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THE STRUGGLE WITH INVERSE FUNCTIONS DOING AND UNDOING PROCESS

Jesus Nolasco

THE STRUGGLE WITH INVERSE FUNCTIONS

DOING AND UNDOING PROCESS

A Thesis

Presented to the

Faculty of

California State University,

San Bernardino

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

in

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by

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ABSTRACT

This study examines why students have difficulty with inverse functions (inverse functions is the process of doing and undoing operations) and what we can do to support their learning. This was a quasi-experimental design in a math classroom in an urban comprehensive high school in California. After two weeks of instruction one group of students was taught the traditional way of inverse functions and another group was taught conceptually. About (N=80) mathematics students in the sampling were assessed before and after the study. Students were given a test to measure their learning of inverse functions and a questionnaire to measure their perspectives on the unit of study of inverse functions. Qualitative and quantitative methods were used to analyze the data. The results will be discussed hoping that in this study students taught conceptually would perform better than the controlled. Also, this study will be useful for teachers and educators to recognize that conceptual teaching yields better results than direct instruction of rote instruction.

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To all my friends and classmates who were always there for me during Saturday sessions of studying, working on our homework, and on our thesis.

Finally, I would like to thank my Creator, without health and patience I would have never made it this far. Without the Lord's guidance and wisdom I would have never made it to the end.

DEDICATION

This thesis would have not been possible without the support of my family and friends. Lastly, I would like to thank God who through with Him all things are possible.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER ONE

INTRODUCTION

Classroom Experience

Bertrand Russell once said, “Mathematics, rightly viewed, possesses not only truth but supreme beauty”. Mathematical ability has always been a debated subject. Many Americans believed that you can be born with the “math gene” or just the opposite. It has become acceptable in the United States to be mediocre or below proficient in mathematics, a social stigma that must be changed in today’s society. “To many people ‘MATH’ is a scary four-letter word: they don’t like it or feel that they are good at it” (Mutawah & Ali, 2015, p.239). Mathematics is an acquirable skill, and with time and determination, students can excel in mathematics (Sandra & Berry, 2016, p.70). Two factors that contribute to the problem are the way that society sometimes portrays mathematics in a negative light and the delivery of instruction. As mathematicians and as educators we must be willing to make the necessary changes in how instruction is delivered if we want to see positive change in our students. But first we teachers must be willing to make the changes in ourselves. I recall as a teacher, the hardest thing for students to do is to have buy-in; sometimes their morale and confidence in mathematics is so low that they are unwilling to even try to solving mathematics problems. To inspire hope in my own students I share my story of how I used to struggle in mathematics when I was in high school. I had a math professor who was able to breach the gap and all of sudden math made sense and it was a beautiful experience. I then press upon my students that math is not a skill you are born with but rather it is a skill that one acquires, and students must practice if they plan on improving. Practice is just one of many factors which comes into play. Similar to learning a new language, if you don't practice it you will lose it. Math is a language. Over my years of teaching I've noticed a pattern in my students. If they don't really understand definitions, they will struggle. If they don't understand notation, they will struggle. If they don't understand the overall concept, they will struggle. If they struggle, fear and anxiety will begin to develop. According to Mutawah and Ali, "levels of anxiety was the highest among those who perceived themselves as low achievers" (Mutawah & Ali, 2015, p.246).

My colleagues and I have continuously observed students' struggle with algebraic concepts. As students pass along to next levels of math courses, their conceptual understanding worsens as they undertake higher level math classes. During my years of teaching Algebra 2, when I would teach the section on inverse functions it was similar to hitting a road block. I would teach right out of the examples of the textbook that was adopted by the school district. After introducing them to $y = 2x + 3$ and then transitioning to $f(x) = 2x + 3$, students began to panic as anxiety kicked in. I would tell my students that $f(x)$ symbolizes that $2x + 3$ is a function. I would use an analogy: a man without a wedding ring is still a man, and when he puts on a wedding ring he is still a man but is now married. $y = 2x + 3$ is a line when graphed. When we use$f(x) = 2x + 3$, it is still a line, but it also tells you that it is a function. I almost could see the light bulbs turn on when the students begin to understand function notation. When I would introduce inverse notation and ask the students to exchange x for y and solve for y and then write their final answers as $f^{-1}(x)$ I could see the struggle they had. I was teaching my students the same way I was taught when I was in high school.

Traditional Teaching

As a student in the public-school system, before the adoption of the Common Core State Standards, I was taught the same way many former students were taught. Too often in secondary education teachers were more focused on providing students with formulas and mathematical procedures rather than letting students self-discover the mathematical concepts or having students derive the formula. Providing students with the means of exploration and the time for investigation is more meaningful and gives students the opportunity for deep conceptual understanding. It is true that these types of problems were very time-consuming and may have taken an entire class period to come up with a solution, but the ends justified the means; it does not matter how we got there, as long as students end up where we wanted them to be.

As I recall in my K-12 education, I was taught mostly through direct instruction in all my math classes. I would come to class, open up my notebook and write down everything the teacher would put on the overhead projector or on the whiteboard. It wasn’t until I was in grad school where I found it very beneficial to teach conceptually.

According to Gray and Tall, the idea and definition of ‘procept’ is defined as, “a process giving a product, or output, represented by the same symbol is seen to occur at all levels of mathematics. It is therefore worth giving this idea a name: we define a procept to be a combined mental objective consisting of a process, a concept produced by that process, and a symbol which may be used to denote either or both” (Gray &Tall, 1992, p.2). Students should strive for “proceptual” understanding, which would allow them the ability to crank out the algorithm but at the same time, be able to understand the concept behind it. Students with proceptual understanding have proven to be more successful in mathematics rather than a student who solely understands the algorithm or solely the concept. “Students experiencing greater learning gains when engaged in activities. All results were statistically significant and also consistent” (LoPresto & Slater, 2016, p. 73). The term “procept” was first introduced to me in the MAT (Masters of Arts in Teaching Mathematics) program at California State University, which was my richest experience in teaching with different strategies. Even from personal experience, after teaching for about a decade in the field of mathematics, I have witnessed firsthand the depth of knowledge a student possesses when they have strong basic skills, such as working out the algebra and being able to understand the concept. These types of students are not only strong mathematicians but excel from the rest of their peers. They are able to manipulate and isolate variables which leads them to growth and flexibility in thought. It is not just the procedural knowledge students must understand but the concept itself. I had students in the past solve a system of equations, the most common problem of a system of two equations goes as follows: a farmer saw some chickens and pigs in a field. He counted 60 heads and 176 legs. Find out how many chickens and how many pigs he saw. Depending on the level of education, students are taught to use variables to symbolically represent the number of heads and the number of legs by using such variables such as x and y. Students were successfully able to solve for x and y. However, when I asked them to put their solutions back into context by telling me what x and y represent, many students were not sure how to respond to that question. Hence, students must be taught how to solve problems not just procedurally, but conceptually as well. Gray and Tall have identified this problem. When students do not understand the conceptual piece of the problem students begin to struggle in mathematics. When students recognize the procedural process and conceptual process and put them together, we would call that a proceptual understanding.

Each year, as part of my curriculum in the San Bernardino City Unified School District, I have been teaching inverse functions to my students, and each year I have students struggle with inverse functions. I teach the way the textbook has shown us, by exchanging x for y and solving for y. Once we have isolated y, we change that notation to f inverse of x. Students can successfully accomplish this by applying procedural knowledge, which would just require algebraic manipulation to solve for the variable. Some students struggle with the notation. When students read $f^{-1}(x) = 2x$, they would believe that the notation meant $\frac{1}{f(x)}$.

Other students were having problems with isolating y because of issues with the order of operations, or once they did isolate y, they would forget to replace y with f inverse of x. I have also observed that after a few weeks, when I would introduce different types of functions (power functions, rational functions, radical functions, etc.), the students who were successful at first would struggle to find the inverse of these new functions. It wasn't until I began my master's in teaching mathematics degree program that I learned to teach conceptually rather than procedurally. I was taught that multiple representations is a key factor in teaching conceptually, which leads back to my goals and research question.

Goals and Research Question

The purpose of this study was to investigate why students have difficulty with inverse functions and what can be done to support their learning. In this study I made an attempt to answer the following research questions:

1) What skills and conceptual understanding of functions among students can serve as possible predictors of misconceptions of inverse functions?
2) Which learning skills help foster students' growth in mathematics?

By analyzing existing research literature regarding inverse functions, I hope to identify common themes throughout the literature review as well as find supporting evidence of conceptual teaching which will yield better results for my students in the process of doing and undoing inverse functions.

CHAPTER TWO

LITERATURE REVIEW

Students' Misconceptions of Mathematical Concepts

Effective teachers are aware of the importance of student learning, as well as the struggle they face in the classroom. Students' prior knowledge and their cognitive behavior come along with them when they enter the classroom. So why do students struggle with inverse functions? One of the reasons may be due to the fact they have a limited understanding of functions. According to Froelich, Bartkovic, and Foerreester (1991), the concept of functions is one of the most important concepts in mathematics. Within a study conducted in Sweden composed of 17 engineering students attending a university, one instructor taught his students inverse functions as an "undoing" operation, and a different instructor taught his students with algorithmic and procedural skills. Both instructors gave a pre-test before classroom instruction and a post-test after. The results of that study presented an improvement within the students that learned inverse functions with the "undoing" operation. Furthermore, the researchers wanted to know if using technology as a pedagogical tool would make a difference in helping students understand inverse functions. By using the dynamic software GeoGebra during the lecture, the instructor was able to manipulate a function while the students observed the screen. While there was minor growth, the researches mentioned that this was an ongoing study which required more investigation on the relationship between best instructional practices and student learning.

In another study conducted in Ireland by Breen, Larson, O'Shea, and Pettersson (2016), data was collected from first-year undergraduate students on their understanding of concept images of inverse functions. "A concept image is defined to be the cognitive structure associated to a concept and includes interpretations of characteristics and processes that the individual connects to the concept" (Pettersson et al., 2016, p. 229). Some of the students gave straightforward answers in an open-ended questionnaire by using "undoing" operations. "The results showed that several students did not draw on their conceptual knowledge of the inverse property of undoing…this tendency to calculate instead of using conceptual meaning of inverse function may be related to weak conceptual knowledge" (Evan, 1995). In this article the authors embellished the three concepts of inverse functions as inverse as algebra (exchanging x for y), inverse as geometry, and reversal process ("undoing"). In contemporary education there is a large emphasis on 'multiple representations', which targets a larger population of our students. In their findings, none of the Irish students mentioned the necessity of 1 to 1 correspondence.

Furthermore, another concept which I believe is critical in students' comprehension of inverse functions is the relationship between x and y. In most high school math textbooks in the classroom the definition of a function does not really specify that the relationship between the x and y axis can be interchangeable. We also know the concept of a function has evolved over the centuries as stated by Evens (1993):

> Developments in mathematics have changed the concept of function from a curve description by a motion (17^{th} century) to an analytic expression made up of variables and constants representing the relation between two variables with its graph having no "sharp corner" (18^{th} century). Then, new discoveries and rigorization led to the modern conception of a function as univalent correspondence between two sets. More formally, a function f from A to B is defined as any subset of the Cartesian product of A and B, such that for every $a \in A$ there is exactly one $b \in B$ such that $(a, b) \in f$. The evolution of the function concept is sometimes described as a move from a dynamic-dependency notion to a static-set theoretic one. (p.95)

As can be seen in the excerpt the concept of a function has evolved over the course of time. The more modern definition of a function is defined to be a correspondence between two nonempty sets that assigns to every element in the first set (the domain) exactly one element in the second set (the codomain). This modern concept of function is called the Dirichlet-Bourbaki concept (Vinner & Dreyfus,1989). In a study conducted in Hebrew University of Jerusalem, Vinner and Dreyfus investigated to see whether college students were able to exhibit the cognitive schemes of the Dirichlet-Bourbaki definition of a function and perform constructive problems. In their study, they gave a questionnaire to 271 first-year college students who majored in biology, economics, agriculture, physics, chemistry, mathematics, technological education, and industrial design, which also included 36 high school teachers. Vinner and Dreyfus created six questions which were then administered during instructional time for a duration of 20 minutes. The subjects of the study were asked to explain their rationale. In their findings, they discovered, “the percentage of students giving some version of this definition increased with the level of the mathematics course the students were taking.” (Vinner & Dreyfus p. 360). The subjects were then presented three graphical images. One of the images was of a piecewise graph. Some of the students gave a negative response, saying that the graph is not a function since it is discontinuous; other students gave a positive response saying that it is a function with a split domain. Vinner and Dreyfus concluded that a lack of conceptual understanding of the definition of a function corresponding to images was present. They also discovered that several students didn’t recognize a piecewise graph to be a function with restrictions in the domain. Furthermore, they also concluded that complex concepts are not acquired in one step. “Several stages precede the complete acquisition and mastery of a complex concept” (Vinner & Dreyfus p. 365).

In another research study conducted by Ruhama Even (1998), 152 college mathematics students were given a questionnaire about the different representation of functions. Even (1998) noted that:

> The ability to identify and represent the same thing in different representations, and flexibility in moving from one representation to

> another, allows one to see rich relationships, develop a better conceptual understanding, broaden and deepen one's understanding, and strengthen one's ability to solve problems. (p.105)

This relationship about which Even talks about is what we call multiple representation and is a powerful tool in today's classroom. If students are able to demonstrate a mathematical solution in more than one representation–graphically, tabularly, analytically, and pictorially–then they demonstrate proficiency of mathematical concepts. Evans concluded that students were still having difficulty understanding functions, in particular piecewise functions; "many students deal with functions pointwise; i.e., they can plot and read points, but cannot think of a function as it behaves over intervals or in a global way "(Evans, 1998, p. 119). Furthermore, Nevin ORHUN investigated how students find connections between the graphs of derived function and its original function. ORHUN worked with 102 eleventh grade students from two calculus classes. Students were asked about graphs of derived functions and some of their characteristics, such as the change of slope, local maximum, and local minimum. "Students were not successful in analyzing derivative functions. This case could be the result of traditional teaching method" (ORHUN, 2012, p. 684).

Moreover, Cansiz, Küçük, and Isleyen (2011) investigated students' misconceptions about functions. Their study included 61 students that were in the 9th grade, 10th grade, and 11th grade. The students were given an assessment and an interview. Cansiz et al., (2011) recognized the importance of identifying and correcting the mistakes that students make. They concluded that students are having a hard time understanding the concept of function and where unable to make connections between different type of mathematical representation. In order to address the issue, we must first be able to see where the problem lies. “We know about students’ prior knowledge and their cognitive features that come along with them when we are educating them” (Cansiz et al., 2011, p. 3837). The authors also stress the fact that functions are probably one of the most important concepts in mathematics. In their study, students were asked to explain their thought process on functions by taking a function knowledge test. Students were asked whether or not a given graph is a function. They concluded that since the students were not able to match the algebraic representation with the graphs, they have misconceptions about the concept of functions. “Students had some misconceptions like failure to understand whether or not the given graphs are function graphs, failure to correlate verbal expressions with the concept of functions, experience confusion regarding whether or not the given algebraic expression are functions…” (Cansiz et al., 2011, p. 3841). The authors also suggested that “every student having conceptual learning must understand whether or not the given graph represents a function by drawing vertical lines instead of horizontal lines. Otherwise, it’s clear that memorizing this condition as a rule will not earn the student much knowledge” (Cansiz et al., 2011, p. 3841). Cansiz et al. recognized that during their research study, students were having problems with some of the characteristics of functions which led to misconceptions.

Another difficulty students have with the concept of a function is not being able to distinguish between the concept of a function and the concept of an equation. There is a strong relationship between these two ideas, which causes students to struggle (Memnun et al., 2015, p.50). Another research study by Memnun et al. conducted in Turkey included 182 volunteering 11th grade students from two comprehensive high schools. Memnun et al. aimed to examine the struggles of 11th grade students in regards to functions and quadratic equations. The 11th grade students were asked ten open-ended questions, the first seven pertaining to quadratic equations and the last three pertaining to quadratic functions and their graphs. “It took these students about 50 minutes to answer these problems included in the probability test. It was assumed that these participants eleventh grade students made use of their real knowledge and skills in the solution of the research problems” (Menmun et al., 2015, p. 52). They discovered that nearly half of the participants were not able to answer some of the questions relating to functions. “Furthermore, about drawing the graphs of quadratic equations and functions, nearly none of the students became successful” (Menmun et al., 2015, p. 59). From this study they concluded that students were still experiencing difficulty with the concept of a function and Dreyfuss(1991) suggested that the understanding of functions requires relational thinking in order to support higher level mathematical thinking and reasoning (Celik & Guzel, 2017, p.122).

How can we expect students to perform inverse functions when they are still struggling with the overall concept of a function? We must first set goals to be concrete and measurable; “goals can become the observable units of analysis, which can be the basis for problem-solving discussions” (Garbacz, et al., 2015). So we must begin by changing the way we approach teaching; by pushing towards a more conceptual change and “away from information transmission/teacher-focused” (Struyven, Dochy, and Janssens, p.49). Lecture-based instruction and rote memorization of exchanging x for y to find inverse functions has not been very successful in the classroom. We must change the way we approach teaching. Struyven et al. investigated with a pre-test and post-test to measure active learning versus lecture-based learning. Their study included more than 800 participants and it “did not simply draw students' approaches to teaching towards conceptual change/student focused teaching and away from information transmission” (Struyven et al., 2010. p. 59). It is true, even from personal experience, that it is much easier to teach traditionally by exchanging of x for y when we solve for inverses of functions. “We acknowledge that the current system may give rise to perceptions that traditional teaching is easier. It cannot be expected that every teaching academic will balance… nor make effort to change even when benefits are obvious” (McLaren and Kenny, 2015, p.32). Although non-traditional teaching is more work since it takes more time to plan a lesson and to research better ways to best deliver instruction, research has shown that non-traditional instruction promotes conceptual understanding. When students don't understand mathematical concepts, the end result is an increase of math anxiety, which will hinder students' growth. In another study from a vocation high school, Yüksel and Geban measured math achievement (self-efficacy) and anxiety. They concluded that "academic self-efficacy and state anxiety were observed to be the variables predicting math achievement… academic self-efficacy was found to be an important predictor of math achievement" (Yüksel and Geban, 2016, p.96). I've also discovered over a decade of teaching that students who are confident have less anxiety and tend to outperform students who have math anxiety.

Inverse Function

Once students have been exposed to functions, the next mathematical concept to follow is the concept of inverse functions. According to Breen, Larson, O'Shea, and Pettersson (2016) the function concept causes problems for students and some are unable to "conceive a function as a process (rather than taking an 'action' view) [which leads to] difficulties [in] inverting functions. The concepts of function and inverses are essential for representing and interpreting the changing nature of a wide array of situations" (Breen et al., 2016, p. 2228). Students don't see functions as a process of doing and undoing, but rather as an algorithmic process of exchanging the variable x for y and isolating one of the variables. “The conception of undoing is not the only way to look upon inverse function” (Breen et al., 2016, p. 2228). We can use the composition of functions and to get the identity. In order to be able to solve for the inverse of a function, one must know what a function is. How can you expect students to find the inverse of a function when they have no idea what a function is, much less how to graph a function? Carlson and Oehrtman (2005) categorize three different concepts of inverse functions: “inverse as algebra (swap x and y and solve for y), inverse as geometry (the reflection in the line y=x) and inverse as a reversal process (the process of ‘undoing’)” (Breen et al., 2016, p. 2228). They were also striving to see whether using various components may or may not enrich students’ conceptual understanding of inverse functions. Breen et al. collected data from assessments taken by first-year students within two Irish universities. They discovered that several students had a concept image of inverse functions containing the algebraic, the geometric, and the formal definition. However, very few gave a comprehensive explanation of the formal definition of an inverse function.

The participants of another study conducted in Turkey were 9th grade students along with two teachers. Bayazit and Gray (2004) administered open-ended pre-test and post-test questionnaires in which they measured their understanding of functions. One of the teachers taught their students with the notion of the “undoing” process while the other teacher “focused on teaching algorithmic skills and acquisition of procedural rules” (Bayazit and Gray, 2004, p.105). They concluded that students have “difficulty in attaining a meaningful understanding of inverse functions without experiencing it through conceptually focused and cognitively challenging tasks using a variety of representations” Bayazit and Gray, 2004, p.105). Although the students who were taught the “undoing” process performed better, there were other factors in play such as the teacher’s teaching experience, students’ prior knowledge, and the teaching style. As teachers:

> We do not assume that the inclusion of the inverse function and composition will result in students understanding the relationships between inverses. Teachers must engage students in reasoning abstractly, constructing arguments, using structure, and looking for and expressing regularity in repeated reasoning. (Edenfield, 2016, p. 676)

Edenfield suggests rote memorization is not enough for students to have a conceptual understanding of inverse functions. We must develop a more profound understanding and engage in appropriate mathematical practices for the sake of our students. Barrera (2016) even demonstrated to his students how to find all inverse trigonometric functions by using the unit circle.

Attorps, Björk, Radic, and Viirman (2013) investigated the relationship between teachers’ instructional practice and student learning on functions and inverse functions. Attrops et al. used a sample of 17 students who were given a pre and post-test. The assessment included both conceptual and procedural questions with a duration of 30 minutes to complete five questions. After teaching the students concept image (more conceptual) of learning, students developed a better understanding of inverse functions. We must be careful with the notation and the concept of functions since it can confuse students (Wilson, Adamson, Cox, and O'Bryan, 2011).

In another case study, Mike Thomas investigated teachers' understanding of functions. Thomas (2003) worked with 34 pre-service secondary mathematics teacher trainees at The University of Auckland. "The idea that a teacher's content knowledge base will influence the quality of the understanding that students develop in the area of mathematics…" (Thomas, 2003, p. 291). The teachers were given a questionnaire comprised of 13 questions having to do with algebraic, graphical, or tabular representation. They were asked whether or not they were functions. At the end of the questionnaire, Thomas concluded that some of the pre-service teachers were lacking some of the principal elements of function concepts. Pettersson (2012) noted that limited understanding of the function concept has shown to have adverse effects on student learning as they transition to university.

The finding above suggests that one of the reasons why students struggle with inverse functions is because students lack a conceptual understanding of a function. Therefore, this study will focus on teaching inverse function by applying arrow diagrams to enhance the delivery of instruction by comparing it to traditional teaching of inverse function.

CHAPTER THREE

METHODOLOGY

According to the research, factors such as students' prior knowledge, students' comprehension of functions, and degree of instruction all played a pivotal role in students' comprehension of inverse functions. The goal of this research study were stated in the introduction and are restated here:

1. What gaps in skills and conceptual understanding of functions can serve as possible predictors of misconceptions of inverse functions?

Research Methodology

In this study, I expanded and conducted an in-depth study to examine if teaching inverse functions conceptually would promote a better understanding of inverse functions and, consequently, of functions itself. For so many years we have been teaching inverse functions with the operation of exchanging x for y and solving for y. This traditional way of teaching inverse functions has not been as fruitful as I had expected it to be during the years I've been teaching. Although, there is limited research on the effects of teaching on students' success with inverse functions, there is extensive research on teaching and learning of functions, as stated in the literature review. In Wilson et al., (2011) they discussed in their journal article the implication of using arrow diagrams as a visual for teaching the doing and undoing operations while using context in their diagrams. Furthermore, in the master's graduate program, Dr. Wallace taught how to use diagrams and pictures as a means to approach and solve math problems. That is why I decided to see if using arrow diagrams for doing and undoing would help students understand inverse functions more conceptually. In order to do this, I decided to teach inverse functions by exchanging x for y to one class of student and teach another class inverse functions with arrow diagrams.

Demographics

The participants in this study were 80 11^{th} and 12^{th} grade students enrolled in regular college-preparatory pre-calculus classes with the same teacher during the entire duration of the investigation. Two classes from an urban high school in Southern California were selected for the study. One class was randomly assigned to treatment or control group. The following rules were applied to determine the assignment of students to treatment or control groups: If the output was an even number, period two would be taught with arrow diagrams (treatment group), but if the program generated an odd number, then period two would be taught by exchanging x with y (control group). The program produced an odd number and therefore period two was taught by exchanging x with y and period four was taught with the use of arrow diagrams. The comprehensive high school had a population of 1,406 students with approximately 73% of the students being Hispanic, 15% African American, 12% White, and 4% Asian. Of those 1,406 students, 1,322 were socioeconomically disadvantaged according to the 2013 APR (Accountability Progress Reporting) from the California

Department of Education. Approximately 34% of the students' parents did not have a high school diploma, 36% had a high school diploma, 20% had some college education, 8% were college graduates, and 3% attended graduate school (U.S. Census, 2018). Furthermore, the school's Academic Performance Index (API) for the year of study was over 700. English learners made up 33% of the school's population and 10% were identified as students with disabilities.

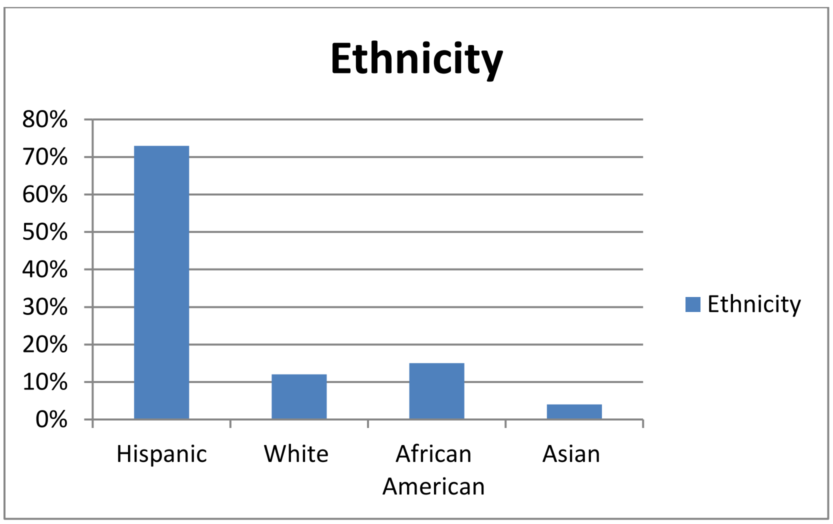


Figure 1. Demographics of School Population.

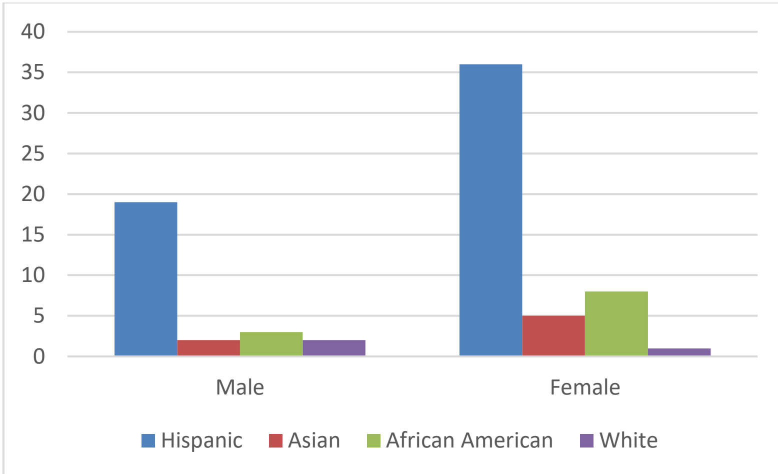


Figure 2. Sample Demographics of Participants by Gender.

Data Collection

Before data collection was conducted for this study, informed consent forms were sent home and parents' permission was obtained. The consent form included a description of the study, the its risks and benefits to the student. In addition, recruitment flyers were sent home informing parents or guardians about the nature of this study. Furthermore, the district approval and principal's approval were also obtained for this study. Finally, participant assent forms were provided and read aloud. Students had already been exposed to the concept of functions and inverse functions from former math classes.

Participant Questionnaire

An open-ended paper and pencil questionnaire was deployed to all 80 participants at the end of the lesson. The questionnaire consisted of four questions (See Appendix B). Both groups of students received the same set of four questions which took them approximately 15 minutes to answer. The first question pertained to what students learned from the lesson, students were asked to see if they understood the relationship between x and y as ordered pairs and its inverse. The second pertained to students' cognitive process of inverse functions. The third question pertained to students' perception of inverse functions, and the last question pertained to students' experience with the overall lesson.

Assessment

The pre-and post-assessments included the same set of questions. Working with my advisor, we created items to assess students' ability and conceptual knowledge of inverse functions. The assessment contained five questions pertaining to functions (See Appendix A). In the first of five items, students were given two blank tables i.e., Table C and Table D. The students were instructed that both tables were functions but that Table D was the inverse of Table C. Students were asked to place values into the tables so that each statement in Table C and Table D were 'true' statements. Students were then asked to generate their own values (see Figure 3). Students were asked to see if they understood the relationship between x and y as ordered pairs and its inverse. What made this question unique were students created their own ordered pairs, rather than the instructor providing the ordered pairs for them.

1. Add values into the tables below so that the following statements about your tables are true:

Table C is a function.
Table D is a function.
Table D is the inverse of Table C.

Table C

x	y

Table D

x	y

Figure 3. Question One on Assessment.

For the second item, students were given three functions to work with: $f(x) = 2x, g(x) = x^2,$ and $h(x) = \sqrt{x}$. Students were asked to find the composition of $f(g(x))$, $g(f(x))$ and $g\big(h(x)\big)$. Students should recognize that two of the functions were inverses of each other. By finding their composition, students should have gotten x as a result

For the third item, students were asked to identify which of the following compositions were inverses of each other and to explain their reasoning.

For the fourth item, students were given a word problem where they had to work backwards in order to find the solution. For the fifth item, students were given diagram of “function machines” and they were required to identify the output of each machine. (See Figure 4). In addition, students were given function notation and were asked to identify the output. Question four was a key point of my study because at this point, I would see a big difference between the students who learned from arrow diagrams and the students who learned to switch x for y. Question four is a word problem in which working backwards was key for students to solve the problem successfully. Using arrow diagrams in this problem was a great strategy in solving the problem.

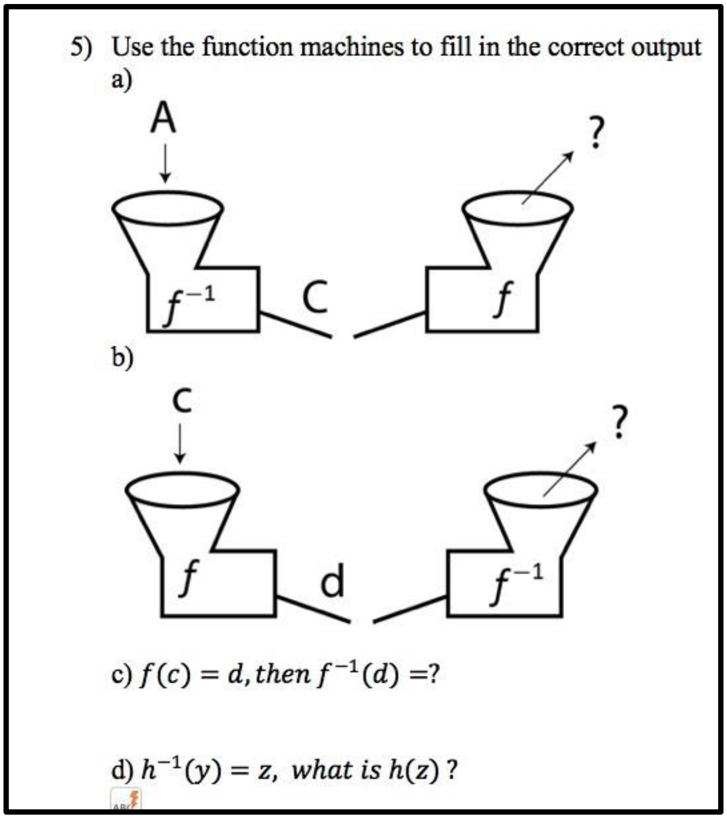


Figure 4. Question Five on Assessment.

Question five was not heavy with algebraic manipulation, but rather a conceptual question where students would demonstrate their knowledge of inverse functions with notation only. Both groups received the same questionnaire to fill out at the end of the lesson. The assessment bore no weight on their class grade; I made it clear from the beginning that the lesson could not count against them and it was only voluntary for the students to participate in my research study.

Positionality

I am a secondary math instructor, and my reasoning for selecting this topic is because I found it to be an interesting topic in mathematics. I used to struggle with the concept of inverse function when I was a student in high school. It didn't make any sense at the time why we would exchange x for y. Since I didn't know the conceptual reason, I struggled with the idea of inverse functions. It wasn't until I was an undergraduate student that I had a better understanding of functions, which made the transition of inverse functions a lot easier for me to understand. Using knowledge I gained as an undergraduate and as a graduate student, I wanted to see if teaching my students inverse functions by using arrow diagrams would help to promote students' comprehension of inverse functions.

Research Design

In order to determine which class would be taught with arrow diagrams (treatment group) and which class would be taught by exchanging x for y (control group), I used a random generator program. The utilization of qualitative/quantitative methodology was used in an in-depth exploration for acquiring a better understanding of how well students understood the concept of inverse functions; this is a quasi-experimental design. One way of measuring was through the use of a questionnaire. For the treatment and to minimize bias in this study I randomly selected two classes, one was taught inverse functions with the assistance of arrow diagrams (treatment group). The other group was taught inverse functions by exchanging x for y (control group). Both groups were taught the same curriculum unit of study. Both groups were given same set of math problems. Another data collection tool used was a pre-and post-assessments; both groups were given the same assessment. On the first day of instruction I gave both groups an assessment on inverse functions. For the intervention each group received the same dosage of instruction on inverse functions for a duration of two weeks. At the conclusion of the two-week lessons I assessed and measured students' growth.

Data Analyses

All data collected were gathered with the intent and purpose of answering the research questions. All data collected were secured in a locked file cabinet. At the end of the lesson a questionnaire was given to each student in order to get a better understanding of students' perception of the lesson. In order to generalize the findings of this study, similar studies must be conducted in similar settings by surrounding districts with similar characteristics of student population.

Limitations

This study was designed to measure the growth of two groups of students, to determine if teaching inverse functions with the use of arrow diagrams will strengthen students' understanding of inverse functions. One of the limitations of this study was the small sample size upon which this study was conducted. To further improve upon this study, many more teachers must conduct the same study and produce similar results. Another limitation is that students may or may not have taken the assessment and questionnaire seriously. In addition, Question 4 was designed to promote the use arrow diagrams in order to solve the problem, however the problem not sufficiently challenging for the students.

Terminology

Throughout this study, the following terms will be used:

1. Arrow diagram group will be used to describe the group in which they will be taught conceptually by using arrow diagrams to enhance instruction and retention of inverse functions. This group is also known as the treatment group.
2. Traditional group will be used to describe the group in which they will be taught by exchanging x for y. This group is also known as the control group.

CHAPTER FOUR

DATA ANALYSIS AND FINDINGS

To address the research question, two major analyses were conducted in this study: (a) an analysis of the pre-and post-assessment and (b) an analysis of the open-ended questionnaire. The objective of this study was to investigate whether using arrow diagrams as a teaching strategy would help students better understand inverse functions rather than teaching students the traditional way of exchanging x with y. Data were obtained from students' responses on the questionnaire and assessments. Data were collected from students in periods two and four.

Pre-assessment

Prior to the lesson all students were given a pre-assessment where students were asked to demonstrate their ability in functions/inverse functions. In Question 1 (see Figure 3) students who were able to input ordered pairs as a function received one point, students who were able to input the inverse of those ordered pairs received another point. The maximum score for the first question was two points. Some of the students were partially correct and earned one point out of the two possible points (see Table 3).

In the post-assessment for Question 1, 63% from both groups answered correctly. Five more students answered partially correct in the control group than the treatment group and 12 students in the control group answered incorrect vs. 8 who answered innocent in the treatment group.

Table 1

Score Distribution for Question 1.

	Period 2		Period 4	
	Control		Treatment	
	Pre	Post	Pre	Post
Correct= 2 points	12(30*)	25(63)	7(17)	25(63)
Partially Correct = 1 point	15(38)	1(2)	16(40)	6(15)
Incorrect Response = 0 points	4(10)	12(30)	15(38)	8(20)
No Response = 0 points	9(22)	2(5)	2(5)	1(2)
Total (N)	40		40	

*The numbers in parentheses are the percentage out of N.

In Question 2, students performed composition of functions for three items. Each sub-part of the three-part item was worth 1 point. In both groups one half of the students were able to answer the question correctly. Yet, seven students in period 2 received partial credit due to the fact that those students were treating a composition of a function as a product of two functions. In period 4, 15 students made a mistake by not squaring the 2 in part 2b of the item. The correct response was $g\left(f(x)\right) = 4x^2$ not $g\left(f(x)\right) = 2x^2$. I was not able to determine if this was due to an incorrect understanding of number sense or because of not recognizing how to apply properties of exponents correctly. Further investigation is required in order to find out the reasons for the error.

The post-assessment for Question 2 showed that the difference in the percentage of correct responses was small i.e., 78% for the treatment group to 70% for the control group. In the control group only eight students left the question blank verses two students who left the question blank from the treatment group (see Table 2).

Table 2

Score Distribution for Question 2.

	Period 2		Period 4	
	Control		Treatment	
	Pre	Post	Pre	Post
Correct = 3 points	19(48*)	28(70)	20(50)	31(78)
Partially Correct =1-2 points	7(17)	1(2)	3(7)	5(12)
Incorrect Response = 0 points	2(5)	3 (8)	15(38)	2(5)
No Response =0 points	12(30)	8(20)	2(5)	2(5)
Total (N)	40		40	

*The numbers in parentheses are the percentage out of N.

In Question 3, students were asked to determine which of the functions were inverses of each other and why. This question was more difficult than the previous two questions. It was interesting to note that approximately one half of the students in the control group did not answer the question. By taking the average correct from both groups, approximately 12.5% were able to identity the correct functions that were inverses of each other but were not able to explain why they were inverses of each other. From going over students' work from both groups, 29 out of 80 students were able to recognize the relationship between a function and its inverse. One point was awarded for identifying the correct inverse function and another point for its explanation.

On the post-assessment, more students attempted to answer Question 3 than on the pre-assessment. Students who received the partial score of 1 point increased significantly for the treatment group but this was not true for the control group. Furthermore, there was an increase by 1 person in the control group who received the maximum score for the item.

Table 3

Score Distribution for Question 3.

	Period 2		Period 4	
	Control		Treatment	
	Pre	Post	Pre	Post
Correct = 2 points	9(23*)	10(25)	11(27)	14(35)
Partially Correct = 1 point	7(17)	1(2)	3(7)	10(25)
Incorrect Response = 0 points	7(17)	22(55)	14(35)	11(27)
No Response = 0 points	17(43)	7(18)	12(30)	5(13)
Total (N)	40		40	

*The numbers in parentheses are the percentage out of N.

From the treatment group 10% of the students were more successful in answering the item than the control group. In the control group, fewer students were able to give a partially correct answer than on the pre-assessment. Further investigation is required to determine why the regression of partial correct responses occurred in the post-assessment.

In Question 4, students were given a word problem that asked them to find out how much money Jasmin won if she ended up with $500. The key in solving the world problem was working backwards. One point was awarded for correct answer. See Table 6 for samples of student work.

On the post-assessment, the treatment and control of students were more successful on answering the word problem. However, while the control group had an increase of 15 percentage points, the treatment group increased by 45% points. The majority of the students in the treatment group were successful in answering the word problem by using the arrow diagrams and working backwards in answering the question. However, a handful of students in the treatment group used mental math without using arrow diagrams.

Table 4

Students Work.

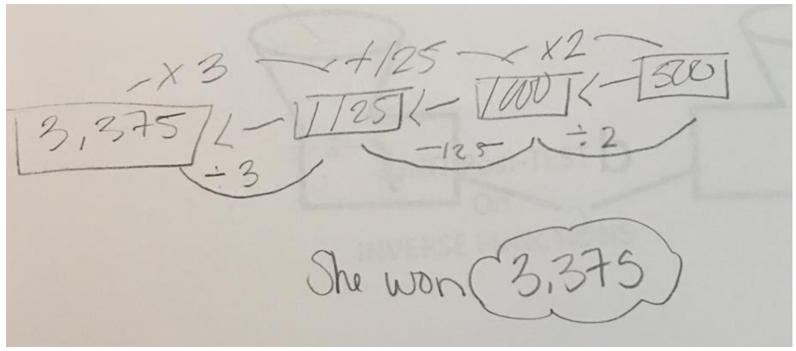
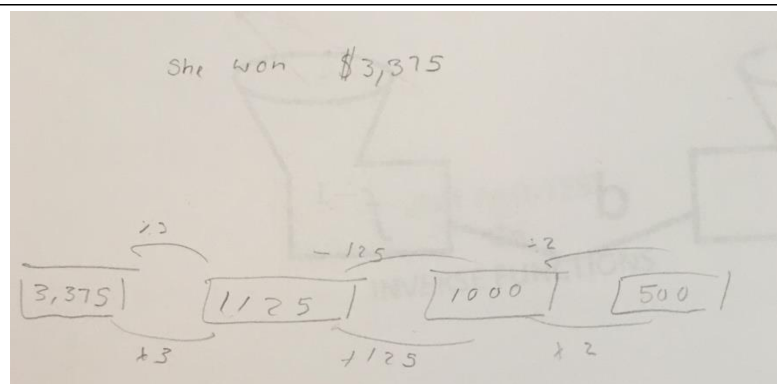
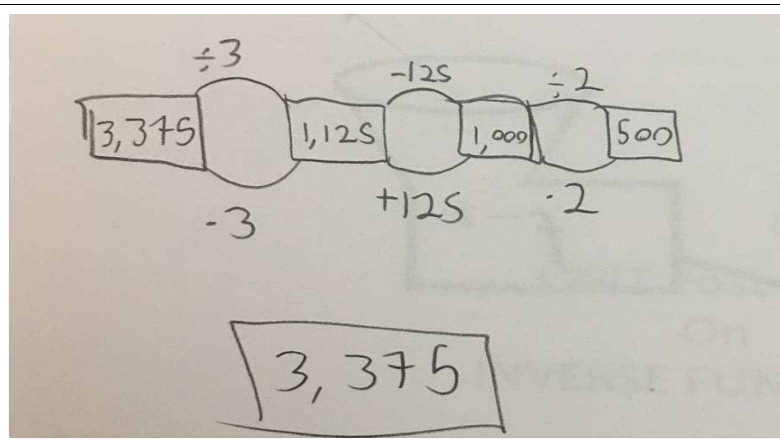
Student	Item 4
A	
B	
C	

Table 5

Score Distribution for Question 4.

	Period 2		Period 4	
	Control		Treatment	
	Pre	Post	Pre	Post
Correct = 2 points	12(30*)	18(45)	12(30)	30(75)
Partially Correct = 1 point	2(5)	1(2)	3(7)	6(15)
Incorrect Response = 0 points	11(27)	16(40)	15(37)	1(2)
No Response = 0 points	15(25)	5(12)	10(25)	3(8)
Total (N)	40		40	

*The numbers in parentheses are the percentage out of N.

Question 5 was a more conceptual question, in which students analyzed a function machine by inputting variables rather than numbers. Only seven students answered question 5 correctly since question 5 was heavy on notation. On the post-assessment, more students from both groups of answered Question 5 correctly since students were more comfortable with math notation.

Table 6

Score Distribution for Question 5.

	Period 2		Period 4	
	Control		Treatment	
	Pre	Post	Pre	Post
Correct= 4 points	4(10*)	18(45)	2(5)	21(53)
Partially Correct = 1-3 point	7(17)	1(2)	3(7)	2(5)
Incorrect Response = 0 points	8(20)	16(40)	13(33)	10(25)
No Response = 0 points	21(53)	5(12)	22(55)	7(17)
Total (N)	40		40	

*The numbers in parentheses are the percentage out of N.

Of all five items, the most difficult item was question 3, where students were asked not only to successfully complete the composition of functions, but to also identify which of the functions were inverses of each other. As I suspected, neither group performed well on the pre-assessment, since neither group had a solid understanding of functions and inverse functions.

Comparison Means

After two weeks of intensive instruction, the same assessment was given again to both groups at the end of the unit of lessons. Both groups showed overall growth. However, the arrow diagram group showed more growth compared to the control group.

Consistent with the pre-test the post-assessment was worth 13 points. The mean score for the Pre-Post assessment for both groups of students was computed. The pre-post mean difference was computed for each individual to measure growth (see Figures 5 and 6). Figure 5 shows the mean of the pre-post assessment scores. The mean for the pre-assessment was $\bar{x}_{pre} = 2.8$ and the post-assessment $\bar{x}_{post} = 8.1$

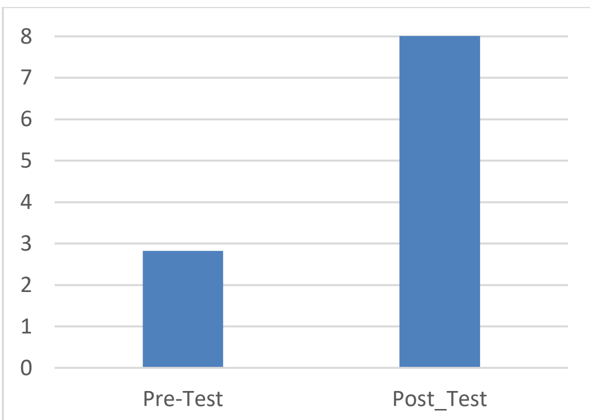


Figure 5. Means of the Treatment Group.

A closer examination of growth by item indicated that effective scaffolding in the treatment group and revisiting the concept of functions using arrow diagrams, students were more comfortable with the concept of inverse functions, since students had experience with composition of functions during the first quarter. Question 2 received the most correct responses from the rest of the items, which asked students to evaluate the composition of functions. On the pre-assessment, several students did not give responses on question 4. Other students attempted to answer the question but were unsuccessful. On the post-assessment, several students used the strategy of arrow diagrams to tackle the question and several were successful in answering the question. In Table 8 I randomly selected three students from the pile of 40 assessments which used arrow diagrams to work backwards. On Question five, 21 more students answered the question correctly compared to the pre-assessment from the arrow diagram group.

For the control group of students, students also showed overall growth. Similar to the arrow diagram group, question 2 was the item that most students answered correctly. Considering that I was the instructor for both groups since the beginning of the academic year, that began in August, both groups had prior knowledge on the composition of functions. In question 1, 13 more students responded correctly. In question 2, 9 more students responded correctly in which students were able to answer the question which asked them to create their own set of ordered pairs and to find its inverse. Questions 3, 4, and 5 showed the least growth of the 5 items: On question 3, only 1 more student answered correctly since question 3 was tied to question 2, if students were unsuccessful in answering question 2, they would have had difficulty in answering question 3. Figure 6 shows the mean of the pre-post assessment scores, the mean for the pre-assessment was $\bar{x}_{pre} = 4.7$ and the post-assessment $\bar{x}_{post} = 6.7$.

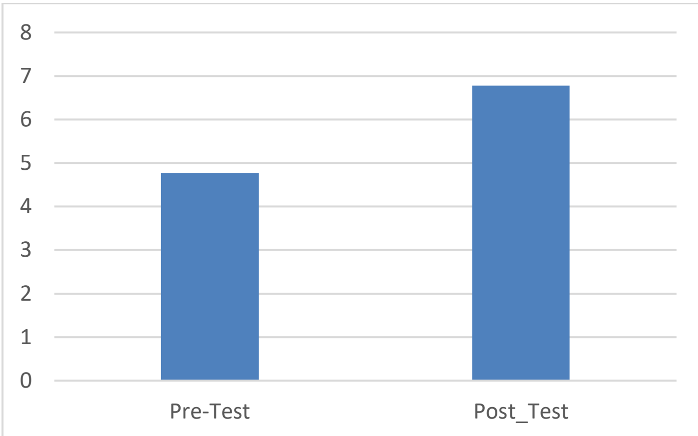


Figure 6. Means of the Control Group.

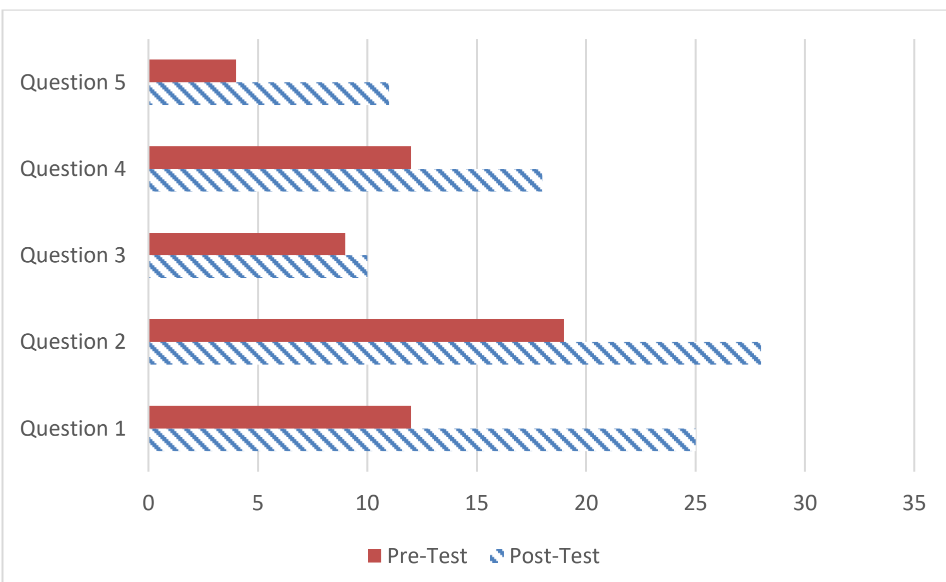


Figure 7. Control Group Assessment by Item.

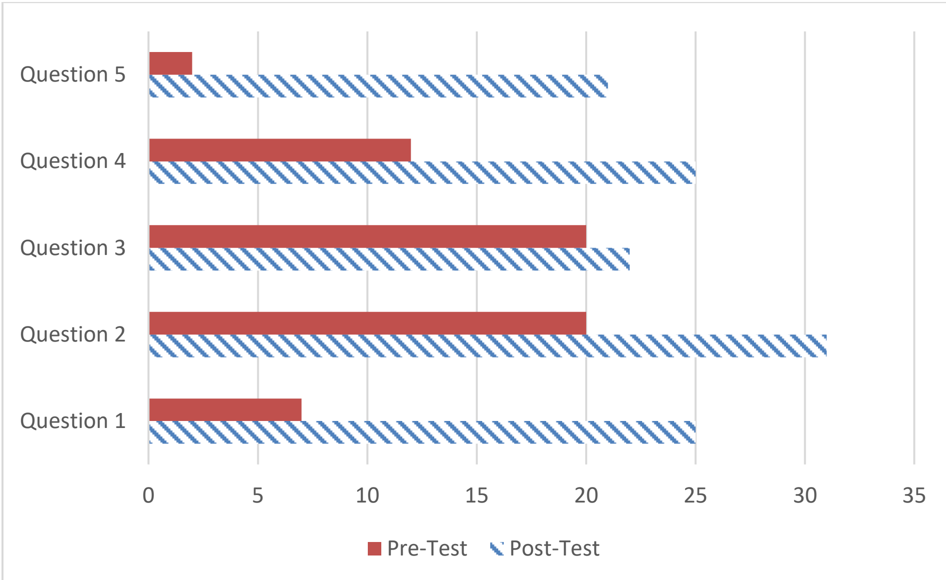


Figure 8. Treatment Group Assessment by Item.

On question 1, 25 students from each group answered correctly. On question 2, three more students from the arrow diagram group answered correctly. On question 3, twelve more students from the arrow diagram group answered correctly. In question 4, 29 students used arrow diagrams to answer the word problem, the rest of the students used number sense to answer the problem. In the traditional group tried using number sense or some sort of algebraic manipulation in order to answer the question. In question 5, ten more students were successful in answering all four items in working with function machines and function notation.

Both groups showed overall progress in the post assessment, however the arrow diagram group showed the most growth by comparing the average means of both groups see figure 9.

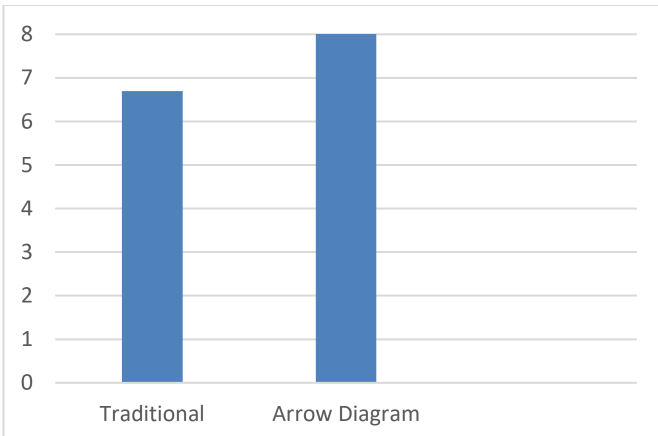


Figure 9. Comparison of Both Groups Means.

Inferential Statistics

By preforming an item analysis of each question, by finding the mean score for each item. I was able to conclude which items were difficult and which ones were easy. In the control group for the pre-assessment item 5 was the most difficult question and in the post-assessment item 5 was still the most difficult question.

In the treatment group, for the pre-assessment item 3 and 5 were the most difficult question and in the post- assessment item 3 was the most difficult question.

The range of scores were from 0-13 points, by using Microsoft Excel I was able to find the mean scores from the pre-and post-assessments for comparisons. Since there were 13 points possible, I was able to compare each individual student's pre- assessment and compared it to their post-assessment. I was able to perform an analysis of covariance (ANCOVA) to test the differences between the means of the treatment and control groups. The pretest scores were used as the covariate to adjust for any initial differences in the two groups.

The null hypothesis: $H_0\colon \mu_{adj.treatment} - \mu_{adj.control} = 0$, if teaching inverse functions by using the arrow diagram has the same result as teaching inverse functions as the traditional way. Then the end result would be the same and the null hypothesis would be 0. The alternative hypothesis was $H_a\colon \mu_{adj.treatment} - \mu_{adj.control} \neq 0$ with $\alpha = 0.05$. To find out if there is a statistical significance which means the p-value is less than 5 %. The ANCOVA results showed that the p-value was less than .01 (see Figure 10) which suggested that there was a statistically significant difference between the adjusted means of the two groups. As shown below in Figure 10, I rejected the null hypothesis in favor of the alternative hypothesis. By teaching inverse functions with the use of arrow diagrams, students were able to have a better conceptual understanding of inverse functions compared to the control group.

Source	SS	df	MS	F	P
adjusted means	90.64	1	90.64	9.42	0.002963
adjusted error	740.64	77	9.62		
adjusted total	831.28	78			

Figure 10. ANCOVA Summary.

Table 7

Student Responses of Questionnaire.

Questions	Arrow Diagram group	Traditional group
What did you learn from this lesson?	71%	52%
Can you explain the process on how to find the inverse of a function?	65%	42%
How can you determine whether the inverse of a function is a function?	55%	38%
What did you enjoy most about this lesson? Why?	71%	52%

Student Questionnaire

The questionnaire consisted of 4 items as shown in Table 2, in which I entered the percentage of positive responses from the questionnaires. On item 1, about 71% of students in the arrow diagram group were able to successfully describe what they learned. I was able to determine this by seeing the results in their post test to see if they used arrow diagrams to solve the word problem. On Item 2 about 65% of the arrow diagram group was able to explain the process. For example, one response from a student was "working backwards for example undoing and doing the opposite of an operation to get your answer" (Participant 1, December 2017). On item 4, a higher percentage of students that were taught conceptually gave positive feedback in comparison to the control group. One student's remark who answered correctly said, "I enjoyed learning the concept of working backwards to find the solution! Also, you have multiple steps to get your answer. I actually knew what was going on and I understood it" (Participant 2, December 2017). As for the control group, about half of the students enjoyed the lesson since the concept of exchanging x for y was not really new to them.

CHAPTER FIVE

DISCUSSIONS AND CONCLUSIONS

Summary

This study provided an insight of a group of students who participated in a pre/post-assessment as an evaluation tool to measure preparedness and performance of students’ knowledge on inverse functions. In addition to measuring how much students improved during the duration of the unit lesson, the pre/post-assessment was a valuable diagnostic tool for more effective teaching. On the pre-assessment, students weren’t expected to know all the answers to every item, but they were expected to utilize prior knowledge to predict logical answers. With the post-assessment, the amount of learning a student had acquired during the two-week unit was measured. The results of the assessment may have been influenced by a variety of factors, such as student’s prior knowledge of the content, considering I had seniors who were repeating the course from last year. As an instructor, I know that students tend to perform better when they are aware that an activity, lesson, or even an assessment will impact their grade. It was challenging to keep the students engaged and for them to take the lesson seriously since students were aware of the fact that the lesson will not impact their grade and equally significant was the fact that the Thanksgiving break was nearing. Nonetheless, the assessment and questionnaire provided pertinent information on the use of arrow diagrams versus traditional strategies to find inverse functions.

Implications for Further Study

This study showed that students achieved higher scores when taught with arrow diagrams rather than being taught with exchange of x for y method. There was a 31% growth difference between the arrow diagram group and the traditional group. There was less student engagement when students were taught to exchange x for y without telling them the reasoning behind what they did. When I introduced functions and inverse functions I needed to explain to the class the relationship they have with each other, not just analytically, but tabularly, graphically, and with arrow diagrams; in short, multiple representations. However, the focus for my research was on the effectiveness of the arrow diagrams, a more visual approach that allows students to easily envision. Furthermore, findings suggest that students have a disconnected concept image of functions and inverse functions, based on what DeMarois (1996) implicated as a mishmash of disconnected procedures since students try to memorize so many steps with little understanding; hence, the reason why the control group underperformed. Another reason that may have contributed to students' inability to perform better would be students' lack of motivation for trying their best.

Future Research

Further research needs to be conducted in the same environment to improve the validity of my research in order to strengthen my pilot study. The four basic principles of experimental design must be met: comparison, random assignment, control, and replication. If many other instructors are able to reproduce the same results over and over again, students would have a better chance of understanding inverse functions. In addition, more research should investigate students' knowledge of inverse functions. There is a lot of research on functions but very limited research on inverse functions.

Policy for Mathematics

Districts should formulate math/strategies textbooks that promote multiple representation in the classrooms. Teachers should receive professional development on using multiple representation as instrumental tools to better enhance student learning. Using arrow diagrams is an example of that tool box. Publishers who work and design the framework of math books should work alongside with districts.

Conclusion

Finding in this study support the use of arrow diagrams as a means to promote students' conceptual understanding of inverse functions. These findings are consistent with those researchers such as Wilson et al., (2011), Akkus et al., (2008) Edenfield (2012), and Pettersson (2012) who all support the idea of arrow diagrams as a means to promote students' conceptual understanding of inverse functions.

In conclusion, this paper explored whether teaching inverse functions with the use of arrow diagrams would improve students' conceptual understanding of inverse functions. All findings support the conclusion that teaching inverse functions by using arrow diagrams does in fact improve students' understanding of inverse functions. Furthermore, the students' responses on the questionnaires confirmed that they enjoyed learning inverse functions by using arrow diagrams, and that it helped them understand inverse functions, as well as functions. I suggest that further exploration on this topic, including student interviews and groups of teachers willing to teach my lesson on inverse functions via arrow diagrams, would only strengthen my study.

APPENDIX A

INVERSE FUNCTIONS ASSESSMENT

Instructions: Please read all instructions carefully. All work must be provided in order to receive full credit.

1. Add values into the tables below so that the following statements about your tables are true:

Table C is a function.
Table D is a function.
Table D is the inverse of Table C.

Table C

x	y

Table D

x	y

2. $f(x) = 2x$ $g(x) = x^2$ $h(x) = \sqrt{x}$

Find the following

a. $f(g(x))$ b. $g(f(x))$ c. $g(h(x))$

3. Which of the functions listed above are inverses of one another? Explain.

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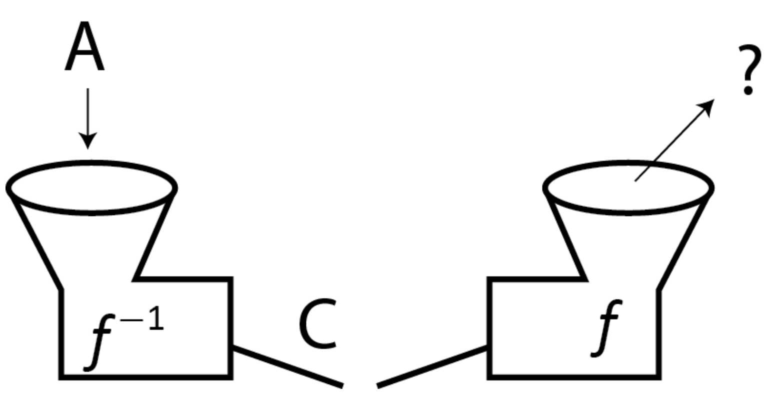
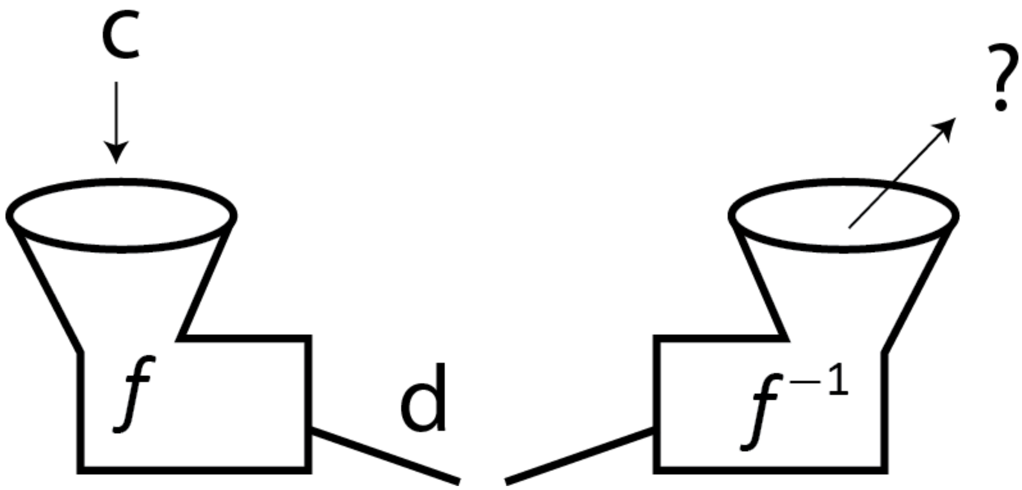
4. Jasmin won a lot of money in the lottery. After her travel expense, she had 1/3 left of it. Then she spent $125 on souvenirs for her family, then she had ½ of the remaining money on her tuition for the next quarter. The remaining money was $500, she put in her savings account. How much money did she win?

5. Use the function machines to fill in the correct output

a)



b)



c) $f(c) = d, then\ f^{-1}(d) =$?

d) $h^{-1}(y) = z,\ \ what\ is\ h(z)$?

APPENDIX B

QUESTIONNAIRE

Questionnaire

1. What did you learn from this lesson?

2. Can you explain the process on how to find the inverse of a function?

3. How can you determine whether the inverse of a function is a function?

4. What did you enjoy most about this lesson? Why?

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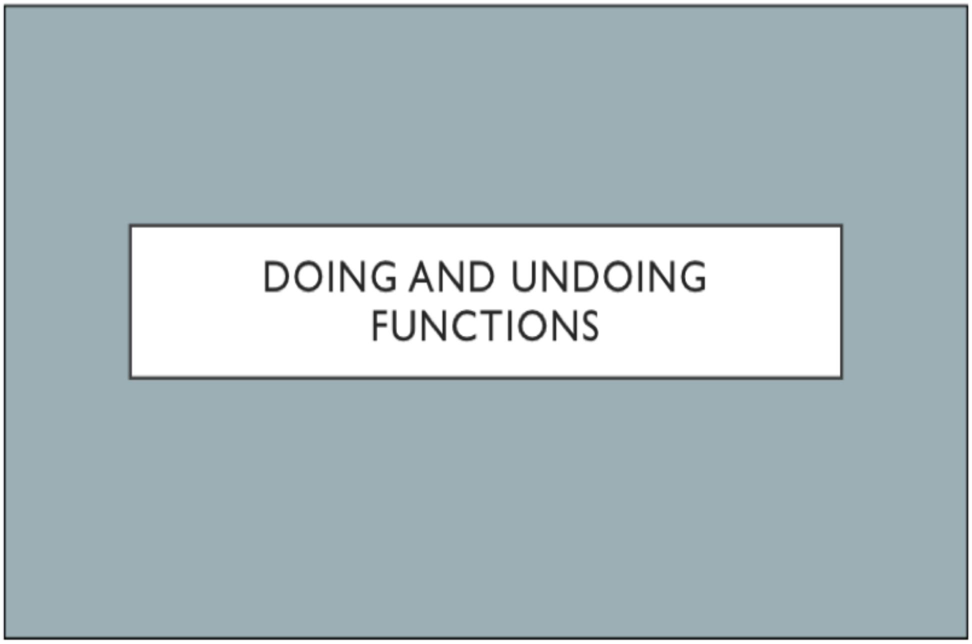
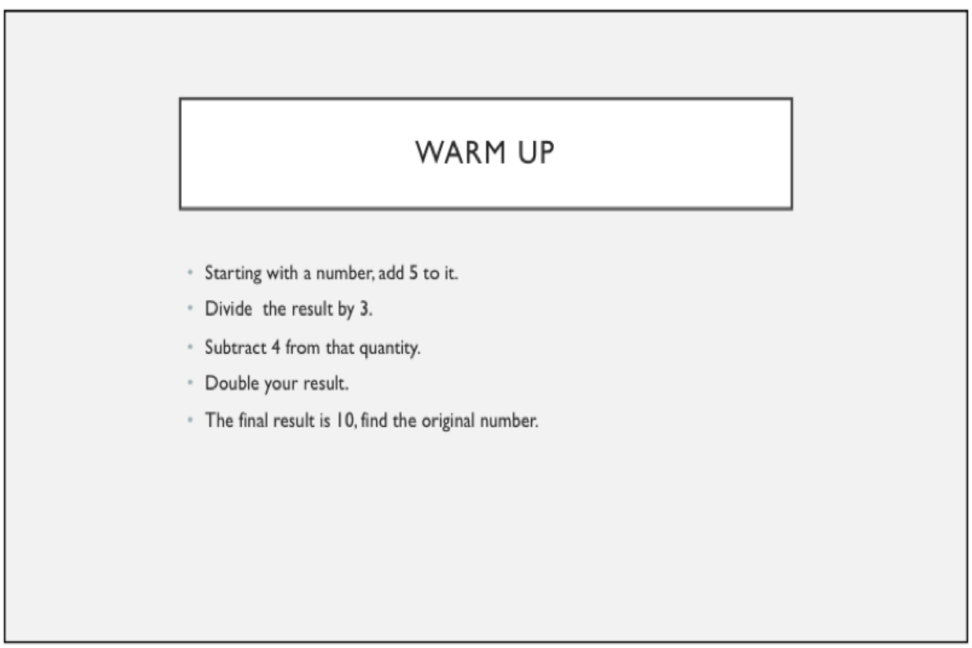
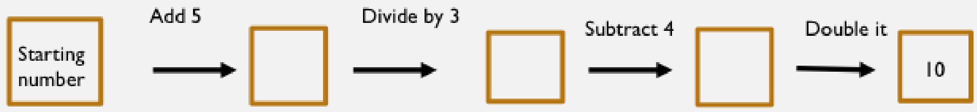
APPENDIX C

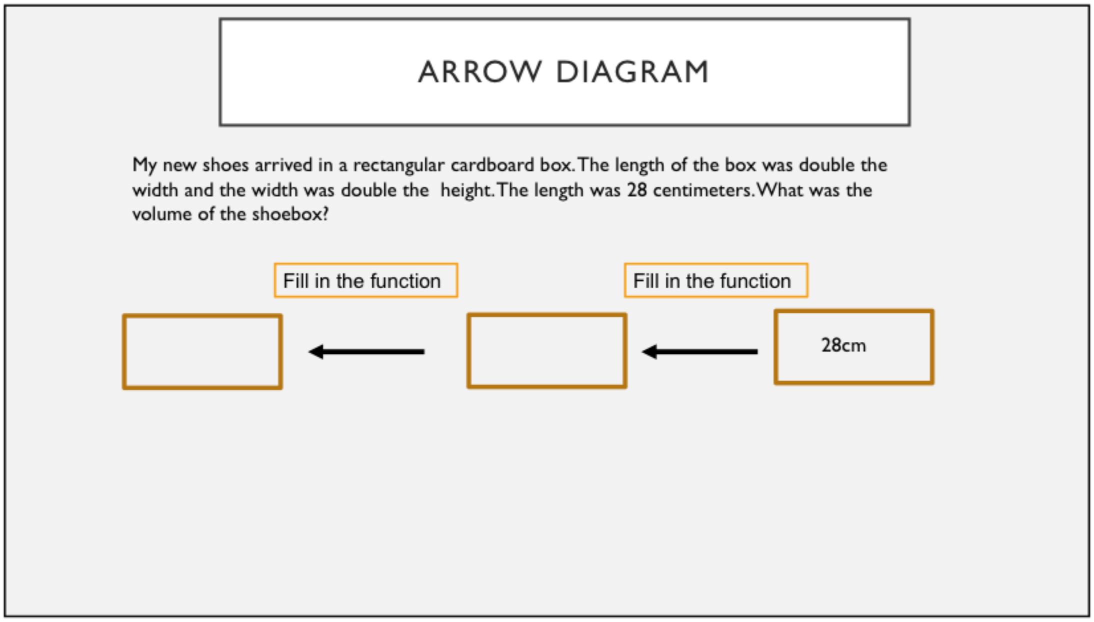
LESSON

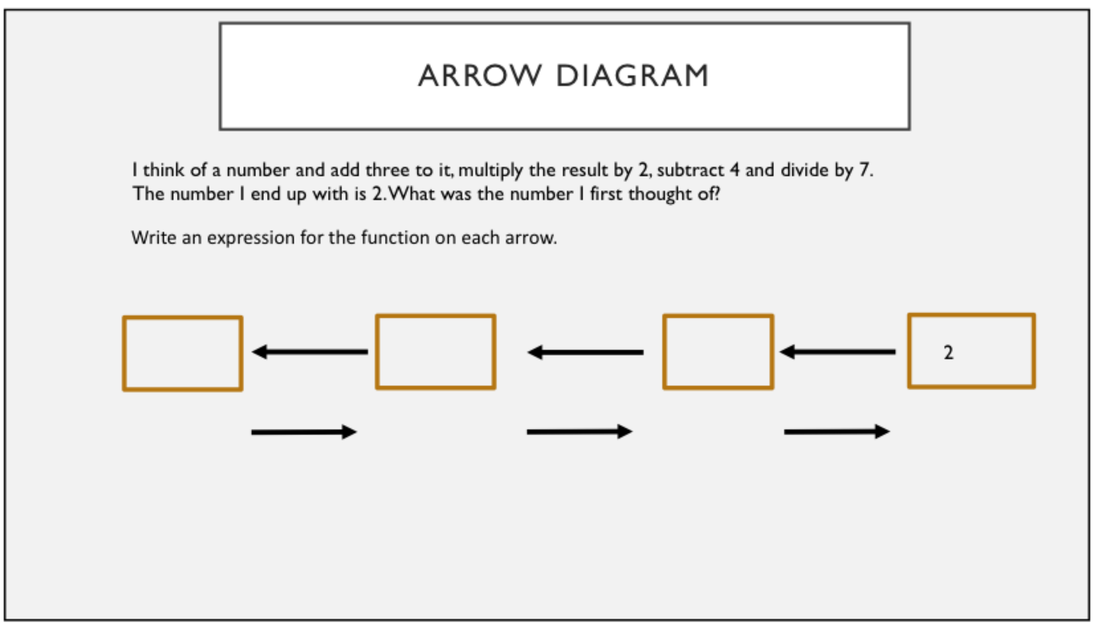

DOING AND UNDOING
FUNCTIONS


WARM UP
Starting with a number, add 5 to it.
Divide the result by 3.
Subtract 4 from that quantity.
Double your result.
The final result is 10, find the original number.



- 1. x+5
- (x+5)/3
- (x+5)/3 -4
- ((x+5)/3 -4)(2)
- 10





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APPENDIX D

IRB APPROVAL LETTER

October 09, 2017

CSUSB INSTITUTIONAL REVIEW BOARD
Full Board Review
IRB# FY2017-132
Status: Approved

Mr. Jesus Nolasco and Prof. Susan Addington
Department of Mathematics
California State University, San Bernardino
5500 University Parkway
San Bernardino, California 92407

Dear Mr. Nolasco and Prof. Addington:

Your application to use human subjects, titled "Inverse Functions in High School Settings" has been reviewed and approved by the Institutional Review Board (IRB). The informed consent document submitted with your IRB application is the official version for use in your study and cannot be changes without prior IRB approval. A change in your informed consent (no matter how minor the change) requires resubmission of your protocol as amended through the Cayuse IRB system protocol change form.

Your application is approved for one year from October 09, 2017 through October 08, 2018. Please note the Cayuse IRB system will notify you when your protocol is due for renewal. Ensure you file your protocol renewal and continuing review form through the Cayuse IRB system to keep your protocol current and active unless you have completed your study.

Your responsibilities as the researcher/investigator reporting to the IRB Committee include the following 4 requirements as mandated by the Code of Federal Regulations 45 CFR 46 listed below. Please note that the protocol change form and renewal form are located on the IRB website under the forms menu. Failure to notify the IRB of the above may result in disciplinary action. You are required to keep copies of the informed consent forms and data for at least three years. Please notify the IRB Research Compliance Officer for any of the following:

1) Submit a protocol change form if any changes (no matter how minor) are proposed in your research protocol for review and approval of the IRB before implemented in your research,
2) If any unanticipated/adverse events are experienced by subjects during your research,
3) To apply for renewal and continuing review of your protocol one month prior to the protocols end date,
4) When your project has ended by emailing the IRB Research Compliance Officer.

The CSUSB IRB has not evaluated your proposal for scientific merit, except to weigh the risk to the human participants and the aspects of the proposal related to potential risk and benefit. This approval notice does not replace any departmental or additional approvals which may be required. If you have any questions regarding the IRB decision, please contact Michael Gillespie, the IRB Compliance Officer. Mr. Michael Gillespie can be reached by phone at (909) 537-7588, by fax at (909) 537-7028, or by email at mgillesp@csusb.edu. Please include your application approval identification number (listed at the top) in all correspondence.

Best of luck with your research.

Sincerely,

Caroline Vickers

Caroline Vickers, Ph.D., IRB Chair
CSUSB Institutional Review Board

CV/MG

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