GRAPHIC MATHEMATICAL MEDIATED STRUCTURE: THE LINK FOR HISPANIC/LATINO AND ENGLISH LEARNERS' MATHEMATICAL SUCCESS

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GRAPHIC MATHEMATICAL MEDIATED STRUCTURE: THE MATH LINK FOR HISPANIC/LATINO AND ENGLISH LEARNERS' MATHEMATICAL SUCCESS

A Project
Presented to the
Faculty of
California State University,
San Bernardino

In Partial Fulfillment
of the Requirements for the Degree
Master of Arts
in
Education:
Bilingual/Cross-Cultural Education

by
Sylvia Martinez Casteloes

September 2018
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Approved by:

Dr. Barbara Flores, First Reader
Dr. Enrique Murillo, Second Reader
ABSTRACT

This project’s goal is to promote and improve the mathematical literacy of fourth-grade Hispanic/Latino and English learners through the use of a graphic mathematical mediated structure. Current California Common Core data finds fourth-grade Hispanic/Latino and English learners significantly behind White and Asian students in mathematics, especially in understanding written word problems.

Research supports the assumption that as a tool, a graphic mathematical mediated structure could: 1) foster conceptual understanding; 2) build content terminology; 3) allow students opportunities to justify their solutions; 4) integrate writing in math; and 5) provide a platform for discourse. This innovative pedagogical project specifically focused on how fourth-grade Hispanic/Latino and English learners could navigate through a graphic math organizer in order to understand how to add and subtract fractions in word problems.

The work presents six teacher models of graphic mathematical mediated structures. Each model provides a fourth-grade word problem related to fractions. Respectively, teacher and student templates, lists of content vocabulary, and suggestions to teach each problem-solving exercise using the graphic mathematical mediated structures that were created and developed are included.
Struggling Hispanic/Latino and English learners’ need a pedagogical structure and process to succeed in solving math word problems. Hence, the need for a graphic mathematical mediated structure to diminish the groups’ prevalent mathematical achievement gap and to increase their achievement in mathematics.
ACKNOWLEDGMENTS

With gratitude and admiration, I would like to honor Dr. Barbara Flores, un tesoro para la educación de niños y maestros y la inspiración de este proyecto. I would also like to express my gratitude to Latino educator and advocate, Dr. Enrique Murillo, for being my second reader. Thank you both for ensuring my success.

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Drs. Juan Gutierrez, Kathryn Howard and Stephen Bronack, thank you for your consideration and encouraging words, “You need to finish your thesis.”

To my amigas and once upon-a-time colleagues, Corinne and Denise, thank you for lifting my spirits and cheering me to the finish line.

Finally, to my husband and my son, thank you for traveling along with unfailing love, support, and patience.
DEDICATION

This project is dedicated to my mom, who encouraged my education and to my son who will soon be staring his master's program.
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## SUMMARY

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CHAPTER ONE

BACKGROUND

Purpose of the Project

This qualitative, pedagogical, and generative research is intended to expand the pedagogic knowledge about teaching elementary Hispanic/Latino and English learners’ fractions and problem solving through the use of a tool, a graphic mathematical mediational structure.

Two significant reasons define the purpose for this project. First, the underachievement in mathematics of Hispanic/Latino and English learners, especially in California, is of great concern. The second reason for this work speaks about the pedagogy derived from state and district curriculum that were considered in order to develop a mathematical tool, the graphic mathematical mediational structure, to improve the problem-solving skills for Hispanic/Latino and predominantly Spanish-speaking English learners.

The vested interest for this project lies in almost 20 years of primary classroom experience working with chiefly Hispanic and Spanish-speaking English learners. Years of text-driven math instruction did not work for my Latino and English learners, especially for word problems. Neither did the isolated English Language Development lessons, as these did not provide a specific format to address problem solving.

Therefore, yearly the California State Test (CST) results showed that the majority of my minority students lacked reasoning skills. The idea of a
graphic math organizer/graphic mathematical mediated structure to address the needs of the two focused populations was the result of a class by Dr. Barbara Flores. Her pedagogy was based on Vygotsky’s idea that knowledge is socially constructed. Her class lectures on Vygotsky shifted my pedagogy paradigm from text-driven instruction and high-stakes state assessments to instruction based on the needs of my students, specifically focusing on Hispanic/Latino and English learners. Hence, an innovative strategy, the graphic mathematical mediational structure, was developed to blend the California English Development Standards, the Common Core Standards in Mathematics, and the Standards for Mathematical Practice.

The change in my mathematical instructional pedagogy allowed the incorporation of the graphic mathematical mediational structure, which significantly impacted not only my Hispanic/Latino and Spanish-English learners, but also the whole class. The end-of-the-year California State Test (CST) scores showed that 75% of my students were proficient or above in mathematics. Another significant CST result was that almost every student made one or two math levels’ growth, from the previous year.

Scope of the Project

The scope of this project is fivefold: 1) To address the mathematical issues that Latino fourth graders face in solving word problems involving fractions; 2) To provide a historical background regarding Latino children’s lack of academic skills in math; 3) To review the literature in the following
areas: The instruction of mathematics using the English Language Development domains, the Common Core State Standards for fourth grade, and the Standards for Mathematical Practices; 4) To create innovative pedagogy using mediated structures that not only embed the complexity of the mathematical concepts, but also teach a patterned interactive procedure that mediates the internalization for solving mathematical word problems; and 5) To show teachers how to concretely use a teaching pedagogy that works and ensures success in how Latino students learn to solve mathematical word problems.

In summary, the graphic mathematical mediational structure embodies all three of the state’s programs to create an innovative instructional program. The graphic mathematical mediational structure unifies these programs' standards on a visual structure. It is a tool that educators could use to fine-tune the pedagogy of problem solving for Hispanic/Latino and English learners. The outcome could be a diminished mathematical achievement gap for thousands of California’s fourth-grade Hispanic/Latino and English learners.

Significance of the Project for Education

Historically, Hispanic/Latino and English learners have dawdled behind the achievement of Anglo children in two of the most important academic subjects, language arts and mathematics. National, state, and local educational entities report the gaps in the mathematical achievements of
Hispanics, English language learners, and African-American students (California Department of Education, 2018). The motivation behind this study is the long-standing mathematics achievement gap that is prevalent of California’s Latino and English Language learners.

Traditional values and cultural biases from the dominant society have influenced instructional pedagogy. What some mathematicians call the heart of mathematics, problem solving, has been inaccessible to generations of Hispanic/Latino and English learners. Consequently, a pattern of mathematically lagging behind White students validates the assumption that Hispanic/Latino and English language learners lack ability in problem solving.

Substantial to this qualitative project for education, is the presentation of an innovative pedagogy in combination with a tool, a graphic mathematical mediated structure designed with the following objectives:

1. Narrow the mathematical achievement gap for Hispanic/Latino and English learners, in the area of adding and subtracting fractions with problem solving.

2. Develop the academic language and communication skills Hispanic/Latino and English learners by incorporating the four English Language Domains: listening, speaking, reading, and writing.

3. Build conceptual understanding of fractions in word problems by including the Standards for Mathematical Practices.
4. Employ the use of the graphic mathematical mediational structure to give Hispanic/Latino and English learners access to grade-level content, including problem solving.

The Common Core State Standards in mathematics, places the bulk of instruction of fractions in grades three through seventh (California Department of Education, 2018). At the primary level, conceptual understanding of fractions is crucial as research shows that they are a prerequisite to being successful with middle and later high school algebra. The graphic mathematical mediated structure could allow teachers to scaffold/mediate instruction of written word problems imbedded with fractions.

The significance of this qualitative and innovative project was that I expanded on the original graphic mathematical organizer students used for geometry, to specifically target the lessons on adding and subtracting fractions and that mediated structures can provide teachers a scaffold that will build conceptual understanding to use when teaching students problem solving in adding and subtracting fractions. By organizing the teaching deliberately and engaging the students in participating in the co-construction of the mathematical knowledge by using oral/written language through the use of the mediated structures, the learning process in solving problems is made visible.

The lessons reviewed for the purpose of developing the organizers are found in Envisions’ fourth-grade Common Core for California textbook. The models offer teachers’ ideas for students to engage in processes that can help
them develop conceptual understanding in adding and subtracting fractions with like denominators. Additionally, the format of the lessons allow teachers to manipulate the placement of information to meet the needs of their underachieving Hispanic and English language learners.

In summary, the Hispanic/Latino and English learner mathematical gap has become a pattern (McNulty, 2004). The assumption is that these groups are failing due to the difficult problem-solving tasks the state assessments present. Traditional instruction of word problems hinders the achievement of Hispanic/Latino and English learners as evidenced on nationwide Common Core State data (NCES, website). These students need educators willing to try an innovative pedagogy and another tool, the graphic mathematical mediational structure, to ensure their access to grade level curriculum.

Limitations of the Project

An important limitation to this project was the time it took to compile the readings specific to the use of graphic organizers for mathematics. The task to disseminate each state program in order to support the use of the graphic mathematical mediated structure was daunting. Another limitation was the mathematics score reported by the California’s Common Core State Standards for Hispanic, Latino, and English learner students. These scores were generalized for all the learners without referencing individual ethnic group scores or the English learners’ specific language. A final limitation was the validity of the research. Scores may be skewed due to the newness of the
Common Core State Standards for some states as well as the variety of the reporting data by each state’s agencies.

Definitions of Key Terms

This project’s key terms and their definitions sprang from the literature researched.

*English learner* (EL) refers to a student whose first language is not English, but who is receiving instruction in the English Language Development (ELD) Standards: reading, writing, listening, and speaking to become a proficient English speaker.

*Hispanic* is a moniker used to identify a person who has ancestral ties to any one of the twenty-one Spanish-speaking countries.

*Hispanic/Latino* is an umbrella term used by the United States Census Bureau and the California Department of Education to publish national data for Hispanic subgroups: Mexican, Puerto Rican, Cuban, or anyone who has ancestral ties to one of the twenty-one Spanish-speaking countries.

*KWL* is a three-column graphic organizer used with expository text. The three letters are acronyms: K-what students know about the topic, W-what students want to learn about the topic, and L-what students learned about a topic.

Graphic mathematical mediated structures, mathematical graphic mediational structures, math mediated structures and math graphic organizers/mediated structures all reference the visual graphic math organizer developed for this project.
Zone of Proximal Development (ZPD), a term used by Vygotsky that states, “It is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (p. 86).
CHAPTER TWO
REVIEW OF THE LITERATURE

The backdrop for this literature review is a short history of the beginning of Hispanic/Latino education in the United States in order to provide the foundation for the assumption: The reason fourth-grade Hispanic/Latino (a term used in state and national data) and English Learners (ELs) are failing in mathematics is because they share a common thread, the lack of ability to problem solve written word problems. Additionally, this review will specifically look at the math achievement of California’s fourth-grade Hispanic/Latino and English Learners. In response to the prevalent mathematical gap of the two groups, this endeavor presents a potential solution, the use of a graphic mathematical mediated structure (graphic mediated structure, graphic structure, mediated structure) to help Hispanic/Latino and English Learners succeed with fourth-grade problem solving in mathematics.

The Identity Labels: Hispanic, Latino, Mexican

This project begins by explaining how the Hispanic/Latino population is referenced. The general moniker, Hispanic/Latino, identifies this population as one and the same by both the United States Census Bureau (2010), and by the California Department of Education (2017). The U.S. Census broadly identifies and clumps populations who are Hispanic/Latino, as having origins from Cuba, Mexico, Puerto Rico, Spain, Honduras, Venezuela, or other Spanish-speaking countries (US Department of Commerce, 1998).

Likewise, without distinguishing between the ethnicities of the populations, the California Department of Education (2017) gathers and releases data for Hispanic/Latino students. This report acknowledges that Hispanic/Latino are not a homogenous population and that subgroups within this population include, Mexican Americans, Puerto Ricans, and Cubans.

Due to the bundling of data for the two focused populations by both the United States Census and the California Department of Education, as well as the lack of specific mathematical results for each ethnic group, the generalized terms, “Hispanic”, “Latino”, and “Mexican”, will be used interchangeably throughout this report. Additionally, this work focuses on English Learners (ELs) who might also be identified under the umbrella of Hispanic: Latino, Mexican, Puerto Rican, or Cuban.
The Hispanic/Latino/Mexican Education: Historical Context

In order to understand the current mathematical underachievement of Latino and ELs, it is important to begin with an historical glimpse of their education within the United States. Walsh (1992) claims, “The education provided to bilingual children in general, and Latino students in particular, is tied to a complex series of historical, sociocultural, and political relations, concerns, and conditions…which include issues of race, class, colonialism, and power” (as cited in Diaz-Greenberg, 2003, p. 2). Marginalization of Mexicans began with the invasion of the Spaniards, but the roots of educational inequalities for Latinos began when the United States accumulated the following Mexican-occupied territories as a result of its war on Mexico: Texas, California, Nevada, Arizona, New Mexico, parts of Utah, and Colorado (Verdugo, 2006, p. 14). According to Jimenez (1994), approximately 100,000 Mexicans had been living in these areas prior to the Texas War, and the U.S.-Mexican War. The signing of the Treaty of Guadalupe Hidalgo, on February 2, 1848, gave the Mexicans who were living from California to Texas American citizenship and rights (Jimenez, 1994).

But, the dominant Americans violated the agreement and treated the Mexicans, not as citizens, but as foreigners in their native land (Jimenez, 1994). For instance, in Texas, between 1836-1900, the establishment of schools by Catholics, Protestants, and political leaders was meant to educate those at the top of the social class and to extend the hierarchy of those in
power over women and ethnic groups (San Miguel, 2010). Therefore, the beginning of the Latinos’ formal schooling appears to have destined them to be treated like immigrants and denied the right to the identical education Anglo children received, as evidenced in the following pedagogy:

Local administrators developed discriminatory measures, as reflected in assessments and placement practices and in the interaction among Mexican-American students, their peers, and the teaching staff. School officials consistently channeled Tejano children into low-track classes with similar-working class, immigrant, and racially different children. Some based their assessments of mental, emotional, and language abilities on biased tests and classified Tejanos as intellectually inferior, culturally backwards, and linguistically deprived. The children were systematically placed in “developmentally appropriate” instructional groups or curricular tracks. At the elementary level, they were assigned to slow-learning or non-academic classes. At the secondary level, they were put in vocational or general-education classes. These policies set the Tejanos apart and deprived them of opportunities to succeed. (San Miguel, 2010, Para. 5)

Therefore, it can be assumed that between 1836-1900, traditional instructional pedagogy was influenced by the ideology and values of the Catholics, Protestants and British (San Miguel, 2010). Their biased
instructional practices promoted the heritage of Anglos consequently impacting the future education of Mexican-American children.

In support of the predictable impact, San Miguel (2010) further reveals that by 1940, the segregation of minorities had become a generalized establishment and Mexican-Texan (Tejano) children were channeled to be educated in Americanization schools at both the elementary and secondary levels. Americanization schools were meant to assimilate the Spanish-speaking children into the American culture by using the Three Cs Curriculum that encapsulated: a) common cultural norms; b) civics instruction; and c) command of English (San Miguel, 2010). The focus on erasing the Mexicans' ethnic culture, their Spanish language and their Mexican identities became fixed in the belief system of the decision-making administrators and educators.

The results of providing Mexican students with an inferior education yielded underfunded schools (San Miguel, 2010). These learning environments provided Mexicans with: 1) older and dilapidated buildings; 2) minimal and substandard recreation space; 3) inadequate school equipment and most detrimental 4) teachers who were untrained, and lacked experience or credentials (San Miguel, 2010).

As a result of an oppressive historical education, the 1960's Hispanic students were performing poorly on standardized tests (Jimenez, 1994). In 1961, while at the University of Southern California, Fred W. Marcoux wrote his master's thesis titled Handicaps of Bilingual Mexican Children (Jimenez,
Marcoux’s paper assessed the intellectual abilities of the Latino children. He confidently concluded:

- Mexican children had a lower average intelligence rating than the non-Mexican children and a higher frequency of mental retardation. They also are deficient on intelligence tests. This is due to environmental and cultural factors…such as a lack of rewarding human relationships which incites feelings of inadequacy. (Jimenez, 1994, p. 264)

The idea that the Mexican children’s’ culture and home environment was the reason for their poor school performance embedded the assumption that Hispanic parents are to blame for their children’s lack of literacy. Unfortunately, Marcoux’s perspective on the aptitudes of Latino children and his postulations echoed the opinions (as previously noted) voiced by the Texas school administrators and supported the educational practices already in place during the period between 1836-1900 (Jimenez, 1994).

Consequently, racist views reinforced by unjust educational policies contributed to the beginning of academic gaps for the Mexicans in English and mathematics. The children’s’ underdeveloped literacy, in these two essential subjects, greatly impacted the future schooling of Hispanic children. The historically inferior education of Hispanics was challenged under the umbrella of the Chicano Movement.

In 1968, the Los Angeles School District’s high school students who identified as Hispanics, Latinos, Mexicans, and Chicanos along with the
support of their families became a united front for change against what they felt was an inferior and substandard education (Jimenez, 1994). The majority of Mexican-American students in the district were enrolled in either industrial arts or shop classes for males or in-home education classes for females (Jimenez, 1994). Disheartened by their educational situation, Mexican students organized “walkouts”. Their efforts became El Movimiento or the Chicano Movement and their slogan, “Chicano Power”, brought their substandard education to the attention of the country.

Latinos involved in the Chicano Movement demanded the following:

- Mexican history taught in their history classes
- Mexican American literature to read
- More Mexican American teachers and administrators
- School facilities and equipment similar to the White schools
- College preparatory classes, not just industrial trades. (Jimenez, 1994)

Ruben Salazar, a Mexican reporter and columnist, for the Los Angeles Times described what Chicanos wanted: “Mexican Americans, though indigenous to the Southwest, are on the lowest rung scholastically, economically, socially and politically. Chicanos feel cheated. They want to effect change. Now” (as cited in Jimenez, 1994, p. 254). “Fifty years later, their bold action has reaped educational gains for Latinos, but they haven’t come fast enough, advocates say” (Romero, 2018).
Fifty years since the Chicano Movement called for an equitable education, and a pathway to college (Romero, 2018), Hispanic/Latino students continue to develop a pattern of trailing Anglos in state assessments. Romero (2018) states: “There’s not a single county in California where a majority of Latino students are proficient in math or English language arts” (as cited in The Majority Report, Education Trust West, 2017).

California’s Hispanic/Latino and English Learners’ Mathematical Gap

Hispanic/Latino and ELs are now faced with the consequences of a deep-rooted inferior education. According to Elias (2015), California’s biggest problem is not its water shortage, but the educational gap belonging to the largest ethnic group, Latinos. Gandara laments, “Latinos are performing at levels that will soon put the entire country at risk and consign these young people to a permanent underclass” (2009, p. 1).

The release of the 2017 Common Core mathematical data for over 2 million California Hispanic/Latino and English learners in grades three through eight and grade 11 revealed their mathematical gap (California Department of Education, 2017). The scores also confirmed the assumption that fourth-grade Hispanic/Latino and English learners are failing in mathematics because they lack the ability to solve written word problems. A mere 28% of California’s fourth-grade Hispanic/Latino students met grade-level mathematics standards
while 23% of the eighth-grade Hispanic/Latino students met grade-level mathematics standards.

The math scores for the fourth-grade ELs, who are predominately Spanish-speakers, show 15% and 6% of the eighth-grade ELs (also predominately Spanish-speaking) met grade-level mathematic standards (Ruffalo, 2017). In comparison, the fourth-grade White students almost doubled the Hispanic/Latino score with a respective 58% having met grade-level mathematics standards and almost tripled the ELs math score (California Department of Education, 2017).

California educates more than three million Hispanic/Latino students and it is predicted that by 2020, these students will become the majority (Education Trust-West, 2010). Research shows that most of these students attend the state’s lowest academically performing schools. Latinos also have the highest high school dropout rate (Elias, 2015).

Decades of traditional practice and a variety of adopted English language development programs that are thought of as supplementary are not working for these students. Hispanic/Latino and ELs need educators to understand their needs and to advocate for them, especially in the area of mathematics. One way is for teachers to begin to plan the scaffolding support for struggling Hispanic/Latino and ELs is by letting go of established doctrine and textbook-driven instruction especially in mathematics.
The Proposal: A Graphic Mathematical Mediated Structure

This project proposes a means that could help Hispanic/Latino and ELs attain mathematical literacy early in their education through the use of a tool to teach problem solving, a graphic mathematical mediated structure.

The agency of learning for Hispanic/Latino and English learners could be enhanced with a graphic mathematical mediated structure. The graphic organizer can be a powerful learning base for the learners to build conceptual understanding of fourth-grade fractions while building all the literacy domains needed to succeed in problem solving.

Underachieving Hispanic/Latinos and ELs who do not have a solid grasp of academic English are challenged to learn both English and content at the same time. Graphic mathematical mediated structures are similar to the KWL organizers used in language arts. The mediated structure is designed to dismantle a word problem in steps. Each step allows the teacher to integrate the ELD standards and the SMP’s with the Common Core standards. The literacy platforms in listening, speaking, reading, and writing could help prepare Hispanic/Latino and EL students for the demands of the Common Core mathematical state standards. The standards require students to comprehend multi-step story problems and explain and justify their solutions in written form.
Setting the pedagogy stage, begins with lessons that include:

(a) Methods of explicit and direct instruction that teaches conceptual understanding of math concepts and principles of a word problem, (b) Visual representation techniques designed to bridge a connection from verbal information to symbolic understanding by creating a mental model, (c) Using instructional feedback with peer–assisted learning strategies during instruction, and (d) Small group instruction, instructional modeling, corrective feedback, and student verbalizations. (as cited by Orosco, 2013, p. 46)

“Mediated structures are visual representations depicting the depth and complexity of scientific concepts, aspects, and detailed relationships” (Flores, 2010, p. 92). English vocabulary skills in math concepts impact the ability to problem solve (Orosco, 2010). The scaffolding/mediation provided by the visual organizer ensures students will build academic language and fractional concepts in a structured manner. As students, especially ELs, process through each part of the problem using the visual organizer, they will: practice and understand the vocabulary in context, make sense of problems, and persevere in solving them.

Almost two decades ago, the National Council of Teachers of Mathematics emphasized a writing component in the math education of students (as cited by Wilcox & Monroe, 2011). “Still many teachers struggle to
link writing and mathematics and honor the integrity of both disciplines at the
same time” (Wilcox & Monroe, 2011, p. 521).

Now, the state’s Common Core math standards emphasize problem solving; therefore, early development of critical thinking skills is necessary for students. Learners must be able to analyze story problems and explain in writing how they know their solution is correct (Jones & Duty, 2013).

Integrating writing and math is a common theme in the Common Core assessments.

Use of Graphic Organizers in the Field

Graphic organizers are widely used in classrooms. “The graphic organizer can make information more apparent, distinct, and articulate for the learner” (Egan, 1999, p. 641). According to Egan (1999), graphic organizers visually represent knowledge, structures information, arrange important ideas or topics into patterns with labels; thus there is little dispute of their versatile applications.

Additionally, Egan (1999, p. 642) reports that on the conceptual level, “When used appropriately, organizers can activate the learning schema, set purposes for learning, and increase the learner’s desire and capability to pursue additional information.” In the pre-reading stage the interaction with the organizer lets the reader know what he truly knows and what he thinks he knows (Egan, 1999). Egan offers teacher the following instructional suggestions in the use of organizers:
1. Model the organizer- Prepare students well Catch glitches - Complete the task on the organizer yourself Questions- Decide what questions might arise Experience the process- To be fully prepared to teach with organizers.

2. Promote interaction among students-Through individual, paired, or grouped Paired or grouped (best arrangement)- Encourages cooperation & social interaction- students learn to respect one another through creating, sharing, and disagreeing

3. Expand use of graphic organizer: Explore its use across the curriculum Pearson and Spiro (1982), state, “Under a teacher’s guidance they can derive great benefit from helping one another activate prior knowledge selectively” (as cited in Egan, 1999, p. 643).

Brennan, Dunlap, Cuyler, and Thomas, state that "Mathematics is the most difficult content area material to read because there are more concepts per word, per sentence, and per paragraph than in any other subject" (as cited in Braselton & Decker, 1994, p. 276). In addition, the text shifts from numbers to symbols to letters to graphics; requiring the reader to switch from one text type to another (Braselton & Decker, 19941). But, Clarke, Flood, Lapp & Farnan & Piccolo recommend an effective strategy to build math comprehension: the use of a graphic organizer (as cited in Braselton & Decker, 1994, p. 276). “The strength of graphic organizers lies in their ability to
visually relate elements of a story (or story problem), layout and design are important.” (Braselton & Decker, 1994, p. 276) Conceptualization of the problem has occurred when the student can describe it in his or her own words (Braselton & Decker, 1994, p. 276)

Graphic organizers provide a versatile structure that can contribute to content knowledge, build organization, comprehension, language, social interaction and depth of content. “Language plays an integral role in the processing of concepts, and mathematics is no exception” (Braselton & Decker, 1994, p. 281). “In other words, they visually show the learner how all the parts are connected to the whole and visually depict the potential” (Flores, 2010, p. 92.) Flores’s article specifically describes mediated/structures that can be used in science. The strategies she explains are deliberate actions educators can use to teach other disciplines in order to teach to students’ potential (2010). All disciplines require the proficiency and fluency of the accorded academic language: mathematics is no exception (Flores, 2010).

Theoretical Frameworks: Constructivism, Critical, and Sociocultural Theories

The theoretical frameworks for this project were a combination of constructivism, critical, and sociocultural theories. A constructivist lens reviewed the education of Hispanic/Latino and English learners within an historical context. The lens of critical theory was applied in interpreting the reasons for the Hispanic/Latino and English learners’ mathematical
achievement gap. The sociocultural theory guided the creation of an innovative pedagogy designed to mediate the learning/teaching process in acquiring mathematical knowledge and skills to solve mathematical word problems that contained the addition and subtraction of fractions.

This project presents a tool, a graphic mathematical mediated structure, to help build the cognitive abilities of underachieving Hispanic/Latino and English Learners. The design of the graphic mathematical mediated structure considered Vygotsky’s sociocultural idea that learning occurs in social situations (Vygotsky, 1978). Thus, the collaborative co-construction of solving mathematical word problems guided by the use of the graphic mathematical mediated structure, engages the students in social interactions (dialogues) that eventually become internalized/learned.

In other words, it is the social interactions that constitute a ZPD of what is internalized or learned by the individual. It is not only WHAT is carried out but also HOW the student eventually can solve the problem independently. (Newman, Griffin, & Cole, 1985)

Since the students do not have the skill or knowledge to solve mathematical word problems, a Zone of Proximal Development (ZPD) (Vygotsky, 1978) is created when the teacher engages and guides the students collectively in the step-by-step problem solving using the graphic mathematical mediated structure. The ZPD refers to the collaborative effort
where a teacher or more capable student works on a problem with a student who cannot yet solve it by herself or himself (Newman, Griffin, & Cole, 1985).

The Vygotskian sociocultural perspective holds that learning is a process of appropriating ‘tools’ for thinking, that are made available by teachers who create a zone of proximal development (Rogoff, 1990). Therefore, the graphic mathematical mediated structure is a tool that could be used to model word problems embedded with fractions to underachieving Hispanic/Latino and English learners in fourth grade. The benefit of the mediated structure is that it could help prepare the students for the rigor of the Common Core State Standards for Mathematics.

The mediated structure allows educators to plan a pedagogy that will incorporate the multiple California curriculum frameworks: The ELD Standards; the Common Core State Standards in Mathematics for fourth grade; and the Standards for Mathematical Practice. Providing Hispanic/Latino and ELs access to rigorous grade-level word problems to build their problem-solving abilities will help prepare the learners for the word problems of subsequent grade levels as well as narrow the groups’ mathematical achievement gap.

But more importantly, are the multiple roles of the teacher: 1) understands how the mathematical concepts for problem solving are learned; 2) embeds the ELD standards, the standards for mathematical practice and common core standards in the mediated structure; 3) creates a
“zone of proximal development;” 4) guides and mediates students collectively through the process of solving word problems using the graphic mathematical mediated structure; and 5) knows that social interactions become internalized thoughts (thinking). Thus, “the goal of the teacher or more capable peer should be to be to guide/mediate learners to do more and solve more difficult problems than they can do independently” (Díaz & Flores, 2001). And most importantly, the teacher becomes a sociocultural mediator, one who organizes teaching/learning as a social co-construction (Díaz & Flores, 2001).

Application of Critical Theory: Empowerment Across the Frameworks with Mediated Structure

The critical theory applicable to ELD is empowerment, as Hispanic/Latino and ELs have the highest high school dropout rate (Elias, 2015). Well-planned teacher lessons that model written word problems using the graphic mathematical mediated structure will help build English proficiency and literacy in underachieving Hispanic/Latino and English learners. Shifting the traditional paradigm of text-driven instruction, teachers can be empowered to use the graphic mediated structure to teach the ELD’s four domains: listening, speaking, reading, and writing. These key standards are vital to the success of these students. Proficiency in elementary fractions is a prerequisite of middle school algebra courses. Continued success in succeeding math courses will empower students to stay in school and motivate them to pursue well-paying careers that require mathematics.
The critical theory application for the Standards for Mathematical Practice, under the umbrella of focus, coherence, and rigor, is empowerment as well. The graphic mathematical mediated structure can facilitate the focus and depth required of the grade level standards - instruction of fractions and problem solving. Mathematical Practice 4 (Model with mathematics) requires students to be able to use multiple strategies to represent word problems visually through drawings, graphs, and equations. In addition, need to be able to explain and justify their solutions in writing.

Coherence refers to the vertical progression of content over time. (California Department of Education, 2015) According to Wu, adding and subtracting fractions is the beginning of elementary algebra (as cited in Brown & Quinn, 2007, p. 8). Fourth-graders can develop conceptual understanding of unit fractions through problem solving and the use of the graphic mediated structure. “Elementary algebra is the gate for higher mathematics and the ability to understand and manipulate fractions might be vital to success in algebra” (Brown & Quinn, 2007, p. 9).

Rigor comprises three components: conceptual understanding, procedural skill and fluency, and application. The graphic mediated structure encourages students to develop deep conceptual understanding of adding and subtracting unit fractions in the context of problem solving.
Conclusion

The review of the literature’s first theme discussed the identifying labels that fall under the umbrella of “Hispanic”. While having to use the moniker, “Hispanic/Latino”, “Latino”, “Mexican” and “English Learner” to represent the people I was writing about, I realized how offensive it is to lump diverse populations as one (Zaslavsky, 1994, p. 3) thus contributing to the deficit ideology that created the second theme to this review: The education of the Hispanic/Latino/Mexican, examined under an historic lens. Almost two hundred years ago, administrators and educators’ opinions were recorded of the Mexican school children as “inferior and retarded.” These labels and others have contributed to the long history of educationally underserving Hispanics, Latinos, Mexicans, and English Learners who speak Spanish.

The third theme in this review is the prevalent mathematical gap hindering almost 5 million California Hispanic/Latino and English Learner students. The math skill most needed by the groups is problem solving. The fourth theme addresses this need with a proposal to teach word problems using a graphic mathematical mediated structure. Vygotsky’s idea that children learn through the guidance of a more capable other such as a teacher and the collective knowledge within groups, is the principle idea behind the structure’s design. These students need to develop literacy skills in problem solving via a non-traditional instructional approach. The mediated structure is similar to the KWL chart and it is discussed in the fifth theme of this review: The use of
graphic organizers in the field. Teachers have successfully used the KWL organizer to teach language arts, so it is assumed that this project’s idea of a graphic mathematical mediated structure to teach word problems will also benefit the students and help to narrow the math gap. The sixth theme, the theoretical frameworks: sociocultural, constructivism and critical theory specifically speak to guiding the pedagogy Hispanic/Latino and English Learners need to succeed in mathematics. The final theme, applies Critical Theory across the frameworks: the need for a paradigm shift from text-driven instruction to one that not only incorporates the literacy domains during instruction and practice with word problems by addressing the specific to the needs of Hispanic/Latino and English learners; but also organizes the teaching/learning process in a systematic, coherent and deliberate way via the mathematical mediated structure that establishes a visual and social routine for solving mathematical word problems.
CHAPTER THREE

METHODOLOGY

Introduction

This project was designed to help fourth-grade educators plan for innovative instruction of problem-solving lessons embedded with fractions. The design of the pedagogy unifies the four domains of the English Language Development Standards (ELD), the Common Core State Standards (CCSS) in mathematics, particular to fourth grade problem solving, and the Standards for Mathematical Practices (SMPs).

The combination of these standards and programs, with a scaffolding tool, a graphic mathematical mediated structure, produces an innovative pedagogy that could help close the mathematical gap of underachieving Hispanic/Latino and English learners. The graphic mathematical mediated structure is not just an organizer, but a unique visual tool that supports the way instruction of fractions embedded in word problems can be read, discussed, drawn, organized, understood, justified, and ultimately solved.

It should be noted that it is not just the graphic mathematical mediated structure that organizes the goal, solving word problems with fractions, but it is the social collaborative interaction that mediates the learning and internalization of the process to solve the word problems in math that is most significant. This is based on Vygotsky’s (1978) theoretical tenet that knowledge is socially constructed based on social interaction.
Fourth Grade was chosen for this project because the majority of fifth-grade Hispanic and English learners studied had dismal fourth-grade CST scores from the previous year. Although fraction instruction begins in third grade, the students needed lengthy review of basic concepts, for example on the conceptual understanding of unit fractions. The traditional lesson of the visual pie chart divided into different parts had not given students enough depth to be able to manipulate fractions, especially when they were embedded in word problems. Many were adding both the numerator and denominator and believing this was the procedure but did not understand that fractions with unlike denominators represent different portions.

As mentioned earlier, while at California State San Bernardino, in Dr. Barbara Flores’s class, I had learned about mediated structures. One of the cooperative assignments was to list the elements of different fairy tales on a matrix. The completed complicated, and co-constructed graphic meditational structure was visually informative, colorful, and organized. It built literary elements, content vocabulary, group collaboration, and it generated a lot of discussion amongst the participants. Dr. Flores then assigned students to develop a matrix in a content area. Hence, I borrowed the professor’s idea to expand my students’ ability in the area of problem solving.

Geometry Matrix Development

In English language arts, teachers use a variety of graphic organizers/mediated structures to teach reading and writing processes. My
students and I had been struggling through the math word problems in the geometry chapter of the textbook, so I decided to develop a mathematics graphic organizer/mediated structure modeled after the language arts matrix I had learned from Dr. Flores.

Dr. Flores’s suggestions helped me design the following 4-column mediated structure. Then, as each triangle and its corresponding traits were introduced, the class helped fill it in. The matrix was displayed on the classroom math wall, throughout its completion and it became an anchor chart for students to reference. The class later developed other concept matrices, similar to the ones developed for this project.

For the purpose of this undertaking, the following is a shortened version of the original geometry matrix collectively built by the students and me. Previous to this problem-solving graphic mathematical mediated structure, the triangles had each been introduced and extensive works with manipulatives had been completed. Protractors, hands-on construction of triangles using orange peels and toothpicks, literature pertaining to triangles had been read, anchor charts had been developed in group settings after initial graphic organizers had given students experience processing through them. Before students were asked the following question, they had background knowledge on: Equilateral, Isosceles, Scalene, Right, Acute, and Obtuse triangles.
Problem—Write it!

The measure of two angles of a triangle equal 120°.

What is the measure of the third angle?

Is the triangle acute, equilateral, or right?

Use math vocabulary in your explanation.

Draw it!

Math it!

Explain it!

<table>
<thead>
<tr>
<th>Problem—Write it!</th>
<th>Draw it!</th>
<th>Math it!</th>
<th>Explain it!</th>
</tr>
</thead>
<tbody>
<tr>
<td>The measure of two angles of a triangle equal 120°.</td>
<td></td>
<td>60° + 60° + 60° = 180°</td>
<td>1. The measure of the third angle is 60°.</td>
</tr>
<tr>
<td>What is the measure of the third angle?</td>
<td></td>
<td></td>
<td>2. The triangle is an equilateral because it has three congruent sides. All the angles have to be 60° because two are given and the triangle can only measure 180°</td>
</tr>
<tr>
<td>Is the triangle acute, equilateral, or right?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use math vocabulary in your explanation.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. Geometry Mediated Structure

Planning the Pedagogy

I specifically planned for each problem that I modeled or that the students and I would co-construct. Ahead of the task, I would work the problem out on the same template that I would use to model with the document camera. The students would get a copy of the template with the word problem on it, but the rest of the template had to be co-constructed during instruction.

Planning ahead allowed me to: 1) plan a review of background knowledge that students would need in order to be successful; 2) preview and select the academic vocabulary; 3) plan the visuals; 4) form the math
sentence; 5) explain the solution in written form using the academic vocabulary; 6) plan for appropriate manipulatives or tools to insure students would have the opportunity to understand as much of the problem the first time it was modeled; and 7) anticipate what part of the lesson would be confusing for the students that needed additional support, as well as plan for those that might need to be challenged.

As I developed instruction for each word problem, I became aware of the reasons my underachieving Latino and English learners might find them difficult: language forms (reading), too much information (multi-step), abstract symbols, and not enough examples of the variety of problems. Therefore, it became a habit of mine to work out the word problems from the review section of the chapter and teach those that aligned to the lessons in that chapter before the students arrived at the problem-solving review. Usually, by the end of the chapter, the class and I had already completed most of the problem-solving exercises in this section.

Solving word problems became a daily habit/routine for the class. As the year progressed, the graphic mathematical mediated structures that the students and I constructed for each chapter, as well as those completed in groups, or individually became anchor charts and were added to the math wall, or wherever space allowed until the end of that chapter’s instruction. For homework exercises, only problems similar to what we had already solved, would be assigned.
I would make double-sided homework pages for the students. On one side would be the problems to work out, and on the other side would be a copy of a similar problem that we had solved together. My goal was for the students to be successful with problem solving, so they would not fear them.
CHAPTER FOUR

SUMMARY, IMPLICATIONS, AND RECOMMENDATIONS

SUMMARY

Having years of experience working with Hispanic/Latino and English learners (ELs), I found this topic worthy of pedagogical and generative research. Two major reasons paved my commitment to become fully engaged with this project. The first one addressed the underachievement, of Hispanic/Latino and ELs, in mathematics. The second one was about the state and district curriculum that were considered, in order to develop a mathematical tool, the mediated structure, to improve the problem-solving skills for these students.

This project was developed under the assumption that the reason my fifth-grade Hispanic and English learners did not do well in the reasoning portion of the CST’s at the end of their fourth year was because they lacked problem-solving abilities. The students generally loathed problem-solving exercises and exhibited anxiety when attempting them.

The literature review confirmed that Hispanic/Latino and ELs are not succeeding in mathematics. They needed their mathematics education drastically altered from its current customary delivery. The literal misconception that math is a universal language has historically permeated in education and thus has contributed to the traditional teaching pedagogy; one size fits all. By-the-book math instruction cements the idea that the language
of math is an ordinary dialect. As a result, Hispanic/Latino and ELs are assumed to be able to understand the given lexicon of math.

Yet, the math scores analyzed from this research attests to one of the reasons I was dedicated to this project; it revealed the current bleak math outcomes Hispanic/Latino and ELs are garnishing in state tests, across the United States. This project was particularly concerned with the mathematics’ instruction in a state that educates one third of the nation’s ELs and millions of Hispanic/Latino students in California.

The consensus, in the literature examined, was that all students need English literacy to be successful with the new Common Core State Standards (CCSS) in Mathematics. Therefore, an assumption can be stated: a component of math instruction should be English support, for struggling learners. Incorporating the four English language development domains (ELD) in mathematical lessons with Hispanic/Latino and ELs would give these students opportunities to understand story problems. The learners must be able to listen, interpret, speak, and transcribe the meaning of the problems in English. Additional agreement within the literature review found that the Standards for Mathematical Practice (SMP’s) are skills that will further develop math literacy in all students.

The research further implicated the validity of my concern. A reason for the underperformance of Hispanic/Latino and ELs in math was that they lack comprehension skills to problem solve. California’s Department of Education
(website) math data clearly show low scores for these groups of students as evidenced by Latino/Hispanic 74.8% and English learners 87.7% not meeting proficiency. The three claim areas that students must score well in are: 1) Concepts and procedures; 2) Problem solving/ modeling/ and data analysis; and 3) Communicating reasoning. Each of these three regions present major hurdles for students who lack the literacy skills the CCSS word problems require (CDE).

Confirmation for the development of the mathematical graphic organizer was also validated as research placed conceptual understanding of mathematics concepts under the literacy umbrella. Listening, speaking, reading, and writing in math are essential components that a mathematical mediated structure can scaffold. This tool could definitely help fourth-grade Hispanic/Latino and ELs gain fraction knowledge at the conceptual level, so that they could begin to build mathematical discourse skills in order to understand more complex problems in subsequent grades.

The literature review continually supported the assumption that instructional practices that incorporated the four ELD domains and the Standards for Mathematical Practice would positively impact the acquisition of problem solving skills the targeted groups appeared to lack. The mathematical graphic mediational structure developed/used with the teaching/learning of the addition and subtraction of fractions in word problems definitely mediated and taught the students to their potential—solving word problems with fractions.
with proficiency as evidenced by 75% of my students scoring at proficient and advanced levels on the California State Test (CST).

Implications

This project was intuitive and interpretive at best. The design and use of a math-mediated structure for my students was a spontaneous response to their reaction when math lessons included word problems. The anxiety that traditional instruction was instilling and perpetuating in my Latino and English learners during word problem exercises was their call for help. Perhaps this was the reason their math scores on the previous year’s CST showed an imbalance between procedural skills and problem solving. Their distress was a signal—to try a different pedagogical approach to teaching story problems as opposed to the traditional way.

As the year progressed, many mathematical concept graphic organizers were created with teacher guidance, in cooperative groups and by individual students. I began to see a huge difference in the way students were engaging with the word problems, sharing information in groups, and justifying their solutions through class presentations. In addition, they were also succeeding in chapter tests that included problem solving. Vygotsky’s (1978) sociocultural theory in action was at work, i.e., “knowledge is socially constructed through social interaction.”

The class’s end-of-the-year results on the California Standards Test (CST) in mathematics indicated that the strategy of using a mathematical
graphic organizer was a success. It appeared to have helped all of my students. The Hispanic/Latino and ELs showed growth not only on procedural knowledge, but also conceptual understanding; two targeted goals. As already noted, 75% of my students scored proficient or advanced in the state exam.

In addition, most Latino, ELs, African-American, and Special Education students scored one or two levels above the previous years’ state mathematics assessment. The growth the targeted learners made in math problem solving implies that a mathematical graphic organizer/mediated structure along with the pedagogy used can enhance Hispanic/Latino and second language speakers’ math abilities, as well as further develop their mastery of the four English Language Development domains: listening, speaking, reading, and writing.

Recommendations

The 2017 California state data has shown the existence of a wide mathematical gap in Latino and English learners as compared to White and Asian students. Reforming the way teachers and students feel about problem solving is paramount. The mathematical graphic organizer/mediated structure and the pedagogy can offer students and teachers successful techniques to synchronize mathematical knowledge to add and subtract fractions, while also building their mathematical reasoning skills, vital to written word problems. Likewise, the mediated structure embeds the complexity of the mathematical
concepts as well as organizes the teaching of a patterned interactive procedure that mediates the internalization for solving word problems.

In addition, the mathematics’ achievement gap that continues to shadow California’s Hispanic/Latino/EL students implies that educators need to review pedagogical practices that are in the best interest of students. The use of graphic mediational structures to tackle math word problems could transform current practices of instruction that are not working, in narrowing the achievement gap. The use of this viable tool in the teaching of math problem solving is especially valid today.

The idea of using graphic organizers to support the acquisition of math concepts is not new, but their use is not a widespread practice. Most of the articles read on their implementation of graphic organizers have been positive.

The investment of teacher created mathematical mediational structures specific to understanding fraction word problems can pay off in an enormous fashion; thus, narrowing the math gap prevalent of Hispanic/Latino and English learners. Furthermore, the benefits of scaffolding problem solving with an interactive structure are manifold: 1) conceptual understanding; 2) content vocabulary attainment; 3) perseverance in problem solving; 4) creating visual models of abstract concepts; 5) providing written justifications to defend an answer using the exclusive vocabulary in context; 6) verbally using the lexis of mathematics in discourse; and 7) intrinsic acquisition of the Standards for Mathematical Practices.
The visual scaffolding developed in a mathematical graphic organizer can help students understand fraction concepts. By using mediational structures, underperforming students can be prepared for the demands made by the state’s rigorous Common Core math curriculum. Unless pedagogy that works for them are designed, Hispanic/Latino and English learners will not ready for the requirements set by Common Core. Teachers are challenged to provide students both a balanced English and Mathematics’ education. All instructors: primary, middle and high school, must evaluate their own math practices and ensure that instruction addresses the needs of Latino/Hispanics and English learners.

The literature review offered an astronomical amount of suggestions for strategies that work with English learners, for example, in the area of vocabulary development. But, mathematical pedagogy specifically modeling word problems that extend the literacy domains or that address how to incorporate the Standards for Mathematical Practices is needed for educators. Math pedagogy tailored to Hispanic/Latino students who have been reclassified but are not doing well in mathematics is also a topic for research development.

In conclusion, the pedagogy along with the graphic mathematical mediated structure appears to have worked for all the demographic groups in my class; White, Hispanic/Latino, English learners, African-American, and Special Education students. Without foundational problem-solving skills
struggling Hispanic/Latino and English learners are unlikely to reach their mathematical potential, thus excluding them from higher math courses, college, and higher-level occupations, where mathematical knowledge and problem solving is a prerequisite.
APPENDIX A

PROBLEM 1
**Problem A: Teacher/Student Template**
Addition of Fractions with Like Denominators

If a painter poured $\frac{1}{4}$ gallon of yellow paint and $\frac{1}{4}$ gallon of red paint into a bucket. 
How much paint is in the bucket?

<table>
<thead>
<tr>
<th>K</th>
<th>What do I know?</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>What question do I need to answer?</td>
</tr>
<tr>
<td>C</td>
<td>Conditions that I notice...</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>What is my plan?</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>Show the work with a model</td>
</tr>
</tbody>
</table>
| J | Justify 
Write an equation that matches the model. |

**Summarize:**

**Extension/Explain:** Is the answer $>$ or $<$ than 1? Can you write a similar problem for your group to solve? Can you find an equivalent fraction for the answer?

---

Suggested Steps for Teaching with Math Organizers Problem A

Explain that they will use a math organizer that will make a word problem seem easier to understand and easier to solve.

Materials needed: Document camera, Problem A Teacher Template & Student Template, sheet of paper (for group summary)

- Pass out student template for Word Problem A.
- Use the document camera to project the teacher template.
- Preview each section of the organizer with students before working through it.
- Word Problem A: Model reading the word problems to students then, have them echo read it back to you.
- Engage students to discuss information that the problem reveals such as, is there more yellow paint being poured than red or is it an equal amount? Are they being poured into the same bucket or different buckets?
- Discuss with students the subjects' attributes: For example, there is $\frac{1}{4}$ gallon of red paint and $\frac{1}{4}$ gallon of yellow paint
- Model writing this information in the K (What do I know?) section. Then, give students time to add it to their organizer.
- Next, guide students to find the question in the problem.
- Model adding the question to the W (What question do I want to answer?) section and underline How much paint
- Explain that there is more than one way to ask the same question.
- Then, model Rephrasing the question so that the word “total” can be used in context:
How much total paint is in the bucket?

- Give students time to add the questions to their organizer.
- Explain C (Conditions that I notice…) section and emphasize what you notice: For example, you notice that the denominators are the same. You also notice that the problem asks for the total amount of paint in the bucket. You notice that because it is a problem asking for a total, you can use addition as the order of operation, to find the answer to the question.
- Model writing what you noticed, in the C section. Then, give students time to add this information to their organizer.
- Explain that in the P (What is my plan?) section you need to describe what you will do to solve the question.
- Model referencing the information, in the C section as you plan for the P section. You might explain: The two fractions have the same denominator so they can be easily added together. The question wants to know how much total paint has been mixed so I will add the fraction of a gallon of the red paint to the fraction of a gallon of the yellow paint.
- Discuss the unit of measure (fourths)
- Model how students can add fractions of the same denominator and add this information to your P section. Give students time to do the same.
- Explain that in the Show section, a model is drawn to show the amount of red paint and yellow paint being added to the bucket.
- Model by using a number line that represents the units of measure (fourths).
• Explain that you will draw a line to the fraction $\frac{1}{4}$ for the red paint and then another $\frac{1}{4}$ for the yellow paint, giving you a total of $\frac{2}{4}$.

• Show that this can be represented by a pie chart that is split into fourths as well.

• Explain that to show that you can use a pie chart shade in one of the fourths and label it red and shade in another fourth and label it yellow.

• Be sure to show all work then, wait for students to fill in their Show section.

• Explain the Justify section. Elaborate that the equation you will write here will explain the model in the “Show” section. Then, write an equation that matches the number line.

• Group students in groups of four and assign the following:

A. Discuss the sections in your organizers using the math vocabulary to explain each section to each other.

B. On a sheet of paper, the group will write a summary that explains the answer to the question using the math vocabulary.

   C. Students will practice reading the summary to their group.

   D. Each group selects one student to read the summary to the class.

• Summarize what students share in the Summarize section of the teacher template then, give students time to write their summary in their organizer.

• Use the Extension/Explain section to further prompt students.
**Problem A: TEACHER MODEL**

Addition of Fractions with Like Denominators

<table>
<thead>
<tr>
<th>K</th>
<th>What do I know?</th>
<th>W</th>
<th>What question do I need to answer?</th>
<th>C</th>
<th>Conditions that I notice...</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• There is yellow and red paint in the bucket.</td>
<td>• How much paint is in the bucket?</td>
<td>• The denominators are the same so they are the same size of parts.</td>
<td></td>
<td>• There is one bucket.</td>
</tr>
<tr>
<td></td>
<td>• There is $\frac{3}{4}$ of a gallon of yellow paint and $\frac{1}{4}$ of a gallon of red paint in the bucket.</td>
<td><strong>Rephrase:</strong> How much total paint is in the bucket?</td>
<td>• I am putting in 2 different paints.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**P** What is my plan?

I will add the $\frac{3}{4}$ yellow paint and the $\frac{1}{4}$ red paint because I am joining them together.

**Show the work with a model**

![Model Diagram]

**Justify**

Write an equation that matches the model.

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$$

**Summarize:** I added $\frac{1}{4}$ of a gallon of yellow paint and $\frac{1}{4}$ of a gallon of red paint. These became mixed and now there is $\frac{2}{4}$ of a gallon of paint in the bucket.

**Extension/Explain:** Is the answer $>$ or $<$ than 1? Can you write a similar problem for your group to solve? Can you find an equivalent fraction for the answer?

---

APPENDIX B

PROBLEM 2
Problem B: Teacher/Student Template
Addition of Fractions with Like Denominators

Justin read $\frac{2}{6}$ of his book on Monday, $\frac{1}{6}$ of his book on Tuesday, and $\frac{2}{6}$ of his book on Saturday. How much of his book did Justin read in all?

<table>
<thead>
<tr>
<th>K</th>
<th>What do I know?</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>What question do I want to answer?</td>
</tr>
<tr>
<td>C</td>
<td>Conditions that I notice…</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>What is my plan?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Show the work with a model</td>
<td></td>
</tr>
<tr>
<td>Justify Write an equation that matches the model.</td>
<td></td>
</tr>
</tbody>
</table>

Summarize:

Extension/Explain: Is the answer $>$ or $<$ than 1? Can you write a similar problem for your group to solve? Can you find an equivalent fraction for the answer?
Suggested Teaching Steps

Explain that they will use a math organizer that will make a word problem seem easier to understand and easier to solve.

Materials needed: Document camera, **Problem B** Teacher Template & Student Template, vocabulary cards, sheet of paper or chart paper (1 per group summary)

- Pass out student template for Word Problem B.
- Use the document camera to project the teacher template.
- Preview each section of the organizer with students before working through it.
- **Word Problem B:** Model reading the word problem to students, then have them *echo read* it back to you.
- Engage students to discuss information that the problem reveals such as. *Did Justin read his entire book in one day or did he split up his reading into multiple days? Is he done with his book yet or does he still have some more to read?*
- Discuss with students the subjects' attributes: For example, Justin read $\frac{2}{6}$ of his book on Monday, $\frac{1}{6}$ of his book on Tuesday and another $\frac{2}{6}$ of his book on Saturday.
- Model writing this information in the *K (What do I know?)* section. Then, give students time to add it to their organizer.
- Next, guide students to find the question in the problem.
- Model reading the question again. Have students echo read it back to you.
- Model adding the question to the *W (What question do I want to answer?)* section and underline *How much of his book did Justin read in all?*
- Rephrase the question if needed.
- Give students time to add the question to their organizer.
- Explain *C (Conditions that I notice...)* section and emphasize what you notice: For example, you *notice* that the denominators are the same. You *also notice* that the problem *asks for the total amount of reading that Justin did*. You *notice* that because it is a problem asking for a total (in all), you can use *addition* as the order of operation, to find the answer to the question.
• Model writing what you noticed, in the C-section. Then, give students time to add this information to their organizer.

• Explain that in the P (What is my plan?) section you need to describe what you will do to solve the question: Make a plan or plan a strategy.

• Model referencing the information, in the C-section as you plan for the P section. You might explain: The three fractions have “like” denominators, or the denominators are alike (same), so they can be added. You might explain: The question wants to know how much of his book Justin has read in total, so I will add the fraction of the book he read on Monday to the fraction of the book he read on Tuesday and then add the fraction of the book, he read on Saturday

• Discuss the unit of measure (Sixths).

• Model how students can add fractions of the same denominator and add this information to your P section. Give students time to do the same.

• Explain that in the Show section, a model is needed to show the fractions of the book Justin read.

• Model by using a number line that represents the units of measure (sixths).

• Explain that you will draw a line to the fraction $\frac{2}{6}$ to show the amount read on Monday, and another $\frac{1}{6}$ to show the amount read on Tuesday, and another $\frac{2}{6}$ to the amount read on Saturday giving you a total of $\frac{5}{6}$. Label these: M, T, S.

• You may show that this can be represented by a pie chart that is split into sixths as well, or use any other manipulative.

• Explain that to show that you can use a pie chart shade in one of the sixths and label it Tuesday and shade in another fourth sixths and label them Monday and Saturday.

• Be sure to show all work then, wait for students to fill in their Show section

• Explain the Justify section. Elaborate that the equation you will write here will explain the model in the “Show” section. Then, write an equation that matches the number line.

• Summarize what students share in the Summarize section of the teacher template then, give students time to write their summary in their organizer.
• Or
• Group students in groups of four and assign the following:
  o A. Ask them to discuss the sections in their organizers using the math vocabulary to explain each section to each other.
  o B. On a sheet of paper or chart paper, the group will co-construct a summary that explains the answer to the question using the math vocabulary.
  o C. Ask students to practice reading the summary to their group.
  o D. Ask each group to select one student to read the summary to the class.
• Use the Extension/Explain section to further prompt students.

When students are ready to solve word problems individually, with each other, or in a cooperative group, it is important to allow them to co-construct.

Give students a blank template or they can create their own. Assign a word problem, or students may write one their own, or mimic one already solved.

• Group students or allow them to choose their partner/s (no more than 3 or 4)
• Ask them to repeat the steps to work the problem:
  o A. Discuss the sections in the organizer using the math vocabulary to explain each section to each other.
  o B. On their organizer, each member of the group will write a summary that explains the answer to the question using the math vocabulary.
  o C. Students will practice reading the summaries to each other.
  o D. Each group selects one student to read the summary to the class.
Be sure to give students feedback on their work, participation, and effort.
**Problem B: TEACHER MODEL**
Addition of Fractions with Like Fractions

<table>
<thead>
<tr>
<th>K</th>
<th>What do I know?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Justin read $\frac{2}{6}$ of his book on Monday, $\frac{1}{6}$ of it on Tuesday, and $\frac{2}{6}$ of it on Saturday.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>W</th>
<th>What question do I want to answer?</th>
</tr>
</thead>
<tbody>
<tr>
<td>How much of his book did Justin read <strong>in all</strong>?</td>
<td></td>
</tr>
</tbody>
</table>

**Rephrase:** How much of the book has Justin read so far?

<table>
<thead>
<tr>
<th>C</th>
<th>Conditions that I notice...</th>
</tr>
</thead>
<tbody>
<tr>
<td>• &quot;Like&quot; denominators</td>
<td></td>
</tr>
<tr>
<td>• Justin read parts of the book.</td>
<td></td>
</tr>
<tr>
<td>• &quot;In all&quot; means altogether, so it is an addition problem.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>What is my plan?</th>
</tr>
</thead>
<tbody>
<tr>
<td>The denominators are the alike, so I can only add the numerators</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Show</th>
<th>the work with a model</th>
</tr>
</thead>
<tbody>
<tr>
<td>M T S</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Justify</th>
<th>Write an equation that matches the model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{2}{6} + \frac{1}{6} + \frac{2}{6}$</td>
<td>$2 + 1 + 2 = \frac{5}{6}$</td>
</tr>
</tbody>
</table>

**Summarize:** I added the three parts $\frac{2}{6}$, $\frac{1}{6}$, and $\frac{2}{6}$ that Justin read over three days. I then set up my equation: $\frac{2}{6} + \frac{1}{6} + \frac{2}{6}$. Since the denominators were "like"(same), I only needed to add the numerators. Hence, the answer is $\frac{5}{6}$. My work shows that Justin read more than half the book, but less than the whole book.

**Extension/Explain:** Is the answer $>$ or $<$ than $1$? Can you write a similar problem for your group to solve? Can you find an equivalent fraction for the answer?

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Graphic Mathematical Mediated Structure © Sylvia Casteloes 2018

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APPENDIX C:

PROBLEM 3
### Problem C: Teacher/Student Template

Addition of Fractions with Like Denominators

Jose rode his bicycle \(\frac{1}{5}\) mile to Angel's house, and they both rode \(\frac{3}{5}\) mile to the soccer field. How far did Jose ride his bicycle? Explain how you found your answer.

<table>
<thead>
<tr>
<th></th>
<th>What do I know?</th>
<th>What question do I want to answer?</th>
<th>Conditions that I notice...</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>What is my plan?</th>
<th>Show the work with a model</th>
<th>Justify</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Write an equation that matches the model.</td>
</tr>
</tbody>
</table>

**Summarize:**

**Extension/Explain:** Is the answer > or < than 1? Can you write a similar problem for your group to solve? Can you find an equivalent fraction for the answer?

---

Suggested Teaching Steps  Problem C

Explain that they will use a math organizer that will make a word problem seem easier to understand and easier to solve.

Materials needed: Document camera, Problem C Teacher Template & Student Template, vocabulary cards, sheet of paper or chart paper (1 per group summary)

- Pass out student template for Word Problem C.
- Use the document camera to project the teacher template.
- Preview each section of the organizer with students before working through it.
- Word Problem C: Model reading the word problem to students then have them echo read it back to you.
- Engage students to discuss information that the problem reveals such as, who is the subject? How far did Jose ride his bike, and did he ride to the soccer field before he went to Angel’s house?
- Discuss with students the subjects’ attributes: For example, Jose rode $\frac{1}{5}$ of a mile to Angel’s house and together they rode $\frac{3}{5}$ of a mile to the soccer field.
- Model writing this information in the K (What do I know?) section. Then, give students time to add it to their organizer.
- Next, guide students to find the question in the problem.
- Model reading the question again and have the students echo read it back.
- Model adding the question to the W (What question do I want to answer?) section and underline How far did Jose ride his bike?
- Rephrase the question if needed.
- Give students time to add the question/s to their organizer.
- Explain C (Conditions that I notice…) section and emphasize what you notice: For example, you notice that the denominators are the same. You notice the problem only asks about Jose. You also notice that the problem asks for the total distance that Jose rode. You notice that because it is a problem asking for a total, you can use addition as the order of operation, to find the answer to the question.
• Model writing what you noticed, in the C-section. Then, give students time to add this information to their organizer.
• Explain that in the P (What is my plan?) section you need to describe what you will do to solve the question: Make a plan or plan a strategy.
• Model referencing the information, in the C-section as you plan for the P section. You might explain: The two fractions have the same denominator so they can be added. The question wants to know how far Jose rode his bike, so I must add the distance that he rode to Angel’s house, to the distance that he and Angel traveled together to the soccer field.
• Discuss the unit of measure (Fifths).
• Model how students can add fractions of the same denominator and add this information to your P section. Give students time to do the same.
• Explain that in the Show section, a model is drawn to show the total distance that Jose traveled. Use manipulatives if necessary.
• Model by using a number line that represents the units of measure (Fifths).
• Explain that you will draw a line in blue to the fraction \(\frac{1}{5}\) for the distance that Jose traveled to Angel’s house. From the end of the blue line you will count \(\frac{3}{5}\) to the right and end at \(\frac{4}{5}\); draw this line in red. The red line represents the distance from Angel’s house to the soccer field.
• Be sure to show all work then, wait for students to fill in their Show section.
• Explain the Justify section. Elaborate that the equation you will write here will explain the model in the “Show” section. Then, write an equation that matches the number line.
• Summarize what students share in the Summarize section of the teacher template then, give students time to write their summary in their organizer.
• Or
• Group students in groups of four and assign the following:
  o A. Ask them to discuss the sections in their organizers using the math vocabulary to explain each section to each other.
B. On a sheet of paper or chart paper, the group will co-construct a summary that explains the answer to the question using the math vocabulary.

C. Ask students to practice reading the summary to their group.

D. Ask each group to select one student to read the summary to the class.

- Use the Extension/Explain section to further prompt students.

When students are ready to solve word problems individually, with each other, or in a cooperative group, it is important to allow them to co-construct.

Give students a blank template or they can create their own. Assign a word problem, or students may write one heir own, or mimic one already solved.

- Group students or allow them to choose their partner/s (no more than 3 or 4)
- Ask them to repeat the steps to work the problem:
  A. Discuss the sections in the organizer using the math vocabulary to explain each section to each other.
  B. On their organizer, each member of the group will write a summary that explains the answer to the question using the math vocabulary.
  C. Students will practice reading the summaries to each other.
  D. Each group selects one student to read the summary to the class.
- Be sure to give students feedback on their work, participation, and effort.
**Problem C: TEACHER MODEL**
Addition of Fractions with Like Denominators

<table>
<thead>
<tr>
<th>K What do I know?</th>
<th>W What question do I want to answer?</th>
<th>C Conditions that I notice...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jose rode (\frac{1}{5}) of a mile to Angel’s house and He rode an additional (\frac{3}{5}) of a mile to the soccer field.</td>
<td>How far did Jose ride his bicycle? <strong>Rephrase:</strong> What was the total distance Jose rode his bike?</td>
<td>I notice that the problem only wants to know about Jose. I notice that there are two parts to the problem. I notice Jose rode to two different places. I notice miles is the unit of measure.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P What is my plan?</th>
<th>Show the work with a model</th>
<th>Justify</th>
</tr>
</thead>
</table>
| I have to add the two distances Jose rode his bike. The fractions have a common denominator, so I can add the numerators to find out how far Jose rode his bicycle. | ![Fraction Model Diagram](image) to Angie’s house | Write an equation that matches the model. 

\[
\frac{1}{5} + \frac{3}{5} = \frac{4}{5}
\]


**Summarize:** I understand that Jose rode to two different places, so I added the two distances. First he rode \(\frac{1}{5}\) of a mile to Angie’s house. Then, he rode an additional \(\frac{3}{5}\) of a mile to the soccer field. In all, Jose rode his bike \(\frac{4}{5}\) of a mile.

**Extension/Explain:** Is the answer \(>\) or \(<\) than 1? Can you write a similar problem for your group to solve? Can you find an equivalent fraction for the answer?

The smallest mammals on Earth are the bumblebee bat and the Etruscan pygmy shrew. The length of a bumblebee bat is $1 \frac{1}{2}$ inches. The length of an Etruscan pygmy shrew is $1 \frac{3}{5}$ inches. How much smaller is the bat than the shrew?

<table>
<thead>
<tr>
<th>K</th>
<th>W</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Show the work with a model Justify

Summarize:

Extension/Explain: Is the answer $>$ or $<$ than 1? Can you write a similar problem for your group to solve? Can you find an equivalent fraction for the answer?
**Suggested Teaching Steps**

Explain that they will use a math organizer that will make a word problem seem easier to understand and easier to solve.

**Problem D**

Materials needed: Document camera, Problem D Teacher Template & Student Template, vocabulary cards, sheet of paper or chart paper (1 per group summary)

- Pass out student template for Word Problem D.
- Use the document camera to project the teacher template.
- Preview each section of the organizer with students before working through it.
- *Word Problem D:* Model reading the word problem to students then have them *echo read* it back to you.
- Engage students to discuss information that the problem reveals such as, *the bat is smaller than the shrew or the shrew is larger than the bat.*
- Discuss with students the subjects’ attributes: For example, the bat is $1 \frac{1}{5}$ inches long and the shrew is $1 \frac{2}{5}$ inches long.
- Model writing this information in the *K (What do I know?)* section. Then, give students time to add it to their organizer.
- Next, guide students to find the question in the problem.
- Model reading the question again and have the students echo read it back.
- Model adding the question to the *W (What question do I want to answer?)* section and underline: *How much smaller* part of the question.
- Explain that there is more than one way to ask the same question.
- Give students time to add the question to their organizer.
- Explain *C (Conditions that I notice…)* section and emphasize what you notice: For example, you *notice* that the bat is smaller than the shrew. You also *notice* that the problem compares the difference in length, of the two subjects. You *notice* that because it a comparing problem, you can
use *subtraction* as the order of operation, to find the answer to the question.

- Model writing what you noticed, in the C-section. Then, give students time to add this information to their organizer.
- Explain that in the *P* (What is my plan?) section you need to describe what you will do to solve the question: Make a plan or plan a strategy.
- Model referencing the information, in the *C-section* as you plan for the *P* section. You might explain:
The bat and the shrew are being compared. The question wants to know how much smaller the bat is, so I will take the shrew’s size and subtract it from the bat’s size to find out.
- Discuss the unit of measure (fifths).
- Model how students can decompose the mixed fraction \(1 \frac{2}{5}\) to get \(\frac{7}{5}\).
  Remind students that you will rename “1” to \(\frac{5}{5}\) and then add \(\frac{2}{5}\) to get \(\frac{7}{5}\).
The length of the shrew is \(\frac{7}{5}\) inches.
- Model how students can decompose the mixed fraction \(1 \frac{2}{5}\) to get \(\frac{6}{5}\). The bat is \(\frac{6}{5}\) inches in length. Add this information to your *P* section. Give students time to do the same.
- Explain that in the *Show* section, a model is drawn to compare the lengths of the bat and the shrew.
- Model by drawing a number line that represents the units of measure (fifths).
- Explain that you will draw a line to the fraction \(\frac{7}{5}\) because this represents the length of the shrew. Then label it “shrew”.
- Explain that to show that you are going to subtract the length of the bat from the length of the shrew, you will have to count backwards six fifths \((\frac{6}{5})\) from \(\frac{7}{5}\). Draw your line and label it “bat”.
- Be sure to show all work then, *wait* for students to fill in their *Show* section.
• Explain the Justify section. Elaborate that the equation you will write here will explain the model in the “Show” section. Then, write an equation that matches the number line.

• Explain that you will use the CHECK method to be sure your equation is correct. Show students how this method works. Then give students time to add this to their organizer.

• **Summarize** what students share in the **Summarize** section of the teacher template then, give students time to write their summary in their organizer.

• **Or**

• Group students in groups of four and assign the following:
  o A. Ask them to discuss the sections in their organizers using the math vocabulary to explain each section to each other.
  o B. On a sheet of paper or chart paper, the group will co-construct a summary that explains the answer to the question using the math vocabulary.
  o C. Ask students to practice reading the summary to their group.
  o D. Ask each group to select one student to read the summary to the class.

• Use the Extension/Explain section to further prompt students.

When students are ready to solve word problems individually, with each other, or in a cooperative group, it is important to allow them to co-construct.

Give students a blank template or they can create their own. Assign a word problem, or students may write one heir own, or mimic one already solved.

• Group students or allow them to choose their partner/s (no more than 3 or 4)

• Ask them to repeat the steps to work the problem:
  o A. Discuss the sections in the organizer using the math vocabulary to explain each section to each other.
B. On their organizer, each member of the group will write a summary that explains the answer to the question using the math vocabulary.
C. Students will practice reading the summaries to each other.
D. Each group selects one student to read the summary to the class.
- Be sure to give students feedback on their work, participation, and effort.
### Problem D: TEACHER MODEL

**Subtraction of Fractions with like Denominators**

The smallest mammals on Earth are the bumblebee bat and the Etruscan pygmy shrew. The length of a bumblebee bat is $1 \frac{1}{5}$ inches. The length of an Etruscan pygmy shrew is $1 \frac{2}{5}$ inches. How much smaller is the bat than the shrew?

<table>
<thead>
<tr>
<th>K</th>
<th>What do I know?</th>
</tr>
</thead>
<tbody>
<tr>
<td>The bat is $1 \frac{1}{5}$ inches.</td>
<td></td>
</tr>
<tr>
<td>The shrew is $1 \frac{2}{5}$ inches.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>W</th>
<th>What question do I want to answer?</th>
</tr>
</thead>
<tbody>
<tr>
<td>How much smaller is the bat than the shrew?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C</th>
<th>Conditions that I notice…</th>
</tr>
</thead>
<tbody>
<tr>
<td>- I notice the problem says that the bat is smaller than the shrew.</td>
<td></td>
</tr>
<tr>
<td>- I notice that I can compare the size of the bat to the size of the shrew, so I can use subtraction.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>What is my plan?</th>
</tr>
</thead>
<tbody>
<tr>
<td>I will subtract $1 \frac{1}{5}$ from $1 \frac{2}{5}$</td>
<td></td>
</tr>
</tbody>
</table>

```
\[
\begin{align*}
1 \frac{2}{5} & \equiv \frac{7}{5} \\
1 \frac{1}{5} & \equiv \frac{6}{5} \\
\frac{7}{5} - \frac{6}{5} & = \frac{1}{5}
\end{align*}
\]
```

<table>
<thead>
<tr>
<th>Show</th>
<th>the work with a model</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Model Diagram]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Justify</th>
<th>Write an equation that matches the model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \frac{2}{5}$</td>
<td></td>
</tr>
<tr>
<td>$- 1 \frac{1}{5}$</td>
<td></td>
</tr>
</tbody>
</table>

**Summarize:** The bat is $\frac{1}{5}$ inch smaller than the shrew.

**Extension/Explain:** Is the answer > or < than 1? Can you write a similar problem for your group to solve? Can you find an equivalent fraction for the answer?

---

APPENDIX E

PROBLEM 5
Problem E: Teacher/Student Template

Subtraction of Fractions with Like Denominators

A golf ball measures $1 \frac{2}{5}$ inches across its center. A tennis ball measures $2 \frac{3}{5}$ inches across its center. How many more inches does the tennis ball measure across its center than the golf ball?

<table>
<thead>
<tr>
<th>K</th>
<th>What do I know?</th>
<th>W</th>
<th>What question do I need to answer?</th>
<th>C</th>
<th>Conditions that I notice…</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>What is my plan?</th>
<th>Show my work</th>
<th>Justify: Write an equation that matches your model.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Summarize:**

**Extension/Explain:** Is the answer > or < than 1? Can you write a similar problem for your group to solve? Can you find an equivalent fraction for the answer?

Suggested Teaching Steps

Problem E

Explain that they will use a math organizer that will make a word problem seem easier to understand and easier to solve.

Materials needed: Appendix for Problem E, a tennis ball & a golf ball. Document camera, Problem E Teacher Template & Student Template, vocabulary cards, sheet of paper or chart paper (1 per group summary)

- Pass out student template for Word Problem E.
- Use the document camera to project the teacher template.
- Preview each section of the organizer with students before working through it.
- Word Problem E: Model reading the word problem to students then have them echo read it back to you.
- Model the golf ball and the tennis ball to students.
- Engage students to discuss information that the problem reveals such as, which ball is wider/bigger?
- Discuss with students the subjects’ attributes: For example, the diameter of the golf ball is 1 \(\frac{1}{2}\) inches and the diameter of the tennis ball is 2 \(\frac{5}{6}\) inches; this means the tennis ball is wider than the golf ball.
- Model writing this information in the K (What do I know?) section. Then, give students time to add it to their organizer.
- Next, guide students to find the question in the problem.
- Model reading the question again. Have students echo read it back to you.
- Model adding the question to the W (What question do I want to answer?) section and underline How many more inches part of the question.
- Explain that there is more than one way to ask the same question.
- Then, model Rephrasing the question so that the word “difference” and “inches” can be used in context: What is the difference in inches between the two objects?
- Give students time to add the question/s to their organizer.
- Explain C (Conditions that I notice . . .) section and emphasize what you notice: For example, you notice that the tennis ball has a longer measurement across its center (diameter) than the golf ball does. You notice that the two mixed fractions have the same denominator. You also notice that the problem compares the difference in diameter of the two balls, therefore, you can use subtraction as the order of operation, to find the answer to the question.
- Model writing what you noticed, in the C-section. Then, give students time to add this information to their organizer.
- Explain that in the P (What is my plan?) section you need to describe what you will do to solve the question: Make a plan or plan a strategy.
- Model referencing the information, in the C-section as you plan for the P section. You might explain: The two fractions are mixed fractions, but we must decompose \(2 \frac{3}{5}\) since its fractional part is smaller than the fractional part of \(1 \frac{4}{6}\). Then they can be subtracted from each other. Use manipulatives as necessary.
- Discuss the unit of measurement (sixths)
- Model how students can decompose the mixed fraction: \(2 \frac{3}{5}\) and rename it \(1 \frac{8}{6}\).
• Explain that in the Show section, we are subtracting the two improper fractions from each other to answer the question.
• Model by using a number line that represents the units of measure (sixths).
• Explain that you will draw a line to the fraction $1 \frac{5}{6}$ because this represents the diameter of the tennis ball. Then label it “tennis ball”.
• Explain that to show that you are going to subtract the diameter of the golf ball from the diameter of the tennis ball you will have to count backwards four sixths ($\frac{4}{6}$) from $\frac{5}{6}$. Modeling drawing your number line and label it “golf ball”.
• The work on the number line should show that the answer is $\frac{5}{6}$.
• Explain that the tennis ball and the golf ball differ by $\frac{5}{6}$ inches across their diameter.
• Be sure to show all work then, wait for students to fill in their Show section
• Explain the Justify section. Elaborate that the equation you will write here will explain the model in the “Show” section. Then, write an equation that matches the number line.
• Summarize what students share in the Summarize section of the teacher template then, give students time to write their summary in their organizer.
• Or
• Group students in groups of four and assign the following:
  o A. Ask them to discuss the sections in their organizers using the math vocabulary to explain each section to each other.
  o B. On a sheet of paper or chart paper, the group will co-construct a summary that explains the answer to the question using the math vocabulary.
  o C. Ask students to practice reading the summary to their group.
  o D. Ask each group to select one student to read the summary to the class.
• Use the Extension/Explain section to further prompt students.

When students are ready to solve word problems individually, with each other, or in a cooperative group, it is important to allow them to co-construct.

Give students a blank template or they can create their own. Assign a word problem, or students may write one heir own, or mimic one already solved.

• Group students or allow them to choose their partner/s (no more than 3 or 4)
• Ask them to repeat the steps to work the problem:
  o A. Discuss the sections in the organizer using the math vocabulary to explain each section to each other.
  o B. On their organizer, each member of the group will write a summary that explains the answer to the question using the math vocabulary.
  o C. Students will practice reading the summaries to each other.
  o D. Each group selects one student to read the summary to the class.
• Be sure to give students feedback on their work, participation, and effort.
Problem D: Teacher/Student Template
Subtraction of Fractions with Like Denominators

<table>
<thead>
<tr>
<th>K</th>
<th>W</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P</th>
<th>Show the work with a model</th>
<th>Justify</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Summarize:

Extension/Explain: Is the answer > or < than 1? Can you write a similar problem for your group to solve? Can you find an equivalent fraction for the answer?

The smallest mammals on Earth are the bumblebee bat and the Etruscan pygmy shrew. The length of a bumblebee bat is 1 $\frac{1}{2}$ inches. The length of an Etruscan pygmy shrew is $1 \frac{2}{5}$ inches. How much smaller is the bat than the shrew?
APPENDIX F

PROBLEM 6
**Problem F: Teacher/Student Template**

Subtraction with Like Denominators

| K | What do I know? | W | What question do I need to answer? | C | Conditions that I notice...
|---|-----------------|---|-----------------------------------|---|-----------------
|   |                 |   |                                   |   |                  |

<table>
<thead>
<tr>
<th>P</th>
<th>What is my plan?</th>
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<th>Justify by Checking</th>
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**Summarize:**

**Extension/Explore:** Is the answer > or < than 1? Can you write a similar problem for your group to solve? Can you find an equivalent fraction for the answer?

---

The average weight of a basketball is $21\frac{1}{8}$ ounces. The average weight of a baseball is $5\frac{2}{8}$ ounces. How many more ounces does the basketball weigh?
Suggested Teaching Steps

**Problem F**

Explain that they will use a math organizer that will make a word problem seem easier to understand and easier to solve.

**Materials needed:** Document camera, Problem F Teacher Template & Student Template, sheet of paper (for group summary)

- Pass out student template for Word Problem F.
- Use the document camera to project the teacher template.
- Preview each section of the organizer with students before working through it.
- _Word Problem F:_ Model reading the word problem to students then have them _echo read_ it back to you.
- Engage students to discuss information that the problem reveals such as _which ball is heavier, the basketball or the baseball?_
- Discuss with students the subjects’ attributes: For example, the weight of the basketball is $21\frac{1}{8}$ ounces and the weight of the baseball is $5\frac{3}{8}$ ounces. We know that the basketball weighs _more_.
- Model writing this information in the _K (What do I know?)_ section. Then, give students time to add it to their organizer.
- Next, guide students to find the question in the problem.
- Model reading the question again. Have students echo read it back to you.
- Model adding the question to the _W (What question do I want to answer?)_ section and underline _How many more ounces_.
- Explain that there is more than one way to ask the same question.
- Then, model _Rephrasing_ the question so that the word “difference” and “weight” can be used in context: _What is the difference in weight of the two objects?_
- Give students time to add the question/s to their organizer.
- Explain _C (Conditions that I notice...)_ section and emphasize what you notice: For example, you _notice_ that you are comparing the difference in weight of the basketball to the difference in weight to the baseball. You also _notice_ that the two objects are being compared, so you use subtraction to find their difference in weight.
- Model writing what you noticed, in the _C_-section. Then, give students time to add this information to their organizer.
- Explain that in the _P (What is my plan?)_ section you need to describe what you will do to solve the question: Make a plan or plan a strategy.
- Model referencing the information, the _C_-section as you plan for the _P_ section. You might explain: the two numbers we are given are mixed fractions and in order to solve this we will need to decompose $21\frac{1}{8}$ so that we are able to subtract $5\frac{3}{8}$ from it, because
the numerators are not the same. You might also say: The fractional parts are not equal, so we will need to decompose $21 \frac{1}{8}$. Use manipulatives.

- Discuss the unit of measurement (eighths)
- Model this problem on a number line that begins at 15 and ends at 22 to show students how the fractions can be subtracted from each other. Be sure to divide the number line by eights. Use a forward arrow to the mixed fraction $21 \frac{1}{8}$. Then draw a backward arrow to subtract the mixed fraction $5 \frac{3}{8}$.

- Next, model how they can decompose the mixed fraction: $21 \frac{1}{8}$ by borrowing a "1" from the 21 and turning it into $\frac{8}{8}$ so that it becomes $20 \frac{9}{8}$.

- Model how to subtract the fractional parts first. Then how to subtract the whole numbers.

- Explain that in the Show section, we are subtracting the two improper fractions from each other to answer the question $(20 \frac{2}{8} - 5 \frac{3}{8})$.

- Be sure to show all work then, wait for students to fill in their Show section
- Explain the Justify section. Elaborate that the equation you will write here will verify the work done in the "Show" section. Write an equation to show the difference of the two weights.
- Summarize what students share in the in the Summarize section of the teacher template then, give students time to write their summary in their organizer.

- Or
- Group students in groups of four and assign the following:
  - A. Ask them to discuss the sections in their organizers using the math vocabulary to explain each section to each other.
  - B. On a sheet of paper or chart paper, the group will co-construct a summary that explains the answer to the question using the math vocabulary.
  - C. Ask students to practice reading the summary to their group.
  - D. Ask each group to select one student to read the summary to the class.
- Use the Extension/Explain section to further prompt students.

When students are ready to solve word problems individually, with each other, or in a cooperative group, it is important to allow them to co-construct.

Give students a blank template or they can create their own. Assign a word problem, or students may write one of their own, or mimic one already solved.
- Group students or allow them to choose their partner/s (no more than 3 or 4)
- Ask them to repeat the steps to work the problem:
  - A. Discuss the sections in the organizer using the math vocabulary to explain each section to each other.
  - B. On their organizer, each member of the group will write a summary that explains the answer to the question using the math vocabulary.
  - C. Students will practice reading the summaries to each other.
  - D. Each group selects one student to read the summary to the class.
- Be sure to give students feedback on their work, participation, and effort.
**Problem F:** *(Advanced Problem) TEACHER MODEL*

Subtraction with Like Denominators

The average **weight** of a basketball is **21 \( \frac{1}{8} \) ounces.** The average **weight** of a baseball is **5 \( \frac{2}{8} \) ounces.** How many more ounces does the basketball weigh?

### K What do I know?

The basketball is **21 \( \frac{1}{8} \) ounces.**

The baseball is **5 \( \frac{2}{8} \) ounces.**

*The basketball **weighs** more.

### W What question do I need to answer?

How many more ounces does the basketball weigh than the baseball?

**Rephrase:** What is the **difference in weight** of the two objects?

### C Conditions that I notice...

I notice that I am comparing the **difference in weight** of the basketball to the **weight** of the baseball.

To find their **difference in weight,** I need to use subtraction.

### P What is my plan?

First, I will need to rename **21 \( \frac{1}{8} \)** by decomposing the **1** into eighths.

Then, I will subtract **5 \( \frac{2}{8} \).**

### Show the work with a model

\[
21 \frac{1}{8} = 20 + 1 + \frac{1}{8}
\]

= \(20 + \frac{8}{8} + \frac{1}{8} = 20 \frac{9}{8}\)

- \(5 \frac{2}{8}\)

\[
\begin{array}{c}
20 \frac{9}{8} \\
- 5 \frac{2}{8} \\
\hline
15 \frac{7}{8}
\end{array}
\]

**basketball**

**baseball**

**difference in weight**

### Justify by Checking

\[
\begin{align*}
15 \frac{7}{8} & = \frac{15}{8} + \frac{7}{8} \\
20 \frac{9}{8} & = 20 + \frac{9}{8} \\
20 + \frac{8}{8} & + \frac{1}{8} \\
20 & + 1 + \frac{1}{8} \rightarrow
\end{align*}
\]

= \(21 \frac{1}{8} \)

**Summarize:** First, I borrowed a "1" from 21 and decomposed it to \( \frac{8}{8} \). Next, I added the \( \frac{8}{8} \) to \( \frac{1}{8} \) and to get \( \frac{9}{8} \). Then, I subtracted \( \frac{2}{8} \) from \( \frac{9}{8} \) to get \( \frac{7}{8} \). Lastly, I subtracted the whole numbers: 20 minus 5 to get 15. The basketball weighs **15 \( \frac{7}{8} \)** ounces more than the baseball.

**Extension/Explain:** Is the answer > or < than 1? Can you write a similar problem for your group to solve? Can you find an equivalent fraction for the answer?
APPENDIX G

PERMISSION TO USE
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Content to be included is:
292-297 Topic 12, lessons 1 - 5

Please credit our material as follows:

Sincerely,
Michael Prince,
Permissions Granting Analyst
APPENDIX H

CONTENT VOCABULARY FOR PROBLEMS A- F
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<th>CONTENT VOCABULARY FOR PROBLEMS:</th>
<th>A, B, C, D, E, F</th>
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