos6

G<x,y,z,t>:=Group<x,y,z,t| x^3 , y^4 , z^2 , (x, y) , $(x^{-1} * z)^2$, $(y^{-1} * z)^2$, t^{37}, $(t^{(x^{-1}*y^{-1})}=t^{29})$,
$((y * x^{-1})*t^{(y^2*z)})^a$,
$((x * y)*t^{(y^2*x^{-1})})^b$,
$((y^2 * x^{-1})*t^{(y*z*x)})^c$,
$((x)*t^{(x*z*y)})^d$,

((y * z)*t$^{(x*y^2)}$)^e,
((x * y)*t$^{(x^{-1})}$)^f,
((x)*t$^{(y^2)}$)^g,
((y * z)*t$^{(x*y^2*z)}$)^h,
((y^2)*t$^{(x*y^2)}$)^i,
((z)*t$^{(y*z*x)}$)^j >

Hotcheetos6										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	3	PSL(2,37)

10.16 2^{*12}:((4:3):2)

We have the following information
S:=Sym(24)
x ∼ (1,17,5)(2,6,18)(3,9,11)(4,12,10)(7,19,23)(8,24,20)(13,21,15)(14,16,22)
y ∼ (1,9,23,15)(2,16,24,10)(3,19,21,5)(4,6,22,20)(7,13,17,11)(8,12,18,14)
zz ∼ (1,2)(3,4)(5,6)(7,8)(9,10)(11,12)(13,14)(15,16)(17,18)(19,20)(21,22)(23,24)
#N = 24
Progenitor of mangos1
G<x,y,z,t>:=Group<x,y,z,t| x^3 , y^4 , z^2 , (x, y) , (x^{-1} * z)^2 , (y^{-1} * z)^2 , t^{13}, (t$^{(x^2)}$ = t^3), (t$^{(y^3)}$ = t^8),
((y * x^{-1})*(t$^{(x*y)}$))^a,
((x * y)*(t$^{(z*x*y)}$))^b,
((y^2 * x^{-1})*(t$^{(x*z*y)}$))^c,
((x)*(t$^{(z*y)}$))^d,
((y * z)*(t$^{(y^2)}$))^e,
((z)*(t$^{(x*y^2*z)}$))^f,
((y^2)*(t$^{(x*y^2)}$))^g,
((y * z)*(t$^{(z*x)}$))^h,
((y^2 * x^{-1})*(t$^{(y)}$))^i,
((y * x^{-1})*(t$^{(y*z*x)}$))^j >

Mangos										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	4	0	0	2:(3:2):(PSL(2,13):2)
0	0	0	0	0	0	4	3	0	0	PSL(2,13)×2
0	0	0	0	3	0	0	0	0	0	PSL(2,13)×2
0	0	0	3	4	0	0	0	0	0	2:(3:2):(PSL(2,13):2)

Progenitor of mangos2

G<x,y,z,t>:=Group<x,y,z,t| x^3 , y^4 , z^2 , (x, y) , $(x^{-1} * z)^2$, $(y^{-1} * z)^2$, t^{13}, $(t^{(x^2)} = t^3)$, $(t^{(y^3)} = t^8)$,

$((y * x^{-1})*(t^{(y)}))^a$,

$((x * y)*(t^{(x*y^2*z)}))^b$,

$((y^2 * x^{-1})*(t^{(y*z*x)}))^c$,

$((y)*(t^{(x*y*z)}))^d$,

$((x)*(t^{(z*y)}))^e$,

$((y * z)*(t^{(x*z)}))^f$,

$((z)*(t^{(x*y*z)}))^g$,

$((y^2)*(t^{(x^{-1})}))^h$,

$((y * z)*(t^{(z)}))^i$,

$((y)*(t^{(y*z*x)}))^j$ >

Mangos2										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	3	0	PSL(2,13)×2
0	0	0	0	0	0	0	0	4	0	2:(3:2):(PSL(2,13):2)

Progenitor of mangos3

G<x,y,z,t>:=Group<x,y,z,t| x^3 , y^4 , z^2 , (x, y) , $(x^{-1} * z)^2$, $(y^{-1} * z)^2$, t^{13}, ($t^{(x^2)} = t^3$), ($t^{(y^3)} = t^8$),
$((x)*(t^{(y^2*z*x)}))^a$,
$((y * z)*(t^{(x)}))^b$,
$((y^2 * x^{-1})*(t^{(x*y^2)}))^c$,
$((y)*(t^{(z*x*y)}))^d$,
$((y * x^{-1})*(t^{(y*z)}))^e$,
$((x * y)*(t^{(y^2)}))^f$,
$((y^2 * x^{-1})*(t^{(x*y^{-1})}))^g$,
$((y)*(t^{(x*z)}))^h$,
$((x)*(t^{(y^2*z*x)}))^i$,
$((y * z)*(t^{(z*y)}))^j$ >

Mangos3										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	3	PSL(2,13)×2
0	0	0	0	0	0	0	0	0	4	2:(3:2):(PSL(2,13):2)
0	3	0	0	0	0	0	0	0	0	PSL(2,13)×2
0	4	0	0	0	0	0	0	0	0	2:(3:2):(PSL(2,13):2)

Progenitor of mangos4

G<x,y,z,t>:=Group<x,y,z,t| x^3 , y^4 , z^2 , (x, y) , $(x^{-1} * z)^2$, $(y^{-1} * z)^2$, t^{13}, ($t^{(x^2)} = t^3$), ($t^{(y^3)} = t^8$),
$((x)*(t^{(y*z)}))^a$,
$((y * z)*(t^{(z*y)}))^b$,
$((z)*(t^{(x^{-1}*y^{-1})}))^c$,
$((y^2)*(t^{(y^2*z*x)}))^d$,

$((y * x^{-1})*(t^{(x*y^2)}))^e$,
$((x * y)*(t^{(z*x*y)}))^f$,
$((y^2 * x^{-1})*(t^{(x^{-1}*y^{-1})}))^g$,
$((y)*(t^{(y^{-1})}))^h$,
$((x)*(t^{(z*y)}))^i$,
$((y * z)*(t^{(x)}))^j >$

Mangos4										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	0	3	PSL(2,13)×2
0	0	0	0	0	0	0	0	0	4	2:(3:2):(PSL(2,13):2)
0	3	0	0	0	0	0	0	0	0	PSL(2,13)×2
0	4	0	0	0	0	0	0	0	0	2:(3:2):(PSL(2,13):2)

Progenitor of mangos5

G<x,y,z,t>:=Group<x,y,z,t| x^3 , y^4 , z^2 , (x, y) , $(x^{-1} * z)^2$, $(y^{-1} * z)^2$, t^{13}, $(t^{(x^2)} = t^3)$, $(t^{(y^3)} = t^8)$,
$((y * x^{-1})*(t^{(x*y^2*z)}))^a$,
$((x * y)*(t^{(x)})*(t^{(x*z)}))^b$,
$((x)*(t^{(y^{-1})}))^c$,
$((z)*(t^{(y^2*z)}))^d$,
$((y * z)*(t^{(z*x)}))^e$,
$((y^2)*(t^{(y)}))^f$,
$((y * z)*(t^{(x*y^2*z)}))^g$,
$((x)*(t^{(y*z)}))^h$,
$((y)*(t^{(x)})*(t^{(x*z)}))^i$,
$((x * y)*(t^{(y*z*x)}))^j >$

Mangos5										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	0	0	2	0	3:(2:(PSL(2,13):2))
0	0	0	0	0	0	3	0	0	0	PSL(2,13)×2
0	0	0	0	0	0	4	0	0	0	2:(3:2):(PSL(2,13):2)
0	2	0	0	0	0	0	0	0	0	2:(PSL(2,13):2)
0	0	9	3	0	0	0	0	0	0	PSL(2,13)×2
0	0	3	4	0	0	0	0	0	0	2:(3:2):(PSL(2,13):2)

Progenitor of mangos6

G<x,y,z,t>:=Group<x,y,z,t| x^3 , y^4 , z^2 , (x, y) , $(x^{-1} * z)^2$, $(y^{-1} * z)^2$, t^{13}, $(t^{(x^2)} = t^3)$, $(t^{(y^3)} = t^8)$,

$((y * x^{-1})*(t^{(y*z)}))^a$,

$((x * y)*(t^{(y^2*z*x)}))^b$,

$((y)*(t^{(y^{-1})})*(t^{(z*y)}))^c$,

$((x)*(t^{(y*z)}))^d$,

$((z)*(t^{(y^2*x^{-1})}))^e$,

$((y^2)*(t^{(x^{-1})}))^f$,

$((y * z)*(t^{(x*y^2*z)}))^g$,

$((y)*(t^{(y*z*x)}))^h$,

$((x)*(t^{(x*z)}))^i$,

$((x * y)*(t^{(x*y^2*z)}))^j$ >

Mangos6										
a	b	c	d	e	f	g	h	i	j	G
0	0	0	0	0	0	3	4	0	0	PSL(2,13)×2
0	0	0	0	0	0	4	0	9	0	2:(3:2):(PSL(2,13):2)

10.17 2^{*80}:2:$^{\cdot}$(5:6)

We have the following information

S:=Sym(80)

x ∼ (1, 2, 4, 8, 15, 27, 40, 59, 73, 80)(3, 6, 12, 22, 14, 25, 13, 24, 37, 56)(5, 10, 19, 32, 47, 60, 61, 71, 75, 16)(7, 11, 21, 35, 52, 38, 57, 20, 9, 17)(18, 31, 46, 63, 42, 53, 23, 33, 49, 66)(26, 39, 58, 72, 55, 70, 79, 29, 43, 51)(28, 41, 48, 64, 36, 54, 68, 78, 44, 34)(30, 45, 62, 74, 65, 69, 76, 50, 67, 77)

y ∼ (1, 3, 7, 14, 26, 37)(2, 5, 11, 19, 33, 50)(4, 9, 18)(6, 13)(8, 16, 29, 44, 17, 30)(10, 20, 34, 51, 68, 72)(12, 23)(15, 28, 42, 45, 49, 67)(21, 36, 55, 41, 43, 61)(22, 27, 25, 38, 56, 70)(24, 31)(32, 48, 65, 75, 78, 77)(35, 53, 59)(40, 60, 57, 71, 46, 62)(47, 58, 64, 52, 69, 73)(54, 66, 76, 63, 74, 80)

#N = 1920

Progenitor of Eggs

G<x,y,t>:=Group<x,y,t | x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,$(y * x^{-2} * y)$), (t,$(y^{-1} * x^{-2} * y * x^{-1} * y^{-1})$),

$((x^2 * y^{-1} * x^{-1} * y)*t^{(x^{-1}*y^{-1}*x^{-1}*y*x^{-1}*y^{-2})})^a$,

$(((x * y^{-1})^3)*t^{(x*y^{-1}*x^{-4}*y)})^b$,

$((x^2 * y^{-1} * x^{-1} * y)*t^{((y*x*y^{-1})^4)})^c$,

$((y * x^4 * y * x^{-1})*t^{(x*y*x^2*y^2*x^{-1})})^d$,

$((y * x^4 * y * x^{-1})*t^{(x*y*x^2*y^2*x^{-1})})^e$,

$(((x * y * x^{-1})^3)*t^{(y^{-1}*x^{-1}*y*x^{-3}*y^{-1})})^f$,

$(((x * y * x^{-1})^3)*t^{(x^{-1}*y*x*y^{-1}*x*y^{-1}*x^{-1})})^g$,

$((y * x^4 * y * x^{-1})*t^{(y^{-1}*x^{-1}*y^{-1}*x^{-1}*y*x^{-1})})^h$,

$((x^4)*t^{(x^{-1}*y^{-1}*x^{-1}*y*x^{-1}*y^{-2})})^i$,

$((x * y^{-1} * x^{-1})*t^{(x^{-2}*y^{-1}*x*y^{-2})})^j$ >

Eggs										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	0	4	A_6 ×2
0	0	0	0	0	0	2	0	7	0	J_1

Progenitor of Eggs2

G<x,y,t>:=Group<x,y,t | x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,

t^2, (t,(y * x^{-2} * y)), (t,(y^{-1} * x^{-2} * y * x^{-1} * y^{-1})),
((x * y^{-1} * x^{-1})*$t^{(x*y^{-2}*x^{-1}*y^2*x)}$)^a,
((x * y * x^{-1})*$t^{(y*x^{-1}*y^{-1}*x^{-1}*y^2)}$)^b,
((x * y^{-1})*$t^{((y*x*y^{-1})^4)}$)^c,
((x * y^{-1})*$t^{((x*y^2)^2)}$)^d,
((x^2 * y)*$t^{(x^{-1}*y^{-1}*x*y^{-1}*x*y^{-1})}$)^e,
((x * y * x^{-1})*$t^{(y^3*x*y*x^{-1})}$)^f,
(((x * y^{-1})^2)*$t^{(y*x*y*x*y^{-1}*x*y^{-1})}$)^g,
((x^4)*$t^{(x^{-2}*y^{-1}*x*y^{-2})}$)^h,
((x^2)*$t^{(x^3*y*x^{-1}*y^2)}$)^i,
((x^2 * y^{-1} * x^{-1} * y)*$t^{(y^{-1}*x*y*x*y)}$)^j >

Eggs2										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	5	0	4	0	2^5:S(4,3)
2	0	0	4	0	0	0	0	0	0	S(4,3)×2

Progenitor of Eggs 3

G<x,y,t>:=Group<x,y,t | x^{10} , y^6 , (x * y^{-2} * x)^2 , (x * y^2 * x^2)^2 , (y^{-1} * x^{-1})^5 , (x * y^2 * x^{-1} * y^{-1})^2 , x^{-1} * y^{-1} * x^5 * y * x^{-4} , y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1} ,
t^2, (t,(y * x^{-2} * y)), (t,(y^{-1} * x^{-2} * y * x^{-1} * y^{-1})),
((x^5)*$t^{(x^{-1}*y^{-1}*x*y^{-1}*x*y^{-1})}$)^a,
((y * x^4 * y * x^{-1})*$t^{(x*y*x*y*x^{-2}*y)}$)^b,
((x^2 * y^{-1} * x^{-1} * y)*$t^{(y^2*x*y*x)}$)^c,
((x^4)*$t^{(x^{-1}*y^{-2}*x^{-1}*y)}$)^d,
(((x * y^{-1})^2)*$t^{(y*x^{-1}*y^{-2})}$)^e,
((x * y * x^{-1})*$t^{(y*x^{-2}*y*x^2)}$)^f,
((x * y^{-1} * x^{-1})*$t^{(y*x^2*y^{-1}*x*y*x)}$)^g,
((x^2 * y)*$t^{(y^{-1}*x^{-1}*y^{-1}*x^{-1}*y*x^{-1})}$)^h,
((x)*$t^{((y*x*y^{-1})^4)}$)^i,
((x^3)*$t^{((x*y^2)^2)}$)^j >

Eggs3										
a	b	c	d	e	f	g	h	i	j	Index
2	0	0	4	0	0	0	0	0	0	S(4,3)×2

Progenitor of Eggs4

G<x,y,t>:=Group<x,y,t | x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$, t^2, $(t,(y * x^{-2} * y))$, $(t,(y^{-1} * x^{-2} * y * x^{-1} * y^{-1}))$,

$((x)*t^{(y^3*x*y*x^{-1})})^a$,

$((x^3)*t^{(y*x*y^{-1}*x^3*y^{-1})})^b$,

$((x^2 * y)*t^{(y^3*x*y*x^{-1})})^c$,

$((x * y^{-1} * x^{-1})*t^{(y^3*x^{-1}*y*x*y)})^d$,

$((x * y * x^{-1})*t^{(x*y*x^{-1}*y*x^{-1}*y^{-1}*x)})^e$,

$((x^4)*t^{(x^2*y^3*x*y*x)})^f$,

$((x^2)*t^{(x^2*y*x*y*x^3)})^h$,

$((x^2 * y^{-1} * x^{-1} * y)*t^{(x^{-1}*y^{-1}*x^{-1}*y*x^{-1}*y^{-2})})^i$,

$(((x * y^{-1})^3)*t^{(x*y*x*y^2)})^j$ >

Eggs4										
a	b	c	d	e	f	g	h	i	j	Index
0	0	0	0	0	0	0	0	8	2	PSL(2,49)

Progenitor of Eggs5

G<x,y,t>:=Group<x,y,t | x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$, t^2, $(t,(y * x^{-2} * y))$, $(t,(y^{-1} * x^{-2} * y * x^{-1} * y^{-1}))$,

$((x * y^{-1})*t^{(x*y*x^2*y^2*x^{-1})})^a$,

$((x)*t^{(x*y*x^2*y^{-1}*x^{-1}*y^{-1})})^b$,

$((x^2 * y)*t^{((y*x*y^{-1})^4)}*t^{(y^{-1}*x*y^{-1}*x^3)})^c$,

$((x * y^{-1} * x^{-1})*t^{(x*y^{-1}*x^{-3}*y)})^d$,

$((x * y * x^{-1})*t^{(y^{-1}*x^{-1}*y^3*x^{-1})})^e$,

$(((x * y^{-1})^2)*t^{(x^{-1}*y^{-1}*x^{-1}*y*x^{-1})}*t^{(y^{-2})})^f$,

$((x^4)*t^{((x*y^2)^2)})^g$,
$((x^2)*t^{(y^2*x*y*x^2)})^h$,
$((x^2 * y^{-1} * x^{-1} * y)*t^{(x*y^{-1}*x^{-1}*y^2*x*y^{-1})})^i$,
$((x * y^{-2} * x)*t^{(x*y^{-1}*x^{-4}*y)}*t^{(x^2*y*x*y*x^3)})^j >$

Eggs5										
a	b	c	d	e	f	g	h	i	j	Index
3	0	3	0	0	0	0	0	8	2	S(4,5)

Progenitor of Eggs6

G<x,y,t>:=Group<x,y,t | x^{10} , y^6 , $(x * y^{-2} * x)^2$, $(x * y^2 * x^2)^2$, $(y^{-1} * x^{-1})^5$, $(x * y^2 * x^{-1} * y^{-1})^2$, $x^{-1} * y^{-1} * x^5 * y * x^{-4}$, $y * x^{-2} * y^{-1} * x^3 * y * x * y^3 * x^{-1}$,
t^2, $(t,(y * x^{-2} * y))$, $(t,(y^{-1} * x^{-2} * y * x^{-1} * y^{-1}))$,
$((x^2 * y)*t^{(x*y^{-1}*x*y)})^a$,
$((x^2)*t^{((x*y^2)^2)})^b$,
$((y * x^4 * y * x^{-1})*t^{(x^{-1}*y^{-1}*x^{-1}*y^{-1}*x*y*x)})^c$,
$((x * y^{-2} * x)*t^{(x^2*y*x*y*x^3)})^d$,
$((x^5)*t^{(y^{-2})})^e$,
$((x^2 * y^{-1} * x^{-1} * y)*t^{(x*y*x^{-1}*y*x*y^{-1})})^f$,
$((x^4)*t^{(y^{-1}*x^3*y^{-1}*x)})^g$,
$((x * y * x^{-1})*t^{(y*x^{-2}*y*x^2)})^h$,
$((x * y^{-1})*t^{(x*y*x^2*y^2*x^{-1})})^i$,
$((x^2 * y)*t^{(y*x^{-1}*y*x^2*y^{-1})})^j >$

Eggs6										
a	b	c	d	e	f	g	h	i	j	Index
3	0	8	0	0	0	0	0	8	2	S(4,7)

Appendix A

Magma Code

$$\frac{2^{*32}{:}(2^5{:}A_5)}{(xy^{-2}xy)^6 t_2 t_9 t_{13} t_{15} t_4 t_2, (x^5 * y^3) t_1 t_4 t_1} = \mathbf{J}_2$$

```
S:=Sym(32);
xx:=S!(1, 2)(3, 5, 7, 11, 17, 4, 6, 9, 14, 22)(8, 13, 20, 29,
    23, 10, 16, 25, 30,18)(12, 19, 28, 32, 26, 15, 24, 27, 31,
    21) ;
yy:=S!(1, 3)(2, 4)(5, 8)(6, 10)(7, 12, 17, 27, 14, 23) (9, 15,
    22, 28, 11, 18)(13, 21, 24, 30, 32, 20)(16, 26, 19, 29, 31,
    25);
N:=sub<S|xx,yy>;
NN<a,b>:=Group<a,b|a^10 ,
b^6 ,
(a * b^-2 * a)^2 ,
(a * b^2 * a^2)^2 ,
(b^-1 * a^-1)^5 ,
(a * b^2 * a^-1 * b^-1)^2 ,
a^-1 * b^-1 * a^5 * b * a^-4 ,
b * a^-2 * b^-1 * a^3 * b * a * b^3 * a^-1>;
G1:=N;G:=NN;
```

```
word:=function(A)
Sch:=SchreierSystem(G,sub<G|Id(G)>);
for i in [2..#G] do
P:=[Id(G1): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=xx; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=xx^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=yy; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=yy^-1; end if;
end for;
PP:=Id(G1);
for k in [1..#P] do
PP:=PP*P[k]; end for;
if A eq PP then B:=Sch[i]; end if;
end for;
return B;
end function;
/*TAKING OFF COMMENTS HERE ENDS AT TTT*/

T:={2..32};
TT:=Setseq(Set(G1));
TTT:=[Id(G) : i in [1..32]];
for j in T do for i in [1..#G1] do   if 1^TT[i] eq j then TTT[j
    ]:= word(TT[i]); break; break; end if;
T:= T diff {j};end for; end for;
TTT;
G<x,y,t>:=Group<x,y,t|x^10 ,
y^6 ,
(x * y^-2 * x)^2 ,
(x * y^2 * x^2)^2 ,
(y^-1 * x^-1)^5 ,
(x * y^2 * x^-1 * y^-1)^2 ,
```

```
x^-1 * y^-1 * x^5 * y * x^-4 ,
y * x^-2 * y^-1 * x^3 * y * x * y^3 * x^-1 ,
t^2,
(t,x^2),
(t,y^2),
((x * y^-2 * x * y)*t^(x))^6,
(x * t)^5>;
f,G1,k:=CosetAction(G,sub<G|x,y>);
A:=[ Id(G), y^2 * x^-1 * y^2 * x^2, x * y^2 * x * y, x * y^-2 *
    x^-2 * y^-1, y^3 * x
* y^-2, y^3 * x^-1 * y^-2 * x^-1, x^-2 * y * x^2, x * y * x^-2 *
    y * x, y * x^-1 *
y^-2, x * y^-1 * x^-1 * y^3 * x, x^3 * y * x^2 * y^-2, x * y^-1
    * x * y * x^-2 *
y^-1 * x^-1, y * x^-1 * y * x^-1 * y * x^-1 * y, y * x * y^-1 *
    x^-1 * y * x, y^-1
* x^-1 * y^-1, x^-1 * y * x * y * x, y * x * y^-1 * x^-1 * y * x
    ^-3, x^2 * y^-1 *
x * y * x^-1, x^-2 * y^-1 * x^2 * y * x, y^-1 * x * y^-1 * x * y
    ^-1, y^-1 * x^-1 *
y * x^-3, x * y^-2 * x^-1 * y^-1 * x^-1, y^2 * x * y^-1 * x * y
    ^-1 * x^-1, y^2 * x
* y * x^-1 * y * x^-1, x * y^-1 * x^-1 * y * x^-1 * y * x^-1, y
    ^-1 * x^-1 * y *
x^2, y^2 * x^-1 * y * x^-1 * y, x^4 * y * x^-1 * y, x * y * x^-1
    * y * x * y^-1,
y^-1 * x^-2 * y^-1 * x^-1 * y, x^2 * y^-1 * x^-2 * y^-1 * x, y
    ^-1 * x * y^-1 * x *
y^-2 ];
ts:=[Id(G1) : i in [1..32]];
ts[1]:=f(t);
for i in [2..32] do ts[i]:=f(t^A[i]); end for;
```

```
S:=Sym(32);
xx:=S!(1, 2)(3, 5, 7, 11, 17, 4, 6, 9, 14, 22)(8, 13, 20, 29,
    23, 10, 16, 25,
30,18)(12, 19, 28, 32, 26, 15, 24, 27, 31, 21) ;
yy:=S!(1, 3)(2, 4)(5, 8)(6, 10)(7, 12, 17, 27, 14, 23) (9, 15,
    22, 28, 11, 18)(13, 21, 24, 30, 32, 20)(16, 26, 19, 29, 31,
    25);
N:=sub<S|xx,yy>;
NN<a,b>:=Group<a,b|a^10 ,b^6 ,
(a * b^-2 * a)^2 ,
(a * b^2 * a^2)^2 ,
(b^-1 * a^-1)^5 ,
(a * b^2 * a^-1 * b^-1)^2 ,
a^-1 * b^-1 * a^5 * b * a^-4 ,
b * a^-2 * b^-1 * a^3 * b * a * b^3 * a^-1>;
G1:=N;
G:=NN;
word:=function(A)
Sch:=SchreierSystem(G,sub<G|Id(G)>);
for i in [2..#G] do
P:=[Id(G1): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=xx; end if;
 if Eltseq(Sch[i])[j] eq -1 then P[j]:=xx^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=yy; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=yy^-1; end if;
end for;
PP:=Id(G1);
for k in [1..#P] do
PP:=PP*P[k]; end for;
if A eq PP then B:=Sch[i]; end if;
```

```
end for;
return B;
end function;
/*TAKING OFF COMMENTS HERE AT TTT*/

T:={2..32};
 TT:=Setseq(Set(G1));
 TTT:=[Id(G) : i in [1..32]];
 for j in T do for i in [1..#G1] do   if 1^TT[i] eq j then TTT[j
     ]:= word(TT[i]);
 break; break; end if; T:= T diff {j};end for; end for;
TTT;
G<x,y,t>:=Group<x,y,t|x^10 ,
 y^6 ,
 (x * y^-2 * x)^2 ,
 (x * y^2 * x^2)^2 ,
 (y^-1 * x^-1)^5 ,
 (x * y^2 * x^-1 * y^-1)^2 ,
 x^-1 * y^-1 * x^5 * y * x^-4 ,
 y * x^-2 * y^-1 * x^3 * y * x * y^3 * x^-1 ,
 t^2,
 (t,x^2),
 (t,y^2),
 ((x * y^-2 * x * y)*t^(x))^6,
 (x * t)^5>;
 f,G1,k:=CosetAction(G,sub<G|x,y>);
 A:=[ Id(G), y^2 * x^-1 * y^2 * x^2, x * y^2 * x * y, x * y^-2 *
      x^-2 * y^-1,y^3 * x
 * y^-2, y^3 * x^-1 * y^-2 * x^-1, x^-2 * y * x^2, x * y * x^-2
     * y * x, y *x^-1 *
 y^-2, x * y^-1 * x^-1 * y^3 * x, x^3 * y * x^2 * y^-2, x * y^-1
      * x * y * x^-2 *
```

```
 y^-1 * x^-1, y * x^-1 * y * x^-1 * y * x^-1 * y, y * x * y^-1 *
     x^-1 * y * x
, y^-1
 * x^-1 * y^-1, x^-1 * y * x * y * x, y * x * y^-1 * x^-1 * y *
    x^-3, x^2 * y^-1 *
 x * y * x^-1, x^-2 * y^-1 * x^2 * y * x, y^-1 * x * y^-1 * x *
    y^-1, y^-1 *
x^-1 * y * x^-3, x * y^-2 * x^-1 * y^-1 * x^-1, y^2 * x * y^-1 *
    x * y^-1 * x^-1, y^2 * x
 * y * x^-1 * y * x^-1, x * y^-1 * x^-1 * y * x^-1 * y * x^-1, y
    ^-1 * x^-1 *
y *
x^2, y^2 * x^-1 * y * x^-1 * y, x^4 * y * x^-1 * y, x * y * x^-1
    * y * x * y^-1,
 y^-1 * x^-2 * y^-1 * x^-1 * y, x^2 * y^-1 * x^-2 * y^-1 * x, y
    ^-1 * x * y^-1
 * x * y^-2 ];
 ts:=[Id(G1) : i in [1..32]];
 ts[1]:=f(t);
 for i in [2..32] do ts[i]:=f(t^A[i]); end for;

/*INSERTING OTHER STUFF*/
a:=0;b:=0;c:=0;d:=0;e:=0;f:=0;g:=0;h:=6;i:=0;j:=0;
G<x,y,t>:=Group<x,y,t|x^10 ,
y^6 ,
(x * y^-2 * x)^2 ,
(x * y^2 * x^2)^2 ,
(y^-1 * x^-1)^5 ,
(x * y^2 * x^-1 * y^-1)^2 ,
x^-1 * y^-1 * x^5 * y * x^-4 ,
y * x^-2 * y^-1 * x^3 * y * x * y^3 * x^-1 ,
t^2,
```

```
(t,x^2),
(t,y^2),
((x * y^-1 * x * y^-2 * x * y)*t^(x^4 * y^-1 * x))^a,
((x * y^-1 * x * y^-2 * x * y)*t^(x))^b,
((x * y^-1 * x * y^-2 * x * y)*t^(x^2 * y^2 * x * y * x^-1))^c,
((x^2)*t^(y^-1 * x^-1 * y^-1 * x^-3))^d,
((x^2)*t^(x^2 * y^2 * x * y * x^-1))^e,
((x^2)*t^(y * x * y^-2 * x * y * x))^f,
((x * y^-2 * x * y)*t^(x^2 * y^-1 * x * y * x^-2 * y^-1))^g,
((x * y^-2 * x * y)*t^(x))^h,
((x * y^-2 * x * y)*t^(x^2 * y^2 * x * y * x^-1))^i,
(((y * x^-1)^2)*t^(y * x^-2 * y * x * y^-1 * x))^j,
(x * t)^5>;

Index(G,sub<G|x,y>);
/*315*/
f,G1,k:=CosetAction(G,sub<G|x,y>);

#sub<G1|f(x),f(y)>;
/*1920*/
CompositionFactors(G1);
/* G
     |   J2
     1
*/
Index(G,sub<G|x,y>);
/*315*/
f,G1,k:=CosetAction(G,sub<G|x,y>);
#sub<G1|f(x),f(y)>;
CompositionFactors(G1);
#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);
/*6*/
```

```
DC:=[f(Id(G)), f(t), f(t * y * t), f(t * x * t),f(t * y * x * y
    * t),f(t * x * t * y* t)];
IN:=sub<G1|f(x),f(y)>;
Index(G1,IN);
/*315*/
cst := [null : i in [1 .. Index(G1,IN)]] where null is [Integers
    () | ];
prodim := function(pt, Q, I)
v := pt;
for i in I do
     v := v^(Q[i]);
    end for;
return v;
end function;
for i := 1 to 32 do
      cst[prodim(1, ts, [i])] := [i];
     end for;
m:=0; for i in [1..315] do if cst[i] ne [] then m:=m+1; end if;
   end for;m;
/*32*/
Orbits(N);
/*
GSet{@ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,
   17, 18, 19,
    20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32 @}
]
*/
for i in [1..#DC] do for m,n in IN do if ts[1] eq m*(DC[i])^n
   then i; break;end if; end for;end for;
/*2*/
N1:=Stabiliser(N,1);
Generators(N1);
```

```
/*{
    (3, 7, 17, 6, 14)(4, 9, 22, 5, 11)(8, 20, 23, 16, 30)(10,
        25, 18, 13,
         29)(12, 28, 26, 24, 31)(15, 27, 21, 19, 32),
    (7, 17, 14)(9, 22, 11)(12, 27, 23)(13, 24, 32)(15, 28, 18)
        (16, 19, 31)(20,
         21, 30)(25, 26, 29)
}
*/
Orbits(N1);
/*

[
    GSet{@ 1 @},
    GSet{@ 2 @},
    GSet{@ 3, 7, 17, 6, 14 @},
    GSet{@ 4, 9, 22, 5, 11 @},
    GSet{@ 8, 20, 23, 21, 16, 12, 19, 30, 28, 27, 32, 31, 26,
        18, 15, 13, 24,
    29, 10, 25 @}
]
*/
#N/#N1;
/*32*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[1] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*1*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[3] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
```

```
/*3*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[4] eq m*(DC[i
   ])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[8] eq m*(DC[i
   ])^n then i; break; end if; end for;end for;
/*5*/

/*THE BEGINNING OF 1,2*/
S:={[1,2]};
SS:=S^N;SS;
SSS:=Setseq(SS);
for i in [1..#SSS] do
for g in IN do if ts[1]*ts[2]
eq g*ts[Rep(SSS[i])[1]]*ts[Rep(SSS[i])[2]]
 then print SSS[i];
end if; end for; end for;
/*
{
    [ 1, 2 ]
}
*/
N12:=Stabiliser(N,[1,2]);
#N12;
/*60*/
N12s:=N12;
#N12s;
/*60*/
Generators(N12s);
/*
(3, 7, 17, 6, 14)(4, 9, 22, 5, 11)(8, 20, 23, 16, 30)(10, 25,
   18, 13,
```

```
        29)(12, 28, 26, 24, 31)(15, 27, 21, 19, 32),
    (7, 17, 14)(9, 22, 11)(12, 27, 23)(13, 24, 32)(15, 28, 18)
       (16, 19, 31)(20,
        21, 30)(25, 26, 29)
*/

tr1:=Transversal(N,N12s);
for i:=1 to #tr1 do
ss:=[1,2]^tr1[i];
cst[prodim(1,ts,ss)]:=ss;
end for;
m:=0; for i in [1..315] do if cst[i] ne []
then m:=m+1;
end if; end for;m;
/*64*/
Orbits(N12s);
/*[
    GSet{@ 1 @},
    GSet{@ 2 @},
    GSet{@ 3, 7, 17, 6, 14 @},
    GSet{@ 4, 9, 22, 5, 11 @},
    GSet{@ 8, 20, 23, 21, 16, 12, 19, 30, 28, 27, 32, 31, 26,
       18, 15, 13, 24, 29,
    10, 25 @}
*/
#N/#N12s;
/*32*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[1] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[2] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
```

```
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[3] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*3*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[4] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*6*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[8] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*3*/

/*THE BEGINNIG OF 1,3*/
S:={[1,3]};
SS:=S^N;SS;
SSS:=Setseq(SS);
for i in [1..#SSS] do
for g in IN do if ts[1]*ts[3]
eq g*ts[Rep(SSS[i])[1]]*ts[Rep(SSS[i])[2]]
 then print SSS[i];
end if; end for; end for;
/*[1,3]*/
N13:=Stabiliser(N,[1,3]);
#N13;
/*12*/
N13s:=N13;
#N13s;
/*12*/
Generators(N13s);
/*
    (7, 17, 14)(9, 22, 11)(12, 27, 23)(13, 24, 32)(15, 28, 18)
       (16, 19, 31)(20, 21,
        30)(25, 26, 29),
```

```
    (5, 22, 11)(6, 17, 14)(8, 18, 15)(10, 23, 12)(13, 26, 20)
        (16, 21, 25)(19, 29,
         32)(24, 30, 31)
}
*/
tr1:=Transversal(N,N13s);
for i:=1 to #tr1 do
ss:=[1,3]^tr1[i];
cst[prodim(1,ts,ss)]:=ss;
end for;
m:=0; for i in [1..315] do if cst[i] ne []
then m:=m+1;
end if; end for;m;
/*224*/
Orbits(N13s);
/*      GSet{@ 1 @},
    GSet{@ 2 @},
    GSet{@ 3 @},
    GSet{@ 4 @},
    GSet{@ 5, 22, 11, 9 @},
    GSet{@ 6, 17, 14, 7 @},
    GSet{@ 8, 18, 15, 28 @},
    GSet{@ 10, 23, 12, 27 @},
    GSet{@ 13, 24, 26, 32, 30, 29, 20, 19, 31, 25, 21, 16 @}
*/
#N/#N13s;
/*160*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[3]*ts[1] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*3*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[3]*ts[2] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
```

```
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[3]*ts[3] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[3]*ts[4] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*6*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[3]*ts[5] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*5*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[3]*ts[6] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*3*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[3]*ts[8] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*5*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[3]*ts[10] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[3]*ts[13] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*3*/

/*THE BEGINNING OF 1,8*/
S:={[1,8]};
SS:=S^N;SS;
SSS:=Setseq(SS);
for i in [1..#SSS] do
for g in IN do if ts[1]*ts[8]
eq g*ts[Rep(SSS[i])[1]]*ts[Rep(SSS[i])[2]]
 then print SSS[i];
end if; end for; end for;
```

```
/*
    [ 1, 8 ]
}
{
    [ 3, 5 ]
}
{
    [ 29, 23 ]
}
{
    [ 31, 7 ]
}
{
    [ 19, 14 ]
}
{
    [ 16, 17 ]
}
{
    [ 25, 12 ]
}
{
    [ 26, 27 ]
}
*/
N18:=Stabiliser(N,[1,8]);
#N18;
/*3*/
N18s:=N18;
#N18s;
/*3*/
Generators(N18s);
```

```
/*
{
    (7, 17, 14)(9, 22, 11)(12, 27, 23)(13, 24, 32)(15, 28, 18)
       (16, 19, 31)(20, 21,
        30)(25, 26, 29)
}
*/
for n in N do if [1,8]^n eq [3,5] then N18s:=sub<N|N18s,n>; end
   if; end for;
#N18s;
/*6*/
[1,8]^N18s;
/*[ 1, 8 ],
    [ 3, 5 ]
*/
for n in N do if [1,8]^n eq [29,23] then N18s:=sub<N|N18s,n>;
   end if; end for;
#N18s;
/*24*/
[1,8]^N18s;
/*      [ 1, 8 ],
    [ 3, 5 ],
    [ 29, 23 ],
    [ 16, 17 ],
    [ 25, 12 ],
    [ 31, 7 ],
    [ 19, 14 ],
    [ 26, 27 ]
@}
*/
Generators(N18s);
tr1:=Transversal(N,N18s);
```

```
for i:=1 to #tr1 do
ss:=[1,8]^tr1[i];
cst[prodim(1,ts,ss)]:=ss;
end for;
m:=0; for i in [1..315] do if cst[i] ne []
then m:=m+1;
end if; end for;m;
/*304*/
Orbits(N18s);
/*  GSet{@ 1, 3, 29, 16, 25, 31, 19, 26 @},
    GSet{@ 2, 4, 30, 13, 20, 32, 24, 21 @},
    GSet{@ 5, 8, 17, 23, 14, 27, 12, 7 @},
    GSet{@ 6, 10, 22, 18, 11, 28, 15, 9 @}
*/
#N/#N18s;
/*80*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[8]*ts[1] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*3*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[8]*ts[2] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*5*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[8]*ts[8] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[8]*ts[6] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*3*/

/*THE BEGINNING OF 1,2,4*/
S:={[1,2,4]};
SS:=S^N;SS;
```

```
SSS:=Setseq(SS);
for i in [1..#SSS] do
for g in IN do if ts[1]*ts[2]*ts[4]
eq g*ts[Rep(SSS[i])[1]]*ts[Rep(SSS[i])[2]]*ts[Rep(SSS[i])[3]]
 then print SSS[i];
end if; end for; end for;
/*
    [ 1, 2, 4 ]
    [ 3, 4, 2 ]

    [ 2, 1, 3 ]

    [ 4, 3, 1 ]

    [ 24, 19, 25 ]

    [ 25, 20, 24 ]

    [ 30, 29, 16 ]

    [ 16, 13, 30 ]

    [ 20, 25, 19 ]

    [ 19, 24, 20 ]

    [ 13, 16, 29 ]

    [ 29, 30, 13 ]

    [ 32, 31, 26 ]
    [ 26, 21, 32 ]
```

```
      [ 21, 26, 31 ]

      [ 31, 32, 21 ]

*/
N124:=Stabiliser(N,[1,2,4]);
#N124;
/*12*/
N124s:=N124;
#N124s;
/*12*/
Generators(N124s);
/*(5, 11, 9)(6, 14, 7)(8, 15, 28)(10, 12, 27)(13, 31, 25)(16,
    32, 20)(19, 30,
          26)(21, 24, 29),
      (5, 22, 11)(6, 17, 14)(8, 18, 15)(10, 23, 12)(13, 26, 20)
          (16, 21, 25)(19, 29,
          32)(24, 30, 31),
      (7, 14, 17)(9, 11, 22)(12, 23, 27)(13, 32, 24)(15, 18, 28)
          (16, 31, 19)(20, 30,
          21)(25, 29, 26)
}
*/
for n in N do if [1,2,4]^n eq [3,4,2] then N124s:=sub<N|N124s,n
    >; end if; end for;
#N124s;
/*24*/
[1,2,4]^N124s;
/*[ 1, 2, 4 ],
      [ 3, 4, 2 ]
*/
```

```
for n in N do if [1,2,4]^n eq [2,1,3] then N124s:=sub<N|N124s,n
   >; end if; end for;
#N124s;
/*48*/
[1,2,4]^N124s;
/*    [ 1, 2, 4 ],
     [ 3, 4, 2 ],
     [ 2, 1, 3 ],
     [ 4, 3, 1 ]
*/
for n in N do if [1,2,4]^n eq [24,19,25] then N124s:=sub<N|N124s
   ,n>; end if; end for;
#N124s;
/*192*/
[1,2,4]^N124s;
/* [ 1, 2, 4 ],
     [ 3, 4, 2 ],
     [ 2, 1, 3 ],
     [ 24, 19, 25 ],
     [ 4, 3, 1 ],
     [ 20, 25, 19 ],
     [ 19, 24, 20 ],
     [ 13, 16, 29 ],
     [ 29, 30, 13 ],
     [ 30, 29, 16 ],
     [ 21, 26, 31 ],
     [ 32, 31, 26 ],
     [ 16, 13, 30 ],
     [ 25, 20, 24 ],
     [ 26, 21, 32 ],
     [ 31, 32, 21 ]
*/
```

```
Generators(N124s);
/*{
    (1, 3)(2, 4)(5, 15)(6, 12)(7, 23)(8, 11)(9, 18)(10, 14)(13,
       16)(17, 27)(21,
        31)(22, 28)(26, 32)(29, 30),
    (1, 24, 13)(2, 19, 16)(3, 20, 30)(4, 25, 29)(7, 22, 15)(9,
       17, 12)(11, 27,
        18)(14, 28, 23),
    (5, 22, 11)(6, 17, 14)(8, 18, 15)(10, 23, 12)(13, 26, 20)
       (16, 21, 25)(19, 29,
        32)(24, 30, 31),
    (1, 2)(3, 4)(5, 14, 9, 6, 11, 7)(8, 12, 28, 10, 15, 27)(13,
       32, 25, 16, 31,
        20)(17, 22)(18, 23)(19, 29, 26, 24, 30, 21),
    (1, 24, 4, 25)(2, 19, 3, 20)(5, 27, 10, 9)(6, 28, 8, 7)(11,
       17, 12, 18)(13,
        31, 29, 21)(14, 22, 15, 23)(16, 32, 30, 26),
    (1, 24, 3, 20)(2, 19, 4, 25)(5, 15, 8, 11)(6, 12, 10, 14)(7,
        22, 27, 18)(9,
        17, 28, 23)(13, 26, 30, 31)(16, 21, 29, 32),
    (5, 11, 9)(6, 14, 7)(8, 15, 28)(10, 12, 27)(13, 31, 25)(16,
       32, 20)(19, 30,
        26)(21, 24, 29),
    (1, 2)(3, 4)(5, 7, 22, 6, 9, 17)(8, 27, 18, 10, 28, 23)(11,
       14)(12, 15)(13,
        26, 19, 16, 21, 24)(20, 30, 31, 25, 29, 32),
    (1, 3)(2, 4)(5, 28)(6, 27)(7, 10)(8, 9)(11, 18)(12, 17)(13,
       29)(14, 23)(15,
        22)(16, 30)(19, 24)(20, 25),
    (1, 24, 29, 3, 20, 16)(2, 19, 30, 4, 25, 13)(5, 17, 28, 8,
       23, 9)(6, 22, 27,
        10, 18, 7)(11, 15)(12, 14)(21, 32)(26, 31),
```

(1, 3)(2, 4)(5, 28, 11, 8, 9, 15)(6, 27, 14, 10, 7, 12)(13,
19, 31, 30, 25,
26)(16, 24, 32, 29, 20, 21)(17, 23)(18, 22),
(1, 24, 2, 19)(3, 20, 4, 25)(5, 17, 6, 22)(7, 12, 9, 15)(8,
23, 10, 18)(11,
27, 14, 28)(13, 21, 16, 26)(29, 31, 30, 32),
(1, 3)(2, 4)(5, 8)(6, 10)(7, 27)(9, 28)(11, 15)(12, 14)(13,
30)(16, 29)(17,
23)(18, 22)(19, 25)(20, 24)(21, 32)(26, 31),
(1, 24, 26, 2, 19, 21)(3, 20, 31, 4, 25, 32)(5, 15, 7, 6,
12, 9)(8, 11, 27,
10, 14, 28)(13, 16)(17, 22)(18, 23)(29, 30),
(1, 3)(2, 4)(5, 18)(6, 23)(7, 12)(8, 22)(9, 15)(10, 17)(11,
28)(14, 27)(19,
20)(21, 26)(24, 25)(31, 32),
(1, 2)(3, 4)(5, 6)(7, 22, 14, 9, 17, 11)(8, 10)(12, 28, 23,
15, 27, 18)(13,
19, 32, 16, 24, 31)(20, 26, 30, 25, 21, 29),
(1, 3)(2, 4)(5, 15, 22, 8, 11, 18)(6, 12, 17, 10, 14, 23)(7,
27)(9, 28)(13,
24, 26, 30, 20, 31)(16, 19, 21, 29, 25, 32),
(1, 2)(3, 4)(5, 14, 22, 6, 11, 17)(7, 9)(8, 12, 18, 10, 15,
23)(13, 25, 26,
16, 20, 21)(19, 31, 29, 24, 32, 30)(27, 28),
(1, 24, 32)(2, 19, 31)(3, 20, 21)(4, 25, 26)(7, 12, 18)(9,
15, 23)(11, 17,
28)(14, 22, 27),
(7, 14, 17)(9, 11, 22)(12, 23, 27)(13, 32, 24)(15, 18, 28)
(16, 31, 19)(20, 30,
21)(25, 29, 26),
(1, 2)(3, 4)(5, 7, 11, 6, 9, 14)(8, 27, 15, 10, 28, 12)(13,
20, 31, 16, 25,

32)(17, 22)(18, 23)(19, 21, 30, 24, 26, 29),
(1, 3)(2, 4)(5, 18, 9, 8, 22, 28)(6, 23, 7, 10, 17, 27)(11, 15)(12, 14)(13,
25, 21, 30, 19, 32)(16, 20, 26, 29, 24, 31),
(1, 2)(3, 4)(5, 17, 11, 6, 22, 14)(7, 9)(8, 23, 15, 10, 18, 12)(13, 21, 20,
16, 26, 25)(19, 30, 32, 24, 29, 31)(27, 28),
(1, 24)(2, 19)(3, 20)(4, 25)(7, 28)(9, 27)(11, 15)(12, 14)(13, 32)(16, 31)(17,
22)(18, 23)(21, 30)(26, 29),
(1, 2)(3, 4)(5, 7)(6, 9)(8, 27)(10, 28)(11, 17)(12, 18)(14, 22)(15, 23)(19,
25)(20, 24)(21, 31)(26, 32),
(1, 2)(3, 4)(5, 6)(7, 9)(8, 10)(11, 14)(12, 15)(13, 16)(17, 22)(18, 23)(19,
24)(20, 25)(21, 26)(27, 28)(29, 30)(31, 32),
(1, 2)(3, 4)(5, 14)(6, 11)(7, 22)(8, 12)(9, 17)(10, 15)(13, 30)(16, 29)(18,
27)(19, 20)(23, 28)(24, 25),
(1, 24, 30, 2, 19, 29)(3, 20, 13, 4, 25, 16)(5, 27, 14, 6, 28, 11)(7, 12, 10,
9, 15, 8)(17, 22)(18, 23)(21, 26)(31, 32),
(1, 2)(3, 4)(5, 17)(6, 22)(7, 11)(8, 23)(9, 14)(10, 18)(12, 28)(13, 29)(15,
27)(16, 30)(21, 32)(26, 31),
(1, 24, 16, 4, 25, 30)(2, 19, 13, 3, 20, 29)(5, 15, 23, 10, 14, 22)(6, 12, 18,
8, 11, 17)(7, 28)(9, 27)(21, 31)(26, 32),
(1, 3)(2, 4)(5, 15, 9, 8, 11, 28)(6, 12, 7, 10, 14, 27)(13, 26, 25, 30, 31,
19)(16, 21, 20, 29, 32, 24)(17, 23)(18, 22),
(1, 3)(2, 4)(5, 8)(6, 10)(7, 23, 14, 27, 17, 12)(9, 18, 11,

```
        28, 22, 15)(13,
         20, 32, 30, 24, 21)(16, 25, 31, 29, 19, 26),
    (1, 3)(2, 4)(5, 8)(6, 10)(7, 12, 17, 27, 14, 23)(9, 15, 22,
        28, 11, 18)(13,
         21, 24, 30, 32, 20)(16, 26, 19, 29, 31, 25),
    (1, 24, 31, 3, 20, 26)(2, 19, 32, 4, 25, 21)(5, 27, 18, 8,
        7, 22)(6, 28, 23,
         10, 9, 17)(11, 15)(12, 14)(13, 30)(16, 29),
    (1, 3)(2, 4)(5, 18, 11, 8, 22, 15)(6, 23, 14, 10, 17, 12)(7,
         27)(9, 28)(13,
         31, 20, 30, 26, 24)(16, 32, 25, 29, 21, 19),
    (1, 24, 21, 4, 25, 31)(2, 19, 26, 3, 20, 32)(5, 17, 12, 10,
        18, 11)(6, 22, 15,
         8, 23, 14)(7, 28)(9, 27)(13, 29)(16, 30),
    (1, 2)(3, 4)(5, 6)(7, 11, 17, 9, 14, 22)(8, 10)(12, 18, 27,
        15, 23, 28)(13,
         31, 24, 16, 32, 19)(20, 29, 21, 25, 30, 26),
    (1, 3)(2, 4)(5, 28, 22, 8, 9, 18)(6, 27, 17, 10, 7, 23)(11,
        15)(12, 14)(13,
         32, 19, 30, 21, 25)(16, 31, 24, 29, 26, 20),
    (1, 2)(3, 4)(5, 17, 9, 6, 22, 7)(8, 23, 28, 10, 18, 27)(11,
        14)(12, 15)(13,
         24, 21, 16, 19, 26)(20, 32, 29, 25, 31, 30)
}
*/
tr1:=Transversal(N,N124s);
for i:=1 to #tr1 do
ss:=[1,2,4]^tr1[i];
cst[prodim(1,ts,ss)]:=ss;
end for;
m:=0; for i in [1..315] do if cst[i] ne []
then m:=m+1;
```

```
end if; end for;m;
/*314*/
Orbits(N124s);
/*
    GSet{@ 1, 3, 2, 24, 4, 20, 19, 13, 29, 30, 21, 32, 16, 25,
       26, 31 @},
    GSet{@ 5, 11, 22, 8, 28, 18, 15, 6, 17, 14, 7, 27, 9, 10,
       23, 12 @}
*/
#N/#N124s;
/*10*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[4]*ts[4]
    eq m*(DC[i])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[4]*ts[5]
    eq m*(DC[i])^n then i; break; end if; end for;end for;
/*3*/

/*RELATION*/
f(xx * ty^-2 * x * y)^6*ts[2]*ts[9]*ts[13]*ts[15]*ts[4] eq ts
   [2];
/*true*/
f(xx * yy^-2 * xx * yy)^6*ts[2]*ts[9]*ts[13]*ts[15] eq ts[2]*ts
   [4];
/*true*/
f(x * y^-2 * x * y)^6*ts[2]*ts[9]*ts[13] eq ts[2]*ts[4]*ts[15];
/*true*/
f(x * y^-2 * x * y)^6*ts[2]*ts[9] eq ts[2]*ts[4]*ts[15]*ts[13];
/*true*/
f(x^5)*ts[1]*ts[2]*ts[1] eq ts[1]*ts[2];
/*true*/
```

```
/*AT THE END INSERT THE FOLLOWING 5 LINES ONLY ONCE*/
L<u,v>:=Group<u,v| u^10 ,
v^6 ,
(u * v^-2 * u)^2 ,
(u * v^2 * u^2)^2 ,
(v^-1 * u^-1)^5 ,
(u * v^2 * u^-1 * v^-1)^2 ,
u^-1 * v^-1 * u^5 * v * u^-4 ,
v * u^-2 * v^-1 * u^3 * v * u * v^3 * u^-1>;
Sch:=SchreierSystem(L,sub<L|Id(L)>);
h:=hom<L->N|u->xx,v->yy>;
g:=hom<IN->N|f(x)->xx,f(y)->yy>;
A:=(xx * yy^-2 * xx * yy)^6;
B:=(xx)^5;


SS:=[2,9,13,15,4,2];
for n in N do
SS:= SS^n;
for i in [1..5] do
if SS[i] eq 1
and SS[i+1] eq 8
and SS[i+2] eq 23
and SS[i+3] eq 29
then A^n, SS^n; end if; end for; end for;

/*to check if 1, 8 and 23 are equal do [1..4] */
/*to check 2( [1..5] */

/*TWO NUMBERS IN BACK*/
for n in N do
if {23,} subset Set([2,9,13,15,2,4]^n) then A^n,
```

```
   [2,9,13,15,2,4]^n;
end if; end for;

/*IF WANT ONE NUMBER IN THE BACK also works*/
for n in N do
if 1 in Set([2,9,13,15,4,2]^n)  then A^n, [2,9,13,15,4,2]^n;
end if; end for;

/*IF WANT ONE NUMBER IN THE BACK also works*/
for n in N do
if 8 in Set([15,4,2,9,13,15]^n)  then A^n, [15,4,2,9,13,15]^n;
end if; end for;

for n in N do
if {7} subset Set([1,2,1,2,1]^n)  then B^n, [1,2,1,2,1]^n;
end if; end for;


/*SECOND RELATION*/
for n in N do
if 8 in Set([1,2,1,2,1]^n)  then B^n, [1,2,1,2,1]^n;
end if; end for;

/*TWO NUMBERS IN BACK, JUST USE THIS ONE IT WORKS*/
for n in N do
if {1,14,7} subset Set([2,9,13,15,4,2]^n)  then A^n,
   [2,9,13,15,4,2]^n;
end if; end for;

for n in N do
if {7} subset Set([1,2,1,2,1]^n)  then B^n, [1,2,1,2,1]^n;
end if; end for;
```

```
for n in N do
if {2,11,7} subset Set([15,4,2,9,13,15]^n)  then A^n,
   [15,4,2,9,13,15]^n;
end if; end for;


for n in N do
if {3} subset Set([1,2,1,2,1]^n)  then B^n, [1,2,1,2,1]^n;
end if; end for;


/*WORK CHECK*/

for n in IN do if ts[1]*ts[8] eq n*ts[30]*ts[8]*ts[4]*ts[28]*ts
   [7]*ts[25] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x *
   y^-1*/

ts[1]*ts[8] eq f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y
    * x * y * x * y^-1)*ts[30]*ts[8]*ts[4]*ts[28]*ts[7]*ts[25];
/*true*/


for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y * x * y * x
   * y * x * y * x * y^-1)*ts[30]*ts[8]*ts[4]*ts[28]*ts[7]*ts
   [25] eq n*ts[30]*ts[8]*ts[4]*ts[28]*ts[7]*ts[25] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
end for; end if; end for;


/* for n in IN do if ts[45]*ts[74]*ts[96] eq n*ts[45]*ts[109]*ts
   [46] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for;
end if; end for;      */

for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y * x * y * x
   * y * x * y * x * y^-1)*ts[30]*ts[8]*ts[4]*ts[28]*ts[7]*ts
   [25] eq n*ts[12]*ts[13]*ts[30]*ts[11]*ts[25]*ts[25]*ts[7]*ts
   [21]*ts[22]*ts[23] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(y * x)^3 */
/*(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x *
    y^-1)*(y * x)^3;
/*y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x^2
   * y * x * y * x */

f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x *
   y^-1)*ts[30]*ts[8]*ts[4]*ts[28]*ts[7]*ts[25] eq f((y * x)^3)*
   ts[12]*ts[13]*ts[30]*ts[11]*ts[25]*ts[25]*ts[7]*ts[21]*ts
   [22]*ts[23];
/*true*/
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x *
   y^-1)*ts[30]*ts[8]*ts[4]*ts[28]*ts[7]*ts[25] eq f((y * x)^3)*
   ts[12]*ts[13]*ts[30]*ts[11]*ts[7]*ts[21]*ts[22]*ts[23];
/*true*/
for n in IN do if f((y * x)^3)*ts[12]*ts[13]*ts[30]*ts[11]*ts
   [7]*ts[21]*ts[22]*ts[23] eq n*ts[27]*ts[13]*ts[6]*ts[32]*ts
```

```
    [11]*ts[30]*ts[30]*ts[11]*ts[7]*ts[21]*ts[22]*ts[23] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;

f((y * x)^3)*ts[12]*ts[13]*ts[30]*ts[11]*ts[7]*ts[21]*ts[22]*ts
    [23] eq f((y * x)^3)*ts[27]*ts[13]*ts[6]*ts[32]*ts[11]*ts
    [30]*ts[30]*ts[11]*ts[7]*ts[21]*ts[22]*ts[23];


/*USING SCHRIER SYSTEM TO FIND WHAT TAKES WHAT TO WHAT*/
/*(A COSET BELONGS TO A DOUBLE COSET)*/
/*FIRST DOUBLE COSET*/
L<u,v>:=Group<u,v|u^10 ,
v^6 ,
(u * v^-2 * u)^2 ,
(u * v^2 * u^2)^2 ,
(v^-1 * u^-1)^5 ,
(u * v^2 * u^-1 * v^-1)^2 ,
u^-1 * v^-1 * u^5 * v * u^-4 ,
v * u^-2 * v^-1 * u^3 * v * u * v^3 * u^-1>;
Sch:=SchreierSystem(L,sub<L|Id(L)>);
h:=hom<L->N|u->xx,v->yy>;
g:=hom<IN->N|f(x)->xx,f(y)->yy>;
for m,n in IN do if ts[1]*ts[1] eq m*(ts[1]*ts[3])^n then
                    for i in [1..#Sch] do if g(m) eq h(Sch[i]) then
    Sch[i]; end if; end for;
for i in [1..#Sch] do if g(m) eq h(Sch[i]) then Sch[i]; end if;
    end for;
break; end if;
end for;

L<u,v>:=Group<u,v|u^10 ,
```

```
v^6 ,
(u * v^-2 * u)^2 ,
(u * v^2 * u^2)^2 ,
(v^-1 * u^-1)^5 ,
(u * v^2 * u^-1 * v^-1)^2 ,
u^-1 * v^-1 * u^5 * v * u^-4 ,
v * u^-2 * v^-1 * u^3 * v * u * v^3 * u^-1>;
Sch:=SchreierSystem(L,sub<L|Id(L)>);
h:=hom<L->N|u->xx,v->yy>;
g:=hom<IN->N|f(x)->xx,f(y)->yy>;
for m,n in IN do if ts[1]*ts[2] eq m*(ts[1]*ts[2])^n then
                    for i in [1..#Sch] do if g(m) eq h(Sch[i]) then
   Sch[i]; end if; end for;
for i in [1..#Sch] do if g(m) eq h(Sch[i]) then Sch[i]; end if;
   end for;
break; end if;
end for;
/*m= Id(L), n=Id(L)*/
/*MAGMA CHECK*/
ts[1]*ts[2] eq ts[1]*ts[2];

L<u,v>:=Group<u,v|u^10 ,
v^6 ,
(u * v^-2 * u)^2 ,
(u * v^2 * u^2)^2 ,
(v^-1 * u^-1)^5 ,
(u * v^2 * u^-1 * v^-1)^2 ,
u^-1 * v^-1 * u^5 * v * u^-4 ,
v * u^-2 * v^-1 * u^3 * v * u * v^3 * u^-1>;
Sch:=SchreierSystem(L,sub<L|Id(L)>);
h:=hom<L->N|u->xx,v->yy>;
g:=hom<IN->N|f(x)->xx,f(y)->yy>;
```

```
for m,n in IN do if ts[1]*ts[3] eq m*(ts[1]*ts[3])^n then
                    for i in [1..#Sch] do if g(m) eq h(Sch[i]) then
    Sch[i]; end if; end for;
for i in [1..#Sch] do if g(m) eq h(Sch[i]) then Sch[i]; end if;
    end for;
break; end if;
end for;

L<u,v>:=Group<u,v|u^10 ,
v^6 ,
(u * v^-2 * u)^2 ,
(u * v^2 * u^2)^2 ,
(v^-1 * u^-1)^5 ,
(u * v^2 * u^-1 * v^-1)^2 ,
u^-1 * v^-1 * u^5 * v * u^-4 ,
v * u^-2 * v^-1 * u^3 * v * u * v^3 * u^-1>;
Sch:=SchreierSystem(L,sub<L|Id(L)>);
h:=hom<L->N|u->xx,v->yy>;
g:=hom<IN->N|f(x)->xx,f(y)->yy>;
for m,n in IN do if ts[1]*ts[4] eq m*(ts[1])^n then
                    for i in [1..#Sch] do if g(m) eq h(Sch[i]) then
    Sch[i]; end if; end for;
for i in [1..#Sch] do if g(m) eq h(Sch[i]) then Sch[i]; end if;
    end for;
break; end if;
end for;
/*
m=u^5 * v^3
/*n=Identity*/

x^5*y^3=(1, 4)(2, 3)(5, 10)(6, 8)(7, 28)(9, 27)(11, 12)(13, 29)
    (14, 15)(16, 30)(17, 18)(19,
```

```
    20)(21, 31)(22, 23)(24, 25)(26, 32)
*/
/*MAGMA CHECK*/
ts[1]*ts[4] eq f(x^5*y^3)*ts[1];
/*true*/


L<u,v>:=Group<u,v|u^10 ,
v^6 ,
(u * v^-2 * u)^2 ,
(u * v^2 * u^2)^2 ,
(v^-1 * u^-1)^5 ,
(u * v^2 * u^-1 * v^-1)^2 ,
u^-1 * v^-1 * u^5 * v * u^-4 ,
v * u^-2 * v^-1 * u^3 * v * u * v^3 * u^-1>;
Sch:=SchreierSystem(L,sub<L|Id(L)>);
h:=hom<L->N|u->xx,v->yy>;
g:=hom<IN->N|f(x)->xx,f(y)->yy>;
for m,n in IN do if ts[1]*ts[8] eq m*(ts[1]*ts[8])^n then
                 for i in [1..#Sch] do if g(m) eq h(Sch[i]) then
   Sch[i]; end if; end for;
for i in [1..#Sch] do if g(m) eq h(Sch[i]) then Sch[i]; end if;
   end for;
break; end if;
end for;


/*SECOND DOUBLE COSET 1,2*/
(A COSET BELONGS TO A DOUBLE COSET)
L<u,v>:=Group<u,v|u^10 ,
v^6 ,
(u * v^-2 * u)^2 ,
```

```
(u * v^2 * u^2)^2 ,
(v^-1 * u^-1)^5 ,
(u * v^2 * u^-1 * v^-1)^2 ,
u^-1 * v^-1 * u^5 * v * u^-4 ,
v * u^-2 * v^-1 * u^3 * v * u * v^3 * u^-1>;
Sch:=SchreierSystem(L,sub<L|Id(L)>);
h:=hom<L->N|u->xx,v->yy>;
g:=hom<IN->N|f(x)->xx,f(y)->yy>;
for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1]*ts[2])^n then
                for i in [1..#Sch] do if g(m) eq h(Sch[i]) then
   Sch[i]; end if; end for;
for i in [1..#Sch] do if g(m) eq h(Sch[i]) then Sch[i]; end if;
   end for;
break; end if;
end for;
/*

/*CHECK IN MAGMA*/
ts[1]*ts[2]*ts[1] eq f(x^5)*ts[1]*ts[2];
/*true*/

L<u,v>:=Group<u,v|u^10 ,
v^6 ,
(u * v^-2 * u)^2 ,
(u * v^2 * u^2)^2 ,
(v^-1 * u^-1)^5 ,
(u * v^2 * u^-1 * v^-1)^2 ,
u^-1 * v^-1 * u^5 * v * u^-4 ,
v * u^-2 * v^-1 * u^3 * v * u * v^3 * u^-1>;
Sch:=SchreierSystem(L,sub<L|Id(L)>);
h:=hom<L->N|u->xx,v->yy>;
g:=hom<IN->N|f(x)->xx,f(y)->yy>;
```

```
for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1])^n then
                  for i in [1..#Sch] do if g(m) eq h(Sch[i]) then
   Sch[i]; end if; end for;
for i in [1..#Sch] do if g(m) eq h(Sch[i]) then Sch[i]; end if;
   end for;
break; end if;
end for;
/*

/*(A COSET BELONGS TO A DOUBLE COSET)*/
L<u,v>:=Group<u,v|u^10 ,
v^6 ,
(u * v^-2 * u)^2 ,
(u * v^2 * u^2)^2 ,
(v^-1 * u^-1)^5 ,
(u * v^2 * u^-1 * v^-1)^2 ,
u^-1 * v^-1 * u^5 * v * u^-4 ,
v * u^-2 * v^-1 * u^3 * v * u * v^3 * u^-1>;
Sch:=SchreierSystem(L,sub<L|Id(L)>);
h:=hom<L->N|u->xx,v->yy>;
g:=hom<IN->N|f(x)->xx,f(y)->yy>;
for m,n in IN do if ts[1]*ts[2]*ts[3] eq m*(ts[1]*ts[3])^n then
                  for i in [1..#Sch] do if g(m) eq h(Sch[i]) then
   Sch[i]; end if; end for;
for i in [1..#Sch] do if g(m) eq h(Sch[i]) then Sch[i]; end if;
   end for;
break; end if;
end for;
/*
u^5 * v^3
u^5 * v^3
*/
```

```
 xx^5*yy^3;
/*(1, 4)(2, 3)(5, 10)(6, 8)(7, 28)(9, 27)(11, 12)(13, 29)(14,
   15)(16, 30)(17,
    18)(19, 20)(21, 31)(22, 23)(24, 25)(26, 32)*/
ts[1]*ts[2]*ts[3] eq f(x^5*y^3)*ts[4]*ts[2];
/*true*/


/*Code to help prove relations*/
a:=0;b:=0;c:=0;d:=0;e:=0;f:=0;g:=0;h:=6;i:=0;j:=0;
G<x,y,t>:=Group<x,y,t|x^10 ,
y^6 ,
(x * y^-2 * x)^2 ,
(x * y^2 * x^2)^2 ,
(y^-1 * x^-1)^5 ,
(x * y^2 * x^-1 * y^-1)^2 ,
x^-1 * y^-1 * x^5 * y * x^-4 ,
y * x^-2 * y^-1 * x^3 * y * x * y^3 * x^-1 ,
t^2,
(t,x^2),
(t,y^2),
((x * y^-1 * x * y^-2 * x * y)*t^(x^4 * y^-1 * x))^a,
((x * y^-1 * x * y^-2 * x * y)*t^(x))^b,
((x * y^-1 * x * y^-2 * x * y)*t^(x^2 * y^2 * x * y * x^-1))^c,
((x^2)*t^(y^-1 * x^-1 * y^-1 * x^-3))^d,
((x^2)*t^(x^2 * y^2 * x * y * x^-1))^e,
((x^2)*t^(y * x * y^-2 * x * y * x))^f,
((x * y^-2 * x * y)*t^(x^2 * y^-1 * x * y * x^-2 * y^-1))^g,
((x * y^-2 * x * y)*t^(x))^h,
((x * y^-2 * x * y)*t^(x^2 * y^2 * x * y * x^-1))^i,
(((y * x^-1)^2)*t^(y * x^-2 * y * x * y^-1 * x))^j,
(x * t)^5,
```

```
t*t^(x * y^-2 * x^-2 * y^-1)=(x^5 * y^3)*t,
t*t^(x * y * x^-2 * y * x) = (y^-1 * x^-3)*t^(x * y * x^-1 * y *
    x * y^-1)*t^(y^2 * x * y^-1 * x * y^-1 * x^-1)>;

/*AT THE END INSERT THE FOLLOWING 5 LINES ONLY ONCE*/
L<u,v>:=Group<u,v| u^10 ,
v^6 ,
(u * v^-2 * u)^2 ,
(u * v^2 * u^2)^2 ,
(v^-1 * u^-1)^5 ,
(u * v^2 * u^-1 * v^-1)^2 ,
u^-1 * v^-1 * u^5 * v * u^-4 ,
v * u^-2 * v^-1 * u^3 * v * u * v^3 * u^-1>;
Sch:=SchreierSystem(L,sub<L|Id(L)>);
h:=hom<L->N|u->xx,v->yy>;
g:=hom<IN->N|f(x)->xx,f(y)->yy>;
A:=(xx * yy^-2 * xx * yy)^6;
B:=(xx)^5;

for n in N do
if {10,26} subset Set([1,4,1]^n) then ( (xx^5 * yy^3)^-1)^n,
   [1,4,1]^n;
end if; end for;

for n in N do
if {10,21} subset Set([29,23,8,1]^n) then ( yy^-1 * xx^-3)^n,
   [29,23,8,1]^n;
end if; end for;

/*TWO NUMBERS IN BACK, JUST USE THIS ONE IT WORKS*/
for n in N do
if {2,4,3} subset Set([2,9,13,15,4,2]^n) then A^n,
```

```
  [2,9,13,15,4,2]^n;
end if; end for;

for n in N do
if {4,3} subset Set([1,2,1,2,1]^n)  then B^n, [1,2,1,2,1]^n;
end if; end for;
```

PERMUTATION CONVERSION

```
NN<a,b>:=Group<a,b|a^10 ,b^6 ,
(a * b^-2 * a)^2 ,
(a * b^2 * a^2)^2 ,
(b^-1 * a^-1)^5 ,
(a * b^2 * a^-1 * b^-1)^2 ,
a^-1 * b^-1 * a^5 * b * a^-4 ,
 b * a^-2 * b^-1 * a^3 * b * a * b^3 * a^-1>;

word:=function(A)
Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
for i in [2..#N] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=xx; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=xx^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=yy; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=yy^-1; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
if A eq PP then B:=Sch[i]; end if;
end for;
```

```
return B;
end function;

EXAMPLE:
/*word(N!(1, 2)(3, 4)(5, 14, 9, 6, 11, 7)(8, 12, 28, 10, 15, 27)
   (13, 32, 25, 16, 31, 20)(17, 22)(18,  23)(19, 29, 26, 24, 30,
    21) );
a^-2 * b^-2 * a
*/

/*GIVEN RELATIONS*/
f(x^5)*ts[1]*ts[2]*ts[1] eq ts[1]*ts[2];
/*true*/
f(xx * yy^-2 * xx * yy)^6*ts[2]*ts[9]*ts[13]*ts[15] eq ts[2]*ts
   [4];
/*true*/

/*THE BEGINNING OF PROVING RELATIONS*/
/*DOUBLE COSET [1,2]*/
1,2,1 belongs to [1,2]
for n in IN do if ts[1]*ts[2]*ts[1] eq n*ts[1]*ts[2]*ts[1]*ts
   [2]*ts[2]*ts[1] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x^5*/
ts[1]*ts[2]*ts[1] eq f(x^5)*ts[1]*ts[2]*ts[1]*ts[2]*ts[2]*ts[1];
/*true*/
ts[1]*ts[2]*ts[1] eq f(x^5)*ts[1]*ts[2];
/*true*/

[1,2,3] belongs to [1,3]
for n in IN do if ts[1]*ts[2]*ts[1]*ts[1]*ts[3] eq n*ts[2]*ts
```

```
[1]*ts[2]*ts[1]*ts[2]*ts[1]*ts[2]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x^4*/
ts[1]*ts[2]*ts[1]*ts[1]*ts[3] eq f(x^5)*ts[2]*ts[1]*ts[2]*ts[1]*
   ts[2]*ts[1]*ts[2]*ts[3];
/*true*/
for n in IN do if f(x^5)*ts[2]*ts[1]*ts[2]*ts[1]*ts[2]*ts[1]*ts
   [2]*ts[3] eq
n*ts[2]*ts[1]*ts[2]*ts[1]*ts[1]*ts[2]*ts[1]*ts[2]*ts[1]*ts[2]*ts
   [3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
f(x^5)*ts[2]*ts[1]*ts[2]*ts[1]*ts[2]*ts[1]*ts[2]*ts[3] eq ts[2]*
   ts[1]*ts[2]*ts[1]*ts[1]*ts[2]*ts[1]*ts[2]*ts[1]*ts[2]*ts[3];
/*true*/
f(x^5)*ts[2]*ts[1]*ts[2]*ts[1]*ts[2]*ts[1]*ts[2]*ts[3] eq ts[1]*
   ts[2]*ts[3];
/*true*/
for n in IN do if ts[1]*ts[2]*ts[3] eq n*ts[4]*ts[2] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x^5 * y^3*/
ts[1]*ts[2]*ts[3] eq f(x^5 * y^3)*ts[4]*ts[2];
/*true*/

[1,2,8] belongs to [1,3]
for n in IN do if ts[1]*ts[2]*ts[8] eq n*ts[22]*ts[25]*ts[16]
   then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
```

```
/*x * y^2 * x * y^-1 * x^-1*/
ts[1]*ts[2]*ts[8] eq f(x * y^2 * x * y^-1 * x^-1)*ts[22]*ts[25]*
    ts[16];
/*true*/
for n in IN do if f(x * y^2 * x * y^-1 * x^-1)*ts[22]*ts[25]*ts
    [16] eq
n*ts[7]*ts[6]*ts[16] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x^2 * y * x * y*/
f(x * y^2 * x * y^-1 * x^-1)*ts[22]*ts[25]*ts[16] eq f(x * y * x
    ^2 * y * x * y)*ts[7]*ts[6]*ts[16];
/*true*/
for n in IN do if f(x * y * x^2 * y * x * y)*ts[7]*ts[6]*ts[16]
    eq
n*ts[1]*ts[6] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x^-1 * y^3 * x * y*/
f(x * y * x^2 * y * x * y)*ts[7]*ts[6]*ts[16] eq f(x^-1 * y^3 *
    x * y)*ts[1]*ts[6];
/*true*/

/*DOUBLE COSET [1,3]*/
[1,3,1] belongs to [1,3] (FOUND!!)
for n in IN do if ts[1]*ts[3]*ts[1] eq
n*ts[5]*ts[5]*ts[1]*ts[3]*ts[1] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*Id(L)*/
ts[1]*ts[3]*ts[1] eq
ts[5]*ts[5]*ts[1]*ts[3]*ts[1];
```

```
/*true*/
for n in IN do if ts[5]*ts[5]*ts[1]*ts[3]*ts[1] eq
n*ts[1]*ts[5]*ts[3]*ts[1] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(y * x^-1 * y * x * y^-1)^3*/
ts[5]*ts[5]*ts[1]*ts[3]*ts[1] eq
f((y * x^-1 * y * x * y^-1)^3)*ts[1]*ts[5]*ts[3]*ts[1];
/*true*/
for n in IN do if f((y * x^-1 * y * x * y^-1)^3)*ts[1]*ts[5]*ts
   [3]*ts[1] eq
n*ts[3]*ts[5]*ts[8]*ts[3]*ts[1] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^2 * x * y^-1 * x * y^-1*/
f((y * x^-1 * y * x * y^-1)^3)*ts[1]*ts[5]*ts[3]*ts[1] eq
f(x * y^2 * x * y^-1 * x * y^-1)*ts[3]*ts[5]*ts[8]*ts[3]*ts[1];
/*true*/
for n in IN do if f(x * y^2 * x * y^-1 * x * y^-1)*ts[3]*ts[5]*
   ts[8]*ts[3]*ts[1] eq
n*ts[8]*ts[2]*ts[8]*ts[1] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^3*/
f(x * y^2 * x * y^-1 * x * y^-1)*ts[3]*ts[5]*ts[8]*ts[3]*ts[1]
   eq
f(y^3)*ts[8]*ts[2]*ts[8]*ts[1];
/*true*/
for n in IN do if f(y^3)*ts[8]*ts[2]*ts[8]*ts[1] eq
n*ts[21]*ts[30]*ts[14]*ts[18] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
```

```
/*x * y * x^-1 * y^-2 * x^-1 * y*/
f(y^3)*ts[8]*ts[2]*ts[8]*ts[1] eq
f(x * y * x^-1 * y^-2 * x^-1 * y)*ts[21]*ts[30]*ts[14]*ts[18];
/*true*/
for n in IN do if f(x * y * x^-1 * y^-2 * x^-1 * y)*ts[21]*ts
    [30]*ts[14]*ts[18] eq
n*ts[9]*ts[30]*ts[18] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/* y * x^-1 * y * x^-1 * y^-2 * x */
f(x * y * x^-1 * y^-2 * x^-1 * y)*ts[21]*ts[30]*ts[14]*ts[18] eq
f(y * x^-1 * y * x^-1 * y^-2 * x)*ts[9]*ts[30]*ts[18];
/*true*/
for n in IN do if f(y * x^-1 * y * x^-1 * y^-2 * x)*ts[9]*ts
    [30]*ts[18] eq
n*ts[17]*ts[3]*ts[18] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y^2 * x * y^-1*/
f(y * x^-1 * y * x^-1 * y^-2 * x)*ts[9]*ts[30]*ts[18] eq
f(x * y^-1 * x * y^2 * x * y^-1)*ts[17]*ts[3]*ts[18];
/*true*/
for n in IN do if f(x * y^-1 * x * y^2 * x * y^-1)*ts[17]*ts[3]*
    ts[18] eq
n*ts[1]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^3*/
f(x * y^-1 * x * y^2 * x * y^-1)*ts[17]*ts[3]*ts[18] eq
f(y^3)*ts[1]*ts[3];
/*true*/
```

```
[1,3,2] belongs to [1,2]
for n in IN do if ts[1]*ts[3]*ts[2] eq n*ts[4]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x^5 * y^3*/
ts[1]*ts[3]*ts[2] eq f(x^5 * y^3)*ts[4]*ts[3];
/*true*/

[1,3,4] belongs to [1,2,4] (FINISHED!!!!)
for n in IN do if ts[1]*ts[3]*ts[4] eq
n*ts[31]*ts[28]*ts[16]*ts[26]*ts[5] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x^3 * y^-1 * x*/
ts[1]*ts[3]*ts[4] eq
f(x * y * x^3 * y^-1 * x)*ts[31]*ts[28]*ts[16]*ts[26]*ts[5];
/*true*/
for n in IN do if f(x * y * x^3 * y^-1 * x)*ts[31]*ts[28]*ts
   [16]*ts[26]*ts[5] eq
n*ts[21]*ts[12]*ts[10]*ts[26]*ts[5] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x^-1 * y * x^-1 * y * x^-1 * y^-1*/
f(x * y * x^3 * y^-1 * x)*ts[31]*ts[28]*ts[16]*ts[26]*ts[5] eq
f(x^-1 * y * x^-1 * y * x^-1 * y^-1)*ts[21]*ts[12]*ts[10]*ts
   [26]*ts[5];
/*true*/
for n in IN do if f(x^-1 * y * x^-1 * y * x^-1 * y^-1)*ts[21]*ts
   [12]*ts[10]*ts[26]*ts[5] eq
n*ts[21]*ts[10]*ts[26]*ts[5] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
```

```
/*x * y^-1 * x * y * x^-2 * y^-1 * x^-1*/
f(x^-1 * y * x^-1 * y * x^-1 * y^-1)*ts[21]*ts[12]*ts[10]*ts
    [26]*ts[5] eq
f(x * y^-1 * x * y * x^-2 * y^-1 * x^-1)*ts[21]*ts[10]*ts[26]*ts
    [5];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x^-2 * y^-1 * x^-1)*ts
    [21]*ts[10]*ts[26]*ts[5] eq
n*ts[8]*ts[10]*ts[5] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y*/
f(x * y^-1 * x * y * x^-2 * y^-1 * x^-1)*ts[21]*ts[10]*ts[26]*ts
    [5] eq
f(y)*ts[8]*ts[10]*ts[5];
/*true*/


[1,3,5] belongs to [1,8]
for n in IN do if ts[1]*ts[3]*ts[5] eq
n*ts[4]*ts[19]*ts[14] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x^-3 * y^-1*/
ts[1]*ts[3]*ts[5] eq f(x^-3 * y^-1)*ts[4]*ts[19]*ts[14];
/*true*/
for n in IN do if f(x^-3 * y^-1)*ts[4]*ts[19]*ts[14] eq
n*ts[25]*ts[31]*ts[14] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x^4 * y^-2*/
f(x^-3 * y^-1)*ts[4]*ts[19]*ts[14] eq f(x^4 * y^-2)*ts[25]*ts
```

```
    [31]*ts[14];
/*true*/
for n in IN do if f(x^4 * y^-2)*ts[25]*ts[31]*ts[14] eq
n*ts[26]*ts[2]*ts[14] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x^-1 * y * x * y * x^-1 * y * x*/
f(x^4 * y^-2)*ts[25]*ts[31]*ts[14] eq
f(x^-1 * y * x * y * x^-1 * y * x)*ts[26]*ts[2]*ts[14];
/*true*/
for n in IN do if f(x^-1 * y * x * y * x^-1 * y * x)*ts[26]*ts
    [2]*ts[14] eq
n*ts[10]*ts[2] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^2*/
f(x^-1 * y * x * y * x^-1 * y * x)*ts[26]*ts[2]*ts[14] eq
f(y^2)*ts[10]*ts[2];
/*true*/

[1,3,6] belongs to [1,3] (FINISHED!!)
for n in IN do if ts[1]*ts[3]*ts[6] eq
n*ts[1]*ts[3]*ts[6]*ts[2]*ts[2] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*Id(L)*/
ts[1]*ts[3]*ts[6] eq
ts[1]*ts[3]*ts[6]*ts[2]*ts[2];
/*true*/
for n in IN do if ts[1]*ts[3]*ts[6]*ts[2]*ts[2] eq
n*ts[5]*ts[10]*ts[6]*ts[2] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
   end for; end if; end for;
/*(v * u^-1 * v * u * v^-1)^3*/
ts[1]*ts[3]*ts[6]*ts[2]*ts[2] eq
f((y * x^-1 * y * x * y^-1)^3)*ts[5]*ts[10]*ts[6]*ts[2];
/*true*/
for n in IN do if f((y * x^-1 * y * x * y^-1)^3)*ts[5]*ts[10]*ts
   [6]*ts[2] eq
n*ts[8]*ts[10]*ts[2] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^2 * x * y^-1 * x * y^-1*/
f((y * x^-1 * y * x * y^-1)^3)*ts[5]*ts[10]*ts[6]*ts[2] eq
f(x * y^2 * x * y^-1 * x * y^-1)*ts[8]*ts[10]*ts[2];
/*true*/
for n in IN do if f(x * y^2 * x * y^-1 * x * y^-1)*ts[8]*ts[10]*
   ts[2] eq
n*ts[30]*ts[9]*ts[12] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^3 * x * y^-1 * x^-2*/
f(x * y^2 * x * y^-1 * x * y^-1)*ts[8]*ts[10]*ts[2] eq
f(y^3 * x * y^-1 * x^-2)*ts[30]*ts[9]*ts[12];
/*true*/
for n in IN do if f(y^3 * x * y^-1 * x^-2)*ts[30]*ts[9]*ts[12]
   eq
n*ts[22]*ts[25]*ts[12]*ts[4] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x^-3 * y^2*/
f(y^3 * x * y^-1 * x^-2)*ts[30]*ts[9]*ts[12] eq
f(y * x^-3 * y^2)*ts[22]*ts[25]*ts[12]*ts[4];
/*true*/
```

```
for n in IN do if f(y * x^-3 * y^2)*ts[22]*ts[25]*ts[12]*ts[4]
   eq
n*ts[8]*ts[4] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-2*/
f(y * x^-3 * y^2)*ts[22]*ts[25]*ts[12]*ts[4] eq
f(y^-2)*ts[8]*ts[4];
/*true*/


[1,3,8] belongs to [1,8]
for n in IN do if ts[1]*ts[3]*ts[8] eq
n*ts[6]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(x^-1 * y * x)^3*/
ts[1]*ts[3]*ts[8] eq
f((x^-1 * y * x)^3)*ts[6]*ts[3];
/*true*/


[1,3,10] belongs to [1,2] (FINISHED!!!!)
for n in IN do if ts[1]*ts[3]*ts[10] eq
n*ts[3]*ts[1]*ts[10]*ts[6] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^3*/
ts[1]*ts[3]*ts[10] eq
f(y^3)*ts[3]*ts[1]*ts[10]*ts[6];
/*true*/
for n in IN do if f(y^3)*ts[3]*ts[1]*ts[10]*ts[6] eq
```

```
n*ts[10]*ts[5]*ts[3]*ts[6]*ts[2] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x^-1 * y * x^-1 * y^-2 * x^-1*/
f(y^3)*ts[3]*ts[1]*ts[10]*ts[6] eq
f(y * x^-1 * y * x^-1 * y^-2 * x^-1)*ts[10]*ts[5]*ts[3]*ts[6]*ts
   [2];
/*true*/
for n in IN do if f(y * x^-1 * y * x^-1 * y^-2 * x^-1)*ts[10]*ts
   [5]*ts[3]*ts[6]*ts[2] eq
n*ts[11]*ts[26]*ts[19]*ts[6]*ts[2] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x^-1 * y * x^2 * y^-2*/
f(y * x^-1 * y * x^-1 * y^-2 * x^-1)*ts[10]*ts[5]*ts[3]*ts[6]*ts
   [2] eq
f(x^-1 * y * x^2 * y^-2)*ts[11]*ts[26]*ts[19]*ts[6]*ts[2];
/*true*/
for n in IN do if f(x^-1 * y * x^2 * y^-2)*ts[11]*ts[26]*ts[19]*
   ts[6]*ts[2] eq
n*ts[29]*ts[7]*ts[19]*ts[2] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x^-2 * y^-1 * x * y^-1*/
f(x^-1 * y * x^2 * y^-2)*ts[11]*ts[26]*ts[19]*ts[6]*ts[2] eq
f(y * x^-2 * y^-1 * x * y^-1)*ts[29]*ts[7]*ts[19]*ts[2];
/*true*/
for n in IN do if f(y * x^-2 * y^-1 * x * y^-1)*ts[29]*ts[7]*ts
   [19]*ts[2] eq
n*ts[1]*ts[2] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
```

```
/*x^-1 * y^3 * x * y*/
f(y * x^-2 * y^-1 * x * y^-1)*ts[29]*ts[7]*ts[19]*ts[2] eq
f(x^-1 * y^3 * x * y)*ts[1]*ts[2];
/*true*/



[1,3,13] belongs to [1,3]
for n in IN do if ts[1]*ts[3]*ts[13] eq
n*ts[22]*ts[30]*ts[24] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-2 * x^-1 * y * x^-1*/
ts[1]*ts[3]*ts[13] eq
f(x * y^-2 * x^-1 * y * x^-1)*ts[22]*ts[30]*ts[24];
/*true*/
for n in IN do if f(x * y^-2 * x^-1 * y * x^-1)*ts[22]*ts[30]*ts
   [24] eq
n*ts[22]*ts[24] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x^-2*/
f(x * y^-2 * x^-1 * y * x^-1)*ts[22]*ts[30]*ts[24] eq
f(y^-1 * x * y^-1 * x^-2)*ts[22]*ts[24];
/*true*/

/*DOUBLE COSET [1,8]*/
[1,8,1] belongs to [1,3] (FINISHED!!!!!!)
for n in IN do if ts[1]*ts[8]*ts[1] eq
n*ts[1]*ts[5]*ts[8]*ts[1] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
```

```
/*(y * x^-1 * y * x * y^-1)^3*/
ts[1]*ts[8]*ts[1] eq
f((y * x^-1 * y * x * y^-1)^3)*ts[1]*ts[5]*ts[8]*ts[1];
/*true*/
for n in IN do if f((y * x^-1 * y * x * y^-1)^3)*ts[1]*ts[5]*ts
   [8]*ts[1] eq
n*ts[3]*ts[5]*ts[1] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^2 * x * y^-1 * x * y^-1*/
f((y * x^-1 * y * x * y^-1)^3)*ts[1]*ts[5]*ts[8]*ts[1] eq
f(x * y^2 * x * y^-1 * x * y^-1)*ts[3]*ts[5]*ts[1];
/*true*/
for n in IN do if f(x * y^2 * x * y^-1 * x * y^-1)*ts[3]*ts[5]*
   ts[1] eq
n*ts[10]*ts[5] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x^5 * y^3*/
f(x * y^2 * x * y^-1 * x * y^-1)*ts[3]*ts[5]*ts[1] eq
f(x^5 * y^3)*ts[10]*ts[5];
/*true*/




[1,8,2] belongs to [1,8]
for n in IN do if ts[1]*ts[8]*ts[2] eq
n*ts[30]*ts[22]*ts[32] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x^-2*/
```

```
ts[1]*ts[8]*ts[2] eq
f(y^-1 * x * y^-1 * x^-2)*ts[30]*ts[22]*ts[32];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x^-2)*ts[30]*ts[22]*ts[32]
     eq
n*ts[30]*ts[32] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-2 * x^-1 * y * x^-1*/
f(y^-1 * x * y^-1 * x^-2)*ts[30]*ts[22]*ts[32] eq
f(x * y^-2 * x^-1 * y * x^-1)*ts[30]*ts[32];
/*true*/

[1,8,6] belongs to [1,3]
for n in IN do if ts[1]*ts[8]*ts[6] eq
n*ts[26]*ts[27]*ts[6] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^3 * x^3*/
ts[1]*ts[8]*ts[6] eq
f(y^3 * x^3)*ts[26]*ts[27]*ts[6];
/*true*/
for n in IN do if f(y^3 * x^3)*ts[26]*ts[27]*ts[6] eq
n*ts[10]*ts[26]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x^-1 * y * x * y * x^-1 * y * x*/
f(y^3 * x^3)*ts[26]*ts[27]*ts[6] eq
f(x^-1 * y * x * y * x^-1 * y * x)*ts[10]*ts[26]*ts[3];
/*true*/
for n in IN do if f(x^-1 * y * x * y * x^-1 * y * x)*ts[10]*ts
   [26]*ts[3] eq
```

```
n*ts[10]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^2*/
f(x^-1 * y * x * y * x^-1 * y * x)*ts[10]*ts[26]*ts[3] eq
f(y^2)*ts[10]*ts[3] ;
/*true*/

/*DOUBLE COSET [1,2,4]*/
[1,2,4,5] belongs to [1,3]
for n in IN do if ts[1]*ts[2]*ts[4]*ts[5] eq
n*ts[1]*ts[2]*ts[1]*ts[4]*ts[5] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x^5*/
ts[1]*ts[2]*ts[4]*ts[5] eq
f(x^5)*ts[1]*ts[2]*ts[1]*ts[4]*ts[5];
/*true*/
for n in IN do if f(x^5)*ts[1]*ts[2]*ts[1]*ts[4]*ts[5] eq
n*ts[4]*ts[3]*ts[1]*ts[5] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^3*/
f(x^5)*ts[1]*ts[2]*ts[1]*ts[4]*ts[5] eq
f(y^3)*ts[4]*ts[3]*ts[1]*ts[5];
/*true*/
for n in IN do if f(y^3)*ts[4]*ts[3]*ts[1]*ts[5] eq
n*ts[8]*ts[10]*ts[1] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x^-1 * y * x^-1 * y^-2 * x^-1*/
f(y^3)*ts[4]*ts[3]*ts[1]*ts[5] eq
```

```
f(y * x^-1 * y * x^-1 * y^-2 * x^-1)*ts[8]*ts[10]*ts[1];
/*true*/
for n in IN do if f(y * x^-1 * y * x^-1 * y^-2 * x^-1)*ts[8]*ts
    [10]*ts[1] eq
n*ts[30]*ts[7]*ts[19] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x^-1 * y * x^-3 * y^-1 * x^-1*/
f(y * x^-1 * y * x^-1 * y^-2 * x^-1)*ts[8]*ts[10]*ts[1] eq
f(x^-1 * y * x^-3 * y^-1 * x^-1)*ts[30]*ts[7]*ts[19];
/*true*/
for n in IN do if f(x^-1 * y * x^-3 * y^-1 * x^-1)*ts[30]*ts[7]*
    ts[19] eq
n*ts[21]*ts[23]*ts[19] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x^-1 * y^2*/
f(x^-1 * y * x^-3 * y^-1 * x^-1)*ts[30]*ts[7]*ts[19] eq
f(y^-1 * x^-1 * y^2)*ts[21]*ts[23]*ts[19];
/*true*/
for n in IN do if f(y^-1 * x^-1 * y^2)*ts[21]*ts[23]*ts[19] eq
n*ts[25]*ts[6]*ts[19] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^2 * x * y^3 * x^-1*/
f(y^-1 * x^-1 * y^2)*ts[21]*ts[23]*ts[19] eq
f(y^2 * x * y^3 * x^-1)*ts[25]*ts[6]*ts[19];
/*true*/
for n in IN do if f(y^2 * x * y^3 * x^-1)*ts[25]*ts[6]*ts[19] eq
n*ts[8]*ts[6] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
```

```
/*y^2*/
f(y^2 * x * y^3 * x^-1)*ts[25]*ts[6]*ts[19] eq
f(y^2)*ts[8]*ts[6];
/*true*/

/*LEMMA 1*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq n*ts[5]*ts
    [6]*ts[5]*ts[2]*ts[6]*ts[1]*ts[5]*ts[1]*ts[1]*ts[5] then for
    i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
(y * x^-1 * y * x * y^-1)^3
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f((y * x^-1 * y * x * y^-1)^3)*
    ts[5]*ts[6]*ts[5]*ts[2]*ts[6]*ts[1]*ts[5]*ts[1]*ts[1]*ts[5];
/*true*/
for n in IN do if f((y * x^-1 * y * x * y^-1)^3)*ts[5]*ts[6]*ts
    [5]*ts[2]*ts[6]*ts[1]*ts[5]*ts[1]*ts[1]*ts[5] eq n*ts[1]*ts
    [2]*ts[1]*ts[6]*ts[2]*ts[1]*ts[5] then for i in [1..#Sch] do
    if g(n) eq h(Sch[i]) then Sch[i]; end if; end for; end if;
    end for;
/*Id(L)*/
f((y * x^-1 * y * x * y^-1)^3)*ts[5]*ts[6]*ts[5]*ts[2]*ts[6]*ts
    [1]*ts[5]*ts[1]*ts[1]*ts[5] eq ts[1]*ts[2]*ts[1]*ts[6]*ts[2]*
    ts[1]*ts[5];
/*true*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq ts[1]*ts[2]*ts[1]*ts[6]*ts[2]*
    ts[1]*ts[5];
/*true*/
/*inverse*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq ts[5]*ts[1]*ts[2]*ts[6]*ts[1]*
    ts[2]*ts[1];
/*true*/
5,1,\underline{2,6},1,2,1
```

```
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq n*ts[1]*ts
    [5]*ts[2]*ts[1]*ts[2]*ts[1] then for i in [1..#Sch] do if g(n
    ) eq h(Sch[i]) then Sch[i]; end if; end for; end if; end for;
(y * x^-1 * y * x * y^-1)^3
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f((y * x^-1 * y * x * y^-1)^3)*
    ts[1]*ts[5]*ts[2]*ts[1]*ts[2]*ts[1];
/*true*/
for n in IN do if (ts[1]*ts[2]*ts[1]*ts[2]*ts[1])^-1 eq n*ts[1]*
    ts[2]*ts[1]*ts[2]*ts[5]*ts[1] then for i in [1..#Sch] do if g
    (n) eq h(Sch[i]) then Sch[i]; end if; end for; end if; end
    for;
(v * u^-1 * v * u * v^-1)^3
(ts[1]*ts[2]*ts[1]*ts[2]*ts[1])^-1 eq f((y * x^-1 * y * x * y
    ^-1)^3)*ts[1]*ts[2]*ts[1]*ts[2]*ts[5]*ts[1];
/*true*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f((y * x^-1 * y * x * y^-1)^3)*
    ts[1]*ts[2]*ts[1]*ts[2]*ts[5]*ts[1];
/*true*/


/*LEMMA 3 WORK*/
for n in N do if [2,7,2] eq [1,4,1]^n then [1,2,1]^(xx^5*yy^3)^n
    ; end if; end for;
/*[ 9, 7, 9 ]*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2] eq n*ts[9]*ts[7]*ts
    [9]*ts[2]*ts[7] then for i in [1..#Sch] do if g(n) eq h(Sch[i
    ]) then Sch[i]; end if; end for; end if; end for;
/*(y^-1 * x * y * x^-1 * y)^3*/
ts[1]*ts[2]*ts[1]*ts[2] eq f((y^-1 * x * y * x^-1 * y)^3)*ts[9]*
```

```
    ts[7]*ts[9]*ts[2]*ts[7];
/*true*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2] eq n*ts[2]*ts[1]*ts
    [9]*ts[27]*ts[19]*ts[5]*ts[7] then for i in [1..#Sch] do if g
    (n) eq h(Sch[i]) then Sch[i]; end if; end for; end if; end
    for;
/*x^-1 * y^-2 * x^-1 * y^2 * x*/
ts[1]*ts[2]*ts[1]*ts[2] eq f(x^-1 * y^-2 * x^-1 * y^2 * x)*ts
    [2]*ts[1]*ts[9]*ts[27]*ts[19]*ts[5]*ts[7];
/*true*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2] eq n*ts[30]*ts[29]*ts
    [17]*ts[27]*ts[3]*ts[10]*ts[7] then for i in [1..#Sch] do if
    g(n) eq h(Sch[i]) then Sch[i]; end if; end for; end if; end
    for;
/*x^-2 * y^-1 * x^-1 * y^-1 * x * y*/
ts[1]*ts[2]*ts[1]*ts[2] eq f(x^-2 * y^-1 * x^-1 * y^-1 * x * y)*
    ts[30]*ts[29]*ts[17]*ts[27]*ts[3]*ts[10]*ts[7];
/*true*/
for n in N do if [29,17,29] eq [1,4,1]^n then [30]^(xx^5*yy^3)^n
    ; end if; end for;
/*[22]*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2] eq n*ts[22]*ts[29]*ts
    [27]*ts[3]*ts[10]*ts[7] then for i in [1..#Sch] do if g(n) eq
     h(Sch[i]) then Sch[i]; end if; end for; end if; end for;
/*x^-1 * y^-2 * x * y * x^-1*/
ts[1]*ts[2]*ts[1]*ts[2] eq f(x^-1 * y^-2 * x * y * x^-1)*ts[22]*
    ts[29]*ts[27]*ts[3]*ts[10]*ts[7];
/*true*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2] eq n*ts[22]*ts[23]*ts
    [10]*ts[7] then for i in [1..#Sch] do if g(n) eq h(Sch[i])
    then Sch[i]; end if; end for; end if; end for;
/*x^-1 * y^-1 * x^-1 * y^3 * x^-1*/
```

```
ts[1]*ts[2]*ts[1]*ts[2] eq f(x^-1 * y^-1 * x^-1 * y^3 * x^-1)*ts
    [22]*ts[23]*ts[10]*ts[7];
/*true*/

/*LEMMA 4*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f((y^-1 * x * y^-1 * x^-1 * y
    ^-1 * x )^-1)*ts[7]*ts[27]*ts[25]*ts[32]*ts[22]*ts[2];
/*true*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq n*ts[7]*ts
    [25]*ts[32]*ts[22]*ts[2] then for i in [1..#Sch] do if g(n)
    eq h(Sch[i]) then Sch[i]; end if; end for; end if; end for;
/*y * x^2 * y * x^-1 * y * x*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f(y * x^2 * y * x^-1 * y * x)*
    ts[7]*ts[25]*ts[32]*ts[22]*ts[2];
/*true*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq n*ts[7]*ts
    [21]*ts[22]*ts[23]*ts[32]*ts[22]*ts[2] then for i in [1..#Sch
    ] do if g(n) eq h(Sch[i]) then Sch[i]; end if; end for; end
    if; end for;
/*y^-1 * x * y^-2 * x * y^-2*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f(y^-1 * x * y^-2 * x * y^-2)*
    ts[7]*ts[21]*ts[22]*ts[23]*ts[32]*ts[22]*ts[2];
/*true*/
for n in N do if [23,32,23] eq [1,4,1]^n then [7,21,22]^(xx^5*yy
    ^3)^n; end if; end for;
/*[ 2, 22, 21 ]*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq n*ts[2]*ts
    [22]*ts[21]*ts[23]*ts[22]*ts[2] then for i in [1..#Sch] do if
     g(n) eq h(Sch[i]) then Sch[i]; end if; end for; end if; end
    for;
/*y * x^-1 * y^-1 * x * y^-2 * x*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f(y * x^-1 * y^-1 * x * y^-2 *
```

```
   x)*ts[2]*ts[22]*ts[21]*ts[23]*ts[22]*ts[2];
/*true*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq n*ts[2]*ts
   [5]*ts[10]*ts[23]*ts[22]*ts[2] then for i in [1..#Sch] do if
   g(n) eq h(Sch[i]) then Sch[i]; end if; end for; end if; end
   for;
/*x * y^3 * x * y^-1 * x*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f(x * y^3 * x * y^-1 * x)*ts
   [2]*ts[5]*ts[10]*ts[23]*ts[22]*ts[2];
/*true*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq n*ts[2]*ts
   [5]*ts[10]*ts[31]*ts[5]*ts[2] then for i in [1..#Sch] do if g
   (n) eq h(Sch[i]) then Sch[i]; end if; end for; end if; end
   for;
/*y^-2 * x * y * x^-2 * y*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f(y^-2 * x * y * x^-2 * y)*ts
   [2]*ts[5]*ts[10]*ts[31]*ts[5]*ts[2];
/*true*/
for n in N do if [2,7,2] eq [1,4,1]^n then [2,5,10,31,5]^(xx^5*
   yy^3)^n; end if; end for;
/*[ 7, 16, 29, 18, 16 ]*/

or n in IN do if ts[2]*ts[5]*ts[10]*ts[31]*ts[5]*ts[2]*ts[7]*ts
   [7] eq n*ts[7]*ts[1]*ts[14]*ts[18]*ts[16]*ts[2]*ts[7] then
   for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end
   if; end for; end if; end for;
/* (x * y * x^-1 * y * x^2 * y^-1) */

for n in IN do if ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq n*ts[7]*ts
   [16]*ts[29]*ts[18]*ts[16]*ts[2]*ts[7] then for i in [1..#Sch]
    do if g(n) eq h(Sch[i]) then Sch[i]; end if; end for; end if
   ; end for;
```

```
/*y^-1 * x * y * x * y * x^-1*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f(y^-1 * x * y * x * y * x^-1)*
    ts[7]*ts[16]*ts[29]*ts[18]*ts[16]*ts[2]*ts[7];
/*true*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq n*ts[7]*ts
    [1]*ts[14]*ts[18]*ts[16]*ts[2]*ts[7] then for i in [1..#Sch]
    do if g(n) eq h(Sch[i]) then Sch[i]; end if; end for; end if;
     end for;
/*x * y^-1 * x * y * x^-1 * y^-1 * x^2*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f(x * y^-1 * x * y * x^-1 * y
    ^-1 * x^2)*ts[7]*ts[1]*ts[14]*ts[18]*ts[16]*ts[2]*ts[7];
/*true*/
for n in N do if [2,7,2] eq [1,4,1]^n then [7,1,14,18,16]^(xx^5*
    yy^3)^n; end if; end for;
/*[ 2, 9, 20, 31, 5 ]*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq n*ts[2]*ts
    [9]*ts[20]*ts[31]*ts[5]*ts[2] then for i in [1..#Sch] do if g
    (n) eq h(Sch[i]) then Sch[i]; end if; end for; end if; end
    for;
/*x^-1 * y * x * y^-1 * x^-1 * y^-2*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f(x^-1 * y * x * y^-1 * x^-1 *
    y^-2)*ts[2]*ts[9]*ts[20]*ts[31]*ts[5]*ts[2];
/*true*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq n*ts[4]*ts
    [18]*ts[20]*ts[31]*ts[28]*ts[4] then for i in [1..#Sch] do if
     g(n) eq h(Sch[i]) then Sch[i]; end if; end for; end if; end
    for;
/*y * x^-1 * y * x^-1 * y^-1 * x^-1*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f(y * x^-1 * y * x^-1 * y^-1 *
    x^-1)*ts[4]*ts[18]*ts[20]*ts[31]*ts[28]*ts[4];
/*true*/
for n in IN do if ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq n*ts[4]*ts
```

```
    [8]*ts[26]*ts[31]*ts[28]*ts[4] then for i in [1..#Sch] do if
    g(n) eq h(Sch[i]) then Sch[i]; end if; end for; end if; end
    for;
/*x^-2 * y * x^-1 * y^-1 * x*/
ts[1]*ts[2]*ts[1]*ts[2]*ts[1] eq f(x^-2 * y * x^-1 * y^-1 * x)*
    ts[4]*ts[8]*ts[26]*ts[31]*ts[28]*ts[4];
/*true*/



/*LEMMA 5*/
ts[2]*ts[5]*ts[10]*ts[31]*ts[5]*ts[2] eq ts[2]*ts[5]*ts[10]*ts
    [31]*ts[5]*ts[2]*ts[7]*ts[7];
/*true*/
ts[2]*ts[5]*ts[10]*ts[31]*ts[5]*ts[2]*ts[7]*ts[7] eq f((y^-1 * x
     * y * x^-1 * y)^3)*ts[7]*ts[16]*ts[29]*ts[18]*ts[16]*ts[2]*
    ts[7];
/*true*/

for n in IN do if f((y^-1 * x * y * x^-1 * y)^3)*ts[7]*ts[16]*ts
    [29]*ts[18]*ts[16]*ts[2]*ts[7] eq n*ts[7]*ts[1]*ts[14]*ts
    [18]*ts[16]*ts[2]*ts[7] then for i in [1..#Sch] do if g(n) eq
     h(Sch[i]) then Sch[i]; end if; end for; end if; end for;
/*x * y * x^-1 * y * x^2 * y^-1*/
f((y^-1 * x * y * x^-1 * y)^3)*ts[7]*ts[16]*ts[29]*ts[18]*ts
    [16]*ts[2]*ts[7] eq f(x * y * x^-1 * y * x^2 * y^-1)*ts[7]*ts
    [1]*ts[14]*ts[18]*ts[16]*ts[2]*ts[7];
/*true*/

for n in IN do if f(x * y * x^-1 * y * x^2 * y^-1)*ts[7]*ts[1]*
    ts[14]*ts[18]*ts[16]*ts[2]*ts[7] eq n*ts[2]*ts[9]*ts[20]*ts
    [31]*ts[5]*ts[2] then for i in [1..#Sch] do if g(n) eq h(Sch[
```

```
    i]) then Sch[i]; end if; end for; end if; end for;
/*x^3 * y^3 * x*/
f(x * y * x^-1 * y * x^2 * y^-1)*ts[7]*ts[1]*ts[14]*ts[18]*ts
    [16]*ts[2]*ts[7] eq
f(x^3 * y^3 * x)*ts[2]*ts[9]*ts[20]*ts[31]*ts[5]*ts[2];
/*true*/
for n in IN do if f(x^3 * y^3 * x)*ts[2]*ts[9]*ts[20]*ts[31]*ts
    [5]*ts[2] eq n*ts[4]*ts[18]*ts[20]*ts[31]*ts[28]*ts[4] then
    for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end
    if; end for; end if; end for;
/*(x * y^-2)^2*/
f(x^3 * y^3 * x)*ts[2]*ts[9]*ts[20]*ts[31]*ts[5]*ts[2] eq f((x *
     y^-2)^2)*ts[4]*ts[18]*ts[20]*ts[31]*ts[28]*ts[4];
/*true*/
for n in IN do if f((x * y^-2)^2)*ts[4]*ts[18]*ts[20]*ts[31]*ts
    [28]*ts[4] eq n*ts[4]*ts[8]*ts[26]*ts[31]*ts[28]*ts[4] then
    for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end
    if; end for; end if; end for;
/*x^2 * y^2*/
f((x * y^-2)^2)*ts[4]*ts[18]*ts[20]*ts[31]*ts[28]*ts[4] eq f(x^2
     * y^2)*ts[4]*ts[8]*ts[26]*ts[31]*ts[28]*ts[4];
/*true*/
for n in IN do if f(x^2 * y^2)*ts[4]*ts[8]*ts[26]*ts[31]*ts[28]*
    ts[4] eq n then for i in [1..#Sch] do if g(n) eq h(Sch[i])
    then Sch[i]; end if; end for; end if; end for;
/*x^2 * y * x * y * x * y^-1*/
f(x^2 * y^2)*ts[4]*ts[8]*ts[26]*ts[31]*ts[28]*ts[4] eq f(x^2 * y
     * x * y * x * y^-1);
/*true*/
for n in IN do if ts[2]*ts[5]*ts[31]*ts[23]*ts[22]*ts[2] eq n
    then for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i];
    end if; end for; end if; end for;
```

```
/*x * y * x * y^2*/
ts[2]*ts[5]*ts[31]*ts[23]*ts[22]*ts[2] eq f(x * y * x * y^2);
/*true*/
```

——————MEETING QUESTION LEMMA——————

```
Lemma 6:
t$_1$,t$_8$,\underline{t$_{23}$},t$_{29}$
\\= (y * x$^2$ * y$^{-2}$ * x$^{-2}$)t$_6$,t$_9$,t$_{23}$,t$_
    {13}$,t$_6$,\underline{t$_1$,t$_3$},t$_{29}$ (R2)
\\= ((y * x * y$^{-1}$)$^4$)t$_{12}$,t$_2$,t$_{30}$,t$_{11}$,t$_
    {12}$,t$_1$,t$_7$,t$_{21}$,\underline{t$_{10}$,t$_{29}$}
\\= (x$^2$ * y * x$^2$ * y$^{-1}$ * x$^{-1}$ * y$^{-1}$)t$_{19}$
    ,t$_7$,t$_8$,t$_{25}$,t$_{19}$,\underline{t$_9$,t$_2$,t$_{22}
    $},t$_{10}$ (R2)
\\= (x * y * x$^4$ * y$^{-1}$)t$_{15}$,t$_{30}$,t$_{32}$,t$_{21}
    $,t$_{15}$,t$_9$,t$_{27}$,\underline{t$_{29}$,t$_{10}$}
\\= (y * x * y * x$^{-1}$ * y$^{-1}$ * x * y * x$^{-1}$)t$_{24}$
    ,t$_8$,t$_{23}$,t$_{22}$,t$_{24}$,t$_1$,t$_3$,t$_{29}$
\\= (y * x * y * x$^{-1}$ * y$^{-1}$ * x * y * x$^{-1}$)t$_{24}$
    ,t$_8$,\underline{t$_{23}$,t$_{22}$,t$_{24}$},t$_1$,t$_3$,t$_
    {29}$ (R2)
\\= (x$^4$ * y * x$^{-1}$ * y)t$_{28}$,t$_8$,t$_{23}$,t$_{31}$,
    t$_{28}$,t$_{1}$,t$_3$,t$_{29}$
\\= (y * x * y$^{-2}$ * x)t$_7$,t$_{22}$,t$_{23}$,t$_{25}$,\
    underline{t$_1$,t$_3$},t$_{29}$
\\= ((y * x * y * x$^{-1}$ * y$^{-1}$)$^2$)t$_1$,t$_{28}$,t$_
    {30}$,t$_{17}$,t$_7$,t$_{21}$,t$_{10}$,t$_{29}$
\\= (y$^2$ * x$^{-1}$ * y$^{-1}$ * x$^2$)t$_9$,t$_4$,t$_8$,t$_
    {26}$,t$_2$,t$_{22}$,t$_{10}$
\\= (x * y$^2$ * x$^{-1}$ * y$^{-2}$ * x)t$_2$,t$_4$,t$_2$,t$_9$
    ,t$_2$,t$_{22}$,t$_{10}$
\\= (y$^2$ * x * y$^3$ * x$^{-1}$ * y)t$_{32}$,t$_4$,t$_2$,t$_
```

{11}$,t$_2$,t$_{22}$,t$_{10}$

\\= (y * x^−1 * y^2 * x)t$_{17}$,t$_7$,t$_{16}$,t$_{11}$,t$_2$,t$_5$,t$_{10}$

\\= ((y * x * y$^{−2}$)^2)t$_{32}$,t$_{24}$,t$_{20}$,t$_2$, t$_2$,t$_{11}$,t$_{30}$

\\= (x^4 * y * x * y * x$^{−1}$)t$_{18}$,t$_{12}$,t$_{20}$,t$_ {30}$

——————————————11,2,22 = 11,2,5 ————————————————

t$_{11}$,t$_2$,t$_{22}$ = t$_1$,t$_8$,t$_{23}$,t$_{29}$

\\= (y * x$^{−1}$ * y^2 * x)t$_{17}$,t$_7$,t$_{16}$,t$_{11}$,\ underline{t$_2$,t$_5$,t$_{10}$}

---

lemma 7:

1,8,23,29=(x^4 * y * x * y * x^−1)18,12,20,30 (from lemma 1)

\implies (1,8,23,29=(x^4 * y * x * y * x^−1)18,12,20,30)^−1

\implies 29,23,8,1 = 30,20,12,18(x^4 * y * x * y * x^−1)^−1

\implies 29,23,8,1 = (x * y * x * y^2 * x * y * x)8,14,23,1

Since (1)^2=e

\implies 29,23,8 = (x * y * x * y^2 * x * y * x)8,14,23

\implies (x^4 * y * x * y * x^−1)29,23,8 = 8,14,23 (true)

conjugated by (x^−2 * y^−1 * x^−1 * y^2)

\implies (y * x * y * x^−3 * y)24,18,28 = 28,12,18

conjugate by (y^−1 * x^−1 * y * x^−1 * y^−1 * x)

\implies (y^−1 * x^2 * y * x * y^−1 * x) 30,20,28 = 28,17,20

conjugate by (y^2 * x^3 * y^−1 * x^−1)

\implies (x^−3 * y^−1 * x^−1 * y * x^−1) 4,14,28 = 28,18,14

Move 28,18,14 to the other side

\implies (x^−3 * y^−1 * x^−1 * y * x^−1)4,\underline

```
    {14,28,14},18,28
\implies (x^-2 * y^-1 * x^-1 * y^-1 * x^2)25,32,19,\underline
    {24,18,28}
\implies (y * x * y^-1 * x^-2 * y^-1)30,20,28,28,12,18
\implies (y * x * y^-1 * x^-2 * y^-1)30,20,12,18

f(x^-3 * y^-1 * x^-1 * y * x^-1)*ts[4]*ts[14]*ts[28]*ts[14]*ts
    [18]*ts[28] eq f(x^-2 * y^-1 * x^-1 * y^-1 * x^2)*ts[25]*ts
    [32]*ts[19]*ts[24]*ts[18]*ts[28];
/*true*/
f(x^-2 * y^-1 * x^-1 * y^-1 * x^2)*ts[25]*ts[32]*ts[19]*ts[24]*
    ts[18]*ts[28] eq f(y * x * y^-1 * x^-2 * y^-1)*ts[30]*ts[20]*
    ts[28]*ts[28]*ts[12]*ts[18];
/*true*/
f(y * x * y^-1 * x^-2 * y^-1)*ts[30]*ts[20]*ts[28]*ts[28]*ts
    [12]*ts[18] eq f(y * x * y^-1 * x^-2 * y^-1)ts[30]*ts[20]*ts
    [12]*ts[18];
```

Appendix B

Magma Code

$$\frac{2^{*110}:PSL(2,11)}{(xy^{-1}xy^{-1}xy^{-1}xyxy^{-1}xyx)^3t_{49}t_{105}t_{25}} = \mathbf{M}_{12}$$

```
a:=0; b:=0; c:=0; d:=0; e:=0; f:=0; g:=0; h:=0; i:=0; j:=3;
S:=Sym(110);
xx:=S!(1, 2)(3, 8)(4, 11)(5, 13)(6, 17)(7, 20)(9, 26)(10, 23)
    (12, 34)(14, 40)(15,
          42)(16, 45)(18, 48)(19, 50)(21, 52)(22, 54)(24, 58)(25,
              61)(27, 57)(28,
          51)(29, 68)(30, 32)(31, 59)(33, 44)(35, 76)(36, 73)(38,
              71)(39, 79)(41,
          85)(43, 66)(46, 65)(47, 89)(49, 83)(53, 93)(55, 90)(56,
              96)(60, 78)(62,
          92)(63, 101)(64, 102)(70, 86)(72, 94)(74, 104)(75, 77)
              (80, 108)(81,
          109)(82, 103)(84, 106)(87, 88)(91, 95)(99, 107)(100,
              110);
yy:=S!(1, 3, 9)(2, 5, 14)(4, 12, 35)(6, 18, 49)(7, 21, 53)(8,
    23, 56)(10, 29,
          69)(11, 31, 71)(13, 37, 74)(15, 43, 88)(16, 22, 55)(17,
              46, 58)(19, 32,
```

```
        48)(20, 51, 41)(24, 59, 98)(25, 45, 65)(26, 64, 92)(27,
           67, 36)(30, 70,
        104)(33, 72, 95)(34, 73, 85)(38, 50, 78)(39, 80, 105)
           (40, 83, 84)(42,
        86, 75)(44, 89, 96)(52, 63, 66)(54, 94, 101)(57, 61,
           100)(60, 79,
        91)(62, 97, 99)(68, 77, 82)(76, 102, 108)(81, 110, 87)
           (90, 109, 103)(93,
        106, 107);
N:=sub<S|xx,yy>;

G<x,y,t>:=Group<x,y,t|x^2, y^3, (y^-1 * x * y * x)^5, (x * y^-1)
    ^11,
(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x)^2,
t^2, (t,x * y * x * y * x * y^-1 * x * y^-1 * x),
(t,x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x
    * y),
((x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x)*t
   ^(x * y * x * y * x * y * x * y * x))^a,
((y^-1 * x * y * x * y * x * y * x * y^-1 * x * y * x)*t^(x * y
   * x * y * x * y^-1 * x * y * x * y * x * y * x))^b,
((y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)
   *t^(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1 * x * y * x * y))^c,
((y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)
   *t^( y^-1 * x * y^-1 * x * y * x * y^-1 * x * y))^d,
((y * x * y * x * y * x * y * x * y * x * y^-1 * x * y^-1 * x *
   y)*t^(y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x * y
    * x * y^-1))^e,
((y * x * y * x * y * x * y * x * y * x * y^-1 * x * y^-1 * x *
   y)*t^(y * x * y * x * y * x * y^-1 * x * y * x * y * x))^f,
((y * x * y * x * y * x * y * x * y * x * y^-1 * x * y^-1 * x *
```

```
    y)*t^(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y^-1)
    )^g,
((x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x)
    *t^( y * x * y * x * y * x * y^-1 * x * y * x * y * x))^h,
((x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x)
    *t^(x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x))^i,
((x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x)
    *t^(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x *
    y * x))^j>;
#G;
/*95040*/
Index(G,sub<G|x,y>);
/*144*/
#sub<G|x,y>;
/*660*/
144*660;
/*95040*/
f,G1,k:=CosetAction(G,sub<G|x,y>);
#k;
/*1*/
CompositionFactors(G1);
/*
     G
     |   M12
     1
*/
#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);
/*5*/
/*NOTE PUT IN THE TS IN SMALL GROUPS TO NOT OVERWHELM THE MAGMA
    */
IN:=sub<G1|f(x),f(y)>;
ts:=[ Id(G1): i in [1 .. 110] ];
```

```
ts[1]:=f(t);
ts[2]:=f(t^( y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
   y));
ts[3]:=f(t^( x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y
  ^-1));
ts[4]:=f(t^( x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x
  * y^-1 * x * y));
ts[5]:=f(t^( y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
   y^-1));
ts[6]:=f(t^( y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
  x * y));
ts[7]:=f(t^( y^-1 * x * y^-1 * x * y * x * y * x * y * x * y));
ts[8]:=f(t^( y * x * y * x * y^-1 * x * y * x * y^-1));
ts[9]:=f(t^( y^-1));
ts[10]:=f(t^( (y * x)^2));
ts[11]:=f(t^( y^-1 * x * y * x * y * x * y * x * y * x * y^-1 *
  x * y * x * y));
ts[12]:=f(t^( y^-1 * x * y * x * y^-1 * x * y^-1));
ts[13]:=f(t^( y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
  * y^-1 * x));
ts[14]:=f(t^( x * y * x * y^-1 * x * y * x * y * x * y * x * y
  ^-1 * x));
ts[15]:=f(t^( (y * x)^5));
ts[16]:=f(t^( y * x * y^-1 * x * y * x * y * x * y^-1 * x * y
  ^-1));
ts[17]:=f(t^( x * y * x * y * x * y^-1 * x * y * x * y * x * y *
   x));
ts[18]:=f(t^( x * y * x * y * x * y * x * y * x * y * x));
ts[19]:=f(t^( y^-1 * x * y * x * y * x * y * x * y * x * y^-1 *
  x * y^-1 * x));
ts[20]:=f(t^( x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x
   * y^-1 * x * y^-1));
```

```
ts[21]:=f(t^( y^-1 * x * y^-1 * x * y * x * y * x * y * x * y
   ^-1));
ts[22]:=f(t^( x * y^-1 * x * y * x * y * x * y * x * y^-1 * x *
   y));
ts[23]:=f(t^( y * x * y));
ts[24]:=f(t^( x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x
    * y));
ts[25]:=f(t^( y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y
    * x * y * x));
ts[26]:=f(t^( y^-1 * x * y^-1 * x * y * x * y^-1 * x * y));
ts[27]:=f(t^( x * y^-1 * x * y * x * y * x * y * x * y * x * y
   ^-1 * x));
ts[28]:=f(t^( y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x
   * y^-1 * x));
ts[29]:=f(t^( x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x
   ));
ts[30]:=f(t^( x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1
   * x * y^-1 * x * y * x * y));
ts[31]:=f(t^( y^-1 * x * y * x * y * x * y * x * y * x * y^-1 *
   x * y * x * y^-1));
ts[32]:=f(t^( x * y * x * y * x * y * x * y * x));
ts[33]:=f(t^( y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x));
ts[34]:=f(t^( y^-1 * x * y^-1 * x * y * x * y * x * y * x * y *
   x * y^-1 * x * y));
ts[35]:=f(t^( y^-1 * x * y * x * y^-1 * x));
ts[36]:=f(t^( y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y
   ^-1 * x * y^-1 * x));
ts[37]:=f(t^( x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1
   * x * y^-1));
ts[38]:=f(t^( x * y^-1 * x * y * x * y * x * y^-1 * x * y * x *
   y * x));
ts[39]:=f(t^( y^-1 * x * y * x * y * x * y * x * y));
```

```
ts[40]:=f(t^( y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y
   ));
ts[41]:=f(t^( y^-1 * x * y^-1 * x * y * x * y * x * y * x * y *
   x * y^-1));
ts[42]:=f(t^( y * x * y * x * y * x * y * x * y));
ts[43]:=f(t^( y * x * y^-1 * x * y * x * y * x * y * x * y * x))
   ;
ts[44]:=f(t^( y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1));
ts[45]:=f(t^( y * x * y * x * y * x * y^-1 * x * y * x * y * x))
   ;
ts[46]:=f(t^( y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
    x * y * x * y));
ts[47]:=f(t^( y * x * y * x * y^-1 * x * y * x * y * x * y^-1 *
   x * y));
ts[48]:=f(t^( (x * y)^5));
ts[49]:=f(t^( y * x * y * x * y * x * y * x * y^-1 * x * y^-1 *
   x * y * x * y));
ts[50]:=f(t^( x * y * x * y * x * y * x * y * x * y^-1 * x));
ts[51]:=f(t^( y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x
   * y^-1));
ts[52]:=f(t^( x * y * x * y * x * y * x * y^-1 * x * y^-1 * x *
   y * x * y));
ts[53]:=f(t^( y^-1 * x * y^-1 * x * y * x * y * x * y * x));
ts[54]:=f(t^( y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x
   * y * x * y^-1));
ts[55]:=f(t^( y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x
   * y * x * y^-1 * x * y));
ts[56]:=f(t^( y * x * y^-1));
ts[57]:=f(t^( y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y
    * x));
ts[58]:=f(t^( y^-1 * x * y * x * y^-1 * x * y * x * y * x * y^-1
    * x));
```

```
ts[59]:=f(t^( y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
    x * y * x * y^-1 * x * y));
ts[60]:=f(t^( y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
   ^-1 * x * y));
ts[61]:=f(t^( x * y^-1 * x * y * x * y * x * y * x * y * x));
ts[62]:=f(t^( x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x
    * y^-1 * x * y * x * y^-1));
ts[63]:=f(t^( y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x
   * y * x * y * x));
ts[64]:=f(t^( x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1
   * x * y^-1));
ts[65]:=f(t^( x * y * x * y^-1 * x * y^-1 * x * y * x * y * x *
   y * x * y * x * y));
ts[66]:=f(t^( y * x * y^-1 * x * y * x * y * x * y * x * y));
ts[67]:=f(t^( y * x * y * x * y * x * y^-1 * x * y^-1 * x * y *
   x * y * x * y));
ts[68]:=f(t^( (y * x)^3));
ts[69]:=f(t^( x * y * x * y * x * y^-1 * x * y^-1 * x * y * x *
   y * x * y^-1));
ts[70]:=f(t^( y * x * y * x * y * x * y * x * y^-1 * x));
ts[71]:=f(t^( y^-1 * x * y * x * y^-1 * x * y * x * y^-1));
ts[72]:=f(t^( y^-1 * x * y * x * y * x * y^-1 * x * y * x * y
   ^-1));
ts[73]:=f(t^( x * y^-1 * x * y * x * y * x * y * x * y * x * y
   ^-1 * x * y^-1 * x));
ts[74]:=f(t^( y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
   * y^-1 * x * y^-1));
ts[75]:=f(t^( x * y * x * y * x * y * x * y^-1 * x * y));
ts[76]:=f(t^( x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x
    * y^-1));
ts[77]:=f(t^( y * x * y * x * y * x * y));
ts[78]:=f(t^( x * y^-1 * x * y * x * y * x * y^-1 * x * y * x *
```

```
    y * x * y^-1));
ts[79]:=f(t^( y^-1 * x * y * x * y * x * y * x * y * x));
ts[80]:=f(t^( y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y
    ));
ts[81]:=f(t^( y * x * y * x * y * x * y * x * y * x * y^-1 * x *
     y));
ts[82]:=f(t^( x * y * x * y^-1 * x * y^-1 * x * y * x * y));
ts[83]:=f(t^( y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1))
    ;
ts[84]:=f(t^( x * y * x * y^-1 * x * y * x * y * x * y * x * y))
    ;
ts[85]:=f(t^( y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y
    ^-1 * x));
ts[86]:=f(t^( y * x * y^-1 * x * y * x * y * x * y * x * y * x *
     y^-1 * x * y));
ts[87]:=f(t^( y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
    ^-1 * x * y^-1));
ts[88]:=f(t^( x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)
    );
ts[89]:=f(t^( y * x * y * x * y^-1 * x * y * x * y * x * y^-1));
ts[90]:=f(t^( (y * x * y^-1 * x * y * x)^2));
ts[91]:=f(t^( x * y * x * y * x * y * x * y * x * y^-1 * x * y *
     x * y^-1));
ts[92]:=f(t^( x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1
    * x));
ts[93]:=f(t^( (x * y^-1)^3));
ts[94]:=f(t^( y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
     * x));
ts[95]:=f(t^( y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
    ^-1));
ts[96]:=f(t^( x * y * x * y * x * y^-1 * x * y^-1 * x * y * x *
    y^-1 * x));
```

```
ts[97]:=f(t^( x * y^-1 * x * y^-1 * x * y * x * y^-1));
ts[98]:=f(t^( y^-1 * x * y * x * y^-1 * x * y * x * y * x * y));
ts[99]:=f(t^( x * y^-1 * x * y^-1 * x * y * x));
ts[100]:=f(t^( y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x *
    y * x * y^-1));
ts[101]:=f(t^( y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x
     * y * x));
ts[102]:=f(t^( x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 *
    x));
ts[103]:=f(t^( x * y * x * y^-1 * x * y^-1 * x * y * x * y * x))
    ;
ts[104]:=f(t^( x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1
     * x * y^-1 * x * y * x));
ts[105]:=f(t^( y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
    y^-1));
ts[106]:=f(t^( y^-1 * x * y^-1 * x * y^-1 * x * y^-1));
ts[107]:=f(t^( x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1 *
    x * y^-1 * x * y * x * y * x));
ts[108]:=f(t^( x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 *
    x * y));
ts[109]:=f(t^( y * x * y * x * y * x * y * x * y * x * y^-1 * x
    * y * x));
ts[110]:=f(t^( x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1
     * x * y^-1));
#DoubleCosets(G,sub<G|x,y>, sub<G|x,y>);
/*5*/
DoubleCosets(G,sub<G|x,y>, sub<G|x,y>);
DC:=[f(Id(G)), f(t), f(t * y^-1 * x * t), f(t * x* t), f(t * x *
     y * t)];
Index(G1,IN);
/*144*/
cst := [null : i in [1 .. Index(G1,IN)]] where null is [Integers
```

```
    () | ];
prodim := function(pt, Q, I)
v := pt;
for i in I do
      v := v^(Q[i]);
     end for;
return v;
end function;
for i := 1 to 110 do
       cst[prodim(1, ts, [i])] := [i];
      end for;
m:=0; for i in [1..144] do if cst[i] ne [] then m:=m+1; end if;
   end for;m;
/*55*/
Orbits(N);
/*
     GSet{@ 1, 2, 3, 5, 8, 9, 13, 14, 23, 26, 37, 40, 10, 56, 64,
         74, 83,
     29, 96, 102, 92, 104, 49, 84, 68, 69, 44, 108, 62, 30, 6,
        106, 77,
     33, 89, 80, 76, 97, 32, 70, 17, 18, 107, 75, 82, 72, 47,
        105, 35,
     99, 48, 86, 46, 93, 42, 103, 94, 95, 39, 4, 19, 65, 58, 53,
        15, 90,
     101, 91, 79, 11, 12, 50, 25, 24, 7, 43, 55, 109, 63, 54, 60,
         31, 34,
     78, 61, 45, 59, 20, 21, 66, 88, 16, 81, 22, 71, 73, 38, 100,
         98, 51,
     52, 87, 110, 36, 85, 57, 28, 41, 27, 67 @}
*/
 Generators(N);
/*{
```

```
    (1, 2)(3, 8)(4, 11)(5, 13)(6, 17)(7, 20)(9, 26)(10, 23)(12,
       34)(14, 40)(15,
        42)(16, 45)(18, 48)(19, 50)(21, 52)(22, 54)(24, 58)(25,
           61)(27, 57)(28,
        51)(29, 68)(30, 32)(31, 59)(33, 44)(35, 76)(36, 73)(38,
           71)(39, 79)(41,
        85)(43, 66)(46, 65)(47, 89)(49, 83)(53, 93)(55, 90)(56,
           96)(60, 78)(62,
        92)(63, 101)(64, 102)(70, 86)(72, 94)(74, 104)(75, 77)
           (80, 108)(81,
        109)(82, 103)(84, 106)(87, 88)(91, 95)(99, 107)(100,
           110),
    (1, 3, 9)(2, 5, 14)(4, 12, 35)(6, 18, 49)(7, 21, 53)(8, 23,
       56)(10, 29,
        69)(11, 31, 71)(13, 37, 74)(15, 43, 88)(16, 22, 55)(17,
           46, 58)(19, 32,
        48)(20, 51, 41)(24, 59, 98)(25, 45, 65)(26, 64, 92)(27,
           67, 36)(30, 70,
        104)(33, 72, 95)(34, 73, 85)(38, 50, 78)(39, 80, 105)
           (40, 83, 84)(42,
        86, 75)(44, 89, 96)(52, 63, 66)(54, 94, 101)(57, 61,
           100)(60, 79,
        91)(62, 97, 99)(68, 77, 82)(76, 102, 108)(81, 110, 87)
           (90, 109, 103)(93,
        106, 107)
}*/

for i in [1..#DC] do for m,n in IN do if ts[1] eq m*(DC[i])^n
   then i; break;end if; end for;end for;
/*2*/
/*THE BEGINNING OF 1*/
S:={[1]};
```

```
SS:=S^N;SS;
SSS:=Setseq(SS);
for i in [1..#SSS] do
for g in IN do if ts[1]
eq g*ts[Rep(SSS[i])[1]]
 then print SSS[i];
end if; end for; end for;
/*[ 1 ]
    [ 4 ]
*/

N1:=Stabiliser(N,1);
Generators(N1);
/*
 (2, 105)(3, 38)(5, 44)(6, 94)(7, 68)(8, 75)(9, 81)(10, 82)(11,
    97)(12, 40)(13,
        27)(14, 61)(15, 16)(17, 99)(18, 108)(19, 73)(20, 31)(21,
            79)(22, 93)(23,
        49)(24, 106)(25, 34)(26, 76)(28, 78)(29, 59)(30, 39)(32,
            52)(33, 57)(35,
        109)(36, 100)(37, 96)(41, 98)(42, 102)(43, 91)(45, 64)
           (46, 53)(47, 84)(48,
        92)(50, 110)(51, 104)(54, 65)(55, 67)(58, 89)(60, 74)
           (62, 80)(63, 101)(66,
        86)(69, 88)(70, 95)(71, 77)(72, 107)(83, 103),
    (2, 22, 7)(3, 28, 9)(5, 20, 16)(6, 18, 19)(8, 63, 25)(10,
       27, 30)(11, 43,
        33)(12, 47, 35)(13, 82, 39)(14, 21, 41)(15, 31, 44)(17,
           36, 29)(23, 92,
        52)(24, 62, 60)(26, 65, 66)(32, 48, 49)(34, 101, 75)(37,
            107, 77)(38, 81,
        78)(40, 109, 84)(42, 102, 85)(45, 64, 56)(46, 104, 67)
```

```
        (50, 110, 87)(51, 53,
      55)(54, 76, 86)(57, 91, 97)(58, 69, 70)(59, 100, 99)(61,
          98, 79)(68, 93,
      105)(71, 72, 96)(73, 108, 94)(74, 80, 106)(83, 103, 90)
        (88, 89, 95)

*/

N1:=Stabiliser(N,[1]);
#N1;
/*6*/
N1s:=N1;
#N1s;
/*6*/
Generators(N1s);
/*(2, 68)(3, 81)(5, 15)(6, 73)(7, 93)(8, 34)(9, 78)(10, 39)(11,
    57)(12, 109)(13, 30)(14,
      98)(16, 31)(17, 59)(18, 94)(19, 108)(20, 44)(21, 61)(22,
          105)(23, 32)(24, 74)(25,
      101)(26, 86)(27, 82)(28, 38)(29, 100)(33, 91)(35, 84)
        (36, 99)(37, 71)(40, 47)(41,
      79)(42, 85)(43, 97)(45, 56)(46, 55)(48, 52)(49, 92)(50,
        87)(51, 67)(53, 104)(54,
      66)(58, 95)(60, 80)(62, 106)(63, 75)(65, 76)(69, 89)(70,
          88)(72, 77)(83, 90)(96,
      107),
    (2, 105)(3, 38)(5, 44)(6, 94)(7, 68)(8, 75)(9, 81)(10, 82)
      (11, 97)(12, 40)(13, 27)(14,
      61)(15, 16)(17, 99)(18, 108)(19, 73)(20, 31)(21, 79)(22,
          93)(23, 49)(24, 106)(25,
      34)(26, 76)(28, 78)(29, 59)(30, 39)(32, 52)(33, 57)(35,
        109)(36, 100)(37, 96)(41,
```

```
        98)(42, 102)(43, 91)(45, 64)(46, 53)(47, 84)(48, 92)(50,
            110)(51, 104)(54, 65)(55,
        67)(58, 89)(60, 74)(62, 80)(63, 101)(66, 86)(69, 88)(70,
            95)(71, 77)(72, 107)(83,
        103)*/
for n in N do if [1]^n eq [4] then N1s:=sub<N|N1s,n>; end if;
    end for;
#N1s;
/*12*/
[1]^N1s;
/*[ 1 ],
    [ 4 ]*/

Orbits(N1);
/* GSet{@ 1 @},
    GSet{@ 4 @},
    GSet{@ 42, 102, 85 @},
    GSet{@ 45, 64, 56 @},
    GSet{@ 50, 110, 87 @},
    GSet{@ 83, 103, 90 @},
    GSet{@ 2, 105, 93, 7, 22, 68 @},
    GSet{@ 3, 38, 78, 9, 28, 81 @},
    GSet{@ 5, 44, 31, 16, 20, 15 @},
    GSet{@ 6, 94, 108, 19, 18, 73 @},
    GSet{@ 8, 75, 101, 25, 63, 34 @},
    GSet{@ 10, 82, 13, 30, 27, 39 @},
    GSet{@ 11, 97, 91, 33, 43, 57 @},
    GSet{@ 12, 40, 84, 35, 47, 109 @},
    GSet{@ 14, 61, 79, 41, 21, 98 @},
    GSet{@ 17, 99, 100, 29, 36, 59 @},
    GSet{@ 23, 49, 48, 52, 92, 32 @},
    GSet{@ 24, 106, 80, 60, 62, 74 @},
```

```
    GSet{@ 26, 76, 54, 66, 65, 86 @},
    GSet{@ 37, 96, 72, 77, 107, 71 @},
    GSet{@ 46, 53, 51, 67, 104, 55 @},
    GSet{@ 58, 89, 88, 70, 69, 95 @}
*/
#N/#N1;
/*110*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[1] eq m*(DC[i
   ])^n then i; break; end if; end for;end for;
/*1*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[4] eq m*(DC[i
   ])^n then i; break; end if; end for;end for;
/*1*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[42] eq m*(DC[i
   ])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[45] eq m*(DC[i
   ])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[50] eq m*(DC[i
   ])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[83] eq m*(DC[i
   ])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2] eq m*(DC[i
   ])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[3] eq m*(DC[i
   ])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[5] eq m*(DC[i
```

```
    ])^n then i; break; end if; end for;end for;
/*3*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[6] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[8] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[10] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[11] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[12] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[14] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[17] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[23] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[24] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[26] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*5*/
```

```
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[37] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[46] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[58] eq m*(DC[i
    ])^n then i; break; end if; end for;end for;
/*2*/
/*THE BEGINNING OF 1,2*/
S:={[1,2]};
SS:=S^N;SS;
SSS:=Setseq(SS);
for i in [1..#SSS] do
for g in IN do if ts[1]*ts[2]
eq g*ts[Rep(SSS[i])[1]]*ts[Rep(SSS[i])[2]]
 then print SSS[i];
end if; end for; end for;
/*
     [ 1, 2 ]
     [ 64, 83 ]
     [ 92, 107 ]
     [ 108, 40 ]
     [ 57, 77 ]
     [ 4, 18 ]
     [ 90, 45 ]
     [ 87, 69 ]
     [ 68, 100 ]
     [ 102, 46 ]

*/
N12:=Stabiliser(N,[1,2]);
```

```
#N12;
/*1*/
N12s:=N12;
#N12s;
/*1*/
Generators(N12s);
/*{}*/
for n in N do if [1,2]^n eq [64,83] then N12s:=sub<N|N12s,n>;
   end if; end for;
#N12s;
/*2*/

[1,2]^N12s;
/* [ 1, 2 ],
     [ 64, 83 ]
*/
for n in N do if [1,2]^n eq [92,107] then N12s:=sub<N|N12s,n>;
   end if; end for;
#N12s;
/*10*/
Generators(N12s);
/*
{
     (1, 92)(2, 107)(3, 56)(4, 108)(5, 103)(7, 25)(8, 17)(9, 20)
        (10,
         37)(11, 79)(12, 85)(13, 80)(14, 28)(15, 23)(16, 73)(18,
            40)(19,
         58)(21, 74)(22, 84)(24, 72)(26, 94)(27, 29)(30, 34)(31,
            110)(33,
         75)(35, 44)(36, 98)(38, 48)(39, 86)(42, 67)(43, 78)(45,
            69)(46,
         100)(47, 71)(49, 104)(50, 61)(51, 101)(52, 76)(54, 88)
```

```
            (55,
        66)(57, 64)(59, 62)(60, 106)(63, 89)(65, 99)(68, 102)
            (70,
        82)(77, 83)(81, 91)(87, 90)(93, 109)(97, 105),
    (1, 64)(2, 83)(3, 5)(4, 102)(6, 84)(7, 106)(8, 44)(9, 23)
       (11,
        50)(12, 31)(13, 30)(14, 37)(15, 81)(16, 109)(17, 59)(18,
            46)(19,
        49)(20, 34)(21, 51)(22, 65)(25, 58)(26, 97)(27, 85)(28,
           41)(29,
        56)(32, 78)(33, 60)(35, 73)(38, 79)(40, 107)(42, 82)(43,
            86)(45,
        100)(47, 96)(48, 70)(52, 88)(53, 63)(54, 66)(55, 94)(57,
            87)(61,
        67)(62, 93)(68, 90)(69, 77)(71, 98)(72, 89)(75, 104)(76,
        105)(80, 91)(92, 108)(95, 101)(103, 110)
*/
[1,2]^N12s;
/*
    [ 1, 2 ],
    [ 64, 83 ],
    [ 92, 107 ],
    [ 57, 77 ],
    [ 108, 40 ],
    [ 87, 69 ],
    [ 4, 18 ],
    [ 90, 45 ],
    [ 102, 46 ],
    [ 68, 100 ]
*/
tr1:=Transversal(N,N12s);
for i:=1 to #tr1 do
```

```
ss:=[1,2]^tr1[i];
cst[prodim(1,ts,ss)]:=ss;
end for;
m:=0; for i in [1..144] do if cst[i] ne []
then m:=m+1;
end if; end for;m;
/*121*/
Orbits(N12s);
/*GSet{@ 6, 84, 22, 65, 99 @},
    GSet{@ 10, 37, 14, 28, 41 @},
    GSet{@ 21, 51, 74, 101, 95 @},
    GSet{@ 24, 72, 89, 63, 53 @},
    GSet{@ 32, 78, 43, 86, 39 @},
    GSet{@ 36, 98, 71, 47, 96 @},
    GSet{@ 1, 64, 92, 57, 108, 87, 4, 90, 102, 68 @},
    GSet{@ 2, 83, 107, 77, 40, 69, 18, 45, 46, 100 @},
    GSet{@ 3, 5, 56, 103, 29, 110, 27, 31, 85, 12 @},
    GSet{@ 7, 106, 25, 60, 58, 33, 19, 75, 49, 104 @},
    GSet{@ 8, 44, 17, 35, 59, 73, 62, 16, 93, 109 @},
    GSet{@ 9, 23, 20, 15, 34, 81, 30, 91, 13, 80 @},
    GSet{@ 11, 50, 79, 61, 38, 67, 48, 42, 70, 82 @},
    GSet{@ 26, 97, 94, 105, 55, 76, 66, 52, 54, 88 @}
*/
#N/#N12s;
/*66*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[6] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*5*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[10] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[21] eq m
```

```
   *(DC[i])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[24] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[32] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*5*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[36] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[1] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[2] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[3] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[7] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*3*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[8] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[9] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[11] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*2*/
```

```
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[2]*ts[26] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/4*/

/*THE BEGINNING OF 1,26*/
S:={[1,26]};
SS:=S^N;SS;
SSS:=Setseq(SS);
for i in [1..#SSS] do
for g in IN do if ts[1]*ts[26]
eq g*ts[Rep(SSS[i])[1]]*ts[Rep(SSS[i])[2]]
 then print SSS[i];
end if; end for; end for;
/*
    [ 1, 26 ]
    [ 2, 9 ]
    [ 64, 97 ]
    [ 83, 23 ]
    [ 102, 97 ]
    [ 49, 10 ]
    [ 4, 26 ]
    [ 19, 10 ]
    [ 11, 9 ]
    [ 50, 23 ]
    [ 109, 102 ]
    [ 42, 2 ]
    [ 16, 4 ]
    [ 81, 64 ]
    [ 82, 50 ]
    [ 82, 83 ]
    [ 15, 1 ]
    [ 9, 42 ]
```

[45, 11]
[42, 11]
[103, 19]
[103, 49]
[23, 82]
[26, 15]
[15, 4]
[23, 100]
[10, 103]
[109, 64]
[9, 45]
[10, 110]
[83, 73]
[16, 1]
[81, 102]
[76, 16]
[26, 16]
[50, 73]
[2, 35]
[49, 36]
[76, 15]
[45, 2]
[36, 110]
[105, 109]
[35, 45]
[64, 105]
[100, 50]
[110, 49]
[110, 19]
[102, 105]
[11, 35]
[19, 36]

```
    [ 1, 76 ]
    [ 105, 81 ]
    [ 35, 42 ]
    [ 97, 81 ]
    [ 36, 103 ]
    [ 73, 100 ]
    [ 100, 83 ]
    [ 4, 76 ]
    [ 97, 109 ]
    [ 73, 82 ]
*/
N126:=Stabiliser(N,[1,26]);
#N126;
/*1*/
N126s:=N126;
for n in N do if [1,26]^n eq [2,9] then N126s:=sub<N|N126s,n>;
  end if; end for;
#N126s;
/*2*/
[1,26]^N126s;
/*[ 1, 26 ],
    [ 2, 9 ]
*/
Generators(N126s);
  /*  (1, 2)(3, 8)(4, 11)(5, 13)(6, 17)(7, 20)(9, 26)(10, 23)
     (12, 34)(14, 40)(15,
        42)(16, 45)(18, 48)(19, 50)(21, 52)(22, 54)(24, 58)(25,
           61)(27, 57)(28,
        51)(29, 68)(30, 32)(31, 59)(33, 44)(35, 76)(36, 73)(38,
           71)(39, 79)(41,
        85)(43, 66)(46, 65)(47, 89)(49, 83)(53, 93)(55, 90)(56,
           96)(60, 78)(62,
```

```
        92)(63, 101)(64, 102)(70, 86)(72, 94)(74, 104)(75, 77)
            (80, 108)(81,
        109)(82, 103)(84, 106)(87, 88)(91, 95)(99, 107)(100,
            110)*/

for n in N do if [1,26]^n eq [64,97] then N126s:=sub<N|N126s,n>;
    end if; end for;
#N126s;
/*10*/
[1,26]^N126s;
/*      [ 1, 26 ],
    [ 2, 9 ],
    [ 64, 97 ],
    [ 83, 23 ],
    [ 102, 97 ],
    [ 49, 10 ],
    [ 4, 26 ],
    [ 19, 10 ],
    [ 11, 9 ],
    [ 50, 23 ]
*/

for n in N do if [1,26]^n eq [109,102] then N126s:=sub<N|N126s,n
    >; end if; end for;
#N126s;
/*60*/
Generators(N126s);
/*
{
    (1, 4)(2, 19)(3, 30)(5, 44)(6, 7)(8, 60)(9, 10)(11, 49)(12,
        17)(13,
        78)(15, 16)(18, 22)(20, 31)(21, 41)(23, 97)(24, 25)(27,
```

28)(29,
47)(32, 33)(34, 106)(35, 36)(37, 107)(38, 39)(40, 99)
(42,
110)(43, 48)(45, 103)(46, 70)(50, 102)(51, 88)(52, 57)
(53,
95)(54, 86)(55, 89)(56, 90)(58, 67)(59, 84)(62, 63)(64,
83)(65,
66)(68, 94)(69, 104)(72, 96)(73, 105)(74, 75)(79, 98)
(80,
101)(81, 82)(85, 87)(91, 92)(93, 108)(100, 109),
(1, 2)(3, 8)(4, 11)(5, 13)(6, 17)(7, 20)(9, 26)(10, 23)(12,
34)(14,
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61)(27, 57)(28, 51)(29, 68)(30, 32)(31, 59)(33, 44)(35,
76)(36,
73)(38, 71)(39, 79)(41, 85)(43, 66)(46, 65)(47, 89)(49,
83)(53,
93)(55, 90)(56, 96)(60, 78)(62, 92)(63, 101)(64, 102)
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86)(72, 94)(74, 104)(75, 77)(80, 108)(81, 109)(82, 103)
(84,
106)(87, 88)(91, 95)(99, 107)(100, 110),
(1, 102, 11, 19, 83)(2, 49, 50, 4, 64)(3, 13, 32, 60, 44)(5,
8, 33,
78, 30)(6, 106, 20, 12, 59)(7, 84, 17, 31, 34)(9, 10,
23, 26,
97)(14, 37, 40, 99, 107)(15, 109, 45, 110, 82)(16, 81,
42, 103,
100)(18, 65, 54, 43, 70)(21, 28, 85, 57, 88)(22, 46, 48,
86,
66)(24, 58, 61, 67, 25)(27, 41, 51, 52, 87)(29, 96, 89,

94,
90)(35, 36, 73, 76, 105)(38, 39, 79, 71, 98)(47, 56, 68,
55,
72)(53, 101, 91, 108, 62)(63, 93, 92, 80, 95)(69, 75,
74, 104,
77),
(1, 49)(2, 102)(3, 33)(4, 19)(5, 32)(6, 31)(7, 12)(8, 13)(9,
97)(10,
26)(11, 64)(14, 99)(15, 103)(16, 110)(17, 106)(18, 86)
(20,
84)(21, 87)(22, 43)(24, 61)(25, 67)(27, 88)(28, 52)(29,
55)(30,
60)(34, 59)(35, 105)(36, 76)(37, 40)(38, 98)(39, 71)(41,
57)(42,
109)(44, 78)(45, 81)(46, 54)(47, 94)(48, 65)(50, 83)(51,
85)(53,
92)(56, 89)(62, 80)(63, 91)(66, 70)(68, 96)(69, 75)(72,
90)(74,
77)(82, 100)(93, 101)(95, 108),
(1, 50)(2, 11)(3, 78)(4, 83)(5, 60)(6, 34)(7, 59)(8, 32)(10,
97)(12,
84)(13, 33)(14, 107)(15, 100)(16, 82)(17, 20)(18, 66)
(19,
64)(21, 27)(22, 70)(23, 26)(24, 67)(28, 87)(29, 72)(30,
44)(31,
106)(36, 105)(37, 99)(39, 98)(41, 88)(42, 45)(43, 46)
(47,
90)(48, 54)(49, 102)(51, 57)(52, 85)(53, 80)(55, 96)(56,
94)(58,
61)(62, 95)(63, 108)(65, 86)(68, 89)(69, 74)(71, 79)(73,
76)(77,
104)(81, 110)(91, 93)(92, 101)(103, 109),

(1, 64)(2, 83)(3, 5)(4, 102)(6, 84)(7, 106)(8, 44)(9, 23)
(11,
50)(12, 31)(13, 30)(14, 37)(15, 81)(16, 109)(17, 59)(18,
46)(19,
49)(20, 34)(21, 51)(22, 65)(25, 58)(26, 97)(27, 85)(28,
41)(29,
56)(32, 78)(33, 60)(35, 73)(38, 79)(40, 107)(42, 82)(43,
86)(45,
100)(47, 96)(48, 70)(52, 88)(53, 63)(54, 66)(55, 94)(57,
87)(61,
67)(62, 93)(68, 90)(69, 77)(71, 98)(72, 89)(75, 104)(76,
105)(80, 91)(92, 108)(95, 101)(103, 110),
(1, 83, 19, 11, 102)(2, 64, 4, 50, 49)(3, 44, 60, 32, 13)(5,
30, 78,
33, 8)(6, 59, 12, 20, 106)(7, 34, 31, 17, 84)(9, 97, 26,
23,
10)(14, 107, 99, 40, 37)(15, 82, 110, 45, 109)(16, 100,
103, 42,
81)(18, 70, 43, 54, 65)(21, 88, 57, 85, 28)(22, 66, 86,
48,
46)(24, 25, 67, 61, 58)(27, 87, 52, 51, 41)(29, 90, 94,
89,
96)(35, 105, 76, 73, 36)(38, 98, 71, 79, 39)(47, 72, 55,
68,
56)(53, 62, 108, 91, 101)(63, 95, 80, 92, 93)(69, 77,
104, 74,
75),
(1, 11, 83, 102, 19)(2, 50, 64, 49, 4)(3, 32, 44, 13, 60)(5,
33, 30,
8, 78)(6, 20, 59, 106, 12)(7, 17, 34, 84, 31)(9, 23, 97,
10,
26)(14, 40, 107, 37, 99)(15, 45, 82, 109, 110)(16, 42,

100, 81,
103)(18, 54, 70, 65, 43)(21, 85, 88, 28, 57)(22, 48, 66,
46,
86)(24, 61, 25, 58, 67)(27, 51, 87, 41, 52)(29, 89, 90,
96,
94)(35, 73, 105, 36, 76)(38, 79, 98, 39, 71)(47, 68, 72,
56,
55)(53, 91, 62, 101, 108)(63, 92, 95, 93, 80)(69, 74,
77, 75,
104),
(1, 19, 102, 83, 11)(2, 4, 49, 64, 50)(3, 60, 13, 44, 32)(5,
78, 8,
30, 33)(6, 12, 106, 59, 20)(7, 31, 84, 34, 17)(9, 26,
10, 97,
23)(14, 99, 37, 107, 40)(15, 110, 109, 82, 45)(16, 103,
81, 100,
42)(18, 43, 65, 70, 54)(21, 57, 28, 88, 85)(22, 86, 46,
66,
48)(24, 67, 58, 25, 61)(27, 52, 41, 87, 51)(29, 94, 96,
90,
89)(35, 76, 36, 105, 73)(38, 71, 39, 98, 79)(47, 55, 56,
72,
68)(53, 108, 101, 62, 91)(63, 80, 93, 95, 92)(69, 104,
75, 77,
74),
(1, 109, 82)(2, 42, 9)(3, 60, 85)(4, 81, 100)(5, 98, 104)(6,
72,
24)(7, 25, 96)(8, 30, 87)(10, 110, 19)(11, 45, 35)(12,
106,
56)(13, 38, 88)(14, 29, 47)(15, 97, 73)(16, 105, 23)(17,
90,
34)(18, 62, 66)(20, 67, 107)(21, 74, 32)(22, 65, 63)(26,

```
        102,
    50)(27, 28, 71)(31, 37, 58)(33, 75, 41)(36, 103, 49)(39,
        78,
    51)(40, 55, 59)(43, 86, 101)(44, 69, 79)(46, 91, 108)
       (48, 80,
    54)(52, 77, 57)(61, 68, 94)(64, 83, 76)(70, 93, 92)(84,
       89, 99)
}
*/
[1,26]^N126s;
/* [ 1, 26 ],
   [ 2, 9 ],
   [ 64, 97 ],
   [ 83, 23 ],
   [ 102, 97 ],
   [ 49, 10 ],
   [ 4, 26 ],
   [ 19, 10 ],
   [ 11, 9 ],
   [ 50, 23 ],
   [ 109, 102 ],
   [ 42, 2 ],
   [ 83, 73 ],
   [ 76, 16 ],
   [ 50, 73 ],
   [ 36, 110 ],
   [ 81, 102 ],
   [ 10, 110 ],
   [ 45, 2 ],
   [ 26, 16 ],
   [ 81, 64 ],
   [ 16, 4 ],
```

```
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[ 110, 19 ],
[ 103, 49 ],
[ 82, 50 ],
[ 9, 42 ],
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[ 64, 105 ],
[ 11, 35 ],
[ 102, 105 ],
[ 4, 76 ],
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[ 36, 103 ],
[ 35, 42 ],
[ 73, 82 ],
[ 26, 15 ],
[ 103, 19 ],
[ 109, 64 ],
[ 15, 4 ],
[ 16, 1 ],
[ 42, 11 ],
[ 100, 83 ],
[ 110, 49 ],
```

```
[ 23, 100 ],
[ 10, 103 ],
[ 9, 45 ],
[ 23, 82 ],
[ 97, 109 ],
[ 97, 81 ]
*/
tr1:=Transversal(N,N126s);
for i:=1 to #tr1 do
ss:=[1,26]^tr1[i];
cst[prodim(1,ts,ss)]:=ss;
end for;
m:=0; for i in [1..144] do if cst[i] ne []
then m:=m+1;
end if; end for;m;
/*132*/
#N/#N126s;
/*11*/
Orbits(N126s);
/*
[
GSet{@ 18, 48, 46, 70, 65, 86, 22, 43, 54, 66, 62, 80, 91,
   93, 63,
101, 92, 108, 53, 95 @},
GSet{@ 1, 2, 64, 83, 102, 49, 4, 19, 11, 50, 109, 42, 76,
   36, 81,
10, 45, 26, 16, 15, 100, 82, 110, 103, 9, 35, 105, 73, 23,
   97 @},
GSet{@ 3, 8, 5, 44, 13, 33, 30, 60, 32, 78, 98, 69, 38, 75,
   87, 85,
21, 51, 71, 79, 39, 104, 77, 74, 88, 41, 57, 52, 27, 28 @},
GSet{@ 6, 17, 84, 59, 106, 31, 7, 12, 20, 34, 72, 90, 89,
```

```
        40, 56,
     37, 25, 67, 94, 55, 47, 96, 68, 29, 24, 99, 14, 107, 58, 61
        @}
]
*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[26]*ts[18] eq
   m*(DC[i])^n then i; break; end if; end for;end for;
/*5*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[26]*ts[1] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[26]*ts[3] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*4*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[26]*ts[6] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*4*/

/*THE BEGINNING OF 1,5*/
S:={[1,5]};
SS:=S^N;SS;
SSS:=Setseq(SS);
for i in [1..#SSS] do
for g in IN do if ts[1]*ts[5]
eq g*ts[Rep(SSS[i])[1]]*ts[Rep(SSS[i])[2]]
 then print SSS[i];
end if; end for; end for;
/*
     [ 1, 5 ]
     [ 2, 13 ]
     [ 69, 18 ]
     [ 13, 70 ]
```

[69, 48]
[89, 93]
[5, 86]
[70, 2]
[47, 53]
[86, 1]
[91, 10]
[59, 46]
[95, 23]
[73, 47]
[31, 65]
[70, 11]
[89, 91]
[36, 89]
[18, 109]
[86, 4]
[81, 67]
[47, 95]
[1, 31]
[23, 28]
[51, 93]
[46, 2]
[48, 81]
[93, 10]
[109, 69]
[28, 53]
[109, 67]
[48, 109]
[2, 59]
[53, 23]
[10, 51]
[95, 73]

```
    [ 51, 91 ]
    [ 65, 1 ]
    [ 11, 59 ]
    [ 81, 69 ]
    [ 10, 89 ]
    [ 5, 65 ]
    [ 18, 81 ]
    [ 65, 4 ]
    [ 91, 36 ]
    [ 28, 95 ]
    [ 4, 31 ]
    [ 59, 70 ]
    [ 23, 47 ]
    [ 13, 46 ]
    [ 67, 18 ]
    [ 46, 11 ]
    [ 11, 13 ]
    [ 31, 86 ]
    [ 53, 73 ]
    [ 36, 51 ]
    [ 67, 48 ]
    [ 4, 5 ]
    [ 93, 36 ]
    [ 73, 28 ]
*/
N15:=Stabiliser(N,[1,5]);
#N15;
/*1*/
N15s:=N15;
for n in N do if [1,5]^n eq [2,13] then N15s:=sub<N|N15s,n>; end
    if; end for;
#N15s;
```

```
/*2*/
Generators(N15s);
/* (1, 2)(3, 8)(4, 11)(5, 13)(6, 17)(7, 20)(9, 26)(10, 23)(12,
   34)(14,
        40)(15, 42)(16, 45)(18, 48)(19, 50)(21, 52)(22, 54)(24,
           58)(25,
        61)(27, 57)(28, 51)(29, 68)(30, 32)(31, 59)(33, 44)(35,
           76)(36,
        73)(38, 71)(39, 79)(41, 85)(43, 66)(46, 65)(47, 89)(49,
           83)(53,
        93)(55, 90)(56, 96)(60, 78)(62, 92)(63, 101)(64, 102)
           (70,
        86)(72, 94)(74, 104)(75, 77)(80, 108)(81, 109)(82, 103)
           (84,
        106)(87, 88)(91, 95)(99, 107)(100, 110)
*/
[1,5]^N15s;
/*[ 1, 5 ],
    [ 2, 13 ]
*/
for n in N do if [1,5]^n eq [69,18] then N15s:=sub<N|N15s,n>;
   end if; end for;
#N15s;
/*60*/
Generators(N15s);
/* (1, 69, 89)(2, 13, 70)(3, 76, 99)(4, 67, 51)(5, 18, 93)(6,
   38,
        104)(7, 94, 15)(8, 82, 29)(9, 74, 84)(10, 86, 109)(11,
           59,
        46)(12, 26, 39)(14, 56, 92)(16, 33, 52)(17, 57, 55)(19,
           42,
        37)(21, 83, 97)(22, 87, 77)(23, 95, 47)(24, 78, 35)(25,
```

```
        101,
      103)(27, 34, 100)(28, 73, 53)(30, 68, 88)(31, 48, 91)
        (32, 40,
      58)(36, 65, 81)(41, 66, 107)(43, 90, 61)(45, 98, 49)(50,
         105,
      72)(54, 79, 96)(60, 102, 80)(62, 106, 64)(63, 110, 75)
        (71, 85,
      108),
  (1, 2)(3, 8)(4, 11)(5, 13)(6, 17)(7, 20)(9, 26)(10, 23)(12,
     34)(14,
      40)(15, 42)(16, 45)(18, 48)(19, 50)(21, 52)(22, 54)(24,
        58)(25,
      61)(27, 57)(28, 51)(29, 68)(30, 32)(31, 59)(33, 44)(35,
        76)(36,
      73)(38, 71)(39, 79)(41, 85)(43, 66)(46, 65)(47, 89)(49,
        83)(53,
      93)(55, 90)(56, 96)(60, 78)(62, 92)(63, 101)(64, 102)
        (70,
      86)(72, 94)(74, 104)(75, 77)(80, 108)(81, 109)(82, 103)
        (84,
      106)(87, 88)(91, 95)(99, 107)(100, 110)
*/
[1,5]^N15s;
/*
  [ 1, 5 ],
    [ 2, 13 ],
    [ 69, 18 ],
    [ 13, 70 ],
    [ 69, 48 ],
    [ 89, 93 ],
    [ 5, 86 ],
    [ 70, 2 ],
```

```
[ 89, 91 ],
[ 47, 53 ],
[ 18, 109 ],
[ 86, 1 ],
[ 47, 95 ],
[ 1, 31 ],
[ 23, 28 ],
[ 48, 81 ],
[ 93, 10 ],
[ 109, 69 ],
[ 23, 47 ],
[ 2, 59 ],
[ 10, 51 ],
[ 95, 73 ],
[ 91, 36 ],
[ 53, 23 ],
[ 81, 69 ],
[ 10, 89 ],
[ 95, 23 ],
[ 13, 46 ],
[ 86, 4 ],
[ 31, 65 ],
[ 28, 95 ],
[ 36, 89 ],
[ 91, 10 ],
[ 5, 65 ],
[ 70, 11 ],
[ 109, 67 ],
[ 59, 46 ],
[ 51, 91 ],
[ 73, 47 ],
[ 65, 1 ],
```

```
    [ 31, 86 ],
    [ 18, 81 ],
    [ 81, 67 ],
    [ 46, 11 ],
    [ 4, 31 ],
    [ 46, 2 ],
    [ 59, 70 ],
    [ 48, 109 ],
    [ 93, 36 ],
    [ 36, 51 ],
    [ 65, 4 ],
    [ 11, 59 ],
    [ 67, 48 ],
    [ 11, 13 ],
    [ 53, 73 ],
    [ 73, 28 ],
    [ 67, 18 ],
    [ 4, 5 ],
    [ 28, 53 ],
    [ 51, 93 ]
*/
tr1:=Transversal(N,N15s);
for i:=1 to #tr1 do
ss:=[1,5]^tr1[i];
cst[prodim(1,ts,ss)]:=ss;
end for;
m:=0; for i in [1..144] do if cst[i] ne []
then m:=m+1;
end if; end for;m;
/*143*/
#N/#N15s;
/*11*/
```

```
Orbits(N15s);
/*GSet{@ 7, 20, 94, 72, 15, 50, 42, 19, 105, 37 @},
    GSet{@ 16, 45, 33, 98, 44, 52, 49, 21, 83, 97 @},
    GSet{@ 1, 2, 69, 13, 89, 5, 70, 47, 18, 86, 23, 48, 93, 109,
        10, 95,
    91, 53, 81, 31, 28, 36, 59, 51, 73, 65, 46, 4, 11, 67 @},
    GSet{@ 3, 8, 76, 82, 35, 99, 103, 29, 24, 107, 25, 68, 58,
       78, 41,
    61, 101, 88, 32, 60, 85, 66, 43, 63, 87, 30, 40, 102, 108,
       90, 110,
    77, 14, 64, 80, 71, 55, 100, 75, 22, 56, 62, 38, 17, 27, 54,
        96, 92,
    106, 104, 6, 57, 34, 79, 84, 74, 12, 39, 9, 26 @}
*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[5]*ts[7] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*3*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[5]*ts[16] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*3*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[5]*ts[5] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*2*/
for i in [1..#DC] do for m,n in IN do if ts[1]*ts[5]*ts[3] eq m
   *(DC[i])^n then i; break; end if; end for;end for;
/*4*/


/*RELATION*/
(xx * yy^-1 * xx * yy^-1 * xx * yy^-1 * xx * yy * xx * yy^-1 *
   xx * yy * xx)^3;
/*(1, 6, 12, 39, 57)(2, 35, 101, 98, 59)(3, 99, 68, 4, 32)(5,
```

```
   87, 22, 108, 85)(7,
    88, 76, 58, 52)(8, 36, 107, 83, 95)(9, 63, 37, 13, 11)(10,
       79, 50, 53, 34)(14,
    61, 91, 20, 48)(15, 38, 89, 67, 30)(16, 40, 51, 69, 17)(18,
       71, 77, 93,
    44)(19, 75, 106, 97, 66)(21, 62, 100, 73, 47)(23, 28, 72,
       80, 82)(24, 70, 42,
    41, 81)(25, 60, 105, 54, 49)(26, 104, 94, 33, 55)(27, 64,
       110, 84, 86)(29,
    102, 103, 78, 65)(31, 90, 43, 92, 56)(45, 96, 109, 74, 46)*/

f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x)
   ^3*ts[49]*ts[105] eq ts[25];
/*true*/
/*RELATION CONJUGATED*/
for n in N do  ((xx * yy^-1 * xx * yy^-1 * xx * yy^-1 * xx * yy
   * xx * yy^-1 * xx * yy * xx)^3)^n;endfor;   [49,105,25]^n;
   end for;

/*AT THE END INSERT THE FOLLOWING 5 LINES ONLY ONCE*/
L<u,v>:=Group<u,v|u^2,v^3,(v^-1 * u * v * u)^5,
(u * v^-1)^11,
(v * u * v * u * v * u * v^-1 * u * v^-1 * u * v^-1 * u)^2 >;
Sch:=SchreierSystem(L,sub<L|Id(L)>);
h:=hom<L->N|u->xx,v->yy>;
g:=hom<IN->N|f(x)->xx,f(y)->yy>;
A:=(xx * yy^-1 * xx * yy^-1 * xx * yy^-1 * xx * yy * xx * yy^-1
   * xx * yy*xx)^3;

for n in N do
if {55,13} subset Set([49,105,25]^n)  then A^n, [49,105,25]^n;
end if; end for;
```

```
/*THEN DO SOMETHING SIMILAR TO THIS*/
for n in IN do if n*ts[49]*ts[105] eq ts[25] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for;
break; end if; end for;
/*SHOULD GIVE SOMETHING SIMILAR TO THIS*/
/* > L<u,v>:=Group<u,v|u^2,v^3,(v^-1 * u * v * u)^5      ,(u * v
    ^-1)^11, (v * u * v * u * v * u * v^-1 * u * v^-1 * u * v^-1
    * u)^2 >;
> Sch:=SchreierSystem(L,sub<L|Id(L)>);
> h:=hom<L->N|u->xx,v->yy>;
> g:=hom<IN->N|f(x)->xx,f(y)->yy>;
> /* For example */
> for n in IN do if n*ts[49]*ts[105] eq ts[25] then
for|if> for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i];
    end if; end for;
for|if> break; end if; end for;
v * u * v * u * v * u * v * u * v^-1 * u * v^-1 * u * v * u * v
    ^-1
*/
According to Magama
f(y * x * y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x *
    y^-10)*ts[49]*ts[105] eq ts[25];
/*true*/

/*RELATIONS*/
f((x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x
    )^3)*ts[49]*ts[105]*ts[25];
```

```
f((x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x
    )^3)*ts[49]*ts[105]eq ts[25];
/*true*/
A:=(xx * yy^-1 * xx * yy^-1 * xx * yy^-1 * xx * yy * xx * yy^-1
    * xx * yy*xx)^3;
for n in N do
if 1 in Set([49,105,25]^n)  then A^n, [49,105,25]^n;
end if; end for;

f((x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x
    )^4)*ts[102]*ts[65] eq ts[29]*ts[103];
/*true*/
f((x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x
    )^4)*ts[102]*ts[65]*ts[103]*ts[29];
B:=(xx * yy^-1 * xx * yy^-1 * xx * yy^-1 * xx * yy * xx * yy^-1
    * xx * yy * xx)^4;
for n in N do
if 1 in Set([102,65,29,103]^n)  then B^n, [102,65,29,103]^n;
end if; end for;


f((x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x
    )^6)*ts[45]*ts[74]*ts[96]*ts[46]*ts[109]*ts[45];
f((x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x
    )^6)*ts[45]*ts[74]*ts[96] eq ts[45]*ts[109]*ts[46];
/*true*/
C:=(xx * yy^-1 * xx * yy^-1 * xx * yy^-1 * xx * yy * xx * yy^-1
    * xx * yy * xx)^6;
for n in N do
if 92 in Set([45,74,96,45,109,46]^n)  then C^n,
    [45,74,96,45,109,46]^n;
end if; end for;
```

```
/*CODE TO HELP PROVE RELATIONS*/
L<u,v>:=Group<u,v|u^2,v^3,(v^-1 * u * v * u)^5,
(u * v^-1)^11,
(v * u * v * u * v * u * v^-1 * u * v^-1 * u * v^-1 * u)^2 >;
Sch:=SchreierSystem(L,sub<L|Id(L)>);
h:=hom<L->N|u->xx,v->yy>;
g:=hom<IN->N|f(x)->xx,f(y)->yy>;
A:=(xx * yy^-1 * xx * yy^-1 * xx * yy^-1 * xx * yy * xx * yy^-1
    * xx * yy*xx)^3;
B:=(xx * yy^-1 * xx * yy^-1 * xx * yy^-1 * xx * yy * xx * yy^-1
    * xx * yy * xx)^4;
C:=(yy * xx * yy^-1 * xx * yy * xx * yy * xx * yy^-1 * xx * yy
    ^-1 * xx * yy);
D:=(yy * xx * yy^-1 * xx * yy * xx * yy^-1 * xx * yy^-1 * xx *
    yy^-1 * xx * yy^-1);
E:=(yy^-1 * xx * yy * xx * yy^-1 * xx * yy^-1 * xx * yy * xx *
    yy * xx * yy);

for n in N do
if {45,30} subset Set([49,105,25]^n)  then A^n, [49,105,25]^n;
end if; end for;

for n in N do
if {1,18} subset Set([102,65,103,29]^n)  then B^n,
    [102,65,103,29]^n;
end if; end for;

for n in N do
if {3} subset Set([1,4]^n)  then [1,4]^n;
```

```
end if; end for;

for n in N do
if {19,37} subset Set([92,107,2,1]^n)  then C^n, [92,107,2,1]^n;
end if; end for;

for n in N do
if {19,37} subset Set([69,18,5,1]^n)  then D^n, [69,18,5,1]^n;
end if; end for;

for n in N do
if {19,37} subset Set([109,102,26,1]^n)  then E^n,
   [109,102,26,1]^n;
end if; end for;


/*PERMUTATION TO WORD CONVERSION CODE*/
NN<a,b>:=Group<a,b|a^2,b^3,(b^-1 * a * b * a)^5,
(a * b^-1)^11,
(b * a * b * a * b * a * b^-1 * a * b^-1 * a * b^-1 * a)^2 >;

word:=function(A)
Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
for i in [2..#N] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=xx; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=xx^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=yy; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=yy^-1; end if;
end for;
PP:=Id(N);
```

```
for k in [1..#P] do
PP:=PP*P[k]; end for;
if A eq PP then B:=Sch[i]; end if;
end for;
return B;
end function;
```

EXAMPLE:

```
/*word(N!(1, 2)(3, 4)(5, 14, 9, 6, 11, 7)(8, 12, 28, 10, 15, 27)
    (13, 32, 25, 16, 31, 20)(17, 22)(18, 23)(19, 29, 26, 24, 30,
     21) );
a^-2 * b^-2 * a
*/
/*PROVED RELATIONS CODE*/
/*Relation*/
for n in IN do if ts[1]*ts[4] eq
n*ts[55]*ts[13]*ts[4] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x
    */
ts[1]*ts[4] eq
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x)
    *ts[55]*ts[13]*ts[4];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
     y * x * y^-1 * x)*ts[55]*ts[13]*ts[4] eq
n*ts[38]*ts[81]*ts[100]*ts[4] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
    * y^-1 * x*/
```

```
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x)
   *ts[55]*ts[13]*ts[4] eq
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
   * y^-1 * x)*ts[38]*ts[81]*ts[100]*ts[4];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y^-1 * x * y * x * y^-1 * x)*ts[38]*ts[81]*ts[100]*ts[4] eq
n*ts[100]*ts[88]*ts[38] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
    x * y^-1*/
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
   * y^-1 * x)*ts[38]*ts[81]*ts[100]*ts[4] eq
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
    x * y^-1)*ts[100]*ts[88]*ts[38];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
    y^-1 * x * y^-1 * x * y^-1)*ts[100]*ts[88]*ts[38] eq
n*ts[95]*ts[89]*ts[88]*ts[38] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
   * y^-1 * x*/
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
    x * y^-1)*ts[100]*ts[88]*ts[38] eq
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
   * y^-1 * x)*ts[95]*ts[89]*ts[88]*ts[38];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y^-1 * x * y * x * y^-1 * x)*ts[95]*ts[89]*ts[88]*ts[38] eq
n*ts[99]*ts[59]*ts[38] then
```

```
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x*/
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
   * y^-1 * x)*ts[95]*ts[89]*ts[88]*ts[38] eq
f(y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x)*ts[99]*ts[59]*
   ts[38];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y * x * y^-1 *
    x)*ts[99]*ts[59]*ts[38] eq
n*ts[81]*ts[109] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
f(y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x)*ts[99]*ts[59]*
   ts[38] eq
ts[81]*ts[109];
/*true*/
```

```
/*FIRST DOUBLE COSET [1]*/
1,4=1 (DONE)
for n in IN do if ts[1]*ts[4] eq
n*ts[1]*ts[1] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[4] eq ts[1]*ts[1];
/*true*/
```

```
1,42 belongs to [1] (DONE)
for n in IN do if ts[1]*ts[42] eq
n*ts[62]*ts[92]*ts[44] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x
   * y * x * y^-1*/
ts[1]*ts[42] eq
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x
   * y * x * y^-1)*ts[62]*ts[92]*ts[44];
/*true*/
for n in IN do if f(x * y * x * y * x * y^-1 * x * y^-1 * x * y
   ^-1 * x * y^-1 * x * y * x * y^-1)*ts[62]*ts[92]*ts[44] eq
n*ts[46]*ts[69]*ts[35]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y*/
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x
   * y * x * y^-1)*ts[62]*ts[92]*ts[44] eq
f(x * y^-1 * x * y * x * y * x * y)*ts[46]*ts[69]*ts[35]*ts[50];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y * x * y)*ts[46]*ts
   [69]*ts[35]*ts[50] eq
n*ts[46]*ts[69]*ts[9]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y*/
f(x * y^-1 * x * y * x * y * x * y)*ts[46]*ts[69]*ts[35]*ts[50]
   eq
f(x * y^-1 * x * y * x * y * x * y)*ts[46]*ts[69]*ts[9]*ts[50];
/*true*/
```

```
for n in IN do if f(x * y^-1 * x * y * x * y * x * y)*ts[46]*ts
   [69]*ts[9]*ts[50] eq
n*ts[1]*ts[4]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y * x * y * x * y)*ts[46]*ts[69]*ts[9]*ts[50]
   eq
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[1]*ts
   [4]*ts[50];
/*true*/
for n in IN do if f(y * x * y * x * y^-1 * x * y * x * y^-1 * x
   * y^-1)*ts[1]*ts[4]*ts[50] eq
n*ts[4]*ts[4]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1*/
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[1]*ts
   [4]*ts[50] eq
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[4]*ts
   [4]*ts[50];
/*true*/
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[1]*ts
   [4]*ts[50] eq
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[50];
/*true*/

[1,45] belongs to [1] (DONE)
for n in IN do if ts[1]*ts[45] eq
n*ts[96]*ts[101]*ts[30] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
```

```
/*y * x * y^-1 * x*/
ts[1]*ts[45] eq
f(y * x * y^-1 * x)*ts[96]*ts[101]*ts[30];
/*true*/
for n in IN do if f(y * x * y^-1 * x)*ts[96]*ts[101]*ts[30] eq
n*ts[96]*ts[63]*ts[30] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x*/
f(y * x * y^-1 * x)*ts[96]*ts[101]*ts[30] eq
f(y * x * y^-1 * x)*ts[96]*ts[63]*ts[30];
/*true*/
for n in IN do if f(y * x * y^-1 * x)*ts[96]*ts[63]*ts[30] eq
n*ts[83] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1*/
f(y * x * y^-1 * x)*ts[96]*ts[63]*ts[30] eq
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[83];
/*true*/

[1,50] belongs to [1] (DONE)
for n in IN do if ts[1]*ts[50] eq
n*ts[34]*ts[57]*ts[16] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y^-1 * x*/
ts[1]*ts[50] eq
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y^-1 * x)*ts[34]*ts[57]*ts[16];
/*true*/
```

```
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
    ^-1 * x * y^-1 * x * y^-1 * x)*ts[34]*ts[57]*ts[16] eq
n*ts[6]*ts[16] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x*/
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
     y^-1 * x)*ts[34]*ts[57]*ts[16] eq
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x)*ts[6]*
    ts[16];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
     y^-1 * x)*ts[6]*ts[16] eq
n*ts[30]*ts[17]*ts[42] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x)*ts[6]*
    ts[16] eq
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[30]*ts
    [17]*ts[42];
/*true*/
for n in IN do if f(y * x * y * x * y^-1 * x * y * x * y^-1 * x
    * y^-1)*ts[30]*ts[17]*ts[42] eq
n*ts[17]*ts[17]*ts[42] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1*/
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[30]*ts
    [17]*ts[42] eq
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[17]*ts
    [17]*ts[42];
```

```
/*true*/
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[30]*ts
    [17]*ts[42] eq
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[42];
/*true*/


[1,83] belongs to [1] /*COPIED AND PASTED THE ABOVE SINCE
    83=50*/ (DONE)
for n in IN do if ts[1]*ts[83] eq
n*ts[1]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*Id(L)*/
for n in IN do if ts[1]*ts[50] eq
n*ts[34]*ts[57]*ts[16] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
     y^-1 * x*/
ts[1]*ts[50] eq
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
     y^-1 * x)*ts[34]*ts[57]*ts[16];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
    ^-1 * x * y^-1 * x * y^-1 * x)*ts[34]*ts[57]*ts[16] eq
n*ts[6]*ts[16] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x*/
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
     y^-1 * x)*ts[34]*ts[57]*ts[16] eq
```

```
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x)*ts[6]*
    ts[16];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
     y^-1 * x)*ts[6]*ts[16] eq
n*ts[30]*ts[17]*ts[42] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x)*ts[6]*
    ts[16] eq
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[30]*ts
    [17]*ts[42];
/*true*/
for n in IN do if f(y * x * y * x * y^-1 * x * y * x * y^-1 * x
    * y^-1)*ts[30]*ts[17]*ts[42] eq
n*ts[17]*ts[17]*ts[42] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1*/
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[30]*ts
    [17]*ts[42] eq
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[17]*ts
    [17]*ts[42];
/*true*/
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[30]*ts
    [17]*ts[42] eq
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[42];
/*true*/


[1,2] belongs to [1,2]
```

```
[1,3] belongs to [1] (DONE)
for n in IN do if ts[1]*ts[3] eq
n*ts[104]*ts[58]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y*/
ts[1]*ts[3] eq
f(y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y)*ts
   [104]*ts[58]*ts[70];
/*true*/
for n in IN do if f(y * x * y * x * y^-1 * x * y^-1 * x * y * x
   * y^-1 * x * y)*ts[104]*ts[58]*ts[70] eq
n*ts[58]*ts[58]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y*/
f(y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y)*ts
   [104]*ts[58]*ts[70] eq
f(y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y)*ts
   [58]*ts[58]*ts[70];
/*true*/
f(y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y)*ts
   [104]*ts[58]*ts[70] eq
f(y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y)*ts
   [70];
/*true*/


[1,5] belongs to [1,5]
[1,6] belongs to [1,2]  (DONE)
for n in IN do if ts[1]*ts[6] eq
n*ts[1]*ts[32] then
```

```
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[6] eq
ts[1]*ts[32];
/*true*/
for n in IN do if ts[1]*ts[32] eq
n*ts[66]*ts[52]*ts[107] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x*/
ts[1]*ts[32] eq
f(x * y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x)*ts
   [66]*ts[52]*ts[107];
/*true*/
for n in IN do if f(x * y * x * y * x * y * x * y^-1 * x * y^-1
   * x * y * x)*ts[66]*ts[52]*ts[107] eq
n*ts[66]*ts[94]*ts[107] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x*/
f(x * y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x)*ts
   [66]*ts[52]*ts[107] eq
f(x * y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x)*ts
   [66]*ts[94]*ts[107];
/*true*/
for n in IN do if f(x * y * x * y * x * y * x * y^-1 * x * y^-1
   * x * y * x)*ts[66]*ts[94]*ts[107] eq
n*ts[97]*ts[107] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1*/
```

```
f(x * y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x)*ts
   [66]*ts[94]*ts[107] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)*ts
   [97]*ts[107];
/*true*/
[97,107] in [1,2]^N;
/*true*/
```

[1,8] belongs to [1,2] (DONE)

```
for n in IN do if ts[1]*ts[8] eq
n*ts[1]*ts[34] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[8] eq
ts[1]*ts[34];
/*true*/
for n in IN do if ts[1]*ts[34] eq
n*ts[33]*ts[74]*ts[65] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x * y^-1*/
ts[1]*ts[34] eq
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x * y^-1)*ts[33]*ts[74]*ts[65];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
   x * y * x * y^-1 * x * y^-1)*ts[33]*ts[74]*ts[65] eq
n*ts[31]*ts[78]*ts[65] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y*/
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x * y^-1)*ts[33]*ts[74]*ts[65] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y)*ts[31]*
   ts[78]*ts[65];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
   x * y * x * y^-1 * x * y^-1)*ts[33]*ts[74]*ts[65] eq
n*ts[5]*ts[78]*ts[65] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y*/
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x * y^-1)*ts[33]*ts[74]*ts[65] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y)*ts[5]*
   ts[78]*ts[65]
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
   y * x * y)*ts[5]*ts[78]*ts[65] eq
n*ts[5]*ts[84]*ts[65] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y)*ts[5]*
   ts[78]*ts[65] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y)*ts[5]*
   ts[84]*ts[65];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
   y * x * y)*ts[5]*ts[84]*ts[65] eq
n*ts[20]*ts[68]*ts[65] then
```

```
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(x * y)^3*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y)*ts[5]*
   ts[84]*ts[65] eq
f((x * y)^3)*ts[20]*ts[68]*ts[65];
/*true*/
for n in IN do if f((x * y)^3)*ts[20]*ts[68]*ts[65] eq
n*ts[30]*ts[51] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x *
   y^-1*/
f((x * y)^3)*ts[20]*ts[68]*ts[65] eq
f(y * x * y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x *
   y^-1)*ts[30]*ts[51];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y * x * y^-1 * x * y
   ^-1 * x * y * x * y^-1)*ts[30]*ts[51] eq
n*ts[30]*ts[89] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x *
   y^-1*/
f(y * x * y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x *
   y^-1)*ts[30]*ts[51] eq
f(y * x * y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x *
   y^-1)*ts[30]*ts[89];
/*true*/
[30,89] in [1,2]^N;
/*true*/
```

```
[1,10] belongs to [1] (DONE)
for n in IN do if ts[1]*ts[10] eq
n*ts[46]*ts[46]*ts[104] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y
   ^-1*/
ts[1]*ts[10] eq
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)
   *ts[46]*ts[46]*ts[104];
/*true*/
ts[1]*ts[10] eq
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)
   *ts[104];
/*true*/


[1,11] belongs to [1,2] (DONE)
for n in IN do if ts[1]*ts[11] eq
n*ts[1]*ts[2] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[11] eq
ts[1]*ts[2];
/*true*/

[1,12] belongs to [1] (DONE)
for n in IN do if ts[1]*ts[12] eq
n*ts[1]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[12] eq
ts[1]*ts[3];
/*true*/
for n in IN do if ts[1]*ts[3] eq
n*ts[104]*ts[58]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y*/
ts[1]*ts[3] eq
f(y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y)*ts
   [104]*ts[58]*ts[70];
/*true*/
for n in IN do if f(y * x * y * x * y^-1 * x * y^-1 * x * y * x
   * y^-1 * x * y)*ts[104]*ts[58]*ts[70] eq
n*ts[58]*ts[58]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y*/
f(y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y)*ts
   [104]*ts[58]*ts[70]  eq
f(y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y)*ts
   [58]*ts[58]*ts[70];
/*true*/
f(y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y)*ts
   [104]*ts[58]*ts[70]  eq
f(y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y)*ts
   [70];
/*true*/
```

```
[1,14] belongs to [1,2] {68,22} (DONE)
for n in IN do if ts[1]*ts[14] eq
n*ts[1]*ts[71] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[14] eq
ts[1]*ts[71];
/*true*/
for n in IN do if ts[1]*ts[71] eq
n*ts[20]*ts[47]*ts[91] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x*/
ts[1]*ts[71] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x)*ts[20]*ts[47]*
   ts[91];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y *
    x)*ts[20]*ts[47]*ts[91] eq
n*ts[67]*ts[40]*ts[95]*ts[22] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x * y*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x)*ts[20]*ts[47]*
   ts[91] eq
f(x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x * y)*
   ts[67]*ts[40]*ts[95]*ts[22];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y * x * y * x * y * x
    * y^-1 * x * y)*ts[67]*ts[40]*ts[95]*ts[22] eq
n*ts[67]*ts[38]*ts[95]*ts[22] then
```

```
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x * y*/
f(x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x * y)*
   ts[67]*ts[40]*ts[95]*ts[22] eq
f(x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x * y)*
   ts[67]*ts[38]*ts[95]*ts[22];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y * x * y * x * y * x
    * y^-1 * x * y)*ts[67]*ts[38]*ts[95]*ts[22] eq
n*ts[68]*ts[89]*ts[22] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y
   ^-1*/
f(x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x * y)*
   ts[67]*ts[38]*ts[95]*ts[22] eq
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y
   ^-1)*ts[68]*ts[89]*ts[22];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
    y * x * y * x * y^-1)*ts[68]*ts[89]*ts[22] eq
n*ts[24]*ts[32]*ts[89]*ts[22] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y * x * y * x * y^-1 * x * y*/
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y
   ^-1)*ts[68]*ts[89]*ts[22] eq
f(y^-1 * x * y * x * y * x * y * x * y^-1 * x * y)*ts[24]*ts
   [32]*ts[89]*ts[22];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y * x * y * x * y^-1 * x
```

```
   * y)*ts[24]*ts[32]*ts[89]*ts[22] eq
n*ts[76]*ts[8]*ts[22] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y * x * y*/
f(y^-1 * x * y * x * y * x * y * x * y^-1 * x * y)*ts[24]*ts
   [32]*ts[89]*ts[22] eq
f(y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y * x * y)*ts
   [76]*ts[8]*ts[22];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y * x * y^-1 *
    x * y * x * y)*ts[76]*ts[8]*ts[22] eq
n*ts[68]*ts[22] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y^-1 * x*/
f(y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y * x * y)*ts
   [76]*ts[8]*ts[22] eq
f(y * x * y^-1 * x * y^-1 * x * y^-1 * x)*ts[68]*ts[22];
/*true*/
[68,22] in [1,2]^N;
/*true*/


[1,17] belongs to [1] (DONE)
for n in IN do if ts[1]*ts[17] eq
n*ts[1]*ts[30] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[17] eq
ts[1]*ts[30];
```

```
/*true*/
for n in IN do if ts[1]*ts[30] eq
n*ts[46] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(y^-1 * x * y * x * y^-1 * x)^2*/
ts[1]*ts[30] eq
f((y^-1 * x * y * x * y^-1 * x)^2)*ts[46];
/*true*/
[46] in [1]^N;
/*true*/

[1,23] belongs to [1,2] (DONE)
for n in IN do if ts[1]*ts[23] eq
n*ts[4]*ts[73] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[23] eq ts[4]*ts[73];
/*true*/
for n in IN do if ts[4]*ts[73] eq
n*ts[43]*ts[96] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
    y * x*/
ts[4]*ts[73] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
    y * x)*ts[43]*ts[96];
/*true*/
[43,96] in [1,2]^N;
/*true*/
```

```
[1,24] belongs to [1,2] {47,48} (DONE)
for n in IN do if ts[1]*ts[24] eq
n*ts[14]*ts[20]*ts[22] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y
   ^-1*/
ts[1]*ts[24] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y
   ^-1)*ts[14]*ts[20]*ts[22];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
   * x * y * x * y^-1)*ts[14]*ts[20]*ts[22] eq
n*ts[37]*ts[12]*ts[66]*ts[48] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y
   ^-1)*ts[14]*ts[20]*ts[22] eq
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*
   ts[37]*ts[12]*ts[66]*ts[48];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
    y^-1 * x * y^-1)*ts[37]*ts[12]*ts[66]*ts[48] eq
n*ts[98]*ts[12]*ts[66]*ts[48] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*
```

```
   ts[37]*ts[12]*ts[66]*ts[48] eq
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*
   ts[98]*ts[12]*ts[66]*ts[48] ;
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
    y^-1 * x * y^-1)*ts[98]*ts[12]*ts[66]*ts[48] eq
n*ts[38]*ts[66]*ts[48] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1*/
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*
   ts[98]*ts[12]*ts[66]*ts[48] eq
f(y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1)*ts
   [38]*ts[66]*ts[48];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y * x * y * x
   * y * x * y^-1)*ts[38]*ts[66]*ts[48] eq
n*ts[38]*ts[54]*ts[48] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1*/
f(y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1)*ts
   [38]*ts[66]*ts[48] eq
f(y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1)*ts
   [38]*ts[54]*ts[48];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y * x * y * x
   * y * x * y^-1)*ts[38]*ts[54]*ts[48] eq
n*ts[89]*ts[95]*ts[54]*ts[48] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x
```

```
* y*/
f(y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1)*ts
  [38]*ts[54]*ts[48] eq
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x
   * y)*ts[89]*ts[95]*ts[54]*ts[48];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y^-1 * x
  * y * x * y^-1 * x * y)*ts[89]*ts[95]*ts[54]*ts[48] eq
n*ts[14]*ts[91]*ts[48] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
  end for; end if; end for;
/*y^-1 * x * y * x * y * x * y * x*/
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x
   * y)*ts[89]*ts[95]*ts[54]*ts[48] eq
f(y^-1 * x * y * x * y * x * y * x)*ts[14]*ts[91]*ts[48];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y * x * y * x)*ts[14]*ts
  [91]*ts[48] eq
n*ts[71]*ts[91]*ts[48] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
  end for; end if; end for;
/*y^-1 * x * y * x * y * x * y * x*/
f(y^-1 * x * y * x * y * x * y * x)*ts[14]*ts[91]*ts[48] eq
f(y^-1 * x * y * x * y * x * y * x)*ts[71]*ts[91]*ts[48];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y * x * y * x)*ts[71]*ts
  [91]*ts[48] eq
n*ts[47]*ts[48] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
  end for; end if; end for;
/*x * y * x * y * x * y^-1*/
f(y^-1 * x * y * x * y * x * y * x)*ts[71]*ts[91]*ts[48] eq
```

```
f(x * y * x * y * x * y^-1)*ts[47]*ts[48];
/*true*/
[47,48] in [1,2]^N;
/*true*/
```

```
[1,26] belongs to [1,26]
[1,37] belongs to [1,2] {2,105} (DONE)
for n in IN do if ts[1]*ts[37] eq
n*ts[44]*ts[40]*ts[33] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1*/
ts[1]*ts[37] eq
f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*ts[44]*ts[40]*
   ts[33];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1)*ts[44]*ts[40]*ts[33] eq
n*ts[51]*ts[47]*ts[58]*ts[105] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y * x*/
f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*ts[44]*ts[40]*
   ts[33] eq
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y * x)*ts[51]*ts[47]*ts[58]*ts[105];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y
```

```
 ^-1 * x * y^-1 * x * y * x)*ts[51]*ts[47]*ts[58]*ts[105] eq
n*ts[51]*ts[28]*ts[58]*ts[105] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
  end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
   y * x*/
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
   y * x)*ts[51]*ts[47]*ts[58]*ts[105] eq
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
   y * x)*ts[51]*ts[28]*ts[58]*ts[105] ;
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y
  ^-1 * x * y^-1 * x * y * x)*ts[51]*ts[28]*ts[58]*ts[105] eq
n*ts[2]*ts[69]*ts[105] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
  end for; end if; end for;
/*x*/
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
   y * x)*ts[51]*ts[28]*ts[58]*ts[105] eq
f(x)*ts[2]*ts[69]*ts[105];
/*true*/
for n in IN do if f(x)*ts[2]*ts[69]*ts[105] eq
n*ts[74]*ts[23]*ts[69]*ts[105] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
  end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1
  * x*/
f(x)*ts[2]*ts[69]*ts[105] eq
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1
  * x)*ts[74]*ts[23]*ts[69]*ts[105];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y * x * y * x
```

```
   * y^-1 * x * y^-1 * x)*ts[74]*ts[23]*ts[69]*ts[105] eq
n*ts[65]*ts[34]*ts[105] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1*/
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1
   * x)*ts[74]*ts[23]*ts[69]*ts[105] eq
f(x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1)*ts[65]*ts[34]*
   ts[105];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y * x * y
   ^-1)*ts[65]*ts[34]*ts[105] eq
n*ts[2]*ts[105] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y
   ^-1*/
f(x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1)*ts[65]*ts[34]*
   ts[105] eq
f(y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)
   *ts[2]*ts[105];
/*true*/
[2,105] in [1,2]^N;
/*true*/

[1,46] belongs to [1] {4} (DONE)
for n in IN do if ts[1]*ts[46] eq
n*ts[30]*ts[10]*ts[29] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
    x * y * x * y*/
```

```
ts[1]*ts[46] eq
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
    x * y * x * y)*ts[30]*ts[10]*ts[29];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
    y^-1 * x * y^-1 * x * y * x * y)*ts[30]*ts[10]*ts[29] eq
n*ts[36]*ts[35]*ts[28]*ts[4] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
    x*/
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
    x * y * x * y)*ts[30]*ts[10]*ts[29] eq
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
    x)*ts[36]*ts[35]*ts[28]*ts[4];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
    y^-1 * x * y^-1 * x)*ts[36]*ts[35]*ts[28]*ts[4] eq
n*ts[10]*ts[35]*ts[28]*ts[4] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
    x*/
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
    x)*ts[36]*ts[35]*ts[28]*ts[4] eq
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
    x)*ts[10]*ts[35]*ts[28]*ts[4] ;
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
    y^-1 * x * y^-1 * x)*ts[10]*ts[35]*ts[28]*ts[4] eq
n*ts[28]*ts[28]*ts[4] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
   end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
     x * y * x * y^-1*/
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
     x)*ts[10]*ts[35]*ts[28]*ts[4] eq
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
     x * y * x * y^-1)*ts[28]*ts[28]*ts[4];
/*true*/
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
     x)*ts[10]*ts[35]*ts[28]*ts[4] eq
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
     x * y * x * y^-1)*ts[4];
/*true*/


[1,58] belongs to [1] {4} (DONE)
for n in IN do if ts[1]*ts[58] eq
n*ts[47]*ts[3]*ts[35] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(y * x * y * x * y^-1 * x)^2*/
ts[1]*ts[58] eq
f((y * x * y * x * y^-1 * x)^2)*ts[47]*ts[3]*ts[35];
/*true*/
for n in IN do if f((y * x * y * x * y^-1 * x)^2)*ts[47]*ts[3]*
   ts[35] eq
n*ts[3]*ts[29]*ts[30]*ts[1] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y * x * y * x * y * x*/
f((y * x * y * x * y^-1 * x)^2)*ts[47]*ts[3]*ts[35] eq
f(y * x * y^-1 * x * y^-1 * x * y * x * y * x * y * x)*ts[3]*ts
```

```
   [29]*ts[30]*ts[1];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y * x * y * x
   * y * x)*ts[3]*ts[29]*ts[30]*ts[1] eq
n*ts[30]*ts[30]*ts[1] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x*/
f(y * x * y^-1 * x * y^-1 * x * y * x * y * x * y * x)*ts[3]*ts
   [29]*ts[30]*ts[1] eq
f(x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x)*ts
   [30]*ts[30]*ts[1];
/*true*/
f(y * x * y^-1 * x * y^-1 * x * y * x * y * x * y * x)*ts[3]*ts
   [29]*ts[30]*ts[1] eq
f(x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x)*ts
   [1];
/*true*/




/*SECOND DOUBLE COSET [1,2]*/
1,2=64,83 (relation) (DONE)
for n in IN do if ts[1]*ts[2] eq
n*ts[26]*ts[28]*ts[103] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y*/
ts[1]*ts[2] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y)*ts[26]*ts[28]*ts
```

```
    [103];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y)
    *ts[26]*ts[28]*ts[103] eq
n*ts[54]*ts[104]*ts[28]*ts[103] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y * x * y*/
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y)*ts[26]*ts[28]*ts
    [103] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y)*ts[54]*ts[104]*ts
    [28]*ts[103];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y)*
    ts[54]*ts[104]*ts[28]*ts[103] eq
n*ts[54]*ts[104]*ts[47]*ts[103] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y * x * y*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y)*ts[54]*ts[104]*ts
    [28]*ts[103] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y)*ts[54]*ts[104]*ts
    [47]*ts[103];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y)*
    ts[54]*ts[104]*ts[47]*ts[103] eq
n*ts[37]*ts[1]*ts[103] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y)*ts[54]*ts[104]*ts
    [47]*ts[103] eq
```

```
f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*ts[37]*ts[1]*ts
    [103];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
    ^-1)*ts[37]*ts[1]*ts[103] eq
n*ts[110]*ts[55]*ts[98]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1*/
f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*ts[37]*ts[1]*ts
    [103] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1)*ts
    [110]*ts[55]*ts[98]*ts[50];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y * x * y^-1)*ts[110]*ts[55]*ts[98]*ts[50] eq
n*ts[110]*ts[88]*ts[98]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1)*ts
    [110]*ts[55]*ts[98]*ts[50] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1)*ts
    [110]*ts[88]*ts[98]*ts[50];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y * x * y^-1)*ts[110]*ts[88]*ts[98]*ts[50] eq
n*ts[87]*ts[97]*ts[7]*ts[98]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1)*ts
```

```
   [110]*ts[88]*ts[98]*ts[50] eq
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x)*ts
   [87]*ts[97]*ts[7]*ts[98]*ts[50];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
   x * y^-1 * x)*ts[87]*ts[97]*ts[7]*ts[98]*ts[50] eq
n*ts[29]*ts[28]*ts[68]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x*/
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x)*ts
   [87]*ts[97]*ts[7]*ts[98]*ts[50] eq
f(y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x)*ts
   [29]*ts[28]*ts[68]*ts[50];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y * x * y * x
   * y^-1 * x)*ts[29]*ts[28]*ts[68]*ts[50] eq
n*ts[58]*ts[30]*ts[28]*ts[68]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1*/
f(y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x)*ts
   [29]*ts[28]*ts[68]*ts[50] eq
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[58]*ts
   [30]*ts[28]*ts[68]*ts[50];
/*true*/
for n in IN do if f(y * x * y * x * y^-1 * x * y * x * y^-1 * x
   * y^-1)*ts[58]*ts[30]*ts[28]*ts[68]*ts[50] eq
n*ts[58]*ts[30]*ts[47]*ts[68]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1*/
```

```
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[58]*ts
    [30]*ts[28]*ts[68]*ts[50] eq
f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[58]*ts
    [30]*ts[47]*ts[68]*ts[50];
/*true*/
for n in IN do if f(y * x * y * x * y^-1 * x * y * x * y^-1 * x
    * y^-1)*ts[58]*ts[30]*ts[47]*ts[68]*ts[50] eq
n*ts[35]*ts[3]*ts[68]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y*/
 f(y * x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1)*ts[58]*ts
     [30]*ts[47]*ts[68]*ts[50] eq
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y)*ts[35]*ts
    [3]*ts[68]*ts[50];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y *
     x * y)*ts[35]*ts[3]*ts[68]*ts[50] eq
n*ts[29]*ts[68]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y)*ts[35]*ts
    [3]*ts[68]*ts[50] eq
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[29]*ts[68]*
    ts[50];
/*true*/
for n in IN do if f(x * y * x * y * x * y^-1 * x * y^-1 * x * y
    ^-1)*ts[29]*ts[68]*ts[50] eq
n*ts[29]*ts[57]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
```

```
/*x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[29]*ts[68]*
    ts[50] eq
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[29]*ts[57]*
    ts[50];
/*true*/
for n in IN do if f(x * y * x * y * x * y^-1 * x * y^-1 * x * y
    ^-1)*ts[29]*ts[57]*ts[50] eq
n*ts[27]*ts[57]*ts[50] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[29]*ts[57]*
    ts[50] eq
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[27]*ts[57]*
    ts[50];
/*true*/
for n in IN do if f(x * y * x * y * x * y^-1 * x * y^-1 * x * y
    ^-1)*ts[29]*ts[57]*ts[50] eq
n*ts[27]*ts[57]*ts[83] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[29]*ts[57]*
    ts[50] eq
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[27]*ts[57]*
    ts[83];
/*true*/
for n in IN do if f(x * y * x * y * x * y^-1 * x * y^-1 * x * y
    ^-1)*ts[27]*ts[57]*ts[83] eq
n*ts[9]*ts[12]*ts[57]*ts[83] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
   end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y*/
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[27]*ts[57]*
   ts[83] eq
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y)*ts[9]*ts
   [12]*ts[57]*ts[83];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y *
    x * y)*ts[9]*ts[12]*ts[57]*ts[83] eq
n*ts[25]*ts[96]*ts[83] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x*/
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y)*ts[9]*ts
   [12]*ts[57]*ts[83] eq
f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x)*ts
   [25]*ts[96]*ts[83];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x * y * x)*ts[25]*ts[96]*ts[83] eq
n*ts[75]*ts[96]*ts[83] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x*/
f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x)*ts
   [25]*ts[96]*ts[83] eq
f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x)*ts
   [75]*ts[96]*ts[83];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x * y * x)*ts[75]*ts[96]*ts[83] eq
n*ts[64]*ts[83] then
```

```
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(x * y^-1 * x * y)^2*/
f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x)*ts
   [75]*ts[96]*ts[83] eq
f((x * y^-1 * x * y)^2)*ts[64]*ts[83];
/*true*/

1,2=92,107 (RELATION) (DONE)
for n in IN do if ts[1]*ts[2] eq
n*ts[54]*ts[93]*ts[79] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y
   ^-1*/
ts[1]*ts[2] eq
f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y
   ^-1)*ts[54]*ts[93]*ts[79];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1 * x * y * x * y^-1)*ts[54]*ts[93]*ts[79] eq
n*ts[108]*ts[79] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y*/
f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y
   ^-1)*ts[54]*ts[93]*ts[79] eq
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [108]*ts[79];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y^-1 * x * y)*ts[108]*ts[79] eq
```

```
n*ts[92]*ts[79] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y*/
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [108]*ts[79] eq
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[79];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y^-1 * x * y)*ts[92]*ts[79] eq
n*ts[92]*ts[107] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y*/
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[79] eq
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107];
/*true*/
```

```
1,2,6 belongs to [1,26] (DONE)
for n in IN do if ts[1]*ts[2]*ts[6] eq
n*ts[1]*ts[11]*ts[6] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[2]*ts[6] eq
```

```
ts[1]*ts[11]*ts[6];
/*true*/
for n in IN do if ts[1]*ts[11]*ts[6] eq
n*ts[42]*ts[86] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(x * y^-1 * x * y * x * y^-1)^2*/
ts[1]*ts[11]*ts[6] eq
f((x * y^-1 * x * y * x * y^-1)^2)*ts[42]*ts[86];
/*true*/
[42,86] in [1,26]^N;
/*true*/

1,2,10 belongs to [1] {66} (DONE)
for n in IN do if ts[1]*ts[2]*ts[10] eq
n*ts[71]*ts[33]*ts[18]*ts[75] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(x * y^-1 * x * y * x * y)^2*/
ts[1]*ts[2]*ts[10] eq
f((x * y^-1 * x * y * x * y)^2)*ts[71]*ts[33]*ts[18]*ts[75];
/*true*/
for n in IN do if f((x * y^-1 * x * y * x * y)^2)*ts[71]*ts[33]*
   ts[18]*ts[75] eq
n*ts[18]*ts[94]*ts[44]*ts[106]*ts[66] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y * x*/
f((x * y^-1 * x * y * x * y)^2)*ts[71]*ts[33]*ts[18]*ts[75] eq
f(x * y * x * y^-1 * x * y * x * y * x)*ts[18]*ts[94]*ts[44]*ts
   [106]*ts[66];
/*true*/
```

```
for n in IN do if f(x * y * x * y^-1 * x * y * x * y * x)*ts
    [18]*ts[94]*ts[44]*ts[106]*ts[66] eq
n*ts[2]*ts[105]*ts[44]*ts[106]*ts[66] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y * x * y*/
f(x * y * x * y^-1 * x * y * x * y * x)*ts[18]*ts[94]*ts[44]*ts
    [106]*ts[66] eq
f(x * y^-1 * x * y * x * y)*ts[2]*ts[105]*ts[44]*ts[106]*ts[66];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y)*ts[2]*ts[105]*ts
    [44]*ts[106]*ts[66] eq
n*ts[60]*ts[90]*ts[106]*ts[66] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*(y * x * y^-1 * x * y^-1 * x)^2*/
f(x * y^-1 * x * y * x * y)*ts[2]*ts[105]*ts[44]*ts[106]*ts[66]
    eq
f((y * x * y^-1 * x * y^-1 * x)^2)*ts[60]*ts[90]*ts[106]*ts[66];
/*true*/
for n in IN do if f((y * x * y^-1 * x * y^-1 * x)^2)*ts[60]*ts
    [90]*ts[106]*ts[66] eq
n*ts[47]*ts[107]*ts[91]*ts[106]*ts[66] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1*/
f((y * x * y^-1 * x * y^-1 * x)^2)*ts[60]*ts[90]*ts[106]*ts[66]
    eq
f(y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*ts[47]*
    ts[107]*ts[91]*ts[106]*ts[66];
/*true*/
for n in IN do if f(y * x * y * x * y^-1 * x * y^-1 * x * y^-1 *
```

```
  x * y^-1)*ts[47]*ts[107]*ts[91]*ts[106]*ts[66] eq
n*ts[79]*ts[11]*ts[109]*ts[66] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(y * x * y^-1 * x)^2*/
f(y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*ts[47]*
   ts[107]*ts[91]*ts[106]*ts[66] eq
f((y * x * y^-1 * x)^2)*ts[79]*ts[11]*ts[109]*ts[66];
/*true*/
for n in IN do if f((y * x * y^-1 * x)^2)*ts[79]*ts[11]*ts[109]*
   ts[66] eq
n*ts[109]*ts[109]*ts[66] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1*/
f((y * x * y^-1 * x)^2)*ts[79]*ts[11]*ts[109]*ts[66] eq
f(x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1)*ts
   [109]*ts[109]*ts[66];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y * x * y
   * x * y^-1)*ts[109]*ts[109]*ts[66] eq
n*ts[66] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1*/
f(x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1)*ts
   [109]*ts[109]*ts[66] eq
f(x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1)*ts
   [66];
/*true*/
```

```
1,2,21 belongs to [1,2] {58,71} (DONE)
for n in IN do if ts[1]*ts[2]*ts[21] eq
n*ts[92]*ts[107]*ts[21] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y*/
ts[1]*ts[2]*ts[21] eq
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[21];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y^-1 * x * y)*ts[92]*ts[107]*ts[21] eq
n*ts[92]*ts[79]*ts[21] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y*/
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[21] eq
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[79]*ts[21];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y^-1 * x * y)*ts[92]*ts[79]*ts[21] eq
n*ts[87]*ts[45]*ts[94]*ts[21] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
   */
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[79]*ts[21] eq
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
   )*ts[87]*ts[45]*ts[94]*ts[21];
```

```
/*true*/
for n in IN do if f(x * y * x * y * x * y^-1 * x * y^-1 * x * y
   * x * y^-1 * x * y)*ts[87]*ts[45]*ts[94]*ts[21] eq
n*ts[12]*ts[39]*ts[6] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x
    * y*/
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
   )*ts[87]*ts[45]*ts[94]*ts[21] eq
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x
    * y)*ts[12]*ts[39]*ts[6];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
    x * y * x * y * x * y)*ts[12]*ts[39]*ts[6] eq
n*ts[48]*ts[79] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x*/
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x
    * y)*ts[12]*ts[39]*ts[6] eq
f(y * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x)*ts[48]*ts
   [79];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y * x * y
   ^-1 * x)*ts[48]*ts[79] eq
n*ts[48]*ts[107] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x*/
f(y * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x)*ts[48]*ts
   [79] eq
```

```
f(y * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x)*ts[48]*ts
    [107];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y * x * y
    ^-1 * x)*ts[48]*ts[107] eq
n*ts[100]*ts[109]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y*/
f(y * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x)*ts[48]*ts
    [107] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y)*ts[100]*
    ts[109]*ts[11];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
    * x * y)*ts[100]*ts[109]*ts[11] eq
n*ts[82]*ts[109]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y*/
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y)*ts[100]*
    ts[109]*ts[11] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y)*ts[82]*ts
    [109]*ts[11];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
    * x * y)*ts[82]*ts[109]*ts[11] eq
n*ts[78]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y
    ^-1*/
```

```
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y)*ts[82]*ts
    [109]*ts[11] eq
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y
    ^-1)*ts[78]*ts[11];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
    ^-1 * x * y * x * y^-1)*ts[78]*ts[11] eq
n*ts[61]*ts[91]*ts[71] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y^-1 * x*/
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y
    ^-1)*ts[78]*ts[11] eq
f(x * y^-1 * x * y^-1 * x * y * x * y^-1 * x)*ts[61]*ts[91]*ts
    [71];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y^-1 * x)*
    ts[61]*ts[91]*ts[71] eq
n*ts[61]*ts[93]*ts[71] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y^-1 * x*/
f(x * y^-1 * x * y^-1 * x * y * x * y^-1 * x)*ts[61]*ts[91]*ts
    [71] eq
f(x * y^-1 * x * y^-1 * x * y * x * y^-1 * x)*ts[61]*ts[93]*ts
    [71];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y^-1 * x)*
    ts[61]*ts[93]*ts[71] eq
n*ts[77]*ts[93]*ts[71] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
```

```
/*x * y^-1 * x * y^-1 * x * y * x * y^-1 * x*/
f(x * y^-1 * x * y^-1 * x * y * x * y^-1 * x)*ts[61]*ts[93]*ts
    [71] eq
f(x * y^-1 * x * y^-1 * x * y * x * y^-1 * x)*ts[77]*ts[93]*ts
    [71];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y^-1 * x)*
    ts[77]*ts[93]*ts[71] eq
n*ts[41]*ts[31]*ts[93]*ts[71] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 *
    x*/
f(x * y^-1 * x * y^-1 * x * y * x * y^-1 * x)*ts[77]*ts[93]*ts
    [71] eq
f(x * y * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 *
    x)*ts[41]*ts[31]*ts[93]*ts[71];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y * x * y * x * y * x
     * y * x * y^-1 * x)*ts[41]*ts[31]*ts[93]*ts[71] eq
n*ts[88]*ts[60]*ts[71] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
    ^-1*/
f(x * y * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 *
    x)*ts[41]*ts[31]*ts[93]*ts[71] eq
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
    ^-1)*ts[88]*ts[60]*ts[71];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
     x * y^-1 * x * y^-1)*ts[88]*ts[60]*ts[71] eq
```

```
n*ts[55]*ts[60]*ts[71] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1*/
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1)*ts[88]*ts[60]*ts[71] eq
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1)*ts[55]*ts[60]*ts[71];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
    x * y^-1 * x * y^-1)*ts[55]*ts[60]*ts[71] eq
n*ts[58]*ts[71] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y*/
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1)*ts[55]*ts[60]*ts[71] eq
f(y * x * y * x * y^-1 * x * y)*ts[58]*ts[71];
/*true*/
[58,71] in [1,2]^N;
/*true*/
```

```
1,2,24 belongs to [1,2] {1,68} (DONE)
for n in IN do if ts[1]*ts[2]*ts[24] eq
n*ts[1]*ts[2]*ts[74] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[2]*ts[24] eq
```

```
ts[1]*ts[2]*ts[74];
/*true*/
for n in IN do if ts[1]*ts[2]*ts[74] eq
n*ts[94]*ts[65]*ts[34]*ts[65] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x*/
ts[1]*ts[2]*ts[74] eq
f(y * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x)*ts
   [94]*ts[65]*ts[34]*ts[65];
/*true*/
for n in IN do if f(y * x * y * x * y^-1 * x * y * x * y * x * y
    * x * y * x)*ts[94]*ts[65]*ts[34]*ts[65] eq
n*ts[40]*ts[2]*ts[65] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y^-1 * x * y * x*/
f(y * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x)*ts
   [94]*ts[65]*ts[34]*ts[65] eq
f(x * y * x * y * x * y^-1 * x * y * x)*ts[40]*ts[2]*ts[65];
/*true*/
for n in IN do if f(x * y * x * y * x * y^-1 * x * y * x)*ts
   [40]*ts[2]*ts[65] eq
n*ts[59]*ts[32]*ts[23]*ts[68] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y
   */
f(x * y * x * y * x * y^-1 * x * y * x)*ts[40]*ts[2]*ts[65] eq
f(x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y
   )*ts[59]*ts[32]*ts[23]*ts[68];
/*true*/
```

```
for n in IN do if f(x * y * x * y^-1 * x * y * x * y * x * y^-1
    * x * y^-1 * x * y)*ts[59]*ts[32]*ts[23]*ts[68] eq
n*ts[59]*ts[6]*ts[23]*ts[68] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y
    */
f(x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y
    )*ts[59]*ts[32]*ts[23]*ts[68] eq
f(x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y
    )*ts[59]*ts[6]*ts[23]*ts[68];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y * x * y * x * y^-1
    * x * y^-1 * x * y)*ts[59]*ts[6]*ts[23]*ts[68] eq
n*ts[1]*ts[69]*ts[68] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y * x * y^-1 * x * y^-1 * x*/
f(x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y
    )*ts[59]*ts[6]*ts[23]*ts[68] eq
f(x * y * x * y * x * y^-1 * x * y^-1 * x)*ts[1]*ts[69]*ts[68];
/*true*/
for n in IN do if f(x * y * x * y * x * y^-1 * x * y^-1 * x)*ts
    [1]*ts[69]*ts[68] eq
n*ts[104]*ts[10]*ts[69]*ts[68] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y*/
f(x * y * x * y * x * y^-1 * x * y^-1 * x)*ts[1]*ts[69]*ts[68]
    eq
f(x * y^-1 * x * y * x * y * x * y)*ts[104]*ts[10]*ts[69]*ts
    [68];
```

```
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y * x * y)*ts[104]*ts
   [10]*ts[69]*ts[68] eq
n*ts[46]*ts[12]*ts[68] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x*/
f(x * y^-1 * x * y * x * y * x * y)*ts[104]*ts[10]*ts[69]*ts[68]
    eq
f(x * y * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x)*ts
   [46]*ts[12]*ts[68];
/*true*/
for n in IN do if f(x * y * x * y * x * y * x * y^-1 * x * y * x
    * y^-1 * x)*ts[46]*ts[12]*ts[68] eq
n*ts[1]*ts[68] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1*/
f(x * y * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x)*ts
   [46]*ts[12]*ts[68] eq
f(y * x * y * x * y^-1)*ts[1]*ts[68];
/*true*/
[1,68] in [1,2]^N;
/*true*/
```

```
1,2,32 belongs to [1,26] {37,59} (DONE)
for n in IN do if ts[1]*ts[2]*ts[32] eq
n*ts[92]*ts[107]*ts[32] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y*/
ts[1]*ts[2]*ts[32] eq
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[32];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y^-1 * x * y)*ts[92]*ts[107]*ts[32] eq
n*ts[67]*ts[99] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
    y^-1*/
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[32] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
    y^-1)*ts[67]*ts[99];
/*true*/
[67,99] in [1,26]^N;
/*true*/



1,2,36 belongs to [1] {66}
for n in IN do if ts[1]*ts[2]*ts[36] eq
n*ts[92]*ts[107]*ts[36] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y*/
ts[1]*ts[2]*ts[36] eq
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[36];
```

```
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y^-1 * x * y)*ts[92]*ts[107]*ts[36] eq
n*ts[92]*ts[107]*ts[10] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y*/
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[36] eq
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[10];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y^-1 * x * y)*ts[92]*ts[107]*ts[10] eq
n*ts[35]*ts[54]*ts[18]*ts[75] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y
   * x*/
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[10] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y
   * x)*ts[35]*ts[54]*ts[18]*ts[75];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y *
    x * y^-1 * x * y * x)*ts[35]*ts[54]*ts[18]*ts[75] eq
n*ts[105]*ts[43]*ts[75] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y
   * x)*ts[35]*ts[54]*ts[18]*ts[75] eq
```

```
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x)*ts[105]*ts
    [43]*ts[75];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y *
     x)*ts[105]*ts[43]*ts[75] eq
n*ts[60]*ts[90]*ts[106]*ts[66] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*(y * x * y^-1 * x * y^-1 * x)^2*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x)*ts[105]*ts
    [43]*ts[75] eq
f((y * x * y^-1 * x * y^-1 * x)^2)*ts[60]*ts[90]*ts[106]*ts[66];
/*true*/
for n in IN do if f((y * x * y^-1 * x * y^-1 * x)^2)*ts[60]*ts
    [90]*ts[106]*ts[66] eq
n*ts[31]*ts[93]*ts[90]*ts[106]*ts[66] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
    */
f((y * x * y^-1 * x * y^-1 * x)^2)*ts[60]*ts[90]*ts[106]*ts[66]
    eq
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x)
    *ts[31]*ts[93]*ts[90]*ts[106]*ts[66];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
     y^-1 * x * y * x)*ts[31]*ts[93]*ts[90]*ts[106]*ts[66] eq
n*ts[103]*ts[81]*ts[106]*ts[66] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y * x * y * x * y * x * y * x * y*/
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x)
```

```
    *ts[31]*ts[93]*ts[90]*ts[106]*ts[66] eq
f(y^-1 * x * y * x * y * x * y * x * y * x * y)*ts[103]*ts[81]*
    ts[106]*ts[66];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y * x * y * x * y * x * y
    )*ts[103]*ts[81]*ts[106]*ts[66] eq
n*ts[60]*ts[106]*ts[66] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1*/
f(y^-1 * x * y * x * y * x * y * x * y * x * y)*ts[103]*ts[81]*
    ts[106]*ts[66] eq
f(x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1)*ts
    [60]*ts[106]*ts[66];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y * x * y
    * x * y^-1)*ts[60]*ts[106]*ts[66] eq
n*ts[106]*ts[106]*ts[66] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1*/
f(x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1)*ts
    [60]*ts[106]*ts[66] eq
f(x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1)*ts
    [106]*ts[106]*ts[66];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y * x * y
    * x * y^-1)*ts[60]*ts[106]*ts[66] eq
n*ts[66] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1*/
```

```
f(x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1)*ts
   [60]*ts[106]*ts[66] eq
f(x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1)*ts
   [66];
/*true*/


1,2,1 belongs to [1] {37} (DONE)
for n in IN do if ts[1]*ts[2]*ts[1] eq
n*ts[92]*ts[107]*ts[1] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y*/
ts[1]*ts[2]*ts[1] eq
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[1];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y^-1 * x * y)*ts[92]*ts[107]*ts[1] eq
n*ts[103]*ts[86]*ts[67]*ts[27] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y * x * y * x * y^-1*/
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[1] eq
f(x * y * x * y^-1 * x * y * x * y * x * y * x * y^-1)*ts[103]*
   ts[86]*ts[67]*ts[27];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y * x * y * x * y * x
    * y^-1)*ts[103]*ts[86]*ts[67]*ts[27] eq
n*ts[103]*ts[65]*ts[67]*ts[27] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
   end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y * x * y * x * y^-1*/
f(x * y * x * y^-1 * x * y * x * y * x * y * x * y^-1)*ts[103]*
   ts[86]*ts[67]*ts[27] eq
f(x * y * x * y^-1 * x * y * x * y * x * y * x * y^-1)*ts[103]*
   ts[65]*ts[67]*ts[27];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y * x * y * x * y * x
    * y^-1)*ts[103]*ts[65]*ts[67]*ts[27] eq
n*ts[36]*ts[76]*ts[27] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x*/
f(x * y * x * y^-1 * x * y * x * y * x * y * x * y^-1)*ts[103]*
   ts[65]*ts[67]*ts[27] eq
f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x)*ts[36]*ts[76]*
   ts[27];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y *
    x)*ts[36]*ts[76]*ts[27] eq
n*ts[93]*ts[94]*ts[44]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y*/
f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x)*ts[36]*ts[76]*
   ts[27] eq
f(y^-1 * x * y * x * y)*ts[93]*ts[94]*ts[44]*ts[37];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y)*ts[93]*ts[94]*ts[44]*
   ts[37] eq
n*ts[91]*ts[94]*ts[44]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
   end for; end if; end for;
/*y^-1 * x * y * x * y*/
 f(y^-1 * x * y * x * y)*ts[93]*ts[94]*ts[44]*ts[37] eq
f(y^-1 * x * y * x * y)*ts[91]*ts[94]*ts[44]*ts[37];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y)*ts[91]*ts[94]*ts[44]*
   ts[37] eq
n*ts[91]*ts[52]*ts[44]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y*/
f(y^-1 * x * y * x * y)*ts[91]*ts[94]*ts[44]*ts[37] eq
f(y^-1 * x * y * x * y)*ts[91]*ts[52]*ts[44]*ts[37];
/*true*/f(y^-1 * x * y * x * y)*ts[91]*ts[52]*ts[44]*ts[37]
for n in IN do if f(y^-1 * x * y * x * y)*ts[91]*ts[94]*ts[44]*
   ts[37] eq
n*ts[11]*ts[107]*ts[52]*ts[44]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
   * y^-1 * x*/
f(y^-1 * x * y * x * y)*ts[91]*ts[94]*ts[44]*ts[37] eq
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
   * y^-1 * x)*ts[11]*ts[107]*ts[52]*ts[44]*ts[37];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y^-1 * x * y * x * y^-1 * x)*ts[11]*ts[107]*ts[52]*ts[44]*ts
   [37] eq
n*ts[82]*ts[42]*ts[44]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1*/
```

```
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
    * y^-1 * x)*ts[11]*ts[107]*ts[52]*ts[44]*ts[37] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)*ts
    [82]*ts[42]*ts[44]*ts[37];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
    ^-1 * x * y^-1)*ts[82]*ts[42]*ts[44]*ts[37] eq
n*ts[100]*ts[42]*ts[44]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)*ts
    [82]*ts[42]*ts[44]*ts[37] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)*ts
    [100]*ts[42]*ts[44]*ts[37];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
    ^-1 * x * y^-1)*ts[100]*ts[42]*ts[44]*ts[37] eq
n*ts[44]*ts[44]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)*ts
    [100]*ts[42]*ts[44]*ts[37] eq
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[44]*ts
    [44]*ts[37];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y^-1 * x
    * y^-1)*ts[44]*ts[44]*ts[37] eq
n*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
```

```
/*y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[44]*ts
   [44]*ts[37] eq
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[37];
/*true*/

1,2,2 belongs to [1] (DONE)
1,2,3 belongs to [1,2] {90,45} (DONE)
for n in IN do if ts[1]*ts[2]*ts[3] eq
n*ts[4]*ts[2]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
ts[1]*ts[2]*ts[3] eq
ts[4]*ts[2]*ts[3];
/*true*/
for n in IN do if ts[4]*ts[2]*ts[3] eq
n*ts[64]*ts[50]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(x * y^-1 * x * y)^2*/
ts[4]*ts[2]*ts[3] eq
f((x * y^-1 * x * y)^2)*ts[64]*ts[50]*ts[3];
/*true*/
for n in IN do if f((x * y^-1 * x * y)^2)*ts[64]*ts[50]*ts[3] eq
n*ts[64]*ts[83]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(x * y^-1 * x * y)^2*/
f((x * y^-1 * x * y)^2)*ts[64]*ts[50]*ts[3] eq
f((x * y^-1 * x * y)^2)*ts[64]*ts[83]*ts[3];
/*true*/
for n in IN do if f((x * y^-1 * x * y)^2)*ts[64]*ts[83]*ts[3] eq
```

```
n*ts[28]*ts[68] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y * x * y^-1*/
f((x * y^-1 * x * y)^2)*ts[64]*ts[83]*ts[3] eq
f(x * y * x * y^-1 * x * y * x * y * x * y^-1)*ts[28]*ts[68];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y * x * y * x * y^-1)
   *ts[28]*ts[68] eq
n*ts[28]*ts[57] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y * x * y^-1*/
f(x * y * x * y^-1 * x * y * x * y * x * y^-1)*ts[28]*ts[68] eq
f(x * y * x * y^-1 * x * y * x * y * x * y^-1)*ts[28]*ts[57];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y * x * y * x * y^-1)
   *ts[28]*ts[57] eq
n*ts[97]*ts[33]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y^-1*/
f(x * y * x * y^-1 * x * y * x * y * x * y^-1)*ts[28]*ts[57] eq
f(x * y * x * y * x * y^-1)*ts[97]*ts[33]*ts[37];
/*true*/
for n in IN do if f(x * y * x * y * x * y^-1)*ts[97]*ts[33]*ts
   [37] eq
n*ts[105]*ts[33]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y^-1*/
f(x * y * x * y * x * y^-1)*ts[97]*ts[33]*ts[37] eq
```

```
f(x * y * x * y * x * y^-1)*ts[105]*ts[33]*ts[37];
/*true*/
for n in IN do if f(x * y * x * y * x * y^-1)*ts[105]*ts[33]*ts
   [37] eq
n*ts[55]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
    y^-1 * x*/
f(x * y * x * y * x * y^-1)*ts[105]*ts[33]*ts[37] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
    y^-1 * x)*ts[55]*ts[37];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
   y^-1 * x * y * x * y^-1 * x)*ts[55]*ts[37] eq
n*ts[1]*ts[85]*ts[45] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
    y^-1 * x)*ts[55]*ts[37] eq
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y)*ts[1]*
   ts[85]*ts[45];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
    y * x * y)*ts[1]*ts[85]*ts[45] eq
n*ts[1]*ts[56]*ts[45] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y*/
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y)*ts[1]*
   ts[85]*ts[45] eq
```

```
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y)*ts[1]*
   ts[56]*ts[45];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
    y * x * y)*ts[1]*ts[56]*ts[45] eq
n*ts[67]*ts[27]*ts[56]*ts[45] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y)*ts[1]*
   ts[56]*ts[45] eq
f(x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[67]*ts[27]*ts[56]*
   ts[45];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts
   [67]*ts[27]*ts[56]*ts[45] eq
n*ts[97]*ts[44]*ts[45] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x*/
f(x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[67]*ts[27]*ts[56]*
   ts[45] eq
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x)*ts[97]*ts
   [44]*ts[45];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y^-1 * x
   * y * x)*ts[97]*ts[44]*ts[45] eq
n*ts[105]*ts[44]*ts[45] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x*/
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x)*ts[97]*ts
```

```
   [44]*ts[45] eq
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x)*ts[105]*
   ts[44]*ts[45];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y^-1 * x
   * y * x)*ts[105]*ts[44]*ts[45] eq
n*ts[90]*ts[45] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x*/
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x)*ts[105]*
   ts[44]*ts[45] eq
f(y * x)*ts[90]*ts[45];
/*true*/
[90,45] in [1,2]^N;
/*true*/
```

```
1,2,7 belongs to [1,5] {38,61} (DONE)
for n in IN do if ts[1]*ts[2]*ts[7] eq
n*ts[92]*ts[107]*ts[7] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y*/
ts[1]*ts[2]*ts[7] eq
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[7];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y^-1 * x * y)*ts[92]*ts[107]*ts[7] eq
n*ts[92]*ts[107]*ts[33] then
```

```
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y*/
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[7] eq
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[33];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y^-1 * x * y)*ts[92]*ts[107]*ts[33] eq
n*ts[87]*ts[45]*ts[94]*ts[33] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
   */
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [92]*ts[107]*ts[33] eq
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
   )*ts[87]*ts[45]*ts[94]*ts[33];
/*true*/
for n in IN do if f(x * y * x * y * x * y^-1 * x * y^-1 * x * y
   * x * y^-1 * x * y)*ts[87]*ts[45]*ts[94]*ts[33] eq
n*ts[56]*ts[51]*ts[25] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   * x*/
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
   )*ts[87]*ts[45]*ts[94]*ts[33] eq
f(y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   * x)*ts[56]*ts[51]*ts[25];
/*true*/
```

```
for n in IN do if f(y * x * y * x * y^-1 * x * y^-1 * x * y^-1 *
     x * y^-1 * x * y * x)*ts[56]*ts[51]*ts[25] eq
n*ts[56]*ts[51]*ts[75] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   * x*/
f(y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   * x)*ts[56]*ts[51]*ts[25] eq
f(y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   * x)*ts[56]*ts[51]*ts[75];
/*true*/
for n in IN do if f(y * x * y * x * y^-1 * x * y^-1 * x * y^-1 *
     x * y^-1 * x * y * x)*ts[56]*ts[51]*ts[75] eq
n*ts[25]*ts[47]*ts[85]*ts[61] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1*/
f(y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   * x)*ts[56]*ts[51]*ts[75] eq
f(x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)*ts
   [25]*ts[47]*ts[85]*ts[61];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y * x * y * x * y^-1
   * x * y^-1)*ts[25]*ts[47]*ts[85]*ts[61] eq
n*ts[94]*ts[104]*ts[47]*ts[85]*ts[61] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y * x*/
f(x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)*ts
   [25]*ts[47]*ts[85]*ts[61] eq
f(y * x * y * x * y * x * y^-1 * x * y * x)*ts[94]*ts[104]*ts
```

```
    [47]*ts[85]*ts[61];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y * x)*ts
    [94]*ts[104]*ts[47]*ts[85]*ts[61] eq
n*ts[108]*ts[1]*ts[85]*ts[61] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y * x*/
f(y * x * y * x * y * x * y^-1 * x * y * x)*ts[94]*ts[104]*ts
    [47]*ts[85]*ts[61] eq
f(x * y^-1 * x * y * x)*ts[108]*ts[1]*ts[85]*ts[61];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x)*ts[108]*ts[1]*ts[85]*
    ts[61] eq
n*ts[108]*ts[1]*ts[56]*ts[61] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y * x*/
f(x * y^-1 * x * y * x)*ts[108]*ts[1]*ts[85]*ts[61] eq
f(x * y^-1 * x * y * x)*ts[108]*ts[1]*ts[56]*ts[61];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x)*ts[108]*ts[1]*ts[56]*
    ts[61] eq
n*ts[110]*ts[67]*ts[27]*ts[56]*ts[61] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x*/
f(x * y^-1 * x * y * x)*ts[108]*ts[1]*ts[56]*ts[61] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x)*ts[110]*ts
    [67]*ts[27]*ts[56]*ts[61];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
```

```
    y * x)*ts[110]*ts[67]*ts[27]*ts[56]*ts[61] eq
n*ts[101]*ts[97]*ts[44]*ts[61] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x)*ts[110]*ts
    [67]*ts[27]*ts[56]*ts[61] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y)*ts
    [101]*ts[97]*ts[44]*ts[61];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
    ^-1 * x * y)*ts[101]*ts[97]*ts[44]*ts[61] eq
n*ts[101]*ts[105]*ts[44]*ts[61] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y*/
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y)*ts
    [101]*ts[97]*ts[44]*ts[61] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y)*ts
    [101]*ts[105]*ts[44]*ts[61];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
    ^-1 * x * y)*ts[101]*ts[105]*ts[44]*ts[61] eq
n*ts[18]*ts[90]*ts[61] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y*/
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y)*ts
    [101]*ts[105]*ts[44]*ts[61] eq
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y)*ts[18]*ts
    [90]*ts[61];
/*true*/
```

```
for n in IN do if f(x * y * x * y * x * y^-1 * x * y^-1 * x * y
   * x * y)*ts[18]*ts[90]*ts[61] eq
n*ts[18]*ts[87]*ts[61] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y*/
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y)*ts[18]*ts
   [90]*ts[61] eq
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y)*ts[18]*ts
   [87]*ts[61];
/*true*/
for n in IN do if f(x * y * x * y * x * y^-1 * x * y^-1 * x * y
   * x * y)*ts[18]*ts[87]*ts[61] eq
n*ts[101]*ts[91]*ts[87]*ts[61] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x * y * x *
   y*/
f(x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y)*ts[18]*ts
   [87]*ts[61] eq
f(y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x * y * x *
   y)*ts[101]*ts[91]*ts[87]*ts[61];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y * x * y * x * y * x * y
   ^-1 * x * y * x * y)*ts[101]*ts[91]*ts[87]*ts[61] eq
n*ts[51]*ts[109]*ts[61] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x*/
f(y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x * y * x *
   y)*ts[101]*ts[91]*ts[87]*ts[61] eq
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x)*ts[51]*
```

```
ts[109]*ts[61];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y^-1 * x)*ts[51]*ts[109]*ts[61] eq
n*ts[89]*ts[109]*ts[61] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x*/
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x)*ts[51]*
   ts[109]*ts[61] eq
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x)*ts[89]*
   ts[109]*ts[61]
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y^-1 * x)*ts[89]*ts[109]*ts[61] eq
n*ts[38]*ts[61] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y*/
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x)*ts[89]*
   ts[109]*ts[61] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y)*ts[38]*
   ts[61];
/*true*/
[38,61] in [1,5]^N;
/*true*/


1,2,8 belongs to [1,2] {48,42} (DONE)
for n in IN do if ts[1]*ts[2]*ts[8] eq
n*ts[1]*ts[11]*ts[8] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[2]*ts[8] eq
ts[1]*ts[11]*ts[8];
/*true*/
for n in IN do if ts[1]*ts[11]*ts[8] eq
n*ts[33]*ts[86] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x * y^-1*/
ts[1]*ts[11]*ts[8] eq
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x * y^-1)*ts[33]*ts[86];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
   x * y * x * y^-1 * x * y^-1)*ts[33]*ts[86] eq
n*ts[7]*ts[86] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x * y^-1*/
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x * y^-1)*ts[33]*ts[86] eq
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x * y^-1)*ts[7]*ts[86];
/*true*/
[7,86] in [1,2]^N;
/*true*/
```

1,2,9 belongs to [1,2] {82,70} (DONE)

```
for n in IN do if ts[1]*ts[2]*ts[9] eq
n*ts[1]*ts[2]*ts[35] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[2]*ts[9] eq
ts[1]*ts[2]*ts[35];
/*true*/
for n in IN do if ts[1]*ts[2]*ts[35] eq
n*ts[47]*ts[48]*ts[29]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(y * x * y * x * y^-1 * x)^2*/
ts[1]*ts[2]*ts[35] eq
f((y * x * y * x * y^-1 * x)^2)*ts[47]*ts[48]*ts[29]*ts[3];
/*true*/
for n in IN do if f((y * x * y * x * y^-1 * x)^2)*ts[47]*ts[48]*
   ts[29]*ts[3] eq
n*ts[9]*ts[53]*ts[30]*ts[58]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y * x*/
f((y * x * y * x * y^-1 * x)^2)*ts[47]*ts[48]*ts[29]*ts[3] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x)*ts[9]*ts[53]*
   ts[30]*ts[58]*ts[70];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
   * x)*ts[9]*ts[53]*ts[30]*ts[58]*ts[70] eq
n*ts[17]*ts[11]*ts[35]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y
```

```
    */
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x)*ts[9]*ts[53]*
    ts[30]*ts[58]*ts[70] eq
f(x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y
    )*ts[17]*ts[11]*ts[35]*ts[70];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y * x * y^-1 * x * y
    ^-1 * x * y * x * y)*ts[17]*ts[11]*ts[35]*ts[70] eq
n*ts[30]*ts[11]*ts[35]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y
    */
f(x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y
    )*ts[17]*ts[11]*ts[35]*ts[70] eq
f(x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y
    )*ts[30]*ts[11]*ts[35]*ts[70];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y * x * y^-1 * x * y
    ^-1 * x * y * x * y)*ts[30]*ts[11]*ts[35]*ts[70] eq
n*ts[15]*ts[107]*ts[11]*ts[35]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
     x*/
f(x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y
    )*ts[30]*ts[11]*ts[35]*ts[70] eq
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
     x)*ts[15]*ts[107]*ts[11]*ts[35]*ts[70];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
    x * y * x * y^-1 * x)*ts[15]*ts[107]*ts[11]*ts[35]*ts[70] eq
```

```
n*ts[61]*ts[109]*ts[35]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x*/
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x)*ts[15]*ts[107]*ts[11]*ts[35]*ts[70] eq
f(x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x)*ts
   [61]*ts[109]*ts[35]*ts[70];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y * x * y * x * y^-1
   * x * y^-1 * x)*ts[61]*ts[109]*ts[35]*ts[70] eq
n*ts[8]*ts[91]*ts[87]*ts[35]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y * x*/
f(x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x)*ts
   [61]*ts[109]*ts[35]*ts[70] eq
f(x * y^-1 * x * y * x * y * x * y * x)*ts[8]*ts[91]*ts[87]*ts
   [35]*ts[70];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y * x * y * x)*ts[8]*
   ts[91]*ts[87]*ts[35]*ts[70] eq
n*ts[41]*ts[52]*ts[97]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y * x * y^-1 * x * y*/
f(x * y^-1 * x * y * x * y * x * y * x)*ts[8]*ts[91]*ts[87]*ts
   [35]*ts[70] eq
f(x * y * x * y * x * y * x * y^-1 * x * y)*ts[41]*ts[52]*ts
   [97]*ts[70];
/*true*/
for n in IN do if f(x * y * x * y * x * y * x * y^-1 * x * y)*ts
```

```
    [41]*ts[52]*ts[97]*ts[70] eq
n*ts[41]*ts[94]*ts[97]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y * x * y * x * y^-1 * x * y*/
f(x * y * x * y * x * y * x * y^-1 * x * y)*ts[41]*ts[52]*ts
    [97]*ts[70] eq
f(x * y * x * y * x * y * x * y^-1 * x * y)*ts[41]*ts[94]*ts
    [97]*ts[70];
/*true*/
for n in IN do if f(x * y * x * y * x * y * x * y^-1 * x * y)*ts
    [41]*ts[94]*ts[97]*ts[70] eq
n*ts[10]*ts[76]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x*/
f(x * y * x * y * x * y * x * y^-1 * x * y)*ts[41]*ts[94]*ts
    [97]*ts[70] eq
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x)*
    ts[10]*ts[76]*ts[70];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y * x * y * x
    * y * x * y * x)*ts[10]*ts[76]*ts[70] eq
n*ts[10]*ts[26]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x*/
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x)*
    ts[10]*ts[76]*ts[70] eq
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x)*
    ts[10]*ts[26]*ts[70];
/*true*/
```

```
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y * x * y * x
    * y * x * y * x)*ts[10]*ts[26]*ts[70] eq
n*ts[46]*ts[104]*ts[26]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x
     * y*/
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y * x)*
    ts[10]*ts[26]*ts[70] eq
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x
     * y)*ts[46]*ts[104]*ts[26]*ts[70];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
     x * y * x * y * x * y)*ts[46]*ts[104]*ts[26]*ts[70] eq
n*ts[98]*ts[49]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1*/
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x
     * y)*ts[46]*ts[104]*ts[26]*ts[70] eq
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*
    ts[98]*ts[49]*ts[70];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
     y^-1 * x * y^-1)*ts[98]*ts[49]*ts[70] eq
n*ts[37]*ts[49]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1*/
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*
    ts[98]*ts[49]*ts[70] eq
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*
```

```
   ts[37]*ts[49]*ts[70];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y^-1 * x * y^-1)*ts[37]*ts[49]*ts[70] eq
n*ts[100]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y * x*/
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*
   ts[37]*ts[49]*ts[70] eq
f(y * x * y * x * y^-1 * x * y * x)*ts[100]*ts[70];
/*true*/
for n in IN do if f(y * x * y * x * y^-1 * x * y * x)*ts[100]*ts
   [70] eq
n*ts[82]*ts[70] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y * x*/
f(y * x * y * x * y^-1 * x * y * x)*ts[100]*ts[70] eq
f(y * x * y * x * y^-1 * x * y * x)*ts[82]*ts[70];
/*true*/
[82,70] in [1,2]^N;
/*true*/


1,2,11 belongs to [1] {4} (DONE)
for n in IN do if ts[1]*ts[2]*ts[11] eq
n*ts[1]*ts[2]*ts[2] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[2]*ts[11] eq
```

```
ts[1]*ts[2]*ts[2];
/*true*/


1,2,26 belongs to [1,2] {92,107} (DONE)
for n in IN do if ts[1]*ts[2]*ts[26] eq
n*ts[1]*ts[11]*ts[26] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[2]*ts[26] eq
ts[1]*ts[11]*ts[26];
/*true*/
for n in IN do if ts[1]*ts[11]*ts[26] eq
n*ts[90]*ts[69] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(y * x * y^-1 * x * y * x)^2*/
ts[1]*ts[11]*ts[26] eq
f((y * x * y^-1 * x * y * x)^2)*ts[90]*ts[69];
/*true*/
for n in IN do if f((y * x * y^-1 * x * y * x)^2)*ts[90]*ts[69]
   eq
n*ts[87]*ts[69] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(y * x * y^-1 * x * y * x)^2*/
f((y * x * y^-1 * x * y * x)^2)*ts[90]*ts[69] eq
f((y * x * y^-1 * x * y * x)^2)*ts[87]*ts[69];
/*true*/
[87,69] in [1,2]^N;
/*true*/
```

```
/*THIRD DOUBLE COSET 1,26*/ /*COMPLETE*/
1,26=2,9 (relation) (DONE)
for n in IN do if ts[1]*ts[26] eq
n*ts[90]*ts[86]*ts[69] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(y * x * y^-1 * x * y * x)^2*/
ts[1]*ts[26] eq
f((y * x * y^-1 * x * y * x)^2)*ts[90]*ts[86]*ts[69];
/*true*/
for n in IN do if f((y * x * y^-1 * x * y * x)^2)*ts[90]*ts[86]*
   ts[69] eq
n*ts[62]*ts[39]*ts[4]*ts[9] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y*/
f((y * x * y^-1 * x * y * x)^2)*ts[90]*ts[86]*ts[69] eq
f(x * y^-1 * x * y^-1 * x * y * x * y)*ts[62]*ts[39]*ts[4]*ts
   [9];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y)*ts[62]*
   ts[39]*ts[4]*ts[9] eq
n*ts[62]*ts[99]*ts[4]*ts[9] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y*/
f(x * y^-1 * x * y^-1 * x * y * x * y)*ts[62]*ts[39]*ts[4]*ts[9]
```

```
    eq
f(x * y^-1 * x * y^-1 * x * y * x * y)*ts[62]*ts[99]*ts[4]*ts
   [9];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y)*ts[62]*
   ts[99]*ts[4]*ts[9] eq
n*ts[87]*ts[81]*ts[9] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y^-1 * x * y * x * y)*ts[62]*ts[99]*ts[4]*ts[9]
    eq
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[87]*ts
   [81]*ts[9];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y^-1 * x
   * y^-1)*ts[87]*ts[81]*ts[9] eq
n*ts[90]*ts[81]*ts[9] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[87]*ts
   [81]*ts[9] eq
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[90]*ts
   [81]*ts[9]
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y^-1 * x
   * y^-1)*ts[87]*ts[81]*ts[9] eq
n*ts[2]*ts[9] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y * x*/
```

```
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[87]*ts
   [81]*ts[9] eq
f(y^-1 * x * y^-1 * x * y * x * y * x)*ts[2]*ts[9];
/*true*/

1,26=64,97 (relation) (DONE)
for n in IN do if ts[1]*ts[26] eq
n*ts[1]*ts[76] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
ts[1]*ts[26] eq
ts[1]*ts[76];
/*true*/
for n in IN do if ts[1]*ts[76] eq
n*ts[102]*ts[94]*ts[97] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1 * x * y^-1*/
ts[1]*ts[76] eq
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1 * x * y^-1)*ts[102]*ts[94]*ts[97];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
    x * y^-1 * x * y^-1 * x * y^-1)*ts[102]*ts[94]*ts[97] eq
n*ts[101]*ts[79]*ts[94]*ts[97] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x*/
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1 * x * y^-1)*ts[102]*ts[94]*ts[97] eq
```

```
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x)*ts[101]*ts[79]*ts[94]*ts[97];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 *
   x * y * x * y^-1 * x)*ts[101]*ts[79]*ts[94]*ts[97] eq
n*ts[72]*ts[45]*ts[97] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x)*ts[101]*ts[79]*ts[94]*ts[97] eq
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)*ts[72]*ts
   [45]*ts[97];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y^-1)*ts[72]*ts[45]*ts[97] eq
n*ts[102]*ts[97] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x*/
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)*ts[72]*ts
   [45]*ts[97] eq
f(y^-1 * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x)*ts
   [102]*ts[97] ;
/*true*/
for n in IN do if f(y^-1 * x * y * x * y * x * y^-1 * x * y * x
   * y^-1 * x)*ts[102]*ts[97] eq
n*ts[64]*ts[97] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x*/
f(y^-1 * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x)*ts
```

```
   [102]*ts[97] eq
f(y^-1 * x * y * x * y * x * y^-1 * x * y * x * y^-1 * x)*ts
   [64]*ts[97];
/*true*/

1,26 = 109,102 (relation) (DONE)
for n in IN do if ts[1]*ts[26] eq
n*ts[83]*ts[57]*ts[73] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1
    * x * y*/
ts[1]*ts[26] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1
    * x * y)*ts[83]*ts[57]*ts[73];
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
   * x * y * x * y^-1 * x * y)*ts[83]*ts[57]*ts[73] eq
n*ts[83]*ts[57]*ts[23] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1
    * x * y*/
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1
    * x * y)*ts[83]*ts[57]*ts[73] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1
    * x * y)*ts[83]*ts[57]*ts[23];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
   * x * y * x * y^-1 * x * y)*ts[83]*ts[57]*ts[23] eq
n*ts[81]*ts[91]*ts[59]*ts[102] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
```

```
/*y * x * y * x * y * x * y * x * y * x * y^-1 * x * y*/
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1
    * x * y)*ts[83]*ts[57]*ts[23] eq
f(y * x * y * x * y * x * y * x * y * x * y^-1 * x * y)*ts[81]*
   ts[91]*ts[59]*ts[102];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y * x * y * x * y^-1
    * x * y)*ts[81]*ts[91]*ts[59]*ts[102] eq
n*ts[31]*ts[71]*ts[91]*ts[59]*ts[102] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y * x * y * x * y * x*/
f(y * x * y * x * y * x * y * x * y * x * y^-1 * x * y)*ts[81]*
   ts[91]*ts[59]*ts[102] eq
f(y^-1 * x * y * x * y * x * y * x * y * x)*ts[31]*ts[71]*ts
   [91]*ts[59]*ts[102];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y * x * y * x * y * x)*ts
   [31]*ts[71]*ts[91]*ts[59]*ts[102] eq
n*ts[79]*ts[47]*ts[59]*ts[102] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y * x * y^-1*/
f(y^-1 * x * y * x * y * x * y * x * y * x)*ts[31]*ts[71]*ts
   [91]*ts[59]*ts[102] eq
f(y^-1 * x * y^-1 * x * y * x * y * x * y^-1)*ts[79]*ts[47]*ts
   [59]*ts[102];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y * x * y^-1)*
   ts[79]*ts[47]*ts[59]*ts[102] eq
n*ts[81]*ts[59]*ts[102] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
   end for; end if; end for;
/*y^-1 * x * y * x * y * x * y * x * y * x * y*/
f(y^-1 * x * y^-1 * x * y * x * y * x * y^-1)*ts[79]*ts[47]*ts
   [59]*ts[102] eq
f(y^-1 * x * y * x * y * x * y * x * y * x * y)*ts[81]*ts[59]*ts
   [102];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y * x * y * x * y * x * y
   )*ts[81]*ts[59]*ts[102] eq
n*ts[95]*ts[88]*ts[59]*ts[102] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y * x * y * x * y^-1 * x * y * x * y*/
f(y^-1 * x * y * x * y * x * y * x * y * x * y)*ts[81]*ts[59]*ts
   [102] eq
f(x * y * x * y * x * y * x * y * x * y^-1 * x * y * x * y)*ts
   [95]*ts[88]*ts[59]*ts[102];
/*true*/
for n in IN do if f(x * y * x * y * x * y * x * y * x * y^-1 * x
    * y * x * y)*ts[95]*ts[88]*ts[59]*ts[102] eq
n*ts[99]*ts[4]*ts[102] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1*/
f(x * y * x * y * x * y * x * y * x * y^-1 * x * y * x * y)*ts
   [95]*ts[88]*ts[59]*ts[102] eq
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*
   ts[99]*ts[4]*ts[102];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
    y^-1 * x * y^-1)*ts[99]*ts[4]*ts[102] eq
n*ts[81]*ts[102] then
```

```
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y*/
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*
   ts[99]*ts[4]*ts[102] eq
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y)*ts
   [81]*ts[102];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y *
    x * y * x * y)*ts[81]*ts[102] eq
n*ts[109]*ts[102] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y*/
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y)*ts
   [81]*ts[102] eq
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y)*ts
   [109]*ts[102];
/*true*/
```

```
1,26,18 belongs to [1,26] (DONE)
for n in IN do if ts[1]*ts[26]*ts[18] eq
n*ts[91]*ts[14]*ts[35] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(y * x * y^-1 * x)^2*/
ts[1]*ts[26]*ts[18] eq
f((y * x * y^-1 * x)^2)*ts[91]*ts[14]*ts[35];
/*true*/
for n in IN do if f((y * x * y^-1 * x)^2)*ts[91]*ts[14]*ts[35]
```

```
   eq
n*ts[2]*ts[35] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y^-1 * x * y * x * y * x *
   y*/
f((y * x * y^-1 * x)^2)*ts[91]*ts[14]*ts[35] eq
f(y * x * y^-1 * x * y * x * y * x * y^-1 * x * y * x * y * x *
   y)*ts[2]*ts[35];
/*true*/
[2,35] in [1,26]^N;
/*true*/
```

```
1,26,26
1,26,3 belongs to [1,2] (DONE)
for n in IN do if ts[1]*ts[26]*ts[3] eq
n*ts[109]*ts[102]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y*/
ts[1]*ts[26]*ts[3] eq
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y)*ts
   [109]*ts[102]*ts[3];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y *
    x * y * x * y)*ts[109]*ts[102]*ts[3] eq
n*ts[55]*ts[30]*ts[58]*ts[97]*ts[62] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1*/
```

```
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y)*ts
    [109]*ts[102]*ts[3] eq
f(y * x * y * x * y^-1)*ts[55]*ts[30]*ts[58]*ts[97]*ts[62];
/*true*/
for n in IN do if f(y * x * y * x * y^-1)*ts[55]*ts[30]*ts[58]*
    ts[97]*ts[62] eq
n*ts[71]*ts[35]*ts[97]*ts[62] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x*/
f(y * x * y * x * y^-1)*ts[55]*ts[30]*ts[58]*ts[97]*ts[62] eq
f(y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x)*ts
    [71]*ts[35]*ts[97]*ts[62];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y * x * y * x
    * y^-1 * x)*ts[71]*ts[35]*ts[97]*ts[62] eq
n*ts[15]*ts[72]*ts[62] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y * x * y*/
f(y * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x)*ts
    [71]*ts[35]*ts[97]*ts[62] eq
f(y^-1 * x * y * x * y)*ts[15]*ts[72]*ts[62];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y)*ts[15]*ts[72]*ts[62]
    eq
n*ts[16]*ts[72]*ts[62] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y * x * y*/
f(y^-1 * x * y * x * y)*ts[15]*ts[72]*ts[62] eq
f(y^-1 * x * y * x * y)*ts[16]*ts[72]*ts[62];
```

```
/*true*/
for n in IN do if f(y^-1 * x * y * x * y)*ts[15]*ts[72]*ts[62]
    eq
n*ts[39]*ts[62] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(y^-1 * x * y * x * y)*ts[15]*ts[72]*ts[62] eq
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts
    [39]*ts[62];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
    ^-1 * x * y^-1)*ts[39]*ts[62] eq
n*ts[99]*ts[62] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts
    [39]*ts[62] eq
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts
    [99]*ts[62];
/*true*/
[99,62] in [1,2]^N;
/*true*/



1,26,6 belongs to [1,2] {51,37} (DONE)
for n in IN do if ts[1]*ts[26]*ts[6] eq
n*ts[91]*ts[57]*ts[34]*ts[57] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
```

```
/*(y * x * y^-1 * x * y^-1 * x)^2*/
ts[1]*ts[26]*ts[6] eq
f((y * x * y^-1 * x * y^-1 * x)^2)*ts[91]*ts[57]*ts[34]*ts[57];
/*true*/
for n in IN do if f((y * x * y^-1 * x * y^-1 * x)^2)*ts[91]*ts
   [57]*ts[34]*ts[57] eq
n*ts[13]*ts[26]*ts[57] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y^-1*/
f((y * x * y^-1 * x * y^-1 * x)^2)*ts[91]*ts[57]*ts[34]*ts[57]
   eq
f(x * y * x * y * x * y^-1)*ts[13]*ts[26]*ts[57];
/*true*/
for n in IN do if f(x * y * x * y * x * y^-1)*ts[13]*ts[26]*ts
   [57] eq
n*ts[92]*ts[51]*ts[33]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1*/
f(x * y * x * y * x * y^-1)*ts[13]*ts[26]*ts[57] eq
f(y^-1 * x * y^-1)*ts[92]*ts[51]*ts[33]*ts[37];
/*true*/
for n in IN do if f(y^-1 * x * y^-1)*ts[92]*ts[51]*ts[33]*ts[37]
    eq
n*ts[100]*ts[34]*ts[51]*ts[33]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x*/
f(y^-1 * x * y^-1)*ts[92]*ts[51]*ts[33]*ts[37] eq
f(y * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x)*
   ts[100]*ts[34]*ts[51]*ts[33]*ts[37];
```

```
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y * x * y
    * x * y^-1 * x)*ts[100]*ts[34]*ts[51]*ts[33]*ts[37] eq
n*ts[36]*ts[6]*ts[33]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y * x*/
f(y * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x)*
   ts[100]*ts[34]*ts[51]*ts[33]*ts[37] eq
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y * x)*ts[36]*ts[6]*ts[33]*ts[37];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
    y * x * y^-1 * x * y * x)*ts[36]*ts[6]*ts[33]*ts[37] eq
n*ts[36]*ts[32]*ts[33]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y * x*/
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y * x)*ts[36]*ts[6]*ts[33]*ts[37] eq
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y * x)*ts[36]*ts[32]*ts[33]*ts[37];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
    y * x * y^-1 * x * y * x)*ts[36]*ts[32]*ts[33]*ts[37] eq
n*ts[5]*ts[102]*ts[32]*ts[33]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y
   */
```

```
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x
    * y * x)*ts[36]*ts[32]*ts[33]*ts[37] eq
f(x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y
    )*ts[5]*ts[102]*ts[32]*ts[33]*ts[37];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y * x * y * x * y^-1
    * x * y^-1 * x * y)*ts[5]*ts[102]*ts[32]*ts[33]*ts[37] eq
n*ts[5]*ts[102]*ts[32]*ts[7]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y
    */
f(x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y
    )*ts[5]*ts[102]*ts[32]*ts[33]*ts[37] eq
f(x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y
    )*ts[5]*ts[102]*ts[32]*ts[7]*ts[37];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y * x * y * x * y^-1
    * x * y^-1 * x * y)*ts[5]*ts[102]*ts[32]*ts[7]*ts[37] eq
n*ts[54]*ts[72]*ts[7]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*(x * y^-1 * x * y)^2*/
f(x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y
    )*ts[5]*ts[102]*ts[32]*ts[7]*ts[37] eq
f((x * y^-1 * x * y)^2)*ts[54]*ts[72]*ts[7]*ts[37];
/*true*/
for n in IN do if f((x * y^-1 * x * y)^2)*ts[54]*ts[72]*ts[7]*ts
    [37] eq
n*ts[49]*ts[79]*ts[16]*ts[7]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
```

```
/*y * x * y^-1 * x * y^-1 * x * y * x * y * x*/
f((x * y^-1 * x * y)^2)*ts[54]*ts[72]*ts[7]*ts[37] eq
f(y * x * y^-1 * x * y^-1 * x * y * x * y * x)*ts[49]*ts[79]*ts
    [16]*ts[7]*ts[37];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y * x * y * x)
    *ts[49]*ts[79]*ts[16]*ts[7]*ts[37] eq
n*ts[87]*ts[70]*ts[80]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y*/
f(y * x * y^-1 * x * y^-1 * x * y * x * y * x)*ts[49]*ts[79]*ts
    [16]*ts[7]*ts[37] eq
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y)*ts
    [87]*ts[70]*ts[80]*ts[37] ;
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y * x * y * x
    * y * x * y)*ts[87]*ts[70]*ts[80]*ts[37] eq
n*ts[87]*ts[46]*ts[80]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y*/
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y)*ts
    [87]*ts[70]*ts[80]*ts[37] eq
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y)*ts
    [87]*ts[46]*ts[80]*ts[37];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y * x * y * x
    * y * x * y)*ts[87]*ts[46]*ts[80]*ts[37] eq
n*ts[38]*ts[95]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
```

```
/*y * x * y * x * y * x * y * x * y^-1 * x * y * x * y^-1*/
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x * y)*ts
   [87]*ts[46]*ts[80]*ts[37] eq
f(y * x * y * x * y * x * y * x * y^-1 * x * y * x * y^-1)*ts
   [38]*ts[95]*ts[37];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y * x * y^-1 * x * y
    * x * y^-1)*ts[38]*ts[95]*ts[37] eq
n*ts[89]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1 * x*/
f(y * x * y * x * y * x * y * x * y^-1 * x * y * x * y^-1)*ts
   [38]*ts[95]*ts[37] eq
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1 * x)*ts[89]*ts[37];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
    x * y^-1 * x * y^-1 * x)*ts[89]*ts[37] eq
n*ts[51]*ts[37] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1 * x)*ts[89]*ts[37] eq
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1 * x)*ts[51]*ts[37];
/*true*/
[51,37] in [1,2]^N;
/*true*/

/*THESE ARE COMPLETE*/
```

```
/*FOURTH DOUBLE COSET*/
1,5=2,13 (relation) (DONE)
for n in IN do if ts[1]*ts[5] eq
n*ts[80]*ts[25]*ts[48] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(y^-1 * x * y^-1 * x * y * x)^2*/
ts[1]*ts[5] eq
f((y^-1 * x * y^-1 * x * y * x)^2)*ts[80]*ts[25]*ts[48];
/*true*/
for n in IN do if f((y^-1 * x * y^-1 * x * y * x)^2)*ts[80]*ts
   [25]*ts[48] eq
n*ts[26]*ts[36]*ts[102]*ts[59] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x*/
f((y^-1 * x * y^-1 * x * y * x)^2)*ts[80]*ts[25]*ts[48] eq
f(y * x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x)*
   ts[26]*ts[36]*ts[102]*ts[59];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y * x * y * x
   * y * x * y * x)*ts[26]*ts[36]*ts[102]*ts[59] eq
n*ts[87]*ts[5]*ts[59] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(y * x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x)*
   ts[26]*ts[36]*ts[102]*ts[59] eq
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[87]*ts
   [5]*ts[59];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y^-1 * x
```

```
   * y^-1)*ts[87]*ts[5]*ts[59] eq
n*ts[90]*ts[5]*ts[59] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[87]*ts
   [5]*ts[59] eq
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[90]*ts
   [5]*ts[59];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y^-1 * x
   * y^-1)*ts[90]*ts[5]*ts[59] eq
n*ts[2]*ts[59] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y
   */
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[90]*ts
   [5]*ts[59] eq
f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y)
   *ts[2]*ts[59];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x * y^-1 * x * y)*ts[2]*ts[59] eq
n*ts[2]*ts[13] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y
   */
f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y)
   *ts[2]*ts[59] eq
f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y)
```

```
   *ts[2]*ts[13];
/*true*/

1,5=69,18 (relation) (DONE)
for n in IN do if ts[1]*ts[5] eq
n*ts[1]*ts[5]*ts[18]*ts[18] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[5] eq
ts[1]*ts[5]*ts[18]*ts[18];
/*true*/
for n in IN do if ts[1]*ts[5] eq
n*ts[1]*ts[31]*ts[18]*ts[18] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[5] eq
ts[1]*ts[31]*ts[18]*ts[18];
/*true*/
for n in IN do if ts[1]*ts[5] eq
n*ts[80]*ts[75]*ts[18] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(y^-1 * x * y^-1 * x * y * x)^2*/
ts[1]*ts[5] eq
f((y^-1 * x * y^-1 * x * y * x)^2)*ts[80]*ts[75]*ts[18];
/*true*/
for n in IN do if f((y^-1 * x * y^-1 * x * y * x)^2)*ts[80]*ts
   [75]*ts[18] eq
n*ts[16]*ts[7]*ts[75]*ts[18] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
   end for; end if; end for;
/*x * y * x * y * x * y * x * y^-1 * x * y * x * y * x*/
f((y^-1 * x * y^-1 * x * y * x)^2)*ts[80]*ts[75]*ts[18] eq
f(x * y * x * y * x * y * x * y^-1 * x * y * x * y * x)*ts[16]*
   ts[7]*ts[75]*ts[18];
/*true*/
for n in IN do if f(x * y * x * y * x * y * x * y^-1 * x * y * x
    * y * x)*ts[16]*ts[7]*ts[75]*ts[18] eq
n*ts[2]*ts[26]*ts[18] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y*/
f(x * y * x * y * x * y * x * y^-1 * x * y * x * y * x)*ts[16]*
   ts[7]*ts[75]*ts[18] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y)*
   ts[2]*ts[26]*ts[18];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
   * x * y * x * y)*ts[2]*ts[26]*ts[18] eq
n*ts[2]*ts[76]*ts[18] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y*/
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y)*
   ts[2]*ts[26]*ts[18] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y)*
   ts[2]*ts[76]*ts[18];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
   * x * y * x * y)*ts[2]*ts[76]*ts[18] eq
n*ts[67]*ts[18] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1*/
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y * x * y * x * y)*
   ts[2]*ts[76]*ts[18] eq
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1)*ts[67]*ts[18];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
    x * y^-1 * x * y^-1)*ts[67]*ts[18] eq
n*ts[69]*ts[18] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1*/
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1)*ts[67]*ts[18] eq
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
   ^-1)*ts[69]*ts[18];
/*true*/

1,5,7 belongs to [1,5] {9,11} (DONE)
for n in IN do if ts[1]*ts[5]*ts[7] eq
n*ts[1]*ts[31]*ts[7] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[5]*ts[7] eq
ts[1]*ts[31]*ts[7];
/*true*/
for n in IN do if ts[1]*ts[31]*ts[7] eq
n*ts[110]*ts[59]*ts[41]*ts[7] then
```

```
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y * x * y * x * y^-1 * x * y^-1*/
ts[1]*ts[31]*ts[7] eq
f(y * x * y * x * y * x * y * x * y * x * y^-1 * x * y^-1)*ts
   [110]*ts[59]*ts[41]*ts[7];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y * x * y * x * y^-1
    * x * y^-1)*ts[110]*ts[59]*ts[41]*ts[7] eq
n*ts[40]*ts[70]*ts[83] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x*/
f(y * x * y * x * y * x * y * x * y * x * y^-1 * x * y^-1)*ts
   [110]*ts[59]*ts[41]*ts[7] eq
f(x * y^-1 * x)*ts[40]*ts[70]*ts[83];
/*true*/
for n in IN do if f(x * y^-1 * x)*ts[40]*ts[70]*ts[83] eq
n*ts[3]*ts[11]*ts[41]*ts[110] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x
   * y^-1*/
f(x * y^-1 * x)*ts[40]*ts[70]*ts[83] eq
f(x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x
   * y^-1)*ts[3]*ts[11]*ts[41]*ts[110];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y^-1 * x *
    y^-1 * x * y * x * y^-1)*ts[3]*ts[11]*ts[41]*ts[110] eq
n*ts[92]*ts[47]*ts[101]*ts[71]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
```

```
/*y^-1 * x * y^-1 * x * y * x * y^-1 * x*/
f(x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x
    * y^-1)*ts[3]*ts[11]*ts[41]*ts[110] eq
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x)*ts[92]*ts[47]*ts[101]*
    ts[71]*ts[11];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y^-1 * x)*ts
    [92]*ts[47]*ts[101]*ts[71]*ts[11] eq
n*ts[92]*ts[28]*ts[63]*ts[71]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y^-1 * x*/
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x)*ts[92]*ts[47]*ts[101]*
    ts[71]*ts[11] eq
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x)*ts[92]*ts[28]*ts[63]*
    ts[71]*ts[11];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y^-1 * x)*ts
    [92]*ts[28]*ts[63]*ts[71]*ts[11] eq
n*ts[90]*ts[29]*ts[46]*ts[63]*ts[71]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y*/
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x)*ts[92]*ts[28]*ts[63]*
    ts[71]*ts[11] eq
f(x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y)*ts[90]*ts
    [29]*ts[46]*ts[63]*ts[71]*ts[11];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y^-1 * x * y * x * y
    * x * y)*ts[90]*ts[29]*ts[46]*ts[63]*ts[71]*ts[11] eq
n*ts[24]*ts[59]*ts[95]*ts[71]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
   end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y*/
f(x * y * x * y^-1 * x * y^-1 * x * y * x * y * x * y)*ts[90]*ts
   [29]*ts[46]*ts[63]*ts[71]*ts[11] eq
f(x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [24]*ts[59]*ts[95]*ts[71]*ts[11];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y * x * y^-1 * x * y
   ^-1 * x * y)*ts[24]*ts[59]*ts[95]*ts[71]*ts[11] eq
n*ts[24]*ts[13]*ts[95]*ts[71]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y*/
f(x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [24]*ts[59]*ts[95]*ts[71]*ts[11] eq
f(x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [24]*ts[13]*ts[95]*ts[71]*ts[11];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y * x * y^-1 * x * y
   ^-1 * x * y)*ts[24]*ts[13]*ts[95]*ts[71]*ts[11] eq
n*ts[102]*ts[84]*ts[71]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y * x * y * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1 * x * y)*ts
   [24]*ts[13]*ts[95]*ts[71]*ts[11] eq
f(y^-1 * x * y * x * y * x * y * x * y^-1 * x * y^-1)*ts[102]*ts
   [84]*ts[71]*ts[11];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y * x * y * x * y^-1 * x
   * y^-1)*ts[102]*ts[84]*ts[71]*ts[11] eq
n*ts[28]*ts[100]*ts[81]*ts[71]*ts[11] then
```

```
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y*/
f(y^-1 * x * y * x * y * x * y * x * y^-1 * x * y^-1)*ts[102]*ts
   [84]*ts[71]*ts[11] eq
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y)*ts
   [28]*ts[100]*ts[81]*ts[71]*ts[11];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
    y * x * y)*ts[28]*ts[100]*ts[81]*ts[71]*ts[11] eq
n*ts[5]*ts[17]*ts[31]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1*/
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y)*ts
   [28]*ts[100]*ts[81]*ts[71]*ts[11] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1)*ts
   [5]*ts[17]*ts[31]*ts[11];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
   y * x * y^-1)*ts[5]*ts[17]*ts[31]*ts[11] eq
n*ts[31]*ts[17]*ts[31]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1)*ts
   [5]*ts[17]*ts[31]*ts[11] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1)*ts
   [31]*ts[17]*ts[31]*ts[11];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
   y * x * y^-1)*ts[31]*ts[17]*ts[31]*ts[11] eq
```

```
n*ts[47]*ts[79]*ts[17]*ts[31]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1)*ts
   [31]*ts[17]*ts[31]*ts[11] eq
f(y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*ts[47]*
   ts[79]*ts[17]*ts[31]*ts[11];
/*true*/
for n in IN do if f(y * x * y * x * y^-1 * x * y^-1 * x * y^-1 *
    x * y^-1)*ts[47]*ts[79]*ts[17]*ts[31]*ts[11] eq
n*ts[83]*ts[39]*ts[31]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y*/
f(y * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*ts[47]*
   ts[79]*ts[17]*ts[31]*ts[11] eq
f(y * x * y * x * y * x * y^-1 * x * y)*ts[83]*ts[39]*ts[31]*ts
   [11];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y)*ts
   [83]*ts[39]*ts[31]*ts[11] eq
n*ts[83]*ts[99]*ts[31]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
f(y * x * y * x * y * x * y^-1 * x * y)*ts[83]*ts[39]*ts[31]*ts
   [11] eq
f(y * x * y * x * y * x * y^-1 * x * y)*ts[83]*ts[99]*ts[31]*ts
   [11];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y)*ts
   [83]*ts[99]*ts[31]*ts[11] eq
```

```
n∗ts[49]∗ts[88]∗ts[82]∗ts[31]∗ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/∗y ∗ x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y^−1 ∗ x ∗ y^−1 ∗ x ∗ y^−1 ∗ x ∗ y
    ^−1 ∗ x ∗ y∗/
f(y ∗ x ∗ y ∗ x ∗ y ∗ x ∗ y^−1 ∗ x ∗ y)∗ts[83]∗ts[99]∗ts[31]∗ts
    [11] eq
f(y ∗ x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y^−1 ∗ x ∗ y^−1 ∗ x ∗ y^−1 ∗ x ∗ y
    ^−1 ∗ x ∗ y)∗ts[49]∗ts[88]∗ts[82]∗ts[31]∗ts[11];
/∗true∗/
for n in IN do if f(y ∗ x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y^−1 ∗ x ∗ y^−1 ∗
     x ∗ y^−1 ∗ x ∗ y^−1 ∗ x ∗ y)∗ts[49]∗ts[88]∗ts[82]∗ts[31]∗ts
    [11] eq
n∗ts[25]∗ts[82]∗ts[31]∗ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/∗x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y ∗ x∗/
f(y ∗ x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y^−1 ∗ x ∗ y^−1 ∗ x ∗ y^−1 ∗ x ∗ y
    ^−1 ∗ x ∗ y)∗ts[49]∗ts[88]∗ts[82]∗ts[31]∗ts[11] eq
f(x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y ∗ x)∗ts[25]∗ts
    [82]∗ts[31]∗ts[11] ;
/∗true∗/
for n in IN do if f(x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y
    ∗ x)∗ts[25]∗ts[82]∗ts[31]∗ts[11] eq
n∗ts[66]∗ts[92]∗ts[85]∗ts[31]∗ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/∗y ∗ x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y ∗ x ∗ y ∗ x ∗ y ∗ x ∗ y^−1 ∗ x ∗
    y∗/
f(x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y ∗ x)∗ts[25]∗ts
    [82]∗ts[31]∗ts[11] eq
f(y ∗ x ∗ y^−1 ∗ x ∗ y ∗ x ∗ y ∗ x ∗ y ∗ x ∗ y ∗ x ∗ y^−1 ∗ x ∗
```

```
   y)*ts[66]*ts[92]*ts[85]*ts[31]*ts[11];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y * x * y
    * x * y^-1 * x * y)*ts[66]*ts[92]*ts[85]*ts[31]*ts[11] eq
n*ts[103]*ts[7]*ts[23]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y * x * y * x * y*/
f(y * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x *
   y)*ts[66]*ts[92]*ts[85]*ts[31]*ts[11] eq
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y)*ts[103]*ts
   [7]*ts[23]*ts[11];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y * x * y * x
   * y)*ts[103]*ts[7]*ts[23]*ts[11] eq
n*ts[62]*ts[41]*ts[7]*ts[23]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y^-1 * x * y * x * y * x*/
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y)*ts[103]*ts
   [7]*ts[23]*ts[11] eq
f(y * x * y * x * y^-1 * x * y * x * y * x)*ts[62]*ts[41]*ts[7]*
   ts[23]*ts[11];
/*true*/
for n in IN do if f(y * x * y * x * y^-1 * x * y * x * y * x)*ts
   [62]*ts[41]*ts[7]*ts[23]*ts[11] eq
n*ts[37]*ts[83]*ts[23]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y^-1*/
f(y * x * y * x * y^-1 * x * y * x * y * x)*ts[62]*ts[41]*ts[7]*
   ts[23]*ts[11] eq
```

```
f(y^-1 * x * y * x * y^-1 * x * y^-1)*ts[37]*ts[83]*ts[23]*ts
    [11];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y^-1)*ts[37]*
    ts[83]*ts[23]*ts[11] eq
n*ts[110]*ts[23]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y*/
f(y^-1 * x * y * x * y^-1 * x * y^-1)*ts[37]*ts[83]*ts[23]*ts
    [11] eq
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x * y)*ts
    [110]*ts[23]*ts[11];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y^-1)*ts[37]*
    ts[83]*ts[23]*ts[11] eq
n*ts[62]*ts[41]*ts[23]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
    * y^-1 * x * y*/
f(y^-1 * x * y * x * y^-1 * x * y^-1)*ts[37]*ts[83]*ts[23]*ts
    [11] eq
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
    * y^-1 * x * y)*ts[62]*ts[41]*ts[23]*ts[11];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y^-1)*ts[37]*
    ts[83]*ts[23]*ts[11] eq
n*ts[62]*ts[41]*ts[73]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
```

```
    * y^-1 * x * y*/
f(y^-1 * x * y * x * y^-1 * x * y^-1)*ts[37]*ts[83]*ts[23]*ts
    [11] eq
f(x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x
    * y^-1 * x * y)*ts[62]*ts[41]*ts[73]*ts[11];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y^-1)*ts[37]*
    ts[83]*ts[23]*ts[11] eq
n*ts[27]*ts[10]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x*/
f(y^-1 * x * y * x * y^-1 * x * y^-1)*ts[37]*ts[83]*ts[23]*ts
    [11] eq
f(x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x)*ts[27]*ts
    [10]*ts[11];
/*true*/
for n in IN do if f(x * y * x * y^-1 * x * y * x * y^-1 * x * y
    ^-1 * x)*ts[27]*ts[10]*ts[11] eq
n*ts[27]*ts[36]*ts[11] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x*/
f(x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x)*ts[27]*ts
    [10]*ts[11] eq
f(x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x)*ts[27]*ts
    [36]*ts[11];
/*true*/

for n in IN do if f(x * y * x * y^-1 * x * y * x * y^-1 * x * y
    ^-1 * x)*ts[27]*ts[36]*ts[11] eq
n*ts[9]*ts[11] then
```

```
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1*/
f(x * y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x)*ts[27]*ts
   [36]*ts[11] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1)*ts
   [9]*ts[11];
/*true*/
[9,11] in [1,5]^N;
/*true*/



1,5,16 belongs to [1,5] {4,20} (done)
for n in IN do if ts[1]*ts[5]*ts[16] eq
n*ts[1]*ts[31]*ts[16] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
ts[1]*ts[5]*ts[16] eq
ts[1]*ts[31]*ts[16];
/*true*/
for n in IN do if ts[1]*ts[31]*ts[16] eq
n*ts[61]*ts[81]*ts[103]*ts[16] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y * x * y * x*/
ts[1]*ts[31]*ts[16] eq
f(x * y^-1 * x * y * x * y * x * y * x * y * x)*ts[61]*ts[81]*ts
   [103]*ts[16];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y * x * y * x * y * x
   )*ts[61]*ts[81]*ts[103]*ts[16] eq
n*ts[89]*ts[18]*ts[7] then
```

```
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y * x * y * x * y * x * y * x)*ts[61]*ts[81]*ts
   [103]*ts[16] eq
f(y * x * y^-1 * x * y^-1)*ts[89]*ts[18]*ts[7];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1)*ts[89]*ts[18]*ts[7]
    eq
n*ts[41]*ts[74]*ts[26]*ts[49] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y*/
f(y * x * y^-1 * x * y^-1)*ts[89]*ts[18]*ts[7] eq
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y)*ts[41]*ts[74]*ts[26]*ts[49];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x *
    y * x * y^-1 * x * y)*ts[41]*ts[74]*ts[26]*ts[49] eq
n*ts[60]*ts[34]*ts[49] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y * x * y * x * y * x * y * x * y^-1 * x * y * x*/
f(x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x
   * y)*ts[41]*ts[74]*ts[26]*ts[49] eq
f(x * y * x * y * x * y * x * y * x * y^-1 * x * y * x)*ts[60]*
   ts[34]*ts[49];
/*true*/
for n in IN do if f(x * y * x * y * x * y * x * y * x * y^-1 * x
    * y * x)*ts[60]*ts[34]*ts[49] eq
n*ts[4]*ts[56]*ts[85]*ts[20] then
```

```
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x
    * y*/
f(x * y * x * y * x * y * x * y * x * y^-1 * x * y * x)*ts[60]*
   ts[34]*ts[49] eq
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x
    * y)*ts[4]*ts[56]*ts[85]*ts[20];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 *
    x * y * x * y * x * y)*ts[4]*ts[56]*ts[85]*ts[20] eq
n*ts[4]*ts[85]*ts[85]*ts[20] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x
    * y*/
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x
    * y)*ts[4]*ts[56]*ts[85]*ts[20] eq
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x
    * y)*ts[4]*ts[85]*ts[85]*ts[20];
/*true*/
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x
    * y)*ts[4]*ts[56]*ts[85]*ts[20] eq
f(y * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y * x
    * y)*ts[4]*ts[20];
/*true*/
[4,20] in [1,5]^N;
/*true*/

1,5,5

1,5,3 belongs to [1,2] {60,78} (done)
```

```
for n in IN do if ts[1]*ts[5]*ts[3] eq
n*ts[1]*ts[31]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*Id(L)*/
ts[1]*ts[5]*ts[3] eq
ts[1]*ts[31]*ts[3] ;
/*true*/
for n in IN do if ts[1]*ts[31]*ts[3] eq
n*ts[61]*ts[81]*ts[103]*ts[3] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y * x * y * x*/
ts[1]*ts[31]*ts[3] eq
f(x * y^-1 * x * y * x * y * x * y * x * y * x)*ts[61]*ts[81]*ts
   [103]*ts[3];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y * x * y * x * y * x
   )*ts[61]*ts[81]*ts[103]*ts[3] eq
n*ts[100]*ts[97]*ts[80] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x*/
f(x * y^-1 * x * y * x * y * x * y * x * y * x)*ts[61]*ts[81]*ts
   [103]*ts[3] eq
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x)*ts[100]*
   ts[97]*ts[80];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y^-1 * x
   * y * x)*ts[100]*ts[97]*ts[80] eq
n*ts[76]*ts[18]*ts[50]*ts[40] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
```

```
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(y * x * y * x * y * x * y^-1 * x * y^-1 * x * y * x)*ts[100]*
    ts[97]*ts[80] eq
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts
    [76]*ts[18]*ts[50]*ts[40];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y
    ^-1 * x * y^-1)*ts[76]*ts[18]*ts[50]*ts[40] eq
n*ts[101]*ts[10]*ts[19]*ts[99]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x*/
f(y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts
    [76]*ts[18]*ts[50]*ts[40] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x)*ts
    [101]*ts[10]*ts[19]*ts[99]*ts[78];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y
    ^-1 * x * y * x)*ts[101]*ts[10]*ts[19]*ts[99]*ts[78] eq
n*ts[63]*ts[10]*ts[19]*ts[99]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x)*ts
    [101]*ts[10]*ts[19]*ts[99]*ts[78] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x)*ts
    [63]*ts[10]*ts[19]*ts[99]*ts[78];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y
    ^-1 * x * y * x)*ts[63]*ts[10]*ts[19]*ts[99]*ts[78] eq
n*ts[42]*ts[17]*ts[10]*ts[19]*ts[99]*ts[78] then
```

```
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x
   */
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y * x)*ts
   [63]*ts[10]*ts[19]*ts[99]*ts[78] eq
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x)
   *ts[42]*ts[17]*ts[10]*ts[19]*ts[99]*ts[78];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
    y * x * y^-1 * x)*ts[42]*ts[17]*ts[10]*ts[19]*ts[99]*ts[78]
   eq
n*ts[38]*ts[69]*ts[19]*ts[99]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y * x * y*/
f(x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x)
   *ts[42]*ts[17]*ts[10]*ts[19]*ts[99]*ts[78] eq
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y * x * y)*ts[38]*ts[69]*ts[19]*ts[99]*ts[78];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y
   ^-1 * x * y^-1 * x * y * x * y)*ts[38]*ts[69]*ts[19]*ts[99]*
   ts[78] eq
n*ts[47]*ts[51]*ts[34]*ts[19]*ts[99]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y * x * y * x * y^-1 * x * y^-1*/
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
    y * x * y)*ts[38]*ts[69]*ts[19]*ts[99]*ts[78] eq
f(y * x * y * x * y * x * y^-1 * x * y^-1)*ts[47]*ts[51]*ts[34]*
```

```
    ts[19]*ts[99]*ts[78];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y^-1 * x * y^-1)*ts
    [47]*ts[51]*ts[34]*ts[19]*ts[99]*ts[78] eq
n*ts[78]*ts[102]*ts[44]*ts[99]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x*/
f(y * x * y * x * y * x * y^-1 * x * y^-1)*ts[47]*ts[51]*ts[34]*
    ts[19]*ts[99]*ts[78] eq
f(x * y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x)*ts
    [78]*ts[102]*ts[44]*ts[99]*ts[78];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y^-1 * x * y * x * y
    * x * y * x)*ts[78]*ts[102]*ts[44]*ts[99]*ts[78] eq
n*ts[78]*ts[102]*ts[20]*ts[99]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*x * y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x*/
f(x * y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x)*ts
    [78]*ts[102]*ts[44]*ts[99]*ts[78] eq
f(x * y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x)*ts
    [78]*ts[102]*ts[20]*ts[99]*ts[78];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y^-1 * x * y * x * y
    * x * y * x)*ts[78]*ts[102]*ts[20]*ts[99]*ts[78] eq
n*ts[44]*ts[13]*ts[84]*ts[21]*ts[99]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y * x * y^-1 * x * y * x * y * x * y * x)*ts
    [78]*ts[102]*ts[20]*ts[99]*ts[78] eq
```

```
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[44]*ts[13]*
    ts[84]*ts[21]*ts[99]*ts[78];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y
    ^-1)*ts[44]*ts[13]*ts[84]*ts[21]*ts[99]*ts[78] eq
n*ts[106]*ts[98]*ts[103]*ts[30]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*(y^-1 * x)^5*/
f(y^-1 * x * y * x * y^-1 * x * y^-1 * x * y^-1)*ts[44]*ts[13]*
    ts[84]*ts[21]*ts[99]*ts[78] eq
f((y^-1 * x)^5)*ts[106]*ts[98]*ts[103]*ts[30]*ts[78];
/*true*/
for n in IN do if f((y^-1 * x)^5)*ts[106]*ts[98]*ts[103]*ts[30]*
    ts[78] eq
n*ts[97]*ts[19]*ts[98]*ts[103]*ts[30]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y
    ^-1*/
f((y^-1 * x)^5)*ts[106]*ts[98]*ts[103]*ts[30]*ts[78] eq
f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y
    ^-1)*ts[97]*ts[19]*ts[98]*ts[103]*ts[30]*ts[78];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y
    ^-1 * x * y * x * y^-1)*ts[97]*ts[19]*ts[98]*ts[103]*ts[30]*
    ts[78] eq
n*ts[102]*ts[108]*ts[103]*ts[30]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x *
    y^-1*/
```

```
f(y * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x * y
    ^-1)*ts[97]*ts[19]*ts[98]*ts[103]*ts[30]*ts[78] eq
f(y * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x *
    y^-1)*ts[102]*ts[108]*ts[103]*ts[30]*ts[78];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y * x * y
     * x * y^-1 * x * y^-1)*ts[102]*ts[108]*ts[103]*ts[30]*ts[78]
     eq
n*ts[102]*ts[108]*ts[110]*ts[17]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x *
    y^-1*/
f(y * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x *
    y^-1)*ts[102]*ts[108]*ts[103]*ts[30]*ts[78] eq
f(y * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x *
    y^-1)*ts[102]*ts[108]*ts[110]*ts[17]*ts[78];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y * x * y * x * y
     * x * y^-1 * x * y^-1)*ts[102]*ts[108]*ts[110]*ts[17]*ts[78]
     eq
n*ts[22]*ts[30]*ts[106]*ts[79]*ts[17]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y^-1*/
f(y * x * y^-1 * x * y * x * y * x * y * x * y * x * y^-1 * x *
    y^-1)*ts[102]*ts[108]*ts[110]*ts[17]*ts[78] eq
f(y^-1 * x * y^-1 * x * y * x * y^-1)*ts[22]*ts[30]*ts[106]*ts
    [79]*ts[17]*ts[78];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y^-1)*ts[22]*
    ts[30]*ts[106]*ts[79]*ts[17]*ts[78] eq
```

```
n*ts[58]*ts[15]*ts[44]*ts[39]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
    y*/
f(y^-1 * x * y^-1 * x * y * x * y^-1)*ts[22]*ts[30]*ts[106]*ts
   [79]*ts[17]*ts[78] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
    y)*ts[58]*ts[15]*ts[44]*ts[39]*ts[78];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y
   ^-1 * x * y^-1 * x * y)*ts[58]*ts[15]*ts[44]*ts[39]*ts[78] eq
n*ts[22]*ts[99]*ts[21]*ts[20]*ts[39]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y * x * y^-1*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x *
    y)*ts[58]*ts[15]*ts[44]*ts[39]*ts[78] eq
f(y^-1 * x * y^-1 * x * y * x * y^-1)*ts[22]*ts[99]*ts[21]*ts
   [20]*ts[39]*ts[78];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y * x * y^-1)*ts[22]*
   ts[99]*ts[21]*ts[20]*ts[39]*ts[78] eq
n*ts[74]*ts[36]*ts[77]*ts[39]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*(y^-1 * x * y * x)^2*/
f(y^-1 * x * y^-1 * x * y * x * y^-1)*ts[22]*ts[99]*ts[21]*ts
   [20]*ts[39]*ts[78] eq
f((y^-1 * x * y * x)^2)*ts[74]*ts[36]*ts[77]*ts[39]*ts[78];
/*true*/
for n in IN do if f((y^-1 * x * y * x)^2)*ts[74]*ts[36]*ts[77]*
```

```
    ts[39]*ts[78] eq
n*ts[74]*ts[10]*ts[77]*ts[39]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*(y^-1 * x * y * x)^2*/
f((y^-1 * x * y * x)^2)*ts[74]*ts[36]*ts[77]*ts[39]*ts[78] eq
f((y^-1 * x * y * x)^2)*ts[74]*ts[10]*ts[77]*ts[39]*ts[78];
/*true*/
for n in IN do if f((y^-1 * x * y * x)^2)*ts[74]*ts[10]*ts[77]*
    ts[39]*ts[78]eq
n*ts[55]*ts[31]*ts[39]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y * x * y * x * y * x * y^-1 * x * y*/
f((y^-1 * x * y * x)^2)*ts[74]*ts[10]*ts[77]*ts[39]*ts[78]eq
f(y * x * y * x * y * x * y * x * y^-1 * x * y)*ts[55]*ts[31]*ts
    [39]*ts[78];
/*true*/
for n in IN do if f(y * x * y * x * y * x * y * x * y^-1 * x * y
    )*ts[55]*ts[31]*ts[39]*ts[78] eq
n*ts[42]*ts[10]*ts[64]*ts[39]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
    end for; end if; end for;
/*y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y
    ^-1*/
f(y * x * y * x * y * x * y * x * y^-1 * x * y)*ts[55]*ts[31]*ts
    [39]*ts[78] eq
f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y
    ^-1)*ts[42]*ts[10]*ts[64]*ts[39]*ts[78];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
     x * y^-1 * x * y^-1)*ts[42]*ts[10]*ts[64]*ts[39]*ts[78] eq
```

```
n*ts[21]*ts[49]*ts[63]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x*/
f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y
   ^-1)*ts[42]*ts[10]*ts[64]*ts[39]*ts[78] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x)*
   ts[21]*ts[49]*ts[63]*ts[78];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 *
    x * y^-1 * x * y^-1)*ts[42]*ts[10]*ts[64]*ts[39]*ts[78] eq
n*ts[72]*ts[49]*ts[63]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x*/
f(y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^-1 * x * y
   ^-1)*ts[42]*ts[10]*ts[64]*ts[39]*ts[78] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x)*
   ts[72]*ts[49]*ts[63]*ts[78];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
   y^-1 * x * y * x)*ts[72]*ts[49]*ts[63]*ts[78] eq
n*ts[17]*ts[52]*ts[49]*ts[63]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y * x)*
   ts[72]*ts[49]*ts[63]*ts[78] eq
f(y * x * y^-1 * x * y * x * y)*ts[17]*ts[52]*ts[49]*ts[63]*ts
   [78];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y)*ts[17]*ts[52]*
```

```
   ts[49]*ts[63]*ts[78] eq
n*ts[17]*ts[52]*ts[19]*ts[63]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y^-1 * x * y * x * y*/
f(y * x * y^-1 * x * y * x * y)*ts[17]*ts[52]*ts[49]*ts[63]*ts
   [78] eq
f(y * x * y^-1 * x * y * x * y)*ts[17]*ts[52]*ts[19]*ts[63]*ts
   [78];
/*true*/
for n in IN do if f(y * x * y^-1 * x * y * x * y)*ts[17]*ts[52]*
   ts[19]*ts[63]*ts[78] eq
n*ts[59]*ts[55]*ts[63]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y*/
f(y * x * y^-1 * x * y * x * y)*ts[17]*ts[52]*ts[19]*ts[63]*ts
   [78] eq
f(y * x * y)*ts[59]*ts[55]*ts[63]*ts[78];
/*true*/
for n in IN do if f(y * x * y)*ts[59]*ts[55]*ts[63]*ts[78] eq
n*ts[13]*ts[55]*ts[63]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y * x * y*/
f(y * x * y)*ts[59]*ts[55]*ts[63]*ts[78] eq
f(y * x * y)*ts[13]*ts[55]*ts[63]*ts[78];
/*true*/
for n in IN do if f(y * x * y)*ts[13]*ts[55]*ts[63]*ts[78] eq
n*ts[51]*ts[63]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
```

```
/*x * y^-1 * x * y * x * y * x * y * x * y * x*/
f(y * x * y)*ts[13]*ts[55]*ts[63]*ts[78] eq
f(x * y^-1 * x * y * x * y * x * y * x * y * x)*ts[51]*ts[63]*ts
   [78];
/*true*/
for n in IN do if f(y * x * y)*ts[13]*ts[55]*ts[63]*ts[78] eq
n*ts[51]*ts[101]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y * x * y * x * y * x * y * x*/
f(y * x * y)*ts[13]*ts[55]*ts[63]*ts[78] eq
f(x * y^-1 * x * y * x * y * x * y * x * y * x)*ts[51]*ts[101]*
   ts[78];
/*true*/
for n in IN do if f(x * y^-1 * x * y * x * y * x * y * x * y * x
   )*ts[51]*ts[101]*ts[78] eq
n*ts[68]*ts[65]*ts[101]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y
   ^-1*/
f(x * y^-1 * x * y * x * y * x * y * x * y * x)*ts[51]*ts[101]*
   ts[78] eq
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)
   *ts[68]*ts[65]*ts[101]*ts[78];
/*true*/
for n in IN do if f(y^-1 * x * y * x * y^-1 * x * y * x * y * x
   * y^-1 * x * y^-1)*ts[68]*ts[65]*ts[101]*ts[78] eq
n*ts[31]*ts[91]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y*/
```

```
f(y^-1 * x * y * x * y^-1 * x * y * x * y * x * y^-1 * x * y^-1)
   *ts[68]*ts[65]*ts[101]*ts[78] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y)*ts
   [31]*ts[91]*ts[78];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
   ^-1 * x * y)*ts[31]*ts[91]*ts[78] eq
n*ts[5]*ts[91]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y*/
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y)*ts
   [31]*ts[91]*ts[78] eq
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y)*ts[5]*
   ts[91]*ts[78];
/*true*/
for n in IN do if f(x * y^-1 * x * y^-1 * x * y * x * y * x * y
   ^-1 * x * y)*ts[5]*ts[91]*ts[78] eq
n*ts[106]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1*/
f(x * y^-1 * x * y^-1 * x * y * x * y * x * y^-1 * x * y)*ts[5]*
   ts[91]*ts[78] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*ts[106]*ts
   [78];
/*true*/
for n in IN do if f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x *
   y^-1)*ts[106]*ts[78] eq
n*ts[60]*ts[78] then
for i in [1..#Sch] do if g(n) eq h(Sch[i]) then Sch[i]; end if;
   end for; end if; end for;
```

```
/*y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1*/
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*ts[106]*ts
    [78] eq
f(y^-1 * x * y^-1 * x * y^-1 * x * y^-1 * x * y^-1)*ts[60]*ts
    [78];
/*true*/
[60,78] in [1,2]^N;
/*true*/
```

Appendix C

Magma Isomorphism Type, (6:2^2):(7×3)

```
T:=TransitiveGroups(42);
N:=T[87];
T[87];
/*Permutation group N acting on a set of cardinality 42
Order = 504 = 2^3 * 3^2 * 7
    (1, 26, 41)(2, 25, 42)(3, 27, 38)(4, 28, 37)(5, 30, 40)(6,
      29, 39)(7, 12, 10)(8, 11,
       9)(13, 36, 23)(14, 35, 24)(15, 31, 19)(16, 32, 20)(17,
          33, 22)(18, 34, 21)
    (1, 7, 20, 2, 8, 19)(3, 9, 21, 4, 10, 22)(5, 11, 24, 6, 12,
      23)(13, 32, 29)(14, 31,
       30)(15, 34, 25)(16, 33, 26)(17, 36, 28)(18, 35, 27)(37,
          38)(39, 40)(41, 42) */
S:=Sym(42);
xx:=S!(1, 26, 41)(2, 25, 42)(3, 27, 38)(4, 28, 37)(5, 30, 40)(6,
    29, 39)(7, 12, 10)(8, 11,
       9)(13, 36, 23)(14, 35, 24)(15, 31, 19)(16, 32, 20)(17,
          33, 22)(18, 34, 21);
yy:=S!(1, 7, 20, 2, 8, 19)(3, 9, 21, 4, 10, 22)(5, 11, 24, 6,
```

```
    12, 23)(13, 32, 29)(14, 31,
        30)(15, 34, 25)(16, 33, 26)(17, 36, 28)(18, 35, 27)(37,
            38)(39, 40)(41, 42);
N:=sub<S|xx,yy>;
#N;
/*504*/
SL:=Subgroups(N);
T:={X` subgroup: X in SL};
#T;
/*36*/
TrivCore := {H:H in T| #Core(N,H) eq 1};
mdeg := Min ({Index(N,H):H in TrivCore});
Good  := {H:H in TrivCore | Index(N,H) eq mdeg};
#Good;
/*3*/
H:= Rep(Good);
#H;
/*21*/
f2,N1,K2:= CosetAction(N,H);
N1;
/*Permutation group N1 acting on a set of cardinality 24
Order = 504 = 2^3 * 3^2 * 7
    (2, 3, 5)(4, 7, 11)(8, 13, 20)(9, 15, 22)(10, 12, 18)(14,
        21, 19)
    (1, 2, 4, 8, 14, 22)(3, 6, 10, 16, 11, 17)(5, 9, 13, 21, 24,
        18)(7, 12, 19, 15, 20, 23)
*/
N;
/*Permutation group N acting on a set of cardinality 42
Order = 504 = 2^3 * 3^2 * 7
    (1, 26, 41)(2, 25, 42)(3, 27, 38)(4, 28, 37)(5, 30, 40)(6,
        29, 39)(7, 12, 10)(8, 11,
```

```
        9)(13, 36, 23)(14, 35, 24)(15, 31, 19)(16, 32, 20)(17,
            33, 22)(18, 34, 21)
    (1, 7, 20, 2, 8, 19)(3, 9, 21, 4, 10, 22)(5, 11, 24, 6, 12,
        23)(13, 32, 29)(14, 31,
        30)(15, 34, 25)(16, 33, 26)(17, 36, 28)(18, 35, 27)(37,
            38)(39, 40)(41, 42)*/
Order(xx);
/*3*/
CompositionFactors(N);
/*G
    |  Cyclic(3)
    *
    |  Cyclic(7)
    *
    |  Cyclic(3)
    *
    |  Cyclic(2)
    *
    |  Cyclic(2)
    *
    |  Cyclic(2)
    1
*/
NL:= NormalLattice(N);
NL;
/*Normal subgroup lattice
-------------------------------


[10]  Order 504  Length 1  Maximal Subgroups: 6 7 8 9
---
[ 9]  Order 168  Length 1  Maximal Subgroups: 5
[ 8]  Order 168  Length 1  Maximal Subgroups: 4 5
```

```
[ 7]  Order 168  Length 1  Maximal Subgroups: 5
[ 6]  Order 168  Length 1  Maximal Subgroups: 5
---
[ 5]  Order 56   Length 1  Maximal Subgroups: 3
[ 4]  Order 24   Length 1  Maximal Subgroups: 2 3
---
[ 3]  Order 8    Length 1  Maximal Subgroups: 1
---
[ 2]  Order 3    Length 1  Maximal Subgroups: 1
---
[ 1]  Order 1    Length 1  Maximal Subgroups:
*/

for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if; end for
   ;
/*1
2
3
4
*/
Generators (NL[4]);
/*{
    (1, 3, 6, 2, 4, 5)(7, 9, 12, 8, 10, 11)(13, 15, 17, 14, 16,
       18)(19, 21, 24)(20, 22,
        23)(25, 28, 30, 26, 27, 29)(31, 34, 35)(32, 33, 36)(37,
           39, 41)(38, 40, 42),
    (13, 14)(15, 16)(17, 18)(19, 20)(21, 22)(23, 24)(25, 26)(27,
        28)(29, 30)(37, 38)(39,
        40)(41, 42),
    (7, 8)(9, 10)(11, 12)(25, 26)(27, 28)(29, 30)(31, 32)(33,
       34)(35, 36)(37, 38)(39, 40)(41,
        42),
```

```
    (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(19, 20)(21, 22)(23,
       24)(37, 38)(39, 40)(41, 42)
}
*/
A:=N!(1, 3, 6, 2, 4, 5)(7, 9, 12, 8, 10, 11)(13, 15, 17, 14, 16,
    18)(19, 21, 24)(20, 22,
        23)(25, 28, 30, 26, 27, 29)(31, 34, 35)(32, 33, 36)(37,
           39, 41)(38, 40, 42);
B:=N!(13, 14)(15, 16)(17, 18)(19, 20)(21, 22)(23, 24)(25, 26)
   (27, 28)(29, 30)(37, 38)(39,
        40)(41, 42);
C:=N!(7, 8)(9, 10)(11, 12)(25, 26)(27, 28)(29, 30)(31, 32)(33,
   34)(35, 36)(37, 38)(39, 40)(41,
        42);
D:=N!(1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(19, 20)(21, 22)(23,
    24)(37, 38)(39, 40)(41, 42);
NL4:=sub<N|A,B,C,D>;
NL[4] eq NL4;
/*true*/
Order(NL4);
/*24*/
Order(N);
/*504*/
504/24;
/*21*/
q,ff:=quo<N|NL4>;
T:=Transversal(N,NL4);
T[2] eq xx;
/*true*/
T[3] eq yy;
/*true*/
#T;
```

```
/*21*/
ff(T[2]) eq q.1;
/*true*/
ff(T[3]) eq q.2;
/*true*/
q;
/*Permutation group q acting on a set of cardinality 7
Order = 21 = 3 * 7
    (2, 3, 5)(4, 6, 7)
    (1, 2, 4)(3, 5, 6)
*/
q,ff:=quo<N|NL4>;
T:=Transversal(N,NL4);
q;
/*Permutation group q acting on a set of cardinality 7
Order = 21 = 3 * 7
    (2, 3, 5)(4, 6, 7)
    (1, 2, 4)(3, 5, 6)*/
IsAbelian(q);
/*false*/
ff(T[2])eq q.1;
/*true*/
ff(T[3])eq q.2;
/*true*/
Order(T[2]);
/*3*/
Order(T[3]);
/*6*/
FPGroup(q);
/*Finitely presented group on 2 generators
Relations
    x^3 ,
```

```
    y^3 ,
    y^-1 * x^-1 * y^-1 * x * y * x^-1 ,*/

for i in [1..6] do for j in [1..2] do for k in [1..2] do
if A^T[2] eq A^i*B^j*C^k then i,j,k;
end if; end for; end for; end for;
/*4 2 1*/
for i in [1..6] do for j,k in [1..2] do
if A^T[3] eq A^i*B^j*C^k then i,j,k;
end if; end for; end for;
/*4 1 1 */
for i in [1..6] do for j,k in [1..2] do
if B^T[2] eq A^i*B^j*C^k then i,j,k;
end if; end for; end for;
/*3 2 1 */
for i in [1..6] do for j,k in [1..2] do
if B^T[3] eq A^i*B^j*C^k then i,j,k;
end if; end for; end for;
/*3 2 1 */
for i in [1..6] do for j,k in [1..2] do
if C^T[2] eq A^i*B^j*C^k then i,j,k;
end if; end for; end for;
/*3 1 2 */
for i in [1..6] do for j,k in [1..2] do
if C^T[3] eq A^i*B^j*C^k then i,j,k;
 end if; end for; end for;
/*6 1 2*/

qqq<d,e>:=Group<d,e|d^3,e^3,e^-1*d^-1*e^-1*d*e*d^-1>;
#qqq;
/*21*/
G<a,b,c,d,e>:=Group<a,b,c,d,e|a^6,b^2,c^2,(a,b),(a,c),(b,c),d^3,
```

```
    e^3,
e^-1*d^-1*e^-1*d*e*d^-1,a^d=a^4*b^2*c,a^e=a^4*b*c, b^d=a^3*b^2*c
    ,b^e=a^3*b^2*c, c^d=a^3*b*c^2,c^e=a^6*b*c^2>;
#G;
/*504*/

/*ISOMORPHISM TYPE OF Q*/
q;
S:=Sym(7);
/*
Permutation group q acting on a set of cardinality 7
Order = 21 = 3 * 7
     (2, 3, 5)(4, 6, 7)
     (1, 2, 4)(3, 5, 6)
*/
a:=S!(2, 3, 5)(4, 6, 7);
b:=S!(1, 2, 4)(3, 5, 6);
q1:=sub<S|a,b>;
nl:=NormalLattice(q1);
nl;
/*Normal subgroup lattice
----------------------------


[3]  Order 21  Length 1  Maximal Subgroups: 2
----
[2]  Order 7   Length 1  Maximal Subgroups: 1
----
[1]  Order 1   Length 1  Maximal Subgroups:
*/
nl[2];
/*Permutation group acting on a set of cardinality 42
Order = 7
```

```
    (1, 3, 2, 6, 5, 7, 4)
*/
A:=q1!(1, 3, 2, 6, 5, 7, 4);
nl2:=nl[2];
T:=Transversal(q1,nl2);
qq,ff:=quo<q1|nl2>;
qq;
/*Permutation group qq acting on a set of cardinality 3
Order = 3
    (1, 2, 3)
    (1, 3, 2)*/
qq.2= qq.1^-1;
/*(1, 3, 2) = (1, 3, 2)*/
 /* qq.2 is redundant*/
ff(T[2]) eq qq.1;
/*true*/
for i in [1..7] do if A^T[2] eq A^i then i; end if; end for;
/*4*/
q2<x,y>:=Group<x,y|x^7,y^3,x^y=x^4>;
f,qq2,k:=CosetAction(q2,sub<q2|Id(q2)>);
IsIsomorphic(q1,qq2);
/*true Mapping from: GrpPerm: q1 to GrpPerm: qq2
Composition of Mapping from: GrpPerm: q1 to GrpPC and
Mapping from: GrpPC to GrpPC and
Mapping from: GrpPC to GrpPerm: qq2
*/
/*N~(6:2^2):(7x3)*/
```

Appendix D

Magma Building Progenitor, 2^{*18}:(3:A_6)

```
S:=Sym(80);
xx:=S!(1, 2, 4, 8, 15, 27, 40, 59, 73, 80)(3, 6, 12, 22, 14, 25,
    13, 24, 37,
        56)(5, 10, 19, 32, 47, 60, 61, 71, 75, 16)(7, 11, 21,
           35, 52, 38, 57,
        20, 9, 17)(18, 31, 46, 63, 42, 53, 23, 33, 49, 66)(26,
           39, 58, 72, 55,
        70, 79, 29, 43, 51)(28, 41, 48, 64, 36, 54, 68, 78, 44,
           34)(30, 45, 62,
        74, 65, 69, 76, 50, 67, 77) ;
yy:=S!(1, 3, 7, 14, 26, 37)(2, 5, 11, 19, 33, 50)(4, 9, 18)(6,
    13)(8, 16, 29, 44,
        17, 30)(10, 20, 34, 51, 68, 72)(12, 23)(15, 28, 42, 45,
           49, 67)(21, 36,
        55, 41, 43, 61)(22, 27, 25, 38, 56, 70)(24, 31)(32, 48,
           65, 75, 78,
        77)(35, 53, 59)(40, 60, 57, 71, 46, 62)(47, 58, 64, 52,
           69, 73)(54, 66,
        76, 63, 74, 80);
```

```
N:=sub<S|xx,yy>;
CompositionFactors(N);
/*    G
      |   Alternating(5)
      *
      |   Cyclic(2)
      *
      |   Cyclic(2)
      *
      |   Cyclic(2)
      *
      |   Cyclic(2)
      *
      |   Cyclic(2)
      1
*/
#N;
/*1920*/
N1:=Stabiliser(N,1);
Generators(N1);
/*(2, 78)(3, 18)(4, 71)(5, 30)(6, 26)(7, 23)(8, 46)(9, 67)(10,
    56)(12, 45)(13, 70)(14,
        61)(15, 17)(16, 22)(19, 59)(20, 77)(21, 65)(24, 76)(25,
            53)(28, 62)(29, 72)(31,
        38)(32, 42)(33, 73)(34, 44)(35, 74)(36, 64)(37, 47)(39,
            68)(40, 48)(41, 79)(43,
        58)(49, 63)(50, 54)(51, 55)(52, 80)(60, 69)(66, 75),
    (2, 53, 72, 68, 75, 22)(3, 77, 45, 42, 4, 33)(5, 21, 39, 80,
        19, 78)(6, 34, 26, 35, 7,
        74)(8, 30, 29, 24, 52, 16)(9, 38, 67, 13, 36, 70)(10,
            14, 54, 61, 56, 28)(11, 51,
        49)(12, 17, 47, 73, 69, 58)(15, 71, 48, 60, 20, 79)(18,
```

```
            43, 41, 32, 37, 40)(23,
        44)(25, 65, 76, 66, 59, 46)(31, 64)(50, 62)(55, 63, 57)
*/
FPGroup(N);
/*a^10 ,
     b^6 ,
     (a * b^-2 * a)^2 ,
     (a * b^2 * a^2)^2 ,
     (b^-1 * a^-1)^5 ,
     (a * b^2 * a^-1 * b^-1)^2 ,
     a^-1 * b^-1 * a^5 * b * a^-4 ,
     b * a^-2 * b^-1 * a^3 * b * a * b^3 * a^-1 ,*/
NN<a,b>:=Group<a,b|a^10 ,
     b^6 ,
     (a * b^-2 * a)^2 ,
     (a * b^2 * a^2)^2 ,
     (b^-1 * a^-1)^5 ,
     (a * b^2 * a^-1 * b^-1)^2 ,
     a^-1 * b^-1 * a^5 * b * a^-4 ,
     b * a^-2 * b^-1 * a^3 * b * a * b^3 * a^-1 >;
#NN;
/*1920*/
word:=function(A)
Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
for i in [2..#NN] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=xx; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=xx^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=yy; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=yy^-1; end if;
end for;
```

```
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
if A eq PP then B:=Sch[i]; end if;
end for;
return B;
end function;
A:=N!(2, 78)(3, 18)(4, 71)(5, 30)(6, 26)(7, 23)(8, 46)(9, 67)
    (10, 56)(12, 45)(13, 70)(14,
          61)(15, 17)(16, 22)(19, 59)(20, 77)(21, 65)(24, 76)(25,
             53)(28, 62)(29, 72)(31,
          38)(32, 42)(33, 73)(34, 44)(35, 74)(36, 64)(37, 47)(39,
             68)(40, 48)(41, 79)(43,
          58)(49, 63)(50, 54)(51, 55)(52, 80)(60, 69)(66, 75);
B:=N!(2, 53, 72, 68, 75, 22)(3, 77, 45, 42, 4, 33)(5, 21, 39,
    80, 19, 78)(6, 34, 26, 35, 7,
          74)(8, 30, 29, 24, 52, 16)(9, 38, 67, 13, 36, 70)(10,
             14, 54, 61, 56, 28)(11, 51,
          49)(12, 17, 47, 73, 69, 58)(15, 71, 48, 60, 20, 79)(18,
             43, 41, 32, 37, 40)(23,
          44)(25, 65, 76, 66, 59, 46)(31, 64)(50, 62)(55, 63, 57);
word(A);
/*b * a^-2 * b*/
word(B);
/*b^-1 * a^-2 * b * a^-1 * b^-1*/
G<x,y,t>:=Group<x,y,t|x^10 ,
     y^6 ,
     (x * y^-2 * x)^2 ,
     (x * y^2 * x^2)^2 ,
     (y^-1 * x^-1)^5 ,
     (x * y^2 * x^-1 * y^-1)^2 ,
     x^-1 * y^-1 * x^5 * y * x^-4 ,
```

```
    y * x^-2 * y^-1 * x^3 * y * x * y^3 * x^-1 ,
t^2,
(t,(y * x^-2 * y)),
(t,(y^-1 * x^-2 * y * x^-1 * y^-1))>;
Orbits(N1);
/*GSet{@ 1 @},
    GSet{@ 27 @},
    GSet{@ 11, 51, 49, 55, 63, 57 @},
    GSet{@ 6, 34, 26, 44, 35, 23, 7, 74 @},
    GSet{@ 9, 38, 67, 31, 13, 64, 36, 70 @},
    GSet{@ 10, 14, 56, 54, 61, 28, 50, 62 @},
    GSet{@ 2, 53, 78, 72, 25, 5, 68, 29, 65, 21, 30, 75, 39, 24,
        76, 22, 66, 80, 52, 16, 59,
    19, 8, 46 @},
    GSet{@ 3, 77, 18, 45, 20, 43, 42, 12, 79, 41, 58, 4, 32, 17,
        15, 33, 71, 37, 47, 73, 48,
    40, 69, 60 @}*/
temp:={27,11,6,9,10,2,3};
for n in N do for j in temp do if 1^n eq j then word(n),j; temp
   :=temp diff {j}; end if;  end for;
end for;
/*
x 2
x^2 * y 9
x * y * x 10
x * y^2 11
x^5 27
y 3
y * x 6
*/
/*PUT INTO A T CYCLE*/
(t,t^x)
```

```
(t,t^(x^2 * y))
(t,t^(x * y * x))
(t,t^(x * y^2))
t,t^(x^5))
(t,t^(y))
(t,t^(y * x))

/*progenitor CHECK: EggsCheck*/
G<x,y,t>:=Group<x,y,t|x^10 ,
     y^6 ,
     (x * y^-2 * x)^2 ,
     (x * y^2 * x^2)^2 ,
     (y^-1 * x^-1)^5 ,
     (x * y^2 * x^-1 * y^-1)^2 ,
     x^-1 * y^-1 * x^5 * y * x^-4 ,
     y * x^-2 * y^-1 * x^3 * y * x * y^3 * x^-1 ,
t^2,
(t,(y * x^-2 * y)),
(t,(y^-1 * x^-2 * y * x^-1 * y^-1)),
(t,t^x),
(t,t^(x^2 * y)),
(t,t^(x * y * x)),
(t,t^(x * y^2)),
(t,t^(x^5)),
(t,t^(y)),
(t,t^(y * x))>;
print Index(G,sub<G|x,y>: CosetLimit:=9^10, Hard:=true, Print
    :=2);


/*LEMMA 3.3 WORK*/
N12:=Stabiliser(N,[1,2]);
```

```
N12;
/*
Permutation group N12 acting on a set of cardinality 80
Order = 1
*/

/*BEGINNING OF LOOKING FOR CLASSES*/
C:=Classes(N);
C;
for i in [2..#C] do i, C[i][3]; word(C[i][3]); Orbits(
   Centraliser(N,C[i][3]));end for;
/*
2 (1, 27)(2, 40)(3, 25)(4, 59)(5, 60)(6, 13)(7, 38)(8, 73)(9,
   35)(10, 61)(11, 57)(12, 24)(14,
    56)(15, 80)(16, 47)(17, 52)(18, 53)(19, 71)(20, 21)(22, 37)
       (23, 31)(26, 70)(28, 54)(29,
    58)(30, 69)(32, 75)(33, 46)(34, 36)(39, 79)(41, 68)(42, 66)
       (43, 72)(44, 64)(45, 76)(48,
    78)(49, 63)(50, 62)(51, 55)(65, 77)(67, 74)
x^5
[
    GSet{@ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,
       16, 17, 18, 19, 20, 21, 22, 23,
    24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38,
       39, 40, 41, 42, 43, 44, 45,
    46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60,
       61, 62, 63, 64, 65, 66, 67,
    68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80 @}
]
3 (1, 10)(3, 25)(4, 78)(5, 43)(6, 53)(7, 76)(8, 30)(9, 75)(11,
   28)(12, 52)(13, 18)(14,
    55)(15, 37)(16, 23)(17, 24)(19, 71)(22, 80)(27, 61)(29, 64)
```

```
        (31, 47)(32, 35)(33, 34)(36,
    46)(38, 45)(39, 65)(41, 68)(44, 58)(48, 59)(49, 62)(50, 63)
        (51, 56)(54, 57)(60, 72)(67,
    74)(69, 73)(77, 79)
(x * y * x^-1)^3
[
    GSet{@ 2, 66, 21, 20, 42, 40, 70, 26 @},
    GSet{@ 3, 25, 68, 19, 41, 71, 74, 67 @},
    GSet{@ 1, 7, 10, 16, 47, 73, 23, 76, 53, 61, 9, 31, 69, 38,
        51, 27, 55, 49, 14, 34, 45,
    56, 6, 75, 13, 5, 30, 4, 72, 65, 62, 33, 18, 35, 80, 12, 8,
        15, 24, 46, 64, 52, 37, 17,
    78, 32, 22, 43, 60, 39, 44, 11, 50, 79, 29, 54, 36, 59, 77,
        63, 57, 28, 48, 58 @}
]
4 (1, 49)(2, 17)(3, 12)(4, 7)(5, 78)(6, 22)(8, 66)(9, 43)(11,
   51)(13, 37)(15, 79)(16, 67)(18,
    46)(19, 44)(20, 31)(21, 23)(24, 25)(26, 58)(27, 63)(28, 54)
        (29, 70)(30, 41)(32, 76)(33,
    53)(34, 77)(35, 72)(36, 65)(38, 59)(39, 80)(40, 52)(42, 73)
        (45, 75)(47, 74)(48, 60)(50,
    62)(55, 57)(64, 71)(68, 69)
x * y^-2 * x
[
    GSet{@ 10, 61 @},
    GSet{@ 14, 56 @},
    GSet{@ 28, 54 @},
    GSet{@ 50, 62 @},
    GSet{@ 1, 27, 49, 11, 51, 63, 57, 55 @},
    GSet{@ 2, 41, 17, 34, 4, 30, 38, 65, 77, 7, 69, 59, 36, 52,
        68, 40 @},
    GSet{@ 3, 66, 12, 72, 6, 8, 37, 9, 35, 22, 73, 13, 43, 24,
```

```
        42, 25 @},
    GSet{@ 5, 15, 78, 21, 44, 79, 71, 31, 23, 19, 39, 64, 20,
        48, 80, 60 @},
    GSet{@ 16, 58, 67, 18, 76, 26, 75, 33, 46, 32, 70, 45, 53,
        74, 29, 47 @}
]
5 (1, 51, 63)(2, 39, 12)(3, 65, 23)(4, 64, 66)(5, 37, 34)(6, 17,
    71)(7, 80, 43)(8, 41, 48)(9,
    30, 21)(13, 52, 19)(14, 28, 50)(15, 72, 38)(16, 26, 53)(18,
        47, 70)(20, 35, 69)(22, 36,
    60)(24, 40, 79)(25, 77, 31)(27, 55, 49)(29, 74, 75)(32, 58,
        67)(42, 59, 44)(54, 62,
    56)(68, 78, 73)
y * x^4 * y * x^-1
[
    GSet{@ 10, 61, 57, 11 @},
    GSet{@ 33, 46, 45, 76 @},
    GSet{@ 1, 51, 27, 28, 63, 55, 50, 54, 49, 14, 62, 56 @},
    GSet{@ 5, 37, 60, 80, 34, 22, 43, 15, 36, 7, 72, 38 @},
    GSet{@ 2, 39, 40, 68, 74, 12, 79, 78, 75, 41, 67, 70, 26,
        24, 73, 29, 48, 32, 18, 53, 8,
    58, 47, 16 @},
    GSet{@ 3, 65, 25, 42, 20, 23, 77, 59, 35, 66, 21, 19, 71,
        31, 44, 69, 4, 9, 13, 6, 64,
    30, 52, 17 @}
]
6 (1, 54, 27, 28)(2, 26, 40, 70)(3, 71, 25, 19)(4, 35, 59, 9)(5,
    80, 60, 15)(6, 77, 13,
    65)(7, 36, 38, 34)(8, 29, 73, 58)(10, 57, 61, 11)(12, 16,
        24, 47)(14, 63, 56, 49)(17, 31,
    52, 23)(18, 39, 53, 79)(20, 42, 21, 66)(22, 72, 37, 43)(30,
        64, 69, 44)(32, 48, 75,
```

```
    78)(33, 76, 46, 45)(41, 74, 68, 67)(50, 51, 62, 55)
(x * y^-1)^3
[
    GSet{@ 1, 14, 54, 33, 22, 28, 63, 5, 7, 57, 27, 76, 72, 56,
        11, 60, 38, 55, 49, 45, 43,
    80, 36, 61, 62, 37, 10, 34, 50, 46, 15, 51 @},
    GSet{@ 2, 19, 26, 73, 31, 3, 64, 44, 16, 40, 58, 52, 71, 70,
         78, 24, 74, 20, 77, 69, 30,
    59, 8, 25, 4, 17, 21, 67, 18, 23, 32, 47, 68, 42, 13, 29,
        48, 65, 66, 41, 39, 12, 35, 9,
    53, 6, 75, 79 @}
]
7 (1, 76, 50, 72)(2, 32, 25, 4)(3, 59, 40, 75)(5, 14, 7, 57)(6,
   41, 39, 66)(8, 23)(9, 70, 78,
    71)(10, 15, 49, 34)(11, 60, 56, 38)(12, 30, 24, 69)(13, 68,
        79, 42)(16, 64, 47, 44)(17,
    29)(18, 21, 65, 67)(19, 35, 26, 48)(20, 77, 74, 53)(22, 28,
        33, 55)(27, 45, 62, 43)(31,
    73)(36, 61, 80, 63)(37, 54, 46, 51)(52, 58)
x^2 * y^-1 * x^-1 * y
[
    GSet{@ 8, 23, 73, 58, 31, 52, 29, 17 @},
    GSet{@ 12, 24, 30, 47, 69, 16, 44, 64 @},
    GSet{@ 1, 50, 76, 27, 28, 72, 62, 55, 45, 33, 54, 43, 22,
        51, 46, 37 @},
    GSet{@ 2, 25, 32, 40, 70, 4, 3, 71, 75, 78, 26, 59, 9, 19,
        48, 35 @},
    GSet{@ 5, 7, 14, 60, 15, 57, 38, 34, 56, 49, 80, 11, 10, 36,
         63, 61 @},
    GSet{@ 6, 39, 41, 13, 65, 66, 79, 18, 68, 67, 77, 42, 21,
        53, 74, 20 @}
]
```

8 (1, 4, 15, 40, 73)(2, 8, 27, 59, 80)(3, 12, 14, 13, 37)(5, 19, 47, 61, 75)(6, 22, 25, 24,
56)(7, 21, 52, 57, 9)(10, 32, 60, 71, 16)(11, 35, 38, 20, 17)(18, 46, 42, 23, 49)(26, 58,
55, 79, 43)(28, 48, 36, 68, 44)(29, 51, 39, 72, 70)(30, 62, 65, 76, 67)(31, 63, 53, 33,
66)(34, 41, 64, 54, 78)(45, 74, 69, 50, 77)

x^2

[
GSet{@ 1, 4, 27, 15, 59, 40, 80, 73, 2, 8 @},
GSet{@ 3, 12, 25, 14, 24, 13, 56, 37, 6, 22 @},
GSet{@ 5, 19, 60, 47, 71, 61, 16, 75, 10, 32 @},
GSet{@ 7, 21, 38, 52, 20, 57, 17, 9, 11, 35 @},
GSet{@ 18, 46, 53, 42, 33, 23, 66, 49, 31, 63 @},
GSet{@ 26, 58, 70, 55, 29, 79, 51, 43, 39, 72 @},
GSet{@ 28, 48, 54, 36, 78, 68, 34, 44, 41, 64 @},
GSet{@ 30, 62, 69, 65, 50, 76, 77, 67, 45, 74 @}
]

9 (1, 15, 73, 4, 40)(2, 27, 80, 8, 59)(3, 14, 37, 12, 13)(5, 47, 75, 19, 61)(6, 25, 56, 22,
24)(7, 52, 9, 21, 57)(10, 60, 16, 32, 71)(11, 38, 17, 35, 20)(18, 42, 49, 46, 23)(26, 55,
43, 58, 79)(28, 36, 44, 48, 68)(29, 39, 70, 51, 72)(30, 65, 67, 62, 76)(31, 53, 66, 63,
33)(34, 64, 78, 41, 54)(45, 69, 77, 74, 50)

x^4

[
GSet{@ 1, 15, 27, 73, 80, 4, 8, 40, 59, 2 @},
GSet{@ 3, 14, 25, 37, 56, 12, 22, 13, 24, 6 @},
GSet{@ 5, 47, 60, 75, 16, 19, 32, 61, 71, 10 @},
GSet{@ 7, 52, 38, 9, 17, 21, 35, 57, 20, 11 @},
GSet{@ 18, 42, 53, 49, 66, 46, 63, 23, 33, 31 @},

```
    GSet{@ 26, 55, 70, 43, 51, 58, 72, 79, 29, 39 @},
    GSet{@ 28, 36, 54, 44, 34, 48, 64, 68, 78, 41 @},
    GSet{@ 30, 65, 69, 67, 77, 62, 74, 76, 50, 45 @}
]
10 (1, 49, 51, 27, 63, 55)(2, 24, 39, 40, 12, 79)(3, 31, 65, 25,
    23, 77)(4, 42, 64, 59, 66,
    44)(5, 36, 37, 60, 34, 22)(6, 19, 17, 13, 71, 52)(7, 72, 80,
        38, 43, 15)(8, 78, 41, 73,
    48, 68)(9, 20, 30, 35, 21, 69)(10, 61)(11, 57)(14, 62, 28,
        56, 50, 54)(16, 18, 26, 47,
    53, 70)(29, 32, 74, 58, 75, 67)(33, 46)(45, 76)
(x * y^-1)^2
[
    GSet{@ 10, 61, 57, 11 @},
    GSet{@ 33, 46, 45, 76 @},
    GSet{@ 1, 49, 28, 51, 56, 54, 27, 50, 14, 63, 62, 55 @},
    GSet{@ 5, 36, 80, 37, 38, 15, 60, 43, 7, 34, 72, 22 @},
    GSet{@ 2, 24, 68, 74, 39, 8, 58, 70, 26, 40, 78, 75, 16, 47,
        67, 41, 12, 18, 53, 29, 73,
    79, 32, 48 @},
    GSet{@ 3, 31, 42, 20, 65, 64, 30, 19, 71, 25, 59, 35, 17,
        52, 21, 66, 23, 13, 6, 69, 44,
    77, 9, 4 @}
]
11 (1, 16, 7, 10, 23, 76)(2, 20, 66)(3, 25)(4, 75, 79, 78, 9,
   77)(5, 57, 44, 43, 54, 58)(6,
    53)(8, 34, 63, 30, 33, 50)(11, 64, 72, 28, 29, 60)(12, 15,
        14, 52, 37, 55)(13, 18)(17,
    22, 51, 24, 80, 56)(19, 41, 74, 71, 68, 67)(21, 42, 40)(27,
        47, 38, 61, 31, 45)(32, 39,
    48, 35, 65, 59)(36, 49, 69, 46, 62, 73)
x * y * x^-1
```

```
[
    GSet{@ 3, 25 @},
    GSet{@ 26, 70 @},
    GSet{@ 6, 53, 18, 13 @},
    GSet{@ 2, 20, 42, 66, 40, 21 @},
    GSet{@ 19, 71, 41, 74, 68, 67 @},
    GSet{@ 1, 10, 16, 45, 23, 38, 7, 27, 76, 61, 47, 31 @},
    GSet{@ 4, 78, 75, 65, 9, 39, 79, 59, 77, 48, 32, 35 @},
    GSet{@ 5, 43, 57, 29, 54, 64, 44, 60, 58, 72, 11, 28 @},
    GSet{@ 8, 30, 34, 62, 33, 49, 63, 73, 50, 69, 36, 46 @},
    GSet{@ 12, 52, 15, 51, 37, 56, 14, 24, 55, 17, 80, 22 @}
]
12 (1, 76, 23, 10, 7, 16)(2, 66, 20)(3, 25)(4, 77, 9, 78, 79,
   75)(5, 58, 54, 43, 44, 57)(6,
    53)(8, 50, 33, 30, 63, 34)(11, 60, 29, 28, 72, 64)(12, 55,
       37, 52, 14, 15)(13, 18)(17,
    56, 80, 24, 51, 22)(19, 67, 68, 71, 74, 41)(21, 40, 42)(27,
       45, 31, 61, 38, 47)(32, 59,
    65, 35, 48, 39)(36, 73, 62, 46, 69, 49)
x * y^-1 * x^-1
[
    GSet{@ 3, 25 @},
    GSet{@ 26, 70 @},
    GSet{@ 6, 53, 18, 13 @},
    GSet{@ 2, 66, 21, 20, 40, 42 @},
    GSet{@ 19, 71, 67, 68, 74, 41 @},
    GSet{@ 1, 10, 76, 47, 7, 31, 23, 27, 16, 61, 45, 38 @},
    GSet{@ 4, 78, 77, 32, 79, 35, 9, 59, 75, 48, 65, 39 @},
    GSet{@ 5, 43, 58, 11, 44, 28, 54, 60, 57, 72, 29, 64 @},
    GSet{@ 8, 30, 50, 36, 63, 46, 33, 73, 34, 69, 62, 49 @},
    GSet{@ 12, 52, 55, 80, 14, 22, 37, 24, 15, 17, 51, 56 @}
]
```

```
13 (1, 9, 14, 6, 27, 35, 56, 13)(2, 16, 20, 30, 40, 47, 21, 69)
   (3, 23, 67, 8, 25, 31, 74,
    73)(4, 28, 65, 63, 59, 54, 77, 49)(5, 33, 76, 15, 60, 46,
        45, 80)(7, 36, 72, 22, 38, 34,
    43, 37)(10, 48, 55, 79, 61, 78, 51, 39)(11, 53, 50, 32, 57,
        18, 62, 75)(12, 26, 64, 66,
    24, 70, 44, 42)(17, 19, 58, 41, 52, 71, 29, 68)
x^2 * y
[
    GSet{@ 1, 9, 14, 6, 27, 35, 56, 13 @},
    GSet{@ 2, 16, 20, 30, 40, 47, 21, 69 @},
    GSet{@ 3, 23, 67, 8, 25, 31, 74, 73 @},
    GSet{@ 4, 28, 65, 63, 59, 54, 77, 49 @},
    GSet{@ 5, 33, 76, 15, 60, 46, 45, 80 @},
    GSet{@ 7, 36, 72, 22, 38, 34, 43, 37 @},
    GSet{@ 10, 48, 55, 79, 61, 78, 51, 39 @},
    GSet{@ 11, 53, 50, 32, 57, 18, 62, 75 @},
    GSet{@ 12, 26, 64, 66, 24, 70, 44, 42 @},
    GSet{@ 17, 19, 58, 41, 52, 71, 29, 68 @}
]
14 (1, 2, 4, 8, 15, 27, 40, 59, 73, 80)(3, 6, 12, 22, 14, 25,
   13, 24, 37, 56)(5, 10, 19, 32,
    47, 60, 61, 71, 75, 16)(7, 11, 21, 35, 52, 38, 57, 20, 9,
        17)(18, 31, 46, 63, 42, 53, 23,
    33, 49, 66)(26, 39, 58, 72, 55, 70, 79, 29, 43, 51)(28, 41,
        48, 64, 36, 54, 68, 78, 44,
    34)(30, 45, 62, 74, 65, 69, 76, 50, 67, 77)
x
[
    GSet{@ 1, 2, 4, 8, 15, 27, 40, 59, 73, 80 @},
    GSet{@ 3, 6, 12, 22, 14, 25, 13, 24, 37, 56 @},
    GSet{@ 5, 10, 19, 32, 47, 60, 61, 71, 75, 16 @},
```

```
    GSet{@ 7, 11, 21, 35, 52, 38, 57, 20, 9, 17 @},
    GSet{@ 18, 31, 46, 63, 42, 53, 23, 33, 49, 66 @},
    GSet{@ 26, 39, 58, 72, 55, 70, 79, 29, 43, 51 @},
    GSet{@ 28, 41, 48, 64, 36, 54, 68, 78, 44, 34 @},
    GSet{@ 30, 45, 62, 74, 65, 69, 76, 50, 67, 77 @}
]
15 (1, 8, 40, 80, 4, 27, 73, 2, 15, 59)(3, 22, 13, 56, 12, 25,
   37, 6, 14, 24)(5, 32, 61, 16,
    19, 60, 75, 10, 47, 71)(7, 35, 57, 17, 21, 38, 9, 11, 52,
       20)(18, 63, 23, 66, 46, 53, 49,
    31, 42, 33)(26, 72, 79, 51, 58, 70, 43, 39, 55, 29)(28, 64,
       68, 34, 48, 54, 44, 41, 36,
    78)(30, 74, 76, 77, 62, 69, 67, 45, 65, 50)
x^3
[
    GSet{@ 1, 8, 40, 80, 4, 27, 73, 2, 15, 59 @},
    GSet{@ 3, 22, 13, 56, 12, 25, 37, 6, 14, 24 @},
    GSet{@ 5, 32, 61, 16, 19, 60, 75, 10, 47, 71 @},
    GSet{@ 7, 35, 57, 17, 21, 38, 9, 11, 52, 20 @},
    GSet{@ 18, 63, 23, 66, 46, 53, 49, 31, 42, 33 @},
    GSet{@ 26, 72, 79, 51, 58, 70, 43, 39, 55, 29 @},
    GSet{@ 28, 64, 68, 34, 48, 54, 44, 41, 36, 78 @},
    GSet{@ 30, 74, 76, 77, 62, 69, 67, 45, 65, 50 @}
]
16 (1, 50, 49, 54, 51, 14, 27, 62, 63, 28, 55, 56)(2, 18, 24,
   26, 39, 47, 40, 53, 12, 70, 79,
    16)(3, 13, 31, 71, 65, 52, 25, 6, 23, 19, 77, 17)(4, 30, 42,
        35, 64, 21, 59, 69, 66, 9,
    44, 20)(5, 72, 36, 80, 37, 38, 60, 43, 34, 15, 22, 7)(8, 67,
        78, 29, 41, 32, 73, 74, 48,
    58, 68, 75)(10, 11, 61, 57)(33, 45, 46, 76)
x * y^-1
```

```
[
    GSet{@ 10, 11, 61, 57 @},
    GSet{@ 33, 45, 46, 76 @},
    GSet{@ 1, 51, 50, 63, 14, 49, 28, 27, 54, 55, 62, 56 @},
    GSet{@ 2, 39, 18, 12, 47, 24, 70, 40, 26, 79, 53, 16 @},
    GSet{@ 3, 65, 13, 23, 52, 31, 19, 25, 71, 77, 6, 17 @},
    GSet{@ 4, 64, 30, 66, 21, 42, 9, 59, 35, 44, 69, 20 @},
    GSet{@ 5, 37, 72, 34, 38, 36, 15, 60, 80, 22, 43, 7 @},
    GSet{@ 8, 41, 67, 48, 32, 78, 58, 73, 29, 68, 74, 75 @}
*/

for j in [2..#N] do for i in [1..#Setseq(Set(N))] do if 1^Setseq
   (Set(N))[i] eq j then j, word(Setseq(Set(N))[i]); break;end
   if; end for;end for;

/*TS[i] CODE*/
ts[2]:=f(t^( y^-1 * x^-1 * y^-1 * x^-1 * y * x^-1));
ts[3]:=f(t^( x * y * x^2 * y^-1 * x^-1 * y^-1));
ts[4]:=f(t^( y * x^-2 * y * x^2));
ts[5]:=f(t^( x^2 * y^3 * x * y * x));
ts[6]:=f(t^( y * x^2 * y^2 * x^-1));
ts[7]:=f(t^( (y * x * y^-1)^4));
ts[8]:=f(t^( x^-1 * y^-1 * x * y * x * y^-1));
ts[9]:=f(t^( y^-1 * x * y^-1 * x^3));
ts[10]:=f(t^( y * x^-2 * y * x * y * x));
ts[11]:=f(t^( y * x * y^2 * x^-1 * y * x));
ts[12]:=f(t^( y * x^2 * y^-2));
ts[13]:=f(t^( x^2 * y^2 * x * y * x^-1));
ts[14]:=f(t^( y^-1 * x * y^2 * x));
ts[15]:=f(t^( x * y^-1 * x * y));
ts[16]:=f(t^( y * x^-1 * y * x^2 * y^-1));
ts[17]:=f(t^( (y^-2 * x^-1 * y)^2));
```

```
ts[18]:=f(t^( y^-1 * x^-1 * y^3 * x^-1));
ts[19]:=f(t^( x^2 * y * x^-1 * y^-1 * x^-1 * y^2));
ts[20]:=f(t^( x * y * x^-1 * y^2 * x * y^-1));
ts[21]:=f(t^( x * y^2 * x));
ts[22]:=f(t^( x^-1 * y * x * y^-1 * x * y^-1 * x^-1));
ts[23]:=f(t^( x * y^-2 * x^-1 * y^-2));
ts[24]:=f(t^( y^-1 * x^3 * y^-1 * x));
ts[25]:=f(t^( y * x^-1 * y^-2));
ts[26]:=f(t^( y^-2));
ts[27]:=f(t^( x^-1 * y^-1 * x^-1 * y * x^-1));
ts[28]:=f(t^( x^-1 * y^-2 * x * y^-1));
ts[29]:=f(t^( x * y * x^-1 * y * x^-1 * y^-1 * x));
ts[30]:=f(t^( x * y^-1 * x * y^-2 * x^-1));
ts[31]:=f(t^( y * x * y^-1 * x * y));
ts[32]:=f(t^( y^2 * x * y * x));
ts[33]:=f(t^( x * y^-2 * x^-1 * y^2 * x));
ts[34]:=f(t^( x * y * x * y^2));
ts[35]:=f(t^( y * x^-1 * y^-1 * x^-2));
ts[36]:=f(t^( y^-1 * x * y^-1 * x^-1 * y^-1 * x));
ts[37]:=f(t^( y^-1 * x^-1 * y^2 * x));
ts[38]:=f(t^( y * x * y^-1 * x^3 * y^-1));
ts[39]:=f(t^( x * y * x * y^-1 * x^-2 * y));
ts[40]:=f(t^( y * x * y^-1 * x^-1 * y^-1 * x));
ts[41]:=f(t^( x * y * x^-1 * y * x * y^-1));
ts[42]:=f(t^( y * x^2 * y^-1 * x^-2));
ts[43]:=f(t^( y^-1 * x * y * x^-1 * y^2));
ts[44]:=f(t^( x^3 * y^-1 * x^-1 * y^-1 * x));
ts[45]:=f(t^( y^-1 * x^-1 * y * x^-3 * y^-1));
ts[46]:=f(t^( x^2 * y^-1 * x * y^2 * x));
ts[47]:=f(t^( y^2 * x * y * x^2));
ts[48]:=f(t^( x * y^-1 * x^-1 * y^2 * x * y^-1));
ts[49]:=f(t^( y^-1 * x^-1 * y * x * y * x^-1 * y));
```

```
ts[50]:=f(t^( x * y^-1 * x^-1 * y^-1 * x^-2 * y));
ts[51]:=f(t^( x^4 * y * x * y * x));
ts[52]:=f(t^( y^3 * x * y * x^-1));
ts[53]:=f(t^( y * x^2 * y^3 * x^-1));
ts[54]:=f(t^( x^-2 * y^-1 * x * y^-2));
ts[55]:=f(t^( (x * y^2)^2));
ts[56]:=f(t^( x * y * x * y * x^-2 * y));
ts[57]:=f(t^( x * y^-1 * x^-2 * y^-1 * x^2));
ts[58]:=f(t^( x^-1 * y^-1 * x * y^-1 * x * y^-1));
ts[59]:=f(t^( y * x^2 * y * x^-1 * y^-2));
ts[60]:=f(t^( y^3 * x^-1 * y * x * y));
ts[61]:=f(t^( y^-1 * x^-1 * y^-1 * x * y^-1 * x^-1));
ts[62]:=f(t^( x^-1 * y^-2 * x^-1 * y));
ts[63]:=f(t^( y * x^-1 * y^-1 * x^-1 * y * x * y));
ts[64]:=f(t^( y * x^-1 * y * x^-3 * y));
ts[65]:=f(t^( x^-1 * y^-1 * x^-1 * y^-1 * x * y * x));
ts[66]:=f(t^( x^3 * y^-2 * x^-1 * y * x^-1));
ts[67]:=f(t^( y * x^2 * y^-1 * x * y * x));
ts[68]:=f(t^( x * y^3 * x^-1 * y^-2));
ts[69]:=f(t^( x^-1 * y * x^-1 * y^-1 * x^2 * y));
ts[70]:=f(t^( x^-1 * y^-1 * x^-1 * y * x^-1 * y^-2));
ts[71]:=f(t^( y * x * y * x * y^-1 * x * y^-1));
ts[72]:=f(t^( y * x^-1 * y * x^-2));
ts[73]:=f(t^( y * x^-1 * y^-1 * x^-1 * y^2));
ts[74]:=f(t^( x^2 * y * x * y * x^3));
ts[75]:=f(t^( x * y^-1 * x^-3 * y));
ts[76]:=f(t^( x * y * x^2 * y^2 * x^-1));
ts[77]:=f(t^( x^3 * y * x^-1 * y^2));
ts[78]:=f(t^( x^-2 * y * x^-1 * y^-2));
ts[79]:=f(t^( y^-1 * x * y * x * y));
ts[80]:=f(t^( x * y^-1 * x^-4 * y));
```

```
/*FIRST ORDER RELATION EXAMPLE*/
/*CLASS 2*/
((x^5)*t^( y^-1 * x^-1 * y^-1 * x^-1 * y * x^-1))
((x^5)*t^( x * y * x^2 * y^-1 * x^-1 * y^-1))
((x^5)*t^( y * x^-2 * y * x^2))
((x^5)*t^( x^2 * y^3 * x * y * x))
((x^5)*t^( y * x^2 * y^2 * x^-1))
((x^5)*t^( (y * x * y^-1)^4))
((x^5)*t^( x^-1 * y^-1 * x * y * x * y^-1))
((x^5)*t^( y^-1 * x * y^-1 * x^3))
((x^5)*t^( y * x^-2 * y * x * y * x))
```

Appendix E

Magma Monomial Progenitor, $17^{*2}:_m((2:8):2)$

```
T:=TransitiveGroups(16);
G:=T[16];
S:=Subgroups(G);
H:=S[25]`subgroup;
CH:=CharacterTable(H);
CG:=CharacterTable(G);
T:=Transversal(G,H);
#T;
/*2*/
H;
/*Permutation group H acting on a set of cardinality 16
Order = 16 = 2^4
    (1, 16, 2, 15)(3, 10, 4, 9)(5, 12, 6, 11)(7, 13, 8, 14)
    (1, 14, 6, 9, 2, 13, 5, 10)(3, 15, 8, 12, 4, 16, 7, 11)
    (1, 6, 2, 5)(3, 8, 4, 7)(9, 13, 10, 14)(11, 15, 12, 16)
    (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16)
*/
G;
/*Permutation group G acting on a set of cardinality 16
```

```
Order = 32 = 2^5
     (1, 14, 6, 9, 2, 13, 5, 10)(3, 15, 8, 12, 4, 16, 7, 11)
     (1, 6, 2, 5)(3, 8, 4, 7)(9, 14, 10, 13)(11, 16, 12, 15)
     (1, 16)(2, 15)(3, 10)(4, 9)(5, 12)(6, 11)(7, 13)(8, 14)*/
H:=sub<G| (1, 16, 2, 15)(3, 10, 4, 9)(5, 12, 6, 11)(7, 13, 8,
    14),
     (1, 14, 6, 9, 2, 13, 5, 10)(3, 15, 8, 12, 4, 16, 7, 11),
     (1, 6, 2, 5)(3, 8, 4, 7)(9, 13, 10, 14)(11, 15, 12, 16),
     (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16)>;
S:=Sym(16);
xx:=S!(1, 14, 6, 9, 2, 13, 5, 10)(3, 15, 8, 12, 4, 16, 7, 11);
yy:=S!(1, 6, 2, 5)(3, 8, 4, 7)(9, 14, 10, 13)(11, 16, 12, 15);
zz:=S!(1, 16)(2, 15)(3, 10)(4, 9)(5, 12)(6, 11)(7, 13)(8, 14);
G:=sub<G|xx,yy,zz>;
CG;
Induction(CH[7],G) eq CG[18];
/*true*/
CH[7];
/*( 1, -1, 1, -1, zeta(8)_8^2, -zeta(8)_8^2, -zeta(8)_8^2, zeta
    (8)_8^2,
     zeta(8)_8^3, zeta(8)_8, -zeta(8)_8^3, -zeta(8)_8, -zeta(8)_8
        , -zeta(8)_8^3,
zeta(8)_8, zeta(8)_8^3 )*/
C:=CyclotomicField(8);
A:=[[C.1,0] : i in [1..2]];
for i,j in [1..2] do A[i,j]:=0; end for;
for i,j in [1..2] do if T[i]*xx*T[j]^-1 in H then
A[i,j]:=CH[8](T[i]*xx*T[j]^-1);end if; end for;
GG:=GL(2,C);
GG!A;
/*
[ zeta_8^3           0]
```

```
[               0 -zeta_8^3]
*/
Order(xx);
/*8*/
Order(GG!A);
/*8*/
B:=[[C.1,0] : i in [1..2]];
for i,j in [1..2] do B[i,j]:=0; end for;
for i,j in [1..2] do if T[i]*yy*T[j]^-1 in H then
B[i,j]:=CH[8](T[i]*yy*T[j]^-1);end if; end for;
GG!B;
/*
[ 0  1]
[-1  0]
*/
Order(yy);
/*4*/
Order(GG!B);
/*4*/
D:=[[C.1,0] : i in [1..2]];
for i,j in [1..2] do D[i,j]:=0; end for;
for i,j in [1..2] do if T[i]*zz*T[j]^-1 in H then
D[i,j]:=CH[8](T[i]*zz*T[j]^-1);end if; end for;

GG!D;
/*
[ 0 -1]
[-1  0]
*/
Order(zz);
/*2*/
Order(GG!D);
```

```
/*2*/
GGG:=GL(2,C);
IsIsomorphic(sub<GGG|GGG!A,GGG!B,GGG!D>,G);
/*true Mapping from: MatrixGroup(2, C) of order 2^5 to GrpPerm:
    G
Composition of Mapping from: MatrixGroup(2, C) of order 2^5 to
    GrpPC and
Mapping from: GrpPC to GrpPC and
Mapping from: GrpPC to GrpPerm: G
*/
for i,j in [1..2] do if (GGG!A)[i,j] ne 0 then i,j, (GGG!A)[i,j
    ]; end if; end for;
/*
1 1 zeta_8^3
2 2 -zeta_8^3 */
/*SMALLEST PRIME P SUCH THAT 8|P-1 IS P=17 Z17 HAS AN ELEMENT
    GENERATOR OF ORDER 2 */
PrimitiveRoot(17);
/*3*/
Modorder(3,17);
/*16*/
Modorder(3^2,17);
/*8*/
Modorder(3^3,17);
/*16*/

/*KEY INFORMATION*/
/*
zeta(8)_8= 9
-zeta(8)_8= -9 mod 17 = 8
zeta(8)_8^2= 9^2= 81 mod 17 = 4
-zeta(8)_8^2= -9^2 = 81 mod 17 = 4
```

```
zeta(8)_8^3= 9^3 =729 mod 17 = 8
-zeta(8)_8^3= -9^3 = -729 mod 17 = 8
*/

/*A MATRIX
[8   0]
[0   8]
a11=t1^8
a22=t2^8
t1 (1),              t1^8 (15)
t2 (2),              t2^8 (16)
t1^2 (3),         (t1^8)^2=16 =t1^16 (31)
t2^2 (4),         (t2^8)^2 =16= t2^16 (32)
t1^3 (5),         (t1^8)^3 =24= t1^7 (13)
t2^3 (6),         (t2^8)^3 =24 = t2^7 (14)
t1^4 (7),         (t1^8)^4 = 32 = t1^15 (29)
t2^4 (8),         (t2^8)^4 = 32= t2^15 (30)
t1^5 (9),         (t1^8)^5 = 40= t1^6 (11)
t2^5 (10),         (t2^8)^5 = 40= t2^6 (12)
t1^6 (11),         (t1^8)^6 =48= t1^14 (27)
t2^6 (12),         (t2^8)^6 =48= t2^14 (28)
t1^7 (13),         (t1^8)^7 =56=t1^5 (9)
t2^7 (14),         (t2^8)^7 =56= t2^5 (10)
t1^8 (15),         (t1^8)^8 =64=t1^13 (25)
t2^8 (16),         (t2^8)^8 =64= t2^13 (26)
t1^9 (17),         (t1^8)^9 =72= t1^4 (7)
t2^9 (18),         (t2^8)^9 =72= t2^4 (8)
t1^10 (19),         (t1^8)^10 =80= t1^12 (23)
t2^10 (20),         (t2^8)^10 =80= t2^12 (24)
t1^11 (21),         (t1^8)^11 =88= t1^3 (5)
t2^11 (22),         (t2^8)^11 =88= t2^3 (6)
t1^12 (23),         (t1^8)^12 =96= t1^11 (21)
```

```
t2^12 (24),          (t2^8)^12 =96= t2^11 (22)
t1^13 (25),          (t1^8)^13= 104 =t1^2 (3)
t2^13 (26),          (t2^8)^13= 104 =t2^2 (4)
t1^14 (27),          (t1^8)^14 = 112 =t1^10 (19)
t2^14 (28),          (t2^8)^14= 112 =t2^10 (20)
t1^15 (29),           (t1^8)^15= 120 =t1 (1)
t2^15 (30),          (t2^8)^15= 120 =t2 (2)
t1^16 (31),           (t1^8)^16= 128 =t1^9 (17)
t2^16 (32),          (t2^8)^16= 128 =t2^9 (18)
t1^17 (33),           (t1^8)^17= identity
t2^17 (34),          (t2^8)^17 = idenitity
Permutation:
(1,15,25,3,31,17,7,29)(2,16,26,4,32,18,8,30)
    (5,13,9,11,27,19,23,21)(6,14,10,12,28,20,24,22)


B matrix:
[ 0   1]
[-1   0]
a12=t2
a21=t1^(-1)
t1 (1),               t2  (2)
t2 (2),               t1^(-1)=t1^16 (31)
t1^2 (3),          (t2)^2  (4)
t2^2 (4),          (t1^-1)^2=-2 = t1^15 (29)
t1^3 (5),          (t2)^3  (6)
t2^3 (6),          (t1^-1)^3=-3 = t1^14 (27)
t1^4 (7),          (t2)^4  (8)
t2^4 (8),          (t1^-1)^4=-4 = t1^13 (25)
t1^5 (9),          (t2)^5  (10)
t2^5 (10),          (t1^-1)^5=-5= t1^12 (23)
t1^6 (11),          (t2)^6  (12)
```

```
t2^6 (12),          (t1^-1)^6=-6 = t1^11 (21)
t1^7 (13),          (t2)^7  (14)
t2^7 (14),          (t1^-1)^7=-7 =t1^10 (19)
t1^8 (15),          (t2)^8  (16)
t2^8 (16),          (t1^-1)^8=-8 = t1^9 (17)
t1^9 (17),          (t2)^9  (18)
t2^9 (18),          (t1^-1)^9=-9 = t1^8 (15)
t1^10 (19),          (t2)^10  (20)
t2^10 (20),          (t1^-1)^10=-10 =t1^7 (13)
t1^11 (21),          (t2)^11  (22)
t2^11 (22),          (t1^-1)^11=-11 = t1^6 (11)
t1^12 (23),          (t2)^12  (24)
t2^12 (24),          (t1^-1)^12=-12= t1^5 (9)
t1^13 (25),          (t2)^13 (26)
t2^13 (26),          (t1^-1)^13=-13= t1^4 (7)
t1^14 (27),          (t2)^14  (28)
t2^14 (28),          (t1^-1)^14=-14=t1^3 (5)
t1^15 (29),           (t2)^15 (30)
t2^15 (30),          (t1^-1)^15=-15=t1^2 (3)
t1^16 (31),           (t2)^16 (32)
t2^16 (32),          (t1^-1)^16=-16=t1 (1)
t1^17 (33),           (t2)^17= identity
t2^17 (34),          (t1^-1)^17 = idenitity
PERMUTATION:
(1,2,31,32)(3,4,29,30)(5,6,27,28)(7,8,25,26)(9,10,23,24)
   (11,12,21,22)(13,14,19,20)(15,16,17,18)


D MATRIX:
[ 0 -1]
[-1  0]
a12=-1 -> t2^-1
```

a21=–1 –> t1ˆ–1

t1 (1), t2ˆ(–1)= t2ˆ16 (32)
t2 (2), t1ˆ(–1)=t1ˆ16 (31)
t1ˆ2 (3), (t2ˆ–1)ˆ2=–2= t2ˆ15 (30)
t2ˆ2 (4), (t1ˆ–1)ˆ2=–2 = t1ˆ15 (29)
t1ˆ3 (5), (t2ˆ–1)ˆ3=–3= t2ˆ14 (28)
t2ˆ3 (6), (t1ˆ–1)ˆ3=–3 = t1ˆ14 (27)
t1ˆ4 (7), (t2ˆ–1)ˆ4=–4= t2ˆ13 (26)
t2ˆ4 (8), (t1ˆ–1)ˆ4=–4 = t1ˆ13 (25)
t1ˆ5 (9), (t2ˆ–1)ˆ5=–5= t2ˆ12 (24)
t2ˆ5 (10), (t1ˆ–1)ˆ5=–5= t1ˆ12 (23)
t1ˆ6 (11), (t2ˆ–1)ˆ6=–6= t2ˆ11 (22)
t2ˆ6 (12), (t1ˆ–1)ˆ6=–6 = t1ˆ11 (21)
t1ˆ7 (13), (t2ˆ–1)ˆ7=–7= t2ˆ10 (20)
t2ˆ7 (14), (t1ˆ–1)ˆ7=–7 =t1ˆ10 (19)
t1ˆ8 (15), (t2ˆ–1)ˆ8=–8= t2ˆ9 (18)
t2ˆ8 (16), (t1ˆ–1)ˆ8=–8 = t1ˆ9 (17)
t1ˆ9 (17), (t2ˆ–1)ˆ9=–9= t2ˆ8 (16)
t2ˆ9 (18), (t1ˆ–1)ˆ9=–9 = t1ˆ8 (15)
t1ˆ10 (19), (t2ˆ–1)ˆ10=–10= t2ˆ7 (14)
t2ˆ10 (20), (t1ˆ–1)ˆ10=–10 =t1ˆ7 (13)
t1ˆ11 (21), (t2ˆ–1)ˆ1=–11= t1ˆ6 (12)
t2ˆ11 (22), (t1ˆ–1)ˆ11=–11 = t1ˆ6 (11)
t1ˆ12 (23), (t2ˆ–1)ˆ12=–12= t1ˆ5 (10)
t2ˆ12 (24), (t1ˆ–1)ˆ12=–12= t1ˆ5 (9)
t1ˆ13 (25), (t2ˆ–1)ˆ13=–13= t2ˆ4 (8)
t2ˆ13 (26), (t1ˆ–1)ˆ13=–13= t1ˆ4 (7)
t1ˆ14 (27), (t2ˆ–1)ˆ14=–14= t2ˆ3 (6)
t2ˆ14 (28), (t1ˆ–1)ˆ14=–14=t1ˆ3 (5)
t1ˆ15 (29), (t2ˆ–1)ˆ15=–15= t2ˆ2 (4)
t2ˆ15 (30), (t1ˆ–1)ˆ15=–15=t1ˆ2 (3)

```
t1^16 (31),          (t2^-1)^16=-16= t2 (2)
t2^16 (32),         (t1^-1)^16=-16=t1 (1)
t1^17 (33),          (t2^-1)^17= identity
t2^17 (34),         (t1^-1)^17 = idenitity
PERMUTATION:
(1,32)(2,31)(3,30)(4,29)(5,28)(6,27)(7,26)(8,25)(9,24)(10,23)
   (11,22)(12,21)(13,20)(14,19)(15,18)(16,17)
*/
S:=Sym(32);
xx:=S!(1,15,25,3,31,17,7,29)(2,16,26,4,32,18,8,30)
   (5,13,9,11,27,19,23,21)(6,14,10,12,28,20,24,22);
yy:=S!(1,2,31,32)(3,4,29,30)(5,6,27,28)(7,8,25,26)(9,10,23,24)
   (11,12,21,22)(13,14,19,20)(15,16,17,18);
zz:=S!(1,32)(2,31)(3,30)(4,29)(5,28)(6,27)(7,26)(8,25)(9,24)
   (10,23)(11,22)(12,21)(13,20)(14,19)(15,18)(16,17);
N:=sub<S|xx,yy,zz>;
#N;
/*32*/
IsIsomorphic(sub<GG|GG!A,GG!B,GG!D>,N);
/*true Mapping from: MatrixGroup(2, C) of order 2^5 to GrpPerm:
   N
Composition of Mapping from: MatrixGroup(2, C) of order 2^5 to
   GrpPC and
Mapping from: GrpPC to GrpPC and
Mapping from: GrpPC to GrpPerm: N
*/
Orbits(Stabilizer(N,1));
/*
 GSet{@ 1 @},
    GSet{@ 3 @},
    GSet{@ 5 @},
    GSet{@ 7 @},
```

```
     GSet{@ 9 @},
     GSet{@ 11 @},
     GSet{@ 13 @},
     GSet{@ 15 @},
     GSet{@ 17 @},
     GSet{@ 19 @},
     GSet{@ 21 @},
     GSet{@ 23 @},
     GSet{@ 25 @},
     GSet{@ 27 @},
     GSet{@ 29 @},
     GSet{@ 31 @},
     GSet{@ 2, 32 @},
     GSet{@ 4, 30 @},
     GSet{@ 6, 28 @},
     GSet{@ 8, 26 @},
     GSet{@ 10, 24 @},
     GSet{@ 12, 22 @},
     GSet{@ 14, 20 @},
     GSet{@ 16, 18 @}*/
FPGroup(N);
/*
Finitely presented group on 3 generators
Relations
     b^4 ,
     c^2 ,
     (a, b) ,
     a^-1 * c * a * c ,
     (b^-1 * c)^2 ,
     a^-3 * b^2 * a^-1 ,
*/
NN<a,b,c>:=Group<a,b,c|
```

```
b^4 ,
     c^2 ,
     (a, b) ,
     a^-1 * c * a * c ,
     (b^-1 * c)^2 ,
     a^-3 * b^2 * a^-1 >;
Nt:= Stabiliser(N,{1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31});
Nt;
/*
Permutation group Nt acting on a set of cardinality 32
Order = 16 = 2^4
     (1, 31)(3, 29)(5, 27)(7, 25)(9, 23)(11, 21)(13, 19)(15, 17)
     (1, 29, 7, 17, 31, 3, 25, 15)(2, 4, 8, 16, 32, 30, 26, 18)
         (5, 21, 23, 19,
          27, 11, 9, 13)(6, 12, 24, 14, 28, 22, 10, 20)
*/
W:=WordGroup(N);
rho:=InverseWordMap(N);
A:=N!(1, 31)(3, 29)(5, 27)(7, 25)(9, 23)(11, 21)(13, 19)(15, 17)
    ;
B:=N!(1, 29, 7, 17, 31, 3, 25, 15)(2, 4, 8, 16, 32, 30, 26, 18)
    (5, 21, 23, 19,
          27, 11, 9, 13)(6, 12, 24, 14, 28, 22, 10, 20);
A@rho;
/*
function(W)
     w4 := W.2 * W.3; w5 := W.2^-2; w6 := w4 * w5; w8 := w6 * W
         .2; w9 := w8 *
     W.2; return w9;
end function
*/
wA:=function(W)
```

```
    w4 := W.2 * W.3; w5 := W.2^-2; w6 := w4 * w5; w8 := w6 * W
        .2; w9 := w8 *
    W.2; return w9;
end function;
B@rho;
/*function(W)
    w4 := W.2 * W.3; w5 := W.2^-2; w6 := w4 * w5; w1 := W.1^-1;
        w10 := w6 * w1;
    return w10;
end function*/
wB:=function(W)
    w4 := W.2 * W.3; w5 := W.2^-2; w6 := w4 * w5; w1 := W.1^-1;
        w10 := w6 * w1;
    return w10;
end function;
wA(NN);
/* y * z */
wB(NN);
/* y * z * y^-2 * x^-1 */
/*CHECK*/
1^(yy * zz);
/*31 = t1^16*/
1^(yy * zz * yy^-2 * xx^-1);
/*29 = t1^15*/

Nt:= Stabiliser(N,{1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31});
/*CHECKING TO FIND WHAT THESE PERMUTATIONS ARE (WE USE THIS
   INFORMATION IN THE PROGEITOR)
zz*yy=(1, 31)(3, 29)(5, 27)(7, 25)(9, 23)(11, 21)(13, 19)(15,
   17)
yy^5*xx^5*zz=(1, 15, 25, 3, 31, 17, 7, 29)(2, 18, 26, 30, 32,
   16, 8, 4)(5, 13, 9, 11, 27,
```

```
        19, 23, 21)(6, 20, 10, 22, 28, 14, 24, 12)
1^(zz*yy);
1
1^(yy^5*xx^5*zz);
15
1=t1, t1^8=15
*/
/*THINKING IT SHOULD BE: 17^2 *32*/
/*GRANDSTAFF LEMMA*/
G<x,y,z,t>:=Group<x,y,z,t|
y^4 ,
    z^2 ,
    (x, y) ,
    x^-1 * z * x * z ,
    (y^-1 * z)^2 ,
    x^-3 * y^2 * x^-1,
t^17,
(t^(z*y)=t),(t^(y^5*x^5*z)=t^8),
(t,t^z)>;
#G;
/*9248*/
Index(G,sub<G|x,y,z>);
/*289*/
#N;
/*32*/
17^2*32;
/*9248*/
/*PROGENITOR:*/
/*
G<x,y,z,t>:=Group<x,y,z,t|
y^4 ,
    z^2 ,
```

```
        (x,  y)   ,
        x^-1  *  z  *  x  *  z   ,
        (y^-1  *  z)^2   ,
        x^-3  *  y^2  *  x^-1,
t^17,
(t^(z*y)=t),(t^(y^5*x^5*z)=t^8),
*/
```

Appendix F

Magma Maximal DCE, S_6 Over S_5 and (5:4)

```
/* For DCE over N see CSUSB Thesis, February 2020, by Mayra
   McGrath*/
S:=Sym(5);
 xx:=S!(1, 2, 3, 4, 5);
yy:=S!(1, 2, 4, 3);
N:=sub<S|xx,yy>;
a1:=0; b1:=0; c1:=0; d1:=5; e1:=0; f1:=0; a2:=0;
 G<x,y,t>:=Group<x,y,t|y^4,x^-5,y^-1*x^-2*y*x^-1,t^2,
t^(y*x*y)=t,t^(y*x^-1)=t,(x*y^-1*t)^a1,
(x*y^-1*t^(x))^b1,(y*x^-1*t)^c1,(y*x^-1*t^(x))^d1,
(y*x*y*t^(x))^e1,(y*x*y*t)^f1,(x*t)^a2>;
#G;
/* 720 */
f,G1,k:=CosetAction(G,sub<G|x,y>);
CompositionFactors(G1);
/*     G
     |   Cyclic(2)
     *
     |   Alternating(6)
```

```
      1
*/
#sub<G|x,y>;
/* 20 */
#k;
/* 1 */
M:=MaximalSubgroups(G1);
 for i in [1..#M] do #M[i]`subgroup/20; end for;
/*
12/5
12/5
18/5
6
6
18
*/
 M4:=M[4]`subgroup;
 f(x) in M4 and f(y) in M4;
 /* false */
 D:=Conjugates(G1,M4);
DD:=Setseq(D);
for i in [1..#D] do if f(x) in DD[i] and f(y) in DD[i] then i;
    end if; end for;
/* 4 */
for g in  DD[4] do if DD[4] eq sub<G1|f(x),f(y),g> then
                              \
Sch:=SchreierSystem(G, sub<G|Id(G)>); for i in [1..#Sch] do if g
      eq f(Sch[i])
then Sch[i]; end if; end for; break g; end if; end for;
/* x * t * y * t * y^-1 * t * x^-1 */
H:=sub<G|x,y,x * t * y * t * y^-1 * t * x^-1>;
 #DoubleCosets(G,H,sub<G|x,y>);
```

```
/*2*/
#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);
/* 5 */



/*------------------------BEGIN HERE------------------------------------*/
S:=Sym(5);
 xx:=S!(1, 2, 3, 4, 5);
yy:=S!(1, 2, 4, 3);
N:=sub<S|xx,yy>;
 G<x,y,t>:=Group<x,y,t|y^4,x^-5,y^-1*x^-2*y*x^-1,t^2,
t^(y*x*y)=t,t^(y*x^-1)=t,(y*x^-1*t^(x))^5>;
#G;
/*720*/
H:=sub<G|x,y,x * t * y * t * y^-1 * t * x^-1>;
f,G1,k:=CosetAction(G,H);
IH:=sub<G1|f(x),f(y),f(x * t * y * t * y^-1 * t * x^-1)>;
Index(G,H);
/*6*/
IN:=sub<G1|f(x),f(y)>;
ts:=[ Id(G1): i in [1 .. 5] ];
ts[1]:=f(t);
ts[2]:=f(t^(y));
ts[3]:=f(t^( x * y^2));
ts[4]:=f(t^( x * y));
ts[5]:=f(t^( x^-1 * y^-1));
DC:=[f(Id(G)), f(t)];

cst:=[null : i in [1..6]] where null is
[Integers() |];
prodim := function(pt, Q, I)
```

```
v:= pt;
for i in I do
v:=v^(Q[i]);
end for;
return v;
end function;
for i :=1 to 5 do
cst[prodim(1,ts,[i])]:=[i];
end for;
m:=0;
for i in [1..6] do if cst[i] ne [] then m:=m+1;
end if; end for;m;
/*5*/
Orbits(N);
/*GSet{@ 1, 2, 3, 4, 5 @}*/
Generators(N);
/*(1, 2, 3, 4, 5),
    (1, 2, 4, 3)*/
for i in [1..5] do
for g in IH do
for h in IN do
if ts[1] eq g*(DC[i])^h then i;
break;break;end if;end for;end for;end for;
/*2*/

/*THE BEGINNING OF DOUBLE COSET [1] LABELLED 2*/
S:={[1]};
SS:=S^N;SS;
SSS:=Setseq(SS);
for i in [1..#SSS] do
for g in IH do if ts[1]
eq g*ts[Rep(SSS[i])[1]]
```

```
 then print SSS[i];
end if; end for; end for;
/*[ 1 ]*/
N1:=Stabiliser(N,1);
Generators(N1);
/*2, 4, 5, 3)*/
N1:=Stabiliser(N,[1]);
#N1;
/*4*/
N1s:=N1;
#N1s;
/*4*/
Generators(N1s);
/*(2, 5)(3, 4),
    (2, 4, 5, 3)*/
Orbits(N1);
/*GSet{@ 1 @},
    GSet{@ 2, 5, 4, 3 @}*/
for i in [1..5] do
for g in IH do
for h in IN do
if ts[1]*ts[1] eq g*(DC[i])^h then i;
break;break;end if;end for;end for;end for;
/*1*/

for i in [1..5] do
for g in IH do
for h in IN do
if ts[1]*ts[2] eq g*(DC[i])^h then i;
break;break;end if;end for;end for;end for;
/*2*/
```

```
/*————————————————BEGINNIGN OF SCHRIER SYSTEM
    ——————————————*/
L<u,v>:=Group<u,v|v^4,u^-5,v^-1*u^-2*v*u^-1>;
Sch:=SchreierSystem(L,sub<L|Id(L)>);
h:=hom<L->N|u->xx,v->yy>;
g:=hom<IN->N|f(x)->xx,f(y)->yy>;


/*ts[1]*ts[2] belongs to [1]*/
for m in IH do for n in IN do if ts[1]*ts[2] eq m*(ts[1])^n then
      D:=m; E:=n;end if; end for; end for;
for i in [1..#Sch] do if g(E) eq h(Sch[i]) then Sch[i]; end if;
    end for;
/*(x * y^-1) = (2, 4, 5, 3)*/
Sch1:=SchreierSystem(G,sub<G|Id(G)>); for i in [1..#Sch1] do if
    D eq f(Sch1[i]) then Sch1[i]; end if; end for;
/*t * x^-1 * t * x * t*/

/*SOLVING CANNONICAL RELATION*/
ts[1]*ts[2] eq f(t * x^-1 * t * x * t)*ts[1];
/*true*/
for n in IN do for i,j,k in [1..5] do if n*ts[i]*ts[j]*ts[k] eq
    f(t * x^-1 * t * x * t) then i,j,k; A:=n; break; break; end
    if; end for;end for;
/*1 2 1
2 1 2
3 5 3
5 3 5
*/
for n in IN do if f(t * x^-1 * t * x * t) eq
n*ts[1]*ts[2]*ts[1] then B:=n; end if; end for;
Order(B);
```

```
/*1*/
for i in [1..#Sch] do if g(B) eq h(Sch[i]) then Sch[i]; end if;
   end for;
/*Id(L)*/
f(t * x^-1 * t * x * t) eq ts[1]*ts[2]*ts[1];
/*true*/
/*THUS*/
ts[1]*ts[2] eq (ts[1]*ts[2]*ts[1])*ts[1];
/*true*/
 Set(N);
{
    (1, 3)(4, 5),
    (1, 4, 5, 2),
    (1, 3, 2, 5),
    (1, 4, 3, 5),
    (1, 2, 4, 3),
    (1, 3, 4, 2),
    (1, 3, 5, 2, 4),
    (1, 2, 5, 4),
    (1, 5, 2, 3),
    (1, 5, 4, 3, 2),
    (1, 5)(2, 4),
    (1, 4, 2, 5, 3),
    (1, 4)(2, 3),
    (1, 5, 3, 4),
    (1, 2, 3, 4, 5),
    (2, 4, 5, 3),
    Id(N),
    (1, 2)(3, 5),
    (2, 5)(3, 4),
    (2, 3, 5, 4)
}
```

```
> ts[1]*ts[2] eq ts[1];
false
> ts[1]*ts[2]*ts[1] in IH;
true




/*SOLVING IH RELATION: (x * t * y * t * y^-1 * t * x^-1) */

for n in IN do for i,j,k in [1..5] do if n*ts[i]*ts[j]*ts[k] eq
   f(x * t * y * t * y^-1 * t * x^-1) then i,j,k; A:=n; break;
   break; end if; end for;end for;
/*2 5 2
5 2 5
3 4 3
4 3 4*/
for n in IN do if f(x * t * y * t * y^-1 * t * x^-1) eq
n*ts[2]*ts[5]*ts[2] then B:=n; end if; end for;
Order(B);
/*1*/

f(x * t * y * t * y^-1 * t * x^-1) eq ts[2]*ts[5]*ts[2];
/*true*/


/*ISOMORPHISM TYPE OF N*/
NL:=NormalLattice(N);
NL;
/*Normal subgroup lattice
----------------------------
```

```
[4]   Order 20   Length 1   Maximal Subgroups: 3
---
[3]   Order 10   Length 1   Maximal Subgroups: 2
---
[2]   Order 5    Length 1   Maximal Subgroups: 1
---
[1]   Order 1    Length 1   Maximal Subgroups:
*/
for i in [1..\#NL] do if IsAbelian(NL[i]) then i; end if;end for
    ;
/*2*/
NL[2];
/*Permutation group acting on a set of cardinality 5
Order = 5
     (1, 2, 3, 4, 5)
*/
A:=N!(1, 2, 3, 4, 5);
IsIsomorphic(NL[2],AbelianGroup(GrpPerm,[5]));
/*true*/
q,ff:=quo<N|NL[2]>;
T:=Transversal(N,NL[2]);
q;
/*Permutation group q acting on a set of cardinality 4
Order = 4 = 2^2
     Id(q)
     (1, 2, 3, 4)*/
FPGroup(q);
/*
     c^4 ,
     b ,
*/
ff(T[2]) eq q.2;
```

```
/*true*/
T[2];
/*(1, 2, 4, 3)*/
T2:=N!(1, 2, 4, 3);

for i in [0..5] do if
 A^T[2] eq A^i then i; end if; end for;
/*2*/

G<a,b,c>:=Group<a,b,c|a^5,
c^4 ,
     b , a^c=a^2>;
#G;
/*20*/
f,G1,k:=CosetAction(G,sub<G|Id(G)>);
IsIsomorphic(G1,N);
/*true*/
/*Semi-direct product: (5:4)


/*------------------------FOR G------------------------*/
NL:=NormalLattice(G1);
NL;
/*
Normal subgroup lattice
-----------------------


[3]  Order 720  Length 1  Maximal Subgroups: 2
---
[2]  Order 360  Length 1  Maximal Subgroups: 1
---
[1]  Order 1    Length 1  Maximal Subgroups:
```

```
*/
for i in [1..\#NL] do if IsAbelian(NL[i]) then i; end if;end for
    ;
/*1*/
IsSimple(G1);
/*false*/
NL[1];
/*Permutation group acting on a set of cardinality 6
Order = 1
    Id($)*/
CompositionFactors(G1);
/*    G
    |  Cyclic(2)
    *
    |  Alternating(6)
    1
*/
IsIsomorphic(G1, Sym(6));
/*true Isomorphism of GrpPerm: G1, Degree 6, Order 2^4 * 3^2 * 5
    into GrpPerm: $, Degree 6,
Order 2^4 * 3^2 * 5 induced by
    (2, 3, 5, 6, 4) |--> (2, 3, 5, 6, 4)
    (2, 3, 6, 5) |--> (2, 3, 6, 5)
    (1, 2) |--> (1, 2)
*/
q,ff:=quo<G1|NL[1]>;
T:=Transversal(G1,NL[1]);
q;
/*Permutation group q acting on a set of cardinality 10
Order = 720 = 2^4 * 3^2 * 5
    (1, 2, 4, 6, 10)(3, 7, 5, 9, 8)
    (1, 3, 4, 8)(2, 5)(6, 7, 10, 9)
```

```
    (2, 6)(3, 7)(4, 9)*/

 IH;
Permutation group IH acting on a set of cardinality 6
    (2, 3, 5, 6, 4)
    (2, 3, 6, 5)
    (3, 4)
> NL:=NormalLattice(IH);
> NL;

Normal subgroup lattice
---------------------------


[3]  Order 120  Length 1  Maximal Subgroups: 2
---
[2]  Order 60   Length 1  Maximal Subgroups: 1
---
[1]  Order 1    Length 1  Maximal Subgroups:

CompositionFactors(IH);
    G
    |  Cyclic(2)
    *
    |  Alternating(5)
    1
> IsIsomorphic(IH,Sym(5));
true Homomorphism of GrpPerm: IH, Degree 6, Order 2^3 * 3 * 5
   into GrpPerm: S, Degree 5,
Order 2^3 * 3 * 5 induced by
    (2, 3, 5, 6, 4) |--> (1, 3, 4, 2, 5)
    (2, 3, 6, 5) |--> (2, 3, 4, 5)
    (3, 4) |--> (1, 4)
```

```
/*DCE S_6 over S_5 and 5:4*/

2^{*5}:(5:4) = Sym(6)
```

Appendix G

Magma Symmetric Presentation, 2^{*12}:$(2^3$:$2^2)$

```
N:=TransitiveGroup(12,21);
#N;
/*48*/
for g,h in N do if sub<N|g,h> eq N then A:=g; B:=h; end if; end
    for;
A;
/* (1, 8)(2, 7)(3, 6)(4, 5)(9, 12)(10, 11)*/
B;
/* (1, 6, 7, 12)(2, 11)(3, 10, 9, 4)(5, 8) */

N12:=Stabiliser(N,[1,2]);
N12;
/*Permutation group N12 acting on a set of cardinality 12
Order = 2
     (3, 9)(6, 12)
*/
FPGroup(N);
/*     x^2 ,
     y^3 ,
```

```
    $.3^2 ,
    (y^-1 * $.3)^2 ,
    (x * $.3)^2 ,
    (y * x * y^-1 * x)^2 ,
*/
NN<a,b,c>:=Group<a,b,c|     a^2 ,
    b^3 ,
    c^2 ,
    (b^-1 * c)^2 ,
    (a * c)^2 ,
    (b * a * b^-1 * a)^2 >;
CompositionFactors(N);
/*G
    |  Cyclic(2)
    *
    |  Cyclic(3)
    *
    |  Cyclic(2)
    *
    |  Cyclic(2)
    *
    |  Cyclic(2)
    1
*/
NL:=NormalLattice(N);
/*Normal subgroup lattice
-----------------------------


[9]  Order 48  Length 1  Maximal Subgroups: 6 7 8
---
[8]  Order 24  Length 1  Maximal Subgroups: 5
[7]  Order 24  Length 1  Maximal Subgroups: 5
```

```
[6]  Order 24  Length 1  Maximal Subgroups: 4 5
---
[5]  Order 12  Length 1  Maximal Subgroups: 3
[4]  Order 8   Length 1  Maximal Subgroups: 2 3
---
[3]  Order 4   Length 1  Maximal Subgroups: 1
---
[2]  Order 2   Length 1  Maximal Subgroups: 1
---
[1]  Order 1   Length 1  Maximal Subgroups:
*/
IsIsomorphic(N,DirectProduct(NL[7],NL[2]));
/*true Mapping from: GrpPerm: N to GrpPerm: $, Degree 24, Order
   2^4 * 3
Composition of Mapping from: GrpPerm: N to GrpPC and
Mapping from: GrpPC to GrpPC and
Mapping from: GrpPC to GrpPerm: $, Degree 24, Order 2^4 * 3
*/
/*OUR PROGENITOR IS 2^(*12):()*/
NN<x,y,z>:=Group<x,y,z|    x^2 ,
     y^3 ,
     z^2 ,
     (y^-1 * z)^2 ,
     (x * z)^2 ,
     (y * x * y^-1 * x)^2 >;
#NN;
/*48*/
G<x,y,z>:=Group<x,y,z|    x^2 ,
     y^3 ,
     z^2 ,
     (y^-1 * z)^2 ,
     (x * z)^2 ,
```

```
     (y * x * y^-1 * x)^2,
t^2,
(t,N) >;
N1:=Stabiliser(N,1);
N1;
/*
Permutation group N1 acting on a set of cardinality 12
Order = 4 = 2^2
     (3, 9)(6, 12)
     (2, 8)(5, 11)
*/
N;
/*Permutation group N acting on a set of cardinality 12
Order = 48 = 2^4 * 3
     (3, 9)(6, 12)
     (1, 5, 9)(2, 6, 10)(3, 7, 11)(4, 8, 12)
     (1, 2)(3, 12)(4, 11)(5, 10)(6, 9)(7, 8)
*/
S:=Sym(12);
xx:=S!(1, 8)(2, 7)(3, 6)(4, 5)(9, 12)(10, 11);
yy:=S!(1, 6, 7, 12)(2, 11)(3, 10, 9, 4)(5, 8);
N:=sub<S|xx,yy>;
FPGroup(N);
/*    a^2 ,
     b^4 ,
     b^-1 * a * b^-2 * a * b^-1 * a * b^2 * a ,
     b^-1 * a * b^-1 * a * b^-1 * a * b * a * b * a * b * a ,*/
NN<a,b>:=Group<a,b|
     a^2 ,
     b^4 ,
     b^-1 * a * b^-2 * a * b^-1 * a * b^2 * a ,
     b^-1 * a * b^-1 * a * b^-1 * a * b * a * b * a * b * a >;
```

```
N1:=Stabiliser(N,1);
N1;
N1;
/*Permutation group N1 acting on a set of cardinality 12
Order = 4 = 2^2
    (2, 8)(3, 9)(5, 11)(6, 12)
    (3, 9)(6, 12)
*/

word:=function(A)
Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
for i in [2..#NN] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=xx; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=xx^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=yy; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=yy^-1; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
if A eq PP then B:=Sch[i]; end if;
end for;
return B;
end function;


word(N!(3, 9)(6, 12));
/* (x * y * x * y^-1 * x * y) */
word(N!(2, 8)(3, 9)(5, 11)(6, 12));
/* (x * y^2 * x) */
```

```
Orbits(N1);
/*
GSet{@ 1 @},
     GSet{@ 4 @},
     GSet{@ 7 @},
     GSet{@ 10 @},
     GSet{@ 2, 8 @},
     GSet{@ 3, 9 @},
     GSet{@ 5, 11 @},
     GSet{@ 6, 12 @}
*/
1^(xx*yy*xx*yy);
/*3*/
1^(xx*yy*xx);
/*4*/
1^(xx*yy);
/*5*/
1^(yy);
/*6*/
1^(yy^2);
/*7*/
1^(xx);
/*8*/
1^(xx*yy*xx*yy^2);
/*10*/
/*
(t,t^(x*y*x*y)),
(t,t^(x*y*x)),
(t,t^(x*y)),
(t,t^(y)),
(t,t^(y^2)),
```

```
(t,t^(x)),
(t,t^(x*y*x*y^2))*/

G<x,y,t>:=Group<x,y,t|     x^2  ,
    y^4  ,
    y^-1 * x * y^-2 * x * y^-1 * x * y^2 * x  ,
    y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y * x  ,
t^2,(t,(x * y * x * y^-1 * x * y)),(t,(x * y^2 * x)),
(t,t^(x*y*x*y)),
(t,t^(x*y*x)),
(t,t^(x*y)),
(t,t^(y)),
(t,t^(y^2)),
(t,t^(x)),
(t,t^(x*y*x*y^2))>;
Index(G,sub<G|x,y>);
/*4096*/
#G;
/*196608*/

2^(12)*48;
/*196608*/

/*TRYING TO APPLY LEMMA 3.3*/

N12:=Stabiliser(N,[1,2]);
N12;
/*Permutation group N12 acting on a set of cardinality 12
Order = 2
    (3, 9)(6, 12)
*/
C:=Centraliser(N,N12);
```

```
/* (2, 8)(5, 11)
     (1, 7)(4, 10)
     (3, 9)(6, 12)
     (1, 2)(3, 6)(4, 11)(5, 10)(7, 8)(9, 12)
*/
Set(C);
/* (1, 8, 7, 2)(3, 12)(4, 5, 10, 11)(6, 9),
     (1, 7)(2, 8)(4, 10)(5, 11),
     (1, 8)(2, 7)(3, 12)(4, 5)(6, 9)(10, 11),
     (3, 9)(6, 12),
     (1, 2)(3, 6)(4, 11)(5, 10)(7, 8)(9, 12),
     (1, 7)(3, 9)(4, 10)(6, 12),
     (2, 8)(3, 9)(5, 11)(6, 12),
     (1, 2, 7, 8)(3, 6)(4, 11, 10, 5)(9, 12),
     (1, 7)(2, 8)(3, 9)(4, 10)(5, 11)(6, 12),
     (1, 2)(3, 12)(4, 11)(5, 10)(6, 9)(7, 8),
     Id(C),
     (1, 2, 7, 8)(3, 12)(4, 11, 10, 5)(6, 9),
     (2, 8)(5, 11),
     (1, 8, 7, 2)(3, 6)(4, 5, 10, 11)(9, 12),
     (1, 7)(4, 10),
     (1, 8)(2, 7)(3, 6)(4, 5)(9, 12)(10, 11)
*/
/*ONLY ONES APPLICABLE (t_1t_2)^k*/
(3, 9)(6, 12) = (x * y * x * y^-1 * x * y)
(1, 2)(3, 6)(4, 11)(5, 10)(7, 8)(9, 12) = (y^2 * x * y^2)
(1, 2)(3, 12)(4, 11)(5, 10)(6, 9)(7, 8) = (y * x * y * x * y)

/*CHECKING IF EACH RELATION WORKS*/
/*CASE: (x * y * x * y^-1 * x * y) EVEN */
for k in [2..10 by 2] do
G<x,y,t>:=Group<x,y,t|     x^2 ,
```

```
    y^4 ,
    y^-1 * x * y^-2 * x * y^-1 * x * y^2 * x ,
    y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y * x ,
t^2,(t,(x * y * x * y^-1 * x * y)),(t,(x * y^2 * x)),
(t*t^(x*y*x*y^-2*x*y))^k=(x * y * x * y^-1 * x * y)>;
k, #G; end for;
/*3 48
5 48
7 48
9 48
*/


/*CASE: (y^2 * x * y^2) ODD*/
for k in [3..9 by 2] do
G<x,y,t>:=Group<x,y,t|    x^2 ,
    y^4 ,
    y^-1 * x * y^-2 * x * y^-1 * x * y^2 * x ,
    y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y * x ,
t^2,(t,(x * y^2 * x)), (t,(y^2 * x * y^2)),
((y^2 * x * y^2)*t)^k>;
k, #G; end for;
/*3 12
5 12
7 12
9 12
*/


/*CASE: (y * x * y * x * y) ODD*/
for k in [3..9 by 2] do
G<x,y,t>:=Group<x,y,t|    x^2 ,
```

```
    y^4  ,
    y^-1 * x * y^-2 * x * y^-1 * x * y^2 * x  ,
    y^-1 * x * y^-1 * x * y^-1 * x * y * x * y * x * y * x  ,
t^2,(t,(x * y * x * y^-1 * x * y)),(t,(x * y^2 * x)), (t,(y * x
   * y * x * y)),
((y * x * y * x * y)*t)^k>;
k, #G; end for;
/*3 12
5 12
7 12
9 12
*/
```

-

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