How to improve students' problem solving skills: K-4

Chuong Hoang Pham
HOW TO IMPROVE STUDENTS' PROBLEM SOLVING SKILLS

K-4

A Project
Presented to the
Faculty of
California State University,
San Bernardino

In Partial Fulfillment
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Master of Arts
in
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by
Chuong Hoang Pham
December 1994
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Ruth Sandlin, Second Reader
Problem solving has been neglected by many elementary teachers because it was difficult to teach to children who were used to depending on teachers' clues and not their own reasoning. The writer of this project felt that problem solving teaching should be given a more prominent place in elementary mathematics education.

The purpose of this project was to examine the traditional way of teaching problem solving in elementary schools in the 1970s and the 1980s and to propose, perhaps, a new teaching way composed of a 4-step plan and effective problem solving strategies supported by current literature in the field.

Many contemporary mathematics educators agreed that problem solving was not an application of addition, subtraction, multiplication and division facts. It required understanding of the content and reasoning skills. It required planning and strategies. In addition, verifying solutions, verbalizing reasoning and creating new word problems should be what the student is expected to be able to do.

This writer also produced a small booklet which was of benefit to himself and
other elementary teachers who found teaching problem solving challenging and frustrating. The problem solving strategies and guidelines included in the handbook were essentially necessary to both teachers and students. The sample problems, the point system, the divided-page format and other documents in the handbook were also very helpful to teachers in making their students feel more comfortable with problem solving if not become successful problem solvers.
ACKNOWLEDGEMENTS

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INTRODUCTION

Since the late nineteenth century, business and commerce had already increased to the point where mathematics was considered important for everyone. However, in our rapidly changing world, the memorization of facts, rules and procedures are not enough. Business, industry and government increasingly need workers capable of using the power of mathematics to solve new problems (Lappan & Schram, 1989). Carl (1989), President of the National Council of Supervisors of Mathematics, argues that students must understand mathematical principles and develop proficiency in problem solving and higher order thinking because skill in whole number computation is not an adequate indicator of mathematical achievement. Baroody (1993), in his book titled Problem solving, reasoning, and communicating, also maintains that what is really needed are people who can solve problems, not computers. Computers are designed to take care of computation only. What is needed are people who can analyze and think logically about new situations, devise unspecified solution procedures, and communicate their solution clearly and convincingly to others. Krulik and Rudnick (1988) also emphasize the importance of problem solving. They believe that problem-solving is exactly the basic skill of mathematics education and the primary reason for teaching mathematics.

Actually, problem solving has been promoted by mathematics educators as a goal in mathematics education since the late 1970's with focused attention
arising from the National Council of Teachers of Mathematics (NCTM)'s *An Agenda for action* (1980). Study after study continues to cite the area of problem solving as the number-one concern in the classrooms of America (Talton, 1988). However, according to Ford (1990), many students dislike word problems in math and many teachers report feeling frustration and discouragement in helping their students learn how to solve such problems. Talton (1988) states that some students who can perform arithmetical computation quite well often have difficulty solving word problems that require the application of those same computational skills. Why is this? What is so difficult about solving word problems? The answer may lie in the discrepancy that exists between the skills commonly taught as problem-solving skills and the critical thinking skills actually needed to solve word problems.

Traditional mathematics teaching has encouraged children to be dependent rule-followers rather than independent thinkers. According to Mahlios (1988), the researchers found that primary grade children chose different strategies to solve addition and subtraction story problems on the basis of the structure of the problems. Huinker (1989) argues that students find it difficult to solve word problems because they are unsure of how to think about deciding what operation to use. Many students resort to guessing or using other inappropriate strategies, such as key words or number size, as they attempt to solve word problems. For example, if the problem says "together", the solution involves addition. If the problem says "less" or "left
over", subtraction must be used. Teachers also taught children to guess the required operation from the size of the numbers. If the problem involves 2 large numbers, you can add or subtract. If one of the numbers is too small, you then multiply or divide. Such "tips" are not really helpful when children are faced with non-routine or more complex problems. Billstein (1981), on the other hand, believes that many students and teachers dislike doing problems mainly because they have not patience in solving problems. Not seeing an immediate solution to a problem, they often give up, convinced that the problem is beyond their capabilities. Problem solving, in many cases, does take time.

For the above mentioned reasons, the curriculum reforms proposed by the National Council of Teachers of Mathematics in Curriculum and Evaluation Standards for School Mathematics (1989) called for a reorientation of the school mathematics curriculum so that a greater emphasis is placed on helping students become mathematical problem solvers. According to the 1992 Mathematics Framework for California Public Schools, instructions and methods used in mathematics education must reflect the shift in content to avoid "spoon-feeding" students recipes for getting answers. Instead, students must become more responsible for formulating, solving problems, thinking about and communicating important mathematical ideas.

Hyde (1991) maintains that problem solving is a way of thinking and doing mathematics and not a set of problems at the end of a chapter. Problem solving activities introduce students to new mathematical ideas, provide
exciting experiences that develop deeper understanding, and also help students apply what they understand to their lives. Wisner (1977) points out that the development of problem solving strategies is a necessary ingredient of elementary mathematics. According to him, strategies are many and varied, therefore the teacher must present the skill or idea in a setting that children can enjoy. Kilpatrick (1985), on the other hand, emphasizes the fact that problem solving instruction should focus on communicating. He argues that mathematics is for children a second language and therefore, they need to practice talking mathematics everyday. Kilpatrick advocates using cooperative learning to teach problem solving, because “discussing problems in a small group of peers can help children analyze and solve problems thoughtfully” (Kilpatrick, 1985). Baroody (1993) also highlights the importance of understanding, reasoning, and communicating in problem solving. He argues that children need to do mathematics to develop mathematical thinking and the autonomy to solve challenging mathematical problems. Doing mathematics, according to Baroody, does not mean doing rows and rows of computational “problems”. It entails:

* solving challenging problems;
* exploring patterns;
* formulating educated guesses and checking them out;
* drawing conclusions;

* communicating ideas, patterns, conjectures, conclusions, and reasons.
As for teachers, they must not only be knowledgeable about and feel comfortable "doing" mathematics, but also have to have an understanding of the learner and of how to motivate and guide them (Charles and Silver, 1988). Baroody (1993) also recommends that they should develop their own problem-solving skills so that they are better able to guide the development of their students' problem-solving skills and to appreciate the difficulties children may have in their problem-solving efforts as well as the value of problem-solving strategies they use.

To improve elementary school teachers' problem-solving skills and put them in a position to help students develop theirs, this project will explore general problem solving strategies proposed by well-known problem-solving experts. Teachers will be presented with basic problem-solving strategies that have been found to be effective in solving a variety of elementary school story problems.

It is the goal of this project to develop a handbook for elementary teachers to implement some of the strategies of problem solving that have been proven to be effective and interesting. The handbook can also help elementary students learn many strategies to solve non-routine word problems.

It is hoped that this handbook will serve both teachers and their students and that it will inspire teachers to explore the joys of teaching math to their students. It is also hoped that students will gain the benefits of improving their mathematical skills by gaining a positive attitude towards mathematics.
An universal belief system is that mathematics is important. It is often the key to many opportunities. The National Research Council (1989) confirms that mathematics is no longer just the language of science, it contributes now in direct and fundamental ways to business, finance, health, and defense. For students, it can open doors to careers. For citizens, it can enable informed decisions. For nations, it can provide knowledge to compete in technological economy. (National Research Council, 1989). Jack Price (1981), current president of NCTM, believes that most careers requiring postsecondary education, whether in college or technical institutions, demand some mathematical background and that mathematics is the principal problem-solving tool for today's society.

However, in our post-industrial, information-based, and technological-oriented age, mathematical literacy requires more than mastery of basic mathematical skills. Business, industry, and government need workers capable of using the power of mathematics to solve new problems (Lappan & Schram, 1989). Krulik and Rudnick (1988) argue that problem solving is the basic skill of mathematics education. It is the primary reason for teaching mathematics. Fennel & Ammon (1985) state that the decade of problem solving is upon educators, because the recommendation provided by the 1978 National Assessment of Educational Progress (NAEP), NCTM, in its An Agenda for Action, and the Priorities in School Mathematics (PRISM) project, all
point toward greater emphasis on Problem Solving in elementary school mathematics. According to Baroody (1993), instruction today needs to focus on the development of mathematical thinking since primary school level. Young children need to be taught how to construct their own understanding of mathematics as well as the cultivation of critical thinking, reasoning, and problem solving skills. The Curriculum and Evaluation Standards for School Mathematics, issued by the National Council of Teachers of Mathematics (NTCM) in March 1989, says "the study of mathematics, for K-4, should emphasize problem solving so that students can:

* Use problem-solving approaches to investigate and understand mathematical content;

* Formulate problems from everyday and mathematical situations;

* Develop and apply strategies to solve a wide variety of problems;

* Verify and Interpret results with respect to the original problem;

* Acquire confidence in using mathematics meaningfully." (p.23)

To examine the evolution of problem solving instruction and its importance today, this study will first examine the nature of problem solving by defining problem-solving. Second, the literature review will explain why traditional instructional practices have failed miserably to interest students in problem solving or develop their potential for problem solving. Third, the strategies proposed by this project will help teachers develop their students' problem solving skills and prove that teaching problem-solving is not, as many teachers think, an overwhelmingly difficult task that, on the contrary, can be
exciting for teachers as well as students.

First of all, the term “problem-solving" should be appropriately defined and clarified because working on a word problem may or may not involve problem solving. Kennedy and Tipps (1991), authors of the textbook titled *Guiding children's learning of Mathematics*, state that a new notion of what problem solving is has evolved in recent years. In the past, “problem-solving” was just a topic, like addition, geometry, or measurement. It was studied following another topic. Now, problem solving is viewed as mathematics. Mathematics is, to Kennedy and Tipps (1991), not merely a collection of concepts and facts to be learned and then applied to the solution of problems. It is problem-solving. Baroody (1993), a professor at University of Illinois at Urbana-Champaign, argues that word problems in which a child can easily identify and apply a solution strategy are more like routine practice exercises. Real problems require a thoughtful analysis and an extended effort. Problems are not puzzled either. Some people may have no interest in solving puzzles. Because of this, Charles and Lester (1982) found it useful to distinguish between puzzles that do and do not motivate interest and action. They state that a problem entails (a) a desire to know something, (b) the lack of an obvious way to find a solution, and (c) an effort to find the solution.

Baroody (1993), in his *Problem solving, reasoning, and communicating*, painstakingly clarifies a variety of types and uses of word problems. He suggests teachers to use and create nonroutine types of problems rather than routine types, because they require a thoughtful analysis. Following are 7
main types of problems teachers should use:

1. **One-step translation problems**: Routine word problems are called one-step translation problems because (a) they involve a single operation and (b) they can be solved directly by translating the wording into a concrete model or a number sentence. Such problems are useful for introducing the arithmetic operations with whole numbers, fractions, and decimals.

2. **Multistep translation problems**: This type of nonroutine problem entails the application of 2 or more arithmetic operations. These operations, such as the following problem, can require a thoughtful analysis of the unknown, the data, and the solution method.

Ex: “Mary bought 6 ice-cream cones for $1.80 and 6 doughnuts for $2.00. If she used a $5.00 bill to pay, how much change should she get?”

3. **Other modifications of translation problems**: One-step translation problems can be modified in other ways to require a more thoughtful analysis.

   a. Problems that require finding a missing addend or factor. Consider, for example, this problem:

   “Mary distributed 12 candies to her sisters. Each of them got 4 candies. How many sisters did Mary have?”

   b. Problems that require applying the answer.

   “Mary set out a dozen doughnuts for the party. Ruth ate 4. Does Mary have enough left to serve 9 people?”

   c. Problems with too much, too little, or incorrect data.

   “Mary set out a dozen doughnuts for her party of 6 children. Ruffus
consumed 8. How many doughnuts did Mary have left?

d. Problems that can be solved in more than one way.

"Mary decided to buy something for her dog. She had $12. A muzzle cost $7 and a chain cost $4. Does she have enough to buy both?

7+4=11 and 11 is less than 12.

Or 12-7=5 and 5-4=1

e. Problems with more than one answer.

"Mary buys some doughnuts. Sugar-coated ones cost 20c each. Cream-filled ones cost 30c each. Cream-filled ones with icing cost 40c each. Mary has 100c. What can she buy?

4. Process problems: Nonroutine problems that require the use of general problem solving processes (strategies) are called process problems.

5. Puzzle problems: Nonroutine problems that involve a trick and require special insight, or luck are called puzzle problems. Such problems can serve as a source of entertainment for those who feel challenged rather than frustrated by them.

6. Applied problems: Nonroutine problems that stem from the real world or that are realistic are called applied problems. Consider the problem below:

"What portion of the school’s waste could be recycled rather than hauled to the landfill?

Such a problem serves as an opportunity to practice general problem solving strategies and also illustrates how mathematics can be integrated in
other subjects (e.g., social studies, science) and a tool for solving "real" problems.

7. **Strategy problems:** Unlike problems that ask students for an answer or answers, strategy problems ask students for a solution strategy or strategies. For example:

   “In a parade, 21 bicycles and tricycles passed in review. Jane counted a total of 51 wheels. How many different ways could you find out how many bicycles were in the parade?

Schroeder and Lester (1989), in "New directions for elementary school Mathematics: 1989 NCTM yearbook", state that there are 3 approaches to deal with problem solving: teaching VIA problem solving, ABOUT problem solving, and FOR problem solving. The first approach is to use problem solving as a means for teaching subject-matter content such as basic computational skills. The second approach is to teach general problem solving strategies. The third approach focuses on teaching strategies by giving children the opportunity to solve problems. In practice, teaching via, about, and for problem solving often overlap. And that is the so-called **integrated approach:** to combine content instruction about problem solving strategies and other subject matters with process-oriented instruction that focuses on how to solve genuine problems.

Actually, problems have a long history in the mathematics curriculum. Charles and Silver (1988) reveal that within the last century, discussions of the teaching of problem solving have moved from advocating that students
simply be presented with problems or with rules for solving particular problems to developing more general approaches to problem solving. However, according to Talton (1988), even in the 1960s and the 1970s, many textbook authors suggested that students do the following to solve word problems:

1. Read the problem.
2. Determine what is asked in the problem.
3. Determine what facts are given.
4. Choose the operation.
5. Solve the problem.

Talton (1988) believes that many students find the fourth step most difficult. To help them with this step, many teachers tended to have them focus on key words in a problem to decide what operation to choose. In his "Children's arithmetics", Ginsburg (1989) states that the main problem-solving strategy children learned from teachers is limited to key-word training. He wrote, "For example, if the problem says "altogether", the solution involves addition. If the problem says "less" or "left", use subtraction. Children also even guess the required operation from the size of the numbers. If the problem involves 2 large numbers, they add or subtract them. If one of the numbers is small, they multiply or divide."

Ginsburg (1989) argues that while tricks like those produce a certain amount of success, they do not involve real understanding and also do not always work. Baroody (1993) believes that this skill approach not only
undermines children's interest to solve problems and gives children an inaccurate notion of real problem solving, it also "encourages passivity and reliance on external authorities such as a teacher or textbook." He argues that, without training, many U.S. students know how to use their everyday knowledge and common sense to solve simple routine word problems in textbooks. Routine word problems are a common feature of traditional instruction, and solving such problems is not a good indication of problem solving skill, because when dealing with nonroutine word problems that include unneeded information and require some analysis or thinking, most children failed.

Charles and Silver (1988) also state that early in the learning of arithmetic, children detect the importance of obeying rules, and as they want to please the adults, who, it seems, make the rules for arithmetic, they watch for affirmative nods from their teachers. Teachers, trying to be helpful, narrow the field of possible rules: shall we add or subtract? Multiply or divide?

After the broad decline in student achievement that characterized the 1970's, math achievement appears to have taken a slight upturn in the 1980's. Carpenter & Lindquist (1988) state that the curricular reforms proposed by the NTCM (1987) called for a reorientation of the school mathematics curriculum so that a greater emphasis is placed on helping students become mathematical problem solvers. Baroody (1993) also reveals that in 1989, the NTCM Curriculum and Evaluation Standards recommended again making problem solving a focus of mathematics instruction.

According to the 1992 Mathematics Framework for California Public
Schools, the procedures used in problem-solving should be divided into the following components to guide the instruction:

- Formulating problems.
- Analyzing problems and selecting strategies.
- Finding solutions.
- Verifying and interpreting solutions.

Baroody (1993) believes that in a program that emphasizes problem-solving, teachers must be prepared as mathematical thinkers or reflective practitioners. They need to make decisions themselves and have considerable insight into the three cornerstones of instruction: cognitive, affective, and metacognitive factors. Cognitive factors include conceptual knowledge and problem solving strategies. Affective factors influence children’s interest to solve problems. Metacognition includes self-regulation: the ability to think through problems on your own. Baroody also emphasizes the importance of reasoning and communicating in problem solving. According to him, intuitive, inductive, and deductive reasoning all play important role in the development and application of mathematics. Communicating in problem solving, or the use of cooperative learning, also greatly helps foster mathematical knowledge.

Corde (1964), meanwhile, emphasizes that developing thought process is essentially important in teaching problem-solving. Problems require thought. Teachers must teach pupils to think and give them adequate time to prepare an intelligent reply. Corde also maintains that teachers must give them an
atmosphere of testing, or trying ideas or hypotheses. Boys and girls must feel that there is nothing shameful about presenting a wrong solution, for errors sometimes point the way to correct thinking. They are encouraged to express ideas because verbalizing an idea or listening to the thoughts of other people may clarify a problem and speed the solution of it. Teachers, according to Corde (1964), also must give them freedom to make evaluations. Children love to talk and present many varied ideas about a problem. Once this has been done, they will feel some responsibility for their intellectual production.

Fennel & Ammon (1985) recommend teachers have children write their own word problems and read to their classmates. According to Fennel & Ammon, this process involves pupils in an expressive and fundamental teaching method that combines reading, critical thinking, and the collection and organization of data. Ford (1988) also argues that when asked to write their own problems, the students were actively recording their own thinking. Their problems reflected real situations. They applied their understanding of mathematics they had learned to solve problems that were unique to them and their friends. Problem solving, therefore, was becoming meaningful to them.

Vocabulary difficulty, meanwhile, is an important element that may affect problem solving skills (Corde, 1964). The vocabulary of numbers must grow with the computational skills, because if children do not understand the content of the problem, they will not be able to solve it. The problems themselves according to Corde (1964) also must be realistic, interesting, related to the cultural patterns of their communities and bear some relationship
to students' experiences. For primary grade children, the best problems should be developed around the classroom activities. After the children outgrow the housekeeping chores, the teacher may select her problems from the neighborhood or from the playground activities of the children. Krulik and Rudnick (1988) agree with Corde that good problems can be found in every aspect of daily living as well as in traditional mathematical settings but they maintain that problems need not always be word problems in order to be good problems.

Charles and Silver (1988) reveal, based on a study conducted in 1985 by Thompson while at Illinois State University, that 11 out of 16 elementary teachers described the nature of problem solving as "challenging" and "frustrating". According to Charles and Silver (1988), those teachers only complained about the nature of problem solving that appeared to influence their performance and instructional actions, and forgot that their perceptions of their own competence as problem solvers and of their ability to teach problem solving might also affect them. Charles and Silver (1988) demand that teachers must not only be knowledgeable about and feel comfortable "doing" mathematics, but also have to have an understanding of the learners, and of how to motivate and guide them.

In the 1980s, owing largely to research by Polya (1973) and others, textbooks have begun to provide a more detailed plan for problem-solving that is designed to promote the higher-order thinking skills. According to Talton (1988), this plan is similar to the following:
1-Read
2-Plan (make a table, think backward, guess and list and so on..)
3-Solve
4-Check

Similarly, Baroody (1993), as well as Kennedy and Tipps (1991), on the other hand, propose a four-phase scheme for approaching problems in a systematic fashion. This scheme was originally designed by George Polya (1945) in his book titled "How to solve it" and now is widely recognized as a useful tool for the problem solving efforts of even primary-age children.

Phase 1: UNDERSTAND THE PROBLEM.

A clear understanding of the question and the unknown is essential for deciding what information is needed, which solution strategies are appropriate, and what answers are reasonable.

Phase 2: DEVISE A PLAN.

Once a problem is understood, it is time to consider how to determine the answer. A thoughtful analysis will lead to considering alternative solution strategies and picking the most appropriate plan.

Phase 3: CARRY OUT THE PLAN.

The third phase entails carrying out the plan devised in Phase 2 and carefully monitoring the solution procedure. This monitoring is important not merely to check whether the procedure is executed accurately but to gauge whether the plan is doing the job intended.

Phase 4: LOOK BACK.
Once a solution is determined, it is important to check the results. Does the solution make sense? Does it answer the original question? Is there any other way the problem could be solved and does this solution method produce the same answer? Many students feel that once they have determined an answer, their job is done.

In phase one, understanding the problem, students have to carefully read carefully the problem and think about it to understand it. Whimbey & Lochhead (1986) complained that most students read the material too rapidly and consequently, missed words, ideas or facts. They were also not consistent in the way they interpreted words and tended to solve the problem in a mechanical manner, without very much thought. Krulik and Rudnick (1986) advise teachers to have students identify necessary information and unnecessary information (or distracters) in the problem. According to Baroody (1993) and Kennedy and Tipps (1991), phase 2, devising a plan, in which we have to select the right problem-solving strategy is the most important. Wisner (1978) conceives that a strategy must comprise three interrelated components: choice, concept, and skill. To solve a problem, one must first choose which mathematical idea will best fit the problem at hand. Then it is necessary to conceptualize and express this idea in mathematical terms. Finally, one must have the skill to be able to derive the solution of the problem. According to Wisner (1978), in the elementary grades, the first strategic component, choice, is normally indicated by the teacher, who then proceeds
with the children to the concepts and skills. It is through numerous experiences with well-selected problems that pupils eventually learn which strategic choices to make. Wisner (1978) states that the most basic problem solving strategy is straightforward computation. It appears very early in the elementary curriculum and continues throughout mathematics studies. Everyone uses direct computation to solve problems in everyday living. In the elementary grades, children use it in situations involving sales taxes, distances, costs, areas, and numerous other real world situations. Here is a very simple example:

Emma’s father gives her 3 dimes. Her mother gives her 2 more. How much did Emma’s receive?

The choice in this problem is, according to Wisner (1978), direct computation that, of course, involves the concept of addition and the skill of doing addition correctly. However, for advanced and nonroutine problems, direct computation is not enough (Baroody, 1993). Students need to be equipped with some basic strategies to solve all general problems. Baroody (1993), Kennedy and Tipps (1991), therefore, propose the following basic problem-solving strategies:

1. REPRESENT THE PROBLEM PICTORIALLY OR CONCRETELY so that he has something to experiment with and reflect on. For a primary grade child, drawing a picture is the simplest way that helps find out the solution for the problem.

"5 chickens were eating corn. 4 went into the barn. How many chickens
For a more complex problem, older children have to make a chart or simplify the problem. In simplifying the problem strategies, a good problem solver, according to Whimbey and Lochhead (1986), will identify similarities and differences among the entries and make mental notes of relationships that he/she sees. Once the analogies or the patterns are seen, the solution is easy to find. Consider this problem for instance:

"If each person in our class shook hands with everyone else in the class, how many handshakes would occur?"

Let’s consider a much simpler case: a class of 2 students. Then a class of 3. Then a class of 4. Organizing the data into a chart can help here:

<table>
<thead>
<tr>
<th>Number of People</th>
<th>Number of Shakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

When the child notices the pattern: 2(3-1), 3(6-3), 4(10-6), he/she can then extend the chart:

<table>
<thead>
<tr>
<th>Number of People</th>
<th>Number of Shakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>8</td>
<td>28</td>
</tr>
<tr>
<td>9</td>
<td>36</td>
</tr>
<tr>
<td>10</td>
<td>45</td>
</tr>
<tr>
<td>11</td>
<td>55</td>
</tr>
<tr>
<td>12</td>
<td>66</td>
</tr>
<tr>
<td>13</td>
<td>78</td>
</tr>
<tr>
<td>14</td>
<td>91</td>
</tr>
</tbody>
</table>

Then extend the chart:

When the child notices the pattern: 2(3-1), 3(6-3), 4(10-6), he/she can then extend the chart:

For a more complex problem, older children have to make a chart or simplify the problem.
2. LOOK FOR A PATTERN:

Whimbey & Lochhead (1986) argue that patterns are found very frequently in the physical and social sciences, as well as in mathematics and all areas where regular, periodic observation is made. According to them, verbalizing an accurate pattern description will greatly challenge our communication skills. Consider the following problem:

"If toothpicks are arranged as a row of triangles as shown below, how many toothpicks will be needed to make a row of 100 triangles?"

Let's examine simpler cases. Organize the data in a table and look for a pattern.

<table>
<thead>
<tr>
<th>Toothpicks</th>
<th>Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
</tr>
</tbody>
</table>

The difference between the toothpicks is consistently 2.

3. GUESS-AND-CHECK:

Sometimes, the child can guess and check. Baroody (1993) argues that a random-guess-and-check strategy is a trial-and-error approach to problem-solving. It is particularly useful where there are only a few
possibilities and a desired outcome can be estimated. Consider the following problem which would be appropriate for primary children and it could be adapted to older children:

"Cross out 1 or more numbers in the numbers in the number sentence below so that the sum is correct: 10+27+35+42=87."

4. USE "IF-THEN" REASONING to eliminate possibilities:

For advanced problems as the one below, Baroody (1993) recommends teachers to use "if-then" reasoning.

"A farmer had to take a wolf, a goat, and a cabbage to the market. He had to cross the river and could only take one thing across the river at a time. How could he take all of them to market safely?"

5. WORK BACKWARDS:

Working backwards is particularly useful when a problem specifies the result of a sequence of events and the task is to determine the starting condition.

"A boy collected golf balls. On his way home, he was stopped by a gang member who demanded half his balls plus 2. After paying him off, another gang member made the same demand. The boy has now 2 balls left. How many balls did he begin with?"

6. WRITE AN EQUATION:

It sometimes helps to write out the information given in a problem in the form of a number sentence or equation. A missing element can be denoted by a box or letter. If more than one element is missing, various shapes (circle,
square, triangle..) or letters can be used.

“3 friends, Al, Bob, and John earned $320 for a trip. Al earned 8 times the amount Bob earned. John earned the difference between Al and Bob. How much did each earn?

\[
\text{Al} + \text{Bob} + \text{John} = 320
\]

\[
8 \text{Bob} + \text{Bob} + 7 \text{Bob} = 320
\]

\[
16 \text{Bob} = 320
\]

\[
\text{Bob} = 20
\]

7.RELATE A NEW PROBLEM TO FAMILIAR PROBLEMS:

Some problems share the same structure and solution. Baroody (1993) advises teachers to encourage students to consider how new problems may be like familiar ones. Let’s compare the two problems we mentioned above, “shake hands” and “toothpicks”. They have similar structures and solutions. If the child already worked with the former, he or she could solve the latter easily.

In the third phase, carrying out the plan, Baroody (1993) recommends students decide whether or not their chosen plan is going to get them where they want to go. If they find out that their solution is wrong, they have to reconsider the problem, decide if a new point of view is needed, and design a new strategy. If they find themselves on the right track, they should determine whether all the relevant information has been used. If they find themselves on a detour, they should consider more solution strategies.

In the fourth phase, looking back, to Baroody, the children must first
decide whether the solution is reasonable, or the answer makes sense. In the following fraction addition: $\frac{2}{3} + \frac{1}{2} = \frac{7}{6}$, the sum is greater than one, and unreflective students may simply record this impossible answer. More reflective students may be baffled by the answer, but unsure what to do. Second, to Baroody (1993), they must make sure whether the solution answers the question. In the following 4th grade problem: “Rose arrived at the bus stop number 1 to go to school just as a bus was pulling away. She saw 72 students in line in front of her waiting to take the next bus. If the maximum capacity of the bus was 24, and a bus left the stop every 10 minutes, how long before Rose can board a bus?”, if the answer is 30 minutes, consider if this answer really addresses the question posed by the problem.

Finally, children should decide if there are other solutions. Sometimes, reflection reveals that there is more than one correct answer to a problem. Drawing a picture, in many cases, can help facilitate reflection.

“Lacy, Marc, and Phil all lived on the same straight road. Lacy’s house and Marc’s house were 3 miles apart, and Marc’s house and Phil’s house were 4 miles apart. How far apart were Lacy’s house and Phil’s house?”

If you answer 7 miles, consider if this is the only possibility.

In reality, according to Kennedy and Tipps (1991), it is seldom that a single strategy is used to solve a problem. Usually, a combination of strategies is used instead. Kennedy and Tipps recommend that during the time children are at school, they should have frequent opportunities to work with the above mentioned strategies in a variety of problems. Teachers should, at all times,
avail themselves of real problems that require students to use mathematics. Krulik and Rudnick (1986) go further by suggesting teachers to encourage students to create their own problems. They argue that nothing helps children to become better problem solvers than having them make up their own problems. In order to create a problem, they must know the ingredients. They must relate settings, facts, questions, and distracters.

Recently, the above four-phase approach designed by Polya has been considered most effectively and commonly used by elementary school teachers. However, diagnosis of the skills children use in their work with problems is a continuing responsibility of the teacher. Devault (1961), in his "Improving Mathematics programs", presented practical suggestions for evaluation. To him, for the purpose of group diagnosis, standard tests and tests constructed by teachers are useful, but for individual diagnosis, day-by-day observation and personal reviews are absolutely necessary. Corde (1964)suggested that the teacher should schedule a personal interview with each pupil at least twice a year: in the middle and at the end of the year. The teacher should prepare a half dozen problems comparable to some the pupils have been working. He/she should give the pupil one problem at a time and let him/her work on it until he/she completes (or abandons) his/her solution. The teacher then explores the pupil's solution, asking why he/she thinks it is right or wrong. He/she begins to work in the next problem, and discusses with the teacher each problem as soon as it is completed.

Evaluation of problem solving skills, according to Corde (1964), also
gives the teacher a chance to appraise his/her own teaching procedures. If undesirable kinds of problem solving behavior occurred less frequently at the end of the year than they did in the middle, he/she should know what teaching techniques might have caused the undesirable behavior to disappear?
GOAL AND LIMITATIONS

As was mentioned previously, the goal of this study is to show elementary school teachers how to extend children's problem solving skills in an effective way. Teachers should guide the children to understand the concepts and to use their natural intuition to relate the problem's structure to the arithmetic operation. The proper approach is not just to calculate but to think about the meaning of the problem. Children must understand the underlying structure and its relation to the required arithmetic operation. The calculation part is easy, identifying the structure presents difficulties. In addition, teachers should give children experience with a variety of word problems and of such structures.

The goal of this project is to analyze the shortcomings of the traditional way of teaching problem solving in elementary grades and examine how teachers can effectively teach problem solving and reasoning skills to elementary school students, using strategies and methods that have been approved by many experts in this field. To assist in the implementation of a problem solving approach in the classroom, a teacher handbook will be developed. The goal of this handbook is to provide elementary teachers in grades 1 through 4 with a broad overview of effective problem-solving methods and reasoning strategies that can be easily implemented into the daily classroom curriculum.
The information in the handbook will be based on the work of many mathematics educators as well as the National Research Council. Other research will also be incorporated into the development of this handbook.

The objectives of this handbook are two-fold. They relate not only to the teachers' actions and attitudes, but also to the actions and attitudes of their students. The problem-solving Handbook for Teachers aims to provide teachers with the following skills:

1. The teacher will effectively convey the problem-solving strategies to help the students successfully solve story problem.

2. The teacher will teach the students the divided-page technique for solving problems, and based on the word problem samples presented in this handbook, will be able to create more story problems of different strands of the curriculum of his/her students.

3. The teacher will enjoy applying the new techniques and strategies and feel more comfortable with teaching problem-solving skills.

The objectives for students who are provided with these problem-solving experiences will be:

1. The students will express a greater enjoyment in solving word-problems with new techniques and thinking strategies.

2. The students will feel more self-confident in solving problems and gain a positive attitude towards mathematics.
This handbook is limited in service to K-4 teachers and students only. Fifth and sixth grade teachers and students may use the same problem solving strategies suggested in this handbook but they need to try more challenging problems in other strands such as fractions and pre-algebra that are not included in this project.
This project is composed of a handbook for use by first, second, third, and fourth grade teachers.

The first portion of the handbook will introduce some guidelines and rules for teachers to prepare their students for problem solving activities.

The second section of the handbook will present background information about basic problem-solving strategies, some instructional schedule models and a point system for assessing students’ work.

The third section will propose a divided-page format for students to solve the problems. This format will help them improve their critical thinking, their math language expression along with their facts computation.

The fourth segment of the handbook will present a number of problem samples accompanied by suggestions of instruction for teachers and rules for small group problem solving.

Finally, the handbook will conclude with a bibliography of books relating to problem solving strategies. This will be included to serve as a reference for teachers who like to extend their knowledge and skill.
PREFACE

As memorizing basic arithmetic skills does not foster a meaningful understanding of mathematics, I write this mini handbook on problem solving in an attempt to help elementary school teachers focus their mathematics instruction on the development of mathematical thinking so that their students can construct their own understanding of mathematics as well as the cultivation of critical thinking, reasoning, and problem-solving skills.

This handbook aims to enable teachers to implement the reforms recommended by the National Council of Teachers of Mathematics (1989) in the Curriculum and Evaluation Standards for School Mathematics (NCTM Curriculum Standards). My hope is that you will become a more capable mathematical and pedagogical problem solver than you might be if you simply read a traditional textbook.

To accomplish this goal, I have designed guidelines for both teachers and their students to follow before, when, and after solving a problem, proposed instructional models, a point system for teachers to assess students’ work, a divided-page format for problem solving, rules for small group problem solving and strategies that help solve fast and ingenuously problems. I also list typical activities and problems for K-4 grades and accompany them with suggested strategies so teachers can save their thinking time when explaining and assigning them to students in class.
For kindergarten, I propose only hands-on activities because they fit the level of young children. Problems for first grade students also center on drawing, coloring, and counting. Second graders still have a chance to draw pictures to solve problems but they are required to start using their minds for adding and subtracting, making a chart, or guessing and checking. Third graders no longer need to draw; they have to think of what strategies to use to solve problems: making a chart, look for a pattern, graphing, or reasoning. Fourth graders have a large variety of kinds of problems to solve, so they need to find the best strategy among all those they learned including making a chart, looking for a pattern, working backwards, and reasoning. They also have to find out other possible solutions for the same problem and discuss them with their partners.

This problem solving methods handbook is still very much a work in progress. Your reaction to this minibook version will be an important indication as to whether or not my experimental format is successful. Therefore please take time to send me your comments and reactions to the following address:

Chuong Pham

25677 Jane st

San Bernardino, Ca. 92404

We are grateful for the opportunity to learn from all of you.
GUIDELINES FOR TEACHERS AND STUDENTS

SUGGESTIONS FOR TEACHERS:

According to Baroody (1993), teachers have to do five things when extending children's problem-solving skills: create a spirit of inquiry, build on children's understanding to maximize the chances of success, foster a positive disposition toward problem solving, foster autonomy, and encourage flexibility.

1. To create a spirit of inquiry, teachers have to encourage children to discover as much for themselves as possible.

2. To build on children's understanding to maximize the chance of success:
   - Word problems should be used to introduce arithmetic operations.
   - Children's intuitive strategies should be encouraged.
   - The use of manipulatives should be allowed.
   - Problems need to be carefully selected to match the developmental level of children.

3. To foster a positive disposition toward problem solving, teachers have to:
   - Model interest in and excitement about problem solving.
   - Use a variety of problems, particularly nonroutine
problems, to challenge students' thinking on a regular basis.
- Create a safe, nonthreatening atmosphere.
- Encourage risk-taking.
- Give children time to work on challenging problems.
- Foster beliefs conducive to problem solving.

4. To foster autonomy, teachers have to:
   - Encourage students to write their own word problems and share them.
   - Encourage them to evaluate their own answers.

5. To encourage flexibility, teachers should:
   - Encourage students to continually examine their assumptions.
   - Foster an openness to different ideas.

According to Whimbey and Lochhead (1986), teachers have to:

1/ SET UP THE CLASSROOM CLIMATE:
   * Be enthusiastic about problem solving.
   * Have students bring in problems from their personal experiences.
   * Recognize and reinforce willingness and perseverance.
   * Reward risk takers.
   * Emphasize persistence rather than speed.
*Emphasize the selection and use of problem solving strategies.

2/ BEFORE SOLVING THE PROBLEM:

*Read or have students read the problem. Discuss words.

*Use a whole-class discussion about understanding the problem.

Ask questions to help students understand the problem.

*Direct students' attention to the list of strategies and ask students which strategies might be helpful for finding a solution.

3/DURING:

*Let students work in groups for practice or if the problem is challenging. Let them work individually in other cases.

*Observe and question students about their work.

*Ask students who obtain a solution to check their work and answer the problem.

*Give a problem extension to students who complete the original problem sooner than others.

4/AFTER:

*Show and discuss students' solutions. Have students name the strategies used.

*Relate the problem to previous problems or solve an extension of the original problem.
SUGGESTIONS FOR STUDENTS

A/ As for students, Whimbey and Lochhead (1986) recommend that they:

* Think aloud or vocalize their thoughts.
* Have a positive attitude.
* Break the ideas into small steps.

B/ For students who work in pairs or in small groups, if they found out their partners apply formulas that are inappropriate and lead to wrong answers, Whimbey and Lochhead recommend them to:

* STOP their partners and ASK to see a diagram which illustrates, step-by-step, the relationships between the facts in the problem.
* REQUEST a full explanation of why he/she is performing such computations.

C/ Krulik and Rudnick (1988), more specific, recommend students to follow 5 steps when solving a problem:

* READ
* EXPLORE
* SELECT A STRATEGY
* SOLVE
* LOOK BACK AND EXTEND

D/ Baroody (1993), as well as Kennedy and Tipps (1991), based on Polya’s design, suggest 4 steps:

* Understand the problem (Read and explore)
*Devise a plan (Select a strategy)

*Carry out the plan (Solve)

*Look back (Check and extend)

All guidelines we mention above are very important for teachers and students to keep in mind before teaching problem solving or solving a problem. Without following them, it will be more likely that students have to do and redo the problem before they find the right solution.
SOME INSTRUCTIONAL MODELS FOR A PROBLEM SOLVING PROGRAM

1. TEACH AND PRACTICE EACH STRATEGY:

   Week 1: Teach "Draw a picture"
   Week 2-3: Practice "Draw a picture"
   Week 4: Teach "Guess and check"
   Week 5-6: Practice "Guess and check"
   etc....

2. PRACTICE SOLVING PROBLEMS RANDOMLY MIXED BY SOLUTION STRATEGIES:

   Week 1: Problem 1 (guess and check)
            Problem 2 (make a table)
   Week 2: Problem 3 (make an organized list)
            Problem 4 (look for a pattern)
   Week 3: Problem 5 (guess and check)
            Problem 6 (work backwards)
   etc....

3. TEACH PROBLEM SOLVING SKILLS:

   Unit 1: Teach student how to "understand the question"
   Unit 2: Teach students how to "find the needed data"
   Unit 3: Teach students how to "design a plan"
   etc....
4. COMBINATION OF THE ABOVE:  A weekly approach

Week 1-3:  
a/Teach “guess and check”
b/Practice “guess and check”
c/Teach problem solving skills

Week 4-6:
a/Teach “drawing a picture”
b/Practice “guess and check” and “drawing a picture”
c/Teach problem solving skills

Week 7-9:
a/Teach “make an organized list”
b/Practice “guess and check”, “drawing a picture”, and “make an organized list”

etc.......
PROBLEM SOLVING STRATEGIES

1. DRAW A PICTURE:

Drawing a picture is a commonly used problem solving strategy. It is helpful for having students visualize a problem in order to be able to solve it. Students should be reminded that detailed drawings are not necessary. Problems that seem difficult at first often become very easy when the problem situation is shown pictorially.

2. MAKE AN ORGANIZED TABLE, LIST, OR CHART:

This strategy is a way for students to pick out information data in a problem, record these data, and systematically look at the various possibilities for solution.

3. LOOK FOR A PATTERN:

In problem-solving, pattern recognition becomes more of an active rather than a passive observation. The student is encouraged to interact with the data and discover a pattern.

4. GUESS AND CHECK:

The student is encouraged to make an educated guess based on the information that he/she has been given in the problem. The student then checks the guess to see if a correct answer has been obtained or if he/she must make an adjustment in the original guess. The critical point to keep in mind when students use this strategy is the checking back to evaluate the
hypotheses for corrections.

5. LOGICAL REASONING:

This strategy might employ various strategies for obtaining a correct solution. When using logical reasoning, students might record data in table form, draw a picture, or act out the situation in order to understand the problem.

6. ACTING OUT (OR USING MANIPULATIVES):

The main obstacles posed by a problem can often be overcome when the student is able to visualize the situation. The difficulty lies in being able to picture how the actions occur and how they are related.

7. WORKING BACKWARDS:

Sometimes, the most efficient way to solve a problem is to start at the end and work toward the beginning. The strategy that involves retracing the steps that were necessary to get the final answer is called working backwards.
A POINT SYSTEM FOR ASSESSING STUDENTS' WORK

Understanding the problem:

0-Completely misinterprets the problem.
1-Misinterprets part of the problem.
2-Complete understanding of the problem.

Solving the problem:

0-No attempt or a totally inappropriate plan.
1-Partly correct procedure based on part of the problem interpreted correctly.
2-A plan that could lead to a correct solution with no arithmetic errors.

Answering the problem:

0-No answer or wrong answer based on an inappropriate plan.
1-Copying error.
  -Computational error.
-Partial answer for problem with multiple answers.
-Answer labeled incorrectly.

2-Correct answer.

**DIVIDED-PAGE FORMAT FOR PROBLEM-SOLVING**

**PROBLEM SAMPLE:**

Paul owns 4 more than one-half as many books as Pete. Paul owns 32 books. How many books does Pete own?

<table>
<thead>
<tr>
<th>Computations</th>
<th>Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>32 books</td>
<td>If 4 books are taken from Paul, he will have exactly one-half as many as Pete:</td>
</tr>
<tr>
<td>4 books</td>
<td>32 b - 4 b = 28 books</td>
</tr>
<tr>
<td>28 books</td>
<td>One half the number of books owned by Pete is 28. So Pete owns twice this number:</td>
</tr>
<tr>
<td>56 books</td>
<td>Solution: Pete owns 56 books.</td>
</tr>
</tbody>
</table>
RULES FOR SMALL GROUP PROBLEM SOLVING

1. All should participate in solving the problem.

2. Be considerate of others.

3. Do not be afraid to ask others in the group to explain their thinking and work.

4. Do not be afraid to disagree with others in the group.

5. Help any group member who asks for help.

6. Do not interact with other groups.

7. Everyone in the group should agree on an answer.

8. Help others in the group understand solution strategies.

9. Ask the teacher for assistance only when everyone in the group has the same question.
KINDERGARTEN

ORGANIZE A LIST

1. HEIGHT RECORDS

SKILLS_________ Solving problems
Ordering
Comparing
Seeing relationships

MATERIALS_______ Butcher paper cut in 9" strips

ACTIVITY_______ This activity extends over several weeks. Begin with two children the 9th day, having them stand against a strip of butcher paper to be measured, one at a time. Cut off a separate piece of paper indicating each child's height. On each following day, one additional child is measured while the group watches and the new height is compared to the earlier height measurement. In this way, at the end of several weeks, every child has been measured and his or her height is recorded, compared, and then arranged in order with the rest of the class.

*This same activity can be done to compared the weights of several rocks or similar-sized toys. The key is to make
one or two additional comparisons each session, gradually completing the whole series.

2. BODY MEASUREMENTS

SKILLS __________ Measuring

   Comparing

   Seeing relationships

MATERIALS ______ String, scissors

ACTIVITY ________ The children measure the circumference of their head with a piece of string and cut off the length. Experimenting with the string, the children measure other parts of their body and try to find some part that is the same measurement. Kids should be shown how to fold their string in half and search for a body part of this measurement. The same thing can be done by folding the string in thirds or fourths, and so on, although this is more difficult and will need to be done in smaller groups.

*Source:

DRAW A PICTURE

BALLOONS

Topic: Problem-solving

Objective: The students will listen to and draw a picture about a story.

Materials: Worksheets, pencils and crayons.

Problem:

Debbie and Allen are blowing balloons. Allen blew 6 green balloons and Debbie blew 3 red ones. Draw the balloons that Allen blew on the sticks and the ones Debbie blew on the strings. How many balloons are there?

Activities:

Understanding the problem

What are Debbie and Allen doing?

How many balloons did Allen blow? (6)

How many balloons did Debbie blow? (3)

What colors were the balloons?

Planning the solution

Help students find the sticks and the strings.

Help students identify that green balloons go on sticks and red ones go on strings.
Finding the answer

Draw/color the green and red balloons.
Count the green ones and red ones together.(9)

Problem extension

If one of Allen's balloons flew away, how many balloons would they have?
FIRST GRADE

MAKE A TABLE

Vocabulary: Menu, orders.

Materials: Worksheets, pencils.

Problem:

Barbara's mother said she could order 1 sandwich and 1 drink for lunch. How many different lunches can she order from this menu?

<table>
<thead>
<tr>
<th>Sandwich</th>
<th>Drinks</th>
</tr>
</thead>
<tbody>
<tr>
<td>hot dog</td>
<td>milk</td>
</tr>
<tr>
<td>hamburger</td>
<td>juice</td>
</tr>
<tr>
<td>grilled cheese</td>
<td>soft drink</td>
</tr>
</tbody>
</table>

Understanding the problem:

What 2 things can Barbara have for lunch? (2: one sandwich and one drink)

How many drinks (sandwiches) can she have for lunch?

How many different kinds of sandwiches are there? Name them.

What kinds of drink can Barbara pick from the menu?

Planning a solution:

Name 1 sandwich and 1 drink Barbara can have? (Answers vary)

If she picks a hamburger, what can she drink with it? (milk, juice, or soft drink)

If she drinks milk, what sandwich can she pick? (hot dog, hamburger, grilled cheese)
Finding the answer:

Complete this table:

<table>
<thead>
<tr>
<th>sandwich</th>
<th>drink</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<tr>
<td></td>
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<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Answer: There are 9 lunch combinations in this problem.

Problem extension:

Suppose Barbara had a peanut butter sandwich added to the menu.

Now there are 12 different lunches she can have. Can you find them?
FIRST GRADE

MAKE A TABLE

Background: Knowledge of counting by 2

Materials: Worksheets and pencils.

Problem:


Understanding the problem:

What did Ted buy? (pencils)

How much did one pencil cost? (2)

How many pencils did he buy? (4)

What are the names for the 2 rows of numbers in the table?

(number of pencils, cost)

Planning the solution:

If 1 pencil costs 2¢, how much will 2 pencils cost? (4¢)

Can you point to the table to show this?

How much do 3 pencils cost? (6¢)

Can you show this on the table?

Count the pennies. How many pennies are there? (6)

How much do 4 pencils cost? (8¢)

Can you draw the pennies?

Finding the answer:

53
Look at the table. Can you see a pattern in the cost?

(increases by 2)

What did Ted pay for 4 pencils? Write the amount that he paid and draw the pennies.

**Problem extension:**

Suppose you can buy 1 pencil for 2c, how many pencils can you buy for 12c. Can you draw the pencils and pennies?
1/ A farmer had 16 sheep. All but 7 died. How many does the farmer have left. (Draw a picture and count)

2/ Gene colored 9 eggs. He colored 5 blue and the rest red. How many eggs did he color red? (Draw a picture and color)

3/ Ana had 75c. She bought 2 pencils for 28c. Did she have enough money left to buy a candy bar costing 45c. (logical reasoning)

4/ John is taller than Mary. Mary is taller than Peter. Who is the shortest of the 3 children? (Draw a picture)

5/ Janice has fewer than 10 baseballs cards. If she put them into piles of 3, she has none left over. But when she puts them into piles of 4, there is 1 left over. How many baseball cards does Janice have? (using manipulatives)
SECOND GRADE

Problem 1: Lucy and Carla leave school at 3:00 p.m. and go home. Their homes are on the same street, and lie in the same direction from the school. Lucy lives 5 miles from the school. How far apart are their homes?

Discussion:
1/ What does 3:00 p.m. have to do with the problem?
2/ The key word is “same direction”.

Strategy:
Draw a diagram.

Problem 2: Lucy and Carla leave school at 3:00 p.m. and go home. Their homes are on the same street, but lie in opposite directions from miles from the school. Lucy lives 3 miles from the school, while Carla lives 2 miles from the school. How far apart are their homes?

Discussion:
1/ What makes this problem different from the preceding one?
2/ The key word is “opposite direction”.

Strategy
Draw a diagram.
Problem 3:

Find the difference in the number of apples in a 5-pound bag that contains 27 apples and a basket that contains 2 dozen apples.

Discussion:

1. Notice that 5-pound bag is extra information.
2. How many apples are there in a dozen?
3. There is no question mark in this problem. What is the question?

Strategy:

Draw a picture.

Problem 4:

Albert weighs 50 pounds. Together, Bennett and Carlos weigh 100 pounds. If Carlos weighs more than Bennett, arrange the 3 boys from heaviest to lightest.

Discussion:

If Carlos and Bennett weigh the same, each would weigh exactly 50 pounds. If Carlos weighs more than Bennett, he weighs more than 50 pounds and Bennett, of course, weighs less than 50 pounds.

Strategy:

Logical reasoning.
Other typical problems:

1/ Mary set out a dozen doughnuts for a party. John ate 4. Does Mary have enough left to serve 9 people? (Drawing a picture and reasoning).

2/ Mary bought a candy for 29c. She gave the clerk a $1 bill and received coins in change. What 5 coins did she receive? (Guess and check).

3/ Ricardo's gold fish had baby fish. He gave 6 to Marlene. He gave 5 to Sonya. If there were 18 baby fish to start, how many does Ricardo keep? (Draw a picture).

4/ Which of the numbers 4, 7 and 9 is the mystery number?
   a/ It is more than 3.
   b/ It is less than 8.
   c/ It is more than 5. (Logical reasoning)

5/ How far is from Corcoran to Milville?

(Draw a diagram)
6/ Mitch bought 3 different toys for his children. The gifts cost him $12.00.

What did he buy? 
- Football $6.00
- Soccer ball $4.00
- Book $5.00
- Puppet $2.00

(Guess and check)

7/ Which of the following sums of money could you pay with exactly 3 coins?

Tell how you would do it. 7c 16c 22c 56c

(Using manipulatives)

8/ Alim, Brenda, and Carol are all selling fruit at the school carnival. They sold oranges, apples, and pears.
   a/ Alim and the orange seller are sisters.
   b/The apples seller is older than Brenda.
   c/ Carol sold the pears.

Who sold which kind of fruit?

(logical reasoning)

9/ Waiting in line to buy movie tickets, Lois was behind Nan. Mary was in front of Nan and behind Ann. Lois was in between Nan and Brad. Who is in the middle of the line?

(Draw a diagram)

10/ Put 10 pennies in a row on your table. Now replace every other coin with a nickel. Next replace every third coin with a dime. What is the value of the 10 coins now on the table? (Make a chart)
THIRD GRADE

Problem 1:

Glady, Jeanette, Jesse and Steve went fishing. Glady caught 16 fish. Jeanette caught 13 fish, Jesse caught 7 fish, and Steve caught 14 fish. How many more fish did Jesse and Steve catch than Glady and Jeanette?

Discussion:

1/ The data in the problem can best be organized with a simple table:

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of Fish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glady</td>
<td>16</td>
</tr>
<tr>
<td>Jeanette</td>
<td>13</td>
</tr>
<tr>
<td>Jesse</td>
<td>17</td>
</tr>
<tr>
<td>Steve</td>
<td>14</td>
</tr>
</tbody>
</table>

2/ Solve the problem by adding and then subtracting:

\[
17 + 14 = 31 \\
16 + 13 = 29 \\
31 - 29 = 2
\]

Problem 2:

In the Watkins family, there are 4 children. Cynthia is 6 years old. Her twin sister, Andrea, is 3 years older than her brother, Matthew. Their brother James is the oldest, and is 3 times as old as Matthew. How old is Matthew? How old is James?

Discussion:
1/ What does “twin” mean? How old is the twin sister then?

2/ Students have to be able to discriminate between necessary and unnecessary information.

**Strategy:** Graphing

![Graph](image)

**Problem 3:**
How many different ways can you make change for a 50c piece without using pennies? *(Guess and check)*

**Problem 4:**
David woke up at 7:00 a.m.
Barbara woke up 1 hour after David.
Susie woke up 2 hours before Barbara.

At what time did Susie wake up? *(Draw a diagram)*

**Problem 5:**
I am an even number.
I am more than 10 and less than 120.
The sum of my digits is 6.
What is the number? *(Guess and check)*
Problem 6:
I have 9 bills in my wallet. 5 of them are $1 bills, and the rest of them are $5 bills. How much money do I have in my wallet?
(Make a chart)

Problem 7:
Arthur is making lunch. He makes sandwiches with white bread or rye bread. He uses either cheese, jelly, or lunch meat. How many different sandwiches can he make?
(Make a chart)

Problem 8:
A bus with 53 people on it makes 2 stops. At the first stop, 17 people get off and 19 people get on. At the second stop, 28 get off and 23 get on. How many people are now on the bus?
(Make a chart, using subtraction and addition)

Problem 9:
Ursula is in training. She did 5 sit-ups the first day. She did 6 sit-ups the second day. 7 the third day, and so on. How many sit-ups did she do on the 14th day?
(Make a table and look for a pattern)

Problem 10:
Nicole has a package of 48 silver stars. She wants to arrange them in rows, so that each row has the same number of stars. How can she arrange them so that the number of stars in each
row is an odd number?

(List all the odd numbers from 1 to 23, i.e. almost half of 48. Make a chart with all the possibilities. Also, use “Estimation” and “Guess and check”)
FOURTH GRADE

Problem 1:

I put 10 checkers into 2 stacks. One stack has 4 more checkers than the other has. How many checkers are in each stack?

(Act it out, use manipulatives, guess and test)

Problem 2:

Peter, Paul, and Mary have 5 cookies. How many ways can they divide the cookies if each person must get at least 1 cookie?

(Make a table)

<table>
<thead>
<tr>
<th>Peter</th>
<th>Paul</th>
<th>Mary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

There are a total of 6 ways it can be done.

Problem 3:

If \[ \begin{array}{c}
\hline
\text{□}
\hline
\end{array} = 18 \quad \text{and} \quad \begin{array}{c}
\hline
\text{□}
\hline
\end{array} = 54
\]

then \[ \begin{array}{c}
\hline
\text{□}
\hline
\end{array} = ? \]

(logical reasoning)
If 2 squares = 18, then each square = 9
Thus, 3 circles + 18 will = 54, and each circle = 12.
Then 3 squares + 4 circles = 3(9) + 4(12) = 75

Problem 4:
Lucy has a dog, a parrot, a goldfish, and a Siamese cat. Their names are Lou, Dottey, Rover, and Sam. The parrot talks to Rover and Dotty. Sam cannot walk or fly. Rover runs away from the dog. What is the name of each of Lucy’s pets? (Logical reasoning)
Prepare a logic matrix as shown. As each clue is given, record the information in the matrix.

<table>
<thead>
<tr>
<th></th>
<th>Dog</th>
<th>Parrot</th>
<th>Goldfish</th>
<th>Cat</th>
</tr>
</thead>
<tbody>
<tr>
<td>LOU</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DOTTY</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ROVER</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAM</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Problem 5: Put a single digit into each box and make the problem correct:

```
□ □ □
```

```
X · □
```

1 0 9 0

65
(Guess and test)

There are 2 possible answers:

\[
\begin{array}{cc}
542 & 218 \\
\times 2 & \times 5 \\
1090 & 1090 \\
\end{array}
\]

**Problem 6:**

Stanley makes extra money by buying and selling comic books. He buys them for 7c each and sells them for 10c each. Stanley needs 54c to buy some batteries for his calculator. How many comic books must Stanley buy and sell to earn the 54c? (Make a table)

<table>
<thead>
<tr>
<th>Number of coins</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>3c</td>
<td>6c</td>
<td>9c</td>
<td>12c</td>
<td>15c</td>
<td>18c</td>
<td>21c</td>
</tr>
</tbody>
</table>

**Problem 7:**

Mitch and his sister Pauline went to visit a friend who lived 12 blocks away. They walked 6 blocks when they realized that they had dropped a book. They walked back and found the book. Then they walked the blocks to their friend’s house. How far from their home did they drop the book? (Work backwards)

Subtract the 8 blocks they finally walked from the 12 block-trip. They dropped the book 4 blocks from their home.
Sample Ratio Problem 8:

(Generally there is more than one way to solve ratio problems).

John can run 7 feet the time Fred runs 5 feet. How far will John run in the time that Fred runs 15 feet?

1/ Make a chart:

<table>
<thead>
<tr>
<th>Fred</th>
<th>John</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>15</td>
<td>21</td>
</tr>
</tbody>
</table>

2/ Logical reasoning and mathematical solution:

If Fred runs 5 feet, John runs 7 feet.

So if Fred runs 15 feet, how far does John run?

7 : 5 \times 15 = 21 feet

This is a shortcut method of spelling out the entire situation that has been proved to be very effective when solving all kinds of ratio problems.

3/ Ratio reasoning:

Another way to solve the problem is to think in terms of ratios that are equal to each other. This approach requires more mathematical background and experience.
Cross multiply: $5x = 7 \times 15$

$$x = 21$$

**Ratio Problem 9:**

A car travels 40 miles an hour and a plane travels 10 miles a minute. How far will the car travel while the plane travels 450 miles?

(Be careful to notice the difference between hour and minute in this problem)

**Ratio Problem 10:**

A man runs 1 mile in 10 minutes and a car goes 50 miles an hour. At these rates, how far does the man go when the car goes 150 miles?

(Notice that 1 mile in 10 minutes = 6 miles win 1 hour)

**Problem 11:**

Nina asked her Dad how old he was. He told her, "If I add 10 to my age and double the result, I will get 84". How old is Nina's Dad?

(Work backwards)

**Problem 12:**

The cost of a concert ticket and a football ticket is $14. The cost
of a movie ticket and a football ticket is $11. The cost of a concert ticket and a movie ticket is $7. Find the cost of each ticket?

(“Logical reasoning” and “Guess and test”. There is more than 1 solution)

1/Add $14 + $11 + $7 = $32.

Divide $32 by 2 = $16 (the cost of 3 different tickets).

Subtract $14 from $16 = $2 (the cost of the movie ticket) etc....

2/Compare the cost of a concert ticket and a football ticket ($14) and the cost of a movie and a football ticket ($11), we see that the concert ticket is $3 more than the movie one. Since the concert and the movie ticket cost $7, the movie ticket must cost $2 and the concert one $5.


REFERENCES


