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A two and three dimensional high school geometry unit implementing recommendations in the National Council of Teachers of Mathematics curriculum and evaluation standards

Stella Sloan

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A TWO AND THREE DIMENSIONAL HIGH SCHOOL GEOMETRY UNIT
IMPLEMENTING
RECOMMENDATIONS IN THE NATIONAL COUNCIL OF TEACHERS OF
MATHEMATICS CURRICULUM AND EVALUATION STANDARDS

A Project
Presented to the
Faculty of
California State University,
San Bernardino

In Partial Fulfillment
of the Requirement for the Degree
Master of Arts
in
Education

by
Stella Sloan
June 1993
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ABSTRACT

The following was a project on spatial visualization and high school geometry. The current geometry curriculum problems addressed were that of students' inability to see the usefulness of geometry in their lives, in professions, and their inability to see relevance to reality. Other curriculum problems addressed included teaching methods, textbook approaches used, and the failure rate of high school geometry students. The problems with spatial visualization included the presentation of problems in textbooks assuming students were able to spatially visualize and the neglect of spatial visualization exercises.

The goals of this unit were to implement the National Council of Teachers of Mathematics Curriculum and Evaluation Standards (NCTM Standards) in a field tested unit as they related to spatial visualization and real world applications with a hands-on approach of two and three dimensional geometry.

The research design objectives were to develop a high school geometry unit that would focus on the applicable skills and pedagogical strategies from the NCTM Standards. Another design objective was to improve student attitudes toward geometry and its application.

The research design of this unit was to develop and assess a curriculum using hands-on and direct application skills, incorporating pedagogical strategies from the NCTM Standards, and
addressing attitudinal outcomes through the relevance of the topic to the students' world.

The literature regarding mathematics and geometry achievement was related to spatial visualization, cognitive structure related to learning geometry, spatial visualization development skills, and the use of manipulatives. The field tested unit was designed based upon the literature and the NCTM Standards. Upon completion, the data from each lesson was analyzed. The student assessment was analyzed. The assessment included a spatial visualization test and student journal entries.
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INTRODUCTION

The National Council of Teachers of Mathematics (NCTM) has long provided leadership in curriculum, instruction, and teacher education (Crosswhite, 1990). History of school mathematics in this country has been characterized by movements to extremes, such as "basic skills or concepts, the concrete or the abstract, intuition or formalism, structure or problem solving, and induction or deduction" (Crosswhite, 1990). According to Crosswhite, there should be a reasonable balance between these elements.

Motivated by this need for a balance, in 1980, the NCTM developed a set of recommendations for school mathematics that became An Agenda for Action. The Agenda was written at a level of generality that was difficult to translate into criteria for program evaluation. An attempt to evaluate the NCTM Agenda was a motivation for development of the NCTM Curriculum and Evaluation Standards for School Mathematics (NCTM Standards). The Standards were intended to define criteria of excellence.

Thomas A. Romberg cites two sources of concern and evidence that the current system is not working. First is the bleak national performance data. The National Assessment of Educational Progress results show that most students are proficient in computational skills, but are unable to apply those skills. Secondly, our schools are failing to educate students to be productive employees in the current work place. The industrial era has ended and our school mathematics curriculum still reflects the industrial needs of the
According to John B. Walsh, the objectives found in the NCTM Standards address inadequacies in the teaching of mathematics which result in problems such as, 1) a nation of mathematical illiterates, 2) early childhood interest in mathematics disappears and 3) bright students are not nurtured. Walsh states that these problems are serious to our industry and our nation as a whole (Walsh, 1990).

In the development of the NCTM Standards, the commission of standards for school mathematics had two tasks in mind. The first was to create a coherent vision of what it means to be mathematically literate in a world where mathematics is rapidly growing and is extensively being applied in diverse fields. The second was to create a set of standards to guide the revision of the school mathematics curriculum toward the above mentioned vision. The commission was also concerned that the "final product represent a consensus of the mathematical science education community on what students in American schools should know and be able to do as a result of their study of school mathematics" (Dossey, 1989). NCTM indicated that ensuring quality, indicating goals, and promoting change were three important reasons to formally adopt a set of standards. A past president of NCTM referred to the NCTM Standards as the completion of a task that culminated five years of planning and development (Crosswhite, 1989).

The NCTM Standards describe a vision for school mathematics, but they do not prescribe a curriculum (Crosswhite, 1989). The
researcher of this unit proposes to implement the NCTM Standards related to problems identified in the high school geometry classroom. Geometry has always been an important strand of school mathematics. The NCTM Standards focus on making sense of real world situations and problems and emphasize that geometry not be addressed as a collection of abstract ideas and procedures to be memorized (Hirsch, 1990).

One of the standards in the 9th through 12th grade section (p. 157) deals with geometry from a synthetic perspective. Synthetic geometry is based on segments, angles, triangles, quadrilaterals, polyhedra, and so on. This standard focuses on providing experiences that deepen the students' understanding of shapes and their properties. According to the standards examples should be provided on how geometry is used in recreation, practical tasks, sciences, and the arts. Activities to develop spatial skill are mentioned in the standards as fundamental to everyday life.
RESEARCH QUESTION AND RESEARCH DESIGN

Problems in Current Geometry Curriculum

Many high school students drop out of mathematics when they reach geometry. Students are turned off to mathematics because they do not see the usefulness of geometry in their everyday life or in the professional careers they are contemplating in pursuing. Teaching methods in most high school geometry classes consist of lecture and independent practice in textbooks that give little or no attention to spatial visualization skills.

During October 1992, at the beginning of a year long geometry course in a Southern California high school, the author of this study surveyed two geometry classes. Fifty four students were asked to complete the following phrase, "I feel that geometry is ...." (Refer to table 1). Of the fifty four students that responded, six percent responded that geometry is necessary for college. They saw the course as one of many prerequisites necessary to reach their goal. There were fifteen percent who responded that they enjoyed geometry and found it useful. Eight percent were already turned off to geometry. The students responded by stating that they didn't like geometry, they didn't understand it and they didn't care. The majority of the students, sixty three percent, did not respond that they liked or disliked the subject; they simply expressed their feelings of frustration, confusion, lack of understanding, and inability to see the relevance of geometry to real world situations.
These surveys suggested that the NCTM Standard focusing on wide applicability of geometry in human activity was a neglected area in these students experience.

The NCTM Standards recommend that instructional methods such as proofs (p. 127), receive decreased attention at the high school level. This type of instructional method is still in practice in most of our high school geometry classes. The teacher and textbook are exclusive sources of knowledge. Students are still required to memorize theorems, postulates, and definitions. Individual seat work practicing routine proofs is still found in the geometry classroom. The teacher still does most of the instructing in the form of lecture, and testing is still the preferred method used to give a grade. Under these current methods the NCTM Standards refers to the teacher as "director" and "one who dispenses information". The Standards stress that by using alternative methods of instruction, students can approach learning more creatively and independently which will strengthen their confidence and skill in doing mathematics (p. 128).

During the 1991-92 school year at a Southern California high school, located in a rural area in Riverside County, California, with a student population of approximately three-thousand students, one-hundred-eighty-three signed up for geometry. Of those one-hundred-eighty-three students in geometry one-hundred-seventeen completed the course, of those one-hundred-seventeen that finished the course, twenty-four failed and seventeen received a grade of "D"
(Refer to table 2). Of the one-hundred-eighty-three students that took the course, approximately forty-one percent completed it successfully, suggesting too many students were being lost at this level of mathematics.

**Problem with Student Ability to Picture Three Dimensional Figures**

Current textbooks such as *Geometry* by Houghton Mifflin and *Geometry* by Merrill have lessons on area, surface area and volume of geometrical solids with the assumption that students can mentally picture the object at different angles. The Merrill book has three sections (pp. 387-398) on finding areas and volumes of spheres, pyramids, prisms, cylinders, cones, pyramids within prisms and cones within cylinders. All of these are interesting and very useful for the student in the future and in future mathematics courses. However, the text does not address the spatial visualization skills necessary for the student to be successful in these sections involving areas and volumes. The Houghton Mifflin book devotes chapter 10 (pp. 422-459) to areas and volumes of usual and unusual three dimensional solids. By teaching students to memorize a formula, the subject matter is reduced to a lower cognitive level and understanding does not occur. Students need to fully understand what they are doing and why they are using the formulas provided by the textbooks. The student needs to apply his/her knowledge of finding areas and volumes to many other situations such as finding
the volume of an unusual solid where the direct application of a formula does not suffice.

**Project Goals**

The NCTM Standards directly address the need to provide students with mathematical experiences that are applicable to human activity and that provide opportunities with three dimensional objects to develop spatial visualization skills (p. 157). The goals of this project are, 1) to implement the National Standards as they relate to spatial visualization and hands on real world applications in geometry, and 2) to develop a field tested high school geometry unit focused on developing spatial visualization skills in the context of real world applications with a hands on approach of two and three dimensional geometry.

**Research Design Objective**

The research design objectives of this project are to develop a field tested unit that will improve the students' spatial visualization skills and focus on other applicable skills and pedagogical strategies found in the National Standards. The Standards refer to the development of spatial skills as fundamental to everyday life and careers (p.157). Developing the students' skill to solve problems in their environment is another skill that will be a focus of this field-tested unit. The Standards indicate that the work involved in the students solving problems in their own
environment enables them to approach learning independently and creatively (p. 128). Other applicable National Standards that will be addressed are active involvement of the students in the construction and application of geometrical ideas (p. 129), and interpreting and drawing three dimensional objects.

Traditionally, learning has been conceived as passive absorption of information. Skills are taught as a precursor to solving problems. The NCTM Standards state that students approach a new task with prior learning and if the problem is presented first the student will recognize the need to apply a concept or procedure (p. 10). One of the research design objectives of this project makes use of a variety of pedagogical strategies such as small group work (cooperative learning), project work, and active involvement of students constructing and applying geometry.

The attitudinal objectives are, 1) to empower the student to break from inappropriate mind sets through the development of spatial visualization skills, 2) to make students comfortable in taking risks when attempting to solve problems, 3) to show students the importance of geometry in a variety of professional fields, and 4) to improve the students attitude toward geometry.

Research Design

In focusing on the active involvement of students in constructions and applications of geometrical ideas this field tested unit will include, construction of two dimensional polygons with
compass and straight-edge and paper folding activities to verify and review geometrical concepts. Students will move from two dimensions to three dimensions by using previously constructed two dimensional polygons to make three dimensional polyhedra. The polyhedra models created by the students will be used to verify and/or discover properties of three dimensional solids. Students will also use their own solids to sketch two dimensional representations of the models on isometric paper. Students will use isometric and orthogonal representations to create three dimensional models. Using dot paper, students will move back to two dimensional geometry through discovery learning of perimeter and area of triangles and quadrilaterals. Surface area and volume of polyhedra will be determined through the use of two dimensional isometric drawings and construction of three dimensional models with plastic cubes.

Students inquire naturally when they are puzzled. Puzzles directly related to spatial visualization and real world applications will be used as introductions throughout this unit. Students will be allowed to discover new properties or concepts on their own through their constructions or models. Cooperative learning groups will be used to give students the opportunity to discuss, formulate and compare ideas. Curriculum textbook authors agree that learners need to apply what they learn. Kinesthetic hands on and minds on learning is important. Daniel Tanner and Laurel N. Tanner (1980) say that, "there is no mastery without intelligent application"(p. 640).
Bruce Joyce and Marsha Weil (1986) put it this way, "It is the learner's activity that results in the learning. Practice is important for the learner to make the necessary connection" (p.430). This field tested unit will focus on kinesthetic hands on learning. Cooperative learning groups will be used for the students to create projects.

Emotional effects of certain kinds of experiences and direct intellectual processes affect student attitudes. The hands-on activities of constructing and working with blocks in this unit will give the students the opportunity to develop their ability to spatially visualize. Developing this skill could provide the students with the ability to break from an inappropriate mind set when presented with a problem related to area and volume. Satisfying experiences in a particular connection improves attitudes. A satisfying experience can come from success. In turn, success can make the students more comfortable in taking risks. The activities of this field-tested unit will provide students with opportunities to succeed.

This project will focus on applying the topics to the students' world by connecting their interest and creativity to the concepts and procedures. Students will develop an understanding of the need for spatial visualization in their own world and in a variety of professional fields. The Standards make reference to the importance of relating topics to situations in the students' world and encouraging them to explore, formulate, prove, and discuss.
According to the Standards, this allows the students to approach learning mathematics creatively and independently. This, in turn, strengthens the students' confidence in doing mathematics (p. 128). Student attitudes improve when they develop an understanding.

At the conclusion of this field tested unit, the researcher will determine student outcomes by giving students a spatial visualization test. The same test will be given to a control group and the results compared between groups. Attitudinal outcomes will be measured through reporting in student journals. Students will be asked to react to the need and uses of geometry in their own life and in professions/careers. The field tested unit and each lesson will be analyzed as to its effectiveness in accomplishing goals and objectives. The effectiveness in accomplishing the goals of this unit will be analyzed by the student journal responses on the usefulness of geometry in their every day life and in professions/careers.
Mathematics Achievement Related to Spatial Visualization

Spatial visualization and mathematics achievement have been shown to correlate (Bishop, 1980). Researchers like Fennema and Tartre (1985) disagree on the reasons for this correlation. They point out that studies showing a relationship between spatial visualization and problem solving use spatial components to establish that relationship. This reinforces the concept that there is a direct relationship between spatial visualization and mathematics that involves spatial components such as measurement and geometry. Fennema and Tartre claim that the relationship of spatial visualization and a broader spectrum of mathematics (such as functions, statistics, algebra, probability, and discrete mathematics) is unclear.

According to Werdelin (1961), an individual who is able to attack a mathematical problem verbally or spatially is more apt to solve it than an individual who has low spatial or verbal skills. In their study, Fennema and Tartre (1985) addressed five concerns in students with discrepant spatial visualization and verbal skill: 1) Ability to solve mathematical problems; 2) Ability to verbalize relevant information; 3) Ability to translate symbols into pictorial representation; 4) Ability to use spatial visualization skills overtly during mathematical problem solving; and, 5) Ability to use pictorial representations during mathematical problem solving. Their study
involved using a problem-solving process emphasizing the use of spatial visualization skills. Between students with high verbal skills and students with high spatial skills, the researchers found there was no statistically significant difference in their ability to solve mathematical problems. In verbalizing relevant data, the students with high verbal skills outperformed students with high spatial visualization. The students with high spatial visualization were able to translate symbols into pictures better than the students with high verbal skills. They also used more mental movement than the students with high verbal skills. Students with high spatial visualization were better able to draw and use pictorial representations. As a conclusion to their study, Fennema and Tartre suggested that emphasizing spatial visualization may not be an effective way to get correct solutions.

Five years later Tartre (1990) made a distinction in the different types of spatial skills. She named three spatial abilities as identified by Linn and Peterson (1985). The three spatial skills were spatial perception, mental rotation, and spatial visualization. Tartre's study focused on the relationship between spatial orientation (rather than spatial visualization) and mathematics. Any mental movement of the object was considered to be spatial visualization. Spatial orientation described the tasks necessary for the individual to mentally readjust his or her perspective, such as seeing the object from a different angle. Tartre referred to spatial orientation as an "intuitive or insightful spatial organizational
process”. Her study identified ways in which students with high or low spatial orientation skills behave differently when solving mathematical problems. Tartre gave students ten mathematical problems to solve. Seven of the problems concerned geometric content. She used the Gestalt Completion Test to measure spatial orientation skill. Estimate error and drew relation were two coded categories that Tartre hypothesized to be related to spatial orientation skill. Estimate error was used to indicate the difference between the correct answer and the student's estimate. A zero would indicate that the student’s estimate was the exact answer. Drew relation indicated that the student used markings or drawings to show a mathematical relation. The mean for the high spatial orientation group was lower than the mean for the low spatial orientation group for estimate error. This meant that the group with high spatial orientation was better able to estimate. There was a significant difference favoring the high spatial orientation group in drew relation.

Tartre concluded that students with low spatial orientation skills are not flexible in changing a formed perceptual mind set. The students with high spatial orientation skills could demonstrate a way to analyze a problem by adding marks or drawing. Students with high spatial orientation skill were better able to estimate answers in geometry by breaking up the big picture into little parts.
Geometry Achievement Related To Spatial Visualization

Studies that contradict Bishop's (1980) research have shown there may not be a direct relationship between spatial visualization and mathematics achievement (Fennema and Tarte, 1985). However, when studying geometry, high school students with high spatial visualization skills tended to do better than those geometry students with low spatial visualization skills (Battista, 1990). Spatial visualization is defined as the ability to mentally manipulate a pictorially presented object related to mathematics.

Battista (1990) reported on a study that investigated the role that spatial visualization plays in the performance in high school geometry. His study involved one-hundred-forty-five high school students. Tests were given in spatial visualization, logical reasoning, geometry knowledge, and geometric problem solving strategies. Scores were recorded for each of the following nine areas; spatial visualization (SV), logical reasoning (LR), geometry achievement (GEOM), geometric problem solving (PS), drawing strategy (D), visualization without drawing (V), nonspatial strategy (N), correct drawings (DRAW), and discrepancy between a students' spatial score and logical reasoning score (DISCREP). Intercorrelations between scores were calculated to determine relationships. Battista found that spatial visualization and logical reasoning were significantly related to geometry achievement and geometric problem solving. Spatial visualization was significantly related to D, V, N, and DRAW. Battista's (1990) study also showed
that males scored significantly higher than females on spatial visualization, geometry achievement, and geometric problem solving.

Fennema and Sherman (1977) disagreed with findings that indicated males score higher than females in mathematics achievement. They found that such studies had not controlled for the individual's previous study of mathematics. They gathered information for mathematics achievement, verbal ability, and spatial visualization. They also gathered information on eight affective variables as well as the number of mathematics related courses taken, number of space related courses taken, and the amount of time spent outside of school in mathematics related activities. Fennema and Sherman found that by covarying out the differences in affective measures eliminated the sex-related differences in mathematics achievement. They found that spatial visualization was as important to achievement as verbal ability. When spatial visualization was covaried out, it eliminated existing sex-related differences. However, when the difference between the sexes in the number of space related courses taken was covaried out, it eliminated the sex-related differences in spatial visualization. The authors concluded that practice and relevant experience are the factors in the differences between the sexes in spatial visualization.

Cooper and Sweller (1989) examined the ability of students to interpret different representations of three-dimensional solids. In
an earlier study Gaulin (1985) found that children represent three dimensional solids in a variety of verbal, graphic, and mixed ways. Cooper and Sweller gave seventh, nineth and eleventh grade students eight different modes of instruction to assemble a three dimensional solid. The eight modes were as follows; 1) layer plan instructions, 2) coded plan instructions, 3) coordinates instructions, 4) elevation instructions, 5) verbal descriptions, 6) perspective drawing, 7) verbal quasi-coordinate description instructions, and 8) instruction card for a prototype. They found that students more easily interpreted verbal descriptions, perspective drawings, and actual solids. Perspective drawings and actual solids were more easily interpreted than verbal instructions. There was no evidence that students interpreted more easily prototypes than perspective drawings. The authors suggested that teachers and textbook writers needed to keep this in mind when choosing representations of three dimensional solids. They did state that instruction and practice should be considered for orthogonal (such as layer plan, coded plan, and elevation instructions) representations of three-dimensional solids since they are common.

Cognitive Structure In Learning Geometry

Students structure geometric content differently. McDonald (1989) determined the relationship between students' cognitive development level and the way in which they structured geometric content. Her study involved forty tenth grade geometry students. Twenty were classified as formal operational students and the other
twenty as concrete operational students. Formal operators were defined as those students who were "able to structure the abstract principles within a given domain". The forty students were drawn from a pool of one-hundred-sixty-one students who were given a Piagetian formal reasoning test to determine their cognitive level. To measure their cognitive structure they were given two tasks to measure their understanding of similarity. The twenty students with the highest combined score were identified as formal operational and the twenty students with the lowest combined score were identified as concrete operational. Five mathematics educators were used as the expert group. The expert group was also given the two tasks. To identify the way in which an individual organizes subject matter, the results were recorded in map form (matrices). The expert matrix was then compared to that of the concrete and formal operational groups. The results of this study showed significant differences in the way in which the students organized subject matter. The concrete operational group showed confusion with terms in similarity that could lead to confusion of the concept. They also assumed that "relationships among terms persist regardless of context". The formal operational group was able to organize subject matter more like the expert group. As a result of this study, McDonald warned that content may not be properly assimilated if teaching occurs at a level beyond that of the students. Teachers should look for ways to help students organize a framework for the concepts and relationships.
Burger and Shaughnessy (1986) investigated the van Hiele levels of development in geometry. The van Hiele levels include level 0 (Visualization), 1 (Analysis), 2 (Abstraction), 3 (Deduction), 4 (Rigor). The researchers addressed three concerns, 1) whether the van Hiele levels were useful in describing students' thinking process on geometry tasks, 2) whether the levels could be characterized operationally by student behavior, and 3) whether an interview procedure could be developed to reveal predominant levels of reasoning on specific geometry tasks. Burger and Shaughnessy's study involved forty-five students from three different states. They ranged from early primary students to college mathematics majors. The subjects of this study were interviewed in depth on triangle and quadrilateral concepts. During the interviews the students were asked to complete tasks which included drawing shapes, identifying and defining shapes, sorting shapes, determining a mystery shape, establishing properties of a parallelogram, and comparing components of the mathematical system. The students were taped during the interviews. Three researchers reviewed each tape and assigned a van Hiele level from 0 to 3 for each task to each student. The researchers then assigned a van Hiele level of reasoning to each student. The researchers found that the levels were useful in describing students' thinking process on polygon tasks. They stated that it would be necessary to investigate student responses on other geometry tasks to determine if the van Hiele levels were useful in
other studies of geometry. Consistent behavior was found among
students assigned the same level on specific tasks.

Berger and Shaughnessy (1986) found that secondary students
were not grounded in basic geometry concepts. Students that were
at different levels used different problem solving approaches and
different language. According to the authors, if teachers were
teaching at a different level than that of a student, the result could
be a lack of understanding and consequently frustration and
In her study, Senk found that high school students' achievement on
standard nonproof geometry content was positively related to van
Hiele's levels of geometric thought. She also claimed that it was
possible for students, teacher, and textbook to be at different levels
in any one class.

Developing Spatial Visualization Skills

In their article, Talsma and Hersberger (1990) described a
course for geometry students, including instructional approaches
and exemplary materials. They found that even the brightest pre-
geometry students lacked adequate background in geometry. They
stated that lack of geometry experiences was one reason that
students did not progress from one van Hiele level to the next. One
of their goals was to enhance the students' spatial ability. Their
curriculum outlined activities using work with rotational and
reflectional symmetry using paper folding and three dimensional
solids.
In their article on implementing the standards Senk and Hirschhorn (1990) described how to teach similarity consistent with the van Hiele model. They started with the visual model where students drew figures and enlarged them by multiplying each side by a constant. They also showed how to do this with coordinates. They moved to the theoretical level, where similarity was done again by enlarging but transformation and rotation was included. The authors then showed how the lesson could move to show connections and extensions. They modeled this by using the parabola.

**Manipulatives**

Friedman (1978) reported on the findings of several researchers on the topic of manipulatives in the learning of mathematics. Most of the results reported no significant differences between scores of students instructed with manipulatives and scores of students instructed with a non-manipulative approach. These studies involved the use of manipulatives in the learning of algorithms such as multiplication. The only studies that showed a significant difference in favor of the manipulatives approach were those involving children in the first grade or younger. Friedman stated that educators should not careen after the latest trend. However, he did mention that we should continue efforts to determine the situations in which manipulatives used was most effective.

Sowell (1989) compared the results of several studies to determine the effects of using manipulatives in the learning of
mathematics. He found that there was a significant difference in favor of the group using manipulatives when the use of manipulatives lasted the entire year. He also found that studies where the use of manipulatives were favored were taught by teachers with extensive training or college professors. Cooper and Sweller (1989) found that "perspective drawings of three dimensional objects are internalized in the same way as the objects themselves."
Lesson On The Need For Developing Spatial Visualization Skills

LESSON I: THE NEED FOR SPATIAL VISUALIZATION SKILLS

Objective: Through group and class discussion, students will determine the need for spatial visualization and spatial orientation skills.

1. Through class discussion, the definition of spatial visualization as the ability to mentally manipulate an object will be determined.

2. In their groups students will have the task of determining as many professions or activities that use spatial visualization. Students will be asked to explain why the chosen professions or activities involve spatial visualization and spatial orientation skills.

3. As a class, the results will be put on the board and discussed.
Lessons on Developing Spatial Visualization Skills

LESSON II: POLYGONS

Objective: Students will discover the sum of the interior angles of any n sided polygon and the measure of each interior angle when the polygon is regular.

1. Review: Practice of previously learned material will include using only a straight edge and a compass to construct congruent segments, congruent angles, and perpendicular bisectors.

2. Exploring sum of the measures of the angles in various triangles: Students will be given the task of constructing several different triangles, acute, obtuse, isosceles, right, equilateral and scalene. Using a protractor, students will be asked to determine the sum of the angles of each triangle.

3. Verifying the sum of the measures of the angles in a triangle: The teacher will ask students what other geometrical figures have 180°. Students will be guided to discuss the straight line and supplementary angles. Students will be asked to conclude from their experiments, what the sum of the angles in a triangle is. Teacher will verify that the sum of the measures of the angles of a triangle appear to be 180°. In their groups, students will be given task of
verifying the sum of the angles in a triangle through paper folding. Students will do this with several different types of triangles. See figure 1 for example of paper folding.

4. **Students will verify and explain** the formulas for the sum of all the angles in any convex polygon and the measure of each interior angle of a regular polygon. Students will determine that the sum of the measures of the angles of any n sided convex polygon is \((n-2)180\) and each angle of a regular polygon is \([(n-2)180]/n\) by completing worksheet 1 (See worksheet 1). Teacher will guide students with the first 2 (or 3).

5. **Extension (Student exercises):**
   
a. Students will be asked to make a model of as many regular polygons as they can using only a straight edge and compass.

   b. Students will construct six congruent squares and four congruent regular triangles using only a compass, a straight-edge, and a pair of scissors.

6. **Application:** Students will be given the task of noting the different road signs along their way home. These signs will be discussed in class the following day. In their groups, students will determine the measure of each angle of some of the signs and
discuss the possible reasons for selecting a particular sign for a particular purpose.

7. **Evaluation:** Students will be asked to complete a log. The log will contain three parts. In part 1, the student will express what he/she has learned (Today I learned...). In part 2, the student will express what puzzles or seems unclear to him/her (Something that puzzles me is...). In part 3, the student will write what new questions he/she has concerning the lesson (Something I would like to know more about is...).

8. **Teacher Recommendations:** In order to connect this lesson to the students' world an application problem and/or question should precede the lesson. The problem and/or question should relate to the students' world and the objective of this lesson. One such question could be as follows:

   What types of polygons are a part of our everyday life?
FIGURE 1
<table>
<thead>
<tr>
<th># OF SIDES</th>
<th>NAME</th>
<th>SKETCH</th>
<th># OF Δs</th>
<th>SUM OF ALL INT. ANGLES</th>
<th>(REGULAR) EACH INT. ANGLE</th>
<th>(REGULAR) EACH EXT. ANGLE</th>
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<tbody>
<tr>
<td>3</td>
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LESSON III: POLYHEDRA

Objective: Students will practice their spatial visualization skills.

1. Review from lesson I: Students will have the task of creating three dimensional solids with the six squares and four triangles from lesson I. The teacher will point out to students that the tetrahedron and the hexahedron (cube) are referred to as platonic solids or regular polyhedra because their faces are all congruent regular polygons and their polyhedral angles are all congruent.

2. Exploring the Collapsed Cube: In their groups, students will sketch pattern layouts of the collapsed cube on orthogonal paper for the cube. The class will discuss some examples of pattern layouts that are the same, but flipped and/or rotated. Students will be asked to make as many different layouts as possible. (See figure 2)

4. Exercising visualization with two dimensional sketches of cube: Students will sketch the cube on isometric paper. Students will be asked to:
   B. imagine a cut through the midpoints of RS, ST, and SX (See Figure 3).
   C. describe the shape of the intersection and sketch the new cut solid on isometric paper.
D. imagine a cut through the midpoints of RS, ST and XY (See Figure 4).
E. describe the shape of the intersection and sketch the new cut solid on isometric paper.
F. imagine a cut through the midpoints of RU, UT, TW, WX, XY, and YR (See Figure 5).
G. describe the shape of the intersection and sketch the new cut solid on isometric paper.

5. **Exercising visualization with two dimensional sketches of tetrahedron**: Students will sketch the tetrahedron on isometric paper. Students will be asked to:
   A. label their vertices A, B, C, and D on the tetrahedron.
   B. imagine a cut through the midpoint of AD, BD, and CD (See Figure 6).
   C. describe the shape of the intersection and sketch the new cut solid on isometric paper.
   D. imagine a cut through the midpoints of AD, BD, and BC (See Figure 7).
   E. describe the shape of the intersection and sketch the new cut solid on isometric paper.
   F. do an extension of steps D and E by connecting the midpoints of every edge (See Figure 8).
   G. describe the solid within the tetrahedron.
6. **Teacher Recommendations:** Connect the students' world with the objective of this lesson. The following are two suggestions for lesson questions:

Name geometrical solids that are a part of our everyday life from the moment that we wake up to the moment we go to sleep.

As a class or in cooperative groups discuss and list geometrical solids in our world that appear to be cubes with vertices cut off.
LESSON IV: EULER'S FORMULA

Objective: Students will discover Euler's formula for convex polyhedra.

1. From lesson II: Reference will be made to the isometric sketches they did in lesson II. Students will count vertices, edges and faces on the two dimensional sketches they made of tetrahedrons and hexahedrons with cuts made through different midpoints. A list of V, E, and F will be made on the board. Students will be asked to check for any possible pattern.

2. Exploring Euler's formula: Students should make their own three dimensional models of the pyramids and prisms listed in Worksheet 2. Using these models, students will start entering data on Worksheet 2 by counting faces, edges, and vertices. Students will be asked to come up with and make three other polyhedra of their own and complete the table using the information obtained from their own polyhedra.

3. Explain: Students will explain their conclusion from the data they gathered in the above table.

4. Extension and verification with spatial visualization: Students will be asked to use their cubes from lesson II and imagine
a small piece cut off of each of the corners. Students will then count the number of vertices, edges, and faces and determine that the results support Euler's formula.

5. Evaluation: Students will be asked to add to their student log, addressing each of the three parts, (1) Today I learned... , (2) Something that puzzles me is... , (3) Something I would like to know more about is...

6. Teacher Recommendations: If time is short, this lesson could be considered optional. This lesson will help students consider and recognize the various parts of polyhedra but the overall goals and objectives of this unit can be met without this lesson.
<table>
<thead>
<tr>
<th>Polyhedron</th>
<th>Number of Vertices</th>
<th>Number of Faces</th>
<th>Number of Edges</th>
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<tbody>
<tr>
<td>Square Prism</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Square Pyramid</td>
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<td></td>
<td></td>
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<tr>
<td>Hexagonal Pyramid</td>
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</tbody>
</table>
LESSON V: PLATONIC SOLIDS

1. **From lesson II:** Reference will be made to the original cube and tetrahedron constructed by the students. The teacher will point out that there are three other platonic solids besides the cube and the tetrahedron.

2. **Exploring and explaining the number of faces and the shapes of the faces in the remaining platonic solids:** Teacher will inform the students of the names of the remaining three platonic solids, octahedron, dodecahedron and icosahedron. Students will determine the number of faces of each by their name. Worksheet 3 will be completed by the class through discussion:

3. **Extension:** In their groups, students will be asked to construct the octahedron, icosahedron and dodecahedron by developing the best possible layouts of them, cutting them and taping them together.

4. **Evaluate:** Students will be asked to add to their logs.

5. **Teacher Recommendations:** If time is short, this lesson could be considered optional. This lesson can help students identify uses of the platonic solids in the future. For example, Platonic solids are used to describe the shape of some matter. However, the goals and objectives of this unit can be met without this lesson.
# WORKSHEET 3
## PLATONIC SOLIDS

<table>
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<tr>
<th>Regular Polyhedra</th>
<th>Number of Faces</th>
<th>Shapes of Faces</th>
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<tbody>
<tr>
<td>Tetrahedron</td>
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<tr>
<td>Hexahedron (Cube)</td>
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<tr>
<td>Octahedron</td>
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<tr>
<td>Dodecahedron</td>
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<tr>
<td>Icosahedron</td>
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</table>
LESSON VI: APPLICATION AND ASSESSMENT

The Geodesic Dome is a network of triangles joined together to form an enclosed space resembling a sphere. Spheres hold a greater volume than other containers with the same surface area, but they are also stronger even though they have no internal support. The Geodesic Dome was patented by R. Buckminster in 1947 (Jugenson, Brown & Jugenson, p.450-451).

Question for discussion: If Geodesic domes are stronger without internal support and they hold a greater volume, why are they not as popular as homes?

ASSESSMENT: In their groups, students will complete the following project:

1. Using the octahedron from lesson V, make an isometric drawing of it.
2. On the drawing cut each of the six corners off, as if a plane were driven through each corner.
3. Make a three dimensional representation of your new solid.
4. Have students discuss the name of the solid created.
Lessons on perimeter and area of triangles and quadrilaterals

LESSON VII: PERIMETER AND AREA OF SQUARES AND RECTANGLES

Objective: Students will discover how to find the area and perimeter of triangles and four sided polygons such as squares, rectangles, parallelograms, and trapezoids. Students will learn the meaning of area and perimeter.

1. Review: In class discussion, students will review the following:

   or or

   Length = 1 unit   Length = $\sqrt{2}$ units   Length = $\sqrt{5}$

   Area = 1 sq unit   Area = 1/2 sq unit   Area = 1 sq unit

2. Students will explore the length of the sides, perimeter and area on Worksheet 4. They will record their findings on Worksheet 5.
3. **Students will explain** their findings and discover formulas for area and perimeter of a square and rectangle from the above exploration.

LESSON VIII: PERIMETER AND AREA OF TRIANGLES

1. **Students will explore** the length of each side, height, perimeter, and area for each triangle on worksheet 6. They will record their findings on Worksheet 7.

5. **Students will explain** their findings and discover formulas for area and perimeter of a triangle from the figures on worksheet 6.

LESSON IX: PERIMETER AND AREA OF PARALLELOGRAMS

1. **Students will explore** the length of each side, height, perimeter, and area for each parallelogram on worksheet 8. They will record their findings on worksheet 9.

2. **Students will explain** their findings and discover formulas for the area and perimeter of a parallelogram for the figures on worksheet 8.
LESSON X: PERIMETER AND AREA OF TRAPEZOIDS

1. Students will explore the length of each side, height, perimeter, and area of each trapezoid on worksheet 10. They will record their findings on worksheet 11.

2. Students will explain their findings and discover formulas for the area and perimeter of trapezoids from the figures on worksheet 10.

3. Extension: Students will find the perimeters and areas of unusual figures. See worksheet 12.

7. Evaluation: Students will write in their learning logs.

8. Teacher Recommendations: Prior to starting this lesson on areas and perimeter ask a question that will connect this lesson to the students' world. One such question could be as follows:

   What is area? Why do we need to know what area is? Discuss in your groups four different things that take up area and determine how many of them you would need to cover the back wall of this classroom.
WORKSHEET 4
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<td>FIGURE #7</td>
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Formula for perimeter of square__________

Formula for area of square______________
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<th>Length of c</th>
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Formula for perimeter of a triangle __________ area of a triangle __________
### WORKSHEET 9
PERIMETER AND AREA OF PARALLELOGRAMS

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Formula for perimeter of a parallelogram

Formula for area of a parallelogram
**WORKSHEET 11**  
**PERIMETER AND AREA OF TRAPEZOIDS**

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Formula for perimeter of a trapezoid

Formula for area of a trapezoid
## WORKSHEET 12
PERIMETER AND AREA OF UNUSUAL FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
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<tr>
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<tr>
<td>Figure 4</td>
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</tbody>
</table>
LESSON XI: APPLICATIONS AND ASSESSMENT

Office space is leased by square foot. An art gallery on a limited budget would want to rent space with the greatest possible perimeter without increasing the square footage to display art on the wall.

Question for discussion: If the art gallery could only afford 600 square feet, what would be the ideal layout for a maximum perimeter?
In their groups, students will determine other applications. These will be put on the board and discussed as a class.

ASSESSMENT: You have found the perfect paint for your room. It sells for $18.95 per gallon, each gallon paints 400 square feet. Make an isometric sketch of your bedroom with all dimensions. Make a separate sketch for the closet. Determine how much it would cost you to paint your bedroom (& closet). Students will work in their groups, but each student will turn in a project.
Lessons on volume and surface area of polyhedra

LESSON XII: VOLUME AND SURFACE AREA WITH MORE SPATIAL VISUALIZATION

Objective: Students will learn the meaning of volume and how to make two dimensional representations of three dimensional rectangular objects with a given volume.

Lesson Question: Our school doesn't have any lockers. Where in our school is there unused space (volume) that could be used for lockers? How many lockers could we fit in that space? How big is each locker? How much wall space would it take? How much floor space would it take? How much would it cost the district?

1. Review: Students will use their cubes from lesson II to review the area of each face, the total surface area and the volume of the cube if each side had a measure of 1 unit.

2. Explore: In their groups (groups of 4), using plastic cubes, students will build three dimensional representations of the isometric drawings from Worksheet 13. After building, students will draw a duplicate isometric representation of the solid. Students will make net layouts of each figure to verify surface area.
Students will log surface area and volume for each (See worksheets 13 & 14).

3. **Explain:** Each group will log their findings on the board. Class will discuss the relationship (or lack thereof) of surface area with volume.

4. **Extension:** Each group will be given a different volume. Each group will make a three dimensional model of the given volume by using plastic cubes. Each group will log volume and surface area of their model on the board. They will be instructed to double every dimension (length, width, and height). Students will then log their new surface area and volume. The class will discuss the effects of doubling the dimensions on the surface area and volume.

5. **Evaluation:** Students will complete their learning logs.
WORKSHEET 13

1)  

2)  

3)  

WORKSHEET 13
# WORKSHEET 14
SURFACE AREA AND VOLUME OF SOLIDS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Perimeter of Base</th>
<th>Height</th>
<th>Lateral Area</th>
<th>Area of Base</th>
<th>Total Area</th>
<th>Volume</th>
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</table>
LESSON XIII: APPLICATIONS AND ASSESSMENT

A six foot man weighs 175 pounds (volume) and a cross section of his leg bone is approximately .75 square inch (area). If he were twice as large, how much would he weigh and what would be the area of the cross section of his leg bone?

Question for discussion: What kinds of physical problems would giants have?

In their groups, students will discuss and record other applications of surface area and volume. These will be put on the board and discussed.

ASSESSMENT: In their groups, students will discuss what would be an ideal swimming pool. They will make an isometric drawing with all dimensions, determine how many gallons of water it would hold and how much it would cost to fill it.
Overall Assessment

1. A short spatial visualization test will be given to the students in four high school geometry classes (See table 3). Two of the classes will be students with whom this field-tested unit was taught and two classes will be control groups.

2. Students will be asked to complete the same phrase they completed earlier in the year (4 months prior to this field tested unit). That phrase is: "I feel that geometry is..."
ANALYSIS OF DATA

This field-tested unit was taught in two high school geometry classes in a Southern California high school in a rural area of Riverside County, California during four weeks of the spring semester of the 1992/93 school year. The high school had a student population of approximately three-thousand students. The total number of students in the two geometry classes was sixty-seven.

Lesson I: The Need for Spatial Visualization Skills

The objective of lesson I was for students to determine activities and professions/careers that make use of spatial visualization skills. The class started with a discussion of spatial visualization. Some students expressed spatial visualization as the ability to change one's visual angle. They pointed to posters of visual illusions in the room. Students in the class who were also enrolled in drafting were able to explain that spatial visualization was also the ability to move objects and picture them in your mind. This supported one of the conclusions made by Fennema and Sherman (1977). Students enrolled in other classes that involve spatial visualization do better in activities involving spatial visualization. In determining the activities and professions/careers that make use of spatial visualization, students started by listing applications such as architecture and careers such as geometry teaching. It did not take long for students to pick up ideas from each other. They realized on their own the many applications of spatial visualization in professions/careers and in their everyday life activities such as
video games and driving. (See Table 4). This lesson was effective in achieving the objective because the need for spatial visualization was determined by the students themselves.

**Lesson II: Polygons**

The objective of this lesson was to have students discover on their own the formula for finding the sum of the measures of the interior angles of any convex polygon, the measure of each interior angle of a regular polygon and the measure of each exterior angle of a regular polygon. Students had the opportunity to do some hands-on activities. They measured angles and verified the sum of the angles of a triangle through paper folding. Traditionally, sum of angles in a triangle was taught by presenting it verbally or written as a theorem. In doing so, we deprived the students of geometry experiences (Talsma and Hersberger, 1990). Most of the students were already aware that the sum of the angles in a triangle is 180°. After doing the actual measuring and paper folding students were confident and were eager to help each other in determining how to fold the obtuse triangles. They used these verified facts and Worksheet 1 to determine the equations for angles of polygons. Since they were confident in the sum of the measure of the angles in a triangle, they were able to easily apply that knowledge to determine facts of any polygon. See Worksheet 1. Students were able to note applications of this by making note of road signs, their shapes, and their angles.
Lesson III: Polyhedra

The objective of this lesson was to have the students practice their spatial visualization skills. Students created three dimensional solids and sketched two dimensional representations of the solids on isometric dot paper. Students practiced visualizing planes cutting into the solids. Once again two dimensional representations were drawn of the cut solids. Students created three dimensional solids of the sketched cut solids. Cooper and Sweller (1989) stated in their study that students more easily interpreted perspective drawings and actual solids. Students did a great deal of interacting and helping each other in their groups during this lesson. Some students were better at visualizing three dimensional objects and sketching representations of them. Most of the students that were better at this were also enrolled in drafting or art. This supported findings by Fennema and Sherman (1977) that students that tend to do better at spatial visualization tasks have had other classes or experiences that involve spatial visualization. In their learning logs, students reported that they learned and enjoyed practicing spatially visualizing. Some students questioned (in their journals) why the beginning of geometry could not be more "like this". The first semester of this geometry course emphasized definitions, theorems and formal proofs. This lesson appeared to be successful in achieving the objective based on the student journal entries. Students experienced geometry by direct hands-on constructing and sketching.
Lesson IV and V: Euler's Formula and Platonic Solids

The objectives of these lessons were to have students discover Euler’s formula (vertices + faces = edges + 2) and to have them determine the number of faces, edges and vertices of the five platonic solids. After making the table of vertices, faces, and edges students were able to determine that \( v + f = e + 2 \). See worksheet 2. Students were then given the opportunity to create a three dimensional solid of their own to verify the formula. Students applied this to determine the number of faces, edges and vertices in the octahedron, dodecahedron, and icosahedron. Students formulated on their own. They also constructed three dimensional solids. According to the NCTM Standards, students can approach learning creatively and independently through these methods of instruction (p. 128). The students expressed that they enjoyed learning by "making things". One question asked by several students was, "Can we do things like this for the rest of the year?"

Lesson VII - X: Lessons on area and perimeter of triangles and quadrilaterals

Students were given worksheets with triangles, squares, rectangles, parallelograms and trapezoids on dot paper. Students had to determine perimeters by counting along the edges and areas by counting squares within the polygon. The developing of formulas was left up to the students. Students were at different cognitive levels. Some students were still at the first van Hiele level of visualization. These students were not sure what it was they were
suppose to be counting to determine area. Some students wanted to
count the dots while other students wanted to count the segments
connecting the dots. There was a great deal of interaction during
these lessons. Students were helping each other. Traditionally,
lessons on area and perimeter were taught by introducing and then
applying a formula. Students that were not sure what area meant
may have been able to apply a formula and not learn what it was they
were doing. After completing this lesson, all students were able to
explain the difference between area and length. They understood
that square units referred to area because it described the number
of squares it took to fill in the polygon. See worksheets 4 through
12. These lessons appeared to be successful in achieving the
objectives based on entries in student learning logs where students
were able to express their understanding of area and perimeter.
They also served as a review and an application of the pythagorean
theorem.

Lesson XI: Applications and Assessment

The objective of this lesson was to have the students apply
what they had learned and to assess what they had learned through a
project. Students were given the task of determining how many
square feet would need to be painted in their bedroom, make a
sketch, and determine the cost in paint. The next day students came
in with their sketches. Most of the students had everything already
calculated, square footage and cost of paint. Some students wanted
to know how to determine square footage on walls where there was
a window or a permanent bookcase. Students worked in groups to help each other answer these questions. Based on the number of students who completed this project, this lesson appeared to be successful in achieving the objective. All students eventually completed the assignment. Students applied finding area to something in their own world and they were successful in completing a project that they would actually have to carry out someday.

Lesson XII: Volume and Surface Area

The objective of this lesson was for students to learn the meaning of volume and surface area. This lesson was started with a problem that directly related to the students world. The high school didn't have any lockers. Students were asked where in the school could lockers be placed and take up unused space (volume). They were asked questions relating to the problem such as: a) How many lockers could fit in that space? b) How much wall and floor space would they take? and, c) How much would it cost? This problem started a class discussion and generated other questions. Measuring tapes and the trundle wheel were taken out within minutes in an attempt to get more information. The discussion in the classroom turned from a hypothetical geometry problem to one which was real and could be solved.

Students determined surface area and volume of sketched three dimensional solids (See worksheet 13). Students who struggled were given plastic cubes to build the solid. Once they
built the solid, they were easily able to complete the task of determining surface area and volume. Some students who were able to determine the surface area and volume without the cubes wanted to use the cubes to verify their numbers. This supported the findings by Cooper and Sweller (1989) that actual solids were interpreted more easily by students. This lesson appeared to be successful at meeting the objective based on the number of students on task and the number of students that completed the project. Through some practice, students were able to determine the difference between surface area and volume. This lesson also made the connection between their world and geometry through the applied projects.

**Lesson XIII: Applications and Assessment**

The objective of this lesson was for the students to apply what they had learned throughout the unit and assess this learning through a project. Students were given the task of designing their ideal swimming pool, determining the cost of plastering or painting the walls, and determining the cost of filling the pool with water. Some students still did not dare to explore and they created a rectangular pool with one depth throughout the entire pool. Most students wanted to create something different. There were rectangular pools with lap pools extending from them. There were multi-level pools. One student created a pool to form a dollar sign. There was a triangular pool. Students were coming into the classroom during lunch wanting help in creating an idea still in their
mind. Questions were generated that were discussed as a class. Examples of some of those questions were: What is the cost of water?, How do we convert gallons to cubic feet?, What is the cost of plaster?, What is the cost of pool paint?, What if I want an Olympic size pool?, What are the dimensions of an Olympic size pool? Students were asking questions of their parents, the science teachers, and the swimming coaches.

This lesson was successful in connecting the material learned throughout this unit with the students world through application in the project.
ASSESSMENT ANALYSIS

Description of Assessment

The unit developed for this study was taught to two high school geometry classes and was assessed with a multiple choice spatial visualization test (See table 3) (*Mechanical Aptitude and Spatial Relations Test*, Joan U. Levy and Norman Levy, pp. 72-73). The test was used with permission from Simon and Schuster (See appendix B). The test consisted of eight box unfolding exercises administered to both the intervention classes and to two other geometry classes from the same high school as a control. Fifty-one students were tested from the intervention group. Forty-five students were tested from the control group. (See table 5). Fifty-two percent of the students in the control group and thirty-five percent of the students who received the intervention had a perfect score on the test. However, eighty percent of the students in the control group and eighty-two percent of the students from the intervention group responded incorrectly to two or less of the problems. Based on the number of students who answered two or less problems incorrectly there appeared to be no difference between the control group and the students taught from this unit. However, the students taught from this unit and the control group were not matched demographically and the results were not tested for significance. The findings provided a preliminary indication that there is no apparent difference in spatial visualization skills between a traditionally
taught high school geometry class and a class taught with this NCTM-based unit. These findings should be verified with further study using larger matched samples, longer NCTM-based interventions, and analyzed for statistical significance.

Journal Entries

Students were asked to write in their journals and express their opinion as to whether or not geometry is useful. The control group was asked to complete the same journal writing assignment. A total of fifty-three students responded from the group that received intervention and a total of fifty-two students responded from the control group (See table 6). The responses were grouped into one of three categories. Those categories were, 1) yes, geometry is useful in everyday life, 2) geometry is useful only in certain professions, and 3) no, geometry is not useful at all.

From the control group twenty-five percent responded that geometry is useful in everyday life. Most of these students were able to cite experiences such as helping their father lay cement or helping build a barn. Fifty-four percent responded that geometry is useful only if you plan to be an architect or a geometry teacher. Nineteen percent responded that geometry is a waste of time and there is no need for it. One student responded that he/she was not sure as to whether it is useful or not. From the intervention group of students, sixty-four percent responded that geometry is useful in their everyday life. Twenty-five percent responded that geometry is
only useful for certain professions, and eleven percent responded that geometry was not useful at all.

These preliminary findings in journal entries appear to indicate that the unit was successful in helping students see the usefulness of geometry in their lives. One student that responded positively to the usefulness of geometry expressed that she liked geometry because she liked the environment in which she lives. These preliminary findings that indicate positive results in student attitudes toward geometry were not tested for significance. However, they do suggest that further research be done to determine if there is a significant difference in student attitudes toward geometry when the learning is student centered as suggested by the NCTM Standards.


The most often asked student question did not come up in mathematics class during this unit: "When are we ever going to use this?!" It was also observed that students did a lot of interacting with each other. The most feared problem of being off task was not present when the learning was connected to the students' world. In observing the students doing and experiencing geometry, it was evident that students often apply formulas without knowing what they were doing. Student responses, such as "I like this" or "This is fun", to learning that traditionally had been greeted with complaining, verified that learning can be connected to the students'
world and accomplish the goals of knowledge and skills traditionally associated with a geometry course.
CONCLUSIONS

The preliminary findings in the assessment of this field-tested unit indicated that there was no difference in spatial visualization skills as measured by a multiple choice spatial test between classes taught geometry concepts using the traditional method and classes taught with this field-tested unit. However, there are three factors that need to be addressed. First, this field tested unit was only four weeks long. It is possible that there could have been a significant improvement in spatial visualization skills if students were taught with an emphasis on the NCTM Standards throughout the year. Second, the test used for assessment was box unfolding. There is more to spatial visualization than the ability to mentally fold or unfold a box. Last, the results from this study were not tested for significance. These are preliminary findings with preliminary indications to be studied further.

The student journals indicated a difference in student attitudes toward geometry and its usefulness. A greater percentage of the students who received the intervention expressed opinions that geometry was relevant to their lives than did the students in the control group. A difference in student attitude could mean a difference in a choice of careers or in electing future mathematics classes. According to the NCTM Standards, businesses are no longer looking for strong backs and hands. New jobs require that employees see the applicability of mathematical ideas to common problems and that they believe in the utility and value of mathematics (pp. 3-4).
The goal of this study was to field-test a geometry unit based on the NCTM Standards. Preliminary findings of this study suggest positive results in student attitude toward geometry. Other findings suggest that there is no difference in spatial visualization skills when compared to a control. Further study to confirm these findings and establish statistically significant differences are recommended.

This research suggest two questions to be studied further. First, do students taught geometry skills with an emphasis on the NCTM Standards (three-dimensional geometry and real world applications) improve significantly in spatial visualization skills compared to students taught traditionally? Secondly, is there a significant improvement in student attitude toward the usefulness of geometry when the course is taught with an emphasis in real-world applications and student-centered instruction as suggested by the NCTM Standards?
APPENDIX A

PROJECT OUTLINE

Project Design:

I. Introduction: National Council of Teachers of Mathematics Curriculum and Evaluation Standards (NCTM Standards)

II. Research Question and Research Design
   A. Problems in current geometry curriculum
      1. Inability to see the usefulness of geometry in professions
      2. Inability to see relevance to reality
      3. Current teaching methods used
      4. Textbook
      5. Geometry failure rate
   B. Problem with student ability to picture three dimensional figures
      1. Current textbook and curricula methods of addressing spatial visualization skills
      2. Lack of exercises to develop a student's spatial visualization skills
   C. Project Goals
      1. Implement the NCTM Standards as they relate to spatial visualization and hands on real world applications in geometry from a synthetic approach
      2. Develop a field tested high school geometry unit focused on developing spatial visualization skills in the context of real world applications with a hands on approach of two and three dimensional geometry
   D. Research Design Objective
      1. Develop a high school geometry unit that will focus on the applicable National Standards which are:
         a. The skill of:
            (1) Enhancing spatial visualization.
(2) Improving ability to solve problems in their environment
(3) Interpreting and drawing three dimensional objects

b. Make use of a variety of pedagogical strategies, specific reference to:
   (1) Small groups
   (2) Project work
   (3) Active involvement of students in constructions and applications of geometrical ideas

2. Improve student attitudes which include:
   a. Empowering the student to break from inappropriate mind sets as a result of developing spatial visualization skills
   b. Making student comfortable in taking risks when attempting to solve problems
   c. Showing students the importance of geometry in a variety of professional fields
   d. Improving the students attitude toward geometry

E. Research Design: To Develop and Assess a Curriculum using the following criteria:
   1. Mathematical Skills
      a. Construction of two dimensional polygons with compass and straight-edge
      b. Creating three dimensional models
      c. Making two dimensional sketches from three dimensional models
      d. Constructing three dimensional models from two dimensional sketches
      e. Discovering area and perimeter formulas for triangles and quadrilaterals
      f. Discovering formulas for surface area and volume of rectangular solids
2. Pedagogical Strategies
   a. Cooperative Learning: Small groups
   b. Kinesthetic: Hands on Minds on active involvement of students in constructions and applications of geometrical ideas
   c. Project work

3. Attitudinal Outcomes
   a. Empower students to break from inappropriate mind sets
   b. Emotional effects from experiences: Making students comfortable in taking risks
   c. Connecting student interest and creativity to spatial visualization: Showing students the importance of geometry in a variety of professional fields
   d. Improving student attitudes toward geometry

4. Student Assessment
   a. Student Outcomes
      (1) Spatial Visualization Test
      (2) Attitudinal results through journal
   b. Analysis of Lessons
      (1) Each lesson
      (2) Overall unit

III. Review of the Literature
    A. Mathematics achievement related to spatial visualization
    B. Geometry achievement related to spatial visualization
    C. Cognitive structure in learning geometry
    D. Developing spatial visualization skills
    E. Manipulatives
IV. Project Design and Procedure

A. Lesson on the need for developing spatial visualization skills

B. Lessons on developing spatial visualization skills
   1. Polygons
      a. Properties of polygons
      b. Constructing polygons
   2. Polyhedra
      a. Constructing a model of a cube and a tetrahedron
      b. Sketching pattern layouts of the collapsed cube and collapsed tetrahedron
      c. Making two dimensional isometric sketches of models

   3. Euler's Formula
   4. Platonic Solids
   5. Application
   6. Assessment

C. Lessons on perimeter and area of triangles and quadrilaterals
   1. Perimeter and area of squares and rectangles using dot paper
   2. Perimeter and area of triangles and parallelograms using dot paper
   3. Perimeter and area of trapezoids.
   4. Application
   5. Assessment

D. Lessons on volume and surface area of polyhedra
   1. Drawing two dimensional sketches on isometric paper of different three dimensional models.
      a. Sketching net patterns of models to represent surface area
      b. Determining volume from models and surface area from net patterns

   2. Application
   3. Assessment
E. Overall Assessment
1. Spatial Visualization Test
2. Attitudinal results through journal

V. Analysis of Data
A. Lesson I: The need for spatial visualization skills
B. Lessons II: Polygons
C. Lessons III: Polyhedra
D. Lesson IV and V: Euler's formula and platonic solids
E. Lesson VII - X: Lessons on area and perimeter of triangles and quadrilaterals
F. Lesson XI: Applications and assessment
G. Lesson XII: Volume and surface area
H. Lesson XIII: Applications and assessment

VI. Assessment Analysis
A. Description of assessment
B. Journal entries
C. Report on researcher's observations on the implementation of project design

VII. Conclusion
Table 1

Geometry Student Journal Responses

Student Completion of the statement; "I feel geometry is..."

<table>
<thead>
<tr>
<th>Response</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Necessary for College</td>
<td>3</td>
</tr>
<tr>
<td>Enjoyable and Useful</td>
<td>8</td>
</tr>
<tr>
<td>Useless</td>
<td>4</td>
</tr>
<tr>
<td>Frustrating and Difficult</td>
<td>34</td>
</tr>
<tr>
<td>Other Responses</td>
<td>5</td>
</tr>
</tbody>
</table>

Total Number of Students Surveyed | 54*   |

*Number of Students Differ in Tables According to Daily Attendance
**Table 2**

**A Southern California High School Geometry Enrollment 1991/1992**

<table>
<thead>
<tr>
<th>Description</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students enrolled in Geometry in September 1991</td>
<td>183</td>
</tr>
<tr>
<td>Number of students that dropped Geometry in January 1992</td>
<td>66</td>
</tr>
<tr>
<td>Number of students remaining in Geometry in February 1992</td>
<td>117</td>
</tr>
<tr>
<td>Number of &quot;D&quot;s issued in Geometry in June 1992</td>
<td>17</td>
</tr>
<tr>
<td>Number of &quot;F&quot;s issued in Geometry in June 1992</td>
<td>24</td>
</tr>
</tbody>
</table>
Table 3

Professions and Activities Requiring Spatial Visualization Skills

1. Video Games
2. Driving
3. Sports
   a. Baseball Players
   b. Football Players
   c. Tennis Players
4. Photography
5. Landsaping
6. Art
7. Furnishing rooms (Interior Design)
8. Drafting
9. Clothes Designer
10. Engineering
11. Mechanical Repair
12. Electrical Work
13. Operators of Complex Machines
14. Medical
   a. Surgeons
   b. CAT Scans
   c. Sonograms
   d. MRIs
   e. Angiography
   f. X-Rays
15. Hollograms
16. Three dimensional children's books

Students concluded that the places we live in, the cars we drive, the malls we shop in and even some games we play are a result of somebody's ability to spatially visualize.
Table 4

Results of Spatial Visualization Multiple Choice Eight-Problem Test

<table>
<thead>
<tr>
<th>Number of Incorrect Responses</th>
<th>Group taught with field-tested unit</th>
<th>Control group</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>18</td>
<td>24</td>
</tr>
<tr>
<td>1</td>
<td>18</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Total Number of Students*</td>
<td>51</td>
<td>45</td>
</tr>
</tbody>
</table>

*Number of Students Differ in Tables According to Daily Attendance
Table 5

Results of Survey on Usefulness of Geometry

<table>
<thead>
<tr>
<th>Response Group</th>
<th>Field-Tested Unit</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Useful in everyday life</td>
<td>34</td>
<td>13</td>
</tr>
<tr>
<td>Useful only in certain professions/careers</td>
<td>13</td>
<td>28</td>
</tr>
<tr>
<td>Not useful at all</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Don't Know</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Total Number of Students*</td>
<td>53</td>
<td>52</td>
</tr>
</tbody>
</table>

*Number of Students Differ in Table According to Daily Attendance
TEST IV. BOX UNFOLDING

DIRECTIONS: Each question in this test consists of a numbered picture showing a box that is to be unfolded. If the box were unfolded it would look like one of the four cardboard patterns, lettered A, B, C, or D, in the other frames in the row. Choose the cardboard pattern that is unfolded from the lettered picture, and blacken your answer sheet accordingly.
April 15, 1993

Ms. Alice Corring,

Per our phone conversation on April 15, 1993 at approx. 8:45 a.m. Pacific time, I would like to request permission to copy the following:

Title of Book: Mechanical Aptitude and Spatial Relations Tests

Authors: Levy, Joan U. and Levy, Norman

Pages to copy: Pages 72 and 73

Purpose: Give test to four high school geometry classes of 36 students each at a Southern California high school. The results of the test and a copy of the test will be published in a Masters' thesis at California State University, San Bernardino.

Date Needed: I would appreciate it if I could get this permission as soon as possible. I am willing to pay a fee to expedite.

Thank you and Sincerely,

Stella Sloan

11972 Kingston
Grand Terrace, California 92324
Phone (909) 783-0550
Fax (909) 783-6714

MS. SLOAN:

Permission is granted for above use.

Alice Corring

Contracts Administrator, Arco Publishing-A Division of Simon & Schuster
April 15, 1993
REFERENCES


