6-2018

Images of Permutation and Monomial Progenitors

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California State University - San Bernardino

Images of Permutation and Monomial Progenitors

A Thesis

Presented to the

Faculty of

California State University,

San Bernardino

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

in

Mathematics

---

by

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Abstract

We have conducted a systematic search for finite homomorphic images of several permutation and monomial progenitors, including $2^{*20} : (2^4 : S_5)$, $2^{*20} : ((5 \times 4) : S_4)$, $2^{*20} : D_{20}$, $2^{*11} : (2 : 11)$, $2^{*11} : L_2(11)$, $2^{*6} : (2 \times S_3)$, $2^{*6} : (S_3 \times S_3)$, $2^{*36} : (3^2 : D_4)$, $2^{*110} : L_2(11)$, $2^{*6} : D_{12}$, $2^{*10} : S_5$, $11^{*4} :_m (4 : 5)$, and $11^{*2} :_m D_{10}$. We have found original symmetric presentations for several important groups such as the Mathieu sporadic simple groups M_{11} and M_{12}, Suzuki simple group sz_8, unitary group $U(3,4)$, Janko group J_1, simplectic groups $S(4,4)$ and $S(4,3)$, and projective special linear groups $L_3(4)$ and $L_3(7)$. We have also constructed, using the technique of double coset enumeration, the following groups, $L_2(11)$, $S(4,3) : 2$, M_{11}, and $PGL(2,11)$. The isomorphism class of each of the finite images is also given.

Acknowledgements

First and foremost I would like to thank my advisor Dr. Zahid Hasan. The countless hours you have spent aiding me in my education are greatly appreciated and will never be forgotten. I thank you for the education, guidance, wisdom, and support you have given me in both my undergraduate as well as graduate work. I would like to also thank my committee members Dr. Hajrudin Fejzic and Dr. Joseph Chavez for helping me throughout my studies. To Dr. Charles Stanton and Dr. Corey Dunn, I thank you both, as well as the entire math department at CSUSB for your guidance and support throughout this wonderful journey.

I also want to thank my fellow graduates, Kevin Baccari, Angelica Baccari, Charles Baccari, Diana Aguirre, and Joana Luna for always offering to help when I had questions. To Joel Webster, Adam Manriquez,and Charles Seager, I thank you for your help, support, and all the laughs we shared. To Erica Fernandez, I thank you for helping me throughout my research. You have always been just a phone call away whenever I have needed help. To Sandra Bahena, words cannot express the amount of gratitude I have towards you. I thank you for all of your support, guidance, friendship, and love. I am happy to say that I leave here with a new sister.

To my parents, I thank you for all that you have done for me to get me to this point. You are and have always been my number one cheerleaders and none of this would be possible without your support. I want to also thank my children, my M & Ms, Marina and Max. The both of you bring such joy to my life and you have been my inspiration to pursue my education. I only want the best for the both of you!

Lastly, I would like to thank Pablo Fonseca. You are the love of my life and I am thankful for you and the support you have given, to not only me, but to our family through this process. I thank you for holding down the fort while I was busy with school. I also thank you for your motivation and for leading by example. You made me realize that anything was possible. You have pushed me when I needed to be pushed, supported me when I needed to be supported, and most of all, loved me through it all.

Table of Contents

Abstract **iii**

Acknowledgements **iv**

List of Tables **vii**

List of Figures **ix**

Introduction **1**

1 Preliminaries **2**
1.1 Definitions, Theorems, and Lemmas 2

2 Writing Progenitors **6**
2.1 Preliminaries 6
2.2 Permutation Progenitor (15 : 4) 7
2.3 Writing Relations 7
2.3.1 First Order Relations 7
2.3.2 Factoring by Famous Lemma 10

3 Extension Problems **16**
3.1 Preliminaries 16
3.2 Direct Product 17
3.3 Semi-Direct Product 20
3.4 Central Extension 23
3.5 Mixed Extension 28

4 Monomial Progenitors **35**
4.1 Preliminaries 35
4.2 Monomial Progenitor $11^{*4} :_m (4 : 5)$ 36
4.3 Monomial Progenitor $11^{*2} :_m D_{10}$ 45

5 Double Coset Enumeration **54**
5.1 Preliminaries 54
5.2 $L_2(11)$ as a Homomorphic Image of $2^{*6} : D_{12}$ 54
5.2.1 The Construction of $L_2(11)$ Over D_{12} 54
5.3 $PGL_2(11)$ as a Homomorphic Image of $11^{*2} :_m D_{10}$ 81
5.3.1 The Construction of $PGL_2(11)$ Over D_{10} 81
5.3.2 Proof of the Isomorphism 100
5.3.3 Building a Map 104
5.4 M_{11} as a Homomorphic Image of $11^{*4} :_m (4 : 5)$ 108
5.4.1 Manual Double Coset Enumeration over a Maximal Subgroup of Order 120 109
5.4.2 Manual Double Coset Enumeration over a Maximal Subgroup of Order 720 225
5.5 $(S(4,3) : 2)$ as a Homomorphic Image of $2^{*10} : S_5$ 237
5.5.1 Factor by Center of G 237
5.5.2 The Construction of $(S(4,3) : 2)$ Over S_5 242

6 Transitive Groups **280**
6.1 Transitive Groups on 20 Letters 280
6.1.1 Transitive Group(20,222) 280
6.1.2 Transitive Group(20,102) 286
6.1.3 Transitive Group(20,121) 291
6.1.4 Transitive Group(20,10) 296
6.1.5 Transitive Group(20,11) 298
6.1.6 Transitive Group(20,12) 302
6.1.7 Transitive Group(20,13) 303
6.2 Transitive Groups on 19 Letters 308
6.2.1 Transitive Group(19,2) 308
6.3 Transitive Groups on 11 Letters 310
6.3.1 Transitive Group(11,2) 310
6.3.2 Transitive Group(11,5) 313
6.4 Transitive Groups on 6 Letters 314
6.4.1 Transitive Group(6,3) 314
6.4.2 Transitive Group(6,9) 315

7 More Progenitors **318**
7.1 $2^{*36} : (3^2 : D_4)$ 318
7.2 $2^{*110} : L_2(11)$ 318
7.3 $2^{*15} : (C_{15} : C_4)$ 319

8 MAGMA Code **320**
8.1 Double Coset Enumeration of $(S(4,3) : 2)$ 320

Bibliography **395**

List of Tables

2.1 Conjugacy Classes of $N = (15 : 4)$ . . . 8
2.2 Orbits of Centraliser(N,Rep) . . . 9
2.3 Orbits of Centraliser(N,Rep) . . . 9
2.4 $2^{*110} : L_2(11)$ . . . 15

4.1 Conjugacy Classes of $(C_4 : C_5)$ . . . 37
4.2 Conjugacy Classes of $H = Z_5$ . . . 37
4.3 Character Table of $H = Z_5$ . . . 38
4.4 Character Table of $G = (C_4 : C_5)$ . . . 38
4.5 Conjugacy Classes of D_{10} . . . 46
4.6 Conjugacy Classes of $H = Z_{10}$ . . . 46
4.7 Character Table of $H = Z_{10}$ . . . 47
4.8 Character Table of $G = D_{10}$ . . . 47

5.1 Single Coset Action of $L_2(11)$ Over D_{12} . . . 78
5.2 Single Coset Action of $PGL(2, 11)$ Over D_{10} . . . 101
5.3 $\alpha : x \mapsto x + 1$ . . . 105
5.4 $\beta : x \mapsto 3x$ . . . 106
5.5 $\gamma : x \mapsto -\frac{1}{x}$ . . . 107
5.6 $aut : x \mapsto \frac{1}{x}$ . . . 108

6.1 $2^{*20} : (2^4 : S_5)$ . . . 283
6.2 $2^{*20} : (5^2 :^{\bullet} 4^2)$ . . . 286
6.3 $2^{*20} : ((5 : 4) \times S_4)$ . . . 291
6.4 $2^{*20} : D_{20}$ . . . 296
6.5 $2^{*20} : D_{20}$ continued . . . 297
6.6 $2^{*20} : 2^{\bullet}D_{10}$ . . . 298
6.7 $2^{*20} : (5 \times D_4)$ . . . 302
6.8 $2^{*20} : (4^{\bullet} : 10)$ . . . 304
6.9 $2^{*20} : (4^{\bullet} : 10)$ continued . . . 305
6.10 $2^{*20} : (2 : 19)$ . . . 308
6.11 $2^{*11} : (2 : 11)$ . . . 311
6.12 $2^{*11} : (L_2(11))$ . . . 313

6.13 $2^{*6} : (2 \times S_3)$ . . . 314
6.14 $2^{*6} : (S_3 \times S_3)$ . . . 316

7.1 $2^{*36} : (3^2 : D_4)$ . . . 318
7.2 $2^{*110} : L_2(11)$ . . . 319
7.3 $2^{*15} : (C_{15} : C_4)$ . . . 319

List of Figures

5.1 Cayley Diagram : $L_2(11)$ Over D_{12} 77
5.2 Cayley Diagram : $PGL(2, 11)$ Over D_{10} 100
5.3 M_{11} Over $(C_4 : C_5)$. 225
5.4 M_{11} Over $(C_4 : C_5)$. 237
5.5 $(S(4, 3) : 2)$ Over $2^{*10} : S_5$ 279

Introduction

The aim of group theory is the discovery and classification of groups. Symmetric presentations give a uniform method for constructing finite groups. Since finite groups are composed of simple groups, we are most interested in simple groups. In Chapter 1 we will discuss some important definitions, lemmas, and theorems. In Chapter 2 we will begin to explore progenitors and the methods used to write them. In Chapter 3 we will solve extension problems in order to define our isomorphism types. Chapter 4 will focus on monomial progenitors and methods used to write these. Chapter 5 is dedicated to Double Coset Enumeration, both manual and computer based. In Chapter 6 we discuss Transitive Groups and explore certain transitive groups written on 20, 19, 11, and 6 letters.

Chapter 1

Preliminaries

1.1 Definitions, Theorems, and Lemmas

Definition 1.1. *[Rot95] If X is a nonempty set, a* **permutation** *of X is a bijection $\alpha : X \longrightarrow X$. We denote the set of all permutations of X by S_x.*

Definition 1.2. *[Rot95] If $x \in X$ and $\alpha \in S_x$, then α* **fixes** *x if $\alpha(x) = x$ and α* **moves** *x if $\alpha(x) \neq x$.*

Definition 1.3. *[Rot95] A (binary)* **operation** *on a nonempty set G is a function $\mu : G \times G \Longrightarrow G$.*

Definition 1.4. *[Rot95] A* **semigroup** *$(G, *)$ is a nonempty set G equipped with an associative operation $*$.*

Definition 1.5. *[Rot95] A* **group** *is a semigroup G containing an element e such that:*

(i) $e * a = a = a * e$ *for all* $a \in G$;

(ii) *for every* $a \in G$, *there is an element* $b \in G$ *with* $a * b = e = b * a$.

Definition 1.6. *[Rot95] A pair of elements a and b in a semigroup* **commutes** *if $a * b = b * a$. A group (or a semigroup) is* **abelian** *if every pair of its elements comutes.*

Theorem 1.7. *[Rot95] If G is a group, there is a unique element e with $e*a = a = a*e$ for all $a \in G$. Moreover, for each $a \in G$, there is a unique $b \in G$ with $a*b = e = b*a$.*

We call e the **identity** *of G and, if $a * b = e = b * a$, then we call b the* **inverse** *of a and denote it by a^{-1}.*

Corollary 1.8. *[Rot95] If G is a group and $a \in G$, then*

$$(a^{-1})^{-1} = a.$$

Definition 1.9. *[Rot95] Let $(G, *)$ and $(H, \circ)$ be groups. A function $f : G \Longrightarrow H$ is a* **homomorphism** *if, for all $a, b \in G$,*

$$f(a * b) = f(a) \circ f(b).$$

Definition 1.10. *[Rot95] An* **isomorphism** *is a homomorphism that is also a bijection. We say that G is* **isomorphic** *to H, denoted by $G \cong H$, if there exists an isomorphism $f : G \Longrightarrow H$.*

Theorem 1.11. *[Rot95] Let $f : (G, *) \Longrightarrow (G', \circ)$ be a homorphism.*

(i) $f(e) = e'$, *where e' is the identity in G'*

(ii) *If $a \in G$, then $f(a^{-1}) = f(a)^{-1}$.*

(iii) *If $a \in G$ and $n \in \mathbb{Z}$, then $f(a^n) = f(a)^n$.*

Definition 1.12. *[Rot95] A nonempty subset S of a group G is a* **subgroup** *of G if $s \in G$ implies $s^{-1} \in G$ and $s, t \in G$ imply $st \in G$.*

Theorem 1.13. *[Rot95] If $S \leq G$ (i.e., if S is a subgroup of G), then S is a group in its own right.*

Theorem 1.14. *[Rot95] A subset S of a group G is a subgroup if and only if $1 \in S$ and $s, t \in S$ imply $st^{-1} \in S$.*

Definition 1.15. *[Rot95] If G is a group and $a \in G$, then the* **cyclic subgroup generated by a**, *denoted by $< a >$, is the set of all powers of a. A group G is called* **cyclic** *if there is $a \in G$ with $G =< a >$; that is, G consists of all the powers of a.*

Theorem 1.16. *[Rot95] If S is a subgroup of G and if $t \in G$, then a* **right coset** *of S in G is the subset of G*

$$St = \{st : s \in S\}$$

(a **left coset** *is $tS = \{ts : s \in S\}$). One calls t a* **representative** *of St (and also of tS).*

Definition 1.17. *[Rot95] If $S \leq G$, then the* **index** *of S in G, denoted by $[G : S]$, is the number of right cosets of S in G.*

Definition 1.18. *[Rot95] If G is a group, then the* **order** *of G, denoted by $|G|$, is the number of elements in G.*

Theorem 1.19. *[Rot95]* **(Lagrange)**

If G is a finite group and $S \leq G$, then $|S|$ divides $|G|$ and $[G : S] = |G|/|S|$.

Corollary 1.20. *[Rot95] If G is a finite group and $a \in G$. Then the order of a divides $|G|$.*

Corollary 1.21. *[Rot95] If p is a prime and $|G| = p$, then G is a cyclic group.*

Definition 1.22. *[Rot95] A subgroup $K \leq G$ is a* **normal subgroup***, denoted by $K \lhd G$, if $gKg^{-1} = K$ for every $g \in G$.*

Definition 1.23. *[Rot95] A* **projective special linear group***, $PSL(n, \mathbb{F})$ is the set of all $n \times n$ matrices with determinant 1 over field $\mathbb{F}$ factored by its center:*

$$PSL(n, \mathbb{F}) = L_n(\mathbb{F}) = \frac{SL(n, \mathbb{F})}{Z(SL(n, \mathbb{F})}.$$

Definition 1.24. *[Rot95] A* **projective general linear group***, $PGL(n, \mathbb{F})$ is the set of all $n \times n$ matrices with nonzero determinant over field $\mathbb{F}$ factored by its center:*

$$PGL(n, \mathbb{F}) = \frac{GL(n, \mathbb{F})}{Z(GL(n, \mathbb{F})}.$$

Definition 1.25. *[Rot95] A* **special linear group***, $SL(n, \mathbb{F})$ is the set of all $n \times n$ matrices with determinant 1 over field $\mathbb{F}$.*

Definition 1.26. *[Rot95] A* **general linear group***, $GL(n, \mathbb{F})$ is the set of all $n \times n$ matrices with nonzero determinant over field $\mathbb{F}$.*

Theorem 1.27. *[Rot95]* **(First Isomorphism Theorem).**
Let $f : g \Longrightarrow H$ *be a homorphism with kernal* K*. Then* K *is a normal subgroup of* G *and* $G/K \cong$ *im* f*.*

Theorem 1.28. *[Rot95]* **(Second Isomorphism Theorem).**
Let N *and* T *be subgroups of* G *with* N *normal. Then* $N \cap T$ *is normal in* T *and* $T/(N \cap T) \cong NT/N$*.*

Theorem 1.29. *[Rot95]* **(Third Isomorphism Theorem).**
Let $K \leq H \leq G$*, where both* K *and* H *are normal subgroups of* G*. Then* H/K *is a normal subgroup of* G/K *and*

$$(G/K)/(H/K) \cong G/H$$

Chapter 2

Writing Progenitors

2.1 Preliminaries

Definition 2.1. *[Rot95] Let X be a set and Δ by a family of words on X. A group G has* **generators** *X and* **relations** *Δ if $G \cong F/R$, where F is a free group with basis X and R is the normal subgroup of F generated by Δ. We say $< X|\Delta >$ is a* **presentation** *of G.*

Definition 2.2. *[Rot95] Let G be a group. If $H \leq G$, the* **normalizer** *of H in G is defined by $N_G(H) = \{a \in G|aHa^{-1} = H\}$*

Definition 2.3. *[Rot95] Let G be a group. If $H \leq G$, the* **centralizer** *of H in G is:*

$$C_G(H) = \{x \in G : [x, h] = 1 \text{ for all } h \in H\}.$$

Definition 2.4. *[Rot95] Let N be a group. The* **point stabiliser** *of w in N is given by:*

$$N^w = \{n \in N|w^n = w\}, \text{ where } w \text{ is a word in the } t_i\text{'s}.$$

Definition 2.5. *[Rot95] Let $a \in G$, where G is a group. The* **conjugacy class** *of a is given by $a^G = \{a^g|g \in G\} = \{g^{-1}ag|g \in G\}$*

Definition 2.6. *[Rot95] Let G be a group and X be a G-set. For $x \in X$, the set $x^G = \{x^g|g \in G\}$ is a* **G-Orbit***.*

Definition 2.7. *[Rot95] If $x \in G$, then a* **conjugate** *of x in G is an element of the form axa^{-1} for some $a \in G$; equivalently, x and y are conjugate if $y = \gamma_a(x)$ for some $a \in G$.*

Lemma 2.8. *[Gri15]* **(The Factoring Lemma)** *Factoring the progenitor $m^{*n} : N$ by (t_i, t_j) for $1 \leq i \leq j \leq n$ gives the group $m^n : N$.*

2.2 Permutation Progenitor $(15 : 4)$

In this section we will write a presentation for the progenitor $2^{*15} : (15 : 4)$. Our control group $N = (15 : 4)$ has the following presentation.
$N =< w, x, y, z | w^4, x^2, z^3, w^{-2}x, w^{-1}z^{-1}wz^{-1}, (y, z), wy^{-1}w^{-1}y^2, xy^{-1}w^2y^{-1}z >$
Since we have 2^{*15}, we will have 15 t's of order 2. We let $t \sim t_1$, which means that t will commute with the stabilizer of 1 in N. We use MAGMA to find these permutations that stabilize 1 in N.

```
N1:=Stabiliser(N,1);
Permutation group N1 acting on a set of cardinality 15
Order = 4 = 2^2
    (2, 11, 12, 8)(3, 14, 9, 13)(4, 7)(5, 6, 15, 10)
```

Using our Schreier System, we see that this permutation is wy^{-1} So we add this, as well as t, to our presentation of N to get a presentation for $2^{*15} : (15 : 4)$.

$$G =< w, x, y, z, t | w^4, x^2, z^3, w^{-2}x, w^{-1}z^{-1}wz^{-1}, (y, z), wy^{-1}w^{-1}y^2, xy^{-1}w^2y^{-1}z, t^2, (t, wy^{-1}) >$$

Our progenitor is infinite, in order to make it finite we must factor by relations.

2.3 Writing Relations

2.3.1 First Order Relations

First order relations are written in the form $(\pi t_i^a)^b = 1$, where $a \in N$ and w is a word in the t_i's. We can exhaust all possible relations by computing the orbits of the centralizers of Conjugacy Classes of N. Continuing with our example from above, we find the classes of $N = (15:4)$.

Table 2.1: Conjugacy Classes of $N = (15:4)$

Class	Representative of the class	# of elements in the class
C_1	e	1
C_2	$x^y = (1,9)(2,7)(3,14)(4,10)(6,8)(12,15)$	5
C_3	$z = (1,4,7)(2,9,10)(3,6,12)(5,13,11)(8,15,14)$	2
C_4	$yw(1,3,9,14)(2,8,7,6)(4,12,10,15)(5,11)$	15
C_5	$w^{-1}y^{-1} = (1,14,9,3)(2,6,7,8)(4,15,10,12)(5,11)$	15
C_6	$y^3 = (1,13,9,3,14)(2,12,15,7,5)(4,11,10,6,8)$	4
C_7	$yx = (1,5,4,13,7,11)(2,8,9,15,10,14)(3,12,6)$	10
C_8	$y = (1,2,8,13,12,4,9,15,11,3,7,10,14,5,6)$	4
C_9	$y^2z = (1,15,6,9,5,4,14,12,10,13,7,8,3,2,11)$	4

Now, we need to find the centraliser of each Class representative as well as the orbit of each centraliser that we find.

```
CL:=Classes(N);
for ii in [2..NumberOfClasses(N)] do
for i in [1..#N] do
if ArrayP[i] eq CL[ii][3] then Sch[i]; end if; end for;
C12:=Centraliser(N,CL[ii][3]);
Orbits(C12);
end for;
```

The output we have is given in the following table.

Table 2.2: Orbits of Centraliser(N,Rep)

Class	Representative	Centraliser(N,Rep)	Orbits of Centraliser(N,Rep)
C_2	x^y	$<(1,9)(2,7)(3,14)(4,10)(6,8)(12,15)>$	$\{5,11,13\}$, $\{1,9,3,12,14,15,2,10,7,4,8,6\}$
C_3	z	$<(1,4,7)(2,9,10)(3,6,12)(5,13,11)(8,15,14)>$	$\{1,3,4,15,6,5,7,14,12,13,2,8,11,9,10\}$
C_4	yw	$<(1,3,9,14)(2,8,7,6)(4,12,10,15)(5,11)>$	$\{13\}$, $\{5,11\}$, $\{1,3,9,14\}$, $\{2,8,7,6\}$, $\{4,12,10,15\}$
C_5	$w^{-1}y^{-1}$	$<(1,14,9,3)(2,6,7,8)(4,15,10,12)(5,11)>$	$\{13\}$, $\{5,11\}$, $\{1,14,9,3\}$, $\{2,6,7,8\}$, $\{4,15,10,12\}$
C_6	y^3	$<(1,13,9,3,14)(2,12,15,7,5)(4,11,10,6,8)>$	$\{1,13,4,9,11,7,3,10,5,14,6,2,8,12,15\}$
C_7	yx	$<(1,5,4,13,7,11)(2,8,9,15,10,14)(3,12,6)>$	$\{3,12,6\}$, $\{1,13,5,7,4,11\}$, $\{2,15,8,10,9,14\}$
C_8	y	$<(1,2,8,13,12,4,9,15,11,3,7,10,14,5,6)>$	$\{1,2,8,13,12,4,9,15,11,3,7,10,14,5,6\}$
C_9	y^2z	$<(1,15,6,9,5,4,14,12,10,13,7,8,3,2,11)>$	$\{1,15,6,9,5,4,14,12,10,13,7,8,3,2,11\}$

Thus we have the following relations

Table 2.3: Orbits of Centraliser(N,Rep)

Class	Relation
C_2	$x^y t_5$,$x^y t_1$
C_3	zt_1
C_4	ywt_{13}, ywt_5, ywt_1,ywt_2,ywt_4
C_5	$w^{-1}y^{-1}t_{13}$,$w^{-1}y^{-1}t_5$,$w^{-1}y^{-1}t_1$,$w^{-1}y^{-1}t_2$,$w^{-1}y^{-1}t_4$
C_6	y^3t_1
C_7	yxt_3, yxt_1,yxt_2
C_8	yt_1
C_9	y^2zt_1

Note that $t_1 \sim t$, and, since $y = (1,2,8,13,12,4,9,15,11,3,7,10,14,5,6)$, then

$t_2 \sim t^y$

$t_3 \sim t^{y^9}$

$t_4 \sim t^{y^5}$

$t_5 \sim t^{y^{13}}$

$t_{13} \sim t^{y^3}$.

Now we add these relations to our progenitor to obtain homomorphic images of G.
$G =< w, x, y, z, t | w^4, x^2, z^3, w^{-2}x, w^{-1}z^{-1}wz^{-1}, (y, z), wy^{-1}w^{-1}y^2, xy^{-1}w^2y^{-1}z,$
$t^2, (t, wy^{-1}),$
$(x^y t^{y^{13}})^{r1}, (x^y t)^{r2}, (zt)^{r3}, (ywt^{y^5})^{r4}, (ywtt^{y^{13}})^{r5}, (ywt)^{r6}, (ywt^y)^{r7}, (ywt^{y^5})^{r8},$
$(w^{-1}y^{-1}t^{y^5})^{r9}, (w^{-1}y^{-1}t^{y^{13}})^{r10}, (w^{-1}y^{-1}t)^{r11}, (w^{-1}y^{-1}t^y)^{r12},$
$(w^{-1}y^{-1}t^{y^5})^{r13}, (y^3t)^{r14}, (yxtt^{y^9})^{r15}, (yxt_1)^{r16}, (yxt^y)^{r17}, (yt)^{r18}, (y^2zt)^{r19} >.$

2.3.2 Factoring by Famous Lemma

We use the Famous Lemma [Cur07] as another method of finding relations. Factoring our progenitor by this lemma guarantees the non-collapse of groups.

Theorem 2.9. *[Cur07] Famous Lemma*

Let $N \cap < t_i, t_j > \leq C_N(N_{ij})$, where N_{ij} denotes the stabilizer in N of the two points i and j.

Proof. Let $w \in N \cap < t_i, t_j >$. We need to show $w \in Cent(N, N^{ij})$.
Let $\pi \in N^{ij}$.
$\pi^w = w.$
$\Longrightarrow \pi^{-1}w\pi = w.$
$\Longrightarrow \pi\pi^{-1}w\pi = \pi w.$
$\Longrightarrow w\pi = \pi w.$
Thus π commutes with every element of N^{ij}.

□

Note that $|t_i| = |t_j| = 2$, and $|t_it_j| = n$, thus $< t_i, t_j >= D_{2n}$ is Dihedral.

$$Z(D_{2n}) = \begin{cases} 1 & \text{if } n \text{ is odd.} \\ < (t_it_j)^{\frac{n}{2}} > & \text{if } n \text{ is even.} \end{cases}$$

So for each two point stabilizer in N we will compute the centralizer of the two point stabilizer in N and then write elements of N in terms of $< t_i, t_j >$ in the following way given by the Famous Lemma.

$$\begin{cases} (xt_1)^m = 1 & \text{where } m \text{ is odd and } x \text{ sends 1 to 2} \\ (t_i t_j)^n = x & \text{where } n \text{ is even and } x \text{ fixes both 1 and 2} \end{cases}$$

Let $x = (1,2)(3,5)(4,7)(6,10)(8,12)(9,13)(11,16)(14,19)(17,22)(18,23)(20,26)$ $(21,27)(24,31)(25,32)(28,36)(29,37)(30,39)(33,43)(34,44)(35,46)(41,49)(42,51)$ $(45,55)(47,57)(48,59)(50,62)(52,64)(53,65)(54,67)(56,69)(58,72)(60,71)(61,74)$ $(66,78)(68,80)(70,82)(73,85)(75,87)(76,83)(77,89)(79,90)(81,92)(84,94)(86,93)$ $(88,97)(91,99)(95,102)(96,103)(98,105)(100,106)(104,107)(109,110)$ and
$y = (1,3,6)(2,4,8)(5,9,14)(7,11,17)(10,15,20)(12,18,24)(16,21,28)(19,25,33)(22,$ $29,38)(23,30,40)(26,34,45)(27,35,39)(31,41,50)(32,42,52)(36,47,58)(37,48,60)$ $(43,53,66)(44,54,68)(46,56,70)(49,61,75)(51,63,62)(55,59,73)(57,71,83)(64,76,$ $88)(65,77,80)(67,79,91)(69,81,78)(72,84,95)(74,86,96)(82,93,100)(85,90,92)(87,$ $97,104)(89,98,102)(94,101,107)(99,105,109)(103,108,110)$.
$N =< x, y >= L_2(11)$.

To find our relations using the Famous Lemma we must first find the Centraliser of N_{ij}, where N_{ij} is the stabiliser of the two points i and j, which we will say 1 and 2 respectively.

```
S:=Sym(110);
xx:=S!(1, 2)(3, 5)(4, 7)(6, 10)(8, 12)(9, 13)(11, 16)(14, 19)
(17, 22)(18, 23)(20,26)(21, 27)(24, 31)(25, 32)(28, 36)(29,
37)(30, 39)(33, 43)(34, 44)(35,46)(41, 49)(42, 51)(45, 55)
(47, 57)(48, 59)(50, 62)(52, 64)(53, 65)(54,67)(56, 69)(58,
72)(60, 71)(61, 74)(66, 78)(68, 80)(70, 82)(73, 85)(75,87)
(76, 83)(77, 89)(79, 90)(81, 92)(84, 94)(86, 93)(88, 97)(91,
99)(95,102)(96, 103)(98, 105)(100, 106)(104, 107)(109, 110);
yy:=S!(1, 3, 6)(2, 4, 8)(5, 9, 14)(7, 11, 17)(10, 15, 20)(12,
18, 24)(16, 21, 28)(19, 25, 33)(22, 29, 38)(23, 30, 40)(26,
34, 45)(27, 35, 39)(31, 41, 50)(32, 42, 52)(36, 47, 58)(37,
48, 60)(43, 53, 66)(44, 54, 68)(46, 56, 70)(49, 61, 75)(51,
63, 62)(55, 59, 73)(57, 71, 83)(64, 76, 88)(65, 77, 80)(67,
79, 91)(69, 81, 78)(72, 84, 95)(74, 86, 96)(82, 93, 100)(85,
```

```
90, 92)(87, 97, 104)(89, 98, 102)(94, 101, 107)(99, 105,
109)(103, 108, 110);
N:=sub<S|xx,yy>;
N12:=Stabiliser(N,[1,2]);
C:=Centraliser(N,N12);
Set(C);
{
(1, 74)(2, 67)(3, 96)(4, 91)(5, 107)(6, 86)(7, 20)(8, 79)
       (9, 101)(10, 17)(11, 15)(12, 50)(14, 94)(18, 41)(19,
       46)(21, 28)(22, 93)(23, 80)(24, 31)(25, 70)(26, 99)
       (27, 95)(29, 82)(30, 77)(32, 37)(33, 56)(34, 109)(35,
       84)(36, 102)(38, 100)(39, 72)(40, 65)(42, 60)(43, 66)
       (44, 75)(45, 105)(47, 98)(48, 52)(49, 68)(51, 85)(54,
       61)(55, 57)(58, 89)(59, 83)(62, 90)(63, 92)(64, 76)
      (69, 78)(71, 73)(87, 110)(97, 108)(103, 104),
Id(C),
(1, 2)(3, 5)(4, 7)(6, 10)(8, 12)(9, 13)(11, 16)(14, 19)(17,
      22)(18, 23)(20, 26)(21, 27)(24, 31)(25, 32)(28, 36)
      (29, 37)(30, 39)(33, 43)(34, 44)(35, 46)(41, 49)(42,
      51)(45, 55)(47, 57)(48, 59)(50, 62)(52, 64)(53, 65)
      (54,67)(56, 69)(58, 72)(60, 71)(61, 74)(66, 78)(68,
      80)(70, 82)(73, 85)(75, 87)(76, 83)(77, 89)(79, 90)
      (81, 92)(84, 94)(86, 93)(88, 97)(91, 99)(95, 102)(96,
      103)(98, 105)(100, 106)(104, 107)(109, 110),
(1, 2)(3, 72)(4, 99)(5, 58)(6, 85)(7, 91)(8, 83)(9, 16)(10,
      73)(11, 13)(12, 76)(14, 37)(15, 101)(17, 42)(18, 95)
      (19, 29)(21, 80)(22, 51)(23, 102)(25, 46)(27, 68)(28,
      41)(30, 107)(32, 35)(33, 98)(34, 75)(36, 49)(38, 108)
      (39, 104)(40, 63)(43, 105)(44, 87)(45, 69)(47, 78)(48,
      50)(52, 79)(53, 92)(54, 67)(55, 56)(57, 66)(59, 62)
      (60, 86)(61, 74)(64, 90)(65, 81)(70, 94)(71, 93)(77,
      96)(82, 84)(88, 100)(89, 103)(97, 106),
(1, 67, 61)(2, 74, 54)(3, 107, 103)(4, 20, 99)(5, 96, 104)
      (6, 17, 93)(7, 91, 26)(8, 50, 90)(9, 13, 101)(10,
      86, 22)(11, 16, 15)(12, 79, 62)(14, 46, 84)(18,
      80, 49)(19, 94, 35)(21, 95, 36)(23, 41, 68)(25,
      37, 82)(27, 28, 102)(29, 32, 70)(30, 72, 89)(33,
      66, 69)(34, 75, 110)(38, 100, 106)(39, 77, 58)(40,
      65, 53)(42, 85, 71)(43, 56, 78)(44, 109, 87)(45, 57,
      98)(47, 55, 105)(48, 83, 64)(51, 60, 73)(52, 76, 59)
      (63, 92, 81)(88, 108, 97),
(3, 58)(4, 91)(5, 72)(6, 73)(7, 99)(8, 76)(9, 11)(10, 85)
      (12, 83)(13, 16)(14, 29)(15, 101)(17, 51)(18, 102)
```

(19, 37)(20, 26)(21, 68)(22, 42)(23, 95)(24, 31)(25, 35)(27, 80)(28, 49)(30, 104)(32, 46)(33, 105)(34, 87)(36, 41)(38, 108)(39, 107)(40, 63)(43, 98)(44, 75)(45, 56)(47, 66)(48, 62)(50, 59)(52, 90)(53, 81)(55, 69)(57, 78)(60, 93)(64, 79)(65, 92)(70, 84)(71, 86)(77, 103)(82, 94)(88, 106)(89, 96)(97, 100)(109, 110),

(1, 74)(2, 67)(3, 89)(5, 39)(6, 71)(7, 26)(8, 64)(9, 15)(10, 51)(11,101)(12, 59)(13, 16)(14, 82)(17, 85)(18, 36)(19, 32)(20, 99)(21, 49)(22, 60)(23, 27)(25, 84)(28, 68)(29, 94)(30, 103)(33, 45)(34, 110)(35, 70)(37, 46)(38, 97)(40, 92)(41, 102)(42, 93)(43, 47)(48, 90)(50, 83)(52, 62)(53, 81)(54, 61)(55, 78)(56, 105)(57, 69)(58, 96)(63, 65)(66, 98)(72, 107)(73, 86)(76, 79)(77, 104)(80, 95)(87, 109)(88, 106)(100, 108),

(1, 54)(2, 61)(3, 104)(4, 26)(5, 103)(6, 22)(7, 99)(8, 62)(10, 93)(12, 90)(13, 101)(14, 35)(15, 16)(17, 86)(18, 68)(19, 84)(20, 91)(21, 102)(23, 49)(24, 31)(25, 29)(27, 36)(28, 95)(30, 58)(32, 82)(33, 78)(34, 87)(37, 70)(38, 106)(39, 89)(40, 53)(41, 80)(42, 73)(43, 69)(44, 110)(45, 47)(46, 94)(48, 76)(50, 79)(51, 71)(52, 83)(55, 98)(56, 66)(57, 105)(59, 64)(60, 85)(63, 81)(67, 74)(72, 77)(75, 109)(88, 108)(96, 107),

(1, 67, 61)(2, 74, 54)(3, 39, 103, 58, 107, 77)(4, 26, 99, 91, 20, 7)(5, 89, 104, 72, 96, 30)(6, 51, 93, 73, 17, 60)(8, 59, 90, 76, 50, 52)(9, 16, 101, 11, 13, 15)(10, 71, 22, 85, 86, 42)(12, 64, 62, 83, 79, 48)(14, 32, 84, 29, 46, 70)(18, 27, 49, 102, 80, 28)(19, 82, 35, 37, 94, 25)(21, 23, 36, 68, 95, 41)(24, 31)(33, 47, 69, 105, 66, 55)(34, 44, 110, 87, 75, 109)(38, 97, 106, 108, 100, 88)(40, 92, 53, 63, 65, 81)(43, 45, 78, 98, 56, 57),

(1, 61, 67)(2, 54, 74)(3, 77, 107, 58, 103, 39)(4, 7, 20, 91, 99, 26)(5, 30, 96, 72, 104, 89)(6, 60, 17, 73, 93, 51)(8, 52, 50, 76, 90, 59)(9, 15, 13, 11, 101, 16)(10, 42, 86, 85, 22, 71)(12, 48, 79, 83, 62, 64)(14, 70, 46, 29, 84, 32)(18, 28, 80, 102, 49, 27)(19, 25, 94, 37, 35, 82)(21, 41, 95, 68, 36, 23)(24, 31)(33, 55, 66, 105, 69, 47)(34, 109, 75, 87, 110, 44)(38, 88, 100, 108, 106, 97)(40, 81, 65, 63, 53, 92)(43, 57, 56, 98, 78, 45),

(1, 54)(2, 61)(3, 30)(4, 20)(5, 77)(6, 42)(8, 48)(9, 11)(10, 60)(12, 52)(13, 15)(14, 25)(16, 101)(17, 71)(18, 21)(19, 70)(22, 73)(23, 28)(26, 91)(27, 41)(29, 35)(32, 94)(33, 57)(36, 80)(37, 84)(38, 88)(39, 96)(40, 81)(43, 55)(44,

```
        109)(45, 66)(46, 82)(47, 56)(49, 95)(50, 64)(51, 86)(53,
        63)(58, 104)(59, 79)(62, 76)(65, 92)(67, 74)(68, 102)(69,
        98)(72, 103)(75, 110)(78, 105)(83, 90)(85, 93)(89, 107)
        (97, 100)(106, 108),
(1, 61, 67)(2, 54, 74)(3, 103, 107)(4, 99, 20)(5, 104, 96)(6,
        93, 17)(7, 26, 91)(8, 90, 50)(9, 101, 13)(10, 22, 86)(11,
        15, 16)(12, 62, 79)(14, 84, 46)(18, 49, 80)(19, 35, 94)
        (21, 36, 95)(23, 68, 41)(25, 82, 37)(27, 102, 28)(29, 70,
        32)(30, 89, 72)(33, 69, 66)(34, 110, 75)(38, 106, 100)
        (39, 58, 77)(40, 53, 65)(42, 71, 85)(43, 78, 56)(44, 87,
        109)(45, 98, 57)(47, 105, 55)(48, 64, 83)(51, 73, 60)(52,
        59, 76)(63, 81, 92)(88, 97, 108)
}
```

Consider the permutation $x = (1,2)(3,5)(4,7)(6,10)(8,12)(9,13)(11,16)(14,19)(17,22)(18,23)(20,26)(21,27)(24,31)(25,32)(28,36)(29,37)(30,39)(33,43)(34,44)(35,46)(41,49)(42,51)(45,55)(47,57)(48,59)(50,62)(52,64)(53,65)(54,67)(56,69)(58,72)(60,71)(61,74)(66,78)(68,80)(70,82)(73,85)(75,87)(76,83)(77,89)(79,90)(81,92)(84,94)(86,93)(88,97)(91,99)(95,102)(96,103)(98,105)(100,106)(104,107)(109,110)$. Note that x sends 1 to 2 and 2 to 1. Therefore, by the Famous Lemma, we have the following relation $(xt_1)^m = 1$. Now, since $t_1 \sim t$, then we have $(xt)^m = 1$.

We have another permutation in C that also sends 1 to 2 and 2 to 1. We use our Schreier System to find this permutation in terms of x and y. Thus, we see that $yxy^{-1}xyxyxy^{-1}xyxyxy = (1,2)(3,72)(4,99)(5,58)(6,85)(7,91)(8,83)(9,16)(10,73)(11,13)(12,76)(14,37)(15,101)(17,42)(18,95)(19,29)(21,80)(22,51)(23,102)(25,46)(27,68)(28,41)(30,107)(32,35)(33,98)(34,75)(36,49)(38,108)(39,104)(40,63)(43,105)(44,87)(45,69)(47,78)(48,50)(52,79)(53,92)(54,67)(55,56)(57,66)(59,62)(60,86)(61,74)(64,90)(65,81)(70,94)(71,93)(77,96)(82,84)(88,100)(89,103)(97,106)$. Our second relation found by the Famous Lemma is $(yxy^{-1}xyxyxy^{-1}xyxyxyt)^n = 1$.

Consider our next permutation $(3,58)(4,91)(5,72)(6,73)(7,99)(8,76)(9,11)(10,85)(12,83)(13,16)(14,29)(15,101)(17,51)(18,102)(19,37)(20,26)(21,68)(22,42)(23,95)(24,31)(25,35)(27,80)(28,49)(30,104)(32,46)(33,105)(34,87)(36,41)(38,108)(39,107)(40,63)(43,98)(44,75)(45,56)(47,66)(48,62)(50,59)(52,90)(53,81)(55,69)(57,78)(60,93)(64,79)(65,92)(70,84)(71,86)(77,103)(82,94)(88,106)(89,96)(97,100)(109,110)$, which we find by

Schrier System is equal to $yxyxyxy^{-1}xyxyxy^{-1}xy$. Now, since $yxyxyxy^{-1}xyxyxy^{-1}xy$ fixes 1 and 2, we have that $(t_1t_2)^k = yxyxyxy^{-1}xyxyxy^{-1}xy$. Note that $t_1 \sim t$ and $t_2 \sim t^x$, so our final relation found by the Famous Lemma is $(tt^x)^k = yxyxyxy^{-1}xyxyxy^{-1}xy$.

We add these relations, as well as some first order relations to our progenitor to produce the following isomorphic images.
$G =< x, y, t|x^2, y^3, (y^{-1}xyx)^5, (xy^{-1})^{11}, (yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2,$
$t^2, (t, yxyxy^{-1}xy^{-1}xy^{-1}), (t, yxyxyxy^{-1}xyxyxy^{-1}xy),$
$(tx)^k, (tyxy^{-1}xyxyxy^{-1}xyxyxy)^l, (t * t^x)^m = yxyxyxy^{-1}xyxyxy^{-1}xy,$
$(xyt^{yx})^{r1}, ((xy)^2t)^{r2} >.$

Table 2.4: $2^{*110} : L_2(11)$

r1	r2	k	l	m	Order	G
10	5	8	8	11	7920	M_{11}

Chapter 3

Extension Problems

3.1 Preliminaries

Definition 3.1. *[Rot95] Let* G *be a group such that* $K \leq G$*.* K *is* **normal** *in* G *if* $gKg^{-1} = K$*, for every* $g \in G$*. We will use* $K \lhd G$ *to denote* K *as being normal in* G*.*

Definition 3.2. *[Rot95] If* $N \lhd G$*, then the cosets of* N *in* G *form a group, denoted by* G/N*, of order* $[G : N]$*.*

Definition 3.3. *[Rot95] Let* G *be a group. A* **normal series** G *is a sequence of subgroups*

$$G = G_0 \geq G_1 \geq \cdots \geq G_n = 1$$

with $G_{i+1} \lhd G_i$*. Furthermore, the* **factor groups** *of* G *are given by* G_i/G_{i+1} *for* $i = 0, 1, \ldots, n-1$*.*

Definition 3.4. *[Rot95] Let* G *be a group. A* **composition series** *of* G *given by:*

$$G = G_0 \geq G_1 \geq \cdots \geq G_n = 1$$

is a normal series where, for all i*, either* G_{i+1} *is a maximal normal subgroup of* G_i *or* $G_{i+1} = G_i$*.*

Definition 3.5. *[Rot95] If group* G *has a composition series, the factor groups of its series are the* **composition factors** *of* G*.*

Definition 3.6. *[Rot95] Let* G *be a group. We say* G *is a* **direct product** *of two subgroups* H *and* K *if:*

1. $H \trianglelefteq G$, $K \trianglelefteq G$;

2. $G = HK$;

3. $H \cap K = 1$,

Definition 3.7. *[Rot95]* G *is a* **semi-direct product** *of two subgroups* H *and* K *if:*

1. $K \trianglelefteq G$, $Q \leq G$;

2. $G = KQ$;

3. $K \cap Q = 1$.

Definition 3.8. *[Rot95] Let* G *be a group. The* **center** *of* G, $Z(G)$, *is the set of all elements in* G *that commute with all elements of* G.

Definition 3.9. *[Rot95] Let* G *be a group and* H, $N \leq G$ *such that* $|G| = |N||H|$. G *is a* **central extension** *by* H *if* N *is the center of* G. *We denote this by* $G \cong N^{\bullet}H$.

Definition 3.10. *[Rot95] Let* G *be a group and* H, $N \leq G$ *such that* $|G| = |N||H|$. G *is a* **mixed extension** *by* H *if it is a combination of both central extensions and semi-direct products, where* N *is the normal subgroup of* G *but not central. We denote this by* $G \cong N^{\bullet} : H$.

3.2 Direct Product

Consider the group $\frac{2^{*20}:L_2(11)}{(tt^x)^1=(yx)^3y^{-1}(xy)^2xy^{-1}xy,(yxy^{-1}(xy)^4xy^{-1}t)^6}$.

G has the following presentation,
$G =< x, y, t | x^2, y^3, (y^{-1}xyx)^5, (xy^{-1})^{11}, (yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2,$
$t^2, (t, yxyxy^{-1}xy^{-1}xy^{-1}), (t, yxyxyxy^{-1}xyxyxy^{-1}xy),$
$(tt^x)^1 = yxyxyxy^{-1}xyxyxy^{-1}xy, (yxy^{-1}xyxyxyxyxy^{-1}t)^6 >.$

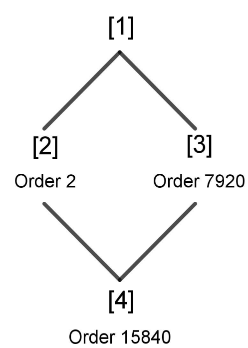
The compositon series of G is below.

```
G
|  M11
*
|  Cyclic(2)
1
```

$G = G_1 \supseteq 1$, where $G = (G/G_1)(G_1/1) = C_2 M_{11}$.

We have C_2 by M_{11}. In order for this to be a direct product we need to have M_{11} and C_2 normal in G.

The Normal Lattice of G is



M_{11} is of order 7920. We verify that our normal subgroup [3] is in fact M_{11}.

```
> load m11;
Loading "/usr/local/MAGMA/libs/pergps/m11"
M11 - Mathieu group on 11 letters - degree 11
Order 7 920 = 2^4 * 3^2 * 5 * 11;  Base 1,2,3,4
Group: G
> M11:=G;
> Order(M11);
```

```
7920
> s:=IsIsomorphic(NL[3],M11);
> s;
true
```

Therefore by definition of a direct product we have that G is isomorphic to $M_{11} \times C_2$. To verify this, we will first need to write a presentation for M_{11}. We use FPGroup in MAGMA to get a presentation for M_{11}.

```
> FPGroup(M11);
Finitely presented group on 2 generators
Relations
    $.1^2 = Id($)
    $.2^4 = Id($)
    $.2^-1 * $.1 * $.2^-2 * $.1 * $.2^-2 * $.1 * $.2^2 *
    $.1 * $.2^2 * $.1 * $.2^2 * $.1 * $.2^-1 = Id($)
    $.1 * $.2 * $.1 * $.2 * $.1 * $.2^-1 * $.1 * $.2^-1 *
    $.1 * $.2 * $.1 * $.2^-1 * $.1 * $.2^2 * $.1 * $.2 *
    $.1 * $.2^-1 = Id($)
    $.1 * $.2^-2 * $.1 * $.2^-1 * $.1 * $.2 * $.1 * $.2^-2 *
    $.1 * $.2^-1 * $.1 * $.2 * $.1 * $.2^2 * $.1 * $.2^-1 *
    $.1 * $.2 = Id($)
    ($.1 * $.2^-1)^11 = Id($)
```

A presentation for M_{11} is $K =< x, y|x^2, y^4, y^{-1}xy^{-2}xy^{-2}xy^2xy^2xy^2xy^{-1},$
$xyxyxy^{-1}xy^{-1}xyxy^{-1}xy^2xyxy^{-1}, xy^{-2}xy^{-1}xyxy^{-2}xy^{-1}xyxy^2xy^{-1}xy, (xy^{-1})^{11} >$

A presentation for C_2 is $H =< z|z^2 >$.

Thus a presentation for $G = H \times K$ is $< z, x, y|z^2, (x, z), (y, z),$
$x^2, y^4, y^{-1}xy^{-2}xy^{-2}xy^2xy^2xy^2xy^{-1},$
$xyxyxy^{-1}xy^{-1}xyxy^{-1}xy^2xyxy^{-1}, xy^{-2}xy^{-1}xyxy^{-2}xy^{-1}xyxy^2xy^{-1}xy, (xy^{-1})^{11} >$

We verify that this presentation is isomorphic to $2 \times M_{11}$.

```
> G<x,y,t>:=Group<x,y,t|x^2,y^3,(y^-1*x*y*x)^5,(x*y^-1)^11,
> (y*x*y*x*y*x*y^-1*x*y^-1*x*y^-1*x)^2,
```

```
> t^2,(t,y * x * y * x * y^-1 * x * y^-1 * x * y^-1),
> (t,y * x * y * x * y * x * y^-1 * x * y * x * y * x *
> y^-1 * x * y),(t*t^x)^1=y * x * y * x * y * x * y^-1 *
> x * y * x * y * x * y^-1 * x * y,(y * x * y^-1 * x * y *
> x * y * x * y * x * y * x * y^-1*t)^6>;
> #G;
15840
> f,G1,k:=CosetAction(G,sub<G|x,y>);
> #k1;
1
> GG<x,y,z>:=Group<x,y,z|x^2,y^4, y^-1 * x * y^-2 * x *
> y^-2 * x * y^2 * x * y^2 * x * y^2 * x * y^-1,x * y * x *
> y * x * y^-1 * x * y^-1 * x * y * x * y^-1 * x * y^2 * x *
> y * x * y^-1,x * y^-2 * x * y^-1 * x * y * x * y^-2 * x *
> y^-1 * x * y * x * y^2 * x * y^-1 * x * y,(x * y^-1)^11,
> z^2,(x,z),(y,z)>
> #GG;
15840
> f,G2,k2:=CosetAction(GG,sub<GG|Id(GG)>);
> #k2;
1
> s:=IsIsomorphic(G1,G2);
s;
> s;
true
```

Therefore, $\frac{2^{*20}:L_2(11)}{(tt^x)^1=(yx)^3y^{-1}(xy)^2xy^{-1}xy,(yxy^{-1}(xy)^4xy^{-1}t)^6} \cong (2 \times M_{11})$.

3.3 Semi-Direct Product

Consider the group $\frac{2^{*20}:(2^4:S_5)}{(x^2y^2x^{-1}y^{-1}t^{x^2y}t^{x^2})^3}$.

G has the following presentation,

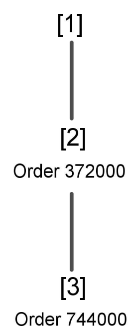
$G =< x, y, t|x^6, (xy^{-1})^4, yxy^{-1}x^{-2}y^{-1}xyx^{-1}, y^{-1}x^{-1}yx^{-1}y^{-1}xy^3x,$
$(x^{-1}y^2x^{-1}y^{-1})^2,$
$t^2, (t, yx^2y^{-2}x^{-1}yx^{-1}), (t, x^{-1}y^{-1}x^{-1}y^{-3}xy^{-1}),$
$(t, (yxy^{-1})^3), (t, y^{-1}x^3y^{-2}), (x^2y^2x^{-1}y^{-1}t^{x^2y}t^{x^2})^3 >.$

The compositon series of G is below.

```
G
|  Cyclic(2)
*
|  A(2, 5)                    = L(3, 5)
1
```

$G = G_1 \supseteq 1$, where $G = (G/G_1)(G_1/1) = L_3(5)C_2$.

The Normal Lattice of G is



We have a normal subgroup of order 372000. Since $L_3(5)$ is of order 372000, we verify that NL[2] is isomoprhic to $L_3(5)$.

```
> s:=IsIsomorphic(NL[2],L_3(5));
> s;
true
```

Recall from the previous section, that in order to have a direct product we must have C_2 as well as $L_3(5)$ normal in G. Since our normal subgroup lattice does not show a subgroup of order 2, we know that C_2 is not normal in G. Thus we cannot have a direct product.

This extension must be a semi-direct product. We need to find an element of order 2 that will extend $L_3(5)$ to G. To do this, we first must write a presentation for $L_3(5)$.

```
> H<x,y>:=Group<x,y|x^4,
>      x^-1 * y^-1 * x^-2 * y * x * y^-1 * x^2 * y,
>      y^-1 * x^-1 * y * x^-2 * y^-2 * x^-1 * y^-1 * x^2 * y^3 *
>      x^-1,
>      y^-1 * x^-2 * y * x^-1 * y * x^-1 * y^-1 * x * y^-1 *
>      x^2 * y^2 * x * y^-1,
>      (x * y^-1 * x^-1 * y^2)^3,
>      y^-2 * x^-2 * y^-1 * x^-1 * y^-2 * x^2 * y^-1 * x^-1 *
>      y^-2 * x^2 * y^-1 * x^-1,
>      y * x^-1 * y^2 * x^-2 * y^-1 * x^-1 * y^-1 * x^-1 * y^-3 *
>      x^2 * y^-1 * x * y^-1 * x^-1>;
> f,H1,k:=CosetAction(H,sub<H|Id(H)>);
> s:=IsIsomorphic(NL[2],H1);
s;
> s;
true
```

Now we find an element of order 2, which we will label C, that will extend $L_3(5)$ to G.

```
for i in NL[3] do if i notin NL[2] and Order(i) eq 2 and
sub<G1|i,NL[2]> eq G1 then C:=i;
break; end if; end for;
```

Now that we have this element C, of order 2, we find the action of C on the generators of H.

Below we use MAGMA Schreier System and the following loop,

```
> for i in [1..#N1] do if ArrayP[i] eq A^C then print Sch[i];
for|if> end if; end for;
y * x * y^-2 * x * y^-2 * x^2 * y^-1 * x * y * x
> for i in [1..#N1] do if ArrayP[i] eq B^C then print Sch[i];
for|if> end if; end for;
x * y^2 * x^-1 * y^-1 * x^-1 * y^2 * x * y^2 * x * y * x
```

So now we know that $x^C = yxy^{-2}xy^{-2}x^2y^{-1}xyx$, and $y^C = xy^2x^{-1}y^{-1}x^{-1}y^2xy^2xyx$.

Now we will add this element of order 2, say z, to our presentation, along with the action of this element on the generators of $L_3(5)$ to obtain the following presentation.
$G2 =< x, y, z|x^4, x^{-1}y^{-1}x^{-2}yxy^{-1}x^2y, y^{-1}x^{-1}yx^{-2}y^{-2}x^{-1}y^{-1}x^2y^3x^{-1},$
$y^{-1}x^{-2}yx^{-1}yx^{-1}y^{-1}xy^{-1}x^2y^2xy^{-1}, (xy^{-1}x^{-1}y^2)^3,$
$y^{-2}x^{-2}y^{-1}x^{-1}y^{-2}x^2y^{-1}x^{-1}y^{-2}x^2y^{-1}x^{-1},$
$yx^{-1}y^2x^{-2}y^{-1}x^{-1}y^{-1}x^{-1}y^{-3}x^2y^{-1}xy^{-1}x^{-1},$
$z^2, x^z = yxy^{-2}xy^{-2}x^2y^{-1}xyx, y^z = xy^2x^{-1}y^{-1}x^{-1}y^2xy^2xyx >.$

We then verify that this presentation is isomorphic to G.

```
> G<x,y,z>:=Group<x,y,z|x^4,
>      x^-1 * y^-1 * x^-2 * y * x * y^-1 * x^2 * y,
>      y^-1 * x^-1 * y * x^-2 * y^-2 * x^-1 * y^-1 * x^2 * y^3 *
>      x^-1,
>      y^-1 * x^-2 * y * x^-1 * y * x^-1 * y^-1 * x * y^-1 *
>      x^2 * y^2 * x * y^-1,
>      (x * y^-1 * x^-1 * y^2)^3,
>      y^-2 * x^-2 * y^-1 * x^-1 * y^-2 * x^2 * y^-1 * x^-1 *
>      y^-2 * x^2 * y^-1 * x^-1,
>      y * x^-1 * y^2 * x^-2 * y^-1 * x^-1 * y^-1 * x^-1 * y^-3 *
>      x^2 * y^-1 * x * y^-1 * x^-1,z^2,x^z=y * x * y^-2 * x *
>      y^-2 * x^2 * y^-1 * x * y * x,y^z=x * y^2 * x^-1 * y^-1 *
>      x^-1 * y^2 * x * y^2 * x * y * x>;
> f2,G2,k1:=CosetAction(G,sub<G|Id(G)>);
> s,t:=IsIsomorphic(G2,G1);s;
true
```

Therefore, $\frac{2^{*20}:(2^4:S_5)}{(x^2y^2x^{-1}y^{-1}t^{x^2yt^{x^2}})^3} \cong (C_2 : L_3(5))$.

3.4 Central Extension

Consider the group $\frac{2^{*20}:((C_4:C_5)\times S_4)}{(x^2yxy^3t^{y^3x})^2,((xy)^2t^y)^7,((xy)^2t)^6}$.

G has the following presentation,
$G =< x, y, t|G < x, y, t >:= Group < x, y, t|x^4, yx^{-2}y^2x^2y, yx^{-1}y^{-2}xy^3, x^{-1}y^{-1}x^{-2}y^{-1}$

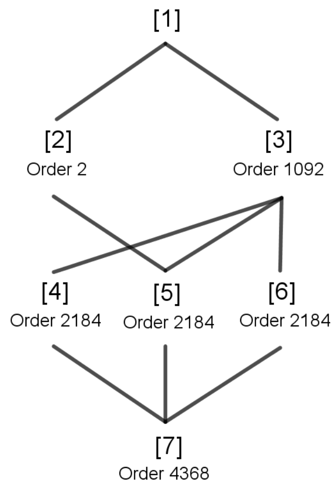
$x^2yx^2yx^{-1}, x^{-1}y^{-1}x^{-1}y^{-1}x^{-1}y^{-1}xy^{-1}xyxy,$
$t^2, (t, yx^{-1}y^2), (t, yxyx^{-1}y), (x^2yxy^3t^{y^3x})^2, ((xy)^2t^y)^7, ((xy)^2t)^6 >.$

The composition series of G is below.

```
CompositionFactors(G1);
    G
    |  Cyclic(2)
    *
    |  A(1, 13)                    = L(2, 13)
    *
    |  Cyclic(2)
    1
```

$G = G_1 \supseteq 1$, where $G = (G/G_1)(G_1/G_2)(G_2/1) = C_2L_2(13)C_2$.

The normal lattice of G is



$NL[2]$ is of order 2. We will check to see if this is our center.

```
> Center(G1) eq NL[2];
true
```

It is possible that we may have a central extension. If there is a larger abelian subgroup then we will instead have a mixed extension.

The following loop will list all of our abelian subgroups.

```
> for i in [1..11] do if IsAbelian(NL[i]) then i;end if;end for;
1
2
```

We now know that $NL[2]$, our center, is a maximal abelian subgroup, thus we will have a central extension. Now we factor G by our center to form the quotient group q, and look at the Composition Factors of q.

```
> q,ff:=quo<G1|NL[2]>;
> CompositionFactors(q);
    G
    |  Cyclic(2)
    *
    |  A(1, 13)                    = L(2, 13)
    1
```

It looks like q may be Isomorphic to $PGL_2(13)$.

```
> s:=IsIsomorphic(q,PGL(2,13));s;
true
```

Thus we will have a central extension of C_2 by $PGL_2(13)$.

Now we need to write a presentation for $PGL_2(13)$.

```
> FPGroup(PGL(2,13));
Finitely presented group on 2 generators
Relations
    $.2^3 = Id($)
    ($.1^-1 * $.2^-1)^4 = Id($)
    $.1^12 = Id($)
    $.2 * $.1 * $.2^-1 * $.1^4 * $.2 * $.1^2 * $.2^-1 *
    $.1^-1 = Id($)
    $.1^2 * $.2 * $.1^4 * $.2^-1 * $.1 * $.2 * $.1^-1 *
    $.2^-1 = Id($)
    $.1^-1 * $.2 * $.1^-1 * $.2^-1 * $.1^3 * $.2 * $.1^3 *
    $.2^-1 = Id($)
```

Thus our presentation for $PGL_2(13)$ is

$H < a, b >:= Group < a, b | b^3, (a^{-1}b^{-1})^4, a^12, bab^{-1}a^4ba^2b^{-1}a^{-1}, a^2ba^4b^{-1}aba^{-1}b^{-1},$
$a^{-1}ba^{-1}b^{-1}a^3ba^3b^{-1} >$.

Now we compute the coset action of $PGL_2(13)$, which we have labeled as H, and check if our presentation is isomorphic to q.

```
> H<a,b>:=Group<a,b|b^3,(a^-1 * b^-1)^4,a^12,
> b * a * b^-1 * a^4 * b * a^2 * b^-1 * a^-1,
> a^2 * b * a^4 * b^-1 * a * b * a^-1 * b^-1,
> a^-1 * b * a^-1 * b^-1 * a^3 * b * a^3 * b^-1>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,q);s;
true
```

To write our presentation we need to write the generators of $PGL_2(13)$ in terms of our center, which we will label c.

```
> T:=Transversal(G1,NL[2]);
> ff(T[2]) eq q.1;
true
> ff(T[3]) eq q.2;
true
> A:=T[2];
> B:=T[3];
> c:=NL[2].2;
> for i in [1..2] do if B^3 eq c^i then i; end if; end for;
> for i in [1..2] do if (A^-1*B^-1)^4 eq c^i then i;
> end if; end for;
> for i in [1..2] do if A^12 eq c^i then i; end if; end for;
2
> for i in [1..2] do if (A^-1*B^-1)^4 eq c^i then i;
> end if; end for;
> for i in [1..2] do if B*A*B^-1*A^4*B*A^2*B^-1*A^-1 eq c^i
> then i; end if; end for;
> for i in [1..2] do if A^2*B*A^4*B^-1*A*B*A^-1*B^-1 eq c^i
> then i; end if; end for;
> for i in [1..2] do if A^-1*B*A^-1*B^-1*A^3*B*A^3*B^-1 eq c^i
> then i;
> end if; end for;
```

Now we can write a presentation for G by including c, our generator of the center C_2, and writing $PGL_2(13)$ in terms of c.

```
HH<c,a,b>:=Group<c,a,b|c^2,(c,a),(c,b),b^3,(a^-1 * b^-1)^4,
> a^12=c^2,b * a * b^-1 * a^4 * b * a^2 * b^-1 * a^-1,
> a^2 * b * a^4 * b^-1 * a * b * a^-1 * b^-1,
> a^-1 * b * a^-1 * b^-1 * a^3 * b * a^3 * b^-1>;
```

```
> f2,H2,k2:=CosetAction(HH,sub<HH|Id(HH)>);
> s:=IsIsomorphic(H2,G1);
> s;
true
```

Thus $G \cong C_2^{\bullet} PGL_2(13)$

3.5 Mixed Extension

Consider the group $\frac{2^{*20}:L_2(11)}{(xt^{y^{14}})^3,(y^5tt^{y^3})^2}$.

G has the following presentation,

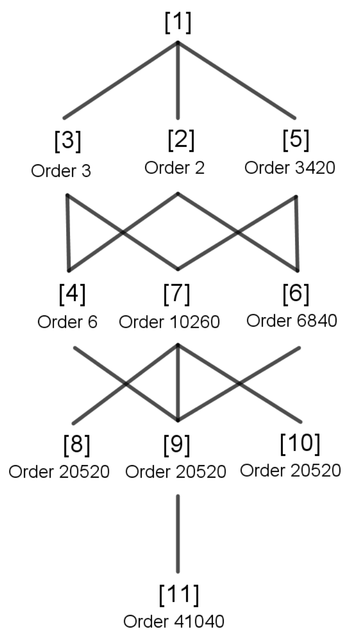
$G =< x, y, t|x^2, (y^{-1}x)^2, y^{20}, t^2, (t, xy^{-9}), (xt^{(y^{14})})^3, (y^5tt^{(y^3)})^2 >$.

The composition series of G is below.

```
CompositionFactors(G1);
    G
    |  Cyclic(2)
    *
    |  A(1, 19)                    = L(2, 19)
    *
    |  Cyclic(3)
    *
    |  Cyclic(2)
    1
```

$G = G_1 \supseteq G_2 \supseteq G_3 \supseteq 1$, where $G = (G/G_1)(G_1/G_2)(G_2/G_3)(G_3/1) = C_2L_2(19)C_3C_2$.

The Normal Lattice of G is



$NL[2]$ is of order 2. We will check to see if this is our center.

```
> Center(G1) eq NL[2];
true
```

It is possible that we may have a central extension. If there is a larger abelian subgroup then we will instead have a mixed extension.

The following loop will list all of our abelian subgroups.

```
> for i in [1..11] do if IsAbelian(NL[i]) then i;
> end if;end for;
1
2
```

```
3
4
```

We now know that $NL[2]$, our center, is not a maximal abelian subgroup. $NL[4]$, which is of order 6, is our maximal abelian subgroup. Below we confirm that C_6 is the isomorphism type of $NL[4]$.

```
> X:=AbelianGroup(GrpPerm,[6]);
> s:=IsIsomorphic(X,NL[4]);s;
true
```

We will have a mixed extension of $NL[4]$ by q where q is the isomorphic image of $G/G2 = G/NL[4]$. Now we need to look at the normal lattice and composition factors of q to solve its isomorphism type.

```
> q,ff:=quo<G1|NL[4]>;
> nl:=NormalLattice(q);
> nl;
Normal subgroup lattice
-----------------------

[3]  Order 6840  Length 1  Maximal Subgroups: 2
---
[2]  Order 3420  Length 1  Maximal Subgroups: 1
---
[1]  Order 1     Length 1  Maximal Subgroups:

> CompositionFactors(q);

    G
    |  Cyclic(2)
    *
    |  A(1, 19)                    = L(2, 19)
    1
```

By looking at the composition series of q, it seems that our extension problem of q may be $PGL(2,19)$.

```
> s:=IsIsomorphic(q,PGL(2,19));
> s;
true
```

So now that we know that q is isomorphic to $PGL(2,19)$, we need to find a presentation for q.

```
> FPGroup(q);
Finitely presented group on 3 generators
Relations
    $.1^2 = Id($)
    $.3^2 = Id($)
    ($.2^-1 * $.1)^2 = Id($)
    ($.2 * $.3 * $.2^-1 * $.3 * $.2)^2 = Id($)
    ($.2 * $.3 * $.2^-1 * $.3 * $.2^-1 * $.3)^2 = Id($)
    ($.1 * $.3 * $.2 * $.3 * $.2^-1 * $.3)^2 = Id($)
    $.2^8 * $.1 * $.3 * $.2^-1 * $.3 * $.2 * $.3 = Id($)
    $.1 * $.3 * $.1 * $.3 * $.2^4 * $.3 * $.2^-1 * $.3 *
    $.2^-2 * $.3 = Id($)
> H<x,y,z>:=Group<x,y,z|x^2,z^2,(y^-1*x)^2,
> (y*z*y^-1*z*y)^2,(y*z*y^-1*z*y^-1*z)^2,
> (x*z*y*z*y^-1*z)^2,y^8*x*z*y^-1*z*y*z,
> x*z*x*z*y^4*z*y^-1*z*y^-2*z>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,q); s;
true
```

Now our next step is to write the generators of H into elements of q. In order to do this we will need to look at the transversals of $NL[4]$.

```
> T:=Transversal(G1,NL[4]);
> #T;
6840
> T[2];
```

Note that $T[2]$ will give us a permutation that we will have to store in MAGMA. We will store this permutation as A. Similarily we will store the permutation for $T[3]$ as

B and the permutation for $T[4]$ as C.

```
> ff(A) eq q.1;
true
> ff(B) eq q.2;
true
> ff(C) eq q.3;
true
```

We want to look at our presentation of H to see which elements have changed by the action of q.

$H =< x, y, z | x^2, z^2, (y^{-1}x)^2, (yzy^{-1}zy)^2, (yzy^{-1}zy^{-1}z)^2, (xzyzy^{-1}z)^2, y^8xzy^{-1}zyz,$
$xzxzy^4zy^{-1}zy^{-2}z >$

Our first relation in the presentation, x^2 tells us that $x^2 = e$, therefore the order of x is 2. We want to see what the order of x is when we apply the action of q.

```
> Order(A);
2
```

So we see that a does not change. We will check the rest of the relations in our presentation and look for any changes.

```
> Order(C);
2
> Order(B^-1*A);
2
> Order(B*C*B^-1*C*B^-1*C);
2
> Order(A*C*B*C*B^-1*C);
2
> Order(B^8*A*C*B^-1*C*B*C);
1
> Order(A*C*A*C*B^4*C*B^-1*C*B^-2*C);
```

```
6
```

The order of our last relation has changed. We will need to write this relation in terms of q.

We will need to find a generator of $NL[4]$. Note that $NL[4]$, of order 6, is cyclic. So if we obtain an element of order 6 then this element will generate the whole group. We will name this element of order 6, D.

```
> IsCyclic(NL[4]);
true
> Order(NL[4].1);
6
> D:=NL[4].1;
```

Now we go back to our relation that has changed and write this relation in terms of D.

```
> A*C*A*C*B^4*C*B^-1*C*B^-2*C eq D;
false
> A*C*A*C*B^4*C*B^-1*C*B^-2*C eq D^2;
false
> A*C*A*C*B^4*C*B^-1*C*B^-2*C eq D^3;
false
> A*C*A*C*B^4*C*B^-1*C*B^-2*C eq D^4;
false
> A*C*A*C*B^4*C*B^-1*C*B^-2*C eq D^5;
true
```

MAGMA tells us that this relation is equal to D^5.

Next we need to check to see if D commutes with x,y, or z.

```
> for i in [0..6] do if D^A eq D^i
> for|if> then i; break; end if; end for;
5
```

The above loop confirms that $D^x = D^5$.

```
> for i in [0..6] do if D^B eq D^i
for|if> then i; break; end if; end for;
5
> for i in [0..6] do if D^C eq D^i
for|if> then i; break; end if; end for;
5
```

We have confirmed that $D^y = D^5$ and $D^z = D^5$. We can now write a presentation for H, where w will be our element of order 6, and check to see if H is isomorphic to G.

```
> H<w,x,y,z>:=Group<w,x,y,z|w^6,x^2,z^2,(y^-1*x)^2,
> (y*z*y^-1*z*y)^2,(y*z*y^-1*z*y^-1*z)^2,
> (x*z*y*z*y^-1*z)^2,y^8*x*z*y^-1*z*y*z,
> x*z*x*z*y^4*z*y^-1*z*y^-2*z=w^5,
> w^x=w^5,
> w^y=w^5,
> w^z=w^5>;
> #H;
41040
> #G1;
41040
> f,h,k:=CosetAction(H,sub<H|Id(H)>);
> #h;
41040
> #G1;
41040
> s:=IsIsomorphic(h,G1);
> s;
true
```

Therefore $G = \frac{2^{*20}:L_2(11)}{(xt^{y^{14}})^3,(y^5tt^{y^3})^2} \cong 6^{\bullet} : PGL(2,19)$.

Chapter 4

Monomial Progenitors

4.1 Preliminaries

Definition 4.1. *[Cur07] A* **monomial representation** *of a group G is a homomorphism from G into $GL(n, F)$, the group of nonsingular $n \times n$ matrices over the field F, in which the image of every element of G is a monomial matrix over F.*

Definition 4.2. *[**?**] (***Monomial Character***) Let G be a finite group and $H \leq G$. The character X of G is monomial if $X = \lambda^G$, where λ is a linear character of H.*

Definition 4.3. *[**?**] A matrix in which there is precisely one non-zero term in each row and in each column is said to be* **monomial**.

Definition 4.4. *[**?**] Let $A(x) = (a_{ij}(x))$ be a matrix representation of G of degree m. We consider the characteristic polynomial of $A(x)$, namely*

$$det(\lambda I - A(x)) = \begin{pmatrix} \lambda - a_{11}(x) & -a_{12}(x) & ... & -a_{1m}(x) \\ -a_{21}(x) & \lambda - a_{22}(x) & ... & -a_{2m}(x) \\ ... & ... & & ... \\ -a_{m1}(x) & -a_{m2}(x) & ... & \lambda - a_{mm}(x) \end{pmatrix}$$

This is a polynomial of degree m in λ, and inspection shows that the coefficient of $-\lambda^{m-1}$ is equal to

$$\phi(x) = a_1(x) + a_{22}(x) + ... + a_{mm}(x).$$

It is customary to call the right-hand side of this equation the **trace** *of $A(x)$, abbreviated to $trA(x)$, so that*

$$\phi(x) = trA(x).$$

Definition 4.5. *[?] The sum of squares of the degrees of the distinct irreducible characters of G is equal to $|G|$. The* **degree of a character** *χ is $\chi(1)$. Note that a character whose degree is 1 is called a linear character.*

Definition 4.6. *[Isa76] Let $H \leq G$ and $\phi(u)$ be a charcter of H and define $\phi(x) = 0$ if $x \in H$, then*

$$\phi^G(x) = \begin{cases} \phi(x), x \in H \\ 0x \notin H \end{cases}$$

is an induced character of G.

Definition 4.7. Formula for Induced Character

[Isa76] Let G be a finite group and H be a subgroup such that $[G : H] = \frac{|G|}{|H|} = n$. Let C_α, $\alpha = 1, 2, ..., m$ be the conjugacy classes of G with $|C_\alpha| = h_\alpha$, $\alpha = 1, 2, ..., m$. Let ϕ be a character of H and ϕ^G be the character of G induced from the character ϕ of H up to G. The values of ϕ^G on the m classes of G are given by:

$$\phi_\alpha^G = \frac{n}{h_\alpha} \sum_{w \in C_\alpha \cap H} \phi(w),\ \alpha = 1, 2, 3, ..., m.$$

4.2 Monomial Progenitor $11^{*4} :_m (4 : 5)$

Consider $11^{*4} :_m (4 : 5)$. $G = (4 : 5)$ is given by
$G = (4 : 5) =< x, y | x^4, xy^4x^3y^3, y^3x^3yx >$, where

$x = (1,4,17,15)(2,3,18,16)(5,12,14,7)(6,11,13,8)(9,19,10,20)$, and
$y = (1,6,10,14,18)(2,5,9,13,17)(3,8,12,15,19)(4,7,11,16,20)$.

$G = (C_4 : C_5)$ has monomial irreducible representation in dimension 5. We will write a progenitor for $11^{*4} :_m (C_4 : C_5)$. Since $\frac{|G|}{|H|} = 5 \Rightarrow \frac{20}{|H|} = 5 \Rightarrow |H| = 4$, we need to find a subgroup H of order 4 and induce a linear character of H up to G to obtain the irreducible character of degree 5 of G.

The conjugacy classes of group $(C_4 : C_5)$ are given in the table below.

Table 4.1: Conjugacy Classes of $(C_4 : C_5)$

Class	Representative of the class	# of elements in the class
C_1	e	1
C_2	$x^2 = (1,17)(2,18)(3,16)(4,15)(5,14)(6,13)(7,12)(8,11)(9,10)(19,20)$	5
C_3	$x = (1,4,17,15)(2,3,18,16)(5,12,14,7)(6,11,13,8)(9,19,10,20)$	5
C_4	$x^3 = (1,15,17,4)(2,16,18,3)(5,7,14,12)(6,8,13,11)(9,20,10,19)$	5
C_5	$y = (1,6,10,14,18)(2,5,9,13,17)(3,8,12,15,19)(4,7,11,16,20)$	4

Consider the subgroup $H = Z_5$ of G given below.
$H = \{e, (1,18,14,10,6)(2,17,13,9,5)(3,19,15,12,8)(4,20,16,11,7), (1,14,6,18,10)(2,13,5,17,9)(3,15,8,19,12)(4,16,7,20,11), (1,10,18,6,14)(2,9,17,5,13)(3,12,19,8,15)(4,11,20,7,16), (1,6,10,14,18)(2,5,9,13,17)(3,8,12,15,19)(4,7,11,16,20)\}$. The conjugacy classes of Z_5 are given in the table below.

Table 4.2: Conjugacy Classes of $H = Z_5$

Class	Representative of the class	# of elements in the class
D_1	e	1
D_2	$y^4 = (1,18,14,10,6)(2,17,13,9,5)(3,19,15,12,8)(4,20,16,11,7)$	1
D_3	$y^3 = (1,14,6,18,10)(2,13,5,17,9)(3,15,8,19,12)(4,16,7,20,11)$	1
D_4	$y^2 = (1,10,18,6,14)(2,9,17,5,13)(3,12,19,8,15)(4,11,20,7,16)$	1
D_5	$y = (1,6,10,14,18)(2,5,9,13,17)(3,8,12,15,19)(4,7,11,16,20)$	1

Consider the irreducible characters ϕ (of H) and χ (of G) given below.

Table 4.3: Character Table of $H = Z_5$

Class	D_1	D_2	D_3	D_4	D_5
Size	1	1	1	1	1
Order	1	5	5	5	5
ϕ_1	1	1	1	1	1
ϕ_2	1	Z	Z^2	Z^3	Z^4
ϕ_3	1	Z^2	Z^4	Z	Z^3
ϕ_4	1	Z^3	Z	Z^4	Z^2
ϕ_5	1	Z^4	Z^3	Z^2	Z

where Z is the 5th root of unity.

Table 4.4: Character Table of $G = (C_4 : C_5)$

Class	C_1	C_2	C_3	C_4	C_5
Size	1	5	5	5	4
Order	1	2	4	4	5
χ_1	1	1	1	1	1
χ_2	1	1	-1	-1	1
χ_3	1	-1	-I	I	1
χ_4	1	-1	I	-I	1
χ_5	4	0	0	0	-1

where I is the 4th root of unity.

Next we must find a non-trivial linear character of H to induce up to G. Note that each character of H is linear since they all have degree 1. We will induce χ_2 up to G.

Now, $G = He \cup Hx \cup Hx^2 \cup Hx^3$
Let $t_1 = e$, $t_2 = x$, $t_3 = x^2$, and $t_4 = x^3$.

Then

$$A(xx) = \begin{bmatrix} \phi(t_1xt_1^{-1}) & \phi(t_1xt_2^{-1}) & \phi(t_1xt_3^{-1}) & \phi(t_1xt_4^{-1}) \\ \phi(t_2xt_1^{-1}) & \phi(t_2xt_2^{-1}) & \phi(t_2xt_2^{-1}) & \phi(t_2xt_4^{-1}) \\ \phi(t_3xt_1^{-1}) & \phi(t_3xt_2^{-1}) & \phi(t_3xt_3^{-1}) & \phi(t_3xt_4^{-1}) \\ \phi(t_4xt_1^{-1}) & \phi(t_4xt_2^{-1}) & \phi(t_4xt_3^{-1}) & \phi(t_4xt_4^{-1}) \end{bmatrix}$$

$$= \begin{bmatrix} \phi(exe^{-1}) & \phi(ex(x)^{-1}) & \phi(ex(x^2)^{-1}) & \phi(ex(x^3)^{-1}) \\ \phi(xxe^{-1}) & \phi(xx(x)^{-1}) & \phi(xx(x^2)^{-1}) & \phi(xx(x^3)^{-1}) \\ \phi(x^2xe^{-1}) & \phi(x^2x(x)^{-1}) & \phi(x^2x(x^2)^{-1}) & \phi(x^2x(x^3)^{-1}) \\ \phi(x^3xe^{-1}) & \phi(x^3x(x)^{-1}) & \phi(x^3x(x^2)^{-1}) & \phi(x^3x(x^3)^{-1}) \end{bmatrix}$$

$$= \begin{bmatrix} \phi(x) & \phi(e) & \phi(x^3) & \phi(x^2) \\ \phi(x^2) & \phi(x) & \phi(e) & \phi(x) \\ \phi(x^3) & \phi(x^2) & \phi(x) & \phi(e) \\ \phi(e) & \phi(x^3) & \phi(x^2) & \phi(x) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Similarily,

$$A(yy) = \begin{bmatrix} \phi(t_1yt_1^{-1}) & \phi(t_1yt_2^{-1}) & \phi(t_1yt_3^{-1}) & \phi(t_1yt_4^{-1}) \\ \phi(t_2yt_1^{-1}) & \phi(t_2yt_2^{-1}) & \phi(t_2yt_2^{-1}) & \phi(t_2yt_4^{-1}) \\ \phi(t_3yt_1^{-1}) & \phi(t_3yt_2^{-1}) & \phi(t_3yt_3^{-1}) & \phi(t_3yt_4^{-1}) \\ \phi(t_4yt_1^{-1}) & \phi(t_4yt_2^{-1}) & \phi(t_4yt_3^{-1}) & \phi(t_4yt_4^{-1}) \end{bmatrix}$$

$$= \begin{bmatrix} \phi(eye^{-1}) & \phi(ey(x)^{-1}) & \phi(ey(x^2)^{-1}) & \phi(ey(x^3)^{-1}) \\ \phi(xye^{-1}) & \phi(xy(x)^{-1}) & \phi(xy(x^2)^{-1}) & \phi(xy(x^3)^{-1}) \\ \phi(x^2ye^{-1}) & \phi(x^2y(x)^{-1}) & \phi(x^2y(x^2)^{-1}) & \phi(x^2y(x^3)^{-1}) \\ \phi(x^3ye^{-1}) & \phi(x^3y(x)^{-1}) & \phi(x^3y(x^2)^{-1}) & \phi(x^3y(x^3)^{-1}) \end{bmatrix}$$

$$= \begin{bmatrix} \phi(y) & \phi(yx^3) & \phi(yx^2) & \phi(yx) \\ \phi(xy) & \phi(y^3) & \phi(xyx^2) & \phi(xyx) \\ \phi(x^2y) & \phi(x^2yx^3) & \phi(y^4) & \phi(x^2yx) \\ \phi(x^3y) & \phi(x^3yx^3) & \phi(x^3yx^2) & \phi(y^2) \end{bmatrix}$$

$$= \begin{bmatrix} z^4 & 0 & 0 & 0 \\ 0 & z^2 & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & z^3 \end{bmatrix}$$

Now, z is the 5th root of unity. To find the value of z we must find the smallest possible field that has a fifth root of unity. We look for the smallest prime p such that $5|p-1$. Therefore $p = 11$.

Now 2 is a primitive root of 11 ; that is, Order(2) $= \phi(11-1) = 10 \Rightarrow$Order(2) $= 10$.
It follows that Order(2^2) $= 5$, because if Order(a) $= n$ and d is a positive divisor of n, then

$$\text{Order}(a^{\frac{n}{d}}) = d.$$

Or generally,

$$\text{Order}(a^d) = \frac{\text{Order}(a)}{\gcd(d, \text{Order}(a))}.$$

Hence, $|4| = 5$.

Now the elements of order 5 in $\mathbb{Z}_{11}$ are $4, 4^2 \equiv_{11} 5$, $4^3 \equiv_{11} 9$, and $4^4 \equiv_{11} 3$. We will choose $z = 3$.

Then we have

$z^2 = z \cdot z = 3 \cdot 3 = 9$

$z^3 = z \cdot z^2 = 3 \cdot 9 = 27 \equiv_{11} 5$ $z^4 = z^2 \cdot z^2 = 9 \cdot 9 = 81 \equiv_{11} 4$

Thus,

$$A(yy) = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}.$$

We verify these matrices by running the following loop.

```
> C:=CyclotomicField(5);
> GG:=GL(4,C);
> T:=Transversal(G,H);
> #T;
4
> A:=[[C.1,0,0,0] : i in [1..4]];
> for i,j in [1..4] do A[i,j]:=0; end for;
> GG:=GL(4,C);
> for i,j in [1..4] do if T[i]*xx*T[j]^-1 in H then
for|if> A[i,j]:=CH[2](T[i]*xx*T[j]^-1);
for|if> end if; end for;
> B:=[[C.1,0,0,0] : i in [1..4]];
> for i,j in [1..4] do B[i,j]:=0; end for;
> for i,j in [1..4] do if T[i]*yy*T[j]^-1 in H then
for|if> B[i,j]:=CH[2](T[i]*yy*T[j]^-1);
for|if> end if; end for;
> HH:=sub<GG|A,B>;
> #HH, #G;
20 20
> GG!A;
[0 1 0 0]
[0 0 1 0]
[0 0 0 1]
[1 0 0 0]
> GG!B;
[-zeta_5^3 - zeta_5^2 - zeta_5 - 1 0 0 0]
[0 zeta_5^2 0 0]
[0 0 zeta_5 0]
[0 0 0 zeta_5^3]
```

The order of $A(x)$ is 4 and the order of $A(y)$ is 5. Also, the order of $A(x)A(y)$

is 4. Thus, $< A(x), A(y) >$ is a faithful representation of $G = (C_4 : C_5)$, since $|x| = 4 = |A(x)|$, $|y| = 5 = A(y)$, and $|xy| = |A(x)A(y)| = 4$.

Now we must convert these matrices into permutations.

$a_{ij} = a \Longleftrightarrow t_1 \longrightarrow t_j^a$, where a_{ij} stands for the ith row and jth column of the matrix.

Then for $A(x) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$, we have

$a_{12} = 1 \Longrightarrow t_1 \rightarrow t_2$,
$a_{23} = 1 \Longrightarrow t_2 \rightarrow t_3$,
$a_{34} = 1 \Longrightarrow t_3 \rightarrow t_4$,
$a_{41} = 1 \Longrightarrow t_4 \rightarrow t_1$.

We have 4 t's since $[G : H] = 4$, and our t's are of order 11 since $\mathbb{Z}_{11}$ is the smallest finite field that has 5th roots of unity.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
t_1	t_2	t_3	t_4	t_1^2	t_2^2	t_3^2	t_4^2	t_1^3	t_2^3	t_3^3	t_4^3	t_1^4	t_2^4	t_3^4	t_4^4
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_2	t_3	t_4	t_1	t_2^2	t_3^2	t_4^2	t_1^2	t_2^3	t_3^3	t_4^3	t_1^3	t_2^4	t_3^4	t_4^4	t_1^4
2	3	4	1	6	7	8	5	10	11	12	9	14	15	16	13

17	18	19	20	21	22	23	24	25	26	27	28	29	30
t_1^5	t_2^5	t_3^5	t_4^5	t_1^6	t_2^6	t_3^6	t_4^6	t_1^7	t_2^7	t_3^7	t_4^7	t_1^8	t_2^8
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_2^5	t_3^5	t_4^5	t_1^5	t_2^6	t_3^6	t_4^6	t_1^6	t_2^7	t_3^7	t_4^7	t_1^7	t_2^8	t_3^8
18	19	20	17	22	23	24	21	26	27	28	25	30	31

31	32	33	34	35	36	37	38	39	40
t_3^8	t_4^8	t_1^9	t_2^9	t_3^9	t_4^9	t_1^{10}	t_2^{10}	t_3^{10}	t_4^{10}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_4^8	t_1^8	t_2^9	t_3^9	t_4^9	t_1^9	t_2^{10}	t_3^{10}	t_4^{10}	t_1^{10}
32	29	34	35	36	33	38	39	40	37

Therefore, our permutation representation of $A(x)$ is

$x = (1,2,3,4)(5,6,7,8)(9,10,11,12)(13,14,15,16)(17,18,19,20)$

$(21,22,23,24)(25,26,27,28)(29,30,31,32)(33,34,35,36)(37,38,39,40)$.

Similarily, for $A(y) = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$, we have

$a_{11} = 4 \Longrightarrow t_1 \to t_1^4$,

$a_{22} = 9 \Longrightarrow t_2 \to t_2^9$,

$a_{33} = 3 \Longrightarrow t_3 \to t_3^3$,

$a_{44} = 5 \Longrightarrow t_4 \to t_4^5$.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
t_1	t_2	t_3	t_4	t_1^2	t_2^2	t_3^2	t_4^2	t_1^3	t_2^3	t_3^3	t_4^3	t_1^4	t_2^4	t_3^4	t_4^4
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_1^4	t_2^9	t_3^3	t_4^5	t_1^8	t_2^7	t_3^6	t_4^{10}	t_1	t_2^5	t_3^9	t_4^4	t_1^5	t_2^3	t_3	t_4^9
13	34	11	20	29	26	23	40	1	18	35	16	17	10	3	36

17	18	19	20	21	22	23	24	25	26	27	28	29	30
t_1^5	t_2^5	t_3^5	t_4^5	t_1^6	t_2^6	t_3^6	t_4^6	t_1^7	t_2^7	t_3^7	t_4^7	t_1^8	t_2^8
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_1^9	t_2	t_3^4	t_4^3	t_1^2	t_2^{10}	t_3^7	t_4^8	t_1^9	t_2^8	t_3^{10}	t_4^2	t_1^4	t_2^6
33	2	15	12	5	38	27	32	21	30	39	8	37	22

31	32	33	34	35	36	37	38	39	40
t_3^8	t_4^8	t_1^9	t_2^9	t_3^9	t_4^9	t_1^{10}	t_2^{10}	t_3^{10}	t_4^{10}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_3^2	t_4^8	t_1	t_2^4	t_3^5	t_4	t_1^6	t_2^2	t_3^8	t_4^6
7	28	9	14	19	4	25	6	31	24

Therefore, our permutation representation of $A(y)$ is

$y = (1,13,17,33,9)(5,29,37,25,21)(6,26,30,22,38)(2,34,14,10,18)$

$(3,11,35,19,15)(4,20,12,16,36)(7,23,27,39,31)(8,40,24,32,28)$

A presentation for $(4:5)$ is $< x, y | x^4, xy^4x^3y^3, y^3x^3yx >$.

Thus, by definition, our presentation of the monomial progenitor $11^{*4} :_m (4:5)$ will be

$$< x, y, z, t | x^4, xy^4x^3y^3, y^3x^3yx, t^m, Normaliser(N, < t >) >$$

Since our t's are of order 11, we will have $t^m = t^{11}$.

Now we must find the $Normaliser(N, < t >)$, that is, the permutations that stabilize all the powers of t_1, {1,5,9,13,17,21,25,29,33,37}.
Let $t \sim t_1$.
We then find these permutations.

```
> Normaliser:=Stabiliser(N,{1,5,9,13,17,21,25,29,33,37});
> Generators(Normaliser);
{
    (1, 13, 17, 33, 9)(2, 34, 14, 10, 18)(3, 11, 35, 19, 15)
    (4, 20, 12, 16, 36)(5, 29, 37, 25, 21)(6, 26, 30, 22, 38)
    (7, 23, 27, 39, 31)(8, 40, 24, 32, 28)
}
```

Therefore, $Normaliser(N, < t_1 >) =< (1, 13, 17, 33, 9)(2, 34, 14, 10, 18)(3, 11, 35, 19, 15)$ $(4, 20, 12, 16, 36)(5, 29, 37, 25, 21)(6, 26, 30, 22, 38)(7, 23, 27, 39, 31)(8, 40, 24, 32, 28) >$.

Since $y = (1, 13, 17, 33, 9)(2, 34, 14, 10, 18)(3, 11, 35, 19, 15)(4, 20, 12, 16, 36)$ $(5, 29, 37, 25, 21)(6, 26, 30, 22, 38)(7, 23, 27, 39, 31)(8, 40, 24, 32, 28)$, and y sends t_1 to t_1^4, we have that $t^y = t^4$.

Thus, the presentation of the monomial progenitor is given by
$11^{*4} :_m (4:5) =< x, y, t | x^4, xy^4x^3y^3, y^3x^3yx, t^{11}, t^y = t^4 >$.

Next we add the following first order relations to our progenitor to find finite homomorphic images.

$$(x^2t^{y^3})^3, (x^3t)^8, (yt^x)^5, (xt^{y^4})^3$$

```
> G<x,y,t>:=Group<x,y,t|x^4,x*y^4*x^3*y^3,
> y^3*x^3*y*x,t^11,t^y=t^4,(x^2*t^(y^3))^3,(x^3*t)^8,
> (y*t^x)^5,
> (x*t^(y^4))^3>;
> #G;
7920
> /*
> 7920
> */
> S:=Sym(40);
> xx:=S!(1,2,3,4)(5,6,7,8)(9,10,11,12)(13,14,15,16)
> (17,18,19,20)(21,22,23,24)(25,26,27,28)
> (29,30,31,32)(33,34,35,36)(37,38,39,40);
> yy:=S!(1,13,17,33,9)(5,29,37,25,21)
> (6,26,30,22,38)(2,34,14,10,18)(3,11,35,19,15)
> (4,20,12,16,36)
> (7,23,27,39,31)(8,40,24,32,28);
> N:=sub<S|xx,yy>;
> f,G1,k:=CosetAction(G,sub<G|x,y>);
> #k;
1
> CompositionFactors(G1);
    G
    |  M11
    1
```

Manual Double Coset enumeration will follow in a later chapter.

4.3 Monomial Progenitor $11^{*2} :_m D_{10}$

Consider $11^{*2} :_m D_{10}$. $G = D_{10}$ is given by
$G = D_{10} =< x, y|x^{10}, y^2, (x^{-1}y)^2 >$, where
$x = (1, 4, 6, 8, 10, 12, 14, 16, 18, 19)(2, 3, 5, 7, 9, 11, 13, 15, 17, 20)$, and
$y = (1, 15)(2, 16)(3, 14)(4, 13)(5, 12)(6, 11)(7, 10)(8, 9)(17, 19)(18, 20)$.

$G = D_{10}$ has monomial irreducible representation in dimension 2. We will write a progenitor for $11^{*11} :_m D_{10}$. Since $\frac{|G|}{|H|} = 2 \Rightarrow \frac{20}{|H|} = 2 \Rightarrow |H| = 10$, we need to find a subgroup H of order 10 and induce a linear character of H up to G to obtain the irreducible character of degree 2 of G.

The conjugacy classes of group D_{10} are given in the table below.

Table 4.5: Conjugacy Classes of D_{10}

Class	Representative of the class	# of elements in the class
C_1	e	1
C_2	$(1,12)(2,11)(3,13)(4,14)(5,15)(6,16)(7,17)(8,18)(9,20)(10,19)$	1
C_3	$(1,15)(2,16)(3,14)(4,13)(5,12)(6,11)(7,10)(8,9)(17,19)(18,20)$	5
C_4	$(1,13)(2,14)(3,12)(4,11)(5,10)(6,9)(7,8)(15,19)(16,20)(17,18)$	5
C_5	$(1,6,10,14,18)(2,5,9,13,17)(3,7,11,15,20)(4,8,12,16,19)$	2
C_6	$(1,10,18,6,14)(2,9,17,5,13)(3,11,20,7,15)(4,12,19,8,16)$	2
C_7	$(1,4,6,8,10,12,14,16,18,19)(2,3,5,7,9,11,13,15,17,20)$	2
C_8	$(1,8,14,19,6,12,18,4,10,16)(2,7,13,20,5,11,17,3,9,15)$	2

Consider the subgroup $H = Z_{10}$ of G given below.

$H = \{e, (1,12)(2,11)(3,13)(4,14)(5,15)(6,16)(7,17)(8,18)(9,20)(10,19), (1,6,10,14,18)(2,5,9,13,17)(3,7,11,15,20)(4,8,12,16,19), (1,10,18,6,14)(2,9,17,5,13)(3,11,20,7,15)(4,12,19,8,16), (1,18,14,10,6)(2,17,13,9,5)(3,20,15,11,7)(4,19,16,12,8), (1,19,18,16,14,12,10,8,6,4)(2,20,17,15,13,11,9,7,5,3), (1,4,6,8,10,12,14,16,18,19)(2,3,5,7,9,11,13,15,17,20), (1,8,14,19,6,12,18,4,10,16)(2,7,13,20,5,11,17,3,9,15), (1,16,10,4,18,12,6,19,14,8)(2,15,9,3,17,11,5,20,13,7), (1,14,6,18,10)(2,13,5,17,9)(3,15,7,20,11)(4,16,8,19,12)\}$.

The conjugacy classes of Z_{10} are given in the table below.

Table 4.6: Conjugacy Classes of $H = Z_{10}$

Class	Representative of the class	# of elements in the class
D_1	e	1
D_2	$(1,12)(2,11)(3,13)(4,14)(5,15)(6,16)(7,17)(8,18)(9,20)(10,19)$	1
D_3	$(1,6,10,14,18)(2,5,9,13,17)(3,7,11,15,20)(4,8,12,16,19)$	1
D_4	$(1,10,18,6,14)(2,9,17,5,13)(3,11,20,7,15)(4,12,19,8,16)$	1
D_5	$(1,14,6,18,10)(2,13,5,17,9)(3,15,7,20,11)(4,16,8,19,12)$	1
D_6	$(1,18,14,10,6)(2,17,13,9,5)(3,20,15,11,7)(4,19,16,12,8)$	1
D_7	$(1,4,6,8,10,12,14,16,18,19)(2,3,5,7,9,11,13,15,17,20)$	1
D_8	$(1,8,14,19,6,12,18,4,10,16)(2,7,13,20,5,11,17,3,9,15)$	1
D_9	$(1,16,10,4,18,12,6,19,14,8)(2,15,9,3,17,11,5,20,13,7)$	1
D_{10}	$(1,19,18,16,14,12,10,8,6,4)(2,20,17,15,13,11,9,7,5,3)$	1

Consider the irreducible characters ϕ (of H) and χ (of G) given below.

Table 4.7: Character Table of $H = Z_{10}$

Class	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}
Size	1	1	1	1	1	1	1	1	1	1
Order	1	2	5	5	5	5	10	10	10	10
ϕ_1	1	1	1	1	1	1	1	1	1	1
ϕ_2	1	-1	1	1	1	1	-1	-1	-1	-1
ϕ_3	1	1	Z	Z^2	Z^3	Z^4	Z^3	Z^4	Z	Z^2
ϕ_4	1	-1	Z	Z^2	Z^3	Z^4	$-Z^3$	$-Z^4$	$-Z$	$-Z^2$
ϕ_5	1	1	Z^2	Z^4	Z	Z^3	Z	Z^3	Z^2	Z^4
ϕ_6	1	-1	Z^2	Z^4	Z	Z^3	$-Z$	$-Z^3$	$-Z^2$	$-Z^4$
ϕ_7	1	1	Z^3	Z	Z^4	Z^2	Z^4	Z^2	Z^3	Z
ϕ_8	1	-1	Z^3	Z	Z^4	Z^2	$-Z^4$	$-Z^2$	$-Z^3$	$-Z$
ϕ_9	1	1	Z^4	Z^3	Z^2	Z	Z^2	Z	Z^4	Z^3
ϕ_{10}	1	-1	Z^4	Z^3	Z^2	Z	$-Z^2$	$-Z$	$-Z^4$	$-Z^3$

where Z is the 5th root of unity.

Table 4.8: Character Table of $G = D_{10}$

Class	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
Size	1	1	5	5	2	2	2	2
Order	1	2	2	2	5	5	10	10
χ_1	1	1	1	1	1	1	1	1
χ_2	1	1	-1	-1	1	1	1	1
χ_3	1	-1	-1	1	1	1	-1	-1
χ_4	1	-1	1	-1	1	1	-1	-1
χ_5	2	-2	0	0	Z	Z^2	$-Z^2$	$-Z$
χ_6	2	2	0	0	Z	Z^2	Z^2	Z
χ_7	2	2	0	0	Z^2	Z	Z	Z^2
χ_8	2	-2	0	0	Z^2	Z	$-Z$	$-Z^2$

where Z is the 5th root of unity.

Next we must find a non-trivial linear character of H to induce up to G. Note that each character of H is linear since they all have degree 1. We will induce χ_4 up to G.

```
> CH[4];
( 1, -1, zeta(5)_5, zeta(5)_5^2, zeta(5)_5^3,
    -zeta(5)_5^3 - zeta(5)_5^2 - zeta(5)_5 - 1,
    -zeta(5)_5^3, zeta(5)_5^3 + zeta(5)_5^2 +
    zeta(5)_5 + 1, -zeta(5)_5, -zeta(5)_5^2 )
```

We will use a loop to find the two induced representations A(x) and A(y) of degree $\frac{|G|}{|H|} = \frac{20}{10} = 2$. First we must input the values of $z, z^2, z^3, z^4, -z, -z^2, -z^3$, and $-z^4$ into our loop, which we will denote as C.1, C.2, C.3, C.4, -C, -C.1, -C.2, -C.3, and -C.4, respectively.

Now, z is the 5th root of unity. To find the value of z we must find the smallest possible field that has a fifth root of unity. We look for the smallest prime p such that $5|(p-1)$. Therefore $p = 11$. Now, 2 is a primitive root of 11 ; that is,
Order(2) $= \phi(11-1) = 10 \Rightarrow$Order(2) $= 10$. It follows that Order(2^2) $= 5$, because if Order(a) $= n$ and d is a positive divisor of n, then

$$\text{Order}(a^{\frac{n}{d}}) = d.$$

Or generally,

$$\text{Order}(a^d) = \frac{\text{Order}(a)}{\gcd(d, \text{Order}(a))}.$$

Hence, $|4| = 5$.
Now the elements of order 5 in $\mathbb{Z}_{11}$ are $4, 4^2 \equiv_{11} 5$, $4^3 \equiv_{11} 9$, and $4^4 \equiv_{11} 3$. We will choose $z = 4$. Then we have,

$z^2 = z \cdot z = 4 \cdot 4 = 16 \equiv_{11} 5$
$z^3 = z \cdot z^2 = 4 \cdot 16 = 64 \equiv_{11} 9$
$z^4 = z^2 \cdot z^2 = 16 \cdot 16 = 256 \equiv_{11} 3.$

Also, $z^4 + z^3 + z^2 + z + 1 = 0$
$\Longrightarrow z^4 + z^3 + z^2 + 1 = -z$
$\Longrightarrow 3 + 9 + 5 + 1 = -z$
$\Longrightarrow 18 \equiv_{11} 7 = -z$.

Similarily, $z^4 + z^3 + z^2 + z + 1 = 0$
$\Longrightarrow z^4 + z^3 + z + 1 = -z^2$
$\Longrightarrow 3 + 9 + 4 + 1 = -z^2$
$\Longrightarrow 17 \equiv_{11} 6 = -z^2$,

$z^4 + z^3 + z^2 + z + 1 = 0$
$\Longrightarrow z^4 + z^2 + z + 1 = -z^3$
$\Longrightarrow 3 + 5 + 4 + 1 = -z^3 \Longrightarrow 13 \equiv_{11} 2 = -z^3$, and

$z^4 + z^3 + z^2 + z + 1 = 0$
$\Longrightarrow z^3 + z^2 + z + 1 = -z^4$
$\Longrightarrow 9 + 5 + 4 + 1 = -z^4$
$\Longrightarrow 19 \equiv_{11} 8 = -z^4$.

```
> C:=CyclotomicField(5);
> N:=H;
> T:=Transversal(G,H);
> GG:=GL(2,11);
> mat := function(n,p,D,k)
function> for i,j in [1..k] do if T[i]*p*T[j]^-1 in H then
function|for|if> if CH[n](T[i]*p*T[j]^-1) eq C.1
function|for|if> then D[i,j]:=4; end if;
function|for|if> if CH[n](T[i]*p*T[j]^-1) eq -C.1
function|for|if> then D[i,j]:=7; end if;
function|for|if> if CH[n](T[i]*p*T[j]^-1) eq C.1^2
function|for|if> then D[i,j]:=5; end if;
function|for|if> if CH[n](T[i]*p*T[j]^-1) eq -C.1^2
function|for|if> then D[i,j]:=6; end if;
function|for|if> if CH[n](T[i]*p*T[j]^-1) eq C.1^3
function|for|if> then D[i,j]:=9; end if;
function|for|if> if CH[n](T[i]*p*T[j]^-1) eq -C.1^3
```

```
function|for|if> then D[i,j]:=2; end if;
function|for|if> if CH[n](T[i]*p*T[j]^-1) eq C.1^4
function|for|if> then D[i,j]:=3; end if;
function|for|if> if CH[n](T[i]*p*T[j]^-1) eq -C.1^4
function|for|if> then D[i,j]:=8; end if;
function|for|if> if CH[n](T[i]*p*T[j]^-1) eq 1
function|for|if> then D[i,j]:=1; end if;
function|for|if> if CH[n](T[i]*p*T[j]^-1) eq -1
function|for|if> then D[i,j]:=-1; end if;
function|for|if> if CH[n](T[i]*p*T[j]^-1) in {1,-1}
function|for|if> then D[i,j]:=CH[n](T[i]*p*T[j]^-1); end if;
function|for|if> end if; end for;
function> return D;
function> end function;
> A:=[[0,0]: i in [1..2]];
> mat(4,xx,A,2);
[
    [ 2, 0 ],
    [ 0, 6 ]
]
> mat(4,yy,A,2);
[
    [ 0, 1 ],
    [ 1, 0 ]
]
```

Thus
$A(x) = \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}$ and $A(y) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

The order of $A(x)$ is 10 and the order of $A(y)$ is 2. Also, the order of $A(x) \cdot A(y)$ is 2. Thus, $< A(x), A(y) >$ is a faithful representation of $G = D_{10}$, since $|x| = 10 = |A(x)|$, $|y| = 2 = A(y)$, and $|xy| = |A(x) \cdot A(y)| = 2$.

Now we must convert these matrices into permutations.

$a_{ij} = a \Longleftrightarrow t_1 \longrightarrow t_j^a$, where a_{ij} stands for the ith row and jth column of the matrix.

Then for $A(x) = \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}$, we have

$a_{11} = 2 \Longrightarrow t_1 \to t_1^2$,

$a_{22} = 6 \Longrightarrow t_2 \to t_2^6$.

We have 2 t's since $[G : H] = 2$, and our t's are of order 11 since $\mathbb{Z}_{11}$ is the smallest finite field that has 5th roots of unity.

1	2	3	4	5	6	7	8	9	10
t_1	t_2	t_1^2	t_2^2	t_1^3	t_2^3	t_1^4	t_2^4	t_1^5	t_2^5
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_1^2	t_2^6	t_1^4	t_2	t_1^6	t_2^7	t_1^8	t_2^2	t_1^{10}	t_2^8
3	12	7	2	11	14	15	4	19	16

11	12	13	14	15	16	17	18	19	20
t_1^6	t_2^6	t_1^7	t_2^7	t_1^8	t_2^8	t_1^9	t_2^9	t_1^{10}	t_2^{10}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_1	t_2^3	t_1^3	t_2^9	t_1^5	t_2^4	t_1^7	t_2^{10}	t_1^9	t_2^5
1	6	5	18	9	8	13	20	17	10

Therefore, our permutation representation of $A(x)$ is

$x = (1, 3, 7, 15, 9, 19, 17, 13, 5, 11)(2, 12, 6, 14, 18, 20, 10, 16, 8, 4)$

Similarily, for $A(y) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, we have

$a_{12} = 1 \Longrightarrow t_1 \to t_2$,

$a_{21} = 1 \Longrightarrow t_2 \to t_1$.

1	2	3	4	5	6	7	8	9	10
t_1	t_2	t_1^2	t_2^2	t_1^3	t_2^3	t_1^4	t_2^4	t_1^5	t_2^5
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_2	t_1	t_2^2	t_1^2	t_2^3	t_1^3	t_2^4	t_1^4	t_2^5	t_1^5
2	1	4	3	6	5	8	7	10	9

11	12	13	14	15	16	17	18	19	20
t_1^6	t_2^6	t_1^7	t_2^7	t_1^8	t_2^8	t_1^9	t_2^9	t_1^{10}	t_2^{10}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
t_2^6	t_1^6	t_2^7	t_1^7	t_2^8	t_1^8	t_2^9	t_1^9	t_2^{10}	t_1^{10}
12	11	14	13	16	15	18	17	20	19

Therefore, our permutation representation of $A(y)$ is
$y = (1,2)(3,4)(5,6),(7,8),(9,10),(11,12),(13,14),(15,16),(17,18),(19,20)$.

A presentation for D_{10} is $< x, y | x^{10}, y^2, (x^{-1}y)^2 >$.

Thus, by definition, our presentation of the monomial progenitor $11^{*2} :_m D_{10}$ will be

$$< x, y | x^{10}, y^2, (x^{-1}y)^2, t^m, Normaliser(N, < t >) >$$

Since our t's are of order 11, we will have $t^m = t^{11}$.

Now we must find the $Normaliser(N, < t >)$, that is, the permutations that stabilize all the powers of t_1, {1,3,5,7,9,11,13,15,17,19}.
Let $t \sim t_1$.
We then find these permutations.

```
> Normaliser:=Stabiliser(N,{1,3,5,7,9,11,13,15,17,19});
> Generators(Normaliser);
{
(1, 11, 5, 13, 17, 19, 9, 15, 7, 3)(2, 4, 8, 16, 10, 20,
    18, 14, 6, 12)
}
```

Therefore, $Normaliser(N, < t_1 >) =< (1, 11, 5, 13, 17, 19, 9, 15, 7, 3)(2, 4, 8, 16, 10, 20, 18, 14, 6, 12) >$.

Since $x^{-1} = (1, 11, 5, 13, 17, 19, 9, 15, 7, 3)(2, 4, 8, 16, 10, 20, 18, 14, 6, 12)$, and x^{-1} sends t_1 to t_1^6, we have that $t^{x^{-1}} = t^6$.

Thus, the presentation of the monomial progenitor is given by
$11^{*2} :_m D_{10} =< x, y, t | x^{10}, y^2, (x^{-1}y)^2, t^{11}, t^{x^{-1}} = t^6 >$.

Next we add the following first order relations to our progenitor to find finite homomorphic images.

$$(x^5t)^2, (yt)^3$$

```
> G<x,y,t>:=Group<x,y,t|x^10,y^2,(x^-1*y)^2,
> t^11,t^(x^-1)=t^6,(x^5*t)^2,(y*t)^3>;
> #G;
1320
> f,G1,k:=CosetAction(G,sub<G|x,y>);
> #k;
1
> CompositionFactors(G1);

    G
    |  Cyclic(2)
    *
    |  A(1, 11)                    = L(2, 11)
    1
```

Double Coset Enumeration will be performed in a later chapter.

Chapter 5

Double Coset Enumeration

5.1 Preliminaries

Definition 5.1. *[Rot95] The* **Dihedral Group** D_n*, n even and greater than* 2*, groups are formed by two elements, one of order* $\frac{n}{2}$ *and one of order* 2*. A presentation for a Dihedral Group is given by* $< a, b | a^{\frac{n}{2}}, b^2, (ab)^2 >$.

5.2 $L_2(11)$ as a Homomorphic Image of $2^{*6} : D_{12}$

5.2.1 The Construction of $L_2(11)$ Over D_{12}

Consider $2^{*6} : D_{12}$, where $D_{12} =< x, y, z >, x \sim (12)(35)(46), y \sim (134)(256)$, $z \sim (12)(36)(45)$, and $t \sim t_1$.
The progenitor $2^{*6} : D_{12}$ is factored by $(xt)^3, (zt^y)^5, (yt)^5$, and $(xyt)^6$.

$G = \frac{2^{*6}:D_{12}}{(xt)^3,(zt^y)^5,(yt)^5,(xyt)^6}$ has symmetric presentation
$< x, y, z, t | x^2, y^3, z^2, y^2xyx, (y^2z)^2, (xz)^2,$
$t^2, (t, xz),$
$(xt)^3, (zt^y)^5, (yt)^5, (xyt)^6 >$
We will first show that $|G| \leq 336$ by performing manual double coset enumeration of G over N.

Let us expand our additional relation

$$\begin{aligned} (xt)^3 &= 1 \\ (xt_1)^3 &= 1 \\ (x^3 {t_1}^{x^2} {t_1}^{x} t_1) &= 1 \\ (12)(35)(46)t_1t_2t_1 &= 1 \\ (12)(35)(46)t_1 &= t_1t_2 \\ Nt_1 &= Nt_1t_2 \end{aligned} \tag{5.1}$$

$$\begin{aligned} (zt^y)^5 &= 1 \\ (z{t_1}^y)^5 &= 1 \\ (zt_3)^5 &= 1 \\ z^5 {t_3}^{z^4} {t_3}^{z^3} {t_3}^{z^2} {t_3}^{z} t_3 &= 1 \\ (12)(36)(45){t_3}^{e}{t_3}^{(1,2)(3,6)(4,5)}{t_3}^{e}{t_3}^{(1,2)(3,6)(4,5)}t_3 &= 1 \\ (12)(36)(45)t_3t_6t_3t_6t_3 &= 1 \\ (12)(36)(45)t_3t_6 &= t_3t_6t_3 \end{aligned} \tag{5.2}$$

$$\begin{aligned} (yt)^5 &= 1 \\ (yt_1)^5 &= 1 \\ y^5 {t_1}^{y^4} {t_1}^{y^3} {t_1}^{y^2} {t_1}^{y} t_1 &= 1 \\ (143)(265){t_1}^{(134)(256)}{t_1}^{e}{t_1}^{(143)(265)}{t_1}^{(134)(256)}t_1 &= 1 \\ (143)(205)t_3t_1t_4t_3t_1 &= 1 \\ (143)(205)t_3t_1 &= t_1t_3t_4 \end{aligned} \tag{5.3}$$

$$\begin{aligned} (xyt)^6 &= 1 \\ (xyt_1)^6 &= 1 \\ (xy)^6 {t_1}^{xy^5} {t_1}^{xy^4} {t_1}^{xy^3} {t_1}^{xy^2} {t_1}^{xy} t_1 &= 1 \\ e{t_1}^{(163245)}{t_1}^{(134)(256)}{t_1}^{(12)(35)(46)}{t_1}^{(143)(265)}t_1^{(154236)}t_1 &= 1 \\ t_6t_3t_2t_4t_5t_1 &= 1 \\ t_6t_3t_2 &= t_1t_5t_4 \end{aligned} \tag{5.4}$$

Our first double coset, $NeN = \{Ne^n | n \in N\} = \{N\}$,
which we will denote by [*].

N is transitive on $\{1, 2, 3, 4, 5, 6\}$ so it has one single orbit $\{1, 2, 3, 4, 5, 6\}$.

We will take a representative from this orbit, say 1, and find out to which double coset Nt_1 belongs.

Nt_1N is a new double coset which we will denote by [1].

Since the orbit $\{1, 2, 3, 4, 5, 6\}$ contains 6 elements then 6 symmetric generators will go to the new double coset [1].

N^1 = Point Stabiliser in N of $Nt_1 = \{n \in N | t_1^n = t_1\} = \{e, (34)(56)\}$.
$N^{(1)}$ = Coset Stabiliser in N of $Nt_1 = \{n \in N | Nt_1^n = t_1\} = \{e\} = N^1$.

Now $N^{(1)} \geq N^1$.
$N^1 = \{e, (34)(56)\}$.

Since we do not have a relation that will increase the Coset Stabiliser $N^{(1)}$, then $N^{(1)} = N^1 = \{e, (34)(56)\}$.

The number of single cosets in Nt_1N is at most $\frac{|N|}{|N^{(1)}|} = \frac{12}{2} = 6$.

$Nt_1N = \{Nt_1{}^n | n \in N\}$.
$Nt_1N = \{Nt_1, Nt_2, Nt_3, Nt_4, Nt_5, Nt_6\}$.

The orbits of $N^{(1)}$ on $\{1, 2, 3, 4, 5, 6\}$ are $\{1\}, \{2\}, \{3, 4\}$, and $\{5, 6\}$. We take t_1, t_2, t_3, and t_5, from each orbit respectively, and determine to which double coset $Nt_1t_1, Nt_1t_2, Nt_1t_3$, and Nt_1t_5 belong.

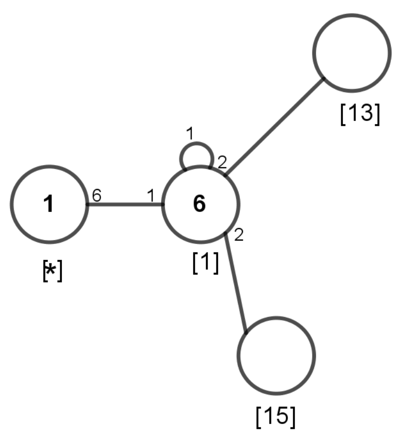
$Nt_1t_1 = N \in [*]$ (Since our t's are of order 2.)
Since the orbit $\{1\}$ contains one element, then one symmetric generator goes back to the double coset $[*]$.

$Nt_1t_2 = Nt_1 \in [1]$ (by Equation 5.1).
One symmetric generator will go back to [1].

Nt_1t_3N is a new double coset which we will denote [13].
Two symmetric generators will go to the new double coset [13].

Nt_1t_5N is a new double coset which we will denote [15].
Two symmetric generators will go to the new double coset [15].

Below is our Cayley Diagram thus far.



$N^{(13)} \geq N^{13}$.
$N^{13} = \{e\}$.
Since we do not have a relation that will increase the Coset Stabiliser $N^{(13)}$, then $N^{(13)} = N^{13} = \{e\}$.

The number of single cosets in Nt_1t_3N is at most $\frac{|N|}{|N^{(13)}|} = \frac{12}{1} = 12$.

$Nt_1t_3N = \{Nt_1t_3{}^n | n \in N\}$.
$Nt_1t_3N = \{Nt_1t_3, Nt_2t_5, Nt_3t_4, Nt_2t_6, Nt_4t_1, Nt_5t_6, Nt_1t_4, Nt_6t_2, Nt_6t_5, Nt_5t_2, Nt_4t_3, Nt_3t_1\}$.

$N^{(15)} \geq N^{15}$.
$N^{15} = \{e\}$.
Since we do not have a relation that will increase the Coset Stabiliser $N^{(15)}$, then $N^{(15)} = N^{15} = \{e\}$.

The number of single cosets in Nt_1t_5N is at most $\frac{|N|}{|N^{(15)}|} = \frac{12}{1} = 12$.

$Nt_1t_5N = \{Nt_1t_5{}^n | n \in N\}$.
$Nt_1t_5N = \{Nt_1t_5, Nt_1t_6, Nt_2t_3, Nt_2t_4, Nt_3t_6, Nt_3t_2, Nt_4t_2, Nt_4t_5, Nt_5t_4, Nt_5t_1, Nt_6t_1, Nt_6t_3\}$.

The orbits of $N^{(13)}$ on $\{1, 2, 3, 4, 5, 6\}$ are $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}$, and $\{6\}$. We take t_1, t_2, t_3, t_4, t_5, and t_6, from each orbit respectively, and determine to which double coset $Nt_1t_3t_1, Nt_1t_3t_2, Nt_1t_3t_3, Nt_1t_3t_4, Nt_1t_3t_5$, and $Nt_1t_3t_6$ belong.

First we will examine $Nt_1t_3t_1$.
$(143)(265)t_3t_1 = t_1t_3t_4$, by Equation 5.3
$\implies (143)(265)t_3t_1t_4 = t_1t_3t_4t_4$
$\implies (143)(265)t_3t_1t_4 = t_1t_3$
$\implies t_4(143)(265)t_3t_1t_4 = t_4t_1t_3$
$\implies (143)(265)t_4^{(143)(265)}t_3t_1t_4 = t_4t_1t_3$
$\implies (143)(265)t_3t_3t_1t_4 = t_4t_1t_3$
$\implies (143)(265)t_1t_4 = t_4t_1t_3$
$\implies (143)(265)t_1t_4t_1 = t_4t_1t_3t_1$
Also,
$(143)(265)t_3t_1 = t_1t_3t_4$, by Equation 5.3
$\implies [(143)(265)t_3t_1]^{(13)(25)} = [t_1t_3t_4]^{(13)(25)}$

$\Longrightarrow (134)(256)t_1t_3 = t_3t_1t_4$

$\Longrightarrow (134)(256)t_1t_3t_4 = t_3t_1t_4t_4$

$\Longrightarrow (134)(256)t_1t_3t_4 = t_3t_1$

Therefore, $(143)(265)t_1t_4t_1 = t_4t_1\underline{t_3t_1}$

$\Longrightarrow (143)(265)t_1t_4t_1 = t_4t_1[(134)(256)t_1t_3t_4]$

$\Longrightarrow (143)(265)t_1t_4t_1 = (134)(256)[t_4t_1]^{(134)(256)}t_1t_3t_4$

$\Longrightarrow (143)(265)t_1t_4t_1 = (134)(256)t_1t_3t_1t_3t_4$

$\Longrightarrow (143)(265)t_1t_4t_1 = (134)(256)[e]t_1t_3t_1t_3t_4$

$\Longrightarrow (143)(265)t_1t_4t_1 = (134)(256)[(12)(35)(46)t_1t_2t_1]t_1t_3t_1t_3t_4$, by Equation 5.1

$\Longrightarrow (143)(265)t_1t_4t_1 = (152436)t_1t_2t_3t_1t_3t_4$. $t_6t_3t_2 = t_1t_5t_4$, by Equation 5.4

$\Longrightarrow [t_6t_3t_2]^{(12)(35)(46)} = [t_1t_5t_4]^{(12)(35)(46)}$

$\Longrightarrow t_4t_5t_1 = t_2t_3t_6$

$\Longrightarrow t_4t_5t_1t_6 = t_2t_3t_6t_6$

$\Longrightarrow t_4t_5t_1t_6 = t_2t_3$

Thus, $(143)(265)t_1t_4t_1 = (154236)t_1\underline{t_2t_3}t_1t_3t_4$

$\Longrightarrow (143)(265)t_1t_4t_1 = (154236)t_1[t_4t_5t_1t_6]t_1t_3t_4$

$(12)(36)(45)t_3t_6 = t_3t_6t_3$, by Equation 5.2

$\Longrightarrow [(12)(36)(45)t_3t_6]^{(13)(25)} = [t_3t_6t_3]^{(13)(25)}$

$\Longrightarrow (35)(16)(42)t_1t_6 = t_1t_6t_1$

$\Longrightarrow (16)(24)(35)t_1t_6 = t_1t_6t_1$

Thus,

$(143)(265)t_1t_4t_1 = (154236)t_1t_4t_5\underline{t_1t_6t_1}t_3t_4$

$\Longrightarrow (143)(265)t_1t_4t_1 = (154236)t_1t_4t_5[(16)(24)(35)t_1t_6]t_3t_4$

$\Longrightarrow (143)(265)t_1t_4t_1 = (154236)(16)(24)(35)[t_1t_4t_5]^{(16)(24)(35)}t_1t_6t_3t_4$

$\Longrightarrow (143)(265)t_1t_4t_1 = (13)(25)t_6t_2t_3t_1t_6t_3t_4$

Also,

$t_6t_3t_2 = t_1t_5t_4$, by Equation 5.4

$\Longrightarrow [t_6t_3t_2]^{(154236)} = [t_1t_5t_4]^{(154236)}$

$\Longrightarrow t_1t_6t_3 = t_5t_4t_2$

Thus,

$(143)(265)t_1t_4t_1 = (13)(25)t_6t_2t_3\underline{t_1t_6t_3}t_4$

$\Longrightarrow (143)(265)t_1t_4t_1 = (13)(25)t_6t_2t_3[t_5t_4t_2]t_4$

$\implies (143)(265)t_1t_4t_1 = (13)(25)t_6t_2t_3t_5\underline{t_4t_2t_4}$

$\implies (143)(265)t_1t_4t_1 = (13)(25)t_6t_2t_3t_5[(16)(24)(35)t_4t_2]$, since

$[(12)(36)(45)t_3t_6]^{(134)(256)} = [t_3t_6t_3]^{(134)(256)} \implies (16)(24)(35)t_4t_2 = t_4t_2t_4$ (Equation 5.2)

$\implies (143)(265)t_1t_4t_1 = (13)(25)(16)(24)(35)(t_6t_2t_3t_5)^{(16)(24)(35)}t_4t_2$

$\implies (143)(265)t_1t_4t_1 = (154236)t_1t_4\underline{t_5t_3}t_4t_2$

$\implies (143)(265)t_1t_4t_1 = (154236)t_1t_4[(12)(35)(46)t_5]t_4t_2$, since

$[(12)(35)(46)t_1]^{(154236)} = [t_1t_2]^{(154236)} \implies (12)(35)(46)t_5 = t_5t_3$ (Equation 5.1)

$\implies (143)(265)t_1t_4t_1 = (154236)(12)(35)(46)(t_1t_4)^{(12)(35)(46)}t_5t_4t_2$

$\implies (143)(265)t_1t_4t_1 = (134)(256)t_2t_6\underline{t_5t_4t_2}$

$\implies (143)(265)t_1t_4t_1 = (134)(256)t_2t_6[t_1t_6t_3]$, Equation 5.4 conjugated by (15)(23)(46)

$\implies (143)(265)t_1t_4t_1 = (134)(256)t_2\underline{t_6t_1t_6}t_3$

$\implies (143)(265)t_1t_4t_1 = (134)(256)t_2[(16)(24)(35)t_6t_1]t_3$, since

$[(12)(36)(45)t_3t_6]^{(13)(25)} = [t_3t_6t_3]^{(13)(25)} \implies (16)(24)(35)t_1t_6 = t_1t_6t_1$ (Equation5.2)

$\implies (143)(265)t_1t_4t_1 = (134)(256)(16)(24)(35)t_2^{(16)(24)(35)}t_6t_1t_3$

$\implies (143)(265)t_1t_4t_1 = (15)(23)(46)\underline{t_4t_6}t_1t_3$

$\implies (143)(265)t_1t_4t_1 = (15)(23)(46)[(12)(35)(46)t_4]t_1t_3$, since

$[(12)(35)(46)t_1]^{(143)(265)} = [t_1t_2]^{(143)(265)} \implies (12)(35)(46)t_4 = t_4t_6$ (Equation 5.1)

$\implies (143)(265)t_1t_4t_1 = (13)(25)\underline{t_4t_1t_3}$

$\implies (143)(265)t_1t_4t_1 = (13)(25)[(143)(265)t_1t_4]$, since

$[(143)(265)t_3t_1]^{(143)(265)} = [t_1t_3t_4]^{(143)(265)} \implies (143)(265)t_1t_4 = t_4t_1t_3$ (Equation 5.3)

$\implies (143)(265)t_1t_4t_1 = (34)(56)t_1t_4$

$\implies [(143)(265)t_1t_4t_1]^{(34)(56)} = [(34)(56)t_1t_4]^{(34)(56)}$

$\implies (134)(256)t_1t_3t_1 = (34)(56)t_1t_3$

Thus, $Nt_1t_3t_1 = Nt_1t_3 \in [13]$.

One symmetric generator will go back to [13].

$Nt_1t_3t_2N$ is a new double coset which we will denote [132].

One symmetric generator will go to [132].

$Nt1t3t3 = Nt1 \in [1]$.

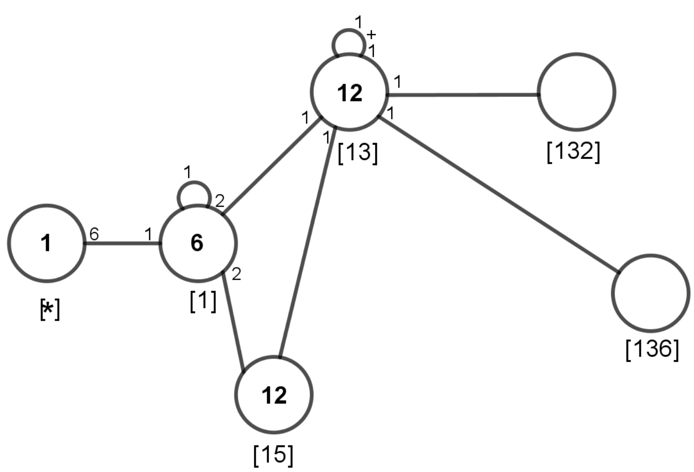
One symmetric generator will go back to [1].

$Nt_1t_3t_4 = Nt_3t_1 \in [13]$, by Equation 5.3.
One symmetric generator will go back to [13].

$Nt_1t_3t_5 = Nt_2t_3 \in [15]$, by Equation 5.1, since
$[(12)(35)(46)t_1]^{(13)(25)} = [t_1t_2]^{(13)(25)} \implies (12)(35)(46)t_3 = t_3t_5$
$\implies t_1(12)(35)(46)t_3 = t_1t_3t_5$
$\implies (12)(35)(46)t_1^{(12)(35)(46)}t_3 = t_1t_3t_5$
$\implies (12)(35)(46)t_2t_3 = t_1t_3t_5.$
One symmetric generator will go to [15].

$Nt_1t_3t_6N$ is a new double coset which we will denote [136].
One symmetric generator will go to [136].

Below is our Cayley Diagram thus far.



The orbits of $N^{(15)}$ on $\{1, 2, 3, 4, 5, 6\}$ are $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}$. We take t_1, t_2, t_3, t_4, t_5, and t_6, from each orbit respectively, and determine to which double coset $Nt_1t_5t_1, Nt_1t_5t_2, Nt_1t_5t_3, Nt_1t_5t_4, Nt_1t_5t_5$, and $Nt_1t_5t_6$ belong.

$Nt_1t_5t_1 = Nt_1t_5 \in [15]$, since
$[(12)(36)(45)t_3t_6]^{(143)(265)} = [t_3t_6t_3]^{(143)(265)}$
$\Longrightarrow (15)(23)(46)t_1t_5 = t_1t_5t_1$, by Equation 5.2.
One symmetric generator will go to [15].

$Nt_1t_5t_2N$ is a new double coset which we will denote [152].
One symmetric generator will go to [152].

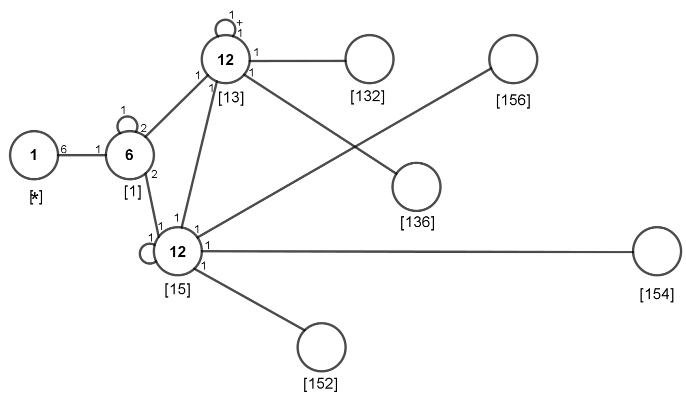
$Nt_1t_5t_3 = Nt_2t_5 \in [13]$, since
$[(12)(35)(46)t_1]^{(15)(23)(46)} = [t_1t_2]^{(15)(23)(46)} \Longrightarrow (12)(35)(46)t_5 = t_5t_3$
$\Longrightarrow t_1(12)(35)(46)t_5 = t_1t_5t_3$
$\Longrightarrow (12)(35)(46)t_1^{(12)(35)(46)}t_5 = t_1t_5t_3$
$\Longrightarrow (12)(35)(46)t_2t_5 = t_1t_5t_3$, by Equation 5.1
One symmetric generator will go back to [13].

$Nt_1t_5t_4N$ is a new double coset which we will denote [154].
One symmetric generator will go to [154].

$Nt_1t_5t_5 = Nt_1 \in [1]$.
One symmetric generator will go to [1].

$Nt_1t_5t_6N$ is a new double coset which we will denote [156].
One symmetric will go to [156].

Below is our Cayley Diagram thus far.



$N^{(132)} \geq N^{132}$.

$N^{132} = \{e\}$.

Next we will check to see if any of our relations will increase the number of elements in the coset stabilizing group $N^{(132)}$.

$t_1 t_3 t_2 = t_1 \underline{t_3 t_2}$

$\implies t_1 t_3 t_2 = t_1 [t_6 t_1 t_5 t_4]$, since

$[t_6 t_3 t_2]^{(163245)} = [t_1 t_5 t_4]^{(163245)}$

$\implies t_3 t_2 t_4 = t_6 t_1 t_5$

$\implies t_3 t_2 t_4 t_4 = t_6 t_1 t_5 t_4$

$\implies t_3 t_2 = t_6 t_1 t_5 t_4$ by Equation 5.4

$t_1 t_3 t_2 = \underline{t_1 t_6 t_1} t_5 t_4$

$t_1 t_3 t_2 = [(16)(24)(35) t_1 t_6] t_5 t_4$, since

$[(12)(36)(45) t_3 t_6]^{(13)(25)} = [t_3 t_6 t_3]^{(13)(25)}$

$\implies (16)(24)(35) t_1 t_6 = t_1 t_6 t_1$, by Equation 5.2

$t_1 t_3 t_2 = (16)(24)(35) t_1 \underline{t_6 t_5} t_4$

$\implies t_1 t_3 t_2 = (16)(24)(35) t_1 [(134)(256) t_5 t_6 t_2] t_4$, since

$[(143)(265) t_3 t_1]^{(16)(24)(35)} = [t_1 t_3 t_4]^{(16)(24)(35)}$

$\implies (134)(256) t_5 t_6 = t_6 t_5 t_2$

$\implies (134)(256) t_5 t_6 t_2 = t_6 t_5 t_2 t_2$

$\implies (134)(256) t_5 t_6 t_2 = t_6 t_5$, by Equation 5.3

$t_1t_3t_2 = (16)(24)(35)(134)(256)t_1^{(134)(256)}t_5t_6t_2t_4$

$\implies t_1t_3t_2 = (12)(36)(45)\underline{t_3t_5}t_6t_2t_4$

$\implies t_1t_3t_2 = (12)(36)(45)[(12)(35)(46)t_3]t_6t_2t_4$, since

$[(12)(35)(46)t_1]^{(13)(25)} = [t_1t_2]^{(13)(25)}$

$\implies (12)(35)(46)t_3 = t_3t_5$, by Equation 5.1

$\implies t_1t_3t_2 = (34)(56)t_3t_6t_2t_4$

$\implies t_1t_3t_2 = (34)(56)t_3t_6\underline{t_3t_3}t_2t_4$

$\implies t_1t_3t_2 = (34)(56)\underline{t_3t_6t_3}t_3t_2t_4$

$\implies t_1t_3t_2 = (34)(56)[(12)(36)(45)t_3t_6]t_3t_2t_4$, by Equation 5.2

$\implies t_1t_3t_2 = (12)(35)(46)t_3t_6\underline{t_3t_2t_4}$

$\implies t_1t_3t_2 = (12)(35)(46)t_3t_6[t_6t_1t_5]$, since

$[t_6t_3t_2]^{(163245)} = [t_1t_5t_4]^{(163245)}$

$\implies t_3t_2t_4 = t_6t_1t_5$, by Equation 5.4

$t_1t_3t_2 = (12)(35)(46)t_3\underline{t_6t_6}t_1t_5$

$\implies t_1t_3t_2 = (12)(35)(46)t_3t_1t_5$

$\implies Nt_1t_3t_2 = Nt_3t_1t_5$.

Now, since $[Nt_1t_3t_2]^{(13)(25)} = Nt_3t_1t_5 = Nt_1t_3t_2$, then $(13)(25) \in N^{(132)}$.

Also, $t_5t_2t_3 = t_5\underline{t_2t_3}$

$\implies t_5t_2t_3 = t_5[(15)(23)(46)t_2t_3t_2]$, since

$[(12)(36)(45)t_3t_6]^{(163245)} = [t_3t_6t_3]^{(163245)}$

$\implies (15)(23)(46)t_2t_3 = t_2t_3t_2$

$\implies (15)(23)(46)t_2t_3t_2 = t_2t_3t_2t_2$

$\implies (15)(23)(46)t_2t_3t_2 = t_2t_3$, by Equation 5.2

$t_5t_2t_3 = (15)(23)(46)t_5^{(15)(23)(46)}t_2t_3t_2$

$\implies t_5t_2t_3 = (15)(23)(46)\underline{t_1t_2}t_3t_2$

$\implies t_5t_2t_3 = (15)(23)(46)\underline{t_1t_2}t_3t_2$

$\implies t_5t_2t_3 = (15)(23)(46)[(12)(35)(46)t_1]t_3t_2$, by Equation 5.1

$\implies t_5t_2t_3 = (13)(25)t_1t_3t_2$

$\implies Nt_5t_2t_3 = Nt_1t_3t_2$.

Since $[Nt_1t_3t_2]^{(15)(23)(46)} = Nt_5t_2t_3 = Nt_1t_3t_2$, then $(15)(23)(46) \in N^{(132)}$.

Also, $[Nt_1t_3t_2]^{(12)(35)(46)} = [Nt_3t_1t_5]^{(12)(35)(46)}$
$\Longrightarrow Nt_2t_5t_1 = Nt_5t_2t_3$

So, $N^{(132)} \geq < (13)(25), (12)(35)(46) >= \{e, (13)(25), (12)(35)(46), (15)(23)(46)\}$.
The number of single cosets in $Nt_1t_3t_2N$ is at most $\frac{|N|}{|N^{(132)}|} = \frac{12}{4} = 3$.

$Nt_1t_3t_2N = \{Nt_1t_3t_2 = Nt_2t_5t_1 = Nt_5t_2t_3 = Nt_3t_1t_5, Nt_3t_4t_5 = Nt_5t_6t_3 = Nt_6t_5t_4 = t_4t_3t_6, Nt_4t_1t_6 = Nt_6t_2t_4 = Nt_2t_6t_1 = Nt_1t_4t_2\}$.

$N^{(136)} \geq N^{136}$.
$N^{136} = \{e\}$.
Next we will check to see if any of our relations will increase the number of elements in the coset stabilizing group $N^{(136)}$.

$t_1t_3t_6 = t_1\underline{t_3t_6}$
$\Longrightarrow t_1t_3t_6 = t_1[(12)(36)(45)t_3t_6t_3]$, by Equation 5.2
$\Longrightarrow t_1t_3t_6 = (12)(36)(45)t_1^{(12)(36)(45)}t_3t_6t_3$
$\Longrightarrow t_1t_3t_6 = (12)(36)(45)\underline{t_2t_3t_6}t_3$
$\Longrightarrow t_1t_3t_6 = (12)(36)(45)[t_4t_5t_1]t_3$, since
$[t_6t_3t_2]^{(12)(35)(46)} = [t_1t_5t_4]^{(12)(35)(46)}$
$\Longrightarrow t_4t_5t_1 = t_2t_3t_6$, by Equation 5.4
$t_1t_3t_6 = (12)(36)(45)t_4\underline{t_5t_1}t_3 \Longrightarrow t_1t_3t_6 = (12)(36)(45)t_4[(15)(23)(46)t_5t_1t_5]t_3$, since
$[(12)(36)(45)t_3t_6]^{(16)(24)(35)} = [t_3t_6t_3]^{(16)(24)(35)}$
$\Longrightarrow (15)(23)(46)t_5t_1 = t_5t_1t_5$
$\Longrightarrow (15)(23)(46)t_5t_1t_5 = t_5t_1t_5t_5$
$\Longrightarrow (15)(23)(46)t_5t_1t_5 = t_5t_1$, by Equation 5.2
$t_1t_3t_6 = (12)(36)(45)t_4(15)(23)(46)t_5t_1t_5t_3$
$\Longrightarrow t_1t_3t_6 = (12)(36)(45)(15)(23)(46)t_4^{(15))(23)(46)}t_5t_1t_5t_3$
$\Longrightarrow t_1t_3t_6 = (134)(256)t_6t_5t_1\underline{t_5t_3}$
$\Longrightarrow t_1t_3t_6 = (134)(256)t_6t_5t_1[(12)(35)(46)t_5]$, since

$[(12)(35)(46)t_1]^{(15)(23)(46)} = [t_1t_2]^{(15)(23)(46)}$

$\Longrightarrow (12)(35)(46)t_5 = t_5t_3$, by Equation 5.1

$t_1t_3t_6 = (134)(256)t_6t_5t_1(12)(35)(46)t_5$

$\Longrightarrow t_1t_3t_6 = (134)(256)(12)(35)(46)[t_6t_5t_1]^{(12)(35)(46)}t_5$

$\Longrightarrow t_1t_3t_6 = (154236)\underline{t_4t_3}t_2t_5$

$\Longrightarrow t_1t_3t_6 = (154236)[(134)(256)t_3t_4t_1]t_2t_5$, since

$[(143)(265)t_3t_1]^{(14)(26)} = [t_1t_3t_4]^{(14)(26)}$

$\Longrightarrow (134)(256)t_3t_4 = t_4t_3t_1$

$\Longrightarrow (134)(256)t_3t_4t_1 = t_4t_3t_1t_1$

$\Longrightarrow (134)(256)t_3t_4t_1 = t_4t_3$, by Equation 5.3

$t_1t_3t_6 = (154236)(134)(256)t_3t_4t_1t_2t_5 \Longrightarrow t_1t_3t_6 = (163245)t_3t_4\underline{t_1t_2}t_5$

$\Longrightarrow t_1t_3t_6 = (163245)t_3t_4[(12)(35)(46)t_1]t_5$, by relation 5.1

$\Longrightarrow t_1t_3t_6 = (163245)(12)(35)(46)[t_3t_4]^{(12)(35)(46)}t_1t_5$

$\Longrightarrow t_1t_3t_6 = (143)(265)t_5\underline{t_6t_1t_5}$

$\Longrightarrow t_1t_3t_6 = (143)(265)t_5[t_3t_2t_4]$, since

$[t_6t_3t_2]^{(13)(25)} = [t_1t_5t_4]^{(13)(25)}$

$\Longrightarrow t_6t_1t_5 = t_3t_2t_4$, by Equation 5.4

$t_1t_3t_6 = (143)(265)\underline{t_5t_3}t_2t_4 \Longrightarrow t_1t_3t_6 = (143)(265)[(12)(35)(46)t_5]t_2t_4$, since

$[(12)(35)(46)t_1]^{(15)(23)(46)} = [t_1t_2]^{(15)(23)(46)}$

$\Longrightarrow (12)(35)(46)t_5 = t_5t_3$, by Equation 5.1

$t_1t_3t_6 = (143)(265)[(12)(35)(46)t_5t_2t_4$

$\Longrightarrow t_1t_3t_6 = (163245)t_5t_2t_4$

$\Longrightarrow Nt_1t_3t_6 = Nt_5t_2t_4$

$[Nt_1t_3t_6]^{(15)(23)(46)} = Nt_5t_2t_4 = Nt_1t_3t_6$

$\Longrightarrow (15)(23)(46) \in N^{(136)}$.

Thus, $N^{(136)} \geq < (15)(23)(46) >= \{e, (15)(23)(46)\}$.

The number of single cosets in $Nt_1t_3t_6N$ is at most $\frac{|N|}{|N^{(136)}|} = \frac{12}{2} = 6$.

$Nt_1t_3t_6N = \{Nt_1t_3t_6 = Nt_5t_2t_4, Nt_2t_5t_4 = Nt_3t_1t_6, Nt_3t_4t_2 = Nt_6t_5t_1, Nt_4t_1t_5 = Nt_2t_6t_3, Nt_5t_6t_1 = Nt_4t_3t_2, = Nt_6t_2t_3 = Nt_1t_4t_5\}$.

$N^{(152)} \geq N^{152}$.

$N^{152} = \{e\}$.

Next we will check to see if any of our relations will increase the number of elements in the coset stabilizing group $N^{(152)}$.

$t_1t_5t_2 = t_1t_5\underline{t_2}$

$\Longrightarrow t_1t_5t_2 = t_1t_5[(12)(35)(46)t_2t_1]$, by Equation 5.1

$\Longrightarrow t_1t_5t_2 = (12)(35)(46)[t_1t_5]^{(12)(36)(45)}t_2t_1$

$\Longrightarrow t_1t_5t_2 = (12)(35)(46)\underline{t_2t_3t_2}t_1$

$\Longrightarrow t_1t_5t_2 = (12)(35)(46)[(15)(23)(46)t_2t_3]t_1$, since

$[(12)(36)(45)t_3t_6]^{(163245)} = [t_3t_6t_3]^{(163245)}$

$\Longrightarrow (15)(23)(46)t_2t_3 = t_2t_3t_2$, by Equation 5.2

$t_1t_5t_2 = (12)(35)(46)(15)(23)(46)t_2t_3t_1$

$\Longrightarrow t_1t_5t_2 = (13)(25)t_2t_3t_1$

$\Longrightarrow Nt_1t_5t_2 = Nt_2t_3t_1$

Then, since $[Nt_1t_5t_2]^{(12)(35)(46)} = Nt_2t_3t_1 = Nt_1t_5t_2$, then $(12)(35)(46) \in N^{(152)}$.

Also,

$t_2t_3t_1 = t_2\underline{t_3t_1}$

$\Longrightarrow t_2t_3t_1 = t_2[(134)(256)t_1t_3t_4]$, since

$[(143)(265)t_3t_1]^{(13)(25)} = [t_1t_3t_4]^{(13)(25)}t_1t_3$

$\Longrightarrow (134)(256)t_3t_1t_4$, by Equation 5.3

$t_2t_3t_1 = t_2(134)(256)t_1t_3t_4$

$\Longrightarrow t_2t_3t_1 = (134)(256)t_2^{(134)(256)}t_1t_3t_4$

$\Longrightarrow t_2t_3t_1 = (134)(256)t_5t_1\underline{t_3}t_4$

$\Longrightarrow t_2t_3t_1 = (134)(256)t_5t_1[(12)(35)(46)t_3t_5]t_4$, since

$[(12)(35)(46)t_1]^{(13)(25)} = [t_1t_2]^{(13)(25)}$

$\Longrightarrow (12)(35)(46)t_3 = t_3t_5$

$\Longrightarrow (12)(35)(46)t_3t_5 = t_3t_5t_5$

$\Longrightarrow (12)(35)(46)t_3t_5 = t_3$, by Equation 5.1

$t_2t_3t_1 = (134)(256)t_5t_1(12)(35)(46)t_3t_5t_4$

$\Longrightarrow t_2t_3t_1 = (134)(256)(12)(35)(46)t_3t_2t_3t_5t_4$

$\Longrightarrow t_2t_3t_1 = (154236)\underline{t_3t_2t_3}t_5t_4$

$\Longrightarrow t_2t_3t_1 = (154236)[(15)(23)(46)t_3t_2]t_5t_4$, since
$[(12)(36)(45)t_3t_6]^{(14)(26)} = [t_3t_6t_3]^{(14)(26)}$
$\Longrightarrow (15)(23)(46)t_3t_2 = t_3t_2t_3$, by Equation 5.2
$t_2t_3t_1 = (154236)(15)(23)(46)t_3t_2t_5t_4$
$\Longrightarrow t_2t_3t_1 = (34)(56)t_3t_2t_5t_4$
$\Longrightarrow t_2t_3t_1 = (34)(56)t_3t_2[t_4t_4]t_5t_4$
$\Longrightarrow t_2t_3t_1 = (34)(56)\underline{t_3t_2t_4}t_4t_5t_4$
$\Longrightarrow t_2t_3t_1 = (34)(56)[t_6t_1t_5]t_4t_5t_4$, since
$t_6t_3t_2]^{(163245)} = [t_1t_5t_4]^{(163245)}$
$\Longrightarrow t_3t_2t_4 = t_6t_1t_5$, by Equation 5.4
$t_2t_3t_1 = (34)(56)t_6t_1t_5\underline{t_4t_5t_4} \Longrightarrow t_2t_3t_1 = (34)(56)t_6t_1t_5[(12)(36)(45)t_4t_5]$, since
$[(12)(36)(45)t_3t_6]^{(34)(56)} = [t_3t_6t_3]^{(34)(56)}$
$\Longrightarrow (12)(36)(45)t_4t_5 = t_4t_5t_4$, Equation 5.4
$t_2t_3t_1 = (34)(56)t_6t_1t_5(12)(36)(45)t_4t_5$
$\Longrightarrow t_2t_3t_1 = (34)(56)(12)(36)(45)[t_6t_1t_5]^{(12)(36)(45)}t_4t_5$
$\Longrightarrow t_2t_3t_1 = (12)(35)(46)t_3t_2t_4t_4t_5$
$\Longrightarrow t_2t_3t_1 = (12)(35)(46)t_3t_2t_5$
$\Longrightarrow Nt_2t_3t_1 = Nt_3t_2t_5$

Thus $[Nt_1t_5t_2]^{(13)(25)(46)} = Nt_3t_2t_5 = Nt_2t_3t_1 = Nt_1t_5t_2$
$\Longrightarrow (13)(25)(46) \in N^{(152)}$.

So, $N^{(152)} \geq < (12)(35)(46), (13)(25)(46) >= \{e, (12)(35)(46), (13)(25)(46), (15)(23)\}$.
The number of single cosets in $Nt_1t_5t_2N$ is at most $\frac{|N|}{|N^{(152)}|} = \frac{12}{4} = 3$.

$Nt_1t_5t_2N = \{Nt_1t_5t_2 = Nt_2t_3t_1 = Nt_5t_3t_1 = Nt_3t_2t_5, Nt_3t_6t_5 = Nt_5t_4t_3 = Nt_6t_3t_4 = t_4t_5t_6, Nt_4t_2t_6 = Nt_6t_1t_4 = Nt_2t_4t_1 = Nt_1t_6t_2\}$.

$N^{(154)} \geq N^{154}$.
$N^{154} = \{e\}$.
Next we will check to see if any of our relations will increase the number of elements in the coset stabilizing group $N^{(154)}$.

$[Nt_1t_5t_4]^{(16)(24)(35)} = Nt_6t_3t_2 = Nt_1t_5t_4$, by Equation 5.4.
Thus, $(16)(24)(35) \in N^{(154)}$.

So, $N^{(154)} \geq < (16)(24)(35) >= \{e, (16)(24)(35)\}$.
The number of single cosets in $Nt_1t_5t_4N$ is at most $\frac{|N|}{|N^{(154)}|} = \frac{12}{2} = 6$.

$Nt_1t_5t_4N = \{Nt_1t_5t_4 = Nt_6t_3t_2, Nt_2t_3t_6 = Nt_4t_5t_1, Nt_3t_6t_1 = Nt_2t_4t_5, Nt_5t_4t_2 = t_1t_6t_3, Nt_4t_2t_3 = Nt_5t_1t_6, Nt_6t_1t_5 = Nt_3t_2t_4\}$.

$N^{(156)} \geq N^{156}$.
$N^{156} = \{e\}$.
Next we will check to see if any of our relations will increase the number of elements in the coset stabilizing group $N^{(156)}$.

$t_1t_5t_6 = \underline{t_1t_5}t_6$
$\Longrightarrow t_1t_5t_6 = [t_6t_3t_2t_4]t_6$, since
$t_6t_3t_2 = t_1t_5t_4 \Longrightarrow t_6t_3t_2t_4 = t_1t_5t_4t_4$
$\Longrightarrow t_6t_3t_2t_4 = t_1t_5$,Equation 5.4
$t_1t_5t_6 = t_6t_3t_2\underline{t_4t_6}$
$\Longrightarrow t_1t_5t_6 = t_6t_3t_2[(12)(35)(46)t_4]$, since
$[(12)(35)(46)t_1]^{(14)(26)} = [t_1t_2]^{(14)(26)}$
$\Longrightarrow (12)(35)(46)t_4 = t_4t_6$, by Equation 5.1
$t_1t_5t_6 = (12)(35)(46)[t_6t_3t_2]^{(12)(35)(46)}t_4 \Longrightarrow t_1t_5t_6 = (12)(35)(46)t_4t_5t_1t_4$
$t_1t_5t_6 = (12)(35)(46)\underline{t_4t_5t_1}t_4 \Longrightarrow t_1t_5t_6 = (12)(35)(46)[t_2t_3t_6]t_4$, since
$[t_6t_3t_2]^{(12)(35)(46)} = t_1t_5t_4]^{(12)(35)(46)}$
$\Longrightarrow t_4t_5t_1 = t_2t_3t_6$, by Equation 5.4
$t_1t_5t_6 = (12)(35)(46)\underline{t_2t_3}t_6t_4$
$t_1t_5t_6 = (12)(35)(46)[(15)(23)(46)t_2t_3t_2]t_6t_4$, since
$[(12)(36)(45)t_3t_6]^{(163245)} = [t_3t_6t_3]^{(163245)}$
$\Longrightarrow (15)(23)(46)t_2t_3 = t_2t_3t_2$
$\Longrightarrow (15)(23)(46)t_2t_3t_2 = t_2t_3t_2t_2$

$\Longrightarrow (15)(23)(46)t_2t_3t_2 = t_2t_3$, by Equation 5.2

$t_1t_5t_6 = (13)(25)t_2t_3t_2\underline{t_6t_4}$

$\Longrightarrow t_1t_5t_6 = (13)(25)t_2t_3t_2[(12)(35)(46)t_6]$, since

$[(12)(35)(46)t_1]^{(163245)} = [t_1t_2]^{(163245)}$

$\Longrightarrow (12)(35)(46)t_6 = t_6t_4$, by Equation 5.1

$t_1t_5t_6 = (13)(25)(12)(35)(46)[t_2t_3t_2]^{(12)(35)(46)}t_6$

$\Longrightarrow t_1t_5t_6 = (15)(23)(46)t_1\underline{t_5t_1t_6}$

$\Longrightarrow t_1t_5t_6 = (15)(23)(46)t_1[t_4t_2t_3]$, since

$[t_6t_3t_2]^{(143)(265)} = [t_1t_5t_4]^{(143)(265)}$

$\Longrightarrow t_5t_1t_6 = t_4t_2t_3$, by Equation 5.4

$t_1t_5t_6 = (15)(23)(46)t_1t_4\underline{t_2t_3}$

$\Longrightarrow t_1t_5t_6 = (15)(23)(46)t_1t_4[(15)(23)(46)t_2t_3t_2]$, since

$[(12)(36)(45)t_3t_6]^{(163245)} = [t_3t_6t_3]^{(163245)}$

$\Longrightarrow (15)(23)(46)t_2t_3 = t_2t_3t_2$

$\Longrightarrow (15)(23)(46)t_2t_3t_2 = t_2t_3t_2t_2$

$\Longrightarrow (15)(23)(46)t_2t_3t_2 = t_2t_3$, by Equation 5.2

$t_1t_5t_6 = (15)(23)(46)(15)(23)(46)[t_1t_4]^{15)(23)(46)}t_2t_3t_2$

$\Longrightarrow t_1t_5t_6 = \underline{t_5t_6t_2}t_3t_2$

$\Longrightarrow t_1t_5t_6 = [(143)(265)t_6t_5]t_3t_2$, since

$[(143)(265)t_3t_1]^{(154236)} = [t_1t_3t_4]^{(154236)}$

$\Longrightarrow (143)(265)t_6t_5 = t_5t_6t_2$, by Equation 5.3

$t_1t_5t_6 = (143)(265)t_6\underline{t_5t_3}t_2$

$\Longrightarrow t_1t_5t_6 = (143)(265)t_6[(12)(35)(46)t_5]t_2$, since

$[(12)(35)(46)t_1]^{(15)(23)(46)} = [t_1t_2]^{(15)(23)(46)}$

$\Longrightarrow (12)(35)(46)t_5 = t_5t_3$, by Equation 5.1

$t_1t_5t_6 = (143)(265)(12)(35)(46)[t_6]^{(12)(35)(46)}t_5t_2$

$\Longrightarrow t_1t_5t_6 = (163245)t_4t_5t_2$

$\Longrightarrow Nt_1t_5t_6 = Nt_4t_5t_2$

Since $[Nt_1t_5t_6]^{(14)(26)} = Nt_4t_5t_2 = Nt_1t_5t_6$, then $(14)(26) \in N^{(156)}$.
So, $N^{(156)} \geq < (14)(26) >= \{e, (14)(26)\}$.
The number of single cosets in $Nt_1t_5t_6N$ is at most $\frac{|N|}{|N^{(156)}|} = \frac{12}{2} = 6$.

$Nt_1t_5t_6N = \{Nt_1t_5t_6 = Nt_4t_5t_2, Nt_2t_3t_4 = Nt_6t_3t_1, Nt_3t_6t_2 = Nt_1t_6t_5, Nt_2t_4t_3 = t_5t_4t_1, Nt_4t_2t_5 = Nt_3t_2t_6, Nt_6t_1t_3 = Nt_5t_1t_4\}$.

The orbits of $N^{(132)}$ on $\{1, 2, 3, 4, 5, 6\}$ are

$$\{1, 2, 5, 3\}, \{4, 6\}.$$

we take t_2 and t_4 from each orbit respectively.

We want to determine to which double coset $Nt_1t_3t_2t_1$ and $Nt_1t_3t_2t_4$ belong.

$Nt_1t_3t_2t_2 = Nt_1t_3 \in [13]$

Thus 4 symmetric generators will go to [13].

$Nt_1t_3t_2t_4 = Nt_1\underline{t_3t_2t_4}$

$\Longrightarrow Nt_1t_3t_2t_4 = Nt_1[t_6t_1t_5]$, since

$[t_6t_3t_2]^{(163245)} = [t_1t_5t_4]^{(163245)}$

$\Longrightarrow t_3t_2t_4 = t_6t_1t_5$, by Equation 5.4

$Nt_1t_3t_2t_4 = N\underline{t_1t_6t_1}t_5 \Longrightarrow Nt_1t_3t_2t_4 = N[(16)(24)(35)t_1t_6]t_5$, since

$[(12)(36)(45)t_3t_6]^{(13)(25)} = [t_3t_6t_3]^{(13)(25)}$

$\Longrightarrow (16)(24)(35)t_1t_6 = t_1t_6t_1$, by Equation 5.2

$Nt_1t_3t_2t_4 = Nt_1t_6t_5 \in [156]$.

Thus 2 symmetric generators will go to [156].

The orbits of $N^{(136)}$ on $\{1, 2, 3, 4, 5, 6\}$ are

$$\{1, 5\}, \{2, 3\}, \{4, 6\}.$$

we take t_1, t_3, and t_6 from each orbit respectively.

We want to determine to which double coset $Nt_1t_3t_6t_1$, $Nt_1t_3t_6t_3$, and $Nt_1t_3t_6t_6$ belong.

$Nt_1t_3t_6t_1 = Nt_1t_3t_6t_1$

$\Longrightarrow Nt_1t_3t_6t_1 = Nt_1[t_2t_4t_5]$, since

$[t_6t_3t_2]^{(12)(36)(45)} = [t_1t_5t_4]^{(12)(36)(45)}$

$\Longrightarrow t_3t_6t_1 = t_2t_4t_5$, by Equation 5.4

$Nt_1t_3t_6t_1 = N\underline{t_1t_2}t_4t_5$

$\Longrightarrow Nt_1t_3t_6t_1 = N[(12)(35)(46)t_1]t_4t_5$, by Equation 5.1

$\Longrightarrow Nt_1t_3t_6t_1 = Nt_1t_4t_5 \in [136]$.

Thus 2 symmetric generators will go to [136].

$Nt_1t_3t_6t_3 = Nt_1\underline{t_3t_6t_3}$

$\Longrightarrow Nt_1t_3t_6t_3 = Nt_1[(12)(36)(45)t_3t_6]$, by Equation 5.3

$\Longrightarrow Nt_1t_3t_6t_3 = N(12)(36)(45)[t_1]^{(12)(36)(45)}t_3t_6$

$\Longrightarrow Nt_1t_3t_6t_3 = Nt_2t_3t_6 \in [154]$.

Thus 2 symmetric generators will go to [154].

$Nt_1t_3t_6t_6 = Nt_1t_3 \in [13]$.

Thus 2 symmetric generators will go to [13].

The orbits of $N^{(152)}$ on $\{1, 2, 3, 4, 5, 6\}$ are

$$\{1, 2, 5, 3\}, \{4, 6\}.$$

we take t_2 and t_6 from each orbit respectively.

We want to determine to which double coset $Nt_1t_5t_2t_2$ and $Nt_1t_5t_2t_6$ belong.

$Nt_1t_5t_2t_2 = Nt_1t_5 \in [15]$.

Thus 4 symmetric generators will go to [15].

$Nt_1t_5t_2t_6 = Nt_1\underline{t_5t_2}t_6$

$\Longrightarrow Nt_1t_5t_2t_6 = Nt_1[(134)(256)t_2t_5t_6]t_6$, since

$[(143)(265)t_3t_1]^{(15)(23)(46)} = [t_1t_3t_4]^{(15)(23)(46)}$

$\Longrightarrow (134)(256)t_2t_5 = t_5t_2t_6$

$\Longrightarrow (134)(256)t_2t_5t_6 = t_5t_2t_6t_6$

$\Longrightarrow (134)(256)t_2t_5t_6 = t_5t_2$, by Equation 5.2

$Nt_1t_5t_2t_6 = N(134)(256)[t_1]^{(134)(256)}t_2t_5t_6t_6$

$\Longrightarrow Nt_1t_5t_2t_6 = Nt_3t_2t_5t \in [152]$.

Thus 2 symmetric generators will go to [152].

The orbits of $N^{(154)}$ on $\{1,2,3,4,5,6\}$ are

$$\{1,6\},\{2,4\},\{3,5\}.$$

We take t_1, t_4, and t_3 from each orbit respectively.

We want to determine to which double coset $Nt_1t_5t_4t_1$, $Nt_1t_5t_4t_4$, and $Nt_1t_5t_4t_3$ belong.

$t_1t_5t_4 = t_6t_3t_2$, by Equation 5.4

$\Longrightarrow t_1t_5t_4t_1 = t_6t_3t_2t_1$

$\Longrightarrow t_1t_5t_4t_1 = t_6t_3\underline{t_2t_1}$

$\Longrightarrow t_1t_5t_4t_1 = t_6t_3[(12)(35)(46)t_2]$, since

$[(12)(35)(46)t_1]^{(12)(35)(46)} = [t_1t_2]^{(12)(35)(46)} \Longrightarrow (120(35)(46)t_2 = t_2t_1$. by Equation 5.1

$t_1t_5t_4t_1 = (12)(35)(46)[t_6t_3]^{(12)(35)(46)}t_2$

$\Longrightarrow t_1t_5t_4t_1 = (12)(35)(46)t_4t_5t_2$

$\Longrightarrow Nt_1t_5t_4t_1 = Nt_4t_5t_2 \in [156]$.

Thus 2 symmetric generators will go to [156].

$Nt_1t_5t_4t_4 = Nt_1t_5t_4 \in [154]$.

Thus 2 symmetric generators will go to [154].

$t_1t_5t_4 = t_6t_3t_2$, by Equation 5.4

$\Longrightarrow t_1t_5t_4t_3 = t_6\underline{t_3t_2t_3}$

$\Longrightarrow t_1t_5t_4t_3 = t_6[(15)(23)(46)t_3t_2]$, since

$[(12)(36)(45)t_3t_6]^{(14)(26)} = [t_3t_6t_3]^{(14)(26)}$

$\Longrightarrow (15)(23)(46)t_3t_2 = t_3t_2t_3$, by relation 5.2

$t_1t_5t_4t_3 = (15)(23)(46)t_6^{(15)(23)(46)}t_3t_2$

$\Longrightarrow t_1t_5t_4t_3 = (15)(23)(46)t_4t_3t_2$

$\Longrightarrow Nt_1t_5t_4t_3 = Nt_4t_3t_2 \in [136]$.

Thus, 2 symmetric generators will go to [136].

The orbits of $N^{(156)}$ on $\{1,2,3,4,5,6\}$ are

$$\{3\},\{5\},\{1,4\},\{2,6\}.$$

we take t_3, t_5, t_1, and t_6 from each orbit respectively.

We want to determine to which double coset $Nt_1t_5t_6t_3$, $Nt_1t_5t_6t_5$, $Nt_1t_5t_6t_1$, and $Nt_1t_5t_6t_6$ belong.

$t_1t_5t_6t_3 = \underline{t_1t_5}t_6t_3$

$\Longrightarrow t_1t_5t_6t_3 = [(15)(23)(46)t_1t_5t_1]t_6t_3$, since

$[(12)(36)(45)t_3t_6]^{(143)(265)} = [t_3t_6t_3]^{(143)(265)}$

$\Longrightarrow (15)(23)(46)t_1t_5 = t_1t_5t_1$

$\Longrightarrow (15)(23)(46)t_1t_5t_1 = t_1t_5t_1t_1 \Longrightarrow (15)(23)(46)t_1t_5t_1 = t_1t_5$, by Equation 5.2

$t_1t_5t_6t_3 = (15)(23)(46)t_1t_5\underline{t_1t_6t_3}$

$\Longrightarrow t_1t_5t_6t_3 = (15)(23)(46)t_1t_5[t_5t_4t_2]$, since

$[t_6t_3t_2]^{(154236)} = [t_1t_5t_4]^{(154236)}$

$\Longrightarrow t_1t_6t_3 = t_5t_4t_2$, by Equation 5.4

$t_1t_5t_6t_3 = (15)(23)(46)t_1\underline{t_5t_5}t_4t_2$

$\Longrightarrow t_1t_5t_6t_3 = (15)(23)(46)t_1t_4t_2$

$\Longrightarrow Nt_1t_5t_6t_3 = Nt_1t_4t_2 \in [132]$.

Thus 1 symmetric generator will go to [132].

$Nt_1t_5t_6t_5 = \underline{Nt_1t_5t_6}t_5$

$\Longrightarrow Nt_1t_5t_6t_5 = [Nt_4t_5t_2]t_5$, since $Nt_1t_5t_6 = Nt_4t_5t_2$

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_4t_5t_2t_5$

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_4[(134)(256)t_2t_5t_6]t_5$, since

$[(143)(265)t_3t_1]^{(15)(23)(46)} = [t_1t_3t_4]^{(15)(23)(46)}$

$\Longrightarrow (134)(256)t_2t_5 = t_5t_2t_6$

$\Longrightarrow (134)(256)t_2t_5t_6 = t_5t_2t_6t_6$

$\Longrightarrow (134)(256)t_2t_5t_6 = t_5t_2$, by Equation 5.3

$Nt_1t_5t_6t_5 = N(134)(256)t_4^{(134)(256)}t_2t_5t_6t_5$

$\Longrightarrow Nt_1t_5t_6t_5 = N\underline{t_1t_2}t_5t_6t_5$

$\Longrightarrow Nt_1t_5t_6t_5 = N[(12)(35)(46)t_1]t_5t_6t_5$, by Equation 5.1

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_1t_5\underline{t_6t_5}$

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_1t_5[(134)(256)t_5t_6t_2]$, since

$[(143)(265)t_3t_1]^{(16)(24)(35)} = [t_1t_3t_4]^{(16)(24)(35)}$

$\Longrightarrow (134)(256)t_5t_6 = t_6t_5t_2$

$\Longrightarrow (134)(256)t_5t_6t_2 = t_6t_5t_2t_2$

$\Longrightarrow (134)(256)t_5t_6t_2 = t_6t_5$, by Equation 5.3

$Nt_1t_5t_6t_5 = N(134)(256)[t_1t_5]^{(134)(256)}t_5t_6t_2$

$\Longrightarrow Nt_1t_5t_6t_5 = N\underline{t_3t_6}t_5t_6t_2$

$\Longrightarrow Nt_1t_5t_6t_5 = N[(12)(36)(45)t_3t_6t_3]t_5t_6t_2$, by Equation 5.2

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_3t_6\underline{t_3t_5}t_6t_2$

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_3t_6[(12)(35)(46)t_3]t_6t_2$, since

$[(12)(35)(46)t_1]^{(134)(256)} = [t_1t_2]^{(134)(256)}$

$\Longrightarrow (12)(35)(46)t_3 = t_3t_5$, by Equation 5.1

$Nt_1t_5t_6t_5 = N(12)(35)(46)[t_3t_6]^{(12)(35)(46)}t_3t_6t_2$

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_5t_4\underline{t_3t_6}t_2$

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_5t_4[(12)(36)(45)t_3t_6t_3]t_2$, by Equation 5.2

$\Longrightarrow Nt_1t_5t_6t_5 = N(12)(36)(45)[t_5t_4]^{(12)(36)(45)}t_3t_6t_3t_2$

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_4\underline{t_5t_3}t_6t_3t_2$

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_4[(12)(35)(46)t_5]t_6t_3t_2$, since

$[(12)(35)(46)t_1]^{(154236)} = [t_1t_2]^{(154236)}$

$\Longrightarrow (12)(35)(46)t_5 = t_5t_3$, by Equation 5.1

$Nt_1t_5t_6t_5 = N(12)(35)(46)t_4^{(12)(35)(46)}t_5t_6t_3t_2$

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_6t_5\underline{t_6t_3t_2}$

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_6t_5[t_1t_5t_4]$, by Equation 5.4

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_6\underline{t_5t_1t_5}t_4$

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_6[(15)(23)(46)t_5t_1]t_4$, since

$[(12)(36)(45)t_3t_6]^{(16)(24)(35)} = [t_3t_6t_3]^{(16)(24)(35)}$

$\Longrightarrow (15)(23)(46)t_5t_1 = t_5t_1t_5$, by Equation 5.2

$Nt_1t_5t_6t_5 = N(15)(23)(46)t_6^{(15)(23)(46)}t_5t_1t_4$

$\Longrightarrow Nt_1t_5t_6t_5 = N\underline{t_4t_5t_1}t_4$

$\Longrightarrow Nt_1t_5t_6t_5 = N[t_2t_3t_6]t_4$, since

$[t_6t_3t_2]^{(12)(35)(46)} = [t_1t_5t_4]^{(12)(35)(46)}$

$\Longrightarrow t_4t_5t_1 = t_2t_3t_6$, by Equation 5.4

$Nt_1t_5t_6t_5 = Nt_2t_3\underline{t_6t_4}$

$\Longrightarrow Nt_1t_5t_6t_5 = Nt_2t_3[(12)(35)(46)t_6$, since

$[(12)(35)(46)t_1]^{(163245)} = [t_1t_2]^{(163245)}$

$\implies (12)(35)(46)t_6 = t_6t_4$, by Equation 5.1

$Nt_1t_5t_6t_5 = N(12)(35)(46)[t_2t_3]^{(12)(35)(46)}t_6$

$\implies Nt_1t_5t_6t_5 = Nt_1t_5t_6 \in [156]$

Thus 1 symmetric generator will go to [156].

$Nt_1t_5t_6 = Nt_4t_5t_2$

$\implies Nt_1t_5t_6t_1 = Nt_4t_5\underline{t_2t_1}$

$\implies Nt_1t_5t_6t_1 = Nt_4t_5[(12)(35)(46)t_2]$, since

$[(12)(35)(46)t_1]^{(12)(35)(46)} = [t_1t_2]^{(12)(35)(46)}$

$\implies (12)(35)(46)t_2 = t_2t_1$, by Equation 5.1

$Nt_1t_5t_6t_1 = N(12)(35)(46)[t_4t_5]^{(12)(35)(46)}t_2]$

$\implies Nt_1t_5t_6t_1 = Nt_6t_3t_2 \in [154]$

Thus 2 symmetric generators will go to [154].

$Nt_1t_5t_6t_6 = Nt_1t_5 \in [15]$.

Thus 2 symmetric generators will go to [15].

Below is our completed Cayley Diagram.

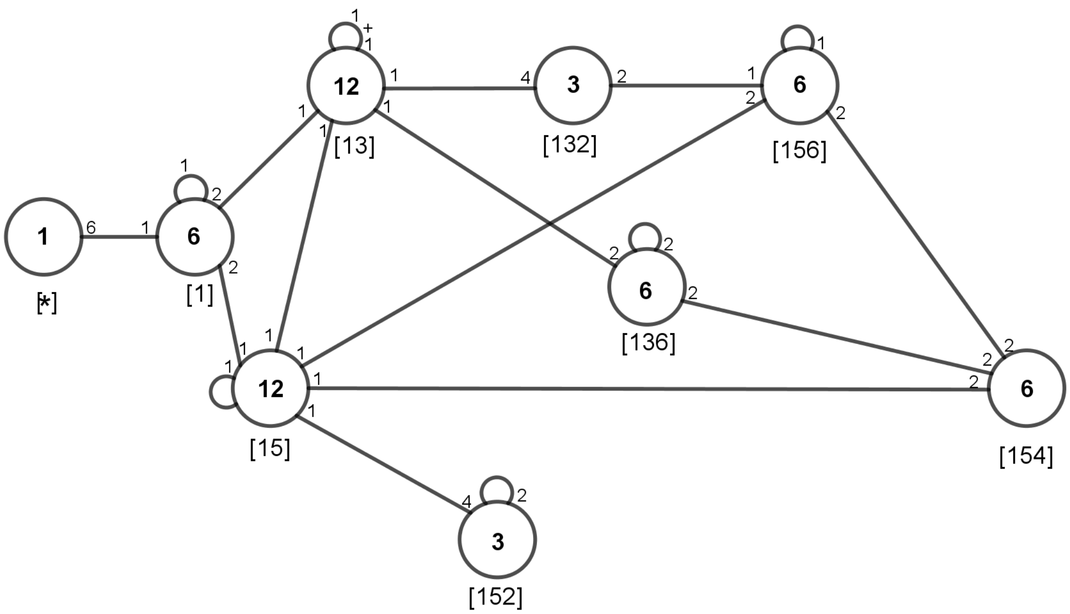


Figure 5.1: Cayley Diagram : $L_2(11)$ Over D_{12}

$$|G| \leq \left(\frac{|N|}{|N|} + \frac{|N|}{|N^{(1)}|} + \frac{|N|}{|N^{(13)}|} + \frac{|N|}{|N^{(15)}|} + \frac{|N|}{|N^{(132)}|} + \frac{|N|}{|N^{(136)}|} + \frac{|N|}{|N^{(152)}|} + \frac{|N|}{|N^{(154)}|} + \frac{|N|}{|N^{(156)}|}\right) \times |N|$$

$$= \left(\frac{12}{12} + \frac{12}{2} + \frac{12}{1} + \frac{12}{1} + \frac{12}{4} + \frac{12}{2} + \frac{12}{4} + \frac{12}{2} + \frac{12}{2}\right) \times 12$$

$$= (12 + 6 + 12 + 12 + 3 + 6 + 3 + 6 + 6) \times 12$$

$$= 55 \times 12$$

$$= 660$$

$$G \leq 660$$

Next we will show that $|G| \geq 660$. We compute the action of G on the 55 cosets in N in G that we have found.

Table 5.1: Single Coset Action of $L_2(11)$ Over D_{12}

Label	**Single Cosets**		x		y		z		t_1
1	N	1	N	1	N	1	N	2	Nt_1
2	Nt_1	3	Nt_2	4	Nt_3	3	Nt_2	1	Nt_1t_1
3	Nt_2	2	Nt_1	6	Nt_5	2	Nt_1	3	Nt_2t_1
4	Nt_3	6	Nt_5	5	Nt_4	7	Nt_6	13	Nt_3t_1
5	Nt_4	7	Nt_6	2	Nt_1	6	Nt_5	14	Nt_4t_1
6	Nt_5	4	Nt_3	7	Nt_6	5	Nt_4	29	Nt_5t_1
7	Nt_6	4	Nt_4	3	Nt_2	4	Nt_3	30	Nt_6t_1
8	Nt_1t_3	10	Nt_2t_5	12	Nt_3t_4	11	Nt_2t_6	8	$Nt_1t_3t_1$
9	Nt_1t_4	11	Nt_2t_6	13	Nt_3t_1	10	Nt_2t_5	9	$Nt_1t_4t_1$
10	Nt_2t_5	8	Nt_1t_3	16	Nt_5t_6	9	Nt_1t_4	32	$Nt_2t_5t_1$
11	Nt_2t_6	9	Nt_1t_4	17	Nt_5t_2	8	Nt_1t_3	34	$Nt_2t_6t_1$
12	Nt_3t_4	16	Nt_5t_6	14	Nt_4t_1	19	Nt_6t_5	15	$Nt_3t_4t_1$
13	Nt_3t_1	17	Nt_5t_2	15	Nt_4t_3	18	Nt_6t_2	4	$Nt_3t_1t_1$
14	Nt_4t_1	18	Nt_6t_2	8	Nt_1t_3	17	Nt_5t_2	5	Nt_4t_1
15	Nt_4t_3	19	Nt_6t_5	9	Nt_1t_4	16	Nt_5t_6	12	$Nt_4t_3t_1$
16	Nt_5t_6	12	Nt_3t_4	18	Nt_6t_2	15	Nt_4t_3	39	$Nt_5t_6t_1$
17	Nt_5t_2	13	Nt_3t_1	19	Nt_6t_5	14	Nt_4t_1	25	$Nt_5t_2t_1$
18	Nt_6t_2	14	Nt_4t_1	10	Nt_2t_5	13	Nt_3t_1	26	$Nt_6t_2t_1$
19	Nt_6t_5	15	Nt_4t_3	11	Nt_2t_6	12	Nt_3t_4	37	$Nt_6t_5t_1$
20	Nt_1t_5	22	Nt_2t_3	24	Nt_3t_6	23	Nt_2t_4	20	$Nt_1t_5t_1$
21	Nt_1t_6	23	Nt_2t_4	25	Nt_3t_2	22	Nt_2t_3	21	$Nt_1t_6t_1$
22	Nt_2t_3	20	Nt_1t_5	28	Nt_5t_4	21	Nt_1t_6	41	$Nt_2t_3t_1$
23	Nt_2t_4	21	Nt_1t_6	29	Nt_5t_1	20	Nt_1t_5	43	$Nt_2t_4t_1$
24	Nt_3t_6	28	Nt_5t_4	26	Nt_4t_2	31	Nt_6t_3	46	$Nt_3t_6t_1$
25	Nt_3t_2	29	Nt_5t_1	27	Nt_4t_5	30	Nt_6t_1	17	$Nt_3t_2t_1$
26	Nt_4t_2	30	Nt_6t_1	20	Nt_1t_5	29	Nt_5t_1	18	$Nt_4t_2t_1$
27	Nt_4t_5	31	Nt_6t_3	21	Nt_1t_6	28	Nt_5t_4	45	$Nt_4t_5t_1$

Continued on next page

Table 5.1 – *Continued from previous page*

Label	Single Cosets		x		y		z		t_1
28	Nt_5t_4	24	Nt_3t_6	30	Nt_6t_1	27	Nt_4t_5	53	$Nt_5t_4t_1$
29	Nt_5t_1	25	Nt_3t_2	31	Nt_6t_3	26	Nt_4t_2	6	$Nt_5t_1t_1$
30	Nt_6t_1	26	Nt_4t_2	22	Nt_2t_3	25	Nt_3t_2	7	$Nt_6t_1t_1$
31	Nt_6t_3	27	Nt_4t_5	23	Nt_2t_4	24	Nt_3t_6	51	$Nt_6t_3t_1$
32	$Nt_1t_3t_2$	32	$Nt_2t_5t_1$	33	$Nt_3t_4t_5$	34	$Nt_4t_1t_6$	10	$Nt_1t_3t_2t_1$
33	$Nt_3t_4t_5$	33	$Nt_5t_6t_3$	34	$Nt_4t_1t_6$	33	$Nt_3t_4t_5$	54	$Nt_3t_4t_5t_1$
34	$Nt_4t_1t_6$	34	$Nt_6t_2t_4$	32	$Nt_1t_3t_2$	32	$Nt_1t_3t_2$	11	$Nt_4t_1t_6t_1$
35	$Nt_1t_3t_6$	36	$Nt_2t_5t_4$	37	$Nt_3t_4t_2$	38	$Nt_4t_1t_5$	40	$Nt_1t_3t_6t_1$
36	$Nt_2t_5t_4$	35	$Nt_1t_3t_6$	39	$Nt_5t_6t_1$	40	$Nt_6t_2t_3$	48	$Nt_2t_5t_4t_1$
37	$Nt_3t_4t_2$	39	$Nt_5t_6t_1$	38	$Nt_4t_1t_5$	37	$Nt_3t_4t_2$	19	$Nt_3t_4t_2t_1$
38	$Nt_4t_1t_5$	55	$Nt_6t_2t_3$	35	$Nt_1t_3t_6$	35	$Nt_1t_3t_6$	49	$Nt_4t_1t_5t_1$
39	$Nt_5t_6t_1$	37	$Nt_3t_4t_2$	40	$Nt_6t_2t_3$	39	$Nt_5t_6t_1$	16	$Nt_5t_6t_1t_1$
40	$Nt_6t_2t_3$	38	$Nt_4t_1t_5$	36	$Nt_2t_5t_4$	36	$Nt_2t_5t_4$	35	$Nt_6t_2t_3t_1$
41	$Nt_1t_5t_2$	41	$Nt_2t_3t_1$	42	$Nt_3t_6t_5$	43	$Nt_4t_2t_6$	22	$Nt_1t_5t_2t_1$
42	$Nt_3t_6t_5$	42	$Nt_5t_4t_3$	43	$Nt_4t_2t_6$	42	$Nt_3t_6t_5$	42	$Nt_3t_6t_5t_1$
43	$Nt_4t_2t_6$	43	$Nt_6t_1t_4$	41	$Nt_1t_5t_2$	41	$Nt_1t_5t_2$	23	$Nt_4t_2t_6t_1$
44	$Nt_1t_5t_4$	45	$Nt_2t_3t_6$	46	$Nt_3t_6t_1$	46	$Nt_3t_6t_1$	50	$Nt_1t_5t_4t_1$
45	$Nt_2t_3t_6$	44	$Nt_1t_5t_4$	47	$Nt_5t_4t_2$	47	$Nt_5t_4t_2$	27	$Nt_2t_3t_6t_1$
46	$Nt_3t_6t_1$	47	$Nt_5t_4t_2$	48	$Nt_4t_2t_3$	44	$Nt_1t_5t_4$	24	$Nt_3t_6t_1t_1$
47	$Nt_5t_4t_2$	46	$Nt_3t_6t_1$	49	$Nt_6t_1t_5$	45	$Nt_2t_3t_6$	52	$Nt_5t_4t_2t_1$
48	$Nt_4t_2t_3$	49	$Nt_6t_1t_5$	44	$Nt_1t_5t_4$	48	$Nt_4t_2t_3$	36	$Nt_4t_2t_3t_1$
49	$Nt_6t_1t_5$	48	$Nt_4t_2t_3$	45	$Nt_2t_3t_6$	49	$Nt_6t_1t_5$	38	$Nt_6t_1t_5t_1$
50	$Nt_1t_5t_6$	51	$Nt_2t_3t_4$	52	$Nt_3t_6t_2$	53	$Nt_2t_4t_3$	44	$Nt_1t_5t_6t_1$
51	$Nt_2t_3t_4$	50	$Nt_1t_5t_6$	53	$Nt_5t_4t_1$	52	$Nt_3t_6t_2$	31	$Nt_2t_3t_4t_1$
52	$Nt_3t_6t_2$	53	$Nt_5t_4t_1$	54	$Nt_4t_2t_5$	51	$Nt_2t_3t_4$	47	$Nt_3t_6t_2t_1$
53	$Nt_2t_4t_3$	52	$Nt_1t_6t_5$	55	$Nt_5t_1t_4$	50	$Nt_1t_5t_6$	28	$Nt_2t_4t_3t_1$
54	$Nt_4t_2t_5$	55	$Nt_6t_1t_3$	50	$Nt_1t_5t_6$	55	$Nt_6t_1t_3$	33	$Nt_4t_2t_5t_1$
55	$Nt_6t_1t_3$	54	$Nt_4t_2t_5$	51	$Nt_2t_3t_4$	54	$Nt_4t_2t_5$	55	$Nt_6t_1t_3t_1$

Thus,

$f(x) = (2,3)(4,6)(5,7)(8,10)(9,11)(12,16)(13,17)(14,18)(15,19)(20,22)(21,23)(24,28)$ $(25,29)(26,30)(27,31)(35,36)(37,39)(38,40)(44,45)(46,47)(48,49)(50,51)(52,53)(54,55)$, $f(y) = (2,4,5)(3,6,7)(8,12,14)(9,13,15)(10,16,18)(17,19,11)(20,24,26)(21,25,27)(22,$ $28,30)(23,29,31)(32,33,34)(35,37,38)(36,39,40)(41,42,43)(44,46,48)(45,47,49)(50,52,$ $54)(53,55,51)$, $f(z) = (2,3)(4,7)(5,6)(8,11)(9,10)(12,19)(13,18)(14,17)(15,16)(20,23)(21,22)$ $(24,31)(25,30)(26,29)(27,28)(32,34)(35,38)(36,40)(41,43)(44,46)(45,47)(50,53)(51,52)(54,55)$, and $f(t) = (1,2)(4,13)(5,14)(6,29)(7,30)(10,32)(11,34)(12,15)(16,39)(17,25)(18,26)(19,37)$ $(22,41)(23,43)(24,46)(27,45)(28,53)(31,51)(33,54)(35,40)(36,48)(38,49)(44,50)(47,52)$.

Now $< f_x, f_y, f_z, f_t > \leq S_{55}$. $f : G \longrightarrow S_{55}$ is a homomorphism, since G acts on $X = \{N, Nt_1, Nt_2, ...Nt_6t_1t_3\}$ with $|X| = 55$. $< f_x, f_y, f_x, f_t >$ is a homomorphic image of the progenitor if

(1) f_t has exactly 6 conjugates under conjugation by $< f_x, f_y, f_z >$ and

(2) $< f_x, f_y, f_z >$ acts on $\{f_{t_1}, f_{t_2}, f_{t_3}, f_{t_4}, f_{t_5}, f_{t_6}\}$ by conjugation as S_{55}.

In addition, if the additional relations $(xt)^3 = (zt^y)^5 = (yt)^5 = 1$ hold in S_{55}, then $< f_x, f_y, f_z, f_t >$ is a homomorphic image of G.

Note that $| < f_x, f_y, f_z, f_t > | = 660$.

$G/Ker_f \cong Im_f$

$\Longrightarrow G/Ker_f \cong < f_x, f_y, f_z, f_t >$

$\Longrightarrow |G/Ker_f| = | < f_x, f_y, f_z, f_t > | = 660$

$\Longrightarrow |G| = | < f_x, f_y, f_z, f_t > | \times Ker_f$

$\Longrightarrow |G| \geq 660$

But, from our Cayley Diagram we saw that $|G| \leq 660$.

Hence, $|G| = 660$.

5.3 $PGL_2(11)$ as a Homomorphic Image of $11^{*2} :_m D_{10}$

5.3.1 The Construction of $PGL_2(11)$ Over D_{10}

Let $G \cong 11^{*2} :_m (D_{10})$ be a symmetric presentation of G given by:
$< x, y, t | x^{10}, y^2, (x^{-1}y)^2, t^{11}, t^{x^{-1}} = t^6, (x^{5t})^2, (yt)^3 > \cong PGL(2, 11)$,
where $N \cong D_{10} =< x, y | x^{10}, y^2, (x^{-1}y)^2 >$,
$x = (1, 3, 7, 15, 9, 19, 17, 13, 5, 11)(2, 12, 6, 14, 18, 20, 10, 16, 8, 4)$, and
$y = (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16)(17, 18)(19, 20)$.

Definition of a double coset: $NwN = \{Nwn | n \in N\}$.
Note: $wn = nn^{-1}wn = nw^n$.
So, $Nwn = \{Nw^n | n \in N\}$.

First we will expand our additional relation.

$$\begin{aligned}(yt)^3 &= e\\ (yt_1)^3 &= e\\ y^3(t_1)^{y^2}(t_1)^y t_1 &= e\\ yt_1t_2t_1 &= e\\ yt_1t_2t_1\underline{t_1^{10}} &= \underline{t_1^{10}}\\ yt_1t_2 &= t_1^{10}\\ yt_1t_2 &= t_{19}\end{aligned} \tag{5.5}$$

Labeling:

$t_1 = t_1$	$t_6 = t_2^3$	$t_{11} = t_1^6$	$t_{16} = t_2^8$
$t_2 = t_2$	$t_7 = t_1^4$	$t_{12} = t_2^6$	$t_{17} = t_1^9$
$t_3 = t_1^2$	$t_8 = t_2^4$	$t_{13} = t_1^7$	$t_{18} = t_2^9$
$t_4 = t_2^2$	$t_9 = t_1^5$	$t_{14} = t_2^7$	$t_{19} = t_1^{10}$
$t_5 = t_1^3$	$t_{10} = t_2^5$	$t_{15} = t_1^8$	$t_{20} = t_2^{10}$

Our first double coset, $NeN = \{Ne^n | n \in N\} = \{N\}$, which we will denote by $[*]$. The orbit of N on {1,2,3,4,5,6,8,7,9,10,11,12,13,14,15,16,17,18,19,20} is {1,2,3,4,5,6,8,7,9, 10,11,12,13,14,15,16,17,18,19,20}. We will take a representative from this orbit, say t_1, and determine to which double coset Nt_1 belongs.

Word of Length 1

Nt_1N is a new double coset which we will denote by [1]. $Nt_1N = \{Nt_1{}^n | n \in N\}$. Since the orbit $\{1, 3, 2, 7, 4, 12, 15, 8, 6, 11, 9, 16, 14, 5, 19, 10, 18, 13, 17, 20\}$ contains 20 elements then 20 symmetric generators will go to the new double coset [1].
Now $N^{(1)} \geq N^1$.
$N^1 = \{e\}$.
$N^{(1)} =$ Coset Stabiliser in N of $Nt_1 = \{n \in N | Nt_1^n = t_1\}$.

We do not have a relation that will increase the Coset Stabiliser $N^{(1)}$.

Now, since $(Nt_1)^e = Nt_1 \Rightarrow e \in N^{(1)}$, then,
$N^1 = N^{(1)} =$ Coset Stabiliser in N of $Nt_1 = \{n \in N | Nt_1^n = t_1\} = \{e\}$.
Furthermore, the number of single cosets in Nt_1N is $\frac{|N|}{|N^{(1)}|} = \frac{20}{1} = 20$.

Therefore, $Nt_1N = \{Nt_1, Nt_2, Nt_3, Nt_4, Nt_5, Nt_6, Nt_7, Nt_8, Nt_9, Nt_{10}, Nt_{11}, Nt_{12}, Nt_{13}, Nt_{14}, Nt_{15}, Nt_{16}, Nt_{17}, Nt_{18}, Nt_{19}, Nt_{20}\}$

The orbits of $N^{(1)}$ on {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20} are {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}, {13}, {14}, {15}, {16}, {17}, {18}, {19}, and {20}.

We want to see to which double coset $Nt_1t_1, Nt_1t_2, Nt_1t_3, Nt_1t_4, Nt_1t_5, Nt_1t_6, Nt_1t_7, Nt_1t_8, Nt_1t_9, Nt_1t_{10}, Nt_1t_{11}, Nt_1t_{12}, Nt_1t_{13}, Nt_1t_{14}, Nt_1t_{15}, Nt_1t_{16}, Nt_1t_{17}, Nt_1t_{18}, Nt_1t_{19}$, and Nt_1t_{20} belong.

$Nt_1t_1 = Nt_1^2 = Nt_3 \in [1]$.
One symmetric generator will go to [1].

$Nt_1t_2 = Nt_1^{10} = Nt_{19} \in [1]$, by Equation 5.5.
One symmetric generator will go to [1].

$Nt_1t_3 = Nt_1t_1^2 = Nt_1^3 = Nt_5 \in [1]$.
One symmetric generator will go to [1].

Nt_1t_4N is a new double coset which we will denote by [1 4].
One symmetric generator will go to [1 4].

$Nt_1t_5 = Nt_1t_1^3 = Nt_1^4 = Nt_7 \in [1]$.
One symmetric generator will go to [1].

Nt_1t_6N is a new double coset which we will denote by [1 6].
One symmetric generator will go to [1 6].

$Nt_1t_7 = Nt_1t_1^4 = Nt_1^5 = Nt_9 \in [1]$.
One symmetric generator will go to [1].

Nt_1t_8N is a new double coset which we will denote by [1 8].
One symmetric generator will go to [1 8].

$Nt_1t_9 = Nt_1t_1^5 = Nt_1^6 = Nt_{11} \in [1]$.
One symmetric generator will go to [1].

$Nt_1t_{10}N$ is a new double coset which we will denote by [1 10].
One symmetric generator will go to [1 10].

$Nt_1t_{11} = Nt_1t_1^6 = Nt_1^7 = Nt_{13} \in [1]$.
One symmetric generator will go to [1].

$Nt_1t_{12}N$ is a new double coset which we will denote by [1 12].
One symmetric generator will go to [1 12].

$Nt_1t_{13} = Nt_1t_1^7 = Nt_1^8 = Nt_{15} \in [1]$.
One symmetric generator will go to [1].

$Nt_1t_{14} = Nt_1t_{14}$
$\Longrightarrow Nt_1t_{14} = Nt_1\underline{t_2^7}$
$\Longrightarrow Nt_1t_{14} = Nt_1[x^4yt_2^4t_1^3]$, by Equation 5.5.
$\Longrightarrow Nt_1t_{14} = Nx^4y[t_1]^{x^4y}t_2^4t_1^3$
$\Longrightarrow Nt_1t_{14} = Nt_2^5t_2^4t_1^3$
$\Longrightarrow Nt_1t_{14} = Nt_2^9t_1^3$
$\Longrightarrow Nt_1t_{14} = Nt_{18}t_5$
Also,
$Nt_1t_{10} \in [110]$
$\Longrightarrow N[t_1t_{10}]^{yx^4} \in [110]$

$\Longrightarrow Nt_{18}t_5 \in [110]$.

Thus, $Nt_1t_{14} \in [110]$.

One symmetric generator will go to [1 10].

$Nt_1t_{15} = Nt_1t_1^8 = Nt_1^9 = Nt_{17} \in [1]$.

One symmetric generator will go to [1].

$Nt_1t_{16} = Nt_1t_{16}$

$\Longrightarrow Nt_1t_{16} = N\underline{t_1}t_2^8$

$\Longrightarrow Nt_1t_{16} = N[yt_1^{10}t_2^{10}]t_2^8$, by Equation 5.5.

$\Longrightarrow Nt_1t_{16} = Nt_1^{10}t_2^7$

$\Longrightarrow Nt_1t_{16} = Nt_{19}t_{14}$

Also,

$Nt_1t_8 \in [18]$

$\Longrightarrow N[t_1t_8]^{x^5} \in [18]$

$\Longrightarrow Nt_{19}t_{14} \in [18]$.

Thus, $Nt_1t_{16} \in [18]$.

One symmetric generator will go to [1 8].

$Nt_1t_{17} = Nt_1t_1^9 = Nt_1^{10} = Nt_{19} \in [1]$.

One symmetric generator will go to [1].

$Nt_1t_{18} = Nt_1t_{18}$

$\Longrightarrow Nt_1t_{18} = N\underline{t_1}t_2^9$

$\Longrightarrow Nt_1t_{18} = N[yt_1^{10}t_2^{10}]t_2^9$, by Equation 5.5.

$\Longrightarrow Nt_1t_{18} = Nt_1^{10}t_2^8$

$\Longrightarrow Nt_1t_{18} = Nt_{19}t_{16}$

Also,

$Nt_1t_6 \in [16]$

$\Longrightarrow N[t_1t_6]^{x^5} \in [16]$

$\Longrightarrow Nt_{19}t_{16} \in [16]$.

Thus, $Nt_1t_{18} \in [16]$.

One symmetric generator will go to [1 6].

$Nt_1t_{19} = Nt_1t_1^{10} = N \in [*]$.
One symmetric generator will go to [*].

$Nt_1t_{20} = Nt_1t_{20}$
$\Longrightarrow Nt_1t_{20} = N\underline{t_1}t_2^{10}$
$\Longrightarrow Nt_1t_{20} = N[yt_1^{10}t_2^{10}]t_2^{10}$, by Equation 5.5.
$\Longrightarrow Nt_1t_{20} = Nt_1^{10}t_2^{9}$
$\Longrightarrow Nt_1t_{20} = Nt_{19}t_{18}$
Also,
$Nt_1t_4 \in [14]$
$\Longrightarrow N[t_1t_4]^{x^5} \in [14]$
$\Longrightarrow Nt_{19}t_{18} \in [16]$.
Thus, $Nt_1t_{20} \in [14]$.
One symmetric generator will go to [1 4].

Word of Length 2

$N^{(14)}$ = Coset Stabiliser in N of $Nt_1t_4 = \{n \in N | (Nt_1t_4)^n = t_1t_4\}$. We will look for a relation that will increase the Coset Stabiliser $N^{(14)}$.
$Nt_1t_2 = Nt_{19}$, by Equation 5.5
$\Longrightarrow Nt_1t_2 = Nt_1^{10}$
$\Longrightarrow Nt_1t_2\underline{t_2} = Nt_1^{10}\underline{t_2}$
$\Longrightarrow Nt_1t_2^2 = Nt_1^{10}t_2$
$\Longrightarrow Nt_1t_4 = Nt_{19}t_2$

Since, $(Nt_1t_4)^e = Nt_1t_4 \Rightarrow e \in N^{(14)}$, and
$(Nt_1t_4)^{x^5} = Nt_{19}t_2 = Nt_1t_4 \Rightarrow x^5 \in N^{(14)}$, then,
$N^{(14)}$ = Coset Stabiliser in N of $Nt_1t_4 = \{n \in N | (Nt_1t_4)^n = t_1t_4\} = \{e, x^5\}$.
Furthermore, the number of single cosets in Nt_1t_4N is $\frac{|N|}{|N^{(14)}|} = \frac{20}{2} = 10$.

We find the equal names by conjugating $t_1t_4 \sim t_{19}t_2$ by elements of N.

$t_1t_4 \sim t_{18}t_{19}$	$t_5t_{16} \sim t_6t_{15}$	$t_8t_{11} \sim t_9t_{14}$
$t_4t_1 \sim t_{19}t_{18}$	$t_{16}t_5 \sim t_{15}t_6$	$t_{11}t_8 \sim t_{14}t_9$
$t_2t_3 \sim t_{17}t_{20}$	$t_7t_{12} \sim t_{10}t_{13}$	
$t_3t_2 \sim t_{20}t_{17}$	$t_{12}t_7 \sim t_{13}t_{10}$	

$N^{(16)}$ = Coset Stabiliser in N of $Nt_1t_6 = \{n \in N|(Nt_1t_6)^n = t_1t_6\}$. We will look for a relation that will increase the Coset Stabiliser $N^{(16)}$.

$Nt_1t_6 = Nt_1t_6$

$\Longrightarrow Nt_1t_6 = Nt_1\underline{t_2^3}$

$\Longrightarrow Nt_1t_6 = Nt_1[yx^4t_2^8t_1^7]$, by Equation 5.5

$\Longrightarrow Nt_1t_6 = Nyx^4[t_1]^{yx^4}t_2^8t_1^7$

$\Longrightarrow Nt_1t_6 = Nt_2^9t_2^8t_1^7$

$\Longrightarrow Nt_1t_6 = N\underline{t_2^6}t_1^7$

$\Longrightarrow Nt_1t_6 = N[yx^2t_2^5t_1^9]t_1^7$, by Equation 5.5

$\Longrightarrow Nt_1t_6 = Nt_2^5t_1^5$

$\Longrightarrow Nt_1t_6 = Nt_{10}t_9$

Since, $(Nt_1t_6)^e = Nt_1t_6 \Rightarrow e \in N^{(16)}$, and

$(Nt_1t_6)^{x^4y} = Nt_{10}t_9 = Nt_1t_6 \Rightarrow x^4y \in N^{(16)}$, then,

$N^{(16)}$ = Coset Stabiliser in N of $Nt_1t_6 = \{n \in N|(Nt_1t_6)^n = t_1t_6\} = \{e, x^4y\}$.

Furthermore, the number of single cosets in Nt_1t_6N is $\frac{|N|}{|N^{(16)}|} = \frac{20}{2} = 10$.

We find the equal names by conjugating $t_1t_6 \sim t_{10}t_9$ by elements of N.

$t_1t_6 \sim t_{10}t_9$	$t_3t_{14} \sim t_{16}t_{19}$	$t_{11}t_{12} \sim t_{20}t_{15}$
$t_6t_1 \sim t_{17}t_8$	$t_{14}t_3 \sim t_{13}t_4$	$t_{12}t_{11} \sim t_{19}t_{16}$
$t_2t_5 \sim t_9t_{10}$	$t_7t_{18} \sim t_8t_{17}$	
$t_5t_2 \sim t_{18}t_7$	$t_{18}t_7 \sim t_5t_2$	

$N^{(18)}$ = Coset Stabiliser in N of $Nt_1t_8 = \{n \in N|(Nt_1t_8)^n = t_1t_8\}$.

We will look for a relation that will increase the Coset Stabiliser $N^{(18)}$.

$Nt_1t_2 = Nt_{19}$, by Equation 5.5

$\Longrightarrow Nt_1t_2 = Nt_1^{10}$

$\Longrightarrow Nt_1t_2\underline{t_2^3} = Nt_1^{10}\underline{t_2^3}$

$\Longrightarrow Nt_1t_2^4 = Nt_1^{10}\underline{t_2^3}$

$\Longrightarrow Nt_1t_2^4 = Nt_1^{10}[yx^4t_2^8t_1^7]$, by Equation 5.5

$\Longrightarrow Nt_1t_2^4 = Nyx^4[t_1^{10}]^{yx^4}t_2^8t_1^7$

$\Longrightarrow Nt_1t_2^4 = Nt_2^2t_2^8t_1^7$

$\Longrightarrow Nt_1t_2^4 = Nt_2^{10}t_1^7$

$\Longrightarrow Nt_1t_8 = Nt_{20}t_{13}$

Since, $(Nt_1t_8)^e = Nt_1t_8 \Rightarrow e \in N^{(18)}$, and
$(Nt_1t_8)^{x^5y} = Nt_{20}t_{13} = Nt_1t_8 \Rightarrow x^5y \in N^{(18)}$, then,
$N^{(18)}$ = Coset Stabiliser in N of $Nt_1t_8 = \{n \in N|(Nt_1t_8)^n = t_1t_8\} = \{e, x^5y\}$.
Furthermore, the number of single cosets in Nt_1t_8N is $\frac{|N|}{|N^{(18)}|} = \frac{20}{2} = 10$.

We find the equal names by conjugating $t_1t_8 \sim t_{20}t_{13}$ by elements of N.

$t_1t_8 \sim t_{20}t_{13}$	$t_3t_4 \sim t_{10}t_5$	$t_{14}t_{19} \sim t_5t_{10}$
$t_8t_1 \sim t_{15}t_{12}$	$t_4t_3 \sim t_9t_6$	$t_6t_9 \sim t_{13}t_{20}$
$t_2t_7 \sim t_{19}t_{14}$	$t_{17}t_{18} \sim t_{12}t_{15}$	
$t_7t_2 \sim t_{16}t_{11}$	$t_{18}t_{17} \sim t_{11}t_{16}$	

$N^{(110)}$ = Coset Stabiliser in N of $Nt_1t_10 = \{n \in N|(Nt_1t_10)^n = t_1t_10\}$.

We will look for a relation that will increase the Coset Stabiliser $N^{(110)}$.
$Nt_1t_2 = Nt_{19}$, by Equation 5.5

$\Longrightarrow Nt_1t_2 = Nt_1^{10}$

$\Longrightarrow Nt_1t_2\underline{t_2^4} = Nt_1^{10}\underline{t_2^4}$

$\Longrightarrow Nt_1t_2^5 = Nt_1^{10}\underline{t_2^4}$

$\Longrightarrow Nt_1t_2^5 = Nt_1^{10}[x^4yt_2^7t_1^8]$, by Equation 5.5

$\Longrightarrow Nt_1t_2^5 = Nx^4y[t_1^{10}]^{x^4y}t_2^7t_1^8$

$\Longrightarrow Nt_1t_2^5 = Nt_2^6t_2^7t_1^8$

$\Longrightarrow Nt_1t_2^5 = Nt_2^2t_1^8$

$\Longrightarrow Nt_1t_{10} = Nt_4t_{15}$

Since, $(Nt_1t_{10})^e = Nt_1t_{10} \Rightarrow e \in N^{(110)}$, and
$(Nt_1t_{10})^{xy} = Nt_4t_{15} = Nt_1t_{10} \Rightarrow xy \in N^{(110)}$, then,
$N^{(110)} =$ Coset Stabiliser in N of $Nt_1t_{10} = \{n \in N|(Nt_1t_{10})^n = t_1t_{10}\} = \{e, xy\}$.
Furthermore, the number of single cosets in $Nt_1t_{10}N$ is $\frac{|N|}{|N^{(110)}|} = \frac{20}{2} = 10$.

We find the equal names by conjugating $t_1t_{10} \sim t_4t_{15}$ by elements of N.

$t_1t_{10} \sim t_4t_{15}$	$t_5t_{18} \sim t_{16}t_3$	$t_7t_8 \sim t_{12}t_{19}$
$t_{10}t_1 \sim t_{13}t_{14}$	$t_{18}t_5 \sim t_{19}t_{12}$	$t_8t_7 \sim t_{11}t_{20}$
$t_2t_9 \sim t_3t_{16}$	$t_6t_{17} \sim t_{15}t_4$	
$t_9t_2 \sim t_{14}t_{13}$	$t_{17}t_6 \sim t_{20}t_{11}$	

$N^{(112)} =$ Coset Stabiliser in N of $Nt_1t_{12} = \{n \in N|(Nt_1t_{12})^n = t_1t_{12}\}$.

We will look for a relation that will increase the Coset Stabiliser $N^{(112)}$.
$Nt_1t_2 = Nt_{19}$, by Equation 5.5
$\Longrightarrow Nt_1t_2 = Nt_1^{10}$
$\Longrightarrow Nt_1t_2\underline{t_2^5} = Nt_1^{10}\underline{t_2^5}$
$\Longrightarrow Nt_1t_2^6 = Nt_1^{10}\underline{t_2^5}$
$\Longrightarrow Nt_1t_{12} = Nt_{19}t_{10}$
Also,
$Nt_1t_{12} = Nt_{19}t_{10}$
$\Longrightarrow Nt_1t_{12} = Nt_1^{10}\underline{t_2^5}$
$\Longrightarrow Nt_1t_{12} = Nt_1^{10}[yx^2t_2^6t_1^2]$, by Equation 5.5
$\Longrightarrow Nt_1t_{12} = Nyx^2[t_1^{10}]^{yx^2}t_2^6t_1^2$
$\Longrightarrow Nt_1t_{12} = Nt_2^8t_2^6t_1^2$
$\Longrightarrow Nt_1t_{12} = Nt_2^3t_1^2$
$\Longrightarrow Nt_1t_{12} = Nt_6t_3$
Also,
$Nt_1t_{12} = Nt_6t_3$

$\Longrightarrow Nt_1t_{12} = N\underline{t_2^3}t_1^2$

$\Longrightarrow Nt_1t_{12} = N[yx^4t_2^8t_1^7]t_1^2$, by Equation 5.5

$\Longrightarrow Nt_1t_{12} = Nt_2^8t_1^9$

$\Longrightarrow Nt_1t_{12} = Nt_{16}t_{17}$

Thus

$Nt_1t_{12} = Nt_{19}t_{10} = Nt_6t_3 = Nt_{16}t_{17}$

Since, $(Nt_1t_{12})^e = Nt_1t_{12} \Rightarrow e \in N^{(112)}$, and
$(Nt_1t_{12})^{x^5} = Nt_{19}t_{10} = Nt_1t_{12} \Rightarrow x^5 \in N^{(112)}$, and
$(Nt_1t_{12})^{yx^2} = Nt_6t_3 = Nt_1t_{12} \Rightarrow yx^2 \in N^{(112)}$, and
$(Nt_1t_{12})^{x^3y} = Nt_{16}t_{17} = Nt_1t_{12} \Rightarrow x^3y \in N^{(112)}$, then,
$N^{(112)} =$ Coset Stabiliser in N of $Nt_1t_{12} = \{n \in N | (Nt_1t_{12})^n = t_1t_{12}\} = \{e, x^5, yx^2, x^3y\}$.
Furthermore, the number of single cosets in $Nt_1t_{12}N$ is $\frac{|N|}{|N^{(112)}|} = \frac{20}{4} = 5$.

We find the equal names by conjugating $t_1t_{12} \sim t_{19}t_{10} \sim t_6t_3 \sim t_{16}t_{17}$ by elements of N.

$t_1t_{12} \sim t_{19}t_{10} \sim t_6t_3 \sim t_{16}t_{17}$

$t_3t_6 \sim t_{17}t_{16} \sim t_{17}t_7 \sim t_8t_{13}$

$t_2t_{11} \sim t_{20}t_9 \sim t_5t_4 \sim t_{15}t_{18}$

$t_{11}t_4 \sim t_9t_{20} \sim t_{12}t_1 \sim t_{10}t_{19}$

$t_7t_6 \sim t_{13}t_8 \sim t_{18}t_{15} \sim t_4t_5$

The orbits of $N^{(14)}$ on {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20} are $\{1, 18\}$, $\{2, 9\}$, $\{3, 14\}$, $\{4, 19\}$, $\{5, 10\}$, $\{6, 7\}$, $\{8, 17\}$, $\{11, 20\}$, $\{12, 15\}$, and $\{13, 16\}$.

We want to see to which double coset $Nt_1t_4t_{18}, Nt_1t_4t_2, Nt_1t_4t_{14}, Nt_1t_4t_4, Nt_1t_4t_{10}, Nt_1t_4t_6,$ $Nt_1t_4t_8, Nt_1t_4t_{20}, Nt_1t_4t_{12}$, and $Nt_1t_4t_{16}$ belong.

$Nt_1t_4t_{18} = Nt_1t_2^2t_2^9$

$\Longrightarrow Nt_1t_4t_{18} = Nt_1 \in [1]$.

Two symmetric generators will go to [1].

$Nt_1t_4t_2 = Nt_1t_2^2t_2$
$\Longrightarrow Nt_1t_4t_2 = Nt_1t_2^3$
$\Longrightarrow Nt_1t_4t_2 = Nt_1t_6 \in [16]$.
Two symmetric generators will go to [1 6].

$Nt_1t_4t_{14} = Nt_1t_2^2t_2^7$
$\Longrightarrow Nt_1t_4t_{14} = Nt_1t_2^9$
$\Longrightarrow Nt_1t_4t_{14} = Nt_1t_{18} \in [14]$.
Two symmetric generators will go to [1 4].

$Nt_1t_4t_4 = Nt_1t_2^2t_2^2$
$\Longrightarrow Nt_1t_4t_4 = Nt_1t_2^4$
$\Longrightarrow Nt_1t_4t_4 = Nt_1t_8 \in [18]$.
Two symmetric generators will go to [1 8].

$Nt_1t_4t_{10} = Nt_1t_2^2t_2^5$
$\Longrightarrow Nt_1t_4t_{10} = N\underline{t_1}t_2^7$
$\Longrightarrow Nt_1t_4t_{10} = N[yt_1^{10}t_2^{10}]t_2^7$, by Equation 5.5
$\Longrightarrow Nt_1t_4t_{10} = Nt_1^{10}t_2^6$
$\Longrightarrow Nt_1t_4t_{10} = Nt_{19}t_{12} \in [110]$.
Two symmetric generators will go to [1 10].

$Nt_1t_4t_6 = Nt_1t_2^2t_2^3$
$\Longrightarrow Nt_1t_4t_6 = Nt_1t_2^5$
$\Longrightarrow Nt_1t_4t_6 = Nt_1t_{10} \in [110]$.
Two symmetric generators will go to [1 10].

$Nt_1t_4t_8 = Nt_1t_2^2t_2^4$
$\Longrightarrow Nt_1t_4t_8 = Nt_1t_2^6$
$\Longrightarrow Nt_1t_4t_8 = Nt_1t_{12} \in [112]$.

Two symmetric generators will go to [1 12].

$Nt_1t_4t_{20} = Nt_1t_2^2t_2^{10}$
$\Longrightarrow Nt_1t_4t_{20} = N\underline{t_1t_2}$.
$\Longrightarrow Nt_1t_4t_{20} = Nt_{19}$, by Equation 5.5
$\Longrightarrow Nt_1t_4t_{20} = Nt_{19} \in [1]$.
Two symmetric generators will go to [1]
$Nt_1t_4t_{12} = Nt_1t_2^2t_2^6$
$\Longrightarrow Nt_1t_4t_{12} = N\underline{t_1}t_2^8$
$\Longrightarrow Nt_1t_4t_{12} = N[yt_1^{10}t_2 10]t_2^8$, by Equation 5.5
$\Longrightarrow Nt_1t_4t_{12} = Nt_1^{10}t_2^7$
$\Longrightarrow Nt_1t_4t_{12} = Nt_{19}t_{14} \in [18]$.
Two symmetric generators will go to [1 8].

$Nt_1t_4t_{16} = Nt_1t_2^2t_2^8$
$\Longrightarrow Nt_1t_4t_{16} = Nt_1\underline{t_2^{10}}$
$\Longrightarrow Nt_1t_4t_{16} = N\underline{t_1t_2}t_2^9$
$\Longrightarrow Nt_1t_4t_{16} = Nt_{19}t_2^9$, by Equation 5.5
$\Longrightarrow Nt_1t_4t_{16} = Nt_{19}t_{18} \in [14]$.
Two symmetric generators will go to [1 4].

The orbits of $N^{(16)}$ on {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20} are $\{1,10\}$, $\{2,17\}$, $\{3,20\}$, $\{4,13\}$, $\{5,8\}$, $\{6,9\}$, $\{7,18\}$, $\{11,16\}$, $\{12,19\}$, and $\{14,15\}$.

We want to see to which double coset $Nt_1t_6t_{10}, Nt_1t_6t_2, Nt_1t_6t_3, Nt_1t_6t_4, Nt_1t_6t_8, Nt_1t_6t_6, Nt_1t_6t_{18}, Nt_1t_6t_{16}, Nt_1t_6t_{12}$, and $Nt_1t_6t_{14}$ belong.

$Nt_1t_6t_{10} = N\underline{t_1}t_2^3t_2^5$
$\Longrightarrow Nt_1t_6t_{10} = N[yt_1^{10}t_2^{10}]t_2^8$, by Equation 5.5
$\Longrightarrow Nt_1t_6t_{10} = Nt_1^{10}t_2^7$
$\Longrightarrow Nt_1t_6t_{10} = Nt_{19}t_{14} \in [18]$.

Two symmetric generators will go to [1 8].

$Nt_1t_6t_2 = Nt_1t_2^3t_2$
$\implies Nt_1t_6t_2 = Nt_1t_2^4$
$\implies Nt_1t_6t_2 = Nt_1t_8 \in [18]$.
Two symmetric generators will go to [1 8].

$Nt_1t_6t_3 = Nt_1\underline{t_2^3}t_1^2$
$\implies Nt_1t_6t_3 = Nt_1[yx^4t_2^8t_1^7]t_1^2$, by Equation 5.5
$\implies Nt_1t_6t_3 = Nyx^4[t_1]^{yx^4}t_2^8t_1^9$
$\implies Nt_1t_6t_3 = Nt_2^9t_2^8t_1^9$
$\implies Nt_1t_6t_3 = N\underline{t_2^6}t_1^9$
$\implies Nt_1t_6t_3 = N[yx^2t_2^5t_1^9]t_1^9$, by Equation 5.5
$\implies Nt_1t_6t_3 = Nt_2^5t_1^7$
$\implies Nt_1t_6t_3 = Nt_{10}t_{13} \in [14]$.
Two symmetric generators will go to [1 4].

$Nt_1t_6t_4 = Nt_1t_2^3t_2^2$
$\implies Nt_1t_6t_4 = Nt_1t_2^5$
$\implies Nt_1t_6t_4 = Nt_1t_{10} \in [110]$.
Two symmetric generators will go to [1 10].

$Nt_1t_6t_8 = N\underline{t_1}t_2^3t_2^4$
$\implies Nt_1t_6t_8 = N[yt_1^{10}t_2^{10}]t_2^7$, by Equation 5.5
$\implies Nt_1t_6t_8 = Nt_1^{10}t_2^6$
$\implies Nt_1t_6t_8 = Nt_{19}t_{12} \in [110]$.
Two symmetric generators will go to [1 10].

$Nt_1t_6t_6 = Nt_1t_2^3t_2^3$
$\implies Nt_1t_6t_6 = Nt_1t_2^6$
$\implies Nt_1t_6t_6 = Nt_1t_{12} \in [112]$.
Two symmetric generators will go to [1 12].

$Nt_1t_6t_{18} = Nt_1t_2^3t_2^9$
$\Longrightarrow Nt_1t_6t_{18} = N\underline{t_1t_2}$
$\Longrightarrow Nt_1t_6t_{18} = Nyt_1^{10}$, by Equation 5.5
$\Longrightarrow Nt_1t_6t_{18} = Nt_{19} \in [1]$.
Two symmetric generators will go to [1].

$Nt_1t_6t_{16} = Nt_1t_2^3t_2^8$
$\Longrightarrow Nt_1t_6t_{16} = Nt_1 \in [1]$.
Two symmetric generators will go to [1].

$Nt_1t_6t_{12} = N\underline{t_1}t_2^3t_2^6$
$\Longrightarrow Nt_1t_6t_{12} = N[yt_1^{10}t_2^{10}]t_2^9$, by Equation 5.5
$\Longrightarrow Nt_1t_6t_{12} = Nt_1^{10}t_2^8$
$\Longrightarrow Nt_1t_6t_{12} = Nt_{19}t_{16} \in [16]$.
Two symmetric generators will go to [1 6].

$Nt_1t_6t_{14} = N\underline{t_1}t_2^3t_2^7$
$\Longrightarrow Nt_1t_6t_{14} = N[yt_1^{10}t_2^{10}]t_2^{10}$, by Equation 5.5
$\Longrightarrow Nt_1t_6t_{14} = Nt_1^{10}t_2^9$
$\Longrightarrow Nt_1t_6t_{14} = Nt_{19}t_{18} \in [14]$.
Two symmetric generators will go to [1 4].

The orbits of $N^{(18)}$ on {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20} are $\{1, 20\}$, $\{2, 19\}$, $\{3, 18\}$, $\{4, 17\}$, $\{5, 16\}$, $\{6, 15\}$, $\{7, 14\}$, $\{8, 13\}$, $\{9, 12\}$, and $\{10, 11\}$.

We want to see to which double coset $Nt_1t_8t_{20}, Nt_1t_8t_2, Nt_1t_8t_{18}, Nt_1t_8t_4, Nt_1t_8t_{16}$, $Nt_1t_8t_6, Nt_1t_8t_{14}, Nt_1t_8t_8, Nt_1t_8t_{12}$, and $Nt_1t_8t_{10}$ belong.

$Nt_1t_8t_{20} = Nt_1t_2^4t_2^{10}$
$\Longrightarrow Nt_1t_8t_{20} = Nt_1t_2^3$

$\Longrightarrow Nt_1t_8t_{20} = Nt_1t_6 \in [16]$.

Two symmetric generators will go to [1 6].

$Nt_1t_8t_2 = Nt_1t_2^4t_2$

$\Longrightarrow Nt_1t_8t_2 = Nt_1t_2^5$

$\Longrightarrow Nt_1t_8t_2 = Nt_1t_{10} \in [110]$.

Two symmetric generators will go to [1 10].

$Nt_1t_8t_{18} = Nt_1t_2^4t_2^9$

$\Longrightarrow Nt_1t_8t_{18} = Nt_1t_2^2$

$\Longrightarrow Nt_1t_8t_{18} = Nt_1t_4 \in [14]$.

Two symmetric generators will go to [1 4].

$Nt_1t_8t_4 = Nt_1t_2^4t_2^2$

$\Longrightarrow Nt_1t_8t_4 = Nt_1t_2^6$

$\Longrightarrow Nt_1t_8t_4 = Nt_1t_{12} \in [112]$.

Two symmetric generators will go to [1 12].

$Nt_1t_8t_{16} = Nt_1t_2^4t_2^8$

$\Longrightarrow Nt_1t_8t_{16} = N\underline{t_1t_2}$

$\Longrightarrow Nt_1t_8t_{16} = Nyt_1^{10}$, by Equation 5.5

$\Longrightarrow Nt_1t_8t_{16} = Nt_{19} \in [1]$.

Two symmetric generators will go to [1].

$Nt_1t_8t_6 = Nt_1t_2^4t_2^3$

$\Longrightarrow Nt_1t_8t_6 = N\underline{t_1}t_2^7$

$\Longrightarrow Nt_1t_8t_6 = N[yt_1^{10}t_2^{10}]t_2^7$, by Equation 5.5

$\Longrightarrow Nt_1t_8t_6 = Nt_1^{10}t_2^6$

$\Longrightarrow Nt_1t_8t_6 = Nt_{19}t_{12} \in [110]$.

Two symmetric generators will go to [1 10].

$Nt_1t_8t_{14} = Nt_1t_2^4t_2^7$

$\Longrightarrow Nt_1t_8t_{14} = Nt_1 \in [1]$.

Two symmetric generators will go to [1].

$Nt_1t_8t_8 = Nt_1t_2^4t_2^4$

$\Longrightarrow Nt_1t_8t_8 = N\underline{t_1}t_2^8$

$\Longrightarrow Nt_1t_8t_8 = N[yt_1^{10}t_2^{10}]t_2^8$, by Equation 5.5

$\Longrightarrow Nt_1t_8t_8 = Nt_1^{10}t_2^7$

$\Longrightarrow Nt_1t_8t_8 = Nt_{19}t_{14} \in [18]$.

Two symmetric generators will go to [1 8].

$Nt_1t_8t_{12} = Nt_1t_2^4t_2^6$

$\Longrightarrow Nt_1t_8t_{12} = N\underline{t_1}t_2^{10}$

$\Longrightarrow Nt_1t_8t_{12} = N[yt_1^{10}t_2^{10}]t_2^{10}$, by Equation 5.5

$\Longrightarrow Nt_1t_8t_{12} = Nt_1^{10}t_2^9$

$\Longrightarrow Nt_1t_8t_{12} = Nt_{19}t_{18} \in [14]$.

Two symmetric generators will go to [1 4].

$Nt_1t_8t_{10} = Nt_1t_2^4t_2^5$

$\Longrightarrow Nt_1t_8t_{10} = N\underline{t_1}t_2^9$

$\Longrightarrow Nt_1t_8t_{10} = N[yt_1^{10}t_2^{10}]t_2^9$, by Equation 5.5

$\Longrightarrow Nt_1t_8t_{10} = Nt_1^{10}t_2^8$

$\Longrightarrow Nt_1t_8t_{10} = Nt_{19}t_{16} \in [16]$.

Two symmetric generators will go to [1 6].

The orbits of $N^{(110)}$ on {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20} are $\{1,4\}$, $\{2,11\}$, $\{3,8\}$, $\{5,12\}$, $\{6,13\}$, $\{7,16\}$, $\{9,20\}$, $\{10,15\}$, $\{14,17\}$, and $\{18,19\}$.

We want to see to which double coset $Nt_1t_{10}t_4, Nt_1t_{10}t_2, Nt_1t_{10}t_8, Nt_1t_{10}t_{12}, Nt_1t_{10}t_6$, $Nt_1t_{10}t_{16}, Nt_1t_{10}t_{20}, Nt_1t_{10}t_{10}, Nt_1t_{10}t_{14}$, and $Nt_1t_{10}t_{18}$ belong.

$Nt_1t_{10}t_4 = Nt_1t_2^5t_2^2$

$\Longrightarrow Nt_1t_{10}t_4 = N\underline{t_1}t_2^7$

$\Longrightarrow Nt_1t_{10}t_4 = N[yt_1^{10}t_2^{10}]t_2^7$, by Equation 5.5

$\Longrightarrow Nt_1t_{10}t_4 = Nt_1^{10}t_2^6$

$\Longrightarrow Nt_1t_{10}t_4 = Nt_{19}t_{12} \in [110]$.

Two symmetric generators will go to [1 10].

$Nt_1t_{10}t_2 = Nt_1t_2^5t_2$

$\Longrightarrow Nt_1t_{10}t_2 = Nt_1t_2^6$

$\Longrightarrow Nt_1t_{10}t_2 = Nt_1t_{12} \in [112]$.

Two symmetric generators will go to [1 12].

$Nt_1t_{10}t_8 = Nt_1t_2^5t_2^4$

$\Longrightarrow Nt_1t_{10}t_8 = N\underline{t_1}t_2^9$

$\Longrightarrow Nt_1t_{10}t_8 = N[yt_1^{10}t_2^{10}]t_2^9$, by Equation 5.5

$\Longrightarrow Nt_1t_{10}t_8 = Nt_1^{10}t_2^8$

$\Longrightarrow Nt_1t_{10}t_8 = Nt_{19}t_{16} \in [16]$.

Two symmetric generators will go to [1 6].

$Nt_1t_{10}t_{12} = Nt_1t_2^5t_2^6$

$\Longrightarrow Nt_1t_{10}t_{12} = Nt_1 \in [1]$.

Two symmetric generators will go to [1 12].

$Nt_1t_{10}t_6 = Nt_1t_2^5t_2^3$

$\Longrightarrow Nt_1t_{10}t_6 = N\underline{t_1}t_2^8$

$\Longrightarrow Nt_1t_{10}t_6 = N[yt_1^{10}t_2^{10}]t_2^8$, by Equation 5.5

$\Longrightarrow Nt_1t_{10}t_6 = Nt_1^{10}t_2^7$

$\Longrightarrow Nt_1t_{10}t_6 = Nt_{19}t_{14} \in [18]$.

Two symmetric generators will go to [1 8].

$Nt_1t_{10}t_{16} = Nt_1t_2^5t_2^8$

$\Longrightarrow Nt_1t_{10}t_{16} = Nt_1t_2^2$

$\Longrightarrow Nt_1t_{10}t_{16} = Nt_1t_4 \in [14]$.

Two symmetric generators will go to [1 4].

$Nt_1t_{10}t_{20} = Nt_1t_2^5t_2^{10}$
$\Longrightarrow Nt_1t_{10}t_{20} = Nt_1t_2^4$
$\Longrightarrow Nt_1t_{10}t_{20} = Nt_1t_8 \in [18]$.
Two symmetric generators will go to [1 8].

$Nt_1t_{10}t_{10} = Nt_1t_2^5t_2^5$
$\Longrightarrow Nt_1t_{10}t_{10} = N\underline{t_1t_2^{10}}$
$\Longrightarrow Nt_1t_{10}t_{10} = N[yt_1^{10}t_2^{10}]t_2^{10}$, by Equation 5.5
$\Longrightarrow Nt_1t_{10}t_{10} = Nt_1^{10}t_2^9$
$\Longrightarrow Nt_1t_{10}t_{10} = Nt_{19}t_{18} \in [14]$.
Two symmetric generators will go to [1 4].

$Nt_1t_{10}t_{14} = Nt_1t_2^5t_2^7$
$\Longrightarrow Nt_1t_{10}t_{14} = N\underline{t_1t_2}$
$\Longrightarrow Nt_1t_{10}t_{14} = Nyt_1^{10}$, by Equation 5.5
$\Longrightarrow Nt_1t_{10}t_{14} = Nt_{19} \in [1]$.
Two symmetric generators will go to [1].

$Nt_1t_{10}t_{18} = Nt_1t_2^5t_2^9$
$\Longrightarrow Nt_1t_{10}t_{18} = Nt_1t_2^3$
$\Longrightarrow Nt_1t_{10}t_{18} = Nt_1t_6 \in [16]$.
Two symmetric generators will go to [1 6].

The orbits of $N^{(112)}$ on {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20} are $\{1, 6, 16, 19\}$, $\{2, 7, 13, 20\}$, $\{3, 12, 10, 17\}$, $\{4, 15, 5, 18\}$, and $\{8, 9, 11, 14\}$.

We want to see to which double coset $Nt_1t_{12}t_{16}, Nt_1t_{12}t_{20}, Nt_1t_{12}t_{10}, Nt_1t_{12}t_{18}$, and $Nt_1t_{12}t_{14}$ belong.

$Nt_1t_{12}t_{16} = Nt_1t_2^6t_2^8$

$\Longrightarrow Nt_1t_{12}t_{16} = Nt_1t_2^3$

$\Longrightarrow Nt_1t_{12}t_{16} = Nt_1t_6 \in [16]$.

Two symmetric generators will go to [1 6].

$Nt_1t_{12}t_{20} = Nt_1t_2^6t_2^{10}$

$\Longrightarrow Nt_1t_{12}t_{20} = Nt_1t_2^5$

$\Longrightarrow Nt_1t_{12}t_{20} = Nt_1t_{10} \in [110]$.

Two symmetric generators will go to [1 10].

$Nt_1t_{12}t_{10} = Nt_1t_2^6t_2^5$

$\Longrightarrow Nt_1t_{12}t_{10} = Nt_1$

$\Longrightarrow Nt_1t_{12}t_{10} = Nt_1 \in [1]$.

Two symmetric generators will go to [1].

$Nt_1t_{12}t_{18} = Nt_1t_2^6t_2^9$

$\Longrightarrow Nt_1t_{12}t_{18} = Nt_1t_2^4$

$\Longrightarrow Nt_1t_{12}t_{18} = Nt_1t_8 \in [18]$.

Two symmetric generators will go to [1 8].

$Nt_1t_{12}t_{14} = Nt_1t_2^6t_2^7$

$\Longrightarrow Nt_1t_{12}t_{14} = Nt_1t_2^2$

$\Longrightarrow Nt_1t_{12}t_{14} = Nt_1t_4 \in [14]$.

Two symmetric generators will go to [1 4].

Below is our completed Cayley Diagram.

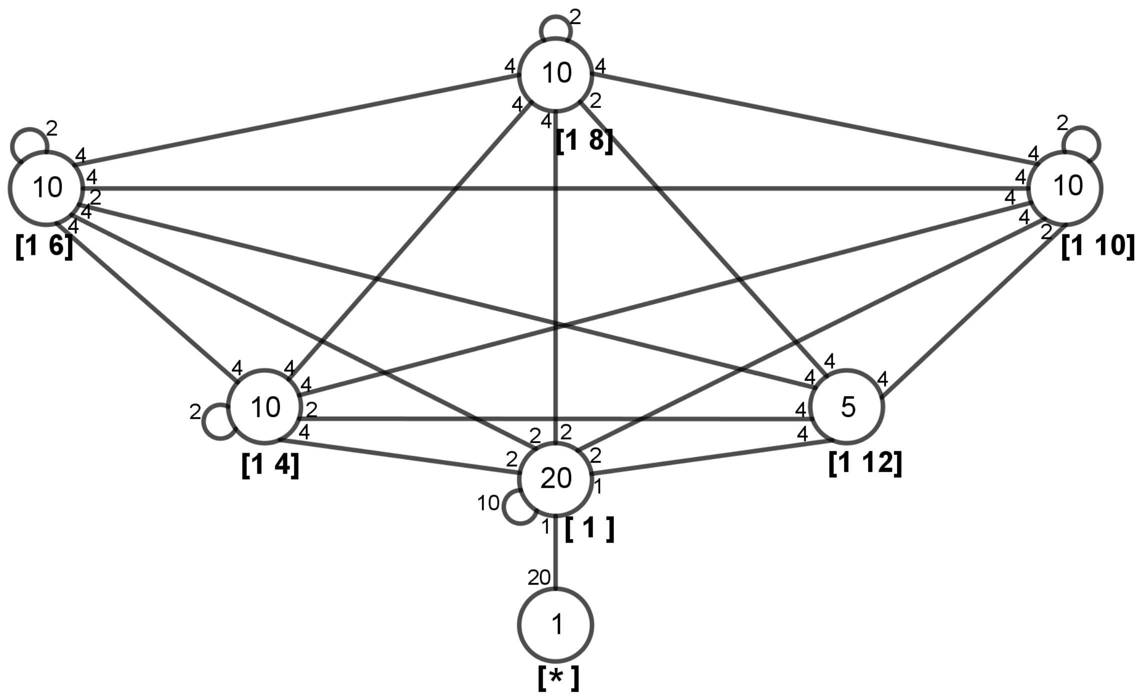


Figure 5.2: Cayley Diagram : $PGL(2,11)$ Over D_{10}

Thus,

$$
\begin{aligned}
|G| &\leq \left(\frac{|N|}{|N|} + \frac{|N|}{|N^{(1)}|} + \frac{|N|}{|N^{(14)}|} + \frac{|N|}{|N^{(16)}|} + \frac{|N|}{|N^{(18)}|} + \frac{|N|}{|N^{(110)}|} + \frac{|N|}{|N^{(112)}|}\right) \times |N| \\
&= \left(\frac{20}{20} + \frac{20}{1} + \frac{20}{2} + \frac{20}{2} + \frac{20}{2} + \frac{20}{2} + \frac{20}{4}\right) \times 20 \\
&= (1 + 20 + 10 + 10 + 10 + 10 + 5) \times 20 \\
&= 66 \times 20 \\
&= 1320 \\
G &\leq 1320
\end{aligned}
$$

5.3.2 Proof of the Isomorphism

Table 5.2: Single Coset Action of $PGL(2,11)$ Over D_{10}

Label	**Single Cosets**		x		y		t_1
1	N	1	N	1	N	2	Nt_1
2	Nt_1	4	Nt_3	3	Nt_2	4	Nt_3
3	Nt_2	13	Nt_{12}	2	Nt_1	21	Nt_{20}
4	Nt_3	8	Nt_7	5	Nt_4	6	Nt_5
5	Nt_4	3	Nt_2	4	Nt_3	23	Nt_4t_1
6	Nt_5	12	Nt_{11}	7	Nt_6	8	Nt_7
7	Nt_6	15	Nt_{14}	6	Nt_5	33	Nt_6t_1
8	Nt_7	16	Nt_{15}	9	Nt_8	10	Nt_9
9	Nt_8	5	Nt_4	8	Nt_7	43	Nt_8t_1
10	Nt_9	20	Nt_{19}	11	Nt_{10}	12	Nt_{11}
11	Nt_{10}	17	Nt_{16}	10	Nt_9	53	$Nt_{10}t_1$
12	Nt_{11}	2	Nt_1	13	Nt_{12}	14	Nt_{13}
13	Nt_{12}	7	Nt_6	12	Nt_{11}	63	$Nt_{12}t_1$
14	Nt_{13}	6	Nt_5	15	Nt_{14}	16	Nt_{15}
15	Nt_{14}	19	Nt_{18}	14	Nt_{13}	61	Nt_8t_7
16	Nt_{15}	10	Nt_9	17	Nt_{16}	18	Nt_{17}
17	Nt_{16}	9	Nt_8	16	Nt_{15}	51	Nt_6t_9
18	Nt_{17}	14	Nt_{13}	19	Nt_{18}	20	Nt_{19}
19	Nt_{18}	21	Nt_{20}	18	Nt_{17}	39	$Nt_{15}t_{20}$
20	Nt_{19}	18	Nt_{17}	21	Nt_{20}	1	N
21	Nt_{20}	11	Nt_{10}	20	Nt_{19}	24	Nt_2t_3
22	Nt_1t_4	25	Nt_3t_2	24	Nt_2t_3	19	Nt_{18}
23	Nt_4t_1	24	Nt_2t_3	25	Nt_3t_2	47	Nt_4t_3
24	Nt_2t_3	29	$Nt_{12}t_7$	22	Nt_1t_4	34	Nt_2t_5
25	Nt_3t_2	28	Nt_7t_{12}	23	Nt_4t_1	3	Nt_2
26	Nt_5t_{16}	31	$Nt_{11}t_8$	26	Nt_5t_{16}	58	Nt_6t_{17}
27	$Nt_{16}t_5$	30	Nt_8t_{11}	27	$Nt_{16}t_5$	26	Nt_5t_{16}

Continued on next page

Table 5.2 – *Continued from previous page*

Label	Single Cosets		x		y		t_1
28	Nt_7t_{12}	27	$Nt_{16}t_5$	30	Nt_8t_{11}	60	Nt_7t_8
29	$Nt_{12}t_7$	26	Nt_5t_{16}	31	$Nt_{11}t_8$	46	Nt_3t_4
30	Nt_8t_{11}	23	Nt_4t_1	28	Nt_7t_{12}	66	Nt_3t_6
31	$Nt_{11}t_8$	22	Nt_1t_4	29	$Nt_{12}t_7$	38	Nt_7t_{18}
32	Nt_1t_6	36	Nt_3t_{14}	34	Nt_2t_5	48	$Nt_{17}t_{18}$
33	Nt_6t_1	37	$Nt_{14}t_3$	35	Nt_5t_2	62	Nt_1t_{12}
34	Nt_2t_5	41	$Nt_{12}t_{11}$	32	Nt_1t_6	44	Nt_2t_7
35	Nt_5t_2	40	$Nt_{11}t_{12}$	33	Nt_6t_1	5	Nt_4
36	Nt_3t_{14}	38	Nt_7t_{18}	39	$Nt_{15}t_{20}$	17	Nt_{16}
37	$Nt_{14}t_3$	35	Nt_5t_2	37	$Nt_{14}t_3$	30	Nt_8t_{11}
38	Nt_7t_{18}	39	$Nt_{15}t_{20}$	38	Nt_7t_{18}	55	Nt_9t_2
39	$Nt_{15}t_{20}$	34	Nt_2t_5	36	Nt_3t_{14}	52	Nt_1t_{10}
40	$Nt_{11}t_{12}$	32	Nt_1t_6	41	$Nt_{12}t_{11}$	25	Nt_3t_2
41	$Nt_{12}t_{11}$	33	Nt_6t_1	40	$Nt_{11}t_{12}$	32	Nt_1t_6
42	Nt_1t_8	46	Nt_3t_4	44	Nt_2t_7	40	$Nt_{11}t_{12}$
43	Nt_8t_1	47	Nt_4t_3	45	Nt_7t_2	50	$Nt_{14}t_{19}$
44	Nt_2t_7	48	$Nt_{17}t_{18}$	42	Nt_1t_8	54	Nt_2t_9
45	Nt_7t_2	43	Nt_8t_1	43	Nt_8t_1	7	Nt_6
46	Nt_3t_4	45	Nt_7t_2	47	Nt_4t_3	41	$Nt_{12}t_{11}$
47	Nt_4t_3	44	Nt_2t_7	46	Nt_3t_4	64	Nt_4t_5
48	$Nt_{17}t_{18}$	51	Nt_6t_9	49	$Nt_{18}t_{17}$	28	Nt_7t_{12}
49	$Nt_{18}t_{17}$	42	Nt_1t_8	48	$Nt_{17}t_{18}$	22	Nt_1t_4
50	$Nt_{14}t_{19}$	49	$Nt_{18}t_{17}$	51	Nt_6t_9	15	Nt_{14}
51	Nt_6t_9	50	$Nt_{14}t_{19}$	50	$Nt_{14}t_{19}$	56	Nt_5t_{18}
52	Nt_1t_{10}	54	Nt_2t_9	54	Nt_2t_9	57	$Nt_{18}t_5$
53	$Nt_{10}t_1$	56	Nt_5t_{18}	55	Nt_9t_2	29	$Nt_{12}t_7$
54	Nt_2t_9	60	Nt_7t_8	52	Nt_1t_{10}	65	Nt_5t_4
55	Nt_9t_2	57	$Nt_{18}t_5$	53	$Nt_{10}t_1$	9	Nt_8

Continued on next page

Table 5.2 – *Continued from previous page*

Label	**Single Cosets**		x		y		t_1
56	Nt_5t_{18}	61	Nt_8t_7	58	Nt_6t_{17}	27	$Nt_{16}t_5$
57	$Nt_{18}t_5$	59	$Nt_{17}t_6$	59	$Nt_{17}t_6$	35	Nt_5t_2
58	Nt_6t_{17}	55	Nt_9t_2	56	Nt_5t_{18}	45	Nt_7t_2
59	$Nt_{17}t_6$	53	$Nt_{10}t_1$	57	$Nt_{18}t_5$	42	Nt_1t_8
60	Nt_7t_8	58	Nt_6t_{17}	61	Nt_8t_7	13	Nt_{12}
61	Nt_8t_7	52	Nt_1t_{10}	60	Nt_7t_8	37	$Nt_{14}t_3$
62	Nt_1t_{12}	66	Nt_3t_6	65	Nt_5t_4	36	Nt_3t_{14}
63	$Nt_{12}t_1$	62	Nt_1t_{12}	63	$Nt_{12}t_1$	11	Nt_{10}
64	Nt_4t_5	65	Nt_5t_4	66	Nt_3t_6	49	$Nt_{18}t_{17}$
65	Nt_5t_4	63	$Nt_{12}t_1$	62	Nt_1t_{12}	59	$Nt_{17}t_6$
66	Nt_3t_6	64	Nt_4t_5	64	Nt_4t_5	31	$Nt_{11}t_8$

Thus,
$f(x) = (2,4,8,16,10,20,18,14,6,12)(3,13,7,15,19,21,11,17,9,5)$
$(22,25,28,27,30,23,24,29,26,31)(32,36,38,39,34,41,33,37,35,40)$
$(42,46,45,43,47,44,48,51,50,49)(52,54,60,58,55,57,59,53,56,61)$
$(62,66,64,65,63)$, $f(y) = (2,3)(4,5)(6,7)(8,9)(10,11)(12,13)(14,15)$
$(16,17)(18,19)(20,21)(22,24)(23,25)(28,30)(29,31)(32,34)(33,35)$
$(36,39)(40,41)(42,44)(43,45)(46,47)(48,49)(50,51)(52,54)(53,55)$
$(56,58)(57,59)(60,61)(62,65)(64,66)$, and $f(t) = (1,2,4,6,8,10,12,$
$14,16,18,20)(3,21,24,34,44,54,65,59,42,40,25)(5,23,47,64,49,22,19,$
$39,52,57,35)(7,33,62,36,17,51,56,27,26,58,45)(9,43,50,15,61,37,30,$
$66,31,38,55)(11,53,29,46,41,32,48,28,60,13,63)$.

Now $< f_x, f_y, f_t > \leq S_{66}$. $f : G \longrightarrow S_{66}$ is a homomorphism, since G acts on $X = \{N, Nt_1, Nt_2, ...Nt_3t_6\}$ with $|X| = 66$. $< f_x, f_y, f_t >$ is a homomorphic image of the progenitor if

(1) f_t has exactly 2 conjugates under conjugation by $< f_x, f_y >$ and

(2) $< f_x, f_y >$ acts on $\{f_{t_1}, f_{t_2}\}$ by conjugation as S_{66}.
In addition, if the additional relations $(yt)^3 = 1$ hold in S_{66}, then $< f_x, f_y, f_t >$ is a homomorphic image of G.
Note that $| < f_x, f_y, f_t > | = 1320$.
$G/Ker_f \cong Im_f$
$\Longrightarrow G/Ker_f \cong < f_x, f_y, f_t >$
$\Longrightarrow |G/Ker_f| = | < f_x, f_y, f_t > | = 1320$
$\Longrightarrow |G| = | < f_x, f_y, f_t > | \times Ker_f$
$\Longrightarrow |G| \geq 1320$

But, from our Cayley Diagram we saw that $|G| \leq 1320$.
Hence, $|G| = 1320$.

5.3.3 Building a Map

$L_2(11) =$
$\left\{x \mapsto \frac{a+bx}{c+dx}, x \in F_{11} \cup \{\infty\}, a, b, c, d \in F_{11} | ad - bc = 1, \text{ equivalently a nonzero square}\right\}$
$=< \alpha, \beta, \gamma >$, where $\alpha = x \mapsto x + 1, \beta = x \mapsto kx$, and $\gamma = x \mapsto -\frac{1}{x}$.

Let us start with our mapping for alpha.

$\alpha : x \mapsto x + 1.$

$0 \mapsto 0 + 1 = 1$
$1 \mapsto 1 + 1 = 2$
$2 \mapsto 2 + 1 = 3$
$3 \mapsto 3 + 1 = 4$
$4 \mapsto 4 + 1 = 5$
$5 \mapsto 5 + 1 = 6$
$6 \mapsto 6 + 1 = 7$
$7 \mapsto 7 + 1 = 8$
$8 \mapsto 8 + 1 = 9$

$9 \mapsto 9 + 1 = 10$

$10 \mapsto 10 + 1 = 11 \equiv 1 \mod 11$

$\infty \mapsto \infty + 1 = \infty$

Table 5.3: $\alpha : x \mapsto x + 1$

0	1	2	3	4	5	6	7	8	9	10	∞
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
1	2	3	4	5	6	7	8	9	10	0	∞

$\alpha = (\infty)(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$

Next, to find our mapping for beta, we will first need to find k. Our nonzero squares in F_{11} are {1,3,4,5,9}.

$1^2 = 1$

$2^2 = 4$

$3^2 = 9$

$4^2 = 16 \equiv 5 \mod 11$

$5^2 = 25 \equiv 3 \mod 11$

$6^2 = 36 \equiv 3 \mod 11$

$7^2 = 49 \equiv 5 \mod 11$

$8^2 = 64 \equiv 9 \mod 11$

$9^2 = 81 \equiv 4 \mod 11$

$10^2 = 100 \equiv 1 \mod 11$

To find k, we need a nonzero square that generates all of the other nonzero squares. Note that

$3^0 = 1$

$3^1 = 3$

$3^2 = 9$

$3^3 = 27 \equiv 5 \mod 11$

$3^4 = 81 \equiv 4 \mod 11$

$3^5 = 243 \equiv 1 \mod 11$

Thus, $k = 3$. Therefore, $\beta : x \mapsto 3x$.

$0 \mapsto 3 \cdot 0 = 0$

$1 \mapsto 3 \cdot 1 = 3$

$2 \mapsto 3 \cdot 2 = 6$

$3 \mapsto 3 \cdot 3 = 9$

$4 \mapsto 3 \cdot 4 = 12 \equiv 1 \mod 11$

$5 \mapsto 3 \cdot 5 = 15 \equiv 4 \mod 11$

$6 \mapsto 3 \cdot 6 = 18 \equiv 7 \mod 11$

$7 \mapsto 3 \cdot 7 = 21 \equiv 10 \mod 11$

$8 \mapsto 3 \cdot 8 = 24 \equiv 2 \mod 11$

$9 \mapsto 3 \cdot 9 = 27 \equiv 5 \mod 11$

$10 \mapsto 3 \cdot 10 = 30 \equiv 8 \mod 11$

$\infty \mapsto 3 \cdot \infty = \infty$

Table 5.4: $\beta : x \mapsto 3x$

0	1	2	3	4	5	6	7	8	9	10	∞
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
0	3	6	9	1	4	7	10	2	5	8	∞

$\beta = (\infty)(0)(1,3,9,5,4)(2,6,7,10,8)$

$\gamma : x \mapsto -\frac{1}{x}$

$0 \mapsto -\frac{1}{0} = \infty$

$1 \mapsto -\frac{1}{1} = -1 \cdot 1^{-1} = -1 \cdot 1 = -1 \equiv 10 \mod 11$

$2 \mapsto -\frac{1}{2} = -1 \cdot 2^{-1} = -1 \cdot 6 = -6 \equiv 5 \mod 11$

$3 \mapsto -\frac{1}{3} = -1 \cdot 3^{-1} = -1 \cdot 4 = -4 \equiv 7 \mod 11$

$4 \mapsto -\frac{1}{4} = -1 \cdot 4^{-1} = -1 \cdot 3 = -3 \equiv 8 \mod 11$

$5 \mapsto -\frac{1}{5} = -1 \cdot 5^{-1} = -1 \cdot 9 = -9 \equiv 2 \mod 11$

$6 \mapsto -\frac{1}{6} = -1 \cdot 6^{-1} = -1 \cdot 2 = -2 \equiv 9 \mod 11$

$7 \mapsto -\frac{1}{7} = -1 \cdot 7^{-1} = -1 \cdot 8 = -8 \equiv 3 \mod 11$
$8 \mapsto -\frac{1}{8} = -1 \cdot 8^{-1} = -1 \cdot 7 = -7 \equiv 4 \mod 11$
$9 \mapsto -\frac{1}{9} = -1 \cdot 9^{-1} = -1 \cdot 5 = -5 \equiv 6 \mod 11$
$10 \mapsto -\frac{1}{10} = -1 \cdot 10^{-1} = -1 \cdot 10 = -10 \equiv 1 \mod 11$
$\infty \mapsto -\frac{1}{\infty} = 0$

Table 5.5: $\gamma : x \mapsto -\frac{1}{x}$

0	1	2	3	4	5	6	7	8	9	10	∞
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
∞	10	5	7	8	2	9	3	4	6	1	0

$\gamma = (0, \infty)(1, 10)(2, 5)(3, 7)(4, 8)(6, 9)$

Now alpha, beta, and gamma generate $L_2(11)$. However, since we have $PGL(2, 11)$ we must find an automorphism for the element of order 2 which is not normal in $PGL(2, 11)$. Note that
$\frac{a(x)+b}{c(x)+d} = \frac{1+0}{0+1x} = \frac{1}{x}$, where $ad - bc = 1 - 0 = 1$, is a non-zero square. Therefore our mapping for this automorphism will be

$aut : x \mapsto \frac{1}{x}$

$0 \mapsto \frac{1}{0} = \infty$
$1 \mapsto \frac{1}{1} = 1$
$2 \mapsto \frac{1}{2} = 1 \cdot 2^{-1} = 1 \cdot 6 = 6$
$3 \mapsto \frac{1}{3} = 1 \cdot 3^{-1} = 1 \cdot 4 = 4$
$4 \mapsto \frac{1}{4} = 1 \cdot 4^{-1} = 1 \cdot 3 = 3$
$5 \mapsto \frac{1}{5} = 1 \cdot 5^{-1} = 1 \cdot 9 = 9$
$6 \mapsto \frac{1}{6} = 1 \cdot 6^{-1} = 1 \cdot 2 = 2$
$7 \mapsto \frac{1}{7} = 1 \cdot 7^{-1} = 1 \cdot 8 = 8$
$8 \mapsto \frac{1}{8} = 1 \cdot 8^{-1} = 1 \cdot 7 = 7$
$9 \mapsto \frac{1}{9} = 1 \cdot 9^{-1} = 1 \cdot 5 = 5$
$10 \mapsto \frac{1}{10} = 1 \cdot 10^{-1} = 1 \cdot 10 = 10$

$\infty \mapsto \frac{1}{\infty} = 0$

Table 5.6: $aut : x \mapsto \frac{1}{x}$

0	1	2	3	4	5	6	7	8	9	10	∞
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
∞	1	6	4	3	9	2	8	7	5	10	0

$aut = (0, \infty)(2, 6)(3, 4)(5, 9)(7, 8)$

```
> G<x,y,t>:=Group<x,y,t|x^10,y^2,(x^-1*y)^2,t^11,
> t^(x^-1)=t^6,(x^5*t)^2,(y*t)^3>;
> #G;
1320
> f,G1,k:=CosetAction(G,sub<G|x,y>);
> S:=Sym(12);
> alpha:=S!(11,1,2,3,4,5,6,7,8,9,10);
> beta:=S!(1,3,9,5,4)(2,6,7,10,8);
> gamma:=S!(11,12)(1,10)(2,5)(3,7)(4,8)(6,9);
> #sub<S|alpha,beta,gamma>;
660
> aut:=S!(11,12)(2,6)(3,4)(5,9)(7,8);
> #sub<S|alpha,beta,gamma,aut>;
1320
> P:=sub<S|alpha,beta,gamma,aut>;
> s:=IsIsomorphic(G1,P);s;
true
```

5.4 M_{11} as a Homomorphic Image of $11^{*4} :_m (4 : 5)$

Let $G \cong 11^{*4} :_m (4 : 5)$ be a symmetric presentation of G given by:
$< x, y, t | x^4, xy^4x^3y^3, y^3x^3yx, t^{11}, t^y = t^4, (x^2t^{y^3})^3, (x^3t)^8, (yt^x)^5, (xt^{y^4})^3 > \cong M_{11}$,
where $N \cong (4 : 5) =< x, y | x^4, xy^4x^3y^3, y^3x^3yx >$,
$x = (1, 2, 3, 4)(5, 6, 7, 8)(9, 10, 11, 12)(13, 14, 15, 16)(17, 18, 19, 20)(21, 22, 23, 24)$
$(25, 26, 27, 28)(29, 30, 31, 32)(33, 34, 35, 36)(37, 38, 39, 40)$, and $y = (1, 13, 17, 33, 9)$
$(5, 29, 37, 25, 21)(6, 26, 30, 22, 38)(2, 34, 14, 10, 18)(3, 11, 35, 19, 15)(4, 20, 12, 16, 36)$
$(7, 23, 27, 39, 31)(8, 40, 24, 32, 28)$.

G is of order 7920, and N is of order 20. The number of single cosets is equal to $\frac{|G|}{|N|} = \frac{7920}{20} = 396$. In the next section we will show how this double coset enumeration of G over $(C_4 : C_5)$ can be done by performing a double coset enumeration of G over a maximal subgroup, say H, and N.

5.4.1 Manual Double Coset Enumeration over a Maximal Subgroup of Order 120

We will find a maximal subgroup, M, or a conjugate of M which contains $f(x), f(y)$ to perform double coset enumeration.

```
> G<x,y,t>:=Group<x,y,t|x^4,x*y^4*x^3*y^3,
> y^3*x^3*y*x,t^11,t^y=t^4,(x^2*t^(y^3))^3,(x^3*t)^8,
> (y*t^x)^5,
> (x*t^(y^4))^3>;
> #G;
7920
> S:=Sym(40);
> xx:=S!(1,2,3,4)(5,6,7,8)(9,10,11,12)(13,14,15,16)
> (17,18,19,20)(21,22,23,24)(25,26,27,28)
> (29,30,31,32)(33,34,35,36)(37,38,39,40);
> yy:=S!(1,13,17,33,9)(5,29,37,25,21)
> (6,26,30,22,38)(2,34,14,10,18)(3,11,35,19,15)
> (4,20,12,16,36)
> (7,23,27,39,31)(8,40,24,32,28);
> N:=sub<S|xx,yy>;
> #N;
20
> f,G1,k:=CosetAction(G,sub<G|x>);
> CompositionFactors(G1);
    G
    |  M11
    1
> M:=MaximalSubgroups(G1);
> #M;
5
> M;
Conjugacy classes of subgroups
------------------------------
```

```
[1]     Order 48            Length 165
        Permutation group acting on a set of cardinality 1980
        Order = 48 = 2^4 * 3
[2]     Order 120           Length 66
        Permutation group acting on a set of cardinality 1980
        Order = 120 = 2^3 * 3 * 5
[3]     Order 660           Length 12
        Permutation group acting on a set of cardinality 1980
        Order = 660 = 2^2 * 3 * 5 * 11
[4]     Order 144           Length 55
        Permutation group acting on a set of cardinality 1980
        Order = 144 = 2^4 * 3^2
[5]     Order 720           Length 11
        Permutation group acting on a set of cardinality 1980
        Order = 720 = 2^4 * 3^2 * 5
>
```

We have found the maximal subgroups of G. We need to find which maximal subgroup contains $f(x)$ and $f(y)$.

```
> for i in [1..#M] do if f(x) in M[i]`subgroup and f(y) in
for|if> M[i]`subgroup then i; end if; end for;
> for i in [1..#M] do D:=Conjugates(G1,M[i]`subgroup);
for> D:=SetToSequence(D);
for> for j in [1..#D] do if f(x) in D[j] and f(y) in D[j] then i,j;
for|for|if> end if; end for; end for;
2 22
5 5
```

Using the previous loop, we see that there are 2 maximal subgroups that contain $f(x)$ and $f(y)$.

```
> #M[2]`subgroup;
120
> #M[5]`subgroup;
720
```

Let us first examine subgroup number 2, which is of order 120. We need to use our Schreier System to find a representation of this subgroup in words so that we may perform Double Coset Enumeration of G over H.

```
> N:=G1;
> NN:=G;
> Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
> ArrayP:=[Id(N): i in [1..7920]];
> for i in [2..7920] do
for> P:=[Id(N): l in [1..#Sch[i]]];
for> for j in [1..#Sch[i]] do
for|for> if Eltseq(Sch[i])[j] eq 1 then P[j]:=f(x); end if;
for|for> if Eltseq(Sch[i])[j] eq 2 then P[j]:=f(y); end if;
for|for> if Eltseq(Sch[i])[j] eq -1 then P[j]:=f(x^-1); end if;
for|for> if Eltseq(Sch[i])[j] eq -2 then P[j]:=f(y^-1); end if;
for|for> if Eltseq(Sch[i])[j] eq 3 then P[j]:=f(t); end if;
for|for> end for;
for> PP:=Id(N);
for> for k in [1..#P] do
for|for> PP:=PP*P[k]; end for;
for> ArrayP[i]:=PP;
for> end for;
> D:=Conjugates(G1,M[2]`subgroup);
> D:=SetToSequence(D);
> f(x) in D[44] and f(y) in D[44];
true
> for g in D[44] do if sub<D[44]|f(x),f(y),g> eq D[44] then gg:=g;
for|if> end if;
for> end for;
> Order(gg);
2
> if Order(gg) eq 2 then for i in [1..7920] do if ArrayP[i] eq gg
if|for|if> then Sch[i]; end if; end for; end if;
x * y * t * x^2 * y^-1 * t^-1 * y^-1
```

Thus our maximal subgroup is $H =< x, y, xytx^2y^4t^{10}y^4 >$.

We show how this double coset enumeration of G over $(C_4 : C_5)$ can be done using a double coset enumeration of G over H and N.

Definition of a double coset: $HwN = \{Hwn|n \in N\}$.

Note: $wn = nn^{-1}wn = nw^n$.

So, $Hwn = \{Hw^n|n \in N\}$.

First we will expand our additional relations.

$$\begin{aligned}
xytx^2y^4t^{10}y^4 &\in H \\
xyt_1x^2y^4t_1^{10}y^4 &\in H \\
xyt_1x^2y^4t_{37}y^4 &\in H \\
xy[x^2x^{-2}]t_1x^2y^4t_{37}y^4 &\in H \\
xyx^2[x^{-2}t_1x^2]y^4t_{37}y^4 &\in H \\
xyx^2[t_1^{x^2}]y^4t_{37}y^4 &\in H \\
xyx^2[t_3]y^4t_{37}y^4 &\in H \\
xyx^2[y^4y^{-4}]t_3y^4t_{37}y^4 &\in H \\
xyx^2y^4[y^{-4}t_3y^4]t_{37}y^4 &\in H \\
xyx^2y^4[t_3^{y^4}]t_{37}y^4 &\in H \\
xyx^2y^4t_{15}t_{37}y^4 &\in H \\
xyx^2y^4[y^4y^{-4}]t_{15}t_{37}y^4 &\in H \\
xyx^2y^4y^4[y^{-4}t_{15}t_{37}y^4] &\in H \\
xyx^2y^3[t_{15}^{y^4}t_{37}^{y^4}] &\in H \\
xyx^2y^3t_{19}t_{29} &\in H \\
Hxyx^2y^3t_{19}t_{29} &= H \\
Hxyx^2y^3t_3^5t_1^8 &= H \\
Hxyx^2y^3t_3^5t_1^8t_1^3 &= Ht_1^3 \\
Hxyx^2y^3t_3^5 &= Ht_1^3 \\
Hxyx^2y^3t_{19} &= Ht_9 \\
Ht_{19} &= Ht_9
\end{aligned} \tag{5.6}$$

$$
\begin{aligned}
(x^2t^{y^3})^3 &= e \\
(x^2t_1^{y^3})^3 &= e \\
(x^2t_{33})^3 &= e \\
x^2t_{33}x^2t_{33}x^2t_{33} &= e \\
x^2(x^2x^{-2})t_{33}x^2t_{33}x^2t_{33} &= e \\
x^2x^2(x^{-2}t_{33}x^2)t_{33}x^2t_{33} &= e \\
t_{33}^{x^2}t_{33}x^2t_{33} &= e \\
t_{35}t_{33}x^2t_{33} &= e \\
t_{35}(x^2x^{-2})t_{33}x^2t_{33} &= e \\
t_{35}x^2(x^{-2}t_{33}x^2)t_{33} &= e \\
t_{35}x^2t_{33}^{x^2}t_{33} &= e \\
t_{35}x^2t_{35}t_{33} &= e \\
(x^2x^{-2})t_{35}x^2t_{35}t_{33} &= e \\
x^2(x^{-2}t_{35}x^2)t_{35}t_{33} &= e \\
x^2t_{35}^{x^2}t_{35}t_{33} &= e \\
x^2t_{33}t_{35}t_{33} &= e
\end{aligned} \tag{5.7}
$$

$$\begin{aligned}
(xt^{y^4})^3 &= e \\
(xt_1^{y^4})^3 &= e \\
(xt_9)^3 &= e \\
xt_9xt_9xt_9 &= e \\
x(xx^{-1})t_9xt_9xt_9 &= e \\
x^2(x^{-1}t_9x)t_9xt_9 &= e \\
x^2t_9^xt_9xt_9 &= e \\
x^2t_{10}t_9xt_9 &= e \\
x^2(xx^{-1})t_{10}t_9xt_9 &= e \\
x^3(x^{-1})t_{10}t_9x)t_9 &= e \\
x^3[t_{10}t_9]^xt_9 &= e \\
x^3t_{11}t_{10}t_9 &= e
\end{aligned} \tag{5.8}$$

Our first double coset, $HeN = \{He^n | n \in N\} = \{H\}$, which we will denote by [$*$].
The orbits of N on
{1,2,3,4,5,6,8,7,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,
29,30,31,32,33,34,35,36,37,38,39,40}
are {1,2,13,3,34,14,17,4,11,35,15,10,18,33,20,12,36,19,16,9}
and {5,6,29,7,26,30,37,8,23,27,31,22,38,25,40,24,28,39,32,21}.
We will take a representative from each orbit, say t_1 and t_5, and determine to which double coset Ht_1 and Ht_5 belong.

Word of Length 1

Ht_1N is a new double coset which we will denote by [1].
$Ht_1N = \{Ht_1{}^n | n \in N\}$.
Since the orbit $\{1, 2, 13, 3, 34, 14, 17, 4, 11, 35, 15, 10, 18, 33, 20, 12, 36, 19, 16, 9\}$ contains 20 elements then 20 symmetric generators will go to the new double coset [1].
Now $N^{(1)} \geq H^1$.
$N^1 = \{e\}$.

$N^{(1)}$ = Coset Stabiliser in N of $Ht_1 = \{n \in N | Ht_1^n = t_1\}$.

We will look for a relation that will increase the Coset Stabiliser $N^{(1)}$.
$Ht_{19} = Ht_9$, by Equation 5.6
$[Ht_{19}]^{xy^{-1}x} = [Ht_9]^{xy^{-1}x}$
$\implies Ht_1 = Ht_{15}$

Now, since $Ht_1^e = Ht_1 \Rightarrow e \in N^{(1)}$, and
$Ht_1^{x^2y^4} = Ht_{15} = Ht_1 \Rightarrow x^2y^4 \in N^{(1)}$, then,
$N^{(1)}$ = Coset Stabiliser in N of $Ht_1 = \{n \in N | Ht_1^n = t_1\} = \{e, x^2y^4\}$.
Furthermore, the number of single cosets of Ht_1N is $\frac{|N|}{|N^{(1)}|} = \frac{20}{2} = 10$.

Conjugating by elements in N gives us the following equal names.

$t_1 \sim t_{15}$	$t_9 \sim t_{19}$	$t_{33} \sim t_{35}$
$t_2 \sim t_{16}$	$t_{10} \sim t_{20}$	$t_{34} \sim t_{36}$
$t_3 \sim t_{13}$	$t_{11} \sim t_{17}$	
$t_4 \sim t_{14}$	$t_{12} \sim t_{18}$	

Therefore, $Ht_1N = \{Ht_1 = Ht_{15}, Ht_2 = Ht_{16}, Ht_3 = Ht_{13}, Ht_4 = Ht_{14},$
$Ht_9 = Ht_{19}, Ht_{10} = Ht_{20}, Ht_{11} = Ht_{17}, Ht_{12} = Ht_{18}, Ht_{33} = Ht_{35}, Ht_{34} = Ht_{36}\}$

Ht_5N is a new double coset which we will denote by [5].
$Ht_5N = \{Ht_5^n | n \in N\}$.
Since the orbit {5,6,29,7,26,30,37,8,23,27,31,22,38,25,40,24,28,39,32,21} contains 20 elements then 20 symmetric generators will go to the double coset [5].
Now $N^{(5)} \geq H^5$.
$N^5 = \{e\}$.
$N^{(5)}$ = Coset Stabiliser in N of $Ht_5 = \{n \in N | Ht_5^n = t_5\}$.

We will look for a relation that will increase the Coset Stabiliser $N^{(5)}$.

$Ht_{19} = Ht_9$, by Equation 5.6

$\Longrightarrow Ht_3^5 = Ht_1^3$

$\Longrightarrow Ht_3^5\underline{t_1^3} = Ht_1^3\underline{t_1^3}$

$\Longrightarrow H\underline{t_3^5t_1^3} = Ht_1^6$

$\Longrightarrow H[xyxt_3^6] = Ht_1^6$, since by Equation 5.7

$x^2t_{33}t_{35}t_{33} = e$, by Equation 5.7

$[x^2t_{33}t_{35}t_{33}]^{x^2y} = [e]^{x^2y}$

$\Longrightarrow x^2y^2t_{19}t_9t_{19} = e$

$\Longrightarrow x^2y^2t_3^5t_1^3t_3^5 = e$

$\Longrightarrow x^2y^2t_3^5t_1^3t_3^5\underline{t_3^6} = \underline{t_3^6}$

$\Longrightarrow x^2y^2t_3^5t_1^3 = t_3^6$

$Hxyxt_3^6 = Ht_1^6$

$\Longrightarrow Ht_3^6 = Ht_1^6$

$\Longrightarrow Ht_{23} = Ht_{21}$

$\Longrightarrow [Ht_{23}]^{x^2y} = [Ht_{21}]^{x^2y}$

$\Longrightarrow Ht_5 = Ht_{27}$

Now, since $Ht_5^e = Ht_5 \Rightarrow e \in N^{(5)}$, and
$Ht_5^{x^2y^2} = Ht_{27} = Ht_5 \Rightarrow x^2y^2 \in N^{(5)}$, then,
$N^{(5)} =$ Coset Stabiliser in N of $Ht_5 = \{n \in N | Ht_5^n = t_5\} = \{e, x^2y^2\}$.
Furthermore, the number of single cosets of Ht_1N is $\frac{|N|}{|N^{(5)}|} = \frac{20}{2} = 10$.

After conjugating by all the elements of N, we have the following equal names.

$t_5 \sim t_{27}$	$t_{21} \sim t_{23}$	$t_{30} \sim t_{40}$
$t_6 \sim t_{28}$	$t_{22} \sim t_{24}$	$t_{31} \sim t_{37}$
$t_7 \sim t_{25}$	$t_{23} \sim t_{21}$	$t_{32} \sim t_{38}$
$t_8 \sim t_{26}$	$t_{29} \sim t_{39}$	

Therefore, $Ht_5N = \{Ht_5 = Ht_{27}, Ht_6 = Ht_{28}, Ht_7 = Ht_{25}, Ht_8 = Ht_{26},$
$Ht_{21} = Ht_{23}, Ht_{22} = Ht_{24}, Ht_{29} = Ht_{39}, Ht_{30} = Ht_{40}, Ht_{31} = Ht_{37}, Ht_{32} = Ht_{38}\}$

The orbits of $N^{(1)}$ on
{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40}
are $\{1, 15\}$, $\{2, 36\}$, $\{3, 9\}$, $\{4, 18\}$, $\{5, 31\}$, $\{6, 28\}$, $\{7, 21\}$, $\{8, 38\}$, $\{10, 20\}$, $\{11, 33\}$, $\{12, 14\}$, $\{13, 19\}$, $\{16, 34\}$, $\{17, 35\}$, $\{22, 40\}$, $\{23, 25\}$, $\{24, 30\}$, $\{26, 32\}$, $\{27, 37\}$, and $\{29, 39\}$.

We want to see to which double coset $Ht_1t_1, Ht_1t_2, Ht_1t_3, Ht_1t_4, Ht_1t_5, Ht_1t_6, Ht_1t_7, Ht_1t_8, Ht_1t_{10}, Ht_1t_{11}, Ht_1t_{14}, Ht_1t_{13}, Ht_1t_{16}, Ht_1t_{17}, Ht_1t_{22}, Ht_1t_{25}, Ht_1t_{24}, Ht_1t_{32}, Ht_1t_{37}$, and Ht_1t_{29} belong.

$Ht_1t_1 = Ht_1t_1$
$\implies Ht_1t_1 = Ht_1^2$
$\implies Ht_1t_1 = Ht_5$
$\implies Ht_1t_1 \in [5]$, since Ht_5 is in [5].
Two symmetric generators will go to [5].

Ht_1t_2N is a new double coset which we will label [1 2].
Two symmetric generators will go to [1 2].

$Ht_1t_3 = H\underline{t_1}t_3$
$\implies Ht_1t_3 = Ht_{15}t_3$, since $Ht_1 = Ht_{15}$
$Ht_1t_3 = Ht_{15}t_3$
$\implies Ht_1t_3 = Ht_3^4t_3$
$\implies Ht_1t_3 = Ht_3^5$
$\implies Ht_1t_3 = Ht_{19}$
$\implies Ht_1t_3 \in [1]$, since Ht_{19} is in [1].
Two symmetric generators will go to [1].

Ht_1t_4N is a new double coset which we will label [1 4].
Two symmetric generators will go to [1 4].

$Ht_1t_5 = Ht_1t_5$

$\implies Ht_1t_5 = Ht_1t_1^2$

$\implies Ht_1t_5 = Ht_1^3$

$\implies Ht_1t_5 = Ht_9$

$\implies Ht_1t_5 \in [1]$, since Ht_9 is in [1].

Two symmetric generators will go to [1].

Ht_1t_6N is a new double coset which we will label [1 6].

Two symmetric generators will go to [1 6].

$Ht_1t_7 = H\underline{t_1}t_7$

$\implies Ht_1t_7 = Ht_{15}t_7$, since $Ht_1 = Ht_{15}$

$Ht_1t_7 = Ht_{15}t_7$

$\implies Ht_1t_7 = Ht_3^4t_3^2$

$\implies Ht_1t_7 = Ht_3^6$

$\implies Ht_1t_7 = Ht_{23}$

$\implies Ht_1t_7 \in [5]$, since Ht_{23} is in [5].

Two symmetric generators will go to [5].

$Ht_1t_8 = H\underline{t_1}t_8$

$\implies Ht_1t_8 = Ht_{15}t_8$, since $Ht_1 = Ht_{15}$

$Ht_1t_8 = Ht_{15}t_8$

$\implies Ht_1t_8 = Ht_3^4\underline{t_4^2}$

$\implies Ht_1t_8 = Ht_3^4[x^{-1}yt_2t_1^4]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\implies x^{-1}yt_2t_{13}t_{36} = e$

$\implies x^{-1}yt_2t_1^4t_4^9 = e$

$\implies x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\implies x^{-1}yt_2t_1^4 = t_4^2$

$Ht_1t_8 = Ht_3^4x^{-1}yt_2t_1^4$

$\implies Ht_1t_8 = Hx^{-1}y[t_3^4]^{x^{-1}y}t_2t_1^4$

$\Longrightarrow Ht_1t_8 = Ht_2^3t_2t_1^4$

$\Longrightarrow Ht_1t_8 = H\underline{t_2^4}t_1^4$

$\Longrightarrow Ht_1t_8 = H[xyt_4^6t_1^2]t_1^4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y^{-1}} = e^{x^{-1}y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{14}t_{33}t_{20} = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5 = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5\underline{t_4^6} = \underline{t_4^6}$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9\underline{t_1^2} = t_4^6\underline{t_1^2}$

$\Longrightarrow \underline{xy}y^{-1}x^{-1}t_2^4 = \underline{xy}t_4^6t_1^2$

$\Longrightarrow t_2^4 = xyt_4^6t_1^2$

$Ht_1t_8 = Hxyt_4^6t_1^2t_1^4$

$\Longrightarrow Ht_1t_8 = Ht_4^6\underline{t_1^6}$

$\Longrightarrow Ht_1t_8 = Ht_4^6[x^{-1}y^{-1}t_3^4t_2^9]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\Longrightarrow x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9 = t_1^6$

$Ht_1t_8 = Ht_4^6x^{-1}y^{-1}t_3^4t_2^9$

$\Longrightarrow Ht_1t_8 = Hx^{-1}y^{-1}[t_4^6]^{x^{-1}y^{-1}}t_3^4t_2^9$

$\Longrightarrow Ht_1t_8 = Ht_3^2t_3^4t_2^9$

$\Longrightarrow Ht_1t_8 = Ht_3^6t_2^9$

$\Longrightarrow Ht_1t_8 = Ht_{23}t_{34}$

Note that

$Ht_1t_6 = Ht_1t_6$

$\Longrightarrow Ht_1t_6 = H\underline{t_1}t_6$

$\Longrightarrow Ht_1t_6 = Ht_{15}t_6$, since $Ht_1 = Ht_{15}$

$Ht_1t_6 = Ht_{15}t_6$

$\Longrightarrow Ht_1t_6 = H\underline{t_3^4}t_2^2$

$\Longrightarrow Ht_1t_6 = H[yxt_1^6t_2^2]t_2^2$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\Longrightarrow x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9\underline{t_1^2} = t_1^6\underline{t_1^2}$

$\Longrightarrow \underline{yx}y^{-1}x^{-1}t_3^4 = \underline{yx}t_1^6t_1^2$

$\Longrightarrow t_3^4 = yxt_1^6t_1^2$

$Ht_1t_6 = Hyxt_1^6t_2^2t_2^2$

$\Longrightarrow Ht_1t_6 = H\underline{t_1^6}t_2^4$

$\Longrightarrow Ht_1t_6 = Ht_3^6t_2^4$, since $Ht_{21} = Ht_{23}$

$\Longrightarrow Ht_1^6 = Ht_3^6$

$Ht_1t_6 = Ht_3^6\underline{t_2^4}$

$\Longrightarrow Ht_1t_6 = Ht_3^6[xyt_4^6t_1^2]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y^{-1}} = e^{x^{-1}y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{14}t_{33}t_{20} = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5 = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5\underline{t_4^6} = \underline{t_4^6}$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9\underline{t_1^2} = t_4^6\underline{t_1^2}$

$\Longrightarrow \underline{xy}y^{-1}x^{-1}t_2^4 = \underline{xy}t_4^6t_1^2$

$\Longrightarrow t_2^4 = xyt_4^6t_1^2$

$Ht_1t_6 = Ht_3^6xyt_4^6t_1^2$

$\Longrightarrow Ht_1t_6 = Hxy[t_3^6]^{xy}t_4^6t_1^2$

$\Longrightarrow Ht_1t_6 = Ht_4^8t_4^6t_1^2$

$\Longrightarrow Ht_1t_6 = H\underline{t_4^3}t_1^2$

$\Longrightarrow Ht_1t_6 = Ht_2^5t_1^2$, since

$Ht_{12}t_{18} \Longrightarrow Ht_4^3 = Ht_2^5$

$Ht_1t_6 = H\underline{t_2^5}t_1^2$

$\Longrightarrow Ht_1t_6 = H[xy^{-1}t_4^7t_1^{10}]t_1^2$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\implies yx^{-1}t_{18}t_1t_{16} = e$

$\implies yx^{-1}t_2^5t_1t_4^4 = e$

$\implies yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\implies yx^{-1}t_2^5t_1\underline{t_1^{10}} = t_4^7\underline{t_1^{10}}$

$\implies \underline{xy^{-1}}yx^{-1}t_2^5 = \underline{xy^{-1}}t_4^7t_1^{10}$

$\implies t_2^5 = xy^{-1}t_4^7t_1^{10}$

$Ht_1t_6 = Hxy^{-1}t_4^7t_1^{10}t_1^2$

$\implies Ht_1t_6 = Ht_4^7t_1$

$\implies Ht_1t_6 = H\underline{t_{28}}t_1$

$\implies Ht_1t_6 = Ht_6t_1$, since $Ht_6 = Ht_{28}$

$$Ht_1t_6 = Ht_6t_1 \tag{5.9}$$

Thus,

$Ht_1t_8 = Ht_{23}t_{34}$

$\implies Ht_1t_8 = Ht_{34}t_{23}$, since by Equation 5.9

$Ht_1t_6 = Ht_6t_1$

$\implies [Ht_1t_6]^{xy} = [Ht_6t_1]^{xy}$

$\implies Ht_{34}t_{23} = Ht_{23}t_{34}$

$Ht_1t_8 = Ht_{34}t_{23}$

$\implies Ht_1t_8 \in [16]$, since $Ht_{34}t_{23}$ is in [1 6].

Two symmetric generators will go to [1 6].

$Ht_1t_{10}N$ is a new double coset which we will label [1 10].

Two symmetric generators will go to [1 10].

$Ht_1t_{11} = H\underline{t_1}t_{11}$

$\implies Ht_1t_{11} = Ht_{15}t_{11}$, since $Ht_1 = Ht_{15}$

$Ht_1t_{11} = Ht_{15}t_{11}$

$\implies Ht_1t_{11} = Ht_3^4t_3^3$

$\implies Ht_1t_{11} = Ht_3^7$

$\Longrightarrow Ht_1t_{11} = Ht_3^7$

$\Longrightarrow Ht_1t_{11} = Ht_{27}$

$\Longrightarrow Ht_1t_{11} \in [5]$, since Ht_{27} is in [5].

Two symmetric generators will go to [5].

$Ht_1t_{14} = H\underline{t_1}t_{14}$

$\Longrightarrow Ht_1t_{14} = Ht_{15}t_{14}$, since $Ht_1 = Ht_{15}$

$\Longrightarrow Ht_1t_{14} = Ht_3^4\underline{t_2^4}$

$\Longrightarrow Ht_1t_{14} = Ht_3^4[xyt_4^6t_1^2]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y^{-1}} = e^{x^{-1}y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{14}t_{33}t_{20} = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5 = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5\underline{t_4^6} = \underline{t_4^6}$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9\underline{t_1^2} = t_4^6\underline{t_1^2}$

$\Longrightarrow \underline{xy}y^{-1}x^{-1}t_2^4 = \underline{xy}t_4^6t_1^2$

$\Longrightarrow t_2^4 = xyt_4^6t_1^2$

$Ht_1t_{14} = Ht_3^4xyt_4^6t_1^2$

$\Longrightarrow Ht_1t_{14} = Hxy[t_3^4]^{xy}t_4^6t_1^2$

$\Longrightarrow Ht_1t_{14} = Ht_4^9t_4^6t_1^2$

$\Longrightarrow Ht_1t_{14} = H\underline{t_4^4}t_1^2$

$\Longrightarrow Ht_1t_{14} = Ht_2t_1^2$, since

$Ht_2 = Ht_{16}$

$\Longrightarrow Ht_2 = Ht_4^4$

$Ht_1t_{14} = H\underline{t_2}t_1^2$

$Ht_1t_{14} = H[y^{-1}xt_4^2t_1^7]t_1^2$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\Longrightarrow x^{-1}yt_2t_{13}t_{36} = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9 = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\Longrightarrow x^{-1}yt_2t_1^4\underline{t_1^7} = t_4^2\underline{t_1^7}$

$\Longrightarrow \underline{y^{-1}x}x^{-1}yt_2 = \underline{y^{-1}x}t_4^2t_1^7$

$\Longrightarrow t_2 = y^{-1}xt_4^2t_1^7$

$Ht_1t_{14} = Hy^{-1}xt_4^2t_1^7t_1^2$

$\Longrightarrow Ht_1t_{14} = H\underline{t_4^2}t_1^9$

$\Longrightarrow Ht_1t_{14} = Ht_2^7t_1^9$, since

$Ht_8 = Ht_{26}$

$\Longrightarrow Ht_4^2 - Ht_2^7$

$Ht_1t_{14} = Ht_2^7\underline{t_1^9}$

$Ht_1t_{14} = Ht_2^7[xyt_3^{10}t_4^6]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y^{-1}} = e^{x^2y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{33}t_{20}t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5\underline{t_4^6} = t_3^{10}\underline{t_4^6}$

$\Longrightarrow \underline{xy}y^{-1}x^{-1}t_1^9 = \underline{xy}t_3^{10}t_4^6$

$\Longrightarrow t_1^9 = xyt_3^{10}t_4^6$

$Ht_1t_{14} = Hxy[t_2^7]^{xy}t_3^{10}t_4^6$

$\Longrightarrow Ht_1t_{14} = Hxy[t_2^7]^{xy}t_3^{10}t_4^6$

$\Longrightarrow Ht_1t_{14} = Ht_3^{10}t_3^{10}t_4^6$

$\Longrightarrow Ht_1t_{14} = Ht_3^9t_4^6$

$\Longrightarrow Ht_1t_{14} = Ht_{35}t_{24}$

$\Longrightarrow Ht_1t_{14} \in [16]$, since $Ht_{35}t_{24}$ is in [1 6].

Two symmetric generators will go to [1 6].

$Ht_1t_{13} = Ht_1t_{13}$

$\Longrightarrow Ht_1t_{13} = Ht_1t_1^4$

$\Longrightarrow Ht_1t_{13} = Ht_1^5$

$\Longrightarrow Ht_1t_{13} = Ht_{17}$

$\Longrightarrow Ht_1t_{13} \in [1]$, since Ht_{17} is in [1].

Two symmetric generators will go to [1].

$Ht_1t_{16} = Ht_1t_{16}$

$\implies Ht_1t_{16} = H\underline{t_1}t_4^4$

$\implies Ht_1t_{16} = H[xy^{-1}t_3^2t_4^7]t_4^4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\implies yx^{-1}t_1t_{16}t_{35} = e$

$\implies yx^{-1}t_1t_4^4t_3^9 = e$

$\implies yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\implies yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\implies \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\implies t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_{16} = Hxy^{-1}t_3^2t_4^7t_4^4$

$\implies Ht_1t_{16} = Ht_3^2$

$\implies Ht_1t_{16} = Ht_7$

$\implies Ht_1t_{16} \in [5]$, since Ht_7 is in [5].

Two symmetric generators will go to [5].

$Ht_1t_{17} = Ht_1t_{17}$

$\implies Ht_1t_{17} = Ht_1t_1^5$

$\implies Ht_1t_{17} = Ht_1^6$

$\implies Ht_1t_{17} = Ht_{21}$

$\implies Ht_1t_{17} \in [5]$, since Ht_{21} is in [5].

Two symmetric generators will go to [5].

$Ht_1t_{22} = H\underline{t_1}t_{22}$

$\implies Ht_1t_{22} = Ht_{15}t_{22}$, since $Ht_1 = Ht_{15}$

$Ht_1t_{22} = Ht_{15}t_{22}$

$\implies Ht_1t_{22} = H\underline{t_3^4}t_2^6 \implies Ht_1t_{22} = H[yxt_1^6t_2^2]t_2^6$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\implies x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\implies x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9\underline{t_1^2} = t_1^6\underline{t_1^2}$

$\Longrightarrow \underline{yx}y^{-1}x^{-1}t_3^4 = \underline{yx}t_1^6t_1^2$

$\Longrightarrow t_3^4 = yxt_1^6t_1^2$

$Ht_1t_{22} = Hyxt_1^6t_2^2t_2^6$

$\Longrightarrow Ht_1t_{22} = Ht_1^6\underline{t_2^8}$

$\Longrightarrow Ht_1t_{22} = Ht_1^6[x^{-1}t_4^3t_3^3]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^x = e^x$

$\Longrightarrow x^{-1}t_{12}t_{11}t_{10} = e$

$\Longrightarrow x^{-1}t_4^3t_3^3t_2^3 = e$

$\Longrightarrow x^{-1}t_4^3t_3^3t_2^3\underline{t_2^8} = \underline{t_2^8}$

$\Longrightarrow x^{-1}t_4^3t_3^3 = t_2^8$

$Ht_1t_{22} = Ht_1^6x^{-1}t_4^3t_3^3$

$\Longrightarrow Ht_1t_{22} = Hx^{-1}[t_1^6]^{x^{-1}}t_4^3t_3^3$

$\Longrightarrow Ht_1t_{22} = Ht_4^6t_4^3t_3^3$

$\Longrightarrow Ht_1t_{22} = Ht_4^9t_3^3$

$\Longrightarrow Ht_1t_{22} = H\underline{t_{36}}t_{11}$

$\Longrightarrow Ht_1t_{22} = Ht_{34}t_{11}$, since $Ht_{34} = Ht_{36}$

$Ht_1t_{22} = Ht_{34}t_{11}$

$\Longrightarrow Ht_1t_{22} \in [12]$, since $Ht_{34}t_{11}$ is in [1 2].

Two symmetric generators will go to [1 2].

$Ht_1t_{25} = Ht_1t_{25}$

$\Longrightarrow Ht_1t_{25} = Ht_1t_1^7$

$\Longrightarrow Ht_1t_{25} = Ht_1^8$

$\Longrightarrow Ht_1t_{25} = Ht_{29}$

$\Longrightarrow Ht_1t_{25} \in [5]$, since Ht_{29} is in [5].

Two symmetric generators will go to [5].

$Ht_1t_{24} = Ht_1t_{24}$

$\Longrightarrow Ht_1t_{24} = H\underline{t_1}t_4^6$

$\implies Ht_1t_{24} = H[xy^{-1}t_3^2t_4^7]t_4^6$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\implies yx^{-1}t_1t_{16}t_{35} = e$

$\implies yx^{-1}t_1t_4^4t_3^9 = e$

$\implies yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\implies yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\implies \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\implies t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_{30} = Hxy^{-1}t_3^2t_4^7t_4^6$

$\implies Ht_1t_{24} = Hxy^{-1}t_3^2t_4^2$

$\implies Ht_1t_{24} = Ht_3^2\underline{t_4^2}$

$\implies Ht_1t_{24} = Ht_3^2[x^{-1}yt_2t_1^4]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\implies x^{-1}yt_2t_{13}t_{36} = e$

$\implies x^{-1}yt_2t_1^4t_4^9 = e$

$\implies x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\implies x^{-1}yt_2t_1^4 = t_4^2$

$Ht_1t_{24} = Ht_3^2x^{-1}yt_2t_1^4$

$\implies Ht_1t_{24} = Hx^{-1}y[t_3^2]^{x^{-1}y}t_2t_1^4$

$\implies Ht_1t_{24} = Ht_2^7t_2t_1^4$

$\implies Ht_1t_{24} = H\underline{t_2^8}t_1^4$

$\implies Ht_1t_{24} = Ht_4^{10}t_1^4$, since

$Ht_{30} = Ht_{40}$

$\implies Ht_2^8 = Ht_4^{10}$

$Ht_1t_{24} = H\underline{t_4^{10}}t_1^4$

$Ht_1t_{24} = H[x^{-1}y^{-1}t_2^9t_1^5]t_1^4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{yx^{-1}} = e^{yx^{-1}}$

$\implies x^{-1}y^{-1}t_{34}t_{17}t_4 = e$

$\implies x^{-1}y^{-1}t_2^9t_1^5t_4 = e$

$\Longrightarrow x^{-1}y^{-1}t_2^9t_1^5t_4\underline{t_4^{10}} = \underline{t_4^{10}}$

$\Longrightarrow x^{-1}y^{-1}t_2^9t_1^5 = t_4^{10}$

$Ht_1t_{24} = Hx^{-1}y^{-1}t_2^9t_1^5t_1^4$

$\Longrightarrow Ht_1t_{24} = Ht_2^9t_1^9$

$\Longrightarrow Ht_1t_{24} = H\underline{t_{34}}t_{33}$

$\Longrightarrow Ht_1t_{24} = Ht_{36}t_{33}$, since $Ht_{34} = Ht_{36}$

$Ht_1t_{24} = Ht_{36}t_{33}$

$\Longrightarrow Ht_1t_{24} \in [110]$, since $Ht_{36}t_{33}$ is in [1 10].

Two symmetric generators will go to [1 10].

$Ht_1t_{32} = Ht_1t_{32}$

$\Longrightarrow Ht_1t_{32} = H\underline{t_1}t_4^8$

$\Longrightarrow Ht_1t_{32} = H[xy^{-1}t_3^2t_4^7]t_4^8$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\Longrightarrow yx^{-1}t_1t_{16}t_{35} = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9 = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\Longrightarrow yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\Longrightarrow \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\Longrightarrow t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_{32} = Hxy^{-1}t_3^2t_4^7t_4^8$

$\Longrightarrow Ht_1t_{32} = Ht_3^2t_4^4$

$\Longrightarrow Ht_1t_{32} = H\underline{t_7}t_{16}$

$\Longrightarrow Ht_1t_{32} = Ht_{25}t_{16}$, since $Ht_7 = Ht_{25}$

$Ht_1t_{32} = H\underline{t_{25}t_{16}}$

$Ht_1t_{32} = Ht_{16}t_{25}$, since by Equation 5.9

$Ht_1t_6 = Ht_6t_1$

$[Ht_1t_6]^{yx^{-1}} = [Ht_6t_1]^{yx^{-1}}$

$\Longrightarrow Ht_{16}t_{25} = Ht_{25}t_{16}$

$Ht_1t_{32} = Ht_{16}t_{25}$

$\Longrightarrow Ht_1t_{32} \in [16]$, since $Ht_{16}t_{25}$ is in [1 6].

Two symmetric generators will go to [1 6].

$Ht_1t_{37} = Ht_1t_{37}$
$\implies Ht_1t_{37} = Ht_1t_1^{10}$
$\implies Ht_1t_{37} = He$
$\implies Ht_1t_{37} \in [*]$, since He is in [*]
Two symmetric generators will go to [*].

$Ht_1t_{29} = Ht_1t_{29}$
$\implies Ht_1t_{29} = Ht_1t_1^8$
$\implies Ht_1t_{29} = Ht_1^9$
$\implies Ht_1t_{29} = Ht_{33}$
$\implies Ht_1t_{29} \in [1]$, since Ht_{33} is in [1].
Two symmetric generators will go to [1].

The orbits of $N^{(5)}$ on
{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,
35,36,37,38,39,40}
are $\{1,35\}$, $\{2,12\}$, $\{3,17\}$, $\{4,14\}$, $\{5,27\}$, $\{6,24\}$, $\{7,37\}$, $\{8,30\}$, $\{9,19\}$, $\{10,36\}$, $\{11,13\}$, $\{15,33\}$, $\{16,18\}$, $\{20,34\}$, $\{21,39\}$, $\{22,28\}$, $\{23,29\}$, $\{25,31\}$, $\{26,40\}$, and $\{32,38\}$.

We want to see to which double coset $Ht_5t_1, Ht_5t_2, Ht_5t_{17}, Ht_5t_{14}, Ht_5t_5, Ht_5t_6, Ht_5t_{37}, Ht_5t_{30}$, $Ht_5t_9, Ht_5t_{10}, Ht_5t_{13}, Ht_5t_{33}, Ht_5t_{18}, Ht_5t_{20}, Ht_5t_{21}, Ht_5t_{28}, Ht_5t_{29}, Ht_5t_{25}, Ht_5t_{26}$, and Ht_5t_{32} belong.

$Ht_5t_1 = Ht_1^2t_1$
$\implies Ht_5t_1 = Ht_1^3$
$\implies Ht_5t_1 = Ht_9$
$\implies Ht_5t_1 \in [1]$, since Ht_9 is in [1].
Two symmetric generators will go to [1].

$Ht_5t_2 = H\underline{t_1^2}t_2$

$\implies Ht_5t_2 = H[y^{-1}x^{-1}t_3t_2^4]t_2$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}} = e^{y^{-1}}$

$\implies x^{-1}y^{-1}t_3t_{14}t_{33} = e$

$\implies x^{-1}y^{-1}t_3t_2^4t_1^9 = e$

$\implies x^{-1}y^{-1}t_3t_2^4t_1^9\underline{t_1^2} = \underline{t_1^2}$

$\implies x^{-1}y^{-1}t_3t_2^4 = t_1^2$

$Ht_5t_2 = Hy^{-1}x^{-1}t_3t_2^4t_2$

$\implies Ht_5t_2 = Ht_3t_2^5$

$\implies Ht_5t_2 = H\underline{t_3}t_{18}$

$\implies Ht_5t_2 = Ht_{13}t_{18}$, since $Ht_3 = Ht_{13}$.

$Ht_5t_2 = Ht_{13}t_{18}$

$\implies Ht_5t_2 \in [110]$, since $Ht_{13}t_{18}$ is in [1 10].

Two symmetric generators will go to [1 10].

$Ht_5t_{17} = Ht_1^2t_1^5$

$\implies Ht_5t_{17} = Ht_1^7$

$\implies Ht_5t_{17} = Ht_{25}$

$\implies Ht_5t_{17} \in [1]$, since Ht_{25} is in [1].

Two symmetric generators will go to [1].

$Ht_5t_{14} = H\underline{t_5}t_{14}$

$\implies Ht_5t_{14} = Ht_{27}t_{14}$, since $Ht_5 = Ht_{27}$

$Ht_5t_{14} = H\underline{t_{27}t_{14}}$

$\implies Ht_5t_{14} = Ht_{14}t_{27}$, since by Equation 5.9

$Ht_1t_6 = Ht_6t_1$

$\implies [Ht_1t_6]^{yx} = [Ht_6t_1]^{yx}$

$\implies Ht_{14}t_{27} = Ht_{27}t_{14}$

$Ht_5t_{14} = Ht_{14}t_{27}$

$\implies Ht_5t_{14} \in [16]$, since $Ht_{14}t_{27}$ is in [1 6].

Two symmetric generators will go to [1 6].

$Ht_5t_5 = Ht_1^2t_1^2$
$\Longrightarrow Ht_5t_5 = Ht_1^4$
$\Longrightarrow Ht_5t_5 = Ht_{13}$
$\Longrightarrow Ht_5t_5 \in [1]$, since Ht_{13} is in [1].
Two symmetric generators will go to [1].

$Ht_5t_6 = Ht_1^2\underline{t_2^2}$
$\Longrightarrow Ht_5t_6 = Ht_1^2[x^{-1}y^{-1}t_4t_3^4]$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{y^{-1}x} = e^{y^{-1}x}$
$\Longrightarrow x^{-1}y^{-1}t_4t_{15}t_{34} = e$
$\Longrightarrow x^{-1}y^{-1}t_4t_3^4t_2^9 = e$
$\Longrightarrow x^{-1}y^{-1}t_4t_3^4t_2^9\underline{t_2^2} = \underline{t_2^2}$
$\Longrightarrow x^{-1}y^{-1}t_4t_3^4 = t_2^2$
$Ht_5t_6 = Ht_1^2x^{-1}y^{-1}t_4t_3^4$
$\Longrightarrow Ht_5t_6 = Hx^{-1}y^{-1}[t_1^2]^{x^{-1}y^{-1}}t_4t_3^4$
$\Longrightarrow Ht_5t_6 = Ht_4^7t_4t_3^4$
$\Longrightarrow Ht_5t_6 = H\underline{t_4^8}t_3^4$
$\Longrightarrow Ht_5t_6 = Ht_2^{10}t_3^4$, since
$Ht_{32} = Ht_{38}$
$\Longrightarrow Ht_4^8 = Ht_2^{10}$
$Ht_5t_6 = H\underline{t_2^{10}}t_3^4$
$\Longrightarrow Ht_5t_6 = H[x^{-1}yt_4^9t_3^5]t_3^4$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{yx} = e^{yx}$
$\Longrightarrow x^{-1}yt_{36}t_{19}t_2 = e$
$\Longrightarrow x^{-1}yt_4^9t_3^5t_2 = e$
$\Longrightarrow x^{-1}yt_4^9t_3^5t_2\underline{t_2^{10}} = \underline{t_2^{10}}$
$\Longrightarrow x^{-1}yt_4^9t_3^5 = t_2^{10}$
$Ht_5t_6 = Hx^{-1}yt_4^9t_3^5t_3^4$
$\Longrightarrow Ht_5t_6 = Ht_4^9t_3^9$

$\Longrightarrow Ht_5t_6 = H\underline{t_{36}}t_{35}$

$\Longrightarrow Ht_5t_6 = Ht_{34}t_{35}$, since $Ht_{34} = Ht_{36}$

$Ht_5t_6 = Ht_{34}t_{35}$

$\Longrightarrow Ht_5t_6 \in [110]$, since $Ht_{34}t_{35}$ is in [1 10].

Two symmetric generators will go to [1 10].

$Ht_5t_{37} = Ht_1^2t_1^{10}$

$\Longrightarrow Ht_5t_{37} = Ht_1$

$\Longrightarrow Ht_5t_{37} \in [1]$, since Ht_1 is in [1].

Two symmetric generators will go to [1].

$Ht_5t_{30} = H\underline{t_1^2}t_2^8$

$Ht_5t_{30} = H[y^{-1}x^{-1}t_3t_2^4]t_2^8$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}} = e^{y^{-1}}$

$\Longrightarrow x^{-1}y^{-1}t_3t_{14}t_{33} = e$

$\Longrightarrow x^{-1}y^{-1}t_3t_2^4t_1^9 = e$

$\Longrightarrow x^{-1}y^{-1}t_3t_2^4t_1^9\underline{t_1^2} = \underline{t_1^2}$

$\Longrightarrow x^{-1}y^{-1}t_3t_2^4 = t_1^2$

$Ht_5t_{30} = Hy^{-1}x^{-1}t_3t_2^4t_2^8$

$\Longrightarrow Ht_5t_{30} = Ht_3t_2$

$\Longrightarrow Ht_5t_{30} \in [14]$, since Ht_3t_2 is in [1 4].

Two symmetric generators will go to [1 4].

$Ht_5t_9 = Ht_1^2t_1^3$

$\Longrightarrow Ht_5t_9 = Ht_1^5$

$\Longrightarrow Ht_5t_9 = Ht_{17}$

$\Longrightarrow Ht_5t_9 \in [1]$, since Ht_{17} is in [1].

Two symmetric generators will go to [1].

$Ht_5t_{10} = H\underline{t_1^2}t_2^3$

$Ht_5t_{10} = H[y^{-1}x^{-1}t_3t_2^4]t_2^3$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}} = e^{y^{-1}}$

$\Longrightarrow x^{-1}y^{-1}t_3t_{14}t_{33} = e$

$\Longrightarrow x^{-1}y^{-1}t_3t_2^4t_1^9 = e$

$\Longrightarrow x^{-1}y^{-1}t_3t_2^4t_1^9\underline{t_1^2} = \underline{t_1^2}$

$\Longrightarrow x^{-1}y^{-1}t_3t_2^4 = t_1^2$

$Ht_5t_{10} = Hy^{-1}x^{-1}t_3t_2^4t_2^3$

$\Longrightarrow Ht_5t_{10} = Ht_3t_2^7$

$\Longrightarrow Ht_5t_{10} = H\underline{t_3}t_{26}$

$\Longrightarrow Ht_5t_{10} = Ht_{13}t_{26}$, since $Ht_3 = Ht_{13}$

$Ht_5t_{10} = Ht_{13}t_{26}$

$\Longrightarrow Ht_5t_{10} \in [16]$, since $Ht_{13}t_{26}$ is in [1 6].

Two symmetric generators will go to [1 6].

$Ht_5t_{13} = Ht_1^2t_1^4$

$\Longrightarrow Ht_5t_{13} = Ht_1^6$

$\Longrightarrow Ht_5t_{13} = Ht_{21}$

$\Longrightarrow Ht_5t_{13} \in [5]$, since Ht_{21} is in [5].

Two symmetric generators will go to [5].

$Ht_5t_{33} = Ht_1^2t_1^9$

$\Longrightarrow Ht_5t_{33} = H$

$\Longrightarrow Ht_5t_{33} \in [*]$, since He is in [*].

Two symmetric generators will go to [*].

$Ht_5t_{18} = H\underline{t_1^2}t_2^5$

$Ht_5t_{18} = H[y^{-1}x^{-1}t_3t_2^4]t_2^5$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}} = e^{y^{-1}}$

$\Longrightarrow x^{-1}y^{-1}t_3t_{14}t_{33} = e$

$\Longrightarrow x^{-1}y^{-1}t_3t_2^4t_1^9 = e$

$\Longrightarrow x^{-1}y^{-1}t_3t_2^4t_1^9\underline{t_1^2} = \underline{t_1^2}$

$\Longrightarrow x^{-1}y^{-1}t_3t_2^4 = t_1^2$

$Ht_5t_{18} = Hy^{-1}x^{-1}t_3t_2^4t_2^3$

$\Longrightarrow Ht_5t_{18} = Ht_3t_2^9$

$\Longrightarrow Ht_5t_{18} = H\underline{t_3}t_{34}$

$\Longrightarrow Ht_5t_{18} = Ht_{13}t_{34}$, since $Ht_3 = Ht_{13}$

$Ht_5t_{18} = Ht_{13}t_{34}$

$\Longrightarrow Ht_5t_{18} \in [12]$, since $Ht_{13}t_{34}$ is in [1 2].

Two symmetric generators will go to [1 2].

$Ht_5t_{20} = Ht_1^2\underline{t_4^5}$

$Ht_5t_{20} = Ht_1^2[xyt_2^7t_3^{10}]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy^{-1}} = e^{xy^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{20}t_3t_{14} = e$

$\Longrightarrow y^{-1}x^{-1}t_4^5t_3t_2^4 = e$

$\Longrightarrow y^{-1}x^{-1}t_4^5t_3t_2^4\underline{t_2^7} = \underline{t_2^7}$

$\Longrightarrow y^{-1}x^{-1}t_4^5t_3\underline{t_3^{10}} = t_2^7\underline{t_3^{10}}$

$\Longrightarrow \underline{xy}y^{-1}x^{-1}t_4^5 = \underline{xy}t_2^7t_3^{10}$

$\Longrightarrow \underline{t_4^5} = xyt_2^7t_3^{10}$

$Ht_5t_{20} = Ht_1^2xyt_2^7t_3^{10}$

$\Longrightarrow Ht_5t_{20} = Hxy[t_1^2]xyt_2^7t_3^{10}$

$\Longrightarrow Ht_5t_{20} = Ht_2^7t_2^7t_3^{10}$

$\Longrightarrow Ht_5t_{20} = Ht_2^3t_3^{10}$

$\Longrightarrow Ht_5t_{20} = Ht_{10}t_{39}$

$\Longrightarrow Ht_5t_{20} \in [16]$, since $Ht_{10}t_{39}$ is in [1 6].

Two symmetric generators will go to [1 6].

$Ht_5t_{21} = Ht_1^2t_1^6$

$\Longrightarrow Ht_5t_{21} = Ht_1^8$

$\Longrightarrow Ht_5t_{21} = Ht_{29}$

$\Longrightarrow Ht_5t_{21} \in [5]$, since Ht_{29} is in [5].

Two symmetric generators will go to [5].

$Ht_5t_{28} = H\underline{t_5}t_{28}$

$\Longrightarrow Ht_5t_{28} = Ht_{27}t_{28}$, since $Ht_5 = Ht_{27}$

$Ht_5t_{28} = Ht_{27}t_{28}$

$\Longrightarrow Ht_5t_{28} = Ht_3^7\underline{t_4^7}$

$\Longrightarrow Ht_5t_{28} = Ht_3^7[yx^{-1}t_2^5t_1]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\Longrightarrow yx^{-1}t_{18}t_1t_{16} = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4 = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\Longrightarrow yx^{-1}t_2^5t_1 = t_4^7$

$Ht_5t_{28} = Ht_3^7yx^{-1}t_2^5t_1$

$\Longrightarrow Ht_5t_{28} = Hyx^{-1}[t_3^7]^{yx^{-1}}t_2^5t_1$

$\Longrightarrow Ht_5t_{28} = Ht_2^{10}t_2^5t_1$

$\Longrightarrow Ht_5t_{28} = Ht_2^4t_1$

$\Longrightarrow Ht_5t_{28} = H\underline{t_{14}}t_1$

$\Longrightarrow Ht_5t_{28} = Ht_4t_1$, since $Ht_4 = Ht_{14}$

$Ht_5t_{28} = Ht_4t_1$

$\Longrightarrow Ht_5t_{28} \in [12]$, since Ht_4t_1 is in [1 2]

Two symmetric generators will go to [1 2].

$Ht_5t_{29} = Ht_1^2t_1^8$

$\Longrightarrow Ht_5t_{29} = Ht_1^{10}$

$\Longrightarrow Ht_5t_{29} = Ht_{37}$

$\Longrightarrow Ht_5t_{29} \in [5]$, since Ht_{37} is in [5].

Two symmetric generators will go to [5].

$Ht_5t_{25} = Ht_1^2t_1^7$

$\Longrightarrow Ht_5t_{25} = Ht_1^9$

$\Longrightarrow Ht_5t_{25} = Ht_{33}$

$\Longrightarrow Ht_5t_{25} \in [1]$, since Ht_{33} is in [1].

Two symmetric generators will go to [1].

$Ht_5t_{26} = H\underline{t_1^2}t_2^7$
$\Longrightarrow Ht_5t_{26} = H[y^{-1}x^{-1}t_3t_2^4]t_2^7$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{y^{-1}} = e^{y^{-1}}$
$\Longrightarrow x^{-1}y^{-1}t_3t_{14}t_{33} = e$
$\Longrightarrow x^{-1}y^{-1}t_3t_2^4t_1^9 = e$
$\Longrightarrow x^{-1}y^{-1}t_3t_2^4t_1^9\underline{t_1^2} = \underline{t_1^2}$
$\Longrightarrow x^{-1}y^{-1}t_3t_2^4 = t_1^2$
$Ht_5t_{26} = Hy^{-1}x^{-1}t_3t_2^4t_2^7$
$\Longrightarrow Ht_5t_{26} = Ht_3$
$\Longrightarrow Ht_5t_{26} \in [1]$, since Ht_3 is in [1].
Two symmetric generators will go to [1].

$Ht_5t_{32} = H\underline{t_5}t_{32}$
$\Longrightarrow Ht_5t_{32} = Ht_{27}t_{32}$, since $Ht_5 = Ht_{27}$
$Ht_5t_{32} = Ht_{27}t_{32}$
$\Longrightarrow Ht_5t_{32} = Ht_3^7\underline{t_4^8}$
$\Longrightarrow Ht_5t_{32} = Ht_3^7[x^{-1}t_2^3t_1^3]$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{x^{-1}} = e^{x^{-1}}$
$\Longrightarrow x^{-1}t_{10}t_9t_{12} = e$
$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3 = e$
$\Longrightarrow x^{-1}t_2^5t_1t_4^3\underline{t_4^8} = \underline{t_4^8}$
$\Longrightarrow x^{-1}t_2^5t_1 = t_4^8$
$Ht_5t_{28} = Ht_3^7x^{-1}t_2^3t_1^3$
$\Longrightarrow Ht_5t_{32} = Hyx^{-1}[t_3^7]^{yx^{-1}}t_2^3t_1^3$
$\Longrightarrow Ht_5t_{32} = Ht_2^7t_2^3t_1^3$
$\Longrightarrow Ht_5t_{32} = Ht_2^{10}t_1^3$
$\Longrightarrow Ht_5t_{32} = H\underline{t_{38}t_9}$
$\Longrightarrow Ht_5t_{32} = Ht_9t_{38}$, since by Equation 5.9

$Ht_1t_6 = Ht_6t_1$

$\Longrightarrow [Ht_1t_6]^{y^{-1}} = [Ht_6t_1]^{y^{-1}}$

$\Longrightarrow Ht_5t_{38} = Ht_{38}t_5$

$Ht_5t_{32} = Ht_9t_{38}$

$\Longrightarrow Ht_5t_{32} \in [16]$, since Ht_9t_{38} is in [1 6].

Two symmetric generators will go to [1 6].

Word of Length 2

$N^{(12)}$ = Coset Stabiliser in N of $Ht_1t_2 = \{n \in N | (Ht_1t_2)^n = t_1t_2\}$.

We will look for a relation that will increase the Coset Stabiliser $N^{(12)}$.

$Ht_1t_6 = Ht_6t_1$, by Equation 5.9

$\Longrightarrow [Ht_1t_6]^{x^2} = [Ht_6t_1]^{x^2}$

$\Longrightarrow Ht_3t_8 = Ht_8t_3$

$\Longrightarrow Ht_3t_4^2 = Ht_4^2t_3$

$\Longrightarrow Ht_3t_4^2\underline{t_4^{10}} = Ht_4^2t_3\underline{t_4^{10}}$

$\Longrightarrow Ht_3t_4 = Ht_4^2t_3\underline{t_4^{10}}$

$\Longrightarrow Ht_3t_4 = Ht_4^2t_3[xyxt_4t_2^4]$, since by Equation 5.7

$[x^2t_{33}t_{35}t_{33}]^{x^{-1}y} = e^{x^{-1}y}$

$\Longrightarrow xyxt_4t_{14}t_4 = e$

$\Longrightarrow xyxt_4t_2^4t_4 = e$

$\Longrightarrow xyxt_4t_2^4t_4\underline{t_4^{10}} = \underline{t_4^{10}}$

$\Longrightarrow xyxt_4t_2^4 = t_4^{10}$

$Ht_3t_4 = Ht_4^2t_3[xyxt_4t_2^4]$

$\Longrightarrow Ht_3t_4 = Hxyx[t_4^2t_3]^{xyx}t_4t_2^4$

$\Longrightarrow Ht_3t_4 = Ht_2^8\underline{t_1^5}t_4t_2^4$

$\Longrightarrow Ht_3t_4 = Ht_2^8[yxt_3^7t_4^{10}]t_4t_2^4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy^{-1}x} = e^{xy^{-1}x}$

$\Longrightarrow x^{-1}y^{-1}t_{17}t_4t_{15} = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4t_3^4 = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4t_3^4\underline{t_3^7} = \underline{t_3^7}$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4\underline{t_4^{10}} = t_3^7\underline{t_4^{10}}$
$\Longrightarrow \underline{yx}x^{-1}y^{-1}t_1^5 = \underline{yx}t_3^7t_4^{10}$
$\Longrightarrow t_1^5 = yxt_3^7t_4^{10}$
$Ht_3t_4 = Ht_2^8yxt_3^7t_4^{10}t_4t_2^4$
$\Longrightarrow Ht_3t_4 = Hyx[t_2^8]^{yx}t_3^7t_2^4$
$\Longrightarrow Ht_3t_4 = Ht_3^6t_3^7t_2^4$
$\Longrightarrow Ht_3t_4 = H\underline{t_3^2}t_2^4$
$\Longrightarrow Ht_3t_4 = Ht_1^7t_2^4$, since
$Ht_7 = Ht_{25}$
$\Longrightarrow Ht_3^2 = Ht_1^7$
$Ht_3t_4 = H\underline{t_1^7}t_2^4$
$\Longrightarrow Ht_3t_4 = H[x^{-1}yt_3^5t_2]t_2^4$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{y^2} = e^{y^2}$
$\Longrightarrow x^{-1}yt_{19}t_2t_{13} = e$
$\Longrightarrow x^{-1}yt_3^5t_2t_1^4 = e$
$\Longrightarrow x^{-1}yt_3^5t_2t_1^4\underline{t_1^7} = \underline{t_1^7}$
$\Longrightarrow x^{-1}yt_3^5t_2 = t_1^7$
$Ht_3t_4 = Hx^{-1}yt_3^5t_2t_2^4$
$\Longrightarrow Ht_3t_4 = H\underline{t_3^5}t_2^5$
$\Longrightarrow Ht_3t_4 = Ht_1^3t_2^5$, since
$Ht_9 = Ht_{19}$
$\Longrightarrow Ht_1^3 = Ht_3^5$
$Ht_3t_4 = Ht_1^3t_2^5 \Longrightarrow Ht_3t_4 = Ht_9t_{18}$
$\Longrightarrow [Ht_3t_4]^{x^2} = [Ht_9t_{18}]^{x^2}$
$\Longrightarrow Ht_1t_2 = Ht_{11}t_{20}$.

Since, $Ht_1t_2^e = Ht_1t_2 \Rightarrow e \in N^{(12)}$, and
$Ht_1t_2^{x^2y} = Ht_{11}t_{20} = Ht_1t_2 \Rightarrow x^2y \in N^{(12)}$, then,
$N^{(12)} =$ Coset Stabiliser in N of $Ht_1t_2 = \{n \in N|(Ht_1t_2)^n = t_1t_2\} = \{e, x^2y\}$.
Furthermore, the number of single cosets of Ht_1t_2N is $\frac{|N|}{|N^{(12)}|} = \frac{20}{2} = 10$.

We find the equal names by conjugating $t_1t_2 \sim t_{11}t_{20}$ by elements of N.

$t_1t_2 \sim t_{11}t_{20}$

$t_2t_3 \sim t_{12}t_{17}$

$t_4t_1 \sim t_{10}t_{19}$

$t_9t_{18} \sim t_3t_4$

$t_{13}t_{34} \sim t_{35}t_{12}$

$t_{14}t_{35} \sim t_{36}t_9$

$t_{17}t_{14} \sim t_{19}t_{16}$

$t_{18}t_{15} \sim t_{20}t_{13}$

$t_{34}t_{11} \sim t_{16}t_{33}$

$t_{35}t_{10} \sim t_{15}t_{36}$

Therefore, $Ht_1t_2N = \{Ht_1t_2 = Ht_{11}t_{20}, Ht_2t_3 = Ht_{12}t_{17}, Ht_4t_1 = Ht_{10}t_{19},$
$Ht_9t_{18} = Ht_3t_4, Ht_{13}t_{34} = Ht_{35}t_{12}, Ht_{14}t_{35} = Ht_{36}t_9, Ht_{17}t_{14} = Ht_{19}t_{16},$
$Ht_{18}t_{15} = Ht_{20}t_{13}, Ht_{34}t_{11} = Ht_{16}t_{33}, Ht_{35}t_{10} = Ht_{15}t_{36}\}$

$N^{(14)}$ = Coset Stabiliser in N of $Ht_1t_4 = \{n \in N | (Ht_1t_4)^n = t_1t_4\}$.

We will look for a relation that will increase the Coset Stabiliser $N^{(14)}$.

$Ht_1t_4 = H\underline{t_1}t_4$
$\implies Ht_1t_4 = H[xy^{-1}t_3^2t_4^7]t_4$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$
$\implies yx^{-1}t_1t_{16}t_{35} = e$
$\implies yx^{-1}t_1t_4^4t_3^9 = e$
$\implies yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$
$\implies yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$
$\implies \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$
$\implies t_1 = xy^{-1}t_3^2t_4^7$
$Ht_1t_4 = Hxy^{-1}t_3^2t_4^7t_4$
$\implies Ht_1t_4 = Ht_3^2\underline{t_4^8}$
$\implies Ht_1t_4 = Ht_3^2[x^{-1}t_2^3t_1^3]$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{x^{-1}} = e^{x^{-1}}$
$\implies x^{-1}t_{10}t_9t_{12} = e$
$\implies x^{-1}t_2^3t_1^3t_4^3 = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3\underline{t_4^8} = \underline{t_4^8}$

$\Longrightarrow x^{-1}t_2^3t_1^3 = t_4^8$

$Ht_1t_4 = Ht_3^2x^{-1}t_2^3t_1^3$

$\Longrightarrow Ht_1t_4 = Hx^{-1}[t_3^2]^{x^{-1}}t_2^3t_1^3$

$\Longrightarrow Ht_1t_4 = Ht_2^2t_2^3t_1^3$

$\Longrightarrow Ht_1t_4 = Ht_2^5t_1^3$

$\Longrightarrow Ht_1t_4 = Ht_{18}t_9$

Also, $Ht_1t_4 = Ht_{18}t_9$

$\Longrightarrow [Ht_1t_4]^{yx^{-1}} = [Ht_{18}t_9]^{yx^{-1}}$

$\Longrightarrow Ht_{16}t_{19} = Ht_1t_4$

$Ht_1t_4 = Ht_{16}t_{19}$

$\Longrightarrow [Ht_1t_4]^{xyx} = [Ht_{16}t_{19}]^{xyx}$

$\Longrightarrow Ht_{35}t_{14} = Ht_{18}t_9$

$Ht_1t_4 = Ht_{16}t_{19}$

$\Longrightarrow [Ht_1t_4]^{xy^{-1}} = [Ht_{16}t_{19}]^{xy^{-1}}$

$\Longrightarrow Ht_{18}t_9 = Ht_1t_4$

Since, $Ht_1t_4^e = Ht_1t_4 \Rightarrow e \in N^{(14)}$,
$Ht_1t_4^{yx^{-1}} = Ht_{16}t_{19} = Ht_1t_4 \Rightarrow yx^{-1} \in N^{(14)}$,
$Ht_1t_4^{xy^{-1}} = Ht_{18}t_9 = Ht_1t_4 \Rightarrow xyx^{-1} \in N^{(14)}$, and
$Ht_1t_4^{xyx} = Ht_{35}t_{14} = Ht_{18}t_9 = Ht_1t_4 \Rightarrow xyx^{-1} \in N^{(14)}$
then,
$N^{(14)} =$ Coset Stabiliser in N of $Ht_1t_4 = \{n \in N | (Ht_1t_4)^n = t_1t_4\} = \{e, yx^{-1}, xy^{-1}, xyx\}$.
Furthermore, the number of single cosets of Ht_1t_4N is $\frac{|N|}{|N^{(14)}|} = \frac{20}{4} = 5$.

We find the equal names by conjugating $t_1t_4 \sim t_{35}t_{14} \sim t_{18}t_9 \sim t_{16}t_{19}$ by elements of N.

$t_1t_4 \sim t_{35}t_{14} \sim t_{18}t_9 \sim t_{16}t_{19}$

$t_2t_1 \sim t_{36}t_{15} \sim t_{19}t_{10} \sim t_{13}t_{20}$

$t_4t_3 \sim t_{34}t_{13} \sim t_{17}t_{12} \sim t_{15}t_{18}$

$t_9t_{36} \sim t_{11}t_{34} \sim t_{10}t_{33} \sim t_{12}t_{35}$

$t_3t_2 \sim t_{33}t_{16} \sim t_{20}t_{11} \sim t_{14}t_{17}$

Therefore, $Ht_1t_4N = \{Ht_1t_4 = Ht_{35}t_{14} = Ht_{18}t_9 = Ht_{16}t_{19},$
$Ht_2t_1 = Ht_{36}t_{15} = Ht_{19}t_{10} = t_{13}t_{20},$
$Ht_4t_3 = Ht_{34}t_{13} = Ht_{17}t_{12} = Ht_{15}t_{18},$
$Ht_9t_{36} = Ht_{11}t_{34} = Ht_{10}t_{33} = Ht_{12}t_{35},$
$Ht_3t_2 = Ht_{33}t_{16} = Ht_{20}t_{11} = t_{14}t_{17}\}$

$N^{(16)}$ = Coset Stabiliser in N of $Ht_1t_6 = \{n \in N | (Ht_1t_6)^n = t_1t_6\}$.

We do not have a relation that will increase the Coset Stabiliser $N^{(16)}$.

$N^{(16)}$ = Coset Stabiliser in N of $Ht_1t_6 = \{n \in N | (Ht_1t_6)^n = t_1t_6\} = \{e\}$.
Furthermore, the number of single cosets of Ht_1t_6N is $\frac{|N|}{|N^{(16)}|} = \frac{20}{1} = 20$.

Ht_1t_6 conjugated by elements of N gives us the following cosets in Ht_1t_6N.
$Ht_1t_6N = \{Ht_1t_6, Ht_2t_7, Ht_{13}t_{26}, Ht_4t_5, Ht_9t_{38}, Ht_3t_8, Ht_{34}t_{23}, Ht_{18}t_{31}, Ht_{14}t_{27}, Ht_{17}, t_{30},$
$Ht_{16}t_{25}, Ht_{20}t_{29}, Ht_{36}t_{21}, Ht_{10}t_{39}, Ht_{12}t_{37}, Ht_{33}t_{22}, Ht_{11}t_{40}, Ht_{15}t_{28}, Ht_{35}t_{24}, Ht_{19}t_{32}\}$

$N^{(16)}$ = Coset Stabiliser in N of $Ht_1t_6 = \{n \in N | (Ht_1t_6)^n = t_1t_6\} = \{e\}$.
Furthermore, the number of single cosets of Ht_1t_6N is $\frac{|N|}{|N^{(16)}|} = \frac{20}{1} = 20$.

$N^{(110)}$ = Coset Stabiliser in$H$ of $Ht_1t_{10} = \{n \in N | (Ht_1t_{10})^n = t_1t_{10}\}$.

We will look for a relation that will increase the Coset Stabiliser $N^{(110)}$.

$Ht_{15} = Ht_1$
$\implies [Ht_{15}]^{x^2y} = [Ht_1]^{x^2y}$
$\implies Ht_{17} = Ht_{11}$
$\implies Ht_1^5 = Ht_3^3$

$\Longrightarrow Ht_1^5 = Ht_3^3\underline{t_3^{11}}$

$\Longrightarrow Ht_1^5 = Ht_3^3(t_3^6t_3^5)$

$\Longrightarrow Ht_1^5 = Ht_3^9\underline{t_3^5}$

$\Longrightarrow Ht_1^5 = Ht_3^9[y^{-1}xt_1^7t_2^{10}]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^2} = e^{y^2}$

$\Longrightarrow x^{-1}yt_{19}t_2t_{13} = e$

$\Longrightarrow x^{-1}yt_3^5t_2t_1^4 = e$

$\Longrightarrow x^{-1}yt_3^5t_2t_1^4\underline{t_1^7} = \underline{t_1^7}$

$\Longrightarrow x^{-1}yt_3^5t_2\underline{t_2^{10}} = t_1^7\underline{t_2^{10}}$

$\Longrightarrow \underline{y^{-1}x}x^{-1}yt_3^5 = \underline{y^{-1}x}t_1^7t_2^{10}$

$\Longrightarrow t_3^5 = y^{-1}xt_1^7t_2^{10}$

$Ht_1^5 = Ht_3^9y^{-1}xt_1^7t_2^{10}$

$\Longrightarrow Ht_1^5 = Hy^{-1}x[t_3^9]^{y^{-1}x}t_1^7t_2^{10}$

$\Longrightarrow Ht_1^5\underline{t_2} = Ht_4^3t_1^7t_2^{10}\underline{t_2}$

$\Longrightarrow Ht_1^5t_2 = Ht_4^3t_1^7$

$\Longrightarrow Ht_1^5t_2 = Ht_4^3\underline{t_1^7}$

$\Longrightarrow Ht_1^5t_2 = Ht_4^3(t_1^5t_1^2)$

$\Longrightarrow Ht_1^5t_2 = H\underline{t_4^3t_1^5}t_1^2$

$\Longrightarrow Ht_1^5t_2 = H[t_2t_3]t_1^2$, since

$Ht_1t_2 = Ht_{11}t_{20}$

$\Longrightarrow [Ht_1t_2]^x = [Ht_{11}t_{20}]^x$

$\Longrightarrow Ht_2t_3 = Ht_{12}t_{17}$

$\Longrightarrow Ht_2t_3 = Ht_4^3t_1^5$

$Ht_1^5t_2 = Ht_2t_3t_1^2$

$\Longrightarrow Ht_1^5t_2 = Ht_2\underline{t_3^{11}}t_3t_1^2$

$\Longrightarrow Ht_1^5t_2 = Ht_2(t_3^{10}t_3)t_3t_1^2$

$\Longrightarrow Ht_1^5t_2 = Ht_2t_3^{10}(t_3t_3)t_1^2$

$\Longrightarrow Ht_1^5t_2 = Ht_2t_3^{10}\underline{t_3^2}t_1^2$

$\Longrightarrow Ht_1^5t_2 = Ht_2t_3^{10}[x^2t_3^9t_1^9]t_1^2$, since by Equation 5.7

$x^2t_{33}t_{35}t_{33} = e$

$[x^2t_{33}t_{35}t_{33}]^{x^2} = e^{x^2}$

$\Longrightarrow x^2t_{35}t_{33}t_{35} = e$

$\Longrightarrow x^2t_3^9t_1^9t_3^9 = e$

$\Longrightarrow x^2t_3^9t_1^9t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\Longrightarrow x^2t_3^9t_1^9 = t_3^2$

$Ht_1^5t_2 = Ht_2t_3^{10}x^2t_3^9$

$\Longrightarrow Ht_1^5t_2 = Hx^2[t_2t_3^{10}]^{x^2}t_3^9$

$\Longrightarrow Ht_1^5t_2 = H\underline{t_4}t_1^{10}t_3^9$

$\Longrightarrow Ht_1^5t_2 = Ht_2^4t_1^{10}t_3^9$, since

$Ht_4 = Ht_{14}$

$\Longrightarrow Ht_4 = Ht_2^4$

$Ht_1^5t_2 = H\underline{t_2^4}t_1^{10}t_3^9$

$\Longrightarrow Ht_1^5t_2 = H[xyt_4^6t_1^2]t_1^{10}t_3^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y^{-1}} = e^{x^{-1}y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{14}t_{33}t_{20} = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5 = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5\underline{t_4^6} = \underline{t_4^6}$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9\underline{t_1^2} = t_4^6\underline{t_1^2}$

$\Longrightarrow \underline{xy}y^{-1}x^{-1}t_2^4 = \underline{xy}t_4^6t_1^2$

$\Longrightarrow t_2^4 = xyt_4^6t_1^2$

$Ht_1^5t_2 = Hxyt_4^6t_1^2t_1^{10}t_3^9$

$\Longrightarrow Ht_1^5t_2 = Ht_4^6t_1\underline{t_3^9}$

$\Longrightarrow Ht_1^5t_2 = Ht_4^6t_1t_3^3\underline{t_3^6}$

$\Longrightarrow Ht_1^5t_2 = Ht_4^6t_1t_3^3[x^{-1}yt_1^4t_4^9]$, since by Equation 5.8

$[x^3t_{11}t_{10}t_9]^{xyx} = e^{xyx}$

$\Longrightarrow x^{-1}yt_{13}t_{36}t_{19} = e$

$\Longrightarrow x^{-1}yt_1^4t_4^9t_3^5 = e$

$\Longrightarrow x^{-1}yt_1^4t_4^9t_3^5\underline{t_3^6} = \underline{t_3^6}$

$\Longrightarrow x^{-1}yt_1^4t_4^9 = t_3^6$

$Ht_1^5t_2 = Ht_4^6t_1t_3^3x^{-1}yt_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = Hx^{-1y}[t_4^6t_1t_3^3]^{x^{-1}y}t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = H\underline{t_3^7}t_4^5t_2^5t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = H[x^{-1}y^{-1}t_1^5t_4]t_4^5t_2^5t_1^4t_4^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy^{-1}x} = e^{xy^{-1}x}$

$\Longrightarrow x^{-1}y^{-1}t_{17}t_4t_{15} = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4t_3^4 = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4t_3^4\underline{t_3^7} = \underline{t_3^7}$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4 = t_3^7$

$Ht_1^5t_2 = Hx^{-1}y^{-1}t_1^5t_4t_4^5t_2^5t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = Ht_1^5\underline{t_4^6}t_2^5t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = Ht_1^5[x^2y^{-1}t_4^5t_2^3]t_2^5t_1^4t_4^9$, since by Equation 5.7

$x^2t_{33}t_{35}t_{33} = e$

$[x^2t_{33}t_{35}t_{33}]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\Longrightarrow x^2y^{-1}t_{20}t_{10}t_{20} = e$

$\Longrightarrow x^2y^{-1}t_4^5t_2^3t_4^5 = e$

$\Longrightarrow x^2y^{-1}t_4^5t_2^3t_4^5\underline{t_4^6} = \underline{t_4^6}$

$\Longrightarrow x^2y^{-1}t_4^5t_2^3 = t_4^6$

$Ht_1^5t_2 = Ht_1^5x^2y^{-1}t_4^5t_2^3t_2^5t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = Hx^2y^{-1}[t_1^5]^{x^2y^{-1}}t_4^5t_2^8t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = H\underline{t_3^9}t_4^5t_2^8t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = H[t_1^9]t_4^5t_2^8t_1^4t_4^9$, since

$Ht_{33} = Ht_{35}$

$\Longrightarrow Ht_1^9 = Ht_3^9$

$Ht_1^5t_2 = H\underline{t_1^9}t_4^5t_2^8t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = H[xyt_3^{10}t_4^6]t_4^5t_2^8t_1^4t_4^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y^{-1}} = e^{x^2y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{33}t_{20}t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5\underline{t_4^6} = t_3^{10}\underline{t_4^6}$

$\Longrightarrow \underline{xy}y^{-1}x^{-1}t_1^9 = \underline{xy}t_3^{10}t_4^6$

$\Longrightarrow t_1^9 = xyt_3^{10}t_4^6$

$Ht_1^5t_2 = Hxyt_3^{10}t_4^6t_4^5t_2^8t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = H\underline{t_3^{10}}t_2^8t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = H[t_1^8]t_2^8t_1^4t_4^9$, since

$Ht_{29} = Ht_{39}$

$\Longrightarrow Ht_1^8 = Ht_3^{10}$

$Ht_1^5t_2 = H\underline{t_1^8t_2^8}t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = H[x^3t_3^3]t_1^4t_4^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$\Longrightarrow x^3t_3^3t_2^3t_1^3 = e$

$\Longrightarrow x^3t_3^3t_2^3t_1^3\underline{t_1^8} = \underline{t_1^8}$

$\Longrightarrow x^3t_3^3t_2^3\underline{t_2^8} = t_1^8\underline{t_2^8}$

$\Longrightarrow x^3t_3^3 = t_1^8t_2^8$

$Ht_1^5t_2 = Hx^3t_3^3t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = H\underline{t_3^3}t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = H\underline{t_3^2}t_3t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = Ht_1^7t_3t_1^4t_4^9$, since

$Ht_7 = Ht_{25}$

$\Longrightarrow Ht_3^2 = Ht_1^7$

$Ht_1^5t_2 = Ht_1^7\underline{t_3}t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = Ht_1^7[x^2yt_3^{10}t_1^7]t_1^4t_4^9$, since by Equation 5.7

$[x^2t_{33}t_{35}t_{33}]^{xy^{-1}x} = e^{xy^{-1}x}$

$\Longrightarrow x^2yt_3t_13t_3 = e$

$\Longrightarrow x^2yt_3t_1^4t_3 = e$

$\Longrightarrow x^2yt_3t_1^4t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\Longrightarrow x^2yt_3t_1^4\underline{t_1^7} = t_3^{10}\underline{t_1^7}$

$\Longrightarrow \underline{x^2y}x^2yt_3 = \underline{x^2y}t_3^{10}t_1^7$

$\Longrightarrow t_3 = x^2yt_3^{10}t_1^7$

$Ht_1^5t_2 = Ht_1^7x^2yt_3^{10}t_1^7t_1^4t_4^9$

$\Longrightarrow Ht_1^5t_2 = Hx^2y[t_1^7]^{x^2y}t_3^{10}t_4^9$

$\Longrightarrow Ht_1^5t_2 = Ht_3^{10}t_3^{10}t_4^9$

$\Longrightarrow Ht_1^5t_2 = Ht_3^9t_4^9$

$\Longrightarrow Ht_{17}t_2 = Ht_{35}t_{36}$

$\Longrightarrow [Ht_{17}t_2]^{y^{-2}} = [Ht_{35}t_{36}]^{y^{-2}}$
$\Longrightarrow Ht_1t_{10} = Ht_3t_{12}$

Conjugating by elements in N gives us the following equal names.

$t_1t_{10} \sim t_3t_{12}$	$t_{34}t_{35} \sim t_{20}t_1$	$t_{18}t_3 \sim t_{36}t_{33}$
$t_2t_{11} \sim t_4t_9$	$t_{10}t_{15} \sim t_{16}t_{17}$	$t_{19}t_4 \sim t_{33}t_{34}$
$t_{13}t_{18} \sim t_{11}t_{16}$	$t_{12}t_{13} \sim t_{14}t_{19}$	
$t_9t_{14} \sim t_{15}t_{20}$	$t_{17}t_2 \sim t_{35}t_{36}$	

Since, $Ht_1t_{10}^e = Ht_1t_{10} \Rightarrow e \in N^{(110)}$,
$Ht_1t_{10}^{x^2} = Ht_3t_{12} = Ht_1t_{10} \Rightarrow x^2 \in N^{(110)}$, then,
$N^{(110)} =$ Coset Stabiliser in N of $Ht_1t_{10} = \{n \in N | (Ht_1t_{10}^n = t_1t_{10}\} = \{e, x^2\}$.
Furthermore, the number of single cosets of $Ht_1t_{10}N$ is $\frac{|N|}{|N^{(110)}|} = \frac{20}{2} = 10$.

$Ht_1t_{10}N = \{Ht_1t_{10} = Ht_3t_{12}, Ht_2t_{11} = Ht_4t_9, Ht_{13}t_{18} = Ht_{11}t_{16},$
$Ht_9t_{14} = Ht_{15}t_{20}, Ht_{34}t_{35} = Ht_{20}t_1, Ht_{10}t_{15} = Ht_{16}t_{17},$
$Ht_{12}t_{13} = Ht_{14}t_{19}, Ht_{17}t_2 = Ht_{35}t_{36}, Ht_{18}t_3 = Ht_{36}t_{33}, Ht_{19}t_4 = Ht_{33}t_{34}\}$.

The orbits of $N^{(12)}$ are $\{1, 11\}, \{2, 20\}, \{3, 13\}, \{4, 34\}, \{5, 23\}, \{6, 40\}, \{7, 29\}, \{8, 26\},$
$\{9, 35\}, \{10, 16\}, \{12, 18\}, \{14, 36\}, \{15, 17\}, \{19, 33\}, \{21, 27\}, \{22, 32\}, \{24, 38\}, \{25, 39\},$
$\{28, 30\}$, and $\{31, 37\}$.

We will check to see where $t_1t_2t_1, t_1t_2t_2, t_1t_2t_{13}, t_1t_2t_4, t_1t_2t_5, t_1t_2t_6, t_1t_2t_{29}, t_1t_2t_8, t_1t_2t_9, t_1t_2t_{16},$ $t_1t_2t_{18}, t_1t_2t_{14}, t_1t_2t_{17}, t_1t_2t_{33}, t_1t_2t_{21}, t_1t_2t_{22}, t_1t_2t_{24}, t_1t_2t_{25}, t_1t_2t_{28}$, and $t_1t_2t_{37}$ belong.

$Ht_1t_2t_1 = H\underline{t_1}t_2t_1$
$\Longrightarrow Ht_1t_2t_1 = Ht_{15}t_2t_1$, since $Ht_1 = Ht_{15}$
$\Longrightarrow Ht_1t_2t_1 = Ht_3^4\underline{t_2}t_1$
$\Longrightarrow Ht_1t_2t_1 = Ht_3^4[y^{-1}xt_4^2t_1^7]t_1$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\Longrightarrow x^{-1}yt_2t_{13}t_{36} = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9 = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\Longrightarrow x^{-1}yt_2t_1^4\underline{t_1^7} = t_4^2\underline{t_1^7}$

$\Longrightarrow \underline{y^{-1}x}x^{-1}yt_2 = \underline{y^{-1}x}t_4^2t_1^7$

$\Longrightarrow t_2 = y^{-1}xt_4^2t_1^7$

$Ht_1t_2t_1 = Ht_3^4y^{-1}xt_4^2t_1^7t_1$

$\Longrightarrow Ht_1t_2t_1 = Hy^{-1}x[t_3^4]^{y^{-1}x}t_4^2t_1^8$

$\Longrightarrow Ht_1t_2t_1 = Ht_4^5t_4^2t_1^8$

$\Longrightarrow Ht_1t_2t_1 = Ht_4^7\underline{t_1^8}$

$\Longrightarrow Ht_1t_2t_1 = Ht_4^7[x^3t_3^3t_2^3]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$\Longrightarrow x^3t_3^2t_2^3t_1^3 = e$

$\Longrightarrow x^3t_3^2t_2^3t_1^3\underline{t_1^8} = \underline{t_1^8}$

$\Longrightarrow x^3t_3^2t_2^3 = t_1^8$

$Ht_1t_2t_1 = Ht_4^7x^3t_3^3t_2^3$

$\Longrightarrow Ht_1t_2t_1 = Hx^3[t_4^7]^{x^3}t_3^3t_2^3$

$\Longrightarrow Ht_1t_2t_1 = Ht_3^7t_3^3t_2^3$

$\Longrightarrow Ht_1t_2t_1 = Ht_3^{10}t_2^3$

$\Longrightarrow Ht_1t_2t_1 = H\underline{t_{39}t_{10}}$

$\Longrightarrow Ht_1t_2t_1 = Ht_{10}t_{39}$, since $Ht_{10}t_{39} = Ht_{39}t_{10}$

$\Longrightarrow Ht_1t_2t_1 \in [16]$, since $Ht_{10}t_{39}$ is in [16].

2 symmetric generators will go to [1 6].

$Ht_1t_2t_2 = Ht_1t_2^2$

$\Longrightarrow Ht_1t_2t_2 = Ht_1t_6$

$\Longrightarrow Ht_1t_2t_2 \in [16]$, since Ht_1t_6 is in [1 6].

2 symmetric generators will go to [1 6].

$Ht_1t_2t_{13} = Ht_1t_2t_{13}$

$\Longrightarrow Ht_1t_2t_{13} = Ht_1\underline{t_2}t_1^4$

$\Longrightarrow Ht_1t_2t_{13} = Ht_1[y^{-1}xt_4^2t_1^7]t_1^4$, since by Equation 5.8

$x^3 t_{11} t_{10} t_9 = e$

$[x^3 t_{11} t_{10} t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\implies x^{-1} y t_2 t_{13} t_{36} = e$

$\implies x^{-1} y t_2 t_1^4 t_4^9 = e$

$\implies x^{-1} y t_2 t_1^4 t_4^9 \underline{t_4^2} = \underline{t_4^2}$

$\implies x^{-1} y t_2 t_1^4 \underline{t_1^7} = t_4^2 \underline{t_1^7}$

$\implies \underline{y^{-1}x} x^{-1} y t_2 = \underline{y^{-1}x} t_4^2 t_1^7$

$\implies t_2 = y^{-1} x t_4^2 t_1^7$

$Ht_1 t_2 t_{13} = Ht_1 y^{-1} x t_4^2 t_1^7 t_1^4$

$\implies Ht_1 t_2 t_{13} = Hy^{-1} x [t_1]^{y^{-1}x} t_4^2$

$\implies Ht_1 t_2 t_{13} = H\underline{t_2^3} t_4^2$

$\implies Ht_1 t_2 t_{13} = Ht_4^5 t_4^2$, since

$Ht_{20} = Ht_{10}$

$\implies Ht_4^5 = Ht_4^2$

$Ht_1 t_2 t_{13} = Ht_4^5 t_4^2$

$\implies Ht_1 t_2 t_{13} = Ht_4^7$

$\implies Ht_1 t_2 t_{13} = Ht_{28}$

$\implies Ht_1 t_2 t_{13} \in [5]$, since Ht_{28} is in [5]

2 symmetric generators will go to [5].

$Ht_1 t_2 t_4 = H\underline{t_1 t_2} t_4$

$\implies Ht_1 t_2 t_4 = Ht_{11} t_{20} t_4$, since

$Ht_1 t_2 = Ht_{11} t_{20}$

$Ht_1 t_2 t_4 = Ht_{11} t_{20} t_4$

$\implies Ht_1 t_2 t_4 = Ht_3^3 t_4^5 t_4$

$\implies Ht_1 t_2 t_4 = H\underline{t_3^3} t_4^6$

$\implies Ht_1 t_2 t_4 = Ht_1^5 t_4^6$, since

$Ht_{11} = Ht_{17}$

$\implies Ht_3^3 = Ht_1^5$

$Ht_1 t_2 t_4 = H\underline{t_1^5} t_4^6$

$\implies Ht_1 t_2 t_4 = H[yx t_3^7 t_4^{10}] t_4^6$, since by Equation 5.8

$x^3 t_{11} t_{10} t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy^{-1}x} = e^{xy^{-1}x}$

$\implies x^{-1}y^{-1}t_{17}t_4t_{15} = e$

$\implies x^{-1}y^{-1}t_1^5t_4t_3^4 = e$

$\implies x^{-1}y^{-1}t_1^5t_4t_3^4\underline{t_3^7} = \underline{t_3^7}$

$\implies x^{-1}y^{-1}t_1^5t_4\underline{t_4^{10}} = t_3^7\underline{t_4^{10}}$

$\implies \underline{yx}x^{-1}y^{-1}t_1^5 = \underline{yx}t_3^7t_4^{10}$

$\implies t_1^5 = yxt_3^7t_4^{10}$

$Ht_1t_2t_4 = Hyxt_3^7t_4^{10}t_4^6$

$\implies Ht_1t_2t_4 = H\underline{t_3^7}t_4^5$

$\implies Ht_1t_2t_4 = Ht_1^2t_4^5$, since

$Ht_5 = Ht_{27}$

$\implies Ht_1^2 = Ht_3^7$

$Ht_1t_2t_4 = Ht_1^2\underline{t_4^5}$

$Ht_1t_2t_4 = Ht_1^2[xyt_2^7t_3^{10}]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy^{-1}} = e^{xy^{-1}}$

$\implies y^{-1}x^{-1}t_{20}t_3t_{14} = e$

$\implies y^{-1}x^{-1}t_4^5t_3t_2^4 = e$

$\implies y^{-1}x^{-1}t_4^5t_3t_2^4\underline{t_2^7} = \underline{t_2^7}$

$\implies y^{-1}x^{-1}t_4^5t_3\underline{t_3^{10}} = t_2^7\underline{t_3^{10}}$

$\implies \underline{xy}y^{-1}x^{-1}t_4^5 = \underline{xy}t_2^7t_3^{10}$

$\implies t_4^5 = xyt_2^7t_3^{10}$

$Ht_1t_2t_4 = Ht_1^2xyt_2^7t_3^{10}$

$\implies Ht_1t_2t_4 = Hxy[t_1^2]^{xy}t_2^7t_3^{10}$

$\implies Ht_1t_2t_4 = Ht_2^7t_2^7t_3^{10}$

$\implies Ht_1t_2t_4 = Ht_2^3t_3^{10}$

$\implies Ht_1t_2t_4 = Ht_{10}t_{39}$

$\implies Ht_1t_2t_4 \in [16]$, since $Ht_{10}t_{39}$ is in [1 6].

2 symmetric generators will go to [1 6].

$Ht_1t_2t_5 = Ht_1t_2t_5$

$\implies Ht_1t_2t_5 = H\underline{t_1}t_2t_1^2$

$\Longrightarrow Ht_1t_2t_5 = Ht_3^4t_2t_1^2$, since

$Ht_1 = Ht_{15}$

$\Longrightarrow Ht_1 = Ht_3^4$

$Ht_1t_2t_5 = Ht_3^4\underline{t_2}t_1^2$

$\Longrightarrow Ht_1t_2t_5 = Ht_3^4[y^{-1}xt_4^2t_1^7]t_1^2$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\Longrightarrow x^{-1}yt_2t_{13}t_{36} = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9 = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\Longrightarrow x^{-1}yt_2t_1^4\underline{t_1^7} = t_4^2\underline{t_1^7}$

$\Longrightarrow \underline{y^{-1}x}x^{-1}yt_2 = \underline{y^{-1}x}t_4^2t_1^7$

$\Longrightarrow t_2 = y^{-1}xt_4^2t_1^7$

$Ht_1t_2t_5 = Ht_3^4y^{-1}xt_4^2t_1^7t_1^2$

$\Longrightarrow Ht_1t_2t_5 = Hy^{-1}x[t_3^4]^{y^{-1}x}t_4^2t_1^9$

$\Longrightarrow Ht_1t_2t_5 = Ht_4^5t_4^2t_1^9$

$\Longrightarrow Ht_1t_2t_5 = H\underline{t_4^7}t_1^9$

$\Longrightarrow Ht_1t_2t_5 = H[yx^{-1}t_2^5t_1]t_1^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\Longrightarrow yx^{-1}t_{18}t_1t_{16} = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4 = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\Longrightarrow yx^{-1}t_2^5t_1 = t_4^7$

$Ht_1t_2t_5 = Hyx^{-1}t_2^5t_1t_1^9$

$\Longrightarrow Ht_1t_2t_5 = Ht_2^5t_1^{10}$

$\Longrightarrow Ht_1t_2t_5 = H\underline{t_{18}}t_{37}$

$\Longrightarrow Ht_1t_2t_5 = Ht_{12}t_{37}$, since $Ht_{12} = Ht_{18}$

$Ht_1t_2t_5 \in [16]$, since $Ht_{12}t_{37}$ is in [1 6]

2 symmetric generators will go to [1 6].

$Ht_1t_2t_6 = Ht_1t_2t_6$

$\implies Ht_1t_2t_6 = Ht_1t_2t_2^2$

$\implies Ht_1t_2t_6 = Ht_1t_2^3$

$\implies Ht_1t_2t_6 = Ht_1t_{10}$

$\implies Ht_1t_2t_6 \in [110]$

2 symmetric generators will go to [1 10].

$Ht_1t_2t_{29} = Ht_1t_2t_{29}$

$\implies Ht_1t_2t_{29} = H\underline{t_1}t_2t_1^8$

$\implies Ht_1t_2t_{29} = Ht_3^4t_2t_1^8$, since

$Ht_1 = Ht_{15}$

$\implies Ht_1 = t_3^4$

$Ht_1t_2t_{29} = Ht_3^4\underline{t_2}t_1^8$

$\implies Ht_1t_2t_{29} = Ht_3^4[y^{-1}xt_4^2t_1^7]t_1^8$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\implies x^{-1}yt_2t_{13}t_{36} = e$

$\implies x^{-1}yt_2t_1^4t_4^9 = e$

$\implies x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\implies x^{-1}yt_2t_1^4\underline{t_1^7} = t_4^2\underline{t_1^7}$

$\implies \underline{y^{-1}x}x^{-1}yt_2 = \underline{y^{-1}x}t_4^2t_1^7$

$\implies t_2 = y^{-1}xt_4^2t_1^7$

$Ht_1t_2t_{29} = Ht_3^4y^{-1}xt_4^2t_1^7t_1^8$

$\implies Ht_1t_2t_{29} = Hy^{-1}x[t_3^4]^{y^{-1}x}t_4^2t_1^4$

$\implies Ht_1t_2t_{29} = Ht_4^5t_4^2t_1^4$

$\implies Ht_1t_2t_{29} = H\underline{t_4^7}t_1^4$

$\implies Ht_1t_2t_{29} = H[yx^{-1}t_2^5t_1]t_1^4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\implies yx^{-1}t_{18}t_1t_{16} = e$

$\implies yx^{-1}t_2^5t_1t_4^4 = e$

$\implies yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\implies yx^{-1}t_2^5t_1 = t_4^7$

$Ht_1t_2t_{29} = Hyx^{-1}t_2^5t_1t_1^4$

$\Longrightarrow Ht_1t_2t_{29} = Ht_2^5t_1^5$

$\Longrightarrow Ht_1t_2t_{29} = H\underline{t_{18}}t_{17}$

$\Longrightarrow Ht_1t_2t_{29} = H\underline{t_{12}}t_{17}$, since

$Ht_{12} = Ht_{18}$

$Ht_1t_2t_{29} \in [12]$, since $Ht_{12}t_{17}$ is in [1 2].

2 symmetric generators will go to [1 2].

$Ht_1t_2t_8 = Ht_1t_2t_8$

$\Longrightarrow Ht_1t_2t_8 = H\underline{t_1t_2}t_8$

$\Longrightarrow Ht_1t_2t_8 = H\underline{t_{11}t_{20}}t_8$

$\Longrightarrow Ht_1t_2t_8 = Ht_3^3t_4^5t_4^2$

$\Longrightarrow Ht_1t_2t_8 = H\underline{t_3^3}t_4^7$

$\Longrightarrow Ht_1t_2t_8 = Ht_3\underline{t_3^2}t_4^7$

$\Longrightarrow Ht_1t_2t_8 = Ht_3[yx^{-1}t_1t_4^4]t_4^7$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\Longrightarrow yx^{-1}t_1t_{16}t_{35} = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9 = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\Longrightarrow yx^{-1}t_1t_4^4 = t_3^2$

$Ht_1t_2t_8 = Ht_3yx^{-1}t_1t_4^4t_4^7$

$\Longrightarrow Ht_1t_2t_8 = Hyx^{-1}[t_3]^{yx^{-1}}t_1$

$\Longrightarrow Ht_1t_2t_8 = Ht_2^3t_1$

$\Longrightarrow Ht_1t_2t_8 = Ht_{10}t_1$

$\Longrightarrow Ht_1t_2t_8 = Ht_{20}t_1$, since

$Ht_{10} = Ht_{20}$

$Ht_1t_2t_8 \in [110]$, since $Ht_{20}t_1$ is in [1 10].

2 symmetric generators will go to [1 10].

$Ht_1t_2t_9 = H\underline{t_1}t_2t_9$

$\Longrightarrow Ht_1t_2t_9 = Ht_{15}t_2t_9$, since

$Ht_1 = Ht_{15}$

$Ht_1t_2t_9 = Ht_{15}t_2t_9$

$\implies Ht_1t_2t_9 = Ht_3^4\underline{t_2}t_1^3$

$\implies Ht_1t_2t_9 = Ht_3^4[y^{-1}xt_4^2t_1^7]t_1^3$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\implies x^{-1}yt_2t_{13}t_{36} = e$

$\implies x^{-1}yt_2t_1^4t_4^9 = e$

$\implies x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\implies x^{-1}yt_2t_1^4\underline{t_1^7} = t_4^2\underline{t_1^7}$

$\implies \underline{y^{-1}x}x^{-1}yt_2 = \underline{y^{-1}x}t_4^2t_1^7$

$\implies t_2 = y^{-1}xt_4^2t_1^7$

$Ht_1t_2t_9 = Ht_3^4y^{-1}xt_4^2t_1^7t_1^3$

$\implies Ht_1t_2t_9 = Hy^{-1}x[t_3^4]^{y^{-1}x}t_4^2t_1^{10}$

$\implies Ht_1t_2t_9 = Ht_4^5t_4^2t_1^{10}$

$\implies Ht_1t_2t_9 = H\underline{t_4^7}t_1^{10}$

$\implies Ht_1t_2t_9 = H[yx^{-1}t_2^5t_1]t_1^{10}$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\implies yx^{-1}t_{18}t_1t_{16} = e$

$\implies yx^{-1}t_2^5t_1t_4^4 = e$

$\implies yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\implies yx^{-1}t_2^5t_1 = t_4^7$

$Ht_1t_2t_9 = Hyx^{-1}t_2^5t_1t_1^{10}$

$\implies Ht_1t_2t_9 = Ht_2^5$

$\implies Ht_1t_2t_9 \in [1]$, since Ht_2^5 is in [1].

2 symmetric generators will go to [1].

$Ht_1t_2t_{16} = H\underline{t_1t_2}t_{16}$

$\implies Ht_1t_2t_{16} = Ht_{11}t_{20}t_{16}$, since

$Ht_1t_2 = Ht_{11}t_{20}$

$Ht_1t_2t_{16} = Ht_{11}t_{20}t_{16}$

$\Longrightarrow Ht_1t_2t_{16} = Ht_3^3t_4^5t_4^4$

$\Longrightarrow Ht_1t_2t_{16} = H\underline{t_3^3}t_4^9$

$\Longrightarrow Ht_1t_2t_{16} = Ht_1^5t_4^9$, since

$Ht_{11} = Ht_{17}$

$\Longrightarrow Ht_3^3 = Ht_1^5$

$Ht_1t_2t_{16} = H\underline{t_1^5}t_4^9$

$\Longrightarrow Ht_1t_2t_{16} = H[yxt_3^7t_4^{10}]t_4^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy^{-1}x} = e^{xy^{-1}x}$

$\Longrightarrow x^{-1}y^{-1}t_{17}t_4t_{15} = e$

$\Longrightarrow x^{-1}y^{-1}t_{17}t_4t_{15} = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4t_3^4 = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4t_3^4\underline{t_3^7} = \underline{t_3^7}$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4\underline{t_4^{10}} = t_3^7\underline{t_4^{10}}$

$\Longrightarrow \underline{yx}x^{-1}y^{-1}t_1^5 = \underline{yx}t_3^7t_4^{10}$

$\Longrightarrow t_1^5 = yxt_3^7t_4^{10}$

$Ht_1t_2t_{16} = Hyxt_3^7t_4^{10}t_4^9$

$\Longrightarrow Ht_1t_2t_{16} = Ht_3^7\underline{t_4^8}$

$\Longrightarrow Ht_1t_2t_{16} = Ht_3^7[x^{-1}t_2^3t_1^3]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}} = e^{x^{-1}}$

$\Longrightarrow x^{-1}t_{10}t_9t_{12} = e$

$\Longrightarrow x^{-1}t_3^3t_91^3t_4^3 = e$

$\Longrightarrow x^{-1}t_3^3t_91^3t_4^3\underline{t_4^8} = \underline{t_4^8}$

$\Longrightarrow x^{-1}t_3^3t_91^3 = t_4^8$

$Ht_1t_2t_{16} = Ht_3^7x^{-1}t_2^3t_1^3$

$\Longrightarrow Ht_1t_2t_{16} = Hx^{-1}[t_3^7]^{x^{-1}}t_2^3t_1^3$

$\Longrightarrow Ht_1t_2t_{16} = Ht_2^7t_2^3t_1^3$

$\Longrightarrow Ht_1t_2t_{16} = Ht_2^{10}t_1^3$

$\Longrightarrow Ht_1t_2t_{16} = H\underline{t_{38}t_9}$

$\Longrightarrow Ht_1t_2t_{16} = Ht_9t_{38}$, by Equation 5.9

$Ht_1t_6 = Ht_6t_1$

$\Longrightarrow [Ht_1t_6]^{y^{-1}} = [Ht_6t_1]^{y^{-1}}$

$\Longrightarrow Ht_9t_{38} = Ht_{38}t_9$

$Ht_1t_2t_{16} = Ht_9t_{38}$

$\Longrightarrow Ht_1t_2t_{16} \in [16]$, since Ht_9t_{38} is in [1 6]

2 symmetric generators will go to [1 6].

$Ht_1t_2t_{18} = Ht_1t_2t_{18}$

$\Longrightarrow Ht_1t_2t_{18} = Ht_1t_2t_2^5$

$\Longrightarrow Ht_1t_2t_{18} = H\underline{t_1}t_2^6$

$\Longrightarrow Ht_1t_2t_{18} = Ht_3^4t_2^6$, since

$Ht_1 = Ht_{15}$

$\Longrightarrow Ht_1 = Ht_3^4$

$Ht_1t_2t_{18} = H\underline{t_3^4}t_2^6$

$Ht_1t_2t_{18} = H[yxt_1^6t_2^2]t_2^6$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\Longrightarrow x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9\underline{t_2^2} = t_1^6\underline{t_2^2}$

$\Longrightarrow \underline{yx}x^{-1}y^{-1}t_3^4 = \underline{yx}t_1^6t_2^2$

$\Longrightarrow t_3^4 = yxt_1^6t_2^2$

$Ht_1t_2t_{18} = Hyxt_1^6t_2^2t_2^6$

$\Longrightarrow Ht_1t_2t_{18} = Ht_1^6\underline{t_2^8}$

$\Longrightarrow Ht_1t_2t_{18} = Ht_1^6[x^{-1}t_4^3t_3^3]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^x = e^x$

$\Longrightarrow x^{-1}t_{12}t_{11}t_{10} = e$

$\Longrightarrow x^{-1}t_4^3t_3^3t_2^3 = e$

$\Longrightarrow x^{-1}t_4^3t_3^3t_2^3\underline{t_2^8} = \underline{t_2^8}$

$\Longrightarrow x^{-1}t_4^3t_3^3 = t_2^8$

$Ht_1t_2t_{18} = Ht_1^6x^{-1}t_4^3t_3^3$

$\Longrightarrow Ht_1t_2t_{18} = Hx^{-1}[t_1^6]^{x^{-1}}t_4^3t_3^3$

$\Longrightarrow Ht_1t_2t_{18} = Ht_4^6t_4^3t_3^3$

$\Longrightarrow Ht_1t_2t_{18} = H\underline{t_4^9}t_3^3$

$\Longrightarrow Ht_1t_2t_{18} = Ht_2^9t_3^3$

$\Longrightarrow Ht_1t_2t_{18} = Ht_{34}t_{11}$

$\Longrightarrow Ht_1t_2t_{18} \in [12]$, since $Ht_{34}t_{11}$ is in [1 2]

2 symmetric generators will go to [1 2].

$Ht_1t_2t_{14} = H\underline{t_1}t_2t_{14}$

$\Longrightarrow Ht_1t_2t_{14} = Ht_{15}t_2t_{14}$, since

$Ht_1 = Ht_{15}$

$Ht_1t_2t_{14} = Ht_{15}t_2t_{14}$

$\Longrightarrow Ht_1t_2t_{14} = Ht_3^4t_2t_2^4$

$\Longrightarrow Ht_1t_2t_{14} = Ht_3^4t_2^5$

$\Longrightarrow Ht_1t_2t_{14} \in [14]$, since $Ht_3^4t_2^5$ is in [1 4]

2 symmetric generators will go to [1 4].

$Ht_1t_2t_{17} = H\underline{t_1}t_2t_{17}$

$\Longrightarrow Ht_1t_2t_{14} = Ht_{15}t_2t_{17}$, since

$Ht_1 = Ht_{15}$

$Ht_1t_2t_{17} = Ht_{15}t_2t_{17}$

$\Longrightarrow Ht_1t_2t_{17} = Ht_3^4\underline{t_2}t_1^5$

$\Longrightarrow Ht_1t_2t_{17} = Ht_3^4[y^{-1}xt_4^2t_1^7]t_1^5$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\Longrightarrow x^{-1}yt_2t_{13}t_{36} = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9 = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\Longrightarrow x^{-1}yt_2t_1^4\underline{t_1^7} = t_4^2\underline{t_1^7}$

$\Longrightarrow \underline{y^{-1}x}x^{-1}yt_2 = \underline{y^{-1}x}t_4^2t_1^7$

$\Longrightarrow t_2 = y^{-1}xt_4^2t_1^7$

$Ht_1t_2t_{17} = Ht_3^4y^{-1}xt_4^2t_1^7t_1^5$

$\Longrightarrow Ht_1t_2t_{17} = Hy^{-1}x[t_3^4]^{y^{-1}x}t_4^2t_1$

$\Longrightarrow Ht_1t_2t_{17} = Ht_4^5t_4^2t_1$

$\Longrightarrow Ht_1t_2t_{17} = Ht_4^7t_1$

$\Longrightarrow Ht_1t_2t_{17} = H\underline{t_{28}}t_1$

$\Longrightarrow Ht_1t_2t_{17} = H\underline{t_6}t_1$, since

$Ht_6 = Ht_{28}$

$Ht_1t_2t_{17} = Ht_{28}t_1$

$\Longrightarrow Ht_1t_2t_{17} = H\underline{t_6t_1}$

$\Longrightarrow Ht_1t_2t_{17} = Ht_1t_6$

$\Longrightarrow Ht_1t_2t_{17} \in [16]$, since Ht_1t_6 is in [1 6].

2 symmetric generators will go to [1 6].

$Ht_1t_2t_{33} = H\underline{t_1}t_2t_{33}$

$\Longrightarrow Ht_1t_2t_{33} = Ht_{15}t_2t_{33}$, since

$Ht_1 = Ht_{15}$

$Ht_1t_2t_{33} = Ht_{15}t_2t_{33}$

$\Longrightarrow Ht_1t_2t_{33} = Ht_3^4\underline{t_2}t_1^9$

$\Longrightarrow Ht_1t_2t_{33} = Ht_3^4[y^{-1}xt_4^2t_1^7]t_1^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\Longrightarrow x^{-1}yt_2t_{13}t_{36} = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9 = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\Longrightarrow x^{-1}yt_2t_1^4\underline{t_1^7} = t_4^2\underline{t_1^7}$

$\Longrightarrow \underline{y^{-1}x}x^{-1}yt_2 = \underline{y^{-1}x}t_4^2t_1^7$

$\Longrightarrow t_2 = y^{-1}xt_4^2t_1^7$

$Ht_1t_2t_{33} = Ht_3^4y^{-1}xt_4^2t_1^7t_1^9$

$\Longrightarrow Ht_1t_2t_{33} = Hy^{-1}x[t_3^4]^{y^{-1}x}t_4^2t_1^5$

$\Longrightarrow Ht_1t_2t_{33} = Ht_4^5t_4^2t_1^5$

$\Longrightarrow Ht_1t_2t_{33} = H\underline{t_4^7}t_1^5$

$\Longrightarrow Ht_1t_2t_{33} = H[yx^{-1}t_2^5t_1]t_1^5$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\implies yx^{-1}t_{18}t_1t_{16} = e$

$\implies yx^{-1}t_2^5t_1t_4^4 = e$

$\implies yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\implies yx^{-1}t_2^5t_1 = t_4^7$

$Ht_1t_2t_{33} = Hyx^{-1}t_2^5t_1t_1^5$

$\implies Ht_1t_2t_{33} = Ht_2^5\underline{t_1^6}$

$\implies Ht_1t_2t_{33} = Ht_2^5[x^{-1}y^{-1}t_3^4t_2^9]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\implies x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\implies x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\implies x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\implies x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\implies x^{-1}y^{-1}t_3^4t_2^9 = t_1^6$

$Ht_1t_2t_{33} = Ht_2^5x^{-1}y^{-1}t_3^4t_2^9$

$\implies Ht_1t_2t_{33} = Hx^{-1}y^{-1}[t_2^5]^{x^{-1}y^{-1}}t_3^4t_2^9$

$\implies Ht_1t_2t_{33} = Ht_3^9t_3^4t_2^9$

$\implies Ht_1t_2t_{33} = H\underline{t_3^2}t_2^9$

$\implies Ht_1t_2t_{33} = Ht_1^7t_2^9$, since

$Ht_7 = Ht_{25}$

$\implies Ht_3^2 = Ht_1^7$

$\implies Ht_1t_2t_{33} = H\underline{t_1^7}t_2^9$

$\implies Ht_1t_2t_{33} = H[x^{-1}yt_3^5t_2]t_2^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^2} = e^{y^2}$

$\implies x^{-1}yt_{19}t_2t_{13} = e$

$\implies x^{-1}yt_3^5t_2t_1^4 = e$

$\implies x^{-1}yt_3^5t_2t_1^4\underline{t_1^7} = \underline{t_1^7}$

$\implies x^{-1}yt_3^5t_2 = t_1^7$

$Ht_1t_2t_{33} = Hx^{-1}yt_3^5t_2t_2^9$

$\implies Ht_1t_2t_{33} = Ht_3^5t_2^{10}$

$\Longrightarrow Ht_1t_2t_{33} = H\underline{t_{19}}t_{38}$

$\Longrightarrow Ht_1t_2t_{33} = H\underline{t_9}t_{38}$, since

$Ht_9 = Ht_{19}$

$Ht_1t_2t_{33} = Ht_9t_{38}$

$\Longrightarrow Ht_1t_2t_{33} \in [16]$, since Ht_9t_{38} is in [1 6].

2 symmetric generators will go to [1 6].

$Ht_1t_2t_{21} = Ht_1t_2t_{21}$

$\Longrightarrow Ht_1t_2t_{21} = Ht_{15}t_2t_{21}$, since

$Ht_1 = Ht_{15}$

$Ht_1t_2t_{21} = Ht_{15}t_2t_{21}$

$\Longrightarrow Ht_1t_2t_{21} = Ht_3^4\underline{t_2}t_1^6$

$\Longrightarrow Ht_1t_2t_{21} = Ht_3^4[y^{-1}xt_4^2t_1^7]t_1^6$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\Longrightarrow x^{-1}yt_2t_{13}t_{36} = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9 = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\Longrightarrow x^{-1}yt_2t_1^4\underline{t_1^7} = t_4^2\underline{t_1^7}$

$\Longrightarrow \underline{y^{-1}x}x^{-1}yt_2 = \underline{y^{-1}x}t_4^2t_1^7$

$\Longrightarrow t_2 = y^{-1}xt_4^2t_1^7$

$Ht_1t_2t_{21} = Ht_3^4y^{-1}xt_4^2t_1^7t_1^6$

$\Longrightarrow Ht_1t_2t_{21} = Hy^{-1}x[t_3^4]^{y^{-1}x}t_4^2t_1^2$

$\Longrightarrow Ht_1t_2t_{21} = Ht_4^5t_4^2t_1^2$

$\Longrightarrow Ht_1t_2t_{21} = H\underline{t_4^7}t_1^2$

$\Longrightarrow Ht_1t_2t_{21} = H[yx^{-1}t_2^5t_1]t_1^2$

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\Longrightarrow yx^{-1}t_{18}t_1t_{16} = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4 = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\Longrightarrow yx^{-1}t_2^5t_1 = t_4^7$

$Ht_1t_2t_{21} = Hyx^{-1}t_2^5t_1t_1^2$
$\Longrightarrow Ht_1t_2t_{21} = Ht_2^5t_1^3$
$\Longrightarrow Ht_1t_2t_{21} = Ht_{18}t_9$
$\Longrightarrow Ht_1t_2t_{21} \in [14]$, since $Ht_{18}t_9$ is in [1 4]
2 symmetric generators will go to [1 4].

$Ht_1t_2t_{21} = Ht_1t_2t_{21}$
$\Longrightarrow Ht_1t_2t_{21} = Ht_1t_2t_2^6$
$\Longrightarrow Ht_1t_2t_{21} = Ht_1\underline{t_2^7}$
$\Longrightarrow Ht_1t_2t_{21} = Ht_1[y^{-1}x^{-1}t_4^5t_3]$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{xy^{-1}} = e^{xy^{-1}}$
$\Longrightarrow y^{-1}x^{-1}t_{20}t_3t_{14} = e$
$\Longrightarrow y^{-1}x^{-1}t_4^5t_3t_2^4 = e$
$\Longrightarrow y^{-1}x^{-1}t_4^5t_3t_2^4\underline{t_2^7} = \underline{t_2^7}$
$\Longrightarrow y^{-1}x^{-1}t_4^5t_3 = t_2^7$
$Ht_1t_2t_{21} = Ht_1y^{-1}x^{-1}t_4^5t_3$
$\Longrightarrow Ht_1t_2t_{21} = Hy^{-1}x^{-1}[t_1]^{y^{-1}x^{-1}}t_4^5t_3$
$\Longrightarrow Ht_1t_2t_{21} = Ht_4^3t_4^5t_3$
$\Longrightarrow Ht_1t_2t_{21} = H\underline{t_4^8}t_3$
$\Longrightarrow Ht_1t_2t_{21} = Ht_2^{10}t_3$, since
$Ht_{32} = Ht_{38}$
$\Longrightarrow Ht_4^8 = Ht_2^{10}$
$Ht_1t_2t_{21} = Ht_2^{10}t_3$
$\Longrightarrow Ht_1t_2t_{21} = H\underline{t_2^{10}}t_3$
$\Longrightarrow Ht_1t_2t_{21} = H[x^-1yt_4^9t_3^5]t_3$, since by Equation 5.8
$[x^3t_{11}t_{10}t_9]^{yx} = e^{yx}$
$\Longrightarrow x^{-1}yt_{36}t_{19}t_2 = e$
$\Longrightarrow x^{-1}yt_4^9t_3^5t_2 = e$
$\Longrightarrow x^{-1}yt_4^9t_3^5t_2\underline{t_2^{10}} = \underline{t_2^{10}}$
$\Longrightarrow x^{-1}yt_4^9t_3^5 = t_2^{10}$
$Ht_1t_2t_{21} = Hx^-1yt_4^9t_3^5t_3$

$\implies Ht_1t_2t_{21} = Ht_4^9t_3^6$

$\implies Ht_1t_2t_{21} = H\underline{t_{36}}t_{23}$

$\implies Ht_1t_2t_{21} = Ht_{34}t_{23}$, since

$Ht_{34} = Ht_{36}$

$Ht_1t_2t_{21} = Ht_{34}t_{23}$

$\implies Ht_1t_2t_{21} \in [16]$, since $Ht_{34}t_{23}$ is in [1 6].

2 symmetric generators will go to [1 6].

$Ht_1t_2t_{24} = H\underline{t_1t_2}t_{24}$

$\implies Ht_1t_2t_{24} = Ht_{11}t_{20}t_{24}$, since

$Ht_1t_2 = Ht_{11}t_{20}$

$Ht_1t_2t_{24} = Ht_{11}t_{20}t_{24}$

$\implies Ht_1t_2t_{24} = Ht_3^3t_4^5t_4^6$

$\implies Ht_1t_2t_{24} = Ht_3^3$

$\implies Ht_1t_2t_{24} = Ht_{11}$

$\implies Ht_1t_2t_{24} \in [1]$, since Ht_{11} is in [1].

2 symmetric generators will go to [1].

$Ht_1t_2t_{25} = Ht_1t_2t_{25}$

$\implies Ht_1t_2t_{25} = Ht_{15}t_2t_{25}$, since

$Ht_1 = Ht_{15}$

$Ht_1t_2t_{25} = Ht_{15}t_2t_{25}$

$\implies Ht_1t_2t_{25} = Ht_3^4\underline{t_2}t_1^7$

$\implies Ht_1t_2t_{25} = Ht_3^4[y^{-1}xt_4^2t_1^7]t_1^7$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\implies x^{-1}yt_2t_{13}t_{36} = e$

$\implies x^{-1}yt_2t_1^4t_4^9 = e$

$\implies x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\implies x^{-1}yt_2t_1^4\underline{t_1^7} = t_4^2\underline{t_1^7}$

$\implies \underline{y^{-1}x}x^{-1}yt_2 = \underline{y^{-1}x}t_4^2t_1^7$

$\implies t_2 = y^{-1}xt_4^2t_1^7$

$Ht_1t_2t_{25} = Ht_3^4y^{-1}xt_4^2t_1^7t_1^7$

$\implies Ht_1t_2t_{25} = Hy^{-1}x[t_3^4]^{y^{-1}x}t_4^2t_1^3$

$\implies Ht_1t_2t_{25} = Ht_4^5t_4^2t_1^3$

$\implies Ht_1t_2t_{25} = H\underline{t_4^7}t_1^3$

$\implies Ht_1t_2t_{25} = H[yx^{-1}t_2^5t_1]t_1^3$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\implies yx^{-1}t_{18}t_1t_{16} = e$

$\implies yx^{-1}t_2^5t_1t_4^4 = e$

$\implies yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\implies yx^{-1}t_2^5t_1 = t_4^7$

$Ht_1t_2t_{25} = Hyx^{-1}t_2^5t_1t_1^3$

$\implies Ht_1t_2t_{25} = Ht_2^5t_1^4$

$\implies Ht_1t_2t_{25} = H\underline{t_{18}}t_{13}$

$\implies Ht_1t_2t_{25} = Ht_{12}t_{13}$, since

$Ht_{12} = Ht_{18}$

$Ht_1t_2t_{25} = Ht_{12}t_{13}$

$\implies Ht_1t_2t_{25} \in [110]$, since $Ht_{12}t_{13}$ is in [1 10].

2 symmetric generators will go to [1 10].

$Ht_1t_2t_{28} = H\underline{t_1t_2}t_{28}$

$\implies Ht_1t_2t_{28} = Ht_{11}t_{20}t_{28}$, since

$Ht_1t_2 = Ht_{11}t_{20}$

$Ht_1t_2t_{28} = Ht_{11}t_{20}t_{28}$

$\implies Ht_1t_2t_{28} = Ht_3^3t_4^5t_4^7$

$\implies Ht_1t_2t_{28} = H\underline{t_3^3}t_4$

$\implies Ht_1t_2t_{28} = Ht_1^5t_4$, since

$Ht_{11} = Ht17$

$\implies Ht_3^3 = Ht_1^5$

$Ht_1t_2t_{28} = H\underline{t_1^5}t_4$

$\implies Ht_1t_2t_{28} = H[yxt_3^7t_4^{10}]t_4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3 t_{11} t_{10} t_9]^{xy^{-1}x} = e^{xy^{-1}x}$

$\Longrightarrow x^{-1}y^{-1}t_{17}t_4t_{15} = e$

$\Longrightarrow x^{-1}y^{-1}t_{17}t_4t_{15} = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5 t_4 t_3^4 = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5 t_4 t_3^4 \underline{t_3^7} = \underline{t_3^7}$

$\Longrightarrow x^{-1}y^{-1}t_1^5 t_4 \underline{t_4^{10}} = t_3^7 \underline{t_4^{10}}$

$\Longrightarrow \underline{yx}x^{-1}y^{-1}t_1^5 = \underline{yx}t_3^7 t_4^{10}$

$\Longrightarrow t_1^5 = yxt_3^7 t_4^{10}$

$Ht_1t_2t_{28} = Hyxt_3^7 t_4^{10} t_4$

$\Longrightarrow Ht_1t_2t_{28} = Ht_3^7$

$\Longrightarrow Ht_1t_2t_{28} = Ht_{27}$

$\Longrightarrow Ht_1t_2t_{28} \in [5]$, since Ht_{27} is in [5].

2 symmetric generators will go to [5].

$Ht_1t_2t_{37} = Ht_1t_2t_{37}$

$\Longrightarrow Ht_1t_2t_{37} = Ht_{15}t_2t_{37}$, since

$Ht_1 = Ht_{37}$

$Ht_1t_2t_{37} = Ht_{15}t_2t_{37}$

$\Longrightarrow Ht_1t_2t_{37} = Ht_3^4 \underline{t_2} t_1^{10}$

$\Longrightarrow Ht_1t_2t_{37} = Ht_3^4[y^{-1}xt_4^2 t_1^7]t_1^{10}$, since by Equation 5.8

$x^3 t_{11} t_{10} t_9 = e$

$[x^3 t_{11} t_{10} t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\Longrightarrow x^{-1}yt_2t_{13}t_{36} = e$

$\Longrightarrow x^{-1}yt_2t_1^4 t_4^9 = e$

$\Longrightarrow x^{-1}yt_2t_1^4 t_4^9 \underline{t_4^2} = \underline{t_4^2}$

$\Longrightarrow x^{-1}yt_2t_1^4 \underline{t_1^7} = t_4^2 \underline{t_1^7}$

$\Longrightarrow \underline{y^{-1}x}x^{-1}yt_2 = \underline{y^{-1}x}t_4^2 t_1^7$

$\Longrightarrow t_2 = y^{-1}xt_4^2 t_1^7$

$Ht_1t_2t_{37} = Ht_3^4 y^{-1}xt_4^2 t_1^7 t_1^{10}$

$\Longrightarrow Ht_1t_2t_{37} = Hy^{-1}x[t_3^4]^{y^{-1}x} t_4^2 t_1^6$

$\Longrightarrow Ht_1t_2t_{37} = Ht_4^5 t_4^2 t_1^6$

$\Longrightarrow Ht_1t_2t_{37} = H\underline{t_4^7} t_1^6$

$\implies Ht_1t_2t_{37} = H[yx^{-1}t_2^5t_1]t_1^6$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\implies yx^{-1}t_{18}t_1t_{16} = e$

$\implies yx^{-1}t_2^5t_1t_4^4 = e$

$\implies yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\implies yx^{-1}t_2^5t_1 = t_4^7$

$Ht_1t_2t_{37} = Hyx^{-1}t_2^5t_1t_1^6$

$Ht_1t_2t_{37} = H\underline{t_2^5}t_1^7$

$Ht_1t_2t_{37} = Ht_4^3t_1^7$, since

$Ht_{12} = Ht_{18}$

$\implies Ht_4^3 = Ht_2^5$

$Ht_1t_2t_{37} = Ht_4^3\underline{t_1^7}$

$\implies Ht_1t_2t_{37} = Ht_4^3[x^{-1}yt_3^5t_2]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^2} = e^{y^2}$

$\implies x^{-1}yt_{19}t_2t_{13} = e$

$\implies x^{-1}yt_3^5t_2t_1^4 = e$

$\implies x^{-1}yt_3^5t_2t_1^4\underline{t_1^7} = \underline{t_1^7}$

$\implies x^{-1}yt_3^5t_2 = t_1^7$

$Ht_1t_2t_{37} = Ht_4^3x^{-1}yt_3^5t_2$

$\implies Ht_1t_2t_{37} = Hx^{-1}y[t_4^3]^{x^{-1}y}t_3^5t_2$

$\implies Ht_1t_2t_{37} = Ht_3^9t_3^5t_2$

$\implies Ht_1t_2t_{37} = Ht_3^3t_2$

$\implies Ht_1t_2t_{37} = Ht_{11}t_2$

$\implies Ht_1t_2t_{37} = Ht_{17}t_2$, since

$Ht_{11} = Ht_{17}$

$Ht_1t_2t_{37} \in [110]$, since $Ht_{17}t_2$ is in [1 10].

2 symmetric generators will go to [1 10].

The orbits of $N^{(14)}$ are $\{1, 35, 18, 16\}, \{2, 12, 15, 33\}, \{3, 17, 36, 10\}, \{4, 14, 9, 19\}$, $\{5, 27, 38, 32\}, \{6, 24, 31, 25\}, \{7, 37, 28, 22\}, \{8, 30, 21, 39\}, \{11, 13, 20, 34\}$, and $\{23, 29, 40, 26\}$.

We must check to see where $t_1t_4t_{16}, t_1t_4t_{12}, t_1t_4t_{36}, t_1t_4t_4, t_1t_4t_{32}, t_1t_4t_{24}, t_1t_4t_{28}, t_1t_4t_8, t_1t_4t_{20}$, and $t_1t_4t_{40}$ belong.

$Ht_1t_4t_{16} = Ht_1t_4t_{16}$
$\implies Ht_1t_4t_{16} = Ht_1t_4t_4^4$
$\implies Ht_1t_4t_{16} = Ht_1t_4^5$
$\implies Ht_1t_4t_{16} = H\underline{t_1}t_{20}$
$\implies Ht_1t_4t_{16} = Ht_{15}t_{20}$, since
$Ht_1 = Ht_{15}4$
$Ht_1t_4t_{16} = Ht_{15}t_{20}$
$\implies Ht_1t_4t_{16} \in [110]$, since $Ht_{15}t_{20}$ is in [1 10].
4 symmetric generators will go to [1 10].

$Ht_1t_4t_{12} = Ht_1t_4t_{12}$
$\implies Ht_1t_4t_{12} = Ht_1t_4t_4^3$
$\implies Ht_1t_4t_{12} = H\underline{t_1}t_4^4$
$\implies Ht_1t_4t_{12} = H[xy^{-1}t_3^2t_4^7]t_4^4$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$
$\implies yx^{-1}t_1t_{16}t_{35} = e$
$\implies yx^{-1}t_1t_4^4t_3^9 = e$
$\implies yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$
$\implies yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$
$\implies \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$
$\implies t_1 = xy^{-1}t_3^2t_4^7$
$Ht_1t_4t_{12} = Hxy^{-1}t_3^2t_4^7t_4^4$
$\implies Ht_1t_4t_{12} = Ht_3^2$
$\implies Ht_1t_4t_{12} = Ht_7$
$\implies Ht_1t_4t_{12} \in [5]$, since Ht_7 is in [5].
4 symmetric generators will go to [5].

$Ht_1t_4t_{36} = Ht_1t_4t_{36}$

$\Longrightarrow Ht_1t_4t_{36} = Ht_1t_4t_4^9$

$\Longrightarrow Ht_1t_4t_{36} = Ht_1\underline{t_4^{10}}$

$\Longrightarrow Ht_1t_4t_{36} = Ht_1[x^{-1}y^{-1}t_2^9t_1^5]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{yx^{-1}} = e^{yx^{-1}}$

$\Longrightarrow x^{-1}y^{-1}t_{34}t_{17}t_4 = e$

$\Longrightarrow x^{-1}y^{-1}t_2^9t_1^5t_4 = e$

$\Longrightarrow x^{-1}y^{-1}t_2^9t_1^5t_4\underline{t_4^{10}} = \underline{t_4^{10}}$

$\Longrightarrow x^{-1}y^{-1}t_2^9t_1^5 = t_4^{10}$

$Ht_1t_4t_{36} = Ht_1x^{-1}y^{-1}t_2^9t_1^5$

$\Longrightarrow Ht_1t_4t_{36} = Hx^{-1}y^{-1}[t_1]^{x^{-1}y^{-1}}t_2^9t_1^5$

$\Longrightarrow Ht_1t_4t_{36} = Ht_2^9t_2^9t_1^5$

$\Longrightarrow Ht_1t_4t_{36} = H\underline{t_2^7}t_1^5$

$\Longrightarrow Ht_1t_4t_{36} = Ht_4^2t_1^5$, since

$Ht_8 = Ht_{26}$

$\Longrightarrow Ht_4^2 = Ht_2^7$

$Ht_1t_4t_{36} = H\underline{t_4^2}t_1^5$

$\Longrightarrow Ht_1t_4t_{36} = H[x^{-1}yt_2t_1^4]t_1^5$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\Longrightarrow x^{-1}yt_2t_{13}t_{36} = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9 = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\Longrightarrow x^{-1}yt_2t_1^4 = t_4^2$

$Ht_1t_4t_{36} = Hx^{-1}yt_2t_1^4t_1^5$

$\Longrightarrow Ht_1t_4t_{36} = Ht_2t_1^9$

$\Longrightarrow Ht_1t_4t_{36} = H\underline{t_2}t_{33}$

$\Longrightarrow Ht_1t_4t_{36} = Ht_{16}t_{33}$, since

$Ht_2 = Ht_{16}$

$Ht_1t_4t_{36} = Ht_{16}t_{33}$

$\Longrightarrow Ht_1t_4t_{36} \in [12]$, since $Ht_{16}t_{33}$ is in [1 6].

4 symmetric generators will go to [1 6].

$Ht_1t_4t_4 = Ht_1t_4t_4$

$\implies Ht_1t_4t_4 = Ht_1t_4t_4$

$\implies Ht_1t_4t_4 = H\underline{t_1}t_4^2$

$\implies Ht_1t_4t_4 = H[xy^{-1}t_3^2t_4^7]t_4^2$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\implies yx^{-1}t_1t_{16}t_{35} = e$

$\implies yx^{-1}t_1t_4^4t_3^9 = e$

$\implies yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\implies yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\implies \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\implies t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_4t_4 = Hxy^{-1}t_3^2t_4^7t_4^2$

$\implies Ht_1t_4t_4 = H\underline{t_3^2}t_4^9$

$\implies Ht_1t_4t_4 = Ht_1^7t_4^9$, since

$Ht_7 = Ht_{25}$

$\implies Ht_3^2 = Ht_1^7$

$Ht_1t_4t_4 = Ht_1^7\underline{t_4^9}$

$\implies Ht_1t_4t_4 = Ht_1^7[y^{-1}xt_2^{10}t_3^6]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{yx} = e^{yx}$

$\implies x^{-1}yt_{36}t_{19}t_2 = e$

$\implies x^{-1}yt_4^9t_3^5t_2 = e$

$\implies x^{-1}yt_4^9t_3^5t_2\underline{t_2^{10}} = \underline{t_2^{10}}$

$\implies x^{-1}yt_4^9\underline{t_3^6} = t_2^{10}\underline{t_3^6}$

$\implies \underline{y^{-1}x}x^{-1}yt_4^9 = \underline{y^{-1}x}t_2^{10}t_3^6$

$\implies t_4^9 = y^{-1}xt_2^{10}t_3^6$

$Ht_1t_4t_4 = Ht_1^7y^{-1}xt_2^{10}t_3^6$

$\implies Ht_1t_4t_4 = Hy^{-1}x[t_1^7]^{y^{-1}x}t_2^{10}t_3^6$

$\implies Ht_1t_4t_4 = Ht_2^{10}t_2^{10}t_3^6$

$\Longrightarrow Ht_1t_4t_4 = Ht_2^9t_3^6$

$\Longrightarrow Ht_1t_4t_4 = Ht_{34}t_{23}$

$\Longrightarrow Ht_1t_4t_4 \in [16]$, since $Ht_{34}t_{23}$ is in [1 6].

4 symmetric generators will go to [1 6].

$Ht_1t_4t_{32} = Ht_1t_4t_{32}$

$\Longrightarrow Ht_1t_4t_{32} = Ht_1t_4t_4^8$

$\Longrightarrow Ht_1t_4t_{32} = Ht_1t_4^9$

$\Longrightarrow Ht_1t_4t_{32} = H\underline{t_1}t_{36}$

$\Longrightarrow Ht_1t_4t_{32} = Ht_{15}t_{36}$, since

$Ht_1 = Ht_{15}$

$Ht_1t_4t_{32} = Ht_{15}t_{36}$

$\Longrightarrow Ht_1t_4t_{32} \in [12]$, since $Ht_{15}t_{36}$ is in [1 2].

4 symmetric generators will go to [1 2].

$Ht_1t_4t_{24} = Ht_1t_4t_{24}$

$\Longrightarrow Ht_1t_4t_{24} = Ht_1t_4t_4^6$

$\Longrightarrow Ht_1t_4t_{24} = Ht_1t_4^7$

$\Longrightarrow Ht_1t_4t_{24} = Ht_{15}t_{28}$, since

$Ht_1 = Ht_{15}$

$Ht_1t_4t_{24} = Ht_{15}t_{28}$

$\Longrightarrow Ht_1t_4t_{24} \in [16]$, since $Ht_{15}t_{28}$ is in [1 6].

4 symmetric generators will go to [1 6].

$Ht_1t_4t_{28} = Ht_1t_4t_{28}$

$\Longrightarrow Ht_1t_4t_{28} = Ht_1t_4t_4^7$

$\Longrightarrow Ht_1t_4t_{28} = H\underline{t_1}t_4^8$

$\Longrightarrow Ht_1t_4t_{28} = H[xy^{-1}t_3^2t_4^7]t_4^8$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\Longrightarrow yx^{-1}t_1t_{16}t_{35} = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9 = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\Longrightarrow yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\Longrightarrow \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\Longrightarrow t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_4t_{28} = Hxy^{-1}t_3^2t_4^7t_4^8 \Longrightarrow Ht_1t_4t_{28} = Ht_3^2t_4^4$

$\Longrightarrow Ht_1t_4t_{28} = H\underline{t_7}t_{16}$

$\Longrightarrow Ht_1t_4t_{28} = Ht_{25}t_{16}$, since $Ht_7 = Ht_{25}$

$Ht_1t_4t_{28} = H\underline{t_{25}t_{16}}$

$\Longrightarrow Ht_1t_4t_{28} = Ht_{16}t_{25}$, since by Equation 5.9

$Ht_1t_6 = Ht_6t_1$

$\Longrightarrow [Ht_1t_6]^{yx^{-1}} = [Ht_6t_1]^{yx^{-1}}$

$\Longrightarrow Ht_{16}t_{25} = Ht_{25}t_{16}$

$Ht_1t_4t_{28} = Ht_{16}t_{25}$

$\Longrightarrow Ht_1t_4t_{28} \in [16]$, since $Ht_{16}t_{25}$ is in [1 6].

4 symmetric generators will go to [1 6].

$Ht_1t_4t_8 = Ht_1t_4t_8$

$\Longrightarrow Ht_1t_4t_8 = Ht_1t_4t_4^2$

$\Longrightarrow Ht_1t_4t_8 = H\underline{t_1}t_4^3$

$\Longrightarrow Ht_1t_4t_8 = H[xy^{-1}t_3^2t_4^7]t_4^3$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\Longrightarrow yx^{-1}t_1t_{16}t_{35} = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9 = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\Longrightarrow yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\Longrightarrow \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\Longrightarrow t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_4t_8 = Hxy^{-1}t_3^2t_4^7t_4^3 \Longrightarrow Ht_1t_4t_8 = Ht_3^2\underline{t_4^{10}}$

$\Longrightarrow Ht_1t_4t_8 = Ht_3^2[x^{-1}y^{-1}t_2^9t_1^5]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{yx^{-1}} = e^{yx^{-1}}$

$\Longrightarrow x^{-1}y^{-1}t_{34}t_{17}t_4 = e$

$\Longrightarrow x^{-1}y^{-1}t_2^9t_1^5t_4 = e$

$\Longrightarrow x^{-1}y^{-1}t_2^9t_1^5t_4\underline{t_4^{10}} = \underline{t_4^{10}}$

$\Longrightarrow x^{-1}y^{-1}t_2^9t_1^5 = t_4^{10}$

$Ht_1t_4t_8 = Ht_3^2x^{-1}y^{-1}t_2^9t_1^5$

$\Longrightarrow Ht_1t_4t_8 = Hx^{-1}y^{-1}[t_3^2]^{x^{-1}y^{-1}}t_2^9t_1^5$

$\Longrightarrow Ht_1t_4t_8 = Ht_2^{10}t_2^9t_1^5$

$\Longrightarrow Ht_1t_4t_8 = Ht_2^8t_1^5$

$\Longrightarrow Ht_1t_4t_8 = H\underline{t_{30}t_{17}}$

$\Longrightarrow Ht_1t_4t_8 = Ht_{17}t_{30}$, since by Equation 5.9

$Ht_1t_6 = Ht_6t_1$

$\Longrightarrow [Ht_1t_6]^{y^2} = [Ht_6t_1]^{y^2}$

$\Longrightarrow Ht_{17}t_{30} = Ht_{30}t_{17}$

$Ht_1t_4t_8 = Ht_{17}t_{30}$

$\Longrightarrow Ht_1t_4t_8 \in [16]$, since $Ht_{17}t_{30}$ is in [1 6]

4 symmetric generators will go to [1 6].

$Ht_1t_4t_{20} = H\underline{t_1}t_4t_{20}$

$\Longrightarrow Ht_1t_4t_{20} = Ht_{15}t_4t_{20}$, since $Ht_1 = Ht_{15}$

$Ht_1t_4t_{20} = Ht_{15}t_4t_{20}$

$\Longrightarrow Ht_1t_4t_{20} = Ht_3^4t_4t_4^5$

$\Longrightarrow Ht_1t_4t_{20} = Ht_3^4\underline{t_4^6}$

$\Longrightarrow Ht_1t_4t_{20} = Ht_3^4[y^{-1}x^{-1}t_2^4t_1^9]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y^{-1}} = e^{x^{-1}y^{-1}}$

$\Longrightarrow yx^{-1}t_{14}t_{33}t_{20} = e$

$\Longrightarrow yx^{-1}t_2^4t_1^9t_4^5 = e$

$\Longrightarrow yx^{-1}t_2^4t_1^9t_4^5\underline{t_4^6} = \underline{t_4^6}$

$\Longrightarrow yx^{-1}t_2^4t_1^9 = t_4^6$

$Ht_1t_4t_{20} = Ht_3^4y^{-1}x^{-1}t_2^4t_1^9$

$\Longrightarrow Ht_1t_4t_{20} = Hy^{-1}x^{-1}[t_3^4]^{y^{-1}x^{-1}}t_2^4t_1^9$

$\Longrightarrow Ht_1t_4t_{20} = Ht_2^5t_2^4t_1^9$

$\Longrightarrow Ht_1t_4t_{20} = Ht_2^9t_1^9$

$\Longrightarrow Ht_1t_4t_{20} = H\underline{t_{34}}t_{33}$

$\Longrightarrow Ht_1t_4t_{20} = Ht_{36}t_{33}$, since $Ht_{34} = Ht_{36}$

$Ht_1t_4t_{20} = Ht_{34}t_{33} \Longrightarrow Ht_1t_4t_{20} \in [110]$, since $Ht_{34}t_{33}$ is in [1 10].

4 symmetric generators will go to [1 10].

$Ht_1t_4t_{40} = Ht_1t_4t_{40}$

$\Longrightarrow Ht_1t_4t_{40} = Ht_1t_4t_4^{10}$

$\Longrightarrow Ht_1t_4t_{40} = Ht_1$

$\Longrightarrow Ht_1t_4t_{40} \in [1]$, since Ht_1 is in [1].

4 symmetric generators will go to [1].

$N^{(16)}$ has 40 single orbits. We will check to see where

$t_1t_6t_1, t_1t_6t_2, t_1t_6t_3, t_1t_6t_4, t_1t_6t_5, t_1t_6t_6, t_1t_6t_7, t_1t_6t_8, t_1t_6t_9, t_1t_6t_{10},$

$t_1t_6t_{11}, t_1t_6t_{12}, t_1t_6t_{13}, t_1t_6t_{14}, t_1t_6t_{15}, t_1t_6t_{16}, t_1t_6t_{17}, t_1t_6t_{18}, t_1t_6t_{19}, t_1t_6t_{10},$

$t_1t_6t_{21}, t_1t_6t_{32}, t_1t_6t_{23}, t_1t_6t_{24}, t_1t_6t_{25}, t_1t_6t_{26}, t_1t_6t_{27}, t_1t_6t_{28}, t_1t_6t_{29}, t_1t_6t_{30},$

$t_1t_6t_{31}, t_1t_6t_{32}, t_1t_6t_{33}, t_1t_6t_{34}, t_1t_6t_{35}, t_1t_6t_{36}, t_1t_6t_{37}, t_1t_6t_{38}, t_1t_6t_{39},$ and $t_1t_6t_{40}$ belong.

$Ht_1t_6t_1 = H\underline{t_1t_6}t_1$

$\Longrightarrow Ht_1t_6t_1 = Ht_6t_1t_1$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_1 = H\underline{t_6}t_1t_1$

$\Longrightarrow Ht_1t_6t_1 = Ht_{28}t_1^2$, since $Ht_6 = Ht_{28}$

$Ht_1t_6t_1 = Ht_{28}t_1^2$

$\Longrightarrow Ht_1t_6t_1 = Ht_4^7\underline{t_1^2} \Longrightarrow Ht_1t_6t_1 = Ht_4^7[y^{-1}x^{-1}t_3t_2^4]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}} = e^{y^{-1}}$

$\Longrightarrow x^{-1}y^{-1}t_3t_{14}t_{33} = e$

$\Longrightarrow x^{-1}y^{-1}t_3t_2^4t_1^9 = e$

$\Longrightarrow x^{-1}y^{-1}t_3t_2^4t_1^9\underline{t_1^2} = \underline{t_1^2}$

$\Longrightarrow x^{-1}y^{-1}t_3t_2^4 = t_1^2$

$Ht_1t_6t_1 = Ht_4^7y^{-1}x^{-1}t_3t_2^4$

$\Longrightarrow Ht_1t_6t_1 = Hy^{-1}x^{-1}[t_4^7]^{y^{-1}x^{-1}}t_3t_2^4$

$\Longrightarrow Ht_1t_6t_1 = H_3^8t_3t_2^4$

$\implies Ht_1t_6t_1 = H_3^9t_2^4$

$\implies Ht_1t_6t_1 = H_{35}t_{14}$

$\implies Ht_1t_6t_1 \in [14]$, since $Ht_{35}t_{14}$ is in [1 4].

1 symmetric generator will go to [1 4].

$Ht_1t_6t_2 = Ht_1t_6t_2$

$\implies Ht_1t_6t_2 = Ht_1t_2^2t_2$

$\implies Ht_1t_6t_2 = Ht_1t_2^3$

$\implies Ht_1t_6t_2 = Ht_1t_{10}$

$\implies Ht_1t_6t_2 \in [110]$, since Ht_1t_{10} is in [1 10].

1 symmetric generator will go to [1 4].

$Ht_1t_6t_3 = Ht_1t_6t_3$

$\implies Ht_1t_6t_3 = Ht_1\underline{t_2^2}t_3$

$Ht_1t_6t_3 = Ht_1[x^{-1}y^{-1}t_4t_3^4]t_3$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x} = e^{y^{-1}x}$

$\implies x^{-1}y^{-1}t_4t_{15}t_{34} = e$

$\implies x^{-1}y^{-1}t_4t_3^4t_2^9 = e$

$\implies x^{-1}y^{-1}t_4t_3^4t_2^9\underline{t_2^2} = \underline{t_2^2}$

$\implies x^{-1}y^{-1}t_4t_3^4 = t_2^2$

$Ht_1t_6t_3 = Ht_1x^{-1}y^{-1}t_4t_3^4t_3$

$\implies Ht_1t_6t_3 = Hx^{-1}y^{-1}[t_1]^{x^{-1}y^{-1}}t_4t_3^5$

$\implies Ht_1t_6t_3 = Ht_4^9t_4t_3^5$

$\implies Ht_1t_6t_3 = Ht_4^{10}\underline{t_3^5}$

$\implies Ht_1t_6t_3 = Ht_4^{10}[y^{-1}xt_1^7t_4]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^2} = e^{y^2}$

$\implies x^{-1}yt_{19}t_2t_{13} = e$

$\implies x^{-1}yt_3^5t_2t_1^4 = e$

$\implies x^{-1}yt_3^5t_2t_1^4\underline{t_1^7} = \underline{t_1^7}$

$\implies x^{-1}yt_3^5t_2\underline{t_2^{10}} = t_1^7\underline{t_2^{10}}$

$\Longrightarrow \underline{y^{-1}x}x^{-1}yt_3^5 = \underline{y^{-1}x}t_1^7t_2^{10}$

$\Longrightarrow t_3^5 = y^{-1}xt_1^7t_2^{10}$

$Ht_1t_6t_3 = Ht_4^{10}y^{-1}xt_1^7t_4$

$\Longrightarrow Ht_1t_6t_3 = Hy^{-1}x[t_4^{10}]^{y^{-1}x}t_1^7t_4$

$\Longrightarrow Ht_1t_6t_3 = Ht_1^2t_1^7t_4$

$\Longrightarrow Ht_1t_6t_3 = H\underline{t_1^9}t_4$

$\Longrightarrow Ht_1t_6t_3 = H[xyt_3^{10}t_4^6]t_4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y^{-1}} = e^{x^2y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{33}t_{20}t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5\underline{t_4^6} = t_3^{10}\underline{t_4^6}$

$\Longrightarrow \underline{xy}y^{-1}x^{-1}t_1^9 = \underline{xy}t_3^{10}t_4^6$

$\Longrightarrow t_1^9 = xyt_3^{10}t_4^6$

$Ht_1t_6t_3 = Hxyt_3^{10}t_4^6t_4$

$\Longrightarrow Ht_1t_6t_3 = Ht_3^{10}\underline{t_4^7}$

$\Longrightarrow Ht_1t_6t_3 = Ht_3^{10}[yx^{-1}t_2^5t_1]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\Longrightarrow yx^{-1}t_{18}t_1t_{16} = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4 = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\Longrightarrow yx^{-1}t_2^5t_1 = t_4^7$

$Ht_1t_6t_3 = Ht_3^{10}yx^{-1}t_2^5t_1$

$\Longrightarrow Ht_1t_6t_3 = Hyx^{-1}[t_3^{10}]^{yx^{-1}}t_2^5t_1$

$\Longrightarrow Ht_1t_6t_3 = Ht_2^8t_2^5t_1$

$\Longrightarrow Ht_1t_6t_3 = Ht_2^2t_1$

$\Longrightarrow Ht_1t_6t_3 = H\underline{t_6t_1}$

$\Longrightarrow Ht_1t_6t_3 = Ht_1t_6$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_3 = Ht_1t_6$

$\Longrightarrow Ht_1t_6t_3 \in [16]$, since Ht_1t_6 is in [1 6].

1 symmetric generator will go to [1 6].

$Ht_1t_6t_4 = H\underline{t_1t_6}t_4$

$\Longrightarrow Ht_1t_6t_4 = Ht_6t_1t_4$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_4 = Ht_6t_1t_4$

$\Longrightarrow Ht_1t_6t_4 = Ht_2^2\underline{t_1}t_4$

$\Longrightarrow Ht_1t_6t_4 = Ht_2^2\underline{t_1}t_4$

$\Longrightarrow Ht_1t_6t_4 = Ht_2^2[xy^{-1}t_3^2t_4^7]t_4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\Longrightarrow yx^{-1}t_1t_{16}t_{35} = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9 = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\Longrightarrow yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\Longrightarrow \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\Longrightarrow t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_6t_4 = Ht_2^2xy^{-1}t_3^2t_4^7t_4$

$\Longrightarrow Ht_1t_6t_4 = Hxy^{-1}[t_2^2]^{xy^{-1}}t_3^2t_4^8$

$\Longrightarrow Ht_1t_6t_4 = Ht_3^8t_3^2t_4^8$

$\Longrightarrow Ht_1t_6t_4 = H\underline{t_3^{10}}t_4^8$

$\Longrightarrow Ht_1t_6t_4 = H[y^{-1}x^{-1}t_1^9t_4^5]t_4^8$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y^{-1}} = e^{x^2y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{33}t_{20}t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5 = t_3^{10}$

$Ht_1t_6t_4 = Hy^{-1}x^{-1}t_1^9t_4^5t_4^8$

$\Longrightarrow Ht_1t_6t_4 = H\underline{t_1^9}t_4^2$

$\Longrightarrow Ht_1t_6t_4 = Ht_3^9t_4^2$, since

$Ht_{33} = Ht_{35}$

$\Longrightarrow Ht_1^9 = Ht_3^9$

$Ht_1t_6t_4 = Ht_3^9\underline{t_4^2}$

$Ht_1t_6t_4 = Ht_3^9[x^{-1}yt_2t_1^4]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\Longrightarrow x^{-1}yt_2t_{13}t_{36} = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9 = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\Longrightarrow x^{-1}yt_2t_1^4 = t_4^2$

$Ht_1t_6t_4 = Ht_3^9x^{-1}yt_2t_1^4$

$\Longrightarrow Ht_1t_6t_4 = Hx^{-1}y[t_3^9]^{x^{-1}y}t_2t_1^4$

$\Longrightarrow Ht_1t_6t_4 = Ht_2^4t_2t_1^4$

$\Longrightarrow Ht_1t_6t_4 = Ht_2^5t_1^4$

$\Longrightarrow Ht_1t_6t_4 = H\underline{t_{18}}t_{13}$

$\Longrightarrow Ht_1t_6t_4 = Ht_{12}t_{13}$, since $Ht_{12} = Ht_{18}$

$Ht_1t_6t_4 = Ht_{12}t_{13}$

$\Longrightarrow Ht_1t_6t_4 \in [110]$, since $Ht_{12}t_{13}$ is in [1 10].

1 symmetric generator will go to [1 10].

$Ht_1t_6t_5 = H\underline{t_1t_6}t_5$

$\Longrightarrow Ht_1t_6t_5 = Ht_6t_1t_5$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_5 = H\underline{t_6}t_1t_5$

$\Longrightarrow Ht_1t_6t_5 = H28t_1t_5$, since $Ht_6 = Ht_{28}$

$Ht_1t_6t_5 = Ht_{28}t_1t_5$

$\Longrightarrow Ht_1t_6t_5 = Ht_4^7t_1t_1^2$

$\Longrightarrow Ht_1t_6t_5 = H\underline{t_4^7}t_1^3$

$\Longrightarrow Ht_1t_6t_5 = H[yx^{-1}t_2^5t_1]t_1^3$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\Longrightarrow yx^{-1}t_{18}t_1t_{16} = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4 = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\Longrightarrow yx^{-1}t_2^5t_1 = t_4^7$

$Ht_1t_6t_5 = Hyx^{-1}t_2^5t_1t_1^3$

$\Longrightarrow Ht_1t_6t_5 = Ht_2^5t_1^4$

$\Longrightarrow Ht_1t_6t_5 = Ht_{18}t_{13}$

$\Longrightarrow Ht_1t_6t_5 = Ht_{12}t_{13}$, since $Ht_{12} = Ht_{18}$

$Ht_1t_6t_5 = Ht_{12}t_{13}$

$\Longrightarrow Ht_1t_6t_5 \in [110]$, since $Ht_{12}t_{13}$ is in [1 10].

1 symmetric generator will go to [1 10].

$Ht_1t_6t_6 = H\underline{t_1}t_6t_6$

$\Longrightarrow Ht_1t_6t_6 = Ht_{15}t_6t_6$, since $Ht_1 = Ht_{15}$

$Ht_1t_6t_6 = Ht_{15}t_6t_6 \Longrightarrow Ht_1t_6t_6 = Ht_1t_2^2t_2^2$

$\Longrightarrow Ht_1t_6t_6 = Ht_1\underline{t_2^4}$

$\Longrightarrow Ht_1t_6t_6 = Ht_1[xyt_4^6t_1^2]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y^{-1}} = e^{x^{-1}y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{14}t_{33}t_{20} = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5 = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5\underline{t_4^6} = \underline{t_4^6}$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9\underline{t_1^2} = t_4^6\underline{t_1^2}$

$\Longrightarrow \underline{xy}y^{-1}x^{-1}t_2^4 = \underline{xy}t_4^6t_1^2$

$\Longrightarrow t_2^4 = xyt_4^6t_1^2$

$Ht_1t_6t_6 = Ht_1xyt_4^6t_1^2$

$\Longrightarrow Ht_1t_6t_6 = Hxy[t_1]^{xy}t_4^6t_1^2$

$\Longrightarrow Ht_1t_6t_6 = Ht_4^9t_4^6t_1^2$

$\Longrightarrow Ht_1t_6t_6 = H\underline{t_4^4}t_1^2$

$\Longrightarrow Ht_1t_6t_6 = Ht_2t_1^2$, since

$Ht_2 = Ht_{16}$

$\Longrightarrow Ht_2 = Ht_4^4$

$Ht_1t_6t_6 = H\underline{t_2}t_1^2$

$\Longrightarrow Ht_1t_6t_6 = H[y^{-1}xt_4^2t_1^7]t_1^2$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\Longrightarrow x^{-1}yt_2t_{13}t_{36} = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9 = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\Longrightarrow x^{-1}yt_2t_1^4\underline{t_1^7} = t_4^2\underline{t_1^7}$

$\Longrightarrow \underline{y^{-1}x}x^{-1}yt_2 = \underline{y^{-1}x}t_4^2t_1^7$

$\Longrightarrow t_2 = y^{-1}xt_4^2t_1^7$

$Ht_1t_6t_6 = Hy^{-1}xt_4^2t_1^7t_1^2$

$\Longrightarrow Ht_1t_6t_6 = H\underline{t_4^2}t_1^9$

$\Longrightarrow Ht_1t_6t_6 = Ht_2^7t_1^9$, since

$Ht_8 = Ht_{26}$

$\Longrightarrow Ht_4^2 = Ht_2^7$.

$Ht_1t_6t_6 = Ht_2^7\underline{t_1^9}$

$Ht_1t_6t_6 = Ht_2^7[xyt_3^{10}t_4^6]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y^{-1}} = e^{x^2y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{33}t_{20}t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5\underline{t_4^6} = t_3^{10}\underline{t_4^6}$

$\Longrightarrow \underline{xy}y^{-1}x^{-1}t_1^9 = \underline{xy}t_3^{10}t_4^6$

$\Longrightarrow t_1^9 = xyt_3^{10}t_4^6$

$Ht_1t_6t_6 = Ht_2^7xyt_3^{10}t_4^6$

$\Longrightarrow Ht_1t_6t_6 = Hxy[t_2^7]^{xy}t_3^{10}t_4^6$

$\Longrightarrow Ht_1t_6t_6 = Ht_3^{10}t_3^{10}t_4^6$

$\Longrightarrow Ht_1t_6t_6 = Ht_3^9t_4^6$

$\Longrightarrow Ht_1t_6t_6 = Ht_{35}t_{24}$

$\Longrightarrow Ht_1t_6t_6 \in [16]$, since $Ht_{35}t_{24}$ is in [1 6]

1 symmetric generator will go to [1 6].

$Ht_1t_6t_7 = Ht_1t_6t_7$

$\Longrightarrow Ht_1t_6t_7 = Ht_1\underline{t_2^2}t_3^2$

$\Longrightarrow Ht_1t_6t_7 = Ht_1[x^{-1}y^{-1}t_4t_3^4]t_3^2$, since by Equation 5.8

$x^3 t_{11} t_{10} t_9 = e$

$[x^3 t_{11} t_{10} t_9]^{y^{-1}x} = e^{y^{-1}x}$

$\Longrightarrow x^{-1}y^{-1}t_4 t_{15} t_{34} = e$

$\Longrightarrow x^{-1}y^{-1}t_4 t_3^4 t_2^9 = e$

$\Longrightarrow x^{-1}y^{-1}t_4 t_3^4 t_2^9 \underline{t_2^2} = \underline{t_2^2}$

$\Longrightarrow x^{-1}y^{-1}t_4 t_3^4 = t_2^2$

$Ht_1 t_6 t_7 = Ht_1 x^{-1}y^{-1}t_4 t_3^4 t_3^2$

$\Longrightarrow Ht_1 t_6 t_7 = Hx^{-1}y^{-1}[t_1]^{x^{-1}y^{-1}} t_4 t_3^4 t_3^2$

$\Longrightarrow Ht_1 t_6 t_7 = Ht_4^9 t_4 t_3^4 t_3^2$

$\Longrightarrow Ht_1 t_6 t_7 = H\underline{t_4^{10}} t_3^4 t_3^2$

$\Longrightarrow Ht_1 t_6 t_7 = Ht_2^8 t_3^4 t_3^2$, since

$Ht_{30} = Ht_{40}$

$\Longrightarrow Ht_2^8 = Ht_4^{10}$

$Ht_1 t_6 t_7 = Ht_2^8 t_3^4 t_3^2$

$\Longrightarrow Ht_1 t_6 t_7 = H\underline{t_2^8} t_3^6$

$\Longrightarrow Ht_1 t_6 t_7 = H[x^{-1}t_4^3 t_3^3]t_3^6$, since by Equation 5.8

$x^3 t_{11} t_{10} t_9 = e$

$[x^3 t_{11} t_{10} t_9]^x = e^x$

$\Longrightarrow x^{-1}t_{12} t_{11} t_{10} = e$

$\Longrightarrow x^{-1}t_4^3 t_3^3 t_2^3 = e$

$\Longrightarrow x^{-1}t_4^3 t_3^3 t_2^3 \underline{t_2^8} = \underline{t_2^8}$

$\Longrightarrow x^{-1}t_4^3 t_3^3 = t_2^8$

$Ht_1 t_6 t_7 = Hx^{-1}t_4^3 t_3^3 t_3^6$

$\Longrightarrow Ht_1 t_6 t_7 = Ht_4^3 t_3^9$

$\Longrightarrow Ht_1 t_6 t_7 = Ht_{12} t_{35}$

$\Longrightarrow Ht_1 t_6 t_7 \in [14]$, since $Ht_{12} t_{35}$ is in [1 4].

1 symmetric generator will go to [1 4].

$Ht_1 t_6 t_8 = H\underline{t_1 t_6} t_8$

$\Longrightarrow Ht_1 t_6 t_8 = Ht_6 t_1 t_8$, since $Ht_1 t_6 = Ht_6 t_1$

$Ht_1 t_6 t_8 = Ht_6 t_1 t_8$

$\Longrightarrow Ht_1 t_6 t_8 = Ht_2^2 \underline{t_1} t_4^2$

$\Longrightarrow Ht_1t_6t_8 = Ht_2^2[xy^{-1}t_3^2t_4^7]t_4^2$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\Longrightarrow yx^{-1}t_1t_{16}t_{35} = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9 = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\Longrightarrow yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\Longrightarrow \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\Longrightarrow t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_6t_8 = Ht_2^2xy^{-1}t_3^2t_4^7t_4^2$

$\Longrightarrow Ht_1t_6t_8 = Hxy^{-1}[t_2^2]^{xy^{-1}}t_3^2t_4^9$

$\Longrightarrow Ht_1t_6t_8 = Ht_3^8t_3^2t_4^9$

$\Longrightarrow Ht_1t_6t_8 = H\underline{t_3^{10}}t_4^9$

$\Longrightarrow Ht_1t_6t_8 = H[y^{-1}x^{-1}t_1^9t_4^5]t_4^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y^{-1}} = e^{x^2y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{33}t_{20}t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5 = t_3^{10}$

$Ht_1t_6t_8 = Hy^{-1}x^{-1}t_1^9t_4^5t_4^9$

$\Longrightarrow Ht_1t_6t_8 = Ht_1^9t_4^3$

$\Longrightarrow Ht_1t_6t_8 = Ht_{33}t_{12}$

$\Longrightarrow Ht_1t_6t_8 = H\underline{t_{33}}t_{12}$

$\Longrightarrow Ht_1t_6t_8 = Ht_{35}t_{12}$, since $Ht_{33} = Ht_{35}$

$Ht_1t_6t_8 = Ht_{35}t_{12}$

$\Longrightarrow Ht_1t_6t_8\ in[12]$, since $Ht_{35}t_{12}$ is in [1 2]

1 symmetric generator will go to [1 2].

$Ht_1t_6t_9 = H\underline{t_1t_6}t_9$

$\Longrightarrow Ht_1t_6t_9 = Ht_6t_1t_9$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_9 = H\underline{t_6}t_1t_9$

$\implies Ht_1t_6t_9 = Ht_{28}t_1t_9$, since $Ht_6 = Ht_{28}$

$Ht_1t_6t_9 = Ht_{28}t_1t_9 \implies Ht_1t_6t_9 = Ht_4^7t_1t_1^3$

$\implies Ht_1t_6t_9 = H\underline{t_4^7}t_1^4$

$\implies Ht_1t_6t_9 = H[yx^{-1}t_2^5t_1]t_1^4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\implies yx^{-1}t_{18}t_1t_{16} = e$

$\implies yx^{-1}t_2^5t_1t_4^4 = e$

$\implies yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\implies yx^{-1}t_2^5t_1 = t_4^7$

$Ht_1t_6t_9 = Hyx^{-1}t_2^5t_1t_1^4$

$\implies Ht_1t_6t_9 = Ht_2^5t_1^5$

$\implies Ht_1t_6t_9 = H\underline{t_{18}}t_{17}$

$\implies Ht_1t_6t_9 = Ht_{12}t_{17}$, since $Ht_{12} = Ht_{18}$

$Ht_1t_6t_9 = Ht_{12}t_{17}$

$\implies Ht_1t_6t_9 \in [12]$, since $Ht_{12}t_{17}$ is in [1 2].

1 symmetric generator will go to [1 2].

$Ht_1t_6t_{10} = Ht_1t_6t_{10}$

$\implies Ht_1t_6t_{10} = Ht_1t_2^2t_2^3$

$\implies Ht_1t_6t_{10} = Ht_1t_2^5$

$\implies Ht_1t_6t_{10} = H\underline{t_1}t_{18}$

$\implies Ht_1t_6t_{10} = Ht_{15}t_{18}$, since $Ht_1 = Ht_{15}$

$Ht_1t_6t_{10} = Ht_{15}t_{18}$

$\implies Ht_1t_6t_{10} \in [14]$, since $Ht_{15}t_{18}$ is in [1 4]

1 symmetric generator will go to [1 4].

$Ht_1t_6t_{11} = Ht_1t_6t_{11}$

$\implies Ht_1t_6t_{11} = Ht_1\underline{t_2^2}t_3^3$

$\implies Ht_1t_6t_{11} = Ht_1[x^{-1}y^{-1}t_4t_3^4]t_3^3$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x} = e^{y^{-1}x}$

$\Longrightarrow x^{-1}y^{-1}t_4t_{15}t_{34} = e$

$\Longrightarrow x^{-1}y^{-1}t_4t_3^4t_2^9 = e$

$\Longrightarrow x^{-1}y^{-1}t_4t_3^4t_2^9\underline{t_2^2} = \underline{t_2^2}$

$\Longrightarrow x^{-1}y^{-1}t_4t_3^4 = t_2^2$

$Ht_1t_6t_{11} = Ht_1x^{-1}y^{-1}t_4t_3^4t_3^3$

$\Longrightarrow Ht_1t_6t_{11} = Hx^{-1}y^{-1}[t_1]^{x^{-1}y^{-1}}t_4t_3^7$

$\Longrightarrow Ht_1t_6t_{11} = Ht_4^9t_4t_3^7$

$\Longrightarrow Ht_1t_6t_{11} = H\underline{t_4^{10}}t_3^7$

$\Longrightarrow Ht_1t_6t_{11} = Ht_2^8t_3^7$, since

$Ht_{30} = Ht_{40}$

$\Longrightarrow Ht_2^8 = Ht_4^{10}$

$Ht_1t_6t_{11} = H\underline{t_2^8}t_3^7$

$\Longrightarrow Ht_1t_6t_{11} = Ht_2^8\underline{t_3^7}$

$\Longrightarrow Ht_1t_6t_{11} = Ht_2^8[x^{-1}y^{-1}t_1^5t_4]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy^{-1}x} = e^{xy^{-1}x}$

$\Longrightarrow [x^{-1}y^{-1}t_{17}t_4t_{15} = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4t_3^4 = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4t_3^4\underline{t_3^7} = \underline{t_3^7}$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4 = t_3^7$

$Ht_1t_6t_{11} = Ht_2^8x^{-1}y^{-1}t_1^5t_4$

$\Longrightarrow Ht_1t_6t_{11} = Hx^{-1}y^{-1}[t_2^8]^{x^{-1}y^{-1}}t_1^5t_4$

$\Longrightarrow Ht_1t_6t_{11} = Ht_1^2t_1^5t_4$

$\Longrightarrow Ht_1t_6t_{11} = H\underline{t_1^7}t_4$

$\Longrightarrow Ht_1t_6t_{11} = Ht_3^2t_4$, since

$Ht_7 = Ht_{25}$

$\Longrightarrow Ht_3^2 = Ht_1^7$

$Ht_1t_6t_{11} = H\underline{t_3^2}t_4$

$\Longrightarrow Ht_1t_6t_{11} = H[yx^{-1}t_1t_4^4]t_4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\Longrightarrow yx^{-1}t_1t_{16}t_{35} = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9 = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\Longrightarrow yx^{-1}t_1t_4^4 = t_3^2$

$Ht_1t_6t_{11} = Hyx^{-1}t_1t_4^4t_4$

$\Longrightarrow Ht_1t_6t_{11} = Ht_1t_4^5$

$\Longrightarrow Ht_1t_6t_{11} = H\underline{t_1}t_{20}$

$\Longrightarrow Ht_1t_6t_{11} = Ht_{15}t_{20}$, since $Ht_1 = Ht_{15}$

$Ht_1t_6t_{11} = Ht_{15}t_{20} \Longrightarrow Ht_1t_6t_{11} \in [110]$, since $Ht_{15}t_{20}$ is in [1 10].

1 symmetric generator will go to [1 10].

$Ht_1t_6t_{12} = H\underline{t_1t_6}t_{12}$

$\Longrightarrow Ht_1t_6t_{12} = Ht_6t_1t_{12}$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_{12} = Ht_6t_1t_{12}$

$\Longrightarrow Ht_1t_6t_{12} = Ht_2^2\underline{t_1}t_4^3$

$\Longrightarrow Ht_1t_6t_{12} = Ht_2^2[xy^{-1}t_3^2t_4^7]t_4^3$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\Longrightarrow yx^{-1}t_1t_{16}t_{35} = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9 = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\Longrightarrow yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\Longrightarrow \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\Longrightarrow t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_6t_{12} = Ht_2^2xy^{-1}t_3^2t_4^7t_4^3$

$\Longrightarrow Ht_1t_6t_{12} = Hxy^{-1}[t_2^2]^{xy^{-1}}t_3^2t_4^{10}$

$\Longrightarrow Ht_1t_6t_{12} = Ht_3^8t_3^2t_4^{10}$

$\Longrightarrow Ht_1t_6t_{12} = H\underline{t_3^{10}}t_4^{10}$

$\Longrightarrow Ht_1t_6t_{12} = H[y^{-1}x^{-1}t_1^9t_4^5]t_4^{10}$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y^{-1}} = e^{x^2y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{33}t_{20}t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5 = t_3^{10}$

$Ht_1t_6t_{12} = Hy^{-1}x^{-1}t_1^9t_4^5t_4^{10}$

$\Longrightarrow Ht_1t_6t_{12} = Ht_1^9t_4^4$

$\Longrightarrow Ht_1t_6t_{12} = Ht_{33}t_{16}$

$\Longrightarrow Ht_1t_6t_{12} \in [14]$, since $Ht_{33}t_{16}$ is in [1 4].

1 symmetric generator will go to [1 4].

$Ht_1t_6t_{13} = H\underline{t_1t_6}t_{13}$

$\Longrightarrow Ht_1t_6t_{13} = Ht_6t_1t_{13}$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_{13} = H\underline{t_6}t_1t_{13}$

$\Longrightarrow Ht_1t_6t_{13} = Ht_2^2t_1t_1^4$

$\Longrightarrow Ht_1t_6t_{13} = Ht_2^2\underline{t_1^5}$

$\Longrightarrow Ht_1t_6t_{13} = Ht_2^2[yxt_3^7t_4^{10}]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy^{-1}x} = e^{xy^{-1}x}$

$\Longrightarrow x^{-1}y^{-1}t_{17}t_4t_{15} = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4t_3^4 = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4t_3^4\underline{t_3^7} = \underline{t_3^7}$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4\underline{t_4^{10}} = t_3^7\underline{t_4^{10}}$

$\Longrightarrow \underline{yx}x^{-1}y^{-1}t_1^5 = \underline{yx}t_3^7t_4^{10}$

$\Longrightarrow t_1^5 = yxt_3^7t_4^{10}$

$Ht_1t_6t_{13} = Ht_2^2yxt_3^7t_4^{10}$

$\Longrightarrow Ht_1t_6t_{13} = Hyx[t_2^2]^{yx}t_3^7t_4^{10}$

$\Longrightarrow Ht_1t_6t_{13} = Ht_3^7t_3^7t_4^{10}$

$\Longrightarrow Ht_1t_6t_{13} = Ht_3^3t_4^{10}$

$\Longrightarrow Ht_1t_6t_{13} = Ht_{11}t_{40}$

$\Longrightarrow Ht_1t_6t_{13} \in [16]$, since $Ht_{11}t_{40}$ is in [1 6].

1 symmetric generator will go to [1 6].

$Ht_1t_6t_{14} = Ht_1t_6t_{14}$

$\Longrightarrow Ht_1t_6t_{14} = Ht_1t_2^2t_2^4$

$\Longrightarrow Ht_1t_6t_{14} = Ht_1\underline{t_2^6}$

$\Longrightarrow Ht_1t_6t_{14} = Ht_1[yx^{-1}t_4^4t_3^9]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy} = e^{xy}$

$\Longrightarrow yx^{-1}t_{16}t_{35}t_{18} = e$

$\Longrightarrow yx^{-1}t_4^4t_3^9t_2^5 = e$

$\Longrightarrow yx^{-1}t_4^4t_3^9t_2^5\underline{t_2^6} = \underline{t_2^6}$

$\Longrightarrow yx^{-1}t_4^4t_3^9 = t_2^6$

$Ht_1t_6t_{14} = Ht_1yx^{-1}t_4^4t_3^9$

$\Longrightarrow Ht_1t_6t_{14} = Hyx^{-1}[t_1]^{yx^{-1}}t_4^4t_3^9$

$\Longrightarrow Ht_1t_6t_{14} = Ht_4^4t_4^4t_3^9$

$\Longrightarrow Ht_1t_6t_{14} = H\underline{t_4^8}t_3^9$

$\Longrightarrow Ht_1t_6t_{14} = Ht_2^{10}t_3^9$, since

$Ht_{32} = Ht_{38}$

$\Longrightarrow Ht_4^8 = Ht_2^{10}$

$\Longrightarrow Ht_1t_6t_{14} = H\underline{t_2^{10}}t_3^9$

$\Longrightarrow Ht_1t_6t_{14} = H[x^{-1}yt_4^9t_3^5]t_3^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{yx} = e^{yx}$

$\Longrightarrow x^{-1}yt_{36}t_{19}t_2 = e$

$\Longrightarrow x^{-1}yt_4^9t_3^5t_2 = e$

$\Longrightarrow x^{-1}yt_4^9t_3^5t_2\underline{t_2^{10}} = \underline{t_2^{10}}$

$\Longrightarrow x^{-1}yt_4^9t_3^5 = t_2^{10}$

$Ht_1t_6t_{14} = Hx^{-1}yt_4^9t_3^5t_3^9$

$\Longrightarrow Ht_1t_6t_{14} = Ht_4^9t_3^3$

$\Longrightarrow Ht_1t_6t_{14} = H\underline{t_{36}}t_{11}$

$\Longrightarrow Ht_1t_6t_{14} = Ht_{34}t_{11}$, since $Ht_{34} = Ht_{36}$

$Ht_1t_6t_{14} = Ht_{34}t_{11}$

$\Longrightarrow Ht_1t_6t_{14} \in [12]$, since $Ht_{34}t_{11}$ is in [1 2].

1 symmetric generator will go to [1 2].

$Ht_1t_6t_{15} = Ht_1t_6t_{15}$

$\Longrightarrow Ht_1t_6t_{15} = Ht_1\underline{t_2^2}t_3^4$

$\Longrightarrow Ht_1t_6t_{15} = Ht_1[x^{-1}y^{-1}t_4t_3^4]t_3^4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x} = e^{y^{-1}x}$

$\Longrightarrow x^{-1}y^{-1}t_4t_{15}t_{34} = e$

$\Longrightarrow x^{-1}y^{-1}t_4t_3^4t_2^9 = e$

$\Longrightarrow x^{-1}y^{-1}t_4t_3^4t_2^9\underline{t_2^2} = \underline{t_2^2}$

$\Longrightarrow x^{-1}y^{-1}t_4t_3^4 = t_2^2$

$Ht_1t_6t_{15} = Ht_1x^{-1}y^{-1}t_4t_3^4t_3^4$

$\Longrightarrow Ht_1t_6t_{15} = Hx^{-1}y^{-1}[t_1]^{x^{-1}y^{-1}}t_4t_3^8$

$\Longrightarrow Ht_1t_6t_{15} = Ht_4^9t_4t_3^8$

$\Longrightarrow Ht_1t_6t_{15} = H\underline{t_4^{10}}t_3^8$

$\Longrightarrow Ht_1t_6t_{15} = Ht_2^8t_3^8$, since

$Ht_{30} = Ht_{40}$

$\Longrightarrow Ht_2^8 = Ht_4^{10}$

$Ht_1t_6t_{15} = H\underline{t_2^8t_3^8}$

$Ht_1t_6t_{15} = Hx^{-1}t_4^3$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^x = e^x$

$\Longrightarrow x^{-1}t_{12}t_{11}t_{10} = e$

$\Longrightarrow x^{-1}t_4^3t_3^3t_2^3 = e$

$\Longrightarrow x^{-1}t_4^3t_3^3t_2^3\underline{t_2^8} = \underline{t_2^8}$

$\Longrightarrow x^{-1}t_4^3\underline{t_3^8} = t_2^8\underline{t_3^8}$

$\Longrightarrow x^{-1}t_4^3 = t_2^8t_3^8$

$Ht_1t_6t_{15} = Hx^{-1}t_4^3$

$\Longrightarrow Ht_1t_6t_{15} = Ht_{12}$

$\Longrightarrow Ht_1t_6t_{15} \in [1]$, since Ht_{12} is in [1].

1 symmetric generator will go to [1].

$Ht_1t_6t_{16} = H\underline{t_1t_6}t_{16}$

$\Longrightarrow Ht_1t_6t_{16} = Ht_6t_1t_{16}$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_{16} = Ht_6t_1t_{16}$

$\Longrightarrow Ht_1t_6t_{16} = Ht_2^2\underline{t_1}t_4^4$

$\Longrightarrow Ht_1t_6t_{16} = Ht_2^2[xy^{-1}t_3^2t_4^7]t_4^4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\Longrightarrow yx^{-1}t_1t_{16}t_{35} = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9 = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\Longrightarrow yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\Longrightarrow \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\Longrightarrow t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_6t_{16} = Ht_2^2xy^{-1}t_3^2t_4^7t_4^4$

$\Longrightarrow Ht_1t_6t_{16} = Hxy^{-1}[t_2^2]^{xy^{-1}}t_3^2$

$\Longrightarrow Ht_1t_6t_{16} = Ht_3^8t_3^2$

$\Longrightarrow Ht_1t_6t_{16} = Ht_3^{10}$

$\Longrightarrow Ht_1t_6t_{16} = Ht_{39}$

$\Longrightarrow Ht_1t_6t_{16} \in [5]$, since Ht_{39} is in [5].

1 symmetric generator will go to [5].

$Ht_1t_6t_{17} = H\underline{t_1t_6}t_{17}$

$\Longrightarrow Ht_1t_6t_{17} = Ht_6t_1t_{17}$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_{17} = H\underline{t_6}t_1t_{17}$

$\Longrightarrow Ht_1t_6t_{17} = Ht_{28}t_1t_{17}$, since $Ht_6 = Ht_{28}$

$Ht_1t_6t_{17} = Ht_{28}t_1t_{17}$

$\Longrightarrow Ht_1t_6t_{17} = Ht_4^7t_1t_1^5$

$\Longrightarrow Ht_1t_6t_{17} = Ht_4^7\underline{t_1^6}$

$\Longrightarrow Ht_1t_6t_{17} = Ht_4^7[x^{-1}y^{-1}t_3^4t_2^9]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\Longrightarrow x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9 = t_1^6$

$Ht_1t_6t_{17} = Ht_4^7x^{-1}y^{-1}t_3^4t_2^9$

$\Longrightarrow Ht_1t_6t_{17} = Hx^{-1}y^{-1}[t_4^7]^{x^{-1}y^{-1}}t_3^4t_2^9$

$\Longrightarrow Ht_1t_6t_{17} = Ht_3^6t_3^4t_2^9$

$\Longrightarrow Ht_1t_6t_{17} = H\underline{t_3^{10}}t_2^9$

$\Longrightarrow Ht_1t_6t_{17} = Ht_1^8t_2^9$, since

$Ht_{29} = Ht_{39}$

$\Longrightarrow Ht_1^8 = Ht_3^{10}$

$Ht_1t_6t_{17} = H\underline{t_1^8}t_2^9$

$\Longrightarrow Ht_1t_6t_{17} = H[x^3t_3^3t_2^3]t_2^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$\Longrightarrow x^3t_3^3t_2^3t_1^3 = e$

$\Longrightarrow x^3t_3^3t_2^3t_1^3\underline{t_1^8} = \underline{t_1^8}$

$\Longrightarrow x^3t_3^3t_2^3 = t_1^8$

$Ht_1t_6t_{17} = Hx^3t_3^3t_2^3t_2^9$

$\Longrightarrow Ht_1t_6t_{17} = Ht_3^3t_2$

$\Longrightarrow Ht_1t_6t_{17} = H\underline{t_{11}}t_2$

$\Longrightarrow Ht_1t_6t_{17} = Ht_{17}t_2$, since $Ht_{11} = Ht_{17}$

$Ht_1t_6t_{17} = Ht_{17}t_2$

$\Longrightarrow Ht_1t_6t_{17} \in [110]$, since $Ht_{17}t_2$ is in [1 10]

1 symmetric generator will go to [1 10].

$Ht_1t_6t_{18} = H\underline{t_1}t_6t_{18}$

$\Longrightarrow Ht_1t_6t_{18} = Ht_{15}t_6t_{18}$, since $Ht_1 = Ht_{15}$

$Ht_1t_6t_{18} = Ht_{15}t_6t_{18}$

$\Longrightarrow Ht_1t_6t_{18} = Ht_3^4t_2^2t_2^5$

$\Longrightarrow Ht_1t_6t_{18} = H\underline{t_3^4}t_2^7$

$\Longrightarrow Ht_1t_6t_{18} = H[yxt_1^6t_2^2]t_2^7$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\Longrightarrow x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\implies x^{-1}y^{-1}t_3^4t_2^9\underline{t_1^2} = t_1^6\underline{t_1^2}$

$\implies \underline{yx}y^{-1}x^{-1}t_3^4 = \underline{yx}t_1^6t_1^2$

$\implies t_3^4 = yxt_1^6t_1^2$

$Ht_1t_6t_{18} = Hyxt_1^6t_2^2t_2^7$

$\implies Ht_1t_6t_{18} = Ht_1^6t_2^9$

$\implies Ht_1t_6t_{18} = H\underline{t_{21}}t_{34}$

$\implies Ht_1t_6t_{18} = Ht_{23}t_{34}$, since $Ht_{21} = Ht_{23}$

$Ht_1t_6t_{18} = H\underline{t_{23}t_{34}}$

$\implies Ht_1t_6t_{18} = Ht_{34}t_{23}$, since $Ht_{23}t_{34} = Ht_{34}t_{23}$

$Ht_1t_6t_{18} = Ht_{34}t_{23}$

$\implies Ht_1t_6t_{18} \in [16]$, since $Ht_{34}t_{23}$ is in [1 6]

1 symmetric generator will go to [1 6].

$Ht_1t_6t_{19} = Ht_1t_6t_{19}$

$\implies Ht_1t_6t_{19} = Ht_1\underline{t_2^2}t_3^5$

$\implies Ht_1t_6t_{19} = Ht_1[x^{-1}y^{-1}t_4t_3^4]t_3^5$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x} = e^{y^{-1}x}$

$\implies x^{-1}y^{-1}t_4t_{15}t_{34} = e$

$\implies x^{-1}y^{-1}t_4t_3^4t_2^9 = e$

$\implies x^{-1}y^{-1}t_4t_3^4t_2^9\underline{t_2^2} = \underline{t_2^2}$

$\implies x^{-1}y^{-1}t_4t_3^4 = t_2^2$

$Ht_1t_6t_{19} = Ht_1x^{-1}y^{-1}t_4t_3^4t_3^5$

$\implies Ht_1t_6t_{19} = Hx^{-1}y^{-1}[t_1]^{x^{-1}y^{-1}}t_4t_3^9$

$\implies Ht_1t_6t_{19} = Ht_4^9t_4t_3^9$

$\implies Ht_1t_6t_{19} = H\underline{t_4^{10}}t_3^9$

$\implies Ht_1t_6t_{19} = Ht_2^8t_3^9$, since

$Ht_{30} = Ht_{40}$

$\implies Ht_2^8 = Ht_4^{10}$

$Ht_1t_6t_{19} = H\underline{t_2^8}t_3^9$

$\implies Ht_1t_6t_{19} = H[x^{-1}t_4^3t_3^3]t_3^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^x = e^x$

$\implies x^{-1}t_{12}t_{11}t_{10} = e$

$\implies x^{-1}t_4^3t_3^3t_2^3 = e$

$\implies x^{-1}t_4^3t_3^3t_2^3\underline{t_2^8} = \underline{t_2^8}$

$\implies x^{-1}t_4^3t_3^3 = t_2^8$

$Ht_1t_6t_{19} = Hx^{-1}t_4^3t_3^3t_3^9$

$\implies Ht_1t_6t_{19} = Ht_4^3t_3$

$\implies Ht_1t_6t_{19} = H\underline{t_{12}}t_3$

$\implies Ht_1t_6t_{19} = Ht_{18}t_3$, since $Ht_{12} = Ht_{18}$

$Ht_1t_6t_{19} = Ht_{18}t_3$

$\implies Ht_1t_6t_{19} \in [110]$, since $Ht_{18}t_3$ is in [1 10].

1 symmetric generator will go to [1 10].

$Ht_1t_6t_{20} = H\underline{t_1t_6}t_{20}$

$\implies Ht_1t_6t_{20} = Ht_6t_1t_{20}$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_{20} = H\underline{t_6}t_1t_{20}$

$\implies Ht_1t_6t_{20} = Ht_2^2\underline{t_1}t_4^5$

$\implies Ht_1t_6t_{20} = Ht_2^2[xy^{-1}t_3^2t_4^7]t_4^5$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\implies yx^{-1}t_1t_{16}t_{35} = e$

$\implies yx^{-1}t_1t_4^4t_3^9 = e$

$\implies yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\implies yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\implies \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\implies t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_6t_{20} = Ht_2^2xy^{-1}t_3^2t_4$

$\implies Ht_1t_6t_{20} = Hxy^{-1}[t_2^2]^{xy^{-1}}t_3^2t_4$

$\implies Ht_1t_6t_{20} = Ht_3^8t_3^2t_4$

$\implies Ht_1t_6t_{20} = H\underline{t_3^{10}}t_4$

$\implies Ht_1t_6t_{20} = H[y^{-1}x^{-1}t_1^9t_4^5]t_4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y^{-1}} = e^{x^2y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{33}t_{20}t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5 = t_3^{10}$

$Ht_1t_6t_{20} = Hy^{-1}x^{-1}t_1^9t_4^5t_4$

$\Longrightarrow Ht_1t_6t_{20} = Ht_1^9t_4^6$

$\Longrightarrow Ht_1t_6t_{20} = H\underline{t_{33}}t_{24}$

$\Longrightarrow Ht_1t_6t_{20} = Ht_{35}t_{24}$, since $Ht_{33} = Ht_{35}$

$Ht_1t_6t_{20} = Ht_{35}t_{24}$

$\Longrightarrow Ht_1t_6t_{20} \in [16]$, since $Ht_{35}t_{24}$ is in [1 6].

1 symmetric generator will go to [1 6].

$Ht_1t_6t_{21} = H\underline{t_1t_6}t_{21}$

$\Longrightarrow Ht_1t_6t_{21} = Ht_6t_1t_{21}$, since $Ht_1t_6 = Ht_6t_1$

$\Longrightarrow Ht_1t_6t_{21} = H\underline{t_6}t_1t_{21}$

$\Longrightarrow Ht_1t_6t_{21} = Ht_{28}t_1t_{21}$, since $Ht_6 = Ht_{28}$

$Ht_1t_6t_{21} = Ht_4^7t_1t_1^6$

$\Longrightarrow Ht_1t_6t_{21} = H\underline{t_4^7}t_1^7$

$\Longrightarrow Ht_1t_6t_{21} = H[yx^{-1}t_2^5t_1]t_1^7$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\Longrightarrow yx^{-1}t_{18}t_1t_{16} = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4 = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\Longrightarrow yx^{-1}t_2^5t_1 = t_4^7$

$Ht_1t_6t_{21} = Hyx^{-1}t_2^5t_1t_1^7$

$\Longrightarrow Ht_1t_6t_{21} = H\underline{t_2^5}t_1^8$

$\Longrightarrow Ht_1t_6t_{21} = Ht_4^3t_1^8$, since

$Ht_{12} = Ht_{18}$

$\Longrightarrow Ht_4^3 = Ht_2^5$

$Ht_1t_6t_{21} = Ht_4^3\underline{t_1^8}$

$Ht_1t_6t_{21} = Ht_4^3[x^3t_3^3t_2^3]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$\implies x^3t_3^3t_2^3t_1^3 = e$

$\implies x^3t_3^3t_2^3t_1^3\underline{t_1^8} = \underline{t_1^8}$

$\implies x^3t_3^3t_2^3 = t_1^8$

$Ht_1t_6t_{21} = Ht_4^3x^3t_3^3t_2^3$

$\implies Ht_1t_6t_{21} = Hx^3[t_4^3]^{x^3}t_3^3t_2^3$

$\implies Ht_1t_6t_{21} = Ht_3^3t_3^3t_2^3$

$\implies Ht_1t_6t_{21} = H\underline{t_3^6}t_2^3$

$\implies Ht_1t_6t_{21} = Ht_1^6t_2^3$, since

$Ht_{21} = Ht_{23}$

$\implies Ht_1^6 = Ht_3^6$

$Ht_1t_6t_{21} = H\underline{t_1^6}t_2^3$

$Ht_1t_6t_{21} = H[x^{-1}y^{-1}t_3^4t_2^9]t_2^3$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\implies x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\implies x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\implies x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\implies x^{-1}y^{-1}t_3^4t_2^9 = t_1^6$

$Ht_1t_6t_{21} = Hx^{-1}y^{-1}t_3^4t_2^9t_2^3$

$\implies Ht_1t_6t_{21} = Ht_3^4t_2$

$\implies Ht_1t_6t_{21} = H\underline{t_{15}}t_2$

$\implies Ht_1t_6t_{21} = Ht_1t_2$, since $Ht_1 = Ht_{15}$

$Ht_1t_6t_{21} = Ht_1t_2$

$\implies Ht_1t_6t_{21} \in [12]$, since Ht_1t_2 is in [1 2].

1 symmetric generator will go to [1 2].

$Ht_1t_6t_{22} = Ht_1t_6t_{22}$

$\implies Ht_1t_6t_{22} = Ht_1t_2^2t_2^6$

$\implies Ht_1t_6t_{22} = Ht_1\underline{t_2^8}$

$\Longrightarrow Ht_1t_6t_{22} = Ht_1[x^{-1}t_4^3t_3^3]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^x = e^x$

$\Longrightarrow x^{-1}t_{12}t_{11}t_{10} = e$

$\Longrightarrow x^{-1}t_4^3t_3^3t_2^3 = e$

$\Longrightarrow x^{-1}t_4^3t_3^3t_2^3\underline{t_2^8} = \underline{t_2^8}$

$\Longrightarrow x^{-1}t_4^3t_3^3 = t_2^8$

$Ht_1t_6t_{22} = Ht_1x^{-1}t_4^3t_3^3$

$\Longrightarrow Ht_1t_6t_{22} = Hx^{-1}[t_1]^{x^{-1}}t_4^3t_3^3$

$\Longrightarrow Ht_1t_6t_{22} = Ht_4t_4^3t_3^3$

$\Longrightarrow Ht_1t_6t_{22} = Ht_4^4t_3^3$

$\Longrightarrow Ht_1t_6t_{22} = H\underline{t_{16}}t_{11}$

$\Longrightarrow Ht_1t_6t_{22} = Ht_2t_{11}$, since $Ht_2 = Ht_{16}$

$Ht_1t_6t_{22} = Ht_2t_{11}$

$\Longrightarrow Ht_1t_6t_{22} \in [110]$, since Ht_2t_{11} is in [1 10].

1 symmetric generator will go to [1 10].

$Ht_1t_6t_{23} = Ht_1t_6t_{23}$

$\Longrightarrow Ht_1t_6t_{23} = Ht_1\underline{t_2^2}t_3^6$

$\Longrightarrow Ht_1t_6t_{23} = Ht_1[x^{-1}y^{-1}t_4t_3^4]t_3^6$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x} = e^{y^{-1}x}$

$\Longrightarrow x^{-1}y^{-1}t_4t_{15}t_{34} = e$

$\Longrightarrow x^{-1}y^{-1}t_4t_3^4t_2^9 = e$

$\Longrightarrow x^{-1}y^{-1}t_4t_3^4t_2^9\underline{t_2^2} = \underline{t_2^2}$

$\Longrightarrow x^{-1}y^{-1}t_4t_3^4 = t_2^2$

$Ht_1t_6t_{23} = Ht_1x^{-1}y^{-1}t_4t_3^{10}$

$\Longrightarrow Ht_1t_6t_{23} = Hx^{-1}y^{-1}[t_1]^{x^{-1}y^{-1}}t_4t_3^{10}$

$\Longrightarrow Ht_1t_6t_{23} = Ht_4^9t_4t_3^{10}$

$\Longrightarrow Ht_1t_6t_{23} = H\underline{t_4^{10}}t_3^{10}$

$\Longrightarrow Ht_1t_6t_{23} = H\underline{t_4^{10}}t_3^{10}$

$\Longrightarrow Ht_1t_6t_{23} = Ht_2^8t_3^{10}$, since

$Ht_{30} = Ht_{40}$

$\implies Ht_2^8 = Ht_4^{10}$

$Ht_1t_6t_{23} = Ht_2^8\underline{t_3^{10}}$

$Ht_1t_6t_{23} = Ht_2^8[y^{-1}x^{-1}t_1^9t_4^5]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y^{-1}} = e^{x^2y^{-1}}$

$\implies y^{-1}x^{-1}t_{33}t_{20}t_3 = e$

$\implies y^{-1}x^{-1}t_1^9t_4^5t_3 = e$

$\implies y^{-1}x^{-1}t_1^9t_4^5t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\implies y^{-1}x^{-1}t_1^9t_4^5 = t_3^{10}$

$Ht_1t_6t_{23} = Ht_2^8y^{-1}x^{-1}t_1^9t_4^5$

$\implies Ht_1t_6t_{23} = Hy^{-1}x^{-1}[t_2^8]^{y^{-1}x^{-1}}t_1^9t_4^5$

$\implies Ht_1t_6t_{23} = Ht_1^7t_1^9t_4^5$

$\implies Ht_1t_6t_{23} = Ht_1^5t_4^5$

$\implies Ht_1t_6t_{23} = H\underline{t_{17}}t_{20}$

$\implies Ht_1t_6t_{23} = Ht_{11}t_{20}$, since $Ht_{11} = Ht_{17}$

$Ht_1t_6t_{23} = Ht_{11}t_{20}$

$\implies Ht_1t_6t_{23} \in [12]$, since $Ht_{11}t_{20}$ is in [1 2].

1 symmetric generator will go to [1 2].

$Ht_1t_6t_{24} = H\underline{t_1t_6}t_{24}$

$\implies Ht_1t_6t_{24} = Ht_6t_1t_{24}$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_{24} = Ht_6t_1t_{24}$

$\implies Ht_1t_6t_{24} = Ht_2^2\underline{t_1}t_4^6$

$\implies Ht_1t_6t_{24} = Ht_2^2[xy^{-1}t_3^2t_4^7]t_4^6$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\implies yx^{-1}t_1t_{16}t_{35} = e$

$\implies yx^{-1}t_1t_4^4t_3^9 = e$

$\implies yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\implies yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\implies \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\Longrightarrow t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_6t_{24} = Ht_2^2xy^{-1}t_3^2t_4^7t_4^6$

$\Longrightarrow Ht_1t_6t_{24} = Hxy^{-1}[t_2^2]^{xy^{-1}}t_3^2t_4^2$

$\Longrightarrow Ht_1t_6t_{24} = Ht_3^8t_3^2t_4^2$

$\Longrightarrow Ht_1t_6t_{24} = Ht_3^{10}\underline{t_4^2}$

$\Longrightarrow Ht_1t_6t_{24} = Ht_3^{10}[x^{-1}yt_2t_1^4]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\Longrightarrow x^{-1}yt_2t_{13}t_{36} = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9 = e$

$\Longrightarrow x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\Longrightarrow x^{-1}yt_2t_1^4 = t_4^2$

$Ht_1t_6t_{24} = Ht_3^{10}x^{-1}yt_2t_1^4$

$\Longrightarrow Ht_1t_6t_{24} = Ht_3^{10}x^{-1}yt_2t_1^4$

$\Longrightarrow Ht_1t_6t_{24} = Hx^{-1}y[t_3^{10}]^{x^{-1}y}t_2t_1^4$

$\Longrightarrow Ht_1t_6t_{24} = Ht_2^2t_2t_1^4$

$\Longrightarrow Ht_1t_6t_{24} = Ht_2^3t_1^4$

$\Longrightarrow Ht_1t_6t_{24} = H\underline{t_{10}}t_{13}$

$\Longrightarrow Ht_1t_6t_{24} = Ht_{20}t_{13}$, since $Ht_{10} = Ht_{20}$

$Ht_1t_6t_{24} = Ht_{20}t_{13}$

$\Longrightarrow Ht_1t_6t_{24} \in [12]$, since $Ht_{20}t_{13}$ is in [1 2].

1 symmetric generator will go to [1 2].

$Ht_1t_6t_{25} = H\underline{t_1t_6}t_{25}$

$\Longrightarrow Ht_1t_6t_{25} = Ht_6t_1t_{25}$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_{25} = H\underline{t_6}t_1t_{25}$

$\Longrightarrow Ht_1t_6t_{25} = Ht_{28}t_1t_{25}$, since $Ht_6 = Ht_{28}$

$Ht_1t_6t_{25} = Ht_{28}t_1t_{25}$

$\Longrightarrow Ht_1t_6t_{25} = Ht_4^7t_1t_1^7$

$\Longrightarrow Ht_1t_6t_{25} = Ht_4^7\underline{t_1^8}$

$\Longrightarrow Ht_1t_6t_{25} = Ht_4^7[x^3t_3^3t_2^3]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$\Longrightarrow x^3t_3^3t_2^3t_1^3 = e$

$\Longrightarrow x^3t_3^3t_2^3t_1^3\underline{t_1^8} = \underline{t_1^8}$

$\Longrightarrow x^3t_3^3t_2^3 = t_1^8$

$Ht_1t_6t_{25} = Ht_4^7x^3t_3^3t_2^3$

$\Longrightarrow Ht_1t_6t_{25} = Hx^3[t_4^7]^{x^3}t_3^3t_2^3$

$\Longrightarrow Ht_1t_6t_{25} = Ht_3^7t_3^3t_2^3$

$\Longrightarrow Ht_1t_6t_{25} = Ht_3^{10}t_2^3$

$\Longrightarrow Ht_1t_6t_{25} = H\underline{t_{39}t_{10}}$

$\Longrightarrow Ht_1t_6t_{25} = Ht_{10}t_{39}$, since $Ht_{10}t_{39} = Ht_{39}t_{10}$

$Ht_1t_6t_{25} = Ht_{10}t_{39}$

$\Longrightarrow Ht_1t_6t_{25} \in [16]$, since $Ht_{10}t_{39}$ is in [1 6].

1 symmetric generator will go to [1 6].

$Ht_1t_6t_{26} = H\underline{t_1}t_6t_{26}$

$\Longrightarrow Ht_1t_6t_{26} = Ht_{15}t_6t_{26}$, since $Ht_1 = Ht_{15}$

$Ht_1t_6t_{26} = Ht_{15}t_6t_{26}$

$\Longrightarrow Ht_1t_6t_{26} = Ht_3^4t_2^2t_2^7$

$\Longrightarrow Ht_1t_6t_{26} = H\underline{t_3^4}t_2^9$

$\Longrightarrow Ht_1t_6t_{26} = H[yxt_1^6t_2^2]t_2^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\Longrightarrow x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9\underline{t_1^2} = t_1^6\underline{t_1^2}$

$\Longrightarrow \underline{yx}y^{-1}x^{-1}t_3^4 = \underline{yx}t_1^6t_1^2$

$\Longrightarrow t_3^4 = yxt_1^6t_1^2$

$Ht_1t_6t_{26} = Hyxt_1^6t_2^2t_2^9$

$\Longrightarrow Ht_1t_6t_{26} = Ht_1^6$

$\Longrightarrow Ht_1t_6t_{26} = Ht_{21}$

$\Longrightarrow Ht_1t_6t_{26} \in [5]$, since Ht_{21} is in [5].

1 symmetric generator will go to [5].

$Ht_1t_6t_{27} = Ht_1t_6t_{27}$

$\implies Ht_1t_6t_{27} = Ht_1\underline{t_2^2}t_3^7$

$\implies Ht_1t_6t_{27} = Ht_1[x^{-1}y^{-1}t_4t_3^4]t_3^7$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x} = e^{y^{-1}x}$

$\implies x^{-1}y^{-1}t_4t_{15}t_{34} = e$

$\implies x^{-1}y^{-1}t_4t_3^4t_2^9 = e$

$\implies x^{-1}y^{-1}t_4t_3^4t_2^9\underline{t_2^2} = \underline{t_2^2}$

$\implies x^{-1}y^{-1}t_4t_3^4 = t_2^2$

$Ht_1t_6t_{27} = Ht_1x^{-1}y^{-1}t_4t_3^4t_3^7$

$\implies Ht_1t_6t_{27} = Hx^{-1}y^{-1}[t_1]^{x^{-1}y^{-1}}t_4$

$\implies Ht_1t_6t_{27} = Ht_4^9t_4$

$\implies Ht_1t_6t_{27} = Ht_4^{10}$

$\implies Ht_1t_6t_{27} \in [5]$, since Ht_4^{10} is in [5].

1 symmetric generator will go to [5].

$Ht_1t_6t_{28} = H\underline{t_1t_6}t_{28}$

$\implies Ht_1t_6t_{28} = Ht_6t_1t_{28}$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_{28} = Ht_6t_1t_{28}$

$\implies Ht_1t_6t_{28} = Ht_2^2\underline{t_1}t_4^7$

$\implies Ht_1t_6t_{28} = Ht_2^2[xy^{-1}t_3^2t_4^7]t_4^7$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\implies yx^{-1}t_1t_{16}t_{35} = e$

$\implies yx^{-1}t_1t_4^4t_3^9 = e$

$\implies yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\implies yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\implies \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\implies t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_6t_{28} = Ht_2^2xy^{-1}t_3^2t_4^7t_4^7$

$\implies Ht_1t_6t_{28} = Hxy^{-1}[t_2^2]^{xy^{-1}}t_3^2t_4^3$

$\Longrightarrow Ht_1t_6t_{28} = Ht_3^8t_3^2t_4^3$

$\Longrightarrow Ht_1t_6t_{28} = H\underline{t_3^{10}}t_4^3$

$\Longrightarrow Ht_1t_6t_{28} = Ht_1^8t_4^3$, since

$Ht_{29}t_{39}$

$\Longrightarrow Ht_1^8 = Ht_3^{10}$

$Ht_1t_6t_{28} = Ht_1^8\underline{t_4^3}$

$Ht_1t_6t_{28} = Ht_1^8[xt_2^8t_3^8]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^x = e^x$

$\Longrightarrow x^{-1}t_{12}t_{11}t_{10} = e$

$\Longrightarrow x^{-1}t_4^3t_3^3t_2^3 = e$

$\Longrightarrow x^{-1}t_4^3t_3^3t_2^3\underline{t_2^8} = \underline{t_3^8}$

$\Longrightarrow x^{-1}t_4^3t_3^3\underline{t_3^8} = t_2^8\underline{t_3^8}$

$\Longrightarrow \underline{x}x^{-1}t_4^3 = \underline{x}t_2^8t_3^8$

$\Longrightarrow t_4^3 = xt_2^8t_3^8$

$Ht_1t_6t_{28} = Ht_1^8xt_2^8t_3^8$

$\Longrightarrow Ht_1t_6t_{28} = Hx[t_1^8]^xt_2^8t_3^8$

$\Longrightarrow Ht_1t_6t_{28} = Ht_2^8t_2^8t_3^8$

$\Longrightarrow Ht_1t_6t_{28} = Ht_2^5t_3^8$

$\Longrightarrow Ht_1t_6t_{28} = Ht_{18}t_{31}$

$\Longrightarrow Ht_1t_6t_{28} \in [16]$, since $Ht_{18}t_{31}$ is in [1 6].

1 symmetric generator will go to [1 6].

$Ht_1t_6t_{29} = H\underline{t_1t_6}t_{29}$

$\Longrightarrow Ht_1t_6t_{29} = Ht_6t_1t_{29}$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_{29} = H\underline{t_6}t_1t_{29}$

$Ht_1t_6t_{29} = Ht_{28}t_1t_{29}$, since $Ht_6 = Ht_{28}$

$Ht_1t_6t_{29} = Ht_{28}t_1t_{29}$

$\Longrightarrow Ht_1t_6t_{29} = H\underline{t_4^7}t_1t_1^8$

$\Longrightarrow Ht_1t_6t_{29} = H[yx^{-1}t_2^5t_1]t_1^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\Longrightarrow yx^{-1}t_{18}t_1t_{16} = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4 = e$

$\Longrightarrow yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\Longrightarrow yx^{-1}t_2^5t_1 = t_4^7$

$Ht_1t_6t_{29} = Hyx^{-1}t_2^5t_1t_1^9$

$\Longrightarrow Ht_1t_6t_{29} = Ht_2^5t_1^{10}$

$\Longrightarrow Ht_1t_6t_{29} = H\underline{t_{18}}t_{37}$

$\Longrightarrow Ht_1t_6t_{29} = Ht_{12}t_{37}$, since $Ht_{12} = Ht_{18}$

$Ht_1t_6t_{29} = Ht_{12}t_{37}$

$\Longrightarrow Ht_1t_6t_{29} \in [16]$, since $Ht_{12}t_{37}$ is in [1 6].

$\Longrightarrow Ht_1t_6t_{29} = Ht_2^2t_1^9$

1 symmetric generator will go to [1 6].

$Ht_1t_6t_{30} = H\underline{t_1}t_6t_{30}$

$\Longrightarrow Ht_1t_6t_{30} = Ht_{15}t_6t_{30}$, since $Ht_1 = Ht_{15}$

$Ht_1t_6t_{30} = Ht_{15}t_6t_{30}$

$\Longrightarrow Ht_1t_6t_{30} = Ht_3^4t_2^2t_2^8$

$\Longrightarrow Ht_1t_6t_{30} = H\underline{t_3^4}t_2^{10}$

$\Longrightarrow Ht_1t_6t_{30} = H[yxt_1^6t_2^2]t_2^{10}$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\Longrightarrow x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9\underline{t_1^2} = t_1^6\underline{t_1^2}$

$\Longrightarrow \underline{yx}y^{-1}x^{-1}t_3^4 = \underline{yx}t_1^6t_1^2$

$\Longrightarrow t_3^4 = yxt_1^6t_1^2$

$Ht_1t_6t_{30} = Hyxt_1^6t_2^2t_2^{10}$

$\Longrightarrow Ht_1t_6t_{30} = Ht_1^6\underline{t_2}$

$\Longrightarrow Ht_1t_6t_{30} = Ht_1^6[y^{-1}xt_4^2t_1^7]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\implies x^{-1}yt_2t_{13}t_{36} = e$

$\implies x^{-1}yt_2t_1^4t_4^9 = e$

$\implies x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\implies x^{-1}yt_2t_1^4\underline{t_1^7} = t_4^2\underline{t_1^7}$

$\implies \underline{y^{-1}x}x^{-1}yt_2 = \underline{y^{-1}x}t_4^2t_1^7$

$\implies t_2 = y^{-1}xt_4^2t_1^7$

$Ht_1t_6t_{30} = Ht_1^6y^{-1}xt_4^2t_1^7$

$\implies Ht_1t_6t_{30} = Hy^{-1}x[t_1^6]^{y^{-1}x}t_4^2t_1^7$

$\implies Ht_1t_6t_{30} = Ht_4^2t_4^2t_1^7$

$\implies Ht_1t_6t_{30} = Ht_4^4t_1^7$

$\implies Ht_1t_6t_{30} = Ht_{16}t_{25}$

$\implies Ht_1t_6t_{30} \in [16]$, since $Ht_{16}t_{25}$ is in [1 6].

1 symmetric generator will go to [1 6].

$Ht_1t_6t_{31} = Ht_1t_6t_{31}$

$\implies Ht_1t_6t_{31} = Ht_1\underline{t_2^2}t_3^8$

$\implies Ht_1t_6t_{31} = Ht_1[x^{-1}y^{-1}t_4t_3^4]t_3^8$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x} = e^{y^{-1}x}$

$\implies x^{-1}y^{-1}t_4t_{15}t_{34} = e$

$\implies x^{-1}y^{-1}t_4t_3^4t_2^9 = e$

$\implies x^{-1}y^{-1}t_4t_3^4t_2^9\underline{t_2^2} = \underline{t_2^2}$

$\implies x^{-1}y^{-1}t_4t_3^4 = t_2^2$

$Ht_1t_6t_{31} = Ht_1x^{-1}y^{-1}t_4t_3^4t_3^8$

$\implies Ht_1t_6t_{31} = Hx^{-1}y^{-1}[t_1]^{x^{-1}y^{-1}}t_4t_3$

$\implies Ht_1t_6t_{31} = Ht_4^9t_4t_3$

$\implies Ht_1t_6t_{31} = H\underline{t_4^{10}}t_3$

$\implies Ht_1t_6t_{31} = Ht_2^8t_3$, since

$Ht_{30} = Ht_{40}$

$\implies Ht_2^8 = Ht_4^{10}$

$Ht_1t_6t_{31} = H\underline{t_2^8}t_3$

$\implies Ht_1t_6t_{31} = H[x^{-1}t_4^3t_3^3]t_3$, since by Equation 5.8

$x^3 t_{11} t_{10} t_9 = e$

$[x^3 t_{11} t_{10} t_9]^x = e^x$

$\implies x^{-1} t_{12} t_{11} t_{10} = e$

$\implies x^{-1} t_4^3 t_3^3 t_2^3 = e$

$\implies x^{-1} t_4^3 t_3^3 t_2^3 \underline{t_2^8} = \underline{t_2^8}$

$\implies x^{-1} t_4^3 t_3^3 = t_2^8$

$Ht_1 t_6 t_{31} = Hx^{-1} t_4^3 t_3^3 t_3$

$\implies Ht_1 t_6 t_{31} = Ht_4^3 t_3^4$

$\implies Ht_1 t_6 t_{31} = Ht_{12} t_{15}$

$\implies Ht_1 t_6 t_{31} = Ht_{18} t_{15}$, since $Ht_{12} = Ht_{18}$

$Ht_1 t_6 t_{31} = Ht_{18} t_{15}$

$\implies Ht_1 t_6 t_{31} \in [12]$, since $Ht_{18} t_{15}$ is in [1 2].

1 symmetric generator will go to [1 2].

$Ht_1 t_6 t_{32} = H\underline{t_1 t_6} t_{32}$

$\implies Ht_1 t_6 t_{32} = Ht_6 t_1 t_{32}$, since $Ht_1 t_6 = Ht_6 t_1$

$Ht_1 t_6 t_{32} = Ht_6 t_1 t_{32}$

$\implies Ht_1 t_6 t_{32} = Ht_2^2 \underline{t_1} t_4^8$

$\implies Ht_1 t_6 t_{32} = Ht_2^2 [xy^{-1} t_3^2 t_4^7] t_4^8$, since by Equation 5.8

$x^3 t_{11} t_{10} t_9 = e$

$[x^3 t_{11} t_{10} t_9]^{x^2 y} = e^{x^2 y}$

$\implies yx^{-1} t_1 t_{16} t_{35} = e$

$\implies yx^{-1} t_1 t_4^4 t_3^9 = e$

$\implies yx^{-1} t_1 t_4^4 t_3^9 \underline{t_3^2} = \underline{t_3^2}$

$\implies yx^{-1} t_1 t_4^4 \underline{t_4^7} = t_3^2 \underline{t_4^7}$

$\implies \underline{xy^{-1}} yx^{-1} t_1 = \underline{xy^{-1}} t_3^2 t_4^7$

$\implies t_1 = xy^{-1} t_3^2 t_4^7$

$Ht_1 t_6 t_{32} = Ht_2^2 xy^{-1} t_3^2 t_4^7 t_4^8$

$\implies Ht_1 t_6 t_{32} = Hxy^{-1} [t_2^2]^{xy^{-1}} t_3^2 t_4^4$

$\implies Ht_1 t_6 t_{32} = Ht_3^8 t_3^2 t_4^4$

$\implies Ht_1 t_6 t_{32} = H\underline{t_3^{10}} t_4^4$

$\implies Ht_1 t_6 t_{32} = H[y^{-1} x^{-1} t_1^9 t_4^5] t_4^4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y^{-1}} = e^{x^2y^{-1}}$

$\implies y^{-1}x^{-1}t_{33}t_{20}t_3 = e$

$\implies y^{-1}x^{-1}t_1^9t_4^5t_3 = e$

$\implies y^{-1}x^{-1}t_1^9t_4^5t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\implies y^{-1}x^{-1}t_1^9t_4^5 = t_3^{10}$

$Ht_1t_6t_{32} = Hy^{-1}x^{-1}t_1^9t_4^5t_4^4$

$\implies Ht_1t_6t_{32} = Ht_1^9t_4^9$

$\implies Ht_1t_6t_{32} = H\underline{t_{33}}t_{36}$

$\implies Ht_1t_6t_{32} = Ht_{35}t_{36}$, since $Ht_{33} = Ht_{35}$

$Ht_1t_6t_{32} = Ht_{35}t_{36}$

$\implies Ht_1t_6t_{32} \in [110]$, since $Ht_{35}t_{36}$ is in [1 10].

1 symmetric generator will go to [1 10].

$Ht_1t_6t_{33} = H\underline{t_1t_6}t_{33}$

$\implies Ht_1t_6t_{30} = Ht_6t_1t_{33}$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_{33} = H\underline{t_6}t_1t_{33}$

$Ht_1t_6t_{33} = Ht_{28}t_1t_{33}$, since $Ht_6 = Ht_{28}$

$Ht_1t_6t_{33} = Ht_{28}t_1t_{33}$

$\implies Ht_1t_6t_{33} = H\underline{t_4^7}t_1t_1^9$

$\implies Ht_1t_6t_{33} = H[yx^{-1}t_2^5t_1]t_1^{10}$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y} = e^{x^{-1}y}$

$\implies yx^{-1}t_{18}t_1t_{16} = e$

$\implies yx^{-1}t_2^5t_1t_4^4 = e$

$\implies yx^{-1}t_2^5t_1t_4^4\underline{t_4^7} = \underline{t_4^7}$

$\implies yx^{-1}t_2^5t_1 = t_4^7$

$Ht_1t_6t_{33} = Hyx^{-1}t_2^5t_1t_1^{10}$

$\implies Ht_1t_6t_{33} = Ht_2^5$

$\implies Ht_1t_6t_{33} = Ht_{18}$

$\implies Ht_1t_6t_{33} \in [1]$, since Ht_{18} is in [1].

1 symmetric generator will go to [1].

$Ht_1t_6t_{34} = Ht_1t_6t_{34}$
$\Longrightarrow Ht_1t_6t_{34} = Ht_1t_2^2t_2^9$
$\Longrightarrow Ht_1t_6t_{34} = Ht_1$
$\Longrightarrow Ht_1t_6t_{34} \in [1]$, since Ht_1 is in [1].
1 symmetric generator will go to [1].

$Ht_1t_6t_{35} = Ht_1t_6t_{35}$
$\Longrightarrow Ht_1t_6t_{34} = Ht_1\underline{t_2^2}t_3^9$
$\Longrightarrow Ht_1t_6t_{34} = Ht_1[x^{-1}y^{-1}t_4t_3^4]t_3^9$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{y^{-1}x} = e^{y^{-1}x}$
$\Longrightarrow x^{-1}y^{-1}t_4t_{15}t_{34} = e$
$\Longrightarrow x^{-1}y^{-1}t_4t_3^4t_2^9 = e$
$\Longrightarrow x^{-1}y^{-1}t_4t_3^4t_2^9\underline{t_2^2} = \underline{t_2^2}$
$\Longrightarrow x^{-1}y^{-1}t_4t_3^4 = t_2^2$
$Ht_1t_6t_{34} = Ht_1x^{-1}y^{-1}t_4t_3^4t_3^9$
$\Longrightarrow Ht_1t_6t_{34} = Hx^{-1}y^{-1}[t_1]^{x^{-1}y^{-1}}t_4t_3^2$
$\Longrightarrow Ht_1t_6t_{34} = Ht_4^9t_4t_3^2$
$\Longrightarrow Ht_1t_6t_{34} = H\underline{t_4^{10}}t_3^2$
$\Longrightarrow Ht_1t_6t_{34} = Ht_2^8t_3^2$, since
$Ht_{30} = Ht_{40}$
$\Longrightarrow Ht_2^8 = Ht_4^{10}$
$Ht_1t_6t_{34} = Ht_2^8\underline{t_3^2}$
$Ht_1t_6t_{34} = Ht_2^8[yx^{-1}t_1t_4^4]$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$
$\Longrightarrow yx^{-1}t_1t_{16}t_{35} = e$
$\Longrightarrow yx^{-1}t_1t_4^4t_3^9 = e$
$\Longrightarrow yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$
$\Longrightarrow yx^{-1}t_1t_4^4 = t_3^2$
$Ht_1t_6t_{34} = Ht_2^8yx^{-1}t_1t_4^4$

$\Longrightarrow Ht_1t_6t_{34} = Hyx^{-1}[t_2^8]^{yx^{-1}}t_1t_4^4$

$\Longrightarrow Ht_1t_6t_{34} = Ht_1^6t_1t_4^4$

$\Longrightarrow Ht_1t_6t_{34} = Ht_1^7t_4^4$

$\Longrightarrow Ht_1t_6t_{34} = H\underline{t_{25}t_{40}}$

$\Longrightarrow Ht_1t_6t_{34} = Ht_{40}t_{25}$, since $Ht_{25}t_{40} = Ht_{40} = Ht_{25}$

$Ht_1t_6t_{34} = Ht_{40}t_{25}$

$\Longrightarrow Ht_1t_6t_{34} \in [16]$, since $Ht_{40}t_{25}$ is in [1 6].

1 symmetric generator will go to [1 6].

$Ht_1t_6t_{36} = H\underline{t_1t_6}t_{36}$

$\Longrightarrow Ht_1t_6t_{36} = Ht_6t_1t_{36}$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_{36} = Ht_2^2t_1t_4^9$

$Ht_1t_6t_{36} = Ht_2^2\underline{t_1}t_4^9$

$Ht_1t_6t_{36} = Ht_2^2[xy^{-1}t_3^2t_4^7]t_4^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\Longrightarrow yx^{-1}t_1t_{16}t_{35} = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9 = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\Longrightarrow yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\Longrightarrow \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\Longrightarrow t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_6t_{36} = Ht_2^2xy^{-1}t_3^2t_4^7t_4^9$

$\Longrightarrow Ht_1t_6t_{36} = Hxy^{-1}[t_2^2]^{xy^{-1}}t_3^2t_4^5$

$\Longrightarrow Ht_1t_6t_{36} = Ht_3^8t_3^2t_4^5$

$\Longrightarrow Ht_1t_6t_{36} = H\underline{t_3^{10}}t_4^5$

$\Longrightarrow Ht_1t_6t_{36} = Ht_1^8t_4^5$, since

$Ht_{29} = Ht_{39}$

$\Longrightarrow Ht_1^8 = Ht_3^{10}$

$Ht_1t_6t_{36} = Ht_1^8\underline{t_4^5}$

$\Longrightarrow Ht_1t_6t_{36} = Ht_1^8\underline{t_4^5}$

$\Longrightarrow Ht_1t_6t_{36} = Ht_1^8[xyt_2^7t_3^{10}]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy^{-1}} = e^{xy^{-1}}$

$\implies y^{-1}x^{-1}t_{20}t_3t_{14} = e$

$\implies y^{-1}x^{-1}t_4^5t_3t_2^4 = e$

$\implies y^{-1}x^{-1}t_4^5t_3t_2^4\underline{t_2^7} = \underline{t_2^7}$

$\implies y^{-1}x^{-1}t_4^5t_3\underline{t_3^{10}} = t_2^7\underline{t_3^{10}}$

$\implies \underline{xy}y^{-1}x^{-1}t_4^5 = \underline{xy}t_2^7t_3^{10}$

$\implies \underline{t_4^5} = xyt_2^7t_3^{10}$

$Ht_1t_6t_{36} = Ht_1^8xyt_2^7t_3^{10}$

$\implies Ht_1t_6t_{36} = Hxy[t_1^8]^{xy}t_2^7t_3^{10}$

$\implies Ht_1t_6t_{36} = Ht_2^6t_2^7t_3^{10}$

$\implies Ht_1t_6t_{36} = Ht_2^2\underline{t_3^{10}}$

$\implies Ht_1t_6t_{36} = Ht_2^2[y^{-1}x^{-1}t_1^9t_4^5]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y^{-1}} = e^{x^2y^{-1}}$

$\implies y^{-1}x^{-1}t_{33}t_{20}t_3 = e$

$\implies y^{-1}x^{-1}t_1^9t_4^5t_3 = e$

$\implies y^{-1}x^{-1}t_1^9t_4^5t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\implies y^{-1}x^{-1}t_1^9t_4^5 = t_3^{10}$

$Ht_1t_6t_{36} = Ht_2^2y^{-1}x^{-1}t_1^9t_4^5$

$\implies Ht_1t_6t_{36} = Hy^{-1}x^{-1}[t_2^2]^{y^{-1}x^{-1}}t_1^9t_4^5$

$\implies Ht_1t_6t_{36} = Ht_1^{10}t_1^9t_4^5$

$\implies Ht_1t_6t_{36} = Ht_1^8t_4^5$

$\implies Ht_1t_6t_{36} = H\underline{t_{29}t_{20}}$

$\implies Ht_1t_6t_{36} = H\underline{t_{20}t_{29}}$, since $Ht_{20}t_{29} = Ht_{29}t_{20}$

$Ht_1t_6t_{36} = Ht_{20}t_{29}$

$\implies Ht_1t_6t_{36} \in [16]$, since $Ht_{20}t_{29}$ is in [1 6].

1 symmetric generator will go to [1 6].

$Ht_1t_6t_{37} = H\underline{t_1t_6}t_{37}$

$\implies Ht_1t_6t_{37} = Ht_6t_1t_{37}$, since $Ht_1t_6 = Ht_6t_1$

$Ht_1t_6t_{37} = Ht_2^2t_1t_1^{10}$

$Ht_1t_6t_{37} = Ht_2^2$

$Ht_1t_6t_{37} = Ht_6$

$Ht_1t_6t_{37} \in [5]$, since Ht_6 is in [5].

1 symmetric generator will go to [5].

$Ht_1t_6t_{38} = Ht_1t_6t_{38}$

$\implies Ht_1t_6t_{38} = Ht_1t_2^2t_2^{10}$

$\implies Ht_1t_6t_{38} = Ht_1t_2$

$\implies Ht_1t_6t_{38} \in [12]$, since Ht_1t_2 is in [1 2].

1 symmetric generator will go to [1 2].

$Ht_1t_6t_{39} = Ht_1t_6t_{39}$

$\implies Ht_1t_6t_{39} = Ht_1\underline{t_2^2}t_3^{10}$

$\implies Ht_1t_6t_{39} = Ht_1[x^{-1}y^{-1}t_4t_3^4]t_3^{10}$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x} = e^{y^{-1}x}$

$\implies x^{-1}y^{-1}t_4t_{15}t_{34} = e$

$\implies x^{-1}y^{-1}t_4t_3^4t_2^9 = e$

$\implies x^{-1}y^{-1}t_4t_3^4t_2^9\underline{t_2^2} = \underline{t_2^2}$

$\implies x^{-1}y^{-1}t_4t_3^4 = t_2^2$

$Ht_1t_6t_{39} = Ht_1x^{-1}y^{-1}t_4t_3^4t_3^{10}$

$\implies Ht_1t_6t_{39} = Hx^{-1}y^{-1}[t_1]^{x^{-1}y^{-1}}t_4t_3^3$

$\implies Ht_1t_6t_{39} = Ht_4^9t_4t_3^3$

$\implies Ht_1t_6t_{39} = Ht_4^{10}t_3^3$

$\implies Ht_1t_6t_{39} = Ht_{40}t_{11}$

$\implies Ht_1t_6t_{39} = Ht_{11}t_{40}$, since $Ht_{11}t_{40} = Ht_{40}t_{11}$

$Ht_1t_6t_{39} = Ht_{11}t_{40}$

$\implies t_1t_6t_{39} \in [16]$, since $Ht_{11}t_{40}$ is in [1 6].

1 symmetric generator will go to [1 6].

$Ht_1t_6t_{40} = H\underline{t_1t_6}t_{40}$

$\implies Ht_1t_6t_{40} = Ht_6t_1t_40$, since $Ht_1t_6 = Ht_6t_1$

$\implies Ht_1t_6t_{40} = Ht_2^2\underline{t_1}t_4^{10}$

$\implies Ht_1t_6t_{40} = Ht_2^2[xy^{-1}t_3^2t_4^7]t_4^{10}$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\implies yx^{-1}t_1t_{16}t_{35} = e$

$\implies yx^{-1}t_1t_4^4t_3^9 = e$

$\implies yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\implies yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\implies \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\implies t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_6t_{40} = Ht_2^2xy^{-1}t_3^2t_4^7t_4^{10}$

$\implies Ht_1t_6t_{40} = Hxy^{-1}[t_2^2]^{xy^{-1}}t_3^2t_4^6$

$\implies Ht_1t_6t_{40} = Ht_3^8t_3^2t_4^6$

$\implies Ht_1t_6t_{40} = H\underline{t_3^{10}}t_4^6$

$\implies Ht_1t_6t_{40} = H[y^{-1}x^{-1}t_1^9t_4^5]t_4^6$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y^{-1}} = e^{x^2y^{-1}}$

$\implies y^{-1}x^{-1}t_{33}t_{20}t_3 = e$

$\implies y^{-1}x^{-1}t_1^9t_4^5t_3 = e$

$\implies y^{-1}x^{-1}t_1^9t_4^5t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\implies y^{-1}x^{-1}t_1^9t_4^5 = t_3^{10}$

$Ht_1t_6t_{40} = Hy^{-1}x^{-1}t_1^9t_4^5t_4^6$

$\implies Ht_1t_6t_{40} = Ht_1^9$

$\implies Ht_1t_6t_{40} = Ht_{33}$

$\implies Ht_1t_6t_{40} \in [1]$, since Ht_{33} is in [1].

1 symmetric generator will go to [1].

The orbits of $N^{(110)}$ are {1, 3}, {2, 4 {5, 7}, {6, 8}, {9, 11}, {10, 12}, {13, 15}, {14, 16}, {17, 19}, {18, 20}, {21, 23}, {22, 24}, {25, 27}, {26, 28}, {29, 31}, {30, 32}, {33, 35}, {34, 36}, {37, 39}, and {38, 40}. We will check to see where $Ht_1t_{10}t_1, Ht_1t_{10}t_2, Ht_1t_{10}t_5, Ht_1t_{10}t_6, Ht_1t_{10}t_9, Ht_1t_{10}t_{10}, Ht_1t_{10}t_{13}, Ht_1t_{10}t_{14}, Ht_1t_{10}t_{17},$ $Ht_1t_{10}t_{18}, Ht_1t_{10}t_{21}, Ht_1t_{10}t_{22}, Ht_1t_{10}t_{25}, Ht_1t_{10}t_{26}, Ht_1t_{10}t_{29}, Ht_1t_{10}t_{30}, Ht_1t_{10}t_{33},$

$Ht_1t_{10}t_{34}, Ht_1t_{10}t_{37}, Ht_1t_{10}t_{38}$ belong.

$Ht_1t_{10}t_1 = H\underline{t_1}t_{10}t_1$
$\Longrightarrow Ht_1t_{10}t_1 = Ht_{15}t_{10}t_1$, since $Ht_1 = Ht_{15}$
$\Longrightarrow Ht_1t_{10}t_1 = Ht_3^4\underline{t_2^3}t_1$
$\Longrightarrow Ht_1t_{10}t_1 = Ht_3^4[xt_4^8t_1^8]t_1$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{x^{-1}} = e^{x^{-1}}$
$\Longrightarrow x^{-1}t_{10}t_9t_{12} = e$
$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3 = e$
$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3\underline{t_4^8} = \underline{t_4^8}$
$\Longrightarrow x^{-1}t_2^3t_1^3\underline{t_1^8} = t_4^8\underline{t_1^8}$
$\Longrightarrow \underline{x}x^{-1}t_2^3 = \underline{x}t_4^8t_1^8$
$\Longrightarrow t_2^3 = xt_4^8t_1^8$
$Ht_1t_{10}t_1 = Ht_3^4xt_4^8t_1^8t_1$
$\Longrightarrow Ht_1t_{10}t_1 = Hx[t_3^4]^xt_4^8t_1^9$
$\Longrightarrow Ht_1t_{10}t_1 = Ht_4^4t_4^8t_1^9$
$\Longrightarrow Ht_1t_{10}t_1 = H\underline{t_4}t_1^9$
$\Longrightarrow Ht_1t_{10}t_1 = Ht_2^4t_1^9$, since
$Ht_4 = Ht_{14}$
$\Longrightarrow Ht_4 = Ht_2^4$
$Ht_1t_{10}t_1 = H\underline{t_2^4}t_1^9$
$Ht_1t_{10}t_1 = H[xyt_4^6t_1^2]t_1^9$, since by Equation 5.8
$x^3t_{11}t_{10}t_9 = e$
$[x^3t_{11}t_{10}t_9]^{x^{-1}y^{-1}} = e^{x^{-1}y^{-1}}$
$\Longrightarrow y^{-1}x^{-1}t_{14}t_{33}t_{20} = e$
$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5 = e$
$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5\underline{t_4^6} = \underline{t_4^6}$
$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9\underline{t_1^2} = t_4^6\underline{t_1^2}$
$\Longrightarrow \underline{xy}y^{-1}x^{-1}t_2^4 = \underline{xy}t_4^6t_1^2$
$\Longrightarrow t_2^4 = xyt_4^6t_1^2$
$Ht_1t_{10}t_1 = Hxyt_4^6t_1^2t_1^9$

$\Longrightarrow Ht_1t_{10}t_1 = Ht_4^6$

$\Longrightarrow Ht_1t_{10}t_1 = Ht_{24}$

$\Longrightarrow Ht_1t_{10}t_1 \in [5]$, since Ht_{24} is in [5].

2 symmetric generators will go to [5].

$Ht_1t_{10}t_2 = H\underline{t_1}t_{10}t_2$

$\Longrightarrow Ht_1t_{10}t_2 = Ht_{15}t_{10}t_2$, since $Ht_1 = Ht_{15}$

$\Longrightarrow Ht_1t_{10}t_2 = Ht_3^4t_2^3t_2$

$\Longrightarrow Ht_1t_{10}t_2 = H\underline{t_3^4}t_2^4$

$\Longrightarrow Ht_1t_{10}t_2 = H[yxt_1^6t_2^2]t_2^4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\Longrightarrow x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9\underline{t_1^2} = t_1^6\underline{t_1^2}$

$\Longrightarrow \underline{yx}y^{-1}x^{-1}t_3^4 = \underline{yx}t_1^6t_1^2$

$\Longrightarrow t_3^4 = yxt_1^6t_1^2$

$Ht_1t_{10}t_2 = Hyxt_1^6t_2^2t_2^4$

$\Longrightarrow Ht_1t_{10}t_2 = Ht_1^6\underline{t_2^6}$

$\Longrightarrow Ht_1t_{10}t_2 = Ht_1^6[yx^{-1}t_4^4t_3^9]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy} = e^{xy}$

$\Longrightarrow yx^{-1}t_{16}t_{35}t_{18} = e$

$\Longrightarrow yx^{-1}t_4^4t_3^9t_2^5 = e$

$\Longrightarrow yx^{-1}t_4^4t_3^9t_2^5\underline{t_2^6} = \underline{t_2^6}$

$\Longrightarrow yx^{-1}t_4^4t_3^9 = t_2^6$

$Ht_1t_{10}t_2 = Ht_1^6yx^{-1}t_4^4t_3^9$

$\Longrightarrow Ht_1t_{10}t_2 = Hyx^{-1}[t_1^6]^{yx^{-1}}t_4^4t_3^9$

$\Longrightarrow Ht_1t_{10}t_2 = Ht_4^2t_4^4t_3^9$

$\Longrightarrow Ht_1t_{10}t_2 = Ht_4^6t_3^9$

$\Longrightarrow Ht_1t_{10}t_2 = Ht_{24}t_{35}$

$\Longrightarrow Ht_1t_{10}t_2 \in [16]$, since $Ht_{24}t_{35}$ is in [1 6].

2 symmetric generators will go to [1 6].

$Ht_1t_{10}t_5 = H\underline{t_1}t_{10}t_5$

$\Longrightarrow Ht_1t_{10}t_5 = Ht_{15}t_{10}t_5$, since $Ht_1 = Ht_{15}$

$Ht_1t_{10}t_5 = Ht_{15}t_{10}t_5$

$\Longrightarrow Ht_1t_{10}t_5 = Ht_3^4\underline{t_2^3}t_1^2$

$\Longrightarrow Ht_1t_{10}t_5 = Ht_3^4[xt_4^8t_1^8]t_1^2$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}} = e^{x^{-1}}$

$\Longrightarrow x^{-1}t_{10}t_9t_{12} = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3 = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3\underline{t_4^8} = \underline{t_4^8}$

$\Longrightarrow x^{-1}t_2^3t_1^3\underline{t_1^8} = t_4^8\underline{t_1^8}$

$\Longrightarrow \underline{x}x^{-1}t_2^3 = \underline{x}t_4^8t_1^8$

$\Longrightarrow t_2^3 = xt_4^8t_1^8$

$Ht_1t_{10}t_5 = Ht_3^4xt_4^8t_1^8t_1^2$

$\Longrightarrow Ht_1t_{10}t_5 = Hyx[t_3^4]^xt_4^8t_1^{10}$

$\Longrightarrow Ht_1t_{10}t_5 = Ht_4^4t_4^8t_1^{10}$

$\Longrightarrow Ht_1t_{10}t_5 = H\underline{t_4}t_1^{10}$

$\Longrightarrow Ht_1t_{10}t_5 = Ht_2^4t_1^{10}$, since

$Ht_4 = Ht_{14}$

$\Longrightarrow Ht_4 = Ht_2^4$

$Ht_1t_{10}t_5 = H\underline{t_2^4}t_1^{10}$

$Ht_1t_{10}t_5 = H[xyt_4^6t_1^2]t_1^{10}$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y^{-1}} = e^{x^{-1}y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{14}t_{33}t_{20} = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5 = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5\underline{t_4^6} = \underline{t_4^6}$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9\underline{t_1^2} = t_4^6\underline{t_1^2}$

$\Longrightarrow \underline{xy}y^{-1}x^{-1}t_2^4 = \underline{xy}t_4^6t_1^2$

$\implies t_2^4 = xyt_4^6t_1^2$

$Ht_1t_{10}t_5 = Hxyt_4^6t_1^2t_1^{10} \implies Ht_1t_{10}t_5 = H\underline{t_4^6}t_1$

$\implies Ht_1t_{10}t_5 = Ht_2^6t_1$, since

$Ht_{22} = Ht_{24}$

$\implies Ht_2^6 = Ht_4^6$

$Ht_1t_{10}t_5 = Ht_2^6\underline{t_1}$

$Ht_1t_{10}t_5 = Ht_2^6[xy^{-1}t_3^2t_4^7]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\implies yx^{-1}t_1t_{16}t_{35} = e$

$\implies yx^{-1}t_1t_4^4t_3^9 = e$

$\implies yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\implies yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\implies \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\implies t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_{10}t_5 = Ht_2^6xy^{-1}t_3^2t_4^7$

$\implies Ht_1t_{10}t_5 = Hxy^{-1}[t_2^6]^{xy^{-1}}t_3^2t_4^7$

$\implies Ht_1t_{10}t_5 = Ht_3^3t_3^2t_4^7$

$\implies Ht_1t_{10}t_5 = Ht_3^4t_4^7$

$\implies Ht_1t_{10}t_5 = Ht_{15}t_{28}$

$\implies Ht_1t_{10}t_5 \in [16]$, since $Ht_{15}t_{28}$ is in [1 6].

2 symmetric generators will go to [1 6].

$Ht_1t_{10}t_6 = H\underline{t_1}t_{10}t_6$

$\implies Ht_1t_{10}t_6 = Ht_{15}t_{10}t_6$, since $Ht_1 = Ht_{15}$

$Ht_1t_{10}t_6 = Ht_{15}t_{10}t_6$

$\implies Ht_1t_{10}t_6 = Ht_3^4t_2^3t_2^2$

$\implies Ht_1t_{10}t_6 = H\underline{t_3^4}t_2^5$

$\implies Ht_1t_{10}t_6 = H[yxt_1^6t_2^2]t_2^5$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\implies x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\implies x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\implies x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\implies x^{-1}y^{-1}t_3^4t_2^9\underline{t_1^2} = t_1^6\underline{t_1^2}$

$\implies \underline{yx}y^{-1}x^{-1}t_3^4 = \underline{yx}t_1^6t_1^2$

$\implies t_3^4 = yxt_1^6t_1^2$

$Ht_1t_{10}t_6 = Hyxt_1^6t_2^2t_2^5$

$\implies Ht_1t_{10}t_6 = Ht_1^6\underline{t_2^7}$

$\implies Ht_1t_{10}t_6 = Ht_1^6[y^{-1}x^{-1}t_4^5t_3]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy^{-1}} = e^{xy^{-1}}$

$\implies [y^{-1}x^{-1}t_{20}t_3t_{14} = e$

$\implies [y^{-1}x^{-1}t_4^5t_3t_2^4 = e$

$\implies [y^{-1}x^{-1}t_4^5t_3t_2^4\underline{t_2^7} = \underline{t_2^7}$

$\implies [y^{-1}x^{-1}t_4^5t_3 = t_2^7$

$Ht_1t_{10}t_6 = Ht_1^6y^{-1}x^{-1}t_4^5t_3$

$\implies Ht_1t_{10}t_6 = Hy^{-1}x^{-1}[t_1^6]^{y^{-1}x^{-1}}t_4^5t_3$

$\implies Ht_1t_{10}t_6 = Ht_4^7t_4^5t_3$

$\implies Ht_1t_{10}t_6 = Ht_4t_3$

$\implies Ht_1t_{10}t_6 \in [14]$, since Ht_4t_3 is in [1 4].

2 symmetric generators will go to [1 4].

$Ht_1t_{10}t_9 = H\underline{t_1}t_{10}t_9$

$\implies Ht_1t_{10}t_9 = Ht_{15}t_{10}t_9$, since $Ht_1 = Ht_{15}$

$Ht_1t_{10}t_9 = Ht_{15}t_{10}t_9$

$\implies Ht_1t_{10}t_9 = Ht_3^4\underline{t_2^3t_1^3}$

$\implies Ht_1t_{10}t_9 = Ht_3^4[xt_4^8]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}} = e^{x^{-1}}$

$\implies x^{-1}t_{10}t_9t_{12} = e$

$\implies x^{-1}t_2^3t_1^3t_4^3 = e$

$\implies x^{-1}t_2^3t_1^3t_4^3\underline{t_4^8} = \underline{t_4^8}$

$\implies \underline{x}x^{-1}t_2^3t_1^3 = \underline{x}t_4^8$

$\Longrightarrow t_2^3 t_1^3 = x t_4^8$

$Ht_1t_{10}t_9 = Ht_3^4 x t_4^8$

$\Longrightarrow Ht_1t_{10}t_9 = Hx[t_3^4]^x t_4^8$

$\Longrightarrow Ht_1t_{10}t_9 = Ht_4^4 t_4^8$

$\Longrightarrow Ht_1t_{10}t_9 = Ht_4$

$\Longrightarrow Ht_1t_{10}t_9 \in [1]$, since Ht_4 is in [1].

2 symmetric generators will go to [1].

$Ht_1t_{10}t_{10} = H\underline{t_1}t_{10}t_{10}$

$\Longrightarrow Ht_1t_{10}t_{10} = Ht_1t_2^3t_2^3$

$\Longrightarrow Ht_1t_{10}t_{10} = Ht_1\underline{t_2^6}$

$\Longrightarrow Ht_1t_{10}t_{10} = Ht_1[yx^{-1}t_4^4t_3^9]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy} = e^{xy}$

$\Longrightarrow yx^{-1}t_{16}t_{35}t_{18} = e$

$\Longrightarrow yx^{-1}t_4^4t_3^9t_2^5 = e$

$\Longrightarrow yx^{-1}t_4^4t_3^9t_2^5\underline{t_2^6} = \underline{t_2^6}$

$\Longrightarrow yx^{-1}t_4^4t_3^9 = t_2^6$

$Ht_1t_{10}t_{10} = Ht_1yx^{-1}t_4^4t_3^9$

$\Longrightarrow Ht_1t_{10}t_{10} = Hyx^{-1}[t_1]^{yx^{-1}}t_4^4t_3^9$

$\Longrightarrow Ht_1t_{10}t_{10} = Ht_4^4t_4^4t_3^9$

$\Longrightarrow Ht_1t_{10}t_{10} = H\underline{t_4^8}t_3^9$

$\Longrightarrow Ht_1t_{10}t_{10} = Ht_2^{10}t_3^9$, since

$Ht_{32} = Ht_{38}$

$\Longrightarrow Ht_4^8 = Ht_2^{10}$

$Ht_1t_{10}t_{10} = H\underline{t_2^{10}}t_3^9$

$Ht_1t_{10}t_{10} = H[x^{-1}yt_4^9t_3^5]t_3^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{yx} = e^{yx}$

$\Longrightarrow x^{-1}yt_{36}t_{19}t_2 = e$

$\Longrightarrow x^{-1}yt_4^9t_3^5t_2 = e$

$\Longrightarrow x^{-1}yt_4^9t_3^5t_2\underline{t_2^{10}} = \underline{t_2^{10}}$

$\Longrightarrow x^{-1}yt_4^9t_3^5 = t_2^{10}$

$Ht_1t_{10}t_{10} = Hx^{-1}yt_4^9t_3^5t_3^9$

$\Longrightarrow Ht_1t_{10}t_{10} = Ht_4^9t_3^3$

$\Longrightarrow Ht_1t_{10}t_{10} = H\underline{t_{36}}t_{11}$

$\Longrightarrow Ht_1t_{10}t_{10} = Ht_{34}t_{11}$, since $Ht_{34} = Ht_{36}$

$Ht_1t_{10}t_{10} = Ht_{34}t_{11}$

$\Longrightarrow Ht_1t_{10}t_{10} \in [12]$, since $Ht_{34}t_{11}$ is in [1 2].

2 symmetric generators will go to [1 2].

$Ht_1t_{10}t_{13} = H\underline{t_1}t_{10}t_{13}$

$\Longrightarrow Ht_1t_{10}t_{13} = Ht_{15}t_{10}t_{13}$, since $Ht_1 = Ht_{15}$

$Ht_1t_{10}t_{13} = Ht_{15}t_{10}t_{13}$

$\Longrightarrow Ht_1t_{10}t_{13} = Ht_3^4\underline{t_2^3}t_1^4$

$\Longrightarrow Ht_1t_{10}t_{13} = Ht_3^4[xt_4^8t_1^8]t_1^4$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}} = e^{x^{-1}}$

$\Longrightarrow x^{-1}t_{10}t_9t_{12} = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3 = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3\underline{t_4^8} = \underline{t_4^8}$

$\Longrightarrow x^{-1}t_2^3t_1^3\underline{t_1^8} = t_4^8\underline{t_1^8}$

$\Longrightarrow \underline{x}x^{-1}t_2^3 = \underline{x}t_4^8t_1^8$

$\Longrightarrow t_2^3 = xt_4^8t_1^8$

$Ht_1t_{10}t_{13} = Ht_3^4xt_4^8t_1^8t_1^4$

$\Longrightarrow Ht_1t_{10}t_{13} = Hx[t_3^4]^xt_4^8t_1$

$\Longrightarrow Ht_1t_{10}t_{13} = Ht_4^4t_4^8t_1$

$\Longrightarrow Ht_1t_{10}t_{13} = Ht_4t_1$

$\Longrightarrow Ht_1t_{10}t_{13} \in [12]$, since Ht_4t_1 is in [1 2].

2 symmetric generators will go to [1 2].

$Ht_1t_{10}t_{14} = H\underline{t_1}t_{10}t_{14}$

$\Longrightarrow Ht_1t_{10}t_{14} = Ht_{15}t_{10}t_{14}$, since $Ht_1 = Ht_{15}$

$Ht_1t_{10}t_{14} = Ht_{15}t_{10}t_{14}$

$\implies Ht_1t_{10}t_{14} = H\underline{t_3^4}t_2^3t_2^4$

$\implies Ht_1t_{10}t_{14} = H[yxt_1^6t_2^2]t_2^7$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\implies x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\implies x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\implies x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\implies x^{-1}y^{-1}t_3^4t_2^9\underline{t_1^2} = t_1^6\underline{t_1^2}$

$\implies \underline{yx}y^{-1}x^{-1}t_3^4 = \underline{yx}t_1^6t_1^2$

$\implies t_3^4 = yxt_1^6t_1^2$

$Ht_1t_{10}t_{14} = Hyxt_1^6t_2^2t_2^7$

$\implies Ht_1t_{10}t_{14} = Ht_1^6t_2^9$

$\implies Ht_1t_{10}t_{14} = H\underline{t_{21}}t_{34}$

$\implies Ht_1t_{10}t_{14} = Ht_{23}t_{34}$, since $Ht_{21} = Ht_{23}$

$Ht_1t_{10}t_{14} = H\underline{t_{23}t_{34}}$

$\implies Ht_1t_{10}t_{14} = Ht_{34}t_{23}$, since by Equation 5.9

$Ht_1t_6 = Ht_6t_1$

$\implies [Ht_1t_6]^{xy} = [Ht_6t_1]^{xy}$

$\implies Ht_{34}t_{23} = Ht_{23}t_{34}$

$Ht_1t_{10}t_{14} = Ht_{34}t_{23}$

$\implies Ht_1t_{10}t_{14} \in [16]$, since $Ht_{34}t_{23}$ is in [1 6].

2 symmetric generators will go to [1 6].

$Ht_1t_{10}t_{17} = H\underline{t_1}t_{10}t_{17}$

$\implies Ht_1t_{10}t_{17} = Ht_{15}t_{10}t_{17}$, since $Ht_1 = Ht_{15}$

$Ht_1t_{10}t_{17} = Ht_{15}t_{10}t_{17}$

$\implies Ht_1t_{10}t_{17} = Ht_3^4\underline{t_2^3}t_1^5$

$\implies Ht_1t_{10}t_{17} = Ht_3^4[xt_4^8t_1^8]t_1^5$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}} = e^{x^{-1}}$

$\implies x^{-1}t_{10}t_9t_{12} = e$

$\implies x^{-1}t_2^3t_1^3t_4^3 = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3\underline{t_4^8} = \underline{t_4^8}$

$\Longrightarrow x^{-1}t_2^3t_1^3\underline{t_1^8} = t_4^8\underline{t_1^8}$

$\Longrightarrow \underline{x}x^{-1}t_2^3 = \underline{x}t_4^8t_1^8$

$\Longrightarrow t_2^3 = xt_4^8t_1^8$

$Ht_1t_{10}t_{17} = Ht_3^4xt_4^8t_1^8t_1^5$

$\Longrightarrow Ht_1t_{10}t_{17} = Hx[t_3^4]^xt_4^8t_1^2$

$\Longrightarrow Ht_1t_{10}t_{17} = Ht_4^4t_4^8t_1^2$

$\Longrightarrow Ht_1t_{10}t_{17} = Ht_4t_1^2$

$\Longrightarrow Ht_1t_{10}t_{17} = Ht_4t_5$

$\Longrightarrow Ht_1t_{10}t_{17} \in [16]$, since Ht_4t_5 is in [1 6].

2 symmetric generators will go to [1 6].

$Ht_1t_{10}t_{18} = Ht_1t_{10}t_{18}$

$\Longrightarrow Ht_1t_{10}t_{18} = Ht_1t_2^3t_2^5$

$\Longrightarrow Ht_1t_{10}t_{18} = Ht_1\underline{t_2^8}$

$\Longrightarrow Ht_1t_{10}t_{18} = Ht_1[x^{-1}t_4^3t_3^3]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^x = e^x$

$\Longrightarrow x^{-1}t_{12}t_{11}t_{10} = e$

$\Longrightarrow x^{-1}t_4^3t_3^3t_2^3 = e$

$\Longrightarrow x^{-1}t_4^3t_3^3t_2^3\underline{t_2^8} = \underline{t_2^8}$

$\Longrightarrow x^{-1}t_4^3t_3^3 = t_2^8$

$Ht_1t_{10}t_{18} = Ht_1x^{-1}t_4^3t_3^3$

$\Longrightarrow Ht_1t_{10}t_{18} = Hx^{-1}[t_1]^{x^{-1}}t_4^3t_3^3$

$\Longrightarrow Ht_1t_{10}t_{18} = Ht_4t_4^3t_3^3$

$\Longrightarrow Ht_1t_{10}t_{18} = Ht_4^4t_3^3$

$\Longrightarrow Ht_1t_{10}t_{18} = H\underline{t_{16}}t_{11}$

$\Longrightarrow Ht_1t_{10}t_{18} = Ht_2t_{11}$, since $Ht_2 = Ht_{16}$

$Ht_1t_{10}t_{18} = Ht_2t_{11}$

$Ht_1t_{10}t_{18} \in [110]$, since Ht_2t_{11} is in [1 10].

2 symmetric generators will go to [1 10].

$Ht_1t_{10}t_{21} = Ht_1t_{10}t_{21}$

$\Longrightarrow Ht_1t_{10}t_{21} = Ht_{15}t_{10}t_{21}$, since $Ht_1 = Ht_{15}$

$Ht_1t_{10}t_{21} = Ht_{15}t_{10}t_{21}$

$\Longrightarrow Ht_1t_{10}t_{21} = H\underline{t_3^4}t_2^3t_1^6$

$\Longrightarrow Ht_1t_{10}t_{21} = Ht_3^4\underline{t_2^3}t_1^6$

$\Longrightarrow Ht_1t_{10}t_{21} = Ht_3^4[xt_4^8t_1^8]t_1^6$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}} = e^{x^{-1}}$

$\Longrightarrow x^{-1}t_{10}t_9t_{12} = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3 = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3\underline{t_4^8} = \underline{t_4^8}$

$\Longrightarrow x^{-1}t_2^3t_1^3\underline{t_1^8} = t_4^8\underline{t_1^8}$

$\Longrightarrow \underline{x}x^{-1}t_2^3 = \underline{x}t_4^8t_1^8$

$\Longrightarrow t_2^3 = xt_4^8t_1^8$

$Ht_1t_{10}t_{21} = Ht_3^4xt_4^8t_1^8t_1^6$

$\Longrightarrow Ht_1t_{10}t_{21} = Hx[t_3^4]^xt_4^8t_1^3$

$\Longrightarrow Ht_1t_{10}t_{21} = Ht_4^4t_4^8t_1^3$

$\Longrightarrow Ht_1t_{10}t_{21} = Ht_4t_1^3$

$\Longrightarrow Ht_1t_{10}t_{21} = Ht_4t_9$

$\Longrightarrow Ht_1t_{10}t_{21} \in [110]$, since Ht_4t_9 is in [1 10].

2 symmetric generators will go to [1 10].

$Ht_1t_{10}t_{22} = Ht_1t_{10}t_{22}$

$\Longrightarrow Ht_1t_{10}t_{22} = Ht_{15}t_{10}t_{22}$, since $Ht_1 = Ht_{15}$

$Ht_1t_{10}t_{22} = Ht_{15}t_{10}t_{22}$

$\Longrightarrow Ht_1t_{10}t_{22} = Ht_3^4t_2^3t_2^6$

$\Longrightarrow Ht_1t_{10}t_{22} = H\underline{t_3^4}t_2^9$

$\Longrightarrow Ht_1t_{10}t_{22} = H[yxt_1^6t_2^2]t_2^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\Longrightarrow x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9\underline{t_1^2} = t_1^6\underline{t_1^2}$

$\Longrightarrow \underline{yx}y^{-1}x^{-1}t_3^4 = \underline{yx}t_1^6t_1^2$

$\Longrightarrow t_3^4 = yxt_1^6t_1^2$

$Ht_1t_{10}t_{22} = Hyxt_1^6t_2^2t_2^9$

$\Longrightarrow Ht_1t_{10}t_{22} = Hyxt_1^6$

$\Longrightarrow Ht_1t_{10}t_{22} = Hyxt_{21}$

$\Longrightarrow Ht_1t_{10}t_{22} \in [5]$, since $Hyxt_{21}$ is in [5].

2 symmetric generators will go to [5].

$Ht_1t_{10}t_{25} = Ht_1t_{10}t_{25}$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_{15}t_{10}t_{25}$, since $Ht_1 = Ht_{15}$

$Ht_1t_{10}t_{25} = Ht_{15}t_{10}t_{22}$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_3^4\underline{t_2^3}t_1^7$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_3^4[xt_4^8t_1^8]t_1^7$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}} = e^{x^{-1}}$

$\Longrightarrow x^{-1}t_{10}t_9t_{12} = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3 = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3\underline{t_4^8} = \underline{t_4^8}$

$\Longrightarrow x^{-1}t_2^3t_1^3\underline{t_1^8} = t_4^8\underline{t_1^8}$

$\Longrightarrow \underline{x}x^{-1}t_2^3 = \underline{x}t_4^8t_1^8$

$\Longrightarrow t_2^3 = xt_4^8t_1^8$

$Ht_1t_{10}t_{25} = Ht_3^4xt_4^8t_1^8t_1^7$

$\Longrightarrow Ht_1t_{10}t_{25} = Hx[t_3^4]^xt_4^8t_1^4$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_4^4t_4^8t_1^4$

$\Longrightarrow Ht_1t_{10}t_{25} = H\underline{t_4}t_1^4$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_2^4t_1^4$, since

$Ht_4 = Ht_{14}$

$\Longrightarrow Ht_4 = Ht_2^4$

$Ht_1t_{10}t_{25} = Ht_2^4\underline{t_1^4}$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_2^4[y^{-1}xt_3^6t_4^2]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xyx} = e^{xyx}$

$\Longrightarrow x^{-1}yt_{13}t_{36}t_{19} = e$

$\Longrightarrow x^{-1}yt_1^4t_4^9t_3^5 = e$

$\Longrightarrow x^{-1}yt_1^4t_4^9t_3^5\underline{t_3^6} = \underline{t_3^6}$

$\Longrightarrow x^{-1}yt_1^4t_4^9\underline{t_4^2} = t_3^6\underline{t_4^2}$

$\Longrightarrow \underline{y^{-1}x}x^{-1}yt_1^4 = \underline{y^{-1}x}t_3^6t_4^2$

$\Longrightarrow t_1^4 = y^{-1}xt_3^6t_4^2$

$Ht_1t_{10}t_{25} = Ht_2^4y^{-1}xt_3^6t_4^2$

$\Longrightarrow Ht_1t_{10}t_{25} = Hy^{-1}x[t_2^4]^{y^{-1}x}t_3^6t_4^2$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_3^9t_3^6t_4^2$

$\Longrightarrow Ht_1t_{10}t_{25} = H\underline{t_3^4}t_4^2$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_1t_4^2$, since

$Ht_1 = Ht_{15}$

$\Longrightarrow Ht_1 = Ht_3^4$

$Ht_1t_{10}t_{25} = H\underline{t_1}t_4^2$

$\Longrightarrow Ht_1t_{10}t_{25} = H\underline{t_1}t_4^2$

$\Longrightarrow Ht_1t_{10}t_{25} = H[xy^{-1}t_3^2t_4^7]t_4^2$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y} = e^{x^2y}$

$\Longrightarrow yx^{-1}t_1t_{16}t_{35} = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9 = e$

$\Longrightarrow yx^{-1}t_1t_4^4t_3^9\underline{t_3^2} = \underline{t_3^2}$

$\Longrightarrow yx^{-1}t_1t_4^4\underline{t_4^7} = t_3^2\underline{t_4^7}$

$\Longrightarrow \underline{xy^{-1}}yx^{-1}t_1 = \underline{xy^{-1}}t_3^2t_4^7$

$\Longrightarrow t_1 = xy^{-1}t_3^2t_4^7$

$Ht_1t_{10}t_{25} = Hxy^{-1}t_3^2t_4^7t_4^2$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_3^2t_4^9$

$\Longrightarrow Ht_1t_{10}t_{25} = H\underline{t_3^2}t_4^9$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_1^7t_4^9$, since

$Ht_7 = Ht_{25}$

$\Longrightarrow Ht_3^2 = Ht_1^7$

$Ht_1t_{10}t_{25} = Ht_1^7\underline{t_4^9}$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_1^7[y^{-1}xt_2^{10}t_3^6]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{yx} = e^{yx}$

$\Longrightarrow x^{-1}yt_{36}t_{19}t_2 = e$

$\Longrightarrow x^{-1}yt_4^9t_3^5t_2 = e$

$\Longrightarrow x^{-1}yt_4^9t_3^5t_2\underline{t_2^{10}} = \underline{t_2^{10}}$

$\Longrightarrow x^{-1}yt_4^9t_3^5\underline{t_3^6} = t_2^{10}\underline{t_3^6}$

$\Longrightarrow \underline{y^{-1}x}x^{-1}yt_4^9 = \underline{y^{-1}x}t_2^{10}t_3^6$

$\Longrightarrow t_4^9 = y^{-1}xt_2^{10}t_3^6$

$Ht_1t_{10}t_{25} = Ht_1^7y^{-1}xt_2^{10}t_3^6$

$\Longrightarrow Ht_1t_{10}t_{25} = Hy^{-1}x[t_1^7]^{y^{-1}x}t_2^{10}t_3^6$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_2^{10}t_2^{10}t_3^6$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_2^9t_3^6$

$\Longrightarrow Ht_1t_{10}t_{25} = Ht_{34}t_{23}$

$\Longrightarrow Ht_1t_{10}t_{25} \in [16]$, since $Ht_{34}t_{23}$ is in [1 6].

2 symmetric generators will go to [1 6].

$Ht_1t_{10}t_{26} = Ht_1t_{10}t_{26}$

$\Longrightarrow Ht_1t_{10}t_{26} = Ht_{15}t_{10}t_{26}$, since $Ht_1 = Ht_{15}$

$Ht_1t_{10}t_{26} = Ht_{15}t_{10}t_{26}$

$\Longrightarrow Ht_1t_{10}t_{26} = Ht_3^4t_2^3t_2^7$

$\Longrightarrow Ht_1t_{10}t_{26} = H\underline{t_3^4}t_2^{10}$

$\Longrightarrow Ht_1t_{10}t_{26} = H[yxt_1^6t_2^2]t_2^{10}$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\Longrightarrow x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9\underline{t_1^2} = t_1^6\underline{t_1^2}$

$\Longrightarrow \underline{yx}y^{-1}x^{-1}t_3^4 = \underline{yx}t_1^6t_1^2$

$\Longrightarrow t_3^4 = yxt_1^6t_1^2$

$Ht_1t_{10}t_{26} = Hyxt_1^6t_2^2t_2^{10}$

$\implies Ht_1t_{10}t_{26} = H\underline{t_1^6}t_2$

$\implies Ht_1t_{10}t_{26} = Ht_3^6t_2$, since

$Ht_{21} = Ht_{23}$

$\implies Ht_1^6 = Ht_3^6$

$Ht_1t_{10}t_{26} = Ht_3^6\underline{t_2}$

$Ht_1t_{10}t_{26} = Ht_3^6[y^{-1}xt_4^2t_1^7]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-1}x^{-1}} = e^{y^{-1}x^{-1}}$

$\implies x^{-1}yt_2t_{13}t_{36} = e$

$\implies x^{-1}yt_2t_1^4t_4^9 = e$

$\implies x^{-1}yt_2t_1^4t_4^9\underline{t_4^2} = \underline{t_4^2}$

$\implies x^{-1}yt_2t_1^4\underline{t_1^7} = t_4^2\underline{t_1^7}$

$\implies \underline{y^{-1}x}x^{-1}yt_2 = \underline{y^{-1}x}t_4^2t_1^7$

$\implies t_2 = y^{-1}xt_4^2t_1^7$

$Ht_1t_{10}t_{26} = Ht_3^6y^{-1}xt_4^2t_1^7$

$\implies Ht_1t_{10}t_{26} = Hy^{-1}x[t_3^6]^{y^{-1}x}t_4^2t_1^7$

$\implies Ht_1t_{10}t_{26} = Ht_4^2t_4^2t_1^7$

$\implies Ht_1t_{10}t_{26} = Ht_4^4t_1^7$

$\implies Ht_1t_{10}t_{26} = Ht_{16}t_{25}$

$\implies Ht_1t_{10}t_{26} \in [16]$, since $Ht_{16}t_{25}$ is in [1 6].

2 symmetric generators will go to [1 6].

$Ht_1t_{10}t_{29} = Ht_1t_{10}t_{29}$

$\implies Ht_1t_{10}t_{29} = Ht_{15}t_{10}t_{29}$, since $Ht_1 = Ht_{15}$

$Ht_1t_{10}t_{29} = Ht_{15}t_{10}t_{29}$

$\implies Ht_1t_{10}t_{29} = Ht_3^4\underline{t_2^3}t_1^8$

$\implies Ht_1t_{10}t_{29} = Ht_3^4[xt_4^8t_1^8]t_1^8$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}} = e^{x^{-1}}$

$\implies x^{-1}t_{10}t_9t_{12} = e$

$\implies x^{-1}t_2^3t_1^3t_4^3 = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3\underline{t_4^8} = \underline{t_4^8}$

$\Longrightarrow x^{-1}t_2^3t_1^3\underline{t_1^8} = t_4^8\underline{t_1^8}$

$\Longrightarrow \underline{x}x^{-1}t_2^3 = \underline{x}t_4^8t_1^8$

$\Longrightarrow t_2^3 = xt_4^8t_1^8$

$Ht_1t_{10}t_{29} = Ht_3^4xt_4^8t_1^8t_1^8$

$\Longrightarrow Ht_1t_{10}t_{29} = Hx[t_3^4]^xt_4^8t_1^5$

$\Longrightarrow Ht_1t_{10}t_{29} = Ht_4^4t_4^8t_1^5$

$\Longrightarrow Ht_1t_{10}t_{29} = H\underline{t_4}t_1^5$

$\Longrightarrow Ht_1t_{10}t_{29} = Ht_2^4t_1^5$, since

$Ht_4 = Ht_{14}$

$\Longrightarrow Ht_4 = Ht_2^4$

$Ht_1t_{10}t_{29} = Ht_2^4\underline{t_1^5}$

$Ht_1t_{10}t_{29} = Ht_2^4[yxt_3^7t_4^{10}]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{xy^{-1}x} = e^{xy^{-1}x}$

$\Longrightarrow x^{-1}y^{-1}t_{17}t_4t_{15} = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4t_3^4 = e$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4t_3^4\underline{t_3^7} = \underline{t_3^7}$

$\Longrightarrow x^{-1}y^{-1}t_1^5t_4\underline{t_4^{10}} = t_3^7\underline{t_4^{10}}$

$\Longrightarrow \underline{yx}x^{-1}y^{-1}t_1^5 = \underline{yx}t_3^7t_4^{10}$

$\Longrightarrow t_1^5 = yxt_3^7t_4^{10}$

$Ht_1t_{10}t_{29} = Ht_2^4yxt_3^7t_4^{10}$

$\Longrightarrow Ht_1t_{10}t_{29} = Hyx[t_2^4]^{yx}t_3^7t_4^{10}$

$\Longrightarrow Ht_1t_{10}t_{29} = Ht_3^3t_3^7t_4^{10}$

$\Longrightarrow Ht_1t_{10}t_{29} = H\underline{t_3^{10}}t_4^{10}$

$\Longrightarrow Ht_1t_{10}t_{29} = H[y^{-1}x^{-1}t_1^9t_4^5]t_4^{10}$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^2y^{-1}} = e^{x^2y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{33}t_{20}t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3 = e$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5t_3\underline{t_3^{10}} = \underline{t_3^{10}}$

$\Longrightarrow y^{-1}x^{-1}t_1^9t_4^5 = t_3^{10}$

$Ht_1t_{10}t_{29} = Hy^{-1}x^{-1}t_1^9t_4^5t_4^{10}$

$\Longrightarrow Ht_1t_{10}t_{29} = Ht_1^9t_4^4$

$\Longrightarrow Ht_1t_{10}t_{29} = Ht_{33}t_{16}$

$\Longrightarrow Ht_1t_{10}t_{29} \in [14]$, since $Ht_{33}t_{16}$ is in [1 4].

2 symmetric generators will go to [1 4].

$Ht_1t_{10}t_{30} = Ht_1t_{10}t_{30}$

$\Longrightarrow Ht_1t_{10}t_{30} = Ht_1t_2^3t_2^8$

$\Longrightarrow Ht_1t_{10}t_{30} = Ht_1$

$\Longrightarrow Ht_1t_{10}t_{30} \in [1]$, since Ht_1 is in [1].

2 symmetric generators will go to [1].

$Ht_1t_{10}t_{33} = Ht_1t_{10}t_{33}$

$\Longrightarrow Ht_1t_{10}t_{33} = Ht_{15}t_{10}t_{33}$, since $Ht_1 = Ht_{15}$

$Ht_1t_{10}t_{33} = Ht_{15}t_{10}t_{33}$

$\Longrightarrow Ht_1t_{10}t_{33} = Ht_3^4\underline{t_2^3}t_1^9$

$\Longrightarrow Ht_1t_{10}t_{33} = Ht_3^4[xt_4^8t_1^8]t_1^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}} = e^{x^{-1}}$

$\Longrightarrow x^{-1}t_{10}t_9t_{12} = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3 = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3\underline{t_4^8} = \underline{t_4^8}$

$\Longrightarrow x^{-1}t_2^3t_1^3\underline{t_1^8} = t_4^8\underline{t_1^8}$

$\Longrightarrow \underline{x}x^{-1}t_2^3 = \underline{x}t_4^8t_1^8$

$\Longrightarrow t_2^3 = xt_4^8t_1^8$

$Ht_1t_{10}t_{33} = Ht_3^4xt_4^8t_1^8t_1^9$

$\Longrightarrow Ht_1t_{10}t_{33} = Hx[t_3^4]^xt_4^8t_1^6$

$\Longrightarrow Ht_1t_{10}t_{33} = Ht_4^4t_4^8t_1^6$

$\Longrightarrow Ht_1t_{10}t_{33} = Ht_4\underline{t_1^6}$

$\Longrightarrow Ht_1t_{10}t_{33} = Ht_4[x^{-1}y^{-1}t_3^4t_2^9]$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{y^{-2}} = e^{y^{-2}}$

$\Longrightarrow x^{-1}y^{-1}t_{15}t_{34}t_{17} = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5 = e$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9t_1^5\underline{t_1^6} = \underline{t_1^6}$

$\Longrightarrow x^{-1}y^{-1}t_3^4t_2^9 = t_1^6$

$Ht_1t_{10}t_{33} = Ht_4x^{-1}y^{-1}t_3^4t_2^9$

$\Longrightarrow Ht_1t_{10}t_{33} = Hx^{-1}y^{-1}[t_4]^{x^{-1}y^{-1}}t_3^4t_2^9$

$\Longrightarrow Ht_1t_{10}t_{33} = Ht_3^4t_3^4t_2^9$

$\Longrightarrow Ht_1t_{10}t_{33} = H\underline{t_3^8}t_2^9$, since

$Ht_{31} = Ht_{37}$

$\Longrightarrow Ht_3^8 = Ht_1^{10}$

$Ht_1t_{10}t_{33} = H\underline{t_1^{10}}t_2^9$

$\Longrightarrow Ht_1t_{10}t_{33} = H[yx^{-1}t_3^9t_2^5]t_2^9$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^y = e^y$

$\Longrightarrow yx^{-1}t_{35}t_{18}t_1 = e$

$\Longrightarrow yx^{-1}t_3^9t_2^5t_1 = e$

$\Longrightarrow yx^{-1}t_3^9t_2^5t_1\underline{t_1^{10}} = \underline{t_1^{10}}$

$\Longrightarrow yx^{-1}t_3^9t_2^5 = t_1^{10}$

$Ht_1t_{10}t_{33} = Hyx^{-1}t_3^9t_2^5t_2^9$

$\Longrightarrow Ht_1t_{10}t_{33} = Ht_3^9t_2^3$

$\Longrightarrow Ht_1t_{10}t_{33} = H\underline{t_{35}}t_{10}$

$\Longrightarrow Ht_1t_{10}t_{33} = Ht_{33}t_{10}$, since $Ht_{33} = Ht_{35}$

$Ht_1t_{10}t_{33} = Ht_{33}t_{10}$

$\Longrightarrow Ht_1t_{10}t_{33} \in [12]$, since $Ht_{33}t_{10}$ is in [1 2].

2 symmetric generators will go to [1 2].

$Ht_1t_{10}t_{34} = Ht_1t_{10}t_{34}$

$\Longrightarrow Ht_1t_{10}t_{34} = Ht_1t_2^3t_2^9$

$\Longrightarrow Ht_1t_{10}t_{34} = Ht_1$

$\Longrightarrow Ht_1t_{10}t_{34} \in [1]$, since Ht_1 is in [1].

2 symmetric generators will go to [1].

$Ht_1t_{10}t_{37} = Ht_1t_{10}t_{37}$

$\Longrightarrow Ht_1t_{10}t_{37} = Ht_{15}t_{10}t_{37}$, since $Ht_1 = Ht_{15}$

$Ht_1t_{10}t_{37} = Ht_{15}t_{10}t_{37}$

$\Longrightarrow Ht_1t_{10}t_{37} = Ht_3^4\underline{t_2^3}t_1^{10}$

$\Longrightarrow Ht_1t_{10}t_{37} = Ht_3^4[xt_4^8t_1^8]t_1^{10}$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}} = e^{x^{-1}}$

$\Longrightarrow x^{-1}t_{10}t_9t_{12} = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3 = e$

$\Longrightarrow x^{-1}t_2^3t_1^3t_4^3\underline{t_4^8} = \underline{t_4^8}$

$\Longrightarrow x^{-1}t_2^3t_1^3\underline{t_1^8} = t_4^8\underline{t_1^8}$

$\Longrightarrow \underline{x}x^{-1}t_2^3 = \underline{x}t_4^8t_1^8$

$\Longrightarrow t_2^3 = xt_4^8t_1^8$

$Ht_1t_{10}t_{37} = Ht_3^4xt_4^8t_1^8t_1^{10}$

$\Longrightarrow Ht_1t_{10}t_{37} = Hx[t_3^4]^xt_4^8t_1^7$

$\Longrightarrow Ht_1t_{10}t_{37} = Ht_4^4t_4^8t_1^7$

$\Longrightarrow Ht_1t_{10}t_{37} = H\underline{t_4}t_1^7$

$\Longrightarrow Ht_1t_{10}t_{37} = Ht_2^4t_1^7$, since

$Ht_4 = Ht_{14}$

$\Longrightarrow Ht_4 = Ht_2^4$

$Ht_1t_{10}t_{37} = H\underline{t_2^4}t_1^7$

$\Longrightarrow Ht_1t_{10}t_{37} = H[xyt_4^6t_1^2]t_1^7$, since by Equation 5.8

$x^3t_{11}t_{10}t_9 = e$

$[x^3t_{11}t_{10}t_9]^{x^{-1}y^{-1}} = e^{x^{-1}y^{-1}}$

$\Longrightarrow y^{-1}x^{-1}t_{14}t_{33}t_{20} = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5 = e$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9t_4^5\underline{t_4^6} = \underline{t_4^6}$

$\Longrightarrow y^{-1}x^{-1}t_2^4t_1^9\underline{t_1^2} = t_4^6\underline{t_1^2}$

$\Longrightarrow \underline{xy}y^{-1}x^{-1}t_2^4 = \underline{xy}t_4^6t_1^2$

$\Longrightarrow t_2^4 = xyt_4^6t_1^2$

$Ht_1t_{10}t_{37} = Hxyt_4^6t_1^2t_1^7$

$\Longrightarrow Ht_1t_{10}t_{37} = Ht_4^6t_1^9$

$\Longrightarrow Ht_1t_{10}t_{37} = H\underline{t_{24}}t_{33}$

$\Longrightarrow Ht_1t_{10}t_{37} = Ht_{22}t_{33}$, since $Ht_{22} = Ht_{24}$

$\Longrightarrow Ht_1t_{10}t_{37} = H\underline{t_{22}t_{33}}$

$\Longrightarrow Ht_1t_{10}t_{37} = H\underline{t_{33}t_{22}}$, since by Equation 5.9

$Ht_1t_6 = Ht_6t_1$

$\Longrightarrow [Ht_1t_6]^{y^{-2}} = [Ht_6t_1]^{y^{-2}}$

$\Longrightarrow Ht_{33}t_{22} = Ht_{22}t_{33}$

$Ht_1t_{10}t_{37} = Ht_{33}t_{22}$

$\Longrightarrow Ht_1t_{10}t_{37} \in [16]$, since $Ht_{33}t_{22}$ is in [1 6].

2 symmetric generators will go to [1 6].

$Ht_1t_{10}t_{38} = Ht_1t_{10}t_{38}$

$\Longrightarrow Ht_1t_{10}t_{38} = Ht_1t_2^3t_2^{10}$

$\Longrightarrow Ht_1t_{10}t_{38} = Ht_1t_2^2$

$\Longrightarrow Ht_1t_{10}t_{38} = Ht_1t_6$

$\Longrightarrow Ht_1t_{10}t_{38} \in [16]$, since Ht_1t_6 is in [1 6].

2 symmetric generators will go to [1 6].

This concludes our double coset enumeration. Below is our completed Cayley Diagram.

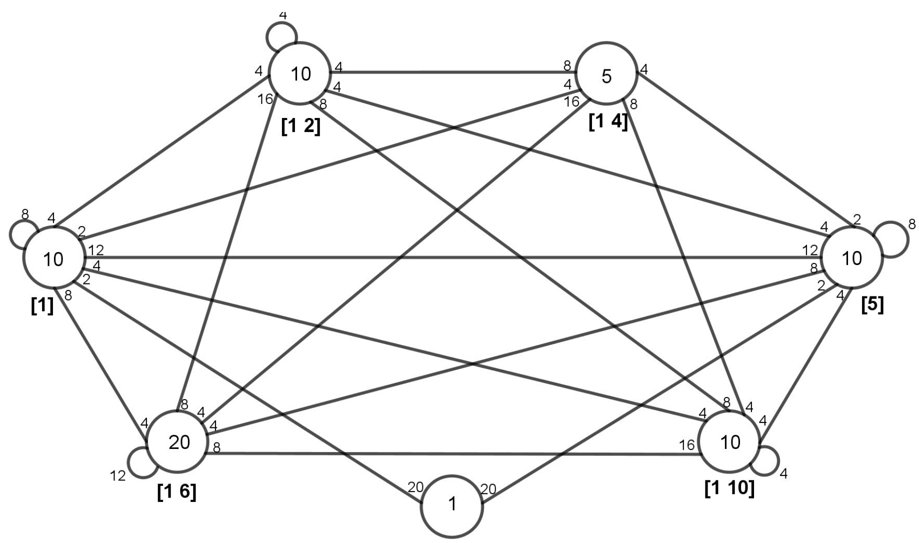


Figure 5.3: M_{11} Over $(C_4 : C_5)$.

5.4.2 Manual Double Coset Enumeration over a Maximal Subgroup of Order 720

Now we will perform Double Coset Enumeration over our other maximal subgroup.

Recall that we had 2 maximal subgroups that contained both $f(x)$ and $f(y)$. We will examine subgroup 5.

We see that the order of this subgroup is 720, which is larger than our other subgroup. Next we find a representation of this larger subgroup in words.

```
> #M[5]`subgroup;
720
> D:=Conjugates(G1,M[5]`subgroup);
> D:=SetToSequence(D);
> f(x) in D[5] and f(y) in D[5];
true
> for g in D[5] do if sub<D[5]|f(x),f(y),g> eq D[5] then gg:=g;
for|if> end if;
for> end for;
> Order(gg);
```

```
4
> if Order(gg) eq 4 then for i in [1..7920] do if ArrayP[i] eq gg
if|for|if> then Sch[i]; end if; end for; end if;
y^-1 * t^-1 * x * t * y^-1 * t
> Order(f(y^-1 * t^-1 * x * t * y^-1 * t));
4
> G<x,y,t>:=Group<x,y,t|x^4,x*y^-1*x^-1*y^-2,
y^-2*x^-1*y*x,t^11,t^y=t^4,(x^-1*t)^8,
(y*t^x)^5,
(x*t^(y^4))^3>;
> H1:=sub<G|x,y>;
> H2:=sub<G|x,y,y^-1 * t^-1 * x * t * y^-1 * t>;
> #DoubleCosets(G,H2,H1);
3
> #G/#H2;
11
```

First we will expand our additional relations.

$$
\begin{aligned}
y^{-1}t^{-1}xty^{-1}t &\in H\\
y^4t_1^{-1}xt_1y^4t_1 &\in H\\
y^4t_1^{10}xt_1y^4t_1 &\in H\\
y^4t_{37}x[y^4y^{-4}]t_1y^4t_1 &\in H\\
y^4t_{37}xy^4[y^{-4}t_1y^4]t_1 &\in H\\
y^4t_{37}xy^4[t_1^{y^4}]t_1 &\in H\\
y^4t_{37}xy^4t_9t_1 &\in H\\
y^4[(xy^4)(xy^4)^{-1}]t_{37}xy^4t_9t_1 &\in H\\
y^4xy^4[(xy^4)^{-1}t_{37}xy^4]t_9t_1 &\in H\\
y^4xy^4[t_{37}^{xy^4}]t_9t_1 &\in H\\
y^4xy^4t_{22}t_9t_1 &\in H\\
xy^2t_{22}t_9t_1 &\in H\\
xy^2t_{22}t_1^3t_1 &\in H\\
xy^2t_{22}t_1^4 &\in H\\
Hxy^2t_{22}t_1^4 &= H\\
Hxy^2t_{22}t_1^4\underline{t_1^7} &= H\underline{t_1^7}\\
Hxy^2t_{22} &= Ht_1^7\\
Hxy^2t_{22} &= Ht_{25}\\
Ht_{22} &= Ht_{25}
\end{aligned}
\tag{5.10}
$$

$$
\begin{aligned}
(x^2 t^{y^3})^3 &= e \\
(x^2 t_1^{y^3})^3 &= e \\
(x^2 t_{33})^3 &= e \\
x^2 t_{33} x^2 t_{33} x^2 t_{33} &= e \\
x^2 (x^2 x^{-2}) t_{33} x^2 t_{33} x^2 t_{33} &= e \\
x^2 x^2 (x^{-2} t_{33} x^2) t_{33} x^2 t_{33} &= e \\
t_{33}^{x^2} t_{33} x^2 t_{33} &= e \\
t_{35} t_{33} x^2 t_{33} &= e \\
t_{35} (x^2 x^{-2}) t_{33} x^2 t_{33} &= e \\
t_{35} x^2 (x^{-2} t_{33} x^2) t_{33} &= e \\
t_{35} x^2 t_{33}^{x^2} t_{33} &= e \\
t_{35} x^2 t_{35} t_{33} &= e \\
(x^2 x^{-2}) t_{35} x^2 t_{35} t_{33} &= e \\
x^2 (x^{-2} t_{35} x^2) t_{35} t_{33} &= e \\
x^2 t_{35}^{x^2} t_{35} t_{33} &= e \\
x^2 t_{33} t_{35} t_{33} &= e
\end{aligned}
\tag{5.11}
$$

$$
\begin{aligned}
(xt^{y^4})^3 &= e \\
(xt_1^{y^4})^3 &= e \\
(xt_9)^3 &= e \\
xt_9xt_9xt_9 &= e \\
x(xx^{-1})t_9xt_9xt_9 &= e \\
x^2(x^{-1}t_9x)t_9xt_9 &= e \\
x^2t_9^xt_9xt_9 &= e \\
x^2t_{10}t_9xt_9 &= e \\
x^2(xx^{-1})t_{10}t_9xt_9 &= e \\
x^3(x^{-1})t_{10}t_9x)t_9 &= e \\
x^3[t_{10}t_9]^xt_9 &= e \\
x^3t_{11}t_{10}t_9 &= e
\end{aligned}
\tag{5.12}
$$

Our first double coset, $HeN = \{He^n|n \in N\} = \{H\}$, which we will denote by $[*]$. The orbits of N on {1,2,3,4,5,6,8,7,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25, 26,27,28,29,30,31,32,33,34,35,36,37,38,39,40} are {1,2,13,3,34,14,17,4,11,35,15,10,18, 33,20,12,36,19,16,9} and {5,6,29,7,26,30,37,8,23,27,31,22,38,25,40,24,28,39,32,21}.

We will take a representative from each orbit, say t_1 and t_5, and determine to which double coset Ht_1 and Ht_5 belong.

Word of Length 1

Ht_1N is a new double coset which we will denote by [1].
$Ht_1N = \{Ht_1{}^n|n \in N\}$.

Since the orbit $\{1, 2, 13, 3, 34, 14, 17, 4, 11, 35, 15, 10, 18, 33, 20, 12, 36, 19, 16, 9\}$ contains 20 elements then 20 symmetric generators will go to the new double coset [1].

Now $N^{(1)} \geq H^1$.

$N^1 = \{e\}$.

$N^{(1)} =$ Coset Stabiliser in N of $Ht_1 = \{n \in N | Ht_1^n = t_1\}$.

We will look for a relation that will increase the Coset Stabiliser $N^{(1)}$.

$Ht_{25} = Ht_{22}$, by Equation 5.10

$\Longrightarrow Ht_1^7 = Ht_2^6$

$\Longrightarrow Ht_1^7 \underline{t_1^5} = Ht_2^6 \underline{t_1^5}$

$\Longrightarrow Ht_1 = Ht_2^6 \underline{t_1^5}$

$\Longrightarrow Ht_1 = Ht_2^6 [yxt_3^7 t_4^{10}]$, since by Equation 5.12

$x^3 t_{11} t_{10} t_9 = e$

$[x^3 t_{11} t_{10} t_9]^{xy^{-1}x} = e^{xy^{-1}x}$

$\Longrightarrow x^{-1} y^{-1} t_{17} t_4 t_{15} = e$

$\Longrightarrow x^{-1} y^{-1} t_1^5 t_4 t_3^4 = e$

$\Longrightarrow x^{-1} y^{-1} t_1^5 t_4 t_3^4 \underline{t_3^7} = \underline{t_3^7}$

$\Longrightarrow x^{-1} y^{-1} t_1^5 t_4 \underline{t_4^{10}} = t_3^7 \underline{t_4^{10}}$

$\Longrightarrow \underline{yx} x^{-1} y^{-1} t_1^5 = \underline{yx} t_3^7 t_4^{10}$

$\Longrightarrow t_1^5 = yxt_3^7 t_4^{10}$

$Ht_1 = Ht_2^6 yxt_3^7 t_4^{10}$

$\Longrightarrow Ht_1 = Hyx[t_2^6]^{yx} t_3^7 t_4^{10}$

$\Longrightarrow Ht_1 = Ht_3^{10} t_3^7 t_4^{10}$

$\Longrightarrow Ht_1 = H\underline{t_3^6} t_4^{10}$

$\Longrightarrow Ht_1 = Ht_4^{10} t_4^{10}$, since by Equation 5.10

$Ht_{22} = Ht_{25}$

$\Longrightarrow [Ht_{22}]^{x^2 y^{-1}} = [Ht_{25}]^{x^2 y^{-1}}$

$\Longrightarrow Ht_{40} = Ht_{23}$

$\Longrightarrow Ht_4^{10} = Ht_3^6$

$Ht_1 = Ht_4^{10} t_4^{10}$

$\Longrightarrow Ht_1 = Ht_4^9$

$\Longrightarrow Ht_1 = Ht_{36}$

Also, $Ht_1 = Ht_{36}$

$\Longrightarrow [Ht_1]^{x^{-1}y^{-1}} = [Ht_{36}]^{x^{-1}y^{-1}}$

$\Longrightarrow Ht_{36} = Ht_{11}$,

and

$Ht_1 = Ht_{36}$

$\Longrightarrow [Ht_1]^{x^2y} = [Ht_{36}]^{x^2y}$

$\Longrightarrow Ht_{11} = Ht_{14}$,

and

$Ht_1 = Ht_{36}$

$\Longrightarrow [Ht_1]^{yx} = [Ht_{36}]^{yx}$

$\Longrightarrow Ht_{14} = Ht_1$.

Thus, $Ht_1 = Ht_{36} = Ht_{11} = Ht_{14}$

Now, since $Ht_1^e = Ht_1 \Rightarrow e \in N^{(1)}$, and
$Ht_1^{x^{-1}y^{-1}} = Ht_{36} = Ht_1 \Rightarrow x^{-1}y^{-1} \in N^{(1)}$,
$Ht_1^{x^2y} = Ht_{11} = Ht_1 \Rightarrow x^2y \in N^{(1)}$,
$Ht_1^{yx} = Ht_{14} = Ht_1 \Rightarrow yx \in N^{(1)}$, then,
$N^{(1)} =$ Coset Stabiliser in N of $Ht_1 = \{n \in N | Ht_1^n = t_1\} = \{e, x^{-1}y^{-1}, x^2y, yx\}$.
Furthermore, the number of single cosets of Ht_1N is $\frac{|N|}{|N^{(1)}|} = \frac{20}{4} = 5$.

Conjugating by elements in N gives us the following equal names.

$t_1 \sim t_{36} \sim t_{11} \sim t_{14}$

$t_2 \sim t_{33} \sim t_{12} \sim t_{15}$

$t_3 \sim t_{34} \sim t_9 \sim t_{16}$

$t_4 \sim t_{35} \sim t_{10} \sim t_{13}$

$t_{17} \sim t_{20} \sim t_{19} \sim t_{18}$

Therefore, $Ht_1N = \{Ht_1 = Ht_{36} = Ht_{11} = Ht_{14}, Ht_2 = Ht_{33} = Ht_{12} = Ht_{15},$
$Ht_3 = Ht_{34} = Ht_9 = Ht_{16}, Ht_4 = Ht_{35} = Ht_{10} = Ht_{13},$
$Ht_{17} = Ht_{20} = Ht_{19} = Ht_{18}\}$

Now $N^{(5)} \geq N^5$.

$N^5 = \{e\}$.

$N^{(5)} =$ Coset Stabiliser in N of $Ht_5 = \{n \in N | Ht_5^n = t_5\}$.

We will look for a relation that will increase the Coset Stabiliser $N^{(5)}$.

$Ht_{22} = Ht_{25}$, by Equation 5.10
$\Longrightarrow [Ht_{22}]^{x^{-1}y} = [Ht_{25}]^{x^{-1}y}$
$\Longrightarrow Ht_5 = Ht_8$.
Also,
$Ht_{22} = Ht_{25}$
$\Longrightarrow [Ht_{22}]^{xy^{-1}x} = [Ht_{25}]^{xy^{-1}x}$
$\Longrightarrow Ht_8 = Ht_7$,
and
$Ht_{22} = Ht_{25}$
$\Longrightarrow [Ht_{22}]^{xy^{-1}} = [Ht_{25}]^{xy^{-1}}$
$\Longrightarrow Ht_7 = Ht_6$,
and
$Ht_{22} = Ht_{25}$
$\Longrightarrow [Ht_{22}]^{y^2} = [Ht_{25}]^{y^2}$
$\Longrightarrow Ht_6 = Ht_5$,

Thus $Ht_5 = Ht_6 = Ht_7 = Ht_8$

Since $Ht_5^e = Ht_5 \Rightarrow e \in N^{(5)}$,
$Ht_5^x = Ht_6 = Ht_5 \Rightarrow x \in N^{(5)}$,
$Ht_5^{x^2} = Ht_7 = Ht_5 \Rightarrow x^2 in N^{(5)}$,
$Ht_5^{x^{-1}} = Ht_8 = Ht_5 \Rightarrow x^3 in N^{(5)}$, then,
$N^{(5)} =$ Coset Stabiliser in N of $Ht_5 = \{n \in N | (Ht_5)^n = t_5\} = \{e, x, x^2, x^{-1}\}$.
Furthermore, the number of single cosets of Ht_5N is $\frac{|N|}{|N^{(5)}|} = \frac{20}{4} = 5$.

Conjugating by elements in N gives us the following equal names.

$t_5 \sim t_6 \sim t_7 \sim t_8$
$t_{29} \sim t_{26} \sim t_{23} \sim t_{40}$
$t_{30} \sim t_{27} \sim t_{24} \sim t_{37}$
$t_{32} \sim t_{25} \sim t_{22} \sim t_{39}$
$t_{31} \sim t_{28} \sim t_{21} \sim t_{38}$

Thus $Ht_5N = \{Ht_5 = Ht_6 = Ht_7 = Ht_8, Ht_{29} = Ht_{26} = Ht_{23} = Ht_{40},$
$Ht_{30} = Ht_{27} = Ht_{24} = Ht_{37}, Ht_{32} = Ht_{25} = Ht_{22} = Ht_{39},$
$Ht_{31} = Ht_{28} = Ht_{21} = Ht_{38}\}$.

The orbits of $N^{(1)}$ are {1, 14, 11, 36}, {2, 35, 20, 9}, {3, 12, 13, 18}, {4, 17, 34, 15}, {5, 30, 23, 28}, {6, 27, 40, 21}, {7, 24, 29, 38}, {8, 37, 26, 31}, {10, 19, 16, 33}, and {22, 39, 32, 25}.

We will check to see where $t_1t_1, t_1t_9, t_1t_{13}, t_1t_{17}, t_1t_5, t_1t_{21}, t_1t_{29}, t_1t_{37}, t_1t_{33},$ and t_1t_{25} belong.

$Ht_1t_1 = Ht_1^2$
$\implies Ht_1t_1 = Ht_5$
$\implies Ht_1t_1 \in [5]$, since Ht_5 is in [5].
4 Symmetric generators will go to [5].

$Ht_1t_9 = Ht_1t_1^3$
$\implies Ht_1t_9 = Ht_1^4$
$\implies Ht_1t_9 = Ht_{13}$
$\implies Ht_1t_9 \in [1]$, since Ht_{13} is in [1].
4 Symmetric generators will go to [1].

$Ht_1t_{13} = Ht_1t_1^4$
$\implies Ht_1t_{13} = Ht_1^5$
$\implies Ht_1t_{13} = Ht_{17}$
$\implies Ht_1t_{13} \in [1]$, since Ht_{17} is in [1].
4 Symmetric generators will go to [1].

$Ht_1t_{17} = Ht_1t_1^5$
$\implies Ht_1t_{17} = Ht_1^6$
$\implies Ht_1t_{17} = Ht_{21}$
$\implies Ht_1t_{17} \in [5]$, since Ht_{21} is in [5].

4 Symmetric generators will go to [5].

$Ht_1t_5 = Ht_1t_1^2$

$\implies Ht_1t_5 = Ht_1^3$

$\implies Ht_1t_5 = Ht_9$

$\implies Ht_1t_5 \in [1]$, since Ht_9 is in [1].

4 Symmetric generators will go to [1].

$Ht_1t_{21} = Ht_1t_1^6$

$\implies Ht_1t_{21} = Ht_1^7$

$\implies Ht_1t_{21} = Ht_{25}$

$\implies Ht_1t_{21} \in [5]$, since Ht_{25} is in [5].

4 Symmetric generators will go to [5].

$Ht_1t_{29} = Ht_1t_1^8$

$\implies Ht_1t_{29} = Ht_1^9$

$\implies Ht_1t_{29} = Ht_{33}$

$\implies Ht_1t_{29} \in [1]$, since Ht_{33} is in [1].

4 Symmetric generators will go to [1].

$Ht_1t_{37} = Ht_1t_1^{10}$

$\implies Ht_1t_{37} = H$

$\implies Ht_1t_{37} \in [1]$, since He is in [*].

4 Symmetric generators will go to [*].

$Ht_1t_{33} = Ht_1t_1^9$

$\implies Ht_1t_{33} = Ht_1^{10}$

$\implies Ht_1t_{33} = Ht_{37}$

$\implies Ht_1t_{33} \in [5]$, since Ht_{37} is in [5].

4 Symmetric generators will go to [5].

$Ht_1t_{25} = Ht_1t_1^7$

$\Longrightarrow Ht_1t_{25} = Ht_1^8$

$\Longrightarrow Ht_1t_{25} = Ht_{29}$

$\Longrightarrow Ht_1t_{25} \in [5]$, since Ht_{29} is in [5].

4 Symmetric generators will go to [5].

The orbits of $N^{(5)}$ are {1,2,3,4}, {5, 6, 7, 8}, {9, 10, 11, 12}, {13, 14, 15, 16}, {17, 18, 19, 20}, {21, 22, 23, 24}, {25, 26, 27, 28}, {29, 30, 31, 32}, {33, 34, 35, 36}, and {37, 38, 39, 40}.

We will check to see where $t_5t_1, t_5t_5, t_5t_9, t_5t_{13}, t_5t_{17}, t_5t_{21}, t_5t_{25}, t_5t_{29}, t_5t_{33}$, and t_5t_{37} belong.

$Ht_5t_1 = Ht_1^2t_1$

$\Longrightarrow Ht_5t_1 = Ht_1^3$

$\Longrightarrow Ht_5t_1 = Ht_9$

$\Longrightarrow Ht_5t_1 \in [1]$, since Ht_9 is in [1].

4 symmetric generators will go to [1].

$Ht_5t_5 = Ht_1^2t_1^2$

$\Longrightarrow Ht_5t_5 = Ht_1^4$

$\Longrightarrow Ht_5t_5 = Ht_{13}$

$\Longrightarrow Ht_5t_5 \in [1]$, since Ht_{13} is in [1].

4 symmetric generators will go to [1].

$Ht_5t_9 = Ht_1^2t_1^3$

$\Longrightarrow Ht_5t_9 = Ht_1^5$

$\Longrightarrow Ht_5t_9 = Ht_{17}$

$\Longrightarrow Ht_5t_9 \in [1]$, since Ht_{17} is in [1].

4 symmetric generators will go to [1].

$Ht_5t_{13} = Ht_1^2t_1^4$

$\Longrightarrow Ht_5t_{13} = Ht_1^6$

$\implies Ht_5t_{13} = Ht_{21}$

$\implies Ht_5t_{13} \in [5]$, since Ht_{21} is in [5].

4 symmetric generators will go to [5].

$Ht_5t_{17} = Ht_1^2t_1^5$

$\implies Ht_5t_{17} = Ht_1^7$

$\implies Ht_5t_{17} = Ht_{25}$

$\implies Ht_5t_{17} \in [5]$, since Ht_{25} is in [5].

4 symmetric generators will go to [5].

$Ht_5t_{21} = Ht_1^2t_1^6$

$\implies Ht_5t_{21} = Ht_1^8$

$\implies Ht_5t_{21} = Ht_{29}$

$\implies Ht_5t_{21} \in [5]$, since Ht_{29} is in [5].

4 symmetric generators will go to [5].

$Ht_5t_{25} = Ht_1^2t_1^7$

$\implies Ht_5t_{25} = Ht_1^9$

$\implies Ht_5t_{25} = Ht_{33}$

$\implies Ht_5t_{25} \in [1]$, since Ht_{33} is in [1].

4 symmetric generators will go to [1].

$Ht_5t_{29} = Ht_1^2t_1^8$

$\implies Ht_5t_{29} = Ht_1^{10}$

$\implies Ht_5t_{29} = Ht_{37}$

$\implies Ht_5t_{29} \in [5]$, since Ht_{37} is in [5].

4 symmetric generators will go to [5].

$Ht_5t_{33} = Ht_1^2t_1^9$

$\implies Ht_5t_{33} = H$

$\implies Ht_5t_{33} \in [*]$, since He is in [*].

4 symmetric generators will go to [*].

$Ht_5t_{37} = Ht_1^2t_1^{10}$
$\implies Ht_5t_{37} = Ht_1$
$\implies Ht_5t_{37} \in [1]$, since Ht_1 is in [1].
4 symmetric generators will go to [1].

This conclude our Double Coset Enumeration. Below is our Cayley Diagram.

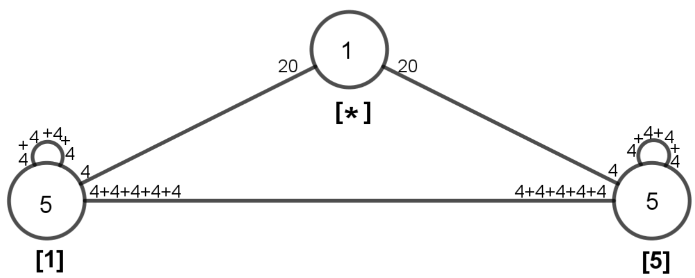


Figure 5.4: M_{11} Over $(C_4 : C_5)$.

5.5 $(S(4,3):2)$ as a Homomorphic Image of $2^{*10} : S_5$

5.5.1 Factor by Center of G

Let $G \cong 2^{*20} : (2^4 : S_5)$ be a symmetric presentation of G given by
$G =< x, y, t | x^6, (xy^{-1})^4, yxy^{-1}x^{-2}y^{-1}xyx^{-1}, y^{-1}x^{-1}yx^{-1}y^{-1}xy^3x,$
$(x^{-1}y^2x^{-1}y^{-1})^2, t^2, (t, yx^2y^{-2}x^{-1}yx^{-1}), (t, x^{-1}y^{-1}x^{-1}y^{-3}xy^{-1}),$
$(t, (yxy^{-1})^3), (t, y^{-1}x^3y^{-2}) >$, where
$x = (1, 16, 9, 2, 15, 10)(3, 18, 11)(4, 17, 12)(5, 6)(7, 14, 19)(8, 13, 20),$
$y = (1, 14, 15, 2, 13, 16)(3, 19, 6, 8, 18, 10, 4, 20, 5, 7, 17, 9),$
$N =< x, y >$, and the order of N is 1920.

Let us factor the progenitor $2^{*20} : (2^4 : S_5)$ by
$[(xyt^{x^3})^6, (xyt^{x^2yx^2})^4, (xyt^{x^2y})^8]$.

The Composition Factors of G are given below.

```
G
|  Cyclic(2)
*
|  C(2, 3)                    = S(4, 3)
*
|  Cyclic(2)
1
```

However, now our control group has changed.

```
> #sub<G|x,y>;
120
```

The order of N is 120 instead of 1920.

We look at our original control group given by
$< x, y|x^6, (xy^{-1})^4, yxy^{-1}x^{-2}y^{-1}xyx^{-1}, y^{-1}x^{-1}yx^{-1}y^{-1}xy^3x, (x^{-1}y^2x^{-1}y^{-1})^2 >$.

Note that $|x| = 6$, $|xy^{-1}| = 4$, $|yxy^{-1}x^{-2}y^{-1}xyx^{-1}| = 1$, $|y^{-1}x^{-1}yx^{-1}y^{-1}xy^3x| = 1$, and $|x^{-1}y^2x^{-1}y^{-1}| = 2$.

We also note that $|f(x)| = 3$, $|f(xy^{-1})| = 4$, $|f(yxy^{-1}x^{-2}y^{-1}xyx^{-1})| = 1$, $|f(y^{-1}x^{-1}yx^{-1}y^{-1}xy^3x)| = 1$, and $|f(x^{-1}y^2x^{-1}y^{-1})| = 2$.

So then we change our control group to the following symmetric representation.
$< x, y|x^3, (xy^{-1})^4, yxy^{-1}x^{-2}y^{-1}xyx^{-1}, y^{-1}x^{-1}yx^{-1}y^{-1}xy^3x, (x^{-1}y^2x^{-1}y^{-1})^2 >$.

```
> NN:=Group<x,y|x^3,(x*y^-1)^4,y*x*y^-1*x^-2*y^-1*x*y*x^-1,
> y^-1*x^-1*y*x^-1*y^-1*x*y^3*x,(x^-1*y^2*x^-1*y^-1)^2>;
> #NN;
120
```

Our new control group of order 120 is a permutation representation of NN over the Stabiliser(N,1), which we will call H.

```
> NN<x,y>:=Group<x,y|x^3,(x*y^-1)^4,y*x*y^-1*x^-2*y^-1*x*y*x^-1,
> y^-1*x^-1*y*x^-1*y^-1*x*y^3*x,(x^-1*y^2*x^-1*y^-1)^2>;
> H:=sub<NN|y*x^2*y^-2*x^-1*y*x^-1,x^-1*y^-1*x^-1*y^-3*x*y^-1,
> (y*x*y^-1)^3,y^-1*x^3*y^-2>;
> #H;
12
> #NN;
120
> f,g1,k:=CosetAction(NN,H);
> #g1;
120
> #k;
1
> g1;
Permutation group g1 acting on a set of cardinality 10
Order = 120 = 2^3 * 3 * 5
    (1, 2, 4)(3, 5, 6)(7, 8, 10)
    (1, 3, 2)(4, 7, 5, 9, 6, 8)
> Stabiliser(g1,1) eq sub<g1|f(H)>;
true
```

Now we check in MAGMA.

```
> S:=Sym(10);
> xx:=S!(1, 2, 4)(3, 5, 6)(7, 8, 10);
> yy:=S!(1, 3, 2)(4, 7, 5, 9, 6, 8);
> N:=sub<S|xx,yy>;
> #N;
120
> #sub<N|yy*xx^2*yy^-2*xx^-1*yy*xx^-1,xx^-1*yy^-1*xx^-1*
> yy^-3*xx*yy^-1,(yy*xx*yy^-1)^3,yy^-1*xx^3*yy^-2>;
12
> Stabiliser(N,1) eq sub<N|yy*xx^2*yy^-2*xx^-1*yy*xx^-1,xx^-1*
> yy^-1*xx^-1*yy^-3*xx*yy^-1,(yy*xx*yy^-1)^3,yy^-1*xx^3*yy^-2>;
```

```
true
> s:=IsIsomorphic(N,Sym(5));s;
true
```

Therefore, $G =< x, y, t | x^3, (xy^{-1})^4, yxy^{-1}x^{-2}y^{-1}xyx^{-1}, y^{-1}x^{-1}yx^{-1}y^{-1}xy^3x,$ $(x^{-1}y^2x^{-1}y^{-1})^2, t^2, (t, yx^2y^{-2}x^{-1}yx^{-1}), (t, x^{-1}y^{-1}x^{-1}y^{-3}xy^{-1}), (t, (yxy^{-1})^3),$ $(t, y^{-1}x^3y^{-2}), (xyt^{x^3})^6, (xyt^{x^2yx^2})^4, (xyt^{x^2y})^8 >$

is isomorphic to

$$\frac{2^{*10}:S_5}{[(xyt^{x^3})^6,(xyt^{x^2yx^2})^4,(xyt^{x^2y})^8]}.$$

The composition factors of G are given below.

```
G
|  Cyclic(2)
*
|  C(2, 3)                         = S(4, 3)
*
|  Cyclic(2)
1
```

Now we want to factor G by C_2 to obtain the following composition series for G, $G = G_1 \supseteq 1$, where $G = (G_1/G_2)(G_2/1) = C_2S(4,3)$.

We find the Normal Lattice of G.

```
> NL:=NormalLattice(G1);
> NL;

Normal subgroup lattice
-----------------------

[7]  Order 103680  Length 1  Maximal Subgroups: 4 5 6
---
[6]  Order 51840   Length 1  Maximal Subgroups: 3
[5]  Order 51840   Length 1  Maximal Subgroups: 2 3
```

```
[4]  Order 51840   Length 1  Maximal Subgroups: 3
---
[3]  Order 25920   Length 1  Maximal Subgroups: 1
---
[2]  Order 2       Length 1  Maximal Subgroups: 1
---
[1]  Order 1       Length 1  Maximal Subgroups:
```

We see that $NL[2]$ is of order 2. We check to see if $NL[2]$ is equal to the center of G.

```
> NL[2] eq Center(G1);
true
```

We use our Schreier System to write the generators of $NL[2]$ in terms of x,y, and t.

```
> IN:=sub<G1|f(x),f(y)>;
> N:=IN;
> #N;
120
> #G;
103680
> N:=G1;
> NN:=G;
> Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
> ArrayP:=[Id(N): i in [1..103680]];
> for i in [2..103680] do
for> P:=[Id(N): l in [1..#Sch[i]]];
for> for j in [1..#Sch[i]] do
for|for> if Eltseq(Sch[i])[j] eq 1 then P[j]:=f(x); end if;
for|for> if Eltseq(Sch[i])[j] eq 2 then P[j]:=f(y); end if;
for|for> if Eltseq(Sch[i])[j] eq -1 then P[j]:=f(x^-1); end if;
for|for> if Eltseq(Sch[i])[j] eq -2 then P[j]:=f(y^-1); end if;
for|for> if Eltseq(Sch[i])[j] eq 3 then P[j]:=f(t); end if;
for|for> end for;
for> PP:=Id(N);
for> for k in [1..#P] do
for|for> PP:=PP*P[k]; end for;
for> ArrayP[i]:=PP;
for> end for;
> for i in [1..103680] do if ArrayP[i] eq NL[2].1 then Sch[i];
for|if> end if; end for;
```

```
Id(G)
> for i in [1..103680] do if ArrayP[i] eq NL[2].2 then Sch[i];
for|if> end if; end for;
x * t * x * y^-1 * x * t * y * x * t * x * y^-1 * x * t * y^-1
* t * x * t * y
```

We now have the following presentation for G.

$G =< x, y, t|x^3, (xy^{-1})^4, yxy^{-1}x^{-2}y^{-1}xyx^{-1}, y^{-1}x^{-1}yx^{-1}y^{-1}xy^3x, (x^{-1}y^2x^{-1}y^{-1})^2,$
$t^2, (t, yx^2y^{-2}x^{-1}yx^{-1}), (t, x^{-1}y^{-1}x^{-1}y^{-3}xy^{-1}), (t, (yxy^{-1})^3), (t, y^{-1}x^3y^{-2}), (xyt^{x^3})^6,$
$(xyt^{x^2yx^2})^4, (xyt^{x^2y})^8, xtxy^{-1}xtyxtxy^{-1}xty^{-1}txty >.$

The Composition Factors of G are,

```
G
|  Cyclic(2)
*
|  C(2, 3)                    = S(4, 3)
1
```

5.5.2 The Construction of $(S(4,3) : 2)$ Over S_5

We are now ready to perform Double Coset Enumeration on the progenitor $2^{*10} : S_5$,
factored by $(xyt^{x^3})^6, (xyt^{x^2yx^2})^4, (xyt^{x^2y})^8, xtxy^{-1}xtyxtxy^{-1}xty^{-1}txty$.

Let $G \cong 2^{*10} : S_5$ be a symmetric presentation of G given by
$< x, y, t|x^3, (xy^{-1})^4, yxy^{-1}x^{-2}y^{-1}xyx^{-1}, y^{-1}x^{-1}yx^{-1}y^{-1}xy^3x, (x^{-1}y^2x^{-1}y^{-1})^2,$
$t^2, (t, yx^2y^{-2}x^{-1}yx^{-1}), (t, x^{-1}y^{-1}x^{-1}y^{-3}xy^{-1}), (t, (yxy^{-1})^3), (t, y^{-1}x^3y^{-2}), (xyt^{x^3})^6,$
$(xyt^{x^2yx^2})^4, (xyt^{x^2y})^8, xtxy^{-1}xtyxtxy^{-1}xty^{-1}txty >$, where
$N \cong S_5 =< x, y >$, $x = (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)$, and $y = (1, 3, 5, 7, 9, 10, 8, 6, 4, 2)$.

We will enter this presentation for G and label our permutations x and y as well as our control group, $N =< x, y >$, then verify that we have $N \cong S_5$.

```
> G<x,y,t>:=Group<x,y,t|x^3,(x*y^-1)^4,y*x*y^-1*x^-2*y^-1*x*y*
> x^-1,y^-1*x^-1*y*x^-1*y^-1*x*y^3*x,(x^-1*y^2*x^-1*y^-1)^2,
> t^2,(t,y*x^2*y^-2*x^-1*y*x^-1),(t,x^-1*y^-1*x^-1*y^-3*x*y^-1),
```

```
> (t,(y*x*y^-1)^3),(t,y^-1*x^3*y^-2),(x*y*t^(x^3))^6,(x*y*t^(x^2*
> y*x^2))^4,(x*y*t^(x^2*y))^8,x*t*x*y^-1*x*t*y*x*t*
> x*y^-1*x*t*y^-1*t*x*t*y>;
> S:=Sym(10);
> xx:=S!(1,2,4)(3,5,6)(7,8,10);
> yy:=S!(1,3,2)(4,7,5,9,6,8);
> N:=sub<S|xx,yy>;
> f,G1,k:=CosetAction(G,sub<G|x,y>);
> CompositionFactors(G1);
    G
    |  Cyclic(2)
    *
    |  C(2, 3)                    = S(4, 3)
    1
> s:=IsIsomorphic(N,Sym(5));s;
true
```

We then use MAGMA to calculate the number of double cosets of G over N as well as to name our t_i^s. Note that when naming our t_i's, $t_1 = t$, then $t_2 = t^x$, since $x = (1,2,4)(3,5,6)(7,8,10)$ takes t_1 to t_2. Similarily, $t_3 = t^y$, since $y = (1,3,2)(4,7,5,9,6,8)$ takes t_1 to t_3, also $t_4 = t^{x^2}$, since $x^2 = (1,4,2)(3,6,5)(7,10,8)$ takes t_1 to t_4. This process is repeated until all of our t_i's have been named.

```
> #DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);
20
> IN:=sub<G1|f(x),f(y)>;
> ts := [Id(G1): i in [1 .. 10] ];
> ts[1]:=f(t); ts[2]:=f(t^x); ts[3]:=f(t^y); ts[4]:=f(t^(x^2));
> ts[5]:=f(t^(y*x)); ts[6]:=f(t^(y*x^2));  ts[7]:=f(t^(x^2*y));
> ts[8]:=f(t^(x^2*y*x)); ts[9]:=f(t^(y*x*y));
> ts[10]:=f(t^(x^2*y*x^2));
```

So we will have 20 double cosets. The number of single cosets is equal to $\frac{|G|}{|N|} = \frac{51840}{120} = 432$. We will use the following loop to keep count of the single cosets. It is important in this loop that we input the number of t_i's that we have, 10, as well as the number of single cosets that we have, 432. The coset counter will give us a running total of how many single cosets we have thus far, starting with our second double coset [1]. It does not keep count of the 1 single coset in [*].

```
> prodim:=function(pt, Q, I)
function> v:=pt;
function> for i in I do
function|for> v := v^(Q[i]);
function|for> end for;
function> return v;
function> end function;
> #G/#N;
432
> cst := [null : i in [1 .. Index(G,sub<G|x,y>)]] where null is
> [Integers() | ];
> for i := 1 to 10 do
for> cst[prodim(1, ts, [i])] := [i];
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
10
```

Words of Length 1

Our first double coset is NeN, denoted by $[*]$.
$[*] = \frac{|N|}{|N|} = \frac{120}{120} = 1$ single coset.

```
> Orbits(N);
[
    GSet{@ 1, 2, 3, 4, 5, 7, 6, 9, 8, 10 @}
]
```

The orbit of N on $\{1,2,3,4,5,6,7,8,9,10\}$ is $\{1,2,3,4,5,6,7,8,9,10\}$. We pick a representative from the orbit, say 1, and determine the double coset that contains Nt_1.

Nt_1N is a new double coset which we will denote by [1]. Since the orbit{1,2,3,4,5,6,7,8,9,10} contains ten elements, then ten symmetric generators will go to the new double coset [1]. Recall that our coset counter, m, was at 10.

We will now examine our double coset [1]. Our representative of this double coset is Nt_1. The following code labels the point stabiliser of 1 in N as N1. Then we label the set SSS, which is made up of t_1 conjugated by all of the elements of N.

```
> N1:=Stabiliser(N,[1]);
> SSS:={[1]};
> SSS:=SSS^N;
> #SSS;
10
> Seqq:=Setseq(SSS);
```

The next loop tells us if we have any equal names, that is, if any of our cosets in [1] are equal to each other.

```
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1 ]
> N1s:=N1;
```

From the above loop we see that there are not any equal names in [1]. Thus the coset stabiliser of 1 in N is equal to the point stabiliser of 1 in N. We compute the transversals of $N^{(1)}$ in N and label this as T1. Then we use our coset counter to see how many single cosets we have thus far.

```
> T1:=Transversal(N,N1s);
> for i in [1..#T1] do
for> ss:=[1]^T1[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
10
```

Now we can use MAGMA to find the elements in the set $N^{(1)}$. To find the distinct single cosets in [1], we first find the transversals, then conjugate Nt_1 by each of the elements in the set of transversals.

```
> #N1s;
12
> Set(N1s);
{
    Id(N1),
    (2, 6, 9, 3, 4, 7)(5, 10, 8),
    (2, 4)(3, 6)(8, 10),
    (2, 9)(3, 7)(5, 10),
    (2, 7, 4, 3, 9, 6)(5, 8, 10),
    (2, 9, 4)(3, 7, 6)(5, 8, 10),
    (4, 9)(5, 8)(6, 7),
    (2, 6)(3, 4)(7, 9)(8, 10),
    (2, 7)(3, 9)(4, 6)(5, 10),
    (2, 4, 9)(3, 6, 7)(5, 10, 8),
    (2, 3)(4, 6)(7, 9),
    (2, 3)(4, 7)(5, 8)(6, 9)
}
> for i in [1..#T1] do ([1]^N1s)^T1[i]; end for;
{@
    [ 1 ]
@}
{@
    [ 2 ]
@}
{@
    [ 3 ]
@}
{@
    [ 4 ]
@}
{@
    [ 5 ]
@}
{@
    [ 6 ]
@}
{@
    [ 7 ]
```

```
@}
{@
    [ 8 ]
@}
{@
    [ 9 ]
@}
{@
    [ 10 ]
@}
```

$N^1 = N^{(1)} = \{e, (2,6,9,3,4,7)(5,10,8), (2,4)(3,6)(8,10), (2,9)(3,7)(5,10), (2,9,4)(3,7,6)$ $(5,8,10), (2,7,4,3,9,6)(5,8,10), (2,6)(3,4)(7,9)(8,10), (4,9)(5,8)(6,7), (2,7)(3,9)(4,6)$ $(5,10), (2,4,9)(3,6,7)(5,10,8), (2,3)(4,6)(7,9), (2,3)(4,7)(5,8)(6,9)\}$.

The number of single cosets in Nt_1N is $\frac{|N|}{|N^{(1)}|} = \frac{120}{12} = 10$.
$Nt_1N = \{Nt_1, Nt_2, Nt_3, Nt_4, Nt_5, Nt_6, Nt_7, Nt_8, Nt_9, Nt_{10}\}$.

Lastly, we need to compute the orbits of $N^{(1)}$.

```
> Orbits(N1s);
[
    GSet{@ 1 @},
    GSet{@ 5, 10, 8 @},
    GSet{@ 2, 7, 4, 6, 9, 3 @}
]
```

The orbits of the coset stabilier $N^{(1)}$ on $\{1,2,3,4,5,6,7,8,9,10\}$ are
$\{1\}, \{5,10,8\}$, and $\{2,7,4,3,6,9\}$.

We take t_1, t_5, and t_2 from each orbit respectively.
We want to determine to which double coset Nt_1t_1, Nt_1t_5, and Nt_1t_2 belong.

$Nt_1t_1 = N \in [*]$ (Since our t's are of order 2.)
Since the orbit $\{1\}$ contains one element, then one symmetric generator goes back to the double coset $[*]$.

Now, so far we have found the double cosets [*] and [1]. We will use the following loop to see if Nt_1t_5, and Nt_1t_2 belong in these double cosets. If not, then then they will go on to a new double coset.

```
> for m,n in IN do if ts[1]*ts[5] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
>
> for m,n in IN do if ts[1]*ts[5] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
>
> for m,n in IN do if ts[1]*ts[2] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
>
> for m,n in IN do if ts[1]*ts[2] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
>
```

Since MAGMA did not return any output with the above loop, then we have two new double cosets.

Nt_1t_5N is a new double coset which we will denote [15].
Since the orbit $\{5, 10, 8\}$ contains three elements, then three symmetric generators will go to the new double coset [15].

Nt_1t_2N is a new double coset which we will denote [12].
Since the orbit $\{2, 7, 4, 3, 6, 9\}$ contains six elements, then six symmetric generators will go to the new double coset [12].

Words of Length 2

Now we move on to our first double coset of length 2. In the same manner as before we look for equal names.

```
> N15:=Stabiliser(N,[1,5]);
> SSS:={[1,5]};
> SSS:=SSS^N;
> #SSS;
30
```

```
> Seqq:=Setseq(SSS);
>
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5 ]
```

Since we do not have a relation that will increase our coset stabiliser, then $N^{15} = N^{(15)}$. We input this and check our coset counter.

```
> T15:=Transversal(N,N15s);
> for i in [1..#T15] do
for> ss:=[1,5]^T15[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
40
```

Our coset counter has increased from 10 to 40. We should have 30 distinct single cosets in [15]. We find the distinct single cosets as well as the orbits of $N^{(15)}$.

```
> [1,5]^N15s;
GSet{@
    [ 1, 5 ]
@}
> for i in [1..#T15] do ([1,5]^N15s)^T15[i]; end for;
{@
    [ 1, 5 ]
@}
{@
    [ 1, 10 ]
@}
{@
    [ 1, 8 ]
@}
{@
    [ 2, 6 ]
@}
```

```
{@
    [ 2, 7 ]
@}
{@
    [ 2, 10 ]
@}
{@
    [ 3, 9 ]
@}
{@
    [ 3, 10 ]
@}
{@
    [ 3, 4 ]
@}
{@
    [ 4, 3 ]
@}
{@
    [ 4, 8 ]
@}
{@
    [ 4, 7 ]
@}
{@
    [ 5, 9 ]
@}
{@
    [ 5, 7 ]
@}
{@
    [ 5, 1 ]
@}
{@
    [ 6, 9 ]
@}
{@
    [ 6, 8 ]
@}
{@
    [ 6, 2 ]
@}
{@
```

```
    [ 7, 2 ]
@}
{@
    [ 7, 4 ]
@}
{@
    [ 7, 5 ]
@}
{@
    [ 8, 1 ]
@}
{@
    [ 8, 6 ]
@}
{@
    [ 8, 4 ]
@}
{@
    [ 9, 6 ]
@}
{@
    [ 9, 5 ]
@}
{@
    [ 9, 3 ]
@}
{@
    [ 10, 1 ]
@}
{@
    [ 10, 2 ]
@}
{@
    [ 10, 3 ]
@}
> Orbits(N15s);
[
    GSet{@ 1 @},
    GSet{@ 5 @},
    GSet{@ 7, 9 @},
    GSet{@ 8, 10 @},
    GSet{@ 2, 6, 4, 3 @}
]
```

$N^{(15)} = \{e, (24)(36)(810), (26)(34)(79)(810), (23)(46)(79)\}$. The number of the single cosets in the double coset Nt_1t_5N is at most $\frac{|N|}{|N^{(15)}|} = \frac{120}{4} = 30$.

$Nt_1t_5N = \{Nt_1t_5, Nt_1t_{10}, Nt_1t_8, Nt_2t_6, Nt_2t_7, Nt_2t_{10}, Nt_3t_9, Nt_3t_{10}, Nt_3t_4, Nt_4t_3, Nt_4t_8, Nt_4t_7, Nt_5t_9, Nt_5t_7, Nt_5t_1, Nt_6t_9, Nt_6t_8, Nt_6t_2, Nt_7t_2, Nt_7t_4, Nt_7t_5, Nt_8t_1, Nt_8t_6, Nt_8t_4, Nt_9t_6, Nt_9t_5, Nt_9t_3, Nt_{10}t_1, Nt_{10}t_2, Nt_{10}t_3\}$.

Similarily we examine our other double coset [12].

```
> N12:=Stabiliser(N,[1,2]);
> SSS:={[1,2]};
> SSS:=SSS^N;
> #SSS;
60
> Seqq:=Setseq(SSS);
>
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[2] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 2 ]
[ 1, 3 ]
```

MAGMA tells us that $Nt_1t_2 = Nt_1t_3$. So we now have a relation that increases $N^{(12)}$. We enter these equal names with the following code and check our coset counter.

```
> N12s:=N12;
> for n in N do if 1^n eq 1 and 2^n eq 3 then
for|if> N12s:=sub<N|N12s,n>; end if; end for;
> N12s; #N12s;
Permutation group N12s acting on a set of cardinality 10
    (4, 9)(5, 8)(6, 7)
    (2, 3)(4, 7)(5, 8)(6, 9)
    (2, 3)(4, 6)(7, 9)
4
> #N/#N12s;
30
> T12:=Transversal(N,N12s);
> for i in [1..#T12] do
```

```
for> ss:=[1,2]^T12[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
70
```

Our coset counter is now at 70, so we must have 30 distinct single cosets in [12]. We will find the distinct single cosets as well as the orbits of $N^{(12)}$.

```
> #N12s;
4
> Set(N12s);
{
    (2, 3)(4, 7)(5, 8)(6, 9),
    (4, 9)(5, 8)(6, 7),
    (2, 3)(4, 6)(7, 9),
    Id(N12s)
}
> for i in [1..#T12] do ([1,2]^N12s)^T12[i]; end for;
{@
    [ 1, 2 ],
    [ 1, 3 ]
@}
{@
    [ 1, 6 ],
    [ 1, 4 ]
@}
{@
    [ 1, 7 ],
    [ 1, 9 ]
@}
{@
    [ 2, 4 ],
    [ 2, 5 ]
@}
{@
    [ 2, 3 ],
    [ 2, 1 ]
@}
{@
    [ 2, 9 ],
```

```
        [ 2, 8 ]
    @}
    {@
        [ 3, 1 ],
        [ 3, 2 ]
    @}
    {@
        [ 3, 8 ],
        [ 3, 7 ]
    @}
    {@
        [ 3, 6 ],
        [ 3, 5 ]
    @}
    {@
        [ 4, 1 ],
        [ 4, 6 ]
    @}
    {@
        [ 4, 5 ],
        [ 4, 2 ]
    @}
    {@
        [ 4, 10 ],
        [ 4, 9 ]
    @}
    {@
        [ 5, 2 ],
        [ 5, 4 ]
    @}
    {@
        [ 5, 10 ],
        [ 5, 8 ]
    @}
    {@
        [ 5, 3 ],
        [ 5, 6 ]
    @}
    {@
        [ 6, 4 ],
        [ 6, 1 ]
    @}
    {@
```

```
    [ 6, 7 ],
    [ 6, 10 ]
@}
{@
    [ 6, 5 ],
    [ 6, 3 ]
@}
{@
    [ 7, 3 ],
    [ 7, 8 ]
@}
{@
    [ 7, 9 ],
    [ 7, 1 ]
@}
{@
    [ 7, 10 ],
    [ 7, 6 ]
@}
{@
    [ 8, 2 ],
    [ 8, 9 ]
@}
{@
    [ 8, 7 ],
    [ 8, 3 ]
@}
{@
    [ 8, 5 ],
    [ 8, 10 ]
@}
{@
    [ 9, 1 ],
    [ 9, 7 ]
@}
{@
    [ 9, 4 ],
    [ 9, 10 ]
@}
{@
    [ 9, 8 ],
    [ 9, 2 ]
@}
```

```
{@
    [ 10, 6 ],
    [ 10, 7 ]
@}
{@
    [ 10, 4 ],
    [ 10, 9 ]
@}
{@
    [ 10, 8 ],
    [ 10, 5 ]
@}
> Orbits(N12s);
[
    GSet{@ 1 @},
    GSet{@ 10 @},
    GSet{@ 2, 3 @},
    GSet{@ 5, 8 @},
    GSet{@ 4, 9, 7, 6 @}
]
```

$N^{(12)} = \{e, (23)(47)(58)(69), (49)(58)(67), (23)(46)(79)\}$. The number of the single cosets in the double coset Nt_1t_2N is at most $\frac{|N|}{|N^{(12)}|} = \frac{120}{4} = 30$.

$Nt_1t_2N = \{Nt_1t_2 = Nt_1t_3, Nt_1t_6 = Nt_1t_4, Nt_1t_7 = Nt_1t_9, Nt_2t_4 = Nt_2t_5,$
$Nt_2t_3 = Nt_2t_1, Nt_2t_9 = Nt_2t_8, Nt_3t_1 = Nt_3t_2, Nt_3t_8 = Nt_3t_7, Nt_3t_6 = Nt_3t_5,$
$Nt_4t_1 = Nt_4t_6, Nt_4t_5 = Nt_4t_2, Nt_4t_{10} = Nt_4t_9, Nt_5t_2 = Nt_5t_4, Nt_5t_{10} = Nt_5t_8,$
$Nt_5t_3 = Nt_5t_6, Nt_6t_4 = Nt_6t_1, Nt_6t_7 = Nt_6t_{10}, Nt_6t_5 = Nt_6t_3, Nt_7t_3 = Nt_7t_8,$
$Nt_7t_9 = Nt_7t_1, Nt_7t_{10} = Nt_7t_6, Nt_8t_2 = Nt_8t_9, Nt_8t_7 = Nt_8t_3, Nt_8t_5 = Nt_8t_{10},$
$Nt_9t_1 = Nt_9t_7, Nt_9t_4 = Nt_9t_{10}, Nt_9t_8 = Nt_9t_2, Nt_{10}t_6 = Nt_{10}t_7, Nt_{10}t_4 = Nt_{10}t_9,$
$Nt_{10}t_8 = Nt_{10}t_5\}$

So far we have $G = N \cup Nt_1N \cup Nt_1t_5N \cup Nt_1t_2N$.
Which gives us $1 + 10 + 30 + 30 = 71$ distinct cosets.

Now we check the orbits of $N^{(15)}$ and $N^{(12)}$ to see where our single cosets go.
The orbits of $N^{(15)}$ on $\{1,2,3,4,5,6,7,8,9,10\}$ are $\{1\}$, $\{5\}$, $\{7,9\}$, $\{8,10\}$, and $\{2,6,4,3\}$. We take t_1, t_5, t_7, t_8, and t_2 from the orbits of $N^{(15)}$. We want to de-

termine to which double coset $Nt_1t_5t_1, Nt_1t_5t_5, Nt_1t_5t_7, Nt_1t_5t_8$ and $Nt_1t_5t_2$ belong.

The double cosets that we have thus far are [*], [1], [15], and [12]. We will use the following loop to see which cosets will go to these double cosets, those that do not, will form new double cosets. We will make sure to add these new double cosets to our loop as we find them.

```
> for m,n in IN do if ts[1]*ts[5]*ts[1] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
>
```

$Nt_1t_5t_1N$ is a new double coset which we will denote [151].

One symmetric generator goes to the new double coset [151].

```
> for m,n in IN do if ts[1]*ts[5]*ts[5] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[5] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
true
> for m,n in IN do if ts[1]*ts[5]*ts[5] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[5] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[5] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
>
```

$Nt_1t_5t_5 \in [1]$.

One symmetric generator goes back to [1].

```
> for m,n in IN do if ts[1]*ts[5]*ts[7] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
```

```
> for m,n in IN do if ts[1]*ts[5]*ts[7] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[7] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[7] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[7] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
>
```

$Nt_1t_5t_7N$ is a new double coset which we will denote [157].

Two symmetric generators will go to the new double coset [157].

```
> for m,n in IN do if ts[1]*ts[5]*ts[8] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[8] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[8] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[8] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[8] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[8] eq m*(ts[1]*ts[5]*ts[7])^n
for|if> then "true"; break; end if; end for;
>
```

$Nt_1t_5t_8N$ is a new double coset which we will denote [158].

Two symmetric generators will go to the new double coset [158].

```
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1]*ts[5]*ts[7])^n
```

```
for|if> then "true"; break; end if; end for;
true
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1]*ts[5]*ts[8])^n
for|if> then "true"; break; end if; end for;
>
```

$Nt_1t_5t_2 \in [157]$.

Four symmetric generators go to [157].

The orbits of $N^{(12)}$ on $\{1,2,3,4,5,6,7,8,9,10\}$ are {1}, {10}, {2,3}, {5,8}, and {4,9,7,6}. We take t_1, t_{10}, t_2, t_5, and t_4 from the orbits of $N^{(12)}$. We want to determine to which double coset $Nt_1t_2t_1, Nt_1t_2t_{10}, Nt_1t_2t_2, Nt_1t_2t_5$, and $Nt_1t_2t_4$ belong.

```
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1]*ts[5]*ts[7])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1]*ts[5]*ts[8])^n
for|if> then "true"; break; end if; end for;
>
```

$Nt_1t_2t_1N$ is a new double coset which we will denote [121].

One symmetric generator will go to the new double coset [121].

```
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[2])^n
```

```
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[5]*ts[7])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[5]*ts[8])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[2]*ts[1])^n
for|if> then "true"; break; end if; end for;
>
```

$Nt_1t_2t_{10}N$ is a new double coset which we will denote [1210].

One symmetric generator will go to the new double coset [1210].

```
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
true
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[5]*ts[7])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[5]*ts[8])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[2]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[2]*ts[10])^n
for|if> then "true"; break; end if; end for;
>
```

$Nt_1t_2t_2 \in [1]$.

Two symmetric generators go back to [1].

```
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1])^n
```

```
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[5]*ts[7])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[5]*ts[8])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[2]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[2]*ts[10])^n
for|if> then "true"; break; end if; end for;
>
```

$Nt_1t_2t_5N$ is a new double coset which we will denote [125].

Two symmetric generators will go to the new double coset [125].

```
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[5]*ts[7])^n
for|if> then "true"; break; end if; end for;
true
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[5]*ts[8])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[2]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[2]*ts[10])^n
for|if> then "true"; break; end if; end for;
>
```

$Nt_1t_2t_4 \in [157]$.

Four symmetric generators go to [157].

Words of Length 3

We continue in the same manner above. Our MAGMA code can be found in appendix.

$N^{(151)} = \{e, (26)(34)(79)(810), (14)(35)(710), (163)(245)(8910), (1256)(34)(79810), (15)(2364)(71098), (164)(235)(7109), (136)(254)(8109), (15)(2463)(78910), (1354)(26)(78109), (16)(25)(910), (1453)(26)(79108), (146)(253)(7910), (24)(36)(810), (13)(45)(89), (15)(26)(78)(910), (12)(56)(78), (123)(456)(798), (23)(46)(79), (124)(356)(7810), (142)(365)(7108), (15)(34)(710)(89), (132)(465)(789), (1652)(34)(71089)\}$. The number of the single cosets in the double coset $Nt_1t_5t_1N$ is at most $\frac{|N|}{|N^{(151)}|} = \frac{120}{24} = 5$.

$Nt_1t_5t_1N = \{Nt_1t_5t_1 = Nt_2t_6t_2 = Nt_4t_3t_4 = Nt_3t_4t_3 = Nt_5t_1t_5 = Nt_6t_2t_6, Nt_3t_9t_3 = Nt_1t_8t_1 = Nt_7t_2t_7 = Nt_2t_7t_2 = Nt_9t_3t_9 = Nt_8t_1t_8, Nt_5t_9t_5 = Nt_2t_{10}Nt_2 = Nt_8t_4t_8 = Nt_4t_8t_4 = Nt_9t_5t_9 = Nt_{10}t_2t_{10}, Nt_6t_9t_6 = Nt_4t_7t_4 = Nt_{10}t_1t_{10} = Nt_1t_{10}t_1 = Nt_9t_6t_9 = Nt_7t_4t_7, Nt_6t_8t_6 = Nt_5t_7t_5 = Nt_{10}t_3t_{10} = Nt_3t_{10}t_3 = Nt_8t_6t_8 = Nt_7t_5t_7\}$.

$N^{(157)} = \{e, (24)(36)(810)\}$. The number of the single cosets in the double coset $Nt_1t_5t_7N$ is at most $\frac{|N|}{|N^{(157)}|} = \frac{120}{2} = 60$.

$Nt_1t_5t_7N = \{Nt_1t_5t_7, Nt_1t_5t_9, Nt_1t_8t_6, Nt_1t_8t_4, Nt_1t_{10}t_3, Nt_1t_{10}t_2, Nt_2t_6t_8, Nt_2t_6t_9, Nt_2t_{10}t_3, Nt_2t_{10}t_1, Nt_2t_7t_5, Nt_2t_7t_4, Nt_3t_9t_5, Nt_3t_9t_6, Nt_3t_4t_8, Nt_3t_4t_7, Nt_3t_{10}t_2, Nt_3t_{10}t_1, Nt_4t_3t_{10}, Nt_4t_3t_9, Nt_4t_7t_5, Nt_4t_7t_2, Nt_4t_8t_6, Nt_4t_8t_1, Nt_5t_9t_6, Nt_5t_9t_3, Nt_5t_1t_{10}, Nt_5t_1t_8, Nt_5t_7t_4, Nt_5t_7t_2, Nt_6t_9t_3, Nt_6t_9t_5, Nt_6t_2t_7, Nt_6t_2t_{10}, Nt_6t_8t_1, Nt_6t_8t_4, Nt_7t_2t_{10}, Nt_7t_2t_6, Nt_7t_5t_9, Nt_7t_5t_1, Nt_7t_4t_8, Nt_7t_4t_3, Nt_8t_1t_{10}, Nt_8t_1t_5, Nt_8t_4t_7, Nt_8t_4t_3, Nt_8t_6t_9, Nt_8t_6t_2, Nt_9t_6t_8, Nt_9t_6t_2, Nt_9t_3t_{10}, Nt_9t_3t_4, Nt_9t_5t_7, Nt_9t_5t_1, Nt_{10}t_1t_8, Nt_{10}t_1t_5, Nt_{10}t_3t_9, Nt_{10}t_3t_4, Nt_{10}t_2t_7, Nt_{10}t_2t_6\}$

$N^{(158)} = \{e, (24)(36)(810), (26)(34)(79)(810), (23)(46)(79)\}$. The number of the single cosets in the double coset $Nt_1t_5t_8N$ is at most $\frac{|N|}{|N^{(158)}|} = \frac{120}{4} = 30$.

$Nt_1t_5t_8N = \{Nt_1t_5t_8 = Nt_1t_5t_{10}, Nt_1t_{10}t_5 = Nt_1t_{10}t_8, Nt_1t_8t_{10} = Nt_1t_8t_5, Nt_2t_6t_{10} = Nt_2t_6t_7, Nt_2t_7t_6 = Nt_2t_7t_{10}, Nt_2t_{10}t_7 = Nt_2t_{10}t_6, Nt_3t_9t_4 = Nt_3t_9t_{10}, Nt_3t_{10}t_9 = Nt_3t_{10}t_4, Nt_3t_4t_{10} = Nt_3t_4t_9, Nt_4t_3t_7 = Nt_4t_3t_8, Nt_4t_8t_3 = Nt_4t_8t_7, Nt_4t_7t_3 = Nt_4t_7t_8, Nt_5t_9t_1 = Nt_5t_9t_7, Nt_5t_7t_1 = Nt_5t_7t_9, Nt_5t_1t_7 = Nt_5t_1t_9, Nt_6t_9t_2 = Nt_6t_9t_8, Nt_6t_8t_2 = Nt_6t_8t_9, Nt_6t_2t_9 = Nt_6t_2t_8, Nt_7t_2t_5 = Nt_7t_2t_4, Nt_7t_4t_5 = Nt_7t_4t_2, Nt_7t_5t_2 = Nt_7t_5t_4, Nt_8t_1t_4 = Nt_8t_1t_6, Nt_8t_6t_1 = Nt_8t_6t_4, Nt_8t_4t_6 = Nt_8t_4t_1, Nt_9t_6t_3 = Nt_9t_6t_5, Nt_9t_5t_3 = Nt_9t_5t_6, Nt_9t_3t_5 = Nt_9t_3t_6, Nt_{10}t_1t_3 = Nt_{10}t_1t_2, Nt_{10}t_2t_1 = Nt_{10}t_2t_3, Nt_{10}t_3t_1 = Nt_{10}t_3t_2\}$.

$N^{(121)} = \{e, (49)(58)(67), (123)(456)(798), (123)(486957), (23)(46)(79), (13)(45)(89), (12)(56)(78), (13)(48)(59)(67), (23)(47)(58)(69), (132)(465)(789), (132)(475968), (12)(49)(57)(68)\}$. The number of the single cosets in the double coset $Nt_1t_2t_1N$ is at most $\frac{|N|}{|N^{(121}|} = \frac{120}{12} = 10$.

$Nt_1t_2t_1N = \{Nt_1t_2t_1 = Nt_3t_1t_3 = Nt_2t_3t_2 = Nt_2t_1t_2 = Nt_1t_3t_1 = Nt_3t_2t_3, Nt_2t_4t_2 = Nt_5t_2t_5 = Nt_4t_5t_4 = Nt_4t_2t_4 = Nt_2t_5t_2 = Nt_5t_4t_5, Nt_4t_1t_4 = Nt_6t_4t_6 = Nt_1t_6t_1 = Nt_1t_4t_1 = Nt_4t_6t_4 = Nt_6t_1t_6, Nt_1t_7t_1 = Nt_9t_1t_9 = Nt_7t_9t_7 = Nt_7t_1t_7 = Nt_1t_9t_1 = Nt_9t_7t_9, Nt_3t_8t_3 = Nt_7t_3t_7 = Nt_8t_7t_8 = Nt_8t_3t_8 = Nt_3t_7t_3 = Nt_7t_8t_7, Nt_8t_2t_8 = Nt_9t_8t_9 = Nt_2t_9t_2 = Nt_2t_8t_2 = Nt_8t_9t_8 = Nt_9t_2t_9, Nt_5t_3t_5 = Nt_6t_5t_6 = Nt_3t_6t_3 = Nt_3t_5t_3 = Nt_5t_6t_5 = Nt_6t_3t_6, Nt_4t_{10}t_4 = Nt_9t_4t_9 = Nt_{10}t_9t_{10} = Nt_{10}t_4t_{10} = Nt_4t_9t_4 = Nt_9t_{10}t_9, Nt_5t_{10}t_5 = Nt_8t_5t_8 = Nt_{10}t_8t_{10} = Nt_{10}t_5t_{10} = Nt_5t_8t_5 = Nt_8t_{10}t_8, Nt_6t_7t_6 = Nt_{10}t_6t_{10} = Nt_7t_{10}t_7 = Nt_7t_6t_7 = Nt_6t_{10}t_6 = Nt_{10}t_7t_{10}\}$.

$N^{(1210)} = \{e, (23)(47)(58)(69), (49)(58)(67), (23)(46)(79)\}$. The number of the single cosets in the double coset $Nt_1t_2t_{10}N$ is at most $\frac{|N|}{|N^{(1210}|} = \frac{120}{4} = 30$.

$Nt_1t_2t_{10}N = \{Nt_1t_2t_{10} = Nt_1t_3t_{10}, Nt_1t_7t_5 = Nt_1t_9t_5, Nt_1t_6t_8 = Nt_1t_4t_8, Nt_2t_4t_7 = Nt_2t_5t_7, Nt_2t_9t_6 = Nt_2t_8t_6, Nt_2t_3t_{10} = Nt_2t_1t_{10}, Nt_3t_1t_{10} = Nt_3t_2t_{10}, Nt_3t_6t_9 = Nt_3t_5t_9, Nt_3t_8t_4 = Nt_3t_7t_4, Nt_4t_1t_8 = Nt_4t_6t_8, Nt_4t_{10}t_3 = Nt_4t_9t_3, Nt_4t_5t_7 = Nt_4t_2t_7, Nt_5t_2t_7 = Nt_5t_4t_7, Nt_5t_3t_9 = Nt_5t_6t_9, Nt_5t_{10}t_1 = Nt_5t_8t_1, Nt_6t_4t_8 = Nt_6t_1t_8, Nt_6t_5t_9 = Nt_6t_3t_9, Nt_6t_7t_2 = Nt_6t_{10}t_2, Nt_7t_3t_4 = Nt_7t_8t_4, Nt_7t_{10}t_2 = Nt_7t_6t_2, Nt_7t_9t_5 = Nt_7t_1t_5, Nt_8t_2t_6 = Nt_8t_9t_6, Nt_8t_5t_1 = Nt_8t_{10}t_1, Nt_8t_7t_4 = Nt_8t_3t_4, Nt_9t_1t_5 = Nt_9t_7t_5, Nt_9t_8t_6 =$

$Nt_9t_2t_6, Nt_9t_4t_3 = Nt_9t_{10}t_3, Nt_{10}t_6t_2 = Nt_{10}t_7t_2, Nt_{10}t_8t_1 = Nt_{10}t_5t_1, Nt_{10}t_4t_3 = Nt_{10}t_9t_3\}$.

$N^{(125)} = \{e, (13)(45)(89), (12)(56)(78), (23)(46)(79), (123)(456)(798), (132)(465)(789)\}$.
The number of the single cosets in the double coset $Nt_1t_2t_5N$ is at most $\frac{|N|}{|N^{(125}|} = \frac{120}{6} = 20$.

$Nt_1t_2t_5N = \{Nt_1t_2t_5 = Nt_2t_3t_6 = Nt_3t_1t_4 = Nt_1t_3t_5 = Nt_3t_2t_4 = Nt_2t_1t_6, Nt_3t_1t_9 = Nt_1t_2t_8 = Nt_2t_3t_7 = Nt_3t_2t_9 = Nt_2t_1t_7 = Nt_1t_3t_8, Nt_5t_2t_1 = Nt_2t_4t_6 = Nt_4t_5t_3 = Nt_5t_4t_1 = Nt_4t_2t_3 = Nt_2t_5t_6, Nt_4t_2t_8 = Nt_2t_5t_{10} = Nt_5t_4t_9 = Nt_4t_5t_8 = Nt_5t_2t_9 = Nt_2t_4t_{10}, Nt_4t_1t_3 = Nt_1t_6t_5 = Nt_6t_4t_2 = Nt_4t_6t_3 = Nt_6t_1t_2 = Nt_1t_4t_5, Nt_6t_1t_9 = Nt_1t_4t_{10} = Nt_4t_6t_7 = Nt_6t_4t_9 = Nt_4t_1t_7 = Nt_1t_6t_{10}, Nt_9t_1t_3 = Nt_1t_7t_8 = Nt_7t_9t_2 = Nt_9t_7t_3 = Nt_7t_1t_2 = Nt_1t_9t_8, Nt_7t_1t_4 = Nt_1t_9t_{10} = Nt_9t_7t_6 = Nt_7t_9t_4 = Nt_9t_1t_6 = Nt_1t_7t_{10}, Nt_7t_3t_2 = Nt_3t_8t_9 = Nt_8t_7t_1 = Nt_7t_8t_2 = Nt_8t_3t_1 = Nt_3t_7t_9, Nt_8t_7t_6 = Nt_7t_3t_5 = Nt_3t_8t_{10} = Nt_8t_3t_6 = Nt_3t_7t_{10} = Nt_7t_8t_5, Nt_8t_2t_1 = Nt_2t_9t_7 = Nt_9t_8t_3 = Nt_8t_9t_1 = Nt_9t_2t_3 = Nt_2t_8t_7, Nt_9t_2t_5 = Nt_2t_8t_{10} = Nt_8t_9t_4 = Nt_9t_8t_5 = Nt_8t_2t_4 = Nt_2t_9t_{10}, Nt_6t_3t_2 = Nt_3t_5t_4 = Nt_5t_6t_1 = Nt_6t_5t_2 = Nt_5t_3t_1 = Nt_3t_6t_4, Nt_5t_3t_7 = Nt_3t_6t_{10} = Nt_6t_5t_8 = Nt_5t_6t_7 = Nt_6t_3t_8 = Nt_3t_5t_{10}, Nt_9t_4t_6 = Nt_4t_{10}t_7 = Nt_{10}t_9t_1 = Nt_9t_{10}t_6 = Nt_{10}t_4t_1 = Nt_4t_9t_7, Nt_{10}t_4t_2 = Nt_4t_9t_8 = Nt_9t_{10}t_5 = Nt_{10}t_9t_2 = Nt_9t_4t_5 = Nt_4t_{10}t_8, Nt_5t_8t_9 = Nt_8t_{10}t_4 = Nt_{10}t_5t_2 = Nt_5t_{10}t_9 = Nt_{10}t_8t_2 = Nt_8t_5t_4, Nt_{10}t_8t_3 = Nt_8t_5t_6 = Nt_5t_{10}t_7 = Nt_{10}t_5t_3 = Nt_5t_8t_7 = Nt_8t_{10}t_6, Nt_{10}t_7t_1 = Nt_7t_6t_4 = Nt_6t_{10}t_9 = Nt_{10}t_6t_1 = Nt_6t_7t_9 = Nt_7t_{10}t_4, Nt_6t_7t_8 = Nt_7t_{10}t_5 = Nt_{10}t_6t_3 = Nt_6t_{10}t_8 = Nt_{10}t_7t_3 = Nt_7t_6t_5\}$.
So far we have $G = N \cup Nt_1N \cup Nt_1t_5N \cup Nt_1t_2N \cup Nt_1t_5t_1N \cup Nt_1t_5t_7N \cup Nt_1t_5t_8N \cup Nt_1t_2t_1N \cup Nt_1t_2t_{10}N \cup Nt_1t_2t_5N$.
Which gives us $1 + 10 + 30 + 30 + 5 + 60 + 30 + 10 + 30 + 20 = 226$ distinct cosets.

The orbits of $N^{(151)}$ on $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ are $\{7, 9, 8, 10\}$ and $\{1, 2, 4, 3, 5, 6\}$.

We take t_7, and t_1 from the orbits of $N^{(151)}$.
We want to determine to which double coset $Nt_1t_5t_1t_7$, and $Nt_1t_5t_1t_1$ belong.

```
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[7] eq
for|if> m*(ts[1]*ts[1])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[7] eq
for|if> m*(ts[1])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[7] eq
for|if> m*(ts[1]*ts[5])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[7] eq
for|if> m*(ts[1]*ts[2])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[7] eq
for|if> m*(ts[1]*ts[5]*ts[1])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[7] eq
for|if> m*(ts[1]*ts[5]*ts[7])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[7] eq
for|if> m*(ts[1]*ts[5]*ts[8])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[7] eq
for|if> m*(ts[1]*ts[2]*ts[1])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[7] eq
for|if> m*(ts[1]*ts[2]*ts[10])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[7] eq
for|if> m*(ts[1]*ts[2]*ts[5])^nthen "true";
for|if> break; end if; end for;
>
```

$Nt_1t_5t_1t_7N$ is a new double coset which we will denote [1517].

Four symmetric generators will go to the new double coset [1517].

```
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[1] eq
for|if> m*(ts[1]*ts[1])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[1] eq
for|if> m*(ts[1])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[1] eq
```

```
for|if> m*(ts[1]*ts[5])^n then "true";
for|if> break; end if; end for;
true
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[1] eq
for|if> m*(ts[1]*ts[2])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[1] eq
for|if> m*(ts[1]*ts[5]*ts[1])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[1] eq
for|if> m*(ts[1]*ts[5]*ts[7])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[1] eq
for|if> m*(ts[1]*ts[5]*ts[8])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[1] eq
for|if> m*(ts[1]*ts[2]*ts[1])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[1] eq
for|if> m*(ts[1]*ts[2]*ts[10])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[1] eq
for|if> m*(ts[1]*ts[2]*ts[5])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1]*ts[1] eq
for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n then "true";
for|if> break; end if; end for;
>
```

$Nt_1t_5t_1t_1 \in [15]$.

Six symmetric generators go back to [15].

The orbits of $N^{(157)}$ on {1,2,3,4,5,6,7,8,9,10} are {1},{5},{7},{9},{2,4},{3,6}, and {8,10}. We take $t_1, t_5, t_7, t_9, t_2, t_3$ and t_8 from the orbits of $N^{(157)}$. We want to determine to which double coset $Nt_1t_5t_7t_1, Nt_1t_5t_7t_5, Nt_1t_5t_7t_7, Nt_1t_5t_7t_9, Nt_1t_5t_7t_2, Nt_1t_5t_7t_3$, and $Nt_1t_5t_7t_8$ belong.

$Nt_1t_5t_7t_1N$ is a new double coset which we will denote [1571].

One symmetric generator will go to the new double coset [1571].

$Nt_1t_5t_7t_5N$ is a new double coset which we will denote [1575].
One symmetric generator will go to the new double coset [1575].

$Nt_1t_5t_7t_7 \in [15]$.
One symmetric generator goes back to [15].

$Nt_1t_5t_7t_9 \in [1517]$.
One symmetric generator goes to [1517].

$Nt_1t_5t_7t_2 \in [12]$.
Two symmetric generators go to [12].

$Nt_1t_5t_7t_3N$ is a new double coset which we will denote [1573].
Two symmetric generators will go to the new double coset [1573].

$Nt_1t_5t_7t_8 \in [15]$.
Two symmetric generators go to [15].

The orbits of $N^{(158)}$ on {1,2,3,4,5,6,7,8,9,10} are {1},{5},{7,9},{8,10}, and {2,3,6,4}. We take t_1, t_5, t_7, t_8, and t_2 from the orbits of $N^{(158)}$. We want to determine to which double coset $Nt_1t_5t_8t_1, Nt_1t_5t_8t_5, Nt_1t_5t_8t_7, Nt_1t_5t_8t_8$, and $Nt_1t_5t_8t_2$ belong.

$Nt_1t_5t_8t_1N$ is a new double coset which we will denote [1581].
One symmetric generator will go to the new double coset [1581].

$Nt_1t_5t_8t_5N$ is a new double coset which we will denote [1585].
One symmetric generator will go to the new double coset [1585].

$Nt_1t_5t_8t_7 \in [1573]$.
Two symmetric generators go to [1573].

$Nt_1t_5t_8t_8 \in [15]$.

Two symmetric generators go back to [15].

$Nt_1t_5t_8t_2 \in [1573]$.
Four symmetric generators go to [1573].

The orbits of $N^{(121)}$ on {1,2,3,4,5,6,7,8,9,10} are {10},{1,3,2}, and {4,9,7,6,8,5}. We take t_{10}, t_1, and t_4 from the orbits of $N^{(121)}$. We want to determine to which double cosets $Nt_1t_2t_1t_{10}, Nt_1t_2t_1t_1$,and $Nt_1t_2t_1t_4$ belong.

$Nt_1t_2t_1t_{10}N$ is a new double coset which we will denote [12110].
One symmetric generator will go to the new double coset [12110].

$Nt_1t_2t_1t_1 \in [12]$.
Three symmetric generators go back to [12].

$Nt_1t_2t_1t_4 \in [1575]$.
Six symmetric generators go to [1575].

Words of Length 4

$N^{(1517)} = \{e, (24)(36)(810), (136)(254)(8109), (13)(45)(89), (163)(245)(8910), (16)(25)(910)\}$.
The number of the single cosets in the double coset $Nt_1t_5t_1t_7N$ is at most $\frac{|N|}{|N^{(1517)}|} = \frac{120}{6} = 20$.

$Nt_1t_5t_1t_7N = \{Nt_1t_5t_1t_7 = Nt_3t_4t_3t_7 = Nt_6t_2t_6t_7, Nt_8t_4t_8t_7 = Nt_9t_5t_9t_7 = Nt_{10}t_2t_{10}t_7,$ $Nt_3t_4t_3t_8 = Nt_2t_6t_2t_8 = Nt_5t_1t_5t_8, Nt_9t_6t_9t_8 = Nt_7t_4t_7t_8 = Nt_{10}t_1t_{10}t_8, Nt_3t_9t_3t_5 =$ $Nt_2t_7t_2t_5 = Nt_8t_1t_8t_5, Nt_4t_7t_4t_5 = Nt_6t_9t_6t_5 = Nt_{10}t_1t_{10}t_5, Nt_5t_1t_5t_{10} = Nt_4t_3t_4t_{10} =$ $Nt_6t_2t_6t_{10}, Nt_9t_3t_9t_{10} = Nt_7t_2t_7t_{10} = Nt_8t_1t_8t_{10}, Nt_2t_7t_2t_4 = Nt_1t_8t_1t_4 = Nt_9t_3t_9t_4,$ $Nt_6t_8t_6t_4 = Nt_{10}t_3t_{10}t_4 = Nt_5t_7t_5t_4, Nt_1t_8t_1t_6 = Nt_3t_9t_3t_6 = Nt_7t_2t_7t_6, Nt_5t_9t_5t_6 =$ $Nt_4t_8t_4t_6 = Nt_{10}t_2t_{10}t_6, Nt_2t_6t_2t_9 = Nt_1t_5t_1t_9 = Nt_4t_3t_4t_9, Nt_7t_5t_7t_9 = Nt_8t_6t_8t_9 =$ $Nt_{10}t_3t_{10}t_9, Nt_1t_{10}t_1t_3 = Nt_6t_9t_6t_3 = Nt_7t_4t_7t_3, Nt_2t_{10}t_2t_3 = Nt_5t_9t_5t_3 = Nt_8t_4t_8t_3,$ $Nt_2t_{10}t_2t_1 = Nt_9t_5t_9t_1 = Nt_4t_8t_4t_1, Nt_3t_{10}t_3t_1 = Nt_7t_5t_7t_1 = Nt_6t_8t_6t_1, Nt_1t_{10}t_1t_2 =$

$Nt_9t_6t_9t_2 = Nt_4t_7t_4t_2, Nt_3t_{10}t_3t_2 = Nt_8t_6t_8t_2 = Nt_5t_7t_5t_2\}$.

$N^{(1571)} = \{e, (24)(36)(810), (19)(24)(310)(68), (19)(38)(610)\}$. The number of the single cosets in the double coset $Nt_1t_5t_7t_1N$ is at most $\frac{|N|}{|N^{(1571)}|} = \frac{120}{4} = 30$.

$Nt_1t_5t_7t_1N = Nt_1t_5t_7t_1 = Nt_9t_5t_7t_9, Nt_9t_5t_1t_9 = Nt_7t_5t_1t_7, Nt_1t_5t_9t_1 = Nt_7t_5t_9t_7,$
$Nt_2t_6t_8t_2 = Nt_9t_6t_8t_9, Nt_9t_6t_2t_9 = Nt_8t_6t_2t_8, Nt_2t_6t_9t_2 = Nt_8t_6t_9t_8, Nt_3t_9t_5t_3 =$
$Nt_6t_9t_5t_6, Nt_6t_9t_3t_6 = Nt_5t_9t_3t_5, Nt_3t_9t_6t_3 = Nt_5t_9t_6t_5, Nt_4t_3t_{10}t_4 = Nt_9t_3t_{10}t_9,$
$Nt_9t_3t_4t_9 = Nt_{10}t_3t_4t_{10}, Nt_4t_3t_9t_4 = Nt_{10}t_3t_9t_{10}, Nt_2t_7t_4t_2 = Nt_5t_7t_4t_5, Nt_4t_7t_2t_4 =$
$Nt_5t_7t_2t_5, Nt_2t_7t_5t_2 = Nt_4t_7t_5t_4, Nt_1t_8t_4t_1 = Nt_6t_8t_4t_6, Nt_4t_8t_1t_4 = Nt_6t_8t_1t_6,$
$Nt_4t_8t_6t_4 = Nt_1t_8t_6t_1, Nt_7t_2t_{10}t_7 = Nt_6t_2t_{10}t_6, Nt_{10}t_2t_7t_{10} = Nt_6t_2t_7t_6, Nt_7t_2t_6t_7 =$
$Nt_{10}t_2t_6t_{10}, Nt_8t_1t_{10}t_8 = Nt_5t_1t_8t_5, Nt_{10}t_1t_8t_{10} = Nt_5t_1t_8t_5, Nt_{10}t_1t_5t_{10} = Nt_8t_1t_5t_8,$
$Nt_1t_{10}t_2t_1 = Nt_3t_{10}t_2t_3, Nt_2t_{10}t_1t_2 = Nt_3t_{10}t_1t_3, Nt_2t_{10}t_3t_2 = Nt_1t_{10}t_3t_1, Nt_3t_4t_8t_3 =$
$Nt_7t_4t_8t_7, Nt_8t_4t_3t_8 = Nt_7t_4t_3t_7, Nt_3t_4t_7t_3 = Nt_8t_4t_7t_8\}$.

$N^{(1575)} = \{e, (24)(36)(810), (294)(376)(5810), (49)(58)(67), (29)(37)(510), (249)(367)(5108)\}$. The number of the single cosets in the double coset $Nt_1t_5t_7t_5N$ is at most $\frac{|N|}{|N^{(1575)}|} = \frac{120}{6} = 20$.

The distinct single cosets in $Nt_1t_5t_7t_5N = \{Nt_1t_5t_7t_5 = Nt_1t_8t_6t_8 = Nt_1t_{10}t_3t_{10}, Nt_1t_5t_9t_5 =$
$Nt_1t_8t_4t_8 = Nt_1t_{10}t_2t_{10}, Nt_2t_6t_8t_6 = Nt_2t_7t_5t_7 = Nt_2t_{10}t_3t_{10}, Nt_2t_6t_9t_6 = Nt_2t_7t_4t_7 =$
$Nt_2t_{10}t_1t_{10}, Nt_3t_9t_5t_9 = Nt_3t_4t_8t_4 = Nt_3t_{10}t_2t_{10}, Nt_3t_9t_6t_9 = Nt_3t_4t_7t_4 = Nt_3t_{10}t_1t_{10},$
$Nt_4t_8t_6t_8 = Nt_4t_3t_{10}t_3 = Nt_4t_7t_5t_7, Nt_4t_3t_9t_3 = Nt_4t_7t_2t_7 = Nt_4t_8t_1t_8, Nt_5t_7t_4t_7 =$
$Nt_5t_1t_{10}t_1 = Nt_5t_9t_6t_9, Nt_5t_9t_3t_9 = Nt_5t_1t_8t_1 = Nt_5t_7t_2t_7, Nt_6t_8t_1t_8 = Nt_6t_2t_7t_2 =$
$Nt_6t_9t_3t_9, Nt_6t_9t_5t_9 = Nt_6t_2t_{10}t_2 = Nt_6t_8t_4t_8, Nt_7t_4t_8t_4 = Nt_7t_2t_{10}t_2 = Nt_7t_5t_9t_5,$
$Nt_7t_5t_1t_5 = Nt_7t_2t_6t_2 = Nt_7t_4t_3t_4, Nt_8t_6t_9t_6 = Nt_8t_1t_{10}t_1 = Nt_8t_4t_7t_4, Nt_8t_4t_3t_4 =$
$Nt_8t_1t_5t_1 = Nt_8t_6t_2t_6, Nt_9t_5t_7t_5 = Nt_9t_3t_{10}t_3 = Nt_9t_6t_8t_6, Nt_9t_6t_2t_6 = Nt_9t_3t_4t_3 =$
$Nt_9t_5t_1t_5, Nt_{10}t_3t_9t_3 = Nt_{10}t_2t_7t_2 = Nt_{10}t_1t_8t_1, Nt_{10}t_2t_6t_2 = Nt_{10}t_1t_5t_1 = Nt_{10}t_3t_4t_3\}$.

$N^{(1573)} = \{e, (13)(45)(89)\}$. The number of the single cosets in the double coset $Nt_1t_5t_7t_3N$ is at most $\frac{|N|}{|N^{(1573)}|} = \frac{120}{2} = 60$.

$Nt_1t_5t_7t_3N = \{Nt_1t_5t_7t_3 = Nt_3t_4t_7t_1, Nt_5t_1t_{10}t_4 = Nt_4t_3t_{10}t_5, Nt_1t_8t_6t_3 = Nt_3t_9t_6t_1, Nt_8t_1t_{10}t_9 = Nt_9t_3t_{10}t_8, Nt_5t_9t_6t_4 = Nt_4t_8t_6t_5, Nt_8t_4t_7t_9 = Nt_9t_5t_7t_8, Nt_2t_6t_8t_5 = Nt_5t_1t_8t_2, Nt_6t_2t_7t_1 = Nt_1t_5t_7t_6, Nt_5t_9t_3t_2 = Nt_2t_{10}t_3t_5, Nt_{10}t_2t_7t_9 = Nt_9t_5t_7t_{10}, Nt_6t_9t_3t_1 = Nt_1t_{10}t_3t_6, Nt_{10}t_1t_8t_9 = Nt_9t_6t_8t_{10}, Nt_3t_9t_5t_2 = Nt_2t_7t_5t_3, Nt_7t_2t_{10}t_9 = Nt_9t_3t_{10}t_7, Nt_3t_4t_8t_2 = Nt_2t_6t_8t_3, Nt_4t_3t_{10}t_6 = Nt_6t_2t_{10}t_4, Nt_9t_6t_8t_7 = Nt_7t_4t_8t_9, Nt_4t_7t_5t_6 = Nt_6t_9t_5t_4, Nt_2t_7t_4t_1 = Nt_1t_8t_4t_2, Nt_7t_2t_{10}t_8 = Nt_8t_1t_{10}t_7, Nt_2t_6t_9t_1 = Nt_1t_5t_9t_2, Nt_5t_1t_{10}t_6 = Nt_6t_2t_{10}t_5, Nt_7t_5t_9t_8 = Nt_8t_6t_9t_7, Nt_6t_8t_4t_5 = Nt_5t_7t_4t_6, Nt_1t_8t_4t_9 = Nt_9t_3t_4t_1, Nt_3t_9t_5t_8 = Nt_8t_1t_5t_3, Nt_9t_6t_2t_1 = Nt_1t_{10}t_2t_9, Nt_6t_9t_5t_{10} = Nt_{10}t_1t_5t_6, Nt_8t_6t_2t_3 = Nt_3t_{10}t_2t_8, Nt_6t_8t_4t_{10} = Nt_{10}t_3t_4t_6, Nt_3t_9t_6t_7 = Nt_7t_2t_6t_3, Nt_2t_7t_4t_9 = Nt_9t_3t_4t_2, Nt_3t_{10}t_1t_7 = Nt_7t_5t_1t_3, Nt_{10}t_3t_4t_5 = Nt_5t_7t_4t_{10}, Nt_2t_{10}t_1t_9 = Nt_9t_5t_1t_2, Nt_5t_9t_6t_{10} = Nt_{10}t_2t_6t_5, Nt_4t_8t_1t_2 = Nt_2t_{10}t_1t_4, Nt_8t_4t_7t_{10} = Nt_{10}t_2t_7t_8, Nt_2t_6t_9t_4 = Nt_4t_3t_9t_2, Nt_3t_4t_7t_6 = Nt_6t_2t_7t_3, Nt_8t_6t_9t_{10} = Nt_{10}t_3t_9t_8, Nt_6t_8t_1t_3 = Nt_3t_{10}t_1t_6, Nt_1t_{10}t_2t_4 = Nt_4t_7t_2t_1, Nt_{10}t_1t_8t_7 = Nt_7t_4t_8t_{10}, Nt_4t_3t_9t_1 = Nt_1t_5t_9t_4, Nt_3t_4t_8t_5 = Nt_5t_1t_8t_3, Nt_{10}t_3t_9t_7 = Nt_7t_5t_9t_{10}, Nt_3t_{10}t_2t_5 = Nt_5t_7t_2t_3, Nt_4t_7t_2t_9 = Nt_9t_6t_2t_4, Nt_6t_9t_3t_7 = Nt_7t_4t_3t_6, Nt_4t_8t_1t_9 = Nt_9t_5t_1t_4, Nt_5t_9t_3t_8 = Nt_8t_4t_3t_5, Nt_7t_5t_1t_6 = Nt_6t_8t_1t_7, Nt_5t_7t_2t_8 = Nt_8t_6t_2t_5, Nt_7t_4t_3t_1 = Nt_1t_{10}t_3t_7, Nt_4t_7t_5t_{10} = Nt_{10}t_1t_5t_4, Nt_1t_8t_6t_7 = Nt_7t_2t_6t_1, Nt_2t_7t_5t_8 = Nt_8t_1t_5t_2, Nt_{10}t_2t_6t_4 = Nt_4t_8t_6t_{10}, Nt_2t_{10}t_3t_8 = Nt_8t_4t_3t_2\}$.

$N^{(1581)} = \{e, (26)(34)(79)(810), (14)(35)(710), (163)(245)(8910), (1256)(34)(79810), (15)(2364)(71098), (164)(235)(7109), (136)(254)(8109), (15)(2463)(78910), (16)(25)(910), (1354)(26)(78109), (1453)(26)(79108), (146)(253)(7910), (24)(36)(810), (13)(45)(89), (15)(26)(78)(910), (12)(56)(78), (123)(456)(798), (23)(46)(79), (124)(356)(7810), (142)(365)(7108), (15)(34)(710)(89), (132)(465)(789), (1652)(34)(71089)\}$. The number of the single cosets in the double coset $Nt_1t_5t_8t_1N$ is at most $\frac{|N|}{|N^{(1581)}|} = \frac{120}{24} = 5$.

$Nt_1t_5t_8t_1N = \{Nt_1t_5t_8t_1 = Nt_2t_6t_{10}t_2 = Nt_4t_3t_7t_4 = Nt_2t_6t_7t_2 = Nt_3t_4t_{10}t_3 = Nt_4t_3t_8t_4 = Nt_5t_1t_9t_5 = Nt_5t_1t_7t_5 = Nt_1t_5t_{10}t_1 = Nt_3t_4t_9t_3 = Nt_6t_2t_9t_6 = Nt_6t_2t_8t_6, Nt_3t_9t_4t_3 = Nt_1t_8t_{10}t_1 = Nt_7t_2t_5t_7 = Nt_1t_8t_5t_1 = Nt_2t_7t_{10}t_2 = Nt_7t_2t_4t_7 = Nt_9t_3t_6t_9 = Nt_9t_3t_5t_9 = Nt_3t_9t_{10}t_3 = Nt_2t_7t_6t_2 = Nt_8t_1t_6t_8 = Nt_8t_1t_4t_8, Nt_5t_9t_1t_5 = Nt_2t_{10}t_7t_2 = Nt_8t_4t_6t_8 = Nt_2t_{10}t_6t_2 = Nt_4t_8t_7t_4 = Nt_8t_4t_1t_8 = Nt_9t_5t_3t_9 = Nt_9t_5t_6t_9 = Nt_5t_9t_7t_5 = Nt_4t_8t_3t_4 = Nt_{10}t_2t_3t_{10} = Nt_{10}t_2t_1t_{10}, Nt_6t_9t_2t_6 = Nt_4t_7t_8t_4 = Nt_{10}t_1t_3t_{10} = Nt_4t_7t_3t_4 =$

$Nt_1t_{10}t_8t_1 = Nt_{10}t_1t_2t_{10} = Nt_9t_6t_5t_9 = Nt_9t_6t_3t_9 = Nt_6t_9t_8t_6 = Nt_1t_{10}t_5t_1 = Nt_7t_4t_5t_7 = Nt_7t_4t_2t_7, Nt_6t_8t_2t_6 = Nt_3t_{10}t_9t_3 = Nt_7t_5t_4t_7 = Nt_3t_{10}t_4t_3 = Nt_5t_7t_9t_5 = Nt_7t_5t_2t_7 = Nt_8t_6t_1t_8 = Nt_8t_6t_4t_8 = Nt_6t_8t_9t_6 = Nt_5t_7t_1t_5 = Nt_{10}t_3t_1t_{10} = Nt_{10}t_3t_2t_{10}\}$.

$N^{(1585)} = \{e, (269347)(5108), (24)(36)(810), (29)(37)(510), (274396)(5810), (294)(376)(5810), (26)(34)(79)(810), (49)(58)(67), (27)(39)(46)(510), (249)(367)(5108), (23)(46)(79), (23)(47)(58)(69)\}$. The number of the single cosets in the double coset $Nt_1t_5t_8t_5N$ is at most $\frac{|N|}{|N^{(1581)}|} = \frac{120}{12} = 10$.

$Nt_1t_5t_8t_5N = \{Nt_1t_5t_8t_5 = Nt_1t_8t_{10}t_8 = Nt_1t_8t_5t_8 = Nt_1t_{10}t_5t_{10} = Nt_1t_5t_{10}t_5 = Nt_1t_{10}t_8t_{10}, Nt_2t_6t_7t_6 = Nt_2t_7t_{10}t_7 = Nt_2t_7t_6t_7 = Nt_2t_{10}t_6t_{10} = Nt_2t_6t_{10}t_6 = Nt_2t_{10}t_7t_{10}, Nt_3t_9t_4t_9 = Nt_3t_4t_{10}t_4 = Nt_3t_4t_9t_4 = Nt_3t_{10}t_9t_{10} = Nt_3t_9t_{10}t_9 = Nt_3t_{10}t_4t_{10}, Nt_4t_8t_3t_8 = Nt_4t_3t_7t_3 = Nt_4t_3t_8t_3 = Nt_4t_7t_8t_7 = Nt_4t_8t_7t_8 = Nt_4t_7t_3t_7, Nt_5t_7t_1t_7 = Nt_5t_1t_9t_1 = Nt_5t_1t_7t_1 = Nt_5t_9t_7t_9 = Nt_5t_7t_9t_7 = Nt_5t_9t_1t_9, Nt_6t_9t_2t_9 = Nt_6t_2t_8t_2 = Nt_6t_2t_9t_3 = Nt_6t_8t_9t_8 = Nt_6t_9t_8t_9 = Nt_6t_8t_2t_8, Nt_7t_4t_2t_4 = Nt_7t_2t_5t_2 = Nt_7t_2t_4t_2 = Nt_7t_5t_4t_5 = Nt_7t_4t_5t_4 = Nt_7t_5t_2t_5, Nt_8t_6t_1t_6 = Nt_8t_1t_4t_1 = Nt_8t_1t_6t_1 = Nt_8t_4t_6t_4 = Nt_8t_6t_4t_6 = Nt_8t_4t_1t_4, Nt_9t_5t_3t_5 = Nt_9t_3t_6t_3 = Nt_9t_3t_5t_3 = Nt_9t_6t_5t_6 = Nt_9t_5t_6t_5 = Nt_9t_6t_3t_6, Nt_{10}t_3t_2t_3 = Nt_{10}t_2t_1t_2 = Nt_{10}t_2t_3t_2 = Nt_{10}t_1t_3t_1 = Nt_{10}t_3t_1t_3 = Nt_{10}t_2t_2t_1\}$.

$N^{(12110)} = \{e, (49)(58)(67), (123)(456)(798), (123)(486957), (23)(46)(79), (13)(45)(89), (12)(56)(78), (13)(48)(59)(67), (23)(47)(58)(69), (132)(465)(789), (132)(475968), (12)(49)(57)(68)\}$. The number of the single cosets in the double coset $Nt_1t_2t_1t_{10}N$ is at most $\frac{|N|}{|N^{(1581)}|} = \frac{120}{12} = 10$.

$Nt_1t_2t_1t_{10}N = \{Nt_1t_2t_1t_{10} = Nt_3t_1t_3t_{10} = Nt_2t_3t_2t_{10} = Nt_2t_1t_2t_{10} = Nt_1t_3t_1t_{10} = Nt_3t_2t_3t_{10}, Nt_2t_4t_2t_7 = Nt_5t_2t_5t_7 = Nt_4t_5t_4t_7 = Nt_4t_2t_4t_7 = Nt_2t_5t_2t_7 = Nt_5t_4t_5t_7, Nt_4t_1t_4t_8 = Nt_6t_4t_6t_8 = Nt_1t_6t_1t_8 = Nt_1t_4t_1t_8 = Nt_4t_6t_4t_8 = Nt_6t_1t_6t_8, Nt_1t_7t_1t_5 = Nt_9t_1t_9t_5 = Nt_7t_9t_7t_5 = Nt_7t_1t_7t_5 = Nt_1t_9t_1t_5 = Nt_9t_7t_9t_5, Nt_3t_8t_3t_4 = Nt_7t_3t_7t_4 = Nt_8t_7t_8t_4 = Nt_8t_3t_8t_4 = Nt_3t_7t_3t_4 = Nt_7t_8t_7t_4, Nt_8t_2t_8t_6 = Nt_9t_8t_9t_6 = Nt_2t_9t_2t_6 = Nt_2t_8t_2t_6 = Nt_8t_9t_8t_6 = Nt_9t_2t_9t_6, Nt_5t_3t_5t_9 = Nt_6t_5t_6t_9 = Nt_3t_6t_3t_9 = Nt_3t_5t_3t_9 = Nt_5t_6t_5t_9 = Nt_6t_3t_6t_9, Nt_4t_{10}t_4t_3 = Nt_9t_4t_9t_3 = Nt_{10}t_9t_{10}t_3 = Nt_{10}t_4t_{10}t_3 = Nt_4t_9t_4t_3 = Nt_9t_{10}t_9t_3, Nt_5t_{10}t_5t_1 = Nt_8t_5t_8t_1 = Nt_{10}t_8t_{10}t_1 = Nt_{10}t_5t_{10}t_1 =$

$Nt_5t_8t_5t_1 = Nt_8t_{10}t_8t_1, Nt_6t_7t_6t_2 = Nt_{10}t_6t_{10}t_2 = Nt_7t_{10}t_7t_2 = Nt_7t_6t_7t_2 = Nt_6t_{10}t_6t_2 = Nt_{10}t_7t_{10}t_2\}$.

So far we have $G = N \cup Nt_1N \cup Nt_1t_5N \cup Nt_1t_2N \cup Nt_1t_5t_1N \cup Nt_1t_5t_7N \cup Nt_1t_5t_8N \cup Nt_1t_2t_1N \cup Nt_1t_2t_{10}N \cup Nt_1t_2t_5N \cup Nt_1t_5t_1t_7N \cup Nt_1t_5t_7t_5N \cup Nt_1t_5t_7t_5N \cup Nt_1t_5t_7t_3N \cup Nt_1t_5t_8t_1N \cup Nt_1t_5t_8t_5N \cup Nt_1t_2t_1t_{10}N$.
Which gives us $1+10+30+30+5+60+30+10+30+20+20+30+20+60+5+10+10 = 381$ distinct cosets.

The orbits of $N^{(1517)}$ on {1,2,3,4,5,6,7,8,9,10} are {7}, {1,3,6}, {2,4,5}, and {8,10,9}. We take t_7, t_1t_2, and t_8 from the orbits of $N^{(1517)}$.

$Nt_1t_5t_1t_7t_7 \in [151]$.
One symmetric generator goes back to [151].

$Nt_1t_5t_1t_7t_1N$ is a new double coset which we will denote [15171].
Three symmetric generators will go to the new double coset [15171].

$Nt_1t_5t_1t_7t_2 \in [125]$.
Three symmetric generators go to [125].

$Nt_1t_5t_1t_7t_8 \in [157]$.
Three symmetric generators go to [157].

The orbits of $N^{(1571)}$ on {1,2,3,4,5,6,7,8,9,10} are {5}, {7}, {1,9}, {2,4} and {3,6,10,8}. We take t_5, t_7, t_1, t_2, and t_3 from the orbits of $N^{(1571)}$.

$Nt_1t_5t_7t_1t_5 \in [1210]$.
One symmetric generator goes back to [1210].

$Nt_1t_5t_7t_1t_7 \in [15171]$.
One symmetric generator goes to [15171].

$Nt_1t_5t_7t_1t_1 \in [157]$.

Two symmetric generators go back to [157].

$Nt_1t_5t_7t_1t_2N$ is a new double coset which we will denote [15712].

Two symmetric generators will go to the new double coset [15712].

$Nt_1t_5t_7t_1t_3 \in [1210]$.

Four symmetric generators go back to [1210].

The orbits of $N^{(1575)}$ on {1,2,3,4,5,6,7,8,9,10} are {1}, {2,4,9}, {3,6,7}, and {5,8,10}. We take t_1, t_2, t_3, and t_5 from the orbits of $N^{(1575)}$.

$Nt_1t_5t_7t_5t_1 \in [15712]$.

One symmetric generator goes to [15712].

$Nt_1t_5t_7t_5t_2 \in [121]$.

Three symmetric generators go to [121].

$Nt_1t_5t_7t_5t_3 \in [1210]$.

Three symmetric generators go to [1210].

$Nt_1t_5t_7t_5t_5 \in [157]$.

Three symmetric generators go to [157].

The orbits of $N^{(1573)}$ on {1,2,3,4,5,6,7,8,9,10} are {2}, {6}, {7}, {10}, {1,3}, {4,5}, and {8,9}. We take $t_2, t_6, t_7, t_{10}, t_1, t_4$ and t_8 from the orbits of $N^{(1573)}$.

$Nt_1t_5t_7t_3t_2 \in [15712]$.

One symmetric generator goes to [15712].

$Nt_1t_5t_7t_3t_6 \in [1210]$.

One symmetric generator goes to [1210].

$Nt_1t_5t_7t_3t_7 \in [158]$.
One symmetric generator goes to [158].

$Nt_1t_5t_7t_3t_{10} \in [125]$.
One symmetric generator goes to [125].

$Nt_1t_5t_7t_3t_1 \in [157]$.
Two symmetric generators go to [157].

$Nt_1t_5t_7t_3t_4 \in [15171]$.
Two symmetric generators go to [15171].

$Nt_1t_5t_7t_3t_8 \in [158]$.
Two symmetric generators go to [158].

The orbits of $N^{(1581)}$ on {1,2,3,4,5,6,7,8,9,10} are {7,9,8,10} and {1,2,4,3,5,6}. We take t_7 and t_1 from the orbits of $N^{(1581)}$.

$Nt_1t_5t_8t_1t_7 \in [125]$.
Four symmetric generators go to [125].

$Nt_1t_5t_8t_1t_1 \in [158]$.
Six symmetric generators go back to [158].

The orbits of $N^{(1585)}$ on {1,2,3,4,5,6,7,8,9,10} are {1}, {5,8,10}, and {2,3,7,9,4,6}. We take t_1,t_5 and t_2 from the orbits of $N^{(1585)}$.

$Nt_1t_5t_8t_5t_1N$ is a new double coset which we will denote [15851].
One symmetric generator will go to the new double coset [15851].

$Nt_1t_5t_8t_5t_5 \in [158]$.
Three symmetric generators go back to [158].

$Nt_1t_5t_8t_5t_2 \in [15171]$.
Six symmetric generators go to [15171].

The orbits of $N^{(12110)}$ on {1,2,3,4,5,6,7,8,9,10} are {10},{1,3,2},and {4,9,7,6,8,5}. We take t_{10},t_1, and t_4 from the orbits of $N^{(12110)}$.

$Nt_1t_2t_1t_{10}t_{10} \in [121]$.
One symmetric generator goes back to [121].

$Nt_1t_2t_1t_{10}t_1 \in [15171]$.
Three symmetric generators go to [15171].

$Nt_1t_2t_1t_{10}t_4 \in [15712]$.
Six symmetric generators go to [15712].

Words of Length 5

$N^{(15171)} = \{e, (24)(36)(810), (19)(24)(310)(68), (19)(38)(610)\}$. The number of the single cosets in the double coset $Nt_1t_5t_1t_7t_1N$ is at most $\frac{|N|}{|N^{(15171)}|} = \frac{120}{4} = 30$.

$Nt_1t_5t_1t_7t_1 = Nt_1t_5t_1t_7t_1 = Nt_9t_5t_9t_7t_9, Nt_9t_5t_9t_1t_9 = Nt_7t_5t_7t_1t_7, Nt_1t_5t_1t_9t_1 = Nt_7t_5t_7t_9t_7, Nt_2t_6t_2t_8t_2 = Nt_9t_6t_9t_8t_9, Nt_9t_6t_9t_2t_9 = Nt_8t_6t_8t_2t_8, Nt_2t_6t_2t_9t_2 = Nt_8t_6t_8t_9t_8, Nt_3t_9t_3t_5t_3 = Nt_6t_9t_6t_5t_6, Nt_6t_9t_6t_3t_6 = Nt_5t_9t_5t_3t_5, Nt_3t_9t_3t_6t_3 = Nt_5t_9t_5t_6t_5, Nt_4t_3t_4t_{10}t_4 = Nt_9t_3t_9t_{10}t_9, Nt_9t_3t_9t_4t_9 = Nt_{10}t_3t_{10}t_4t_{10}, Nt_4t_3t_4t_9t_4 = Nt_{10}t_3t_{10}t_9t_{10}, Nt_2t_7t_2t_4t_2 = Nt_5t_7t_5t_4t_5, Nt_4t_7t_4t_2t_4 = Nt_5t_7t_5t_2t_5, Nt_2t_7t_2t_5t_2 = Nt_4t_7t_4t_5t_4, Nt_1t_8t_1t_4t_1 = Nt_6t_8t_6t_4t_6, Nt_4t_8t_4t_1t_4 = Nt_6t_8t_6t_1t_6, Nt_4t_8t_4t_6t_4 = Nt_1t_8t_1t_6t_1, Nt_7t_2t_7t_{10}t_7 = Nt_6t_2t_6t_{10}t_6, Nt_{10}t_2t_{10}t_7t_{10} = Nt_6t_2t_6t_7t_6, Nt_7t_2t_7t_6t_7 = Nt_{10}t_2t_{10}t_6t_{10}, Nt_8t_1t_8t_{10}t_8 = Nt_5t_1t_5t_{10}t_5, Nt_{10}t_1t_{10}t_8t_{10} = Nt_5t_1t_5t_8t_5, Nt_{10}t_1t_{10}t_5t_{10} = Nt_8t_1t_8t_5t_8, Nt_1t_{10}t_1t_2t_1 = Nt_3t_{10}t_3t_2t_3, Nt_2t_{10}t_2t_1t_2 = Nt_3t_{10}t_3t_1t_3, Nt_2t_{10}t_2t_3t_2 = Nt_1t_{10}t_1t_3t_1, Nt_3t_4t_3t_8t_3 =$

$Nt_7t_4t_7t_8t_7, Nt_8t_4t_8t_3t_8 = Nt_7t_4t_7t_3t_7, Nt_3t_4t_3t_7t_3 = Nt_8t_4t_8t_7t_8\}$.

$N^{(15712)} = \{e, (129)(387)(5106), (29)(37)(510), (19)(38)(610), (192)(378)(5610), (12)(56)(78)\}$. The number of the single cosets in the double coset $Nt_1t_5t_7t_1t_2N$ is at most $\frac{|N|}{|N^{(15712)}|} = \frac{120}{6} = 20$.

$Nt_1t_5t_7t_1t_2 = \{Nt_1t_5t_7t_1t_2 = Nt_9t_6t_8t_9t_1 = Nt_9t_5t_7t_9t_2 = Nt_1t_{10}t_3t_1t_9 = Nt_2t_{10}t_3t_2t_9 = Nt_2t_6t_8t_2t_1, Nt_1t_5t_7t_1t_2 = Nt_9t_6t_8t_9t_1 = Nt_9t_5t_7t_9t_2 = Nt_1t_{10}t_3t_1t_9 = Nt_2t_{10}t_3t_2t_9 = Nt_2t_6t_8t_2t_1, Nt_{10}t_1t_8t_{10}t_6 = Nt_5t_9t_3t_5t_{10} = Nt_5t_1t_8t_5t_6 = Nt_{10}t_2t_7t_{10}t_5 = Nt_6t_2t_7t_6t_5 = Nt_6t_9t_3t_6t_{10}, Nt_2t_7t_5t_2t_9 = Nt_4t_3t_{10}t_4t_2 = Nt_4t_7t_5t_4t_9 = Nt_2t_6t_8t_2t_4 = Nt_9t_6t_8t_9t_4 = Nt_9t_3t_{10}t_9t_2, Nt_7t_2t_{10}t_7t_3 = Nt_6t_9t_5t_6t_7 = Nt_6t_2t_{10}t_6t_3 = Nt_7t_4t_8t_7t_6 = Nt_3t_4t_8t_3t_6 = Nt_3t_9t_5t_3t_7, Nt_3t_9t_5t_3t_1 = Nt_6t_8t_4t_6t_3 = Nt_6t_9t_5t_6t_1 = Nt_3t_{10}t_2t_3t_6 = Nt_1t_{10}t_2t_1t_6 = Nt_1t_8t_4t_1t_3, Nt_9t_3t_4t_9t_8 = Nt_{10}t_1t_5t_{10}t_9 = Nt_{10}t_3t_4t_{10}t_8 = Nt_9t_6t_2t_9t_{10} = Nt_8t_6t_2t_8t_{10} = Nt_8t_1t_5t_8t_9, Nt_9t_3t_{10}t_9t_1 = Nt_4t_8t_6t_4t_9 = Nt_4t_3t_{10}t_4t_1 = Nt_9t_5t_7t_9t_4 = Nt_1t_5t_7t_1t_4 = Nt_1t_8t_6t_1t_9, Nt_3t_9t_6t_3t_8 = Nt_5t_1t_{10}t_5t_3 = Nt_5t_9t_6t_5t_8 = Nt_3t_4t_7t_3t_5 = Nt_8t_4t_7t_8t_5 = Nt_8t_1t_{10}t_8t_3, Nt_2t_7t_4t_2t_3 = Nt_5t_9t_6t_5t_2 = Nt_5t_7t_4t_5t_3 = Nt_2t_{10}t_1t_2t_5 = Nt_3t_{10}t_1t_3t_5 = Nt_3t_9t_6t_3t_2Nt_7t_2t_6t_7t_9 = Nt_{10}t_3t_4t_{10}t_7 = Nt_{10}t_2t_6t_{10}t_9 = Nt_7t_5t_1t_7t_{10} = Nt_9t_5t_1t_9t_{10} = Nt_9t_3t_4t_9t_7, Nt_1t_5t_9t_1t_6 = Nt_7t_2t_{10}t_7t_1 = Nt_7t_5t_9t_7t_6 = Nt_1t_8t_4t_1t_7 = Nt_6t_8t_4t_6t_7 = Nt_6t_2t_{10}t_6t_1, Nt_5t_1t_{10}t_5t_2 = Nt_8t_6t_9t_8t_5 = Nt_8t_1t_{10}t_8t_2 = Nt_5t_7t_4t_5t_8 = Nt_2t_7t_4t_2t_8 = Nt_2t_6t_9t_2t_5, Nt_2t_6t_9t_2t_3 = Nt_8t_4t_7t_8t_2 = Nt_8t_6t_9t_8t_3 = Nt_2t_{10}t_1t_2t_8 = Nt_3t_{10}t_1t_3t_8 = Nt_3t_4t_7t_3t_2, Nt_6t_2t_7t_6t_4 = Nt_{10}t_3t_9t_{10}t_6 = Nt_{10}t_2t_7t_{10}t_4 = Nt_6t_8t_1t_6t_{10} = Nt_4t_8t_1t_4t_{10} = Nt_4t_3t_9t_4t_6, Nt_4t_8t_1t_4t_5 = Nt_6t_9t_3t_6t_4 = Nt_6t_8t_1t_6t_5 = Nt_4t_7t_2t_4t_6 = Nt_5t_7t_2t_5t_6 = Nt_5t_9t_3t_5t_4, Nt_7t_4t_3t_7t_9 = Nt_8t_6t_2t_8t_7 = Nt_8t_4t_3t_8t_9 = Nt_7t_5t_1t_7t_8 = Nt_9t_5t_1t_9t_8 = Nt_9t_6t_2t_9t_7, Nt_3t_4t_8t_3t_1 = Nt_7t_5t_9t_7t_3 = Nt_7t_4t_8t_7t_1 = Nt_3t_{10}t_2t_3t_7 = Nt_1t_{10}t_2t_1t_7 = Nt_1t_5t_9t_1t_3, Nt_4t_3t_9t_4t_5 = Nt_{10}t_1t_8t_{10}t_4 = Nt_{10}t_3t_9t_{10}t_5 = Nt_4t_7t_2t_4t_{10} = Nt_5t_7t_2t_5t_{10} = Nt_5t_1t_8t_5t_4, Nt_1t_8t_6t_1t_2 = Nt_4t_7t_5t_4t_1 = Nt_4t_8t_6t_4t_2 = Nt_1t_{10}t_3t_1t_4 = Nt_2t_{10}t_3t_2t_4 = Nt_2t_7t_5t_2t_1, Nt_8t_1t_5t_8t_7 = Nt_{10}t_2t_6t_{10}t_8 = Nt_{10}t_1t_5t_{10}t_7 = Nt_8t_4t_3t_8t_{10} = Nt_7t_4t_3t_7t_{10} = Nt_7t_2t_6t_7t_8\}$.

$N^{(15851)} = \{e\}$. The number of the single cosets in the double coset $Nt_1t_5t_8t_5t_1N$ is at most $\frac{|N|}{|N^{(15851)}|} = \frac{120}{120} = 1$.

$Nt_1t_5t_8t_5t_1 = \{Nt_1t_5t_8t_5t_1 = Nt_3t_9t_4t_9t_3 = Nt_4t_3t_7t_3t_4 = Nt_1t_8t_{10}t_8t_1 = Nt_5t_9t_1t_9t_5 = Nt_2t_6t_7t_6t_2 = Nt_7t_2t_5t_2t_7 = Nt_2t_{10}t_7t_{10}t_2 = Nt_3t_4t_{10}t_4t_3 = Nt_6t_9t_2t_9t_6 = Nt_9t_6t_3t_6t_9 = Nt_4t_3t_8t_3t_4 = Nt_1t_8t_5t_8t_1 = Nt_8t_4t_6t_4t_8 = Nt_5t_1t_9t_1t_5 = Nt_4t_7t_8t_7t_4 = Nt_1t_{10}t_5t_{10}t_1 = Nt_5t_1t_7t_1t_5 = Nt_2t_7t_{10}t_7t_2 = Nt_8t_6t_1t_6t_8 = Nt_9t_3t_5t_3t_9 = Nt_6t_8t_2t_8t_6 = Nt_1t_5t_{10}t_5t_1 = Nt_7t_2t_4t_2t_7 = Nt_2t_{10}t_6t_{10}t_2 = Nt_3t_4t_9t_4t_3 = Nt_{10}t_1t_3t_1t_{10} = Nt_6t_2t_9t_2t_6 = Nt_9t_3t_6t_3t_9 = Nt_7t_5t_4t_5t_7 = Nt_2t_7t_6t_7t_2 = Nt_3t_{10}t_9t_{10}t_3 = Nt_6t_2t_8t_2t_6 = Nt_4t_8t_7t_8t_4 = Nt_{10}t_3t_2t_3t_{10} = Nt_4t_8t_3t_8t_4 = Nt_9t_5t_6t_5t_9 = Nt_3t_{10}t_4t_{10}t_3 = Nt_8t_4t_1t_4t_8 = Nt_3t_9t_{10}t_9t_3 = Nt_4t_7t_3t_7t_4 = Nt_1t_{10}t_8t_{10}t_1 = Nt_8t_1t_6t_1t_8 = Nt_9t_5t_3t_5t_9 = Nt_5t_9t_7t_9t_5 = Nt_5t_7t_9t_7t_5 = Nt_8t_1t_4t_1t_8 = Nt_7t_4t_5t_4t_7 = Nt_{10}t_2t_1t_2t_{10} = Nt_6t_9t_8t_9t_6 = Nt_5t_7t_1t_7t_5 = Nt_{10}t_1t_2t_1t_{10} = Nt_7t_5t_2t_5t_7 = Nt_{10}t_2t_3t_2t_{10} = Nt_9t_6t_5t_6t_9 = Nt_6t_8t_9t_8t_6 = Nt_8t_6t_4t_6t_8 = Nt_{10}t_3t_1t_3t_{10} = Nt_7t_4t_2t_4t_7 = Nt_2t_6t_{10}t_6t_2\}$.

So now we have the following double cosets, $G = N \cup Nt_1N \cup Nt_1t_5N \cup Nt_1t_2N \cup Nt_1t_5t_1N \cup Nt_1t_5t_7N \cup Nt_1t_5t_8N \cup Nt_1t_2t_1N \cup Nt_1t_2t_{10}N \cup Nt_1t_2t_5N \cup Nt_1t_5t_1t_7N \cup Nt_1t_5t_7t_5N \cup Nt_1t_5t_7t_5N \cup Nt_1t_5t_7t_3N \cup Nt_1t_5t_8t_1N \cup Nt_1t_5t_8t_5N \cup Nt_1t_2t_1t_{10}N \cup Nt_1t_5t_1t_7t_1N \cup Nt_1t_5t_7t_1t_2N \cup Nt_1t_5t_8t_5t_1N$.
Which gives us $1 + 10 + 30 + 30 + 5 + 60 + 30 + 10 + 30 + 20 + 20 + 30 + 20 + 60 + 5 + 10 + 10 + 30 + 20 + 1 = 432$ distinct cosets.

We have now found all of our distinct cosets. Thus, there will be no more new double cosets. To show this we can continue in the same manner as above.

The orbits of $N^{(15171)}$ on {1,2,3,4,5,6,7,8,9,10} are {5}, {7}, {1,9}, {2,4}, and {3,6,10,8}. We take t_5,t_7,t_1,t_2, and t_3 from the orbits of $N^{(15171)}$.

$Nt_1t_5t_1t_7t_1t_5 \in [12110]$.
One symmetric generator will go to [12110] .

$Nt_1t_5t_1t_7t_1t_7 \in [1571]$.
One symmetric generator will go to [1571].

$Nt_1t_5t_1t_7t_1t_1 \in [1517]$.

Two symmetric generators will go to [1517].

$Nt_1t_5t_1t_7t_1t_2 \in [1585]$.
Two symmetric generators will go to [1585].

$Nt_1t_5t_1t_7t_1t_3 \in [1573]$.
Four symmetric generators will go to [1573].

The orbits of $N^{(15712)}$ on {1,2,3,4,5,6,7,8,9,10} are {4}, {1,9,2}, {3,7,8}, and {5,6,10}. We take t_4,t_1,t_3, and t_5 from the orbits of $N^{(15712)}$.

$Nt_1t_5t_7t_1t_2t_4 \in [1575]$.
One symmetric generator will go to [1575].

$Nt_1t_5t_7t_1t_2t_1 \in [1571]$.
Three symmetric generators will go to [1571].

$Nt_1t_5t_7t_1t_2t_3 \in [1573]$.
Three symmetric generators will go to [1573].

$Nt_1t_5t_7t_1t_2t_5 \in [12110]$
Three symmetric generators will go to [12110].

The orbits of $N^{(15851)}$ on {1,2,3,4,5,6,7,8,9,10} are {1,3,4,5,2,7,6,9,8,10}. We take t_1 from the orbit of $N^{(15851)}$.

$Nt_1t_5t_8t_5t_1t_1 \in [1585]$.
Ten symmetric generators will go to [1585].

Below is our completed Cayley Diagram.

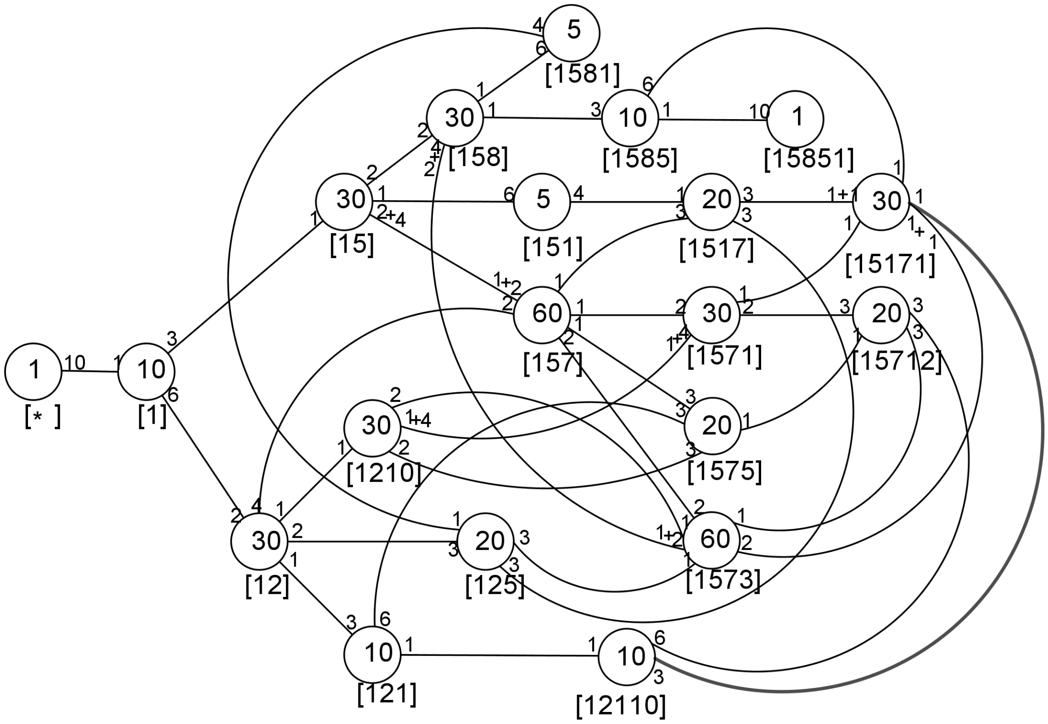


Figure 5.5: $(S(4,3):2)$ Over $2^{*10}:S_5$

Chapter 6

Transitive Groups

In this chapter we will examine several groups that are transitive on n letters by using NumberOfTransitiveGroups(n) command in MAGMA.

6.1 Transitive Groups on 20 Letters

Using the following code we find that there are 1117 transitive groups on 20 letters.

```
> NumberOfTransitiveGroups(20);
1117
```

We will examine some of these groups and write progenitors.

6.1.1 Transitive Group(20,222)

Let N be transitive group 222 on 20 letters. N is of order 1920 and is generated by $xx = (1,16,9,2,15,10)(3,18,11)(4,17,12)(5,6)(7,14,19)(8,13,20)$ and $yy = (1,14,15,2,13,16)(3,19,6,8,18,10,4,20,5,7,17,9)$.

```
> N:=TransitiveGroup(20,222);
> #N;
1920
> Generators(N);
{
(1, 16, 9, 2, 15, 10)(3, 18, 11)(4, 17, 12)(5, 6)
```

```
    (7, 14, 19)(8, 13, 20),
(1, 14, 15, 2, 13, 16)(3, 19, 6, 8, 18, 10, 4, 20,
    5, 7, 17, 9)
}
```

Next we find a presentation for N

```
> FPGroup(N);
Finitely presented group on 2 generators
Relations
$.1^6 = Id($)
($.1 * $.2^-1)^4 = Id($)
$.2 * $.1 * $.2^-1 * $.1^-2 * $.2^-1 * $.1 * $.2 * $.1^-1 = Id($)
$.2^-1 * $.1^-1 * $.2 * $.1^-1 * $.2^-1 * $.1 * $.2^3 * $.1 = Id($)
($.1^-1 * $.2^2 * $.1^-1 * $.2^-1)^2 = Id($)
```

Thus we have the following presentation of N for the progenitor $2^{*20} : (2^4 : S_5)$. (Proof of Isomorphism of N to follow.)

$$N = < x, y | x^6, (xy^5)^4, yxy^5x^4y^5xyx^5, y^5x^5yx^5y^5xy^3x, (x^5y^2x^5y^5)^2 >.$$

Next we will add t. Let $t \sim t_1$. Since our t's are of order 2, we add t^2 to the presentation. Now we need to look at the stabiliser of 1 that commute with t. We label N1 as the stabiliser of 1 in N and find the generators of N1.

```
> N1:=Stabiliser(N,1);
> Generators(N1);
{
    (3, 7, 14)(4, 8, 13)(5, 10, 16)(6, 9, 15)(11, 19, 18)(12, 20, 17),
    (3, 5, 4, 6)(7, 9, 8, 10)(13, 16, 14, 15)(19, 20),
    (13, 14)(15, 16)(17, 18)(19, 20),
    (3, 8, 4, 7)(5, 10, 6, 9)(15, 16)(17, 20, 18, 19)
}
```

We then use our Schreier System to translate these permutations into words.

```
> Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
> ArrayP:=[Id(N): i in [1..1920]];
> for i in [2..1920] do
for> P:=[Id(N): l in [1..#Sch[i]]];
for> for j in [1..#Sch[i]] do
for|for> if Eltseq(Sch[i])[j] eq 1 then P[j]:=xx; end if;
for|for> if Eltseq(Sch[i])[j] eq 2 then P[j]:=yy; end if;
for|for> if Eltseq(Sch[i])[j] eq -1 then P[j]:=xx^-1; end if;
```

```
for|for> if Eltseq(Sch[i])[j] eq -2 then P[j]:=yy^-1; end if;
for|for> end for;
for> PP:=Id(N);
for> for k in [1..#P] do
for|for> PP:=PP*P[k]; end for;
for> ArrayP[i]:=PP;
for> end for;
> for i in [1..1920] do if ArrayP[i] eq N!(3, 7, 14)(4, 8, 13)
for|if> (5, 10, 16)(6, 9, 15)(11, 19, 18)(12, 20, 17) then Sch[i];
for|if> end if; end for;
y * x^2 * y^-2 * x^-1 * y * x^-1
> for i in [1..1920] do if ArrayP[i] eq N!(3, 5, 4, 6)(7, 9, 8, 10)
for|if> (13, 16, 14, 15)(19, 20) then Sch[i]; end if; end for;
x^-1 * y^-1 * x^-1 * y^-3 * x * y^-1
> for i in [1..1920] do if ArrayP[i] eq N!(13, 14)(15, 16)(17, 18)
for|if> (19, 20) then Sch[i]; end if; end for;
(y * x * y^-1)^3
> for i in [1..1920] do if ArrayP[i] eq N!(3, 8, 4, 7)(5, 10, 6, 9)
for|if> (15, 16)(17, 20, 18, 19) then Sch[i]; end if; end for;
y^-1 * x^3 * y^-2
```

Therefore we have the following presentation,

$2^{*20} : (2^4 : S_5) =< x, y, t | x^6, (xy^5)^4, yxy^5x^4y^5xyx^5, y^5x^5yx^5y^5xy^3x, (x^5y^2x^5y^5)^2,$
$t^2, (t, yx^2y^4x^5yx^5), (t, x^5y^5x^5y^3xy^5), (t, (yxy^5)^3), (t, y^5x^3y^4) >.$

Now we add first and second relations to our progenitor in order to find homomorphic images of $2^{*20} : (2^4 : S_5)$.

$G =< x, y, t | x^6, (xy^5)^4, yxy^5x^4y^5xyx^5, y^5x^5yx^5y^5xy^3x, (x^5y^2x^5y^5)^2,$
$t^2, (t, yx^2y^4x^5yx^5), (t, x^5y^5x^5y^3xy^5), (t, (yxy^5)^3), (t, y^5x^3y^4),$
$(xyt)^{r1}, (xyt^{(x^3)})^{r2}, (xyt^{(x^2yx^2)})^{r3}, (xyt^{(x^2y)})^{r4}, ((xy)^3t^{(x^2*y)}t^{(x^5y)})^{r5},$
$((xy)^3t^{(x^2y)}t^{(x^2)})^{r6}, ((xy)^3t^{(x^2y)})^{r7}, ((xy)^3t^{(x^2y)}t)^{r8}, (xy^5xtt^{(x^2y)})^{r9},$
$(xyxtt^{(x^3)})^{r10}, (x^3tt^{(yxy^3x)})^{r11}, (xyxy^5t^{(x^5y)})^{r12} >.$

Table 6.1: $2^{*20} : (2^4 : S_5)$

r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	Order of G	Shape of G
6	6	4	8	0	0	0	0	0	0	0	0	103680	$2^{\bullet}(S(4,3):2)$
0	0	0	0	2	3	0	0	0	0	0	0	744000	$L_3(5):2$
0	0	0	0	2	4	5	9	0	0	0	0	979200	$S(4,4)$
0	0	0	0	0	0	0	0	3	4	0	0	190080	$M_{12}:2$
0	0	0	0	0	0	0	0	0	0	2	8	380160	$(M_{12}\times 2):2$

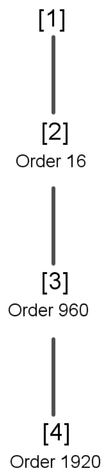
Proof of the Isomorphism for the Shape of N

The composition series of N is given below.

```
G
|  Cyclic(2)
*
|  Alternating(5)
*
|  Cyclic(2)
*
|  Cyclic(2)
*
|  Cyclic(2)
*
|  Cyclic(2)
1
```

$G = G_1 \supseteq G_2 \supseteq G_3 \supseteq\supseteq G_4 \supseteq G_5 \supseteq 1$, where $G = (G/G_1)(G_1/G_2)(G_2/G_3)(G_3/G_4)$ $(G_4/G_5)(G_5/1) = C_2A_5C_2C_2C_2C_2$.

The Normal Lattice of N is



We use the following loop to give us the largest abelian subgroup of N.

```
> NL:=NormalLattice(N);
> for i in [1..4] do if IsAbelian(NL[i]) then i;end if;end for;
1
2
```

$NL[2]$, our largest abelian subgroup of N is of order 16. We will examine possibilities of groups of order 16 to find the isomorphism type of $NL[2]$.

```
> X:=AbelianGroup(GrpPerm,[2,2,2,2]);
> s:=IsIsomorphic(X,NL[2]);s;
true
```

We have verified that $NL[2] = 2^4$. Since 1920/16=120, and we do not have a normal subgroup of order 120, we will have a semi-direct product. We will factor by $NL[2]$ and check the isomorphism type of q, our factor group.

```
> q,ff:=quo<N|NL[2]>;
> s:=IsIsomorphic(q,Sym(5));s;
true
```

So we will have a semi-direct product $2^4 : S_5$. We need to write a presentation of 2^4.

$$H =< w, x, y, z|w^2, x^2, y^2, z^2, (w, x), (w, y), (w, z), (x, y), (x, z), (y, z) >.$$

A presentation for S_5 is $Q =< a, b|a^5, b^2, (a^{-1}b)^4, (aba^{-2}ba)^2 >$. We find an element of order 5 and an element of order 2, F and G, respectively.

```
for i in NL[4] do if i notin NL[2] and Order(i) eq 5 and sub<N|i,NL[4]>
eq N then F:=i; break; end if; end for;
for i in NL[4] do if i notin NL[2] and Order(i) eq 2 and sub<N|i,NL[4]>
eq N then G:=i; break; end if; end for;
```

Now we need to find the action of F and G on the generators of $NL[2]$.

```
> for i in [1..#N1] do if ArrayP[i] eq A^F then print Sch[i];
for|if> end if; end for;
z
> for i in [1..#N1] do if ArrayP[i] eq B^F then print Sch[i];
for|if> end if; end for;
w * x * y * z
> for i in [1..#N1] do if ArrayP[i] eq C^F then print Sch[i];
for|if> end if; end for;
w
> for i in [1..#N1] do if ArrayP[i] eq D^F then print Sch[i];
for|if> end if; end for;
x
> for i in [1..#N1] do if ArrayP[i] eq A^G then print Sch[i];
for|if> end if; end for;
w
> for i in [1..#N1] do if ArrayP[i] eq B^G then print Sch[i];
for|if> end if; end for;
z
> for i in [1..#N1] do if ArrayP[i] eq C^G then print Sch[i];
for|if> end if; end for;
y
> for i in [1..#N1] do if ArrayP[i] eq D^G then print Sch[i];
for|if> end if; end for;
x
```

Finally we will add the presentation of Q, along with the action of a and b on the generators of $H = 2^4$, to our presentation of H.

$G = w, x, y, z, a, b|w^2, x^2, y^2, z^2, (w, x), (w, y), (w, z), (x, y), (x, z), (y, z), a^5, b^2, (a^{-1}b)^4,$

$(aba^{-2}ba)^2, w^a = z, x^a = wxyz, y^a = w, z^a = x, w^b = w, x^b = z, y^b = y, z^b = x >$.

We then verify the isomorphism.

```
> G<w,x,y,z,a,b>:=Group<w,x,y,z,a,b|w^2,x^2,y^2,z^2,(w,x),(w,y),
> (w,z),(x,y),(x,z),(y,z),
> a^5,b^2,(a^-1*b)^4,(a*b*a^-2*b*a)^2,
> w^a=z,x^a=w*x*y*z,y^a=w,z^a=x,
> w^b=w,x^b=z,y^b=y,z^b=x>;
> f1,G1,k1:=CosetAction(G,sub<G|Id(G)>);
> s,t:=IsIsomorphic(G1,N);
> s;
true
```

Thus $N \cong (2^4 : S_5)$.

6.1.2 Transitive Group(20,102)

Let N be transitive group 102 on 20 letters. $N = (5^2 :^{\bullet} 4^2)$ is of order 400 and is generated by $x = (1, 6, 15, 16, 2, 9, 12, 17, 3, 7, 14, 18, 4, 10, 11, 19, 5, 8, 13, 20)$, $y = (1, 8, 12, 17, 4, 10, 11, 18, 2, 7, 15, 19, 5, 9, 14, 20, 3, 6, 13, 16)$, and $z = (1, 7, 15, 16)$ $(2, 8, 13, 19)(3, 9, 11, 17)(4, 10, 14, 20)(5, 6, 12, 18)$. In the same manner as before, we find the following presentation for G.

$G < x, y, z, t | z^4, (y^{-1}z^{-1})^2, (yz^{-1})^2, zx^3z^{-1}x, xz^{-2}x^2z^2x, x^{-1}y^{-2}x^{-1}yz^{-1}$,
$t^2, (t, yxy^2), (t, (y^{-1}, x^{-1}))$,
$(y^2t)^{r1}, (x^3t)^{r2}, (yt)^{r3}, (y^3t)^{r4} >$.

Table 6.2: $2^{*20} : (5^2 :^{\bullet} 4^2)$

r1	r2	r3	r4	Order of G	Shape of G
3	9	9	9	2448	$L_2(17)$
3	0	0	7	672	$4 \times L_2(7)$

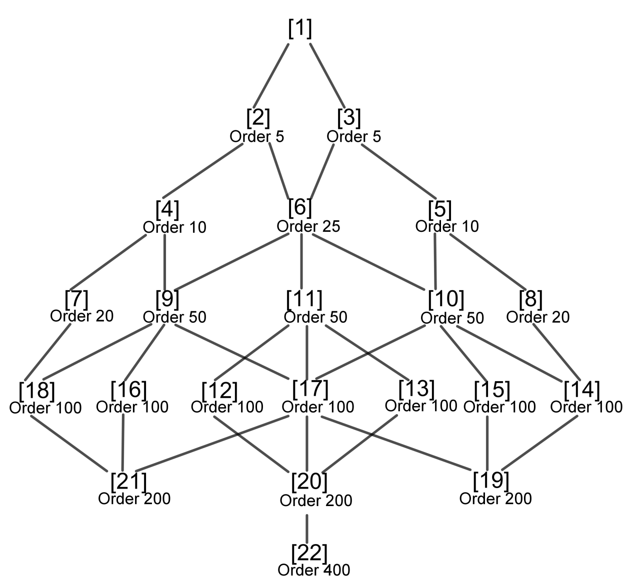
Proof of the Isomorphism for the Shape of N

The composition series of N is given below.

```
G
|   Cyclic(2)
*
|   Cyclic(2)
*
|   Cyclic(2)
*
|   Cyclic(2)
*
|   Cyclic(5)
*
|   Cyclic(5)
1
```

$G = G_1 \supseteq G_2 \supseteq G_3 \supseteq\supseteq G_4 \supseteq G_5 \supseteq 1$, where $G = (G/G_1)(G_1/G_2)(G_2/G_3)(G_3/G_4)$ $(G_4/G_5)(G_5/1) = C_2C_2C_2C_2C_5C_5$.

The Normal Lattice of N is



By looking at the normal lattice we see that we will not have a direct extension since we do not have 2 normal subroups of N whose product will give us $|N| = 400$. We then find the largest abelian subgroup of N.

.

```
> for i in [1..22] do if IsAbelian(NL[i]) then i;end if;end for;
1
2
3
6
> NL[6];
Permutation group acting on a set of cardinality 20
Order = 25 = 5^2
    (1, 2, 3, 4, 5)(6, 7, 8, 9, 10)(16, 20, 19, 18, 17)
    (6, 9, 7, 10, 8)(11, 14, 12, 15, 13)(16, 19, 17, 20, 18)
> X:=AbelianGroup(GrpPerm,[5,5]);
> s:=IsIsomorphic(X,NL[6]);s;
true
```

NL[6]$\cong 5^2$ is an abelian subgroup of N. Therefore we will have a mixed extension $N \cong 5^2 :^{\bullet} Q$. Now we will factor by NL[6] and form a factor group, q. We then check the normal lattice of q to find the isomorphism type of Q.

```
q,ff:=quo<N|NL[6]>;
```

Now, Q is of order 16. We check isomorphism of Q.

```
> s:=IsIsomorphic(q,CyclicGroup(16));s;
false
> s:=IsIsomorphic(q,DirectProduct(CyclicGroup(8),CyclicGroup(2)));s;
false
> s:=IsIsomorphic(q,DirectProduct(CyclicGroup(4),CyclicGroup(4)));s;
true
```

$Q \cong 4^2$. So we should have a mixed exension $5^2 :^{\bullet} (4^2)$. To prove this isomorphism we need a presentation for $Q \cong 4^2$.

```
> Q<a,b>:=Group<a,b|a^4,b^4,(a,b)>;
> f1,Q1,k1:=CosetAction(Q,sub<Q|Id(Q)>);
> s,t:=IsIsomorphic(Q1,q); s;
true
```

Now we need to write the generators of Q into elements of q. We will need to look at the transversals of NL[6].

```
> T:=Transversal(N,NL[6]);
> #T;
16
> A:=t(f1(a));
> B:=t(f1(b));
> for i in [1..#T] do if ff(T[i]) eq A then i; end if; end for;
2
> for i in [1..#T] do if ff(T[i]) eq B then i; end if; end for;
3
```

Now we store T[2] and T[3] as A and B, respectively.

```
> T[2];
(1, 6, 15, 16, 2, 9, 12, 17, 3, 7, 14, 18, 4, 10, 11, 19, 5, 8, 13,
20)
> A:=N!(1, 6, 15, 16, 2, 9, 12, 17, 3, 7, 14, 18, 4, 10, 11, 19, 5,
> 8, 13, 20);
> T[3];
(1, 8, 12, 17, 4, 10, 11, 18, 2, 7, 15, 19, 5, 9, 14, 20, 3, 6, 13,
16)
> B:=N!(1, 8, 12, 17, 4, 10, 11, 18, 2, 7, 15, 19, 5, 9, 14, 20, 3,
> 6, 13, 16);
```

Next we find generators of NL[6] and store them. Recall that NL[6] $\cong 5^2$, so we will need two generators of order 5.

```
> Order(NL[6].1);
5
> Order(NL[6].2);
5
> D:=NL[6].1;
> E:=NL[6].2;
```

Now we look at our presentation for Q to see if anything has changed once we apply the action of q.

```
> Order(A);
20
> Order(B);
20
```

Recall that our presentation for Q was $< a, b|a^4, b^4, (a, b) >$, thus the order of a and the order of b have been changed by the action of q. We will need to write these generators in terms of D and E.

```
> for i,j in [0..5] do if A^4 eq D^i*E^j
for|if> then i,j; break; end if; end for;
1 4
> for i,j in [0..5] do if B^4 eq D^i*E^j
for|if> then i,j; break; end if; end for;
3 3
> for i,j in [0..5] do if (A,B) eq D^i*E^j
for|if> then i,j; break; end if; end for;
1 0
> for n,o in [0..5] do
for> for p,q in [0..20] do
for|for> if D^A eq D^n*E^o*A^p*B^q
for|for|if> then n,o,p,q; break; end if; end for; end for;
0 0 12 8
> for n,o in [0..5] do
for> for p,q in [0..20] do
for|for> if D^B eq D^n*E^o*A^p*B^q
for|for|if> then n,o,p,q; break; end if; end for; end for;
0 0 16 4
> for n,o in [0..5] do
for> for p,q in [0..20] do
for|for> if E^A eq D^n*E^o*A^p*B^q
for|for|if> then n,o,p,q; break; end if; end for; end for;
0 0 8 8
> for n,o in [0..5] do
for> for p,q in [0..20] do
for|for> if E^B eq D^n*E^o*A^p*B^q
for|for|if> then n,o,p,q; break; end if; end for; end for;
0 0 4 4
```

The above loop tell us that $a^4 = de^4, b^4 = d^3e^3, (a, b) = d, d^a = a^{12}b^8, d^b = a^{16}b^4, e^a = a^8b^8$, and $e^b = a^4b^4$. We can now add these relations to our presentation of Q, along with the 2 generators of NL[6], say d and e, to check our isomorphism of N.

```
> NN<a,b,d,e>:=Group<a,b,d,e|a^4=d*e^4,b^4=d^3*e^3,(a,b)=d,d^5,e^5,
> d^a=a^12*b^8,d^b=a^16*b^4,e^a=a^8*b^8,e^b=a^4*b^4>;
```

```
> f2,NN1,k2:=CosetAction(NN,sub<NN|Id(NN)>);
> s:=IsIsomorphic(N,NN1);s;
true
```

Thus $N \cong 5^2 :^{\bullet} (4^2)$

6.1.3 Transitive Group(20,121)

Let N be transitive group 102 on 20 letters. $N = ((5:4) \times S_4)$ is of order 480 and is generated by $x = (1,4,2,3)(5,12,18,15)(6,11,17,16)(7,9,20,14)(8,10,19,13)$ and $y = (1,8,9,16,17,4,5,12,13,20)(2,6,10,14,18)(3,7,11,15,19)$. In the same manner as before, we find the following presentation for G.

$$G =< x,y,t|x^4, yx^{-2}y^2x^2y, yx^{-1}y^{-2}xy^3, x^{-1}y^{-1}x^{-2}y^{-1}x^2yx^2yx^{-1},$$
$$x^{-1}y^{-1}x^{-1}y^{-1}x^{-1}y^{-1}xy^{-1}xyxy,$$
$$t^2, (t, yx^{-1}y^2), (t, yxyx^{-1}y),$$
$$(yx^2y^{-1}x^{-1}y^{-1}x^2t)^{r1}, (yx^2y^{-1}x^{-}1y^{-1}x^2t^{x^2})^{r2}, (y^{-1}x^{-1}y^{-1}xt^{x^3})^{r3},$$
$$(y^{-1}x^{-1}y^{-1}xt)^{r4}, (x^{-1}y^{-1}x^{-1}yx^2y^{-1}t)^{r5}, (xyx^{-1}y^{-1}x^{-1}yt^{y^4})^{r6},$$
$$(xyx^{-1}y^{-1}x^{-1}yt)^{r7}, (x^2yt)^{r8}, (y^3x^{-1}yx^2t)^{r9}, (y^3x^{-1}yx^2t^{x^2})^{r10},$$
$$(x^2yxy^3t^{y^6})^{r11}, (x^2yxy^3t^{y^3x})^{r12}, (y^2t)^{r13}, ((xy)^2t^{y^4})^{r14}, ((xy)^2t^y)^{r15}, ((xy)^2t)^{r16} > .$$

Table 6.3: $2^{*20} : ((5:4) \times S_4)$

r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15	r16	Order	G
0	7	7	7	7	0	0	0	0	0	0	0	0	0	0	0	58240	$2^{\bullet}Sz(8)$
0	7	8	8	8	0	0	0	0	0	0	0	0	0	0	0	161280	$2^{\bullet}(2:L_3(4))$
6	0	6	0	7	0	0	0	0	0	0	0	0	0	0	0	48720	$PGL(2,29) \times 2$
6	0	6	8	6	0	0	0	0	0	0	0	0	0	0	0	1344	$PGL(2,7) \times (2 \times 2)$
6	0	6	0	6	0	0	0	0	0	0	0	0	0	0	0	4032	$PGL(2,7) \times D_{12}$
0	0	0	0	0	5	5	0	0	0	0	0	0	0	0	0	7920	$6^{\bullet} : PGL(2,11)$
0	0	0	0	0	0	0	0	0	0	0	0	2	0	7	0	8736	$PGL(2,13) \times 4$
0	0	0	0	0	0	0	0	0	0	0	0	6	2	8	0	1520640	$2 * M_{12} * 2 * 2 * 2$
0	0	0	0	0	0	0	0	0	0	0	2	0	0	7	6	4368	$2^{\bullet}PGL(2,13)$
0	0	0	0	0	0	0	0	0	0	0	7	0	2	7	7	116480	$(2^{\bullet}Sz(8)) \times 2$
0	0	0	0	0	0	0	0	0	0	7	6	10	0	8	6	235200	$2 : L_2(49)$

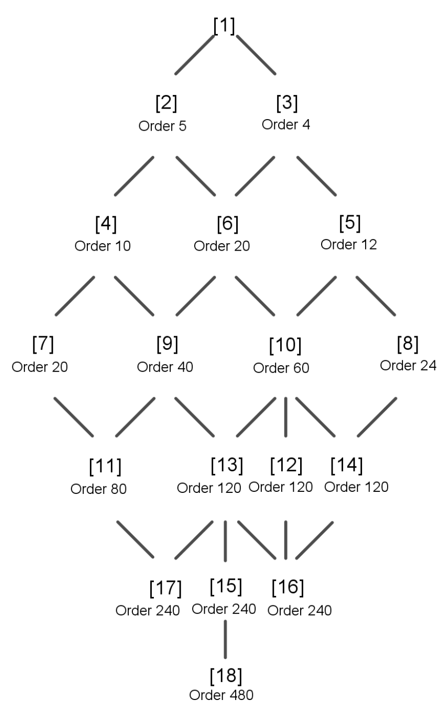
Proof of the Isomorphism for the Shape of N

The composition series of N is given below.

```
G
|  Cyclic(2)
*
|  Cyclic(2)
*
|  Cyclic(5)
*
|  Cyclic(2)
*
|  Cyclic(3)
*
|  Cyclic(2)
*
|  Cyclic(2)
1
```

$G = G_1 \supseteq G_2 \supseteq G_3 \supseteq\supseteq G_4 \supseteq G_5 \supseteq G_6 \supseteq 1$, where
$G = (G/G_1)(G_1/G_2)(G_2/G_3)(G_3/G_4)(G_4/G_5)(G_5/G_6)(G_6/1) = C_2C_2C_5C_2C_3C_2C_2$.

The Normal Lattice of N is



We see that NL[7] is of order 20 and NL[8] is of order 24. Since $20 \cdot 24 = 480$, and both subgroups are normal in N, then we have a direct product.

```
> s:=IsIsomorphic(N,DirectProduct(NL[7],NL[8]));s;
true
```

Now we will need to find the isomorphism of NL[7] and NL[8].

```
> NL[7];
Permutation group acting on a set of cardinality 20
Order = 20 = 2^2 * 5
    (5, 9, 17, 13)(6, 10, 18, 14)(7, 11, 19, 15)(8, 12, 20, 16)
    (5, 17)(6, 18)(7, 19)(8, 20)(9, 13)(10, 14)(11, 15)(12, 16)
    (1, 9, 17, 5, 13)(2, 10, 18, 6, 14)(3, 11, 19, 7, 15)(4, 12,
20, 8, 16)
> FPGroup(NL[7]);
Finitely presented group on 3 generators
Relations
    $.1^4 = Id($)
    $.2^2 = Id($)
    $.1^-2 * $.2 = Id($)
    ($.2 * $.3^-1)^2 = Id($)
    $.1 * $.3^-1 * $.1^-1 * $.3^-2 = Id($)
> G<x,y,z>:=Group<x,y,z|x^4,y^2,x^-2*y,(y*z^-1)^2,x*z^-1*x^-1*z^-2>;
> f1,G1,k1:=CosetAction(G,sub<G|Id(G)>);
> s,t:=IsIsomorphic(G1,NL[7]); s;
> nnl:=NormalLattice(G1);
> nnl;
Normal subgroup lattice
-----------------------

[4]  Order 20  Length 1  Maximal Subgroups: 3
---
[3]  Order 10  Length 1  Maximal Subgroups: 2
---
[2]  Order 5   Length 1  Maximal Subgroups: 1
---
[1]  Order 1   Length 1  Maximal Subgroups:

> Center(G1);
Permutation group acting on a set of cardinality 20
```

```
Order = 1

> for i in [1..4] do if IsAbelian(nnl[i]) then i;end if;end for;
1
2
> s:=IsIsomorphic(nnl[2],CyclicGroup(5));s;
> H<x>:=Group<x|x^5>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,nnl[2]); s;
true
> for i in nnl[4] do if i notin nnl[2] and Order(i) eq 4 and
for|if> sub<G1|i,nnl[4]> eq G1 then F:=i;
for|if> break; end if; end for;
> A:=t(f1(x));
> N1:=sub<nnl[4]|A>;
> NN<x>:=Group<x|x^5>;
> Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
> ArrayP:=[Id(N1): i in [1..#N1]];
> Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
> for i in[2..#N1] do
for> P:=[Id(N1): I in [1..#Sch[i]]];
for> for j in [1..#Sch[i]] do
for|for> if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
for|for> if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
for|for> end for;
for> PP:=Id(N1);
for> for k in [1..#P] do
for|for> PP:=PP*P[k]; end for;
for> ArrayP[i]:=PP;
for> end for;
> for i in [1..#N1] do if ArrayP[i] eq A^F then print Sch[i];
for|if> end if; end for;
x^2
> H<x,y>:=Group<x,y|x^5,y^4,x^y=x^2>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,NL[7]); s;
true
```

Thus NL[7] = (5:4).

```
> NL[8];
Permutation group acting on a set of cardinality 20
Order = 24 = 2^3 * 3
```

```
    (3, 4)(7, 8)(11, 12)(15, 16)(19, 20)
    (2, 4, 3)(6, 8, 7)(10, 12, 11)(14, 16, 15)(18, 20, 19)
    (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16)(17,
18)(19, 20)
    (1, 4)(2, 3)(5, 8)(6, 7)(9, 12)(10, 11)(13, 16)(14, 15)(17,
20)(18, 19)
> s:=IsIsomorphic(NL[8],Sym(4));s;
true
```

Hence NL[8] $= S_4$.

Now we add both presentations together and verify the isomorphism of N.

```
> FPGroup(NL[7]);
Finitely presented group on 3 generators
Relations
    $.1^4 = Id($)
    $.2^2 = Id($)
    $.1^-2 * $.2 = Id($)
    ($.2 * $.3^-1)^2 = Id($)
    $.1 * $.3^-1 * $.1^-1 * $.3^-2 = Id($)
> FPGroup(NL[8]);
Finitely presented group on 4 generators
Relations
    $.1^2 = Id($)
    $.2^3 = Id($)
    $.3^2 = Id($)
    $.4^2 = Id($)
    ($.2^-1 * $.1)^2 = Id($)
    $.2^-1 * $.3 * $.2 * $.4 = Id($)
    ($.1 * $.3)^2 = Id($)
    ($.3 * $.4)^2 = Id($)
    $.2 * $.3 * $.2^-1 * $.3 * $.4 = Id($)
> H<u,v,w,x,y,z,r>:=Group<u,v,w,x,y,z,r|u^4,v^2,u^-2*v,(v*w^-1)^2,
> u*w^-1*u^-1*w^-2,x^2,y^3,z^2,r^2,(y^-1*x)^2,
> y^-1*z*y*r,(x*z)^2,(z*r)^2,
> y*z*y^-1*z*r,(u,x),(u,y),(u,z),(u,r),(v,x),(v,y),(v,z),(v,r),
> (w,x),(w,y),(w,z),(w,r)>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,N); s;
true
```

Hence $N \cong (5:4) \times S_4$.

6.1.4 Transitive Group(20,10)

Let N be transitive group 10 on 20 letters. $N = D_{20}$ is of order 40 and is generated by $x = (1,19)(2,20)(3,18)(4,17)(5,16)(6,15)(7,13)(8,14)(9,12)(10,11)$ and $y = (1,3,5,8,10,11,14,16,18,19,2,4,6,7,9,12,13,15,17,20)$. In the same manner as before, we find the following presentation,

$$G =< x, y, t | x^2, (y^{-1}x)^2, y^{20}, t^2, (t, xy^{-9}), (y^6t)^{r1}, (yt)^{r2}, (y^3t)^{r3}, (y^7t)^{r4}, (y^9t)^{r5}, (xyt^{y^{13}})^{r6}, (xt^{y^{14}}t^{y^6})^{r7}, (y^5t)^{r8}, (y^{10}tt^{y^{12}})^{r9}, (xt)^{r10}, (xt^y)^{r11}, (y^{10}tt^y)^{r12}, (y^{10}tt^{y^{11}})^{r13}, (y^{10}tt^{y^2})^{r14}, (y^9tt^{y^2})^{r15}, (y^4tt^{y^{14}})^{r16}, (xyt^y)^{r17}, (xt^yt^{y^{19}})^{r18}, (xt^{y^{14}}t^{y^9})^{r19}, (xyt^{y^{14}}t^{y^5})^{r20}, (xyt^{y^{14}})^{r21}, (y^9tt^{y^{16}})^{r22}, (xt^{y^{14}})^{r23}, (y^5tt^{y^3})^{r24}, (y^9tt^{y^6})^{r25}, (xyt^{y^2}t^{y^{15}})^{r26}, (xyt^{y^2})^{r27}, (xyt^{y^5}t^{y^4})^{r28}, (y^8tt^{y^2})^{r29}, (xt^{y^{13}}t^{y^{17}})^{r30}, (xyt^{y^{13}}t^{y^9})^{r31}, (xyt)^{r32}, (y^4tt^{y^3})^{r33}, (y^7tt^{y^{15}})^{r34} >.$$

Table 6.4: $2^{*20} : D_{20}$

	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15	r16	r17	r18	r19	r20	r21
1.	3	5	0	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.	4	0	0	5	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3.	4	3	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4.	10	3	10	5	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5.	0	0	0	0	0	3	5	5	5	0	0	0	0	0	0	0	0	0	0	0	0
6.	3	0	0	0	0	0	2	0	0	0	0	0	0	0	7	0	0	0	0	0	0
7.	3	0	0	0	0	0	2	0	0	0	0	0	0	0	9	0	0	0	0	0	0
8.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	5	3	0	0	0
9.	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	9	9	5	0	0	0
10.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	3	10
14.	0	0	0	0	0	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15.	0	0	0	0	0	10	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
16.	0	0	0	0	0	10	0	0	0	0	0	0	0	0	0	0	6	0	0	0	0
17.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
18.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 6.5: $2^{*20} : D_{20}$ continued

	r22	r23	r24	r25	r26	r27	r28	r29	r30	r31	r32	r33	r34	Order	G
1.	0	0	0	0	0	0	0	0	0	0	0	0	0	6840	$2^{\bullet}L_2(19)$
2.	0	0	0	0	0	0	0	0	0	0	0	0	0	13680	$2 \times PGL(2,19)$
3.	0	0	0	0	0	0	0	0	0	0	0	0	0	31680	$2^3 :^{\bullet} (2 : (L_2(11) \times 3))$
4.	0	0	0	0	0	0	0	0	0	0	0	0	0	249600	$2^{\bullet}(2 : U(3,4))$
5.	0	0	0	0	0	0	0	0	0	0	0	0	0	6840	$L_2(19) \times 2$
6.	0	0	0	0	0	0	0	0	0	0	0	0	0	336	$PGL(2,7)$
7.	0	0	0	0	0	0	0	0	0	0	0	0	0	4896	$PGL(2,17)$
8.	0	0	0	0	0	0	0	0	0	0	0	0	0	410400	$PGL(2,19) \times A_5$
9.	0	0	0	0	0	0	0	0	0	0	0	0	0	6840	$PGL(2,19)$
10.	0	3	2	0	0	0	0	0	0	0	0	0	0	41040	$PGL(2,19) \times A_5$
11.	2	5	2	0	0	0	0	0	0	0	0	0	0	68400	$5 :^{\bullet} (PGL(2,19) \times 2)$
12.	2	10	2	0	0	0	0	0	0	0	0	0	0	136800	$10 :^{\bullet} (PGL(2,19) \times 2)$
13.	10	10	10	0	0	0	0	0	0	0	0	0	0	13680	$2^{\bullet}L_2(19)$
14.	0	0	0	0	3	3	10	3	0	0	0	0	0	744000	$L_3(5) : 2$
15.	0	0	0	0	5	9	10	3	0	0	0	0	0	1320	$PGL(2,11)$
16.	0	0	0	2	0	5	10	10	0	0	0	0	0	2640	$PGL(2,11) \times 2$
17.	0	0	0	0	0	0	0	0	3	0	0	0	0	6840	$PGL(2,19)$
18.	0	0	0	0	0	3	0	0	0	0	0	3	2	137760	$PGL(2,41) \times 2$
19.	0	0	0	0	0	3	0	0	0	3	2	7	7	34440	$L_2(41)$
20.	0	0	0	0	0	3	0	0	0	3	2	10	4	1320	$PGL(2,11)$

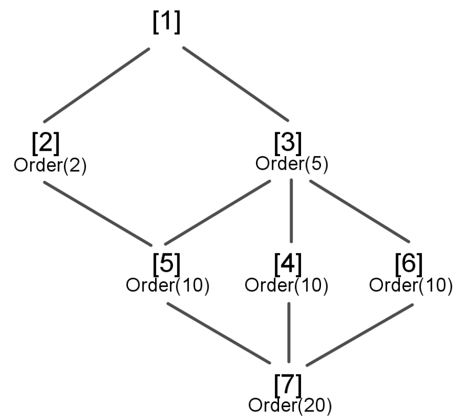
Proof of the Isomorphism for the Shape of N

The composition series of N is given below.

```
G
|  Cyclic(2)
*
|  Cyclic(5)
*
|  Cyclic(2)
*
|  Cyclic(2)
1
```

$G = G_1 \supseteq G_2 \supseteq G_3 \supseteq 1$, where
$G = (G/G_1)(G_1/G_2)(G_2/G_3)(G_3/1) = C_2C_5C_2C_2$.

The Normal Lattice of N is




```
s:=IsIsomorphic(N,DihedralGroup(20));s;
true
```

6.1.5 Transitive Group(20,11)

Let N be transitive group 10 on 20 letters. $N = 2^{\bullet}D_{10}$ is of order 40 and is generated by $x = (1,17,2,18)(3,16,4,15)(5,14,6,13)(7,12,8,11)(9,20,10,19)$ and $y = (1,4,6,7,10)(2,3,5,8,9)(11,13,16,17,20,12,14,15,18,19)$. In the same manner as before, we find the following presentation,

$G =< x, y, t | x^4, (x^{-1}y^{-1})^2, (xy^{-1})^2, y^{10}, t^2, (t, y^5), (y^2 t^{x^2y^3x} t^{y^3})^{r1}, (xt^{x^6y^2})^{r2}, (xtt^{x^2y^3x})^{r3},$
$(xyt^{x^2y^2} t^{x^2yxy^2})^{r4}, (yt^{x^6y^3x})^{r5}, (xt^{y^3} t^{y^4})^{r6}, (x^2y^2 t^{x^2y^3x} t^{x^2y^2})^{r7}, (xyt)^{r8}, (y^4 tt^y)^{r9} >.$

Table 6.6: $2^{*20} : 2^{\bullet}D_{10}$

r1	r2	r3	r4	r5	r6	r7	r8	r9	Order	G
2	3	4	0	0	0	0	0	0	31200	$L_2(25) \times 2^2$
3	3	3	0	0	0	0	0	0	158400	$2^{\bullet}(L_2(11) \times A_5)$
4	2	4	0	0	0	0	0	0	940800	$2^2 \times L_2(49)$
0	0	0	2	4	5	0	0	0	2640	$2^{\bullet}PGL(2,11)$
0	0	0	0	0	0	3	7	9	4368	$2^{\bullet}PGL(2,13)$

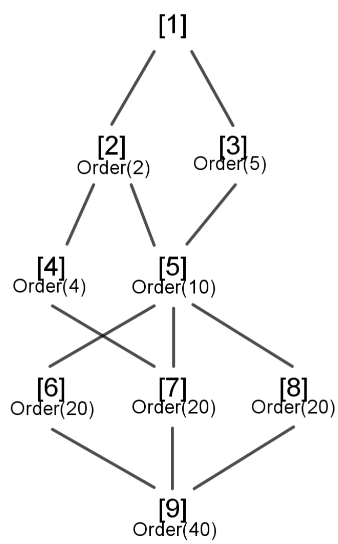
Proof of the Isomorphism for the Shape of N

The composition series of N is given below.

```
G
|  Cyclic(2)
*
|  Cyclic(5)
*
|  Cyclic(2)
*
|  Cyclic(2)
1
```

$G = G_1 \supseteq G_2 \supseteq G_3 \supseteq 1$, where
$G = (G/G_1)(G_1/G_2)(G_2/G_3)(G_3/1) = C_2C_5C_2C_2$.

The Normal Lattice of N is



We find the center of N.

```
> Center(N);
Permutation group acting on a set of cardinality 20
Order = 2
```

```
    (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)(11, 12)(13, 14)(15, 16)
        (17, 18)(19, 20)
> NL[2] eq Center(G1);
true
```

We will factor by the center of N and examine the factor group q.

```
> q,ff:=quo<G1|NL[2]>;
> nl:=NormalLattice(q);
> nl;
Normal subgroup lattice
-----------------------

[7]  Order 20  Length 1  Maximal Subgroups: 4 5 6
---
[6]  Order 10  Length 1  Maximal Subgroups: 3
[5]  Order 10  Length 1  Maximal Subgroups: 2 3
[4]  Order 10  Length 1  Maximal Subgroups: 3
---
[3]  Order 5   Length 1  Maximal Subgroups: 1
[2]  Order 2   Length 1  Maximal Subgroups: 1
---
[1]  Order 1   Length 1  Maximal Subgroups:

> s:=IsIsomorphic(q,DihedralGroup(10));s;
true
```

We see that q is isomorphic to D_{10}. Thus we will have a central extension of 2 by D_{10}. Now we need to write a presentation for $q \cong D_{10}$ and proceed to verify our isomorphism.

```
> FPGroup(q);
Finitely presented group on 2 generators
Relations
    $.1^2 = Id($)
    ($.2^-1 * $.1)^2 = Id($)
    $.2^10 = Id($)
> F<x,y>:=Group<x,y|x^2,(y^-1*x)^2,y^10>;
> f1,F1,k1:=CosetAction(F,sub<F|Id(F)>);
> s,t:=IsIsomorphic(F1,q);
> s;
```

```
> T:=Transversal(G1,NL[2]);
> #T;
20
> T[2];
(1, 2, 6, 4)(3, 9, 13, 8)(5, 11, 14, 7)(10, 16, 21, 17)(12, 15,
    22, 19)(18, 25, 29, 24)(20, 27, 30, 23)(26, 32, 36, 33)(28,
    31, 37, 35)(34, 39, 40, 38)
> A:=G1!(1, 2, 6, 4)(3, 9, 13, 8)(5, 11, 14, 7)(10, 16, 21, 17)
> (12, 15, 22, 19)(18, 25, 29, 24)(20, 27, 30, 23)(26, 32, 36,
> 33)(28, 31, 37, 35)(34, 39, 40, 38);
> T[3];
(1, 3, 10, 18, 26, 34, 28, 20, 12, 5)(2, 7, 15, 23, 31, 38, 32,
    24, 16, 8)(4, 11, 19, 27, 35, 39, 33, 25, 17, 9)(6, 13, 21,
    29, 36, 40, 37, 30, 22, 14)
> B:=G1!(1, 3, 10, 18, 26, 34, 28, 20, 12, 5)(2, 7, 15, 23, 31,
> 38, 32, 24, 16, 8)(4, 11, 19, 27, 35, 39, 33, 25, 17, 9)(6, 13,
> 21, 29, 36, 40, 37, 30, 22, 14);
> q;
> ff(A) eq q.1;
true
> ff(B) eq q.2;
true
> NL[2].1;
> C:=G1!(1, 6)(2, 4)(3, 13)(5, 14)(7, 11)(8, 9)(10, 21)(12, 22)
> (15, 19)(16, 17)(18, 29)(20, 30)(23, 27)(24, 25)(26, 36)(28,
> 37)(31, 35)(32, 33)(34, 40)(38, 39);
> F;
Finitely presented group F on 2 generators
Relations
    x^2 = Id(F)
    (y^-1 * x)^2 = Id(F)
    y^10 = Id(F)
> for i in [1..2] do if A^2 eq C^i then i; end if; end for;
1
> for i in [1..2] do if (B^-1*A)^2 eq C^i then i; end if; end for;
2
> for i in [1..2] do if B^10 eq C^i then i; end if; end for;
2
> H<c,x,y>:=Group<c,x,y|c^2,(y,c),(x,c),x^2=c,(y^-1*x)^2,y^10>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,G1);
> s;
true
```

Thus $N \cong 2^{\bullet}D_{10}$.

6.1.6 Transitive Group(20,12)

Let N be transitive group 10 on 20 letters. $N = (5 \times D_4)$ is of order 40 and is generated by $x = (1, 13, 8, 20, 4, 15, 10, 12, 5, 18, 2, 14, 7, 19, 3, 16, 9, 11, 6, 17)$ and $y = (1, 19, 9, 18, 7, 15, 5, 13, 4, 11)(2, 20, 10, 17, 8, 16, 6, 14, 3, 12)$. In the same manner as before, we find the following presentation,

$$G =< x, y, t | x^4y^2, x^{-1}y^{-1}x^2yx^{-1}, y^{-1}xy^{-1}xy^{-2}, t^2, (t, x^2y^{-1}x), (y^{-1}x^{-1}t)^{r1},$$
$$(y^{-1}x^{-1}t^{y^9})^{r2}, (yxyt)^{r3}, (yx^{-1}y^{-1}t)^{r4}, (xyxtt^{x^{19}})^{r5}, (x^3t)^{r6}, (xyxtt^{x^{17}})^{r7} >.$$

Table 6.7: $2^{*20} : (5 \times D_4)$

r1	r2	r3	r4	r5	r6	r7	Order	G
0	4	0	3	0	0	0	31680	$2 :^{\bullet} (2 : (L_2(11) \times 3))$
2	5	5	0	0	0	0	1320	$PGL(2, 11)$
2	3	0	8	0	0	0	1344	$4 :^{\bullet} (PGL(2, 7))$
0	0	0	3	0	0	4	31680	$2^3 :^{\bullet} (2 : (L_2(11) \times 3))$
0	0	0	3	0	5	10	249600	$2 : U(3, 4)$
0	0	0	3	0	9	5	6840	$L_2(19)$
0	0	0	0	2	4	10	3993600	$C_2 * U(3, 4) * C_2 * C_2 * C_2 * C_2 * C_2$
0	0	0	6	3	0	2	672	$2 \times PGL(2, 7)$
0	0	0	9	3	0	2	4896	$2^{\bullet}L_2(17)$

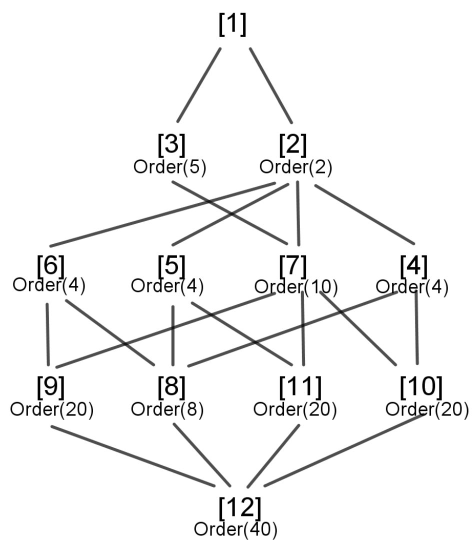
Proof of the Isomorphism for the Shape of N

The composition series of N is given below.

```
G
|  Cyclic(2)
*
|  Cyclic(5)
*
|  Cyclic(2)
*
|  Cyclic(2)
1
```

$G = G_1 \supseteq G_2 \supseteq G_3 \supseteq 1$, where
$G = (G/G_1)(G_1/G_2)(G_2/G_3)(G_3/1) = C_2C_5C_2C_2$.

The Normal Lattice of N is



It is possible that we have a direct extension.

```
> s:=IsIsomorphic(N,DirectProduct(CyclicGroup(5),DihedralGroup(4)));s;
true
```

Thus $N \cong (5 \times D_4)$.

6.1.7 Transitive Group(20,13)

Let N be transitive group 10 on 20 letters. N is of order 40 and is generated by $x = (1, 13, 18, 6)(2, 14, 17, 5)(3, 7, 16, 11)(4, 8, 15, 12)(9, 10)(19, 20)$ and $y = (1, 3, 5, 8, 10, 12, 14, 16, 18, 19)(2, 4, 6, 7, 9, 11, 13, 15, 17, 20)$. In the same manner as before, we find the following presentation,

$$G =< x, y, t | x^4, x^{-1}y^{-1}x^2y^{-1}x^{-1}, y^{-1}x^{-1}y^{-1}xy^{-2}, t^2, (t, x^2y^2), (x^{-1}t)^{r1},$$
$$(x^{-1}t^{xy^2x^2})^{r2}, (x^{-1}t^{yxy}t^{y})^{r3}, (xt^{yxy}t^{y^9x})^{r4}, (x^2y^{-1}t^{x^2y^3})^{r5},$$
$$(xyt^{y^2x}t^{y^3x})^{r6}, (xytt^{y^2x})^{r7}, (x^{-1}t^{yxy}t^{yx})^{r8}, (xyt)^{r9},$$
$$(xt^{y^2x}t^{y^3x})^{r10}, (x^2t^{xy^2x^2})^{r11}, (x^2tt^{yxy})^{r12}, (yt^{xy^2x^2})^{r13},$$
$$(x^2y^{-1}t)^{r14}, (y^{-1}x^{-1}tt^{y^2x})^{r15}, (x^2y^{-1}tt^{y^2x})^{r16}, (x^2t^{y^4x})^{r17},$$
$$(xytt^{y^6})^{r18}, (xt^{xy^2x^2})^{r19}, (y^{-1}x^{-1}t^{y^2x}t^{yx})^{r20}, (yt^{y^2x}t^{y})^{r21},$$
$$(xt^{y^4x})^{r22}, (xyt^{y^2x}t^{x^3})^{r23}, (x^2tt^{y^6})^{r24}, (xyt^{x^3y})^{r25}, (x^2y^{-1}t^{y^2x}t^{y^2})^{r26},$$
$$(xyt^{y^2x}t^{y^6})^{r27}, (x^{-1}t^{y^4x})^{r28}, (x^{-1}tt^{y^2xy})^{r29} >.$$

Table 6.8: $2^{*20} : (4^{\bullet} : 10)$

	r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12	r13	r14	r15	r16
1.	3	5	4	0	0	0	0	0	0	0	0	0	0	0	0	0
2.	0	0	0	3	7	8	0	0	0	0	0	0	0	0	0	0
3.	0	0	0	3	9	7	0	0	0	0	0	0	0	0	0	0
4.	0	0	0	3	9	9	0	0	0	0	0	0	0	0	0	0
5.	0	0	0	5	5	5	0	0	0	0	0	0	0	0	0	0
6.	0	0	0	5	9	4	0	0	0	0	0	0	0	0	0	0
7.	0	4	0	0	0	0	0	5	8	10	2	10	10	0	0	0
8.	0	0	0	0	0	0	0	0	0	0	0	0	4	0	3	5
9.	0	0	0	0	0	0	0	0	0	0	0	0	7	0	3	5
10.	0	0	0	0	0	0	0	0	0	0	0	0	9	0	3	3
11.	0	0	0	0	0	0	0	0	0	0	0	0	6	3	7	7
12.	0	0	0	0	0	0	0	0	0	0	0	0	6	3	7	10
13.	0	0	0	0	0	0	0	0	0	0	0	0	6	3	9	6
14.	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0
15.	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0
16.	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0
17.	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0
18.	0	0	0	0	0	0	0	0	0	0	0	0	7	0	0	0
19.	0	0	0	0	0	0	0	0	0	0	0	0	6	0	0	0
20.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 6.9: $2^{*20} : (4^{\bullet} : 10)$ continued

	r17	r18	r19	r20	r21	r22	r23	r24	r25	r26	r27	r28	r29	Order	G
1.	0	0	0	0	0	0	0	0	0	0	0	0	0	235200	$2 : PGL_2(49)$
2.	0	0	0	0	0	0	0	0	0	0	0	0	0	336	$PGL_2(7)$
3.	0	0	0	0	0	0	0	0	0	0	0	0	0	504	$L_2(8)$
4.	0	0	0	0	0	0	0	0	0	0	0	0	0	3420	$L_2(19)$
5.	0	0	0	0	0	0	0	0	0	0	0	0	0	660	$L_2(11)$
6.	0	0	0	0	0	0	0	0	0	0	0	0	0	6840	$PGL(2, 19)$
7.	0	0	0	0	0	0	0	0	0	0	0	0	0	322560	$2^{3^{\bullet}} : (2 : L_3(4))$
8.	8	8	0	0	0	0	0	0	0	0	0	0	0	1320	$PGL(2, 13)$
9.	2	4	0	0	0	0	0	0	0	0	0	0	0	34440	$L_2(41)$
10.	8	6	0	0	0	0	0	0	0	0	0	0	0	2448	$L_2(17)$
11.	10	10	0	0	0	0	0	0	0	0	0	0	0	1092	$L_2(13)$
12.	10	10	0	0	0	0	0	0	0	0	0	0	0	24360	$PGL(2, 29)$
13.	10	10	0	0	0	0	0	0	0	0	0	0	0	20520	$S_5 : L_2(19)$
14.	0	0	0	0	9	8	2	0	0	0	0	0	0	23336640	$C2 * A5 * L_2(73)$
15.	0	0	0	0	4	9	2	0	0	0	0	0	0	101232	$2 \times PGL_2(37)$
16.	0	0	0	0	5	6	2	0	0	0	0	0	0	13680	$2 \times PGL_2(19)$
17.	0	0	0	0	6	5	6	0	0	0	0	0	0	2640	$2 \times PGL_2(11)$
18.	0	0	5	5	8	10	2	0	0	0	0	0	0	161280	$4^{\bullet}(2 : L_3(4))$
19.	0	0	5	10	5	10	2	0	0	0	0	0	0	6600	$L_2(11) \times D_5$
20.	0	0	0	0	0	4	0	1	10	6	9	8	10	499200	$C2 * U(3, 4) * C2 * C2$

Proof of the Isomorphism for the Shape of N

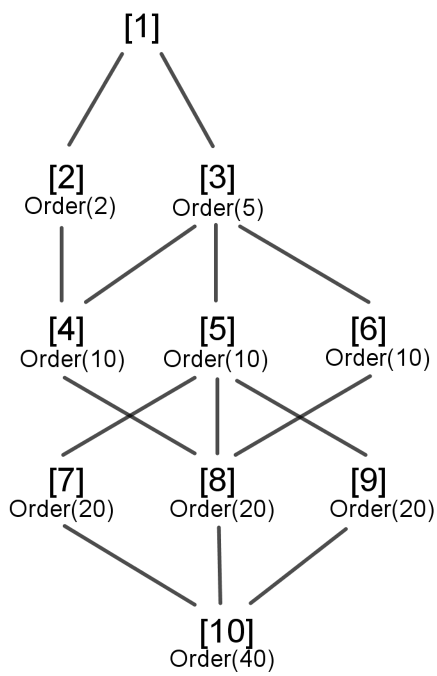
The composition series of N is given below.

```
G
|  Cyclic(2)
*
|  Cyclic(2)
*
|  Cyclic(5)
*
|  Cyclic(2)
1
```

$G = G_1 \supseteq G_2 \supseteq G_3 \supseteq 1$, where

$G = (G/G_1)(G_1/G_2)(G_2/G_3)(G_3/1) = C_2C_5C_2C_2$.

The Normal Lattice of N is



We find that the center of N is NL[2]. However we see that NL[2] is not the largest abelian subgroup. Since NL[4] $= C_{10}$ is the largest abelian subgroup then we will have a mixed extension.

```
> Center(N);
Permutation group acting on a set of cardinality 20
Order = 2
    (1, 12)(2, 11)(3, 14)(4, 13)(5, 16)(6, 15)(7, 17)(8, 18)(9,
     20)(10, 19)
> NL[2] eq Center(N);
true
> for i in [1..10] do if IsAbelian(NL[i]) then i;end if;end for;
1
2
3
4
> q,ff:=quo<N|NL[4]>;
> nl:=NormalLattice(q);
```

```
> nl;

Normal subgroup lattice
-----------------------

[3]  Order 4  Length 1  Maximal Subgroups: 2
---
[2]  Order 2  Length 1  Maximal Subgroups: 1
---
[1]  Order 1  Length 1  Maximal Subgroups:

> s:=IsIsomorphic(q,CyclicGroup(4));
> s;
true
> H<a>:=Group<a|a^4>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,q); s;
true
> T:=Transversal(N,NL[4]);
> #T;
4
> T[2];
(1, 13, 18, 6)(2, 14, 17, 5)(3, 7, 16, 11)(4, 8, 15, 12)(9,
     10)(19, 20)
> A:=N!(1, 13, 18, 6)(2, 14, 17, 5)(3, 7, 16, 11)(4, 8, 15,
> 12)(9, 10)(19, 20);
> q;
Permutation group q acting on a set of cardinality 4
Order = 4 = 2^2
    (1, 2, 3, 4)
    Id(q)
> ff(A) eq q.1;
true
> Order(A);
4
> IsCyclic(NL[4]);
true
> Order(NL[4].1);
10
> NL[4].1;
(1, 3, 5, 8, 10, 12, 14, 16, 18, 19)(2, 4, 6, 7, 9, 11,
     13, 15, 17, 20)
> B:=N!(1, 3, 5, 8, 10, 12, 14, 16, 18, 19)(2, 4, 6, 7, 9,
```

```
> 11, 13, 15, 17, 20);
> for i in [0..10] do if B^A eq B^i
for|if> for|if> then i; break; end if; end for;
7
> H<b,a>:=Group<b,a|b^10,a^4,b^a=b^7>;
> f,h,k:=CosetAction(H,sub<H|Id(H)>);
> #h;
> s:=IsIsomorphic(h,N);
> s;
true
```

Thus $N \cong 4^{\bullet} : 10$.

6.2 Transitive Groups on 19 Letters

Using the following code we find that there are 8 transitive groups on 19 letters.

```
> #TransitiveGroups(19);
8
```

We will examine some of these groups and write progenitors.

6.2.1 Transitive Group(19,2)

Let N be transitive group 10 on 19 letters. $N = (2 : 19)$ is of order 38 and is generated by $x = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19)$ and $y = (2, 19)(3, 18)(4, 17)(5, 16)(6, 15)(7, 14)(8, 13)(9, 12)(10, 11)$. In the same manner as before, we find the following presentation,

$$G =< x, y, t|y^2, (x^{-1}y)^2, x^{-19}, t^2, (t, yx^2), (x^5t)^{r1}, (x^6t)^{r2}, (x^8t)^{r3}, (x^9t)^{r4}, (yt^{x^6})^{r5}, (xt)^{r6} >.$$

Table 6.10: $2^{*20} : (2 : 19)$

r1	r2	r3	r4	r5	r6	Order	G
5	3	5	10	0	0	6840	$PGL_2(19)$
0	0	0	0	3	3	25308	$L_2(37)$

Proof of the Isomorphism for the Shape of N

```
> S:=Sym(19);
> xx:=S!(1,2,3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,
> 18, 19);
> yy:=S!(1, 18)(2, 17)(3, 16)(4, 15)(5, 14)(6, 13)(7, 12)(8,
> 11)(9, 10);
> N:=sub<N|xx,yy>;
> #N;
38
> NL:=NormalLattice(N);
> NL;
Normal subgroup lattice
-----------------------

[3]  Order 38  Length 1  Maximal Subgroups: 2
---
[2]  Order 19  Length 1  Maximal Subgroups: 1
---
[1]  Order 1   Length 1  Maximal Subgroups:

> IsIsomorphic(NL[2],CyclicGroup(19));
true
> H<x>:=Group<x|x^19>;
> f,H1,k:=CosetAction(H,sub<H|Id(H)>);
> s:=IsIsomorphic(NL[2],H1);s;
true
> for i in NL[3] do if i notin NL[2] and Order(i) eq 2 and
for|if> sub<N|i,NL[2]> eq N then C:=i;
for|if> break; end if; end for;
> FPGroup(N);
Finitely presented group on 2 generators
Relations
    $.1^4 = Id($)
    $.1^-1 * $.2^-1 * $.1^2 * $.2^-1 * $.1^-1 = Id($)
    $.2^-1 * $.1^-1 * $.2^-1 * $.1 * $.2^-2 = Id($)
> NN<x,y>:=Group<x,y|y^2,(x^-1*y)^2,x^-19>;
> Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
> ArrayP:=[Id(N): i in [1..38]];
> for i in [2..38] do
for> P:=[Id(N): l in [1..#Sch[i]]];
for> for j in [1..#Sch[i]] do
for|for> if Eltseq(Sch[i])[j] eq 1 then P[j]:=xx; end if;
for|for> if Eltseq(Sch[i])[j] eq 2 then P[j]:=yy; end if;
```

```
for|for> if Eltseq(Sch[i])[j] eq -1 then P[j]:=xx^-1; end if;
for|for> end for;
for> PP:=Id(N);
for> for k in [1..#P] do
for|for> PP:=PP*P[k]; end for;
for> ArrayP[i]:=PP;
for> end for;
> for i in [1..#NN1] do if ArrayP[i] eq A^C then print Sch[i];
for|if> end if; end for;
x^-1
> H<y,x>:=Group<y,x|x^19,y^2,x^y=x^-1>;
> f2,H2,k1:=CosetAction(H,sub<H|Id(H)>);
> IsIsomorphic(H2,N);
true
```

Thus $N \cong (2:19)$.

6.3 Transitive Groups on 11 Letters

Using the following code we find that there are 8 transitive groups on 11 letters.

```
> NumberOfTransitiveGroups(11);
8
```

We will examine some of these groups and write progenitors.

6.3.1 Transitive Group(11,2)

Let N be transitive group 2 on 11 letters. $N = (2:11)$ is of order 22 and is generated by $x = (1,2,3,4,5,6,7,8,9,10,11)$ and $y = (1,10)(2,9)(3,8)(4,7)(5,6)$. In the same manner as before, we find the following presentation,

$$G < x, y, t | y^2, (x^{-1}y)^2, x^{-11}, t^2, (t, yx^2), (xt)^{r1}, (x^2t)^{r2}, (x^3t)^{r3}, (x^4t)^{r4}, (x^5t)^{r5} >.$$

Table 6.11: $2^{*11} : (2 : 11)$

r1	r2	r3	r4	r5	Order	G
3	0	5	10	6	2703360	$2^{11} : PGL(2, 11)$
0	3	5	0	5	1320	$PGL(2, 11)$
0	3	6	0	6	190080	$2 : M_{12}$
0	0	0	4	3	12144	$PGL(2, 23)$

Proof of the Isomorphism of N

```
> S:=Sym(11);
> xx:=S!(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11);
> yy:=S!(1, 10)(2, 9)(3, 8)(4, 7)(5, 6);
> N:=sub<S|xx,yy>;
> #N;
22
> NormalLattice(N);

Normal subgroup lattice
-----------------------

[3]  Order 22  Length 1  Maximal Subgroups: 2
---
[2]  Order 11  Length 1  Maximal Subgroups: 1
---
[1]  Order 1   Length 1  Maximal Subgroups:

> CompositionFactors(N);
    G
    |  Cyclic(2)
    *
    |  Cyclic(11)
    1
> NL:=NormalLattice(N);
> s:=IsIsomorphic(NL[2],CyclicGroup(11));s;
true
> FPGroup(N);
Finitely presented group on 2 generators
Relations
    $.2^2 = Id($)
    ($.1^-1 * $.2)^2 = Id($)
    $.1^-11 = Id($)
> G<x,y>:=Group<x,y|y^2,(x^-1*y)^2,x^11>;
```

```
> #G;
22
> f,G1,k:=CosetAction(G,sub<G|Id(G)>);
> #k;
> NL:=NormalLattice(G1);
> H<x>:=Group<x|x^11>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,NL[2]);s;
true
> for i in NL[3] do if i notin NL[2] and Order(i) eq 2 and
for|if> sub<G1|i,NL[3]> eq G1 then E:=i; break; end if; end for;
> A:=t(f1(x));
> N1:=sub<NL[3]|A>;
> NN<a>:=Group<a|a^11>;
> Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
> ArrayP:=[Id(N1): i in [1..#N1]];
> Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
> for i in[2..#N1] do
for> P:=[Id(N1): I in [1..#Sch[i]]];
for> for j in [1..#Sch[i]] do
for|for> if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
for|for> if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
end for;
for> PP:=Id(N1);
for> for k in [1..#P] do
for|for> PP:=PP*P[k]; end for;
for> ArrayP[i]:=PP;
end for;
for i in [1..#N1] do if ArrayP[i] eq A^E then print Sch[i];
for|if> end if; end for;
a^-1
> H<x,e>:=Group<x,e|x^11,e^2,x^e=x^-1>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,G1);s;
true
```

Thus $N \cong 2 : 11$.

6.3.2 Transitive Group(11,5)

Let N be transitive group 2 on 11 letters. $N = L_2(11)$ is of order 660 and is generated by $x = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11)$ and $y = (2, 10)(3, 4)(5, 9)(6, 7)$. In the same manner as before, we find the following presentation,

$$G < x, y, t | y^2, (yx^{-1})^3, x^{-11}, (xyx^{-3}yx^2)^2, t^2, (t, y^x), (t, x^2yx^{-3}), (t, y),$$
$$((yx^3)^2t^(x^2))^{r1}, (yx^5t)^{r2}, (yx^5t^x)^{r3}, (yx^5t^{x^2})^{r4}, (xt)^{r5}, (x^2t)^{r6} >.$$

Table 6.12: $2^{*11} : (L_2(11))$

r1	r2	r3	r4	r5	r6	Order	G
5	0	6	0	0	0	660	$L_2(11)$
0	0	0	0	6	0	351120	$J_1 \times 2$
0	0	5	0	0	0	175560	J_1

Proof of the Isomorphism of N

```
> N:=TransitiveGroup(11,5);
> #N;
660
> Generators(N);
{
    (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11),
    (2, 10)(3, 4)(5, 9)(6, 7)
}
> S:=Sym(11);
> xx:=S!(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11);
> yy:=S!(2, 10)(3, 4)(5, 9)(6, 7);
> N:=sub<S|xx,yy>;
> #N;
660
> N:=sub<S|xx,yy>;
> #N;
660
> CompositionFactors(N);
    G
    |  A(1, 11)                    = L(2, 11)
    1
> s:=IsIsomorphic(N,PSL(2,11));s;
true
```

6.4 Transitive Groups on 6 Letters

```
> #TransitiveGroups(6);
16
```

We will examine some of these groups and write progenitors.

6.4.1 Transitive Group(6,3)

Let N be transitive group 3 on 6 letters. $N = 2 \times S_3$ is of order 12 and is generated by $x = (1,4)(2,3)(5,6)$ and $y = (1,2,3,4,5,6)$. In the same manner as before, we find the following presentation,

$$G < x, y, t | x^2, (y^{-1}x)^2, y^6,$$
$$t^2, (t, xy^3),$$
$$(xt^y)^r1, (xyt^{y^2})^r2, (xyt)^{r3}, (y^2t)^{r4}, (yt)^{r5} >.$$

Table 6.13: $2^{*6} : (2 \times S_3)$

r1	r2	r3	r4	r5	Order	G
10	10	0	3	10	483840	$(2 \times 6) :^{\cdot} (L_3(4) : 2)$
6	0	5	5	8	241920	$6^{\cdot} : (L_3(4) : 2)$
5	8	5	10	6	1320	$2^{\cdot}L_2(11)$
5	8	6	4	9	51840	$2 : S(4,3)$
5	8	6	8	8	380160	$2^4 \times M_{12}$
4	2	8	7	10	322560	$2^{3^{\cdot}} : L_3(4)$
4	0	0	0	7	4368	$2 \times PGL(2,13)$
0	0	0	3	7	2184	$PGL(2,13)$
0	0	3	0	7	24360	$PGL(2,29)$
0	0	5	0	4	13680	$2 \times PGL(2,19)$
0	0	5	5	4	6840	$PGL(2,19)$

Proof of the Isomorphism of N

```
> N:=TransitiveGroup(6,3);
> #N;
12
> Generators(N);
{
```

```
     (1, 4)(2, 3)(5, 6),
     (1, 2, 3, 4, 5, 6)
}
> S:=Sym(6);
> xx:=S!(1, 4)(2, 3)(5, 6);
> yy:=S!(1, 2, 3, 4, 5, 6);
> ;
> N:=sub<S|xx,yy>;
> #N;
12
> CompositionFactors(N);
     G
     |  Cyclic(2)
     *
     |  Cyclic(3)
     *
     |  Cyclic(2)
     1
> NormalLattice(N);

Normal subgroup lattice
-----------------------

[7]  Order 12  Length 1  Maximal Subgroups: 4 5 6
---
[6]  Order 6   Length 1  Maximal Subgroups: 2 3
[5]  Order 6   Length 1  Maximal Subgroups: 3
[4]  Order 6   Length 1  Maximal Subgroups: 3
---
[3]  Order 3   Length 1  Maximal Subgroups: 1
[2]  Order 2   Length 1  Maximal Subgroups: 1
---
[1]  Order 1   Length 1  Maximal Subgroups:

> s:=IsIsomorphic(N,DirectProduct(Sym(3),CyclicGroup(2)));s;
true
```

6.4.2 Transitive Group(6,9)

Let N be transitive group 9 on 6 letters. $N = S_3 \times S_3$ is of order 36 and is generated by $x = (1,4)(2,5)(3,6)$, $y = (2,4,6)$, and $z = (1,5)(2,4)$. In the same manner as before, we find the following presentation,

$$G =< x, y, z, t | x^2, y^3, z^2, (y^{-1}z)^2, (xz)^2, y^{-1}xy^{-1}xyxyx,$$
$$t^2, (t, y), (t, xzyxy),$$
$$(xyxt)^{r1}, (xyxt^{(xy)^3})^{r2}, (xy^{-1}xyt)^{r3}, (xyt)^{r4}, (xzyt)^{r5}, (tt^{xz})^m = xzyxy^{-1} >$$

Table 6.14: $2^{*6} : (S_3 \times S_3)$

r1	r2	r3	r4	r5	m	Order	G
3	6	0	0	0	4	3753792	$2 * L_3(7)$
8	0	0	0	0	2	451584	$2^2 :^\bullet (L_2(7) \times L_2(7)) : 2^2)$
0	6	4	8	10	0	760320	$(M_{12} : 2) \times 2^2$

Proof of the Isomorphism of N

```
> N:=TransitiveGroup(6,9);
> #N;
36
> NL:=NormalLattice(N);
> NL;
Normal subgroup lattice
-----------------------

[10]  Order 36  Length 1  Maximal Subgroups: 7 8 9
---
[ 9]  Order 18  Length 1  Maximal Subgroups: 5 6
[ 8]  Order 18  Length 1  Maximal Subgroups: 6
[ 7]  Order 18  Length 1  Maximal Subgroups: 4 6
---
[ 6]  Order 9   Length 1  Maximal Subgroups: 2 3
[ 5]  Order 6   Length 1  Maximal Subgroups: 2
[ 4]  Order 6   Length 1  Maximal Subgroups: 3
---
[ 3]  Order 3   Length 1  Maximal Subgroups: 1
[ 2]  Order 3   Length 1  Maximal Subgroups: 1
---
[ 1]  Order 1   Length 1  Maximal Subgroups:

> s:=IsIsomorphic(G1,DirectProduct(NL[5],NL[4]));s;
true
> NL[5];
Permutation group acting on a set of cardinality 36
Order = 6 = 2 * 3
(1, 2)(3, 9)(4, 7)(5, 12)(6, 13)(8, 16)(10, 18)(11, 20)(14, 24)
     (15, 26)(17, 23)(19, 27)(21, 25)(22, 28)(29, 33)(30, 36)(31,
```

```
    35)(32, 34)
(1, 27, 25)(2, 21, 19)(3, 28, 13)(4, 34, 36)(5, 16, 23)(6, 22, 9)
    (7, 30, 32)(8, 12, 17)(10, 24, 35)(11, 33, 26)(14, 18, 31)(15,
    29, 20)
> s:=IsIsomorphic(NL[5],Sym(3));s;
true
s:=IsIsomorphic(G1,DirectProduct(Sym(3),Sym(3)));s;
true
```

Chapter 7

More Progenitors

7.1 $2^{*36} : (3^2 : D_4)$

$G =< v, w, x, y, z, t | v^2, w^4, x^2, y^3, z^3, w^{-2}x, (w^{-1}v)^2, (xy^{-1})^2,$
$vz^{-1}vz, (xz^{-1})^2, (y, z), wy^{-1}w^{-1}yz^{-1},$
$t^2, (t, vy^{-1}w^{-1}), (tvy^{-1}z^{-1})^m,$
$(vt)^{r1}, (vt^2)^{r2}, (vwt^{(wz^2v)})^{r3}, (yt^{(w^3v)})^{r5},$
$(zt^{(w^3v)})^{r6}, (wt^y)^{r7}, (vyt^{(w^3v)})^{r8} >$

Table 7.1: $2^{*36} : (3^2 : D_4)$

r1	r2	r3	m	r5	r6	r7	r8	Order	G
3	8	3	3	0	0	0	0	161280	$4^{\bullet}(2 : L_3(4))$
2	2	4	5	0	0	0	0	3916800	$2^2 \times S(4, 4)$
0	0	0	5	2	10	9	5	6840	$PGL_2(19)$
0	0	0	7	0	2	6	7	4368	$2 \times PGL_2(13)$

7.2 $2^{*110} : L_2(11)$

$G =< x, y, t | x^2, y^3, (y^{-1}xyx)^5, (xy^{-1})^{11}, (yxyxyxy^{-1}xy^{-1}xy^{-1}x)^2,$
$t^2, (t, yxyxy^{-1}xy^{-1}xy^{-1}), (t, yxyxyxy^{-1}xyxyxy^{-1}xy),$
$(tx)^k, (tyxy^{-1}xyxyxy^{-1}xyxyxy)^l, (tt^x)^m = yxyxyxy^{-1}xyxyxy^{-1}xy,$
$(yxy^{-1}xyxyxyxyxy^{-1}t)^{r1}, (yxy^{-1}xyxyxyxyxy^{-1}t^{(yxy^2)})^{r2},$

$(yxyxyxy^{-1}xyxyxy^{-1}xt^{y})^{r3}, (y^{-1}xyxyxy^{-1}xy^{-1}xyxyt^{(yxy^2)})^{r4},$
$(xyt^{(yx)})^{r5}, ((xy)^2t)^{r6} >$

Table 7.2: $2^{*110} : L_2(11)$

r1	r2	k	l	m	r3	r4	r5	r6	Order	G
6	0	4	4	5	0	0	0	0	15840	$C_2 \times M_{11}$
0	0	8	4	3	0	0	0	5	7920	M_{11}

7.3 $2^{*15} : (C_{15} : C_4)$

$G =< a, b, c, d, t | a^4, b^2, d^3, a^{-2}b, a^{-1}d^{-1}ad^{-1}, (c, d), ac^{-1}a^{-1}c^2, bc^{-1}a^2c^{-1}d,$
$t^2, (t, ac^{-1}),$
$(cbt^b)^{r1}, (cbt)^{r2}, (cbt^a)^{r3}, (ct)^{r4}, (c^2dt)^{r5} >$

Table 7.3: $2^{*15} : (C_{15} : C_4)$

r1	r2	r3	r4	r5	Order	G
2	8	8	6	6	161280	$C_2 * M12 * C2 * C2 * C3$

Chapter 8

MAGMA Code

8.1 Double Coset Enumeration of $(S(4,3):2)$

```
> G<x,y,t>:=Group<x,y,t|x^3,(x*y^-1)^4,y*x*y^-1*x^-2*y^-1*x*y*x^-1,
> y^-1*x^-1*y*x^-1*y^-1*x*y^3*x,(x^-1*y^2*x^-1*y^-1)^2,t^2,(t,y*
> x^2*y^-2*x^-1*y*x^-1),(t,x^-1*y^-1*x^-1*y^-3*x*y^-1),(t,(y*x*
> y^-1)^3),(t,y^-1*x^3*y^-2),(x*y*t^(x^3))^6,(x * y*t^(x^2*y*x^2))^4,
> (x * y*t^(x^2*y))^8,x * t * x * y^-1 * x * t * y * x *t * x * y^-1
> * x * t * y^-1 * t * x * t * y>;
> #G;
51840
> S:=Sym(10);
> xx:=S!(1,2,4)(3,5,6)(7,8,10);
> yy:=S!(1,3,2)(4,7,5,9,6,8);
> N:=sub<S|xx,yy>;
> f,G1,k:=CosetAction(G,sub<G|x,y>);
> CompositionFactors(G1);
    G
    |  Cyclic(2)
    *
    |  C(2, 3)                     = S(4, 3)
    1
> s:=IsIsomorphic(N,Sym(5));s
true
> IN:=sub<G1|f(x),f(y)>;
> ts := [Id(G1): i in [1 .. 10] ];
> /* since there are 10 letters */
> ts[1]:=f(t); ts[2]:=f(t^x); ts[3]:=f(t^y); ts[4]:=f(t^(x^2));
> ts[5]:=f(t^(y*x)); ts[6]:=f(t^(y*x^2));  ts[7]:=f(t^(x^2*y));
```

```
> ts[8]:=f(t^(x^2*y*x)); ts[9]:=f(t^(y*x*y));
> ts[10]:=f(t^(x^2*y*x^2));
> prodim:=function(pt, Q, I)
function> v:=pt;
function> for i in I do
function|for> v := v^(Q[i]);
function|for> end for;
function> return v;
function> end function;
> #G/#N;
432
> cst := [null : i in [1 .. Index(G,sub<G|x,y>)]] where null is
> [Integers() | ];
> for i := 1 to 10 do
for> cst[prodim(1, ts, [i])] := [i];
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
10
> #N1s;
12
> Set(N1s);
{
    Id(N1),
    (2, 6, 9, 3, 4, 7)(5, 10, 8),
    (2, 4)(3, 6)(8, 10),
    (2, 9)(3, 7)(5, 10),
    (2, 7, 4, 3, 9, 6)(5, 8, 10),
    (2, 9, 4)(3, 7, 6)(5, 8, 10),
    (2, 6)(3, 4)(7, 9)(8, 10),
    (4, 9)(5, 8)(6, 7),
    (2, 7)(3, 9)(4, 6)(5, 10),
    (2, 4, 9)(3, 6, 7)(5, 10, 8),
    (2, 3)(4, 6)(7, 9),
    (2, 3)(4, 7)(5, 8)(6, 9)
}
> for i in [1..#T1] do ([1]^N1s)^T1[i]; end for;
> Orbits(N1s);
[
    GSet{@ 1 @},
    GSet{@ 5, 10, 8 @},
    GSet{@ 2, 3, 4, 7, 6, 9 @}
]
```

```
> for m,n in IN do if ts[1]*ts[5] eq m*(ts[1]*ts[1])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[5] eq m*(ts[1])^n then "true";
for|if> break; end if; end for;
>
> for m,n in IN do if ts[1]*ts[2] eq m*(ts[1]*ts[1])^n then "true";
for|if> break; end if; end for;
> for m,n in IN do if ts[1]*ts[2] eq m*(ts[1])^n then "true";
for|if> break; end if; end for;
>
> N15:=Stabiliser(N,[1,5]);
> SSS:={[1,5]};
> SSS:=SSS^N;
> #SSS;
30
> Seqq:=Setseq(SSS);
>
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5 ]
>
> N15s:=N15;
> N15s; #N15s;
Permutation group N15 acting on a set of cardinality 10
Order = 4 = 2^2
    (2, 4)(3, 6)(8, 10)
    (2, 6)(3, 4)(7, 9)(8, 10)
4
> #N/#N15s;
30
> T15:=Transversal(N,N15s);
> for i in [1..#T15] do
for> ss:=[1,5]^T15[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
40
> #N15s;
```

```
4
> Set(N15s);
{
    (2, 4)(3, 6)(8, 10),
    (2, 6)(3, 4)(7, 9)(8, 10),
    (2, 3)(4, 6)(7, 9),
    Id(N15)
}
> for i in [1..#T15] do ([1,5]^N15s)^T15[i]; end for;
> Orbits(N15s);
[
    GSet{@ 1 @},
    GSet{@ 5 @},
    GSet{@ 7, 9 @},
    GSet{@ 8, 10 @},
    GSet{@ 2, 4, 6, 3 @}
]
> N12:=Stabiliser(N,[1,2]);
> SSS:={[1,2]};
> SSS:=SSS^N;
> #SSS;
60
> Seqq:=Setseq(SSS);
>
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[2] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 2 ]
[ 1, 3 ]
>
> N12s:=N12;
> for n in N do if 1^n eq 1 and 2^n eq 3 then
for|if> N12s:=sub<N|N12s,n>; end if; end for;
> N12s; #N12s;
Permutation group N12s acting on a set of cardinality 10
    (4, 9)(5, 8)(6, 7)
    (2, 3)(4, 7)(5, 8)(6, 9)
    (2, 3)(4, 6)(7, 9)
4
> #N/#N12s;
```

```
30
>
> T12:=Transversal(N,N12s);
> for i in [1..#T12] do
for> ss:=[1,2]^T12[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
70
> #N12s;
4
> Set(N12s);
{
    (2, 3)(4, 7)(5, 8)(6, 9),
    (4, 9)(5, 8)(6, 7),
    (2, 3)(4, 6)(7, 9),
    Id(N12s)
}
> for i in [1..#T12] do ([1,2]^N12s)^T12[i]; end for;
> Orbits(N12s);
[
    GSet{@ 1 @},
    GSet{@ 10 @},
    GSet{@ 2, 3 @},
    GSet{@ 5, 8 @},
    GSet{@ 4, 9, 7, 6 @}
]
> for m,n in IN do if ts[1]*ts[5]*ts[1] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[1] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
>
> for m,n in IN do if ts[1]*ts[5]*ts[5] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[5] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
true
> for m,n in IN do if ts[1]*ts[5]*ts[5] eq m*(ts[1]*ts[5])^n
```

```
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[5] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[5] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
>
> for m,n in IN do if ts[1]*ts[5]*ts[7] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[7] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[7] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[7] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[7] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
>
> for m,n in IN do if ts[1]*ts[5]*ts[8] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[8] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[8] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[8] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[8] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[8] eq m*(ts[1]*ts[5]*ts[7])^n
for|if> then "true"; break; end if; end for;
>
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1]*ts[5]*ts[7])^n
for|if> then "true"; break; end if; end for;
true
```

```
> for m,n in IN do if ts[1]*ts[5]*ts[2] eq m*(ts[1]*ts[5]*ts[8])^n
for|if> then "true"; break; end if; end for;
>
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1]*ts[5]*ts[7])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[1] eq m*(ts[1]*ts[5]*ts[8])^n
for|if> then "true"; break; end if; end for;
>
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[5]*ts[7])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[5]*ts[8])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[10] eq m*(ts[1]*ts[2]*ts[1])^n
for|if> then "true"; break; end if; end for;
>
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
true
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[2])^n
```

```
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[5]*ts[7])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[5]*ts[8])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[2]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[2] eq m*(ts[1]*ts[2]*ts[10])^n
for|if> then "true"; break; end if; end for;
>
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[5]*ts[7])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[5]*ts[8])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[2]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[5] eq m*(ts[1]*ts[2]*ts[10])^n
for|if> then "true"; break; end if; end for;
>
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[5])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[2])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[5]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[5]*ts[7])^n
for|if> then "true"; break; end if; end for;
```

```
true
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[5]*ts[8])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[2]*ts[1])^n
for|if> then "true"; break; end if; end for;
> for m,n in IN do if ts[1]*ts[2]*ts[4] eq m*(ts[1]*ts[2]*ts[10])^n
for|if> then "true"; break; end if; end for;
>
> N151:=Stabiliser(N,[1,5,1]);
> SSS:={[1,5,1]};
> SSS:=SSS^N;
> #SSS;
30
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5]*ts[1] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5, 1 ]
[ 2, 6, 2 ]
[ 4, 3, 4 ]
[ 3, 4, 3 ]
[ 5, 1, 5 ]
[ 6, 2, 6 ]
> N151s:=N151;
> for n in N do if 1^n eq 2 and 5^n eq 6 and 1^n eq 2 then
for|if> N151s:=sub<N|N151s,n>; end if; end for;
> for n in N do if 1^n eq 4 and 5^n eq 3 and 1^n eq 4 then
for|if> N151s:=sub<N|N151s,n>; end if; end for;
> for n in N do if 1^n eq 3 and 5^n eq 4 and 1^n eq 3 then
for|if> N151s:=sub<N|N151s,n>; end if; end for;
> for n in N do if 1^n eq 5 and 5^n eq 1 and 1^n eq 5 then
for|if> N151s:=sub<N|N151s,n>; end if; end for;
> for n in N do if 1^n eq 6 and 5^n eq 2 and 1^n eq 6 then
for|if> N151s:=sub<N|N151s,n>; end if; end for;
> N151s; #N151s;
Permutation group N151s acting on a set of cardinality 10
    (2, 3)(4, 6)(7, 9)
    (2, 4)(3, 6)(8, 10)
    (1, 2, 3)(4, 5, 6)(7, 9, 8)
```

```
    (1, 2, 4)(3, 5, 6)(7, 8, 10)
    (1, 2, 5, 6)(3, 4)(7, 9, 8, 10)
    (1, 2)(5, 6)(7, 8)
    (1, 4, 6)(2, 5, 3)(7, 9, 10)
    (1, 4, 2)(3, 6, 5)(7, 10, 8)
    (1, 4, 5, 3)(2, 6)(7, 9, 10, 8)
    (1, 4)(3, 5)(7, 10)
    (1, 3, 2)(4, 6, 5)(7, 8, 9)
    (1, 3, 6)(2, 5, 4)(8, 10, 9)
    (1, 3, 5, 4)(2, 6)(7, 8, 10, 9)
    (1, 3)(4, 5)(8, 9)
    (1, 5)(2, 6)(7, 8)(9, 10)
    (1, 5)(3, 4)(7, 10)(8, 9)
    (1, 5)(2, 4, 6, 3)(7, 8, 9, 10)
    (1, 5)(2, 3, 6, 4)(7, 10, 9, 8)
    (1, 6, 4)(2, 3, 5)(7, 10, 9)
    (1, 6, 3)(2, 4, 5)(8, 9, 10)
    (1, 6, 5, 2)(3, 4)(7, 10, 8, 9)
    (1, 6)(2, 5)(9, 10)
24
> #N/#N151s;
5
> T151:=Transversal(N,N151s);
> for i in [1..#T151] do
for> ss:=[1,5,1]^T151[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
75
> #N151s;
24
> Set(N151s);
{
    (2, 6)(3, 4)(7, 9)(8, 10),
    (1, 4)(3, 5)(7, 10),
    (1, 6, 3)(2, 4, 5)(8, 9, 10),
    (1, 2, 5, 6)(3, 4)(7, 9, 8, 10),
    (1, 5)(2, 3, 6, 4)(7, 10, 9, 8),
    (1, 3, 6)(2, 5, 4)(8, 10, 9),
    (1, 6, 4)(2, 3, 5)(7, 10, 9),
    (1, 5)(2, 4, 6, 3)(7, 8, 9, 10),
    (1, 3, 5, 4)(2, 6)(7, 8, 10, 9),
```

```
    (1, 6)(2, 5)(9, 10),
    (1, 4, 5, 3)(2, 6)(7, 9, 10, 8),
    (1, 4, 6)(2, 5, 3)(7, 9, 10),
    Id(N151s),
    (2, 4)(3, 6)(8, 10),
    (1, 3)(4, 5)(8, 9),
    (1, 5)(2, 6)(7, 8)(9, 10),
    (1, 2)(5, 6)(7, 8),
    (1, 2, 3)(4, 5, 6)(7, 9, 8),
    (2, 3)(4, 6)(7, 9),
    (1, 2, 4)(3, 5, 6)(7, 8, 10),
    (1, 4, 2)(3, 6, 5)(7, 10, 8),
    (1, 5)(3, 4)(7, 10)(8, 9),
    (1, 3, 2)(4, 6, 5)(7, 8, 9),
    (1, 6, 5, 2)(3, 4)(7, 10, 8, 9)
}
> for i in [1..#T151] do ([1,5,1]^N151s)^T151[i]; end for;
> Orbits(N151s);
[
    GSet{@ 7, 9, 8, 10 @},
    GSet{@ 1, 2, 4, 3, 5, 6 @}
]
> N157:=Stabiliser(N,[1,5,7]);
> SSS:={[1,5,7]};
> SSS:=SSS^N;
> #SSS;
60
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5]*ts[7] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5, 7 ]
> N157s:=N157;
> N157s; #N157s;
Permutation group N157 acting on a set of cardinality 10
Order = 2
    (2, 4)(3, 6)(8, 10)
2
> #N/#N157s;
```

```
60
> T157:=Transversal(N,N157s);
> for i in [1..#T157] do
for> ss:=[1,5,7]^T157[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
135
> #N157s;
2
> Set(N157s);
{
    (2, 4)(3, 6)(8, 10),
    Id(N157)
}
> for i in [1..#T157] do ([1,5,7]^N157s)^T157[i]; end for;
> Orbits(N157s);
[
    GSet{@ 1 @},
    GSet{@ 5 @},
    GSet{@ 7 @},
    GSet{@ 9 @},
    GSet{@ 2, 4 @},
    GSet{@ 3, 6 @},
    GSet{@ 8, 10 @}
]
> N158:=Stabiliser(N,[1,5,8]);
> SSS:={[1,5,8]};
> SSS:=SSS^N;
> #SSS;
60
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5]*ts[8] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5, 8 ]
[ 1, 5, 10 ]
> N158s:=N158;
```

```
> for n in N do if 1^n eq 1 and 5^n eq 5 and 8^n eq 10 then
for|if> N158s:=sub<N|N158s,n>; end if; end for;
> N158s; #N158s;
Permutation group N158s acting on a set of cardinality 10
    (2, 3)(4, 6)(7, 9)
    (2, 6)(3, 4)(7, 9)(8, 10)
    (2, 4)(3, 6)(8, 10)
4
> #N/#N158s;
30
> T158:=Transversal(N,N158s);
> for i in [1..#T158] do
for> ss:=[1,5,8]^T158[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
165
> #N158s;
4
> Set(N158s);
{
    (2, 4)(3, 6)(8, 10),
    (2, 6)(3, 4)(7, 9)(8, 10),
    (2, 3)(4, 6)(7, 9),
    Id(N158s)
}
> [1,5,8]^N158s;
GSet{@
    [ 1, 5, 8 ],
    [ 1, 5, 10 ]
@}
> for i in [1..#T158] do ([1,5,8]^N158s)^T158[i]; end for;
> Orbits(N158s);
[
    GSet{@ 1 @},
    GSet{@ 5 @},
    GSet{@ 7, 9 @},
    GSet{@ 8, 10 @},
    GSet{@ 2, 3, 6, 4 @}
]
> N121:=Stabiliser(N,[1,2,1]);
> SSS:={[1,2,1]};
```

```
> SSS:=SSS^N;
> #SSS;
60
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[2]*ts[1] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 2, 1 ]
[ 3, 1, 3 ]
[ 2, 3, 2 ]
[ 2, 1, 2 ]
[ 1, 3, 1 ]
[ 3, 2, 3 ]
> N121s:=N121;
> for n in N do if 1^n eq 3 and 2^n eq 1 and 1^n eq 3 then
for|if> N121s:=sub<N|N121s,n>; end if; end for;
> for n in N do if 1^n eq 2 and 2^n eq 3 and 1^n eq 2  then
for|if> N121s:=sub<N|N121s,n>; end if; end for;
> for n in N do if 1^n eq 2 and 2^n eq 1 and 1^n eq 2  then
for|if> N121s:=sub<N|N121s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 2^n eq 3 and 1^n eq 1  then
for|if> N121s:=sub<N|N121s,n>; end if; end for;
> for n in N do if 1^n eq 3 and 2^n eq 2 and 1^n eq 3  then
for|if> N121s:=sub<N|N121s,n>; end if; end for;
> N121s; #N121s;
Permutation group N121s acting on a set of cardinality 10
    (4, 9)(5, 8)(6, 7)
    (1, 3, 2)(4, 7, 5, 9, 6, 8)
    (1, 3, 2)(4, 6, 5)(7, 8, 9)
    (1, 2, 3)(4, 8, 6, 9, 5, 7)
    (1, 2, 3)(4, 5, 6)(7, 9, 8)
    (1, 2)(5, 6)(7, 8)
    (1, 2)(4, 9)(5, 7)(6, 8)
    (2, 3)(4, 7)(5, 8)(6, 9)
    (2, 3)(4, 6)(7, 9)
    (1, 3)(4, 8)(5, 9)(6, 7)
    (1, 3)(4, 5)(8, 9)
12
> #N/#N121s;
```

```
10
>
> T121:=Transversal(N,N121s);
> for i in [1..#T121] do
for> ss:=[1,2,1]^T121[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
175
> #N121s;
12
> Set(N121s);
{
    (4, 9)(5, 8)(6, 7),
    (1, 2, 3)(4, 5, 6)(7, 9, 8),
    (1, 2, 3)(4, 8, 6, 9, 5, 7),
    (2, 3)(4, 6)(7, 9),
    (1, 3)(4, 5)(8, 9),
    (1, 2)(5, 6)(7, 8),
    (1, 3)(4, 8)(5, 9)(6, 7),
    Id(N121s),
    (2, 3)(4, 7)(5, 8)(6, 9),
    (1, 3, 2)(4, 6, 5)(7, 8, 9),
    (1, 3, 2)(4, 7, 5, 9, 6, 8),
    (1, 2)(4, 9)(5, 7)(6, 8)
}
> for i in [1..#T121] do ([1,2,1]^N121s)^T121[i]; end for;
> Orbits(N121s);
[
    GSet{@ 10 @},
    GSet{@ 1, 3, 2 @},
    GSet{@ 4, 9, 7, 6, 8, 5 @}
]
> N1210:=Stabiliser(N,[1,2,10]);
> SSS:={[1,2,10]};
> SSS:=SSS^N;
> #SSS;
60
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[2]*ts[10] eq
```

```
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 2, 10 ]
[ 1, 3, 10 ]
> N1210s:=N1210;
> for n in N do if 1^n eq 1 and 2^n eq 3 and 10^n eq 10 then
for|if> N1210s:=sub<N|N1210s,n>; end if; end for;
> N1210s; #N1210s;
Permutation group N1210s acting on a set of cardinality 10
    (4, 9)(5, 8)(6, 7)
    (2, 3)(4, 7)(5, 8)(6, 9)
    (2, 3)(4, 6)(7, 9)
4
> #N/#N1210s;
30
> T1210:=Transversal(N,N1210s);
> for i in [1..#T1210] do
for> ss:=[1,2,10]^T1210[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
205
> #N1210s;
4
> Set(N1210s);
{
    (2, 3)(4, 7)(5, 8)(6, 9),
    (4, 9)(5, 8)(6, 7),
    (2, 3)(4, 6)(7, 9),
    Id(N1210s)
}
> for i in [1..#T1210] do ([1,2,10]^N1210s)^T1210[i]; end for;
> Orbits(N1210s);
[
    GSet{@ 1 @},
    GSet{@ 10 @},
    GSet{@ 2, 3 @},
    GSet{@ 5, 8 @},
    GSet{@ 4, 9, 7, 6 @}
]
```

```
> N125:=Stabiliser(N,[1,2,5]);
> SSS:={[1,2,5]};
> SSS:=SSS^N;
> #SSS;
120
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[2]*ts[5] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 2, 5 ]
[ 2, 3, 6 ]
[ 3, 1, 4 ]
[ 1, 3, 5 ]
[ 3, 2, 4 ]
[ 2, 1, 6 ]
> N125s:=N125;
> for n in N do if 1^n eq 2 and 2^n eq 3 and 5^n eq 6 then
for|if> N125s:=sub<N|N125s,n>; end if; end for;
> for n in N do if 1^n eq 3 and 2^n eq 1 and 5^n eq 4 then
for|if> N125s:=sub<N|N125s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 2^n eq 3 and 5^n eq 5 then
for|if> N125s:=sub<N|N125s,n>; end if; end for;
> for n in N do if 1^n eq 3 and 2^n eq 2 and 5^n eq 4 then
for|if> N125s:=sub<N|N125s,n>; end if; end for;
> for n in N do if 1^n eq 2 and 2^n eq 1 and 5^n eq 6 then
for|if> N125s:=sub<N|N125s,n>; end if; end for;
> N125s; #N125s;
Permutation group N125s acting on a set of cardinality 10
    (1, 2, 3)(4, 5, 6)(7, 9, 8)
    (1, 3, 2)(4, 6, 5)(7, 8, 9)
    (2, 3)(4, 6)(7, 9)
    (1, 3)(4, 5)(8, 9)
    (1, 2)(5, 6)(7, 8)
6
> #N/#N125s;
20
> T125:=Transversal(N,N125s);
> for i in [1..#T125] do
for> ss:=[1,2,5]^T125[i];
```

```
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
225
> #N125s;
6
> Set(N125s);
{
    (1, 3)(4, 5)(8, 9),
    (1, 2)(5, 6)(7, 8),
    (2, 3)(4, 6)(7, 9),
    (1, 2, 3)(4, 5, 6)(7, 9, 8),
    Id(N125s),
    (1, 3, 2)(4, 6, 5)(7, 8, 9)
}
> for i in [1..#T125] do ([1,2,5]^N125s)^T125[i]; end for;
> Orbits(N125s);
[
    GSet{@ 10 @},
    GSet{@ 1, 2, 3 @},
    GSet{@ 4, 5, 6 @},
    GSet{@ 7, 9, 8 @}
]
/* Checking Orbits */
> Orbits(N151s);
[
    GSet{@ 7, 9, 8, 10 @},
    GSet{@ 1, 2, 4, 3, 5, 6 @}
]

> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[i] eq m*(ts[1]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[i] eq m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[i] eq m*(ts[1]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
1
```

```
1
1
1
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[i] eq m*(ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[i] eq m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[i] eq m*(ts[1]*ts[5]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[i] eq m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[i] eq m*(ts[1]*ts[2]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[i] eq m*(ts[1]*ts[2]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[i] eq m*(ts[1]*ts[2]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> Orbits(N157s);
[
    GSet{@ 1 @},
    GSet{@ 5 @},
    GSet{@ 7 @},
    GSet{@ 9 @},
    GSet{@ 2, 4 @},
    GSet{@ 3, 6 @},
    GSet{@ 8, 10 @}
]
> for m,n in IN do for i in [1,5,7,9,2,3,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[i] eq
```

```
for|for|if> m*(ts[1]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,7,9,2,3,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[i] eq
for|for|if> m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,7,9,2,3,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
7
8
> for m,n in IN do for i in [1,5,7,9,2,3,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
2
> for m,n in IN do for i in [1,5,7,9,2,3,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,7,9,2,3,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,7,9,2,3,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,7,9,2,3,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,7,9,2,3,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
```

```
>
> for m,n in IN do for i in [1,5,7,9,2,3,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,7,9,2,3,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
9
> Orbits(N158s);
[
    GSet{@ 1 @},
    GSet{@ 5 @},
    GSet{@ 8 @},
    GSet{@ 10 @},
    GSet{@ 2, 3 @},
    GSet{@ 4, 6 @},
    GSet{@ 7, 9 @}
]
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
8
10
>
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
>
```

```
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[5])^n then i;
```

```
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,8,10,2,4,7] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[3])^n then i;
for|for|if> break; end if; end for; end for;
4
7
2> Orbits(N121s);
[
    GSet{@ 10 @},
    GSet{@ 1, 3, 2 @},
    GSet{@ 4, 9, 7, 6, 8, 5 @}
]
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
1
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7])^n then i;
```

```
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
4
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[3])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
```

```
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> N1517:=Stabiliser(N,[1,5,1,7]);
> SSS:={[1,5,1,7]};
> SSS:=SSS^N;
> #SSS;
60
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5]*ts[1]*ts[7] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]*ts[Rep(Seqq[i])[4]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5, 1, 7 ]
[ 3, 4, 3, 7 ]
[ 6, 2, 6, 7 ]
> N1517s:=N1517;
> for n in N do if 1^n eq 3 and 5^n eq 4 and 1^n eq 3 and
for|if> 7^n eq 7 then
for|if> N1517s:=sub<N|N1517s,n>; end if; end for;
> for n in N do if 1^n eq 6 and 5^n eq 2 and 1^n eq 6 and
for|if> 7^n eq 7 then
for|if> N1517s:=sub<N|N1517s,n>; end if; end for;
> N1517s; #N1517s;
Permutation group N1517s acting on a set of cardinality 10
    (2, 4)(3, 6)(8, 10)
    (1, 3, 6)(2, 5, 4)(8, 10, 9)
    (1, 3)(4, 5)(8, 9)
    (1, 6, 3)(2, 4, 5)(8, 9, 10)
    (1, 6)(2, 5)(9, 10)
6
> #N/#N1517s;
20
> T1517:=Transversal(N,N1517s);
```

```
> for i in [1..#T1517] do
for> ss:=[1,5,1,7]^T1517[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
245
> #N1517s;
6
> Set(N1517s);
{
    (2, 4)(3, 6)(8, 10),
    (1, 3, 6)(2, 5, 4)(8, 10, 9),
    (1, 3)(4, 5)(8, 9),
    (1, 6, 3)(2, 4, 5)(8, 9, 10),
    Id(N1517s),
    (1, 6)(2, 5)(9, 10)
}
> for i in [1..#T1517] do ([1,5,1,7]^N1517s)^T1517[i]; end for;
> Orbits(N1517s);
[
    GSet{@ 7 @},
    GSet{@ 1, 3, 6 @},
    GSet{@ 2, 4, 5 @},
    GSet{@ 8, 10, 9 @}
]
> N1571:=Stabiliser(N,[1,5,7,1]);
> SSS:={[1,5,7,1]};
> SSS:=SSS^N;
> #SSS;
60
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5]*ts[7]*ts[1] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]*ts[Rep(Seqq[i])[4]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5, 7, 1 ]
[ 9, 5, 7, 9 ]
> N1571s:=N1571;
> for n in N do if 1^n eq 9 and 5^n eq 5 and 7^n eq 7 and
```

```
for|if> 1^n eq 9 then
for|if> N1571s:=sub<N|N1571s,n>; end if; end for;
> N1571s; #N1571s;
Permutation group N1571s acting on a set of cardinality 10
    (2, 4)(3, 6)(8, 10)
    (1, 9)(2, 4)(3, 10)(6, 8)
    (1, 9)(3, 8)(6, 10)
4
> #N/#N1571s;
30
> T1571:=Transversal(N,N1571s);
> for i in [1..#T1571] do
for> ss:=[1,5,7,1]^T1571[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
275
>
> #N1571s;
4
> Set(N1571s);
{
    (2, 4)(3, 6)(8, 10),
    (1, 9)(2, 4)(3, 10)(6, 8),
    (1, 9)(3, 8)(6, 10),
    Id(N1571s)
}
> for i in [1..#T1571] do ([1,5,7,1]^N1571s)^T1571[i]; end for;
> Orbits(N1571s);
[
    GSet{@ 5 @},
    GSet{@ 7 @},
    GSet{@ 1, 9 @},
    GSet{@ 2, 4 @},
    GSet{@ 3, 6, 10, 8 @}
]
> N1575:=Stabiliser(N,[1,5,7,5]);
> SSS:={[1,5,7,5]};
> SSS:=SSS^N;
> #SSS;
60
> Seqq:=Setseq(SSS);
```

```
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5]*ts[7]*ts[5] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]*ts[Rep(Seqq[i])[4]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5, 7, 5 ]
[ 1, 8, 6, 8 ]
[ 1, 10, 3, 10 ]
> N1575s:=N1575;
> for n in N do if 1^n eq 1 and 5^n eq 8 and 7^n eq 6 and
for|if> 5^n eq 8 then
for|if> N1575s:=sub<N|N1575s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 5^n eq 10 and 7^n eq 3 and
for|if> 5^n eq 10 then
for|if> N1575s:=sub<N|N1575s,n>; end if; end for;
> N1575s; #N1575s;
Permutation group N1575s acting on a set of cardinality 10
    (2, 4)(3, 6)(8, 10)
    (4, 9)(5, 8)(6, 7)
    (2, 9, 4)(3, 7, 6)(5, 8, 10)
    (2, 4, 9)(3, 6, 7)(5, 10, 8)
    (2, 9)(3, 7)(5, 10)
6
> #N/#N1575s;
20
> T1575:=Transversal(N,N1575s);
> for i in [1..#T1575] do
for> ss:=[1,5,7,5]^T1575[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
295
> #N1575s;
6
> Set(N1575s);
{
    (2, 4)(3, 6)(8, 10),
    (2, 9, 4)(3, 7, 6)(5, 8, 10),
    (4, 9)(5, 8)(6, 7),
    (2, 9)(3, 7)(5, 10),
```

```
    (2, 4, 9)(3, 6, 7)(5, 10, 8),
    Id(N1575s)
}
> for i in [1..#T1575] do ([1,5,7,5]^N1575s)^T1575[i]; end for;
> Orbits(N1575s);
[
    GSet{@ 1 @},
    GSet{@ 2, 4, 9 @},
    GSet{@ 3, 6, 7 @},
    GSet{@ 5, 8, 10 @}
]
> N1573:=Stabiliser(N,[1,5,7,3]);
> SSS:={[1,5,7,3]};
> SSS:=SSS^N;
> #SSS;
120
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5]*ts[7]*ts[3] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]*ts[Rep(Seqq[i])[4]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5, 7, 3 ]
[ 3, 4, 7, 1 ]
> N1573s:=N1573;
> for n in N do if 1^n eq 3 and 5^n eq 4 and 7^n eq 7 and
for|if> 3^n eq 1 then
for|if> N1573s:=sub<N|N1573s,n>; end if; end for;
> N1573s; #N1573s;
Permutation group N1573s acting on a set of cardinality 10
Order = 2
    (1, 3)(4, 5)(8, 9)
2
> #N/#N1573s;
60
>
> T1573:=Transversal(N,N1573s);
> for i in [1..#T1573] do
for> ss:=[1,5,7,3]^T1573[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
```

```
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
355
> #N1573s;
2
> Set(N1573s);
{
    (1, 3)(4, 5)(8, 9),
    Id(N1573s)
}
> for i in [1..#T1573] do ([1,5,7,3]^N1573s)^T1573[i]; end for;
> Orbits(N1573s);
[
    GSet{@ 2 @},
    GSet{@ 6 @},
    GSet{@ 7 @},
    GSet{@ 10 @},
    GSet{@ 1, 3 @},
    GSet{@ 4, 5 @},
    GSet{@ 8, 9 @}
]
> N1581:=Stabiliser(N,[1,5,8,1]);
> SSS:={[1,5,8,1]};
> SSS:=SSS^N;
> #SSS;
60
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5]*ts[8]*ts[1] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]*ts[Rep(Seqq[i])[4]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5, 8, 1 ]
[ 2, 6, 10, 2 ]
[ 4, 3, 7, 4 ]
[ 2, 6, 7, 2 ]
[ 3, 4, 10, 3 ]
[ 4, 3, 8, 4 ]
[ 5, 1, 9, 5 ]
[ 5, 1, 7, 5 ]
[ 1, 5, 10, 1 ]
```

```
[ 3, 4, 9, 3 ]
[ 6, 2, 9, 6 ]
[ 6, 2, 8, 6 ]
> N1581s:=N1581;
> for n in N do if 1^n eq 2 and 5^n eq 6 and 8^n eq 10
for|if> and 1^n eq 2 then
for|if> N1581s:=sub<N|N1581s,n>; end if; end for;
> for n in N do if 1^n eq 4 and 5^n eq 3 and 8^n eq 7
for|if> and 1^n eq 4 then
for|if> N1581s:=sub<N|N1581s,n>; end if; end for;
> for n in N do if 1^n eq 2 and 5^n eq 6 and 8^n eq 7
for|if> and 1^n eq 2 then
for|if> N1581s:=sub<N|N1581s,n>; end if; end for;
> for n in N do if 1^n eq 3 and 5^n eq 4 and 8^n eq 10
for|if> and 1^n eq 3 then
for|if> N1581s:=sub<N|N1581s,n>; end if; end for;
> for n in N do if 1^n eq 4 and 5^n eq 3 and 8^n eq 8
for|if> and 1^n eq 4 then
for|if> N1581s:=sub<N|N1581s,n>; end if; end for;
> for n in N do if 1^n eq 5 and 5^n eq 1 and 8^n eq 9
for|if> and 1^n eq 5 then
for|if> N1581s:=sub<N|N1581s,n>; end if; end for;
> for n in N do if 1^n eq 5 and 5^n eq 1 and 8^n eq 7
for|if> and 1^n eq 5 then
for|if> N1581s:=sub<N|N1581s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 5^n eq 5 and 8^n eq 10
for|if> and 1^n eq 1 then
for|if> N1581s:=sub<N|N1581s,n>; end if; end for;
> for n in N do if 1^n eq 3 and 5^n eq 4 and 8^n eq 9
for|if> and 1^n eq 3 then
for|if> N1581s:=sub<N|N1581s,n>; end if; end for;
> for n in N do if 1^n eq 6 and 5^n eq 2 and 8^n eq 9
for|if> and 1^n eq 6 then
for|if> N1581s:=sub<N|N1581s,n>; end if; end for;
> for n in N do if 1^n eq 6 and 5^n eq 2 and 8^n eq 8
for|if> and 1^n eq 6 then
for|if> N1581s:=sub<N|N1581s,n>; end if; end for;
> N1581s; #N1581s;
Permutation group N1581s acting on a set of cardinality 10
    (2, 3)(4, 6)(7, 9)
    (1, 2, 4)(3, 5, 6)(7, 8, 10)
    (1, 2, 5, 6)(3, 4)(7, 9, 8, 10)
    (1, 4, 2)(3, 6, 5)(7, 10, 8)
```

```
    (1, 4, 5, 3)(2, 6)(7, 9, 10, 8)
    (1, 2, 3)(4, 5, 6)(7, 9, 8)
    (1, 2)(5, 6)(7, 8)
    (1, 3, 6)(2, 5, 4)(8, 10, 9)
    (1, 3, 5, 4)(2, 6)(7, 8, 10, 9)
    (1, 4, 6)(2, 5, 3)(7, 9, 10)
    (1, 4)(3, 5)(7, 10)
    (1, 5)(3, 4)(7, 10)(8, 9)
    (1, 5)(2, 4, 6, 3)(7, 8, 9, 10)
    (1, 5)(2, 6)(7, 8)(9, 10)
    (1, 5)(2, 3, 6, 4)(7, 10, 9, 8)
    (2, 6)(3, 4)(7, 9)(8, 10)
    (2, 4)(3, 6)(8, 10)
    (1, 3, 2)(4, 6, 5)(7, 8, 9)
    (1, 3)(4, 5)(8, 9)
    (1, 6, 3)(2, 4, 5)(8, 9, 10)
    (1, 6, 5, 2)(3, 4)(7, 10, 8, 9)
    (1, 6, 4)(2, 3, 5)(7, 10, 9)
    (1, 6)(2, 5)(9, 10)
24
> #N/#N1581s;
5
>
> T1581:=Transversal(N,N1581s);
> for i in [1..#T1581] do
for> ss:=[1,5,8,1]^T1581[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
360
> #N1581s;
24
> Set(N1581s);
{
    (2, 6)(3, 4)(7, 9)(8, 10),
    (1, 4)(3, 5)(7, 10),
    (1, 6, 3)(2, 4, 5)(8, 9, 10),
    (1, 2, 5, 6)(3, 4)(7, 9, 8, 10),
    (1, 5)(2, 3, 6, 4)(7, 10, 9, 8),
    (1, 6, 4)(2, 3, 5)(7, 10, 9),
    (1, 3, 6)(2, 5, 4)(8, 10, 9),
    (1, 5)(2, 4, 6, 3)(7, 8, 9, 10),
```

```
    (1, 6)(2, 5)(9, 10),
    (1, 3, 5, 4)(2, 6)(7, 8, 10, 9),
    (1, 4, 5, 3)(2, 6)(7, 9, 10, 8),
    (1, 4, 6)(2, 5, 3)(7, 9, 10),
    Id(N1581s),
    (2, 4)(3, 6)(8, 10),
    (1, 3)(4, 5)(8, 9),
    (1, 5)(2, 6)(7, 8)(9, 10),
    (1, 2)(5, 6)(7, 8),
    (1, 2, 3)(4, 5, 6)(7, 9, 8),
    (2, 3)(4, 6)(7, 9),
    (1, 2, 4)(3, 5, 6)(7, 8, 10),
    (1, 4, 2)(3, 6, 5)(7, 10, 8),
    (1, 5)(3, 4)(7, 10)(8, 9),
    (1, 3, 2)(4, 6, 5)(7, 8, 9),
    (1, 6, 5, 2)(3, 4)(7, 10, 8, 9)
}
> for i in [1..#T1581] do ([1,5,8,1]^N1581s)^T1581[i]; end for;
> Orbits(N1581s);
[
    GSet{@ 7, 9, 8, 10 @},
    GSet{@ 1, 2, 4, 3, 5, 6 @}
]
> N1585:=Stabiliser(N,[1,5,8,5]);
> SSS:={[1,5,8,5]};
> SSS:=SSS^N;
> #SSS;
60
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5]*ts[8]*ts[5] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]*ts[Rep(Seqq[i])[4]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5, 8, 5 ]
[ 1, 8, 10, 8 ]
[ 1, 8, 5, 8 ]
[ 1, 10, 5, 10 ]
[ 1, 5, 10, 5 ]
[ 1, 10, 8, 10 ]
> N1585s:=N1585;
```

```
> for n in N do if 1^n eq 1 and 5^n eq 8 and 8^n eq 10
for|if> and 5^n eq 8 then
for|if> N1585s:=sub<N|N1585s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 5^n eq 8 and 8^n eq 5
for|if> and 5^n eq 8 then
for|if> N1585s:=sub<N|N1585s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 5^n eq 10 and 8^n eq 5
for|if> and 5^n eq 10 then
for|if> N1585s:=sub<N|N1585s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 5^n eq 5 and 8^n eq 10
for|if> and 5^n eq 5 then
for|if> N1585s:=sub<N|N1585s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 5^n eq 10 and 8^n eq 8
for|if> and 5^n eq 10 then
for|if> N1585s:=sub<N|N1585s,n>; end if; end for;
> N1585s; #N1585s;
Permutation group N1585s acting on a set of cardinality 10
    (2, 3)(4, 6)(7, 9)
    (2, 7, 4, 3, 9, 6)(5, 8, 10)
    (2, 9, 4)(3, 7, 6)(5, 8, 10)
    (4, 9)(5, 8)(6, 7)
    (2, 3)(4, 7)(5, 8)(6, 9)
    (2, 4, 9)(3, 6, 7)(5, 10, 8)
    (2, 6, 9, 3, 4, 7)(5, 10, 8)
    (2, 6)(3, 4)(7, 9)(8, 10)
    (2, 4)(3, 6)(8, 10)
    (2, 9)(3, 7)(5, 10)
    (2, 7)(3, 9)(4, 6)(5, 10)
12
> #N/#N1585s;
10
> T1585:=Transversal(N,N1585s);
> for i in [1..#T1585] do
for> ss:=[1,5,8,5]^T1585[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
370
> #N1585s;
12
> Set(N1585s);
{
```

```
    Id(N1585s),
    (2, 6, 9, 3, 4, 7)(5, 10, 8),
    (2, 4)(3, 6)(8, 10),
    (2, 9)(3, 7)(5, 10),
    (2, 7, 4, 3, 9, 6)(5, 8, 10),
    (2, 9, 4)(3, 7, 6)(5, 8, 10),
    (2, 6)(3, 4)(7, 9)(8, 10),
    (4, 9)(5, 8)(6, 7),
    (2, 7)(3, 9)(4, 6)(5, 10),
    (2, 4, 9)(3, 6, 7)(5, 10, 8),
    (2, 3)(4, 6)(7, 9),
    (2, 3)(4, 7)(5, 8)(6, 9)
}
> for i in [1..#T1585] do ([1,5,8,5]^N1585s)^T1585[i]; end for;
> Orbits(N1585s);
[
    GSet{@ 1 @},
    GSet{@ 5, 8, 10 @},
    GSet{@ 2, 3, 7, 9, 4, 6 @}
]
> N12110:=Stabiliser(N,[1,2,1,10]);
> SSS:={[1,2,1,10]};
> SSS:=SSS^N;
> #SSS;
60
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[2]*ts[1]*ts[10] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]*ts[Rep(Seqq[i])[4]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 2, 1, 10 ]
[ 3, 1, 3, 10 ]
[ 2, 3, 2, 10 ]
[ 2, 1, 2, 10 ]
[ 1, 3, 1, 10 ]
[ 3, 2, 3, 10 ]
> N12110s:=N12110;
> for n in N do if 1^n eq 3 and 2^n eq 1 and 1^n eq 3
for|if> and 10^n eq 10 then
for|if> N12110s:=sub<N|N12110s,n>; end if; end for;
```

```
> for n in N do if 1^n eq 2 and 2^n eq 3 and 1^n eq 2
for|if> and 10^n eq 10 then
for|if> N12110s:=sub<N|N12110s,n>; end if; end for;
> for n in N do if 1^n eq 2 and 2^n eq 1 and 1^n eq 2
for|if> and 10^n eq 10 then
for|if> N12110s:=sub<N|N12110s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 2^n eq 3 and 1^n eq 1
for|if> and 10^n eq 10 then
for|if> N12110s:=sub<N|N12110s,n>; end if; end for;
> for n in N do if 1^n eq 3 and 2^n eq 2 and 1^n eq 3
for|if> and 10^n eq 10 then
for|if> N12110s:=sub<N|N12110s,n>; end if; end for;
> N12110s; #N12110s;
Permutation group N12110s acting on a set of cardinality 10
    (4, 9)(5, 8)(6, 7)
    (1, 3, 2)(4, 7, 5, 9, 6, 8)
    (1, 3, 2)(4, 6, 5)(7, 8, 9)
    (1, 2, 3)(4, 8, 6, 9, 5, 7)
    (1, 2, 3)(4, 5, 6)(7, 9, 8)
    (1, 2)(5, 6)(7, 8)
    (1, 2)(4, 9)(5, 7)(6, 8)
    (2, 3)(4, 7)(5, 8)(6, 9)
    (2, 3)(4, 6)(7, 9)
    (1, 3)(4, 8)(5, 9)(6, 7)
    (1, 3)(4, 5)(8, 9)
12
> #N/#N12110s;
10
> T12110:=Transversal(N,N12110s);
> for i in [1..#T12110] do
for> ss:=[1,2,1,10]^T12110[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
380
> #N12110s;
12
> Set(N12110s);
{
    (4, 9)(5, 8)(6, 7),
    (1, 2, 3)(4, 5, 6)(7, 9, 8),
    (1, 2, 3)(4, 8, 6, 9, 5, 7),
```

```
    (2, 3)(4, 6)(7, 9),
    (1, 3)(4, 5)(8, 9),
    (1, 2)(5, 6)(7, 8),
    (1, 3)(4, 8)(5, 9)(6, 7),
    Id(N12110s),
    (2, 3)(4, 7)(5, 8)(6, 9),
    (1, 3, 2)(4, 6, 5)(7, 8, 9),
    (1, 3, 2)(4, 7, 5, 9, 6, 8),
    (1, 2)(4, 9)(5, 7)(6, 8)
}
> for i in [1..#T12110] do ([1,2,1,10]^N12110s)^T12110[i]; end for;
> Orbits(N12110s);
[
    GSet{@ 10 @},
    GSet{@ 1, 3, 2 @},
    GSet{@ 4, 9, 7, 6, 8, 5 @}
]
/*Checking Orbits*/
> Orbits(N1517s);
[
    GSet{@ 7 @},
    GSet{@ 1, 3, 6 @},
    GSet{@ 2, 4, 5 @},
    GSet{@ 8, 10, 9 @}
]
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[2])^n then i;
```

```
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
7
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
8
>
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
2
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
```

```
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[3])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1,2,8] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
>
> Orbits(N1571s);
[
    GSet{@ 5 @},
    GSet{@ 7 @},
    GSet{@ 1, 9 @},
    GSet{@ 2, 4 @},
    GSet{@ 3, 6, 10, 8 @}
]
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
```

```
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
1
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1])^n
for|for|if> then i;break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[10])^n
for|for|if> then i;break; end if; end for; end for;
3
5
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[5])^n
for|for|if> then i;break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
```

```
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n
for|for|if> then i;break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1])^n
for|for|if> then i;break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[5])^n
for|for|if> then i;break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[3])^n
for|for|if> then i;break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[5])^n
for|for|if> then i;break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[1])^n
for|for|if> then i;break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1]*ts[10])^n
for|for|if> then i;break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7]*ts[1])^n
for|for|if> then i;break; end if; end for; end for;
7
> Orbits(N1575s);
[
    GSet{@ 1 @},
    GSet{@ 2, 4, 9 @},
    GSet{@ 3, 6, 7 @},
```

```
    GSet{@ 5, 8, 10 @}
]
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
5
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
2
> for m,n in IN do for i in [1,2,3,5] do if
```

```
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
3
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[3])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
```

```
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,2,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
1
> Orbits(N1573s);
[
    GSet{@ 2 @},
    GSet{@ 6 @},
    GSet{@ 7 @},
    GSet{@ 10 @},
    GSet{@ 1, 3 @},
    GSet{@ 4, 5 @},
    GSet{@ 8, 9 @}
]
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]* ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
```

```
for|for|if> m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
1
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
7
8
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
6
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
10
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
```

```
>
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[3])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]* ts[5]*ts[8]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
4
> for m,n in IN do for i in [2,6,7,10,1,4,8] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[3]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
2
> Orbits(N1581s);
[
    GSet{@ 7, 9, 8, 10 @},
    GSet{@ 1, 2, 4, 5, 3, 6 @}
]
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
```

```
for|for|if> m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
1
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]* ts[2]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]* ts[2]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
7
```

```
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]* ts[5]*ts[7]*ts[5])^n then i;
for|for|if> break; end if;  end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[3])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]* ts[5]*ts[8]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [7,1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1]*ts[2])^n then i;
```

```
for|for|if> break; end if; end for; end for;
>
> Orbits(N1585s);
[
    GSet{@ 1 @},
    GSet{@ 5, 8, 10 @},
    GSet{@ 2, 3, 7, 9, 4, 6 @}
]
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]* ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
5
```

```
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[3])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[1])^n then i;
```

```
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
2
> for m,n in IN do for i in [1,5,2] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
>
> Orbits(N12110s);
[
    GSet{@ 10 @},
    GSet{@ 1, 3, 2 @},
    GSet{@ 4, 9, 7, 6, 8, 5 @}
]
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
>
```

```
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]* ts[5]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
10
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[5])^n then i;
```

```
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[3])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
1
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]* ts[5]*ts[7]*ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
4
> for m,n in IN do for i in [10,1,4] do if
for|for|if> ts[1]*ts[2]*ts[1]*ts[10]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> N15171:=Stabiliser(N,[1,5,1,7,1]);
> SSS:={[1,5,1,7,1]};
> SSS:=SSS^N;
> #SSS;
60
```

```
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5]*ts[1]*ts[7]*ts[1] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]*ts[Rep(Seqq[i])[4]]*
for|for|if> ts[Rep(Seqq[i])[5]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5, 1, 7, 1 ]
[ 9, 5, 9, 7, 9 ]
> N15171s:=N15171;
> for n in N do if 1^n eq 9 and 5^n eq 5 and 1^n eq
for|if> 9 and 7^n eq 7 and 1^n eq 9 then
for|if> N15171s:=sub<N|N15171s,n>; end if; end for;
> N15171s; #N15171s;
Permutation group N15171s acting on a set of cardinality 10
    (2, 4)(3, 6)(8, 10)
    (1, 9)(2, 4)(3, 10)(6, 8)
    (1, 9)(3, 8)(6, 10)
4
> #N/#N15171s;
30
> T15171:=Transversal(N,N15171s);
> for i in [1..#T15171] do
for> ss:=[1,5,1,7,1]^T15171[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
400
>
> [1,5,1,7,1]^N15171s;
GSet{@
    [ 1, 5, 1, 7, 1 ],
    [ 9, 5, 9, 7, 9 ]
@}
> for i in [1..#T15171] do ([1,5,1,7,1]^N15171s)^T15171[i]; end for;
> Orbits(N15171s);
[
    GSet{@ 5 @},
    GSet{@ 7 @},
    GSet{@ 1, 9 @},
```

```
    GSet{@ 2, 4 @},
    GSet{@ 3, 6, 10, 8 @}
]
> #N15171s;
4
>
> N15712:=Stabiliser(N,[1,5,7,1,2]);
> SSS:={[1,5,7,1,2]};
> SSS:=SSS^N;
> #SSS;
120
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5]*ts[7]*ts[1]*ts[2] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]*ts[Rep(Seqq[i])[4]]*
for|for|if> ts[Rep(Seqq[i])[5]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5, 7, 1, 2 ]
[ 9, 6, 8, 9, 1 ]
[ 9, 5, 7, 9, 2 ]
[ 1, 10, 3, 1, 9 ]
[ 2, 10, 3, 2, 9 ]
[ 2, 6, 8, 2, 1 ]
> N15712s:=N15712;
> for n in N do if 1^n eq 9 and 5^n eq 6 and 7^n eq 8
for|if> and 1^n eq 9 and 2^n eq 1 then
for|if> N15712s:=sub<N|N15712s,n>; end if; end for;
> for n in N do if 1^n eq 9 and 5^n eq 5 and 7^n eq 7
for|if> and 1^n eq 9 and  2^n eq 2 then
for|if> N15712s:=sub<N|N15712s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 5^n eq 10 and 7^n eq 3
for|if> and 1^n eq 1 and 2^n eq 9 then
for|if> N15712s:=sub<N|N15712s,n>; end if; end for;
> for n in N do if 1^n eq 2 and 5^n eq 10 and 7^n eq 3
for|if> and 1^n eq 2 and 2^n eq 9 then
for|if> N15712s:=sub<N|N15712s,n>; end if; end for;
> for n in N do if 1^n eq 2 and 5^n eq 6 and 7^n eq 8
for|if> and 1^n eq 2 and 2^n eq 1 then
for|if> N15712s:=sub<N|N15712s,n>; end if; end for;
> N15712s; #N15712s;
```

```
Permutation group N15712s acting on a set of cardinality 10
    (1, 9, 2)(3, 7, 8)(5, 6, 10)
    (1, 9)(3, 8)(6, 10)
    (2, 9)(3, 7)(5, 10)
    (1, 2, 9)(3, 8, 7)(5, 10, 6)
    (1, 2)(5, 6)(7, 8)
6
> #N/#N15712s;
20
> T15712:=Transversal(N,N15712s);
> for i in [1..#T15712] do
for> ss:=[1,5,7,1,2]^T15712[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
420
>
> [1,5,7,1,2]^N15712s;
GSet{@
    [ 1, 5, 7, 1, 2 ],
    [ 9, 6, 8, 9, 1 ],
    [ 9, 5, 7, 9, 2 ],
    [ 1, 10, 3, 1, 9 ],
    [ 2, 10, 3, 2, 9 ],
    [ 2, 6, 8, 2, 1 ]
@}
> for i in [1..#T15712] do ([1,5,7,1,2]^N15712s)^T15712[i]; end for;
> Orbits(N15712s);
[
    GSet{@ 4 @},
    GSet{@ 1, 9, 2 @},
    GSet{@ 3, 7, 8 @},
    GSet{@ 5, 6, 10 @}
]
> #N15712s;
6
>
> N15851:=Stabiliser(N,[1,5,8,5,1]);
> SSS:={[1,5,8,5,1]};
> SSS:=SSS^N;
> #SSS;
60
```

```
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[5]*ts[8]*ts[5]*ts[1] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*
for|for|if> ts[Rep(Seqq[i])[3]]*ts[Rep(Seqq[i])[4]]*
for|for|if> ts[Rep(Seqq[i])[5]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 5, 8, 5, 1 ]
[ 2, 6, 10, 6, 2 ]
[ 3, 9, 4, 9, 3 ]
[ 4, 3, 7, 3, 4 ]
[ 1, 8, 10, 8, 1 ]
[ 5, 9, 1, 9, 5 ]
[ 2, 6, 7, 6, 2 ]
[ 7, 2, 5, 2, 7 ]
[ 2, 10, 7, 10, 2 ]
[ 3, 4, 10, 4, 3 ]
[ 6, 9, 2, 9, 6 ]
[ 9, 6, 3, 6, 9 ]
[ 4, 3, 8, 3, 4 ]
[ 1, 8, 5, 8, 1 ]
[ 8, 4, 6, 4, 8 ]
[ 5, 1, 9, 1, 5 ]
[ 4, 7, 8, 7, 4 ]
[ 1, 10, 5, 10, 1 ]
[ 5, 1, 7, 1, 5 ]
[ 2, 7, 10, 7, 2 ]
[ 8, 6, 1, 6, 8 ]
[ 9, 3, 5, 3, 9 ]
[ 6, 8, 2, 8, 6 ]
[ 1, 5, 10, 5, 1 ]
[ 7, 2, 4, 2, 7 ]
[ 2, 10, 6, 10, 2 ]
[ 3, 4, 9, 4, 3 ]
[ 10, 1, 3, 1, 10 ]
[ 6, 2, 9, 2, 6 ]
[ 9, 3, 6, 3, 9 ]
[ 7, 5, 4, 5, 7 ]
[ 2, 7, 6, 7, 2 ]
[ 3, 10, 9, 10, 3 ]
[ 6, 2, 8, 2, 6 ]
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[ 4, 8, 7, 8, 4 ]
[ 10, 3, 2, 3, 10 ]
[ 4, 8, 3, 8, 4 ]
[ 9, 5, 6, 5, 9 ]
[ 3, 10, 4, 10, 3 ]
[ 8, 4, 1, 4, 8 ]
[ 3, 9, 10, 9, 3 ]
[ 4, 7, 3, 7, 4 ]
[ 1, 10, 8, 10, 1 ]
[ 8, 1, 6, 1, 8 ]
[ 9, 5, 3, 5, 9 ]
[ 5, 9, 7, 9, 5 ]
[ 5, 7, 9, 7, 5 ]
[ 8, 1, 4, 1, 8 ]
[ 7, 4, 5, 4, 7 ]
[ 10, 2, 1, 2, 10 ]
[ 7, 4, 2, 4, 7 ]
[ 6, 9, 8, 9, 6 ]
[ 5, 7, 1, 7, 5 ]
[ 10, 1, 2, 1, 10 ]
[ 7, 5, 2, 5, 7 ]
[ 10, 2, 3, 2, 10 ]
[ 9, 6, 5, 6, 9 ]
[ 6, 8, 9, 8, 6 ]
[ 8, 6, 4, 6, 8 ]
[ 10, 3, 1, 3, 10 ]
> N15851s:=N15851;
> for n in N do if 1^n eq 2 and 5^n eq 6 and 8^n eq 10
for|if> and 5^n eq 6 and 1^n eq 2 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 3 and 5^n eq 9 and 8^n eq 4
for|if> and 5^n eq 9 and 1^n eq 3 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 4 and 5^n eq 3 and 8^n eq 7
for|if> and 5^n eq 3 and 1^n eq 4 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 5^n eq 8 and 8^n eq 10
for|if> and 5^n eq 8 and 1^n eq 1 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 5 and 5^n eq 9 and 8^n eq 1
for|if> and 5^n eq 9 and 1^n eq 5 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 2 and 5^n eq 6 and 8^n eq 7
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for|if> and 5^n eq 6 and 1^n eq 2 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 7 and 5^n eq 2 and 8^n eq 5
for|if> and 5^n eq 2 and 1^n eq 7 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 2 and 5^n eq 10 and 8^n eq 7
for|if> and 5^n eq 10 and 1^neq 2 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 3 and 5^n eq 4 and 8^n eq 10
for|if> and 5^n eq 4 and 1^n eq 3 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 6 and 5^n eq 9 and 8^n eq 2
for|if> and 5^n eq 9 and 1^n eq 6 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 9 and 5^n eq 6 and 8^n eq 3
for|if> and 5^n eq 6 and 1^n eq 9 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 4 and 5^n eq 3 and 8^n eq 8
for|if> and 5^n eq 3 and 1^n eq 4 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 5^n eq 8 and 8^n eq 5
for|if> and 5^n eq 8 and 1^n eq 1 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 8 and 5^n eq 4 and 8^n eq 6
for|if> and 5^n eq 4 and 1^n eq 8 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 5 and 5^n eq 1 and 8^n eq 9
for|if> and 5^n eq 1 and 1^n eq 5 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 4 and 5^n eq 7 and 8^n eq 8
for|if> and 5^n eq 7 and 1^n eq 4 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 5^n eq 10 and 8^n eq 5
for|if> and 5^n eq 10 and 1^neq 1 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 5 and 5^n eq 1 and 8^n eq 7
for|if> and 5^n eq 1 and 1^n eq 5 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 2 and 5^n eq 7 and 8^n eq 10
for|if> and 5^n eq 7 and 1^n eq 2 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 8 and 5^n eq 6 and 8^n eq 1
for|if> and 5^n eq 6 and 1^n eq 8 then
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for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 9 and 5^n eq 3 and 8^n eq 5
for|if> and 5^n eq 3 and 1^n eq 9 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 6 and 5^n eq 8 and 8^n eq 2
for|if> and 5^n eq 8 and 1^n eq 6 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 5^n eq 5 and 8^n eq 10
for|if> and 5^n eq 5 and 1^n eq 1 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 7 and 5^n eq 2 and 8^n eq 4
for|if> and 5^n eq 2 and 1^n eq 7 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 2 and 5^n eq 10 and 8^n eq 6
for|if> and 5^n eq 10 and 1^n eq 2 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 3 and 5^n eq 4 and 8^n eq 9
for|if> and 5^n eq 4 and 1^n eq 3 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 10 and 5^n eq 1 and 8^n eq 3
for|if> and 5^n eq 1 and 1^n eq 10 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 6 and 5^n eq 2 and 8^n eq 9
for|if> and 5^n eq 2 and 1^n eq 6 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 9 and 5^n eq 3 and 8^n eq 6
for|if> and 5^n eq 3 and 1^n eq 9 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 7 and 5^n eq 5 and 8^n eq 4
for|if> and 5^n eq 5 and 1^n eq 7 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 2 and 5^n eq 7 and 8^n eq 6
for|if> and 5^n eq 7 and 1^n eq 2 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 3 and 5^n eq 10 and 8^n eq 9
for|if> and 5^n eq 10 and 1^n eq 3 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end
for> for n in N do if 1^n eq 6 and 5^n eq 2 and 8^n eq 8
for|if> and 5^n eq 2 and 1^n eq 6 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 4 and 5^n eq 8 and 8^n eq 7
for|if> and 5^n eq 8 and 1^n eq 4 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
```

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> for n in N do if 1^n eq 10 and 5^n eq 3 and 8^n eq 2
for|if> and 5^n eq 3 and 1^n eq 10 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 4 and 5^n eq 8 and 8^n eq 3
for|if> and 5^n eq 8 and 1^n eq 4 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 9 and 5^n eq 5 and 8^n eq 6
for|if> and 5^n eq 5 and 1^n eq 9 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 3 and 5^n eq 10 and 8^n eq 4
for|if> and 5^n eq 10 and 1^n eq 3 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 8 and 5^n eq 4 and 8^n eq 1
for|if> and 5^n eq 4 and 1^n eq 8 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 3 and 5^n eq 9 and 8^n eq 10
for|if> and 5^n eq 9 and 1^n eq 3 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 4 and 5^n eq 7 and 8^n eq 3
for|if> and 5^n eq 7 and 1^n eq 4 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 1 and 5^n eq 10 and 8^n eq 8
for|if> and 5^n eq 10 and 1^n eq 1 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 8 and 5^n eq 1 and 8^n eq 6
for|if> and 5^n eq 1 and 1^n eq 8 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 9 and 5^n eq 5 and 8^n eq 3
for|if> and 5^n eq 5 and 1^n eq 9 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 5 and 5^n eq 9 and 8^n eq 7
for|if> and 5^n eq 9 and 1^n eq 5 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 5 and 5^n eq 7 and 8^n eq 9
for|if> and 5^n eq 7 and 1^n eq 5 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 8 and 5^n eq 1 and 8^n eq 4
for|if> and 5^n eq 1 and 1^n eq 8 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 7 and 5^n eq 4 and 8^n eq 5
for|if> and 5^n eq 4 and 1^n eq 7 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 10 and 5^n eq 2 and 8^n eq 1
```

```
for|if> and 5^n eq 2 and 1^n eq 10 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 7 and 5^n eq 2 and 8^n eq 2
for|if> and 5^n eq 4 and 1^n eq 7 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 6 and 5^n eq 9 and 8^n eq 8
for|if> and 5^n eq 9 and 1^n eq 6 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 5 and 5^n eq 7 and 8^n eq 1
for|if> and 5^n eq 7 and 1^n eq 5 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 10 and 5^n eq 1 and 8^n eq 2
for|if> and 5^n eq 1 and 1^n eq 10 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 7 and 5^n eq 5 and 8^n eq 2
for|if> and 5^n eq 5 and 1^n eq 7 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 10 and 5^n eq 2 and 8^n eq 3
for|if> and 5^n eq 2 and 1^n eq 10 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 9 and 5^n eq 6 and 8^n eq 5
for|if> and 5^n eq 6 and 1^n eq 9 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 6 and 5^n eq 8 and 8^n eq 9
for|if> and 5^n eq 8 and 1^n eq 6 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 8 and 5^n eq 6 and 8^n eq 4
for|if> and 5^n eq 6 and 1^n eq 8 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> for n in N do if 1^n eq 10 and 5^n eq 3 and 8^n eq 1
for|if> and 5^n eq 3 and 1^n eq 10 then
for|if> N15851s:=sub<N|N15851s,n>; end if; end for;
> N15851s; #N15851s;
Permutation group N15851s acting on a set of cardinality 10
    (2, 3)(4, 6)(7, 9)
    (1, 3, 2)(4, 7, 5, 9, 6, 8)
    (1, 3)(4, 8)(5, 9)(6, 7)
    (1, 4, 2)(3, 6, 5)(7, 10, 8)
    (1, 4, 5, 3)(2, 6)(7, 9, 10, 8)
    (2, 7, 4, 3, 9, 6)(5, 8, 10)
    (2, 9, 4)(3, 7, 6)(5, 8, 10)
    (1, 5, 9, 3, 4, 8)(6, 10, 7)
    (1, 5, 9, 6, 8)(2, 4, 10, 7, 3)
```

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(1, 2, 3)(4, 5, 6)(7, 9, 8)
(1, 2)(5, 6)(7, 8)
(1, 7, 10, 4)(2, 3, 8, 5)(6, 9)
(1, 7, 6)(2, 8, 5)(4, 9, 10)
(1, 2, 8, 7)(3, 9)(4, 5, 10, 6)
(1, 2, 9)(3, 8, 7)(5, 10, 6)
(1, 3, 6)(2, 5, 4)(8, 10, 9)
(1, 3, 5, 4)(2, 6)(7, 8, 10, 9)
(1, 6, 7, 3)(2, 4, 10, 8)(5, 9)
(1, 6, 10, 8, 2)(3, 4, 7, 5, 9)
(1, 9, 2)(3, 7, 8)(5, 6, 10)
(1, 9, 8, 3)(2, 7)(4, 10, 5, 6)
(1, 4, 6)(2, 5, 3)(7, 9, 10)
(1, 4)(3, 5)(7, 10)
(4, 9)(5, 8)(6, 7)
(2, 3)(4, 7)(5, 8)(6, 9)
(1, 8, 6, 9, 3, 10)(2, 5, 4)
(1, 8, 6, 2, 10)(3, 5, 4, 9, 7)
(1, 5)(3, 4)(7, 10)(8, 9)
(1, 5)(2, 4, 6, 3)(7, 8, 9, 10)
(1, 4)(2, 9)(3, 10)(5, 7)
(1, 4, 6)(2, 10, 3, 9, 5, 7)
(2, 4, 9)(3, 6, 7)(5, 10, 8)
(2, 6, 9, 3, 4, 7)(5, 10, 8)
(1, 5)(2, 6)(7, 8)(9, 10)
(1, 5)(2, 3, 6, 4)(7, 10, 9, 8)
(1, 2, 9, 4)(3, 8, 10, 6)(5, 7)
(1, 2, 8, 10, 6)(3, 9, 5, 7, 4)
(1, 8)(2, 7)(4, 10)(5, 6)
(1, 8)(2, 3, 7, 9)(4, 5, 6, 10)
(1, 9, 4)(3, 8, 5)(6, 7, 10)
(1, 9, 10, 6)(2, 8, 5, 3)(4, 7)
(1, 6, 10, 9)(2, 3, 5, 8)(4, 7)
(1, 6, 7)(2, 5, 8)(4, 10, 9)
(2, 6)(3, 4)(7, 9)(8, 10)
(2, 4)(3, 6)(8, 10)
(1, 7, 10, 5, 2)(3, 9, 6, 8, 4)
(1, 7, 6, 3)(2, 9, 10, 5)(4, 8)
(1, 2, 5, 10, 7)(3, 4, 8, 6, 9)
(1, 2, 4, 9)(3, 5, 10, 7)(6, 8)
(1, 3, 2)(4, 6, 5)(7, 8, 9)
(1, 3)(4, 5)(8, 9)
(1, 10, 2, 6, 9, 5)(3, 7, 8)
```

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(1, 10, 2, 7, 5)(3, 6, 4, 9, 8)
(1, 6, 3)(2, 4, 5)(8, 9, 10)
(1, 6, 5, 2)(3, 4)(7, 10, 8, 9)
(1, 9, 4, 2)(3, 7, 10, 5)(6, 8)
(1, 9, 10, 5, 3)(2, 7, 4, 8, 6)
(1, 7)(2, 10)(3, 6)(4, 8)
(1, 7, 9)(2, 6, 8, 4, 3, 10)
(1, 2, 3)(4, 8, 6, 9, 5, 7)
(1, 2)(4, 9)(5, 7)(6, 8)
(1, 4, 5, 8, 7)(2, 10, 3, 9, 6)
(1, 4, 2, 9)(3, 10)(5, 8, 7, 6)
(1, 10)(2, 8)(3, 5)(4, 7)
(1, 10)(2, 5, 3, 8)(4, 6, 7, 9)
(1, 4, 10, 7)(2, 5, 8, 3)(6, 9)
(1, 4, 9)(3, 5, 8)(6, 10, 7)
(1, 9)(2, 4)(3, 10)(6, 8)
(1, 9, 7)(2, 10, 3, 4, 8, 6)
(1, 3, 5, 10, 9)(2, 6, 8, 4, 7)
(1, 3, 6, 7)(2, 5, 10, 9)(4, 8)
(1, 8)(3, 9)(4, 5)(6, 10)
(1, 8)(2, 9, 7, 3)(4, 10, 6, 5)
(1, 3, 7, 6)(2, 8, 10, 4)(5, 9)
(1, 3, 8, 10, 4)(2, 7, 5, 9, 6)
(1, 4, 9, 2)(3, 6, 10, 8)(5, 7)
(1, 4, 10, 8, 3)(2, 6, 9, 5, 7)
(2, 9)(3, 7)(5, 10)
(2, 7)(3, 9)(4, 6)(5, 10)
(1, 8, 6, 9, 5)(2, 3, 7, 10, 4)
(1, 8, 6, 2, 7, 5)(4, 9, 10)
(1, 9, 7)(2, 8, 3)(4, 10, 6)
(1, 9)(3, 8)(6, 10)
(1, 5, 9, 6, 2, 10)(3, 8, 7)
(1, 5, 9, 3, 10)(2, 8, 7, 6, 4)
(1, 5, 7, 2, 10)(3, 8, 9, 4, 6)
(1, 5, 7, 4, 3, 10)(2, 8, 9)
(1, 8, 4, 3, 9, 5)(6, 7, 10)
(1, 8, 4, 7, 5)(2, 9, 10, 6, 3)
(1, 7, 3, 6)(2, 10)(4, 9, 8, 5)
(1, 7, 8, 5, 4)(2, 6, 9, 3, 10)
(1, 10, 3, 9, 6, 8)(2, 4, 5)
(1, 10, 3, 4, 8)(2, 9, 7, 6, 5)
(1, 6, 4)(2, 7, 5, 9, 3, 10)
(1, 6)(2, 10)(3, 7)(5, 9)
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    (1, 5, 7, 2, 6, 8)(4, 10, 9)
    (1, 5, 7, 4, 8)(2, 3, 6, 10, 9)
    (1, 10, 3, 9, 5)(2, 4, 6, 7, 8)
    (1, 10, 3, 4, 7, 5)(2, 9, 8)
    (1, 7, 9)(2, 3, 8)(4, 6, 10)
    (1, 7)(2, 8)(4, 10)
    (1, 10)(2, 5)(3, 8)(6, 9)
    (1, 10)(2, 8, 3, 5)(4, 9, 7, 6)
    (1, 9, 8, 5, 6)(2, 10, 3, 4, 7)
    (1, 9, 2, 4)(3, 10)(5, 6, 7, 8)
    (1, 6, 5, 8, 9)(2, 7, 4, 3, 10)
    (1, 6, 3, 7)(2, 10)(4, 5, 8, 9)
    (1, 8, 4, 3, 10)(2, 5, 6, 7, 9)
    (1, 8, 4, 7, 2, 10)(3, 5, 6)
    (1, 10, 2, 7, 4, 8)(3, 6, 5)
    (1, 10, 2, 6, 8)(3, 7, 9, 4, 5)
120
> #N/#N15851s;
1
> T15851:=Transversal(N,N15851s);
> for i in [1..#T15851] do
for> ss:=[1,5,8,5,1]^T15851[i];
for> cst[prodim(1, ts, ss)]:=ss;
for> end for;
> m:=0; for i in [1..432] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
421
>
> [1,5,8,5,1]^N15851s;
GSet{@
    [ 1, 5, 8, 5, 1 ],
    [ 3, 9, 4, 9, 3 ],
    [ 4, 3, 7, 3, 4 ],
    [ 1, 8, 10, 8, 1 ],
    [ 5, 9, 1, 9, 5 ],
    [ 2, 6, 7, 6, 2 ],
    [ 7, 2, 5, 2, 7 ],
    [ 2, 10, 7, 10, 2 ],
    [ 3, 4, 10, 4, 3 ],
    [ 6, 9, 2, 9, 6 ],
    [ 9, 6, 3, 6, 9 ],
    [ 4, 3, 8, 3, 4 ],
    [ 1, 8, 5, 8, 1 ],
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[ 8, 4, 6, 4, 8 ],
[ 5, 1, 9, 1, 5 ],
[ 4, 7, 8, 7, 4 ],
[ 1, 10, 5, 10, 1 ],
[ 5, 1, 7, 1, 5 ],
[ 2, 7, 10, 7, 2 ],
[ 8, 6, 1, 6, 8 ],
[ 9, 3, 5, 3, 9 ],
[ 6, 8, 2, 8, 6 ],
[ 1, 5, 10, 5, 1 ],
[ 7, 2, 4, 2, 7 ],
[ 2, 10, 6, 10, 2 ],
[ 3, 4, 9, 4, 3 ],
[ 10, 1, 3, 1, 10 ],
[ 6, 2, 9, 2, 6 ],
[ 9, 3, 6, 3, 9 ],
[ 7, 5, 4, 5, 7 ],
[ 2, 7, 6, 7, 2 ],
[ 4, 8, 7, 8, 4 ],
[ 10, 3, 2, 3, 10 ],
[ 4, 8, 3, 8, 4 ],
[ 9, 5, 6, 5, 9 ],
[ 3, 10, 4, 10, 3 ],
[ 8, 4, 1, 4, 8 ],
[ 3, 9, 10, 9, 3 ],
[ 4, 7, 3, 7, 4 ],
[ 1, 10, 8, 10, 1 ],
[ 8, 1, 6, 1, 8 ],
[ 9, 5, 3, 5, 9 ],
[ 5, 9, 7, 9, 5 ],
[ 5, 7, 9, 7, 5 ],
[ 8, 1, 4, 1, 8 ],
[ 7, 4, 5, 4, 7 ],
[ 10, 2, 1, 2, 10 ],
[ 6, 9, 8, 9, 6 ],
[ 5, 7, 1, 7, 5 ],
[ 10, 1, 2, 1, 10 ],
[ 7, 5, 2, 5, 7 ],
[ 10, 2, 3, 2, 10 ],
[ 9, 6, 5, 6, 9 ],
[ 6, 8, 9, 8, 6 ],
[ 8, 6, 4, 6, 8 ],
[ 10, 3, 1, 3, 10 ],
```

```
    [ 7, 4, 2, 4, 7 ],
    [ 2, 6, 10, 6, 2 ],
    [ 3, 10, 9, 10, 3 ],
    [ 6, 2, 8, 2, 6 ]
@}
> for i in [1..#T15851] do ([1,5,8,5,1]^N15851s)^T15851[i]; end for;
> Orbits(N15851s);
[
    GSet{@ 1, 3, 4, 5, 2, 7, 6, 9, 8, 10 @}
]
> #N15851s;
120
/*Checking Orbits*/
> Orbits(N15171s);
[
    GSet{@ 5 @},
    GSet{@ 7 @},
    GSet{@ 1, 9 @},
    GSet{@ 2, 4 @},
    GSet{@ 3, 6, 10, 8 @}
]
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
```

```
for|for|if> m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
1
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
7
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
```

```
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[3])^n then i;
for|for|if> break; end if; end for; end for;
3
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
2
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1]*ts[10])^n then i;
for|for|if> break; end if; end for; end for;
5
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [5,7,1,2,3] do if
for|for|if> ts[1]*ts[5]*ts[1]*ts[7]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> Orbits(N15712s);
[
    GSet{@ 4 @},
    GSet{@ 1, 9, 2 @},
    GSet{@ 3, 7, 8 @},
    GSet{@ 5, 6, 10 @}
]
> for m,n in IN do for i in [4,1,3,5] do if
```

```
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[5])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[10])^n then i;
```

```
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[5])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
1
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[5])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
4
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[3])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
3
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[5])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[1])^n then i;
for|for|if> break; end if;
```

```
for|for> end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1]*ts[10])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
5
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7]*ts[1])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1]*ts[2])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [4,1,3,5] do if
for|for|if> ts[1]*ts[5]*ts[7]*ts[1]*ts[2]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[5]*ts[1])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> Orbits(N15851s);
[
    GSet{@ 1, 3, 4, 5, 2, 7, 6, 9, 8, 10 @}
]
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5])^n then i;
```

```
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8])^n then i;
for|for|if> break; end if; end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[10])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[5])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7])^n then i;
```

```
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[5])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[3])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[5])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
1
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[1])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[2]*ts[1]*ts[10])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[1]*ts[7]*ts[1])^n then i;
for|for|if> break; end if;
```

```
for|for> end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[7]*ts[1]*ts[2])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
> for m,n in IN do for i in [1] do if
for|for|if> ts[1]*ts[5]*ts[8]*ts[5]*ts[1]*ts[i] eq
for|for|if> m*(ts[1]*ts[5]*ts[8]*ts[5]*ts[1])^n then i;
for|for|if> break; end if;
for|for> end for; end for;
>
```

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