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Non-Abelian Finite Simple Groups as Homomorphic Images

Sandra Bahena

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A Thesis

Presented to the

Faculty of

California State University,

San Bernardino

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

in

Mathematics

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Abstract

The purpose of exploring infinite groups in this thesis was to produce non-abelian finite simple groups as homomorphic images. These infinite groups are semi-direct products known as progenitors. The permutation progenitors studied were: $2^{*8} : 2^2 {}^{\cdot} A_4$, $2^{*10} : D_{20}$, $2^{*4} : C_4$, $2^{*7} : (7 : 6)$, $3^{*3} : S_3$, $2^{*15} : ((5 \times 3) : 2)$, and $2^{*20} : A_5$. When we factored said progenitors by an appropriate number of relations, we produced several original symmetric presentations and constructions of linear groups, other classical groups and sporadic groups. We have found original symmetric presentations of several important groups, including: $PGL_2(7), PSL_2(8), PSL_2(11), PGL_2(11), PGL_2(13)$, $PSL_2(19)$, $PGL_2(29), PSL_2(41), PSL_2(71), J_2, U(3,4), U(3,5), M_{11}$, and M_{22}.
When solving various extension problems, we are able to identify the isomorphism types of the finite images we discovered. We present proofs of the four types of extension problems: Direct Products, Semi-Direct Products, Central Extensions, and Mixed Extensions. We perform manual double coset enumeration with the support of a computer-based program, Magma, to construct Cayley diagrams of the finite groups: $3^2 : S_3$, M_{11}, $PSL_2(19)$, $PSL_2(7)$, S_4, and $U_3(5) : 2$.

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Table of Contents

List of Tables

List of Figures

Introduction

Progenitors factored by one or more relations frequently give non-abelian simple groups and even sporadic groups as their homomorphic images. The main goal of this thesis was to obtain original symmetric presentations and constructions of linear groups, other classical groups and sporadic groups by factoring permutation progenitors. Let G be a group and $T = \{t_o, t_1, ..., t_{n-1}\}$ where $T_i =< t_i >$, with $i = 0, 1, ..., n-1$, is the cyclic subgroup generated by t_i and let N be a subgroup of S_n that acts transitively and faithfully on T called the **control subgroup**.

In the semi-direct product, $2^{*n} : N = \{\pi w | \pi \in N, w$ is a reduced word in $t_i\}$, N acts by conjugation as permutations of the n involutary symmetric generators. Every element of the progenitor can be represented as a word of πw. We want to factor the progenitor by relations of the form $\pi w(t_1, ..., t_n)$, giving us a finite homomorphic image of the infinite progenitor. We will then perform double coset enumeration of some of these finite groups to find the double cosets and determine the number of single cosets each contains. We will use a Cayley Diagram to demonstrate a graphical representation of this process.

Our goal is to factor $2^{*n} : N$ by relations, that equate elements of N to the product of t_is, resulting in finite homomorphic images. Once we find all of the relations, we will perform double coset enumeration of G over N. Hence, we will find all of the double cosets and find the total number of single cosets each double coset contains. We will have completed the double coset enumeration when the set of right cosets obtained is closed under the operation of right multiplication. Thus, we will find the order of G once we find all relations, perform double coset enumeration of G over N, create the Cayley Graph and obtain the index of N in G.

Chapter 1

Preliminaries

1.1 Related Theorems and Definitions

Definition 1.1. ***Permutation***

If X is a nonempty set, a **permutation** *is a bijection mapping $\alpha : X \to X$. (Rot95)*

Definition 1.2. ***Fixes***

If $x \in X$ and $\alpha \in S_X$, then α **fixes** *x if $\alpha(x) = x$ and α* **moves** *x if $\alpha(x) \neq x$. (Rot95)*

Definition 1.3. ***Disjoint***

Two permutations $\alpha, \beta \in S_X$ are **disjoint** *if every x moved by one is fixed by the other. In symbols, if $\alpha(x) \neq x$, then $\beta(x) = x$ and if $\beta(y) \neq y$, then $\alpha(y) = y$. (Rot95)*

Theorem 1.4. *Every permutation $\alpha \in S_n$ is either a cycle or a product of disjoint cycles. (Rot95)*

Definition 1.5. ***Operation***

A (binary) **operation** *on a nonempty set G is a function $\mu : G \times G \to G$. (Rot95)*

Definition 1.6. ***Semigroup***

A **semigroup** $(G, *)$ *is a nonempty set G equipped with an associative operation $*$. (Rot95)*

Definition 1.7. ***Group***
A **group** *is a semigroup G containing an element e such that*

(i) $e * a = a = a * e$ *for all* $a \in G$

(ii) for every $a \in G$*, there is an element* $b \in G$ *with* $a * b = e = b * a$*. (Rot95)*

Definition 1.8. ***Commutes***
A pair of elements a and b in a semigroup **commutes** *if* $a * b = b * a$*. A group (or a semigroup) is* **abelian** *if every pair of its elements commutes. (Rot95)*

Theorem 1.9. ***Identity***
If G is a group, there is a unique element e with $e*a = a = a*e$ for all $a \in G$. Moreover, for each $a \in G$, there is a unique $b \in G$ with $a * b = e = b * a$. We call e the **identity** *if G and, if $a*b = e = b*a$, then we call b the* **inverse** *of a and denote it by a^{-1}. (Rot95)*

Corollary 1.10. ***Inverse of an Inverse***
If G is a group and $a \in G$, then $(a^{-1})^{-1} = a$. (Rot95)

Definition 1.11. ***Homomorphism***
Let $(G, *)$ and $(H, \circ)$ be groups. A function $f : G \to H$ is a **homomorphism** *if, for all $a, b \in G, f(a * b) = f(a) \circ f(b)$. (Rot95)*

Definition 1.12. ***Isomorphism***
An **isomorphism** *is a homomorphism that is also a bijection. We say that G is* **isomorphic** *to H, denoted $G \cong H$, if there exists an isomorphism $f : G \to H$. (Rot95)*

Theorem 1.13. *Let $f : (G, *) \to (G', \circ)$ be a homomorphism.*

(i) $f(e) = e'$*, where e' is the identity in G'*

(ii) If $a \in G$, then $f(a^{-1}) = f(a)^{-1}$.

(iii) If $a \in G$ and $n \in \mathbb{Z}$, then $f(a^n) = f(a)^n$.

(Rot95)

Definition 1.14. ***Subgroup***
A nonempty subset S of a group G is a **subgroup** *of G if $s \in G$ implies $s^{-1} \in G$ and $s, t \in G$ imply $st \in G$. (Rot95)*

Theorem 1.15. *If $S \leq G$ (i.e., if S is a subgroup of G), then S is a group in its own right. (Rot95)*

Theorem 1.16. *Subset*
A subset S of a group G is a subgroup if and only if $1 \in S$ and $s, t \in S$ imply $st^{-1} \in S$. (Rot95)

Definition 1.17. *Cyclic Subgroup*
If G is a group and $a \in G$, then the **cyclic subgroup generated by** *a, denoted by $< a >$, is the set of all the powers of a. A group G is called* **cyclic** *if there is $a \in G$ with $G =< a >$; that is G consists of all the powers of a. (Notice different elements can generate the same cyclic group.) (Rot95)*

Definition 1.18. *Order*
If G is a group and $a \in G$, then the **order** *of a is $| < a > |$, the number of elements in $< a >$. If G is a group, then the* **order** *of G, denoted by $|G|$, is the number of elements in G. (Rot95)*

Definition 1.19. *Involutions*
Elements of order 2 are known as **involutions***. (Rot95)*

Definition 1.20. *Kernel, Image*
Let $f : G \to H$ be a homomorphism and define **kernel *f*** $= \{a \in G : f(a) = 1\}$ *and* **image *f*** $= \{h \in H : h = f(a)$ *for some* $a \in G\}$. *Then $K = kernel f = ker\ f$ is a subgroup of G, $K \leq G$ and image $f = im\ f$ is a subgroup of H, $imf \leq H$. (Rot95)*

Definition 1.21. *Generates*
If X is a subset of a group G, then the smallest subgroup of G containing X, denoted by $< X >$, is called the ***subgroup* generated *by*** *X. One also that X generates $< X >$. (Rot95)*

Theorem 1.22. *Let X be a subset of a group G. If $X = \emptyset$, then $< X > = 1$; if X is nonempty, then $< X >$ is the set of all the words on X. (Rot95)*

Definition 1.23. *Index*
If $S \leq G$, then the **index** *of S in G, denoted by $[G : S]$, is the number of right cosets of S in G. (Rot95)*

Definition 1.24. ***Right Coset***

If S is a subgroup of G and if $t \in G$, then a **right coset** *of S in G is the subset of G*

$$St = \{st : s \in S\}$$

(a ***left coset*** *is $tS = \{tS : s \in S\}$). We understand t is a* ***representative*** *of S_t (and also of tS). (Rot95)*

Theorem 1.25. ***Lagrange***

If G is a finite group and $S \leq G$, then $|S|$ divides $|G|$ and $[G : S] = \frac{|G|}{|S|}$. (Rot95)

Corollary 1.26. *If G is a finite group and $a \in G$. Then the order of a divides $|G|$. (Rot95)*

Corollary 1.27. *If p is a prime and $|G| = p$, then G is a cyclic group. (Rot95)*

Definition 1.28. ***Normal Subgroup***

A subgroup $K \leq G$ is a **normal subgroup***, denoted by $K \lhd G$, if $gKg^{-1} = K$ for every $g \in G$. (Rot95)*

Definition 1.29. ***Maximal Normal Subgroup***

A subgroup $H \leq G$ is a **maximal normal subgroup** *of G if there is no normal subgroup N of G with $H < N < G$. (Rot95)*

Definition 1.30. ***Simple***

A group $G \neq 1$ is **simple** *if it has no normal subgroups other than G and 1. (Rot95)*

Theorem 1.31. *If $N \lhd G$, then the cosets of N in G form a group, denoted by G/N, of order $[G : N]$. (Rot95)*

Theorem 1.32. ***First Isomorphism Theorem***

Let $f : G \to H$ be a homomorphism with kernal K. Then K is a normal subgroup of G and $G/K \cong \text{im } f$. (Rot95)

Theorem 1.33. ***Second Isomorphism Theorem***

Let N and T be subgroups of G with N normal. Then $N \cap T$ is normal in T and $\frac{T}{N \cap T} \cong \frac{NT}{N}$. (Rot95)

Theorem 1.34. ***Third Isomorphism Theorem***

Let $K \leq H \leq G$, where both K and H are normal subgroups of G. Then H/K is a normal subgroup of G/K and $(G/K)/(H/K) \cong G/H$. (Rot95)

Chapter 2

Writing a Progenitor Presentation

2.1 Related Theorems and Definitions

Definition 2.1. ***Progenitor***

A **progenitor** *is a semi-direct product of the following form:*

$$P \cong 2^{*n} : N = \{\pi w \mid \pi \in N,\ w \text{ is a reduced word in the } t_i\},$$

where 2^{*n} denotes a free product of n copies of a cyclic group of order 2 generated by involutions t_i for $i = 1, ..., n$; and N is a transitive permutation group of degree n which acts on the free product by permuting the involutory generators. (Cur07)

Definition 2.2. ***Free Group***

If X is a subset of a group F, then F is a **free group** *with* ***basis*** *X if, for every group G and every function $f : X \to G$, there exists a unique homomorphism $\varphi : F \to G$ extending f. (Rot95)*

Definition 2.3. ***Presentation***

Let X be a set and let Δ be a family of words on X. A group G has ***generators*** *X and* ***relations*** *Δ if $G \cong F/R$, where F is the free group with basis X and R is the normal subgroup of F generated by Δ. The ordered pair $(X|\Delta)$ is called a* **presentation** *of G. (Rot95)*

Definition 2.4. ***Conjugate***

If $H \leq G$ and $g \in G$, then the **conjugate** *gHg^{-1}, denoted as H^g, is $\{ghg^{-1} : h \in H\}$. (Rot95)*

Definition 2.5. ***Conjugation***
If a is a fixed element of a group G, define $\gamma_a : G \to G$ *by* $\gamma_a(x) = a * x * a^{-1}$ *(*γ_a *is called* **conjugation** *by a). (Rot95)*

Definition 2.6. ***Centralizer***
If G is a finite group and $a \in G$*, then the* **centralizer** *of* $a \in G$*, denoted by* $C_G(a)$ *is the set of all* $x \in G$ *which commute with a. (Rot95)*

Definition 2.7. ***Normaliser***
If G is a finite group and $H \leq G$*, then the* **normaliser** *of* $H \in G$*, denoted by* $N_G(H)$ *is*

$$N_G(H) = \{a \in G : aHa^{-1} = H\}.$$

(Rot95)

Definition 2.8. ***Point Stabiliser***
Let N be a group. The **point stabiliser** *of w in N is given by:*

$$N^w = \{n \in N | w^n = w\}, \text{ where } w \text{ is a word in the } t_i s$$

(Rot95)

Definition 2.9. ***Conjugacy Class***
If G is a group, then the equivalence class of $a \in G$ *under the relation "y is a conjugate of x in G" is called the* **conjugacy class** *of a; it is denoted and defined by*
$a^G = \{a^g | g \in G\} = \{g^{-1}ag | g \in G\}$*. (Rot95)*

Definition 2.10. ***Word***
If X is a nonempty subset of a group G, then a **word** *on X is an element* $w \in G$ *of the form* $w = {x_1}^{e_1}{x_2}^{e_2}...{x_n}^{e_n}$*, where* $x_i \in X, e_i = \pm 1$ *and* $n \geq 1$*. (Rot95)*

2.2 Permutation Progenitor

In this section we will discuss the technique we used to write presentations for permutation progenitors. We wish to write a presentation for permutation progenitors of the form $2^{*n} : N$. We first choose a control group transitive on n letters, denoted as N. Once we have a presentation for our control group N, we introduce a symmetric generator typically labeled as t, where t is a generator of a free product group. Throughout this thesis we will label our t as t_1, with the exception of one example. This example will clearly state the labeling for t. In order to give our t the name t_1, we must find the generator of our point stabilizing group, hence N^1. Allowing our t to commute with such a generator ensures our t is t_1. In general, the progenitors $\mathbf{(2)}^{*n} : N$ will take on the following form:

$$< \textit{generators of } N, t \mid \textit{presentation of } N, t^{\mathbf{2}}, (t, N^1) > .$$

In the following example, we will be demonstrating the process of writing a presentation for a permutation progenitor.

Example 2.2.1. ***Writing a Presentation for the Permutation Progenitor,*** $2^{*10} : D_{20}$

We wish to write a presentation for a progenitor of the form $2^{*10} : N$. The 2 in the free product 2^{*10} represents the order of our t_is. The 10 represents the amount of t_is we have of order 2. Then control group N must be transitive on 10 letters. With the help of Magma's database of stored groups, we find our desired transitive group, D_{20}. We will be using Magma to assist with finding the presentation for this control group N.

As stated previously we must include a presentation of the control group in our progenitor presentation. Since we have 10 t_is, it's convenient to select the Symmetric Group 10, a set of size 10, to begin this process. We let our control group be a subgroup of S_{10} generated by the permutations Magma provided us with: a, b and c. We store this information into Magma as follows.

```
S:=Sym(10);
A:=S!(1, 3, 9, 7, 8)(2, 6, 4, 10, 5);
B:=S!(1, 2)(3, 5)(4, 7)(6, 8)(9, 10);
C:=S!(1, 4)(2, 7)(3, 10)(5, 9)(6, 8);
N:=sub<S|A,B,C>;
#N;
```

We discover that the order of N is 20. Now we will construct an FP-Group of N to give a presentation in terms of the generators a, b, and c. (CBFS13) We find the FP-Group with the following command.

```
FPGroup(N);
> Finitely presented group on 3 generators
Relations
$.2^2 = Id($)
$.3^2 = Id($)
($.1^-1 * $.2)^2 = Id($)
$.1^-1 * $.3 * $.1 * $.3 = Id($)
($.2 * $.3)^2 = Id($)
$.1^-5 = Id($)
```

This presents us with relations of the finitely presented group on three generators. The relations given, \$.1, \$.2, and \$.3 are labeled as a, b, and c, respectively. The construction of the presentation for N is now completed:

$$N =< a, b, c|b^2, c^2, (a^{-1}b)^2, a^{-1}cac, (bc)^2, a^{-5} >.$$

The presentation of our control group can now be written at the beginning of the progenitor presentation.

```
G<a,b,c>:=Group<a,b,c|b^2,c^2,(a^-1*b)^2,a^-1*c*a*c,(b*c)^2,a^-5>;
```

One way to verify this group so far is the control group, is to examine the size of the group. Before constructing the FP-Group of N, we discovered the order of N was 20. When we check the size of this group G, we confirm it is indeed 20.

The last step to writing the presentation of the progenitor is to state the order of our t_is and allow t to commute with a point stabilizer in N. Stabilizing 1, $t \sim t_1$, suggests t commutes with the stabilizer 1 in N. We ask Magma to provide us with a permutation group N^1 acting on a set of cardinality 10.

```
N1:=Stabiliser(N,1);
N1;
```

The output given is a permutation in the stabilizer, $(2, 5)(3, 8)(6, 10)(7, 9)$, which needs to be changed into a word in the generators of the control group. The Schreier System allows us to change permutations into words. Generator a is the only generator with a distinct inverse, thus we use the following Schreier System.

```
Sch:=SchreierSystem(G,sub<G|Id(G)>);
ArrayP:=[Id(N): i in [1..#N]];
for i in [2..#N] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;
if Eltseq(Sch[i])[j] eq 3 then P[j]:=C; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if; end for;
PP:=Id(N); for k in [1..#P] do PP:=PP*P[k]; end for; ArrayP[i]:=PP;
end for; for i in [1..#N] do
if ArrayP[i] eq S!(2, 5)(3, 8)(6, 10)(7, 9)
then Sch[i]; end if; end for;
```

The word in the generators of the control group that allows t to commute with the stabilizer 1 in N, is ba^2c. Adding t, the order of t, and this word that commutes with t to the presentation of N, gives us the completed presentation of the permutation progenitor, $2^{*10} : (D_{20})$. We store our presentation in Magma,
$G =< a, b, c, t | b^2, c^2, (a^{-1}b)^2, a^{-1}cac, (bc)^2, a^{-5}, t^2, (t, ba^2c) >$.

```
G<a,b,c,t>:=Group<a,b,c,t|b^2,c^2,(a^-1*b)^2,a^-1*c*a*c,(b*c)^2,a^-5,
t^2,(t,b*a^2*c)>;
```

Curtis' Famous Lemma gives us light on how to find relations in hopes of finding our desired sporadic groups. Two other methods in finding relations are also discussed in the following section.

2.3 Factoring by the Famous Lemma

We wish to factor a progenitor of the form $2^{*n} : N$ by first order relations. First order relations are elements in both the free product group 2^{*n} and control group N. Robert T. Curtis formulated a process to find finite homomorphic images by forming a progenitor and finding the centralizer in N of each 2-point stabilizer, to possibly write elements of the control group as words of symmetric generators. A symmetric presentation can be successfully constructed with the help of Curtis' Famous Lemma. This Lemma applies to progenitors of the form $m^{*n} : N$ where N is transitive on n letters.

Lemma 2.11. ***Famous Lemma***
$N \cap < t_i, t_j > \leq C_N(N_{ij})$, where N_{ij} denotes the stabilizer in N of the two points i and j.

Proof. Let $w \in N \cap < i, j >$ and w be a word in the two symmetric generators t_i and t_j. We must show w belongs to the centralizer, $C = Centralizer(N, N^{i,j})$. We understand $w \in C$ if w commutes with every element of N^{ij}. Let $\pi \in N^{ij}$. Then,

$$\begin{aligned} w^{\pi} &= w \\ \Longrightarrow \pi^{-1} w \pi &= w \\ \Longrightarrow w\pi &= \pi w \end{aligned}$$

Thus π commutes with every element of N^{ij}. (Cur07) □

We note that the Dihedral Group of order $2k$ is $D_{2k} =< t_i, t_j >= \{t_i{}^2, t_j{}^2, (t_i t_j)^k\}$. The center of this Dihedral Group is $< t_i, t_j >$ denoted as follows.

$$Center(D_{2k}) = \begin{cases} 1, & \text{if } k \text{ is odd} \\ < (t_i, t_j)^{\frac{k}{2}} >, & \text{if } k \text{ is even} \end{cases}$$

Once we have our centralizer, we write elements of the control group as words of symmetric generators. In other words, we write the elements of N in terms of $< t_i, t_j >$ in the following way.

$$\begin{cases} (xt_i)^m = 1, & \text{where } m \text{ is odd and } x \text{ sends 1 to 2} \\ (t_i t_j)^m = x, & \text{where } m \text{ is even and } x \text{ fixes both 1 and 2} \end{cases}$$

The Famous Lemma is applied below to the presentation of the progenitor we created in Example 2.2.1.

Example 2.3.1. ***Applying the Famous Lemma***

We let our control group be generated by the following three permutations, $a = (1, 3, 9, 7, 8)(2, 6, 4, 10, 5)$, $b = (1, 2)(3, 5)(4, 7)(6, 8)(9, 10)$, and $c = (1, 4)(2, 7)(3, 10)(5, 9)(6, 8)$, where $N =< a, b, c >= D_{20}$. Our first step is to compute the centralizer of the stabilizer of the points i and j, N_{ij}, to find relations.

```
S:=Sym(10);
A:=S!(1, 3, 9, 7, 8)(2, 6, 4, 10, 5);
B:=S!(1, 2)(3, 5)(4, 7)(6, 8)(9, 10);
C:=S!(1, 4)(2, 7)(3, 10)(5, 9)(6, 8);
N:=sub<S|A,B,C>;
N12:=Stabiliser(N,[1,2]);
C:=Centraliser(N,N12);
```

Magma gives us the permutation group C acting on a set of cardinality 10.

```
(1, 3, 9, 7, 8)(2, 6, 4, 10, 5)
(1, 2)(3, 5)(4, 7)(6, 8)(9, 10)
(1, 4)(2, 7)(3, 10)(5, 9)(6, 8)
```

We notice that in the second permutation, $(1, 2)(3, 5)(4, 7)(6, 8)(9, 10)$, 1 is being sent to 2. Since the centralizer sends 1 to 2, Famous Lemma suggests we use the relation, $(xt_i)^m = 1$ where m is odd and x sends 1 to 2.

In our case, $((1, 2)(3, 5)(4, 7)(6, 8)(9, 10)t)^m = 1$, since $t \sim t_1$. We now convert our relation into $a, b,$, and c terms for our presentation. To do this, we can use the Schreier System to convert our permutation into a word, but notice that our generator b is actually this permutation. Then our relation is $(t^b * t)^m = 1$. Thus adding this relation to our progenitor presentation can assist us in finding finite homomorphic images. We use the following code in Magma to do so.

```
for m in [0,1,3,5,7,9,11,13,15,17,19,21,23,25,27,29] do
G<a,b,c,t>:=Group<a,b,c,t|b^2,c^2,(a^-1*b)^2,a^-1*c*a*c,(b*c)^2,a^-5,
t^2,(t,b*a^2*c),
(t^b*t)^m=1>;
if #G gt 40 then m,
#G; end if; end for;
```

Unfortunately, adding this relation gave us no results. Robert T. Curtis, along with A.N.A Hammas and J.N. Bray developed a second method in finding all possible relations to find finite homomorphic images when factoring a progenitor by relations. (RAJ96)

2.4 First Order Relations

The second technique to finding all possible first order relations is by computing the conjugacy classes of our control group. With the help of Magma, we will be given representatives of each of those classes. We will then be able to find the relations by computing the orbits of the centralizer of each those representatives. This is done by right multiplying every class representative by a t_i. This method is a shortcut to find a smaller amount of relations. The control group D_{20} is of order 20. Since there are 10 $t_i s$, to find the total number of relations we right multiply each t by 20 to obtain 200 relations. Thus right multiplying solely by a representative from each orbit makes this for a simpler presentation. We will continue to use Example 2.2.1 to demonstrate the process.

Example 2.4.1. ***Adding First Order Relations***

Let $N =< a, b, c >= D_{20}$. We begin with computing the conjugacy classes of our control group. The command we use to compute the conjugacy classes and the table produced in Magma are as follows.

```
CL:=Classes(N);
CL;
#CL;
```

Table 2.1: Conjugacy Classes of D_{20}

Class Number	Order	Class Representative	Length
[1]	1	e	1
[2]	2	(1, 4)(2, 7)(3, 10)(5, 9)(6, 8)	1
[3]	2	(1, 2)(3, 5)(4, 7)(6, 8)(9, 10)	5
[4]	2	(1, 7)(2, 4)(3, 9)(5, 10)	5
[5]	5	(1, 3, 9, 7, 8)(2, 6, 4, 10, 5)	2
[6]	5	(1, 9, 8, 3, 7)(2, 4, 5, 6, 10)	2
[7]	10	(1, 10, 9, 2, 8, 4, 3, 5, 7, 6)	2
[8]	10	(1, 2, 3, 6, 9, 4, 7, 10, 8, 5)	2

Next, we compute the orbits of the centralizer of each representative. We ask Magma this with the following command.

```
for ii in [2..NumberOfClasses(N)] do
for i in [1..#N] do
if ArrayP[i] eq CL[ii][3]  then Sch[i]; end if; end for;
C12:=Centraliser(N,CL[ii][3]);
Orbits(C12);
end for;
>   c
    [
    GSet{@ 1, 3, 2, 4, 9, 5, 10, 6, 7, 8 @}
    ]
    b
    [
    GSet{@ 6, 8 @},
    GSet{@ 1, 2, 7, 4 @},
    GSet{@ 3, 5, 9, 10 @}
    ]
    b*c
    [
    GSet{@ 6, 8 @},
    GSet{@ 1, 7, 4, 2 @},
    GSet{@ 3, 9, 10, 5 @}
    ]
    a
    [
    GSet{@ 1, 3, 10, 9, 5, 7, 2, 8, 6, 4 @}
    ]
    a^2
    [
    GSet{@ 1, 9, 10, 8, 2, 3, 4, 7, 5, 6 @}
    ]
    a*c
    [
    GSet{@ 1, 10, 9, 2, 8, 4, 3, 5, 7, 6 @}
    ]
    c * a^-2
    [
    GSet{@ 1, 2, 3, 6, 9, 4, 7, 10, 8, 5 @}
    ]
```

We use our generators to find the relations of the form $(\pi {t_i}^m)^n = 1$, where $\pi \in N$ and w is a word in the t_is. With our orbits, we will right multiply every class representative by a t_i to create all possible first order relations. The most convenient t to select from each orbit if possible is t_1.

Table 2.2: First Order Relations for $2^{*10} : D_{20}$

Class	Rep	Centralizer(N, Rep)	Orbit(Centr(N, Rep))	Relations
C_2	c	$<(1,4)(2,7)(3,10)(5,9)(6,8)>$	$\{1,3,2,4,9,5,10,6,7,8\}$	ct_1
C_3	b	$<(1,2)(3,5)(4,7)(6,8)(9,10)>$	$\{6,8\},\{1,2,7,4\},\{3,5,9,10\}$	bt_8, bt_1, bt_3
C_4	$b*c$	$<(1,7)(2,4)(3,9)(5,10)>$	$\{6,8\},\{1,7,4,2\},\{3,9,10,5\}$	bct_8, bct_1, bct_3
C_5	a	$<(1,3,9,7,8)(2,6,4,10,5)>$	$\{1,3,10,9,5,7,2,8,6,4\}$	at_1
C_6	a^2	$<(1,9,8,3,7)(2,4,5,6,10)>$	$\{1,9,10,8,2,3,4,7,5,6\}$	a^2t_1
C_7	$a*c$	$<(1,10,9,2,8,4,3,5,7,6)>$	$\{1,10,9,2,8,4,3,5,7,6\}$	act_1
C_8	$c*a^{-2}$	$<(1,2,3,6,9,4,7,10,8,5)>$	$\{1,2,3,6,9,4,7,10,8,5\}$	$ca^{-2}t_1$

We must write the t_is in relationship to t_1. Recall $t \sim t_1$, $a = (1,3,9,7,8)(2,6,4,10,5)$, $b = (1,2)(3,5)(4,7)(6,8)(9,10)$, and $c = (1,4)(2,7)(3,10)(5,9)(6,8)$. If we are unable to convert our relations into the terms of our generators, we use the Schreier System to assist with changing the permutation into our desired word.

Examining the representative from each orbit of the centralizer we selected, we observe we can write t_8 and t_3 by hand; t_8 is $t^{a^{-1}}$ and t_3 is t^a. The first order relations are: $(ct), (bt^{a^{-1}}), (bt), (bt^a), (bct^{a^{-1}}), (bct), (bct^a), (at), (a^2t), (act)$, and $(ca^{-2}t)$. Therefore, adding these relations to our progenitor, we have successfully constructed a symmetric presentation that produces several finite homomorphic images,

$$G =< a,b,c,t|b^2,c^2,(a^{-1}b)^2,a^{-1}cac,(bc)^2,a^{-5},t^2,(t,ba^2c),(ct)^{r1},(bt^{a^{-1}})^{r2},(bt)^{r3},$$
$$(bt^a)^{r4},(bct^{a^{-1}})^{r5},(bct)^{r6},(bct^a)^{r7},(at)^{r8},(a^2t)^{r9},(act)^{r10},(ca^{-2}t)^{r11}> .$$

2.4.1 Saving in Nano: Magma's Background

Example 2.2.1 had a few relations added to the progenitor presentation. There are many progenitors we worked with where we added 20^+ relations. When Magma begins to compute these finite homomorphic images, it begins with the very last relation written and works its way up to relation 1. This is a long process and Magma will time out after some time. To compute these images in Magma's background, we save a clean code for each symmetric presentation we produce; this way, the homomorphic images can still be generated. We aim to ensure there are no mistakes in the code in order for the images to be produced successfully. When Magma begins to output the finite images in its server, we understand the file is clean and we can now save it in the background.

Let's say we want to call Example 2.2.1, **2*10:D20**. We want to name the file something that will help us remember what group is being ran. Before logging into Magma, we do the following.

1. We use the command, **nano 2*10:D20**. This opens a blank file named 2*10:D20. We then paste our magma code and exit the file with "ctrl x".

2. Magma will ask if we want to save the modified buffer. We type **y** for yes. Magma will asks us "file name to write: 2*10:D20". Since we've already named it in the beginning, we simply press enter to keep the same name. If we accidentally named the file wrong, this is a great opportunity to change the name now.

3. We are now logged out of the file. To complete the process of saving the file, we type, **nohup magma "2*10:D20"&>2*10:D20.out &**, where the "&" symbol is the last figure in this command. If we edit the file later, we redo steps 2 and 3.

These three steps only need to be done once and are all done outside of Magma (before we type *magma*). We can retrieve these files at any time as long as we are outside of Magma. For example, we are able to see our output for the file we created by using the command: **nano 2*10:D20.out**. If for some reason we need to edit our file, we can see our file by typing **nano 2*10:D20**. Sometimes it's difficult to keep track of the groups that are being generated in the background, thus to see a list of the files we have created, we use the command, **dir**. If we wish to remove a file, we can type **rm 2*10:D20***.
Only use this command if it is completely necessary. If a space is accidentally placed in between the file's name and the last asterisk, ALL FILES will be deleted.

2.4.2 Mixed First Order Relations

We now understand Magma begins to compute these finite homomorphic images beginning with the last relation written until reaching relation 1. As stated previously, some of these symmetric presentations have a copious amount of relations, thus many times Magma will not produce the sporadic groups we are searching for. This is due to the fact that it sometimes takes a substantial amount of time for Magma to get through every relation. To improve this process, we simply mix the first order relations so Magma has the opportunity to reach the relations at the beginning. Let us mix the first order relations in $2^{*10} : D_{20}$ as an example.

$$G =< a, b, c, t|b^2, c^2, (a^{-1}b)^2, a^{-1}cac, (bc)^2, a^{-5}, t^2, (t, ba^2c), (bct)^{r6}, (bct^a)^{r7},$$
$$(bct^{a^{-1}})^{r5}, (at)^{r8}, (bt^a)^{r4}, (a^2t)^{r9}, (bt)^{r3}, (act)^{r10}, (bt^{a^{-1}})^{r2}, (ca^{-2}t)^{r11}, (ct)^{r1} > .$$

These relations can be mixed in any way we wish to do so. The mixture of these relations is recommended for presentations that contain many relations. Lastly, we renumber the relations sequentially.

$$G =< a, b, c, t|b^2, c^2, (a^{-1}b)^2, a^{-1}cac, (bc)^2, a^{-5}, t^2, (t, ba^2c), (bct)^{r1}, (bct^a)^{r2},$$
$$(bct^{a^{-1}})^{r3}, (at)^{r4}, (bt^a)^{r5}, (a^2t)^{r6}, (bt)^{r7}, (act)^{r8}, (bt^{a^{-1}})^{r9}, (ca^{-2}t)^{r10}, (ct)^{r11} > .$$

2.5 Second Order Relations

During our research, we established a process in finding second order relations. These relations are added to the presentation of the progenitors along with the first order relations. Many sporadic groups were developed using second order relations. When we find first order relations, we find the class representative of each. Let's label our first relation from Section 2.4, $R = (c * t)$. To find the second order relations, we find all the permutations in N that commute with c. Of the permutations that commute with C, we want to know which of those permutations stabilize 1. We then find the stabilizer that gives us a list of permutations in N that stabilize 1. Lastly, we find the orbits of said permutations.

```
Orbits(Stabiliser(Centraliser(N,C),1));
GSet{@ 1 @},
GSet{@ 4 @},
GSet{@ 2, 5 @},
GSet{@ 3, 8 @},
GSet{@ 6, 10 @},
GSet{@ 7, 9 @
```

Second order relations will have the form $(\pi {t_i}^m {t_j}^k)^n = 1$, where $\pi \in N$ and w is a word in the t_is and t_js. We will right multiply our relation, R by a t_j to create all possible second order relations. The most convenient t to select from each orbit if possible is t_1. Given the orbits above for relation $R = (c * t)$, our second order relations will be $(c * t * t_1), (c * t * t_4), (c * t * t_2), (c * t * t_3), (c * t * t_6)$, and $(c * t * t_7)$. Note that $t \sim t_1$ and t is 2-transitive, hence the second order relation, $(c * t * t_1)$, will not be produced. We repeat this process with the remaining relations from Section 2.4 and we obtain a symmetric presentation with first order and second order relations.

$$G =< a, b, c, t | b^2, c^2, (a^{-1}b)^2, a^{-1}cac, (bc)^2, a^{-5}, t^2, (t, ba^2c), (ct)^{r1}, (bt^{a^{-1}})^{r2}, (bt)^{r3}, (bt^a)^{r4}, (bct^{a^{-1}})^{r5}, (bct)^{r6}, (bct^a)^{r7}, (at)^{r8}, (a^2t)^{r9}, (act)^{r10}, (ca^{-2}t)^{r11}, (ctt^c)^{r12}, (ctt^{b^2})^{r13}, (ctt^{a^2})^{r14}, (ctt^{b^2a})^{r15}, (ctt^{a^3})^{r16}, (bt^{a^{-1}}t^{b^2a})^{r17}, (bt^{a^{-1}}t^{a^4})^{r18}, (bt^{a^{-1}}t)^{r19}, (bt^{a^{-1}}t^{b^2})^{r20}, (bt^{a^{-1}}t^{a^2})^{r21}, (bt^{a^{-1}}t^{b^2a^4})^{r22}, (btt^{b^2})^{r23}, (btt^{a^2})^{r24}, (btt^{c^2})^{r25}, (btt^{a^2b})^{r26}, (btt^{b^2a})^{r27}, (btt^{a^3})^{r28}, (btt^{a^{-1}})^{r29}, (btt^{a^2})^{r30}, (btt^{a^2b})^{r31}, (bt^at)^{r32}, (bt^at^{b^2})^{r33}, (bt^at^{a^2})^{r34}, (bt^at^{c^2})^{r35}, (bt^at^{a^2b})^{r36}, (bt^at^{b^2a})^{r37}, (bt^at^{a^3})^{r38}, (bt^at^{a^{-1}})^{r39}, (bt^at^{a^2})^{r40}, (bt^at^{a^2b})^{r41} >$$

This symmetric presentation is a perfect example on when we would mix the relations in hopes of finding a sporadic group. In the next section, we will give examples to the four types of extension problems to help find the isomorphism types of our finite homomorphic images and control groups.

Chapter 3

Isomorphism Types

3.1 Related Theorems and Definitions

Definition 3.1. ***Center***
The **center** *of a group G, denoted by $Z(G)$, is the set of all $a \in G$ that commute with every element of G. (Rot95)*

Definition 3.2. ***Centerless***
A group G is **centerless** *if $Z(G) = 1$. (Rot95)*

Definition 3.3. ***Alternating Group***
The set of all even permutations in S_n, is a subgroup with $\frac{n!}{2}$ elements denoted as A_n. This is the **alternating group** *on n letters. (Rot95)*

Definition 3.4. ***Symmetric Group***
The **symmetric group***, denoted S_n is the set of all permutations of the nonempty set $X = \{1, 2, ..., n\}$. S_n is a group of order $n!$ on n letters. (Rot95)*

Definition 3.5. ***Dihedral Group***
The **dihedral group***, denoted as D_n or D_{2n}, for $2n \geq 4$, is a group of order $2n$ which is generated by two elements s and t such that*

$$s^n = 1,\ t^2 = 1 \text{ and } tst = s^{-1}.$$

(Rot95)

Definition 3.6. ***Special Linear Group***
If k is a field, the **special linear group** *over k,* **SL(n,k)***, is the set of all $n \times n$ matrices over k having determinant 1. It is a subgroup of $GL(n,k)$. (Rot95)*

Definition 3.7. ***General Linear Group***
If k is a field, then the set of all $n \times n$ matrices with nonzero determinant over field k is a group, denoted by $\mathrm{GL}(n,k)$, called the **general linear group***. (Rot95)*

Definition 3.8. ***Projective Special Linear Group***
A **projective special linear group** *$PSL(n,\mathbb{F})$, is the set of all $n \times n$ matrices with determinant 1 over field $\mathbb{F}$ factored by its center, denoted by*

$$PSL(n,\mathbb{F}) = L_s(\mathbb{F}) = \tfrac{SL(n,\mathbb{F})}{Z(SL(n,\mathbb{F}))}.$$

(Rot95)

Definition 3.9. ***Projective General Linear Group***
A **projective general linear group***, $PGL(n,\mathbb{F})$ is the set of all $n \times n$ matrices with nonzero determinant over field $\mathbb{F}$ factored by its center, denoted by*

$$PGL(n,\mathbb{F}) = \tfrac{GL(n,\mathbb{F})}{Z(GL(n,\mathbb{F}))}.$$

(Rot95)

Definition 3.10. ***Normal Subgroup***
A subgroup $K \leq G$ is a **normal subgroup** *denoted by $K \lhd G$ if $gKg^{-1} = K$, for every $g \in G$. (Rot95)*

Theorem 3.11. *If $N \lhd G$, then the cosets of N in G form a group, denoted by G/N, of order $[G : N]$. (Rot95)*

Definition 3.12. ***Normal Series***
A **normal series** *of a group G is a sequence of subgroups*

$$G = G_0 \geq G_1 \geq ... \geq G_n = 1$$

in which $G_{i+1} \lhd G_i$ for all i. The ***factor groups*** *of this normal series are the groups G_i/G_{i+1} for $i = 0, 1, ..., n-1$; the* ***length*** *of the normal series is the number of strict inclusions; that is, the length is the number of nontrivial factor groups.*

Definition 3.13. ***Composition Series***
A **composition series** *is a normal series*

$$G = G_0 \geq G_1 \geq ... \geq G_n = 1$$

in which, for all i, either G_{i+1} is a maximal normal subgroup of G_i or $G_{i+1} = G_i$.

Definition 3.14. ***Composition factors***
If G has a composition series, then the factor groups of this series are called the **composition factors** *of G. (Rot95)*

Definition 3.15. ***Extension***
If K and Q are groups, then an **extension** *of K by Q is a group G having a normal subgroup $K_1 \cong K$ with $G/K_1 \cong Q$. (Rot95)*

Definition 3.16. ***Direct Product***
If H and K are groups, then their **direct product**, *denoted by $H \times K$, is the group with elements all ordered pairs (h, k), where $h \in H$ and $k \in K$, and with the operation $(h, k)(h', k') = (hh', kk')$. (Rot95)*

Definition 3.17. ***Semi-Direct Product***
A group G is a **semi-direct product** *of K by Q, denoted by $G = K \rtimes Q$, if $K \lhd G$ and K has a complement $Q_1 \cong Q$. One also says that G* ***splits*** *over K. (Rot95)*

Definition 3.18. ***Central Extension***
A **central extension** *of K by Q is an extension G of K by Q with $K \leq Z(G)$. (Rot95)*

Definition 3.19. ***Mixed-Extension***
If G is an extension of an abelian group not equal to the center of G, then this extension is called a **mixed extension**. *(Rot95)*

Lemma 3.20. *Any finite non-abelian simple group is an image of a progenitor of form $P = 2^{*n} : N$ where N is a transitive subgroup of the symmetric group S_n.*

3.2 Direct Product

Consider the group

$$G = \frac{2^{*10} : D_{20}}{(a^2t)^3, (act)^9, (ca^{-2}t)^5}.$$

Our group G has the following symmetric presentation,

$G = < a, b, c, t | b^2, c^2, (a^{-1}b)^2, a^{-1}cac, (bc)^2, a^{-5}, t^2, (t, ba^2c), (a^2t)^3, (act)^9, (ca^{-2}t)^5 >$.

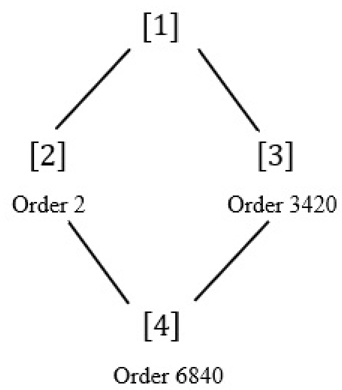
We compute the composition series of group G.

```
CompositionFactors(G1);
>   G
    |  A(1, 19)                     = L(2, 19)
    *
    |  Cyclic(2)
    1
```

The composition series is $G = G_1 \supseteq 1$ where $G = (G/G_1)(G_1/1) = (C_2)(L_2(19))$.

In order to prove G is a direct product of C_2 by $PSL_2(19)$, we must show these subgroups are normal in G, their intersection is the identity, and the subgroups are not contained in one another. We study the normal lattice of G using the following command in Magma.

```
NL:=NormalLattice(G1);
NL;
> Normal subgroup lattice
-----------------------
[4]  Order 6840  Length 1  Maximal Subgroups: 2 3
---
[3]  Order 3420  Length 1  Maximal Subgroups: 1
---
[2]  Order 2     Length 1  Maximal Subgroups: 1
---
[1]  Order 1     Length 1  Maximal Subgroups:
```



This is the graph of the Normal Lattice of group G. We notice that subgroup $NL[2]$ is normal with order 2 and subgroup $NL[3]$ is normal with order 3420. $PSL_2(19)$ has order 3420 and realize this is the same order as subgroup $NL[3]$. There's a possibility $NL[3]$ is isomorphic to $PSL_2 19$. The intersection of $NL[2]$ and $NL[3]$ is the identity, and they are not contained in one another.

```
PSL(2,19);
> Permutation group acting on a set of cardinality 20
Order = 3420 = 2^2 * 3^2 * 5 * 19
(3, 9, 17, 16, 19, 8, 18, 13, 11)(4, 7, 6, 14, 20, 12, 10, 5, 15)
(1, 20, 2)(3, 11, 19)(4, 7, 17)(5, 15, 18)(6, 12, 8)(10, 16, 14)
```

Since the subgroups are not contained in one another, we are considering this symmetric presentation to be a direct product. We ask Magma to verify our hypothesis and conclude it is true.

```
D:=DirectProduct(NL[2],NL[3]);
s,t:=IsIsomorphic(G1,D);
s;
> true
```

In order to confirm that the group G is isomorphic to $2 \times PSL_2(19)$, we must write a presentation for $PSL_2(19)$.

```
FPGroup(PSL(2,19));
> Finitely presented group on 2 generators
Relations
$.1^9 = Id($)
$.2^3 = Id($)
$.1^-1 * $.2 * $.1^-1 * $.2^-1 * $.1^2 * $.2 * $.1^2 * $.2^-1 = Id($)
($.1 * $.2^-1 * $.1 * $.2^-1 * $.1)^2 = Id($)
$.1^3 * $.2^-1 * $.1^-3 * $.2 * $.1 * $.2^-1 * $.1^-2 * $.2 = Id($)
```

As we have done before, we will let \$.1 and \$.2 be a and b, respectively.
Then a presentation for $PSL_2(19)$ is given by,

$$H =< a, b|a^9, b^3, a^{-1}ba^{-1}b^{-1}a^2ba^2b^{-1}, (ab^{-1}ab^{-1}a)^2, a^3b^{-1}a^{-3}bab^{-1}a^{-2}b > .$$

We confirm the presentation we created is correct in Magma.

```
H<a,b>:=Group<a,b|a^9,b^3,a^-1*b*a^-1*b^-1*a^2*b*a^2*b^-1,
(a*b^-1*a*b^-1*a)^2,a^3*b^-1*a^-3*b*a*b^-1*a^-2*b>;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,PSL(2,19));
s;
> true
```

The presentation for C_2 can be easily constructed with a generator of order 2. We let $C_2 =< c \mid c^2 >$. We will now add C_2 to our presentation for $PSL_2(19)$, ensure their generators commute with one another, and verify this presentation is isomorphic to G.

```
H<a,b,c>:=Group<a,b,c|a^9,b^3,a^-1*b*a^-1*b^-1*a^2*b*a^2*b^-1,
(a*b^-1*a*b^-1*a)^2,a^3*b^-1*a^-3*b*a*b^-1*a^-2*b,c^2,(a,c),(b,c)>;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,G1);
s;
> true
```

Hence,

$$G = \frac{2^{*10} : D_{20}}{(a^2t)^3, (act)^9, (ca^{-2}t)^5} \cong (2 \times PSL_2(19)) .$$

3.3 Semi-Direct Product

Consider the group

$$G = \frac{2^{*7} : (7:6)}{(x^{-2}yt)^7, (yxt)^8, (x^{-1}y^{-1}t^y)^6, (yt)^8}.$$

Our group G has the following symmetric presentation,
$G =< x, y, t | x * y^{-1} * x^{-1} * y^{-2}, x^6, y * x^{-1} * y^{-1} * x * y^2, t^2, (t, x * y^{-2}), (t, x^2 * y^{-1}),$
$(x^{-2} * y * t)^7, (y * x * t)^8, (x^{-1} * y^{-1} * t^y)^6, (y * t)^8 >$. We compute the composition series of group G.

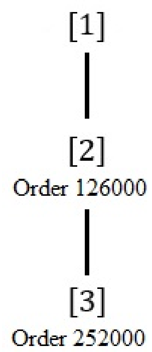
```
CompositionFactors(G1);
>   G
    |  Cyclic(2)
    *
    |  2A(2, 5)                        = U(3, 5)
    1
```

The composition series of the G is $G = G_1 \supseteq 1$ where $G = (G/G_1)(G_1/1) = (U_3(5))(C_2)$. In order for G to be a direct product of $U_3(5)$ by C_2, we must show these subgroups are normal in G. We study the normal lattice of G.

```
NL:=NormalLattice(G1);
NL;
> Normal subgroup lattice
-----------------------

[3]  Order 252000  Length 1  Maximal Subgroups: 2
---
[2]  Order 126000  Length 1  Maximal Subgroups: 1
---
[1]  Order 1       Length 1  Maximal Subgroups:
```

This is the graph of the Normal Lattice of group G.



We notice that subgroup $NL[2]$ is normal with order 126,000, and it is the projective special unitary group, $PSU(3,5)$, also denoted as $U_3(5)$.

```
PSU(3,5);
> Permutation group acting on a set of cardinality 126
Order = 126000 = 2^4 * 3^2 * 5^3 * 7
```

We now verify $U_3(5)$ is indeed isomorphic to $NL[2]$.

```
s:=IsIsomorphic(NL[2],PSU(3,5));
s;
> true
```

We have shown that $NL[2]$ is normal. In order for G to be a direct product, $NL[3]$ must also be normal. The normal lattice of group G does not contain a subgroup of order 2. Hence, C_2 is not normal. We also observe $NL[3]$ is contained in $NL[2]$, thus also confirming this group is not a direct product. We predict the symmetric presentation to be a semi-direct product. There must be an element of order 2 which will extend $U_3(5)$ to G. We begin with writing a presentation for $U_3(5)$.

```
FPGroup(NL[2]);
> Finitely presented group on 3 generators
Relations
$.1^3 = Id($)
$.2^7 = Id($)
$.3^4 = Id($)
$.1 * $.2^-1 * $.1^-1 * $.2^2 = Id($)
$.2 * $.1 * $.3^-1 * $.1^-1 * $.3 * $.2^-1 * $.3 = Id($)
$.3 * $.1 * $.3^-2 * $.1^-1 * $.3^-1 * $.1^-1 * $.3^2 * $.2^-1 *
$.1^-1 * $.3^-1 * $.2^-1 = Id($)
$.3^-1 * $.1 * $.3^-2 * $.1^-1 * $.3 * $.1^-1 * $.3^-1 * $.2^-1 *
$.1^-1 * $.3^2 * $.2^-1 = Id($)
$.3^-1 * $.1^-1 * $.2^-1 * $.3^-2 * $.1^-1 * $.2^-1 * $.1^-1 * $.3
* $.2 * $.1^-1 * $.3^2 * $.1 = Id($)
$.1 * $.3^-1 * $.2^-1 * $.3^-2 * $.1^-1 * $.2^-1 * $.1 * $.3^2 *
$.1^-1 * $.3^2 * $.2 * $.3 = Id($)
```

We let \$.1, \$.2, and \$.3 be a, b, and c, respectively. Then a presentation for $U_3(5)$ is given by,

$$H = < a, b, c | a^3, b^7, c^4, a*b^{-1}*a^{-1}*b^2, b*a*c^{-1}*a^{-1}*c*b^{-1}*c,$$
$$c*a*c^{-2}*a^{-1}*c^{-1}*a^{-1}*c^2*b^{-1}*a^{-1}*c^{-1}*b^{-1},$$
$$c^{-1}*a*c^{-2}*a^{-1}*c*a^{-1}*c^{-1}*b^{-1}*a^{-1}*c^2*b^{-1},$$
$$c^{-1}*a^{-1}*b^{-1}*c^{-2}*a^{-1}*b^{-1}*a^{-1}*c*b*a^{-1}*c^2*a,$$
$$a*c^{-1}*b^{-1}*c^{-2}*a^{-1}*b^{-1}*a*c^2*a^{-1}*c^2*b*c > .$$

We confirm the presentation we created is correct in Magma.

```
H<a,b,c>:=Group<a,b,c|a^3,b^7,c^4,a*b^-1*a^-1*b^2,
b*a*c^-1*a^-1*c*b^-1*c,
c*a*c^-2*a^-1*c^-1*a^-1*c^2*b^-1*a^-1*c^-1*b^-1,
c^-1*a*c^-2*a^-1*c*a^-1*c^-1*b^-1*a^-1*c^2*b^-1,
c^-1*a^-1*b^-1*c^-2*a^-1*b^-1*a^-1*c*b * a^-1*c^2*a,
a*c^-1*b^-1*c^-2*a^-1*b^-1*a*c^2*a^-1*c^2*b*c>;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,NL[2]);
s;
> true
```

We ask Magma to help us find an element of order 2, labeled as C, that extends $U_3(5)$ to G.

```
for i in NL[3] do if i notin NL[2] and Order(i) eq 2
and sub<G1|i,NL[2]> eq G1 then C:=i; break;
end if; end for;
```

Unfortunately, this code did not give us an output. Our next step is to use the Inverse Word Map to find said element.

```
A:=NL[2].1;
B:=NL[2].2;
C:=NL[2].3;
for i in NL[3] do if i notin NL[2] and Order(i) eq 2
and sub<G1|i,NL[2]> eq G1 then D:=i; break;
end if; end for;
E:=A^D;
W:=WordGroup(G1);
rho:=InverseWordMap(G1);
E@rho;
EE:=function(W)
    w1 := W.1^-1; w4 := w1 * W.3; w5 := w4 * W.1; w6 := w5 * W.3;
    w7 := w6 * W.2; w8 := w7 * W.3; w9 := w8 * W.1; w10 := w9 * W.2;
    w11 := w10 * W.3; w2:= W.2^-1; w12 := w11 * w2; w13 := w12 * W.3;
    w14 := w13 * W.1; w15 := w14 * W.2; return w15;
end function;
EE(G);
> a^-1 * c * a * c * b * c * a * b * c * b^-1 * c * a * b
A^D eq A^-1 * C * A * C * B * C * A * B * C * B^-1 * C * A * B;
> false
```

Again, we are unsuccessful. We must find an element that will give us the extension, thus with the help of Atlas, we are provided with a very well known presentation for $U_3(5):2$.
$G =< x, y|x^2, y^4, (x*y)^{10}, (x*y*x*y^{-1}*x*y^2)^2, (x, y*x*y)^4, (x*y*x*y*x*y*x*y^2)^7 > .$
(WPT$^+$) We register this group in Magma as the following.

```
G<c,d>:=Group<c,d|c^2,d^4,(c*d)^10,(c*d*c*d^-1*c*d^2)^2,(c,d*c*d)^4,
(c*d*c*d*c*d*c*d^2)^7>;
```

We find a faithful permutation representation on the cosets M_2, the maximal subgroup S_7, and are able to successfully show that our group is isomorphic to $U_3(5):2$.

```
M2:=sub<G|x,y^-1*x*y*x*y*x*y>;
ff,G2,k:=CosetAction(G,M2);
#G2;
> 252000
s:=IsIsomorphic(G1,G2);
s;
> true
```

Hence,

$$G = \frac{2^{*7}:(7:6)}{(x^{-2}yt)^7, (yxt)^8, (x^{-1}y^{-1}t^y)^6, (yt)^8} \cong (U_3(5):2).$$

3.4 Central Extension

Consider the group

$$G = \frac{2^{*7} : (7:3)}{(xy^{-1}x^{-1}tt^{x^2})^5, (x^{-1}y^{-1}t)^5}.$$

Our group G has the following symmetric presentation,

$G =< x, y, t | x^3, y^{-2} * x^{-1} * y * x, t^2, (t, y * x^{-1}), (x * y^{-1} * x^{-1} * t * t^{x^2})^5, (x^{-1} * y^{-1} * t)^5 >$.

We compute the composition series of group G.

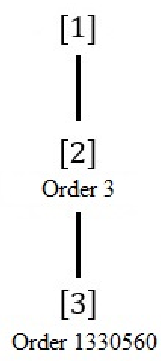
```
CompositionFactors(G1);
>   G
    |  M22
    *
    |  Cyclic(3)
    1
```

The composition series of the G is $G = G_1 \supseteq 1$ where $G = (G/G_1)(G_1/1) = (C_3)(M_{22})$. In order for G to be a direct product of C_3 by M_{22}, we must show these subgroups are normal in G. We study the normal lattice of G.

```
NL:=NormalLattice(G1);
NL;
> Normal subgroup lattice
-----------------------

[3]  Order 1330560  Length 1  Maximal Subgroups: 2
---
[2]  Order 3        Length 1  Maximal Subgroups: 1
---
[1]  Order 1        Length 1  Maximal Subgroups:
```

This is the graph of the Normal Lattice of group G.



We notice that subgroup $NL[3]$ is not normal, but $NL[2]$ is normal.

```
IsAbelian(NL[2]);
> true
IsAbelian(NL[3]);
> false
```

Since $NL[2]$ is the largest abelian subgroup, we wish to identify if this is the center of the group. If $NL[2]$ results in being the center of the group, we will have a central extension.

```
Center(G1);
> Permutation group acting on a set of cardinality 63360
Order = 3

NL[2] eq Center(G1);
> true
```

Since $NL[2]$ is the maximal abelian subgroup and is the center of the group, we will factor our group by its center and look at the composition factors of that quotient. Observing the composition factors of our original group, we understand that this quotient will be isomorphic to the Mathieu group M_{22}.

```
q,ff:=quo<G1|NL[2]>;
q;
> Permutation group q acting on a set of cardinality 672
Order = 443520 = 2^7 * 3^2 * 5 * 7 * 11

CompositionFactors(q);
> M22
```

Therefore, our group is a central extension of C_3 by M_{22}. We begin with writing a presentation for the quotient group, M_{22} using Magma.

```
FPGroup(q);
Finitely presented group on 3 generators
Relations
$.1^3 = Id($)
$.2^7 = Id($)
$.3^2 = Id($)
$.2^-2 * $.1^-1 * $.2 * $.1 = Id($)
$.2^-1 * $.3 * $.2 * $.1^-1 * $.3 * $.1 = Id($)
$.3 * $.1^-1 * $.3 * $.2^-2 * $.3 * $.1^-1
* $.3 * $.2 * $.1 * $.3 * $.2^-2
= Id($)
($.2^-1 * $.3)^8 = Id($)
($.3 * $.1^-1)^11 = Id($)
```

We let \$.1, \$.2, and \$.3 be x, y, and z, respectively. Then a presentation for M_{22} is given by,

$$H = < x, y, z | x^3, y^7, z^2, y^{-2} * x^{-1} * y * x, y^{-1} * z * y * x^{-1} * z * x,$$
$$z * x^{-1} * z * y^{-2} * z * x^{-1} * z * y * x * z * y^{-2}, (y^{-1} * z)^8, (z * x^{-1})^{11} > .$$

We verify with Magma this presentation is correct to proceed with the next step.

```
H<x,y,z>:=Group<x,y,z|x^3,y^7,z^2,y^-2*x^-1*y*x,y^-1*z*y*x^-1*z*x,
z*x^-1*z*y^-2*z*x^-1*z*y*x*z*y^-2,(y^-1*z)^8,(z*x^-1)^11>;
#H;
> 443520
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>); s,t:=IsIsomorphic(H1,q); s;
> true
```

Then the final step to writing our presentation is to convert the generators of M_{22} in terms of our center, D. We study the right transversals that match our presentation of q and understand $ff(T[i])$s are permutations from H_1. Then ff maps the permutations from H_1 to q. A, B, and C are permutations that map H_1 to q. H_1 is generated by A, B, and C, hence we need to find the permutations that will take our H_1 to q.

```
T:=Transversal(G1,NL[2]);
ff(T[2]) eq q.1;
> true
ff(T[3]) eq q.2;
> true
ff(T[4]) eq q.3;
> true
A:=T[2];
B:=T[3];
C:=T[4];
D:=NL[2].2;
for i in [1..3] do if A^3 eq D^i then i; end if; end for;
> 3
for i in [1..3] do if B^7 eq D^i then i; end if; end for;
> 3
for i in [1..3] do if C^2 eq D^i then i; end if; end for;
> 3
for i in [1..3] do if B^-2*A^-1*B*A eq D^i then i; end if; end for;
> 3
for i in [1..3] do if B^-1*C*B*A^-1*C*A eq D^i
then i; end if; end for;
> 3
for i in [1..3] do if C*A^-1*C*B^-2*C*A^-1*C*B*A*C*B^-2 eq D^i
then i; end if; end for;
> 3
for i in [1..3] do if (B^-1*C)^8 eq D^i then i; end if; end for;
for i in [1..3] do if (C*A^-1)^11 eq D^i then i; end if; end for;
```

We discover six of the relations from our quotient group can be written in terms of the center. Then to our presentation, we add the center of the group as a generator of order 3, ensure the generators commute with one another, write the six relations in terms of the center, and verify this presentation is isomorphic to G.

```
HH<x,y,z,c>:=Group<x,y,z,c|x^3=c^3,y^7=c^3,z^2=c^3,y^-2*x^-1*y*x=c^3,
y^-1*z*y*x^-1*z*x=c^3,z*x^-1*z*y^-2*z*x^-1*z*y*x*z*y^-2=c^3,
(y^-1*z)^8,(z*x^-1)^11,c^3,(c,x),(c,y),(c,z)>;
f2,H2,k2:=CosetAction(HH,sub<HH|Id(HH)>);
s,t:=IsIsomorphic(H2,G1);
s;
> true
```

Hence,

$$G = \frac{2^{*7} : (7:3)}{(xy^{-1}x^{-1}tt^{x^2})^5, (x^{-1}y^{-1}t)^5} \cong (3\,^{\cdot}M_{22})\ .$$

3.5 Mixed Extension

Consider the group

$$G = \frac{2^{*10} : D_{20}}{(a^2t)^4, (ca^{-2}t)^3} \ .$$

Our group G has the following symmetric presentation,

$G =< a, b, c, t | b^2, c^2, (a^{-1}b)^2, a^{-1}cac, (bc)^2, a^{-5}, t^2, (t, ba^2c), (a^2t)^4, (ca^{-2}t)^3 >$.

We compute the composition series of group G.

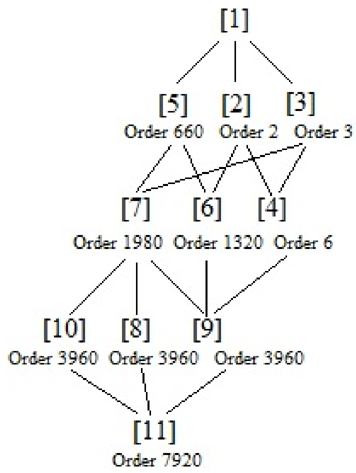
```
CompositionFactors(G1);
>   G
    |  Cyclic(2)
    *
    |  A(1, 11)                    = L(2, 11)
    *
    |  Cyclic(3)
    *
    |  Cyclic(2)
    1
```

The composition series of the group is $G = G_1 \supseteq G_2 \supseteq G_3 \supseteq 1$ where $G = (G/G_1)(G_1/G_2)(G_2/G_3)(G_3/1) = (C_2)(C_3)(L_2(11))(C_2)$. We study the normal lattice of G.

```
Normal subgroup lattice
-----------------------

[11]  Order 7920  Length 1  Maximal Subgroups: 8 9 10
---
[10]  Order 3960  Length 1  Maximal Subgroups: 7
[ 9]  Order 3960  Length 1  Maximal Subgroups: 4 6 7
[ 8]  Order 3960  Length 1  Maximal Subgroups: 7
---
[ 7]  Order 1980  Length 1  Maximal Subgroups: 3 5
[ 6]  Order 1320  Length 1  Maximal Subgroups: 2 5
---
[ 5]  Order 660   Length 1  Maximal Subgroups: 1
---
[ 4]  Order 6     Length 1  Maximal Subgroups: 2 3
---
[ 3]  Order 3     Length 1  Maximal Subgroups: 1
[ 2]  Order 2     Length 1  Maximal Subgroups: 1
---
[ 1]  Order 1     Length 1  Maximal Subgroups:
```

This is the graph of the Normal Lattice of group G.



Since our normal lattice has quite a few subgroups, we will use the following Magma Code to find if there are any normal subgroups. We wish to know if this group has a center, thus we will also ask Magma if the group has a center.

```
for i in [1..11] do if IsAbelian(NL[i]) then i; end if; end for;
> 1
  2
  3
  4
Center(G1);
> Permutation group acting on a set of cardinality 7920
Order = 2
```

We understand now that $NL[2]$ is the center of the group since $NL[2]$ is the only subgroup of order 2 in our lattice. In order for this problem to be a central extension problem, $NL[2]$ must be the largest abelian subgroup of our group. Magma identified $NL[3]$ and $NL[4]$ as abelian subgroups. Since $NL[2]$ is not the maximal abelian subgroup, we have a mixed extension. This means that if the largest abelian subgroup contains the center, then we have a mixed extension. Mixed extensions have characteristics of central extensions and semi-direct products.

From previous knowledge, if we analyze the composition factors of G, we understand that C_2 with $L_2(11)$ will result in being the Projective General Linear Group $(2, 11)$ of order 1320. The product of 1320 with 6 give us the order of the group G.

Now if we look at the normal lattice of G, we observe that $NL[2]$ and $NL[3]$ have order 2 and 3. Thus we make the assumption that C_3 and C_2 will be a direct product.

```
D:=DirectProduct(NL[2],NL[3]);
IsIsomorphic(D,NL[4]);
> true Mapping from: GrpPerm: D to GrpPerm: \$,
Degree 7920, Order 2 * 3
Composition of Mapping from: GrpPerm: D to GrpPC and
Mapping from: GrpPC to GrpPC and
Mapping from: GrpPC to GrpPerm: \$, Degree 7920, Order 2 * 3*/
```

Moving forward, we will now factor the largest abelian subgroup, $NL[4]$, from our group to create the quotient group, q. Recall that *ff* is a function that sends elements from group G into the quotient group. We will then look at the normal lattice of this quotient group.

```
q,ff:=quo<G1|NL[4]>;
q;
nl:=NormalLattice(q);
nl;
> Normal subgroup lattice
-----------------------

[3]  Order 1320  Length 1  Maximal Subgroups: 2
---
[2]  Order 660   Length 1  Maximal Subgroups: 1
---
[1]  Order 1     Length 1  Maximal Subgroups:
```

As presumed, the quotient group q has order 1320. We confirm with Magma that q is indeed $PGL_2(11)$.

```
IsIsomorphic(PGL(2,11),q);
> true Homomorphism of GrpPerm: $,
Degree 12, Order 2^3 * 3 * 5 * 11 into
GrpPerm: q, Degree 12, Order 2^3 * 3 * 5 * 11 induced by
(3, 8, 7, 12, 9, 10, 4, 11, 5, 6) |-->
(1, 2, 11, 6, 3, 5, 7, 12, 10, 9)
(1, 8, 2)(3, 4, 7)(5, 12, 11)(6, 9, 10) |-->
(1, 2, 7)(3, 4, 8)(5, 6, 9)(10, 11, 12)
```

We must now write a presentation for $PGL_2(11)$.

```
FPGroup(q);
> Finitely presented group on 4 generators
Relations
$.1^5 = Id($)
$.2^2 = Id($)
$.3^2 = Id($)
$.4^2 = Id($)
($.1^-1 * $.2)^2 = Id($)
$.1^-1 * $.3 * $.1 * $.3 = Id($)
($.2 * $.3)^2 = Id($)
$.3 * $.4 * $.2 * $.1^2 * $.4 * $.3 * $.4 = Id($)
$.2 * $.4 * $.3 * $.1^-2 * $.4 * $.1 * $.2 * $.4 = Id($)
$.3 * $.1 * $.4 * $.3 * $.1^-1 * $.4 * $.2 * $.1 * $.4 * $.1 * $.3 *
$.4 * $.1^-1 * $.3 * $.4 = Id($)
```

We let \$.1, \$.2, \$.3, and \$.4 be a, b, c, and d, respectively. Then a presentation for $PGL_2(11)$ is given by,

$$H = < a, b, c, d | a^5, b^2, c^2, d^2, (a^{-1}b)^2, a^{-1}cac, (bc)^2, dba^2dcd, bdca^{-2}dabd,$$
$$cadca^{-1}dbadacda^{-1}cd > .$$

We verify with Magma this presentation is correct to proceed with the next step.

```
H<a,b,c,d>:=Group<a,b,c,d| a^5,b^2,c^2,d^2,(a^-1 * b)^2,
a^-1 * c * a * c, (b * c)^2,c * d * b * a^2 * d * c * d,
b * d * c * a^-2 * d * a * b * d,
c * a * d * c * a^-1 * d * b * a * d * a * c * d * a^-1 * c * d>;
#H;
> 1320
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>); s,t:=IsIsomorphic(H1,q); s;
> true
```

Then the final step to writing our presentation is to convert the generators of $PGL_2(11)$ in terms of our center, D. We study the right transversals that match our presentation of q and understand $ff(T[i])$s are permutations from H_1. Then ff maps the permutations from H_1 to q. A, B, and C are permutations that map H_1 to q. H_1 is generated by A, B, and C, hence we need to find the permutations that will take our H_1 to q.

```
T:=Transversal(G1,NL[4]);
ff(T[2]) eq q.1; ff(T[3]) eq q.2; ff(T[4]) eq q.3;
> true
  true
  true

A:=T[2]; B:=T[3]; C:=T[4]; D:=NL[2].1;
```

We will now label the direct product of C_2 and C_3 as J, and verify that the direct product is isomorphic to the largest abelian subgroup, $NL[4]$. The direct product, $J = C_2 \times C_3 =< e^3, f^2, (e, f) >$ where e and f commute. Then, we will ask Magma if any of the generators can be written in terms of the center. Since J contains the center, we will be writing the code with E and F.

```
J:=DirectProduct(NL[2],NL[3]);
s,t:=IsIsomorphic(J,NL[4]);
for e,f in NL[4] do if Order(e) eq 3 and Order(f) eq 2 and e^f eq e
then E:=e; F:=f; end if; end for;
for i in [0..2] do for j in [0..1] do if A^5 eq E^i*F^j
then i,j; break; end if; end for; end for;
/*1 0*/
for i in [0..2] do for j in [0..1] do if B^2 eq E^i*F^j
then i,j; break; end if; end for; end for;
/*0 0*/
for i in [0..2] do for j in [0..1] do if C^2 eq E^i*F^j
then i,j; break; end if; end for; end for;
/*0 0*/
for i in [0..2] do for j in [0..1] do if D^2 eq E^i*F^j
then i,j; break; end if; end for; end for;
/*0 0*/
for i in [0..2] do for j in [0..1] do if (A^-1 * B)^2 eq E^i*F^j
then i,j; break; end if; end for; end for;
/*0 0*/
for i in [0..2] do for j in [0..1] do if A^-1 * C * A * C eq E^i*F^j
then i,j; break; end if; end for; end for;
```

```
/*2 0*/
for i in [0..2] do for j in [0..1] do if (B * C)^2 eq E^i*F^j
then i,j; break; end if; end for; end for;
/*1 0*/
for i in [0..2] do for j in [0..1] do if C*D*B*A^2*D*C*D eq E^i*F^j
then i,j; break; end if; end for; end for;
/*1 1*/
for i in [0..2] do for j in [0..1] do if
B*D*C*A^-2*D*A*B*D eq E^i*F^j
then i,j; break; end if; end for; end for;
/*1 1*/
for i in [0..2] do for j in [0..1] do if
C*A*D*C*A^-1*D*B*A*D*A*C*D*A^-1*C*D eq E^i*F^j
then i,j; break; end if; end for; end for;
/*2 1*/
```

Lastly, we will write the semi-direct product of the entire mixed extension.

```
for i in [0..4] do for j,k,l in [0..1] do if
A^E eq A^i*B^j*C^k*D^l then i;j;k;l;break;end if; end for; end for;
/* 1  0  0  0 */
/*[0..4] because |A|=5 and [0..1] since |B,C,D|=2*/

for i in [0..4] do for j,k,l in [0..1] do if
A^F eq A^i*B^j*C^k*D^l then i;j;k;l;break;end if; end for; end for;
/* 1  0  0  0 */

for i in [0..4] do for j,k,l in [0..1] do if
B^E eq A^i*B^j*C^k*D^l then i;j;k;l;break;end if; end for; end for;
/* 0  1  0  0 */

for i in [0..4] do for j,k,l in [0..1] do if
B^F eq A^i*B^j*C^k*D^l then i;j;k;l;break;end if; end for; end for;

for i in [0..4] do for j,k,l in [0..1] do if
C^E eq A^i*B^j*C^k*D^l then i;j;k;l;break;end if; end for; end for;
/* 0  0  1  0 */

for i in [0..4] do for j,k,l in [0..1] do if
C^F eq A^i*B^j*C^k*D^l then i;j;k;l;break;end if; end for; end for;

for i in [0..4] do for j,k,l in [0..1] do if
D^E eq A^i*B^j*C^k*D^l then i;j;k;l;break;end if; end for; end for;
```

```
for i in [0..4] do for j,k,l in [0..1] do if
D^F eq A^i*B^j*C^k*D^l then i;j;k;l;break;end if; end for; end for;
/* 0  0  0  1 */
```

Then to our presentation, we add J which contains the center of the group in terms of e and f, ensure the generators commute with one another, rewrite some of the relations in terms of the center, and verify this presentation is isomorphic to G.

```
HH<a,b,c,d,e,f>:=Group<a,b,c,d,e,f|a^5=e,b^2,c^2,d^2,(a^-1*b)^2,
a^-1*c*a*c=e^2,(b*c)^2=e,c*d*b*a^2*d*c*d=e*f, b*d*c*a^-2*d*a*b*d=e*f,
c*a*d*c*a^-1*d*b*a*d*a*c*d*a^-1*c*d=e^2*f,
e^3,f^2,(e,f),a^e=a,a^f=a,b^f=b,c^f=c,d^f=d>;
f2,H2,k2:=CosetAction(HH,sub<HH|Id(HH)>);
#H2;
> 7920
s,t:=IsIsomorphic(H2,G1);
s;
> true
```

When possible, we rewrite direct products as one group. In our case, the direct product is isomorphic to C_6.

```
D:=DirectProduct(CyclicGroup(2),CyclicGroup(3));
> IsIsomorphic(D,CyclicGroup(6));
true Mapping from: GrpPerm: D to GrpPerm: $, Degree 6, Order 2 * 3
Composition of Mapping from: GrpPerm: D to GrpPC and
Mapping from: GrpPC to GrpPC and
Mapping from: GrpPC to GrpPerm: $, Degree 6, Order 2 * 3
> s,t:=IsIsomorphic(D,CyclicGroup(6));
> s;
true
```

Hence,

$$G = \frac{2^{*10} : D_{20}}{(a^2t)^4, (ca^{-2}t)^3} \cong (6 : {}^{\cdot}PGL_2(11)) \ .$$

Chapter 4

Double Coset Enumeration

4.1 Related Theorems and Definitions

Definition 4.1. ***Double Coset***

For H and K subgroups of the group G, we define a relation on G as follows:

$$x \sim y \iff \exists\ h \in H \text{ and } k \in K \text{ such that } y = hxk,$$

where ∼ is an equivalence relation and the equivalence classes are sets of the following form:

$$HxK = \{hxk | h \in H, k \in K\} = \bigcup_{k \in K} Hxk = \bigcup_{h \in H} hxK.$$

Such a subset of G, a union of right cosets of H and a union of left cosets of K is called a **double coset***. (Cur07)*

Definition 4.2. ***Coset Stabilizing Group***

The **coset stabilizing group** *of a coset Nw is defined as* $N^w = \{\pi \in N | Nw\pi = Nw\}$ *where* $n \in N$ *and w is a reduced word in the* t_i*s. (Cur07)*

Theorem 4.3. ***Number of Single Cosets in*** NwN

From above we see that,

$$\begin{aligned} N^{(w)} &= \{\pi \in N | Nw\pi = Nw\} = \{\pi \in N | Nw\pi w^{-1} = N\} \\ &= \{\pi \in N | w\pi w^{-1} \in N\} = \{\pi \in N | \pi \in N^w\} \\ &= N \cap N^w \end{aligned}$$

and the number of single cosets in NwN *is given by* $|N : N^{(w)}|$*. (Cur07)*

Definition 4.4. ***G-set***

If X is a set and G is a group, then X is a **G-set** *if there is a function α: $G \times X \to X$ (called an* ***action****), denoted by α: $(g, x) \mapsto gx$, such that:*

(i) $1x = x$ for all $x \in X$; and

(ii) $g(hx) = (gh)x$ for all $g, h \in G$ and $x \in X$.

(Rot95)

Definition 4.5. ***Acts***

G **acts** *on X, if $|X| = n$, then n is called the* ***degree*** *of the G-set X. (Rot95)*

Definition 4.6. ***G-orbit***

If X is a G-set and $x \in X$, then the **G-orbit** *of x is*

$$\vartheta(x) = \{gx : g \in G\} \subset X.$$

(Rot95)

Definition 4.7. ***Stabilizer***

If X is a G-set and $x \in X$, then the **stabilizer** *of x, denoted by G_x, is the subgroup $G_x = \{g \in G : gx = x\} \leq G$. (Rot95)*

Theorem 4.8. *If X is a G-set and $x \in X$, then $|\vartheta(x)| = [G : G_x]$. (Rot95)*

Corollary 4.9. *If a finite group G acts on a set X, then the number of elements in any orbit is a divisor of $|G|$. (Rot95)*

Definition 4.10. ***Transitive***

A G-set X is **transitive** *if it has only one orbit; that is for every x, $y \in X$, there exists $\sigma \in G$ with $y = \sigma x$. (Rot95)*

Definition 4.11. ***Block***

If X is a G-set, then a **block** *is a subset B of X such that, for each $g \in G$, either $gB = B$ or $gB \cap B = \emptyset$, where $gB = \{gx : x \in B\}$. Any other block is* ***nontrivial****. (Rot95)*

Definition 4.12. ***Primitive***

A transitive G-set X is **primitive** *if it contains no nontrivial block; otherwise, it is imprimitive. (Rot95)*

Theorem 4.13. *Let X be a transitive G-set. Then X is primitive if and only if for each $x \in X$, the stabilizer G_x is a maximal subgroup. (Rot95)*

Theorem 4.14.

(i) If X is a faithful primitive G-set of degree $n \geq 2$, if $H \lhd G$ and if $H \neq 1$, then X is a transitive H-set.

(ii) n divides $|H|$.

(Rot95)

Definition 1.1.53. (**Double Coset Algorithm**) *Perform the double coset enumeration of group G over transitive group N, where double cosets take the form*

$$NwN = \{Nwn \mid n \in N\} = \{Nw^n \mid n \in N\}.$$

(i) Compute the point-stabilizer N^w and coset stabilizer of each double coset.
(ii) Compute the number of right cosets by using the formula $\frac{|N|}{|N^{(w)}|}$,
where $N^{(w)} = \{n \in N \mid Nw^n = Nw\}$ is the coset stabilizer of the right coset.
(iii) For each double coset NwN, compute the orbits of $N^{(w)}$. It suffices to determine the double coset of Nwt_i for a single representative of each orbit. Note, $N^{(w)} \geq N^w$ is always true.
(iv) Determine which double coset each coset representative Nwt_i belongs to, (repeat the process until closed by coset multiplication).

4.2 Manual Double Coset Enumeration of $3^2 : S_3$ over $3^{*3} : S_3$

The group $G_1 = 3^{*3} : S_3$ factored by the relations $t * t^x = t^x * t, t^{x^2} * t^x * t$ with order 54 is isomorphic to $3^2 : S_3$. We wish to show that

$$G = \frac{3^{*3} : S_3}{t * t^x = t^x * t, t^{x^2} * t^x * t = e} \cong 3^2 : S_3.$$

Consider the group generated by $x = (0, 1, 2)(\bar{0}, \bar{1}, \bar{2})$ and $y = (0, 1)(\bar{0}, \bar{1})$ such that, $G =< x, y, t | x^3, y^2, (x * y)^2, t^3, (t, y), t * t^x = t^x * t, t^{x^2} * t^x * t >$. We will let $t = t_0$, which we know has order 3. The control group, $N = S_3 =< (0, 1, 2)(\bar{0}, \bar{1}, \bar{2}), (0, 1)(\bar{0}, \bar{1}) >$ is defined as the following.

$$\begin{aligned} N = & \ \{e, (0,1,2)(\bar{0},\bar{1},\bar{2}), (0,2,1)(\bar{0},\bar{2},\bar{1}), (0,1)(\bar{0},\bar{1}), (1,2)(\bar{1},\bar{2}), (0,2)(\bar{0},\bar{2})\} \\ = & \ \{e, x, x^2, y, xy, x^2y\} \\ = & \ \{e, x, x^{-1}, y, xy, x^{-1}y\}. \end{aligned}$$

First we observe the relation $t * t^x = t^x * t$.

$$\begin{aligned} t * t^x &= t^x * t \\ t_0 \ t_0{}^{(0,1,2)(\bar{0},\bar{1},\bar{2})} &= t_0{}^{(0,1,2)(\bar{0},\bar{1},\bar{2})} t_0 \\ t_0 t_1 &= t_1 t_0 \end{aligned}$$

By conjugating this relation by all of the elements in $N = S_3$, we obtain the following equal names.

$$\begin{aligned} 01 &= 10 \\ 12 &= 21 \\ 20 &= 02 \end{aligned}$$

Our second relation, $t^{x^2} * t^x * t = e$, gives us the following.

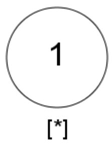
$$\begin{aligned} t^{x^2} * t^x * t &= e \\ t_0{}^{(0,2,1)(\bar{0},\bar{2},\bar{1})} \ t_0{}^{(0,1,2)(\bar{0},\bar{1},\bar{2})} \ t_0 &= e \\ t_2 t_1 t_0 &= e \end{aligned}$$

By conjugating this relation by all of the elements in S_3, we obtain the following relations.

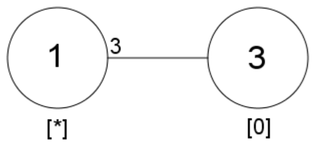
$$210 = e \qquad 021 = e \qquad 102 = e$$

$$120 = e \qquad 012 = e \qquad 201 = e$$

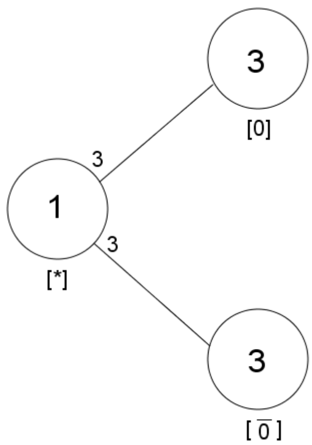
The double coset denoted as [$*$] is $NeN = \{Ne^n|n \in N\} = \{Ne|n \in N\} = \{N\}$, where the coset representative for [$*$] is N. N is transitive on 6 letters.



Three symmetric generators will move forward to a new double coset represented as [0]. The double coset [0] is defined as $Nt_0N = \{Nt_0{}^n|n \in N\} = \{Nt_0, Nt_1, Nt_2\}$. There exists three different single cosets in this double coset.



Similarly three symmetric generators will move forward to a new double coset represented as [$\bar{0}$]. There exists three different single cosets in the double coset [$\bar{0}$], where it is defined as $N\overline{t_0}N = \{N\overline{t_0}{}^n|n \in N\} = \{N\overline{t_0}, N\overline{t_1}, N\overline{t_2}\}$.



We must now determine where the potential six new double cosets that come from the double coset [0] belong to: Nt_0t_0, Nt_0t_1, Nt_0t_2, $Nt_0\overline{t_0}$, $Nt_0\overline{t_1}$, $Nt_0\overline{t_2}$.

Since $t_0{}^2$ is equivalent to $\overline{t_0}$, the first potential new double coset is not new; $Nt_0t_0 = Nt_0{}^2 = N\overline{t_0} \in [\ \bar{0}\ ]$.
The next double coset is also not new as it belongs to $[\ \bar{0}\ ]$: $Nt_0t_1 = N\overline{t_2} \in [\ \bar{0}\ ]$. Right multiplying both sides of our relation from above, $012 = e$, gives us this equivalency.

$$012\overline{2} = e\overline{2}$$

$$01 = \bar{2}$$

The relation, $021 = e$ gives us, $Nt_0t_2 = N\overline{t_1} \in [\ \bar{0}\ ]$, when we right multiply by $\bar{1}$.

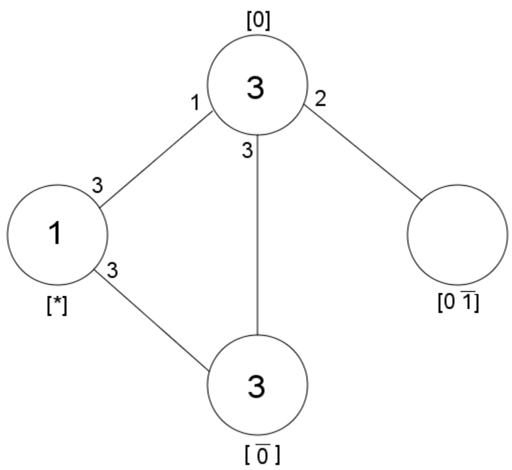
$$021\bar{1} = e\bar{1}$$

$$02 = \bar{1}$$

For the next double coset, $Nt_0\overline{t_0} = N \in [*]$, thus it is not new.

Now, $Nt_0\overline{t_1} \in [0\bar{1}]$ is a new double coset. By the definition of a double coset, $[0\bar{1}]$ is defined as: $Nt_0\overline{t_1}N = \{Nt_0\overline{t_1}{}^n | n \in N\} = \{Nt_0\overline{t_1}, Nt_1\overline{t_2}, Nt_2\overline{t_0}, Nt_1\overline{t_0}, Nt_0\overline{t_2}, Nt_2\overline{t_1}\}$.

Lastly, since the possible new double coset $Nt_0\overline{t_2}N$ belongs in the double coset $[0\bar{1}]$, it is not new. Hence, the two symmetric generators from Nt_0 will move forward to $[0\bar{1}]$.



We must now determine where the potential 6 new double cosets that come from the double coset $[\overline{0}]$ belong to: $N\overline{t_0}t_0$, $N\overline{t_0}t_1$, $N\overline{t_0}t_2$, $N\overline{t_0}\,\overline{t_0}$, $N\overline{t_0}\,\overline{t_1}$, $N\overline{t_0}\,\overline{t_2}$.

Since $\overline{t_0}t_0$ is equivalent to the identity, the first potential new double coset is not new; $N\overline{t_0}t_0 = N \in [*]$.

The next double coset is also not new as it belongs to $[0\bar{1}]$. Left multiplying and then right multiplying both sides of a relation from above, $10 = 01$, gives us the following.

$$10 = 01$$
$$\overline{0}10 = \overline{0}01$$
$$\overline{0}10 = 1$$
$$\overline{0}10\overline{0} = 1\overline{0}$$
$$\overline{0}1 = 1\overline{0}$$

Therefore, $N\overline{t_0}t_1 = Nt_1\overline{t_0} \in [0\bar{1}]$.

The third possible new double coset is not new, $N\overline{t_0}t_2 = Nt_2\overline{t_0} \in [0\bar{1}]$.

$\because 20 = 02$

$\overline{0}20 = \overline{0}02$

$\overline{0}20 = 2$

$\overline{0}20\overline{0} = 2\overline{0}$

$\overline{0}2 = 2\overline{0}$

Also, $N\overline{t_0}\,\overline{t_0} = Nt_0 \in [0]$ is not new.

The next double coset $N\overline{t_0}\,\overline{t_1} = Nt_2 \in [0]$.

$\because 210 = e$

$210\overline{0} = e\,\overline{0}$

$21 = \overline{0}$

$21\overline{1} = \overline{0}\,\overline{1}$

$2 = \overline{0}\,\overline{1}$

Lastly, this double coset is also not new, $N\overline{t_0}\,\overline{t_2} = Nt_1 \in [0]$.

$\because 210 = e$

$210\overline{0} = e\,\overline{0}$

$21 = \overline{0}$

$\overline{2}21 = \overline{2}\,\overline{0}$

$1 = \overline{2}\,\overline{0}$

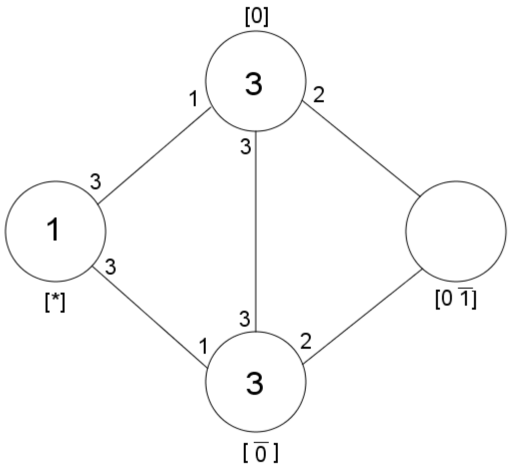
$1 = 22\overline{0}$

$1 = 2\overline{0}2$

$1 = \overline{0}22$

$1 = \overline{0}\,\overline{2}$

We notice there are no new double cosets. One symmetric generator will return to $[*]$, two symmetric generators will return to $[0\overline{1}]$, and three will return to $[0]$.



We must now determine where the potential 6 new double cosets that come from the double coset $[0\overline{1}]$ belong to: $Nt_0\overline{t_1}t_0$, $Nt_0\overline{t_1}t_1$, $Nt_0\overline{t_1}t_2$, $Nt_0\overline{t_1}\,\overline{t_0}$, $Nt_0\overline{t_1}\,\overline{t_1}$, $Nt_0\overline{t_1}\,\overline{t_2}$.

The first double coset, $Nt_0\overline{t_1}t_0 = Nt_2 \in [0]$. From a previous relation above, we obtained $2 = \overline{0}\,\overline{1}$.

Then $2 = 00\overline{1}$.

$\overline{0}2 = \overline{0}00\overline{1}$

$\overline{0}2 = 0\overline{1}$

$\therefore 0\overline{1}0 = \overline{0}20 = \overline{0}02 = 2$

The second double coset, $Nt_0\overline{t_1}t_1 = Nt_0 \in [0]$.

The third double coset, $Nt_0\overline{t_1}t_2 = Nt_1 \in [0]$.

$\because 0\overline{1}2 = 0\overline{1}\,\overline{0}\,\overline{1} = 0\overline{1}0011 = 0\overline{1}0101 = 0\overline{1}1001 = 0001 = 1$, since t is of order 3.

The fourth double coset, $Nt_0\overline{t_1}\,\overline{t_0} = N\overline{t_1} \in [\overline{0}]$.

Since $2 = \overline{0}\,\overline{1}$, then $\overline{0}2 = \overline{0}\,\overline{0}\,\overline{1}$.

Thus, $\overline{0}2 = 0\overline{1}$.

$\therefore 0\overline{1}\,\overline{0} = \overline{0}2\overline{0} = \overline{0}200 = \overline{0}020 = 20 = 02 = 0\overline{0}\,\overline{1} = \overline{1}$

The fifth double coset, $Nt_0\overline{t_1}\,\overline{t_1} = N\overline{t_2} \in [\overline{0}]$.

$\because 0\overline{1}\,\overline{1} = 01 = 0\overline{0}\,\overline{2} = 2$

We know from a previous relation that $1 = \overline{0}\,\overline{2}$.

The sixth double coset, $Nt_0\overline{t_1}\,\overline{t_2} = N\overline{t_0} \in [\overline{0}]$.

We know from a previous relation that $\overline{0}2 = 0\overline{1}$.

Thus, $0\overline{1}\,\overline{2} = \overline{0}2\overline{2} = \overline{0}$.

We can conclude that there are no new double cosets. Then three symmetric generators will return to $[0]$ and three will return to $[\overline{0}]$. Now to complete our Cayley Diagram, we must find the number of single cosets in the double coset $[0\overline{1}]$. We have that, $Nt_0\overline{t_1}N = \{Nt_0\overline{t_1}, Nt_1\overline{t_2}, Nt_2\overline{t_0}, Nt_1\overline{t_0}, Nt_0\overline{t_2}, Nt_2\overline{t_1}\}$. When we were investigating $N\overline{t_0}\,\overline{t_2}$, we discovered $1 = \overline{0}\,\overline{2}$. We find another equivalency using this relation.

$$
\begin{aligned}
1 &= \overline{0}\,\overline{2} \\
1 &= 00\overline{2} \\
\overline{0}1 &= \overline{0}00\overline{2} \\
\overline{0}1 &= 0\overline{2} \\
001 &= 0\overline{2} \\
010 &= 0\overline{2} \\
100 &= 0\overline{2} \\
1\overline{0} &= 0\overline{2} \\
&= 022 \\
&= 202 \\
&= 20\,\overline{0}\,\overline{1} \quad \text{since } \ 2 = \overline{0}\,\overline{1} \\
&= 2\overline{1}
\end{aligned}
$$

Now, $1\overline{0} = 0\overline{2} = 2\overline{1}$. Hence, $Nt_1\overline{t_0} = Nt_0\overline{t_2} = Nt_2\overline{t_1}$. These three equivalent single cosets belong in the double coset, $[0\overline{1}]$.

The relation below gives us the following.

$$
\begin{aligned}
2 &= \overline{0}\,\overline{1} \\
02 &= 0\overline{0}\,\overline{1} \\
02 &= \overline{1} \\
002 &= 0\overline{1} \\
\overline{0}2 &= 0\overline{1} \\
002 &= \\
020 &= \\
200 &= \\
2\overline{0} &= 0\overline{1} \\
&= 011 \\
&= 0\,\overline{0}\,\overline{2}1 \quad \text{since } \ 1 = \overline{0}\,\overline{2} \\
&= \overline{2}1 \\
&= 221 \\
&= 212 \\
&= 122 \\
&= 1\overline{2}
\end{aligned}
$$

Thus $2\overline{0} = 0\overline{1} = 1\overline{2}$. Hence, $Nt_2\overline{t_0} = Nt_0\overline{t_1} = Nt_1\overline{t_2}$. These three equivalent single cosets also belong in the double coset, $[0\overline{1}]$.

We now conclude that there are two distinct single cosets in the double coset $[0\bar{1}]$. Our Cayley Diagram is now complete.

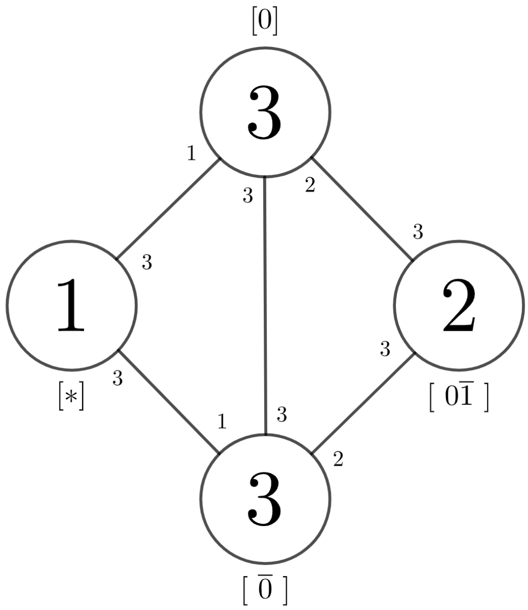


Figure 4.1: Cayley Diagram of $3^2 : S_3$

4.3 M_{11} as a Homomorphic Image of $2^{*8} : 2^2 {}^{\cdot} A_4$

We wish to prove that the progenitor $2^{*8} : (2^2 {}^{\cdot} A_4)$ factored by the relations $(c^{-1}a^{-1}t^b)^3, (c^{-1}a^{-1}t)^6$, and $(adt^b)^5$, is isomorphic to the sporadic simple group, the Mathieu group M_{11} of order 7,920, by performing Double Coset Enumeration. We wish to show that

$$G = \frac{2^{*8} : (2^2 {}^{\cdot} A_4)}{(c^{-1}a^{-1}t^b)^3, (c^{-1}a^{-1}t)^6, (adt^b)^5} \cong M_{11}.$$

Consider the group,

$$G =< a, b, c, d, t | b^4, c^4, d^2, a^3d, b^{-2}d, c^{-1}b^2c^{-1}, b^{-1}c^{-1}bc^{-1}, a^{-1}dad, b^{-1}dbd, c^{-1}dcd,$$
$$c^{-1}a^{-2}b^{-1}a^{-1}, ac^{-1}a^{-1}bc^{-1}, t^2, (t, a^2), (c^{-1}a^{-1}t^b)^3, (c^{-1}a^{-1}t)^6, (adt^b)^5 > .$$

Let $a = (1,2)(3,7,4,5,8,6)$, $b = (1,3,2,5)(4,8,6,7)$, $c = (1,4,2,6)(3,7,5,8)$, and $d = (1,2)(3,5)(4,6)(7,8)$, where $N = (2^2 \cdot A_4) =< a, b, c, d >$. We will let $t = t_1$.

Double Coset Enumeration

The double coset denoted as $[*]$ is $NeN = \{N\}$, where $N = \{1,2,3,4,5,6,7,8\}$. The number of single cosets in $[*]$ is the number of right cosets which can be determined by $\frac{|N|}{|N|} = \frac{24}{24} = 1$. In order to move forward, we choose a representative from the orbit $\{1,2,3,4,5,6,7,8\}$. In this case we will choose 1. Since there are 8 elements in the orbit $\{1,2,3,4,5,6,7,8\}$, 8 symmetric generators will move forward to the new double coset.

The new double coset denoted as $[1]$ is $Nt_1N = \{Nt_1, Nt_2, Nt_3, Nt_4, Nt_5, Nt_6, Nt_7, Nt_8\}$. Now $N^1 =< e, (3,4,8)(5,6,7) >= N^{(1)}$. We note that the inverse of $(3,4,8)(5,6,7)$ is also included in the coset stabiliser. The number of singles cosets in $[1]$ are $\frac{|N|}{|N^{(1)}|} = \frac{24}{3} = 8$. The orbits of $N^{(1)}$ are $\{1\}$, $\{2\}$, $\{3,4,8\}$, and $\{5,6,7\}$. Choosing a representative from each orbit, we have four double cosets. But, $Nt_1t_1 = N \in [*]$ since t is of order 2. There is one element in orbit $\{1\}$, thus one symmetric generator will return to $[*]$. Thus, we have three possible new double cosets: $Nt_1t_2, Nt_1t_3, Nt_1t_5$.

The Relation

We will further investigate our relations that we factored by to determine whether these double cosets are new.

Our first relation is:

$$\begin{aligned}(c^{-1}a^{-1}{t_1}^b)^3 &= c^{-1}\cdot a^{-1}\cdot {t_1}^b\cdot c^{-1}\cdot a^{-1}\cdot {t_1}^b\cdot c^{-1}\cdot a^{-1}\cdot {t_1}^b\\
&= c^{-1}\cdot a^{-1}\cdot {t_1}^b\cdot c^{-1}\cdot a^{-1}\cdot \underline{c^{-1}\cdot c}\cdot {t_1}^b\cdot c^{-1}\cdot a^{-1}\cdot {t_1}^b\\
&= c^{-1}\cdot a^{-1}\cdot {t_1}^b\cdot c^{-1}\cdot a^{-1}\cdot c^{-1}\cdot \underline{c\cdot {t_1}^b\cdot c^{-1}}\cdot a^{-1}\cdot {t_1}^b\\
&= c^{-1}\cdot a^{-1}\cdot {t_1}^b\cdot c^{-1}\cdot a^{-1}\cdot c^{-1}\cdot \underline{({t_1}^b)^{c^{-1}}}\cdot a^{-1}\cdot {t_1}^b\\
&= c^{-1}\cdot a^{-1}\cdot {t_1}^b\cdot c^{-1}\cdot a^{-1}\cdot c^{-1}\cdot \underline{a^{-1}\cdot a}\cdot ({t_1}^b)^{c^{-1}}\cdot a^{-1}\cdot {t_1}^b\\
&= c^{-1}\cdot a^{-1}\cdot {t_1}^b\cdot c^{-1}\cdot a^{-1}\cdot c^{-1}\cdot a^{-1}\cdot \underline{a\cdot ({t_1}^b)^{c^{-1}}\cdot a^{-1}}\cdot {t_1}^b\\
&= c^{-1}\cdot a^{-1}\cdot {t_1}^b\cdot c^{-1}\cdot a^{-1}\cdot c^{-1}\cdot a^{-1}\cdot \underline{({t_1}^b)^{c^{-1}a^{-1}}}\cdot {t_1}^b\end{aligned}$$

Continuing this process we will obtain the following.

$$\begin{aligned}&c^{-1}\cdot a^{-1}\cdot \underline{{t_1}^b\cdot c^{-1}\cdot a^{-1}\cdot c^{-1}\cdot a^{-1}}\cdot ({t_1}^b)^{c^{-1}a^{-1}}\cdot {t_1}^b\\
&= c^{-1}\cdot a^{-1}\cdot \underline{c^{-1}\cdot a^{-1}\cdot c^{-1}\cdot a^{-1}\cdot ({t_1}^b)^{(c^{-1}a^{-1})^2}}\cdot ({t_1}^b)^{c^{-1}a^{-1}}\cdot {t_1}^b\\
&= (c^{-1}\cdot a^{-1})^3\cdot ({t_1}^b)^{(c^{-1}a^{-1})^2}\cdot ({t_1}^b)^{c^{-1}a^{-1}}\cdot {t_1}^b\\
&= (\underline{c^{-1}}\cdot \underline{a^{-1}})^3\cdot ({t_1}^b)^{(c^{-1}a^{-1})^2}\cdot ({t_1}^b)^{c^{-1}a^{-1}}\cdot {t_1}^b\\
&= (\underline{(1,6,2,4)(3,8,5,7)}\ \underline{(1,2)(3,6,8,5,4,7)})^3\cdot ({t_1}^b)^{(c^{-1}a^{-1})^2}\cdot ({t_1}^b)^{c^{-1}a^{-1}}\cdot {t_1}^b\\
&= ((1,8,4,2,7,6)(3,5))^3\cdot ({t_1}^b)^{(c^{-1}a^{-1})^2}\cdot ({t_1}^b)^{c^{-1}a^{-1}}\cdot {t_1}^b\\
&= (1,2)(3,5)(4,6)(7,8)\cdot ({t_1}^b)^{(c^{-1}a^{-1})^2}\cdot ({t_1}^b)^{c^{-1}a^{-1}}\cdot {t_1}^b\\
&= (1,2)(3,5)(4,6)(7,8)\cdot ({t_1}^b)^{((1,8,4,2,7,6)(3,5))^2}\cdot ({t_1}^b)^{(1,8,4,2,7,6)(3,5)}\cdot {t_1}^b\\
&= (1,2)(3,5)(4,6)(7,8)\cdot ({t_1}^b)^{(1,4,7)(2,6,8)}\cdot ({t_1}^b)^{(1,8,4,2,7,6)(3,5)}\cdot {t_1}^b\\
&= (1,2)(3,5)(4,6)(7,8)\cdot ({t_1}^{(1,3,2,5)(4,8,6,7)})^{(1,4,7)(2,6,8)}\\
&\quad\cdot ({t_1}^{(1,3,2,5)(4,8,6,7)})^{(1,8,4,2,7,6)(3,5)}\cdot {t_1}^{(1,3,2,5)(4,8,6,7)}\\
&= (1,2)(3,5)(4,6)(7,8)\cdot (t_3)^{(1,4,7)(2,6,8)}\cdot (t_3)^{(1,8,4,2,7,6)(3,5)}\cdot t_3\\
&= (1,2)(3,5)(4,6)(7,8)\cdot t_3\cdot t_5\cdot t_3\end{aligned}$$

Thus, our relation gives us $(1,2)(3,5)(4,6)(7,8)t_3t_5t_3 = e \Rightarrow Nt_3t_5 = Nt_3$.

Following the same rule of the definition of conjugation, we investigate our second relation.

$$
\begin{aligned}
(c^{-1}a^{-1}t_1)^6 &= (c^{-1}\cdot a^{-1})^6 \cdot {t_1}^{(c^{-1}\cdot a^{-1})^5} \cdot {t_1}^{(c^{-1}\cdot a^{-1})^4} \cdot {t_1}^{(c^{-1}\cdot a^{-1})^3} \\
&\cdot {t_1}^{(c^{-1}\cdot a^{-1})^2} \cdot {t_1}^{(c^{-1}\cdot a^{-1})} \cdot t_1 \\
&= ((1,8,4,2,7,6)(3,5))^6 \cdot {t_1}^{(1,8,4,2,7,6)(3,5)^5} \cdot {t_1}^{(1,8,4,2,7,6)(3,5)^4} \\
&\cdot {t_1}^{(1,8,4,2,7,6)(3,5)^3} \cdot {t_1}^{(1,8,4,2,7,6)(3,5)^2} \cdot {t_1}^{(1,8,4,2,7,6)(3,5)} \cdot t_1 \\
&= e \cdot {t_1}^{(1,6,7,2,4,8)(3,5)} \cdot {t_1}^{(1,7,4)(2,8,6)} \cdot {t_1}^{(1,2)(3,5)(4,6)(7,8)} \cdot {t_1}^{(1,4,7)(2,6,8)} \cdot t_8 \cdot t_1 \\
&= t_6 \cdot t_7 \cdot t_2 \cdot t_4 \cdot t_8 \cdot t_1
\end{aligned}
$$

Thus, our relation gives us $t_6t_7t_2t_4t_8t_1 = e \Rightarrow Nt_6t_7t_2 = Nt_1t_8t_4$.

We conjugate the third relation and obtain the following.

$$
\begin{aligned}
(adt_1{}^b)^5 &= (a\cdot d)^5 \cdot (t_1{}^b)^{(a\cdot d)^4} \cdot (t_1{}^b)^{(a\cdot d)^3} \cdot (t_1{}^b)^{(a\cdot d)^2} \cdot (t_1{}^b)^{(a\cdot d)} \cdot t_1{}^b \\
&= ((3,8,4)(5,7,6))^5 \cdot (t_3)^{((3,8,4)(5,7,6))^4} \cdot (t_3)^{((3,8,4)(5,7,6))^3} \\
&\cdot (t_3)^{((3,8,4)(5,7,6))^2} \cdot (t_3)^{(3,8,4)(5,7,6)} \cdot t_3 \\
&= (3,4,8)(5,6,7) \cdot (t_3)^{(3,8,4)(5,7,6)} \cdot (t_3)^e \cdot (t_3)^{(3,4,8)(5,6,7)} \cdot t_8 \cdot t_3 \\
&= (3,4,8)(5,6,7) \cdot t_8 \cdot t_3 \cdot t_4 \cdot t_8 \cdot t_3
\end{aligned}
$$

Thus, our relation gives us $(3,4,8)(5,6,7)t_8t_3t_4t_8t_3 = e \Rightarrow Nt_8t_3t_4 = Nt_3t_8$.

Investigating Nt_1t_2

Consider the double coset denoted as [12] such that $Nt_1t_2N = \{Nt_1t_2, Nt_2t_1, Nt_3t_5, Nt_4t_6, Nt_5t_3, Nt_6t_4, Nt_7t_8, Nt_8t_7\}$. The coset counter does not increase in Magma, thus we know that Nt_1t_2N is not a new double coset. Using the following code, generated by Leonard Lamp and Dustin Grindstaff, helps us determine whether the proposed new double coset is equal to a previous double coset with one t in our diagram that we have discovered.

```
for a in [1..8] do for g,h in IN do if ts[1]*ts[2] eq g*(ts[a])^{h}
then "1,2=", a; end if; end for; end for;
```

(Lam15)(Gri15)

We obtain that $Nt_1t_2N = Nt_1N$. We wish to determine the permutation that proves $Nt_1t_2N = Nt_1N$. There exists two elements, $g, h \in N$, such that

$$t_1t_2 = g \cdot (t_1)^{\ h} = (1,2)(3,5)(4,6)(7,8) \cdot (t_1)^{\ e} = (1,2)(3,5)(4,6)(7,8)t_1.$$

By taking N of both sides, we have $Nt_1t_2 = Nt_1 \in [1] = Nt_1 \Rightarrow Nt_1t_2N = Nt_1N$. There is one element in $\{2\}$, thus one symmetric generator will return to itself, [1].

Investigating Nt_1t_3

Consider the double coset denoted as [13] where $Nt_1t_3N = \{Nt_1t_3, Nt_2t_7, Nt_3t_2, Nt_4t_7, Nt_2t_5, Nt_1t_4, Nt_5t_4, Nt_6t_5, Nt_1t_8, Nt_7t_1, Nt_7t_6, Nt_5t_1, Nt_8t_4, Nt_6t_8, Nt_3t_8, Nt_4t_2, Nt_2t_6, Nt_8t_5, Nt_8t_2, Nt_3t_6, Nt_4t_3, Nt_7t_3, Nt_5t_7, Nt_6t_1\}$.

The orbit $\{3, 4, 8\}$ had three elements, thus three symmetric generators moved forward to Nt_1t_3. Now $N^{13} =< e >= N^{(13)}$. The number of singles cosets in [13] are $\frac{|N|}{|N^{(13)}|} = \frac{24}{1} = 24$. The orbits of $N^{(13)}$ are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, $\{6\}$, $\{7\}$, and $\{8\}$. Choosing a representative from each orbit, we have eight double cosets. Selecting the orbit $\{3\}$, we realize $Nt_1t_3t_3 = Nt_1 \in [1]$. There is one element in $\{3\}$, thus one symmetric generator will return to [1]. Thus we have seven possible new double cosets: $Nt_1t_3t_1, Nt_1t_3t_2, Nt_1t_3t_4, Nt_1t_3t_5, Nt_1t_3t_6, Nt_1t_3t_7, Nt_1t_3t_8$.

Investigating Nt_1t_5

Consider the double coset denoted as [15] where $Nt_1t_5N = \{Nt_1t_5, Nt_2t_8, Nt_3t_1, Nt_4t_8, Nt_2t_3, Nt_1t_6, Nt_5t_6, Nt_6t_3, Nt_1t_7, Nt_7t_2, Nt_7t_4, Nt_5t_2, Nt_8t_6, Nt_6t_7, Nt_3t_7, Nt_4t_1, Nt_2t_4, Nt_8t_3, Nt_8t_1, Nt_3t_4, Nt_4t_5, Nt_7t_5, Nt_5t_8, Nt_6t_2\}$.

The orbit $\{5, 6, 7\}$ had three elements, thus three symmetric generators moved forward to Nt_1t_5. Now $N^{15} =< e >= N^{(15)}$. The number of singles cosets in [15] are $\frac{|N|}{|N^{(15)}|} = \frac{24}{1} = 24$. The orbits of $N^{(15)}$ are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, $\{6\}$, $\{7\}$, and $\{8\}$. Choosing a representative from each orbit, we have eight double cosets. From orbit $\{5\}$, $Nt_1t_5t_5 = Nt_1 \in [1]$. There is one element in $\{5\}$, thus one symmetric generator will return to [1]. Thus we have seven possible new double cosets: $Nt_1t_5t_1, Nt_1t_5t_2, Nt_1t_5t_3, Nt_1t_5t_4, Nt_1t_5t_6, Nt_1t_5t_7, Nt_1t_5t_8$.

Investigating $Nt_1t_3t_1$

Consider the double coset denoted as [131]; $Nt_1t_3t_1N = \{Nt_1t_3t_1, Nt_2t_7t_2, Nt_3t_2t_3, Nt_4t_7t_4, Nt_2t_5t_2, Nt_1t_4t_1, Nt_5t_4t_5, Nt_6t_5t_6, Nt_1t_8t_1, Nt_7t_1t_7, Nt_7t_6t_7, Nt_5t_1t_5, Nt_8t_4t_8, Nt_6t_8t_6, Nt_3t_8t_3, Nt_4t_2t_4, Nt_2t_6t_2, Nt_8t_5t_8, Nt_8t_2t_8, Nt_3t_6t_3, Nt_4t_3t_4, Nt_7t_3t_7, Nt_5t_7t_5, Nt_6t_1t_6\}$. The orbit $\{1\}$ had one element, thus one symmetric generator moved forward to $Nt_1t_3t_1$. Now $N^{131} =< e >= N^{(131)}$. The number of singles cosets in [131] are $\frac{|N|}{|N^{(131)}|} = \frac{24}{1} = 24$. The orbits of $N^{(131)}$ are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, $\{6\}$, $\{7\}$, and $\{8\}$. Choosing a representative from each orbit, we presume to have eight new double cosets. We have $Nt_1t_3t_1t_1 = Nt_1t_3 \in [13]$. There is one element in the orbit $\{1\}$, thus one symmetric generator will return to [13]. Thus we have seven possible new double cosets: $Nt_1t_3t_1t_2, Nt_1t_3t_1t_3, Nt_1t_3t_1t_4, Nt_1t_3t_1t_5, Nt_1t_3t_1t_6, Nt_1t_3t_1t_7, Nt_1t_3t_1t_8$.

Investigating $Nt_1t_3t_2$

Consider the double coset denoted as [132]; $Nt_1t_3t_2N = \{Nt_1t_3t_2, Nt_2t_7t_1, Nt_3t_2t_5, Nt_4t_7t_6, Nt_2t_5t_1, Nt_1t_4t_2, Nt_5t_4t_3, Nt_6t_5t_4, Nt_1t_8t_2, Nt_7t_1t_8, Nt_7t_6t_8, Nt_5t_1t_3, Nt_8t_4t_7, Nt_6t_8t_4, Nt_3t_8t_5, Nt_4t_2t_6, Nt_2t_6t_1, Nt_8t_5t_7, Nt_8t_2t_7, Nt_3t_6t_5, Nt_4t_3t_6, Nt_7t_3t_8, Nt_5t_7t_3, Nt_6t_1t_4\}$. The orbit $\{2\}$ had one element, thus one symmetric generator moved forward to $Nt_1t_3t_2$. Now $N^{132} =< e >= N^{(132)}$. The number of singles cosets in [132] are $\frac{|N|}{|N^{(132)}|} = \frac{24}{1} = 24$. The orbits of $N^{(132)}$ are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, $\{6\}$, $\{7\}$, and $\{8\}$. Choosing a representative from each orbit, we possibly have eight new double cosets. We have $Nt_1t_3t_2t_2 = Nt_1t_3 \in [13]$. There is one element in the orbit $\{2\}$, thus one symmetric generator will return to [13]. Thus we have seven possible new double cosets: $Nt_1t_3t_2t_1, Nt_1t_3t_2t_3, Nt_1t_3t_2t_4, Nt_1t_3t_2t_5, Nt_1t_3t_2t_6, Nt_1t_3t_2t_7, Nt_1t_3t_2t_8$.

Investigating $Nt_1t_3t_4$

Consider the double coset denoted as [134]; $Nt_1t_3t_4N = \{Nt_1t_3t_4, Nt_2t_7t_5, Nt_3t_2t_8, Nt_4t_7t_2, Nt_2t_5t_6, Nt_1t_4t_8, Nt_5t_4t_1, Nt_6t_5t_8, Nt_1t_8t_3, Nt_7t_1t_6, Nt_7t_6t_3, Nt_5t_1t_7, Nt_8t_4t_5, Nt_6t_8t_1, Nt_3t_8t_6, Nt_4t_2t_3, Nt_2t_6t_7, Nt_8t_5t_2, Nt_8t_2t_4, Nt_3t_6t_2, Nt_4t_3t_7, Nt_7t_3t_1, Nt_5t_7t_4, Nt_6t_1t_5\}$. The orbit $\{4\}$ had one element, thus one symmetric generator moved forward to $Nt_1t_3t_4$. Now $N^{134} =< e >= N^{(134)}$. The number of singles cosets in [134] are $\frac{|N|}{|N^{(134)}|} = \frac{24}{1} = 24$. The orbits of $N^{(134)}$ are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, $\{6\}$, $\{7\}$, and $\{8\}$. Choosing a representative from each orbit, we have eight double cosets. Since t is of order 2, we have $Nt_1t_3t_4t_4 = Nt_1t_3 \in [13]$. There is one element in $\{4\}$, thus one symmetric generator will return to [13]. Thus we have seven possible new double cosets: $Nt_1t_3t_4t_1, Nt_1t_3t_4t_2, Nt_1t_3t_4t_3, Nt_1t_3t_4t_5, Nt_1t_3t_4t_6, Nt_1t_3t_4t_7, Nt_1t_3t_4t_8$.

Investigating $Nt_1t_3t_5$

Consider the double coset denoted as [135]; $Nt_1t_3t_5N = \{Nt_1t_3t_5, Nt_2t_7t_8, Nt_3t_2t_1, Nt_4t_7t_8, Nt_2t_5t_3, Nt_1t_4t_6, Nt_5t_4t_6, Nt_6t_5t_3, Nt_1t_8t_7, Nt_7t_1t_2, Nt_7t_6t_4, Nt_5t_1t_2, Nt_8t_4t_6, Nt_6t_8t_7, Nt_3t_8t_7, Nt_4t_2t_1, Nt_2t_6t_4, Nt_8t_5t_3, Nt_8t_2t_1, Nt_3t_6t_4, Nt_4t_3t_5, Nt_7t_3t_5, Nt_5t_7t_8, Nt_6t_1t_2\}$. Using Magma, the coset counter does not increase, thus we know that $Nt_1t_3t_5N$ is not a new double coset. Using the following code helps us determine whether the proposed new double coset is equal to a previous double coset with word of length one or two in our diagram, that we have discovered.

```
for a in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[5] eq
g*(ts[a])^{h} then "1,3,5=", a;
end if; end for; end for;

for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[5] eq
g*(ts[a]*ts[b])^{h} then "1,3,5=", a,b;
end if; end for; end for;
```

We obtain that $Nt_1t_3t_5N = Nt_1t_5N$. We wish to determine the permutation that proves this. There exists two elements, $g, h \in N$, such that

$$\begin{aligned} t_1t_3t_5 &= g \cdot (t_1t_5)^{\ h} = (1,2)(3,5)(4,6)(7,8) \cdot (t_1t_5)^{\ (1,2)(3,5)(4,6)(7,8)} \\ &= (1,2)(3,5)(4,6)(7,8)t_2t_3. \end{aligned}$$

By taking N of both sides, we have $Nt_1t_3t_5 = Nt_2t_3 \in [15] \Rightarrow Nt_1t_3t_5N = Nt_1t_5N$. The symmetric generator contained in the orbit $\{5\}$, will return to the double coset [15].

Investigating $Nt_1t_3t_6$

Consider the double coset denoted as [136]; $Nt_1t_3t_6N = \{Nt_1t_3t_6, Nt_2t_7t_3, Nt_3t_2t_7, Nt_4t_7t_1, Nt_2t_5t_4, Nt_1t_4t_7, Nt_5t_4t_2, Nt_6t_5t_7, Nt_1t_8t_5, Nt_7t_1t_4, Nt_7t_6t_5, Nt_5t_1t_8, Nt_8t_4t_3, Nt_6t_8t_2, Nt_3t_8t_4, Nt_4t_2t_5, Nt_2t_6t_8, Nt_8t_5t_1, Nt_8t_2t_6, Nt_3t_6t_1, Nt_4t_3t_8, Nt_7t_3t_2, Nt_5t_7t_6, Nt_6t_1t_3\}$. Since the coset counter does not increase in Magma, we know that $Nt_1t_3t_6N$ is not a new double coset.

```
for a in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[6] eq
g*(ts[a])^{h} then "1,3,6=", a; end if; end for; end for;

for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[6] eq
g*(ts[a]*ts[b])^{h} then "1,3,6=", a,b; end if; end for; end for;
```

We obtain that $Nt_1t_3t_6N$ =Nt_1t_5N. We wish to determine the permutation that proves this. There exists two elements, $g, h \in N$, such that

$$t_1t_3t_6 = g \cdot (t_1t_5)^{\ h} = (1,6,3)(2,4,5) \cdot (t_1t_5)^{\ (1,3,2,5)(4,8,6,7)} = (1,6,3)(2,4,5)t_3t_1\ .$$

By taking N of both sides, we have $Nt_1t_3t_6 = Nt_3t_1 \in [15] \Rightarrow Nt_1t_3t_6N = Nt_1t_5N$. The symmetric generator contained in the orbit $\{6\}$ will return to the double coset [15].

Investigating $Nt_1t_3t_7$

Consider the double coset denoted as [137]; $Nt_1t_3t_7N = \{Nt_1t_3t_7, Nt_2t_7t_4, Nt_3t_2t_4, Nt_4t_7t_5, Nt_2t_5t_8, Nt_1t_4t_5, Nt_5t_4t_8, Nt_6t_5t_2, Nt_1t_8t_6, Nt_7t_1t_5, Nt_7t_6t_2, Nt_5t_1t_6, Nt_8t_4t_1, Nt_6t_8t_3, Nt_3t_8t_1, Nt_4t_2t_8, Nt_2t_6t_3, Nt_8t_5t_6, Nt_8t_2t_3, Nt_3t_6t_7, Nt_4t_3t_1, Nt_7t_3t_4, Nt_5t_7t_2, Nt_6t_1t_7\}$. The orbit $\{7\}$ had one element, therefore one symmetric generator moved forward to $Nt_1t_3t_7$. Now $N^{137} =< e >= N^{(137)}$. The number of singles cosets in [137] are $\frac{|N|}{|N^{(137)}|} = \frac{24}{1} = 24$. The orbits of $N^{(137)}$ are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, $\{6\}$, $\{7\}$, and $\{8\}$. Choosing a representative from each orbit, we presumably have eight double cosets. By conjugation, $Nt_1t_3t_7t_7 = Nt_1t_3 \in [13]$. There is one element in $\{7\}$, thus one symmetric generator will return to [13]. Thus we have seven possible new double cosets: $Nt_1t_3t_7t_1, Nt_1t_3t_7t_2, Nt_1t_3t_7t_3, Nt_1t_3t_7t_4, Nt_1t_3t_7t_5, Nt_1t_3t_7t_6, Nt_1t_3t_7t_8$.

Investigating $Nt_1t_3t_8$

Consider the double coset denoted as [138]; $Nt_1t_3t_8N = \{Nt_1t_3t_8, Nt_2t_7t_6, Nt_3t_2t_6, Nt_4t_7t_3, Nt_2t_5t_7, Nt_1t_4t_3, Nt_5t_4t_7, Nt_6t_5t_1, Nt_1t_8t_4, Nt_7t_1t_3, Nt_7t_6t_1, Nt_5t_1t_4, Nt_8t_4t_2, Nt_6t_8t_5, Nt_3t_8t_2, Nt_4t_2t_7, Nt_2t_6t_5, Nt_8t_5t_4, Nt_8t_2t_5, Nt_3t_6t_8, Nt_4t_3t_2, Nt_7t_3t_6, Nt_5t_7t_1, Nt_6t_1t_8\}$. The orbit $\{8\}$ had one element, therefore one symmetric generator moved forward to $Nt_1t_3t_8$. Now $N^{138} =< e >= N^{(138)}$. The number of singles cosets in [138] are $\frac{|N|}{|N^{(138)}|} = \frac{24}{1} = 24$. The orbits of $N^{(138)}$ are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, $\{6\}$, $\{7\}$, and $\{8\}$. Choosing a representative from each orbit, we have eight double cosets. Since t is of order 2, $Nt_1t_3t_8t_8 = Nt_1t_3 \in [13]$. There is one element in $\{8\}$, thus one symmetric generator will return to [13]. Thus we have seven possible new double cosets: $Nt_1t_3t_8t_1, Nt_1t_3t_8t_2, Nt_1t_3t_8t_3, Nt_1t_3t_8t_4, Nt_1t_3t_8t_5, Nt_1t_3t_8t_6, Nt_1t_3t_8t_7$.

Investigating $Nt_1t_5t_1$

Consider the double coset denoted as [151]; $Nt_1t_5t_1N = \{Nt_1t_5t_1, Nt_2t_8t_2, Nt_3t_1t_3, Nt_4t_8t_4, Nt_2t_3t_2, Nt_1t_6t_1, Nt_5t_6t_5, Nt_6t_3t_6, Nt_1t_7t_1, Nt_7t_2t_7, Nt_7t_4t_7, Nt_5t_2t_5, Nt_8t_6t_8, Nt_6t_7t_6, Nt_3t_7t_3, Nt_4t_1t_4, Nt_2t_4t_2, Nt_8t_3t_8, Nt_8t_1t_8, Nt_3t_4t_3, Nt_4t_5t_4, Nt_7t_5t_7, Nt_5t_8t_5, Nt_6t_2t_6\}$. The orbit $\{1\}$ had one element, thus one symmetric generator moved forward to $Nt_1t_5t_1$. Now $N^{151} =< e >= N^{(151)}$. The number of singles cosets in [151] are $\frac{|N|}{|N^{(151)}|} = \frac{24}{1} = 24$. The orbits of $N^{(151)}$ are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, $\{6\}$, $\{7\}$, and $\{8\}$. Choosing a representative from each orbit, we have eight double cosets. Since t is of order 2, $Nt_1t_5t_1t_1 = Nt_1t_5 \in [15]$. There is one element in the orbit $\{1\}$, thus one symmetric generator will return to [15]. Thus we have seven possible new double cosets: $Nt_1t_5t_1t_2, Nt_1t_5t_1t_3, Nt_1t_5t_1t_4, Nt_1t_5t_1t_5, Nt_1t_5t_1t_6, Nt_1t_5t_1t_7, Nt_1t_5t_1t_8$.

Investigating $Nt_1t_5t_2$

Consider the double coset denoted as [152]; $Nt_1t_5t_2N = \{Nt_1t_5t_2, Nt_2t_8t_1, Nt_3t_1t_5, Nt_4t_8t_6, Nt_2t_3t_1, Nt_1t_6t_2, Nt_5t_6t_3, Nt_6t_3t_4, Nt_1t_7t_2, Nt_7t_2t_8, Nt_7t_4t_8, Nt_5t_2t_3, Nt_8t_6t_7, Nt_6t_7t_4, Nt_3t_7t_5, Nt_4t_1t_6, Nt_2t_4t_1, Nt_8t_3t_7, Nt_8t_1t_7, Nt_3t_4t_5, Nt_4t_5t_6, Nt_7t_5t_8, Nt_5t_8t_3, Nt_6t_2t_4\}$. In Magma, the coset counter does not increase, therefore $Nt_1t_5t_2N$ is not a new double coset. Unfortunately, Magma does not give us a previously found new double coset with word of length one or two that [152] is equivalent to. Since there are new double cosets with word of length three, we investigate which double coset [152] is equivalent to.

```
for a in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[2]
eq g*(ts[a])^{h} then "1,5,2=", a; end if; end for; end for;

for a,b in [1..8] do for g,h in IN do if  ts[1]*ts[5]*ts[2]
eq  g*(ts[a]*ts[b])^{h} then "1,5,2=",a,b;
end if; end for; end for;

for a,b,c in [1..8] do for g,h in IN do if  ts[1]*ts[5]*ts[2]
eq  g*(ts[a]*ts[b]*ts[c])^{h} then "1,5,2=",a,b,c;
end if; end for; end for;
```

We discover $Nt_1t_5t_2N = Nt_1t_3t_2N$. We wish to determine the permutation that proves this. There exists two elements, $g, h \in N$, such that

$$t_1t_5t_2 = g \cdot (t_1t_3t_2)^{\ h} = (1,2)(3,5)(4,6)(7,8) \cdot (t_1t_3t_2)^{\ (1,5,2,3)(4,7,6,8)}$$
$$= (1,2)(3,5)(4,6)(7,8)t_5t_1t_3.$$

Then $Nt_1t_5t_2 = Nt_5t_1t_3 \in [132] \Rightarrow Nt_1t_5t_2N = Nt_1t_3t_2N$ when we take N of both sides. There is one symmetric generator contained in the orbit $\{2\}$, that will return to the double coset [132].

Investigating $Nt_1t_5t_3$

Consider the double coset denoted as [153]; The coset counter does not increase in Magma, thus $Nt_1t_5t_3N$ is not a new double coset. We determine this [153] is equivalent to a double coset of word of length two.

```
for a in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[3]
eq g*(ts[a])^{h} then "1,5,3=", a;
end if; end for; end for;

for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[3]
eq g*(ts[a]*ts[b])^{h} then "1,5,3=", a,b;
end if; end for; end for;
```

The code results in $Nt_1t_5t_3N = Nt_1t_3N$. We wish to determine the permutation that proves this. There exists two elements, $g, h \in N$, such that

$$t_1t_5t_3 = g \cdot (t_1t_3)^h = (1,2)(3,5)(4,6)(7,8) \cdot (t_1t_3)^{(1,2)(3,5)(4,6)(7,8)}$$
$$= (1,2)(3,5)(4,6)(7,8)t_2t_5.$$

By taking N of both sides, we have $Nt_1t_5t_3 = Nt_2t_5 \in [13] \Rightarrow Nt_1t_5t_3N = Nt_1t_3N$. The symmetric generator contained in the orbit $\{3\}$, will return to the double coset [13].

We continue the algorithm for performing double coset enumeration for the remaining double cosets of the group. The techniques and Magma Code are shown in the Appendix. Thus, our completed Cayley Diagram is shown below.

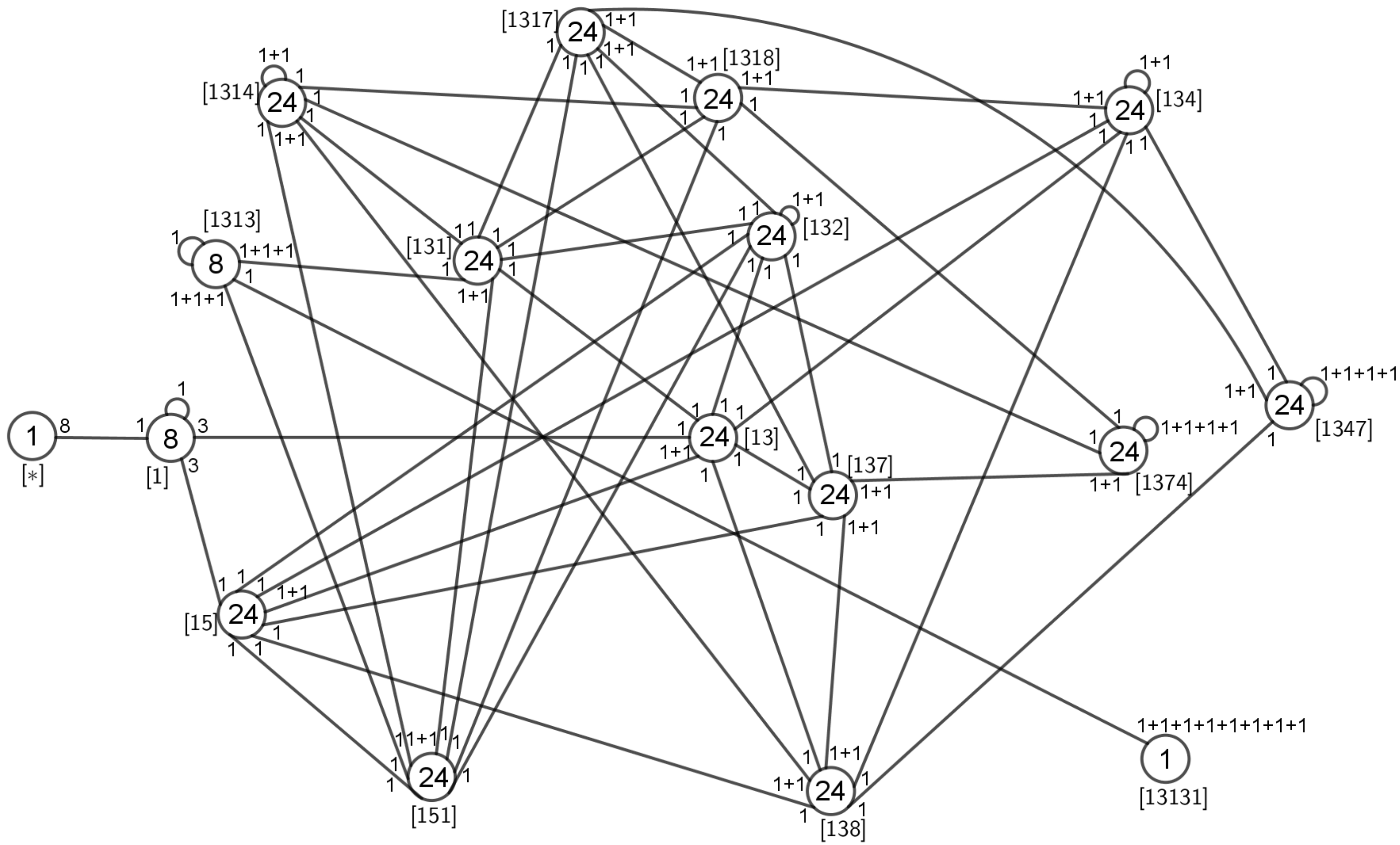


Figure 4.2: Cayley Diagram of M_{11}

4.4 $PSL_2(19)$ as a Homomorphic Image of $2^{*10} : D_{20}$

We will construct the Cayley Diagram of L_219 over $2^{*10} : D_{20}$. We consider the group factored by its center.

$$G = \frac{2^{*10} : D_{20}}{(a^2t)^3, (act)^9, (ca^{-2}t)^5, actata^{-1}tb}$$
$$= < a, b, c, t | b^2, c^2, (a^{-1}b)^2, a^{-1}cac, (bc)^2, a^{-5},$$
$$t^2, (t, ba^2c), (a^2t)^3, (act)^9, (ca^{-2}t)^5, actata^{-1}tb >$$

Now, $N = D_{20} =< a, b, c >$, where $a = (1, 3, 9, 7, 8)(2, 6, 4, 10, 5)$,
$b = (1, 2)(3, 5)(4, 7)(6, 8)(9, 10)$, and $c = (1, 4)(2, 7)(3, 10)(5, 9)(6, 8)$. We will let $t = t_1$.

Double Coset Enumeration

The double coset denoted as $[*]$ is $NeN = \{N\}$, where $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. The number of single cosets in $[*]$ is the number of right cosets which can be determined by $\frac{|N|}{|N|} = \frac{20}{20} = 1$. In order to move forward, we choose a representative from the orbit $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. In this case we will choose 1. Since there are 10 elements in the orbit $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, 10 symmetric generators will move forward to the new double coset.

The new double coset denoted as [1] is $Nt_1N = \{Nt_1, Nt_2, Nt_3, Nt_4, Nt_5, Nt_6, Nt_7, Nt_8, Nt_9, Nt_{10}\}$. Now the point stabilizer is $N^1 =< e, (2, 5)(3, 8)(6, 10)(7, 9) >= N^{(1)}$. The number of single cosets in [1] are $\frac{|N|}{|N^{(1)}|} = \frac{20}{2} = 10$. The orbits of $N^{(1)}$ are $\{1\}$, $\{4\}$, $\{2, 5\}$, $\{3, 8\}$, $\{6, 10\}$, and $\{7, 9\}$. Choosing a representative from each orbit, we have six double cosets. Since t is of order 2, $Nt_1t_1 = N \in [*]$. There is one element in orbit $\{1\}$, thus one symmetric generator will return to $[*]$. Thus, we have five possible new double cosets: $Nt_1t_4, Nt_1t_2, Nt_1t_3, Nt_1t_6, Nt_1t_7$.

The Relation

We factored a finite homomorphic image we had found, by its center, to construct a Cayley diagram with fewer double cosets. We will further investigate our center relation, $actata^{-1}tb$.

$$\begin{aligned}(a \cdot c \cdot t \cdot a \cdot t \cdot a^{-1} \cdot t \cdot b) &= a \cdot c \cdot t_1 \cdot \underline{a \cdot t_1 \cdot a^{-1}} \cdot t_1 \cdot b \\ &= a \cdot c \cdot t_1 \cdot \underline{{t_1}^{a^{-1}}} \cdot t_1 \cdot b \\ &= a \cdot c \cdot t_1 \cdot {t_1}^{a^{-1}} \cdot b \cdot \underline{b^{-1} \cdot t_1 \cdot b} \\ &= a \cdot c \cdot t_1 \cdot {t_1}^{a^{-1}} \cdot b \cdot \underline{{t_1}^b} \\ &= a \cdot c \cdot t_1 \cdot b \cdot \underline{b^{-1} \cdot {t_1}^{a^{-1}} \cdot b} \cdot {t_1}^b a \\ &= a \cdot c \cdot t_1 \cdot b \cdot \underline{\left({t_1}^{a^{-1}}\right)^b} \cdot {t_1}^b \\ &= a \cdot c \cdot b \cdot \underline{b^{-1} \cdot t_1 \cdot b} \cdot \left({t_1}^{a^{-1}}\right)^b \cdot {t_1}^b \\ &= a \cdot c \cdot b \cdot \underline{{t_1}^b} \cdot \left({t_1}^{a^{-1}}\right)^b \cdot {t_1}^b\end{aligned}$$

Then,

$$\begin{aligned}(a \cdot c \cdot t \cdot a \cdot t \cdot a^{-1} \cdot t \cdot b) &= a \cdot c \cdot b \cdot t_1^b \cdot ({t_1}^{a^{-1}})^b \cdot t_1^b \\ &= (1,3,9,7,8)(2,6,4,10,5)(1,4)(2,7)(3,10)(5,9)(6,8) \\ &(1,2)(3,5)(4,7)(6,8)(9,10) \cdot t_1^{(1,2)(3,5)(4,7)(6,8)(9,10)} \\ &\cdot \left({t_1}^{(1,8,7,9,3)(2,5,10,4,6)}\right)^{(1,2)(3,5)(4,7)(6,8)(9,10)} \cdot t_1^{(1,2)(3,5)(4,7)(6,8)(9,10)} \\ &= (1,9)(2,6)(4,5)(7,8)t_2t_6t_2 \\ &= e\end{aligned}$$

Thus, our relation gives us $(1,9)(2,6)(4,5)(7,8)t_2t_6t_2 = e$.

Investigating Nt_1t_4

Consider the double coset denoted as [14]; $Nt_1t_4N = \{Nt_1t_4, Nt_3t_{10}, Nt_2t_7, Nt_4t_1, Nt_9t_5, Nt_5t_9, Nt_{10}t_3, Nt_6t_8, Nt_7t_2, Nt_8t_6\}$. The orbit $\{4\}$ had one element, thus one symmetric generator moved forward to Nt_1t_4. Now the point stabilizer is equivalent to the coset stabilizer and is given by, $N^{14} =< e, (2,5)(3,8)(6,10)(7,9) >= N^{(14)}$. The number of singles cosets in [14] are $\frac{|N|}{|N^{(14)}|} = \frac{20}{2} = 10$.

The orbits of $N^{(14)}$ are $\{1\}$, $\{4\}$, $\{2,5\}$, $\{3,8\}$, $\{6,10\}$, and $\{7,9\}$. Choosing a representative from each orbit, we have 6 double cosets. But, $Nt_1t_4t_4 = Nt_1 \in [1]$. There is one element in orbit $\{4\}$, thus one symmetric generator will return to [1]. Thus, we have five possible new double cosets; $Nt_1t_4t_1, Nt_1t_4t_2, Nt_1t_4t_3, Nt_1t_4t_6, Nt_1t_4t_7$.

Investigating Nt_1t_2

Consider the double coset denoted as [12]; $Nt_1t_2N = \{Nt_1t_2, Nt_3t_6, Nt_2t_1, Nt_4t_7, Nt_9t_4, Nt_5t_8, Nt_{10}t_8, Nt_6t_3, Nt_7t_4, Nt_7t_{10}, Nt_{10}t_7, Nt_5t_1, Nt_9t_6, Nt_4t_9, Nt_8t_5, Nt_8t_{10}, Nt_2t_3, Nt_3t_2, Nt_1t_5, Nt_6t_9\}$. The orbit $\{2,5\}$ had two elements, therefore two symmetric generators moved forward to Nt_1t_2. Now $N^{12} =< e >= N^{(12)}$. The number of singles cosets in [12] are $\frac{|N|}{|N^{(12)}|} = \frac{20}{1} = 20$. The orbits of $N^{(12)}$ are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, $\{6\}$, $\{7\}$, $\{8\}$, $\{9\}$, and $\{10\}$. Choosing a representative from each orbit, we have ten double cosets. But, $Nt_1t_2t_2 = Nt_1 \in [1]$. There is one element in $\{2\}$, thus one symmetric generator will return to [1]. Thus we have nine possible new double cosets: $Nt_1t_2t_1, Nt_1t_2t_3, Nt_1t_2t_4, Nt_1t_2t_5, Nt_1t_2t_6, Nt_1t_2t_7, Nt_1t_2t_8, Nt_1t_2t_9, Nt_1t_2t_{10}$.

Investigating Nt_1t_3

Consider the double coset denoted as [13]; $Nt_1t_3N = \{Nt_1t_3, Nt_3t_9, Nt_2t_5, Nt_4t_{10}, Nt_9t_7, Nt_5t_{10}, Nt_{10}t_5, Nt_6t_2, Nt_7t_9, Nt_7t_8, Nt_{10}t_4, Nt_5t_2, Nt_9t_3, Nt_4t_6, Nt_8t_1, Nt_8t_7, Nt_2t_6, Nt_3t_1, Nt_1t_8, Nt_6t_4\}$. The coset counter did not increase in Magma, thus we know that Nt_1t_3N is not a new double coset. We wish to investigate which double coset Nt_1t_3N is equal to.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[3]
eq g*(ts[a])^{h} then "1,3=", a; end if; end for; end for;
```

We discover $Nt_1t_3N = Nt_1N$. We now wish to determine the permutation that proves this. By conjugating our original relation by an element in the control group, we have

$$[(1,9)(2,6)(4,5)(7,8)t_2t_6t_2]^{(1,5,8,10,7,4,9,6,3,2)} = (5,6)(1,3)(9,8)(4,10)t_1t_3t_1 = e$$

This implies, by right multiplication of t_1 and taking N of both sides, we have

$$(5,6)(1,3)(9,8)(4,10)t_1t_3t_1 = e \Rightarrow Nt_1t_3 = Nt_1 \in [1] \Rightarrow Nt_1t_3N = Nt_1N.$$

Two symmetric generators contained in the orbit $\{3,8\}$ will return to the double coset [1].

Investigating Nt_1t_6

Consider the double coset denoted as [16]; $Nt_1t_6N = \{Nt_1t_6, Nt_3t_4, Nt_2t_8, Nt_4t_8, Nt_9t_{10}, Nt_5t_7, Nt_{10}t_1, Nt_6t_1, Nt_7t_6, Nt_7t_5, Nt_{10}t_9, Nt_5t_3, Nt_9t_2, Nt_4t_3, Nt_8t_2, Nt_8t_4, Nt_2t_9, Nt_3t_5, Nt_1t_{10}, Nt_6t_7\}$. The orbit $\{6, 10\}$ had two elements, therefore two symmetric generators moved forward to Nt_1t_6. Now $N^{16} =< e >= N^{(16)}$. The number of singles cosets in [16] are $\frac{|N|}{|N^{(16)}|} = \frac{20}{1} = 20$. The orbits of $N^{(16)}$ are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, $\{6\}$, $\{7\}$, $\{8\}$, $\{9\}$, and $\{10\}$. Choosing a representative from each orbit, we have ten double cosets. But, $Nt_1t_6t_6 = Nt_1 \in [1]$. There is one element in $\{6\}$, thus one symmetric generator will return to [1]. Thus we have nine possible new double cosets; $Nt_1t_6t_1, Nt_1t_6t_2, Nt_1t_6t_3, Nt_1t_6t_4, Nt_1t_6t_5, Nt_1t_6t_7, Nt_1t_6t_8, Nt_1t_6t_9, Nt_1t_6t_{10}$.

Investigating Nt_1t_7

Consider the double coset denoted as [17]; $Nt_1t_7N = \{Nt_1t_7, Nt_3t_8, Nt_2t_4, Nt_4t_2, Nt_9t_1, Nt_5t_6, Nt_{10}t_6, Nt_6t_{10}, Nt_7t_1, Nt_7t_3, Nt_{10}t_2, Nt_5t_4, Nt_9t_8, Nt_4t_5, Nt_8t_9, Nt_8t_3, Nt_2t_{10}, Nt_3t_7, Nt_1t_9, Nt_6t_5\}$. Using Magma, the coset counter does not increase, thus we know that Nt_1t_7N is not a new double coset. We now determine whether the proposed new double coset is equal to a previous double coset with one t in our diagram, that we have discovered. We obtain that $Nt_1t_7N = Nt_1N$.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[7]
eq g*(ts[a])^{h} then "1,7=", a; end if; end for; end for;
```

We wish to determine the permutation that proves this. Unless stated otherwise, we have used Magma to produce relations throughout the remaining chapter. There exists two elements, $g, h \in N$, such that

$$\begin{aligned} t_1t_7 = g \cdot {t_1}^h &= (1, 8, 7, 9, 3)(2, 5, 10, 4, 6) \cdot {t_1}^{(1,3,9,7,8)(2,6,4,10,5)} \\ &= (1, 8, 7, 9, 3)(2, 5, 10, 4, 6)t_3. \end{aligned}$$

Then by taking N of both sides, we have $Nt_1t_7 = Nt_3 \in [1] = Nt_1 \Rightarrow Nt_1t_7N = Nt_1N$. There are two symmetric generators contained in the orbit $\{7, 9\}$, that will return to itself.

Investigating $Nt_1t_4t_1$

Consider the double coset denoted as [141]; $Nt_1t_4t_1N = \{Nt_1t_4t_1, Nt_3t_{10}t_3, Nt_2t_7t_2, Nt_4t_1t_4, Nt_9t_5t_9, Nt_5t_9t_5, Nt_{10}t_3t_{10}, Nt_6t_8t_6, Nt_7t_2t_7, Nt_8t_6t_8\}$. Since the coset counter does not increase, we know that $Nt_1t_4t_1N$ is not a new double coset. We then determine whether the proposed new double coset is equal to a previous double coset with word of length one or two in our diagram. We obtain that $Nt_1t_4t_1N = Nt_1t_4N$.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[1]
eq g*(ts[a])^{h} then "1,4,1=", a; end if; end for; end for;

for a,b in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[1]
eq g*(ts[a]*ts[b])^{h} then "1,4,1=", a,b; end if; end for; end for;
```

There exists two elements, $g, h \in N$, such that

$$t_1t_4t_1 = g \cdot (t_1t_4)^{\ h} = (1,4)(3,6)(2,9)(8,10)(5,7) \cdot (t_1t_4)^{\ Id(N)}$$
$$= (1,4)(3,6)(2,9)(8,10)(5,7)t_1t_4.$$

Taking N of both sides, we have $Nt_1t_4t_1 = Nt_1t_4 \in [14] \Rightarrow Nt_1t_4t_1N = Nt_1t_4N$. There is one symmetric generator contained in the orbit $\{1\}$ that will return to itself; the double coset [14].

Investigating $Nt_1t_4t_2$

Consider the double coset denoted as [142]; $Nt_1t_4t_2N = \{Nt_1t_4t_2, Nt_3t_{10}t_6, Nt_2t_7t_1, Nt_4t_1t_7Nt_9t_5t_4Nt_5t_9t_8, Nt_{10}t_3t_8, Nt_6t_8t_3, Nt_7t_2t_4, Nt_7t_2t_{10}, Nt_{10}t_3t_7, Nt_5t_9t_1, Nt_9t_5t_6, Nt_4t_1t_9, Nt_8t_6t_5, Nt_8t_6t_{10}, Nt_2t_7t_3, Nt_3t_{10}t_2, Nt_1t_4t_5, Nt_6t_8t_9\}$. $Nt_1t_4t_1N$ is not a new double coset since the coset counter does not increase. We conclude that the double coset $Nt_1t_4t_2N = Nt_1t_2N$ and wish to determine the permutation that proves this. There exists two elements, $g, h \in N$, such that

$$t_1t_4t_2 = g \cdot (t_1t_2)^{\ h} = (1,8,7,9,3)(2,5,10,4,6) \cdot (t_1t_2)^{\ (1,8)(3,7)(2,10)(4,6)}$$
$$= (1,8,7,9,3)(2,5,10,4,6)t_8t_{10}.$$

By taking N of both sides, we have $Nt_1t_4t_2 = Nt_8t_{10} \in [12] = Nt_1t_2$ which implies $Nt_1t_4t_2N = Nt_1t_4N$. There are two symmetric generators contained in the orbit $\{2,5\}$ that will return to the double coset [12].

Investigating $Nt_1t_4t_3$

Consider the double coset denoted as [143];
$Nt_1t_4t_3N = \{N(t_1t_4t_3)^n | n \in N\} = \{Nt_1t_4t_3, Nt_3t_{10}t_9, Nt_2t_7t_5, Nt_4t_1t_{10}, Nt_9t_5t_7, Nt_5t_9t_{10}, Nt_{10}t_3t_5, Nt_6t_8t_2, Nt_7t_2t_9, Nt_7t_2t_8, Nt_{10}t_3t_4, Nt_5t_9t_2, Nt_9t_5t_3, Nt_4t_1t_6, Nt_8t_6t_1, Nt_8t_6t_7, Nt_2t_7t_6, Nt_3t_{10}t_1, Nt_1t_4t_8, Nt_6t_8t_4\}$. The orbit $\{3, 8\}$ had two elements, thus two symmetric generators moved forward to $Nt_1t_4t_3$. Now the coset stabilizer $N^{(143)} \geq N^{143} =< e >$. We find equal names with the help of Magma; we find $Nt_1t_4t_3 = Nt_3t_{10}t_1$. By conjugation $N(t_1t_4t_3)^{(1,3)(4,10)(5,6)(8,9)} = Nt_3t_{10}t_1 = Nt_1t_4t_3$. Thus, $(1, 3)(4, 10)(5, 6)(8, 9) \in N$ belongs in the coset stabilizer.

Now $N^{143} =< e, (1, 3)(4, 10)(5, 6)(8, 9) >= N^{(143)}$. The number of singles cosets in [143] are $\frac{|N|}{|N^{(143)}|} = \frac{20}{2} = 10$. The orbits of $N^{(143)}$ are $\{2\}$, $\{7\}$, $\{1, 3\}$, $\{4, 10\}$, $\{5, 6\}$, and $\{8, 9\}$. Choosing a representative from each orbit, we have six double cosets. But, $Nt_1t_4t_3t_3 = Nt_1t_4 \in [14]$. There are two elements in the orbit $\{1, 3\}$, therefore two symmetric generators will return to [14]. Thus we have five possible new double cosets: $Nt_1t_4t_3t_2, Nt_1t_4t_3t_7, Nt_1t_4t_3t_4, Nt_1t_4t_3t_5, Nt_1t_4t_3t_8$.

Investigating $Nt_1t_4t_6$

Consider the double coset denoted as [146]; $Nt_1t_4t_6N = \{Nt_1t_4t_6, Nt_3t_{10}t_4, Nt_2t_7t_8, Nt_4t_1t_8, Nt_9t_5t_{10}, Nt_5t_9t_7, Nt_{10}t_3t_1, Nt_6t_8t_1, Nt_7t_2t_6, Nt_7t_2t_5, Nt_{10}t_3t_9, Nt_5t_9t_3, Nt_9t_5t_2, Nt_4t_1t_3, Nt_8t_6t_2, Nt_8t_6t_4, Nt_2t_7t_9, Nt_3t_{10}t_5, Nt_1t_4t_{10}, Nt_6t_8t_7\}$.
The coset counter does not increase, thus $Nt_1t_4t_6N$ is not a new double coset.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[6]
eq g*(ts[a])^{h} then "1,4,6=", a; end if; end for; end for;

for a,b in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[6]
eq g*(ts[a]*ts[b])^{h} then "1,4,6=", a,b; end if; end for; end for;
```

We obtain that $Nt_1t_4t_6N = Nt_1t_6N$. We wish to determine the permutation that proves $Nt_1t_4t_6N = Nt_1t_6N$. There exists two elements, $g, h \in N$, such that

$$t_1t_4t_6 = g \cdot (t_1t_6)^{\,h} = (1, 8)(3, 7)(2, 10)(4, 6) \cdot (t_1t_6)^{\,(1,8)(3,7)(2,10)(4,6)}$$
$$= (1, 8)(3, 7)(2, 10)(4, 6)t_8t_4.$$

Taking N of both sides, we have $Nt_1t_4t_6 = Nt_8t_4 \in [16] = Nt_1t_6 \Rightarrow Nt_1t_4t_6N = Nt_1t_6N$. There are two symmetric generators contained in $\{6, 10\}$ that will return to the double coset [16].

Investigating $Nt_1t_4t_7$

Consider the double coset denoted as [147]; $Nt_1t_4t_7N = \{Nt_1t_4t_7, Nt_3t_{10}t_8,$ $Nt_2t_7t_4, Nt_4t_1t_2, Nt_9t_5t_1, Nt_5t_9t_6, Nt_{10}t_3t_6, Nt_6t_8t_{10}, Nt_7t_2t_1, Nt_7t_2t_3, Nt_{10}t_3t_2,$ $Nt_5t_9t_4, Nt_9t_5t_8, Nt_4t_1t_5, Nt_8t_6t_9, Nt_8t_6t_3, Nt_2t_7t_{10}, Nt_3t_{10}t_7, Nt_1t_4t_9, Nt_6t_8t_5\}$.
Since the orbit $\{7, 9\}$ had two elements, thus two symmetric generators moved forward to $Nt_1t_4t_7$. Now $N^{147} =< e >= N^{(147)}$. Notice by conjugation we obtain the following.

$$N(t_1t_4t_7)^{(1,4)(2,7)(3,10)(5,9)(6,8)} = Nt_4t_1t_2 = Nt_1t_4t_7$$

$$N(t_1t_4t_7)^{(1,6)(2,3)(4,8)(5,9)(7,10)} = Nt_6t_8t_{10} = Nt_1t_4t_7$$

$$N(t_1t_4t_7)^{(1,8)(2,10)(3,7)(4,6)} = Nt_8t_6t_3 = Nt_1t_4t_7$$

We find equal names; $Nt_1t_4t_7 = Nt_4t_1t_2 = Nt_6t_8t_{10} = Nt_8t_6t_3$.
Thus, $(1, 4)(2, 7)(3, 10)(5, 9)(6, 8), (1, 6)(2, 3)(4, 8)(5, 9)(7, 10), (1, 8)(2, 10)(3, 7)(4, 6) \in N$ belong in the coset stabilizer. Now $N^{147} =< e, (1, 4)(2, 7)(3, 10)(5, 9)(6, 8),$ $(1, 6)(2, 3)(4, 8)(5, 9)(7, 10), (1, 8)(2, 10)(3, 7)(4, 6) >= N^{(147)}$. The number of singles cosets in [147] are $\frac{|N|}{|N^{(147)}|} = \frac{20}{4} = 5$. The orbits of $N^{(147)}$ are $\{5, 9\}$, $\{1, 4, 6, 8\}$, and $\{2, 7, 3, 10\}$. Choosing a representative from each orbit, we have three double cosets. But, $Nt_1t_4t_7t_7 = Nt_1t_4 \in [14]$. There are four elements in $\{2, 7, 3, 10\}$, so there will be four symmetric generators returning to [14]. Thus we have two possible new double cosets: $Nt_1t_4t_7t_5, Nt_1t_4t_7t_1$.

Investigating $Nt_1t_2t_1$

Consider the double coset denoted as [121]; $Nt_1t_2t_1N = \{Nt_1t_2t_1, Nt_3t_6t_3,$ $Nt_2t_1t_2, Nt_4t_7t_4, Nt_9t_4t_9, Nt_5t_8t_5, Nt_{10}t_8t_{10}, Nt_6t_3t_6, Nt_7t_4t_7, Nt_7t_{10}t_7, Nt_{10}t_7t_{10},$ $Nt_5t_1t_5, Nt_9t_6t_9, Nt_4t_9t_4, Nt_8t_5t_8, Nt_8t_{10}t_8, Nt_2t_3t_2, Nt_3t_2t_3, Nt_1t_5t_1, Nt_6t_9t_6\}$.
Orbit $\{1\}$ had one element, thus one symmetric generator moved forward to $Nt_1t_2t_1$. Thus, $N^{121} =< e >= N^{(121)}$. The number of singles cosets in [121] are $\frac{|N|}{|N^{(121)}|} = \frac{20}{1} = 20$. The orbits of $N^{(121)}$ are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, $\{6\}$, $\{7\}$, $\{8\}$, and $\{9\}$. Since t has order 2, $Nt_1t_2t_1t_1 = Nt_1t_2 \in [12]$. One symmetric generator will return to [12] since there is one element in $\{1\}$. There are nine possible new double cosets: $Nt_1t_2t_1t_2, Nt_1t_2t_1t_3,$ $Nt_1t_2t_1t_4, Nt_1t_2t_1t_5, Nt_1t_2t_1t_6, Nt_1t_2t_1t_7, Nt_1t_2t_1t_8, Nt_1t_2t_1t_9, Nt_1t_2t_1t_{10}$.

Investigating $Nt_1t_2t_3$

Consider the double coset denoted as [123]; $Nt_1t_2t_3N = \{Nt_1t_2t_3\}$. The coset counter does not increase, thus we know that $Nt_1t_2t_3N$ is not a new double coset. We obtain that $Nt_1t_2t_3N = Nt_1t_2N$ using the following code.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[3]
eq g*(ts[a])^{h} then "1,2,3=", a; end if; end for; end for;

for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[3]
eq g*(ts[a]*ts[b])^{h} then "1,2,3=", a,b; end if; end for; end for;
```

We wish to determine the permutation that proves $Nt_1t_2t_3N = Nt_1t_2N$. There exists two elements, $g, h \in N$, such that

$$\begin{aligned} t_1t_2t_3 &= g \cdot (t_1t_2)^{\ h} = (1,4)(3,10)(2,7)(8,6)(9,5) \cdot (t_1t_2)^{\ (1,9)(2,6)(4,5)(8,7)} \\ &= (1,4)(3,10)(2,7)(8,6)(9,5)t_9t_6. \end{aligned}$$

Hence, $Nt_1t_2t_3 = Nt_9t_6 \in [12] = Nt_1t_2 \Rightarrow Nt_1t_2t_3N = Nt_1t_2N$ when we take N of both sides. There is one element in $\{3\}$, thus one symmetric generator will return to itself, [12].

Investigating $Nt_1t_2t_4$

Consider the double coset denoted as [124]; $Nt_1t_2t_4N = \{Nt_1t_2t_4\}$. $Nt_1t_2t_4N$ is not a new double coset; it is equivalent to Nt_1t_6N.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[4]
eq g*(ts[a])^{h} then "1,2,4=", a; end if; end for; end for;

for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[4]
eq g*(ts[a]*ts[b])^{h} then "1,2,4=", a,b; end if; end for; end for;
```

There exists two elements, $g, h \in N$, such that

$$\begin{aligned} t_1t_2t_4 &= g \cdot (t_1t_6)^{\ h} = (1,3,9,7,8)(2,6,4,10,5) \cdot (t_1t_6)^{\ (1,3)(4,10)(8,9)(5,6)} \\ &= (1,3,9,7,8)(2,6,4,10,5)t_3t_5. \end{aligned}$$

Taking N of both sides, we have $Nt_1t_2t_4 = Nt_3t_5 \in [16] = Nt_1t_6 \Rightarrow Nt_1t_2t_4N = Nt_1t_6N$. There is one element in $\{4\}$, thus one symmetric generator will return to [16].

Investigating $Nt_1t_2t_5$

Consider the double coset denoted as [125]; $Nt_1t_2t_5N = \{Nt_1t_2t_5\}$. $Nt_1t_2t_5N$ is equivalent to a previously found double coset, Nt_1t_2N.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[5]
eq g*(ts[a])^{h} then "1,2,5=", a; end if; end for; end for;

for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[5]
eq g*(ts[a]*ts[b])^{h} then "1,2,5=", a,b; end if; end for; end for;
```

There exists two elements, $g, h \in N$, such that

$$\begin{aligned} t_1t_2t_5 &= g \cdot (t_1t_2)^{\ h} = (3,8)(2,5)(9,7)(10,6) \cdot (t_1t_2)^{\ Id(N)} \\ &= (3,8)(2,5)(9,7)(10,6)t_1t_2. \end{aligned}$$

$Nt_1t_2t_5 = Nt_1t_2 \in [12] = Nt_1t_2 \Rightarrow Nt_1t_2t_5N = Nt_1t_2N$ by taking N of both sides. There is one element in $\{5\}$, thus one symmetric generator will return to itself.

Investigating $Nt_1t_2t_6$

Consider the double coset denoted as [126]; $Nt_1t_2t_6N = \{Nt_1t_2t_6\}$. In Magma, the coset counter does not increase, thus we know that $Nt_1t_2t_6N$ is not a new double coset.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[6]
eq g*(ts[a])^{h} then "1,2,6=", a; end if; end for; end for;

for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[6]
eq g*(ts[a]*ts[b])^{h} then "1,2,6=", a,b; end if; end for; end for;
```

We discover $Nt_1t_2t_6N$ $=Nt_1t_6N$. We wish to determine the permutation that proves $Nt_1t_2t_6N$ $=Nt_1t_6N$.There exists two elements, $g, h \in N$, such that

$$\begin{aligned} t_1t_2t_6 &= g \cdot (t_1t_6)^{\ h} = (1,9)(2,6)(4,5)(8,7) \cdot (t_1t_6)^{\ (1,9)(2,6)(4,5)(8,7)} \\ &= (1,9)(2,6)(4,5)(8,7)t_9t_2. \end{aligned}$$

We have $Nt_1t_2t_6 = Nt_9t_2 \in [16] = Nt_1t_6 \Rightarrow Nt_1t_2t_6N = Nt_1t_6N$ by taking N of both sides,. There is one element in $\{6\}$, thus one symmetric generator will return to [16].

Investigating $Nt_1t_2t_7$

Consider the double coset denoted as [127]; $Nt_1t_2t_7N = \{Nt_1t_2t_1, Nt_3t_6t_3, Nt_2t_1t_2, Nt_4t_7t_4, Nt_9t_4t_9, Nt_5t_8t_5, Nt_{10}t_8t_{10}, Nt_6t_3t_6, Nt_7t_4t_7, Nt_7t_{10}t_7, Nt_{10}t_7t_{10}, Nt_5t_1t_5, Nt_9t_6t_9, Nt_4t_9t_4, Nt_8t_5t_8, Nt_8t_{10}t_8, Nt_2t_3t_2, Nt_3t_2t_3, Nt_1t_5t_1, Nt_6t_9t_6\}$.
Since the orbit $\{7\}$ had one element, one symmetric generator moved forward to $Nt_1t_2t_7$. Now $N^{127} =< e >= N^{(127)}$. Notice by conjugation we obtain the following.

$$N(t_1t_2t_7)^{(1,2)(3,5)(4,7)(6,8)(9,10)} = Nt_2t_1t_4 = Nt_1t_2t_7$$

$$N(t_1t_2t_7)^{(1,4)(2,7)(3,10)(5,9)(6,8)} = Nt_4t_7t_2 = Nt_1t_2t_7$$

$$N(t_1t_2t_7)^{(1,7)(2,4)(3,9)(5,10)} = Nt_7t_4t_1 = Nt_1t_2t_7$$

Then the following are equal names are, $Nt_1t_2t_7 = Nt_2t_1t_4 = Nt_4t_7t_2 = Nt_7t_4t_1$. Thus, $(1,2)(3,5)(4,7)(6,8)(9,10), (1,4)(2,7)(3,10)(5,9)(6,8), (1,7)(2,4)(3,9)(5,10) \in N$ belong in the coset stabilizer. Now $N^{127} =< e, (1,2)(3,5)(4,7)(6,8)(9,10), (1,4)(2,7)(3,10)(5,9)(6,8), (1,7)(2,4)(3,9)(5,10) >= N^{(127)}$. The number of singles cosets in [127] are $\frac{|N|}{|N^{(127)}|} = \frac{20}{4} = 5$. The orbits of $N^{(127)}$ are $\{6,8\}$, $\{1,2,4,7\}$, and $\{3,5,10,9\}$. Choosing a representative from each orbit, we have ten double cosets. But, $Nt_1t_2t_7t_7 = Nt_1t_2 \in [12]$. There are four elements in the orbit $\{1,2,4,7\}$, thus four symmetric generators will return to [12]. Thus we have two possible new double cosets: $Nt_1t_2t_7t_6, Nt_1t_2t_7t_3$.

Investigating $Nt_1t_2t_8$

Consider the double coset denoted as [128]; $Nt_1t_2t_8N = \{Nt_1t_2t_8, Nt_3t_6t_1, Nt_2t_1t_6, Nt_4t_7t_6, Nt_9t_4t_3, Nt_5t_8t_2, Nt_{10}t_8t_4, Nt_6t_3t_4, Nt_7t_4t_8, Nt_7t_{10}t_9, Nt_{10}t_7t_5, Nt_5t_1t_{10}, Nt_9t_6t_7, Nt_4t_9t_{10}, Nt_8t_5t_7, Nt_8t_{10}t_1, Nt_2t_3t_5, Nt_3t_2t_9, Nt_1t_5t_3, Nt_6t_9t_2\}$.
One symmetric generator moved forward to $Nt_1t_2t_8$ since the orbit $\{8\}$ had one element. Now $N^{128} =< e >= N^{(128)}$. Notice by conjugation we obtain the following.

$$N(t_1t_2t_8)^{(1,3)(4,10)(5,6)(8,9)} = Nt_3t_2t_9 = Nt_1t_2t_8$$

Since $Nt_1t_2t_8 = Nt_3t_2t_9$, $(1,3)(4,10)(5,6)(8,9) \in N$ belongs in the coset stabilizer. Now $N^{128} =< e, (1,3)(4,10)(5,6)(8,9) >= N^{(128)}$. The number of singles cosets in [128] are $\frac{|N|}{|N^{(128)}|} = \frac{20}{2} = 10$. The orbits of $N^{(128)}$ are $\{2\}$, $\{7\}$, $\{1,3\}$, $\{4,10\}$, $\{5,6\}$, and $\{8,9\}$. Choosing a representative from each orbit, we have six double cosets.

Since t is of order 2, $Nt_1t_2t_8t_8 = Nt_1t_2 \in [12]$. There are two elements in $\{8, 9\}$, thus two symmetric generators will return to [12]. Thus we have five possible new double cosets: $Nt_1t_2t_8t_2, Nt_1t_2t_8t_7, Nt_1t_2t_8t_1, Nt_1t_2t_8t_4, Nt_1t_2t_8t_5$.

Investigating $Nt_1t_2t_9$

Consider the double coset denoted as [129]; $Nt_1t_2t_9N = \{Nt_1t_2t_9\}$. $Nt_1t_2t_9N$ is not a new double coset since the coset counter does not increase. We obtain that $Nt_1t_2t_9N = Nt_1t_2t_1N$.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[9]
eq g*(ts[a])^{h} then "1,2,9=", a; end if; end for; end for;

for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[9]
eq g*(ts[a]*ts[b])^{h} then "1,2,9=", a,b; end if; end for; end for;
```

We wish to determine the permutation that proves $Nt_1t_2t_9N = Nt_1t_2t_1N$. There exists two elements, $g, h \in N$, such that

$$\begin{aligned} t_1t_2t_9 &= g \cdot (t_1t_2t_1)^{\,h} = (1,2)(3,5)(4,7)(8,6)(9,10) \cdot (t_1t_2t_1)^{\,(1,2)(3,5)(4,7)(8,6)(9,10)} \\ &= (1,2)(3,5)(4,7)(8,6)(9,10)t_3t_6t_3. \end{aligned}$$

This implies, by taking N of both sides, $Nt_1t_2t_9 = Nt_3t_6t_3 \in [121] = Nt_1t_2t_1 \Rightarrow Nt_1t_2t_9N = Nt_1t_2t_1N$. There is one element in $\{9\}$, thus one symmetric generator will return to [121].

Investigating $Nt_1t_2t_{10}$

Consider the double coset denoted as [1210]; $Nt_1t_2t_{10}N = \{Nt_1t_2t_{10}\}$. Using Magma, the coset counter does not increase, thus we know that $Nt_1t_2t_{10}N$ is not a new double coset. Using the following code, we conclude $Nt_1t_2t_{10}N = Nt_1t_4N$.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[10]
eq g*(ts[a])^{h} then "1,2,10=", a; end if; end for; end for;

for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[10]
eq g*(ts[a]*ts[b])^{h} then "1,2,10=", a,b; end if; end for; end for;
```

We wish to determine the permutation that proves $Nt_1t_2t_{10}N$ =Nt_1t_4N. There exists two elements, $g, h \in N$, such that

$$t_1t_2t_{10} = g \cdot (t_1t_4)^{\ h} = (1,8,7,9,3)(2,5,10,4,6) \cdot (t_1t_4)^{\ (1,8,7,9,3)(2,5,10,4,6)}$$
$$= (1,8,7,9,3)(2,5,10,4,6)t_8t_6.$$

$Nt_1t_2t_{10} = Nt_8t_6 \in [14] = Nt_1t_4 \Rightarrow Nt_1t_2t_{10}N = Nt_1t_4N$ when we take N of both sides. There is one element in $\{10\}$, thus one symmetric generator will return to [14].

Investigating $Nt_1t_6t_1$

Consider the double coset denoted as [161]; $Nt_1t_6t_1N = \{Nt_1t_6t_1, Nt_3t_4t_3, Nt_2t_8t_2, Nt_4t_8t_4, Nt_9t_{10}t_9, Nt_5t_7t_5, Nt_{10}t_1t_{10}, Nt_6t_1t_6, Nt_7t_6t_7, Nt_7t_5t_7, Nt_{10}t_9t_{10}, Nt_5t_3t_5, Nt_9t_2t_9, Nt_4t_3t_4, Nt_8t_2t_8, Nt_8t_4t_8, Nt_2t_9t_2, Nt_3t_5t_3, Nt_1t_{10}t_1, Nt_6t_7t_6\}$.
9*/The orbit $\{1\}$ had one element, thus one symmetric generator moved forward to $Nt_1t_6t_1$. Now $N^{161} =< e >= N^{(161)}$. Using Magma, we find equal names, $Nt_1t_6t_1 = Nt_5t_7t_5$. Notice by conjugation we obtain the following.

$$N(t_1t_6t_1)^{(1,5)(2,8)(3,10)(4,9)(6,7)} = Nt_5t_7t_5 = Nt_1t_6t_1.$$

Thus, $(1,5)(2,8)(3,10)(4,9)(6,7) \in N$ belongs in the coset stabilizer. The point stabilizer of [161] is defined as $N^{161} =< e, (1,5)(2,8)(3,10)(4,9)(6,7) >= N^{(161)}$. The number of singles cosets in [161] are $\frac{|N|}{|N^{(161)}|} = \frac{20}{2} = 10$. The orbits of $N^{(161)}$ are $\{1,5\}$, $\{2,8\}$, $\{3,10\}$, $\{4,9\}$, and $\{6,7\}$. There are five double cosets when selecting a representative from each orbit. But, $Nt_1t_6t_1t_1 = Nt_1t_6 \in [16]$. There are two elements in $\{1,5\}$, thus two symmetric generators will return to [16]. Thus we have four possible new double cosets: $Nt_1t_6t_1t_2, Nt_1t_6t_1t_3, Nt_1t_6t_1t_4, Nt_1t_6t_1t_6$.

Investigating $Nt_1t_6t_2$

Consider the double coset denoted as [162]; $Nt_1t_6t_2N = \{Nt_1t_6t_2\}$. The coset counter does not increase, thus we know that $Nt_1t_6t_2N$ is not a new double coset. We discover $Nt_1t_6t_2N$ =Nt_1t_2N using the following code.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[2]
eq g*(ts[a])^{h} then "1,6,2=", a; end if; end for; end for;

for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[2]
eq g*(ts[a]*ts[b])^{h} then "1,6,2=", a,b; end if; end for; end for;
```

We wish to determine the permutation that proves $Nt_1t_6t_2N$ =Nt_1t_2N. There exists two elements, $g, h \in N$, such that

$$t_1t_6t_2 = g \cdot (t_1t_2)\ ^h = (1,9)(2,6)(4,5)(8,7) \cdot (t_1t_2)\ ^{(1,9)(2,6)(4,5)(8,7)}$$
$$= (1,9)(2,6)(4,5)(8,7)t_9t_6.$$

This implies, by taking N of both sides, we have $Nt_1t_6t_2 = Nt_9t_6 \in [12] = Nt_1t_2 \Rightarrow Nt_1t_6t_2N = Nt_1t_2N$. There is one element in $\{2\}$, thus one symmetric generator will return to [12].

Investigating $Nt_1t_6t_3$

Consider the double coset denoted as [163]; $Nt_1t_6t_3N = \{Nt_1t_6t_3, Nt_3t_4t_9, Nt_2t_8t_5, Nt_4t_8t_{10}, Nt_9t_{10}t_7, Nt_5t_7t_{10}, Nt_{10}t_1t_5, Nt_6t_1t_2, Nt_7t_6t_9, Nt_7t_5t_8, Nt_{10}t_9t_4, Nt_5t_3t_2, Nt_9t_2t_3, Nt_4t_3t_6, Nt_8t_2t_1, Nt_8t_4t_7, Nt_2t_9t_6, Nt_3t_5t_1, Nt_1t_{10}t_8, Nt_6t_7t_4\}$.
The orbit $\{3\}$ had one element, thus one symmetric generator moved forward to $Nt_1t_6t_3$. Now $N^{163} =< e >= N^{(163)}$. Using Magma, we find equal names, $Nt_1t_6t_3 = Nt_6t_1t_2$. Notice by conjugation we obtain the following.

$$N(t_1t_6t_3)^{(1,6)(2,3)(4,8)(5,9)(7,10)} = Nt_6t_1t_2 = Nt_1t_6t_3.$$

Thus, $(1,6)(2,3)(4,8)(5,9)(7,10) \in N$ belongs in the coset stabilizer. Therefore, the point stabilizer is now defined as $N^{163} =< e, (1,6)(2,3)(4,8)(5,9)(7,10) >= N^{(163)}$. The number of singles cosets in [163] are $\frac{|N|}{|N^{(163)}|} = \frac{20}{2} = 10$. The orbits of $N^{(163)}$ are $\{1,6\}$, $\{2,3\}$, $\{4,8\}$, $\{5,9\}$, and $\{7,10\}$. Choosing a representative from each orbit, we have five double cosets. But, $Nt_1t_6t_3t_3 = Nt_1t_6 \in [16]$. There are two elements in $\{2,3\}$, thus two symmetric generators will return to [16]. Thus we have four possible new double cosets: $Nt_1t_6t_3t_1, Nt_1t_6t_3t_4, Nt_1t_6t_3t_5, Nt_1t_6t_3t_7$.

Investigating $Nt_1t_6t_4$

Consider the double coset denoted as [164]; $Nt_1t_6t_4N = \{Nt_1t_6t_4\}$. $Nt_1t_6t_4N$ is not a new double coset as it is equivalent to Nt_1t_4N.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[4]
eq g*(ts[a])^{h} then "1,6,4=", a; end if; end for; end for;

for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[4]
eq g*(ts[a]*ts[b])^{h} then "1,6,4=", a,b; end if; end for; end for;
```

There exists two elements, $g, h \in N$, such that

$$t_1t_6t_4 = g \cdot (t_1t_4)^{\ h} = (1,8)(3,7)(2,10)(4,6) \cdot (t_1t_4)^{\ (1,8,7,9,3)(2,5,10,4,6)}$$
$$= (1,8)(3,7)(2,10)(4,6)t_8t_6.$$

This implies, by taking N of both sides, we have $Nt_1t_6t_4 = Nt_8t_6 \in [14] = Nt_1t_4 \Rightarrow Nt_1t_6t_4N = Nt_1t_4N$. There is one element in $\{4\}$, thus one symmetric generator will return to [14].

Investigating $Nt_1t_6t_5$

The presumably new double coset [165] is equivalent to Nt_1t_6N.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[5]
eq g*(ts[a])^{h} then "1,6,5=", a; end if; end for; end for;

for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[5]
eq g*(ts[a]*ts[b])^{h} then "1,6,5=", a,b; end if; end for; end for;
```

There exists two elements, $g, h \in N$, such that

$$t_1t_6t_5 = g \cdot (t_1t_6)^{\ h} = (1,8,7,9,3)(2,5,10,4,6) \cdot (t_1t_6)^{\ (1,8)(3,7)(2,10)(4,6)}$$
$$= (1,8,7,9,3)(2,5,10,4,6)t_8t_4.$$

This implies, by taking N of both sides, we have $Nt_1t_6t_5 = Nt_8t_4 \in [16] = Nt_1t_6 \Rightarrow Nt_1t_6t_5N = Nt_1t_6N$. There is one element in $\{5\}$, thus one symmetric generator will return to itself.

Investigating $Nt_1t_6t_7$

Consider the double coset denoted as [167]; $Nt_1t_6t_7N = \{Nt_1t_6t_7, Nt_3t_4t_8, Nt_2t_8t_4, Nt_4t_8t_2, Nt_9t_{10}t_1, Nt_5t_7t_6, Nt_{10}t_1t_6, Nt_6t_1t_{10}, Nt_7t_6t_1, Nt_7t_5t_3, Nt_{10}t_9t_2, Nt_5t_3t_4, Nt_9t_2t_8, Nt_4t_3t_5, Nt_8t_2t_9, Nt_8t_4t_3, Nt_2t_9t_{10}, Nt_3t_5t_7, Nt_1t_{10}t_9, Nt_6t_7t_5\}$.
From the orbit $\{7\}$, a symmetric generator moves forward to $Nt_1t_6t_7$. The double coset's point stabilizer is $N^{167} =< e >= N^{(167)}$. The number of singles cosets in [167] are $\frac{|N|}{|N^{(167)}|} = \frac{20}{1} = 20$. The orbits of $N^{(167)}$ are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$, $\{6\}$, $\{7\}$, $\{8\}$, $\{9\}$, and $\{10\}$. We have $Nt_1t_6t_7t_7 = Nt_1t_6 \in [16]$; a symmetric generator will return to [17]. Then the nine possible new double cosets are: $Nt_1t_6t_7t_1, Nt_1t_6t_7t_2, Nt_1t_6t_7t_3, Nt_1t_6t_7t_4, Nt_1t_6t_7t_5, Nt_1t_6t_7t_6, Nt_1t_6t_7t_8, Nt_1t_6t_7t_9, Nt_1t_6t_7t_{10}$.

Investigating $Nt_1t_6t_8$

Consider the double coset denoted as [168]; $Nt_1t_6t_8N = \{Nt_1t_6t_8, Nt_3t_4t_1, Nt_2t_8t_6, Nt_4t_8t_6, Nt_9t_{10}t_3, Nt_5t_7t_2, Nt_{10}t_1t_4, Nt_6t_1t_4, Nt_7t_6t_8, Nt_7t_5t_9, Nt_{10}t_9t_5, Nt_5t_3t_{10}, Nt_9t_2t_7, Nt_4t_3t_{10}, Nt_8t_2t_7, Nt_8t_4t_1, Nt_2t_9t_5, Nt_3t_5t_9, Nt_1t_{10}t_3, Nt_6t_7t_2\}$.
The orbit $\{8\}$ had one element, thus one symmetric generator moved forward to $Nt_1t_6t_8$. Now $N^{168} =< e >= N^{(168)}$. Using Magma, we find equal names, $Nt_1t_6t_8 = Nt_2t_8t_6$. Notice by conjugation we obtain the following.

$$N(t_1t_6t_8)^{(1,2)(3,5)(4,7)(6,8)(9,10)} = Nt_2t_8t_6 = Nt_1t_6t_8.$$

Therefore, $(1,2)(3,5)(4,7)(6,8)(9,10) \in N$ belongs in the coset stabilizer defined as $N^{168} =< e, (1,2)(3,5)(4,7)(6,8)(9,10) >= N^{(168)}$. The number of singles cosets in [168] are $\frac{|N|}{|N^{(168)}|} = \frac{20}{2} = 10$. The orbits of $N^{(168)}$ are $\{1,2\}$, $\{3,5\}$, $\{4,7\}$, $\{6,8\}$, and $\{9,10\}$. Choosing a representative from each orbit, we have five double cosets. But, $Nt_1t_6t_8t_8 = Nt_1t_6 \in [16]$. Two symmetric generators will return to [16] because there were two elements in $\{6,8\}$. Thus we have four possible new double cosets: $Nt_1t_6t_8t_1, Nt_1t_6t_8t_3, Nt_1t_6t_8t_4, Nt_1t_6t_8t_9$.

Investigating $Nt_1t_6t_9$

The presumably new double coset [169] is equivalent to $Nt_1t_2t_1N$.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[9]
eq g*(ts[a])^{h} then "1,6,9=", a; end if; end for; end for;

for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[9]
eq g*(ts[a]*ts[b])^{h} then "1,6,9=", a,b; end if; end for; end for;
```

There exists two elements, $g, h \in N$, such that

$$t_1t_6t_9 = g \cdot (t_1t_2t_1)^{\ h} = (1,10)(3,4)(2,7)(8,5)(9,6) \cdot (t_1t_2t_1)^{\ (1,4)(3,10)(2,7)(8,6)(9,5)}$$
$$= (1,10)(3,4)(2,7)(8,5)(9,6)t_4t_7t_4.$$

This implies, by taking N of both sides, we have $Nt_1t_6t_9 = Nt_4t_7t_4 \in [121] = Nt_1t_2t_1 \Rightarrow Nt_1t_6t_9N = Nt_1t_2t_1N$. There is one element in $\{9\}$, thus one symmetric generator will return to [121].

Investigating $Nt_1t_6t_{10}$

The presumably new double coset [16 10] is equivalent to Nt_1t_2N.

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[10]
eq g*(ts[a])^{h} then "1,6,10=", a; end if; end for; end for;

for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[10]
eq g*(ts[a]*ts[b])^{h} then "1,6,10=", a,b; end if; end for; end for;
```

There exists two elements, $g, h \in N$, such that

$$t_1t_6t_{10} = g \cdot (t_1t_2)^{\,h} = (1,3,9,7,8)(2,6,4,10,5) \cdot (t_1t_2)^{\,(1,3)(4,10)(8,9)(5,6)}$$
$$= (1,3,9,7,8)(2,6,4,10,5)t_3t_2.$$

This implies, by taking N of both sides, we have $Nt_1t_6t_{10} = Nt_3t_2 \in [12] = Nt_1t_2 \Rightarrow Nt_1t_6t_{10}N = Nt_1t_2N$. There is one element in $\{10\}$, thus one symmetric generator will return to [12].

Investigating $Nt_1t_4t_3t_2$

Consider the double coset denoted as [1432]; $Nt_1t_4t_3t_2N = \{Nt_1t_4t_3t_2\}$. The coset counter does not increase, but Magma does not give us a previous double coset with word of length one or two [1432] is equivalent to. Since there are double cosets with word of length three, we investigate which double coset [1432] is equivalent to.

```
for a,b,c in [1..10] do for g,h in IN do
if ts[1]*ts[4]*ts[3]*ts[2] eq g*(ts[a]*ts[b]*ts[c])^{h}
then "1,4,3,2=", a,b,c; end if; end for; end for;
```

We conclude $Nt_1t_4t_3t_2N = Nt_1t_2t_8N$. We wish to determine the permutation that proves $Nt_1t_4t_3t_2N = Nt_1t_2t_8N$. There exists two elements, $g, h \in N$, such that

$$t_1t_4t_3t_2 = g \cdot (t_1t_2t_8)^{\,h} = (1,4)(3,6)(2,9)(8,10)(5,7) \cdot (t_1t_2t_8)^{\,(1,3)(4,10)(8,9)(5,6)}$$
$$= (1,4)(3,6)(2,9)(8,10)(5,7)t_3t_2t_9.$$

This implies, by taking N of both sides, we have $Nt_1t_4t_3t_2 = Nt_3t_2t_9 \in [128] = Nt_1t_2t_8 \Rightarrow Nt_1t_4t_3t_2N = Nt_1t_2t_8N$. There is one symmetric generator contained in the orbit $\{2\}$, that will return to the double coset [128].

Investigating $Nt_1t_4t_3t_7$

Consider the double coset denoted as [1437]; $Nt_1t_4t_3t_7N = \{Nt_1t_4t_3t_7\}$. Similarly, Magma does not give us a previous new double coset with word of length one or two [1437] is equivalent to when we find that the coset counter does not increase. We further investigate which double coset with three t_is [1437] is equivalent to.

```
for a,b,c in [1..10] do for g,h in IN do
if ts[1]*ts[4]*ts[3]*ts[7] eq g*(ts[a]*ts[b]*ts[c])^{h}
then "1,4,3,7=", a,b,c; end if; end for; end for;
```

This results in $Nt_1t_4t_3t_7N = Nt_1t_4t_3N$. There exists two elements, $g, h \in N$, such that

$$\begin{aligned} t_1t_4t_3t_7 = g \cdot (t_1t_4t_3)^{\ h} &= (1,3,9,7,8)(2,6,4,10,5) \cdot (t_1t_4t_3)^{\ (1,3)(4,10)(8,9)(5,6)} \\ &= (1,3,9,7,8)(2,6,4,10,5)t_3t_{10}t_1. \end{aligned}$$

$Nt_1t_4t_3t_7 = Nt_3t_{10}t_1 \in [143] = Nt_1t_4t_3 \Rightarrow Nt_1t_4t_3t_7N = Nt_1t_4t_3N$ when taking N of both sides. There is one symmetric generator contained in the orbit $\{7\}$, that will return to the double coset [143].

Investigating $Nt_1t_4t_3t_4$

Consider the double coset denoted as [1434]; $Nt_1t_4t_3t_4N = \{Nt_1t_4t_3t_4\}$. This double coset is also equivalent to a double coset with three t_is.

```
for a,b,c in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[4] eq g*(ts[a]*ts[b]*ts[c])^{h}
then "1,4,3,4=", a,b,c; end if; end for; end for;
```

This results in $Nt_1t_4t_3t_4N = Nt_1t_6t_7N$. There exists two elements, $g, h \in N$, such that

$$\begin{aligned} t_1t_4t_3t_4 = g \cdot (t_1t_6t_7)^{\ h} &= (1,8,7,9,3)(2,5,10,4,6) \cdot (t_1t_6t_7)^{\ (1,3)(4,10)(8,9)(5,6)} \\ &= (1,8,7,9,3)(2,5,10,4,6)t_3t_5t_7. \end{aligned}$$

This implies, by taking N of both sides, we have $Nt_1t_4t_3t_4 = Nt_3t_5t_7 \in [167] = Nt_1t_6t_7 \Rightarrow Nt_1t_4t_3t_4N = Nt_1t_6t_7N$.There are two symmetric generators contained in the orbit $\{4, 10\}$, that will return to the double coset [167].

Investigating $Nt_1t_4t_3t_5$

Consider the double coset denoted as [1435]; $Nt_1t_4t_3t_5N = \{Nt_1t_4t_3t_5,$ $Nt_3t_{10}t_9t_2, Nt_2t_7t_5t_3, Nt_4t_1t_{10}t_9, Nt_9t_5t_7t_6, Nt_5t_9t_{10}t_1, Nt_{10}t_3t_5t_7, Nt_6t_8t_2t_9, Nt_7t_2t_9t_{10},$ $Nt_7t_2t_8t_4, Nt_{10}t_3t_4t_8, Nt_5t_9t_2t_8, Nt_9t_5t_3t_4, Nt_4t_1t_6t_7Nt_8t_6t_1t_{10}, Nt_8t_6t_7t_5, Nt_2t_7t_6t_1,$ $Nt_3t_{10}t_1t_6, Nt_1t_4t_8t_2, Nt_6t_8t_4t_3\}$. The orbit $\{5,6\}$ had two elements, thus two symmetric generators moved forward to $Nt_1t_4t_3t_5$. Now $N^{1435} =< e >= N^{(1435)}$. Using Magma, we find equal names, $Nt_1t_4t_3t_5 = Nt_5t_9t_{10}t_1$. Notice by conjugation we obtain the following.

$$N(t_1t_4t_3t_5)^{(1,5)(2,8)(3,10)(4,9)(6,7)} = Nt_5t_9t_{10}t_1 = Nt_1t_4t_3t_5.$$

Therefore, $(1,5)(2,8)(3,10)(4,9)(6,7) \in N$ belongs in the coset stabilizer defined as $N^{1435} =< e, (1,5)(2,8)(3,10)(4,9)(6,7) >= N^{(1435)}$. The number of singles cosets in [1435] are $\frac{|N|}{|N^{(1435)}|} = \frac{20}{2} = 10$. The orbits of $N^{(1435)}$ are $\{1,5\}$, $\{2,8\}$, $\{3,10\}$, $\{4,9\}$, and $\{6,7\}$. Choosing a representative from each orbit, we have four double cosets. But, $Nt_1t_4t_3t_5t_5 = Nt_1t_4t_3 \in [143]$. There are two elements in the orbit $\{1,5\}$, thus two symmetric generators will return to [143]. Thus we have four possible new double cosets: $Nt_1t_4t_3t_5t_2, Nt_1t_4t_3t_5t_3, Nt_1t_4t_3t_5t_4, Nt_1t_4t_3t_5t_6$.

Our coset counter on Magma has determined that all 171 single cosets have been found since the number of single cosets of our group G are: $\frac{|G|}{|N|} = \frac{3420}{20} = 171$. Then all of the possible new double cosets from previous double cosets must return to a double coset that has already been found. We conclude that the previous double cosets that were assumed to be possible new double cosets are not new, and are equal to the following.

Double Cosets from [143]

$Nt_1t_4t_3t_8 = Nt_8t_6t_9 \in [147] = Nt_1t_4t_7 \Rightarrow Nt_1t_4t_3t_8N = Nt_1t_4t_7N.$

There are two elements in $\{8,9\}$; thus two symmetric generators will return to [147].

Double Cosets from [147]

$Nt_1t_4t_7t_5 = Nt_1t_4t_7 \in [147] \Rightarrow Nt_1t_4t_7t_5N = Nt_1t_4t_7N.$

There are two elements in $\{5,9\}$; thus two symmetric generators will return to itself.

$Nt_1t_4t_7t_1 = Nt_3t_{10}t_9 \in [143] \Rightarrow Nt_1t_4t_7t_1N = Nt_1t_4t_3N.$

There are four elements in $\{1,4,6,8\}$; thus four symmetric generators will return to [143].

Double Cosets from [121]

$Nt_1t_2t_1t_2 = Nt_4t_3t_5 \in [167] \Rightarrow Nt_1t_2t_1t_2N = Nt_1t_6t_7N$.

There is one element in $\{2\}$; thus one symmetric generator will return to [167].

$Nt_1t_2t_1t_3 = Nt_8t_5 \in [12] \Rightarrow Nt_1t_2t_1t_3N = Nt_1t_2N$.

There is one element in $\{3\}$; thus one symmetric generator will return to [12].

$Nt_1t_2t_1t_4 = Nt_6t_7t_2 \in [168] \Rightarrow Nt_1t_2t_1t_4N = Nt_1t_6t_8N$.

There is one element in $\{4\}$; thus one symmetric generator will return to [168].

$Nt_1t_2t_1t_5 = Nt_4t_8 \in [16] \Rightarrow Nt_1t_2t_1t_5N = Nt_1t_6N$.

There is one element in $\{5\}$; thus one symmetric generator will return to [16].

$Nt_1t_2t_1t_6 = Nt_{10}t_1t_{10} \in [161] \Rightarrow Nt_1t_2t_1t_6N = Nt_1t_6t_1N$.

There is one element in $\{6\}$; thus one symmetric generator will return to [161].

$Nt_1t_2t_1t_7 = Nt_1t_2t_1 \in [121] \Rightarrow Nt_1t_2t_1t_7N = Nt_1t_2t_1N$.

There is one element in $\{7\}$; thus one symmetric generator will return to [121].

$Nt_1t_2t_1t_8 = Nt_7t_{10}t_9 \in [128] \Rightarrow Nt_1t_2t_1t_8N = Nt_1t_2t_8N$.

There is one element in $\{8\}$; thus one symmetric generator will return to [128].

$Nt_1t_2t_1t_9 = Nt_3t_6t_8 \in [127] \Rightarrow Nt_1t_2t_1t_9N = Nt_1t_2t_7N$.

There is one element in $\{9\}$; thus one symmetric generator will return to [127].

$Nt_1t_2t_1t_{10} = Nt_8t_2t_1 \in [163] \Rightarrow Nt_1t_2t_1t_{10}N = Nt_1t_6t_3N$.

There is one element in $\{10\}$; thus one symmetric generator will return to [163].

Double Cosets from [127]

$Nt_1t_2t_7t_6 = Nt_{10}t_8t_4 \in [128] \Rightarrow Nt_1t_2t_7t_6N = Nt_1t_2t_8N$.

There are two elements in $\{6, 8\}$; thus two symmetric generators will return to [128].

$Nt_1t_2t_7t_3 = Nt_8t_5t_8 \in [121] \Rightarrow Nt_1t_2t_7t_3N = Nt_1t_2t_1N$.

There are four elements in $\{3, 5, 10, 9\}$; thus four symmetric generators will return to [121].

Double Cosets from [128]

$Nt_1t_2t_8t_2 = Nt_3t_{10}t_1 \in [143] \Rightarrow Nt_1t_2t_8t_2N = Nt_1t_4t_3N.$

There is one element in $\{2\}$; thus one symmetric generator will return to [143].

$Nt_1t_2t_8t_7 = Nt_6t_9t_5 \in [127] \Rightarrow Nt_1t_2t_8t_7N = Nt_1t_2t_7N.$

There is one element in $\{7\}$; thus one symmetric generator will return to [127].

$Nt_1t_2t_8t_1 = Nt_8t_{10}t_8 \in [121] \Rightarrow Nt_1t_2t_8t_1N = Nt_1t_2t_1N.$

There are two elements in $\{1, 3\}$; thus two symmetric generators will return to [121].

$Nt_1t_2t_8t_4 = Nt_3t_4t_8 \in [167] \Rightarrow Nt_1t_2t_8t_4N = Nt_1t_6t_7N.$

There are two elements in $\{4, 10\}$; thus two symmetric generators will return to [167].

$Nt_1t_2t_8t_5 = Nt_1t_4t_8t_2 \in [1435] \Rightarrow Nt_1t_2t_8t_5N = Nt_1t_4t_3t_5N.$

There are two elements in $\{5, 6\}$; thus two symmetric generators will return to [1435].

Double Cosets from [161]

$Nt_1t_6t_1t_2 = Nt_3t_5t_9 \in [168] \Rightarrow Nt_1t_6t_1t_2N = Nt_1t_6t_8N.$

There are two elements in $\{2, 8\}$; thus two symmetric generators will return to [168].

$Nt_1t_6t_1t_3 = Nt_5t_3t_2 \in [163] \Rightarrow Nt_1t_6t_1t_3N = Nt_1t_6t_3N.$

There are two elements in $\{3, 10\}$; thus two symmetric generators will return to [163].

$Nt_1t_6t_1t_4 = Nt_{10}t_9t_2 \in [167] \Rightarrow Nt_1t_6t_1t_4N = Nt_1t_6t_7N.$

There are two elements in $\{4, 9\}$; thus two symmetric generators will return to [167].

$Nt_1t_6t_1t_6 = Nt_7t_4t_7 \in [121] \Rightarrow Nt_1t_6t_1t_6N = Nt_1t_2t_1N.$

There are two elements in $\{6, 7\}$; thus two symmetric generators will return to [121].

Double Cosets from [163]

$Nt_1t_6t_3t_1 = Nt_6t_7t_6 \in [161] \Rightarrow Nt_1t_6t_3t_1N = Nt_1t_6t_1N.$

There are two elements in $\{1, 6\}$; thus two symmetric generators will return to [161].

$Nt_1t_6t_3t_4 = Nt_4t_3t_5 \in [167] \Rightarrow Nt_1t_6t_3t_4N = Nt_1t_6t_7N.$

There are two elements in $\{4, 8\}$; thus two symmetric generators will return to [167].

$Nt_1t_6t_3t_5 = Nt_3t_6t_3 \in [121] \Rightarrow Nt_1t_6t_3t_5N = Nt_1t_2t_1N.$

There are two elements in $\{5, 9\}$; thus two symmetric generators will return to [121].

$Nt_1t_6t_3t_7 = Nt_3t_4t_1 \in [168] \Rightarrow Nt_1t_6t_3t_7N = Nt_1t_6t_8N.$

There are two elements in $\{7, 10\}$; thus two symmetric generators will return to [168].

Double Cosets from [167]

$Nt_1t_6t_7t_1 = Nt_3t_4t_9 \in [163] \Rightarrow Nt_1t_6t_7t_1N = Nt_1t_6t_3N$. There is one element in $\{1\}$; thus one symmetric generator will return to [163].

$Nt_1t_6t_7t_2 = Nt_4t_1t_{10}t_9 \in [1435] \Rightarrow Nt_1t_6t_7t_2N = Nt_1t_4t_3t_5N$. There is one element in $\{2\}$; thus one symmetric generator will return to [1435].

$Nt_1t_6t_7t_3 = Nt_8t_2t_8 \in [161] \Rightarrow Nt_1t_6t_7t_3N = Nt_1t_6t_1N$. There is one element in $\{3\}$; thus one symmetric generator will return to [161].

$Nt_1t_6t_7t_4 = Nt_1t_6t_7 \in [167] \Rightarrow Nt_1t_6t_7t_4N = Nt_1t_6t_7N$. There is one element in $\{4\}$; thus one symmetric generator will return to itself.

$Nt_1t_6t_7t_5 = Nt_7t_2t_9t_{10} \in [1435] \Rightarrow Nt_1t_6t_7t_5N = Nt_1t_4t_3t_5N$. There is one element in $\{5\}$; thus one symmetric generator will return to [1435].

$Nt_1t_6t_7t_6 = Nt_8t_5t_7 \in [128] \Rightarrow Nt_1t_6t_7t_6N = Nt_1t_2t_8N$. There is one element in $\{6\}$; thus one symmetric generator will return to [128].

$Nt_1t_6t_7t_8 = Nt_6t_7t_2 \in [168] \Rightarrow Nt_1t_6t_7t_8N = Nt_1t_6t_8N$. There is one element in $\{8\}$; thus one symmetric generator will return to [168].

$Nt_1t_6t_7t_9 = Nt_4t_9t_4 \in [121] \Rightarrow Nt_1t_6t_7t_9N = Nt_1t_2t_1N$. There is one element in $\{9\}$; thus one symmetric generator will return to [121].

$Nt_1t_6t_7t_{10} = Nt_3t_{10}t_1 \in [143] \Rightarrow Nt_1t_6t_7t_{10}N = Nt_1t_4t_3N$. There is one element in $\{10\}$; thus one symmetric generator will return to [143].

Double Cosets from [168]

$Nt_1t_6t_8t_1 = Nt_5t_3t_5 \in [161] \Rightarrow Nt_1t_6t_8t_1N = Nt_1t_6t_1N$.

There are two elements in $\{1, 2\}$; thus two symmetric generators will return to [161].

$Nt_1t_6t_8t_3 = Nt_{10}t_8t_{10} \in [121] \Rightarrow Nt_1t_6t_8t_3N = Nt_1t_2t_1N$.

There are two elements in $\{3, 5\}$; thus two symmetric generators will return to [121].

$Nt_1t_6t_8t_4 = Nt_{10}t_1t_6 \in [167] \Rightarrow Nt_1t_6t_8t_4N = Nt_1t_6t_7N$.

There are two elements in $\{4, 7\}$; thus two symmetric generators will return to [167].

$Nt_1t_6t_8t_9 = Nt_8t_2t_1 \in [163] \Rightarrow Nt_1t_6t_8t_9N = Nt_1t_6t_3N$.

There are two elements in $\{9, 10\}$; thus two symmetric generators will return to [163].

Double Cosets from [1435]

$Nt_1t_4t_3t_5t_2 = Nt_1t_5t_3 \in [128] \Rightarrow Nt_1t_4t_3t_5t_2N = Nt_1t_2t_8N$.

There are two elements in $\{2, 8\}$; thus two symmetric generators will return to [128].

$Nt_1t_4t_3t_5t_3 = Nt_1t_5t_3 \in [167] \Rightarrow Nt_1t_4t_3t_5t_3N = Nt_1t_6t_7N$.

There are two elements in $\{3, 10\}$; thus two symmetric generators will return to [167].

$Nt_1t_4t_3t_5t_4 = Nt_1t_4t_3t_5 \in [1435] \Rightarrow Nt_1t_4t_3t_5t_4N = Nt_1t_4t_3t_5N$.

There are two elements in $\{4, 9\}$; thus two symmetric generators will return to itself.

$Nt_1t_4t_3t_5t_6 = Nt_9t_2t_8 \in [167] \Rightarrow Nt_1t_4t_3t_5t_6N = Nt_1t_6t_7N$.

There are two elements in $\{6, 7\}$; thus two symmetric generators will return to [167].

Our completed Cayley Diagram is shown below.

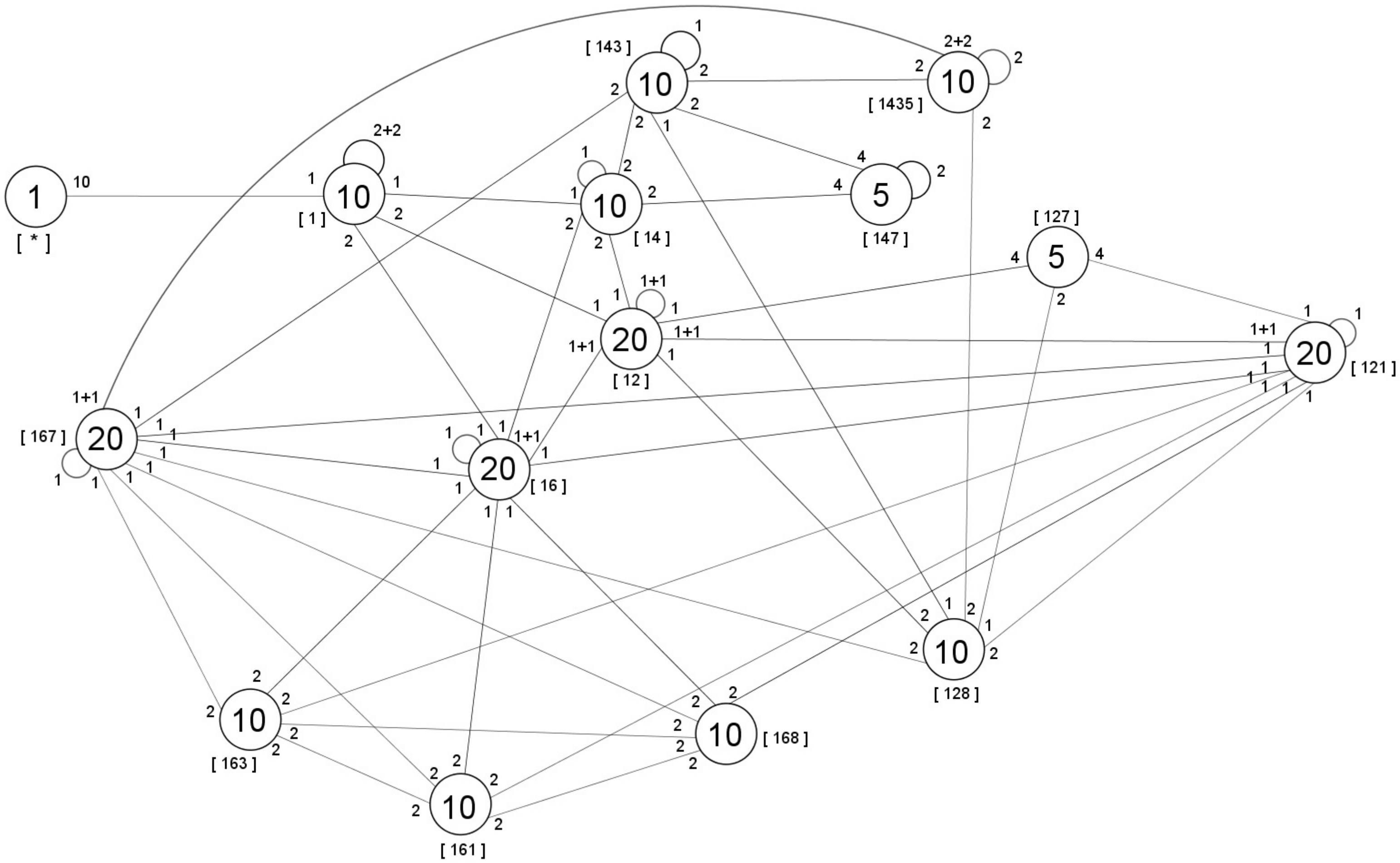


Figure 4.3: Cayley Diagram of $PSL_2(19)$

4.5 Maximal Subgroup $PSL_2(7)$ as a Homomorphic Image of $2^{*4} : C_4$

The group $2^{*4} : C_4$ of order 168, where $C_4 = x = (1, 2, 3, 4)$ is a cyclic group of order 4, is made up of 42 double cosets. When the number of right cosets is too large, we use a suitable maximal subgroup M of G containing to perform Double Coset Enumeration over a maximal subgroup of the image of our progenitor to reduce the number of double cosets in our Cayley diagram. We begin with factoring the progenitor $2^{*4} : C_4$ by the relations $(x * t)^7, (x^{-1} * t)^7$, and $(x^2 * t)^3$.

$$G \cong \frac{2^{*4} : C_4}{(x^{-1} * t)^7 = (x^2 * t)^3 = (x * t)^7}$$

The symmetric presentation of the group is given by

$$G < x, t >:= Group < x, t | x^4, t^2, (x * t)^7, (x^{-1} * t)^7, (x^2 * t)^3 > .$$

Our control group is $N \cong C_4 =< x | x^4 >$ and the action of N on the symmetric generator is given by $x \sim (1, 2, 3, 4)$. We will let $t = t_1$.

The first relation is expanded and is equivalent to

$$(x * t)^7 = x^7 * t^{x^6} * t^{x^5} * t^{x^4} * t^{x^3} * t^{x^2} * t^x * t = x^3 * t^{x^2} * t^x * t * t^{x^3} * t^{x^2} * t^x * t. \quad \text{(Relation 1)}$$

The second relation informs us that

$$(x^{-1} * t)^7 = x * t^{x^2} * t^{x^3} * t * t^x * t^{x^2} * t^{x^3} * t. \quad \text{(Relation 2)}$$

Finally, the third relation states

$$(x^2 * t)^3 = x^2 * t * t^{x^2} * t. \quad \text{(Relation 3)}$$

Hence,

$$G \cong \frac{2^{*4} : C_4}{(1, 4, 3, 2)t_3t_2t_1t_4t_3t_2t_1, (1, 2, 3, 4)t_3t_4t_1t_2t_3t_4t_1, (1, 3)(2, 4)t_1t_3t_1},$$

where $t_3t_2t_1t_4 = x^3t_1t_2t_3$, $t_3t_4t_1t_2 = xt_1t_4t_3$, and $t_1t_3 = x^2t_1$.

Double Coset Enumeration of
G over $M =< f(x), f(x^{-1} * t * x * t * x^{-1} * t * x) >$

There are $\frac{|G|}{|N|} = \frac{168}{4} = 42$ right cosets of G over N. We show how this double coset enumeration of G over C_4 can be done using a double coset enumeration of G over M and N. Performing the double coset enumeration of G over M in N, Magma tells us that there are only 3 double cosets. Since we are constructing the Cayley Diagram of a maximal subgroup, the original definition of double coset will change. The definition of a double coset of a group over a maximal subgroup is given by $MwN = \{Mw^N|n \in N\}$. We begin with the double coset $MeN = \{Me^N|n \in N\} = \{M\}$, such that the coset representative for M is $[*]$. We use the Magma commands to begin double coset enumeration.

```
G<x,t>:=Group<x,t|x^4,t^2,(x^{-1}*t)^7,(x^2*t)^3,(x*t)^7>;
f,G1,k:=CosetAction(G,sub<G|x>);
N:=sub<Sym(4)|(1,2,3,4)>;
H:=sub<G|x,x^{-1} * t * x * t * x^{-1} * t * x>;
IN:=sub<G1|f(x)>;
IH:=sub<G1|f(x),f(x^{-1} * t * x * t * x^{-1} * t * x)>;
W,phi:=WordGroup(G1);
rho:=InverseWordMap(G1);
ts := [Id(G1): i in [1 .. 4] ];
ts[1]:=f(t); ts[2]:=f(t^x); ts[3]:=f(t^(x^2)); ts[4]:=f(t^(x^3));
```

$N =< x >$ is transitive on $\{1, 2, 3, 4\}$, the double coset $[*]$ contains a single orbit, $\{1, 2, 3, 4\}$. The number of single cosets in $[*]$ is the number of right cosets. We right multiply M by a representative from this orbit, say 1. We now have a new double coset Mt_1N, denoted as [1]. The number of single cosets in $[*]$ can be determined by $\frac{|N|}{|N|} = \frac{4}{4} = 1$. Since there are 4 elements in the orbit, 4 symmetric generators will move forward to the new double coset.

The new double coset [1] is $Mt_1N = \{Mt_1{}^n|n \in N\} = \{Mt_1, Mt_2, Mt_3, Mt_4\}$. Now the coset stabiliser $N^{(1)}$ is equivalent to the point stabiliser N^1. $N^{(1)} = N^1 =< e >$. The number of single cosets of Mt_1N is at most $\frac{|N|}{|N^{(1)}|} = \frac{4}{1} = 4$. Using Magma, we input the coset counter code to count the number of single cosets we have as we present new double cosets, excluding the first single coset in $[*]$.

```
cst := [null : i in [1 .. Index(G,sub\<G|x,y>)]]
where null is [Integers() | ];
for i := 1 to 4 do
       cst[prodim(1, ts, [i])] := [i];
    end for;
 m:=0;
for i in [1..10] do if cst[i] ne [] then m:=m+1; end if; end for; m;
```

Magma confirms there are four single cosets in Mt_1N. The generators of $N^{(1)}$ tells us that the orbits of N on $\{1, 2, 3, 4\}$ are $\{1\}$, $\{2\}$, $\{3\}$, and $\{4\}$. Now we select a representative from each orbit and we determine where the four possible new double cosets, Mt_1t_1, Mt_1t_2, Mt_1t_3, Mt_1t_4 belong.

Investigating Mt_1t_1

Our t_is are of order 2, so $Mt_1t_1 = M \in [*]$. There is one element in the orbit $\{1\}$, thus one symmetric generator will return to $[*]$.

Investigating Mt_1t_2

We have a new double coset Mt_1t_2N, denoted as [12]. The double coset [12] is $Mt_1t_2N = \{M(t_1t_2)^n | n \in N\} = \{Mt_1t_2, Mt_2t_3\}$. The orbit $\{2\}$ had one element, thus one symmetric generator will move forward to Mt_1t_2. Now $N^{12} \geq N^{(12)} =< e >$. Using Magma, we find that Mt_1t_2 is equal to Mt_3t_4. There exists a permutation in N that will prove this equivalency. Notice by conjugation $M(t_1t_2)^{(1,3)(2,4)} = Mt_3t_4 = Mt_1t_2$. Thus $(1, 3)(2, 4) \in N$ belongs in the coset stabiliser, $N^{12} \geq N^{(12)} =< e, (1, 3)(2, 4) >$. The number of singles cosets in [12] are $\frac{|N|}{|N^{(12)}|} = \frac{4}{2} = 2$. The orbits of $N^{(12)}$ are $\{1, 3\}$ and $\{2, 4\}$. Choosing a representative from each orbit, we have two double cosets that are not new as there are three double cosets needed to complete our Cayley Diagram. But, $Nt_1t_2t_2 = Nt_1 \in [1]$. There are two elements in $\{2, 4\}$, thus two symmetric generators will return to [1]. Our next found double coset, $Nt_1t_2t_1$ is also not new as it must be equivalent to a double coset we already have. We will prove this below.

Investigating Mt_1t_3

Observing (Relation 3) from above, $t_1t_3 = x^2t_1$. Applying M to both sides of the relation, we obtain $Mt_1t_3 = Mt_1 \in [1]$. There is one element in the orbit $\{3\}$, thus 1 symmetric generator will return to Mt_1.

Investigating Mt_1t_4

We understand that [14] is not a new double coset and is equivalent to a previous double coset. Using the following code, we discover that $Mt_1t_4 = Mt_1$.

```
for a in [1..4] do for g,h in IH do if ts[1]*ts[4]
eq g*(ts[a])^h then "1,4=", a; end if; end for; end for;
```

There is no element in our control group N that will prove this equivalency. Since we are performing double coset enumeration over M in N, we can investigate the words of our maximal subgroup: x and $x^{-1}t_1xt_1x^{-1}t_1x$. We then simplify the second word.

$$
\begin{aligned}
x^{-1}t_1xt_1x^{-1}t_1x &= x^{-1}t_1xt_1x^{-1}\underline{xx^{-1}}t_1x \text{ (Identity)}\\
&= x^{-1}t_1xt_1x^{-1}x\underline{x^{-1}t_1x}\\
&= x^{-1}t_1xt_1x^{-1}x\underline{t_1{}^x} \qquad \text{(Definition of Conjugation)}\\
&= x^{-1}t_1xt_1x^{-1}xt_2\\
&= x^{-1}t_1xt_1t_2\\
&= x^{-1}xt_2t_1t_2\\
&= t_2t_1t_2
\end{aligned}
$$

We conjugate $t_2t_1t_2$ by an element in N to prove $t_1t_4t_1 \in M$; $t_2t_1t_2{}^{(1,4,3,2)} = t_1t_4t_1$. Using (Relation 2), we prove that $t_1t_4 = t_1t_4$.

$$
\begin{aligned}
t_1t_4 = xt_3t_4t_1t_2t_3 &= xt_3t_4\underline{t_1t_2t_3}\\
&= xt_3t_4\underline{x^{-1}t_3t_2t_1t_4} \qquad \text{(Relation 1)}\\
&= xt_3\underline{x^{-1}x}t_4x^{-1}t_3t_2t_1t_4\\
&= xt_3x^{-1}t_4^{x^{-1}}t_3t_2t_1t_4\\
&= xt_3x^{-1}t_3t_3t_2t_1t_4\\
&= x\underline{x^{-1}x}t_3x^{-1}t_3t_3t_2t_1t_4\\
&= xx^{-1}t_3{}^{x^{-1}}t_3t_3t_2t_1t_4\\
&= et_3{}^{x^{-1}}t_3t_3t_2t_1t_4\\
&= t_2t_3t_3t_2t_1t_4\\
&= t_1t_4
\end{aligned}
$$

Now $t_1t_4 = t_1t_4$. Since t is of order 2, we can say that $t_1t_4 = t_1t_4t_1t_1$. From above, we have learned that $t_1t_4t_1 \in M$. Next, we apply our maximal subgroup to both sides.

$$Mt_1t_4 = M(t_1t_4t_1)t_1$$

Thus we observe that $Mt_1t_4 = Mt_1$ since $t_1t_4t_1 \in M$.
There is one element in the orbit $\{4\}$, thus 1 symmetric generator will return to Mt_1.

Investigating $Mt_1t_2t_1$

Since we know this is not a new double coset, we must find out which double coset this is equal to. Using Magma, we find out that [121] = [12].

```
for a,b in [1..4] do for g,h in IH do if ts[1]*ts[2]*ts[1]
eq g*(ts[a]*ts[b])^h then "1,2,1=", a,b; end if; end for; end for;
> 1,2,1= 1 2
```

However, $t_1t_2t_1 = (t_2t_1t_2)t_1t_2 \in [12]$. There are 2 elements in the orbit $\{1{,}3\}$, thus two symmetric generators will return to [12]. Therefore, we have completed our double coset enumeration and the construction of our Cayley Diagram.

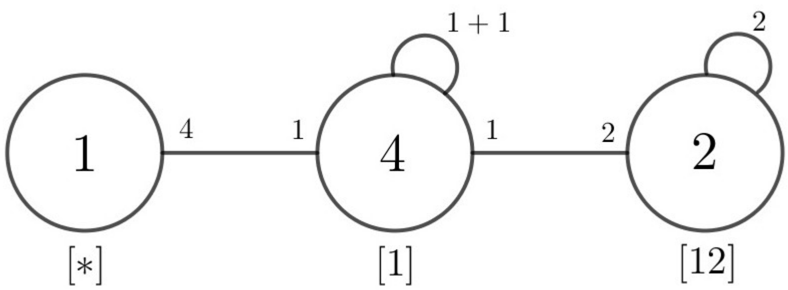


Figure 4.4: Cayley Diagram of $PSL_2(7)$

4.6 Maximal Subgroup S_4 as a Homomorphic Image of S_3

Consider the group
$G < x, y, t >:= Group < x, y, t | x^3, y^2, (x * y)^2, t^2, (y, t), x = t^x * t^{(x^2)} * t * t^x >$, such that $x = (1, 2, 3), y = (1, 2)$, and $t = t_1$.

Double Coset Enumeration over $M =< f(x), f(y), f(t * t^x * t) >$

Magma tells us that this group contains 2 double cosets. A double coset of a maximal subgroup is defined as: $MwN = \{Mw^n | n \in N\}$. The first double coset denoted as $[*]$ is $MeN = \{Me^n | n \in N\} = \{Me | e \in N\} = \{M\}$, where the coset representative for $[*]$ is M. Since N is transitive on $\{1, 2, 3\}$ it contains a single orbit $\{1, 2, 3\}$. The number of single cosets in $[*]$ is the number of right cosets. We right multiply M by a representative from this orbit, say 1. We now have a new double coset Mt_1N, denoted as [1]. The number of single cosets in $[*]$ can be determined by $\frac{|N|}{|N|} = \frac{6}{6} = 1$. Since there are 3 elements in the orbit, 3 symmetric generators will move forward to the new double coset.

The new double coset [1] is $Mt_1N = \{Mt_1{}^n | n \in N\} = \{Mt_1, Mt_2, Mt_3\}$. Now the coset stablilizer $N^{(1)}$ is equivalent to the point stabilizer N^1; $N^{(1)} = N^1 =< (2, 3) >$. The number of single cosets of Mt_1N is at most $\frac{|N|}{|N^{(1)}|} = \frac{6}{2} = 3$. Using the coset counter, we begin to count the number of single cosets we have as we present new double cosets, excluding the first single coset in $[*]$. Magma confirms there are 3 single cosets in Mt_1N.

The generators of $N^{(1)}$ tells us that the orbits on $\{1, 2, 3\}$, are $\{1\}$ and $\{2, 3\}$. Choosing a representative from each orbit, we must determine where the two possible new double cosets Mt_1t_1 and Mt_1t_2 belong. Our t_is are of order 2, so $Mt_1t_1 = M \in [*]$. There is one element in the orbit $\{1\}$, thus one symmetric generator will return to $[*]$. We now have a new double coset Mt_1t_2N, denoted as [12]. There are 2 elements in the orbit $\{2, 3\}$, thus 2 symmetric generators will move forward to Mt_1t_2N.

So far, we have found the two double cosets. Thus we assume that [12] is not a new double coset and is equal to a previous double coset. The double coset [12], $Mt_1t_2N = \{M(t_1t_2)^n | n \in N\} = \{Mt_1t_2, Mt_3t_2, Mt_2t_1, Mt_1t_3, Mt_2t_3, Mt_3t_1\}$. Now, $N^{(12)} \geq N^{12} =< e >$. Using Magma, we find that $Mt_1t_2 = Mt_1t_3$. Notice by conjugation $M(t_1t_2)^{(2,3)} = Mt_1t_3 = Mt_1t_2$ from the equal names. Thus, $(2, 3) \in N$ belongs in the coset stabilizer. $N^{(12)} \geq N^{12} =< e, (2, 3) >$.

The number of single cosets of Mt_1t_2N is at most $\frac{|N|}{|N^{(12)}|} = \frac{6}{2} = 3$. The coset counter has now increased to 6, however, it should have not increased since we know [12] is not a new double coset.

We use the following code, where IH is $M =< f(x), f(y), f(t * t^x * t) >$, to determine whether there's a permutation that will verify if [12] is equivalent to [1].

```
for m in IH do for n in IN do if ts[1]*ts[2]
eq m*(ts[1])^n then m, n; break; end if; end for; end for;
```

The code computes a permutation that states [12] = [12]. Now the question is why the coset counter is increasing. The reason the coset counter is increasing is because we're constructing the double cosets over N. In other words, Magma is informing us that $Nt_1t_2 \neq Nt_1$, hence the reason why the number of single cosets is increasing. But we are constructing the Cayley diagram of our group S_4. We are performing double coset enumeration over M in N. Thus, $Mt_1t_2 = Mt_1$. Mt_1t_2 is not a new double coset. Since we have the orbit $\{2, 3\}$, we understand that Mt_1t_2N returns to [1] on two symmetric generators. Thus we have completed our double coset enumeration and the construction of our Cayley Diagram.

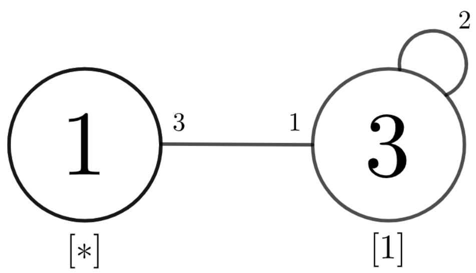


Figure 4.5: Cayley Diagram of S_4

4.7 Maximal Subgroup $U_3(5):2$ as a Homomorphic Image of $2^{*7}:(7:6)$

The group $G_1 = 2^{*7}:(7:6)$ is of order 252,000. We wish to construct the Cayley Diagram of a smaller group. A maximal subgroup of G_1 is $U_3(5):2 \cong S_7$ with order 5,040. The progenitor $2^{*7}:(7:6)$ factored by the relations $(x^{-2}*y*t)^7, (y*x*t)^8$, $(x^{-1}*y^{-1}*t^y)^6$, and $(y*t)^8$ is isomorphic to $U_3(5):2$. We wish to show that

$$G = \frac{2^{*7}:(7:6)}{(x^{-2}*y*t)^7 = (y*x*t)^8 = (x^{-1}*y^{-1}*t^y)^6 = (y*t)^8 = e} \cong U_3(5):2$$

.

Our control group is defined as $N = (C_7 : C_6) =< (1,3,2,6,4,5),(1,2,3,4,5,6,7) >$, such that $x = (1,3,2,6,4,5)$ and $y = (1,2,3,4,5,6,7)$. We let $t = t_1$. Consider the group, $G =< x,y,t|xy^{-1}x^{-1}y^{-2}, x^6, yx^{-1}y^{-1}xy^2, t^2, (t,xy^{-2}), (t,x^2y^{-1}), (x^{-2}yt)^7, (yxt)^8$, $(x^{-1}y^{-1}t^y)^6, (yt)^8 >$.

Double Coset Enumeration over
$M =< f(x), f(y), f(x^2*y^{-1}*x*t*y^{-1}*t*y^{-1}*t*y^{-1}*t*y) >$

We discover this group to have five double cosets. We are constructing the Cayley Diagram of a maximal subgroup, thus the first double coset is denoted as [∗]; $MeN = \{Me^n|n \in N\} = \{Me|n \in N\} = \{M\}$. The coset representative for [∗] is M. Since N is transitive on $\{1,2,3,4,5,6,7\}$ it contains a single orbit $\{1,2,3,4,5,6,7\}$. The number of single cosets in [∗] is the number of right cosets. We right multiply M by a representative from this orbit, say 1. We now have a new double coset Mt_1N, denoted as [1]. The number of single cosets in [∗] can be determined by $\frac{|N|}{|N|} = \frac{42}{42} = 1$. Since there are 7 elements in the orbit, 7 symmetric generators will move forward to the new double coset.

The new double coset [1] is $Mt_1N = \{Mt_1{}^n|n \in N\} = \{Mt_1, Mt_2, Mt_3, Mt_4, Mt_5, Mt_6, Mt_7\}$. Now, the coset stabilizer $N^{(1)}$ is equivalent to the point stabilizer N^1. $N^{(1)} = N^1 =< (2,7)(3,6)(4,5),(2,3,5)(4,7,6) >$. The number of single cosets of Mt_1N is at most $\frac{|N|}{|N^{(1)}|} = \frac{42}{6} = 7$. Next, the coset counter code determines there are 7 single cosets in Mt_1N. The generators of $N^{(1)}$ show the orbits on $\{1,2,3,4,5,6,7\}$ are $\{1\}$ and $\{2,3,6,5,4,7\}$. Selecting a representative from each orbit, we must determine where the two possible new double cosets, Mt_1t_1 and Mt_1t_2 belong. Our t_is are of order 2, so

$Mt_1t_1 = M \in [*]$. There is one element in the orbit $\{1\}$, thus one symmetric generator will return to $[*]$. We now have a new double coset Mt_1t_2N, denoted as [12]. There are six elements in the orbit $\{2, 3, 6, 5, 4, 7\}$, thus six symmetric generators will move forward to Mt_1t_2N.

Consider the double coset [12]; $Mt_1t_2N = \{M(t_1t_2)^n | n \in N\} = \{Mt_1t_2, Mt_3t_6, Mt_2t_3, Mt_2t_4, Mt_4t_7, Mt_6t_2, Mt_3t_4, Mt_6t_5, Mt_3t_5, Mt_5t_7, Mt_5t_1, Mt_4t_6, Mt_7t_3, Mt_2t_5, Mt_4t_5, Mt_4t_1, Mt_7t_6, Mt_2t_1, Mt_1t_7, Mt_6t_1, Mt_1t_3, Mt_5t_4, Mt_7t_2, Mt_1t_4, Mt_5t_6, Mt_5t_3, Mt_5t_2, Mt_7t_4, Mt_6t_3, Mt_3t_2, Mt_3t_7, Mt_4t_3, Mt_1t_5, Mt_6t_7, Mt_6t_4, Mt_1t_6, Mt_7t_5, Mt_4t_2, Mt_2t_6, Mt_2t_7, Mt_3t_1, Mt_7t_1\}$. Now, $N^{(12)} \geq N^{12} =< e >$. Using Magma, we find that $Mt_1t_2 = Mt_4t_3$. Notice by conjugation $M(t_1t_2)^{(1,4)(2,3)(5,7)} = Mt_4t_3 = Mt_1t_2$ from the equal names. Therefore, the permutation $(1, 4)(2, 3)(5, 7) \in N$ belongs in the coset stabilizer; $N^{(12)} \geq N^{12} =< e, (1, 4)(2, 3)(5, 7) >$. The number of single cosets of Mt_1t_2N is at most $\frac{|N|}{|N^{(12)}|} = \frac{42}{2} = 21$. In Magma, the coset counter increases to 28; the single cosets in [1] as well as in [12], confirming our results.

The generators of $N^{(12)}$ informs us that the orbits on $\{1, 2, 3, 4, 5, 6, 7\}$ are $\{6\}$, $\{1, 4\}$, $\{2, 3\}$, and $\{5, 7\}$. The next step is to determine where the four possible new double cosets, $Mt_1t_2t_6$, $Mt_1t_2t_1$, $Mt_1t_2t_2$, and $Mt_1t_2t_5$ belong to by selecting a representative from each orbit. We observe that $Mt_1t_2t_2 = Mt_1 \in [1]$, with two symmetric generators returning to [1]. Then, 1 symmetric generator will move forward to $Mt_1t_2t_6$, 2 symmetric generators will move forward to $Mt_1t_2t_1$, and 2 symmetric generators will move forward to $Mt_1t_2t_5$.

Consider the double coset [121]; $Mt_1t_2t_1N = \{M(t_1t_2t_1)^n | n \in N\}$. This too will have 42 right cosets, according to Magma. Now, $N^{(121)} \geq N^{121} =< e >$. We find that $Mt_1t_2t_1 = Mt_2t_4t_2 = Mt_4t_1t_4$. Conjugating this double coset, we obtain $M(t_1t_2t_1)^{(1,2,4)(3,6,5)} = Mt_2t_4t_2 = Mt_1t_2t_1$ from the equal names. We also conjugate by a second element in N which results in $M(t_1t_2t_1)^{(1,4,2)(3,5,6)} = Mt_4t_1t_4 = Mt_1t_2t_1$ from the equal names. Therefore, we now know that these two elements belong to the coset stabilizer; $N^{(121)} \geq N^{121} =< e, (1, 2, 4)(3, 6, 5), (1, 4, 2)(3, 5, 6) >$. The number of single cosets in $Mt_1t_2t_1N$ is at most $\frac{|N|}{|N^{(121)}|} = \frac{42}{3} = 14$. The coset counter has now increased to 42.

The generators of $N^{(121)}$ tells us that the orbits on $\{1, 2, 3, 4, 5, 6, 7\}$ are $\{7\}$, $\{1, 2, 4\}$, and $\{3, 6, 5\}$. Choosing a representative from each orbit, we must determine where the three possible new double cosets, $Mt_1t_2t_1t_7$, $Mt_1t_2t_1t_1$, $Mt_1t_2t_1t_3$ belong. We observe that $Mt_1t_2t_1t_1 = Mt_1t_2 \in [12]$, with three symmetric generators returning to [12]. Then, 1 symmetric generator will move forward to $Mt_1t_2t_1t_7$ and three symmetric generators will move forward to $Mt_1t_2t_1t_3$.

Consider the double coset [125]; $Mt_1t_2t_5N = \{M(t_1t_2t_5)^n | n \in N\}$, with 42 right cosets, according to Magma. Now, $N^{(125)} \geq N^{125} =< e >$. Using Magma, we find that $Mt_1t_2t_5 = Mt_5t_7t_6 = Mt_2t_5t_7 = Mt_7t_6t_3 = Mt_6t_3t_1 = Mt_3t_1t_2$. Notice by conjugation and the equal names, we have the following.

$$M(t_1t_2t_5)^{(1,5,6)(2,7,3)} = Mt_5t_7t_6 = Mt_1t_2t_5$$

$$M(t_1t_2t_5)^{(1,2,5,7,6,3)} = Mt_2t_5t_7 = Mt_1t_2t_5$$

$$M(t_1t_2t_5)^{(1,7)(2,6)(3,5)} = Mt_7t_6t_3 = Mt_1t_2t_5$$

$$M(t_1t_2t_5)^{(1,6,5)(2,3,7)} = Mt_6t_3t_1 = Mt_1t_2t_5$$

$$M(t_1t_2t_5)^{(1,3,6,7,5,2)} = Mt_3t_1t_2 = Mt_1t_2t_5$$

These elements in N belong in the coset stabilizer; $N^{(125)} \geq N^{125} =< e, (1, 5, 6)(2, 7, 3)$, $(1, 2, 5, 7, 6, 3), (1, 7)(2, 6)(3, 5), (1, 6, 5)(2, 3, 7)(1, 3, 6, 7, 5, 2) >$. We have the number of single cosets in $Mt_1t_2t_5N$ to be at most $\frac{|N|}{|N^{(125)}|} = \frac{42}{6} = 7$. The coset counter has now increased to 49. The generators of $N^{(125)}$ tells us that the orbits on $\{1, 2, 3, 4, 5, 6, 7\}$ are $\{4\}$ and $\{1, 5, 2, 7, 6, 3\}$. Choosing a representative from each orbit, we must determine where the two possible new double cosets, $Mt_1t_2t_5t_4$ and $Mt_1t_2t_5t_5$ belong. We observe that $Mt_1t_2t_5t_5 = Mt_1t_2 \in [12]$, with six symmetric generators returning to [12]. Then one symmetric generator will move forward to $Mt_1t_2t_5t_4$.

We have found a total of five double cosets. Thus we assume that [126] is not a new double coset and is equal to a previous double coset. Consider the double coset [126]; $Mt_1t_2t_6N = \{M(t_1t_2t_6)^n | n \in N\}$, with 42 right cosets, according to Magma. Now, $N^{(126)} \geq N^{126} =< e >$. The coset counter has now increased to 70. The coset counter should have not increased since we know [126] is not a new double coset. We use the following code to determine which double coset [126] is equivalent to where IH is $M =< f(x), f(y), f(x^2 * y^{-1} * x * t * y^{-1} * t * y^{-1} * t * y^{-1} * t * y) >$.

```
for m in IH do for n in IN do if ts[1]*ts[2]*ts[6]
eq m*(ts[1]*ts[2])^n then m, n; break; end if; end for; end for;
```

Magma gives us a permutation that states [126]=[12]. Now the question is why is the coset counter increasing. The reason it's increasing is because we're constructing the double cosets over N. In other words magma is informing us that $Nt_1t_2t_6 \neq Nt_1t_2$, hence the reason why the number of single cosets is increasing. But we are constructing the Cayley diagram of our maximal subgroup $U_3(5) : 2$. We are performing double coset enumeration over M in N. Thus, $Mt_1t_2t_6 = Mt_1t_2$. $Mt_1t_2t_6$ is not a new double coset. One symmetric generator will return to itself, [12]. Similarly, our possible new double cosets $Mt_1t_2t_5t_4$, $Mt_1t_2t_1t_7$, and $Mt_1t_2t_1t_3$ are equal to previous double cosets and therefore not new.

$Mt_1t_2t_5t_4 = Mt_1t_2t_5$; one symmetric generator will return to [125]

$Mt_1t_2t_1t_7 = Mt_1t_2t_1$; one symmetric generator will return to [121]

$Mt_1t_2t_1t_3 = Mt_1t_2t_1$; three symmetric generators will return to [121]

Thus the construction of our Cayley Diagram is now complete.

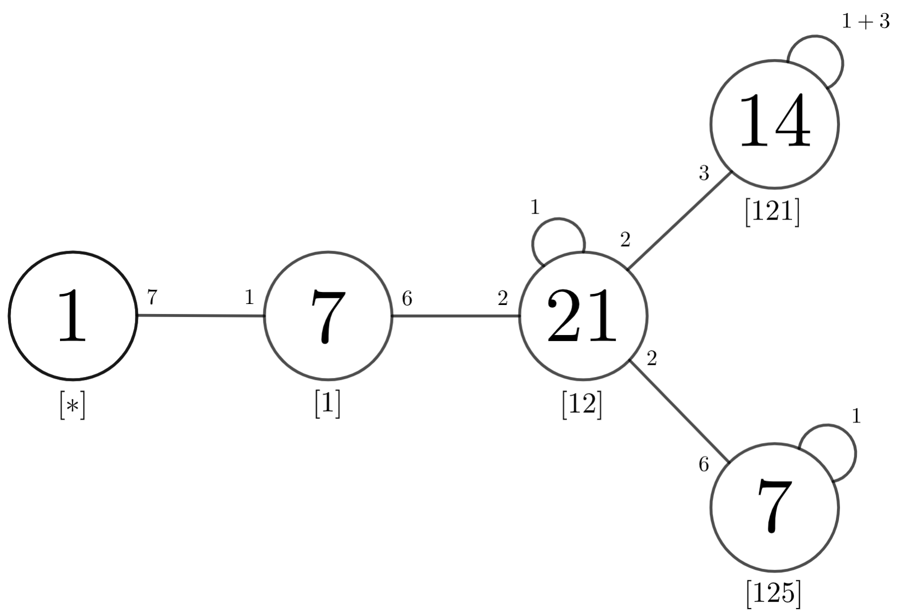


Figure 4.6: Cayley Diagram of $U_3(5) : 2$

Chapter 5

Tables of Progenitor Presentations and their Finite Homomorphic Images

5.1 $2^{*8} : (2^{\cdot} A_4)$

```
G<a,b,c,d,t>:=Group<a,b,c,d,t|b^4,c^4,d^2,a^3*d,b^-2*d,
c^-1*b^2*c^-1,b^-1*c^-1*b*c^-1,a^-1*d*a*d,b^-1*d*b*d,c^-1*d*c*d,
c^-1*a^-2*b^-1*a^-1,a*c^-1*a^-1*b*c^-1,
t^2,(t,a^2),
(c^-1*a^-1*t^b)^r18,(c^-1*a^-1*t)^r19,
(a*d*t)^r20,(a*d*t^b)^r21,(d*a^-1*t)^r22,(d*a^-1*t^b)^r23>;
```

Table 5.1: $2^{*8} : (2^{\cdot} A_4)$

r18	r19	r20	r21	r22	r23	Order	G
3	6	0	5	0	0	7920	M_{11}

5.2 $2^{*10} : D_{20}$

```
G<a,b,c,t>:=Group<a,b,c,t|b^2,c^2,(a^-1*b)^2,a^-1*c*a*c,(b*c)^2,
a^-5,t^2,(t,b*a^2*c),(b*c*t^a)^r7,(a*t)^r8,(a^2*t)^r9,
(a*c*t)^r10,(c*a^-2*t)^r11>;
```

Table 5.2: $2^{*10} : D_{20}$

r7	r8	r9	r10	r11	Order	G
0	0	3	9	5	6840	$2 \times PSL_2(19)$
0	0	4	0	3	7920	$6 :^{\cdot} PGL_2(11)$
0	0	6	6	3	90720	$(3^2 : 2) : S_7$
0	0	10	5	3	249600	$2 \times (U(3,4) : 2)$
3	0	0	4	10	2640	$PGL_2(11) \times 2$
3	0	3	0	5	3420	$PSL_2(19)$
3	0	3	4	0	1320	$PGL_2(11)$
3	0	3	6	8	241920	$6 \times (PSL_3(4) : 2)$
3	0	3	7	0	34440	$PSL_2(41)$
4	0	4	4	6	62400	$2^2 \times (PSL_2(25) : 2)$
4	0	5	0	4	16320	$2 \times (PSL_2(16) : 2)$
4	0	6	4	6	6969600	$2^3 : [PSL_2(11) \times PSL_2(11) : 2]$
5	0	0	3	0	6415200	$[3^4 \times PSL_2(11)] : S_5$
5	0	0	5	4	161280	$2^2 : (PSL_3(4) : 2)$
6	6	0	3	0	748800	$[6 \times U(3,4)] : 2$

5.3 $2^{*15} : ((5 \times 3) : 2)$

```
G<a,b,c,t>:=Group<a,b,c,t|b^3,c^2,(a,b),a^-1*c*a*c,(b^-1*c)^2,
a^-5,t^2,(t,c),(c*a^-1*t)^r14,(c*a^-1*t^b)^r15,(a*b*t)^r16,
(a^2*b^-1*t)^r17,(b*a^-1*t)^r18,(b*a^-2*t)^r19>;
```

Table 5.3: $2^{*15} : ((5 \times 3) : 2)$

r14	r15	r16	r17	r18	r19	Order	G
2	9	0	0	0	10	20520	$PSL_2(19) \times S_3$
2	7	0	0	0	9	504	$PSL_2(8)$
2	7	0	0	8	0	336	$PGL_2(7)$
2	9	0	9	9	9	3420	$PSL_2(19)$

5.4 $2^{*20} : A_5$

```
G<x,y,t>:=Group<x,y,t|x^2,y^3,(x*y^-1)^5,t^2,
(t,x*y*x*y^-1*x*y^-1),(x * y*t^(y^2*x*y*x))^r17,
((x * y)^2*t)^r18,((x * y)^2*t^(y^2*x*y))^r19,
((x * y)^2*t^x)^r20,((x * y)^2*t^(y^2*x*y*x))^r21>;
```

Table 5.4: $2^{*20} : A_5$

r17	r18	r19	r20	r21	Order	G
0	0	0	0	3	660	$PSL_2(11)$
4	0	0	0	4	8160	$PSL_2(16) : 2$

5.5 Transitive Group$(15, 6)$

```
G<x,y,z,t>:=Group<x,y,z,t|x^4,y^2,x^-2*y,z^-2*x^-1*z*x,t^2,
(t,x*z^-1),(z^3*t)^r14,(z * y*t^x)^r15,(z * y*t)^r16,
(z * y*t^y)^r17,(z*t)^r18,(x * z^-1 * x^-1*t)^r19>;
```

Table 5.5: $2^{*15} : (15 : 4)$

r14	r15	r16	r17	r18	r19	Order	G
0	2	0	6	0	8	2280960	$4 \times (M_{12} : 2)$

5.6 Transitive Group$(15, 6)$ with Mixed Relations

```
G<x,y,z,t>:=Group<x,y,z,t|x^4,y^2,x^-2*y,z^-2*x^-1*z*x,t^2,
(t,x*z^-1),(z * x*t)^r16,(z * x*t^(z^2))^r17,((y * z)^2*t)^r18,
((z * x)^2*t^(z^2))^r19>;
```

Table 5.6: $2^{*15} : (15 : 4)$ - Relations Mixed

r16	r17	r18	r19	Order	G
0	7	8	10	336	$PGL_2(7)$
0	7	9	0	504	$PSL_2(8)$
0	9	9	10	3420	$PSL_2(19)$
0	9	10	0	20520	$S_3 \times PSL_2(19)$
2	8	8	10	672	$2 : PGL_2(7)$

5.7 Primitive Group$(7, 3)$

```
G<x,y,t>:=Group<x,y,t|x^3,y^-2*x^-1*y*x,t^2,(t,y*x^-1),
(x^-1 * y^-1*t)^r5,(x^-1 * y^-1*t^y)^r6,(y*t)^r7,
(x * y^-1 * x^-1*t)^r8>;
```

Table 5.7: $2^{*7} : (7 : 3)$

r5	r6	r7	r8	Order	G
0	5	0	8	1774080	$2^2 \,^\cdot M_{22}$

5.8 Primitive Group$(7, 3)$ with Second Order Relations

```
G<x,y,t>:=Group<x,y,t|x^3,y^-2*x^-1*y*x,t^2,(t,y*x^-1),
(x * y^-1 * x^-1*t*t^(x^2))^r7,(x^-1 * y^-1*t^(y^4))^r8,
(x * y^-1 * x^-1*t*t^(y^2))^r9,(x^-1 * y^-1*t)^r10>;
```

Table 5.8: $2^{*7} : (7 : 3)$ - Second Order Relations

r7	r8	r9	r10	Order	G
3	0	0	8	120960	$(2 \times PGL_3(4)) : 2$
4	0	4	6	483840	$(2^2 \,^\cdot PSL_3(4)) : 6$
5	0	0	5	1330560	$3 \,^\cdot M_{22}$
5	6	4	5	443520	M_{22}

5.9 Primitive Group$(7, 2)$

```
G<x,y,t>:=Group<x,y,t|x^2,(y^-1*x)^2,y^-7,t^2,(t,x*y^2),
(x*t^(y^2))^r4,(y*t)^r5,(y^2*t)^r6,(y^3*t)^r7>;
```

Table 5.9: $2^{*7} : D_{14}$

r4	r5	r6	r7	Order	G
0	0	6	3	2184	$PGL_2(13)$
0	0	8	3	21504	$(2^6 : PSL_2(7)) : 2$
3	0	0	9	178920	$PSL_2(71)$
3	0	0	10	24360	$PGL_2(29)$
4	0	0	4	4368	$PGL_2(13) \times 13$
6	5	7	5	161280	$(4 : PSL_3(4)) : 2$
6	7	4	6	43008	$(2^7 : PGL_2(7)) : 2$
6	7	4	9	244944	$(3^6 : PGL_2(7)) : 2$
6	8	4	8	322560	$(4 : PSL_3(2)) : 2^2$

5.10 Primitive Group$(7, 4)$

```
G<x,y,t>:=Group<x,y,t|x*y^-1*x^-1*y^-2,x^6,y*x^-1*y^-1*x*y^2,
t^2,(t,x*y^-2),(t,x^2*y^-1),(x^-2 * y*t)^r6,(y*x*t^y)^r7,
(y*x*t)^r8,(x^-1 * y^-1*t^y)^r9,(x^-1 * y^-1*t)^r10,(y*t)^r11>;
```

Table 5.10: $2^{*7} : (7 : 6)$

r6	r7	r8	r9	r10	r11	Order	G
7	0	0	0	7	0	604800	J_2
7	0	8	6	0	8	252000	$U_3(5) : 2$

5.11 Primitive Group$(7, 4)$ with Second Order Relations

```
G<x,y,t>:=Group<x,y,t|x*y^-1*x^-1*y^-2,x^6,y*x^-1*y^-1*x*y^2,
t^2,(t,x*y^-2),(t,x^2*y^-1),(y * x*t*t^(y^6))^r50,
(x^-1 * y^-1*t^y*t)^r51,(x^-1 * y^-1*t*t^y)^r52>;
```

Table 5.11: $2^{*7} : (7 : 6)$ - Second Order Relations

r50	r51	r52	Order	G
3	0	0	58968	$PSL_2(7) : 6$

Appendix A

Magma Code for Extension Problems

A.1 Direct Product: $(2 \times PSL_2(19))$

$$G = \frac{2^{*10} : D_{20}}{(a^2t)^3, (act)^9, (ca^{-2}t)^5} \cong (2 \times PSL_2(19))$$

```
G<a,b,c,t>:=Group<a,b,c,t|b^2,c^2,(a^-1*b)^2,a^-1*c*a*c,(b*c)^2,a^-5,
t^2,(t,b*a^2*c),(a^2*t)^3,(a*c*t)^9,(c*a^-2*t)^5>;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);
#k;
CompositionFactors(G1);
NL:=NormalLattice(G1);
NL;
D:=DirectProduct(NL[2],NL[3]);
s,t:=IsIsomorphic(G1,D);
s;
FPGroup(PSL(2,19));
H<a,b>:=Group<a,b|a^9,b^3,a^-1*b*a^-1*b^-1*a^2*b*a^2*b^-1,
(a*b^-1*a*b^-1*a)^2,a^3*b^-1*a^-3*b*a*b^-1*a^-2*b>;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,PSL(2,19));
s;
H<a,b,c>:=Group<a,b,c|a^9,b^3,a^-1*b*a^-1*b^-1*a^2*b*a^2*b^-1,
(a*b^-1*a*b^-1*a)^2,a^3*b^-1*a^-3*b*a*b^-1*a^-2*b,c^2,(a,c),(b,c)>;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,G1);s;
```

A.2 Semi-Direct Product: $(U_3(5):2)$

$$G = \frac{2^{*7} : (7:6)}{(x^{-2}yt)^7, (yxt)^8, (x^{-1}y^{-1}t^y)^6, (yt)^8} \cong (U_3(5):2)$$

```
G<x,y,t>:=Group<x,y,t|x*y^-1*x^-1*y^-2,x^6,y*x^-1*y^-1*x*y^2,t^2,
(t,x*y^-2),(t,x^2*y^-1),(x^-2 * y*t)^7,(y*x*t)^8,
(x^-1 * y^-1*t^y)^6,(y*t)^8>;
f,G1,k:=CosetAction(G,sub<G|x,y>);
#k;
CompositionFactors(G1);
NL:=NormalLattice(G1);
NL;

PSU(3,5);
s:=IsIsomorphic(NL[2],PSU(3,5));
s;
FPGroup(NL[2]);
H<a,b,c>:=Group<a,b,c|a^3,b^7,c^4,a*b^-1*a^-1*b^2,
b*a*c^-1*a^-1*c*b^-1*c,
c*a*c^-2*a^-1*c^-1*a^-1*c^2*b^-1*a^-1*c^-1*b^-1,
c^-1*a*c^-2*a^-1*c*a^-1*c^-1*b^-1*a^-1*c^2*b^-1,
c^-1*a^-1*b^-1*c^-2*a^-1*b^-1*a^-1*c*b * a^-1*c^2*a,
a*c^-1*b^-1*c^-2*a^-1*b^-1*a*c^2*a^-1*c^2*b*c>;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,NL[2]);
s;

for i in NL[3] do if i notin NL[2] and Order(i) eq 2
and sub<G1|i,NL[2]> eq G1 then C:=i; break;
end if; end for;

A:=NL[2].1;
B:=NL[2].2;
C:=NL[2].3;
for i in NL[3] do if i notin NL[2] and Order(i) eq 2
and sub<G1|i,NL[2]> eq G1 then D:=i; break;
end if; end for;

E:=A^D;
W:=WordGroup(G1);
rho:=InverseWordMap(G1);
E@rho;
EE:=function(W)
```

```
    w1 := W.1^-1; w4 := w1 * W.3; w5 := w4 * W.1; w6 := w5 * W.3;
    w7 := w6 * W.2; w8 := w7 * W.3; w9 := w8 * W.1; w10 := w9 * W.2;
    w11 := w10 * W.3; w2:= W.2^-1; w12 := w11 * w2; w13 := w12 * W.3;
    w14 := w13 * W.1; w15 := w14 * W.2; return w15;
end function;
EE(G);
A^D eq A^-1 * C * A * C * B * C * A * B * C * B^-1 * C * A * B;

G<c,d>:=Group<c,d|c^2,d^4,(c*d)^10,
(c*d*c*d^-1*c*d^2)^2,(c,d*c*d)^4,(c*d*c*d*c*d*c*d^2)^7>;
M2:=sub<G|x,y^-1*x*y*x*y*x*y>;
ff,G2,k:=CosetAction(G,M2);
#G2;
s:=IsIsomorphic(G1,G2);
s;
```

A.3 Central Extension: $(3\,^{\cdot}M_{22})$

$$G = \frac{2^{*7} : (7:3)}{(xy^{-1}x^{-1}tt^{x^2})^5, (x^{-1}y^{-1}t)^5} \cong (3\,^{\cdot}M_{22})$$

```
G<x,y,t>:=Group<x,y,t|x^3,y^-2*x^-1*y*x,t^2,(t,y*x^-1),
(x * y^-1 * x^-1*t*t^(x^2))^5,
(x^-1 * y^-1*t)^5>;
f,G1,k:=CosetAction(G,sub<G|x,y>);
#k;
CompositionFactors(G1);
NL:=NormalLattice(G1);
NL;

IsAbelian(NL[3]);
IsAbelian(NL[2]);
Center(G1);
NL[2] eq Center(G1);
q,ff:=quo<G1|NL[2]>;
q;
CompositionFactors(q);
FPGroup(q);
H<x,y,z>:=Group<x,y,z|x^3,y^7,z^2,y^-2*x^-1*y*x,y^-1*z*y*x^-1*z*x,
z*x^-1*z*y^-2*z*x^-1*z*y*x*z*y^-2,(y^-1*z)^8,(z*x^-1)^11>;
#H;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,q);s;
```

```
T:=Transversal(G1,NL[2]);
ff(T[2]) eq q.1;
ff(T[3]) eq q.2;
ff(T[4]) eq q.3;
A:=T[2];
B:=T[3];
C:=T[4];
D:=NL[2].2;
for i in [1..3] do if A^3 eq D^i then i; end if; end for;
for i in [1..3] do if B^7 eq D^i then i; end if; end for;
for i in [1..3] do if C^2 eq D^i then i; end if; end for;
for i in [1..3] do if B^-2*A^-1*B*A eq D^i then i; end if; end for;
for i in [1..3] do if B^-1*C*B*A^-1*C*A eq D^i then i; end if; end for;
for i in [1..3] do if C*A^-1*C*B^-2*C*A^-1*C*B*A*C*B^-2 eq D^i
then i; end if; end for;
for i in [1..3] do if (B^-1*C)^8 eq D^i then i; end if; end for;
for i in [1..3] do if (C*A^-1)^11 eq D^i then i; end if; end for;

HH<x,y,z,c>:=Group<x,y,z,c|x^3=c^3,y^7=c^3,z^2=c^3,y^-2*x^-1*y*x=c^3,
y^-1*z*y*x^-1*z*x=c^3,z*x^-1*z*y^-2*z*x^-1*z*y*x*z*y^-2=c^3,(y^-1*z)^8=c,
(z*x^-1)^11=c,c^3,(c,x),(c,y),(c,z)>;
f2,H2,k2:=CosetAction(HH,sub<HH|Id(HH)>);
s,t:=IsIsomorphic(H2,G1);
s;
```

A.4 Mixed Extension: $(6 : {}^{\cdot}PGL_2(11))$

```
G<a,b,c,t>:=Group<a,b,c,t|b^2,c^2,(a^-1*b)^2,a^-1*c*a*c,(b*c)^2,a^-5,t^2,
(t,b*a^2*c),(a^2*t)^4,(c*a^-2*t)^3>;
f,G1,k:=CosetAction(G,sub<G|Id(G)>);
#k;
CompositionFactors(G1);
NL:=NormalLattice(G1);
NL;

for i in [1..11] do if IsAbelian(NL[i]) then i; end if; end for;
Center(G1);
Center(G1) eq NL[2];

D:=DirectProduct(NL[2],NL[3]);
IsIsomorphic(D,NL[4]);
q,ff:=quo<G1|NL[4]>;
```

```
q;
nl:=NormalLattice(q);
nl;

IsIsomorphic(PGL(2,11),q);
FPGroup(q);
H<a,b,c,d>:=Group<a,b,c,d| a^5,b^2,c^2,d^2,(a^-1 * b)^2,a^-1 * c * a * c,
(b * c)^2,c * d * b * a^2 * d * c * d,b * d * c * a^-2 * d * a * b * d,
c * a * d * c * a^-1 * d * b * a * d * a * c * d * a^-1 * c * d>;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H2,q);
A:=T[2];
B:=T[3];
C:=T[4];
D:=NL[2].1;
T:=Transversal(G1,NL[4]);
ff(T[2]) eq q.1;
ff(T[3]) eq q.2;
ff(T[4]) eq q.3;

/*Contains the center*/
J:=DirectProduct(NL[2],NL[3]);
s,t:=IsIsomorphic(J,NL[4]);
for e,f in NL[4] do if Order(e) eq 3 and Order(f) eq 2 and e^f eq e then
E:=e; F:=f; end if; end for;

/*Central part-the normal subgroup*/
for i in [0..2] do for j in [0..1] do if A^5 eq E^i*F^j
then i,j; break;end if;end for;end for;
/*[0..2] because |e|=3 and [0..1] since |f|=2*/

for i in [0..2] do for j in [0..1] do if B^2 eq E^i*F^j
then i,j; break;end if;end for;end for;

for i in [0..2] do for j in [0..1] do if C^2 eq E^i*F^j
then i,j; break;end if;end for;end for;

for i in [0..2] do for j in [0..1] do if D^2 eq E^i*F^j
then i,j; break;end if;end for;end for;

for i in [..2] do for j in [0..1] do if (A^-1 * B)^2 eq E^i*F^j
then i,j; break;end if;end for;end for;
```

```
for i in [0..2] do for j in [0..1] do if A^-1 * C * A * C eq E^i*F^j
then i,j; break;end if;end for;end for;

for i in [0..2] do for j in [0..1] do if (B * C)^2 eq E^i*F^j
then i,j; break;end if;end for;end for;

for i in [0..2] do for j in [0..1] do if C * D * B * A^2 * D * C * D
eq E^i*F^j then i,j; break;end if;end for;end for;

for i in [0..2] do for j in [0..1] do if B * D * C * A^-2 * D * A * B * D
eq E^i*F^j then i,j; break;end if;end for;end for;

for i in [0..2] do for j in [0..1] do if
C * A * D * C * A^-1 * D * B * A * D * A * C * D * A^-1 * C * D
eq E^i*F^j then i,j; break;end if;end for;end for;


/*Semi-direct product of the entire mixed extension*/
for i in [0..4] do for j,k,l in [0..1] do if A^E eq A^i*B^j*C^k*D^l
then i;j;k;l;break;end if; end for; end for;
/*[0..4] because |A|=5 and [0..1] since |B,C,D|=2*/

for i in [0..4] do for j,k,l in [0..1] do if A^F eq A^i*B^j*C^k*D^l
then i;j;k;l;break;end if; end for; end for;

for i in [0..4] do for j,k,l in [0..1] do if B^E eq A^i*B^j*C^k*D^l
then i;j;k;l;break;end if; end for; end for;

for i in [0..4] do for j,k,l in [0..1] do if B^F eq A^i*B^j*C^k*D^l
then i;j;k;l;break;end if; end for; end for;

for i in [0..4] do for j,k,l in [0..1] do if C^E eq A^i*B^j*C^k*D^l
then i;j;k;l;break;end if; end for; end for;

for i in [0..4] do for j,k,l in [0..1] do if C^F eq A^i*B^j*C^k*D^l
then i;j;k;l;break;end if; end for; end for;

for i in [0..4] do for j,k,l in [0..1] do if D^E eq A^i*B^j*C^k*D^l
then i;j;k;l;break;end if; end for; end for;

for i in [0..4] do for j,k,l in [0..1] do if D^F eq A^i*B^j*C^k*D^l
then i;j;k;l;break;end if; end for; end for;
```

```
HH<a,b,c,d,e,f>:=Group<a,b,c,d,e,f|a^5=e,b^2,c^2,d^2,(a^-1*b)^2,
a^-1*c*a*c=e^2,(b*c)^2=e,c*d*b*a^2*d*c*d=e*f,b*d*c*a^-2*d*a*b*d=e*f,
c*a*d*c*a^-1*d*b*a*d*a*c*d*a^-1*c*d=e^2*f,
e^3,f^2,(e,f),a^e=a,a^f=a,b^f=b,c^f=c,d^f=d>;
f2,H2,k2:=CosetAction(HH,sub<HH|Id(HH)>);
#H2;
s,t:=IsIsomorphic(H2,G1);
s;

D:=DirectProduct(CyclicGroup(2),CyclicGroup(3));
IsIsomorphic(D,CyclicGroup(6));
s,t:=IsIsomorphic(D,CyclicGroup(6));
s;
```

Appendix B

Magma Code for Double Coset Enumeration

B.1 Construction of M_{11} over $2^{2\cdot}A_4$

```
S:=Sym(8);
A:=S!(1, 2)(3, 7, 4, 5, 8, 6);
B:=S!(1, 3, 2, 5)(4, 8, 6, 7);
C:=S!(1, 4, 2, 6)(3, 7, 5, 8);
D:=S!(1, 2)(3, 5)(4, 6)(7, 8);
N:=sub<S|A,B,C,D>;
#N;
/*24*/

G<a,b,c,d,t>:=Group<a,b,c,d,t|b^4,c^4,d^2,a^3*d,b^-2*d,
c^-1*b^2*c^-1,b^-1*c^-1*b*c^-1,a^-1*d*a*d,b^-1*d*b*d,
c^-1*d*c*d,c^-1*a^-2*b^-1*a^-1,a*c^-1*a^-1*b*c^-1,t^2,(t,a^2),
(c^-1*a^-1*t^b)^3,(c^-1*a^-1*t)^6,(a*d*t^b)^5>;
f,G1,k:=CosetAction(G,sub<G|a,b,c,d>);
IN:=sub<G1|f(a),f(b),f(c),f(d)>;
NN<x,y,z,w>:=Group<x,y,z,w|y^4,z^4,w^2,x^3*w,y^-2*w,
z^-1*y^2*z^-1,y^-1*z^-1*y*z^-1,x^-1*w*x*w,y^-1*w*y*w,
z^-1*w*z*w,z^-1*x^-2*y^-1*x^-1,x*z^-1*x^-1*y*z^-1>;
Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
ArrayP:=[Id(N): i in [1..#N]];
for i in [2..#N] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
```

```
if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=B^-1; end if;
if Eltseq(Sch[i])[j] eq 3 then P[j]:=C; end if;
if Eltseq(Sch[i])[j] eq -3 then P[j]:=C^-1; end if;
if Eltseq(Sch[i])[j] eq 4 then P[j]:=D; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;
for i in [1..#N] do if ArrayP[i] eq N!(1, 2)(3, 7, 4, 5, 8, 6)
then Sch[i]; end if; end for;

Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
ArrayP:=[Id(N): i in [1..#N]];
for i in [2..#N] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=B^-1; end if;
if Eltseq(Sch[i])[j] eq 3 then P[j]:=C; end if;
if Eltseq(Sch[i])[j] eq -3 then P[j]:=C^-1; end if;
if Eltseq(Sch[i])[j] eq 4 then P[j]:=D; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;
for i in [1..#N] do if ArrayP[i] eq N!(1, 3, 2, 5)(4, 8, 6, 7)
then Sch[i]; end if; end for;

Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
ArrayP:=[Id(N): i in [1..#N]];
for i in [2..#N] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;
```

```
if Eltseq(Sch[i])[j] eq -2 then P[j]:=B^-1; end if;
if Eltseq(Sch[i])[j] eq 3 then P[j]:=C; end if;
if Eltseq(Sch[i])[j] eq -3 then P[j]:=C^-1; end if;
if Eltseq(Sch[i])[j] eq 4 then P[j]:=D; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;
for i in [1..#N] do if ArrayP[i] eq N!(1, 4, 2, 6)(3, 7, 5, 8)
then Sch[i]; end if; end for;

prodim:=function(pt,Q,I)
v:=pt;
for i in I do
v:=v^(Q[i]);
end for;
return v;
end function;
ts:=[Id(G1): i in [1..8]];
ts[1]:=f(t); ts[2]:=f(t^a); ts[3]:=f(t^b); ts[4]:=f(t^c);
ts[5]:=f(t^(b^3)); ts[6]:=f(t^(c^3));ts[7]:=f(t^(b*a));
ts[8]:=f(t^(c*a^2));

cst:=[null: i in [1..Index(G,sub<G|a,b,c,d>)]] where null is
[Integers() | ]; for i:= 1 to 8 do
cst[prodim(1, ts, [i])]:= [i];
end for;
m:=0;
for i in [1..#G/#N] do if cst[i] ne [] then m:=m+1;
end if; end for; m;

N1:=Stabiliser(N,1);
N1;
#N/#N1;
Orbits(N1);
for i in [1..8] do i, cst[i]; end for;
N1:=Stabiliser(N,[1]);
N1;

/*NOT NEW [12]=[1]
N12:=Stabiliser(N,[1,2]);
SSS:={[1,2]};
```

```
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]
then print Rep(Seqq[i]);
end if; end for; end for;
T12:=Transversal(N,N12);
for i in [1..#T12] do
ss:=[1,2]^T12[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..#G/#N] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for i in [1..8] do i, cst[i]; end for;
1 []
2 [ 1 ] (Nt1 is 2)
3 [ 2 ]
4 [ 3 ]
5 [ 4 ]
6 [ 5 ]
7 [ 6 ]
8 [ 7 ]
for a in [1..8] do for g,h in IN do if ts[1]*ts[2] eq g*(ts[a])^h
then "1,2=", a; end if; end for; end for;
for g,h in IN do if ts[1]*ts[2] eq g*(ts[1])^h then g,h; end if;
end for;

/*Choose any combination for g and h. When there are two identitied
we use both to make it easier. If there's one then use one.
g=(2, 3)(4, 6)(5, 7)(8, 10) change to t's
g=(1, 2)(3, 5)(4,6)(7, 8)
h=Id(IN)
h=Id(IN)
12=g*(1)^h
12=(1, 2)(3, 5)(4,6)(7, 8)*(1)^Id(IN)
12=g*(1)*/

Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
ArrayP:=[Id(N): i in [1..#N]];
```

```
for i in [2..#N] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=B^-1; end if;
if Eltseq(Sch[i])[j] eq 3 then P[j]:=C; end if;
if Eltseq(Sch[i])[j] eq -3 then P[j]:=C^-1; end if;
if Eltseq(Sch[i])[j] eq 4 then P[j]:=D; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;
for i in [1..#N] do if ArrayP[i] eq N!(1, 2)(3, 5)(4,6)(7, 8)
then Sch[i]; end if; end for;
ts[1]*ts[2] eq f(d)*ts[1];
TRUUUUUUUUUUUUUUE
*/

N13:=Stabiliser(N,[1,3]);
SSS:={[1,3]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
T13:=Transversal(N,N13);
/* #N/#N13s = #T13 */ #T13;
for i in [1..#T13] do
ss:=[1,3]^T13[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N13);
```

```
N15:=Stabiliser(N,[1,5]);
SSS:={[1,5]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
T15:=Transversal(N,N15);#T15;
for i in [1..#T15] do
ss:=[1,5]^T15[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N15);

N131:=Stabiliser(N,[1,3,1]);
SSS:={[1,3,1]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T131:=Transversal(N,N131);#T131;
for i in [1..#T131] do
ss:=[1,3,1]^T131[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N131);
```

```
N132:=Stabiliser(N,[1,3,2]);
SSS:={[1,3,2]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T132:=Transversal(N,N132);#T132;
for i in [1..#T132] do
ss:=[1,3,2]^T132[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N132);

N134:=Stabiliser(N,[1,3,4]);
SSS:={[1,3,4]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T134:=Transversal(N,N134);#T134;
for i in [1..#T134] do
ss:=[1,3,4]^T134[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N134);
```

```
/*Not a new coset, equal to [15]
N135:=Stabiliser(N,[1,3,5]);
SSS:={[1,3,5]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T135:=Transversal(N,N135);#T135;
for i in [1..#T135] do
ss:=[1,3,5]^T135[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[5] eq
g*(ts[a])^h then "1,3,5=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[5]
eq g*(ts[a]*ts[b])^h then "1,3,5=",a,b; end if; end for; end for;
for g,h in IN do if ts[1]*ts[3]*ts[5] eq g*(ts[1]*ts[5])^h
then g,h; end if; end for;
g=(2, 3)(4, 6)(5, 7)(8, 10)
g=(1, 2)(3, 5)(4,6)(7, 8)
h=(2, 3)(4, 6)(5, 7)(8, 10)
h=(1, 2)(3, 5)(4,6)(7, 8)
135=g*(1)^h
135=(1, 2)(3, 5)(4,6)(7, 8)*(15)^(1, 2)(3, 5)(4,6)(7, 8)
135=g*(23)
ts[1]*ts[3]*ts[5] eq f(d)*ts[2]*ts[3];
true*/


/*Not a new coset, equal to [15]
N136:=Stabiliser(N,[1,3,6]);
SSS:={[1,3,6]};
SSS:=SSS^N;
SSS;
```

```
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T136:=Transversal(N,N136);#T136;
for i in [1..#T136] do
ss:=[1,3,6]^T136[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[6]
eq g*(ts[a])^h then "1,3,6=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[6]
eq g*(ts[a]*ts[b])^h then "1,3,6=",a,b; end if; end for; end for;
for g,h in IN do if ts[1]*ts[3]*ts[6] eq g*(ts[1]*ts[5])^h
then g,h; end if; end for;
g=(2, 7, 4)(3, 5, 6)
g=(1, 6, 3)(2, 4, 5)
h=(2, 4, 3, 6)(5, 10, 7, 8)
h=(1, 3, 2, 5)(4, 8, 6, 7)
136=g*(15)^h
136=(1, 6, 3)(2, 4, 5)*(15)^(1, 3, 2, 5)(4, 8, 6, 7)
136=g*(31)
Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
ArrayP:=[Id(N): i in [1..#N]];
for i in [2..#N] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=B^-1; end if;
if Eltseq(Sch[i])[j] eq 3 then P[j]:=C; end if;
if Eltseq(Sch[i])[j] eq -3 then P[j]:=C^-1; end if;
if Eltseq(Sch[i])[j] eq 4 then P[j]:=D; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
```

```
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;
for i in [1..#N] do if ArrayP[i] eq N!(1, 6, 3)(2, 4, 5)
then Sch[i]; end if; end for;
ts[1]*ts[3]*ts[6] eq f(b*a^-1)*ts[3]*ts[1];
true*/

N137:=Stabiliser(N,[1,3,7]);
SSS:={[1,3,7]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T137:=Transversal(N,N137);#T137;
for i in [1..#T137] do
ss:=[1,3,7]^T137[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N137);

N138:=Stabiliser(N,[1,3,8]);
SSS:={[1,3,8]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T138:=Transversal(N,N138);#T138;
```

```
for i in [1..#T138] do
ss:=[1,3,8]^T138[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N138);

N151:=Stabiliser(N,[1,5,1]);
SSS:={[1,5,1]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T151:=Transversal(N,N151);#T151;
for i in [1..#T151] do
ss:=[1,5,1]^T151[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N151);

\*Not a New Coset, equal to [132]
N152:=Stabiliser(N,[1,5,2]);
SSS:={[1,5,2]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
```

```
T152:=Transversal(N,N152);#T152;
for i in [1..#T152] do
ss:=[1,5,2]^T152[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[2]
eq g*(ts[a])^h then "1,5,2=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[2]
eq g*(ts[a]*ts[b])^h then "1,5,2=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[2]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,5,2=",a,b,c; end if;
end for; end for;
for g,h in IN do if ts[1]*ts[5]*ts[2] eq g*(ts[1]*ts[3]*ts[2])^h
then g,h; end if; end for;
g=(2, 3)(4, 6)(5, 7)(8, 10)
g=(1, 2)(3, 5)(4, 6)(7, 8)
h=(2, 6, 3, 4)(5, 8, 7, 10)
h=(1, 5, 2, 3)(4, 7, 6, 8)
152=g*(132)^h
152=(1, 2)(3, 5)(4, 6)(7, 8)*(132)^(1, 5, 2, 3)(4, 7, 6, 8)
152=g*(513)
Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
ArrayP:=[Id(N): i in [1..#N]];
for i in [2..#N] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=B^-1; end if;
if Eltseq(Sch[i])[j] eq 3 then P[j]:=C; end if;
if Eltseq(Sch[i])[j] eq -3 then P[j]:=C^-1; end if;
if Eltseq(Sch[i])[j] eq 4 then P[j]:=D; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;
for i in [1..#N] do if ArrayP[i] eq N!(1, 2)(3, 5)(4, 6)(7, 8)
then Sch[i]; end if; end for;
ts[1]*ts[5]*ts[2] eq f(d)*ts[5]*ts[1]*ts[3]; true*/
```

```
/*Not New, equal to [13]
N153:=Stabiliser(N,[1,5,3]);
SSS:={[1,5,3]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T153:=Transversal(N,N153);#T153;
for i in [1..#T153] do
ss:=[1,5,3]^T153[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[3]
eq g*(ts[a])^h then "1,5,3=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[3]
eq g*(ts[a]*ts[b])^h then "1,5,3=",a,b; end if; end for; end for;
for g,h in IN do if ts[1]*ts[5]*ts[3] eq g*(ts[1]*ts[3])^h
then g,h; end if; end for;
g=(2, 3)(4, 6)(5, 7)(8, 10)
g=(1, 2)(3, 5)(4, 6)(7, 8)
h=(2, 3)(4, 6)(5, 7)(8, 10)
h=(1, 2)(3, 5)(4, 6)(7, 8)
153=g*(13)^h
153=(1, 2)(3, 5)(4, 6)(7, 8)*(13)^(1, 2)(3, 5)(4, 6)(7, 8)
153=g*(25)
Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
ArrayP:=[Id(N): i in [1..#N]];
for i in [2..#N] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=B^-1; end if;
if Eltseq(Sch[i])[j] eq 3 then P[j]:=C; end if;
```

```
if Eltseq(Sch[i])[j] eq -3 then P[j]:=C^-1; end if;
if Eltseq(Sch[i])[j] eq 4 then P[j]:=D; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;
for i in [1..#N] do if ArrayP[i] eq N!(1, 2)(3, 5)(4, 6)(7, 8)
then Sch[i]; end if; end for;
ts[1]*ts[5]*ts[3] eq f(d)*ts[2]*ts[5];
true*/

/*Not New, equal to [137]
N154:=Stabiliser(N,[1,5,4]);
SSS:={[1,5,4]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T154:=Transversal(N,N154);#T154;
for i in [1..#T154] do
ss:=[1,5,4]^T154[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[4]
eq g*(ts[a])^h then "1,5,4=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[4]
eq g*(ts[a]*ts[b])^h then "1,5,4=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[4]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,5,4=",a,b,c; end if;
end for; end for;*/

/*Not new, equal to [138]
N156:=Stabiliser(N,[1,5,6]);
```

```
SSS:={[1,5,6]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T156:=Transversal(N,N156);#T156;
for i in [1..#T156] do
ss:=[1,5,6]^T156[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[6]
eq g*(ts[a])^h then "1,5,6=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[6]
eq g*(ts[a]*ts[b])^h then "1,5,6=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[6]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,5,6=",a,b,c; end if;
end for; end for;*/

/*Not New, equal to [134]
N157:=Stabiliser(N,[1,5,7]);
SSS:={[1,5,7]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T157:=Transversal(N,N157);#T157;
for i in [1..#T157] do
ss:=[1,5,7]^T157[i];
```

```
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[7] eq
g*(ts[a])^h then "1,5,7=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[7]
eq g*(ts[a]*ts[b])^h then "1,5,7=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[7]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,5,7=",a,b,c; end if;
end for; end for;*/

/*Not new, equal to [13]
N158:=Stabiliser(N,[1,5,8]);
SSS:={[1,5,8]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T158:=Transversal(N,N158);#T158;
for i in [1..#T158] do
ss:=[1,5,8]^T158[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[8]
eq g*(ts[a])^h
then "1,5,8=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[8]
eq g*(ts[a]*ts[b])^h
then "1,5,8=",a,b; end if; end for; end for;*/

/*Not new, equal to [132]
N1312:=Stabiliser(N,[1,3,1,2]);
SSS:={[1,3,1,2]};
SSS:=SSS^N;
```

```
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[1]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1312:=Transversal(N,N1312);#T1312;
for i in [1..#T1312] do
ss:=[1,3,1,2]^T1312[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[1]*ts[2]
eq g*(ts[a])^h
then "1,3,1,2=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[1]*ts[2]
eq g*(ts[a]*ts[b])^h
then "1,3,1,2=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[1]*ts[2]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,1,2=",a,b,c; end if; end for; end for;*/

/*NEW*/
N1313:=Stabiliser(N,[1,3,1,3]);
SSS:={[1,3,1,3]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[1]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N1313s:=N1313;
```

```
for n in N do if 1^n eq 1 and 3^n eq 4 and 1^n eq 1 and 3^n eq 4
then N1313s:=sub<N|N1313s,n>; end if; end for;
for n in N do if 1^n eq 1 and 3^n eq 8 and 1^n eq 1 and 3^n eq 8
then N1313s:=sub<N|N1313s,n>; end if; end for;
N1313s; #N1313s;
#N/#N1313s;
T1313:=Transversal(N,N1313);#T1313;
for i in [1..#T1313] do
ss:=[1,3,1,3]^T1313[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
[1,3,1,3]^N1313s;
for i in [1..#T1313] do ([1,3,1,3]^N1313s)^T1313[i]; end for;
Orbits(N1313);

/*NEW*/
N1314:=Stabiliser(N,[1,3,1,4]);
SSS:={[1,3,1,4]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[1]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1314:=Transversal(N,N1314);#T1314;
for i in [1..#T1314] do
ss:=[1,3,1,4]^T1314[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1314);

/*Not new, equal to [151]
N1315:=Stabiliser(N,[1,3,1,5]);
SSS:={[1,3,1,5]};
```

```
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[1]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1315:=Transversal(N,N1315);#T1315;
for i in [1..#T1315] do
ss:=[1,3,1,5]^T1315[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1315);
for a in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[1]*ts[5]
eq g*(ts[a])^h
then "1,3,1,5=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[1]*ts[5]
eq g*(ts[a]*ts[b])^h
then "1,3,1,5=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[1]*ts[5]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,1,5=",a,b,c; end if; end for; end for;*/

/Not new, equal to [151]
N1316:=Stabiliser(N,[1,3,1,6]);
SSS:={[1,3,1,6]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[1]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
```

```
end if; end for; end for;
T1316:=Transversal(N,N1316);#T1316;
for i in [1..#T1316] do
ss:=[1,3,1,6]^T1316[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1316);
for a in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[1]*ts[6]
eq g*(ts[a])^h
then "1,3,1,6=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[1]*ts[6]
eq g*(ts[a]*ts[b])^h
then "1,3,1,6=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[1]*ts[6]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,1,6=",a,b,c; end if; end for; end for;*/

/*NEW*/
N1317:=Stabiliser(N,[1,3,1,7]);
SSS:={[1,3,1,7]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[1]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1317:=Transversal(N,N1317);#T1317;
for i in [1..#T1317] do
ss:=[1,3,1,7]^T1317[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1317);

/*NEW*/
```

```
N1318:=Stabiliser(N,[1,3,1,8]);
SSS:={[1,3,1,8]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[1]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1318:=Transversal(N,N1318);#T1318;
for i in [1..#T1318] do
ss:=[1,3,1,8]^T1318[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1318);

/*Not new, equal to [131]
N1321:=Stabiliser(N,[1,3,2,1]);
SSS:={[1,3,2,1]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[2]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1321:=Transversal(N,N1321);#T1321;
for i in [1..#T1321] do
ss:=[1,3,2,1]^T1321[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
```

```
then m:=m+1; end if; end for; m;
Orbits(N1321);
for a in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[1]
eq g*(ts[a])^h
then "1,3,2,1=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[1]
eq g*(ts[a]*ts[b])^h
then "1,3,2,1=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[1]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,2,1=",a,b,c; end if;
end for; end for;*/

/*Not new, equal to [151]
N1323:=Stabiliser(N,[1,3,2,3]);
SSS:={[1,3,2,3]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[2]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1323:=Transversal(N,N1323);#T1323;
for i in [1..#T1323] do
ss:=[1,3,2,3]^T1323[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1323);
for a in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[3]
eq g*(ts[a])^h
then "1,3,2,3=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[3]
eq g*(ts[a]*ts[b])^h
then "1,3,2,3=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[3]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,2,3=",a,b,c; end if;
end for; end for;*/
```

```
/*Not new, equal to [137]
N1324:=Stabiliser(N,[1,3,2,4]);
SSS:={[1,3,2,4]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[2]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1324:=Transversal(N,N1324);#T1324;
for i in [1..#T1324] do
ss:=[1,3,2,4]^T1324[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1324);
for a in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[4]
eq g*(ts[a])^h
then "1,3,2,4=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[4]
eq g*(ts[a]*ts[b])^h
then "1,3,2,4=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[4]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,2,4=",a,b,c;
end if; end for; end for;*/

/*Not new, equal to [15]
N1325:=Stabiliser(N,[1,3,2,5]);
SSS:={[1,3,2,5]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
```

```
if ts[1]*ts[3]*ts[2]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1325:=Transversal(N,N1325);#T1325;
for i in [1..#T1325] do
ss:=[1,3,2,5]^T1325[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1325);
for a in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[5]
eq g*(ts[a])^h
then "1,3,2,5=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[5]
eq g*(ts[a]*ts[b])^h
then "1,3,2,5=",a,b; end if; end for; end for;*/

/*Not new, equal to [1317]
N1326:=Stabiliser(N,[1,3,2,6]);
SSS:={[1,3,2,6]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[2]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1326:=Transversal(N,N1326);#T1326;
for i in [1..#T1326] do
ss:=[1,3,2,6]^T1326[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1326);
for a in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[6]
```

```
eq g*(ts[a])^h
then "1,3,2,6=",a; end if; end for; end for;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[6]
eq g*(ts[a]*ts[b])^h
then "1,3,2,6=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[6]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,2,6=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[6]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,3,2,6=",a,b,c,d;
end if; end for; end for;*/

/*Not new, equal to [132]
N1327:=Stabiliser(N,[1,3,2,7]);
SSS:={[1,3,2,7]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[2]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1327:=Transversal(N,N1327);#T1327;
for i in [1..#T1327] do
ss:=[1,3,2,7]^T1327[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1327);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[7]
eq g*(ts[a]*ts[b])^h
then "1,3,2,7=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[7]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,2,7=",a,b,c;
end if; end for; end for;*/

/*Not new, equal to [132]
N1328:=Stabiliser(N,[1,3,2,8]);
```

```
SSS:={[1,3,2,8]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[2]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1328:=Transversal(N,N1328);#T1328;
for i in [1..#T1328] do
ss:=[1,3,2,8]^T1328[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1328);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[8]
eq g*(ts[a]*ts[b])^h
then "1,3,2,8=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[2]*ts[8]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,2,8=",a,b,c;
end if; end for; end for;*/

/*Not new, equal to [134]
N1341:=Stabiliser(N,[1,3,4,1]);
SSS:={[1,3,4,1]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[4]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1341:=Transversal(N,N1341);#T1341;
```

```
for i in [1..#T1341] do
ss:=[1,3,4,1]^T1341[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1341);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[4]*ts[1]
 eq g*(ts[a]*ts[b])^h
then "1,3,4,1=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[4]*ts[1]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,4,1=",a,b,c;
 end if; end for; end for;*/

/*Not new, equal to [1318]
N1342:=Stabiliser(N,[1,3,4,2]);
SSS:={[1,3,4,2]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[4]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1342:=Transversal(N,N1342);#T1342;
for i in [1..#T1342] do
ss:=[1,3,4,2]^T1342[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1342);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[4]*ts[2]
eq g*(ts[a]*ts[b])^h
then "1,3,4,2=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[4]*ts[2]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,4,2=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[4]*ts[2]
```

```
eq g*(ts[a]
*ts[b]*ts[c]*ts[d])^h then "1,3,4,2=",a,b,c,d; end if;
end for; end for;*/

/*Not new, equal to [1318]
N1343:=Stabiliser(N,[1,3,4,3]);
SSS:={[1,3,4,3]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[4]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1343:=Transversal(N,N1343);#T1343;
for i in [1..#T1343] do
ss:=[1,3,4,3]^T1343[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1343);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[4]*ts[3]
eq g*(ts[a]*ts[b])^h
then "1,3,4,3=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[4]*ts[3]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,4,3=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[4]*ts[3]
 eq g*(ts[a]
*ts[b]*ts[c]*ts[d])^h then "1,3,4,3=",a,b,c,d;
end if; end for; end for;*/

/*Not new, equl to [134]
N1345:=Stabiliser(N,[1,3,4,5]);
SSS:={[1,3,4,5]};
SSS:=SSS^N;
SSS;
#SSS;
```

```
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[4]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1345:=Transversal(N,N1345);#T1345;
for i in [1..#T1345] do
ss:=[1,3,4,5]^T1345[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1345);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[4]*ts[5]
eq g*(ts[a]*ts[b])^h
then "1,3,4,5=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[4]*ts[5]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,4,5=",a,b,c; end if; end for; end for;*/

/*Not new, equal to [15]
N1346:=Stabiliser(N,[1,3,4,6]);
SSS:={[1,3,4,6]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[4]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1346:=Transversal(N,N1346);#T1346;
for i in [1..#T1346] do
ss:=[1,3,4,6]^T1346[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
```

```
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1346);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[4]*ts[6]
eq g*(ts[a]*ts[b])^h
then "1,3,4,6=",a,b; end if; end for; end for;*/

/*NEW*/
N1347:=Stabiliser(N,[1,3,4,7]);
SSS:={[1,3,4,7]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[4]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1347:=Transversal(N,N1347);#T1347;
for i in [1..#T1347] do
ss:=[1,3,4,7]^T1347[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1347);

/*Not new, equal to [138]
N1348:=Stabiliser(N,[1,3,4,8]);
SSS:={[1,3,4,8]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[4]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
```

```
then print Rep(Seqq[i]);
end if; end for; end for;
T1348:=Transversal(N,N1348);#T1348;
for i in [1..#T1348] do
ss:=[1,3,4,8]^T1348[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1348);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[4]*ts[8]
eq g*(ts[a]*ts[b])^h
then "1,3,4,8=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[4]*ts[8]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,4,8=",a,b,c; end if; end for; end for;*/

/*Not New, equal to [138]
N1371:=Stabiliser(N,[1,3,7,1]);
SSS:={[1,3,7,1]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[7]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1371:=Transversal(N,N1371);#T1371;
for i in [1..#T1371] do
ss:=[1,3,7,1]^T1371[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1371);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[7]*ts[1]
eq g*(ts[a]*ts[b])^h
then "1,3,7,1=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[7]*ts[1]
```

```
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,7,1=",a,b,c;
 end if; end for; end for;*/

/*Not new, equal to [15]
N1372:=Stabiliser(N,[1,3,7,2]);
SSS:={[1,3,7,2]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[7]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1372:=Transversal(N,N1372);#T1372;
for i in [1..#T1372] do
ss:=[1,3,7,2]^T1372[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1372);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[7]*ts[2]
eq g*(ts[a]*ts[b])^h
then "1,3,7,2=",a,b; end if; end for; end for;*/

/*Not new, equal to [1317]
N1373:=Stabiliser(N,[1,3,7,3]);
SSS:={[1,3,7,3]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[7]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
```

```
end if; end for; end for;
T1373:=Transversal(N,N1373);#T1373;
for i in [1..#T1373] do
ss:=[1,3,7,3]^T1373[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1373);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[7]*ts[3]
eq g*(ts[a]*ts[b])^h
then "1,3,7,3=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[7]*ts[3]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,7,3=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[7]*ts[3]
eq g*(ts[a]
*ts[b]*ts[c]*ts[d])^h then "1,3,7,3=",a,b,c,d;
end if; end for; end for;*/

/*NEW*/
N1374:=Stabiliser(N,[1,3,7,4]);
SSS:={[1,3,7,4]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[7]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1374:=Transversal(N,N1374);#T1374;
for i in [1..#T1374] do
ss:=[1,3,7,4]^T1374[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1374);
```

```
/*Not new, equal to [132]
N1375:=Stabiliser(N,[1,3,7,5]);
SSS:={[1,3,7,5]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[7]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1375:=Transversal(N,N1375);#T1375;
for i in [1..#T1375] do
ss:=[1,3,7,5]^T1375[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1375);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[7]*ts[5]
eq g*(ts[a]*ts[b])^h
then "1,3,7,5=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[7]*ts[5]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,7,5=",a,b,c; end if; end for; end for;*/

/*Not new, equal to [1374]
N1376:=Stabiliser(N,[1,3,7,6]);
SSS:={[1,3,7,6]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[7]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
```

```
end if; end for; end for;
T1376:=Transversal(N,N1376);#T1376;
for i in [1..#T1376] do
ss:=[1,3,7,6]^T1376[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1376);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[7]*ts[6]
eq g*(ts[a]*ts[b])^h
then "1,3,7,6=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[7]*ts[6]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,7,6=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[7]*ts[6]
 eq g*(ts[a]
*ts[b]*ts[c]*ts[d])^h then "1,3,7,6=",a,b,c,d;
end if; end for; end for;*/

/*Not new, equal to [138]
N1378:=Stabiliser(N,[1,3,7,8]);
SSS:={[1,3,7,8]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[7]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1378:=Transversal(N,N1378);#T1378;
for i in [1..#T1378] do
ss:=[1,3,7,8]^T1378[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1378);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[7]*ts[8]
```

```
eq g*(ts[a]*ts[b])^h
then "1,3,7,8=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[7]*ts[8]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,7,8=",a,b,c; end if; end for; end for;*/

/*Not new, equal to [137]
N1381:=Stabiliser(N,[1,3,8,1]);
SSS:={[1,3,8,1]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[8]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1381:=Transversal(N,N1381);#T1381;
for i in [1..#T1381] do
ss:=[1,3,8,1]^T1381[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1381);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[1]
eq g*(ts[a]*ts[b])^h
then "1,3,8,1=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[1]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,8,1=",a,b,c; end if; end for; end for;*/

/*Not new, equal to [15]
N1382:=Stabiliser(N,[1,3,8,2]);
SSS:={[1,3,8,2]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
```

```
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[8]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1382:=Transversal(N,N1382);#T1382;
for i in [1..#T1382] do
ss:=[1,3,8,2]^T1382[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1382);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[2]
eq g*(ts[a]*ts[b])^h
then "1,3,8,2=",a,b; end if; end for; end for;*/

/*Not new, equal to [1314]
N1383:=Stabiliser(N,[1,3,8,3]);
SSS:={[1,3,8,3]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[8]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1383:=Transversal(N,N1383);#T1383;
for i in [1..#T1383] do
ss:=[1,3,8,3]^T1383[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1383);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[3]
eq g*(ts[a]*ts[b])^h
```

```
then "1,3,8,3=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[3]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,8,3=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[3]
eq g*(ts[a]
*ts[b]*ts[c]*ts[d])^h then "1,3,8,3=",a,b,c,d;
end if; end for; end for;*/

/*Not new, equal to [134]
N1384:=Stabiliser(N,[1,3,8,4]);
SSS:={[1,3,8,4]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[8]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1384:=Transversal(N,N1384);#T1384;
for i in [1..#T1384] do
ss:=[1,3,8,4]^T1384[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1384);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[4]
eq g*(ts[a]*ts[b])^h
then "1,3,8,4=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[4]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,8,4=",a,b,c; end if; end for; end for;*/

/*Not new, equal to [1314]
N1385:=Stabiliser(N,[1,3,8,5]);
SSS:={[1,3,8,5]};
SSS:=SSS^N;
SSS;
```

```
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[8]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1385:=Transversal(N,N1385);#T1385;
for i in [1..#T1385] do
ss:=[1,3,8,5]^T1385[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1385);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[5]
eq g*(ts[a]*ts[b])^h
then "1,3,8,5=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[5]
eq g*(ts[a]
*ts[b]*ts[c])^h
then "1,3,8,5=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[5]
eq g*(ts[a]
*ts[b]*ts[c]*ts[d])^h
then "1,3,8,5=",a,b,c,d; end if; end for; end for;*/

/*Not new, equal to [1347]
N1386:=Stabiliser(N,[1,3,8,6]);
SSS:={[1,3,8,6]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[8]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
```

```
end if; end for; end for;
T1386:=Transversal(N,N1386);#T1386;
for i in [1..#T1386] do
ss:=[1,3,8,6]^T1386[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1386);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[6]
eq g*(ts[a]*ts[b])^h
then "1,3,8,6=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[6]
eq g*(ts[a]
*ts[b]*ts[c])^h
then "1,3,8,6=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[6]
eq g*(ts[a]
*ts[b]*ts[c]*ts[d])^h
then "1,3,8,6=",a,b,c,d; end if; end for; end for;*/

/*Not new, equal to [137]
N1387:=Stabiliser(N,[1,3,8,7]);
SSS:={[1,3,8,7]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[8]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1387:=Transversal(N,N1387);#T1387;
for i in [1..#T1387] do
ss:=[1,3,8,7]^T1387[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1387);
```

```
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[7]
eq g*(ts[a]*ts[b])^h
then "1,3,8,7=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[3]*ts[8]*ts[7]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,3,8,7=",a,b,c; end if; end for; end for;*/

/*Not new, equal to [132]
N1512:=Stabiliser(N,[1,5,1,2]);
SSS:={[1,5,1,2]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5]*ts[1]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1512:=Transversal(N,N1512);#T1512;
for i in [1..#T1512] do
ss:=[1,5,1,2]^T1512[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1512);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[2]
eq g*(ts[a]*ts[b])^h
then "1,5,1,2=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[2]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,5,1,2=",a,b,c; end if; end for; end for;*/

/*Not new, equal to [131]
N1513:=Stabiliser(N,[1,5,1,3]);
SSS:={[1,5,1,3]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
```

```
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5]*ts[1]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1513:=Transversal(N,N1513);#T1513;
for i in [1..#T1513] do
ss:=[1,5,1,3]^T1513[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1513);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[3]
eq g*(ts[a]*ts[b])^h
then "1,5,1,3=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[3]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,5,1,3=",a,b,c; end if; end for; end for;*/

/*Not new, equal to [1317]
N1514:=Stabiliser(N,[1,5,1,4]);
SSS:={[1,5,1,4]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5]*ts[1]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1514:=Transversal(N,N1514);#T1514;
for i in [1..#T1514] do
ss:=[1,5,1,4]^T1514[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
```

```
then m:=m+1; end if; end for; m;
Orbits(N1514);
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[4]
eq g*(ts[a]*ts[b])^h
then "1,5,1,4=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[4]
 eq g*(ts[a]
*ts[b]*ts[c])^h then "1,5,1,4=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[4]
eq g*(ts[a]
*ts[b]*ts[c]*ts[d])^h then "1,5,1,4=",a,b,c,d;
end if; end for; end for;*/

/*Not new, equal to [1313]
N1515:=Stabiliser(N,[1,5,1,5]);
SSS:={[1,5,1,5]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5]*ts[1]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1515:=Transversal(N,N1515);#T1515;
for i in [1..#T1515] do
ss:=[1,5,1,5]^T1515[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[5]
eq g*(ts[a]*ts[b])^h
then "1,5,1,5=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[5]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,5,1,5=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[5]
eq g*(ts[a]
*ts[b]*ts[c]*ts[d])^h then "1,5,1,5=",a,b,c,d;
```

```
end if; end for; end for;*/

/*Not new, equal to [1318]
N1516:=Stabiliser(N,[1,5,1,6]);
SSS:={[1,5,1,6]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5]*ts[1]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1516:=Transversal(N,N1516);#T1516;
for i in [1..#T1516] do
ss:=[1,5,1,6]^T1516[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[6]
eq g*(ts[a]*ts[b])^h
then "1,5,1,6=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[6]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,5,1,6=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[6]
eq g*(ts[a]
*ts[b]*ts[c]*ts[d])^h then "1,5,1,6=",a,b,c,d;
end if; end for; end for;*/

/*Not new, equal to [1314]
N1517:=Stabiliser(N,[1,5,1,7]);
SSS:={[1,5,1,7]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
```

```
for n in IN do
if ts[1]*ts[5]*ts[1]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1517:=Transversal(N,N1517);#T1517;
for i in [1..#T1517] do
ss:=[1,5,1,7]^T1517[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[7]
eq g*(ts[a]*ts[b])^h
then "1,5,1,7=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[7]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,5,1,7=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[7]
eq g*(ts[a]
*ts[b]*ts[c]*ts[d])^h then "1,5,1,7=",a,b,c,d;
end if; end for; end for;*/

/*Not new, equal to [131]
N1518:=Stabiliser(N,[1,5,1,8]);
SSS:={[1,5,1,8]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[5]*ts[1]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
T1518:=Transversal(N,N1518);#T1518;
for i in [1..#T1518] do
ss:=[1,5,1,8]^T1518[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
```

```
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a,b in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[8]
eq g*(ts[a]*ts[b])^h
then "1,5,1,8=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[8]
eq g*(ts[a]
*ts[b]*ts[c])^h then "1,5,1,8=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if ts[1]*ts[5]*ts[1]*ts[8]
eq g*(ts[a]
*ts[b]*ts[c]*ts[d])^h then "1,5,1,8=",a,b,c,d;
end if; end for; end for;*/

/*NEW*/
N13131:=Stabiliser(N,[1,3,1,3,1]);
SSS:={[1,3,1,3,1]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[1]*ts[3]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]*ts[Rep(Seqq[i])[5]]
then print Rep(Seqq[i]);
end if; end for; end for;
T13131:=Transversal(N,N13131);#T13131;
for i in [1..#T13131] do
ss:=[1,3,1,3,1]^T13131[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..330] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N13131);

/*[13134]=[1313]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[2]
eq g*(ts[a]*ts[b])^h then "1,3,1,3,2=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[2]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,3,2=",a,b,c; end if;
```

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end for; end for;for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[2] eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h
then "1,3,1,3,2=",a,b,c,d; end if; end for; end for;*/

/*[13134]=[131]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[4]
eq g*(ts[a]*ts[b])^h then "1,3,1,3,4=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[4]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,3,4=",a,b,c;
end if; end for; end for;*/

/*[13135]=[151]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[5]
eq g*(ts[a]*ts[b])^h then "1,3,1,3,5=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[5]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,3,5=",a,b,c;
end if; end for; end for;*/

/*[13136]=[151]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[6]
eq g*(ts[a]*ts[b])^h then "1,3,1,3,6=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[6]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,3,6=",a,b,c;
end if; end for; end for;*/

/*[13137]=[151]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[7]
eq g*(ts[a]*ts[b])^h then "1,3,1,3,7=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[7]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,3,7=",a,b,c;
end if; end for; end for;*/

/*[13138]=[131]
for a,b in [1..8] do for g,h in IN do if
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ts[1]*ts[3]*ts[1]*ts[3]*ts[8]
eq g*(ts[a]*ts[b])^h then "1,3,1,3,8=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[8]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,3,8=",a,b,c;
end if; end for; end for;*/

/*[13141]=[1314]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[1]
eq g*(ts[a]*ts[b])^h then "1,3,1,4,1=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[1]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,4,1=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[1]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,3,1,4,1=",a,b,c,d;
end if; end for; end for;*/

/*[13142]=[138]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[2]
eq g*(ts[a]*ts[b])^h then "1,3,1,4,2=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[2]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,4,2=",a,b,c;
end if; end for; end for;*/

/*[13143]=[138]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[3] eq
g*(ts[a]*ts[b])^h then "1,3,1,4,3=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[3]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,4,3=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[3]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,3,1,4,3=",a,b,c,d;
end if; end for; end for;*/
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/*[13145]=[1314]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[5]
eq g*(ts[a]*ts[b])^h then "1,3,1,4,5=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[5]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,4,5=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[5]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,3,1,4,5=",a,b,c,d;
end if; end for; end for;*/

/*[13146]=[151]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[6]
eq g*(ts[a]*ts[b])^h then "1,3,1,4,6=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[6] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,4,6=",a,b,c;
end if; end for; end for;*/

/*[13147]=[1374]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[7] eq
g*(ts[a]*ts[b])^h then "1,3,1,4,7=",a,b; end if;
end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[7] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,4,7=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[7]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,3,1,4,7=",a,b,c,d;
end if; end for; end for;*/

/*[13148]=[1318]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[8] eq
g*(ts[a]*ts[b])^h then "1,3,1,4,8=",a,b;
end if; end for; end for;
```

```
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[8] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,4,8=",a,b,c; end if; end for;
end for; for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[4]*ts[8]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,3,1,4,8=",a,b,c,d;
end if; end for; end for;*/

/*[13171]=[1318]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[1] eq
g*(ts[a]*ts[b])^h then "1,3,1,7,1=",a,b; end if;
end for;end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[1] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,7,1=",a,b,c; end if;
end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[1]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,3,1,7,1=",a,b,c,d;
end if; end for; end for;*/

/*[13172]=[151]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[2] eq
g*(ts[a]*ts[b])^h then "1,3,1,7,2=",a,b; end if; end for; end
for; for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[2] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,7,2=",a,b,c;
end if; end for; end for;*/

/*[13173]=[137]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[3] eq
g*(ts[a]*ts[b])^h then "1,3,1,7,3=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[3]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,7,3=",a,b,c; end if;
end for; end for;*/

/*[13174]=[1347]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[4] eq
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g*(ts[a]*ts[b])^h then "1,3,1,7,4=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[4] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,7,4=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[4]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h
then "1,3,1,7,4=",a,b,c,d; end if; end for; end for;*/

/*[13175]=[132]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[5] eq
g*(ts[a]*ts[b])^h then "1,3,1,7,5=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[5]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,7,5=",a,b,c;
end if; end for; end for;*/

/*[13176]=[1347]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[6] eq
g*(ts[a]*ts[b])^h then "1,3,1,7,6=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[6] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,7,6=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[6]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,1,7,6=",a,b,c,d; end if; end for; end for;*/

/*[13178]=[1318]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[8] eq
g*(ts[a]*ts[b])^h then "1,3,1,7,8=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[7]*ts[8] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,7,8=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
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ts[1]*ts[3]*ts[1]*ts[7]*ts[8]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,3,1,7,8=",a,b,c,d;
end if; end for; end for;*/

/*[13181]=[1317]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[1] eq
g*(ts[a]*ts[b])^h then "1,3,1,8,1=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[1] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,8,1=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[1]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,1,8,1=",a,b,c,d; end if; end for; end for;*/

/*[13182]=[151]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[2] eq
g*(ts[a]*ts[b])^h then "1,3,1,8,2=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[2] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,8,2=",a,b,c;
end if; end for; end for;*/

/*[13183]=[134]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[3] eq
g*(ts[a]*ts[b])^h then "1,3,1,8,3=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[3] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,8,3=",a,b,c;
end if; end for; end for;*/

/*[13184]=[1314]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[4] eq
g*(ts[a]*ts[b])^h then "1,3,1,8,4=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[4] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,8,4=",a,b,c;
end if; end for; end for;
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for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[4]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,1,8,4=",a,b,c,d; end if; end for; end for;*/

/*[13185]=[134]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[5] eq
g*(ts[a]*ts[b])^h then "1,3,1,8,5=",a,b; end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[5] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,8,5=",a,b,c;
end if; end for; end for;*/

/*[13186]=[1374]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[6] eq
g*(ts[a]*ts[b])^h then "1,3,1,8,6=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[6] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,8,6=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[6]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,1,8,6=",a,b,c,d; end if;
end for; end for;*/

/*[13187]=[1317]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[7] eq
g*(ts[a]*ts[b])^h then "1,3,1,8,7=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[7] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,8,7=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[8]*ts[7] eq
g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,1,8,7=",a,b,c,d; end if; end for; end for;*/

/*[13471]=[1317]
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for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[1] eq
g*(ts[a]*ts[b])^h then "1,3,4,7,1=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[1] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,4,7,1=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[1] eq
g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,4,7,1=",a,b,c,d; end if;
end for; end for;*/

/*[13472]=[1347]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[2] eq
g*(ts[a]*ts[b])^h then "1,3,4,7,2=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[2] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,4,7,2=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[2]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,4,7,2=",a,b,c,d; end if; end for; end for;*/

/*[13473]=[1347]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[3] eq
g*(ts[a]*ts[b])^h then "1,3,4,7,3=",a,b; end if;
end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[3] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,4,7,3=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[3]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,4,7,3=",a,b,c,d; end if;
end for; end for;*/

/*[13474]=[1347]
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for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[4] eq
g*(ts[a]*ts[b])^h then "1,3,4,7,4=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[4] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,4,7,4=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[4]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,4,7,4=",a,b,c,d; end if;
end for; end for;*/

/*[13475]=[1317]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[5] eq
g*(ts[a]*ts[b])^h then "1,3,4,7,5=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[5] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,4,7,5=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[5]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,4,7,5=",a,b,c,d; end if; end for; end for;*/

/*[13476]=[1347]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[6] eq
g*(ts[a]*ts[b])^h then "1,3,4,7,6=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[6] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,4,7,6=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[6]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,4,7,6=",a,b,c,d; end if; end for; end for;*/

/*[13478]=[138]
for a,b in [1..8] do for g,h in IN do if
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ts[1]*ts[3]*ts[4]*ts[7]*ts[8] eq
g*(ts[a]*ts[b])^h then "1,3,4,7,8=",a,b; end if;
end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[4]*ts[7]*ts[8] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,4,7,8=",a,b,c;
end if; end for; end for;*/

/*[13741]=[1314]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[1] eq
g*(ts[a]*ts[b])^h then "1,3,7,4,1=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[1] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,7,4,1=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[1]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,7,4,1=",a,b,c,d; end if; end for; end for;*/

/*[13742]=[1318]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[2] eq
g*(ts[a]*ts[b])^h then "1,3,7,4,2=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[2] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,7,4,2=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[2]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h
then "1,3,7,4,2=",a,b,c,d; end if; end for; end for;*/

/*[13743]=[1374]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[3] eq
g*(ts[a]*ts[b])^h then "1,3,7,4,3=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[3] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,7,4,3=",a,b,c;
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end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[3]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,7,4,3=",a,b,c,d; end if; end for; end for;*/

/*[13745]=[137]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[5] eq
g*(ts[a]*ts[b])^h then "1,3,7,4,5=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[5] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,7,4,5=",a,b,c;
end if; end for; end for;*/

/*[13746]=[1374]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[6] eq
g*(ts[a]*ts[b])^h then "1,3,7,4,6=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[6] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,7,4,6=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[6]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,7,4,6=",a,b,c,d; end if; end for; end for;*/

/*[13747]=[1374]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[7] eq
g*(ts[a]*ts[b])^h then "1,3,7,4,7=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[7] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,7,4,7=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[7]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h
then "1,3,7,4,7=",a,b,c,d; end if; end for; end for;*/
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/*[13748]=[1374]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[8] eq
g*(ts[a]*ts[b])^h then "1,3,7,4,8=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[8] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,7,4,8=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[7]*ts[4]*ts[8]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,7,4,8=",a,b,c,d; end if; end for; end for;*/

/*[131312]=[1313]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[2]
eq g*(ts[a]*ts[b])^h then "1,3,1,3,1,2=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[2]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,3,1,2=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[2] eq
g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,1,3,1,2=",a,b,c,d; end if; end for; end for;*/

/*[131313]=[1313]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[3]
eq g*(ts[a]*ts[b])^h then "1,3,1,3,1,3=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[3]
eq g*(ts[a]*ts[b]*ts[c])^h then
"1,3,1,3,1,3=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[3] eq
g*(ts[a]*ts[b]*ts[c]*ts[d])^h
then "1,3,1,3,1,3=",a,b,c,d; end if; end for; end for;*/

/*[131314]=[1313]
for a,b in [1..8] do for g,h in IN do if
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ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[4]
eq g*(ts[a]*ts[b])^h then "1,3,1,3,1,4=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[4] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,3,1,4=",a,b,c;
end if; end for; end for;for a,b,c,d in [1..8] do
for g,h in IN do if ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[4] eq
g*(ts[a]*ts[b]*ts[c]*ts[d])^h
then "1,3,1,3,1,4=",a,b,c,d; end if; end for; end for;*/

/*[131315]=[1313]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[5]
eq g*(ts[a]*ts[b])^h then "1,3,1,3,1,5=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[5] eq g*(ts[a]*ts[b]*ts[c])^h
then "1,3,1,3,1,5=",a,b,c; end if; end for;
end for;for a,b,c,d in [1..8] do for
g,h in IN do if ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[5] eq
g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,3,1,3,1,5=",a,b,c,d;
end if; end for; end for;*/

/*[131316]=[1313]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[6]
eq g*(ts[a]*ts[b])^h then "1,3,1,3,1,6=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[6]
eq g*(ts[a]*ts[b]*ts[c])^h then
"1,3,1,3,1,6=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[6] eq
g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,3,1,3,1,6=",a,b,c,d;
end if; end for; end for;*/

/*[131317]=[1313]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[7]
eq g*(ts[a]*ts[b])^h then "1,3,1,3,1,7=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
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ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[7] eq g*(ts[a]*ts[b]*ts[c])^h
then "1,3,1,3,1,7=",a,b,c; end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[7] eq
g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,3,1,3,1,7=",a,b,c,d;
end if; end for; end for;*/

/*[131318]=[1313]
for a,b in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[8]
eq g*(ts[a]*ts[b])^h then "1,3,1,3,1,8=",a,b;
end if; end for; end for;
for a,b,c in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[8] eq
g*(ts[a]*ts[b]*ts[c])^h then "1,3,1,3,1,8=",a,b,c;
end if; end for; end for;
for a,b,c,d in [1..8] do for g,h in IN do if
ts[1]*ts[3]*ts[1]*ts[3]*ts[1]*ts[8] eq
g*(ts[a]*ts[b]*ts[c]*ts[d])^h then
"1,3,1,3,1,8=",a,b,c,d; end if; end for; end for;*/
```

B.2 Construction of $PSL_2(19)$ over $2^{*10} : D_{20}$

```
S:=Sym(10);
A:=S!(1, 3, 9, 7, 8)(2, 6, 4, 10, 5);
B:=S!(1, 2)(3, 5)(4, 7)(6, 8)(9, 10);
C:=S!(1, 4)(2, 7)(3, 10)(5, 9)(6, 8);
N:=sub<S|A,B,C>;

r1:=0;r2:=0;r3:=0;r4:=0;r5:=0;r6:=0;r7:=0;r8:=0;r9:=3;r10:=9;r11:=5;
G<a,b,c,t>:=Group<a,b,c,t|b^2,c^2,(a^-1*b)^2,a^-1*c*a*c,(b*c)^2,a^-5,
t^2,(t,b*a^2*c),(c*t)^r1,(b*t^(a^-1))^r2,(b*t)^r3,(b*t^a)^r4,
(b*c*t^(a^-1))^r5,(b*c*t)^r6,(b*c*t^a)^r7,(a*t)^r8,(a^2*t)^r9,
(a*c*t)^r10,(c*a^-2*t)^r11,(a * c * t * a * t * a^-1 * t * b)>;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);
IN:=sub<G1|f(a),f(b),f(c)>;
NN<x,y,z>:=Group<x,y,z|y^2,z^2,(x^-1*y)^2,x^-1*z*x*z,(y*z)^2,x^-5>;
Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
ArrayP:=[Id(N): i in [1..#N]];
for i in [2..#N] do
P:=[Id(N): l in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
```

```
if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;
if Eltseq(Sch[i])[j] eq 3 then P[j]:=C; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;
for i in [1..20] do if ArrayP[i] eq N!(1, 3, 9, 7, 8)(2, 6, 4, 10, 5)
then Sch[i]; end if; end for;

prodim:=function(pt,Q,I)
v:=pt;
for i in I do
v:=v^(Q[i]);
end for;
return v;
end function;
ts:=[Id(G1): i in [1..10]];
ts[1]:=f(t); ts[2]:=f(t^b); ts[3]:=f(t^a); ts[4]:=f(t^c);
ts[5]:=f(t^(a*b)); ts[6]:=f(t^(a^4*b));ts[7]:=f(t^(a^3));
ts[8]:=f(t^(a^4)); ts[9]:=f(t^(a^2));
ts[10]:=f(t^(a^2*b));

cst:=[null: i in [1..Index(G,sub<G|a,b,c>)]] where null is
[Integers() | ];
for i:= 1 to 10 do
cst[prodim(1, ts, [i])]:= [i];
end for;

m:=0;
for i in [1..171] do if cst[i] ne [] then m:=m+1; end if; end for; m;
 /* 171=|G|/|N| */

N1:=Stabiliser(N,1);
#N/#N1;
Orbits(N1);
for i in [1..10] do i, cst[i]; end for;

N14:=Stabiliser(N,[1,4]);
SSS:={[1,4]};

/*conjugating the double coset by N*/
```

```
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;

/*We change it to sequence to check whether any of our singles cosets
are equal to each other inside N14, our double coset*/
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;

/*Transversals are the number of right cosets of N14 in N. In other
words, N has permutations and N(14) has permutations that stabilise
the coset [1,4] and the equal names of N14(for ex, if there's a
permutation that sends (2,3) to (1,4), not this case though).
The Transversals are the permutations that are in N but are NOT in
N(14) and we are right multiplying to all elements in N(14).
(#Tranversals = #single cosets) */

T14:=Transversal(N,N14);
/* #N/#N14 = #T14 (#Tranversals = #single cosets)*/
for i in [1..#T14] do
ss:=[1,4]^T14[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;

/* ([1,4]^N14)^T14 gives all of the different #T14 cosets in the
double coset Nt1t4N These are computed as follows*/
for i in [1..#T14] do ([1,4]^N14)^T14[i]; end for;
Orbits(N14);
#N14;

N12:=Stabiliser(N,[1,2]);
SSS:={[1,2]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
```

```
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
T12:=Transversal(N,N12);
/* #N/#N12s = #T12 */
for i in [1..#T12] do
ss:=[1,2]^T12[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;

/* NOT A NEW COSET, equal to [1]
N13:=Stabiliser(N,[1,3]);
SSS:={[1,3]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N13s:=N13;
for n in N do if 1^n eq 1 and 3^n eq 8 then
N13s:=sub<N|N13s,n>; end if; end for;
N13s; #N13s;
#N/#N13s;
(#N/#N13s gives the number of single coset in the double
cosets in the double coset $Nt1t13 )
T13:=Transversal(N,N13s);
( #N/#N13s = #T13 )
for i in [1..#T13] do
ss:=[1,3]^T13[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
```

```
then m:=m+1; end if; end for; m;
( [1,3]^N13s  gives equal names of the coset Nt1t3 )
[1,3]^N13s;
( ([1,3]^N13s)^T13 gives all of the different #T13 cosets in the
double coset Nt0t1N These are computed as follows)
for i in [1..#T13] do ([1,3]^N13s)^T13[i]; end for;
Orbits(N13s);
for n in IN do if ts[1]*ts[3] eq n*ts[1]*ts[8] then n; end if; end for;
for i in [1..20] do i, cst[i]; end for;

ts:=[Id(G1): i in [1..15]];
ts[1]:=f(t); ts[2]:=f(t^b); ts[3]:=f(t^a); ts[4]:=f(t^c);
ts[5]:=f(t^(a*b)); ts[6]:=f(t^(a^4*b));ts[7]:=f(t^(a^3));
ts[8]:=f(t^(a^4)); ts[9]:=f(t^(a^2));
ts[10]:=f(t^(a^2*b));

ts[1]*ts[3] eq f(a^3)*ts[1]*ts[8];
(true)#N13s;
for a in [1..10] do for g,h in IN do if ts[1]*ts[3] eq g*(ts[a])^h
then "1,3=", a; end if; end for; end for;
*/

N16:=Stabiliser(N,[1,6]);
SSS:={[1,6]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
T16:=Transversal(N,N16);
/* #N/#N16s = #T16 */
for i in [1..#T16] do
ss:=[1,6]^T16[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
```

```
/*NOT A NEW COSET, equal to [1]
N17:=Stabiliser(N,[1,7]);
SSS:={[1,7]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
T17:=Transversal(N,N17);
/* #N/#N17s = #T17 */
for i in [1..#T17] do
ss:=[1,7]^T17[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;

for m,n in IN do if ts[1]*ts[7] eq m*(ts[9]*ts[8])^n
then m, n; end if; end for;
for i in [1..20] do i, cst[i]; end for;
for a in [1..10] do for g,h in IN do if ts[1]*ts[7] eq g*(ts[a])^h
then "1,3=", a; end if; end for; end for;*/

/*NOT NEW, equal to [14]
N141:=Stabiliser(N,[1,4,1]);
SSS:={[1,4,1]};

(conjugating the double coset by N)
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;

(We change it to sequence to check whether any of our singles cosets
are equal to each other inside N141, our double coset)
for i in [1..#SSS] do
for n in IN do
```

```
if ts[1]*ts[4]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;

(Transversals are the number of right cosets of N141 in N. In other
words, N has permutations and N(141) has permutations that stabilise
the coset [1,4,1] and the equal names of N141)

T141:=Transversal(N,N141);
/*( #N/#N14 = #T14 (#Tranversals = #single cosets))*/
for i in [1..#T141] do
ss:=[1,4,1]^T141[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;

for a in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[1] eq g*(ts[a])^h
then "1,4,1=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[1]
eq g*(ts[a]*ts[b])^h then "1,4,1=", a,b; end if; end for; end for;*/

/*Not a new coset, equal to [12]
N142:=Stabiliser(N,[1,4,2]);
SSS:={[1,4,2]};
-conjugating the double coset by N
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
-We change it to sequence to check whether any of our singles
cosets are equal to each other inside N142, our double coset
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[4]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T142:=Transversal(N,N142);
for i in [1..#T142] do
ss:=[1,4,2]^T142[i];
cst[prodim(1, ts, ss)]:=ss;
```

```
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
for a in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[2]
eq g*(ts[a])^h then "1,4,2=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[2]
eq g*(ts[a]*ts[b])^h then "1,4,2=", a,b; end if; end for; end for;
*/

N143:=Stabiliser(N,[1,4,3]);
SSS:={[1,4,3]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[4]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N143s:=N143;
for n in N do if 1^n eq 3 and 4^n eq 10 and 3^n eq 1 then
N143s:=sub<N|N143s,n>; end if; end for;
N143s; #N143s;
#N/#N143s;
T143:=Transversal(N,N143s);
for i in [1..#T143] do
ss:=[1,4,3]^T143[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
[1,4,3]^N143s;
for i in [1..#T143] do ([1,4,3]^N143s)^T143[i]; end for;
Orbits(N143s);

/*NOT A NEW COSET [146], equal to [16]
for a in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[6] eq
g*(ts[a])^h then "1,4,6=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[6] eq
g*(ts[a]*ts[b])^h then "1,4,6=", a,b; end if; end for; end for;
*/
```

```
N147:=Stabiliser(N,[1,4,7]);
SSS:={[1,4,7]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[4]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N147s:=N147;
for n in N do if 1^n eq 4 and 4^n eq 1 and 7^n eq 2 then
N147s:=sub<N|N147s,n>; end if; end for;
for n in N do if 1^n eq 6 and 4^n eq 8 and 7^n eq 10 then
N147s:=sub<N|N147s,n>; end if; end for;
for n in N do if 1^n eq 8 and 4^n eq 6 and 7^n eq 3 then
N147s:=sub<N|N147s,n>; end if; end for;
N147s; #N147s;
#N/#N147s;
T147:=Transversal(N,N147s);
for i in [1..#T147] do
ss:=[1,4,7]^T147[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
[1,4,7]^N147s;
for i in [1..#T147] do ([1,4,7]^N147s)^T147[i]; end for;
Orbits(N147s);

N121:=Stabiliser(N,[1,2,1]);
SSS:={[1,2,1]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[2]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
```

```
then print Rep(Seqq[i]);
end if; end for; end for;
T121:=Transversal(N,N121);
for i in [1..#T121] do
ss:=[1,2,1]^T121[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N121);

/*NOT A NEW COSET [123] equal to [12]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[3]
eq g*(ts[a])^h then "1,2,3=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[3]
eq g*(ts[a]*ts[b])^h then "1,2,3=", a,b; end if; end for; end for;*/

/*Not a new coset [124] equal to [16]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[4]
eq g*(ts[a])^h then "1,2,4=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[4]
eq g*(ts[a]*ts[b])^h then "1,2,4=", a,b; end if; end for; end for;*/

/*Not a new coset [125] equal to [12]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[5]
eq g*(ts[a])^h then "1,2,5=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[5]
eq g*(ts[a]*ts[b])^h then "1,2,5=", a,b; end if; end for; end for;*/

/*Not a new coset [126] goes to [16]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[6] eq
g*(ts[a])^h then "1,2,6=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[6] eq
g*(ts[a]*ts[b])^h then "1,2,6=", a,b; end if; end for; end for;*/

N127:=Stabiliser(N,[1,2,7]);
SSS:={[1,2,7]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
```

```
if ts[1]*ts[2]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N127s:=N127;
for n in N do if 1^n eq 2 and 2^n eq 1 and 7^n eq 4 then
N127s:=sub<N|N127s,n>; end if; end for;
for n in N do if 1^n eq 4 and 2^n eq 7 and 7^n eq 2 then
N127s:=sub<N|N127s,n>; end if; end for;
for n in N do if 1^n eq 7 and 2^n eq 4 and 7^n eq 1 then
N127s:=sub<N|N127s,n>; end if; end for;
N127s; #N127s;
#N/#N127s;
T127:=Transversal(N,N127s);
for i in [1..#T127] do
ss:=[1,2,7]^T127[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
[1,2,7]^N127s;
for i in [1..#T127] do ([1,2,7]^N127s)^T127[i]; end for;
Orbits(N127s);

N128:=Stabiliser(N,[1,2,8]);
SSS:={[1,2,8]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[2]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N128s:=N128;
for n in N do if 1^n eq 3 and 2^n eq 2 and 8^n eq 9 then
N128s:=sub<N|N128s,n>; end if; end for;
N128s; #N128s;
#N/#N128s;
T128:=Transversal(N,N128s);
for i in [1..#T128] do
```

```
ss:=[1,2,8]^T128[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
[1,2,8]^N128s;
for i in [1..#T128] do ([1,2,8]^N128s)^T128[i]; end for;
Orbits(N128s);

/*Not a new coset; it's not equal to anything but the coset
counter does not increase
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[9] eq
g*(ts[a])^h then "1,2,9=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[9]
eq g*(ts[a]*ts[b])^h then "1,2,9=", a,b; end if; end for; end for;
N129:=Stabiliser(N,[1,2,9]);
SSS:={[1,2,9]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[2]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T129:=Transversal(N,N129);
/* #N/#N129s = #T129 */
for i in [1..#T129] do
ss:=[1,2,9]^T129[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N129);*/

/*Not a new coset, [12 10] to [14]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[10]
eq g*(ts[a])^h then "1,2,10=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[10]
eq g*(ts[a]*ts[b])^h then "1,2,10=", a,b; end if; end for; end for;
N1210:=Stabiliser(N,[1,2,10]);
```

```
SSS:={[1,2,10]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[2]*ts[10] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N1210s:=N1210;
for n in N do if 1^n eq 7 and 2^n eq 4 and 10^n eq 5 then
N1210s:=sub<N|N1210s,n>; end if; end for;
N1210s; #N1210s;
#N/#N1210s;
T1210:=Transversal(N,N1210s);
for i in [1..#T1210] do
ss:=[1,2,10]^T1210[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
[1,2,10]^N1210s;
for i in [1..#T1210] do ([1,2,10]^N1210s)^T1210[i]; end for;
Orbits(N1210s);*/

N161:=Stabiliser(N,[1,6,1]);
SSS:={[1,6,1]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[6]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;

N161s:=N161;
for n in N do if 1^n eq 5 and 6^n eq 7 and 1^n eq 5 then
```

```
N161s:=sub<N|N161s,n>; end if; end for;

N161s; #N161s;
#N/#N161s;
T161:=Transversal(N,N161s);
for i in [1..#T161] do
ss:=[1,6,1]^T161[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
[1,6,1]^N161s;
for i in [1..#T161] do ([1,6,1]^N161s)^T161[i]; end for;
Orbits(N161s);

/*Not a new coset, [162] equals to [12]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[2]
eq g*(ts[a])^h then "1,6,2=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[2]
eq g*(ts[a]*ts[b])^h then "1,6,2=", a,b; end if; end for; end for;*/

N163:=Stabiliser(N,[1,6,3]);
SSS:={[1,6,3]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[6]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N163s:=N163;
for n in N do if 1^n eq 6 and 6^n eq 1 and 3^n eq 2 then
N163s:=sub<N|N163s,n>; end if; end for;
N163s; #N163s;
#N/#N163s;
T163:=Transversal(N,N163s);
for i in [1..#T163] do
ss:=[1,6,3]^T163[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
```

```
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
[1,6,3]^N163s;
for i in [1..#T163] do ([1,6,3]^N163s)^T163[i]; end for;
Orbits(N163s);

/*Not a new coset [164] to [14]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[4]
eq g*(ts[a])^h then "1,6,4=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[4]
eq g*(ts[a]*ts[b])^h then "1,6,4=", a,b; end if; end for; end for;*/

/*Not a new coset [165] equals to [16]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[5] eq
g*(ts[a])^h then "1,6,5=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[5]
eq g*(ts[a]*ts[b])^h then "1,6,5=", a,b; end if; end for; end for;*/

N167:=Stabiliser(N,[1,6,7]);
SSS:={[1,6,7]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[6]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
T167:=Transversal(N,N167);
/* #N/#N167s = #T167 */
for i in [1..#T167] do
ss:=[1,6,7]^T167[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N167);

N168:=Stabiliser(N,[1,6,8]);
SSS:={[1,6,8]};
SSS:=SSS^N;
```

```
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[6]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N168s:=N168;
for n in N do if 1^n eq 2 and 6^n eq 8 and 8^n eq 6 then
N168s:=sub<N|N168s,n>; end if; end for;
N168s; #N168s;
#N/#N168s;
T168:=Transversal(N,N168s);
for i in [1..#T163] do
ss:=[1,6,8]^T168[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
[1,6,8]^N168s;
for i in [1..#T168] do ([1,6,8]^N168s)^T168[i]; end for;
Orbits(N168s);

/*not a new coset, goes back to [16]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[9]
eq g*(ts[a])^h then "1,6,9=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[9]
eq g*(ts[a]*ts[b])^h then "1,6,9=", a,b; end if; end for; end for;
N169:=Stabiliser(N,[1,6,9]);
SSS:={[1,6,9]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[6]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
```

```
T169:=Transversal(N,N169);
#N/#N169s = #T169
for i in [1..#T169] do
ss:=[1,6,9]^T169[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N169);*/

/*not a new coset [16 10] goes to [12]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[10]
eq g*(ts[a])^h then "1,6,10=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[10]
eq g*(ts[a]*ts[b])^h then "1,6,10=", a,b; end if; end for; end for;*/

/*Not a new coset [1432] goes back to [128]
for a in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[2]
eq g*(ts[a])^h then "1,4,3,2=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[2]
eq g*(ts[a]*ts[b])^h then "1,4,3,2=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[2]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,4,3,2=", a,b,c;
end if; end for; end for;*/

/*Not a new coset [1434] goes back to [167]
for a in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[4]
eq g*(ts[a])^h then "1,4,3,4=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[4]
eq g*(ts[a]*ts[b])^h then "1,4,3,4=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[4]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,4,3,4=", a,b,c;
end if; end for; end for;*/

/*[1437] goes back to itself [143]
for a in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[7]
eq g*(ts[a])^h then "1,4,3,7=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[7]
eq g*(ts[a]*ts[b])^h then "1,4,3,7=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[7]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,4,3,7=", a,b,c;
end if; end for; end for;*/

/*[1438] goes to [147]
```

```
for a in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[8]
eq g*(ts[a])^h then "1,4,3,8=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[8]
eq g*(ts[a]*ts[b])^h then "1,4,3,8=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[8]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,4,3,8=", a,b,c;
end if; end for; end for;*/

for a in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[5]
eq g*(ts[a])^h then "1,4,3,5=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[5]
eq g*(ts[a]*ts[b])^h then "1,4,3,5=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[3]*ts[5]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,4,3,5=", a,b,c;
end if; end for; end for;
N1435:=Stabiliser(N,[1,4,3,5]);
SSS:={[1,4,3,5]};
SSS:=SSS^N;
SSS;
#SSS;
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[4]*ts[3]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N1435s:=N1435;
for n in N do if 1^n eq 5 and 4^n eq 9 and 3^n eq 10 and 5^n eq
1 then N1435s:=sub<N|N1435s,n>; end if; end for;
N1435s; #N1435s;
#N/#N1435s;
T1435:=Transversal(N,N1435s);
for i in [1..#T1435] do
ss:=[1,4,3,5]^T1435[i];
cst[prodim(1, ts, ss)]:=ss;
end for;
m:=0; for i in [1..171] do if cst[i] ne []
then m:=m+1; end if; end for; m;
[1,4,3,5]^N1435s;
for i in [1..#T1435] do ([1,4,3,5]^N1435s)^T1435[i]; end for;
Orbits(N1435s);
```

```
/* [1475] goes back to itself [147]
for a in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[7]*ts[5]
eq g*(ts[a])^h then "1,4,7,5=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[7]*ts[5]
eq g*(ts[a]*ts[b])^h then "1,4,7,5=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[7]*ts[5]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,4,7,5=", a,b,c;
end if; end for; end for;*/

/*[1472] goes to [14]
for a in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[7]*ts[2]
eq g*(ts[a])^h then "1,4,7,2=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[7]*ts[2]
eq g*(ts[a]*ts[b])^h then "1,4,7,2=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[4]*ts[7]*ts[2]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,4,7,2=", a,b,c;
end if; end for; end for;*/

/*[1212] goes to [167]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[2]
eq g*(ts[a])^h then "1,2,1,2=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[2]
eq g*(ts[a]*ts[b])^h then "1,2,1,2=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[2]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,1,2=", a,b,c;
end if; end for; end for;*/

/*[1213] not a new coset, goes to [12]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[3]
eq g*(ts[a])^h then "1,2,1,3=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[3]
eq g*(ts[a]*ts[b])^h then "1,2,1,3=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[3]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,1,3=", a,b,c;
end if; end for; end for;*/

/*[1214] not new, goes to [168]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[4]
eq g*(ts[a])^h then "1,2,1,4=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[4]
eq g*(ts[a]*ts[b])^h then "1,2,1,4=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[4]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,1,4=", a,b,c;
```

```
end if; end for; end for;*/

/*[1215] goes back to [16]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[5]
eq g*(ts[a])^h then "1,2,1,5=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[5]
eq g*(ts[a]*ts[b])^h then "1,2,1,5=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[5]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,1,5=", a,b,c;
end if; end for; end for;*/

/*[1216] goes back to [161]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[6]
eq g*(ts[a])^h then "1,2,1,6=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[6]
eq g*(ts[a]*ts[b])^h then "1,2,1,6=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[6]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,1,6=", a,b,c;
end if; end for; end for;*/

/*[1217] goes back to [121]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[7]
eq g*(ts[a])^h then "1,2,1,7=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[7]
eq g*(ts[a]*ts[b])^h then "1,2,1,7=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[7]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,1,7=", a,b,c;
end if; end for; end for;*/

/*[1218] goes back to [128]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[8]
eq g*(ts[a])^h then "1,2,1,8=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[8]
eq g*(ts[a]*ts[b])^h then "1,2,1,8=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[8]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,1,8=", a,b,c;
end if; end for; end for;*/

/*[1219] goes back to [127]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[9]
eq g*(ts[a])^h then "1,2,1,9=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[9]
eq g*(ts[a]*ts[b])^h then "1,2,1,9=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[9]
```

```
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,1,9=", a,b,c;
end if; end for; end for;*/

/*[121 10] goes back to [163]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[10]
eq g*(ts[a])^h then "1,2,1,10=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[10]
eq g*(ts[a]*ts[b])^h then "1,2,1,10=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[1]*ts[10]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,1,10=", a,b,c;
end if; end for; end for;*/

/*[1276] goes back to [128]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[7]*ts[6]
eq g*(ts[a])^h then "1,2,7,6=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[7]*ts[6]
eq g*(ts[a]*ts[b])^h then "1,2,7,6=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[7]*ts[6]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,7,6=", a,b,c;
end if; end for; end for;*/

/*[1273] goes back to [121]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[7]*ts[3]
eq g*(ts[a])^h then "1,2,7,3=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[7]*ts[3]
eq g*(ts[a]*ts[b])^h then "1,2,7,3=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[7]*ts[3]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,7,3=", a,b,c;
end if; end for; end for;*/

/*[1282] goes back to [143]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[2]
eq g*(ts[a])^h then "1,2,8,2=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[2]
eq g*(ts[a]*ts[b])^h then "1,2,8,2=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[2]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,8,2=", a,b,c;
end if; end for; end for;*/

/*[1287] goes back to [127]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[7]
eq g*(ts[a])^h then "1,2,8,7=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[7]
eq g*(ts[a]*ts[b])^h then "1,2,8,7=", a,b; end if; end for; end for;
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for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[7]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,8,7=", a,b,c;
end if; end for; end for;*/

/*[1281] goes back to [121]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[1]
eq g*(ts[a])^h then "1,2,8,1=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[1]
eq g*(ts[a]*ts[b])^h then "1,2,8,1=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[1]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,8,1=", a,b,c;
end if; end for; end for;*/

/*[1284] goes back to [167]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[4]
eq g*(ts[a])^h then "1,2,8,4=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[4]
eq g*(ts[a]*ts[b])^h then "1,2,8,4=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[4]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,8,4=", a,b,c;
end if; end for; end for;*/

/*[1285] goes back to [1435]
for a in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[5]
eq g*(ts[a])^h then "1,2,8,5=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[5]
eq g*(ts[a]*ts[b])^h then "1,2,8,5=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[5]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,2,8,5=", a,b,c;
end if; end for; end for;
for a,b,c,d in [1..10] do for g,h in IN do if ts[1]*ts[2]*ts[8]*ts[5]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,2,8,5=", a,b,c,d; end if;
end for; end for;*/

/*[1612] goes back to [168]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[1]*ts[2]
eq g*(ts[a])^h then "1,6,1,2=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[1]*ts[2]
eq g*(ts[a]*ts[b])^h then "1,6,1,2=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[1]*ts[2]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,1,2=", a,b,c;
end if; end for; end for;*/

/*[1613] goes back to [163]
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for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[1]*ts[3]
eq g*(ts[a])^h then "1,6,1,3=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[1]*ts[3]
eq g*(ts[a]*ts[b])^h then "1,6,1,3=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[1]*ts[3]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,1,3=", a,b,c;
end if; end for; end for;*/

/*[1614] goes back to [167]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[1]*ts[4]
eq g*(ts[a])^h then "1,6,1,4=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[1]*ts[4]
eq g*(ts[a]*ts[b])^h then "1,6,1,4=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[1]*ts[4]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,1,4=", a,b,c;
end if; end for; end for;*/

/*[1616] goes back to [121]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[1]*ts[6]
eq g*(ts[a])^h then "1,6,1,6=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[1]*ts[6]
eq g*(ts[a]*ts[b])^h then "1,6,1,6=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[1]*ts[6]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,1,6=", a,b,c;
end if; end for; end for;*/

/*[1631] goes back to [161]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[3]*ts[1]
eq g*(ts[a])^h then "1,6,3,1=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[3]*ts[1]
eq g*(ts[a]*ts[b])^h then "1,6,3,1=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[3]*ts[1]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,3,1=", a,b,c;
end if; end for; end for;*/

/*[1634] goes back to [167]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[3]*ts[4]
eq g*(ts[a])^h then "1,6,3,4=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[3]*ts[4]
eq g*(ts[a]*ts[b])^h then "1,6,3,4=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[3]*ts[4]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,3,4=", a,b,c;
end if; end for; end for;*/
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/*[1635] goes back to [121]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[3]*ts[5]
eq g*(ts[a])^h then "1,6,3,5=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[3]*ts[5]
eq g*(ts[a]*ts[b])^h then "1,6,3,5=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[3]*ts[5]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,3,5=", a,b,c;
end if; end for; end for;*/

/*[1637] goes back to [168]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[3]*ts[7]
eq g*(ts[a])^h then "1,6,3,7=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[3]*ts[7]
eq g*(ts[a]*ts[b])^h then "1,6,3,7=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[3]*ts[7]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,3,7=", a,b,c;
end if; end for; end for;*/

/*[1671] goes back to [168]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[1]
eq g*(ts[a])^h then "1,6,7,1=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[1]
eq g*(ts[a]*ts[b])^h then "1,6,7,1=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[1]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,7,1=", a,b,c;
end if; end for; end for;*/

/*[1672] goes back to [1435]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[2]
eq g*(ts[a])^h then "1,6,7,2=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[2]
eq g*(ts[a]*ts[b])^h then "1,6,7,2=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[2]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,7,2=", a,b,c;
end if; end for; end for;
for a,b,c,d in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[2]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,6,7,2=", a,b,c,d;
end if; end for; end for;*/

/*[1673] goes back to [161]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[3]
eq g*(ts[a])^h then "1,6,7,3=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[3]
eq g*(ts[a]*ts[b])^h then "1,6,7,3=", a,b; end if; end for; end for;
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for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[3]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,7,3=", a,b,c;
end if; end for; end for;*/

/*[1674] goes back to [167]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[4]
eq g*(ts[a])^h then "1,6,7,4=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[4]
eq g*(ts[a]*ts[b])^h then "1,6,7,4=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[4]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,7,4=", a,b,c;
end if; end for; end for;*/

/*[1675] goes back to [1435]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[5]
eq g*(ts[a])^h then "1,6,7,5=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[5]
eq g*(ts[a]*ts[b])^h then "1,6,7,5=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[5]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,7,5=", a,b,c;
end if; end for; end for;
for a,b,c,d in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[5]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,6,7,5=", a,b,c,d;
end if; end for; end for;*/

/*[1676] goes back to [128]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[6]
eq g*(ts[a])^h then "1,6,7,6=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[6]
eq g*(ts[a]*ts[b])^h then "1,6,7,6=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[6]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,7,6=", a,b,c;
end if; end for; end for;*/

/*[1678] goes back to [168]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[8]
eq g*(ts[a])^h then "1,6,7,8=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[8]
eq g*(ts[a]*ts[b])^h then "1,6,7,8=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[8]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,7,8=", a,b,c;
end if; end for; end for;*/

/*[1679] goes back to [121]
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for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[9]
eq g*(ts[a])^h then "1,6,7,9=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[9]
eq g*(ts[a]*ts[b])^h then "1,6,7,9=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[9]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,7,9=", a,b,c;
end if; end for; end for;*/

/*[167 10] goes back to [143]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[10]
eq g*(ts[a])^h then "1,6,7,10=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[10]
eq g*(ts[a]*ts[b])^h then "1,6,7,10=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[7]*ts[10]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,7,10=", a,b,c;
end if; end for; end for;*/

/*[1681] goes back to [161]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[8]*ts[1]
eq g*(ts[a])^h then "1,6,8,1=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[8]*ts[1]
eq g*(ts[a]*ts[b])^h then "1,6,8,1=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[8]*ts[1]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,8,1=", a,b,c;
end if; end for; end for;*/

/*[1683] goes back to [161]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[8]*ts[3]
eq g*(ts[a])^h then "1,6,8,3=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[8]*ts[3]
eq g*(ts[a]*ts[b])^h then "1,6,8,3=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[8]*ts[3]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,8,3=", a,b,c;
end if; end for; end for;*/

/*[1684] goes back to [167]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[8]*ts[4]
eq g*(ts[a])^h then "1,6,8,4=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[8]*ts[4]
eq g*(ts[a]*ts[b])^h then "1,6,8,4=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[8]*ts[4]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,8,4=", a,b,c;
end if; end for; end for;*/
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/*[1689] goes back to [163]
for a in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[8]*ts[9]
eq g*(ts[a])^h then "1,6,8,9=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[8]*ts[9]
eq g*(ts[a]*ts[b])^h then "1,6,8,9=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if ts[1]*ts[6]*ts[8]*ts[9]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,6,8,9=", a,b,c;
end if; end for; end for;*/

/*[14352] goes back to [128]
for a in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[2]
eq g*(ts[a])^h then "1,4,3,5,2=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[2]
eq g*(ts[a]*ts[b])^h then "1,4,3,5,2=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[2]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,4,3,5,2=", a,b,c;
end if; end for; end for;
for a,b,c,d in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[2]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,4,3,5,2=", a,b,c,d;
end if; end for; end for;*/

/*[14353] goes back to [167]
for a in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[3]
eq g*(ts[a])^h then "1,4,3,5,3=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[3]
eq g*(ts[a]*ts[b])^h then "1,4,3,5,3=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[3]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,4,3,5,3=", a,b,c;
end if; end for; end for;
for a,b,c,d in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[3]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,4,3,5,3=", a,b,c,d;
end if; end for; end for;*/

/*[14354] goes back to [1285]
for a in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[4]
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eq g*(ts[a])^h then "1,4,3,5,4=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[4]
eq g*(ts[a]*ts[b])^h then "1,4,3,5,4=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[4]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,4,3,5,4=", a,b,c;
end if; end for; end for;
for a,b,c,d in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[4]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,4,3,5,4=", a,b,c,d;
end if; end for; end for;*/

/*[14356] goes back to [167]
for a in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[6]
eq g*(ts[a])^h then "1,4,3,5,6=", a; end if; end for; end for;
for a,b in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[6]
eq g*(ts[a]*ts[b])^h then "1,4,3,5,6=", a,b; end if; end for; end for;
for a,b,c in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[6]
eq g*(ts[a]*ts[b]*ts[c])^h then "1,4,3,5,6=", a,b,c;
end if; end for; end for;
for a,b,c,d in [1..10] do for g,h in IN do if
ts[1]*ts[4]*ts[3]*ts[5]*ts[6]
eq g*(ts[a]*ts[b]*ts[c]*ts[d])^h then "1,4,3,5,6=", a,b,c,d;
end if; end for; end for;*/
```

B.3 Construction of $PSL_2(7)$ over $2^{*4} : C_4$

```
G<x,t>:=Group<x,t|x^4,t^2,(x^-1*t)^7,(x^2*t)^3,(x*t)^7>;
Index(G,sub<G|x>);

/*When the number of right cosets is too large, we use a
suitable maximal subgroup H of G containing to perform DCE.*/
/* Relation 1:
  (x*t)^7 = x^7*t^(x^6)*t(x^5)*t^(x^4)*t^(x^3)*t^(x^2)*t^x*t
          = x^3*t^(x^2)*t^x*t*t^(x^3)*t^(x^2)*t^x*t
  = (1432)t3t2t1t4t3t2t1 = e
=> (1432)t3t2t1t4 = t1t2t3

Relation 2:
```

```
  (x^(-1)*t)^7 = x*t^(x^2)*t^(x^3)*t*t^(x)*t^(x^2)*t^(x^3)*t
       = (1234)t3t4t1t2t3t4t1 = e
=> (1234)t3t4t1t2 = t1t4t3

Relation 3:
  (x^2*t)^3 = x^2*t*t^(x^2)*t
    = (13)(24)t1t3t1 = e
=>(13)(24)t1 = t1t3
*/

f,G1,k:=CosetAction(G,sub<G|x>);
CompositionFactors(G1);
M:=MaximalSubgroups(G1);
#M;
M;
N:=sub<Sym(4)|(1,2,3,4)>;
/* D:=Conjugates(G1,M[2]`subgroup);
D:=SetToSequence(D);
#D;/*7 groups of order of M[2]*/
for j in [1..#D] do if f(x) in D[j] then j; end if; end for;
/*output*/
A:=D[1]; /*The output was 1, but this number will change*/
#A;/*24*/
f(x) in A; /*NEEDS to be true*/ */
C:=Classes(G1);
#C;
for c in Class(G1,C[2][3]) do if A eq sub<G1|f(x),c>
then B:=c; break;end if; end for;
Order(B); /*2*/
W,phi:=WordGroup(G1);
rho:=InverseWordMap(G1);
B@rho;
/*  function(W)
    w1 := W.1^-1; w3 := w1 * W.2; w4 := w3 * W.1; w5 := w4 * W.2;
w6 := w5 * w1;
    w7 := w6 * W.2; w8 := w7 * W.1; return w8;
end function;
*/
gg:=function(W)
    w1 := W.1^-1; w3 := w1 * W.2; w4 := w3 * W.1; w5 := w4 * W.2;
w6 := w5 * w1;
    w7 := w6 * W.2; w8 := w7 * W.1; return w8;
end function;
gg(G); /*x^-1 * t * x * t * x^-1 * t * x */
```

```
/* H is M[2]'subgroup */
H:=sub<G|x,x^-1 * t * x * t * x^-1 * t * x>;
#H;/*24*/
#DoubleCosets(G,H,sub<G|x>);
IN:=sub<G1|f(x)>;
IH:=sub<G1|f(x),f(x^-1 * t * x * t * x^-1 * t * x)>;
#IH;/*24*/
ts:=[Id(G1): i in [1 .. 4] ];
ts[1]:=f(t); ts[2]:=f(t^x); ts[3]:=f(t^(x^2)); ts[4]:=f(t^(x^3));
IH eq sub<G1|f(x),f(x^2)*f(t^(x^3))*f(t^(x^2))*f(t^(x^3))>;

/*DCE PSL_2(7) as a Homomorphic Image of 2^{4}:C_4*/

S:=Sym(4);
xx:=S!(1,2,3,4);
N:=sub<S|xx>;
G<x,t>:=Group<x,t|x^4,t^2,(x^-1*t)^7,(x^2*t)^3,(x*t)^7>;
f,G1,k:=CosetAction(G,sub<G|x>);
#G1; #k; #sub<G|x>;
H:=sub<G|x,x^-1 * t * x * t * x^-1 * t * x>;
IN:=sub<G1|f(x)>;
IH:=sub<G1|f(x),f(x^-1 * t * x * t * x^-1 * t * x)>;
W,phi:=WordGroup(G1);
rho:=InverseWordMap(G1);
ts := [Id(G1): i in [1 .. 4] ];
ts[1]:=f(t); ts[2]:=f(t^x); ts[3]:=f(t^(x^2)); ts[4]:=f(t^(x^3));
prodim := function(pt, Q, I)/*Return the image of pt under
permutations Q[I] applied sequentially.*/
  v := pt;
   for i in I do
       v := v^(Q[i]);
   end for;
return v;
end function;
cst := [null : i in [1 .. Index(G,sub<G|x>)]] where null is
[Integers() | ];
for i := 1 to 4 do cst[prodim(1, ts, [i])] := [i];
end for; m:=0;
for i in [1..42] do if cst[i] ne [] then m:=m+1; end if; end for; m;
/*4*/

N1:=Stabiliser(N,[1]);
N1;
#N/#N1;
```

```
N1s:=N1;
Orbits(N1s);
for i in [1..4] do i, cst[i]; end for;
SSS:={[1]}; SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;for i in [1..#SSS] do
for n in IH do
if ts[1] eq
n*ts[Rep(Seqq[i])[1]]
then print Rep(Seqq[i]);
end if; end for; end for;

N12:=Stabiliser(N,[1,2]);
SSS:={[1,2]};
SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IH do
if ts[1]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
/* [ 1, 2 ]
[ 3, 4 ]*/
/*

N12:=Stabiliser(N,[1,2]);
N12s:=N12;
for g in N do if 1^g eq 3 and 2^g eq 4 then N12s:=sub<N|N12s,g>;
end if; end for;
T12:=Transversal(N,N12s);
#T12;
for i in [1..#T12] do
ss:=[1,2]^T12[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..Index(G,sub<G|x>)] do if cst[i] ne []
then m:=m+1; end if; end for;  m;
/*6*/
```

```
Orbits(N12s);

/*NOT A NEW DOUBLE COSET N13=N1*/
N13:=Stabiliser(N,[1,3]); SSS:={[1,3]}; SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IH do
if ts[1]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
/* [ 1, 3 ]*/
N13s:=N13;
N13s; #N13s;
#N/#N13s;
T13:=Transversal(N,N13s);
for i in [1..#T13] do
ss:=[1,3]^T13[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..Index(G,sub<G|x>)] do if cst[i] ne []
then m:=m+1; end if; end for;  m; /*did not increase*/
for i in [1..#T13] do ([1,3]^N13s)^T13[i]; end for;
for n in IN do if ts[1]*ts[3] eq n*ts[1] then n; end if; end for;
for i in [1..4] do i, cst[i]; end for;
ts := [Id(G1): i in [1 .. 4] ];
ts[1]:=f(t); ts[2]:=f(t^x); ts[3]:=f(t^(x^2)); ts[4]:=f(t^(x^3));
ts[1]*ts[3] eq f(x^2)*ts[1];
for a in [1..4] do for g,h in IH do if ts[1]*ts[3] eq g*(ts[a])^h
then "1,3=", a; end if; end for; end for;

/*NOT A NEW DOUBLE COSET N14=N1 */
N14:=Stabiliser(N,[1,4]);
SSS:={[1,4]};
SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IH do
```

```
if ts[1]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
/* [ 1, 4 ]*/
N14s:=N14;
N14s; #N14s;
#N/#N14s;
T14:=Transversal(N,N14s);
for i in [1..#T14] do
ss:=[1,4]^T14[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..Index(G,sub<G|x>)] do if cst[i] ne []
then m:=m+1; end if; end for;  m; /*did increase*/
for i in [1..#T14] do ([1,4]^N14s)^T14[i]; end for;
for n in IH do if ts[1]*ts[4] eq n*ts[1] then n; end if; end for;
for i in [1..4] do i, cst[i]; end for;
ts := [Id(G1): i in [1 .. 4] ];
ts[1]:=f(t); ts[2]:=f(t^x); ts[3]:=f(t^(x^2)); ts[4]:=f(t^(x^3));
ts[1]*ts[4] eq f(x)*ts[1];
for a in [1..4] do for g,h in IH do if ts[1]*ts[4] eq g*(ts[a])^h
then "1,4=", a; end if; end for; end for;

/*NOT A NEW DOUBLE COSET N121=N1 */
N121:=Stabiliser(N,[1,2,1]);
SSS:={[1,2,1]};
SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IH do
if ts[1]*ts[2]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
/* [ 1, 2, 1 ]
[ 3, 4, 3 ]
*/
for a,b in [1..4] do for g,h in IH do if ts[1]*ts[2]*ts[1]
eq g*(ts[a]*ts[b])^h
then "1,2,1=", a,b; end if; end for; end for;
```

```
/*1,2,1=1 2*/
```

B.4 Construction of S_4 over S_3

```
S:=Sym(3);
xx:=S!(1, 2, 3);
yy:=S!(1,2);
N:=sub<S|xx,yy>;
G<x,y,t>:=Group<x,y,t|x^3,y^2,(x*y)^2,t^2,(y,t),x=t^x*t^(x^2)*t*t^x>;
#G;
f,G1,k:=CosetAction(G,sub<G|x,y>);
#G1; #k; #sub<G|x,y>;
H:=sub<G|x,y,t*t^x*t>;
#DoubleCosets(G,H,sub<G|x,y>);
IN:=sub<G1|f(x),f(y)>;
IH:=sub<G1|f(x),f(y),f(t*t^x*t)>;
W,phi:=WordGroup(G1);
rho:=InverseWordMap(G1);
ts := [Id(G1): i in [1 .. 3] ];
ts[1]:=f(t); ts[2]:=f(t^x); ts[3]:=f(t^(x^2));
prodim := function(pt, Q, I)
  v := pt;
   for i in I do
       v := v^(Q[i]);
   end for;
return v;
end function;
cst := [null : i in [1 .. Index(G,sub<G|x,y>)]] where null
is [Integers() | ];
SSS:={[1]}; SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;
for i in [1..3] do
for n in IH do
if ts[1] eq
n*ts[Rep(Seqq[i])[1]]
then print Rep(Seqq[i]);
end if; end for; end for;

N1:=Stabiliser(N,[1]); SSS:={[1]}; SSS:=SSS^N;
SSS;
```

```
#(SSS);
Seqq:=Setseq(SSS);
Seqq;for i in [1..#SSS] do
for n in IH do
if ts[1] eq
n*ts[Rep(Seqq[i])[1]]
then print Rep(Seqq[i]);
end if; end for; end for;
N1s:=N1;
Orbits(N1s);
/*  GSet{@ 1 @},
    GSet{@ 2, 3 @}
*/
for i := 1 to 3 do cst[prodim(1, ts, [i])] := [i];
end for; m:=0;
for i in [1..8] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N12:=Stabiliser(N,[1,2]); SSS:={[1,2]}; SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IH do
if ts[1]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
/*[ 1, 2 ]
[ 1, 3 ]*/

N12:=Stabiliser(N,[1,2]);
N12s:=N12;
for g in N do if 1^g eq 1 and 2^g eq 3
then N12s:=sub<N|N12s,g>; end if; end for;

T12:=Transversal(N,N12s);
for i in [1..#T12] do
ss:=[1,2]^T12[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..8] do if cst[i] ne []
then m:=m+1; end if; end for;  m;
/*6*/
```

```
for m in IH do for n in IN do if ts[1]*ts[2] eq m*(ts[1])^n
then m, n; break; end if; end for; end for;
/* (1, 8)(2, 5)(3, 7)(4, 6)
Id(IN)*/
```

B.5 Construction of $U_3(5):2$ over $2^{*7}:(7:6)$

```
/*0 0 0 0 0 7 0 8 6 0 8 252000*/
r1:=0;r2:=0;r3:=0;r4:=0;r5:=0;r6:=7;r7:=0;r8:=8;r9:=6;r10:=0;r11:=8;
G<x,y,t>:=Group<x,y,t|x*y^-1*x^-1*y^-2,x^6,y*x^-1*y^-1*x*y^2,
t^2,(t,x*y^-2), (t,x^2*y^-1),(x^2 * y^-1 * x*t^y)^r1,
(x^2 * y^-1 * x*t)^r2,(y^-1 * x^2*t^y)^r3,(y^-1 * x^2*t)^r4,
(x^-2 * y*t^y)^r5,(x^-2 * y*t)^r6,(y*x*t^y)^r7,(y*x*t)^r8,
(x^-1 * y^-1*t^y)^r9,(x^-1 * y^-1*t)^r10,(y*t)^r11>;
Index(G,sub<G|x,y>);

/*6000*/

f,G1,k:=CosetAction(G,sub<G|x,y>);
CompositionFactors(G1);
/*    G
    |  Cyclic(2)
    *
    |  2A(2, 5)                   = U(3, 5)
    1
*/

M:=MaximalSubgroups(G1);
#M;
/*6*/

/*D:=Conjugates(G1,M[1]`subgroup);
D:=SetToSequence(D);
#D;
for j in [1..#D] do if f(x) in D[j] and f(y)
in D[j] then j; end if; end for;
A:=D[294];
#A;
/*336*/
f(x) in A;
/*true*/
f(y) in A;
```

```
/*true*/
*/

D:=Conjugates(G1,M[1]`subgroup);
D:=SetToSequence(D);
#D;
/*750 groups of order of M[1]*/

for j in [1..#D] do if f(x) in D[j] and f(y)
in D[j] then j; end if; end for;
/*748 - this one will change every time magma runs*/
C:=Classes(G1);
#C;
/*19*/

A:=D[748];  /*12- this number will change
depending on what magma gave us*/
for c in Class(G1,C[2][3]) do if A eq sub<G1|f(x),f(y),c>
then B:=c; break;end if; end for;
Order(B);
/*2*/

D:=Conjugates(G1,M[5]`subgroup);
D:=SetToSequence(D);
#D;
/*50*/

for j in [1..#D] do if f(x) in D[j] and f(y)
in D[j] then j; end if; end for;
/*50*/

A:=D[50];
for c in Class(G1,C[5][3]) do if A eq sub<G1|f(x),f(y),c>
then E:=c; end if; break; end for;
Order(E);
/*4*/

W,phi:=WordGroup(G1);
rho:=InverseWordMap(G1);
B@rho;
/*  function(W)
    w2 := W.2^-1; w4 := W.1 * w2; w5 := w4 * W.3; w6 := w5 * W.1;
    w7 := w6 * W.2; w8 := w7 * W.3; w9 := w8 * W.2; w10 := w9 * W.3;
    w11 := w10 * W.1; w12:= w11 * W.3; w13 := w12 * w2;
```

```
    w14 := w13 * W.3; w15 := w14 * W.1; w16 := w15 * W.3; return w16;
end function
*/

gg:=function(W)
     w2 := W.2^-1; w4 := W.1 * w2; w5 := w4 * W.3; w6 := w5 * W.1;
     w7 := w6 * W.2; w8 := w7 * W.3; w9 := w8 * W.2; w10 := w9 * W.3;
     w11 := w10 * W.1; w12:= w11 * W.3; w13 := w12 * w2;
     w14 := w13 * W.3; w15 := w14 * W.1; w16 := w15 * W.3; return w16;
end function;

gg(G);
/*   x * y^-1 * t * x * y * t * y * t * x * t * y^-1 * t * x * t  */

E@rho;
/*function(W)
    w17 := W.1^2; w2 := W.2^-1; w18 := w17 * w2; w19 := w18 * W.1;
    w20 := w19 * W.3; w21 := w20 * w2; w22 := w21 * W.3;
    w23 := w22 * w2; w24 := w23 * W.3; w25 := w24 * w2;
    w26 := w25 * W.3; w27 := w26 * W.2; return w27;
end function*/

gg:=function(W)
   w17 := W.1^2; w2 := W.2^-1; w18 := w17 * w2; w19 := w18 * W.1;
   w20 := w19 * W.3; w21 := w20 * w2; w22 := w21 * W.3;
   w23 := w22 * w2; w24 :=w23 * W.3; w25 := w24 * w2;
   w26 := w25 * W.3; w27 := w26 * W.2; return w27;
   end function;

gg(G);
/*  x^2 * y^-1 * x * t * y^-1 * t * y^-1 * t * y^-1 * t * y  */

/* H1 is M[1]`subgroup */
H1:=sub<G|x,y,x*y^-1*t*x*y*t*y*t*x*t *y^-1*t*x*t>;
#H1;
/*336*/

#DoubleCosets(G,H1,sub<G|x,y>);
/*26*/

ImH1:=sub<G|f(x),f(y),
f(x * y^-1 * t * x * y * t * y * t* x * t * y^-1 * t * x *t )>;
#ImH1;
/*336*/
```

```
/* H2 is M[5]‘subgroup */
H2:=sub<G|x,y,x^2 * y^-1 * x * t * y^-1 * t * y^-1 * t * y^-1 * t * y>;
#H2;
/*5040*/

#DoubleCosets(G,H2,sub<G|x,y>);
/*5*/

ImH2:=sub<G1|f(x),f(y),
f(x^2 * y^-1 * x * t * y^-1 * t * y^-1 * t * y^-1 * t * y)>;
#ImH2;
/*5040*/

S:=Sym(7);
xx:=S!(1, 3, 2, 6, 4, 5);
yy:=S!(1,2,3,4,5,6,7);
N:=sub<S|xx,yy>;
r1:=7;r2:=8;r3:=6;r4:=8;
G<x,y,t>:=Group<x,y,t|x*y^-1*x^-1*y^-2,x^6,y*x^-1*y^-1*x*y^2,
t^2,(t,x*y^-2),
(t,x^2*y^-1),(x^-2 * y*t)^r1,(y*x*t)^r2,
(x^-1 * y^-1*t^y)^r3,(y*t)^r4>;
f,G1,k:=CosetAction(G,sub<G|x,y>);
#G1; #k; #sub<G|x,y>; /*252000*/
H:=sub<G|x,y,x^2 * y^-1 * x * t * y^-1 * t * y^-1 * t * y^-1 * t * y>;
#DoubleCosets(G,H,sub<G|x,y>);
IN:=sub<G1|f(x),f(y)>;
IH:=sub<G1|f(x),f(y),f(x^2*y^-1*x*t*y^-1*t*y^-1*t*y^-1*t*y)>;
W,phi:=WordGroup(G1);
rho:=InverseWordMap(G1);
ts := [Id(G1): i in [1 .. 7] ];
ts[1]:=f(t); ts[2]:=f(t^y); ts[3]:=f(t^x); ts[4]:=f(t^(y^3));
ts[5]:=f(t^(y^4));ts[6]:=f(t^(x^3));ts[7]:=f(t^(y^6));
prodim := function(pt, Q, I)
/*
Return the image of pt under permutations Q[I] applied sequentially.
*/
  v := pt;
   for i in I do
       v := v^(Q[i]);
   end for;
return v;
end function;
```

```
cst := [null : i in [1 .. Index(G,sub<G|x,y>)]]
where null is [Integers() | ];
SSS:={[1]}; SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IH do
if ts[1] eq
n*ts[Rep(Seqq[i])[1]]
then print Rep(Seqq[i]);
end if; end for; end for;

for i := 1 to 7 docst[prodim(1, ts, [i])] := [i];
end for; m:=0;
for i in [1..10] do if cst[i] ne [] then m:=m+1; end if; end for; m;

N1:=Stabiliser(N,[1]); SSS:={[1]}; SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;for i in [1..#SSS] do
for n in IH do
if ts[1] eq
n*ts[Rep(Seqq[i])[1]]
then print Rep(Seqq[i]);
end if; end for; end for;
N1s:=N1;
Orbits(N1s);

N12:=Stabiliser(N,[1,2]); SSS:={[1,2]}; SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IH do
if ts[1]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
/* [ 1, 2 ]
[ 4, 3 ]*/
```

```
/* for h in IH do if ts[1]*ts[2] eq h*ts[4]*ts[3] then h@rho;
break;end if;end for;
/*function(W)
    w4 := W.2 * W.3; w5 := w4 * W.2; w6 := w5 * W.3; w7 := w6 * W.2;
    w8 := w7 * W.3; w9 := w8 * W.1; w10 := w9 * W.2; w11 := w10 * W.3;
    w1 := W.1^-1; w12 := w11 * w1; return w12;
end function*/
h:=function(W)
w4 := W.2 * W.3; w5 := w4 * W.2; w6 := w5 * W.3; w7 := w6 * W.2;
    w8 := w7 * W.3; w9 := w8 * W.1; w10 := w9 * W.2; w11 := w10 * W.3;
    w1 := W.1^-1; w12 := w11 * w1; return w12;
end function;
h(G);
/* y * t * y * t * y * t * x * y * t * x^-1 */
N12:=Stabiliser(N,[1,2]);
N12s:=N12;
for g in N do if 1^g eq 4 and 2^g eq 3 then N12s:=sub<N|N12s,g>;
end if; end for;
T12:=Transversal(N,N12s);
for i in [1..#T12] do
ss:=[1,2]^T12[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..6000] do if cst[i] ne [] /* 252000/7=36000*/
then m:=m+1; end if; end for;  m;
/*28*/
Orbits(N12s);

N121:=Stabiliser(N,[1,2,1]); SSS:={[1,2,1]}; SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IH do
if ts[1]*ts[2]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
/* [ 1, 2 ,1]
[ 2, 4 ,2]
[4, 1, 4]*/
/*
for h in IH do if ts[1]*ts[2]*ts[1] eq h*ts[2]*ts[4]*ts[2] then h@rho;
```

```
break;end if;end for;
/*function(W)
    w4 := W.1^2; w5 := w4 * W.2; w6 := w5 * W.1; w7 := w6 * W.3;
    w2 := W.2^-1; w8 := w7 * w2; w9:= w8 * W.3; w10 := w9 * W.1;
    w11 := w10 * W.3; w1 := W.1^-1; w12 := w11 * w1;
    w13 := w12 * W.3; w14 := w13 * W.1; w15 := w14 * W.2;
    w16 := w15 * W.3; return w16;
end function */
h:=function(W)
w4 := W.1^2; w5 := w4 * W.2; w6 := w5 * W.1; w7 := w6 * W.3;
    w2 := W.2^-1; w8 := w7 * w2; w9:= w8 * W.3; w10 := w9 * W.1;
    w11 := w10 * W.3; w1 := W.1^-1; w12 := w11 * w1;
    w13 := w12 * W.3; w14 := w13 * W.1; w15 := w14 * W.2;
    w16 := w15 * W.3; return w16;
end function;
h(G);
/* x^2 * y * x * t * y^-1 * t * x * t * x^-1 * t * x * y * t  */

for h in IH do if ts[1]*ts[2]*ts[1] eq h*ts[4]*ts[1]*ts[4] then h@rho;
break;end if;end for;
/*function(W)
w17 := W.2 * W.1; w18 := w17 * W.3; w2 := W.2^-1; w19 := w18 * w2; w20
:= w19 * W.3; w21 := w20 * W.1; w22 := w21 * W.3; w1 := W.1^-1; w23 :=
w22 * w1; w24 := w23 * W.3; w25 := w24 * W.1; w26 := w25 * W.2; w27 :=
w26 * W.3; return w27;
end function */
h:=function(W)
w17 := W.2 * W.1; w18 := w17 * W.3; w2 := W.2^-1; w19 := w18 * w2; w20
:= w19 * W.3; w21 := w20 * W.1; w22 := w21 * W.3; w1 := W.1^-1; w23 :=
w22 * w1; w24 := w23 * W.3; w25 := w24 * W.1; w26 := w25 * W.2; w27 :=
w26 * W.3; return w27;
end function;
h(G);
/* y * x * t * y^-1 * t * x * t * x^-1 * t * x * y * t*/

N121:=Stabiliser(N,[1,2,1]);
N121s:=N121;
for g in N do if 1^g eq 2 and 2^g eq 4 and 1^g eq 2 then
N121s:=sub<N|N121s,g>;
end if; end for;
for g in N do if 1^g eq 4 and 2^g eq 1 and 1^g eq 4 then
N121s:=sub<N|N121s,g>;
end if; end for;
T121:=Transversal(N,N121s);
```

```
for i in [1..#T121] do
ss:=[1,2,1]^T121[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..6000] do if cst[i] ne [] /* 252000/7=36000*/
then m:=m+1; end if; end for;  m;
/*42*/
for i in [1..#T121] do ([1,2,1]^N121)^T121[i]; end for;
Orbits(N121s);

N125:=Stabiliser(N,[1,2,5]); SSS:={[1,2,5]}; SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IH do
if ts[1]*ts[2]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
/* [ 1, 2 ,5]
[ 5, 7 ,6]
[2,5,7]
[7,6,3]
[6,3,1]
[3,1,2]*/
for h in IH do if ts[1]*ts[2]*ts[5] eq h*ts[5]*ts[7]*ts[6] then h@rho;
break;end if;end for;
/*function(W)
    w18 := W.1^3; w17 := w18 * W.2; w1 := W.1^-1; w19 := w17 * w1;
    w20 := w19 * W.3; w21 := w20 * W.1; w22 := w21 * W.3; w2 := W.2^-1;
    w23 := w22 * w2; w24 := w23 * W.3; w25 := w24 * W.2;
    w26:= w25 * W.3; w27 := w26 * w1; w4 := w27 * W.3; w5 := w4 * w2;
    w6 := w5 * W.3; return w6;
end function */
h:=function(W)
w18 := W.1^3; w17 := w18 * W.2; w1 := W.1^-1; w19 := w17 * w1;
    w20 := w19 * W.3; w21 := w20 * W.1; w22 := w21 * W.3; w2 := W.2^-1;
    w23 := w22 * w2; w24 := w23 * W.3; w25 := w24 * W.2;
    w26:= w25 * W.3; w27 := w26 * w1; w4 := w27 * W.3; w5 := w4 * w2;
    w6 := w5 * W.3; return w6;
end function;
h(G);
```

```
/* x^3 * y * x^-1 * t * x * t * y^-1 * t * y * t * x^-1 * t * y^-1 * t*/

for h in IH do if ts[1]*ts[2]*ts[5] eq h*ts[2]*ts[5]*ts[7] then h@rho;
break;end if;end for;
/*function(W)
     w18 := W.2 * W.3; w19 := w18 * W.1; w20 := w19 * W.3;
     w2 := W.2^-1; w21 := w20 * w2; w22 := w21 * W.3;
     w23 := w22 * W.2; w24 := w23 * W.3; w1 := W.1^-1;
     w25:= w24 * w1; w26 := w25 * W.3; w27 := w26 * w2;
     w28 := w27 * W.3; return w28;
end function */
h:=function(W)
w18 := W.2 * W.3; w19 := w18 * W.1; w20 := w19 * W.3;
     w2 := W.2^-1; w21 := w20 * w2; w22 := w21 * W.3;
     w23 := w22 * W.2; w24 := w23 * W.3; w1 := W.1^-1;
     w25:= w24 * w1; w26 := w25 * W.3; w27 := w26 * w2;
     w28 := w27 * W.3; return w28;
end function;
h(G);
/* y * t * x * t * y^-1 * t * y * t * x^-1 * t * y^-1 * t  */

for h in IH do if ts[1]*ts[2]*ts[5] eq h*ts[7]*ts[6]*ts[3] then h@rho;
break;end if;end for;
/*function(W)
w5 := W.2 * W.1; w4 := w5 * W.3; w6 := w4 * W.1; w7 := w6 * W.3;
w8 := w7 *
W.2; w9 := w8 * W.3; w1 := W.1^-1; w10 := w9 * w1; w11 := w10 * w1;
w12 := w11
* W.3; w2 := W.2^-1; w13 := w12 * w2; w14 := w13 * W.3; return w14;
end function */
h:=function(W)
w5 := W.2 * W.1; w4 := w5 * W.3; w6 := w4 * W.1; w7 := w6 * W.3;
w8 := w7 *
    W.2; w9 := w8 * W.3; w1 := W.1^-1; w10 := w9 * w1;
w11 := w10 * w1; w12 := w11
    * W.3; w2 := W.2^-1; w13 := w12 * w2; w14 := w13 * W.3;
return w14;
end function;
h(G);
/* y * x * t * x * t * y * t * x^-2 * t * y^-1 * t  */

for h in IH do if ts[1]*ts[2]*ts[5] eq h*ts[6]*ts[3]*ts[1] then h@rho;
break;end if;end for;
/*function(W)
```

```
    w18 := W.1 * W.2; w19 := w18 * W.2; w20 := w19 * W.3;
w21 := w20 * W.1; w22 :=
    w21 * W.3; w23 := w22 * W.1; w24 := w23 * W.2; w25 := w24 * W.3;
w26 := w25 *
    W.1; w27 := w26 * W.3; w28 := w27 * W.2; w15 := w28 * W.1;
w16 := w15 * W.3;
    w1 := W.1^-1; w17 := w16 * w1; return w17;
end function */
h:=function(W)
w18 := W.1 * W.2; w19 := w18 * W.2; w20 := w19 * W.3;
w21 := w20 * W.1; w22 :=
    w21 * W.3; w23 := w22 * W.1; w24 := w23 * W.2; w25 := w24 * W.3;
w26 := w25 *
    W.1; w27 := w26 * W.3; w28 := w27 * W.2; w15 := w28 * W.1;
w16 := w15 * W.3;
    w1 := W.1^-1; w17 := w16 * w1; return w17;
end function;
h(G);
/* x * y^2 * t * x * t * x * y * t * x * t * y * x * t * x^-1  */

for h in IH do if ts[1]*ts[2]*ts[5] eq h*ts[3]*ts[1]*ts[2] then h@rho;
break;end if;end for;
/*function(W)
    w4 := W.2 * W.3; w5 := w4 * W.2; w6 := w5 * W.3; w7 := w6 * W.2;
w8 := w7 *
    W.3; w9 := w8 * W.1; w10 := w9 * W.2; w11 := w10 * W.3;
w1 := W.1^-1; w12 :=
    w11 * w1; return w12;
end function */
h:=function(W)
w4 := W.2 * W.3; w5 := w4 * W.2; w6 := w5 * W.3; w7 := w6 * W.2;
w8 := w7 *
    W.3; w9 := w8 * W.1; w10 := w9 * W.2; w11 := w10 * W.3;
w1 := W.1^-1; w12 :=
    w11 * w1; return w12;
end function;
h(G);
/* y * t * y * t * y * t * x * y * t * x^-1  */

N125:=Stabiliser(N,[1,2,5]);
N125s:=N125;
for g in N do if 1^g eq 5 and 2^g eq 7 and 5^g eq 6 then
N125s:=sub<N|N125s,g>;
end if; end for;#
```

```
for g in N do if 1^g eq 2 and 2^g eq 5 and 5^g eq 7 then
N125s:=sub<N|N125s,g>;
end if; end for;
for g in N do if 1^g eq 7 and 2^g eq 6 and 5^g eq 3 then
N125s:=sub<N|N125s,g>;
end if; end for;
for g in N do if 1^g eq 6 and 2^g eq 3 and 5^g eq 1 then
N125s:=sub<N|N125s,g>;
end if; end for;
for g in N do if 1^g eq 3 and 2^g eq 1 and 5^g eq 2 then
N125s:=sub<N|N125s,g>;
end if; end for;
T125:=Transversal(N,N125s);
for i in [1..#T125] do
ss:=[1,2,5]^T125[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..6000] do if cst[i] ne [] /* 252000/7=36000*/
then m:=m+1; end if; end for;  m;
/*49*/
for i in [1..#T125] do ([1,2,5]^N125)^T125[i]; end for;
Orbits(N125s);

N126:=Stabiliser(N,[1,2,6]); SSS:={[1,2,6]}; SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IH do
if ts[1]*ts[2]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
/* [ 1, 2 ,6]
[ 4, 3 ,6]*/
for h in IH do if ts[1]*ts[2]*ts[6] eq h*ts[4]*ts[3]*ts[6] then h@rho;
break;end if;end for;
/*function(W)
    w17 := W.2 * W.3; w18 := w17 * W.2; w19 := w18 * W.3;
w20 := w19 * W.2; w21 := w20 * W.3; w22
    := w21 * W.1; w23 := w22 * W.2; w24 := w23 * W.3; w1 := W.1^-1;
w25 := w24 * w1; return w25;
end function */
```

```
h:=function(W)
w17 := W.2 * W.3; w18 := w17 * W.2; w19 := w18 * W.3;
w20 := w19 * W.2; w21 := w20 * W.3; w22
    := w21 * W.1; w23 := w22 * W.2; w24 := w23 * W.3; w1 := W.1^-1;
w25 := w24 * w1; return w25;
end function;
h(G);
/* y * t * y * t * y * t * x * y * t * x^-1  */


N126:=Stabiliser(N,[1,2,6]);
N126s:=N126;
for g in N do if 1^g eq 4 and 2^g eq 3 and 6^g eq 6 then
N126s:=sub<N|N126s,g>;
end if; end for;
N126s; #N126s;
#N/#N126s;
T126:=Transversal(N,N126s);
for i in [1..#T126] do
ss:=[1,2,6]^T126[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..6000] do if cst[i] ne [] /* 252000/7=36000*/
then m:=m+1; end if; end for;  m;
/*70*/
for i in [1..#T126] do ([1,2,6]^N126)^T126[i]; end for;
Orbits(N126s);

for m,n in IN do if ts[1]*ts[2]*ts[6] eq m*(ts[4]*ts[3]*ts[6])^n
then m, n; end if; end for;
for i in [1..7] do i, cst[i]; end for;

for a in [1..7] do for g,h in IH do if ts[1]*ts[2]*ts[6] eq
g*(ts[a])^h then "1,2,6=", a; end if; end for; end for;
for a,b in [1..7] do for g,h in IH  do if ts[1]*ts[2]*ts[6] eq
g*(ts[a]*ts[b])^h then "1,2,6=", a,b; end if; end for; end for;
/*MAGMA SAYS THERE'S NO SUCH PERMUTATION*/

for m in IH do for n in IN do if ts[1]*ts[2]*ts[6] eq
m*(ts[1]*ts[2])^n then m, n; break; end if; end for; end for;
for m in IH do for n in IN do if ts[1]*ts[2]*ts[1]*ts[7] eq
m*(ts[1]*ts[2]*ts[1])^n then m, n; break; end if; end for; end for;
/* [126]=[12] */
```

```
/*NOT NEW*/
N1217:=Stabiliser(N,[1,2,1,7]); SSS:={[1,2,1,7]}; SSS:=SSS^N;
SSS;
#(SSS);
Seqq:=Setseq(SSS);
Seqq;
for i in [1..#SSS] do
for n in IH do
if ts[1]*ts[2]*ts[1]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
*ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
/* [ 1, 2 ,1, 7]
[ 2, 4 ,2, 7]
[4, 1, 4, 7]*/
/*
for h in IH do if ts[1]*ts[2]*ts[1]*ts[7] eq h*ts[2]*ts[4]*ts[2]*ts[7]
 then h@rho;
break;end if;end for;
/*function(W)
    w7 := W.1^2; w8 := w7 * W.2; w9 := w8 * W.1; w10 := w9 * W.3;
w2 := W.2^-1; w11 := w10 * w2;
    w12 := w11 * W.3; w13 := w12 * W.1; w14 := w13 * W.3;
w1 := W.1^-1; w15 := w14 * w1; w16 :=
    w15 * W.3; w28 := w16 * W.1; w18 := w28 * W.2; w17 := w18 * W.3;
return w17;
end function */
h:=function(W)
w7 := W.1^2; w8 := w7 * W.2; w9 := w8 * W.1; w10 := w9 * W.3;
w2 := W.2^-1; w11 := w10 * w2;
w12 := w11 * W.3; w13 := w12 * W.1; w14 := w13 * W.3;
w1 := W.1^-1; w15 := w14 * w1; w16 :=
w15 * W.3; w28 := w16 * W.1; w18 := w28 * W.2; w17 := w18 * W.3;
return w17;
end function;
h(G);
/* x^2 * y * x * t * y^-1 * t * x * t * x^-1 * t * x * y * t  */

for h in IH do if ts[1]*ts[2]*ts[1]*ts[7] eq h*ts[4]*ts[1]*ts[4]*ts[7]
 then h@rho;
break;end if;end for;
/*function(W)
    w17 := W.2 * W.1; w18 := w17 * W.3; w2 := W.2^-1; w19 := w18 * w2;
```

```
w20:= w19 * W.3; w21 := w20 * W.1; w22 := w21 * W.3; w1 := W.1^-1;
w23 :=w22 * w1; w24 := w23 * W.3; w25 := w24 * W.1; w26 := w25 * W.2;
w27 :=w26 * W.3; return w27;
end function */
h:=function(W)
w17 := W.2 * W.1; w18 := w17 * W.3; w2 := W.2^-1; w19 := w18 * w2;
w20:= w19 * W.3; w21 := w20 * W.1; w22 := w21 * W.3; w1 := W.1^-1;
w23 :=w22 * w1; w24 := w23 * W.3; w25 := w24 * W.1; w26 := w25 * W.2;
w27 :=w26 * W.3; return w27;
end function;
h(G);
/* y * x * t * y^-1 * t * x * t * x^-1 * t * x * y * t*/

N1217:=Stabiliser(N,[1,2,1,7]);
N1217s:=N1217;
for g in N do if 1^g eq 2 and 2^g eq 4 and 1^g eq 2 and 7^g eq 7 then
N1217s:=sub<N|N1217s,g>;
end if; end for;
for g in N do if 1^g eq 4 and 2^g eq 1 and 1^g eq 4 and 7^g eq 7 then
N1217s:=sub<N|N1217s,g>;
end if; end for;
T1217:=Transversal(N,N1217s);
for i in [1..#T1217] do
ss:=[1,2,1,7]^T1217[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..6000] do if cst[i] ne [] /* 252000/7=36000*/
then m:=m+1; end if; end for;  m;
/*84*/
for i in [1..#T1217] do ([1,2,1,7]^N1217)^T1217[i]; end for;
Orbits(N1217s);

/* [1217]=[121]*/
for m in IH do for n in IN do if ts[1]*ts[2]*ts[1]*ts[7] eq
m*(ts[1]*ts[2]*ts[1])^n then m, n; break; end if; end for; end for;

/* [1254]=[125]*/
for m in IH do for n in IN do if ts[1]*ts[2]*ts[5]*ts[4] eq
m*(ts[1]*ts[2]*ts[5])^n then m, n; break; end if; end for; end for;

/* [1213]=[121]*/
for m in IH do for n in IN do if ts[1]*ts[2]*ts[1]*ts[3] eq
m*(ts[1]*ts[2]*ts[1])^n
then m, n; break; end if; end for; end for;
```

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