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Indicators of Future Mathematics Proficiency: Literature Review & Synthesis

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INDICATORS OF FUTURE MATHEMATICS PROFICIENCY:

LITERATURE REVIEW AND SYNTHESIS

A Thesis

Presented to the

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In Partial Fulfillment

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in

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by

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ABSTRACT

The beauty of mathematics can arguably be found in the way in which all concepts are interrelated and interwoven to create a massive web of knowledge and in the ways this can be applied to all aspects of life and technology. Given this inextricable interrelationship amongst several mathematical topics, many students encounter issues in learning mathematics due to gaps in their understanding of previously taught material. As a result, mathematics education in the K-12 setting has emphasized the need for interventions in order to help students grasp the progressively complex concepts that are required by our current society and education system as they advance throughout their academic career. This literature review researches effective and non-effective indicators of future mathematics proficiency as an initial step towards identifying the most beneficial cognitive and non-cognitive areas of focus, and consequently early interventions, in order to support student learning especially for underperforming students. Specifically, this research synthesizes research about three essential questions: (1) What skills, conceptual understandings, or student traits can serve as possible predictors of future mathematics proficiency? (2) Which of these identified skills, conceptual understandings, or student traits are stronger predictors of future mathematics proficiency? and (3) What is the degree of accuracy of these predictors? The research was conducted through the review of articles retrieved from diverse research studies.

The literature revealed that the single most effective indicator of future mathematical proficiency is knowledge of fractions, specifically, conceptual understanding of and operations with fractions as well as fluidity with rational operations. Other less effective indicators included early knowledge of whole number division, functional numeracy, students' attitudes and dispositions towards mathematics, gender, early mathematics achievement/ability, and literacy/linguistic ability. Other skills, conceptual understandings and student traits investigated in the relevant research included whole number arithmetic knowledge, number system knowledge, verbal and non-verbal IQ, working memory, and family education and income. These indicators did not exhibit a significant correlation to future mathematics performance and thus were classified as non-effective.

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CHAPTER ONE

INTRODUCTION

Background

"Research has shown that success in math is actually a predictor of success in college" (Gervasi, 2004, p. 3) yet, despite the significance of this correlation, too many students are repeatedly failing or underperforming in mathematics courses at all levels. According to *Algebra Policy in California: Great Expectations and Serious Challenges* (2009):

> Too many students are repeating the course [Algebra 1], sometimes multiple times … [and] data show that 38% of 9th graders who took the Algebra I CST in 2008 had taken the test in a prior year ... [and] more than half of 10th and 11th graders who took the CST were repeating it as well. (p. 8)

Furthermore, the California Department of Education reported that in the 2012-2013 academic school year, on average, 62.5% of students tested from grades 2 thru 7 scored proficient or advanced proficient in mathematics as opposed to 36% of Algebra 1 students who were tested that same year, a similar disparity appears in years 2010-2012("Standardized Testing and Reporting (STAR) Results (CA Dept of Education)," n.d.), thus highlighting the progressively poorer proficiency levels attained by students in mathematics courses. Such data naturally leads educators to question what can be done to improve early mathematics achievement either through more effective initial instruction or, if needed, early interventions in order to better ensure success in future mathematics classes and later in college. Additionally, this data also emphasizes a deeper issue within mathematics education especially considering the Common Core State Standards which call for;

> [L]earning mathematical content in the context of real-world situations, using mathematics to solve problems, and developing "habits of mind" that foster mastery of mathematics content as well as mathematical understanding… [and] reflect the knowledge and skills that are necessary to prepare students for college and careers and productive citizenship. ("California Common Core State Standards for Mathematics," 2013, p. 2).

Therefore, the Common Core State Standards requires students to learn more than algebraic computation skills and are asked to develop a deeper comprehension of algebraic reasoning.

My colleagues and I have consistently observed that students' struggle with algebraic concepts are usually made worse by a lack of understanding of basic mathematical concepts including operations with fractions. I have observed this same issue throughout all levels of high school including in my three years of teaching Pre-Calculus and my experience with tutoring Calculus. Many students were able to grasp the concepts being taught but failed to fully develop them because of their weak knowledge of fractions and other basic concepts. As a result, I began to wonder how student comprehension and proficiency would change if specific skills, operations with fractions in particular, were properly addressed at an earlier age and if this deficiency was, as a result, affecting students' proficiency in the future.

Goals and Research Questions

After an initial search of the relevant literature on fractions and student proficiency, I identified several skills, conceptual understanding, and student traits that had previously been researched and analyzed within the education community in relations to success in mathematics at all K-12 grade levels. The literature also revealed two possible categorizations of these skills based on their analyzed effectiveness: conceptual understanding and student traits. These two categories are differentiated given that the first (conceptual understanding) can be categorized as cognitive skills, the "ability to process…, reason, remember, and relate"("Cognitive Learning Approach | Oxford Learning®," n.d.) information, whereas the second (student traits) can be classified as non-cognitive, a set of attitudes, behaviors, and strategies. These categories then led me to consider the effectiveness of the individual conceptual understanding and student traits for the purpose of early intervention.

The purpose of this synthesis was to investigate these possible indicators of future mathematics proficiency for K-12 students. I have attempted to accomplish this by analyzing the literature in order to answer the following questions:

1) What skills, conceptual understandings, or student traits can serve as possible predictors of future mathematics proficiency?

2) Which of these skills, conceptual understandings, or student traits are stronger predictors of future mathematics proficiency?
3) What is the degree of accuracy of these predictors?

Through an analysis and synthesis of the existing research, I intended to identify and rank the various cognitive and non-cognitive skills and student traits that have been previously found to be effective predictors throughout all levels of mathematical education possibly including at the college level. Additionally, based on observations of my students, I anticipated identifying sufficient supporting evidence indicating a strong correlation between later algebra knowledge and early knowledge of whole number division and fractions as well as with early achievement in mathematics and students' attitudes towards mathematics.

CHAPTER TWO

METHODOLOGY

Search Procedures

Beginning with the hypothesis that knowledge of fractions is vital to future success in mathematics, I began researching related academic articles on fractions and operations with fractions through California State University San Bernardino's Library online resources. I then found additional resources within the previously collected articles through their references and citations. After developing a structure for the content and topics of my research, I extended my search to include specific skills and conceptual understanding of fractions, functional numeracy, and whole number division. I also began searching for articles related to the student traits that previous readings had revealed as effective indicators of future mathematics proficiency as well as skills and traits that were identified as non-effective indicators of future mathematics proficiency. These searches were conducted using California State University San Bernardino's Library online database and Google Scholar.

Criteria for Inclusion

The cited resources were selected based on their relevance to the previously listed students' conceptual understanding, mathematical skills, and traits. These student skills and conceptual understanding included cognitive indicators comprised of knowledge of fractions, functional numeracy, early knowledge of whole number division, whole number arithmetic knowledge, and number system knowledge. There was a wide range of student traits investigated within the research in relation to mathematics achievement; these included a variety of mathematical skills and early mathematics achievement and ability as well as students' attitudes and dispositions towards mathematics, effort (e.g. time on task), gender, literacy/linguistic ability, verbal and non-verbal IQ ("measure [of] general cognition without the confound of language ability" (DeThorne LS & Schaefer BA, 2004, p. 275)), working memory, and family education and income. Each reference was derived from these articles as supporting evidence to their predictive abilities with respects to future mathematics proficiency. Additional references were selected as background or statistical data pertaining to the current state of mathematics education data and policy.

CHAPTER THREE

THEMATIC UNITS ON THE RESULTS OF THE LITERATURE REVIEW

Effective Indicators of Future Mathematics Proficiency

Skills and Conceptual Understanding

For the purpose of this literature review, effective skills were defined as mathematical computational or arithmetic skills performed on real numbers, rational and irrational, which demonstrated a significant statistical correlation to later mathematics performance. Effective conceptual understandings were defined as an in-depth comprehension and critical thinking about the mathematical reasoning behind the previously mentioned computational skills that exhibited a strong correlation to later mathematics performance. The two categories were grouped together since they are closely intertwined in the sense that the underlying conceptual understanding and the identified skills given that "the precision and fluency in the execution of the skills are the requisite vehicles to convey the conceptual understanding… [and] one [cannot] acquire the former without the latter"(Wu, 1999, p. 1). The following indicators listed belong to of one or both categories.

Fractions Knowledge. Fractions knowledge proved to be the most prominently studied indicator of future mathematics proficiency. This is most likely due to the fact that this content area has also shown to be the single most effective indicator of future mathematics proficiency (Siegler et al., 2012, p. 695). Specifically, students' grasp of the concept of fractions and their fluidity with operations with fractions has demonstrated the strongest correlation to its predictive ability. Yet, despite their apparent significance, fractions knowledge and fluidity remains one of the major issues within mathematics education and in several professional career pathways. Gervasi (2004) noted that:

> According to The Nation's Report Card Mathematics 2000 … 31% of America's fourth graders … had difficulty shading a region so as to represent a fraction…34% of America's eight graders … were unable to determine the exact change due back from a purchase …[and] 35% of America's 12th graders … were unable to place a dot on a number line to represent a given fraction. (p. 20)

This fractions deficiency is also evident in professional fields such as nursing in which there is a strong potential for dire consequences based on errors. Sibbald (2005) noted from his research "that nurses are generally less able to function with fractions than high school results suggest they should be" (p. 6). This deficiency and the importance for the need of proper conceptual understanding of fractions substantiates the need for further studies regarding knowledge, in-depth comprehension, and applications of fractions as well as research on the predictive abilities of this indicator.

Robert Siegler conducted several studies researching the correlation between fractions and later mathematics performance. This in turn prompted several other studies in this specific content area of mathematics education. Siegler et al. (2012) argued that "if we can identify specific areas of mathematics that are most consistently predictive of later mathematics proficiency … we can then determine why those types of knowledge are uniquely predictive and can increase efforts to improve instruction and learning in those areas" (p. 691). Siegler et al. (2012) then proceeded to hypothesize "that knowledge of fractions at age 10 would predict algebra knowledge and overall mathematics achievement in high school, above and beyond the effects of general intellectual ability, other mathematical knowledge, and family background" (p. 693). Evidence from the collected data of Siegler et al. (2012) as well as many other research studies support this hypothesis as specified below.

Relevant research data revealed that fractions were the single greatest indicator of future mathematics performance. Ooten (2013) noted that "[p]revious investigation by MDTP has shown that the strongest predictive value by one test problem for success in the appropriate level class is a fraction question" (p. 76) and several of the studies quantified the predictive value of fractions through advancement in students' mathematics achievement across a one to six year comparison. One such study was that conducted by Bailey et al. (2012) in which a cross-lagged analysis, "the most popular procedure in many areas of psychological and educational research for identifying causal effects from longitudinal panel data"(Rogosa, 1980, p. 245), concluded that "performance on the sixth grade fractions concepts measure predicted 1-year gains in mathematics achievement" (Bailey, Hoard, Nugent, & Geary, 2012a, p. 447) meaning that in one academic year, there was a measureable gain in mathematics achievement that correlated to sixth grade fractions performance. This study also concluded that fractional computational fluency demonstrated a much stronger correlation to seventh grade mathematics achievement than "fluency in computational whole number arithmetic, performance on number fluency and number line tasks, central executive span, and intelligence" (Bailey, Hoard, Nugent, & Geary, 2012b, p. 447). The most significant of these correlations was identified by Siegler et al. (2012) in an analysis of national data collected from the United States and the United Kingdom identified that "competence with fractions in the fifth or sixth grade predicted performance on algebra and mathematics achievement tests 5 or 6 years later" (Bailey et al., 2012b, p. 448). According to Siegler at el. (2012):

> In the United Kingdom (U.K.) data …fractions knowledge at age 10 was the strongest of the five mathematics predictors of age-16 algebra knowledge and mathematics achievement…In the U.S. data, after effects of all other variables were statistically controlled, the relations between fractions knowledge at ages 10 and 12 and high school algebra and overall mathematics achievement at ages 15 to 17 were of approximately the same strength as the corresponding relations in the U.K. data… [and], in both data sets, the predictive power of increments to fractions knowledge was equally strong for children lower and higher in fractions knowledge. (p. 693)

The data also revealed a correlation between students' knowledge of fractions and both overall mathematics achievement and algebra knowledge yet “the correlation between high school students’ knowledge of fractions and their overall mathematics achievement was stronger than the correlation between their algebra knowledge and their overall mathematics achievement in both the U.K. … and in the U.S. data” (Siegler et al., 2012, pp. 693). Furthermore, the observed correlations were strongest for fractions and division, as opposed to all other arithmetic operations and specific student traits such as verbal IQ and parent income or education, and “did not differ between students with greater and lesser mathematics achievement in high school” (Siegler et al., 2012, p. 696). Thus, fractions are the single most significant predictor of future mathematics performance based on the compiled data and research conclusions.

Most of the relevant studies then proceeded to analyze the reasons as to why fractions were the most significant of the researched indicators. Siegler et al. (2012) hypothesized that the observed predictive ability might be due to the fact that the procedural competence with fractions and division “stems from those operations being more difficult than addition, subtraction, and multiplication, and thus measuring more advanced thinking”(p. 695) yet observed that they were “not uniquely predictive of most subsequent literacy skills…, as should have been the case if their predictive value was due solely to their greater difficulty… [and] that the[ir] predictive strength … did not differ between students with greater and lesser mathematics achievement in high school”(Siegler et al., 2012, pp. 695).

Siegler et al (2012) ultimately concluded that:

> The unique predictive value of early fractions and division knowledge seems to be due to the combination of many students not mastering fractions and division and to those operations being essential for more advanced mathematics, rather than to the relatively great difficulty of fractions and division per se. (p. 696)

In the case of the MDTP test problem, Ooten (2013) also noted that questions on fractions also involved additional concepts such as "symbolic notation and an understanding of the number line [thus] the abstraction level of this [type of] question is raised to an understanding of algebra, making it no surprise that an understanding of this problem is a strong predictor" (p. 76). Booth and Newton (2012) also concluded that algebra readiness heavily relied on fractions and suggested that knowledge of fraction magnitude, an understanding of a fractional number's relative size and position on the number line, accounted for the strong correlation to future algebra knowledge. According to Booth and Newton, "fraction magnitude knowledge represents a more fundamental understanding of fractions … [and] students' knowledge of the magnitudes of unit fractions seems particularly important for algebra readiness" (p. 251). Furthermore, Bailey et al. (2012) also stated that:

> It is possible … that gains in fractions procedural competence may be responsible for the observed cross-lagged relation between sixth grade fractions comparison scores and seventh grade mathematics

> achievement. However, this is not likely to be the full story given that Hecht and colleagues found that although both computational and conceptual competence with fractions predict outcomes on other mathematic measures, conceptual measures are generally the better predictor (Hecht, 1998; Hecht & Vagi, 2010). If anything, our results may underestimate the importance of conceptual knowledge of fractions because, due to time constraints for testing sessions, the fractions comparison test is only a brief measure of children's understanding of a few key aspects of fractions (e.g., the inverse relation between size of denominator and magnitude), and more intensive and elaborate measures of this conceptual knowledge may yield a more robust pattern (Mazzocco & Devlin, 2008; Siegler et al., 2011). However, this hypothesis remains to be tested. (Bailey et al., 2012b, p. 454)

However, Bailey et al. (2012) also argued that competence with fractions might not be the critical element to future mathematics achievement since "[a] finding that fractions measures predict word reading would suggest that these measures are proxies for more general cognitive abilities (e.g., working memory)" (p.448). Despite this correlation, the research data overwhelmingly points to fractions knowledge, specifically a conceptual understanding of fractions, as the main and strongest predictor of future mathematics performance.

Early Knowledge of Whole Number Division**.** The second most effective student skill and area of conceptual understanding, based on the relevant research, was that of early knowledge of whole number division. Siegler et al.'s (2012) research on the predictive value of fractions also revealed that early knowledge of whole number division consistently correlated to students' proficiency in later mathematics courses as evidenced by the data:

> Among the five mathematics variables derived from the elementary school tests, early division had the second-strongest correlation with later mathematics outcomes in the U.K. data (Table 1) and the strongest correlation with later mathematics outcomes in the U.S. data (Table 2). Concurrent correlations between high school students' knowledge of division and their overall mathematics achievement were also substantial both in the United Kingdom, $r(3675) = .59$, and in the United States, $r(597) = .69$, *p*s < .001. (Siegler et al., 2012, p. 694)

This data further supports early division knowledge as an indicator over others studied, specifically above the remaining three basic arithmetic operations and other student traits, since "[k]nowledge of fractions and whole-number division also had a stronger relation to math achievement than did knowledge of whole-number addition, subtraction, and multiplication; verbal IQ; working memory; and parental income" (Siegler et al., 2012, p. 695). Spline tests, "piecewise polynomials of degree *n* … [used] as approximating functions in mathematics and numerical analysis" (Wold, 1974, p. 1), from this study also showed that students' overall mathematics achievement did not influence fractions or division as an indicator of future mathematics proficiency.

Siegler et al. (2012) proposed that division, like fractions, measures "more advanced thinking" (p. 695) than addition, subtraction, and multiplication given its greater complexity and difficulty. This study also theorized that;

> [This] unique predictive value of early fractions and division knowledge seems to be due to the combination of many students not mastering fractions and division and to those operations being essential for more advanced mathematics, rather than to the relatively great difficulty of fractions and division per se. (Siegler et al., 2012, p. 696).

Previous research has established a distinction between fractions and whole number division despite their closely related cognitive skills requirements and mathematical structure. One possible explanation for the distinction might be that, based on the research data, understanding of fractions appears to have a greater impact as an indicator of future mathematics proficiency over whole number division thus validating the separation. We conjecture that this greater predictive ability may be attributed to fractions and operations with fractions requiring a deeper conceptual understanding of integer and rational numbers as opposed to whole number division. Furthermore, the conceptual understanding required for whole number division is also embedded within that of fractions and operations with fractions therefore possibly accounting for the closely related predictive abilities of the two content areas.

<u>Functional Numeracy.</u> Functional numeracy, also know as quantitative literacy, can be defined as an "individual's capacity and disposition to apply

mathematical knowledge and skills in everyday activities (Steen, 2001; DEECD, 2009, p.7)" ("Numeracy Literature Review for Evidence Based Practices Framework," 2010, p. 5). This capacity is closely related to, yet distinct from, academic mathematics and "[both] should be complementary aspects of the school curriculum [since] … [b]oth are necessary for life and work, and each strengthens the other" (Steen, 2001, p. 108). However, there appear to be varying opinions as to the specific skills required for this "capacity and disposition" as it pertains to everyday life. These skills range from simple arithmetic abilities for basic computations and rudimentary comprehension of percentages for financial decisions, up to more complex statistical abilities that would allow an individual to interpret and conduct basic risk-benefit analysis. These skills have also been shown to influence individual's economic opportunities yet "[a] substantial number of adults have not mastered the mathematics expected of an eighth grader (22% in the U.S.) [1], making them functionally innumerate" (Geary, Hoard, Nugent, & Bailey, 2013, p. 1) see also (United States Center for Educational Statistics, 2007).

An extensive search of the literature indicated a correlation between functional numeracy and future mathematics proficiency thus illustrating the extent of its predictive value. According to Geary, Hoard, Nugent, and Bailey (2013), "functional numeracy measures have been shown to be predictive of important life outcomes in adults" (p. 5) and their data also demonstrated a significant correlation between "Seventh grade mathematics achievement and functional numeracy scores" (p. 5). This correlation though was less significant after controlling for other factors such as "fifth grade working memory (the assessment closest to seventh grade), intelligence, and in-class attentive behavior" (Geary et al., 2013, p. 5).

The relevant literature also indicated two specific areas of study within the scope of functional numeracy, counting competence and number system knowledge. According to Geary et al (2013), "[a]dolescents' scores on the functional numeracy measure were significantly correlated with their beginning of first grade counting competence (r=.31, p,.0001) and number system knowledge (r=0.69, p,.0001) scores" (p. 4). However, the research pointed to a greater significance as a predictive measure for functional numeracy on number system knowledge over counting competence when other variables were included in the data analysis. Specifically, "counting competence did not predict functional numeracy (p=.40), when all other variables were included in the regression equation…[and it] was never significant (ßs = 20.091 to 0.124, ps..12) … [whereas] the number system knowledge variable was always significant (ßs = 0.246 to 0.348, ps,.014)" (Geary et al., 2013, pp. 4-5). Furthermore, Geary et al (2013) claimed that number system knowledge served as a predictor of functional numeracy but not of future mathematics performance when controlling for "seventh grade mathematics achievement [and]… did not predict seventh grade mathematics achievement after controlling for functional numeracy" (p. 5). Their findings indicated that children who performed poorly on functional numeracy, the bottom quartile, also displayed “a lower number system knowledge start point and slower first to fifth grade growth than children in the top and middle quartiles” (Geary et al., 2013, p. 6) thus reinforcing the correlation between the two.

Based on the research, functional numeracy proved to be the less effective of the skills and conceptual understandings presented in this literature review, yet still statistically significant thus suggesting the need for further study. As previously mentioned, a major issue with the use of functional numeracy as an indicator of future mathematics proficiency is the lack of a concrete composition of mathematical skills required within functional numeracy mostly due to its varying definitions throughout the literature. A formal list of skills for functional numeracy might provide grounds for further study of its predictive abilities by specifying which of its attributes account for the desired correlation to future mathematics proficiency.

Skills and Conceptual Understanding Summary

The following table, Table 1. Cognitive Effective Indicators of Future Mathematics Proficiency, provides a comprehensive summary of the effective indicators of future mathematics proficiency that were previously discussed in greater detail throughout this chapter. This summary is presented as an overview of the synthesized data as evidenced by the relevant literature.

Table 1. Cognitive Effective Indicators of Future Mathematics Proficiency

	Indicators	Main Research Results
Skills and Conceptual Understanding	Fractions Knowledge	• "In the United Kingdom (U.K.) data …fractions knowledge at age 10 was the strongest of the five mathematics predictors of age-16 algebra knowledge and mathematics achievement…In the U.S. data, after effects of all other variables were statistically controlled, the relations between fractions knowledge at ages 10 and 12 and high school algebra and overall mathematics achievement at ages 15 to 17 were of approximately the same strength as the corresponding relations in the U.K. data… [and], in both data sets, the predictive power of increments to fractions knowledge was equally strong for children lower and higher in fractions knowledge"(Siegler et al., 2012, p. 693). • "High school students' knowledge of fractions did correlate very strongly with their overall mathematics achievement, in both the United Kingdom, r (3675) = .81, $p < .001$, and in the United States, r (597) = .87, $p < .001$. Their fractions knowledge also was closely related to their knowledge of algebra in both the United Kingdom, r (3675) = .68, $p < .001$, and the United States, r (597) = .65, $p < .001$ … the correlation between high school students' knowledge of fractions and their overall mathematics achievement was stronger than the correlation between their algebra knowledge and their overall mathematics achievement in both the U.K. data, r (3675) = .81 versus .73, $\chi 2$ (1, N = 3,677) = 66.49, $p < .001$, and in the U.S. set, r (597) = .87 versus .80, $\chi 2$ (1, N = 599) = 15.03, $p < .001$" (Siegler et al., 2012, p. 694). • "[T]he unique predictive value of knowledge of fractions and knowledge of division stems from those operations being more difficult than addition, subtraction, and multiplication, and thus measuring more advanced thinking" (Siegler et al., 2012, p. 695).

		• Mastery and fluidity with fractions and whole number division "is critical for learning algebra... based on the mathematical structure of algebra (i.e., fractions are heavily embedded in much of algebra)" (Bailey, Hoard, Nugent, & Geary, 2012, p. 454). • "Scores on the fractions comparison test in sixth grade significantly predicted seventh grade mathematics achievement, controlling for central executive span, intelligence, seventh grade fractions comparison performance, and sixth grade mathematics achievement" (Bailey et al., 2012, p. 452).
	Early Knowledge of Whole Number Division	• "Among the five mathematics variables derived from the elementary school tests, early division had the second-strongest correlation with later mathematics outcomes in the U.K. data (Table 1) and the stron- gest correlation with later mathematics outcomes in the U.S. data (Table 2). Concurrent correlations between high school stu- dents' knowledge of division and their overall mathematics achievement were also substantial both in the United Kingdom, $r(3675) = .59$, and in the United States, $r(597) = .69$, $ps < .001$" (Siegler et al., 2012, p. 694). • "The unique predictive value of knowledge of fractions and knowledge of division stems from those operations being more difficult than addition, subtraction, and multiplication, and thus measuring more advanced thinking" (Siegler et al., 2012, p. 695). • Mastery and fluidity with fractions and whole number division "is critical for learning algebra... based on the mathematical structure of algebra (i.e., fractions are heavily embedded in much of algebra)" (Bailey et al., 2012, p. 454).
	Functional Numeracy	• "Functional numeracy measures have been shown to be predictive of important life outcomes in adults" (Geary et al., 2013, p. 5). • According to Geary et al. (2013), data demonstrated a significant correlation between "Seventh grade mathematics achievement and functional numeracy scores" although it was less

		significant after controlling for other factors such as “fifth grade working memory …, intelligence, and in-class attentive behavior” (p. 5). • “Counting competence did not predict functional numeracy … when all other variables were included in the regression equation…[and it] was never significant … [whereas] the number system knowledge [see below] variable was always significant” (Geary et al., 2013, pp. 4-5). • Geary et al (2013) claimed that number system knowledge served as a predictor of functional numeracy but not of future math performance when controlling for “seventh grade mathematics achievement [and]… did not predict seventh grade mathematics achievement after controlling for functional numeracy” (p. 5).

Student Traits

For the purpose of this literature review, effective student traits were defined as non-cognitive indicators including a student’s attitudes, abilities, effort, and personal characteristics including family background, which demonstrated a significant statistical correlation to later mathematics performance. These indicators were grouped together given that they cannot be qualified as mathematical skills or as conceptual understanding and are better described as attributes that a student possesses or describes aspects of their upbringing or background.

Student’s Attitudes/Dispositions Towards Mathematics. Several student traits were also prevalent within my reading in addition to information on indicators of future mathematics proficiency in the form of mathematical skills.

One of the most commonly mentioned student traits as a possible indicator was that of student's attitudes or dispositions towards mathematics. For the purpose of this literature review, student's attitudes/dispositions towards mathematics was subcategorized as a student's 'liking' or 'interest in' mathematics, expectations of success, achievement-related behaviors, self-efficacy, self-concept, confidence, and performance goals.

According to Hemmings and Kay (2010), "perceived mathematical confidence was a good predictor of mathematical achievement… [and] the more positive the attitude a student holds towards executing mathematical tasks, the better the achievement on those tasks and vice versa" (p. 43). However, in later works, Hemmings, Grootenboer, and Kay (2010) also proceeded to indicate that with respect to a negative attitude towards mathematics it is not clear whether the correlation "causes or is a result of low attainment" (p. 692). The relevant research generally indicated that "'liking of' or 'interest in' a subject has been found to have a statistically significant correlations with academic performance [and] … that 'interest in' a subject predicted achievement [although] … positive attitudes decrease across the years " (Winheller, Hattie, & Brown, 2013, p. 51). Winheller et al. (2013) also concluded that "[a]mong New Zealand elementary students, enhanced liking of mathematics resulted in worse mathematics performance, while among high-school students liking mathematics was irrelevant to performance" (pp. 65-66) thus identifying students' "liking of" or "interest in" mathematics as a poor indicator of future mathematics proficiency.

Winheller et al. (2013) hypothesized that this irrelevant performance was possibly attributed to high school students' awareness of high-stakes testing at that academic level, their "personal engagement in appropriately challenging instruction" (pp. 65-66) as it pertains to their academic achievement, and their development of self-monitoring abilities and perception of quality of instruction.

Although "liking of" and "interest in" mathematics exhibited little to no correlation with future mathematics proficiency, other forms of student's attitudes/dispositions towards mathematics, such as students' expectations of success and achievement-related behaviors such as confidence and performance goals, did demonstrate positive correlation. Long's 2003 research identified several studies in which mathematics achievement and attitudes towards mathematics displayed varying degrees of correlation with later mathematics performance and higher-level mathematics classes completion:

> Tsai and Walberg (1983) applied Walberg's theoretical model in a study of the mathematical learning of 13-year olds [in which]… [t]hey investigated two dependent variables, mathematics achievement and attitude toward mathematics [and]… indicated all factors had a significant (a = 0.05) positive association with mathematics achievement. While it was found to be a positive relationship between the independent variables and mathematics achievement, it was not as strong an association as found in an earlier study of 17-year-old mathematics students (Welch, Anderson, and Harris, 1982). In a later study, Horn and Walberg (1984) found that

> for 17-year old high school mathematics students who participated in the National Assessment of Educational Progress (NAEP), mathematics achievement was most closely related to higher level coursework and number of high school mathematics courses completed [and]… [i]nterest in mathematics was more closely related to student-centered, stimulating high school instruction and "an absence of stimulating objects in the home". (pp. 18-19)

Gervasi (2004) also noted that “non-cognitive variables such as student attitudes about their academic ability, their expectations of success, and their achievement-related behaviors (persistence in math courses for example) have a positive influence on their subsequent mathematics achievement” (p. 43). Furthermore, students’ confidence with respect to solving mathematics problems also demonstrated a positive correlation with mathematics achievement. Pajares and David (1995) identified “[s]tudents’ confidence to solve mathematics problems [as] … a more powerful predictor of their ability to solve those problems than … their confidence to perform math-related tasks or their confidence to earn As or Bs in math-related courses” (p. 196). However, one issue with confidence as an indicator according to Pajares and David (1995) was its cultural interpretation as specifically noted through their example of the contrast between American and Japanese students’ confidence in their mathematical abilities (p. 197). In this study, American students displayed higher levels of confidence despite being outperformed by Japanese students in a greater number of mathematics content areas. Pajares and David (1995) concluded that this effect could be attributed to Japanese students' modesty typical in Japanese culture as opposed to a "genuine lack of self-confidence" (p. 197). Additionally, students' performance goals in relation to their mathematics ability also demonstrated a similar correlation as students' confidence when compared to their future mathematics proficiency. Keys (2013) categorized performance goals as performance-avoidance goals, performance-approach goals, and mastery goals and described each through the following examples; "students may want to improve their math skills (mastery) *and* impress the teacher (performance-approach) *and* not come across as dumb in math class (performance-avoidance)" (p. 16). According to Keys (2013), study results concluded that performance-approach and performance avoidance goal orientations resulted in no or inconsistent association to future mathematics achievement (p. 30). Specifically, performance-approach goal orientation showed drastically inconsistent results "with some studies finding positive associations, other studies finding negative associations, and still other studies finding no associations" (Keys, 2013, p. 30). In contrast, Keys' (2013) research concluded that mastery goals "consistently predicted mathematics achievement" and that "students who maintain higher levels of mastery goals had the largest gains in achievement in this sample" (p. 29). However, this study "was composed of predominantly Hispanic and Vietnamese students" (Keys, 2013, p. 30) whereas other studies mostly focusing on White students did not exhibit as consistent findings.

The final subcategories within students' attitudes/dispositions towards mathematics were that of self-efficacy and self-concept. According to the American Psychological Association's *Self-Efficacy Teaching Tip Sheet* (2015), self-efficacy is defined as "an individual's belief in his or her capacity to execute behaviors necessary to produce specific performance attainments (Bandura, 1977, 1986, 1997) … [and] reflects confidence in the ability to exert control over one's own motivation, behavior, and social environment" (para. 1). Gervasi (2004) academic defined self-concept is as "a person's conception of his or her own ability to learn and perform an expected academic behavior that would lead to successful school achievement" (p. 15).

"Mathematics self-concept has been found to be a positive predictor of mathematics-related educational and career choices (Hackett and Betz, 1989) of persistence in math (Sherman, 1980) and performance on tests of math achievement (Astin, 1993)" (Gervasi, 2004, pp. 36-37). According to Gervasi (2004), the relevant research revealed that self-concept was a "better predictor of success than the … number of years of high school math taken" (Gervasi, 2004, p. 42). Gervasi (2004) also noted that relevant studies demonstrated that students' with higher academic self-concept later earned considerably higher grades in college algebra yet achievement-related expectancies did not display a significant correlation (Gervasi, 2004, p. 39). Similarly, Winheller et al. (2013) identified self-efficacy as an important predictor of academic performance (p. 51) and that “mathematics performance was predicted primarily by self-efficacy ratings” (pp. 63-64). However, Winheller et al. (2013) also stressed that “it is not always clear that … ‘having a sense of self-efficacy’ is more critical to success and engagement in mathematics [since its] … interplay with the learning environment and the joint effect on academic performance should receive greater focus” (p. 51). Ooten (2013) also studied “student self-efficacy beliefs about mathematics skills, understanding of fractions, and reasoning with fractions and higher fraction and decimal subscores on the placement exam (Mathematics Diagnostic Testing Project Level I Algebra Readiness)” as predictors with respects to mathematics in his study comprised of 563 Community college students (abstract). This study concluded that “[s]elf-efficacy beliefs … were stronger predictors than placement exam subscores [and] ... [s]tudent belief about the likelihood of passing the course was [also] a strong predictor” (Ooten, 2013, abstract). Pajares and David (1995) noted that in previous studies;

> The researchers concluded that variables within each model predicted performance to some degree but that self-efficacy was a weak predictor…Cooper and Robinson (1991) found a low but significant correlation between math self-efficacy and performance, but a regression model with math anxiety, the quantitative score on the American College Test, and prior math experience revealed that self-efficacy did not account for a significant portion of the variance in math performance. (p. 192)

According to Pajares and David (1995), self-efficacy, however, displayed "the strongest direct effect on problem-solving performance [when compared to] … self-concept, perceived usefulness of mathematics, math background" (Pajares & David, 1995, p. 190).

Despite the conflicting results from various research studies, student's attitudes/dispositions towards mathematics, or elements within it, appears to be a significant enough indicator of future mathematics proficiency. Specifically, self-concept and self-efficacy displayed the most significant effect on future mathematical performance thus warranting further study.

<u>Gender</u>. One of the more debatable non-cognitive indicators of future mathematics proficiency within the relevant literature was that of gender. Several research studies found data supporting and contradicting the predictive value of gender with respects to mathematics performance. Winheller et al. (2013) cited the 2006 PISA, Programme for International Student Assessment, test results in which "sex differences were observed in 10 of the high-performing countries including New Zealand … [and] on average, boys performed significantly better than girls" (p. 53). Winheller et al. (2013) also referenced other studies in which small gender differences were noted with respects to mathematics performance (p. 51). However, Winheller et al. (2013) also pointed to the data indicating that "girls felt significantly less confident in their mathematics abilities than boys and were less motivated to do well" thus indicating an alternate root to the possible correlation observed with gender and future mathematics performance (p. 51).

Gervasi (2004) indicated that:

> [T]he literature has been fairly consistent in finding that females have lower confidence in their math abilities than males [and] … [s]uch attitudes may explain shy females … are less likely than males to choose higher-level math courses in high school …, take fewer math courses overall than males …, and … avoid majoring in math and science in college. (p. 28)

Gervasi (2004) also noted that male and female students exhibited similar attitudes towards mathematics yet female students tended to develop more negative attitudes over time (p. 27).

Tartre and Fennema (1995) also identified higher means for male students when comparing gender as a significant predictor of mathematics achievement although this observed correlation was only significant for 8th grade students in their study (pp. 202-203). This study also pointed towards other factors, including affective variables, among each gender that could account for the higher means for male students. Tartre and Fennema (1995) identified spatial orientation as one such factor specifically for female students:

> For males, spatial orientation was not positively correlated with mathematics achievement, verbal or spatial visualization skill for any year of the study. However, for females, the 10th grade spatial orientation test was positively correlated with mathematics achievement grade 6 and 12, spatial visualization grade 10 and verbal skill each year. (pp. 209-210)

Tartre and Fennema (1995) also noted that despite previous research, male students' affective variables seemed more predictive of mathematics achievement than female students' affective variables and that this correlation was strongest for younger students (p. 214). Additionally, "[t]he results from this study also suggested that the cognitive variables were more consistently related to mathematics achievement than were the affective factors … [and it] did not find consistent gender differences in levels of mathematics achievement" (Tartre & Fennema, 1995, pp. 213-214).

Overall, conflicting research results would suggest that gender appears to be a poor indicator of mathematics proficiency. However, elements within each gender are worthwhile further investigating in the hopes of pinpointing the root of the observed correlations.

Early Mathematics Achievement/Ability. Another prominent student trait among the relevant research was that of early mathematics achievement or ability. Several of the articles indicated that there was a strong correlation between students' prior achievements in mathematics and their future performance. As Siegler et al. (2012) more simply phrased this observation; "children who start ahead in mathematics generally stay ahead, and children who start behind generally stay behind" (p. 691).

Tartre and Fennema (1995) found that performance in previous mathematics courses was "the single most consistent and strongest predictor of mathematics achievement for both genders at any grade level" among specific cognitive and non-cognitive variables as studied through a random sample of 60 students and their progression through grades six through twelve (pp. 203-204). This pattern was also shown to extend to higher level mathematics including college where data revealed that "taking algebra in eighth grade was strongly associated with taking advanced math courses in high school which in turn was found to be strongly associated with a higher likelihood of going on to college" (Gervasi, 2004, p. 25). Gervasi (2004) also noted that:

> [T]he strongest predictor of success in college algebra was the performance in the last remedial math course [and] the highest math course taken, highest placement test scores for mathematics and for English, and students' self-report of their overall average grade in high school, were significant positive correlates of total math completion ratios. (pp. 21-22)

Adelman (1999) identified the highest level of mathematics completion displayed the strongest and lasting influence on completing a bachelor's degree (p. 16). Specifically, Adelman (1999) identified that completion of algebra 2 more than doubled the likelihood of a student completing a bachelor's degree in the future and that "for each rung of HIGHMATH climbed, the odds of completing a bachelor's degree increased by a factor of 2.59 to 1" (pp. 16-17). Bailey et al. (2012) referenced previous studies that identified a correlation between sixth grade fractions comparison test scores and seventh grade mathematics achievement although "[t]he corresponding cross-lagged path from sixth grade mathematics achievement to seventh grade scores on the fractions comparison test was not statistically significant” (pp. 452-453). Reynolds and Walberg (1992) found that, grade seven mathematics achievement had “unmediated effects on Grade 8 mathematics achievement” along with exposure to mass media and instructional time although “prior achievement was the most dominant” (p. 321).

Although all of the previously mentioned articles identified a correlation between early mathematics achievement/ability and future mathematics performance, many of the authors also acknowledged several issues, mainly highlight racial and socioeconomic factors, within their research pertaining to this indicator’s predictive abilities. According to Siegler et al. (2012), “[m]arked individual and social-class differences in mathematical knowledge are present even in preschool and kindergarten [and] … are stable at least from kindergarten through fifth grade” (p. 691). Gervasi (2004) noted that Tartre and Fennema’s study did not account for ethnicity which is a problem when studying low income colleges and universities (p. 33). Gervasi (2004) also claimed that African-American and Hispanic students can sometimes be discouraged of taking higher mathematics courses by their teachers and counselors because of a language barrier, perceived abilities based on ethnicity, or they might simply have no access to these courses (pp. 33-34). Additionally, Long (2003) pointed out that Reynolds and Walberg's (1992) findings identified that “home environment [was] … more important to mathematics achievement while motivation was more important to mathematics attitude” (p. 21) thus identifying possible indirect influences that could account for their correlation between past and future mathematics performance. All of these extraneous influences can account for the observed correlation between early mathematics achievement/ability and future performance in the available data thus weakening its reliability as a predictor of future mathematics proficiency.

Literacy/Linguistic Ability. Literacy/linguistic ability is defined within the relevant research as the ability to read, write, and understand language. Gervasi (2004) noted that "[h]igher English placement scores contributed to higher total and remedial math completion ratios" (p. 101) thus highlighting a correlation between a student's literacy/linguistic ability and their mathematics performance. Gervasi (2004) also noted that;

> [T]he National Center for Education Statistics on the condition of education 2001 (NCES, 2001) found that students assigned to any remedial reading were less likely to complete a two- or four-year degree compared to students who took two or fewer remedial courses in mathematics only. (p. 101)

Yet these conclusion were based on data collected on a student population in which "nearly half (46%) of the students surveyed responded that English was not their native language" (Gervasi, 2004, p. 101) thus suggesting a less significant correlation given that "mathematics requires the ability to analytically solve problems but it also requires the ability to read the problems" (Gervasi, 2004, p.101). Although literacy/linguistic ability appears to be an external factor, specifically a language barrier, that influences future mathematics performance, it can still be considered a significant indicator given its predictive ability and remediation opportunity.

Other Indicators. Several other indicators emerged when researching the major categories proposed in this literature review yet were not studied as in-depth and thus can be classified as less significant yet a few are still worth noting. The first of these indicators was that of the learning environment. According to Winheller, Hattie, and Brown (2013), "[d]ata from 31 participating countries (including New Zealand) showed that the learning environment of schools, including school climate, was one of the more important contributing factors to students' academic outcomes" (p. 50). Winheller et al. (2013) also stated that "Quality of School Life (QSL) … [has] been shown to be [an] important predictor[ ] of academic performance" (p. 51). Specifically, "the QSL factors of 'satisfaction with' and 'enthusiasm for learning' positively predicted liking of mathematics, while the perception of a caring teacher and positive peer interaction all negatively predicted liking of mathematics" (Winheller et al., 2013, p. 63). The second of these indicators was that of social perception of peers. Winheller et al. (2013) identified research supporting "social perceptions of peers, particularly at high school, … [as] significantly important for students' academic progress and achievement" (pp. 50-51). Although both of these indicators revealed some degree of correlations to future mathematics proficiency, the relevant research did not delve deeper into their significance thus possibly implying a weaker correlation.

Student Traits Summary

The following table, Table 2. Non-Cognitive Effective Indicators of Future Mathematics Proficiency, provides a comprehensive summary of the non-cognitive effective indicators of future mathematics proficiency that were previously discussed in further detail throughout this chapter. This summary is presented as an overview of the synthesized data as evidenced by the relevant literature.

Table 2. Non-Cognitive Effective Indicators of Future Mathematics Proficiency

	Indicators	Main Research Results
Student Traits	Students' Attitudes/Dispositions Towards Mathematics	• "[S]tudents with higher academic self-concept earned significantly higher grades in college algebra but achievement-related expectancies (making at least a B average and graduating with honors) were not significant predictors" (Gervasi, 2004, p. 39). • "'[L]iking of' or 'interest in' a subject has been found to have a statistically significant relationship with academic performance. Marsh et al. (2006) showed that 'interest in' a subject [,i.e. math,] predicted achievement (i.e. b = 0.33 reading, 0.37 mathematics) in 25 countries involved in the PISA studies" (Winheller et al., 2013, p. 51). • "[T]he QSL factors of 'satisfaction with' and 'enthusiasm for learning' positively predicted liking of mathematics … [which] itself had negative or zero impact on mathematics performance" (Winheller et al., 2013, pp. 63-64). • "Study results suggest that mastery was the only

		goal orientation that consistently predicted mathematics achievement. That is, students who maintain higher levels of mastery goals had the largest gains in achievement in this sample" (Keys, 2013, p. 29). • "Students' confidence to solve mathematics problems was a more powerful predictor of their ability to solve those problems than was their confidence to perform math-related tasks or their confidence to earn As or Bs in math-related courses" (Pajares & David, 1995, p. 196). • "No significant associations for performance-approach goal orientation were found… There are inconsistent findings in the literature, with some studies finding positive associations, other studies finding negative associations, and still other studies finding no associations (Harackiewicz, et al., 2002; Linnenbrink-Garcia, et al., 2008; Urdan, 2004)" (Keys, 2013, p. 30).
	Gender	• "[G]irls around the world were not worse at mathematics than boys; on average there were only small gender differences, but girls felt significantly less confident in their mathematics abilities than boys and were less motivated to do well. These results underline girls not trusting in their abilities and the low self-efficacy expectations they have" (Winheller et al., 2013, p. 51). • "In PISA 2006, sex differences were observed in 10 of the high-performing countries including New Zealand; on average, boys performed significantly better than girls (Telford and Caygill 2007; see also Else-Quest et al. 2010)" (Winheller et al., 2013, p. 53). • "Females did not achieve significantly different completion ratios than males at the [community college] transfer or remedial level" (Gervasi, 2004, p. 110).
	Early Mathematics Achievement/Ability	• "[T]aking algebra in eight grade was strongly associated with taking advanced math courses in high school which in turn was found to be strongly associated with a higher likelihood of going to college" (Gervasi, 2004, p. 25).

		• "[T]he strongest predictor of success in college algebra was the performance in the last remedial math course" (Gervasi, 2004, p. 21). • "[T]he highest level of mathematics had the strongest continuing influence on bachelor's degree completion … [and] completing a course beyond algebra 2 (such as trigonometry and pre-calculus) in high school more than doubled the odds that a student who entered college would complete a bachelor's degree" (Gervasi, 2004, p. 25). • "Tartre and Fennema (1995) studied certain cognitive and affective(non-cognitive) variables and the mathematics achievement of students from grade six through grade twelve and found that prior mathematics performance was the strongest single predictor of subsequent mathematics achievement" (Gervasi, 2004, p. 33). • "[C]hildren who start ahead in mathematics generally stay ahead, and children who start behind generally stay behind. (Duncan et al., 2007; Stevenson & Newman, 1986)" (Siegler et al., 2012, p. 691). • "Reynolds and Walberg (1992a) found that, grade eleven mathematics achievement was directly affected by prior (grade ten) mathematics achievement, advanced coursework, number of courses, and instructional quality" (Long, 2003, p. 21).
	Literacy/Linguistic Ability	• "Higher English placement scores contributed to higher total and remedial math completion ratios … [since] mathematics requires the ability to analytically solve problems but it also requires the ability to read the problems" (Gervasi, 2004, p. 101). • "A report by the National Center for Education Statistics on the condition of education 2001 (NCES, 2001) found that students assigned to any remedial reading were less likely to complete a two- or four-year degree compared to students who took two or fewer remedial courses in mathematics only… suggesting that

		when students have difficulty reading, all other skills suffer" (Gervasi, 2004, p. 101).
	Other	• "Data from 31 participating countries (including New Zealand) showed that the learning environment of schools, including school climate, was one of the more important contributing factors to students' academic outcomes" (Winheller et al., 2013, p. 50). • "[S]ocial perceptions of peers, particularly at high school, are significantly important for students academic progress and achievement' (Patrick et al. 1997)" (Winheller et al., 2013, pp. 50-51).

Non-Effective Indicators of Future Mathematics Proficiency

Skills and Conceptual Understanding

For the purpose of this literature review, non-effective skills were defined as mathematical computational or arithmetic skills performed on real numbers, rational and irrational, which demonstrated a weaker statistical correlation to later mathematics performance. Non-effective conceptual understanding was defined as a deeper comprehension and critical thinking of the mathematical reasoning behind the previously mentioned computational skills that did not exhibit a strong correlation to later mathematics performance. The two categories were grouped together given that they are closely intertwined in the sense that the underlying conceptual understanding usually enhances the identified skills and vice versa. The following indicators listed belong to one or both categories.

Whole Number Arithmetic Knowledge. Whole number arithmetic knowledge is a student's knowledge of and fluency to perform operations on whole numbers. Previous data has demonstrated that fractional arithmetic fluency and early knowledge of whole number division has proven to predict mathematics performance throughout several grade levels. However, whole number arithmetic knowledge falls short with its predictive abilities despite being closely related. Both Siegler et al. (2012) and Bailey et al. (2012) compared fractions and whole number arithmetic knowledge and fluency and concluded that knowledge of and computational fluency with fractions was a significantly better predictor of future mathematics performance (p. 695; p. 447). Specifically, Siegler et al. (2012) found that;

> elementary school students' knowledge of fractions and whole-number division predicts their mathematics achievement in high school, above and beyond the contributions of their knowledge of whole-number addition, subtraction, and multiplication; verbal and nonverbal IQ; working memory; family education; and family income. (Siegler et al., 2012, p. 695).

Similarly, Bailey et al. found that "measures of fluency with computational fractions significantly predicted seventh grade mathematics achievement above and beyond the influence of fluency in computational whole number arithmetic, performance on number fluency and number line tasks, central executive span, and intelligence" (Bailey et al., 2012, p. 447). Additionally, Geary et al. (2013) also found that elements within whole number arithmetic knowledge did not display a significant correlation to future mathematics proficiency since "children's skill at using counting procedures to solve addition problems at the beginning of first grade was not predictive of their later functional numeracy scores, holding other factors constant" (p. 6). Geary et al. (2013) also attributed more complex cognitive knowledge skills within whole number arithmetic knowledge as predictive given their use of more intricate procedures (p. 6). Thus, the relevant research reveled that whole number arithmetic knowledge is not as significant an indicator of future mathematics proficiency as specific cognitive skills within it have proven to be.

Number System Knowledge. According to the *Eunice Kennedy Shriver National Institute of Child Health and Human Development* (2013), "[n]umber system knowledge is the ability to relate a quantity to the numerical symbol that represents it, and to manipulate quantities and make calculations" (para. 1). This cognitive indicator has also been researched for its predictive value with respects to students' future performance in mathematics. Although subsets within number system knowledge have been shown to be predictive of mathematics performance, specifically that of fractions and whole number division, this cognitive ability has not proven to be an indicator of future mathematics proficiency as a whole. Geary et al. (2013) concluded that "number system knowledge does not predict mathematics achievement, once functional numeracy is controlled" (pp. 6-7). This study also observed a correlation between number system knowledge and functional numeracy, which has been shown to serve as a predictor of future mathematics proficiency as previously stated.

Geary et al. (2013) identified school entry number system knowledge was an important factor in predicting student's functional numeracy when controlling for achievement in seventh grade mathematics (pp. 5-6). However, "poor number system knowledge scores did not predict the odds of being in the bottom quartile of seventh grade mathematics achievement, [when] controlling for all variables in Table 2 and functional numeracy scores" (Geary et al., 2013, p. 5). Finally, Geary et al. (2013) concluded that children with low first grade number system knowledge have a higher risk of performing low in seventh grade functional numeracy scores (p. 5).

Skills and Conceptual Understanding Summary

The following table, Table 3. Cognitive Non-Effective Indicators of Future Mathematics Proficiency, provides a comprehensive summary of the cognitive non-effective indicators of future mathematics proficiency that were previously discussed in further detail throughout this chapter. This summary is presented as an overview of the synthesized data as evidenced by the relevant literature.

Table 3. Cognitive Non-Effective Indicators of Future Mathematics Proficiency

	Non-Indicators	Main Research Results
Skills and Conceptual Understanding	Whole Number Arithmetic Knowledge	• "[E]lementary school student's knowledge of fractions and division predicts their mathematics achievement in high school, above and beyond the contributions of whole number arithmetic knowledge, verbal & non-verbal IQ, working memory, and family education & income" (Siegler et al., 2012, p. 691). • "[C]hildren's skill using counting procedures to solve addition problems at the beginning of first grade was not predictive of their later functional numeracy scores, holding other factors constant. One potential reason for this is because children who begin school behind their peers in the use of these counting procedures tend to catch up with other children within one or two years" (Bailey et al., 2012, p. 6). • "[M]easures of fluency with computational fractions significantly predict seventh grade mathematics achievement above and beyond the influence of fluency in computational whole number arithmetic, performance on number fluency and number line tasks, central executive span, and intelligence" (Bailey et al., 2012, p. 447).
	Number System Knowledge	• "[P]oor number system knowledge scores did not predict the odds of being in the bottom quartile of seventh grade mathematics achievement, … and functional numeracy scores [odds = 1.28, confidence interval, 0.42-3.90]. x2(1) = 0.19, p = .6664, 95%" (Geary et al., 2013, p. 5). • "[C]hildren who begin first grade with low number system knowledge are at heightened risk for low functional numeracy scores in seventh grade" (Geary et al., 2013, p. 5). • "[N]umber system knowledge does not predict mathematics achievement, once functional numeracy is controlled" (Geary et al., 2013, pp. 6-7). • "[G]rowth in number system knowledge is less

		important for predicting functional numeracy than is school entry number system knowledge" (Geary et al., 2013, p. 7). • "[N]umber system knowledge remained predictive of functional numeracy, after controlling for seventh grade mathematics achievement (ß = 0.195, p = .0014…) but did not predict seventh grade mathematics achievement after controlling for functional numeracy" (Geary et al., 2013, p. 5)

Student Traits

For the purpose of this literature review, non-effective student traits were defined as non-cognitive indicators including verbal and non-verbal IQ, working memory, family education and income, and effort, which did not demonstrate a significant statistical correlation to later mathematics performance. These indicators were grouped together given that they cannot be qualified as mathematical skills or as conceptual understanding and are better described as attributes that a student possesses or describes aspects of their upbringing or background.

Family Education and Income. Unlike other non-indicators of mathematics proficiency, family education and income could possibly be one of the strongest predictors of proficiency in mathematics according to the relevant data. However, the observed correlations can arguably be attributed to alternate factors that result from family education and income as opposed to a direct relationship. Gervasi (2004) identified research data that "found a strong relationship between parents' level of education and mathematics proficiency using the same data" (p. 30). Gervasi (2004) noted that:

> [A]s parents' level of education increased, so did the proportion of their children who demonstrated a high mathematics proficiency score while the proportion of students demonstrating a low proficiency score decreased as their parents' education increased… [and] that students placed in remedial math courses were more likely to come from families with lower incomes and educational levels, receive less encouragement to go to college than those students that placed in non-remedial math courses, and to live in neighborhoods and attend high schools that were predominantly non-minority. (p. 30)

Gervasi (2004) attributed this strong correlation to the larger number of resources available to students whose parents attended college and have a higher socioeconomic background (p. 31). Specifically, Gervasi (2004) hypothesized that a higher family education and income bracket can result in parents being able to properly diagnose and treat learning disabilities at an early age and provide for individual tutoring to help their students when needed (p. 31). Furthermore, "educationally sophisticated parents are more likely to assist their children … [whereas] parents of poor and minority students and first-generation students often lack the education and skills necessary to help their children with the same types of support and encouragement" (Gervasi, 2004, p. 31). This observed correlation could particularly be attributed to parents being able to provide for tutoring since this would deliver early targeted interventions possibly related to previously discussed cognitive variables that have been shown to predict mathematics ability. Thus, despite a strong correlation, family education and income cannot fully be classified as predictive of mathematics performance given the possible ramifications this non-cognitive variable has on a student's education.

Effort. Although it is closely related to student achievement, which has been shown to predict mathematics performance, students' effort, on its own, did not result in a significant correlation to future mathematics performance. Long (2003) referred to research that identified a relationship between student achievement and effort, including data on "time-on-task which could be the length of time in college or the number of hours per week spent on homework and courses" (p. 33). Additionally, this study also found that participating "students attribute[d] their failures, not to lack of effort which they can control, but to ability which is fixed and not under their control" (Long, 2003, p. 38). Long (2003) indicated that students' attitudes were most likely more closely related to students' academic achievement although he did not include this as a variable within his research and that "[s]tudents who like mathematics may be more willing to spend the time and energy necessary to be successful, whereas students who don't like mathematics may not invest the appropriate effort and time needed for successful course completion" (p. 98).

Verbal and Non-Verbal IQ and Working Memory. Verbal and non-verbal IQ and working memory were two of the least mentioned non-cognitive variables within the relevant research. Siegler et al. (2012) was the only scholarly article within my research that specifically mentioned these two variables as they pertain to future mathematics performance. Specifically, Siegler et al. (2012) noted that data from the United Kingdom and the United States consistently showed that:

> [E]lementary school students' knowledge of fractions and whole-number division predicts their mathematics achievement in high school, above and beyond the contributions of their knowledge of whole-number addition, subtraction, and multiplication; verbal and nonverbal IQ; working memory; family education; and family income. (p. 695)

Thus, one can only conclude that Siegler et al. (2012) researched these two non-cognitive variables yet found no significant correlation meriting further study.

Student Traits Summary

The following table, Table 4. Non-Cognitive Non-Effective Indicators of Future Mathematics Proficiency, provides a comprehensive summary of the cognitive non-effective indicators of future mathematics proficiency that were previously discussed in further detail throughout this chapter. This summary is presented as an overview of the synthesized data as evidenced by the relevant literature.

Table 4. Non-Cognitive Non-Effective Indicators of Future Mathematics Proficiency

	Non-Indicators	Main Research Results
Student Traits	Family Education and Income	• "According to the College Board (1999), educationally sophisticated parents are more likely to assist their children early on by reading to their children at young ages, seeking help and guidance early in diagnosing learning disabilities, and finding tutors if necessary from the time their children are in grade school through the high school years. In contrast, parents of poor and minority students and first-generation students often lack the education and skills necessary to help their children with the same types of support and encouragement" (Gervasi, 2004, p. 31).
	Effort	• "Student attitude goes to the heart of the learning situation [since] … [s]tudents who like mathematics may be more willing to spend the time and energy necessary to be successful, whereas students who don't like mathematics may not invest the appropriate effort and time needed for successful course completion" (Long, 2003, p. 98). • "Smith and Price (1996) found, in a study of freshman developmental students at an urban university … [that they] attributed their failures, not to lack of effort which they can control, but to ability which is fixed and not under their control" (Long, 2003, p. 38).
	Verbal and Non-Verbal IQ and Working Memory	• "Knowledge of fractions and whole number division also had a stronger relation to math achievement than did knowledge of whole-number addition, subtraction, and multiplication; verbal IQ; working memory; and parental income. These results were consistent across data sets from the United Kingdom and the United States" (Siegler et al., 2012, p. 695).

CHAPTER FOUR

DISSCUSSION

Conclusion and General Issues

Based on the presented findings, one can only conclude that fractions knowledge and early knowledge of whole number division are the single most significant cognitive predictors of future mathematics proficiency. Similarly, early mathematics achievement or ability also demonstrated to be the most significant non-cognitive indicator of future achievement in mathematics. However, this non-cognitive student trait can also possibly be attributed to the aforementioned cognitive skills of fractions and whole number division knowledge given that if a student has shown to exhibit early mathematics achievement then they are most likely competent with both of these skills. This possible issue mirrors that of other researched indicators, specifically that of family education and income. Such indicators can be considered to have circumstantial causes that render them as predictors such as the previously mentioned early interventions in the form of tutoring for students whose parents can afford to provide private help. This early intervention for students of specific socioeconomic backgrounds can provide remediation opportunities, which most likely would include fractions and whole number division knowledge, thus serving as possible proof that interventions in certain areas will result in better mathematics ability in the future.

CHAPTER FIVE

FUTURE DIRECTION

VanDerHeyden and Burns (2009) noted that “[r]esearch has demonstrated that early identification and intervention may prevent and remediate deficits in mathematics performance [and] sensitive procedures are needed to better identify which students need assistance” (p. 73). This literature review is the first step towards the development and implementation of significant targeted early interventions based on the researched indicators of future mathematics proficiency in order to ensure a stronger foundation in mathematics. I believe that the previously identified predictors of future mathematics proficiency can be used to identify at risk students at an early age in order to attempt to correct one possible factor that might attribute to poor future mathematics performance. Several of the relevant research studies also emphasized this application for the identified predictors such as Siegler et al. (2012):

> If researchers can identify specific areas of mathematics that consistently predict later mathematics proficiency, after controlling for other types of mathematical knowledge, general intellectual ability, and family background variables, they can then determine why those types of knowledge are uniquely predictive, and society can increase efforts to improve instruction and learning in those areas. The educational payoff is

> likely to be the strongest for areas that are strongly predictive of later achievement and in which many children's understanding is poor. (p. 691)

This "educational payoff" is even more significant when considering the possible ramifications throughout a student's entire academic education including college. Gervasi (2004) identified a strong correlation between "taking algebra in eight grade … [and] taking advanced math courses in high school which in turn was found to be strongly associated with a higher likelihood of going to college" (p. 25). Gervasi (2004) also stated that research has also identified that "success in math is actually a predictor of success in college" (p. 3) and that "[d]etermining the factors that predict students' achievement in mathematics is worthy of our consideration as many students fail or drop out of college mathematics courses, more so than any other academic discipline" (p. 6).

Although several of the effective indicators within this literature review demonstrated a strong correlation to future mathematics abilities, they are not the most practical pathways to possible interventions given their immutable nature, such as gender and other such student traits. For this reason, we must address those indicators that can be affected through interventions and classroom modifications such as fractions knowledge, functional numeracy, early knowledge of whole number division and students' attitudes/dispositions towards mathematics. Booth and Newton (2012) also believed that "[i]f fraction knowledge is indeed the doorman for the gatekeeper (Algebra), then it follows that improving students' skill in fractions should lead to more complete mastery of algebra" (p. 252). Siegler et al. (2012) stressed that the high school mathematics education system requires extensive improvements, as evidenced by their findings, and that fractions and division conceptual understanding has to be reinforced not only for students but also for teachers in order to possibly "yield substantial improvements in students' learning … of more advanced mathematics as well" (p. 696). Additionally, according to Winheller et al. (2013):

> Teachers should focus more on enhancing the quality of learning and student self-efficacy than on prioritizing peer or teacher–student relations and liking of mathematics … [since] students who have confidence and belief in their ability to control their engagement and learning activities achieve more. (pp. 65-66)

Ultimately, as Gervasi (2004) stated, "[e]vidence suggests … that completing an advanced mathematics course in high school helps mitigate the disadvantages of first-generation students … further adding to the endorsement of mathematics as an agent of success" (p. 31) and thus emphasizing the importance of additional study in order to help all students succeed.

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