1996

A hypercard stack on exploring single variable equations

Michael Sean Haskins

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A HYPERCARD STACK ON
EXPLORING SINGLE VARIABLE EQUATIONS

A Project
Presented to the
Faculty of
California State University,
San Bernardino

In Partial Fulfillment
of the Requirements for the Degree
Master of Arts
in
Education: Instructional Technology Option

by
Michael Sean Haskins
September 1996
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EXPLORING SINGLE VARIABLE EQUATIONS

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Approved by:

Dr. Rowena Santiago, First Reader

Sylvester Robertson, Second Reader
ABSTRACT

Exploring Single Variable Equations (ESVE) is designed to supplement the California state mathematics frameworks. ESVE is a HyperCard stack designed as a cross between a CAI drill-and-practice and tutorial programs. The stack has been created to be used by Algebra teachers in their efforts to teach students the process of solving one and two step single variable equations.

This thesis is written to describe how the HyperCard stack ESVE was developed to allow teachers the option to use it as a presentation tool. ESVE can also be used on a stand alone computer for individualized instruction, or in a lab setting for the entire class to discover the proper process needed to solve these types of equations.
I would like to acknowledge the enormous amount of help I received from Rowena Santiago in the writing of this thesis. I would also like to thank Sylvester Robertson for his teaching of HyperCard. But most of all I would like to thank my wife Debbie, and son Sean Michael for supporting and encouraging me while in pursuit of this degree.
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CHAPTER 1
INTRODUCTION

Algebra instruction consists of teaching students to do arithmetic operations with an unknown value. Algebra is a fundamental mathematical tool for thinking and communicating symbolically. (Frameworks, 1992) Algebra instruction usually begins with introducing the unknown in the form of a variable. A variable, in mathematics, is a symbolic representations of a number. The process of determining what this unknown is, is solving equations.

A single variable equation is a mathematics sentence that relates one variable (unknown) to a known value, as in $x + 5 = 34$, which can be translated from the statement, what number added to 5 has a result of 34. Single variable equations come in many forms and their complexity also comes in varying degrees. To be able to solve the more complex equations accurately, students must first master the task of solving single variable equations.

Single variable equations can be solved in four stages or steps (Glencoe, 1990; Addison-Wesley, 1987; Houghton Mifflin, 1995; Holt, 1992, South-Western 1995, D.C. Heath 1995) Step 1 would be to clear away parentheses and/or fractions: For example,

$$2(4x - 4) + 3 = 3$$
becomes \( 8x - 8 + 3 = 3 \) by using the distributive property of multiplication over addition or subtraction, (equation 1)
or it becomes \( 4x - 4 + 3/2 = 3/2 \) by using the division property (equation 2) where both sides are divided by 2.

Both the distributive property and the division property are covered in the various Algebra texts, as two ways of clearing away the parentheses in an equation. However, the use of the division property may result in an equation with fractions. This may or may not cause the student difficulty. It is not the recommended approach if students still have difficulty with fractions.

Step 2 is to combine any like terms that may or may not have been created in Step 1. Therefore,

\[
\begin{align*}
\text{Equation 1} & \quad 8x - 8 + 3 = 3 & \quad \text{Equation 2} & \quad 4x - 4 + 3/2 = 3/2 \\
\text{become} & \quad 8x - 5 = 3 & \quad \text{and} & \quad 4x - 5/2 = 3/2, \\
\end{align*}
\]

respectively. This is a result of combining the negative 8 with the positive 3 in the first equation or the negative 4 and positive 3/2 in the second equation.

Step 3 would usually have the student move the variable terms to one side (usually left side) of the equation and the
numerical terms to the other side (usually right side). This is accomplished by adding the correct number or variable term to both sides of the equation. In this example, the number term 5, must be added to both sides of the first equation and, 5/2 in the second equation.

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>8x - 5 + 5 = 3 + 5</td>
<td>4x - 5/2 + 5/2 = 3/2 + 5/2.</td>
</tr>
</tbody>
</table>

Both sides of the equation are then simplified by combining the number terms on either side of the equation to get,

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>8x = 8</td>
<td>4x = 8/2</td>
</tr>
<tr>
<td></td>
<td>or 4x = 4.</td>
</tr>
</tbody>
</table>

Finally, Step 4 would be to divide both sides of the equation if there is a coefficient attached to the variable term. In this example, the coefficients are 8 and 4 respectively.

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>8x/8 = 8/8</td>
<td>4x/4 = 4/4</td>
</tr>
</tbody>
</table>

to get

\[ x = 1 \quad \text{and} \quad x = 1. \]

Using these 4 steps would solve most single variable equations found at the Algebra 1 or Algebra 2 levels. This is by no means the only process that could be used to solve this equation. The student could have added a negative 3 to both sides first and then cleared away the parentheses.
\[2(4x - 4) + 3 = 3\]

add \(-3\) to both sides \[2(4x - 4) + 3 - 3 = 3 - 3\]
to get \[2(4x - 4) = 0\]
by simplifying both sides, and then using the distributive property to get,

\[8x - 8 = 0\]
then add 8 \[8x - 8 + 8 = 0 + 8\]
to get \[8x = 8\]
divide by 8 \[8x/8 = 8/8\]
to get \[x = 1\]

Not all equations have parentheses, have terms to combine, or even have coefficients. However, the 4-step method shown above and the use of inverse operations usually gives students useful strategies for solving equations. Once learned, the student will realize that, done correctly, single variable equations can be solved in more than one way.

Each of the Algebra textbooks used in schools (Glencoe, 1990; Addison-Wesley, 1987; Houghton Mifflin, 1995; Holt, 1992, South-Western 1995, D.C. Heath 1995) start with 1-step equations where the inverse operation, usually addition or subtraction, is needed to find the unknown value of the variable. Knowing what operation is applied to the variable in the first place will help the student to identify the correct inverse operation needed to solve the given equation. The concept of inverse operations is a
prerequisite that the student must bring into solving one-
step equations. For example, in an equation such as
\[ x - 5 = -67, \] adding 5 to both sides of the equation is needed
to solve it because addition is the inverse of subtraction.
However, students do not always do this successfully. The
problems that arise in solving this first type of equations
in Algebra vary from not identifying the right inverse
operation, (thus, applying the wrong inverse operation
altogether) to not performing the basic arithmetic
calculation accurately. To illustrate, students see the
- (minus) sign in \[ x - 5 = -67 \] and may subtract 5 on both
sides of the equation thinking that this is a subtraction
problem (not identifying the right operation) and subtraction
is the operation to be applied (applying the wrong inverse
operation). The other type of mistake is where the student
may recognize that addition is the inverse of subtraction,
but may not calculate the numbers correctly resulting in
\[ x = 72 \] or any other combination of 5 and 67 (e.g. 62).

Because equations are abstract ideas, teachers employ
different strategies to teach them to students. According to
cognitive theorists, the teacher must also take into account
the students’ unique interpretations of these abstract ideas
and know how to introduce formal mathematics by building on
existing abilities, based on current assessment by the
teacher (Piaget, 1973). One method would have the teacher
present the process in a direct method using many examples (or models) that build upon the students’ constructions of mathematical relationships (Kaplan, 1989). Modeling the steps needed to solve a given problem, usually on the chalkboard, is vital and is a common method in the process of teaching Algebra. This is usually followed by assigning a set of problems to be done in class and with the instructor working with as many individual students during this guided practice time. Finally, students are assigned a set of problems as homework. Once these steps are done, the teacher can do some type of evaluation to determine the students’ level of understanding and mastery (Hunter, 1979). The problem associated with the direct method is that teachers do not have the opportunity to provide individual student feedback during the guided practice portion of the class. Teaching proceeds most effectively when an adult mentor takes into account an individual child’s framework and encourages and guides the child’s inquiry (Kaplan, 1989). Without this individualized guidance, students may successfully complete a few problems in class without knowing why. Thus, mastery is not achieved. When faced again with a similar set of problems as homework, success is not experienced, students get frustrated and give up. A related issue in direct instruction involves the modeling of the procedure of solving equations. When teachers present the process, they model the
correct way of solving the equation and the students do not see the effects of applying a wrong operation. Another method used in teaching equation solving is to give the students a set of problems and ask them to find a number that works through experimenting or estimating, until they discover the correct answer (Womack, 1989). The use of manipulatives can help in the experimentation process whereby preventing the student from blindly applying conventional algorithms and thus reducing the number of procedural errors (Kaplan, 1989). The problem with this method is that it takes much more time in class with no guarantee that the time spent will produce the wanted results. Therefore, given the strengths and weaknesses of the two methods (direct instruction with modeling and the discovery method), a technology-based teaching tool is needed so that more students will acquire the necessary skills needed to continue on in Algebra.

Using the authoring tool, HyperCard, this master’s project was designed and developed to address the above need in Algebra instruction. The project will use a HyperCard “stack”, giving students a platform that will have control structures which will lead to identifying the correct inverse operations and in performing the correct calculations. Another goal of the stack is to provide immediate feedback to the students during the process of solving single variable
equations. Students will immediately see on screen the results of the operation they chose to use. With this immediate feedback, they are also given the option to analyze their decision, then go back and select another operation without being penalized for the mistake that was made. The process of learning by discovery is used but with the computer providing the appropriate learning guidance.

In this project, the computer is used to (1) introduce the process of using inverse operation in the solving of single variable equations; (2) as a manipulative tool to guide the student in their practice; and (3) to ensure that the students learn from their mistakes and receive the correct reinforcement during the discovery process. Using the stack in a lab setting as an exploratory tool, students can experiment with different operations until they discover the operations needed to solve this type of equation. The HyperCard stack will show them that if given the equation $x - 5 = -67$, the subtraction of 5 on both sides will only create a new and different equation, not solve it. With the use of technology, teachers can use the discovery method in the lab with less worry that the wrong steps are being learned.

To accomplish the above goals, the stack Exploring Single Variable Equations or ESVE was programmed to generate an infinite supply of different equations. These equations
are grouped according to the operations needed to solve the problem: 1) add or subtract; 2) multiply; 3) divide; 4) add or subtract and then multiply; and 5) add or subtract and then divide.

ESVE has six parts. Part One contains the introduction screens that informs the user what the stack is about, and what the objectives are. Part Two contains has menu screens as well as the user instructions (More Information). Part Three is the equation generation portion where new equations are generated through the random number generating capability of HyperCard©. Part Four is where the users make their choice of operation to solve the given equation. Part Five is the place where all the simplifying is done. Finally, Part Six is the solution card(s) for each type equation.
CHAPTER 2
RELATED LITERATURE

MATHEMATICS CURRICULUM

The National Council of Teachers of Mathematics (NCTM) published the NCTM Curriculum and Evaluation Standards (1991) and then the California Department of Education followed with a revised Frameworks (1992) for California teachers to use in the teaching of mathematics throughout the K-12 public schools. The concepts of Algebra and the teaching of it is well written in the Algebra section of the Frameworks.

Algebra in the core curriculum. The notions of variables and symbol manipulation and the relationships between them are the core concepts of Algebra. Student understanding and use of these concepts should continue in the core program through the description of quantitative situations by means of Algebraic expressions, equations, and inequalities. Students should learn to interpret by means of tables, graphs, diagrams, and verbal description. The ways in which standard forms of expressions, equations, and inequalities can be obtained (by symbolic transformations, using addition, subtraction, multiplication, and division; forming powers and roots; and factoring) should be exploited to increase the variety and breadth of interpretive methods student can use. Students should learn how the interrelationships between algebraic and geometric representation of equations and inequalities can assist in determining specific results in investigation problem situations. By using these techniques in conjunction with problem-solving activities, student will learn to appreciate the power of Algebra as a concise language for describing and modeling problem situation and as a tool for solving specific problems (p. 144).
The State Frameworks tells teachers of mathematics that specific topics are to be taught in Algebra and the NCTM Standards suggest methods on how these topics should be taught. And it is with commercial textbooks and supplementary materials that math teachers teach these courses. The textbooks for Algebra come from different publishing companies, all of whom use the State Frameworks and Standards in the design of their publications for Algebra 1 and Algebra 2. But, regardless of which company publishes the textbook, they all include the topic of solving equations in basically the same sequence. Houghton Mifflins’ Algebra 1 An Integrated Approach, South-Westerns’ Mathmaters Book 1 An Integrated Approach, and D.C. Heaths’ Course One Integrating Algebra, Data Analysis, Functions, and Geometry all start the sequence of instruction with the concept of inverse operations and how they are used to solve the single variable one-step equation. They continue from there with the 2-step equation. After this, the concept of combining like terms is presented. The textbooks go into the removal of parentheses by using the distributive property. From there, they present the types of equations that have variable terms on both sides of the equal sign. After these skills are presented, more complex equations are presented using a combination of the concepts, with the multiple use of parentheses and variables on both sides. From here, one textbook (the Houghton Mifflin
text) continues to teach more complex equations and the South-Western and D.C. Heath books leave these complex operations for their next level. Only Houghton Mifflin, mentioned above, combine the very important topics of signed numbers in the solving of equations. The other texts have the topics or signed numbers separate from the sections of solving equations. All equations use signed numbers, but only the Houghton Mifflin textbook has a chapter dedicated to the combined concepts. Also, the Houghton Mifflin textbook continues by giving students different examples of solving the same equation using different operations in different orders similar to the equations discussed in the introduction of this paper.

The textbooks that would be used in Algebra 2 courses are Holts’ Algebra with Trigonometry and Addison-Wesleys’ Algebra with Trigonometry. Both of these books actually spell out the four steps and introduce the method of clearing fractions from the equations. Both have the students simplify both sides of the equations by clearing the parentheses and using the idea of Least Common Denominator or LCD to clear away the fractions in the equation, and combining like terms.

The State of California, through the Frameworks, and the NCTM with their standards, spell out the topics to be taught in an Algebra class. With the use of the published
mathematics textbooks, Algebra teachers carry out Algebra instruction. Solving equations is a vital aspect of this teaching, and the different texts go about this presentation of the topic in similar ways. The overriding difference in the texts is the readability of the text, the examples used, and the supplementary materials that accompany the texts. Teachers of Algebra must wade through these materials and choose which textbooks best fits their individual states/district/school curriculum. They must also use their own talent and knowledge of the course to make decisions on how specific topics need to be presented to their students. Students have varying abilities and teachers need to have different teaching strategies to suit student needs. Therefore, teachers use materials from the best thought out curriculums from the (state, district, or school levels), from textbooks from various publishers, and computer technology to teach to these varying student abilities.

The ability to recognize basic operations (adding, subtracting, multiplying, and dividing integers) is the underlying skill used in the process of solving equations. This skill is always addressed in the Algebra curriculum and textbooks. The student’s ability to solve equations, whether it is single variable equations or those more complex equations in later courses, depends on the ability to identify which operation to use and the order of operation.
If an Algebra teacher understands how the solving of equations depends on the ability to perform these operations, the teaching of Algebra as separate skills is not likely to happen. This view is validated in the NCTM's "Professional Standards for Teaching Mathematics" (1991) where it states that as professionals, math instructors need to know mathematical pedagogy and, more importantly, must change teaching strategies to reflect this new knowledge (NCTM, 1991). However, teaching the math topics separately or combined is not the issue here rather, it is the understanding of the interrelationship of solving equations and using various operations. Students must have the ability to recognize the order of the operations being applied in an equation to be able to "undo" them when asked to solve the equation as well as simplifying the equations once an operation is applied. (Ederhart, 1994)

COMPUTERS IN ALGEBRA INSTRUCTION

In addition to textbooks, computer assisted instructional materials (CAI materials) also address the NCTM Standards. The idea of using selected application programs as cognitive tools, or mindtools, according to David Jonassen (1996), can be used to engage and enhance thinking in learners. Thinking is engaged by different learning activities, which can be embedded in the tasks and functional
requirements of computer applications. Therefore, mindtools are computer applications that require students to think in meaningful ways in order to use the application to represent what they know (Jonassen, 1996). CAI represents learning from computers, where the computer is programmed to teach the student, to direct the activities of the learner toward the acquisition of prespecified knowledge or skills (Jonassen, 1996). CAI comes in many forms, such as; drill-and-practice, and tutorials (Newby, 1996; Dinkheller, 1989, Jonassen, 1996 & Merrill 1996). According to Dinkheller (1989), a good drill-and-practice has eight aspects: (1) focuses clearly on objectives by working on separate tasks within the curriculum, (2) states questions clearly so students knows what to do and when to do it, (3) refrains from giving irrelevant clues, (4) provides appropriate advice when mistakes are made, (5) gives immediate feedback to correct as well as incorrect answers, (6) provides remedial information when the student makes the same mistake repeatedly, (7) moves from easy to difficult problems or has branching features so that students can avoid unnecessary repetition of previously mastered concepts, and (8) provides continued practice until student demonstrates mastery. According to Newby, (1996) drill-and-practice programs offer some significant advantages to the traditional drill worksheets: (1) Interactivity. The computer can present
many problems and require student responses. (2) Immediate feedback. The computer can immediately inform the learner if an answer is right or wrong, and in a well-designed program tell the learner why. (3) Infinite patience. A computer drill-and-practice program can go all day without getting tired or irritable. (4) Variable level of difficulty. The computer can adjust the problem level of difficulty. This adjustment might be set by the teacher or by the learner, or the program may adjust the difficulty level automatically, based on the student's performance. Well-designed drill-and-practice programs automatically recycle missed items until the are mastered. (5) Motivation. Through the use of challenge and gaming elements, or just because it is on the computer, a computer drill-and-practice program may be more motivating to the student than similar paper-and-pencil exercised. With the immediate feedback, patience, variable levels of difficulty, and the motivating factors inherent in drill-and-practice programs, they can become the deciding factor in enabling students to master the low-level skill needed to be successful in later Algebra skills. Another form of CAI used in mathematics classrooms is the tutorial. A tutorial application program assumes the primary instructional role of teacher or tutor. It presents new content and assesses learning. A tutorial typically contains an organized body of knowledge, one or more pathways through
that knowledge, specific learning objectives, and built-in tests for student learning. Again, according to Dinkheller (1989), a good tutorial program that employs a strategy of branching programmed instruction has three aspects: (1) provides immediate and interactive feedback; (2) provides animated graphics; (3) provide a focus on the topic.

Computer-based tutorials offer a number of advantages (Newby, 1996). These four advantages are as follows: (1) Embedded questions. Like computer drills, tutorials on the computer give the advantage of being interactive. Students must take an active role by answering embedded questions as well as receiving immediate feedback. (2) Branching. While programmed texts can incorporate branching by directing the learner to a particular page, computer tutorials can automatically branch according to the learner’s responses to embedded questions. Remediation or advancement can be built in to meet the needs of the individual learner. (3) Dynamic presentation. The power of the computer can be used to present information dynamically. Important text can be highlighted on the screen to capture the learner’s attention. Processes can be depicted through animated graphics. (4) Record keeping. Computer tutorials can automatically maintain student records. These can be used to inform the learners of their progress. In addition, the teacher can check the records to ensure the students are progressing
satisfactorily. The design of tutorials enables the program to respond to each learner's individual responses and branch accordingly. These features make them ideal for selected students that may need remediation, enrichment or makeup work (Newby, 1996).

CAI programs like the drill-and-practice, and tutorial applications are not designed to replace the teacher but are for use by the teacher to enhance the environment created by the teacher. Drill-and-practice programs can be used to help the student master the basic skills needed to be successful in mathematics. Tutorials can be used to supplement the curriculum, present the curriculum to students that may have missed the teacher's presentation, or use it to enrich or reenforce the curriculum. Both drill-and-practice and tutorial programs help the teacher do the job they are required to accomplish. Both types of programs give the learner feedback when needed, they are motivating to students, and present the different topics to the students in ways that allow mathematics students to be successful.

Drill-and-practice, and tutorials provide the teacher with many different forms of instructional tools. The drill-and-practice form or CAI provides the learner with an opportunity to gain automaticity (Merrill, 1996). Automaticity is necessary, according to Merrill, because learners must first be able to perform the lower-level sub-
skills automatically in order to learn complex, higher-order skills. This does not mean the teacher should use the computer as an electronic ditto sheet (Jonassen, 1996, Merrill, 1996, Newby, 1996). Drill-and-practice programs can be dry and boring or considered drill-and-kill torture devices for most students. They do help some remedial students, but is not the most effective way to use powerful computing technology.

Tutorials, have been described as mere electronic page turners (Newby, 1996). They are usually designed to take the student through a sequence and do not allow a student to proceed to lesson 3 without first going through lessons 1 and 2 (Merrill, 1996). This is a distinct disadvantage to poorly designed tutorials. Another disadvantage to the tutorial form of CAI is that to design a good tutorial the programmer must anticipate every possible student response. This is impossible to do even in the best of circumstances. Finally, tutorial do not allow learners to construct their own meaning, but rather seek to map a single interpretation of the world onto what students know (Jonassen, 1996).

SOFTWARE REVIEW OF ALGEBRA/MATH PROGRAMS

NCTM's Curriculum and Evaluation Standards for School Mathematics (1991) says that Algebra classrooms need to find ways to use technology in the classroom. To this end, many
different publishers have come out with numerous software titles that have this standard in mind. The titles reviewed written the past six years by NCTM’s Mathematics Teacher range from the basics of Arithmetic to the integrated tools used in the advanced college courses and engineering fields. The programs reviewed for this project are restricted to those that involve Algebra topics. The programs will be dealt with in three categories: (1) Integrated tools, (2) Algebra graphing applications, and (3) Algebra Tutorials.

Integrated Mathematical Tools are symbolic manipulators that can solve equations and do complex numerical computations. Two software titles that fit this category are SymbMath by Weiguang Huang (1995), and Mathematica from Wolfram Research (1992 ). SymbMath is a symbolic calculator program that is designed to solve symbolic mathematical problems. It is an expert system that allows user input. The software has graphing, programming, and database capabilities. It is also capable of analytically manipulating matrices and handling data. Teachers will not find SymbMath easy to teach and use with students, especially because of the inadequate documentation and the lack of on-line help. The software would complement high school and college mathematics courses, but the limitations are too great for use by most students (Stiff, 1995). Mathematica (1992), combines symbolic manipulation, numerical
commutation, outstanding graphics capabilities, and a sophisticated programming language. Mathematica takes the busy work out of computations. It enables the user to explore beyond the customary boundaries and asks What if...? It does not remove the need to understand the processes, but it does focus on the thought behind the process rather than the mechanics. Graphing capabilities in 2-dimensions and 3-dimensions are remarkable, and the programming capacity makes it a bit more difficult to learn, but gives it versatility. Both SybMath for IBM and Mathematica for both IBM and Mac platforms are similar to Scalar by Future Graph, and MathCad by MathCad for the IBM platforms. All of these programs are not recommended for the use in beginning Algebra classrooms and do not have the capability to teach the students to solve simple first degree single variable equations. They do evaluate Algebraic expressions and give the solution for an equation, but do not show the Algebraic manipulation used to find the solution.

Algebraic graphing applications, such as APS-II and Algebra Alder from Michael Rieck, Algebra Lessons from Vance, and Algebra II Vol. 1 from Microcomputer Curriculum Project, have topics that include real numbers and linear relations, polynomial and polynomial equations, radical and complex numbers, graphing and solving quadratic equations, basic coordinate geometry, sequences and series, and probability.
Algebra Lessons include lessons in solving systems of linear equations and factoring quadratic equations. These programs would be appropriate for the latter chapters of Algebra I as well as the Algebra II courses but none of these deal with the topic of solving first degree one variable equations.

Other programs reviewed in Mathematics Teacher are Function Probe Academic by Intellimation, which was reviewed by David Thomas and Tammy Hovland (1995), and Algebra Xpresser from Bradford Publishing Co. which was reviewed by Goodman (1993). Functions Probe is an integrated set of three tools for studying mathematical function. A calculator, a table maker, and a grapher. Using this dynamic interaction students can explore families of functions and possibly draw their own conclusions about the roles of coefficients and constants in linear, quadratic and other functions. One potential drawback is the lack of options for manipulating transformed equations. This is a tool to facilitate investigations. Algbrush from MareWare (IBM) is similar, and the reviewer Richard Edgerton (1995) feels it will help advanced Algebra students as their understanding of functions and relations deepen. Algbrush is designed to explore transformations of functions over user selected domains and ranges. Algebra Xpresser for the Mac is a symbolic manipulator combined with Graph Wiz. The program can simplify and transform Algebraic expressions, solve
equations and inequalities, graph Algebraic and trigonometric functions. According to the reviewer, Terry Goodman, teachers will have to monitor students use since it is possible simply to have the program perform all possible simplifications at once. The program does afford opportunities for students to become more actively involved in the process of doing Algebra, because they will be able to do an operation to the equation and then check with the computer to determine if they have done that operation correctly.

The last group of software that was reviewed include applications that were truly aimed at the first year Algebra student and classroom: Math Connection: Algebra I from Wings of Learning, Function Grapher and Function Finder from C. Blohm & Associates, Algebra form Kemeny-Kurtz Math, and Converge: Software for Algebra Through Calculus from JEMware. These programs aim to help students understand Algebra concepts by making the formal methods and symbols of Algebra more concrete. In particular, according to reviewer William Doyle, Math Connections makes realistic connections among fundamental Algebra concepts involving expressions, equations, and table and graphs. The software is mathematically sound in that it features standard math notation and symbols and investigates linear and quadratic functions, operations on polynomials, composite functions,
and graphing concepts. Math Connections can be used as a
demonstration tool, exploration, reinforcement or
remediation. The program is a fun and easy way for students
to explore relationships among formulas, tables and graphs.

Personal Tutor - Algebra I: First Degree Equations (C.
Blohm & Assoc.), according to Phyllis Kritzman Townswick is
the fourth of eleven units planned for the Algebra I series.
The tutorial and worksheets are organized into four sections;
one-step equations, two and three-step equations, multi-step
equations, and formulas. Each section has a menu showing
fifteen to twenty-one equations to be solved. The student
can click on one of these equations. The action menu
includes "General Hints", "Specific Hints", "Answer",
"Similar Examples", "Slow Check", and "Print". The student
is supposed to work on the problem using paper and pencil,
revealing only as many hints and steps as necessary to reach
the solution independently. Almost any textbook or work book
would offer similar utility with greater portability at less
expense.

Expert Algebra Tutor or EAT from TuSoft according to
Dave Becker covers a variety of Algebra topics. The student
is given a problem to solve with pencil and paper. The
questions are multiple choice type and the student selects
one correct response. Missing two or more problems forces
reworking of those problems. If the student is unsuccessful
a second time, EAT shows the solution to the problem. EAT will not correct the misconception but refer the student to a text book. The "Help" option basically repeats comments that could be found in most text books.

Interactive Algebra from Technical Education Consultants is a set of four disks that involves Algebra topics from elementary to intermediate level. According to the reviewer Garland Dovel, Interactive Algebra is designed so that they can be used for homework, for supplementing classwork, or for independent work. This program follows the sequence of a normal Algebra course. The programs contain a short tutorial or review, sample problems, and randomly generated problems at varying levels of difficulty. Problems can be entered by the student and they can get help when needed. Students can work with formulas, graph, and get a printout of the work performed by the student.

Algebra I Homework Tutor from Missing Link Software is designed for students to use as an aid when "stuck" on an Algebraic manipulation, simplifying algebraic expressions, solving equations, and factoring polynomials and quadratic equations. The program, according to reviewer May Crowley, has two modes. The first mode allows the student to enter problems, and the second mode is randomly generated by the computer. There are two options in the first mode; entering the next step or asking for prompts from the computer. At

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each correct step, the user is prompted that another step is required or the solution has been reached. The user is informed about mistakes with hints at the users request. The user controls the operations, variables and constants at each step. At no time does the computer solve the problem independently. Verbal tips are given on what to look for in step-by-step solution processes. Students may ask for similar problems, but students may become frustrated with finding a similar problem or not have the sophistication needed to identify a similar problem. The software does handle fractions, mixed numbers and decimals, but may not recognize the answer as correct. The range of skills covered makes it attractive and the interactive nature of the software is appealing.

Algebra Tutor from Weston Walsh Publisher offers diagnostic and prescriptive testing, tutorials and practice activities in Pre-Algebra and first year Algebra topics. These topics include first-degree equations, signed numbers, simple polynomials, FOIL multiplication, squaring binomials, factoring trinomials, and determining the roots of quadratics. The student initially completes a pretest of 30 problems representing all the available topics. The tutorial section offers drill-and-practice on categories where the student needs help. As problems are randomly generated and the student answers, erroneous responses triggers both a
clue and an opportunity to respond again. No thought has been put into the order in which topics are offered. For example $2x + 3 = 8x - 9$ may come before $6x = 12$. In response to receiving a wrong answer, the tutorial offers a hint rather than a detailed explanation.

Algebra I Second Semester from Britannica Software was reviewed by William Thomas. Unit 1-3 of Algebra I Second Semester cover factoring trinomials and solving quadratic and 2 variable linear equations. Unit 4-6 consist of problems that include operations with fractions and solving fractional equations. Unit 7-9 include functions and graphing, radicals and exponents, and problem solving. The software complements Houghton Mifflin, Addison-Wesley textbooks to name just two first year Algebra I text books. Students could spend 30 minutes going through a tutorial, practice set, and testing on a topic with which they are somewhat familiar with the topic. The time could be considerably more if the student is unfamiliar with the topic. Students have only partial control of the tutorial. In some instances, a problem is given in several steps, which could be confusing to slower readers or a student struggling with the concept. Also, the test cannot be exited except by finishing all 20 questions or by pulling the plug. Overall, the software would be a good addition to a schools mathematics software library. Students
who would like more work on a topic could make good use of this software.

The software available at this time could be used in a variety of classrooms. The tools can be used in courses that have students with a firm grasp and understanding of Algebra topics and are at the level where they are ready to ask “What if...” questions, to find solutions to complicated math questions with time consuming complex calculations. Other programs can be used in Algebra courses where the students are beyond the solving of equations and are now ready to study the interrelationships between similar functions. These programs and also help the students study the relationships among the different parts of a function and there relationship to the behavior of the functions graph when these parts are changed. Still other programs are appropriate for the beginning Algebra classroom. These programs help students learn the beginning concepts of manipulating expressions, working with numbers, and solving linear equations. But applications that deal with solving of one and 2-step single variable equations have their problems. Students must be able to solve single variable one and 2-step equations to be able to go on and benefit from the other software programs. The students need materials at their level that guide them in their study of solving equations. They need a platform that allows them to ask “What if...”
questions at their level of expertise. They need to be able to see what happens to an equation when an operation is applied to it in the solution process. They need an environment that is non-threatening and informative so that if an operation does not "solve" the equation or do what they expected, they can go back and try something that will "solve" the equation. This process is frustrating to the students as well as the teacher if the student continuously makes arithmetic mistakes in their learning to solving equations.

ALGEBRA INSTRUCTION

Teaching methods vary from teacher to teacher. Just as learning styles vary from student to student. It has only been since NCTM has published the Standards in 1991 that the methods of teaching Mathematics has been placed under the microscope. The NCTM has been working for a number of years to address the concept of teaching mathematics to students with methods other than the direct method. One of the main problems mathematics teachers face is that there is an enormous amount of information to get across to the student in an Algebra class. The two most common methods of presenting this information is the expository method and the discovery method. In the expository, or direct method, the teacher presents the information to the students. The
students usually write down what the teacher says', and does, on the chalk board and then use their notes as reference while doing independent work. The other method (discovery or inquiry) is to have the student figure out Algebra for themselves by using different activities that will guide them in the right direction. Either way, it is the teacher that must get the students through the curriculum of Algebra in such a way that the student can use the skills in courses that follow.

The expository mode of instruction assumes that each student has an essential body of knowledge and that all students are at (about) the same ability level. (Womack, 1989). The problem with this according to Womack is that all students are not "level," regardless of any leveling system that may have been attempted and the teacher will face the same pitfalls associated with the old chalkboard method.

The discovery approach enables and encourages students to find “answers” for themselves. The teacher may arrange the environment so the “discovery” can occur. A principle of discovery learning is that students learn best by doing rather than by just hearing and reading about a concept (Newby, 1996). Inquiry mode of instruction is not for all students but at least the problems they do attempt with the stack will not be done wrong. On the other hand those students who do well in self-directed settings will do more
problems correctly and be more willing to take chances with the problem set and be eager to move on to more difficult problems. This mode of instruction will allow the teacher to help those students who need it as well as encourage some with a helpful hint to get them on their way. (Womack, 1989)

Related closely to the instructional method used is mastery learning. Mastery learning derives from the conviction that learners of different abilities can learn or master a particular learning task or body of content if given enough time and opportunity. While a mastery approach to learning can be used in settings that are not strictly individualized, the concept of mastery implies individualization because each student is individually accountable for mastering the content. Mastery learning is not one approach; rather, it forms the foundation for many of the systems of individualized instruction (Newby, 1996). The philosophical premise of mastery learning is that all children can learn when provided with the conditions that are appropriate for their learning (Guskey, 1988). The two aspects of a lesson that fulfills the requirements of mastering learning are: 1) the session must have regular and specific feedback to the student and 2) it must also ensure there is consistency across the major components of the learning process (Guskey, 1988). Teaching and learning cannot occur without the other. Therefore the teaching
device must be willing and able to give appropriate feedback to the learner and the learner needs to be receptive to that feedback.

ROLE OF CAI IN ALGEBRA INSTRUCTION

In its simplest form of CAI consists of drill sessions in which a student would be presented with multiple choice questions. The basic idea was: 1) student learning would be reinforced by correct answers, and 2) students would be able to identify those areas in which their knowledge was inadequate. In addition, instructors would be able, by keeping track of student records to determine topics in which further training was required (Porter, 1988, Newby, 1996, Jonassen, 1996, Merrill, 1996).

Computers are reported to affect teachers' ability to guide more students in the guided practice portion of class, as well as give the gifted students a platform to actively experiment with the curriculum (Grandgenett, 1991). Furthermore according to Dinkheller (1989), the computer can provide an immense advantage in arranging for effective use of Academic Learning Time or ALT. ALT is defined as the amount of time a student spends attending to relevant academic tasks while performing those same tasks with a high rate of success (Cadwell, 1982) Computers can provide the needed practice of objectives tailored to the student needs.
Dinkheller goes on to say that, the student must respond to each problem presented by the computer, in contrast to responding only when called on in class or when the teacher is close by watching. Computers can help teachers and students increase ALT by 1) permitting learners to acquire specific information and practice specific skills, and 2) by helping student develop basic tools of learning that can be applied in later settings (Dinkheller, 1989).

Increasing ALT, while striving to increase the students ability to correctly solve equations, is one of the main goals for this project. ALT can be increased by lengthening the school day or by enabling the teacher to manage a classroom more efficiently and by enabling students to study more efficiently and at a higher rate of success. CAI cannot lengthen the amount of time spent at school, but they can help teachers and students perform their tasks with a higher rate of success. The use of CAI can be an essential bridge to mathematics equity when used as an integrated instructional tool (Carl 1993, Grandgenett, 1991) in that CAI may by the tool needed to increase students ability to master the basic skills and have equal access to higher level skills. When CAI does enhance learning, they do so because the increase effective ALT by permitting learners to acquire specific information and practice specific skills, and by
helping student develop basic tools of learning that they can apply in a wide variety of settings (Dinkheller, 1989).

Basic Algebra skills (including addition, subtraction, multiplication, and division of signed numbers, order of operation, and identifying inverses) have to be mastered if the student is to have success in future mathematics courses. To acquire higher-order or complex rules, learners must have learned some concrete concepts, and to learn these concepts, they must be able to retrieve some previously learned discriminations (Gagne, 1992). These discriminations are those mentioned above. Mastering these skills is a matter of practice and feedback. Even the most rudimentary computer-based systems can offer significant advantages over paper-and-pencil drill-and-practice (Schoenfeld, 1988). When using paper-and-pencil, a student can get many problems wrong before discovering that he or she has a problem. When using an on-line system, the student can get immediate feedback. When students make a mistake, they are brought to their attention. This can nip a problem in the bud, keep the student from wasting time and energy, and prevent bad habits from becoming engrained. Once the students have the basic mathematical skills of Algebra, they can go on to higher level of Algebra skills where they will apply their basic skills to address higher order Algebra questions. These questions come into play only after the student has mastered
manipulating an equation and is at the level of learning where questions related to changing parts of the equation rather than solving and entirely new equation.

AUTHORING TOOLS FOR ALGEBRA INSTRUCTION

An authoring system is an integrated set of tools designed to simplify the programming process and make it possible for non professional programmers, such as teachers, to develop computer-based, hypermedia instructional programs. HyperCard® was the first card-based authoring system that was developed for the Macintosh computers. Card-based authoring systems are designed around the metaphor of a stack of three-by-five cards. The cards forming the stack are displayed on the screen one at a time. Each card contains several different types of objects, such as graphics, fields, and buttons (Merrill, 1996). Fields are rectangular areas on the screen for entering and displaying text. Fields can be of different sizes and placed anywhere on the screen. They can be used to display headings, labels, instruction, or textual information (Jonassen, 1996). Buttons are small areas on the screen indicated by a small picture or icon or by a rectangular box with a textual label. When the user clicks on a button with the mouse pointer, the computer will perform some action. The label or icon on the button generally indicates the action that will be performed when the button
is clicked. Which action is performed depends on the set of computer commands or instructions associated with the button. These instructions are essentially small computer programs referred to as scripts (Jonassen, 1996). Programming in card-based authoring systems is quite different from traditional programming. With traditional programming, the program has a definitive beginning and end, and the program controls the sequence of information presentation and user input. In contrast, programs written in HyperCard are event driven. With an event-driven program, the user controls the actions of the program rather than the program controlling the actions of the user. This student controlled, event-driven aspect of solving equations is ideal for the card-based authoring of HyperCard because each choice taken by the student will change the equation in a specific way and can be shown on a card with feedback specific to that choice and card (Merrill, 1996).

Two HyperCard stacks were found among the software programs that were reviewed for this project. They are Personal Tutor-Algebra 1: Integers and Rational, by Phyllis Kretzmann Townswick, Published by Intellimation, and Euclid's Toolbox-Triangles, by Jerry Beckmann and Charles Friessen, published by Heartland Software.

A workbook accompanies the Personal Tutor software and problems from the workbook can be entered directly into the
stack where all options for checking, explanations, and hints are available to the student. Euclid's Toolbox is exploratory in nature and encourages making and testing conjectures. The reviewer, Bill Blubaugh, believes the Toolbox facilitates (a) quick and easy conjecturing, (b) accurate measurements, (c) clear graphics, and (d) quick and thorough repetition by including measurements as well as constructions. Euclid's Toolbox is a powerful tool because of HyperCards' scripting capabilities that allows for the geometric explorations and conjectures, and the ability to repeat the measurements and constructions performed by the student as many times as the student needs to determine if there is any pattern. This repeating of instructions is programmed into the stack through the use of scripting. The HyperTalk scripting is the programming language of HyperCard. It allows the teacher to program the computer to do certain commands without having the programming getting in the way of the commands.

The scripting in HyperCard provides the programmer the ability to develop a stack that addresses all the aspects of a given problem, in this case, solving equations. As mentioned previously, students have four options when choosing an operation to solve the given equation. Only certain options will correctly solve each equation. The scripting language of HyperTalk allowed the programmer to
address each of these options separately and either program
in an error message, reveal the result of the chosen
operation, or show the new equation generated by the users
choice. Each possible option students could take have been
anticipated, and HyperCards’ card-based platform for
authoring was ideal to reveal to the student what will happen
to the equation when each operation was applied.

Using the HyperCard stacks like Personal Tutor-Algebra
1, and Euclid’s Toolbox as examples, HyperCard has the
capability to provide the following features: repetition,
accuracy, graphics, student input, feedback, and conjectures
or “What If’s...”. Because of these features, the card-based
authoring tool of HyperCard was chosen to develop this
project and create the stack Exploring 1 and 2 Step Single
Variable Equations for use in the Algebra classroom.
CHAPTER 3

STATEMENT OF PROJECT GOALS AND OBJECTIVES

As an instructional technology project, the goal of this project is to design and develop a computer assisted instructional material that can be used in Algebra instruction and support the following:

1) introduce the process of using inverse operation in the solving of single variable equations;

2) use as a manipulative tool to guide the student in their practice; and

3) to ensure that the students learn from their mistakes and receive the correct reinforcement during the discovery process.

The instructional objective of this project is for students to demonstrate the ability to solve one and two step single variable equations. In terms of sub-skills, the students should be able to:

a) identify the inverse operation that will solve the equation.

b) define the number term that will solve the equation.

c) analyze the resulting equation and determine if it is the solution to the equation.

d) initialize on-line help and feedback in discovering the strategy for solving single variable equations.
WHAT IS EXPLORING SINGLE VARIABLE EQUATIONS?

The rules for solving equations can be broken down into four basic steps: clearing parentheses and fractions; combining like terms; moving variable terms to one side of the equation, numbers terms to the other side; and clearing a coefficient. Exploring Single Variable Equations (ESVE) is intended as an introduction to solving equations. The stack was designed to focus on the basic 1-step and 2-step single variable equations which correspond to the last two steps in the equation solving process. If the students understand these foundation steps, the latter solution of more complex equations will be easier to grasp.

ESVE is a collection of HyperCard stacks students can use to discover, and do practice on, the steps needed to solve simple 1 and 2 step equations in Algebra. A 1-step equation is an equation that has 1 operation being applied to the equation, such as addition, subtraction, multiplication, or division. The 2-step equations has a combination of operations being applied to the variable, such as multiplication and addition, or division and subtraction. The project, ESVE, consists of 6 parts: Main Menu; with sub-
menus for the 1-step equations namely: Type one 1-step, Type two 1-step, Type three 1-step; and, the two types of 2-step equations which are: Type one 2-step, and Type two 2-step. The equation type, equation example, description of each type equations, and stack name can be seen in Figure 1.

<table>
<thead>
<tr>
<th>EQUATION TYPE</th>
<th>EXAMPLE</th>
<th>DESCRIPTION</th>
<th>STACK NAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type one 1-step</td>
<td>( x + 5 = 34 )</td>
<td>The equation in this stack have the operation of addition or subtraction being applied to the variable.</td>
<td>Type 1</td>
</tr>
<tr>
<td></td>
<td>( x - 5 = 34 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type two 1-step</td>
<td>( 5x = 34 )</td>
<td>This type equation has a positive or negative coefficient attached to the variable.</td>
<td>Type 2</td>
</tr>
<tr>
<td>Type three 1-step</td>
<td>( x/5 = 34 )</td>
<td>Here is a variation of the type two 1-step equation where the operation is now division instead of multiplication.</td>
<td>Type 3</td>
</tr>
<tr>
<td>Type one 2-step</td>
<td>( 5x + 5 = 34 )</td>
<td>The first of the 2-step equations is a combination of Types one and two 1-step equations.</td>
<td>Type 4</td>
</tr>
<tr>
<td>Type two 2-step</td>
<td>( x/5 - 5 = 34 )</td>
<td>The second of the 2-step equations is a combination of Types one and three 1-step equations.</td>
<td>Type 5</td>
</tr>
</tbody>
</table>

Figure 1. Stack names and descriptions.

The Main Menu stack (see Figure 2) is the launching point for the project. It is from here that the student selects what type of equation they would like to practice solving. Each equation stack is tied directly to the menu item selected and which brought the student to his/her current location in the stack. Through the scripting in the MENU button found in the lower right corner of every card,
the student can get back to the 1-STEP menu from any of the
of the three 1-step equations, or to the 2-STEP menu from any
of the two 2-step equation.

HOW EXPLORING SINGLE VARIABLE EQUATIONS (ESVE) WORKS?

To begin a session of ESVE, the user would need to open
HyperCard Player from the hard drive of their Mac computer.
From there the user would choose Open Stack from the File
Menu located at the menu bar. The user needs to double click
on the Main Menu stack to start the session. Double clicking
calls up the program and reveals the projects' goals and
objective.

ESVE requires the user to click on the NEXT PAGE button
to get to the MAIN MENU screen from the opening screen. The
MAIN MENU is where the user chooses what type of equation to
practice solving. Theses choices are 1-STEP, 2-STEP, and
RANDOM. Users can choose the type of equation to practice
with, or allow the program to choose for them by clicking on
the RANDOM button (see Figure 2).

Clicking on the 1-STEP button will get the user to the
1-Step Menu where they can choose to see how the different
equations are solved (MORE INFORMATION) or click on the
PRACTICE button to solve an equation for themselves (see
Figure 3). Both 1-STEP and 2-STEP menus follow this design.
The menu shown in Figure 3 is that of the 1-STEP menu. The
Choose from the two buttons below to go to a 1 step equation or a 2 step equation. If you feel really ambitious, try the random button.

Solving 1-Step Single Variable Equations

There are 3 different 1-step equations. Click on the More Information buttons to see how that type equation is solved and then click on the Practice button to try that type for yourself.

1. Type 1 equation, \( x + 3 = 5 \).
2. Type 2 equation, \( 2x = 6 \).
3. Type 3 equation, \( x/3 = 9 \).
2-STEP menu is the same except, there are only two choices for 2-step equations.

Each of the MORE INFORMATION buttons is scripted to take the user to appropriate screens that show the user the steps needed to solve the given problem. Or, the user may choose to bypass the instruction screens and go directly to the equations by clicking on the PRACTICE button of the equation of their choice.

Each of the PRACTICE buttons takes the user to an equation of their choice. Through the random number generating capability of HyperCard, the numbers, coefficients, and operations (e.g., add or subtract) for that equation (if applicable) are generated. The random generation in a Type one 1-step equation generates the "+" or "-" sign in the equation, as well as a number between -10 and 10 for the left side of the equation and a number between -100 and 100 for the right side of the equation. In a Type two 1-step equation, the coefficient of the variable is between -10 to 10 and there is no "+" or "-". The Type three 1-step equation, is identical to the Type two 1-step equation, except the coefficient is now a denominator of a variable fraction. For 2-step equations, combinations of the three types of 1-step equations are used to generate the Type two 2-step practice problems.
In each stack, an equation-generating HyperCard card is the first card that shows on screen when PRACTICE is selected by the user. From this equations-generating card, the numbers and operations are created. Then the information is passed to the next card in each stack.

Each equation generated has 7 options the user may choose from: ADD, SUBTRACT, MULTIPLY, DIVIDE, SIMPLIFY, HELP, and MENU (see Figure 4).

![Figure 4. Type one 1-step equation (subtraction).](image)

After generating the equation, the user selects one of the 7 options. Option 1, the ADD button, adds the number the user inputs to each side of the equation. Once the user selects their option, the stack will hide all other option buttons simultaneously and three fields open (see Figure 5).
The top field asks what number is to be added to both sides of the equation. The bottom field instructs the user how that is accomplished. These two fields prompts the user on what needs to be done. The third field is located just above much do you want to add to both sides of the equation?

\[ x - 8 = 75 \]

Figure 5. Instruction fields for ADD option.

The option button that was selected, which in this case is ADD. The third field is where the user enters the number to be added to both sides. Since the RETURN key and the ENTER key in the HyperCard scripting language HyperTalk are different functions and do different things, the user must press the return key (not the key pad's Enter key) to continue.

The stack automatically places the cursor in the third field and waits for the user's input. Once the user enters
the number and presses return, the current card will show the previously hidden buttons before closing the current card. It then passes the equation and input data to the appropriate fields of the next card.

Using the card with the equation \( x - 8 = 75 \) as an example, the user would need to choose the ADD button, enter the value of 8, and then press return (see Figure 4). This takes the field information and transfers it to the appropriate fields in the next card, as well as putting the "+" sign in front of the 8 being added to both sides of the equation. This results in the equation \( x - 8 + 8 = 75 + 8 \). The card in Figure 5 shows a typical message the user would see when an operation button is clicked. The equation in Figure 6 is that of a Type one 1-step where a student chooses to add a number to both sides. In Figure 5, the choice is to add, Figure 6 shows the new equation based on that choice.

To solve the equation, the user must proceed by selecting the SIMPLIFY button. Any choice other than SIMPLIFY at this time would get an error message that says "You must finish the addition you have started before you can continue". The SIMPLIFY option checks to see if the number added will indeed solve the equation, and if it does, the SIMPLIFY button transfers the correct answer to the next appropriate card. In this case, the addition of 8 will result in the subtraction of 8, creating the solution of
\[ x = 83. \] If any other number aside from the correct answer is added, like \(-8\), then the SIMPLIFY option will still do

\[ x - 8 + 8 = 75 + 8 \]

Figure 6. Card showing addition on both sides of the equation.

the indicated addition and transfer the results to the appropriate card. The result however is \( x - 16 = 67 \) instead of the calculated value of the unknown variable. The new equation will require the user to do the same PRACTICE process again. At this point, it is expected that the user will realize that an error has been made, that is, the number chosen to solve the equation was not the correct answer. A later section describes the help that is given the user when an equation is not solved accurately.
Option 2, the SUBTRACT button, subtracts a number from each side of the equation. The SUBTRACT option works exactly the same way as the ADD option except that the user is required to input a number to undo the addition in the equation.

Option 3, the MULTIPLY button, also does the same thing as Option 1 (ADD) and Option 2 (SUBTRACT) except the messages will be specific to the MULTIPLY option. If the user chooses the MULTIPLY button, the entered number is multiplied on both sides of the equation as shown in Figure 7. Figure 7, a Type three 1-step equation, shows the user has multiplied both sides of the equation by -7 to clear the division.

![Figure 7](Image)

Figure 7. Type three 1-step equation after choosing the MULTIPLY option.
After an operation is applied as in Figure 7, the user must choose the SIMPLIFY option to find out if the entered number does indeed solve the equation.

Option 4, the DIVIDE button is used to solve the type of equation shown in Figure 8. Division is performed on both sides of the equation to clear the coefficient of 2. The user then clicks the SIMPLIFY button to finish the operation.

Every equation in the PRACTICE phase of 1-Step equations has the same design and follows the same set of rules: the user decides on an option (an inverse operation), enters a number, and then presses Return. The next 3 figures illustrate this progression of cards.

![Figure 8. Type two 1-step equation.](image)

50
Figure 9. Division messages.

Figure 10. Division to both sides of a Type two 1-step equation.
Option 5 is the SIMPLIFY button. As mentioned in the previous sections, SIMPLIFY calculates the results based on the number input from the user. This calculation is done by the scripting of HyperCard, using the authoring language HyperTalk. The scripting in the SIMPLIFY button starts by checking the operation on the right side of the equation. It then determines if the two numbers should be added or subtracted. SIMPLIFY will place a "-" before the number selected if subtraction was the choice of operation and a "+" is placed for addition. Figure 11 is an example of a user choosing to input 5 to solve the equation. After pressing the return key the screen, provides "+5" for the ADD option (A), a "-5" for the SUBTRACT option (S), the "5" and parentheses for the MULTIPLY option (M), and the fraction/division line and denominators of "5" for the DIVIDE option (D). Figure 11 also shows all of the possible outcomes of a Type one 1-step equation based on the user's choice of inverse operations (Options 1 to 4 and pressing return). As the flow chart indicates, the student can create a new Type one 1-step equation (option 1, ADD), solve the equation (option 2, SUBTRACT), create a Type one 2-step equation (option 3, MULTIPLY), or get an error message (option 4, DIVIDE), depending on the option selected.

If the SIMPLIFY button is chosen before an inverse operation is applied, an error message is displayed telling
the user that there is nothing to simplify at that point. The SIMPLIFY button is where all the complicated scripting is located. This scripting is responsible for the project's ability to tell if an entered number will indeed solve the equation or create a new equation. The scripting also takes care of preventing the occurrence of double signs. For example, if the user subtracted 10 instead of 5 in Figure 11, the result could have been displayed as \( x + -5 = 24 \), this could be very confusing to the user. For an example of this scripting, see Appendix 1.

Option 6 is the HELP button. This button is specific to each card in every stack, and the instructions and hints are dependent on the card from which it was used. The HELP button will display a large field that covers all other buttons.
The user cannot continue solving the equation without seeing and reading the message. The message field closes when the HELP button is pressed a second time. See Figure 12 for an example of a HELP message.

Option 7, the MENU button transfers the user back to the menu from which they came. All 1-step equations will return the user to the 1-step Menu and the 2-step equations will return the user to the 2-step Menu.

![Type 3 Help Screen](image)

Figure 12. Help screen for Type three 1-step equation.

SCREEN DESIGN

The screen design of Exploring Single Variable Equations focuses the students' attention at the information being presented on that card. The information is at the center of the screen in large print so that it is easy to read. If the
card is an information card, then the information is labeled in steps and is separated by blank lines. If the students attention is to be on the equation at hand, then the equation is again at the center of the screen in large print. When a student chooses an operation, the text fields never obscure the equation. The text fields are either above the equation or below the operation buttons. When there is an error message presented to the student, the message does cover the equation so that the student must read that message to be able to continue.

The screen layout has the operation buttons in the same location on all cards. This is to keep the design of the stacks consistent. The ADD button is always first and the SIMPLIFY button last. The order of the buttons is always ADD, SUBTRACT, MULTIPLY, DIVIDE, AND SIMPLIFY. For users who need help, the HELP button is always in the upper right corner of the card. The HELP statements are card specific, because the help that users will require differ for each type of equation. This information was designed to help them analyze their error, help them determine what happened, or what to do next. All of the information and messages are written not to help the student memorize the steps, but to analyze, and apply the process of solving equation. They are not written to tell the student what to do each step of the way. It is the intent, that if the student needs help or
guidance, that they can click on the appropriate button and receive the support they are seeking in the process of solving of equations. This feedback is supported by Dinkheller (1989) in his description of Tutorial programs. Also, keeping the screen design simple enhances the benefits of this instructional tool by focusing on the topic of solving equations.

INSTRUCTIONAL DESIGN

Learning to solve equations is just one of the many topics students need to master in a beginning Algebra course. The textbooks and supplementary materials available to teachers present the topic to students using similar methods. They begin by showing how a 1-step equation involving addition or subtraction is solved, proceeding, usually at a faster pace, to more difficult combinations of operations. The computer software available to Algebra teachers either concentrate on topics students need prior to single variable equations or develop the students skills in analyzing linear equations (usually 2 variables equations), but few concentrate on the basics of solving single variable 1 and 2 step equations. The development of ESVE can be used to fill this gap. The instructional design has but one objective, help students learn the relationship between inverse operations in the solving of basic 1 and 2 step single
variable equations. The goals that guided the instructional design of ESVE are as follows: (1) keep the equations simple and basic by having the variable on the same side of the equation and not allowing the appearance of double sign; (2) instructions were worded to simulate the way a teacher might help a student by giving feedback, and providing (error) messages in a short, clear, and concise manner; (3) students are allowed to choose any operation they think will work and show them the results of their choices, right or wrong; and (4) a platform that has an endless supply of equations and patience was provided.

The first exposure that students get in an Algebra textbook have them solve Type one 1-Step equations of the form $x + 4 = 7$. The variable is on the left with a number being added or subtracted from it and a simple result. Similarly, this project is only interested in the very basic types of equations. That means the variable is always on the left, an operation is being applied to it, and there is a result. The project is intended as a platform for the last two steps in the solving process. It is not the goal to teach them how to solve any given equation in any given form.

To suit the level of students in a first year Algebra class, the project has information cards (screens) that show the student that there is a set of decisions they need to make to be able to proceed. Well designed CAI tutorials can
be used as a stand alone teaching tool, therefore the instructional design of this project had to include these screens so that students could read the instructions, and see an example to determine how the equations could be solved. To solve the given equation, students must decide what operation is being applied to the variable, choose the correct inverse operation and apply it to the equation, and then simplify what they have done. The project was also designed to show students how an equation solution is accomplished by providing an example. This aspect of the project supports good teaching practice (NCTM, 1991) that says modeling a process must accompany any instruction if students are to know what is expected of them.

Each of the 1-step equations have these instruction and example cards as part of the MAIN MENU's MORE INFORMATION cards. The 2-step information cards show them how these decisions are used to solve the 2-step equations in only one screen. These instruction cards can be reached by clicking the MORE INFORMATION buttons of each type equation found on the 1-STEP and 2-STEP MENUS. These instructions are duplicated in the HELP prompts found in the equation solving stacks. The difference is that the HELP prompts guide the student in a particular direction rather that telling them what must be done to solve the equation.
This process of deciding what operation is being applied to an equation, applying the inverse operation to the problem, and simplifying, is done in such a way that the student can get a number of problems incorrect and nobody is the wiser. They simply select MENU and get another equation and try again. If they mess that one up, they select MENU again and continue. This means that the student will have all the opportunity they need to keep trying until they figure it out (guided discovery process). Furthermore, only the student knows how long it took them to learn the process which helps the students gain confidence in their math abilities. However, the teacher will need to monitor this process to make sure the student is progressing at an appropriate rate and is being successful.

Part of being successful with the equation solving process is seeing what happens to an equation when an operation is applied to both sides. Student have difficulty determining what operation is to be used to solve a given problem so they simply apply whatever operation they think will work. This program will accommodate that thinking. It will correctly apply any of the 4 operations as commanded. This way the students will see that if they choose the wrong operation at a given point, the result of that choice is an error message letting them know that the selected operation is not possible at this point. Once the student gets
proficient, they could start playing a "What If..." game and experiment with different methods of solving equations. The flow chart in Figure 11 is just one of the possible lists of choices a student can make. The student's choice to use addition to solve a Type three 1-step equation creates a Type two 2-step equation. This creation of different type equations can happen in all the stacks. Each was designed to deal with any possible choice the student could make, with the intent, that seeing the result of a choice will help them determine that there is only one inverse operation that will solve the given equation.

Instructional design guided the screen design to accomplish the single objective of teaching students to solve basic 1 and 2 step equations. This single objective design is to support the idea of students gaining automaticity in the last two steps of the equation solving process mentioned earlier. The material was kept at a level to ensure success. Guidance and feedback were worded to encourage and steer the student towards the solution. The environment was kept non-threatening to encourage the student to experiment, explore, and play "What If...". But most importantly, the screen, instructional design, and equation-generating cards were designed so that the student would have a never ending supply of equations to practice with and see what happens to an equation when they use an operation to solve it.
FORMATIVE EVALUATION

Evaluation Procedures

This software was evaluated by 2 individual Algebra students in the ninth grade, Algebra classes of 25 or more students, and two Algebra teachers. The purpose of the first evaluation, one class of 25 Algebra students, was to determine if there were any areas that needed to be ironed out, such as, wording on instruction cards, and if the student could navigate the program with little or no teacher intervention. The second round of evaluation by the individuals and two classes of Algebra students was to determine if the program accomplished what it was designed to do, and to tell the author how the program could be improved. Evaluations gave general comments on how the program could be improved, and comments about the programs weaknesses and strengths. The students did this anonymously and their comments are summarized below. The students and teachers used a classroom Macintosh IIIsi computers with 13-inch RGB monitors. Group evaluations were done in the school's computer lab with 27 Macintosh IIIsi computers with 13-inch RGB monitors.

Results of Teacher Evaluation

The two teachers that evaluated the original project were instrumental in pointing out areas that helped make the program as "kid proof" as possible. The original program
allowed the user to select an operation, then select another
operation, and continue to the next card from the second
operation. Leaving the card from the second operation left
the first operations messages fields open. These messages
were still open when the screen was used the next time. That
problem was solved by hiding or covering up the other buttons
so that the student could not click on a second button and
leave the message fields open when they left that screen.
The objects hidden were then un-hidden either by having the
user click on the button a second time or by pressing return
after entering a number in a field.

The teachers had some esthetic comments that were
addressed as well. The screen design for the menus and more
information screens needed to be added or changed to
accommodate the user's ability to read the information
without a distracting background. Also, the teachers
suggested that a "cute" graphic be found to liven up the
screen, hence, the stork at the computer in the upper left
corner.

Results of First Student Evaluation

The first thing that was noticed after the class got
started was that the students did not read, or could not
understand, the MORE INFORMATION cards to determine how the
equations were to be solved. This resulted in them doing
much more guessing at what number was needed to solve the
equation. The numbers used in the repeated guessing got to be very large and the programs input fields could not handle these large numbers. Therefore, the students became more and more confused on how the equation was to be solved. The students did not understand what happened to the equation they began with when they applied an operation. Another difficulty was that the students entered numbers in the correct fields and sometimes pressed the ENTER key rather than the RETURN key.

Revisions Made After First Evaluations

After this session in the computer lab, the program went through a minor re-scripting to address the areas of difficulty mentioned by the students and teachers. The MORE INFORMATION cards were rewritten to make the instruction more step-by-step rather that narrative in style. The 1-Step instructions were divided into two cards where the first card held the step-by-step instruction and the second card held the step-by-step example of the solution process, following the sequence of instructions mentioned on the previous card.

The second area of concern was the students entering numbers too large for the program to handle. For example, using \( x - 2 = 98 \), students would try to enter the answer of 100 into the field to "solve" the equation giving them the new equation of \( x - 98 = 198 \). After the second attempt, the number on the left and right would be too large for the size
of the field programmed for them. Therefore, the length of the numbers used in the solution process were restricted by the HyperTalk Length Function that measures the number of characters in a field. If the student tries to use a number that is too large, the program beeps and gives an error message letting them know the number is too big and give them a hint at to what to do next.

Once the problem of number size was fixed, the problem of the ENTER key and RETURN key was addressed. The ENTER key in the scripting language HyperTalk, passes the cursor to the next field in the cards' hierarchy rather than process the scripting associated with the field. Therefore, the ENTER key was disabled and it will do nothing if the student uses it. This forces the students to use the RETURN key as they were instructed to do.

Finally, the issue of on-screen help was addressed. The students were confused as to what happened to the original equation after they applied an operation, or they would say that after they applied an operation and pressed return, a new equation appeared. This was dealt with by adding the HELP button in the upper right corner of the screens. The HELP field has information that helps the student understand what happened to the original equation and offers a hint on where to go from there. The help field covers everything on the screen except the equation and the HELP button. This
will force the students to read the message to be able to continue as well as not allowing them to leave that screen with the help message still open.

**Results of Second Student Evaluation**

The most common comments about the program was that it was too easy and it needed color. The students felt that the program needed more difficult problems or it needed to be more challenging. Other than being too easy, the students felt the program needed some color to liven it up. A few suggested that the program was too easy for Algebra and might be better suited for Pre-Algebra students. Some students felt that the computer did too much of the problem and the student should do more and suggested removing the SIMPLIFY button so the student does the simplifying.

Other comments the student made in reference to how to make the program better are: the ENTER key should be available to use just like the RETURN key, to utilized the keyboard as well as the mouse to choose the operations, as in; A for ADDITION, S for SUBTRACTION, M for MULTIPLY, and D for DIVISION, and "=" for SIMPLIFY, add sound and/or music, fields are too small to handle 3 digit numbers, to add instructions at the beginning informing users what all the buttons on the equation screens are used for and what they do.
The students felt the directions were clear and easy to follow and shows how to solve the equation in a step-by-step manner. Another common theme in the comments was that the program does most of the problem for you but does not directly teach you what operations and numbers to use in solving equations. That help is available if you needed it was appreciated. This program was much better than a worksheets because they knew their answer was right when they got to the end of the equation.

**STRENGTHS AND LIMITATIONS OF THE PROJECT**

In the introduction, it was stated this project will be used: (1) to introduce the process of using inverse operation in the solving of single variable equation; (2) as a manipulative tool to guide the student in their practice; and (3) to ensue that the student learn from their mistakes and receive the correct reenforcement during the discovery process.

As an introduction to solving single variable equations, the program, according to students comments, serves as an appropriate tool to learn the relationship between the operations and their inverses. It also helps the students determine which numbers they need to use when applying that inverse operations. The only drawback to the program is that it is too easy for most students in Algebra and may be more suited for those in courses prior to Algebra. The form the
equations take on screen could be the reason for this. The variable is always in the same place and the equations do not vary enough from problem to problem.

Once the student figured out the process it became clear they preferred this platform more than the traditional methods of worksheets or problem sets from the textbook. They found the problems from the first types of equations to be of benefit as they continued to the next type of equations. This helps achieve the project goal of providing an instructional computer-based tool that will develop skills, from basic ones that will lead to more complex solutions. They did, however, find the restriction of using only the mouse a bit cumbersome and recommended the use of the keyboard to help speed up the process.

The rewritten instructions in the MORE INFORMATION cards turned out to be worded in such a way that they were able to continue on and learn the process needed to solve single variable 1-Step and 2-Step equations. The HELP button and subsequent help messages also were of benefit to the students. They were able to get the help and feedback they needed to continue. Restricting the size of the numbers the students used in the adding and subtracting equations also helped the students realize that entering what they felt was the solution was not the correct way to solve the equation. This supports the project goal to support guided discovery
learning in math.

Overall the program accomplishes the objective of introducing the concept of solving one and two step single variable equations. It may not be suited for high school students because they felt it was much too easy. It might be better suited for middle school student or as a remedial tool for some high school students. Future programs will have this in mind and will provide equations that advanced students with higher abilities can use. It is also recommended that future programs be designed using the colorized version of HyperCard or an authoring tool that sill provide both the scripting power of HyperCard and the color and graphics that appeal to users.
On mouseUp
  PUT LINE 1 OF CARD FIELD ID 17 INTO X
  PUT LINE 1 OF CARD FIELD ID 19 INTO Y
  PUT LINE 1 OF CARD FIELD ID 3 INTO Z
  IF ITEM 1 OF CARD FIELD id 36 IS "+" and ITEM 1 OF CD FIELD id 26 IS "+" then
    IF X=Y THEN
      PUT Z-Y INTO LINE 1 OF CARD FIELD 3 OF CARD e
      visual effect dissolve slow
      go TO CARD e
    ELSE IF X<Y THEN
      PUT "+" INTO LINE ONE OF CD FIELD id 18 OF CD a1
      PUT Y-X INTO CD FIELD ID 17 OF CD a1
      PUT Z-Y INTO CD FIELD ID 3 OF CD a1
      visual effect dissolve slow
      GO TO CARD a1
    ELSE IF X>Y AND Y<0 THEN
      PUT "+" INTO LINE ONE OF CD FIELD id 18 OF CD a1
      PUT X-Y INTO LINE 1 OF CARD FIELD ID 17 OF CARD a1
      PUT Z-Y INTO LINE 1 OF CARD FIELD ID 3 OF CARD a1
      visual effect dissolve slow
      GO TO CARD a1
    ELSE IF X>Y AND -Y<X THEN
      PUT "+" INTO LINE ONE OF CD FIELD id 18 OF CD a1
      PUT X-Y INTO LINE 1 OF CARD FIELD ID 17 OF CARD a1
      PUT Z-Y INTO LINE 1 OF CARD FIELD ID 3 OF CARD a1
      visual effect dissolve slow
      GO TO CARD a1
    END IF
  ELSE
    IF ITEM 1 OF CARD FIELD id 36 IS "+" AND ITEM 1 OF CD FIELD id 26 IS "+" THEN
      IF X=Y THEN
        PUT Z+Y INTO LINE 1 OF CARD FIELD 3 OF CARD e
        visual effect dissolve slow
        go TO CARD e
      ELSE IF X<Y THEN
        PUT "+" INTO LINE ONE OF CD FIELD id 18 OF CD a1
        PUT Y-X INTO CD FIELD ID 17 OF CD a1
      END IF
  END IF
PUT Z+Y INTO CD FIELD ID 3 OF CD a1
visual effect dissolve slow
go TO CARD a1
ELSE IF X>Y THEN
PUT "." INTO LINE ONE OF CD FIELD id 18 OF CD a1
PUT X-Y INTO LINE 1 OF CARD FIELD ID 17 OF CARD a1
PUT Z+Y INTO LINE 1 OF CARD FIELD ID 3 OF CARD a1
visual effect dissolve slow
GO TO CARD a1
ELSE IF X>Y AND Y<0 THEN
PUT "." INTO LINE ONE OF CD FIELD id 18 OF CD a1
PUT X-Y INTO LINE 1 OF CARD FIELD ID 17 OF CARD a1
PUT Z+Y INTO LINE 1 OF CARD FIELD ID 3 OF CARD a1
visual effect dissolve slow
GO TO CARD a1
ENDIF
ELSE
IF ITEM 1 OF CARD FIELD id 36 IS "+" AND ITEM 1 OF CD FIELD id 26 IS "+" then
IF X=-Y THEN
PUT Z+Y INTO LINE 1 OF CARD FIELD ID 3 OF CARD e
visual effect dissolve slow
go TO CARD e
ELSE IF Y>0 THEN
PUT "+" INTO LINE ONE OF CD FIELD id 18 OF CD a1
PUT Z+Y INTO CD FIELD ID 17 OF CD a1
PUT Z+Y INTO CD FIELD ID 3 OF CD a1
visual effect dissolve slow
go TO CARD a1
ELSE IF Y<0 AND (X+Y)<0 THEN
PUT "." INTO LINE ONE OF CD FIELD id 18 OF CD a1
PUT ABS(X+Y) INTO LINE 1 OF CARD FIELD ID 17 OF CARD a1
PUT Z+Y INTO LINE 1 OF CARD FIELD ID 3 OF CARD a1
visual effect dissolve slow
GO TO CARD a1
ELSE IF Y<0 AND (X+Y)>0 THEN
PUT "+" INTO LINE ONE OF CD FIELD id 18 OF CD a1
PUT Z+Y INTO LINE 1 OF CARD FIELD ID 17 OF CARD a1
PUT Z+Y INTO LINE 1 OF CARD FIELD ID 3 OF CARD a1
visual effect dissolve slow
GO TO CARD a1
ENDIF
ELSE
IF ITEM 1 OF CARD FIELD id 36 IS "." AND ITEM 1 OF CD FIELD id 26 IS "." THEN
IF X=-Y THEN
PUT Z-Y INTO LINE 1 OF CARD FIELD 3 OF CARD e
visual effect dissolve slow
go TO CARD e

ELSE IF X<Y THEN
  PUT "-" INTO LINE ONE OF CD FIELD id 18 OF CD a1
  PUT X+Y INTO CD FIELD ID 17 OF CD a1
  PUT Z-Y INTO CD FIELD ID 3 OF CD a1
  visual effect dissolve slow
  go TO CARD a1
ELSE IF X=Y THEN
  PUT "-" INTO LINE ONE OF CD FIELD id 18 OF CD a1
  PUT X+Y INTO LINE 1 OF CARD FIELD ID 17 OF CARD a1
  PUT Z-Y INTO LINE 1 OF CARD FIELD ID 3 OF CARD a1
  visual effect dissolve slow
  GO TO CARD a1
ELSE IF Y<0 AND (X+Y)<0 THEN
  PUT "+" INTO LINE ONE OF CD FIELD id 18 OF CD a1
  PUT abs(x+y) INTO LINE 1 OF CARD FIELD ID 17 OF CARD a1
  PUT Z-Y INTO LINE 1 OF CARD FIELD ID 3 OF CARD a1
  visual effect dissolve slow
  GO TO CARD a1
ELSE IF X>Y AND Y>0 THEN
  PUT "-" INTO LINE ONE OF CD FIELD id 18 OF CD a1
  PUT X+Y INTO LINE 1 OF CARD FIELD ID 17 OF CARD a1
  PUT Z-Y INTO LINE 1 OF CARD FIELD ID 3 OF CARD a1
  visual effect dissolve slow
  GO TO CARD a1
END IF
END IF
END IF
END IF
END IF
END IF
end mouseUp
REFERENCES


