Teaching and learning through a brain-compatible approach: Implications for junior high school mathematics

Nancy G. O'Kelley

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TEACHING AND LEARNING
THROUGH
A BRAIN-COMPATIBLE APPROACH
Implications for Junior
High School Mathematics

A Project Submitted to
The Faculty of the school of Education
In Partial Fulfillment of the Requirements of the
Degree of

Master of Arts
in
Education: Secondary Option

By

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1989
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Teaching and Learning
Through
A Brain Compatible
Approach

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RATIONALE AND PURPOSE

"I taught them what they needed to know, but they didn't learn it."

That is a statement that every teacher has uttered at one time or another. The feeling is especially frustrating after a teacher has enthusiastically presented an extremely well prepared lesson. Why didn't they learn?

Teachers often take such failures personally or they may look for others to blame. Parents, socio-economic factors, other teachers, lack of support, television, electronic games, the breakdown of the nuclear family, too little respect for authority, stingy taxpayers, and lower moral standards are a few examples of whom we blame for the failure of students to learn. However, we tend to overlook the fact that in our system students are compelled to attend. Therefore the schools must accept them as they are, not as one might wish them to be. The problems are difficult and varied but they are the school's problems to solve.

The pressure is growing on schools and teachers to produce learning results. We must adopt methods and approaches that sharply increase the acquisition of learning. A vastly different approach is needed. In the days of Horace Mann, a teacher's function was to pass along to the students his knowledge of the few books that were available. In the last fifty years an explosion of knowledge has occurred
that has rendered that system of education useless in dealing with present needs. Mann's class and grade factory-type school and educational system had two major flaws. The first flaw was that children are not inert raw material, they are individuals with enormous differences. Some of them do not process well. The second flaw was the belief that children could be processed successfully at a uniform rate. Learning was viewed as a function of time - a child should learn twice as much in two years as in one year. When this did not happen there were attempts to modify the system. But the system has prevailed. Huge numbers of students who have not processed well have dropped out of school. Current drop out rates are as high as 50%. We are obviously not meeting the needs of our students. No crisis has arisen because of the number of drop outs until recently, because there were a variety of jobs for those who did not process well. However those jobs are becoming fewer and fewer.

The central problem faced by educators today is how to bring about learning. Since failure to learn brings about additional problems in discipline, crime, mental health and other societal problems, it affects all of us. With new knowledge of how the brain functions, of what it demands, and of how learning takes place we can at last create school environments that will far more successfully help all learners to achieve. The teacher becomes a "creator" of
learning rather than an instructor. The learning stems from the student's own brain and activities rather than resulting from "being taught".

This project will explain what is meant by brain-compatible education, how the brain works (patterns and programs), the Triune Brain Theory, ideas for designing a brain-compatible school and a brain-compatible approach to teaching and learning mathematics. It will provide sample lessons and teacher ideas for use in the classroom that are in a brain-compatible mode.

Although the ideas presented may seem visionary and futuristic, they do make sense in light of our knowledge of the brain. Brain-compatible schools may not be designed and implemented in the near future, but in the meantime we can utilize as many of the brain-compatible principles and techniques as possible.
REVIEW OF LITERATURE

I. WHAT IS MEANT BY "BRAIN-COMPATIBLE"?

Logical Thinking

The future of teaching and learning lies in the study of the brain. Only through an understanding of the brain can we untangle the mysteries of how people learn. It is essential to understand brain functions and operations before schools can be significantly improved. (Hart, 1983, p. IX)

A new concept in learning has emerged that focuses on the problem of matching settings and instruction with the nature of the brain. The brain has modes of operation that are natural, empowering, and effective in making use of the fantastic power that the brain possesses. If forced to operate in other ways, the brain functions reluctantly, slowly and with substantial error. Brain-compatible is an appropriate term for education designed for the brain. Noticeably better outcomes would be produced by moving from our current brain-antagonistic school settings to brain-compatible schooling and training. (Hart, 1983, p.XIV)

The more knowledge we gain of the brain, the better we can understand humans, human behavior and human learning. Our human brain is the product of hundreds of millions of years of evolution. It is not a logical apparatus, as is a computer, but reflects the illogical events and accidents of history. The brain is the organ for learning, and it is an
organ, made of tissue, not an abstract concept. It took its shape tens of thousands of years before Greek-type sequential logic was invented. No part of the brain is naturally logical. (Hart, 1983, p. 49) As Morton Hunt points out in the The Universe Within:

For many centuries, philosophers and others who have studied the human mind have believed that reasoning takes place according to the laws governing logic. Or rather, that it should, but regrettably often fails to do so .... Such is the tradition that runs unbroken from Aristotle to Piaget. But the finds of cognitive science run counter to it: Logical reasoning is not our usual - or natural - practice, and the technically invalid kinds of reasoning we generally employ work rather well in most of everyday situations in which one might suppose rigorous deductive thinking was essential. (Hunt, 1982, p. 121)

People with good knowledge of the material being presented may benefit from a logical presentation of the material. However, this type of presentation produces consistently very poor learning results with students who are not already familiar with the material. Students prior to learning are unable to perceive the logic that seems so obvious to the instructor who already possesses that logic. (Hart, 1983, p. 57)

Our previous individual experience is the foundation to which more learning is added, and since each of us learns differently, in various sequences, logical group-instruction usually produces a serious degree of failure. Each of us learns in a highly individual and mainly random way, always adding to, sorting out and revising the input from teachers.
and others that we have had up to that point. Therefore, any group instruction that has been logically planned and limited will be wrongly planned for many in the group and will usually prevent or inhibit learning. In fact, upon examining the useful learning that we have acquired as adults, the majority of it has been acquired randomly and from random experiences with no logical sequences being followed. (Hart, 1983, p. 58) Leslie Hart cites an excellent example of this fact by discussing how people learn to drive cars. This is an activity that requires precise knowledge and expertise. From early childhood we learn about steering, braking, parking and obeying traffic signals. We acquire our ability to use and manage cars in a mainly random manner. Once we actually drive the car we don't do each operation separately. We drive holistically - managing many operations at the same time, not by using some pre-programmed logical sequence of steps. (Hart, 1983, p. 54)

Actually, very little in our world has been accomplished by sequential logic. The fact that we view logic as the highest form of achievement is merely a product of tradition. (Hart, 1983, p. 58) Robert Ornstein in *Multimind* responds to this Tradition by writing "... a living human being is usually thought to consist of the pure verbal and logical processes ... It is the dominant view of what is known as the 'the Western intellectual tradition,' a view that might be remembered by its acronym, TWIT." (Ornstein,
The conventional classroom has become a specialized place where aggressive teaching with a sequential, logical curriculum is done. Very few alternatives to this type of school system are even considered. Benjamin S. Bloom states:

Group instruction, as presently used in most countries of the world may approach optimal qualities of instruction for only a small proportion of students in a given class. Even when this is the case, it is likely that the majority of students in the class are paying a heavy price for the ways in which the different qualities of instruction serve the special needs for a few members of the class! (Bloom, 1976, p. 136)

Katz states:

Education has not suffered from any freedom granted teachers to run schools as they see fit; it has suffered from the suffocating atmosphere in which teacher have to work. (Katz, 1971, p. 131)

Learning is a process, not sequential, i.e., the resonating of what we know from experience with the resonating of new experience, particularly "school" experience. This process allows the brain its natural function of randomness; when the new experience (new information or concept) coincides with the old experience (the already learned information), then resonance, the process of learning, takes place. In *Sympathetic Vibrations*, K.C. Cole uses learning to explain resonance.

... resonance may provide a good analogy for the process of learning. It often takes a lot of little pieces of information to add up to a deep understanding, and a lot of little insights to add up to a great idea. But these little bits and pieces need to come at the right time and strike you in the right way. If the information comes along before
you are prepared to offer feedback, the input simply
dissipates like a swing pumped the wrong way.
(Cole, 1985, p 267)

Although the term "brain-compatible education" is
relatively new, many of the beliefs and goals of brain
compatible education have been discussed and promoted for
decades. In his book, Schools of Tomorrow, John Dewey wrote
of many of these concepts way back in 1915. Dewey believed
that education had to be relevant to the daily life
experiences of the students; "knowledge that is worthy of
being called knowledge, training of the intellect that is
sure to amount to anything is obtained only by participating
intimately and actively in activities of societal life.
"(Dewey, 1915, p. 63) He cites the following example; It is
possible for a child to learn the various properties of
squares, rectangles, etc., and to acquire their names. But
unless the squares and rectangles enter into his purposeful
activities, he is merely accumulating scholastic information."
(Dewey, 1915, p. 68) He believes that:

Until a child goes to school he learns nothing that
has not some direct bearing on his life. How
he acquires this knowledge is the question that will
furnish the clue for a natural school method. And
the answer is, not by reading books or listening to
explanations of the nature of fire or food, but by
burning himself and feeding himself; that is, by
doing things. (Dewey, 1915, p. 72)

He also adds,

If textbooks are used as the sole material, the work
is much harder for the teacher, for besides teaching
everything herself she must constantly repress and
cut off the impulses of the child toward action.
Teaching becomes an external presentation lacking meaning and purpose as far as the child is concerned. (Dewey, 1915, p. 73)

Dewey further emphasizes:

Give the child work that he recognizes as interesting and valuable and a chance to play, and his hatred of school will speedily be forgotten. The inflexibility of the ordinary public school tends to push students out of school instead of keeping them in. The curriculum does not fit them, and there is no way of making it fit without upsetting the entire organization of the school. One failure sets a pupil back in all his work and he soon gets the feeling that his own efforts are not important because the school machinery works on at the same rate, regardless of any individual pupil or study. Indifference or dislike is almost surely the result of feeling that work is making no impression, that the machine for which he is working is not after all affected or dependent upon his work. (Dewey, 1915, p. 98)

Dewey described the public school system in Gary, Indiana at that time. In Gary, the schools were organized to fit each individual child. They were extremely flexible, which allowed the child and the school to get along together. Individuals were allowed to spend more or less time on any one subject, or they could drop it altogether. Students who were stronger in one area than in the rest of their work could take that subject at a higher lever - they were allowed to excel. Students who were losing interest in school or falling behind were aided by their teachers. The teacher found out what the child was good at and gave him plenty of time to work at it - to get ahead in it so that his interest in his work was stimulated. Even if his interest in the regular school program was never aroused, at least he was
kept in school long enough to master one thing, probably the one most suited to the child's ability, instead of failing entirely by being held back in everything until even his one strong ability died and the pupil was without either training or the moral stimulus of success. The school program was reorganized every two months and the student could change his entire program at any of these times instead of having to struggle along for a whole year with work that was too hard or too easy. Students were classified not by grade, but rather as "rapid", "average" and "slow" workers. This classification didn't describe the quality of work done, but was used to allow the student's work to keep up with the natural growth of the child. Children at the schools in Gary were happy and hard working. The Gary schools taught concepts through experiences. All learning was relevant and perceived as useful. The community was a large contributor to the educational process. (Dewey, 1915: 164 - 204)

Many of the characteristics of the public school system in Gary in the early 1900's would be termed brain-compatible today. The logic of these ideas was seen at least 70 years ago, but seems to have been forgotten until recently.

This flip-flopping between a sequentialized approach to education and a more holistic approach has occurred for one major reason: the influence of the basic theory promoted at the time. During America's agricultural phase students, in one-room school houses, were taught that which was relevant
and necessary to their daily lives. With the advent of the Industrial Revolution, America was transformed into a time clock oriented society in which sequential motion and productivity had the highest value. In educational settings, children were taught through a production-line, assembly mentality that produced quantity, not necessarily quality. According to Foshay,

Most school practice arises from tradition, ritual and the context in which schools are conducted. Only during this century has scientific learning theory had an influence and then only in a fairly minor way. The school is a kind of subculture in which are preserved the relics of former times, with a few practices added or subtracted because of contemporary thought. (Foshay, 1973, p. 197)

These past theories, the agricultural type and the industrial type, have been countered and weighed back and forth, a veritable flip-flopping between extremes. "...education flounders, often going around in circles by discovering 'new' methods that some historical research may show to have been 'new' 20 and 40 and 60 years before." (Hart, 1983, p. 24) This perspective cannot be tolerated today. As Hart adamantly points out,

With the brain as a focus, however, it becomes possible to get onto genuinely new paths using information never previously available, and to arrive at a kind of theory that does explain what learning is and how it does come about sharply enough to be tested in many ways. (Hart, 1983, p. 24)

Moreover, as Toffler shows, the "Third Wave", The Technological Age in which we are currently living, is more random and non-sequentialized and is aptly termed brain-
compatible.

The Third Way brings with it a genuinely new way of life based on diversified, renewable energy sources, on methods of production that make most factory assembly lines obsolete; on new, non-nuclear families; on a novel institution that might be called the "electronic cottage"; and on radically changed schools and corporations of the future. The emergent civilization writes a new code of behavior for us and carries us beyond standardization, synchronization and centralization, beyond the concentration of energy, money and power. (Toffler, 1980, p. 4)

As Dewey stated, the schools must "deal thoroughly with a small number of typical experiences in such a way as to master the tools of learning, and present situations that make the students hungry to acquire additional knowledge." (Dewey, 1915, p. 16) "To find out how to make knowledge when it is needed is the true end of the acquisition of information in school, not the information itself." (Dewey, 1915, p. 16)

It seems definite that these concepts and ideas used in the early 1900's along with today's current knowledge of the brain may produce a school program that could guarantee success for our students.
II. PATTERNS AND PROGRAMS—HOW THE BRAIN WORKS

The brain is by nature a marvelous pattern detecting device. From birth sensory stimuli trace patterns in our brain tissue—almost like road maps. The message we receive through our sight, hearing, touch, smelling, and tasting have been organized into patterns and stamped into our brains.

These mental pictures are useful and essential because they help us to recognize familiar objects and circumstances. We know what to do without having to stop and figure it out. Although we are usually not aware of it, we all rely on pattern thinking. The brain needs no coaxing, teaching or motivation to construct and interpret patterns. According to David B. Bronson:

Pattern matching is inherently pleasing because that is what our minds are designed (or programmed) for ... Quite apart from anything the teacher does ... the student, being human, is a pattern finder and a pattern-maker. Possibly the greatest obstacle to our making use of this not very startling principle is our ingrained notion that education is the acquisition and mastery of new material. What we 'teach' and they do not 'learn' is the material. (Bronson, 1977, p. 453)

The brain detects characteristics or features and also relationships among these features. This is an extremely flexible ability that is enhanced by the use of clues. The brain uses positive and negative clues in a probabilistic manner as is illustrated in the following example.

The experience one brings to a given situation is what pattern recognition depends on heavily. Children often need
to revise the patterns they have identified to fit new experiences. For example, consider the problem a child has in grasping such patterns as dessert, pie and cake. The child first must realize that meals have a sequence and dessert comes at the end of the meal. A wide variety of dishes may be suitable for dessert. An adult with years of experience has no difficulty understanding the concept of pie, but to a small child an open pecan pie, a crusted apple pie or a whipped creme covered chocolate creme pie have very little in common. The child may decide that pie means round - since that is the most obvious feature. However, many desserts are round, especially cakes - and they vary from pie-like cheesecake to layer cakes to birthday cakes with candles. In a few years, through random exposure and experience, the child has extracted the patterns to identify these treats correctly.

We live by programs that can be acquired in two different ways; either through the genes or by being learned after birth. We switch on one program after another, selecting from those that have been acquired and stored in the brain. We as humans are far more dependent on the thousands of programs acquired after birth in contrast to animals that rely more on genetically transmitted programs. A program is a fixed sequence for accomplishing some end - a goal, objective or outcome that is usually a personal and individual purpose. In order to carry on activities, one must constantly select a program from those stored in the brain.
and implement it. We have one program for walking across the room, another for climbing stairs, another for going down stairs, another for picking up packages etc. A shopping trip can entail the use of many programs. Each time an activity changes, the program in use must switched off and another selected and switched on. The brain does this so smoothly that we are usually not even aware of any switches being made.

However, when getting dressed, we must make a conscious selection of what clothes to wear depending on the perception of the pattern that we will be encountering. If it is a work day, we will dress differently than if it is a day to paint the house. We select the most appropriate program to deal with what is seen as the pattern in effect. This constitutes a basic cycle. We must:

1) Evaluate the situation or need (detect and identify the pattern or patterns).
2) Select the most appropriate program from those stored.
3) Implement the program. (Hart, 1983, p. 84)

If a student is unable to evaluate accurately the need or problem a situation presents, the cycle goes haywire. The student, quite plainly, does not know what to do. A good example of this would be the word problem dilemma. Students often cannot detect a mathematical pattern when it is cushioned with words.
They have no idea what operations to use to solve the problem. George Polya illustrates a brain-compatible learning theory in the following paragraph on problem solving. He has titled this paragraph:

Working From Inside, Working From Outside

Establishing contacts between the proposed problem; and his previous experience is certainly an essential part of the problem solver's performance. He can try to discover such contacts "from inside" or "from outside". He may remain within the problem, examining its elements till he finds one that is capable of attracting some usable element from outside, that is, from his previously acquired knowledge. Or he may go outside the problem examining his previously acquired knowledge until he find some element applicable to his problem. Working from inside, the problem solver scans his problem, its component parts, its aspects. Working from the outside, he surveys his existing knowledge and ransacks the provinces of knowledge that are most likely to be applicable to the present problem. (Polya, 1981, p. 51)

People can use only the programs that they already have stored. There is no way to perform a program unless it has been stored. The person would not know how to do it. A student also cannot implement a program unless given the chance to do so.

If a program is selected and implemented, but does not work, it aborts. "Abortion of a program always causes some degree of emotional shift because the failure of a program to work is in general disturbing and threatening." (Hart, 1983, p. 86) When programs work, an individual's confidence increases, when they do not work, confidence diminishes.

Hart gives a new and much sharper definition of learning:
“Learning is the acquisition of useful programs.” (Hart, 1983, p. 86) Behavioral objectives switch from test-oriented type objectives (what the student will do when ordered to perform in a particular way) to broader concept of behavior, such as: Will the student select an appropriate program from those stored in the brain? Will the program prove to be useful and achieve the intended goal and not abort?

Test, quizzes and examinations would become almost obsolete under this new definition of learning. “We almost never can find out from directed behavior what a student would do if not directed.” (Hart, 1983, p. 87) In real life we don’t give people pencil and paper tests, rather we find out what they can do.

If people are coerced into acquiring a particular program, they may use it under duress but will not use it willingly. By the nature of being human, using a freely learned program satisfies the individual, while using a coerced program brings back the old fears under which it was built. Learning progress can be properly evaluated only through the observation of undirected behavior.

New learning is speeded up by an effective transfer of learning. This entails using stored programs in new applications. The student who can adopt previously built programs to new situations and can see the similarities of patterns involved learns much more rapidly than a student who
cannot. Present learning depends heavily on previous learning and biases stored in the brain of each individual. When individuals are given uniform instruction without regard to what they bring to the learning situation a high degree of failure is virtually guaranteed. Uniform instruction may also result in teaching students what they already know. An example of this is found in a recent study conducted by Educational Products Exchange Institute in Long Island, New York. They tested fourth grader on the contents of a mathematical test before they used it. Sixty percent scored 80 or above on the test! What a wasted year of mathematics for that sixty percent of the class. (Hart, 1983, p. 101)

III. The Triune Brain Theory

Emotions are greatly involved with learning. Paul D. Maclean's Triune Brain Theory (Hart, 1983, p. 36) can aid in understanding how and to what extent emotions affect learning. He shows how our present brain is composed of three brains of different ages. The oldest is known as the Reptilian. It is similar to the kind of brain possessed by agile reptiles that became the ancestors of mammals. It was, and still is, a survival brain - it insures the survival of the body. It is the fastest acting part of the brain and reads non-verbals signals long before anything else. It controls our flight or fight behaviors by taking over in stressful situations. It has a few ancient programs.

The second brain is known as the old mammalian and is a
far more sophisticated brain. This brain deals with the emotion of the individual. It is the brain that gives us a sense of responsibility for others. It combines with the Reptilian brain to form the deeply held value systems which govern our behavior. Our values and attitudes are non-rational and can only be changed through changed experiences.

The third brain is called the New Mammalian. It is many times larger than the other two brains put together, taking up 85% of the brain mass. It is the cognitive part of the brain. It interprets situations and detects patterns. The New Mammalian or Neocortex controls our speech and writing. It acts on the basis of models and problem solving techniques that it has developed. It cannot function effectively, if all, under threat.

Threatening situations have great implications for instruction. The more complex a brain is, the slower it is to make decisions. The oldest brain made survival decisions based on life or death situations. The larger and more complex old mammalian brain was slower to make decisions, but was much more able to detect patterns and interpret situations and to store and implement many more programs. The huge neocortex became much too big and intricate, after it developed, to make quick action decisions, but was fantastic for pattern discrimination and the storage of a vast collection of programs. It had to depend on the older brains for decisions linked to survival.
Thus we have the phenomenon, that Hart refers to as "downshifting". According to Hart, "When the individual detects threat in an immediate situation, full use of the great new cerebral brain is suspended, and faster-acting, simpler brain resources take larger roles." (Hart, 1983, p. 108) This is typical of students who have "test anxiety." The students often know and understand the material, but are so threatened by testing situations that the reptilian brain takes over and only the automatic pre-programmed responses are accessed. The student blanks out.

Since the majority of academic and vocational learning depends primarily on the neocortex, the absence of threat is essential to effective instruction. "Cerebral learning and threat conflict directly and completely." (Hart, 1983, p.110) By threat, Hart means not what is happening as much as what reasonably can be expected to occur in the very near future. It is the noose around one's neck, waiting to be tightened. It is what some person or group with power may do in the near future.

The typical classroom setting can be a threatening situation in that it is a setting of captivity in which the instructor has the power to punish, ridicule or embarrass. Students may feel threatened by having to perform publicly at any given time and face public failure and the ridicule of their peers. The system of grading and issuing marks and report cards has become widely used for threat purposes. Failure to
learn the new information is to be expected in a threatening situation which forces the brain to perform at the reptilian level, thereby not allowing the neocortex to function optimally. The neocortex functions best when we feel secure. A student's ability is best determined by providing opportunity, standing aside, and observing. This method poses no threat to students. Our current class and grade type of school is totally unrelated to our current understanding of brain development; and can be extremely brain antagonistic.

A brain-compatible approach would tend to move our schools in the following directions:

1) Schools would provide a setting with open-ended expectations of student learning that is so organized that the fullest student achievement is welcomed.

2) Schools would provide a non-threatening, non-punitive ambiance that reduces all aspects of captivity to a bare minimum. Students should be allowed much freedom of movement and schedule.

3) Schools would place an emphasis on mastery rather than on passing. The mastery approach "makes 100 percent attainment of the key elements the goal, but leaves the time to achieve it flexible." (Hart, 1983, p. 134) This creates an atmosphere of high standards and reduces the threat of examinations which put undue value on a single performance at one time under set of conditions. The emphasis should be on domination of learning over
the clock, scheduling and processing.

4) Schools would place a much greater emphasis on reality than on "the book". They would avoid the fragmentation caused by courses and would use unifying, holistic ideas. A brain-compatible approach does not define an educated person as one "who knows a lot," but rather as one who knows what to do, one who can detect the patterns involved and implement appropriate programs. (Hart, 1983, p. 136-137) Wayne Jennings and Joe Nathan stated: "For more than 50 years now, studies have been documenting the effectiveness of nontraditional school programs in the United States. This research should cause us to question 95 percent of current education practice." (Jennings, 1977, p. 568)

John Goodlad points out: "For the schools to change, the people in them must change. Changing people is the most difficult of all human enterprises." (Goodlad, 1974, p. 116)

IV. Designing a Brain-Compatible School

A brain-compatible approach provides student activities which produce the desired learning. Through this approach learning means that: "The child should become familiar with and able to operate successfully in the real, complex world, in which he or she now operates to a limited degree and will soon operate to adult degree." (Hart, 1983 p. 168) "The brain-compatible school must be one in which the process of world discovery accelerates and broadens." (Hart, 1983,
Some basic factors of a brain-compatible approach, according to Hart, are the following:

1) Young children must manipulate what they deal with. Activities that involve manipulation are important and helpful in later years also, but are not as essential.

2) We must dominantly address learning to immediate and later uses, instead of to testing. How will the child use it? We must steer clear of statements such as "You will need algebra to get into college."

3) The human brain must have the freedom to function in a natural, intuitive way and its output should be accepted.

4) Rote learning should be achieved only as individually necessary to achieve mastery and only by multimodal, vigorous, fast-paced methods.

5) The idea of curriculum will shift from meaning what teachers do to broadly planned ideas for what students will do.

6) Success will be expected and obtained. This will become obvious through monitoring student activities and progress.

7) Emphasis can be on mastery, rather than partial learning and on actual accomplishment rather than test answer. A threat free environment enables this type of quality learning and excellence.

8) Students can receive feedback from reality, rather than from authority. This can be enhanced by designing and
providing activities with this built-in characteristic.

9) Students will be given many choices in interests in short term goals without the school surrendering control of main, consequential objectives. Students will acquire the ability to take responsibilities only if they are given responsibilities.

10) Students will be exposed to a rich variety of people, situations and settings which will greatly enhance the real world.

11) Instead of completing exercises, students will use oral and written language for communication purposes. (Hart, 1983 p.169)

The student is already a member of society, of the real world and must learn to take responsibilities in his own life.

A 1979 yearbook of the Association for Supervision and Curriculum Development brought forth the following views:

As practiced, schooling is a poor facilitator of learning. Its persistent view of learning as product interferes with significant learning connected to such complex processes as inquiry and appreciation. What often passes for education is noise that interrupts the natural flow of learning. Schooling too often fragments learning into subject areas, substitutes control for the natural desire to learn, co-opts naturally active children for hours in assembly line classroom structures, and ignores both individual and cultural differences... The formal education system often destroys opportunities for learning from elders, from each other, and from the new generation... Much is known about the learning process but little has been applied to education... The American education system is not making use of brain research findings. (Overly, 1979, p. 107)
The possibilities for designing brain-compatible schools are many. Staff retraining would be on-the-job and real, rather than by lecture and inservice. Students would be engaged in realities rather than writing at a desk. Teachers would provide individual guidance to students rather than aggressive teaching. Learning goals would be sharply defined; and would be sought in random order over extended periods of time. The classroom grouping would largely vanish. Students would be grouped temporarily for as long as the grouping served a purpose. Failure would not be acceptable to the school, but the progress of each student would be known in detail and reviewed often. Attendance would not be mandatory, but would reflect individual activities, home circumstances, out of school learning opportunities and explorations as much as student age and maturity would permit.

As previously described, the schools in Gary, Indiana in the early 1900's utilized many of the techniques and concepts found in brain-compatible education. The success rate reflected a system which met the needs of its pupils. Over one third of all the pupils who left the Gary schools during the eight years of their existence went to college. This is admirable, especially when you consider that the population of Gary was principally made up of laborers in the steel mills who were foreign born. School appealed to these students so they wanted to continue their educational pursuits.
Currently, no true brain-compatible schools exist, but the advocates of brain-compatible education feel that if we implemented brain-compatible approaches in our schools today, the following outcomes are ones which we may experience:

1) Most students may go far beyond present grade levels and receive a more solid education.

2) The discipline problem may be reduced.

3) Youth crime and drug problems which are at least partly due to school failure, boredom and age grouping may be reduced.

4) The community would be continuously involved in the educational processes.

5) As the tone of schools changes to a successful, cooperative effort, student and staff may show great improvement.

6) Student failure or serious lagging may be virtually eliminated as far as core learning is concerned.

7) Students may leave secondary school much more prepared for further education or work, with a grasp of and experiences of the real world. (Hart, 1983, p. 183)

Learning is a brain function. As we learn more and more about the brain, we must apply this knowledge to teaching and learning in order to achieve optimum results. Learning and teaching must be done in a brain-compatible approach.

Perhaps the schools described in this paper are a vision of the future. Change takes time, patience and cooperation.
However, teachers can, in their given structured settings, adopt and utilize some brain-compatible teaching strategies and implement them in their classrooms now. Let's look at some brain-compatible teaching approaches for the mathematics classroom.
Rationale for a Brain Compatible Approach to Teaching and Learning Mathematics

To prepare students for life in today's highly technical society, their mathematical training must include and go far beyond providing training in the simple skills of counting, computing, putting numbers into formulas and even solving equations. Learning only rote mathematical rules poorly equips students to apply those rules to solving problems outside the classroom. The mathematics curriculum must focus on what mathematical concepts mean, how they are related and where they apply. The majority of mathematics concepts should be taught in such a way that students understand their application in day to day living and their value in various careers and vocations.

Many students have become completely "turned off" to learning mathematics because of the drudgery they have been forced to suffer through for years. This situation must change so that students will be able to appreciate and enjoy mathematics. "The study of mathematics helps students to develop thinking skills, order their thoughts, develop logical arguments and make valid inferences." (O'Malley, 1985, p. 1) A knowledge of mathematics is essential to many other disciplines. Since technology has caused the role of mathematics in society to change, we must change the content in mathematics education. For example, the mathematics program should emphasize the effective use of
calculators and computers.

According to the new California State Framework, "mathematics education must focus on students' capacity to make use of what they have learned in all settings." (O'Malley, 1985, p. 1) The Framework emphasizes that:

Mathematical power, which involves the ability to discern mathematical relationships, reason logically and use mathematical techniques effectively, must be the central concern of mathematics education and must be the context in which skills are developed. (O'Malley, 1985, p. 1)

Current CAP and SAT test results indicate that the methods that we are now using for teaching and for learning are not producing mathematically powerful students.

Teachers must be committed to developing mathematical understanding by allowing students to experience mathematics as a cumulative unified subject. To emphasize, mathematics can be experienced by using the whole language approach. In this approach, the four language skills are integrated continuously. The skills are speaking, listening, reading and writing. Each skill is interwoven with another and each other simultaneously. In mathematics this would mean that a concept is verbalized by the teacher and the students, is written on the board by the teacher and is written in the notebooks by the students, is read to the students by the teacher and is read by students to other students in small groups, and is heard by the teacher and by the other students. Instruction should be flexible enough to meet the learning needs of each
student, and should stress applying concepts learned rather than memorizing rules and procedures.

A brain-compatible approach to teaching and learning mathematical concepts would aid in meeting the demands of the new State Framework. Concepts would be taught as their need in practical applications arose. The program would stress learning through discovery, situational and open-ended lessons, calculator usage and cooperative learning. Low test scores as well as low enrollment in higher level math classes substantiate the claim that current approaches do not appear to be working, it is time for a new, fresh approach: A brain-compatible approach.

I. Long-Term Goals for the Students

A brain-compatible approach to teaching mathematics should include the following long-term goals listed below. These goals reflect the demands of the new California State Framework and the Model Curriculum Standards. Although many districts currently publish similar goals and objectives, the feeling is that the objectives are not being attained because of our teaching and learning behaviors. These goals are broader, less structured by time, and are reflective of brain-compatible teaching and learning strategies. The emphasis here is to attain these goals through brain-compatible approaches. Then learning will become relevant and worthy of pursuit.
1) The students will have a firm understanding of the significance and use of numbers in counting, measuring, comparing and ordering.

2) The students will have mastery of the basic operations with whole numbers. Whatever other skills and understanding people have, they must have the ability to calculate a precise answer when required.

3) The students will have sufficient familiarity with the number system to avoid applying tedious algorithms in special cases. They should be comfortable seeking shortcuts and realizing that mathematical understanding helps them avoid unnecessary work.

4) The students will have the ability to use the appropriate mathematical operation or operations to solve realistic problems. They should be able to recognize which problems can be solved by mathematics, to select the relevant information, and to choose the most appropriate mathematical model.

5) The students will have an understanding of when an approximate calculation is appropriate and the ability to make such an approximate calculation. Typically decision making is based on approximations and does not require accurate calculation.

6) The students will have an appreciation of the role of estimation in intelligent behavior and the capacity to make reasonable estimates.
They should be able to derive reasonable estimates from (a) rough calculations. (b) intelligent guessing of the quantities that appear in a given mathematical formula. (c) measurement. (d) sampling techniques. (e) preparation and interpretation of simple graphs.

7) The students will have the ability to use a calculator effectively. They should be able to realize when a calculator is useful and when other methods such as mental approximation are more appropriate. They must approximate answers and use numbers intelligently so that they can detect absurd answers that might result from pushing the wrong buttons or using an incorrect calculation.

8) The students will be familiar with the nature and the purpose of computers. They should have the opportunity to use computers. Whether they do use them or not it is essential that they understand the principles on which computers function and the role they are capable of playing in our daily lives - domestic, social and professional.

9) The students will have a firm understanding of magnitude with respect to measurements and of the role of units in assigning numerical magnitudes to physical quantities. They should understand the need for standard units of measurement and know how to use appropriate measurement tools such as rulers, balances, liquid volume measure, thermometers
etc.

10) The students will have the ability to organize and arrange data for greater intelligibility. They should develop the skills of tabulating and graphing results as well as the ability to detect patterns and trends in poorly organized data either before or after reorganization.

11) The students will develop the ability to extrapolate and interpolate from data and from graphic representations. They should know when extrapolation or interpolation is justified and when it is not.

12) The students will have an understanding of the role of functions modeling the real world. They will realize that real world phenomena are in a constant state of change and that the relationship between changing quantities is represented mathematically by a functional relationship between variables. They should be able to draw the graphs of functions (with the aid of calculators) and to derive information about functions from their graphs. They should understand the connection between the study of functions and the solution of equations and inequalities.

13) The students will have an understanding of rational numbers and of the relationship of fractions to decimals. They will be able to do the appropriate calculations with fractions or decimals or both in realistic situations.

14) The students will understand the meaning of rates and their relationship to the arithmetic concept of ratio.
They should be able to calculate ratios, proportions and percentages, understand how to use them intelligently in real life situations; understand the common units in which rates occur (such as kilometers per hour, cents per gram), understand the meaning of per; and be able to express ratios as fractions.

15) The students will develop the ability to use probabilistic ideas in ordinary, elementary applications. They should understand the reasons for (and the dangers of) using sampling techniques; they should have the ability to describe a population terms of some simple statistic (mean, median, range); and they should understand the difference between intelligent risk taking, based on reasonable estimates of probabilities and foolish gambling based on unsupported guesswork or wishful thinking.

16) The students will develop the ability to solve problems involving money. They should understand decimal coinage, be able to use money in ordinary situations, understand the difference between income and expense, know the approximate cost of common articles and understand elementary concepts of economics such as profit, debt and interest.

17) The students will have an understanding of and the ability to use, the geometric concepts of perimeter, area, volume and congruency (as applied to simple figures).

18) The students will have an understanding of and ability to add and subtract signed numbers. They will be able
to appreciate the advantage of using signed numbers in many real situations, for example, in measurement of temperature and other quantities.

19) The students will develop the ability to make, read and use a map. They should understand the symbolic nature of various map representations and learn to interpret scale drawings.

20) The students will develop the ability to think intelligently using numbers. This includes the ability to recognize given answers as absurd, without doing a precise calculation, by observing that they violate experience, common sense, elementary logic or familiar arithmetic patterns. It also includes the use of imagination and insight in using numbers to solve problems. They should be able to recognize when a trial and error method is likely to be easier to use than a standard algorithm.

21) The students will develop a positive attitude toward mathematics. Specific abilities and understandings will be of little value to people unless accompanied by two convictions:

(1) that mathematics does do what it was invented to do - solve real, interesting problems and

(2) that it is a tool that people can use confidently and well. The hope is that the students will find it enjoyable to do and appreciate it aesthetically. (O'Malley, 1985, p.28-32)

These goals will aid in developing mathematical power in
our students, which should be the central concern of mathematics education according to the California Mathematical Framework. Students who are mathematically powerful will have an attitude of curiosity and the willingness and ability to probe, explore, experiment, make conjectures and persevere.

II "The Ten Commandments for Teachers"

Ultimately, the quality of any mathematics program is only as good as the teachers in that program. Each student should be presented with exciting and successful experiences in mathematics, and no one method or approach will work for all students. Each teacher must therefore make use of a full range of strategies and devices that can be matched to the students' learning needs, to the students' expressed interests and to the mathematical goals that have been established.

Differences in learning styles exist simply because of differences in the number and type of programs stored in each individual's brain. Students vary widely with respect to experiences, feelings, interests, capabilities, rates at which they learn and ways in which they prefer to learn. Because of these differences, the instructional materials and processes should be equally diverse. Over a period of time students should be involved in a wide variety of activities, including teacher-led discussions, assignments from textbooks, student-led discussions, small group or individual work on projects, experiments using concrete materials, activities that involve
collecting data and making graphs; use of audiovisual materials; involvement in meaningful games; outdoor experiments; and exposure to musical and artistic elements. However, it is essential that every activity be purposeful and designed to help students learn specific skills or concepts. Diversity of activities just for the sake of variety is of limited value to students. (Smith, 1983, p. 22)

Teachers should make a conscious effort to seek out and make available sources of information that will be resources for students both in and out of the classroom. Teachers can and should use student interests as a way of reinforcing the usefulness of mathematical concepts. Tours of local businesses and industries, presentations by speakers who use mathematics in their work and participation in work-related programs can all serve to reinforce learning and motivate students. (Smith, 1982, p. 23)

Teachers who create an expectation of maximum achievement and provide for success and challenges motivate students to reach their potential achievement levels. The teacher must honor each student's right to work up to his or her maximum potential. Teachers should present interesting projects, ask stimulating questions and pose meaningful problems with equal frequency to students of all ability levels. To be motivated students must be active participants in the learning process. An important part of learning mathematics is doing it. Time must be allocated in class for questions, interacting, and
trying new problems. George Polya suggested that teachers be given the following "Ten Commandments for Teachers" for problem solving.

**TEN COMMANDMENTS FOR TEACHERS**

1. Be interested in your subject.
2. Know your subject.
3. Know about the ways of learning: The best way to learn anything is to discover it by yourself.
4. Try to read the faces of your students, try to see their expectations and difficulties, put yourself in their place.
5. Give them not only information, but "know-how," attitudes of mind, the habit of methodical work.
6. Let them learn guessing.
7. Let them learn proving.
8. Look out for such features of the problem at hand as may be useful in solving the problems to come—try to disclose the general pattern that lies behind the present concrete situation.
9. Do not give away your whole secret at once—let the students guess before you tell it—let them find out by themselves as much as is feasible.
10. Suggest it, don't force it down their throats. (Polya, 1981, p. 51)

These ten commandments reflect a brain-compatible approach to teaching. If obeyed, learning will be maximized and teaching will become a more rewarding, interesting and delightful job.

As we have seen, educators have been aware of brain-compatible approaches to teaching and learning for decades. These ideas are not necessarily new—only the terminology is different. When these strategies were implemented in schools, the results were outstanding—as was witnessed by Dewey in the early 1900's. (Dewey, 1915, chap. 4) However we continue to plod along using the Horace Mann factory type
school which virtually guarantees failure for a large percentage of our students. Natural learning - learning randomly, by doing and experiencing in a social, cooperative atmosphere - is the way the brain was designed to learn. As educators, we must implement as many brain-compatible strategies as we can in our classrooms even if we are functioning in a brain-antagonistic school setting.

The following pages contain a sprinkling of discussion topics and related student activities that portray a brain-compatible mode.
Entry: I like to factor trinomials because it is similar to doing puzzles. I enjoy figuring out what factors of the third term will result in the desired middle term.

Commentary: This excerpt from a math journal typifies a whole language approach to the assimilation of the concept of factoring trinomials. At the outset, a trinomial is a polynomial of three terms which can sometimes be factored into simpler form.

Entry: I know that trinomials have three terms because the prefix "tri" means three. A polynomial has two or more terms since the prefix "poly" means many. Although we deal primarily with polynomials of two, three or four terms, a polynomial could have an infinite number of terms.

Commentary: This excerpt shows that the student has memorized the definitions of "tri" and "poly". Nevertheless, does the student still know how to factor, $x^2 + 5x + 4$?

Entry: To factor a trinomials of this type you list the factors of the third term. Factors are the numbers that divide evenly into a given number. My job is to find a pair of factors, that when multiplied will result in the third term, but when added will result in the middle term. The fun
lies in the fact that it is an easy, yet puzzling skill.

Entry: The first type of puzzle that I ever enjoyed doing was a jigsaw puzzle. I enjoyed putting the pieces together to form the whole picture. The whole picture was important to me because I was anxious to see the final results of my efforts.

Entry: In regard to trinomials, factoring consists of breaking down polynomials into their parts. Once this is done I can use the parts to make the whole again. It is self-satisfying to be able to see the whole picture and its parts and their relationship to each other.

Commentary: The foregoing is an example of the manner in which I would have students develop an honest appraisal of their math knowledge by using my interpretation of a math journal. I modeled these excerpts for my students.

A brain-compatible approach is not memorizing definitions, but is actual feelings, thoughts and ideas toward a mathematical concept.

The interplay of my old information and my new information is the intrinsic reward of the success of being able to figure out the puzzle. Tapping in to students own feelings about their own knowledge in order to build and resonate with prior experience enhances their learning ability.
MATH IN NATURE AND ART

PHOTO ALBUM

In science, all living things are structured in some form of symmetrical pattern. In math, the symmetry concept is essential to geometry. In art, balance, pattern and color are all parts of design symmetry. Have the students photograph or draw pictures reflecting the math found in nature and art. Some examples include: spider webs, snowflakes, leaves, pine cones, flowers, honey combs, butterflies, pineapples and other fruits, not to mention the architecture of houses, shopping centers, schools and so on. The students should mount their pictures on posterboard. Underneath each picture, students should write an essay detailing the patterns, geometrical elements and/or the type of symmetry they see in each figure.
BAROQUE MUSIC

The art of music was once tied to medicine and to bringing about so-called "supernatural feats". Hermes Trismegistus of Ancient Egypt wrote two books devoted to music. He set out the principles of a philosophy relating to music that was passed down for centuries through secret groups and through guilds of musicians, masons and architects.

The gist of his philosophy was that there is a harmony and correspondence among all the different kinds of manifestations in the universe - the circling of the planets, the tides of the earth, the growth of vegetation, the lives of people and animals - all are related. All-that-is in the universe emanates from the same source, according to Hermetic philosophy, and therefore the same laws, principles and characteristics apply to each unit - "As above, so below".

Ancient mathematicians looked out at the universe, noted the ratios of the different planetary cycles, counted the rhythmic periods in nature, calculated the ratios of the human body. They put together a "sacred geometry" - a set of mathematical ratios and they believed that these ratios, if used in the sound of music and the architecture of buildings would resonate with the life forces of the universe and thus enhance life.

When you sound a note on one piano in a hall full of pianos the same note will resonate on the other instruments, enhancing the power of your single note to fill the whole
hall. In the same way, the ancients believed that playing certain harmonies and combinations of notes would resonate with other elements in the universe tuned to the same scale. Through this resonance we could, at will, have our single notes increased in power. This would allow us to harmonize and tune in to the energies of the planet to open our natural powers.

It was believed in the ancient schools of music that music was the bridge linking all things. They used specific harmonies, intervals and proportions in their music. They felt that when people heard sounds made of specific ratios, the rhythms of their cells, bodies and minds would be synchronized to the very same rhythms as the planets and plants, earth and sea. These particular sounds and rhythms would enhance life, making it healthier and more abundant. Music would open our minds to higher powers and would increase our awareness.

The composers of Baroque music were trained and made to use these particular numbers and patterns for harmony, rhythm and tempo in their music. This "mathematical" Baroque music was supposed to affect us by aligning, harmonizing and synchronizing our minds and bodies to more harmonious patterns.

Research shows that listening a few minutes a day to Baroque music will result in expanded awareness, better memory and other health benefits. Listeners felt refreshed,
energized, centered. Tension and stress disappeared. Headaches
and pains went away. Physiological graphs printed out the
proof-lowered blood pressure, lowered muscle tension, slower
pulse. (Ostrander, Schroeder, 1979, p. 141-143)

Try an experiment with plants and music. Each student in
your small group should raise the same plant in the same con-
ditions of light, water and temperature. Each group member
should choose one of the following types of music: rock,
jazz, country-western, Baroque, Indian or no music at all.
The plants should be able to hear only the chosen music.
Keep a daily log of each plant's progress - noting the health
of the plant, its stature (is it leaning or straight, and in
what direction is it leaning?) and its growth rate. After a
few months, report back to the group and the entire class on
your discoveries.

To make this even more accurate, chambers for each plant
could be built and kept in the classroom in the same location
with the music source inside.

An extension of this would be to research the effects of
various types of music on unborn babies.

Research yourself. In class, listen to rock music for
several minutes, take your pulse and write it down. Follow
the same procedure with the other types of music. What
differences do you see? If a blood pressure cuff is available,
follow the same procedure with your blood pressure.

Teachers may wish to play Baroque music in the
background during class. Note the effects it has on student performance and behavior.
Since the earliest times man has used his own bodies as the basis for measurement. Over the centuries man evolved rules of thumb — and of arm and food — to give himself units of length.

The measure that Noah used to build his ark was originally the length from the elbow to the middle fingertip — about 18 inches. It was divided by the ancient Egyptians into fingers and palms. Four finger widths or digits, made a palm, and six palms made a cubit. About 4,000 years later, the Egyptians added a seventh palm and the royal cubit became standardized at about 21 inches.

The inch used to refer to the width of a man's thumb. Eventually, in Rome, it became standardized as one-twelfth of a foot. Later, in England, Edward I defined the yard as three 12-inch feet and decreed that the inch should be equal to three grains of dry barley laid end to end. Shoemakers still use the barleycorn unit of measure.

During the 16th century King Henry VII used his thumb to redefine the yard. He fixed the yard as the distance from the tip of his own nose to the tip of his thumb at the end of his outstretched arm.

The foot, as its name implies, was once the distance from the heel to the tip of the big toe. It came to be used to measure
an acre by Edward I. Before Edward’s time an acre was the amount of land an ox could plow in a day. But since this could vary enormously, Edward pegged an acre at 40 rods by 40 rods — each road being 16 1/2 feet long — a measurement that has survived unchanged to this day.

The first and only measurement system based on logic, rather than physical accident, was worked out during the French Revolution, as part of a general reaction against tradition. Twelve scientists created the metric system, and they decided to find some fundamental length in the natural world — to be called a meter, after the Greek Metron, meaning "measure" — and to make it the basis of a system of multiples of ten.

The length they picked was the circumference of the earth measured on a line through the poles. The scientists measured a quarter of that circumference on a meridian through Paris and for convenience, called it 10 million meters or 10,000 kilometers. So the meter, which is slightly longer than a yard was fixed as 1/40,000,000 of the earth’s circumference through the poles. Today, however, it is defined in terms of the wavelengths of light.

Activities:

1) Check the units of length described here, that were measures of the human body, on yourself. How close were your measures to the measures given in history? How accurate were the ancient measuring systems?
2) Which measurement system do you prefer using today, customary or metric? Why? Is your answer a product of tradition and what you are used to, logic?

3) Design your own measurement system, based on a particular length that you invent. Determine the length of your base and of all other parts of your system. Be prepared to demonstrate measuring in your system.
Maurits Cornelius Escher, a Dutch artist, studied under Samuel Jessurun de Mesquita at the school of Architecture and Ornamental Design in Haarlem in the Netherlands. During this time Escher mastered the graphic techniques of woodcut, wood engraving and lithograph. He originally concentrated his efforts on landscapes and buildings.

The Escher that most of us are familiar with, the artist preoccupied with space filling of a repetitive nature, developed from an interest in the work of Moorish artists. The Moors occupied Spain from 711 to 1492. In 1936, while traveling through southern Spain, Escher visited the Alhambra in Granada. He studied the Moorish mosaics on the walls and the floors of the Alhambra. He copied the motifs of the tiles in his notebook and used them later in his paints. Although he was inspired by the Moorish tiles, Escher preferred recognizable, animate figures to purely geometric patterns in his drawings. He enjoyed tessellating animate figures so much that he said:

The dynamic action of making a symmetric tessellation is done more or less unconsciously. While drawing I sometimes feel as if I were spiritualist medium, controlled by the creatures which I am conjuring up. It is as if they themselves decide on the shape in which they choose to appear. They take little account of my critical opinion during their birth and I cannot exert much influence on the measure of their development. They are usually very difficult and obstinate creatures. (Mac Gillavry, 1965, p. 42)
Escher's style of art changed dramatically after 1937. He began to concentrate more on communicating his own personal ideas and explorations through his art. It is strange that Escher's prints should be so inextricably tied up with mathematics since he was not a trained mathematician. Escher himself expressed surprise at this fact when he said of some of his work:

The ideas that are basic to them often bear witness to my amazement and wonder at the laws of nature which operate in the world around us. He who wonders discovers that this in itself is a wonder. By keenly confronting the enigmas that surround us, and by considering and analyzing the observations that I had made, I ended up in the domain of mathematics. Although I am absolutely without training or knowledge in the exact sciences, I often seem to have more in common with mathematicians than with my fellow artists. (Escher, 1971)

Escher was also surprised to learn that some of the basic rules of periodic space-filling which he had worked so hard to discover and communicate has already been discovered by the science of crystallography.

Crystal formations became another rich source from which Escher drew material for his drawings. Many of his works were based on the principles of crystallography and color symmetry. Escher adapted the Moorish approaches and nature's forms with a remarkable inventiveness of his own. Today Escher's work is very well-known and popular, but is also highly respected by artists, mathematicians and scientists.

M.C. Escher's fanciful repeating patterns are, in essence, tessellations of modified polygons. The underlying
polygon tessellation and the relationships that exist between the congruent polygons in the tessellation are of prime importance in creating "Escher-like" patterns.

Let us select a polygon which tessellates the plane. A square is a convenient choice. Now select a transformation which could be used to generate the complete tessellation from this single polygon. For example, if we label the square as follows

Then the polygon could be rotated four times about B, 90° each time, before it returns to its original position. It could then be translated two square over, both horizontally and vertically, and the rotation repeated. If we keep track of where the numbered sides land as the polygon moves over the plane, we obtain the following schematic:
If we examine the arrangement, we see that a protrusion of "bump" on side 1 is a congruent indentation or "hole" on side 2, both equidistant from point A (and vice versa).

Likewise, a "bump" on side 3 is a congruent "hole" on side 4, both equidistant from point B (and vice versa).

We can summarize these relationships with the following notation:

1 <-> 2  Measure from A
3 <-> 4  Measure from B

By applying the rule several times in succession to the square, we have:
The modified square has exactly the same area as the parent square. It will tessellate the plane in the same manner as did the parent square.

For more practice and information on creating tessellations see *Creating Escher Type Drawings* By E.R. Ranucci, 1977; Creative Publications.

Activities

1) Examine several of the works of M.C. Escher and write down exactly what you feel and what you feel you see. Do you feel that there is a underlying message in the print? Write a conversation between you and a friend (or you and an enemy) about the feelings that you get from looking at it.

2) As you examine each piece of work that you choose, take notice of the geometric elements involved. Describe the work geometrically. Think of the following criteria:

   a) Is it a tessellation?
      1) If so, what is tessellating?
      2) What was the original polygon that Escher distorted to create the tessellation?

   b) If it is not a tessellation, what other geometric elements are involved?
      1) Does it appear to be three-dimensional?
      2) It is an optical illusion of sorts?
      3) Is it a paradox of some sort?
      4) List the interesting and amazing aspects of this
piece of work. What are the "hidden" surprises?

5) What questions would you ask another person about the tessellation?

3) Choose a polygon that tessellates. Create your own tessellation by distorting and rotation the polygon. Remember to start simple, then in subsequent tessellations add more and more distortions. Color your tessellation so as to make the repetitive pattern obvious to the viewer.

4) Using brightly colored ceramic tiles and a tile cutter create a tessellation mosaic of your own design. It may be framed to hang up, or used as a hot plate for foods or any other practical purpose that you can think of. For ideas research Arabic art. Geometric themes often occur in Arabic art. Find out why it is common to see so much geometry in Arabic art, instead of living things. How does science become an art? How is art really a science?

Visitor

At this point it would be a great idea to invite an expert on stained-glass art to visit the classroom. This person could discuss how stained glass is made, the importance of geometry in stained-glass art and the relevance of mathematics in the world of art. It would be especially exciting if he/she could give a demonstration of making stained glass
and allow the students to participate. The students' creations would make lovely mobiles for the classroom.
DOES ANYBODY REALLY KNOW WHAT TIME IT IS?

In prehistoric times human beings lived from day to day and from season to season with little concern for the passing time. They began to watch the movements of the sun, the moon and the stars. They used these regular, natural events to keep track of time. These natural events still govern the way we keep track of time. Our calendar year is based on the time required for the earth to make a full trip around the sun.

Do some research on time. See what you can find out about calendars, clocks and other ways to measure time.

Even though people use different systems for measuring other quantities, nearly everyone uses the same system for time. Why do you think this is so?

Work in small groups. Try to develop a “metric” system for measuring time. Think about the units we now use. Which would have to stay the same and which could be changed.

Which units of time are determined by nature and which are determined by people?

Notes:

The students responses should show that units of time are governed by astrological events that are the same everywhere on earth.

The students should also realize that it would be difficult to produce a completely metric calendar. The day could be divided into 10 hours and each hour divided in 10 units
and each of those divided into 10 minutes; so the minute would be $1/1000$ of a day instead of $1/1440$ of a day. The week could be 10 days, but the year would have to be 36 weeks plus several days.

Which units of time are determined by nature?

1) Day - the time it takes for the earth to rotate on its axis once, or the time it takes for the earth the sun and other stars to make one complete apparent revolution around the earth. The length of the day would be different on a different planet.

2) Year - the length of time it takes the earth to make one complete revolution around the sun. If we changed the length of the year, seasons would occur at different time of the year - and that is what happened before the present calendar was adopted.

3) Month - the revolution of the moon around the earth. To have the full moon always occur on the same day of the month, a month should be approximately 28 days long. Since the months of the calendar currently in use are mostly too long to be consistent with the phases of the moon, it could be argued that the month is not a natural unit. In some calendars the month is based on the moon and is definitely natural.

4) Week - is about $1/4$ of a lunar month and so is about the difference between the phases of the moon (new,
first-quarter, full, last-quarter) so it can be said to the natural, although it may not be quite the right length.

Which units of time are determined by people?
1) Hour
2) Minute
3) Second

An interesting notion to examine is the length of a day and a year (in earth days) on each of the other planets.

Looking at the chart below, try to determine how old you would be on each of the other planets.

Also, what strange phenomenon do you notice in the information about Venus? Why does this strange phenomenon occur?

How many days are in a year:

Well, that depends on what kind of year you are talking about. An earth year has 365 or 366 days. However, an astronomical year (the time between vernal equinoxes, when the sun appears to be in the same plane with respect to the earth) is 365 days, 5 hours, 48 minutes and 6 seconds. A
sidereal year (the length of time it takes the sun to get back to the same apparent position among the stars) is 365 days, 6 hours, 9 minutes and 9 seconds. A Hebrew year has 12 months and 354 days, except in those years when the month of Adar Sheni (29 days) is added to make 383 days. There are many other possibilities depending on the particular culture involved. Research various cultures and discover the differences in the lengths of years.
About 5000 years ago the Egyptians built what is still the world's biggest clock. It was designed to register time not only in hours but in days, seasons and even centuries.

It is the Great Pyramid of Cheops which was built in the 27th century B.C. It is the largest of the group of pyramids at Giza on the Nile. These pyramids are the only surviving example of the Seven Wonders of the Ancient World.

The Egyptians built this pyramid on such a scale, at such a latitude and at such a precise angle that it could indicate the precise day of the year as well as the time of day.

On the ground adjoining the northern and southern faces, wide, level pavement or "shadow floors" were constructed. In winter the pyramid would cast its shadow on the pavement to the north, and in summer the highly polished southern face would reflect a triangle of sunlight onto the pavement to the south.

The paving blocks had been cut in widths similar to the gradations by which each noonday shadow or reflection succeeded its predecessor. So the days could be measured and forecasts could be made for the solstices and the equinoxes.

The equinoxes are the two times each year when the sun crosses the Equator, while the solstices are the two days
when it is farthest from the equator.

The descending passage, leading into the heart of the pyramid was set at an angle of 26 17', the exact alignment to aim it at the Pole Star. Although the Pole Star changes every few thousand years due to the gravitational pull of the sun and the moon, the Pyramid has moved also with the movement of the earth and has aligned itself with the new Pole Star (today being Polaris).

The Pyramid was constructed to be an almost perfect observation from which the movement of the planets and the stars could be recorded.

It is believed now that Stonehenge - the great circle of giant stone slabs on Salisbury Plain in the English county of Wiltshire was begun around 1900 B.C., about 800 years after construction of the Great Pyramid.

Until recently, Stonehenge was popularly believed to have been built by the Druids as a sun worshipping temple and a site for human sacrifice. However it has been deduced that Stonehenge was built before the Druids appeared.

They believe that it was built in several stages over a period of 300 years. In 1963 Professor Gerald Hawkins, the American astronomer, publicized his description of Stonehenge as some kind of a prehistoric computer, whose function was to make intricate calculations of sunrises and sunsets, the movements of the moon and eclipses of both the sun and the moon. Hawkins fed details of the structure's stone alignment
into a computer and found that many related to the positioning of the sun and the moon. He brought forth the idea that by moving a marker stone around an outer circle of stones once a year, priests or astrologers could calculate suitable times for crop planting, predict weather cycles or practice divination.

It is easy to make a sundial that will allow you to tell the approximate time of day without a watch. On a sunny day set a stick that is at least one meter long into the ground so that it is upright. Carefully observe the positions of the shadow each hour and mark them using chalk or stones.

**Discussion questions:**

1) Why did you choose the location that you did?

2) Why is it important that others agree that your markings are accurately placed?

3) Do you think your sundial would still be useful several months from now? How would you need to change it?
THE MOBIUS RING

A recent and interesting field of modern mathematics is known as topology. The word topology means "study of place or position". Topology does not deal with the objects of traditional geometry. It is the geometry of distortion. It studies geometrical figures that retain their mathematical properties even when their size and shape change.

Among the founders of topology was the German mathematician and astronomer Augustus Ferdinand Mobius (1790-1868), who discovered a strange topological feature which became known as the Mobius strip. He described the figure as a "strip without a second side".

We expect a surface to have two sides ordinarily. For instance, a sheet of paper has a front and a back as does any other plane surface. However, Mobius managed to construct a one-sided strip; we cannot distinguish front from back or an upper side from a lower side. To illustrate this visually, take a rectangular strip and glue the ends together to form a ring. It will have a inner and an outer side. If, at a given point we begin to paint the outside green we will soon find that the entire outside is green. Similarly we can paint the inside red. A Mobius ring would be quite different as the two colors would overlap. Let us construct such a ring. Start with a rectangle but, before closing it into a ring, give one of the ends a half-turn. If we now start coloring a side, as we did before, we find there is nothing left
unpainted.

Such properties are called invariant. They concern the single side and single edge of the Mobius ring. Though originally the Mobius strip was just a mathematical curiosity, its features have become very important in theory and in practice. Recently, American industrialists have used the theory to design a new conveyor belt. Vacuum companies have also applied the theories in manufacturing their belts. Why would the Mobius features be beneficial to conveyor belts and vacuum belts?

**Mobius Activities**

1) Make a Mobius strip as described before. Cut the strip along its middle. What are your results. How do they differ from the results you would get if you cut a plain ring down the middle? Take your first results and cut it down the middle again. Now what are your results?

2) Take a rectangular strip of paper and draw three lines lengthwise on the paper. Make a mobius ring. Cut the ring along the lines. What are the results?

3) Try taking a rectangular strip of paper and twisting it twice before taping or gluing the ends together. Cut this loop down the center of the strip. What are the results?

Make another similar loop and cut it 1/3 of the way in from the edge. What are the results?

4) Finally, twist your rectangular strip of paper three
times before gluing the ends together. Cut this loop down the center of the strip. What are the results?

5) In your groups, place one Mobius strip on top of the other. Before you cut down the center of both strips, try to visualize what you predict will happen when you cut.

6) What are some other ways in which the Mobius strip could be used?
BIBLIOGRAPHY


