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Additional activities for Workjobs II: Number activities for early childhood by Mary Baratta-Lorton: supplementary activities for beginning number concepts for learning handicapped students

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Additional Activities for

<u>Workjobs II: Number Activities for Early Childhood</u>

by Mary Baratta-Lorton

Supplementary Activities for Beginning Number Concepts

For Learning Handicapped Students

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The purpose of this project is to provide a resource guide in the development of supplementary activities which can be used by both special education and regular teachers to provide concrete experience that will support the development of mathematical concepts. The guide is based on concepts generated by Mary Baratta-Lorton in her book, Workjobs II: Number Activities for Early Childhood. It will consist of activities developed for this project and selected material from the text.

This project is based on an activity-centered approach which utilizes the child's natural style of learning. How children develop mathematical concepts is crucial in developing experiences for mathematical understandings. The mathematics that is presented at any given time should reflect what is known about children and their development. Children with learning disabilities, because of their disabilities, have not progressed normally through developmental stages. (Glennon and Cruickshank, 1981) An appropriate mathematics curriculum for primary grades would reflect this by providing for the manipulations of concrete objects, multiple embodiments of mathematical ideas, and social interactions of children. Programs that move quickly into the symbolic level using paper and pencil for rote drill are at odds with what is known about how children learn.

As a teacher in a Special Day class, I researched the available programs appropriate for learning disabled students. Most programs are concerned with essentially the same mathematics topics but are inconsistent with the psychological makeup of the student. According to T. Post (1980) in "The Role of Manipulative Materials in the Learning of Mathematical Concepts," "Mathematics programs that are dominated by textbooks are inadvertently creating a mismatch between the nature of the learner's needs and the mode in which content is to be assimilated." For young children, mathematics involves the mental construction of relationships children build by reflecting on concrete objects. (Kamii, 1985) It is this researcher's opinion that the <u>Mathematics Their Way</u> approach and instructional theory emphasizes manipulative learning experiences suitable for the primary grades in special education and regular classes.

For the purpose of this paper, two definitions will be established: "learning disability" and "manipulatives." The term learning disability has had a history of being ambiguous. The following definition was established in 1981 by the National Joint Council for Learning Disabilities:

> Learning disabilities is a generic term that refers to a heterogeneous group of disorders manifested by significant difficulties in the acquisition and use of listening, speaking, reading, writing, reasoning or mathematical abilities. These disorders are intrinsic to the individual and presumed to be due to central nervous system dysfunction. Even though a learning disability may occur concomitantly with other handicapping conditions (e.g., sensory impairment, mental retardation, social and emotional disturbance) or environmental influences (e.g., cultural differences, insufficient/inappropriate

instruction, psychogenic factors), it is not the direct result of those conditions or influences.

This paper will use the definition for manipulative materials presented by Michael Hynes (1986) in Arithmetic Teacher:

> Manipulative materials are concrete models that incorporate mathematical concepts, appeal to several senses, and can be touched and moved by students.
>
> This definition implies that students will have materials in their possession to manipulate. It is not sufficient for students to observe a demonstration of the use of an aid. The act of manipulating the aid allows students to experience the patterns and relations that are the focus of mathematics.
>
> The purpose of using manipulatives is to assist students in bridging the gap from their own concrete environment to the abstract level of mathematics. (p. 11)

This project is designed to fill three general needs. First, it will provide a literature review of the role of manipulatives in mathematics education for the learning disabled student. Second, it will provide an overview of the Mathematics Their Way approach and philosophy. Third, it will provide a guide for the construction of activities that have been developed for the number strand text, Workjobs II. This guide will provide a variety of easily constructed, inexpensive, and durable activities which can be used to develop beginning number concepts.

II(a). The Role of Manipulatives in Mathematics Education

The earliest records of mathematics indicates that it probably arose in response to practical needs in agriculture, business, and industry. Manipulatives in mathematics have their origins in the basic fundamental process of describing the size of sets or groups of things in the idea of one-to-one correspondence. The nomad shepherd kept record of this flock by keeping an equal number of rocks in his pouch. Groups of stones were too ephemeral and cumbersome for activities and prehistoric man sometimes made a number record by cutting notches in a stick or bone. Numerals can be traced to the various designs that were used. Words expressing these numerical ideas were slow in developing because it was easier to cut notches than to identify by words. (Boyer, 1968)

The idea that the learning of mathematics progressed through levels of abstraction has evolved. Pestaozzi is credited with influencing instruction through experiences with concrete objects. Later, Froebel, with his "gifts" of manipulative materials for the kindergarten curriculum, provided a basic model. Montessori used the same principle but added the ingredient of action which was channeled in advance by the nature of the manipulative.

At the turn of the century, Dewey argued for the provision of first-hand experiences in a child's education. In 1935, Brownell wrote of the importance of concrete experience in mathematics. He expressed concern that instruction in counting was immediately followed by drill and that this practice neglected the elements of meaning and complexity during the first stages of arithmetical learning.

The major theoretical rationale for the use of manipulative materials in a laboratory-type setting has been attributed to the works of Piaget, Bruner, and Dienes. They represent a cognitive viewpoint of learning which differs from the connectionist views of Dewey. This modern cognitive approach places great emphasis on the process of learning. It is concerned with the "how" as well as the "what" when dealing with learning. Piaget suggests that concepts are formed by children through reconstruction of reality, not through an imitation of it. Bruner indicates that knowing is a process, not a product; and Dienes, whose work specifically relates to mathematics instruction, suggests that children need to build or construct their own concepts from within rather than having these concepts imposed upon them. They subscribe to the basic tenets of gestalt psychology in that they view the whole as being greater than the sum of its parts. Translated into learning theory, it represents the idea that the learning of large conceptual structure is more important than the mastery of large collections of isolated bits of information. Each concurs that it is the meaning that the individual attaches to an experience which is important, and that it is the physical action on the part of the child that contributes to his or her understanding of ideas encountered. (Post, 1980)

In his early works, Piaget formally develops the stages of intellectual development and the ways in which they relate to cognitive structure. Originally a biologist, Piaget realized that the problem of knowledge could only be studied through psychology. He found that it was necessary to think of intellectual development in terms of an evolution of qualitatively different stages of thought. He conceived of the term genetic epistemology, which is the the study of the nature and origins of knowledge. (Kamii, 1985) Piaget regards intelligence as effective adaptation to one's environment. Just as the infant's motor coordination, respiration, and digestion change to adapt him to his environment, so does his mental structure. The evolution of intelligence goes through stages of development involving assimilation and accommodation. In his book, The Child's Perception of Numbers, Piaget (1961) states in his hypothesis that the construction of numbers go "hand in hand" with the development of logic. He uses the child's response to questions as windows to the level of cognitive structure. J. McVickers Hunt (1975) noted that Piaget's observations suggested how easily observed actions can imply cognitive structure, and how changes in reactions to a given situation can imply a change in these structures. (Uzgiris and Hunt, 1975)

The first stage of development, from birth to about two years of age, is referred to as the sensory-motor period, and is characterized by physical exploration and sensory stimulation. The preoperational stage continues until approximately seven years of age and is characterized by egocentric thinking and illogical intuition based on perception. The third period of concrete operations continues until about age twelve and is characterized by thought processes which are reversible. Reversibility was Piaget's criterion for the separation of preoperational and operational thought. The last stage of formal operations begins at about twelve and is characterized by abstract thought. (Pulaski, 1971) There is a gradual development of structure within each period of each individual and there are inter-individual and intra-individual differences within and among cultures. Piaget assigned an age level to a task when 75 percent of the children tested achieved success. (Labinowicz, 1980)

The stage of preoperational thought is particularly important to educators of young children. Piaget places major emphasis on the role of activity in intellectual development. He perceives that one of the major sources of learning, if not the most essential one, is the intrinsic activity of the child. The young child must act on things to understand them. Ginsburg and Opper (1969) state that this may be the most important single proposition that educators can derive from Piaget's work.

Elkind (1972), who interprets Piaget for early childhood educators, agrees that it is the child's actions upon things that facilitates his or her thinking. He states that, "The young child's actions are progressively internalized until he is able to do in his head what he had to do with his hands." (p. 18) This transfer of action comes about gradually as he develops the mental structure to think about such activities.

Piaget made a distinction between three kinds of knowledge according to their ultimate source: physical knowledge, logical-mathematical knowledge, and social knowledge. These are particularly important in the mathematical learnings of the child. Physical knowledge is the knowledge of objects in external reality and can be observed. Logical-mathematical knowledge is knowledge consisting of relationships constructed by each individual. This refers to the differences or likeness of things. It also refers to the "two-ness" of two objects. The two is not within one object or the other, but in the relationship between them and must be constructed mentally by the child. The ultimate source of social knowledge is convention made by people such as the fact that Christmas comes on December 25th. (Kamii, 1985)

Piaget speaks of more than the learning of mathematics. Dienes, however, has concerned himself exclusively with mathematics learning.

For both the major message is student involvement in the learning process. This involvement routinely involves the use of a vast amount of materials.

Similar to Piaget's structure, Dienes' theory of mathematics learning involves four basic components. The first principle is the play stage. This involves the exploration of new materials by children as they tend to play with it. The second principle is the conceptual learning that takes place when children are exposed to the concept through a variety of physical contexts. The provision of multiple experiences, using a variety of materials, is designed to promote abstraction of the mathematical concept. The third principle is of constructivity and consists of constructive thinkers and analytical thinkers which can be equated with Piaget's concrete and formal operational stages, respectfully. The last principle concerns summary and implications.

Dienes argues that the presentation of mathematical concepts in several different forms or embodiments is the key to mathematics learning. The use of different representations causes children to focus on what is constant about them. The child first transfers a learning from one content to another, and then to generalize across contexts to develop a firmer understanding of mathematics.

The instructional model of Bruner is based on the concepts of structure, readiness, intuition, and motivation. He feels that a key to readiness for learning is intellectual development, and this is supported by a rich and meaningful environment.

Piaget, Dienes, and Bruner agree that meanings of concepts can best be developed when individuals are allowed and encouraged to interact personally with various aspects of their environment. This includes materials, children, and adults. Proper use of manipulative materials would promote these goals. (Post, 1980)

In addition to the mathematical concepts, manipulative materials in mathematics can provide an avenue for language experience. Preoperational children are very egocentric and feel no obligation to be coherent. The necessity to make logical statements grows out of social interaction. For older children, it can provide the experiential base while complementing the symbolic side of mathematics. Lovell (1971) emphasizes that concrete experience involving action on the part of the child, active teacher participation, and the stimulation of discussion must go together for optimum learnings in mathematics.

Research supports Piaget in his views that language is structured by logical thought. Although there is a close relationship between language and thought at the stage of formal operations, most children of five to twelve do not achieve operational thinking or reasoning through words, but must do so through mental constructions of their environment. When using manipulatives, young children can demonstrate their thinking in problems concerning physical materials several years before they can deal with it verbally. For example, children can seriate lengths and color about five years prior to solving verbal seriation problems. (Labinowicz, 1980)

The importance of using manipulatives has long been recognized by mathematics educators. Writing for the Editorial Panel of <u>Arithmetic Teacher</u>, Joan Worth (1986) reviewed the support during the last forty years which shows that the value and importance of using manipulatives has long been recognized. A comprehensive review of research was compiled in 1976 and reported by Suydam and Higgins. In general, the report supports the use of manipulatives and concludes that "across a variety of mathematical topics, studies at every grade level support the importance of the use of manipulative materials." (p. 3) Weaver and Morse refer to a position paper written by the National Council of Supervisors of Mathematics in 1978 which states "to help students fully understand basic mathematical concepts, teachers should utilize the full range of activities and materials available, including objects the students can actually handle." (p. 138) Worth summarizes the 1980 National Council of Teachers of Mathematics report, An Agenda for Action, which continues to support the use of manipulatives. "Teachers should use diverse instructional strategies, materials, and resources, such as . . . the use of manipulatives, where suited, to illustrate or develop a concept or skill." (p. 2)

In a later review, Suydam (1986) found that research continues to support the use of manipulatives in producing mathematic achievement. She cited research by Parham in 1983 in which students of sixty-four elementary schools who had used manipulatives material scored at approximately the eighty-fifth percentile. Included in this review was a 1984 research by Canny which found the fourth graders involved in the study scored higher in problem-solving activities when manipulatives materials were used to introduce the basic skills.

Suydam (1986) conducted a survey in which she reported that most teachers indicated a belief in the materials but this belief is not always translated into the classroom. Her research indicated that first grade teachers make frequent use of manipulatives. From second grade on, however, these materials were used with decreasing frequency. In general, the research showed that thirty-seven percent of K-6 teachers used manipulatives less than once a week and they were never used by nine percent of the teachers. Suydam considers that the lack of manipulative materials is a major cause of learning difficulties of students. She cited a survey conducted in 1983 in which, "few teachers of kindergarten through fifth grade in a large urban district reported using any manipulative materials more than five times a year." (p. 3)

Supported by theory, research, and teachers, the use of manipulatives materials is of definite importance in how well children understand and achieve in mathematics. But, as indicated, research reveals the infrequent use of these materials. There are many factors involved. Classroom teachers suggest the lack of funds as the most important factor. Post (1980), however, suggests that there are other more complex reasons for their absence. He states that systematic use of materials is more difficult for the teacher than presenting a program designed around texts and workbooks. The school-aged child is in the process of learning to be responsible for his or her own actions, learnings, and behaviors. For this reason, Post suggests that classroom management in such a laboratory approach prevents teachers from working with the manipulative materials.

The accountability issue also serves to prevent the use of manipulatives. When accomplishment is viewed in terms of "covering" pages, the use of extra activities are seldom used. When the value of the overall program is determined by the use of standardized instruments which requires children to calculate at the symbolic level, Post concludes that these activities would again appear counterproductive. According to Leinwand (1986), in his article, "Curricular Improvement Versus Standardized Testing" in Arithmetic Teacher, standardized test results are the most important measure of educational success in the eyes of the public. He states,"In this fashion, standardized testing programs, more than texts or local or state curricula, drive mathematics instruction and serve as the greatest single obstacle to the implementation of positive change." (Leinwand, 1986, p. 3)

II(b). Trends in Mathematics Instruction for Learning Disabled Students

A review of the literature was conducted to identify trends in mathematics instruction for learning disabled students. The scope of the research was limited to programs and approaches for the primary grades.

There are limited references to and programs for mathematic instruction, but from these references a definite trend can be identified. In special education, as in general eduction, there are many voices condemning the excessive emphasis on teaching symbolic operations to children who have inadequate foundations in the concrete, manipulative realm.

In their book, Learning Disabilities, Myers and Hammill(1982) stated that in comparison to speaking, reading, and writing disorders, relatively little is known about either the nature or the type of specific disabilities in mathematics. However, according to the authors there exists a great number of children who experience learning problems in mathematics, and these children are handicapped as children who cannot read. Sternberg and Fair (1982) articulate the various hypotheses describing the reasons why handicapped children experience problems in the area of mathematics. The first hypothesis is that these problems stem from deficits in the functioning of various psychological processes such as memory, spatial ability, language awareness, and perceptual ability. A second hypothesis is that they arise from the presences of physiological dysfunctions. The third hypothesis involves the issue of cognitive readiness. Other hypothesis would include the variety of problems that handicapped children experience with mathematics such as poor teaching and the stress upon computational worksheets or rote memorization of number facts.

Myers and Hammill (1982) review the issue of selecting an appropriate instructional resource for the learning disabled student and state that although there are many instructional programs for "teachers who wish to persuade their students that 2 + 2 = 4," there are few that reflect the recent shift from emphasis on mathematical products to programs which support concept development. (p. 259)

Myers and Hammill (1982) cite Children Discover Arithmetic by Stern and Stern, and The Fabric of Mathematics by Laycock and Watson, as being developmentally relevant instructional programs. Children Discover Arithmetic subscribes to a discovery method by the extensive use of manipulative material. The Fabric of Mathematics stresses a broad-based program with "threads" of number, numeration, measurement, geometry, sets, and logic.

Included in the same text are references to programs developed by Glennon and Wilson, and a closely related approach by Reisman and Kaufman which are directed at successfully reteaching specific "unlearned" skills using a systematic diagnosis and carefully prescribed approach. Both programs stress the importance of developmental factors in proper diagnosis. Another diagnostic approach is Ashlock's semi-programmed material oriented for error analysis.

According to Myers and Hammill (1982), the best known and most comprehensive program is John Cawley's, Project Math. This developmental and remedial program was developed specifically for children with special needs. However, the key issue in this program was not the development of a special mathematics for use with the handicapped, but rather, how the logical structure of mathematics should be taught. According to the developers of Project Math, a mathematical program for children with education needs should: (1) provide for a wide range of experiences, (2) minimize the effects of inadequately developed skills, and (3) enhance affective as well as cognitive growth.

In addition to Project Math, John Cawley (1984) is editor of the text, Developmental Teaching of Mathematics for Learning Disabled. The text stresses developmental and concept orientation rather than drill. There are chapters which contain curriculum and instructional activities for preschool - grade 4. The activities are broad-based and include classification, patterns, seriation, one-to-one correspondence, numeration, geometry, and fractions.

In The Mathematical Education of Exceptional Children and Youth, Glennon and Cruickshank (1981) identify two fundamental concepts that are important in the mathematics instruction of children with perceptual and cognitive processing deficits. First, the acquisition of arithmetical and mathematical understandings in children is developmental; and, secondly, children with perceptual and cognitive processing deficits have not progressed normally in these concepts. To ensure success, teaching should begin developmentally at the lowest functional level of the student.

In a separate chapter of the text, J. F. Weaver and William Morse (1981) recommend a diagnostic and remediation plan for socially and emotionally impaired students, but caution against a narrow "basic skills" approach. Accordingly, they state that programs should take into account a broad-based curriculum in mathematics, and instruction should make use of environmental factors.

Tina Bangs (1982), in her text, In Language and Learning Disorders of the Preacademic Child, stresses that mathematics is a language-based science which is an important tool for academic achievement. The author supports affective education and relating mathematics to the real world.

Shiela Swelt (1980) agrees with this concept of using mathematics to support academic achievement. In her article, "Math as a Teaching Tool for the Learning Disabled," in Readings in Learning Disabilities, she supports math as a potentially significant tool for the learning disabled child because of the variety of visual and concrete materials that can be used in the program. She stresses the organizational skills developed with the use of manipulatives and refers to the importance of Piagetian conceptual stages in determining what and how the materials should be used.

In an article for the Journal of Learning Disabilities, Sara Traver (1986) compares three approaches to education of students with learning disabilities. These approaches are designed for general use and serve as directions for implementing the mathematics program. Cognitive Behavior Modification (CBM) has been developed for the purpose of teaching general cognitive and metacognitive strategies rather than specific skills. Leaders in the field are Meichenbaum, Deshler, and Hallahan. The Direct Instruction (DI) programs are more specific and programs in mathematics have been developed. These programs reflect the thinking of Siefried Engleman and proponents of this field include Carnine, Becker, and Rhine. Direct Instruction is concerned with the organization of instruction based on certain assumptions about how individuals learn.

The third approach in the article is holistic. The proponents of holistic education have relied on a language experience approach in reading and advocate an unstructured Piagetian approach in general. Leading supporters of this approach are Brown, Poplin, and Smith. Mary Poplin(1984), a leading advocate of holistic education and former editor of the Learning Disability Quarterly, is critical of the special education field and refers to it as being "deficit driven." She advocates a more positive approach which would assess and teach each student in areas where they may become competent.

According to van Erp and Heshusius (1986) in "Action Psychology: Learning as the Interiorization of Action in Early Instruction of Mathematically Disabled Learners," in the Journal of Learning Disabilities, approaches such as memorization of rules and quick attempts to cure through the use of drill, work sheets, and flash cards are based on the view that the student is a reactive organism rather than a thinking organism and that this view is prevalent in many special education models, and one that is increasingly being criticized.

Recently, the field of remedial math has been criticized by A. Dean Hendrickson.(Hendrickson, Carnine, Elkind, Meichenbaum, Seiben and Smith, 1983). He stated that although many children have mathematical learning difficulties based on some kind of brain dysfunction:

> The probability is just as high that just as many other learning disabilities are caused by a curriculum that is out of step with known intellectual development ideas, by textbooks that mislead

> though word and picture, or by teachers who do not teach accord-
> to recognized psychological principles. (p. 106)

Hendrickson (1983) advocates that a developmental program of sound fundamental mathematics concepts and learner activity, both physical and mental, "is likely to obviate the need for 'band-aid' treatment usually employed in corrective programs." (p. 106)

III. Mathematics Their Way

Mathematics Their Way is a manipulative approach to mathematics. It is developmentally appropriate for young children and provides for a wide range of activities in several strands of mathematics.

Workjobs, the first book by Mary Baratta-Lorton (1972), was originally written as a master's project at the University of California at Berkeley. It follows the strategem for a curriculum developer to use in the preparation of programs in school mathematics proposed by Lloyd F. Scott (1972), Professor of Education at the university. His strategem consisted of three phases through which the presentation of mathematical ideas must pass. The first two phases deal with beginning number concepts.

The first phase would bring mathematical concepts to students in a means other than pure abstraction. Materials would be of the real world, designed to appeal to the senses, and presented in concrete language. It would be experimental, with variety as a key feature, and include experiences in measurement, tables, and graphs.

In the second phase, developed concepts would be represented abstractly and relations between concepts would be further extended through the symbolic mode with the expectation that concepts would have prior form, and the tendency to reveal structure too early would be constrained.

Workjobs provided a resource for the development of various manipulative projects involving basic number concepts. It was created in response to the needs of inner-city children for activities that were concrete and involved familiar objects. The second book, Mathematics Their Way (1976), was developed to present the author's activities in additional areas of mathematics. It includes activities in patterning, counting, comparing, graphing, sorting and classifying, and number concepts. The manual is separated into strands and each strand includes a wide variety of activities in a loosely sequential order for K-2 grades. The manual provides for a rich variety of activities which are experiential and promote the mental construction of numbers and the development of an understanding and insight of the patterns of mathematics. Workjobs II (1979) extends the concepts of Workjobs and consists of activities for greater developmental range. It was designed as a supplement to the number strand of Mathematics Their Way or other mathematics instructional programs. *Math Their Way* is a term used to express a dynamic approach to mathematics for young children. Through workshops, conferences, and newsletters, it provides teachers with new and innovative materials. It incorporates Piaget's emphasis of action on objects and social interactions, Dienes' concept of multiple embodiment of concepts, and Bruner's idea of a rich and meaningful environment.

This approach replaces the traditional syllabus-bound instructional style in which teacher explanation and questioning is followed by student seatwork on paper and pencil assignments. The *Math Their Way* approach utilizes a wide range of manipulative materials and the use of these materials has a profound effect on the role of the teacher. The teacher involved with the active learning of mathematics focuses attention on arranging and facilitating appropriate interactions between students and materials.

Math Their Way effectively uses manipulatives as an intermediary between the real world and the mathematical world. According to Richard Lesh (1979), this is an expanded use of manipulatives, and he contends that such a use promotes problem-solving. Workjobs utilizes this concept with the use of a variety of miniature real-world situations. The text, Mathematics Their Way, utilizes various materials from the real world in addition to representative materials. The problems originate with concrete objects and are transferred to the abstract, symbolic, and more manageable mathematical world. This differs from the traditional system of teaching in which the symbolic system is presented and explained through a limited number of manipulatives.

The *Math Their Way* approach requires a wide range of manipulatives. This provided for a meta-cognitive benefit. In Mathematics Education Research: Implications for the 80s, K. Fuson (1981) stated that young children learn more easily when provided with activities with goals of relatively short duration and stimuli to which they can react. She states that "perceptually present stimuli in the immediate environment will serve to organize children's behavior and keep it focused on the desired stimuli for learning." She concluded by stating that "providing a rich range of mathematical experiences from which children can choose those most consistent with their particular patterns of thinking is critical." She refers to the material of Baratta-Lorton and Nuffield project as being supportive of this need. (p. 65)

Already in use in grades K-2 in the Portland School District, *Math Their Way* was implemented in grade 3 of eight pilot elementary schools in the district in 1983. The results were interpreted as a positive non-significant group achievement gain of 1.5 standard deviation greater than the third grade district average. The interpretation of the GROW value from fall to spring revealed a variation within the classes involved. Three classes indicate a positive significant gain (2.0 and above), one class at a non-significant positive gain (.7 to 1.6), and four classes at no gain/no loss (-.7 to +.7).

Math Their Way was selected by the Portland schools because it was ". . . perceived as a program which provides a firm conceptual foundation for mathematics and emphasizes manipulative learning experiences suitable for early primary grades." (Ingebo, 1984, pp. 2-4) The report from the Portland School District concluded by stating that putting *Math Their Way* instructional theory into practice involved a considerable adaptation in regular classroom management and instructional behavior.

<u>Workjobs II</u>, the supplemental text, deals specifically with the number strand of the *Math Their Way* approach. Workjobs is a term developed by Mary Baratta-Lorton and is used to denote an activity designated by the characteristics of the counters involved. Each workjob consists of eight child-oriented gameboards of the same design and 80-100 counters specifically selected or constructed to support the theme of the gameboard. The workjobs are used by the children to explore the concept of numbers from counting through the four arithmetic operations. They provide a variety of activities, each with essentially the same concepts, in order to provide practice without having it seem repetitious. Each activity is varied in material, but similar in concept. Once the child understands the procedures, teacher time can be focused on guiding the growth, organization and social skills of the students.

Workjobs are activities designed to be completed in about ten minutes. They require specific actions from the child, and it is through these actions that the child develops the mental structures of the concepts involved. Workjobs provide for a wide range of abilities within a group and promotes language development with the rich and meaningful materials.

Counting is a primary skill developed with workjobs. Research has shown that children used counting and counting-on as the principle means of calculating by students until they were in the fourth grade.(Resnick, 1983)

The act of counting rests upon several important principles that include: (1) each object to be counted must be assigned one and only one number, (2) the same number list must be used every time a set of objects is counted, (3) the last number word gives the numerosity of the set, and (4) the order in which the objects are counted does not matter. It also requires a basic classification skill which is the mental or physical grouping of objects counted from those not counted.

As children's proficiencies continue to develop, they begin to establish relations among counting words, allowing them to count forward from a number other than one and the recognition that number words themselves can be used as objects of counting. This represents a change in what is counted. Concrete objects are no longer needed as counters; number words can serve as counting units.(Hiebert, 1981)

In addition to counting, Workjobs II provides a developmental sequence of learning activities for gradual introduction of the conventional symbolism. This is provided through the context of physical situation and oral discussions as children create, verbalize, and demonstrate situations with the gameboards and counters. The author stresses the importance of this connecting level between the concrete and symbolic. Numeral and equation cards are used by the children to represent the quantities before writing a conventional equality sentence to represent the action. Ed Labinowicz (1985), in Learning from Children: New Beginnings for Teaching Numerical Thinking, confirms the importance of this transitional period and refers to Mary Baratta-Lorton as a model of a "teacher as a researcher and curriculum developer." (p. 146)

IV. The Project

IV(a). Statement of Purpose and Design

This guide consists of activities which are adaptations and additions to the supplementary text, Workjobs II: Number Activities for Early Childhood, by Mary Baratta-Lorton. It was developed for special education classes to provide additional workjobs to be used with the strategies presented in the text. They were developed to provide activities that were durable, inexpensive and could be more easily constructed and to provide a greater variety for selection. A strength of the material is that it can be used with students with a wide range of abilities to provide extensive practice necessary for the concept development often required of students with learning handicaps. Although the strategies were originally developed for regular classes, this makes them ideally suited for special day classes. The vocabulary and language development involved support the use of workjobs with special education students. The materials are suitable for older children when the age and interest of the students are considered in the selection. Many of the workjobs included in the guide would be high interest activities for older children in special education who are in the process of developing basic number concepts.

The original intent for the development of this project was to provide activities that would supplement the activities in Workjobs II by providing material that would utilize and extend the experiences of learning handicapped students in Southern California. For example, the workjobs would reflect foods such as tacos and burritos in place of spaghetti and meatballs, and rocks of the mountains and seashore in place of rocks of the river. However, my experiences during district workshops and inservices provided an insight into a more critical need. Simplicity and ease of development became the important criterion. Teachers have the responsibility to change, adapt, and create materials in response to the needs and backgrounds of their students. For teachers of the learning handicapped, this is a critical and time consuming task. Therefore, the materials presented for special education should reflect an understanding of the time constraints on these educators.

In addition to providing simplicity in design, there were concerns regarding the materials and concepts used in some workjobs. For example, the hazardous straight pins used as counters were replaced in a similar workjob that used plastic toothpicks, and the mousetraps were replaced with non-aggressive "cheese" wedges. Some of the original workjobs, e.g. Sea Shells, are excellent for use with special eduction students. They consist of high interest activities and materials which are easily constructed, durable and inexpensive. The workjobs selected from the text and/or this guide should reflect the interest of the class and the availability of the materials.

Students benefit most from active involvement. Therefore, when the development of the materials can include the students, i.e. when students mix, measure and make the play-dough "cookies", collect acorns, bottle caps, etc., learning is further facilitated. Brainstorming possible sources of materials and sorting the collected materials can provide valuable learning experiences. If the class has limited resources, enlisting the assistance of regular classes can provide additional materials. Some of the students in the class may be adequately skilled to color the gameboards. However, observations have indicated that students prefer to work with workjobs that have aesthetic appeal. A regular class of upper grade students should have sufficient skills to provide a source of willing artists. The strategy of having the class "adopted" by an older class of regular students would have beneficial outcomes for both groups.

In the field of mathematics, workjobs promote number skills and problem solving. The materials can be extended for use in comparison, graphing, patterning and estimating. Simultaneously, they provide an excellent avenue for developing organizational skills, an area of difficulty for many learning handicapped students. In addition, workjobs can be used to initiate or stimulate an interest in other areas of the curriculum. For example, the Rock and Sea Shell workjobs can develop from or into sciences units, and story telling and writing experiences can be extensions of the workjobs developed on literature themes such as Caps For Sale and Curious George.

The benefits of the workjob concept are multiple and varied. However, for many children it will be in the area of personality development that the concept of workjobs has its greatest value. The time when young children are beginning to think systematically of the characteristics of numbers is also the time for validation of self. Piaget's work was primarily in the field of cognition and his work indicates the value of activities that support the development of competencies for the operational stage, i.e. decentration, conservation, seriation, reversibility, transitivity, and classification. In his book, Childhood and Society, Erik Erikson (1963) defines the psychosocial development into eight stages. When Erikson's stages are superimposed on Piaget's cognitive stages, we see the need for activities that will also support the development of initiative and industry, the positive outcomes of the stage, and protect against the polarity of guilt and inferiority. Because workjobs provide for the construction of number knowledge in concrete, self-paced, and non-threatening activities, they are ideally suited for the curriculum of learning handicapped students.

Included in this guide are the black line masters for gameboards and instructions for their development. It includes suggested objects to be used for counters and directions when necessary.

The original Workjobs II included 20 activities. This guide provides for a wide variety of workjobs from which to select. The number of workjobs needed will be determined by number of students involved in the activity. For high interest and decision-making opportunities, a minimum of 12 workjobs should be available to the students. Used with the format described in the original text, Workjobs II, they serve to develop the following skills:

Counting
1:1 Correspondence
Numeral Form
Numeral Recognition
Conservation of Number
Relationships Within and Between Numbers
The Process of Addition
The Process of Subtraction
Interpreting Symbols
Writing and Solving Addition and Subtracting Equations

For the purpose of clarification, the materials have been presented in the following sequence:

(1) information regarding the basic structure of the workjobs

(2) a simplified overview of the implementation of the workjobs

(3) directions for the construction of the materials

(4) activities which can be used to extend the use of the counters from the workjobs in patterning, graphing, sorting and classifying, comparison and estimating.

(5) an appendix which provides black line masters for the gameboards, which are identified by workjob name, graphs, samples for patterning activities, vertical problems, and a list of resource books and materials.

The first twenty-two workjobs represent activities developed for this project. An additional four activities were adapted from the text, Workjob II. The changes are suggested to reduce the cost and time of construction or to provide variations on the theme.

IV(b) The Structure of the Activities

(1) Each of the activities is designed in one of two ways:

A) with one kind of counter and two parts to the counting area, or

B) with two kinds of counters and only one part to the counting area.

A		B	
Acorns	on the ground and on the tree	Snowman	2 colors of buttons
Pennies	at the store or at the bank	House	nails and "boards"
Clifford	food for Clifford food for the puppy	Ladybug	2 colors of ladybugs
Apples	stacked on his head fallen on the ground	Flat tire	nuts and bolts
Corduroy	buttons on his clothes, lost buttons on the ground	Donuts	frosted and unfrosted
Rocks	on the mountain on the desert	Pirates	gold nuggets and jewels
Caps	on his head in the tree	Plants	with flowers without flowers
Swords	red area white area	Mice	2 colors of tails
		Eggs	2 colors of eggs

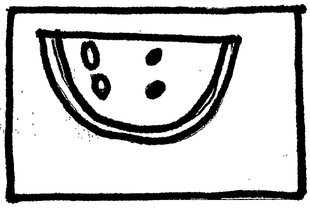
Rainbow	white and dark clouds
Balloons	2 colors of balloons
Flowers	2 colors of flowers
Keys	2 colors of keys
Cars	2 colors of cars
Birds	2 colors of birds
Outer Space	rocket ships space ships
King Midas	gold and plain objects

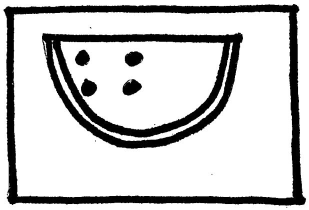
(2) An Overview

A simplified overview of the use of materials moving from concept development of numbers through equations using symbolic forms.

Exploring the numbers from 0-9 at the concept level:

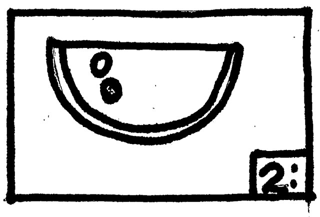
Students count out the same number of objects onto each counting area. No symbols are used at this level.

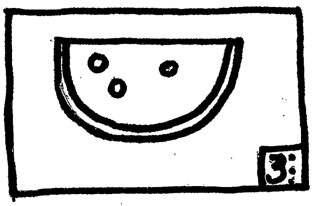




Exploring the numbers from 0-9 at the transition or connecting level:

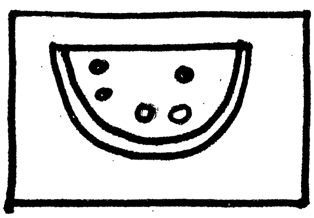
Students count out the appropriate number of counters to match each numeral. Numeral cards 2-9 are given.





Exploring operations at the concept level:

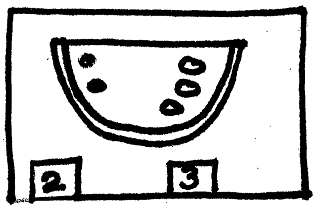
Students work in pairs and verbalize combinations created on the gameboards. No totals or remainders are given at this level.



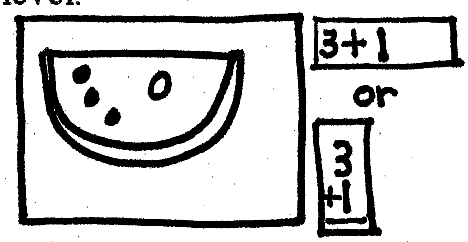
"My watermelon has three black seeds and two white seeds."

Exploring operations at the transition or connecting level:

a. Students work in pairs and verbalize combinations created on the gameboard using numeral cards to represent the quantities. Addition, subtraction and equal signs are not used. Numeral cards are given.

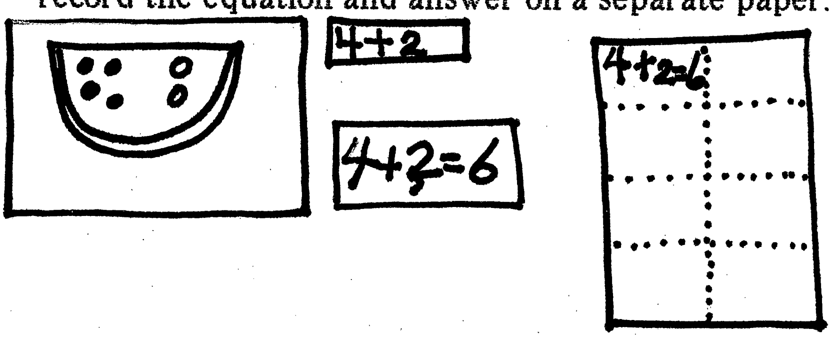


b. Students use counters representing each equation concretely. Equation cards are given. Black line masters for horizontal equations are included in the original text. This guide provides black line masters for experiences with vertical problems at this level.



Exploring operations at the symbolic level:

Students build a problem from an equation card and record the equation and answer on a separate paper.



(3) Directions

Gameboards:

1. Make xerox copies on 8 1/2 x 11" tagboard, or
2. glue copies
3. cut sheets in half (4 1/4 x 5 1/2)
4. decorate
   (a) color with crayons, watercolors, water paints, and/or
   (b) use construction paper, felt, or any combination
5. laminate or cover with Contact, when possible

Counters and additional counting areas:

1. Snowman

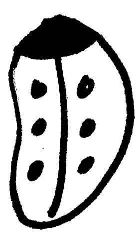
Counting area	8 snowman gameboards
Counters	80-100 buttons - two colors

2. House

Counting area	8 house gameboards
Counters	40-50 small nails
	40-50 popsicle or craft sticks

3. Ladybugs

Counting area	8 flower gameboards
Counters	40-50 lima beans (sprayed yellow)
	40-50 lima beans (sprayed red)
	Mark with permanent marker



4. Acorns

Counting area	8 oak tree/squirrel gameboards
Counters	80-100 acorns

5. Pennies

Counting area	8 store/bank gameboards
Counters	80-100 pennies

6. Flat Tire

Counting area	8 car gameboards
Counters	80-100 small nuts and bolts

7. Donuts

Counting area	Winchell's gameboards
Counters	80-100 tortellina raviolini (a pasta available in supermarkets) "Frost" the tops of half the donuts with spray paint or tempera (use clear acrylic to protect tempera)

8. Pirate

Counting area	8 pirate gameboards
Counters	40-50 small rocks Spray rocks with gold paint for gold nuggets and use 40-50 "jewels" collected from broken necklaces.

9. Plants

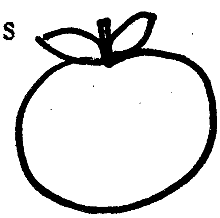
Counting area	8 "Plants for Sale" gameboards
Counters	80-100 potted plants (Use small 1/2" square wooden beads for pots and very small plastic flowers and leaves) Add glue to the holes of the beads and add flowered stems in half the beads and non-flowered stems to half the beads.

10. Clifford — (from the book, Clifford)

Counting area	8 Clifford gameboards
Counters	80-100 pieces of dry dog food sprayed with clear acrylic spray.

11. Apples — (from the book, 10 Apples on Top)

Counting area	8 lion gameboards
Counters	80-100 red apples made from felt or construction paper covered with Contact (leaves are optional)



12. Corduroy — (from the book, Corduroy)

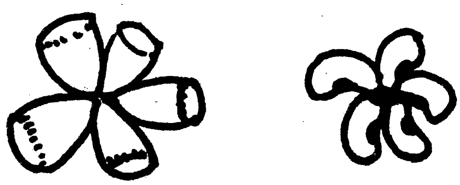
Counting area	8 bear gameboards
Counters	80-100 buttons of various colors and shapes

13. Balloons (from the book, Curious George)

Counting area	8 balloon gameboards
Counters	80-100 plastic lids from juice and milk containers, two colors

14. Flowers

Counting area	8 flower gameboards
Counters	80-100 flowers, two colors



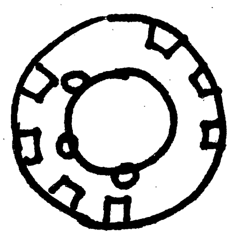
Flowers are made from seeds of corn and white pinto or red kidney beans. For each flower, place a dollop of glue on waxed paper. Arrange seeds as petals and add a contrasting center (using a paper-punch dot)

15. King Midas

Counting area	8 King Midas gameboards
Counters	40-50 small objects, i.e. plastic animals, blocks, sprayed gold 40-50 small unpainted objects

16. Outer Space

Counting area	8 outer space gameboards 40-50 plastic screw anchors for rockets
Counters	40-50 checkers or counters sprayed silver for space ships. mark windows and lights with permanent markers



17. Rainbows

Counting area	8 rainbow gameboards
Counters	40-50 white clouds (cumulus) 40-50 dark clouds (nimbus)

Rainbow colors:

red, orange, yellow, green
blue, indigo violet

Clouds may be made from cotton glued to small varying shaped pieces of tagboard. Spray with clear acrylic and darken 1/2 the clouds with permanent markers. Laminated lace can also be used.

Language development is enhanced if the cloud shapes resemble the ice cream cone, flowers, animals, etc. the the book <u>It Looked Like Spilt Milk</u>

18. Keys

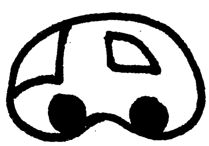
Counting area	8 metal shower rings
Counters	80-100 brass and silver-colored keys (keys made in error are available at locksmiths)

19. Marbles

Counting area	8 marble ring gameboards
Counters	80-100 marbles (to prevent marbles from rolling out of the circle or off the gameboard, squeeze a line of Elmer's glue and allow to dry)

20. Cars

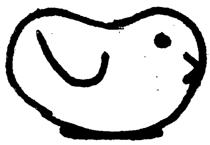
Counting area	8 car counting gameboards
Counters	80-100 lima beans spray painted two colors as shown. Glue to small pieces of tagboard for upright position, or add split peas as wheels



21. Birds

Counting area — 8 bird/park gameboards

Counters — 40-50 lima beans spray painted red



40-50 lima beams spray painted blue (colors are optional) Mark as shown.

Glue to small pieces of tagboard for upright pieces

22. Caps — (from the book, Caps For Sale)

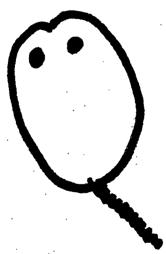
Counting area — 8 caps gameboards

Counters — 80-100 caps from various small containers e.g. toothpaste, perfume, bottle caps

23. Mice

Counting area — 8 cheese gameboards

Counters — 80-100 cowie shells



40-50 1" lengths of pink yarn

40-50 1" lengths of black yarn

Glue "yarn tails" to the bottom of shells. Mark eyes with permanent markers (inexpensive cowie shells are used for jewelry and can be found in thrift stores)

24. Rocks

Counting area	8 rocks gameboards
Counters	80-100 small rocks Rocks of varying colors and shapes increases interest. For older children, smooth rocks are placed on the desert, and rough rocks, on the mountains.

25. Nests

Counting area	8 nests Mold dough into 3" diameter shallow nest. Add feathers if desired. Dough recipe: 1 cup flour, 1/2 cup salt, 1/4 cup with 2T brown tempera added
Counters	40-50 pinto beans 40-50 white or kidney beans If desired, beans may be spray-painted two colors, e.g. blue, yellow or pink

26. Swords

Counting area	8 pieces of plastic styrofoam (approx. 3" x 5" x 1/2") Spray 1/2 of one side with spray paint (any color)
Counters	plastic cocktail toothpicks, to be used as swords

Containers

Containers for storing the workjobs will be determined by teacher preference and the available storage area. Organization of and student access to the workjobs are important factors. Plastic mini-baskets can be purchased at variety stores. The approximate cost is fifty cents for 4 1/2" x 6 1/2" x 9 1/2" baskets. Inexpensive storage boxes can be purchased through the Math Learning Center at a cost of forty-five cents per 9" x 12" x 2" box.

(4) Extending the Materials

Activities for extending use of the manipulative materials from the workjobs in the development of mathematical concepts involving patterning, graphing, sorting and classifying, comparison, and estimating.

Patterning:

Materials: pattern cards, counters

Procedure: Children select a pattern card and use the workjob counters to extend the pattern. Examples of pattern cards are presented in the appendix.

Graphing:

Materials: graphing paper (see Appendix), counters from Section B workjobs

Procedure: The number of counters will be determined by student abilities. Students place like counters in each row.

<u>Level 1</u>: Students verbally express which now has more/less. Encourage the use of complete sentences, e.g., I have more red ladybugs than orange ladybugs.

<u>Level 2</u>: Students verbally express the number in each row before expressing a more/less statement.

<u>Level 3</u>: Students verbally express how many more/less, e.g., I have three more red ladybugs than orange ladybugs on my graph.

<u>Level 4</u>: Students draw a picture of each counter as it is removed, developing a picture graph.

Level 5: Students who are capable of written expression write a statement about their picture graph.

Level 6: Students color the squares as counters are removed to produce a bar graph and write one or more statements about the graph.

Note: Vary procedure by the use of horizontal and vertical graphs.

Sorting and Classifying:

Materials: counters from Rocks, Corduroy, Marbles, jewels from Pirate, ornaments from Trees, nuts or bolts from Flat Tire.

Procedure: Introduce the activity by using 8-10 counters from the same workjob for each student. Students are asked to find the counters that are alike in some way and separate the two groups. Beginning classification groupings would be:

blue - not blue
dark colored - light colored
smooth - rough, etc.

Extend the activity by having students work with partners. The students attempt to define the partner's classifying attribute. Older children may be capable of working with two classifying attributes.

Increase the number of counters used as students develop their skills.

Comparison:

Materials: three clear glass or plastic containers of the following dimensions: one short and wide, one tall and narrow, one regular counter.

Procedure: Pour counters from one container to another. Encourage the students to anticipate whether the level will be higher or lower.

Note: A small tub of rice and various plastic containers will provide students with additional time to explore this concept.

Estimating:

Materials: clear glass or plastic container counters

Procedure: Place the counters in the container and encourage the students to estimate the number. The number of counters should not exceed the number they can accurately count.

<u>Level 1</u>: Introduce the activity by providing options. If five counters are place in the container, provide a choice by asking, "Do you think there are two ladybugs or five ladybugs in the container?" Count together to verify.

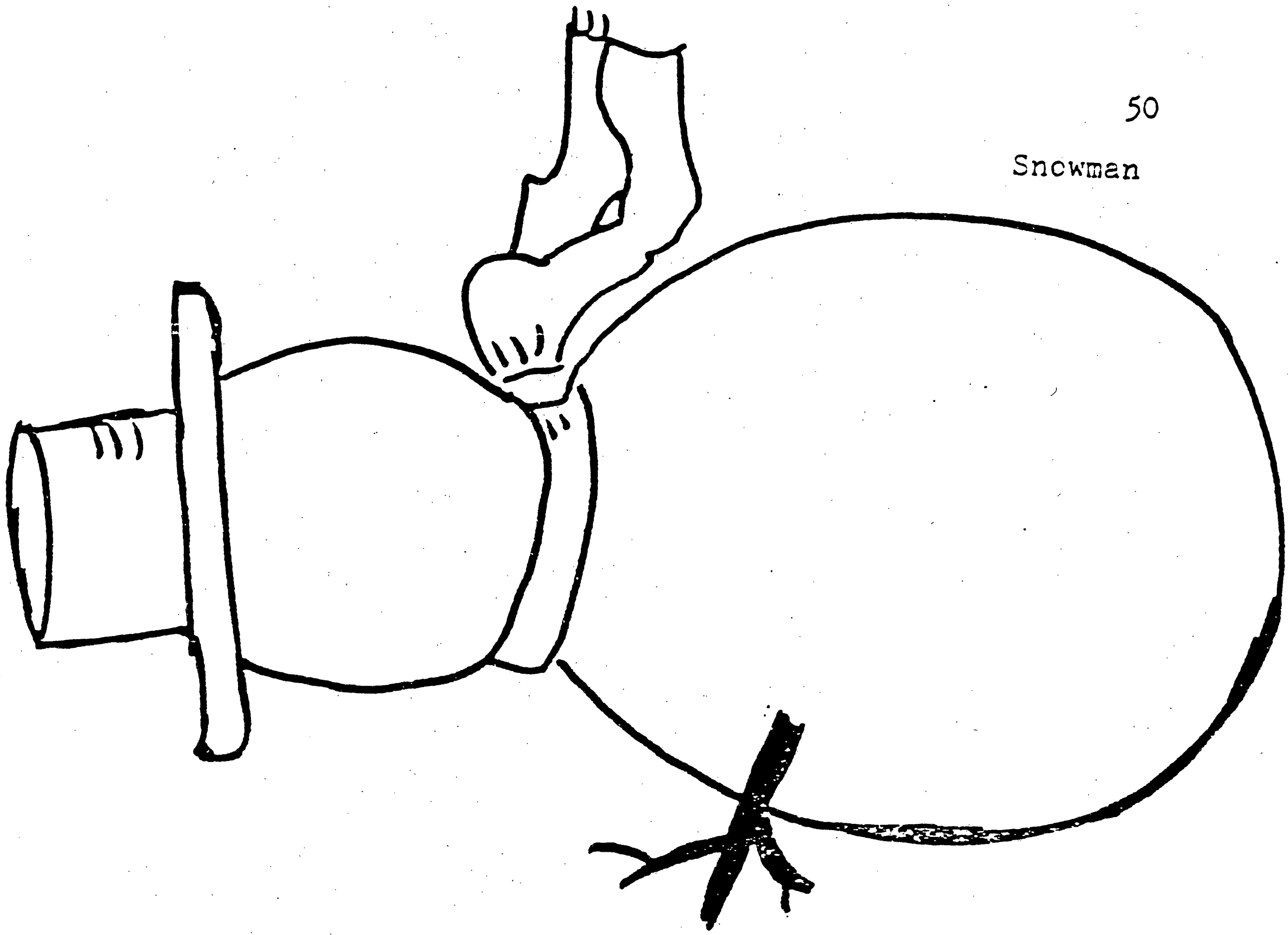
<u>Level 2</u>: Omit the options.

<u>Level 3</u>: Vary with numbers of counters within the counting range. Remind students that this is a "thoughtful guess" and support all reasonable estimations.

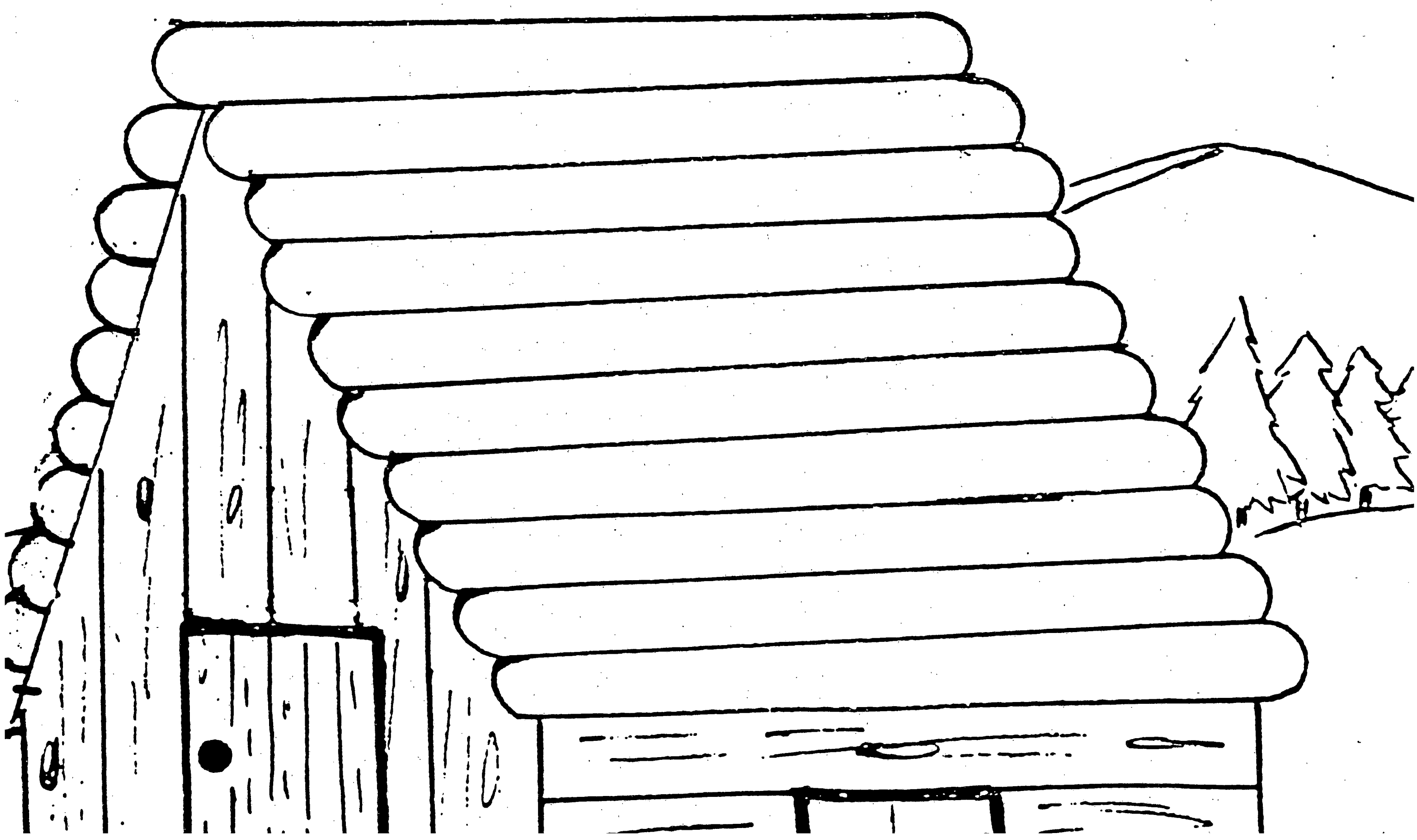
Note: Use the containers from the comparison activities. Varying the shape of the container will expand the student's estimating skills.

(5) Appendix

Snowman

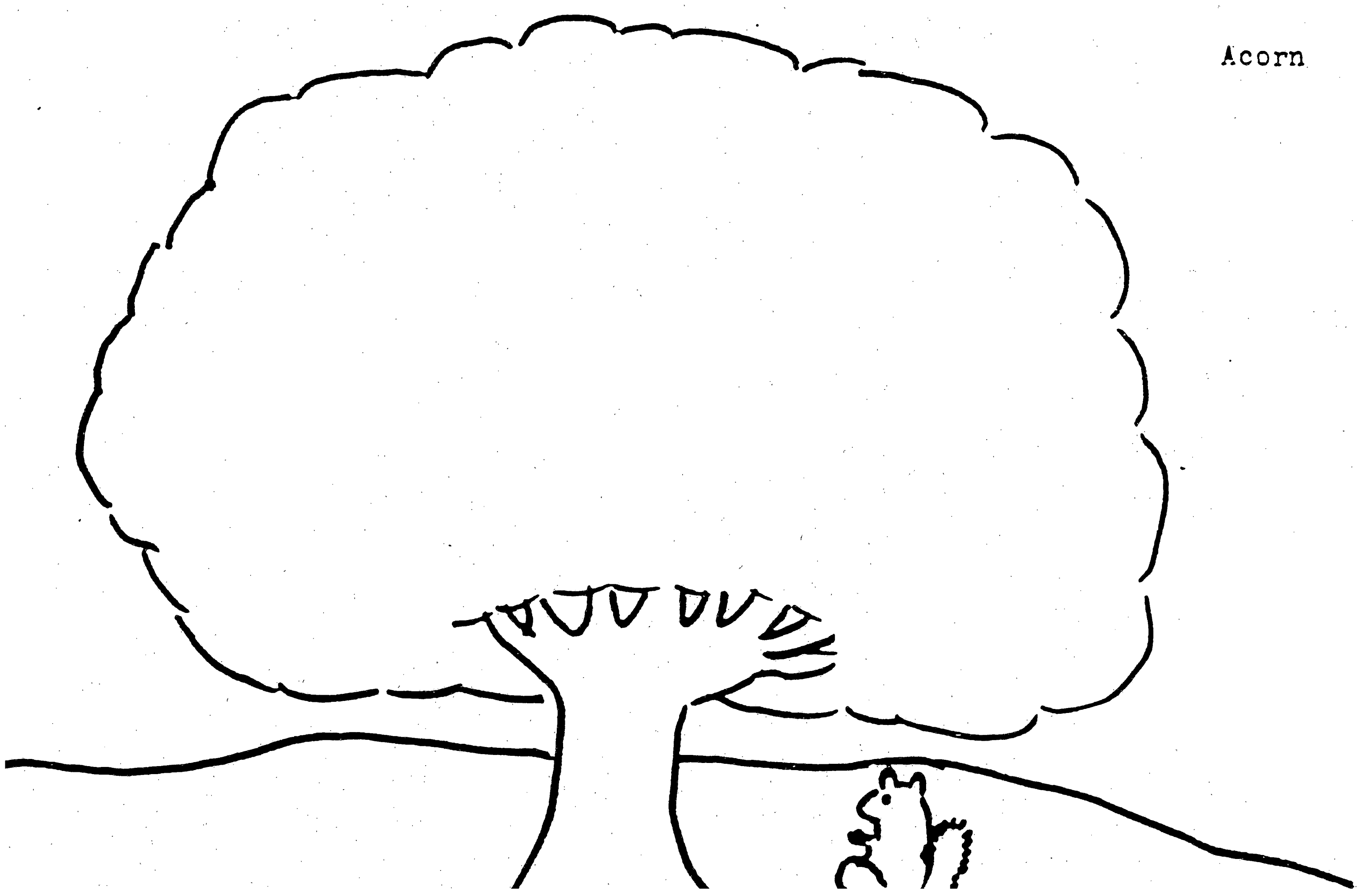


House

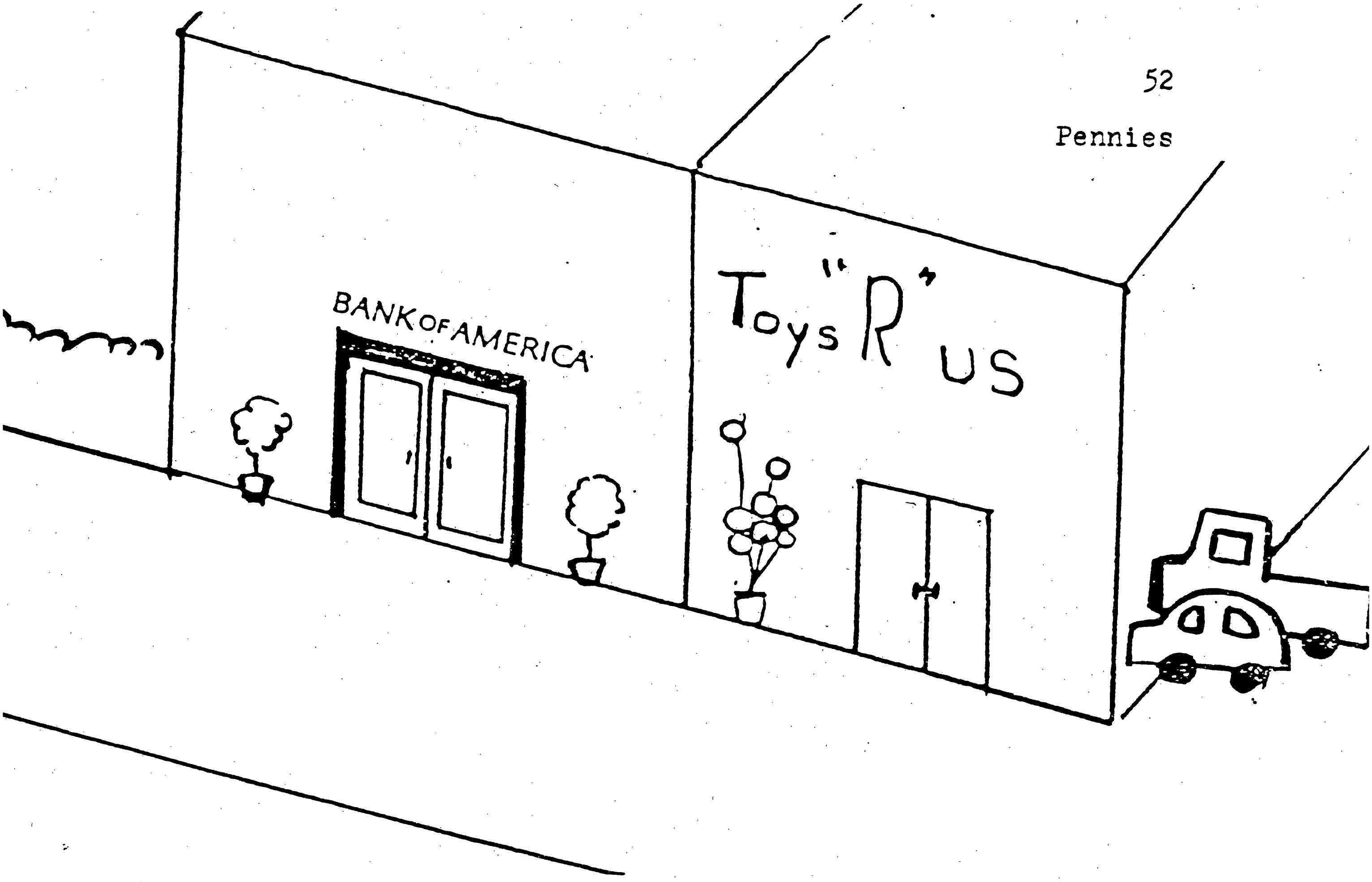


Ladybugs

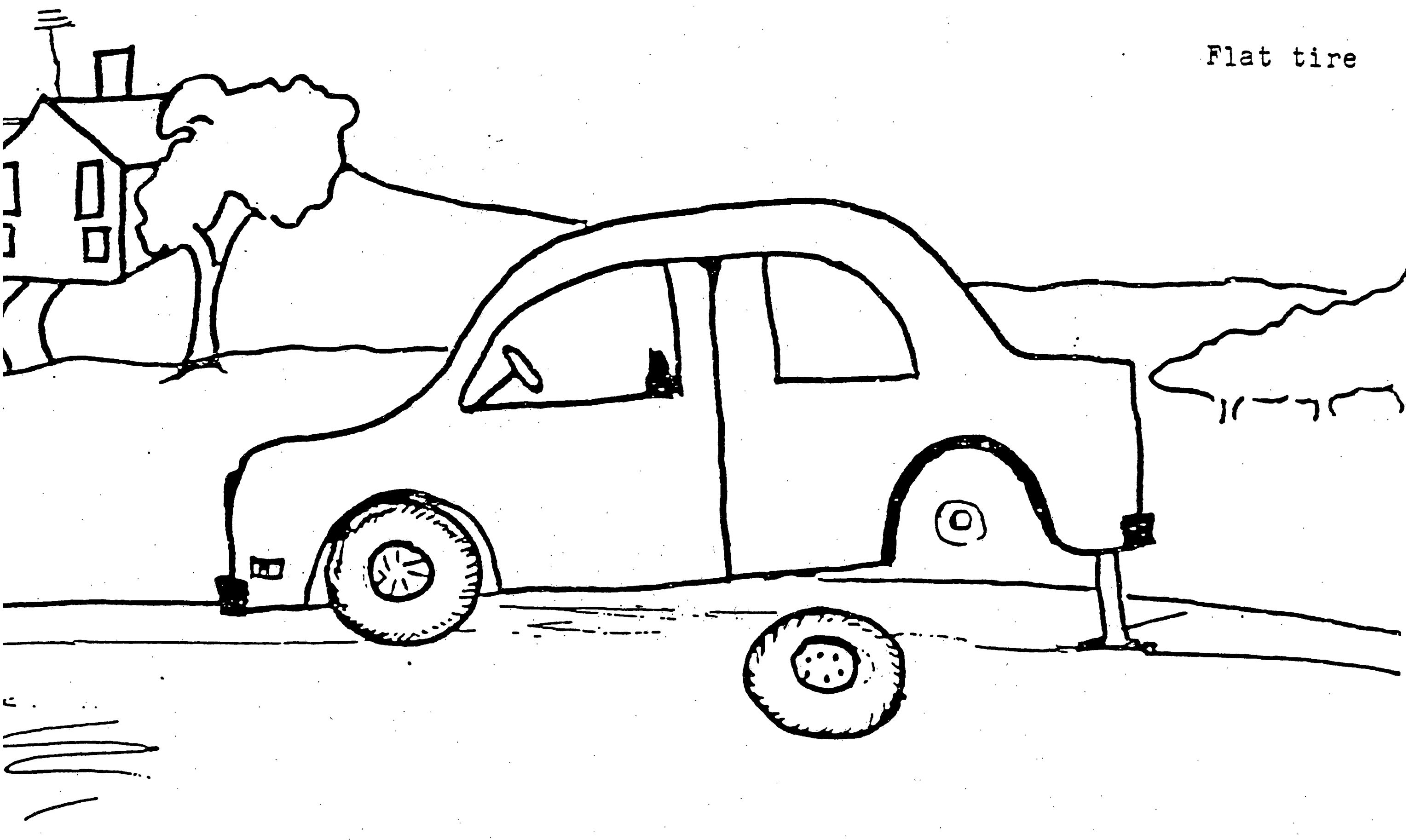
Acorn



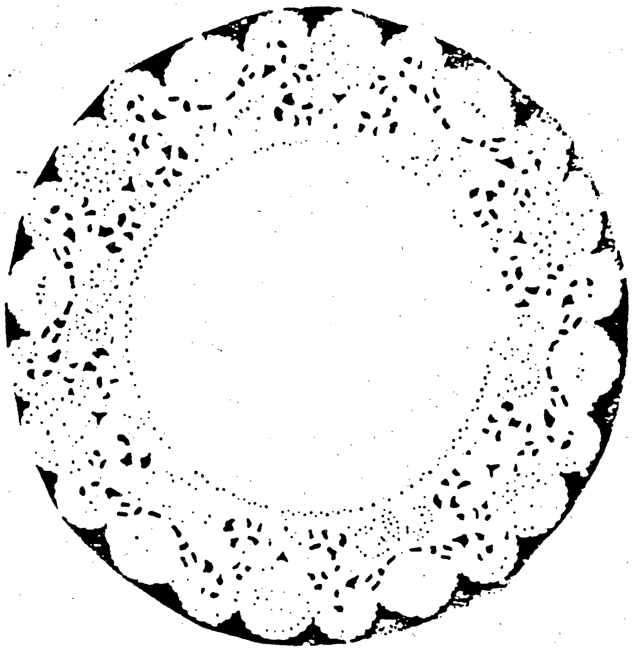
Pennies



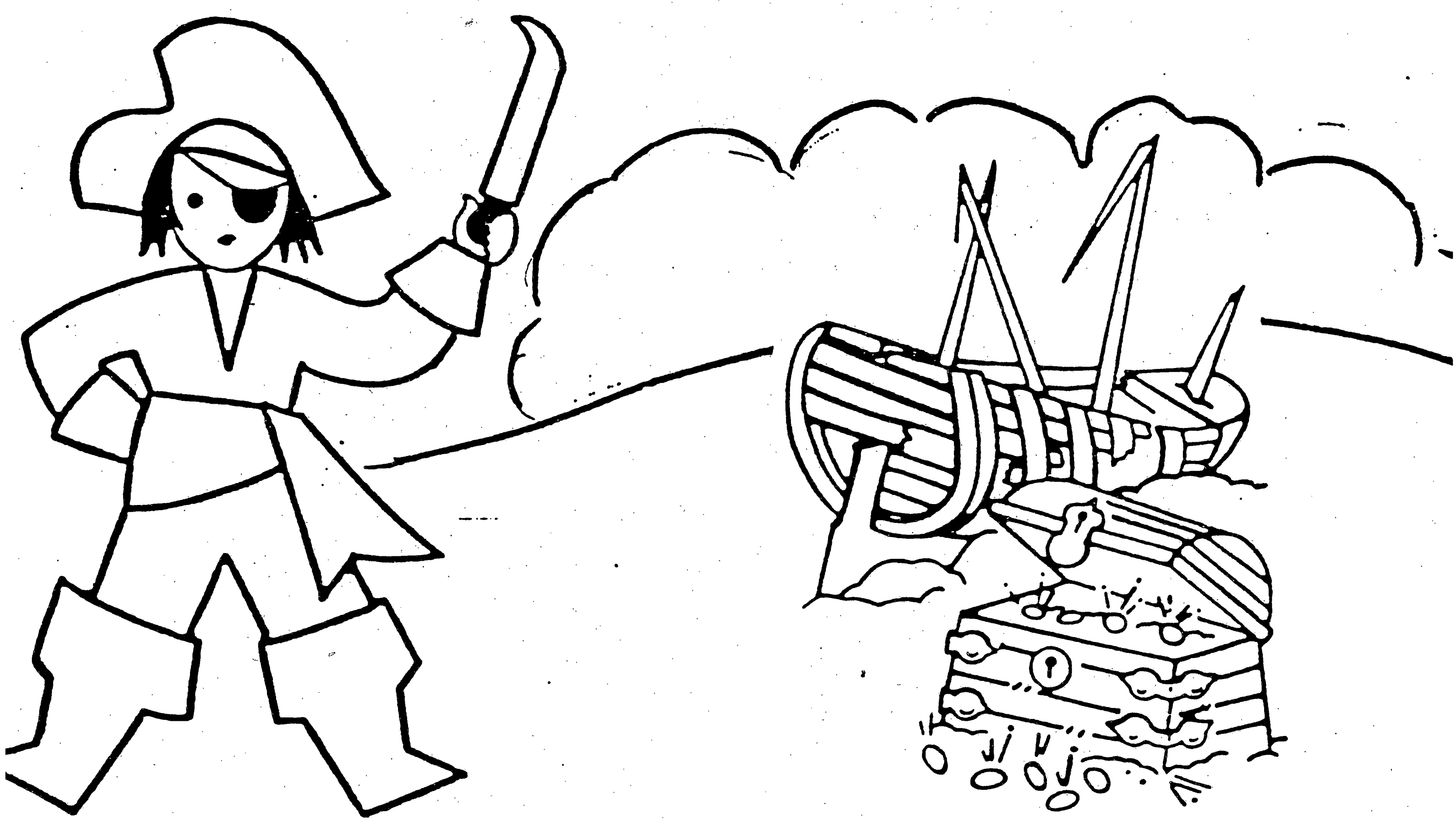
Flat tire



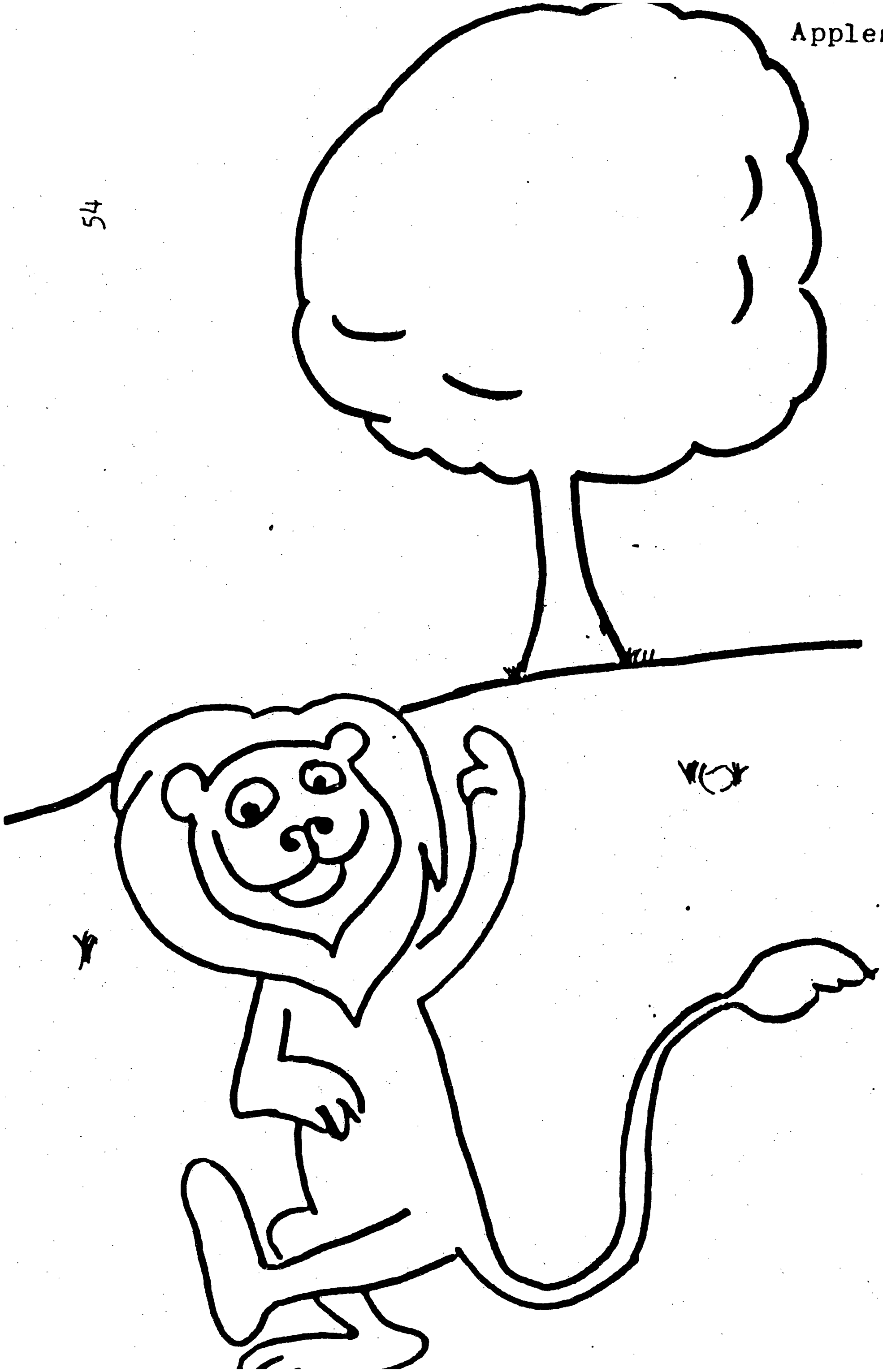
Winchell's®
Donuts



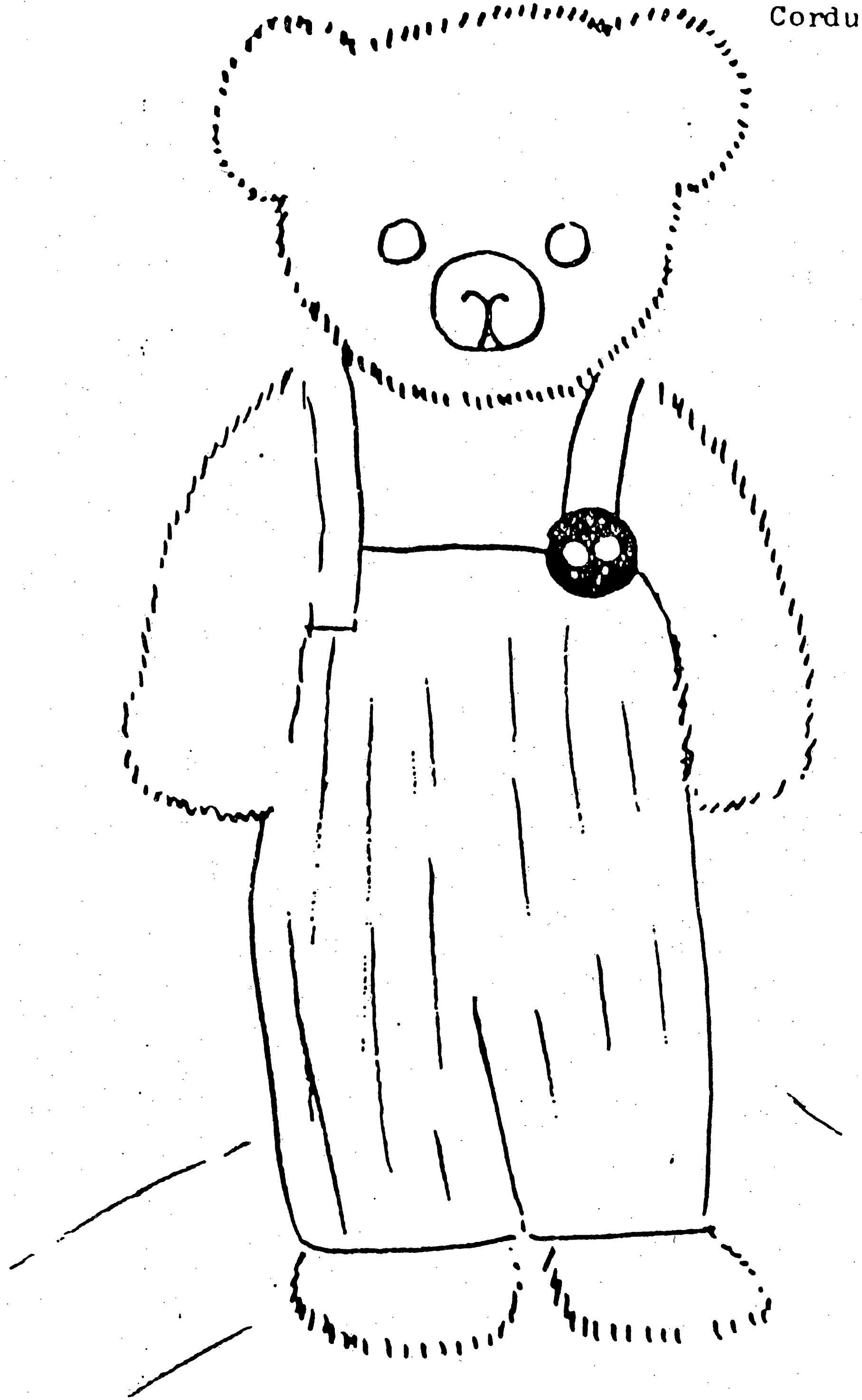
Pirates

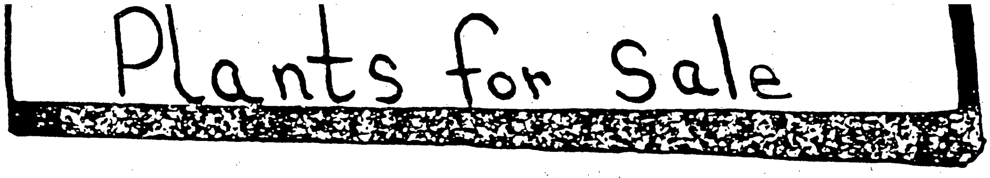


Apples

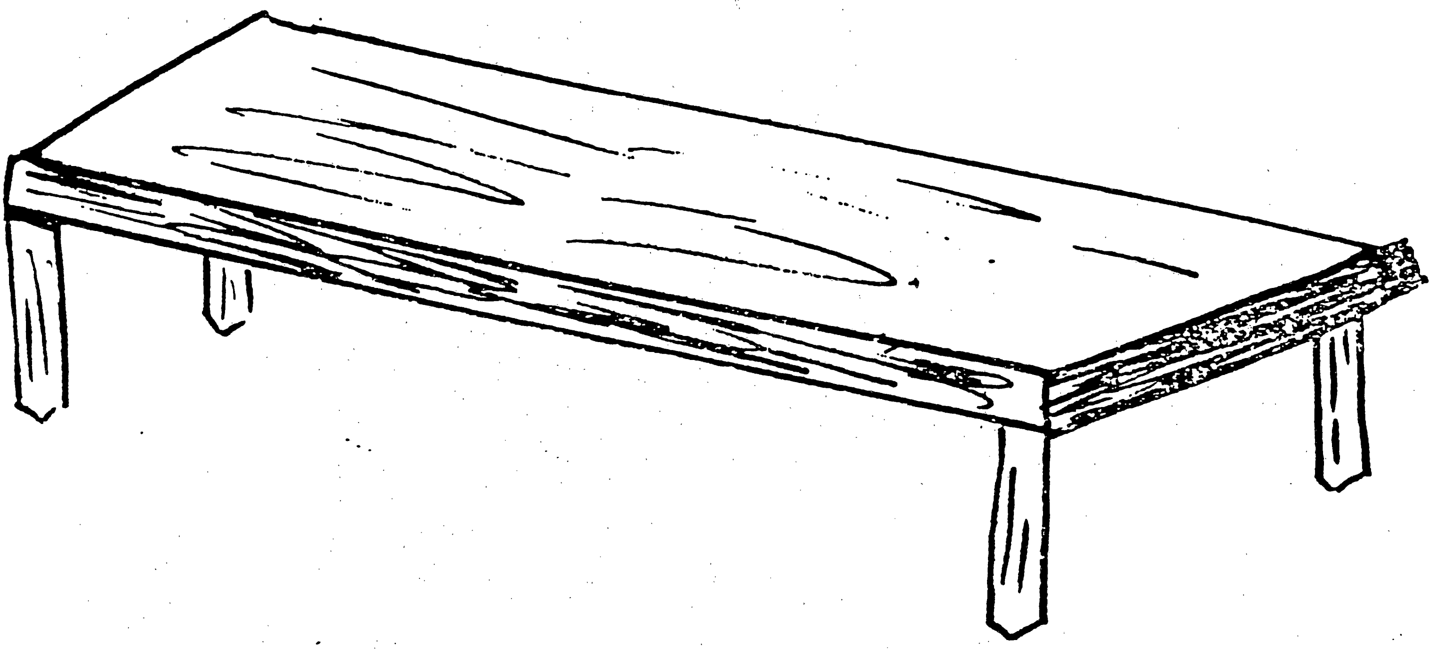


Corduroy

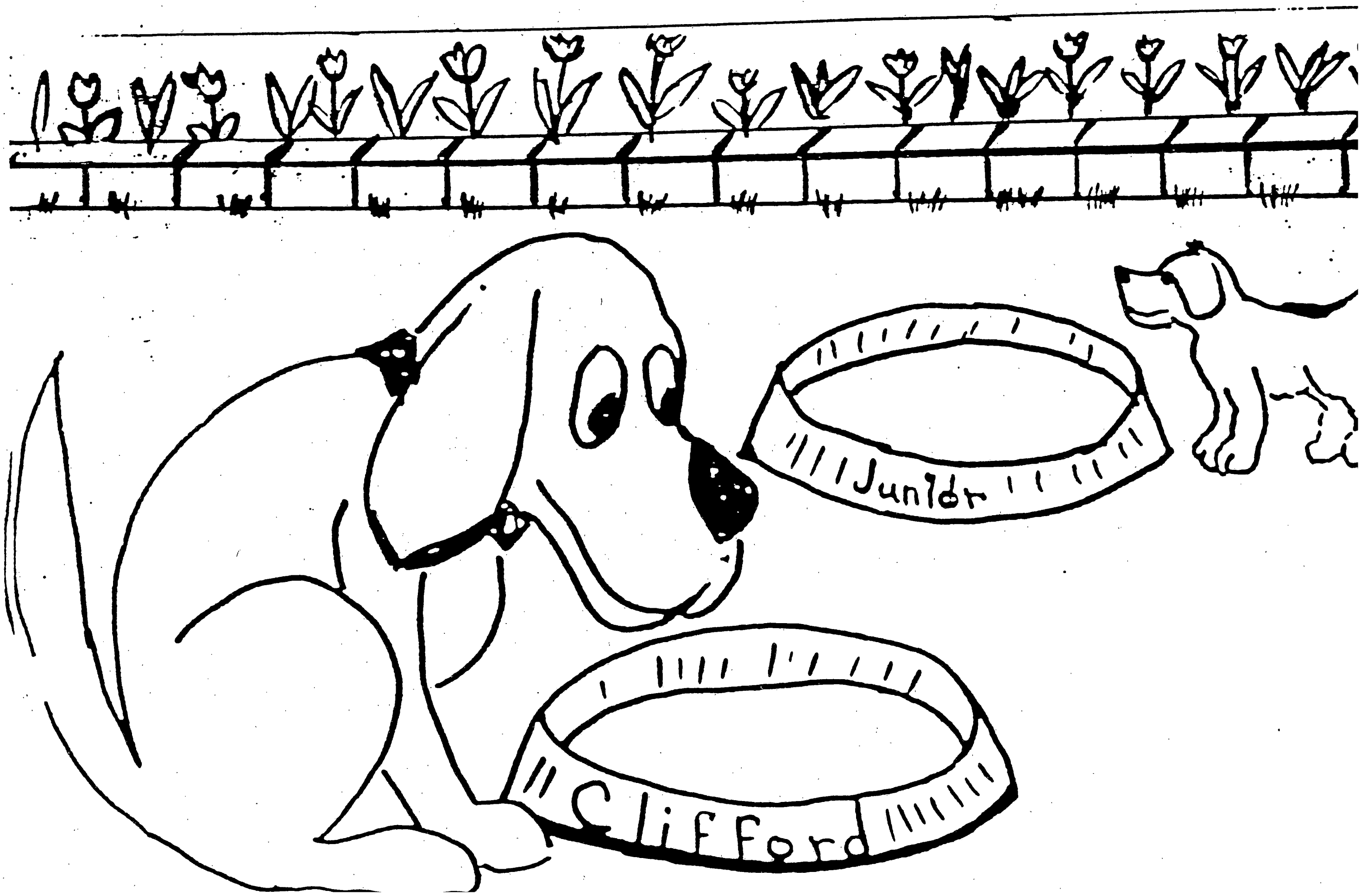




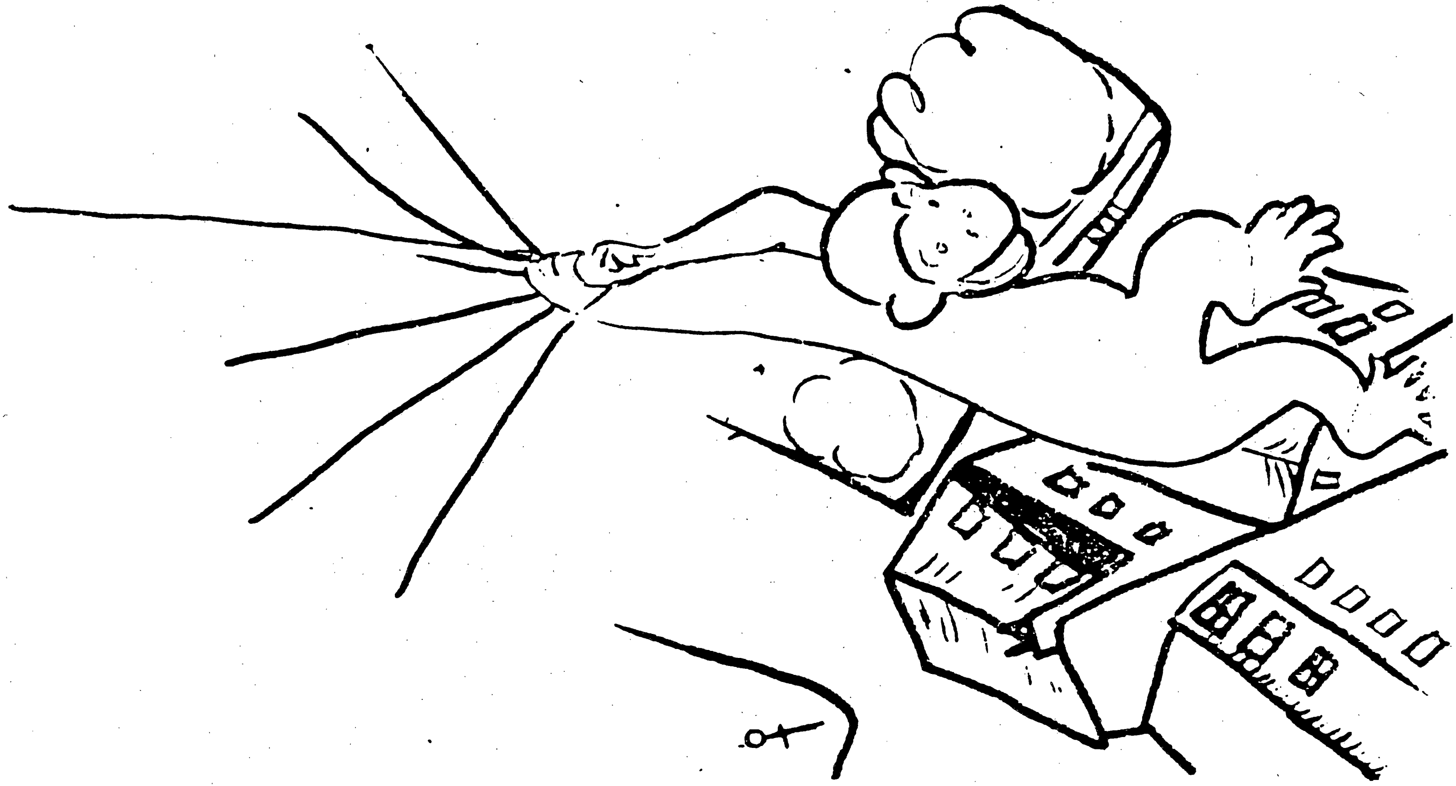
Plants



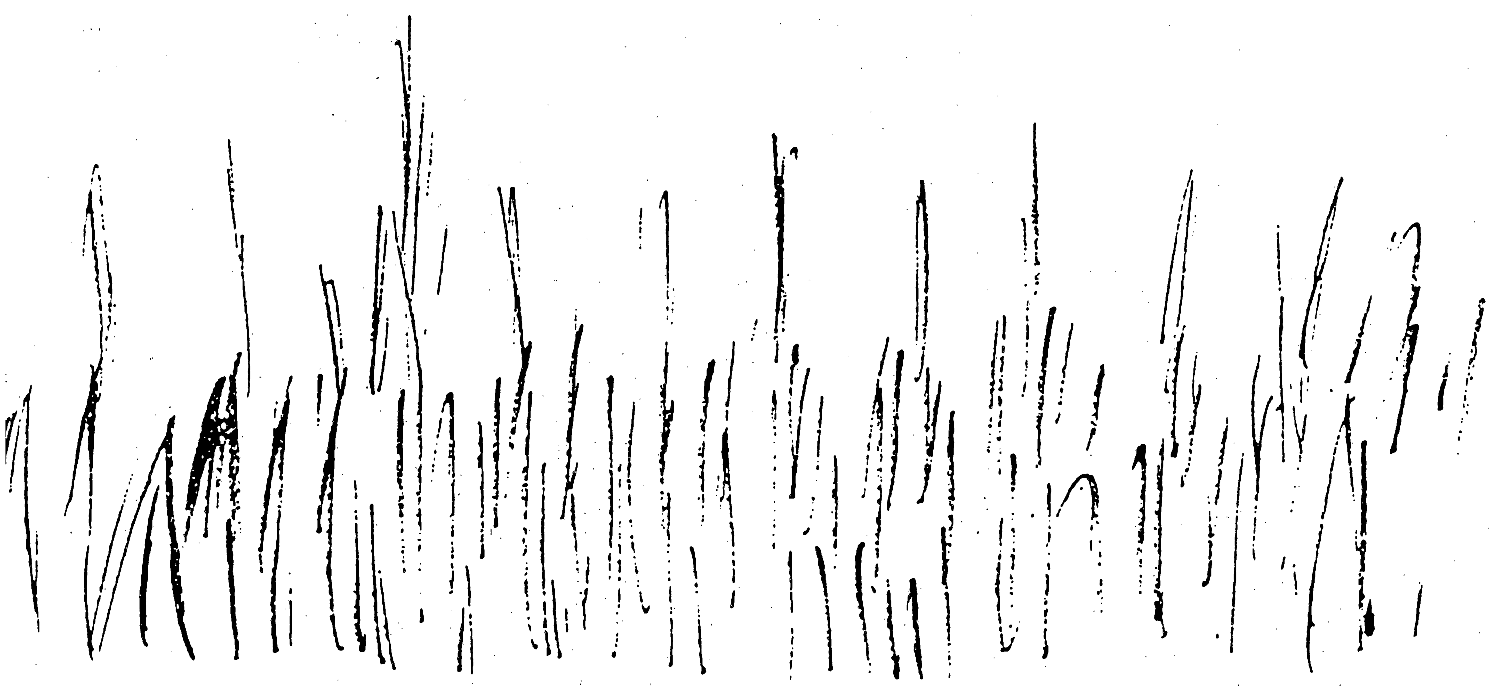
Cliffor



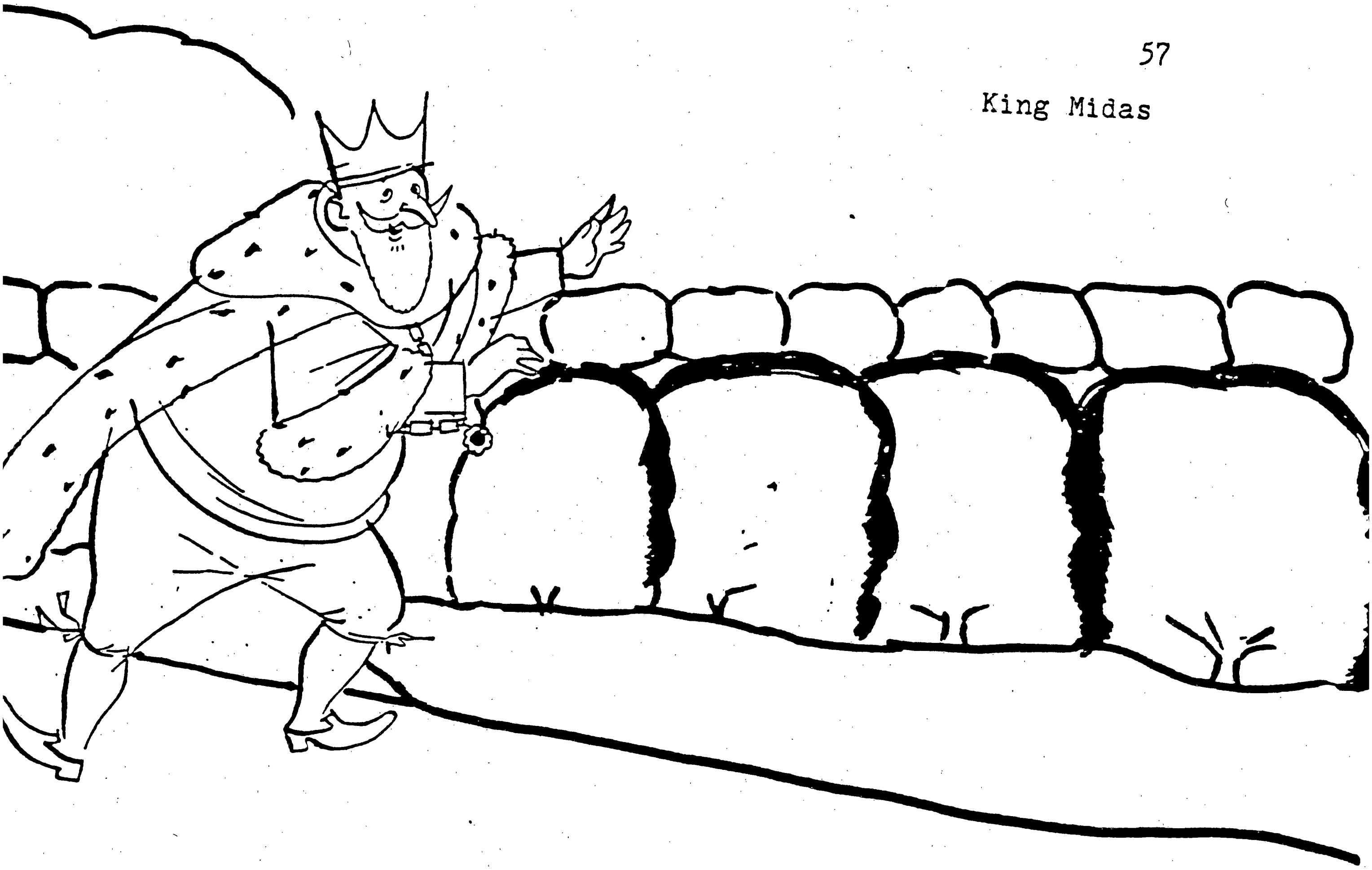
Balloons



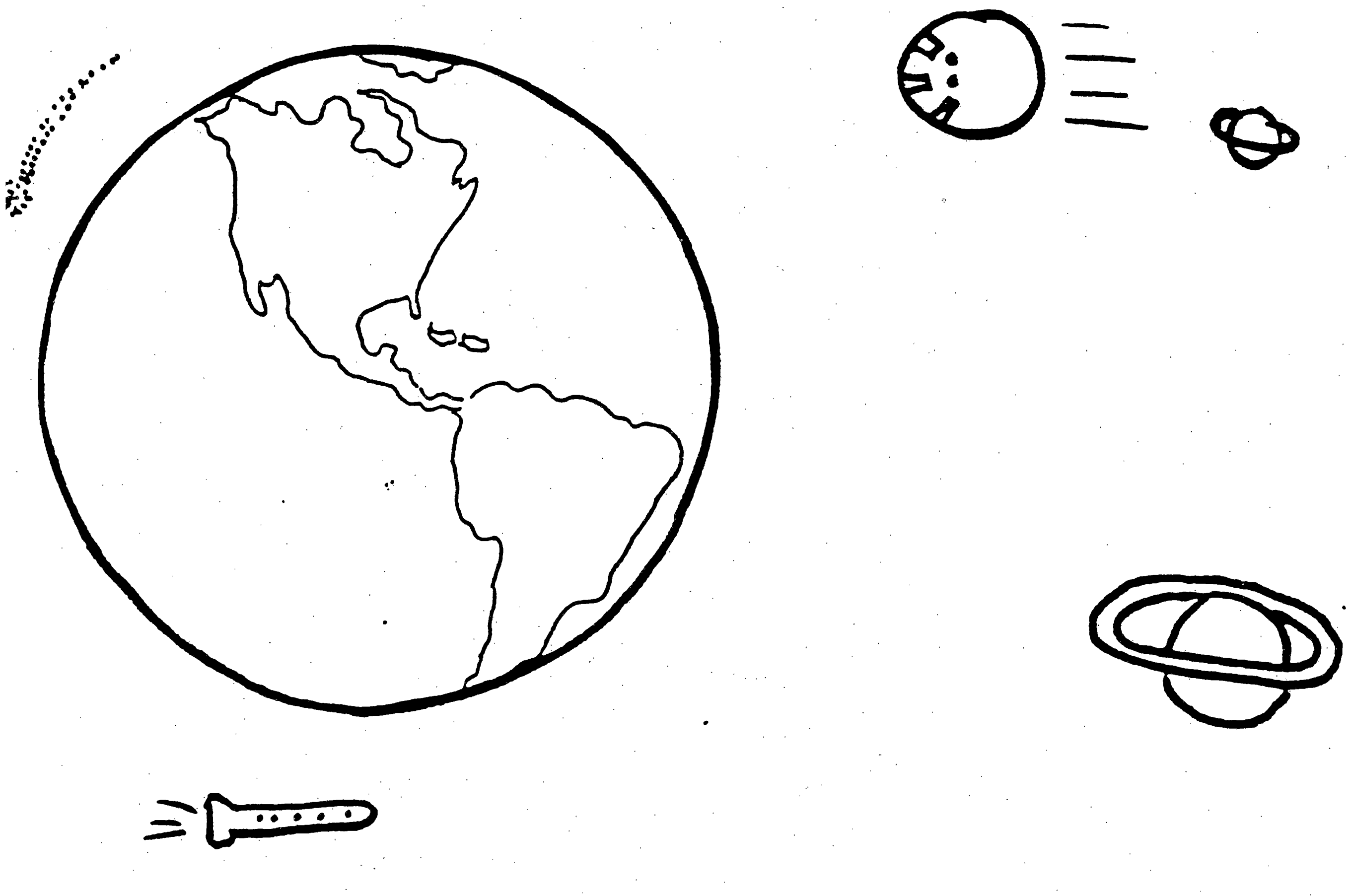
Flowers

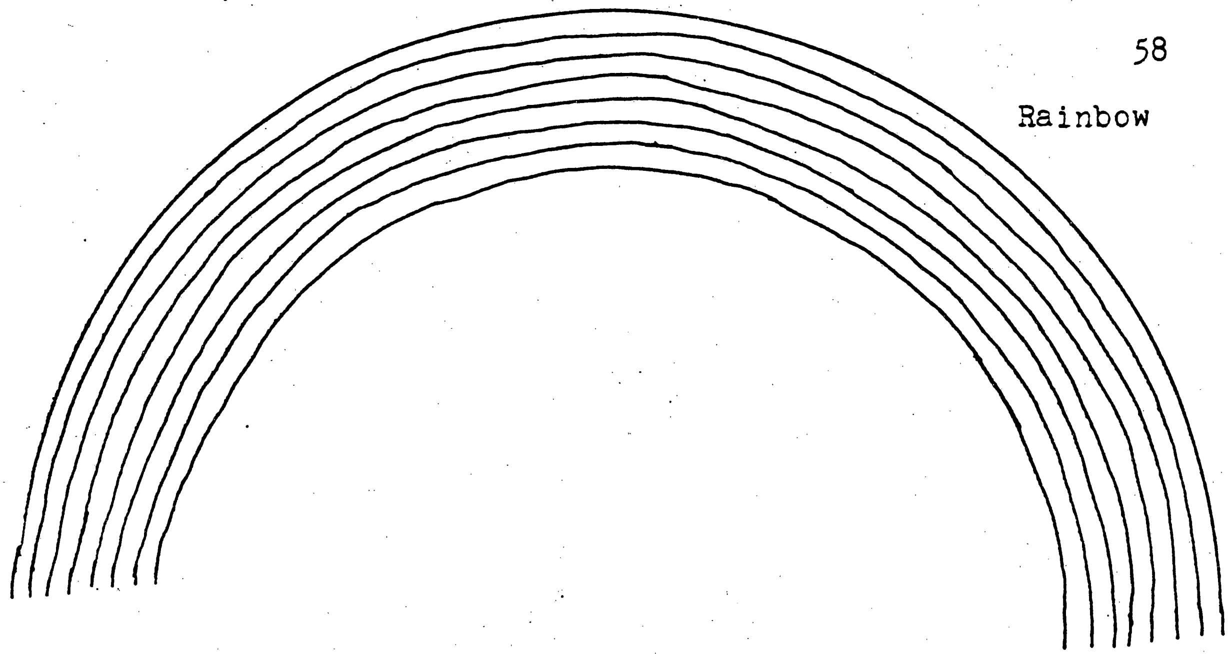


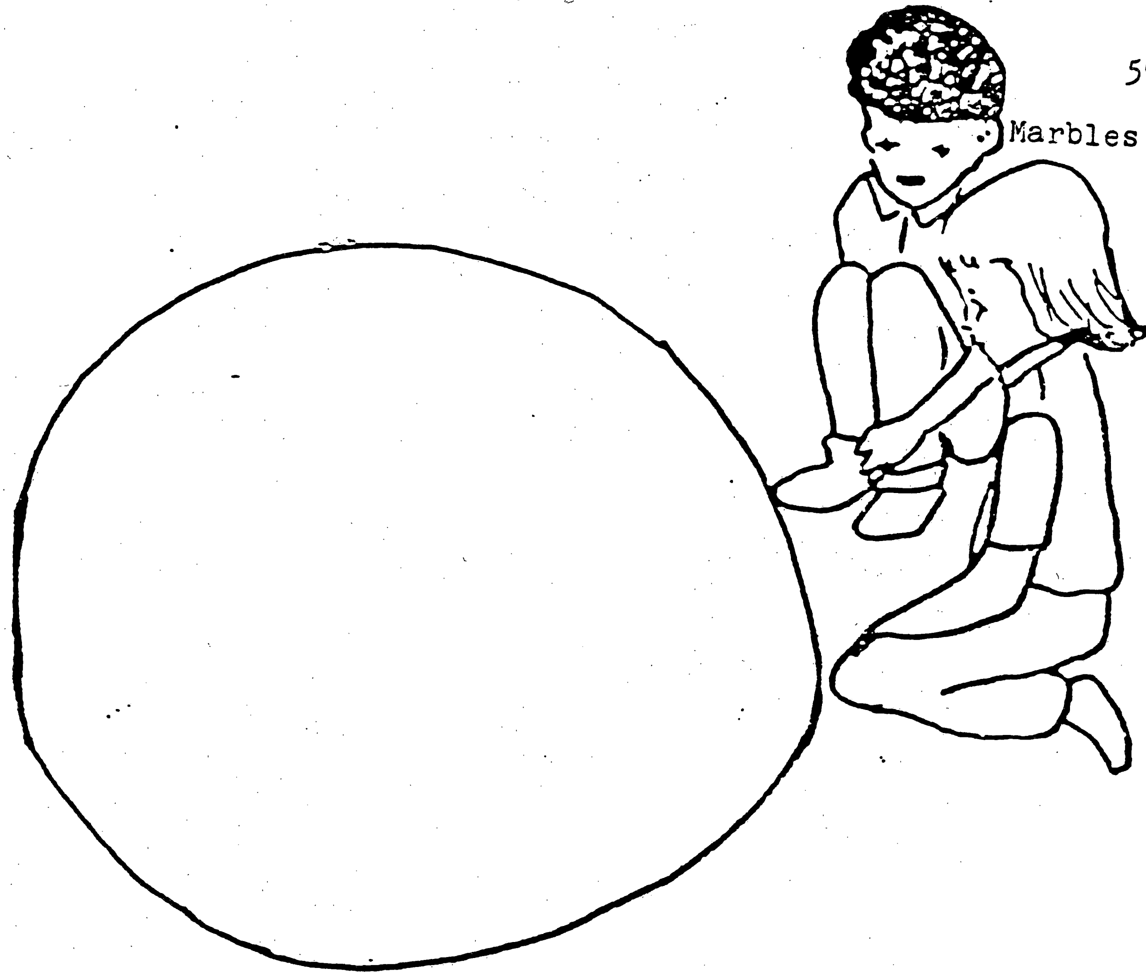
King Midas



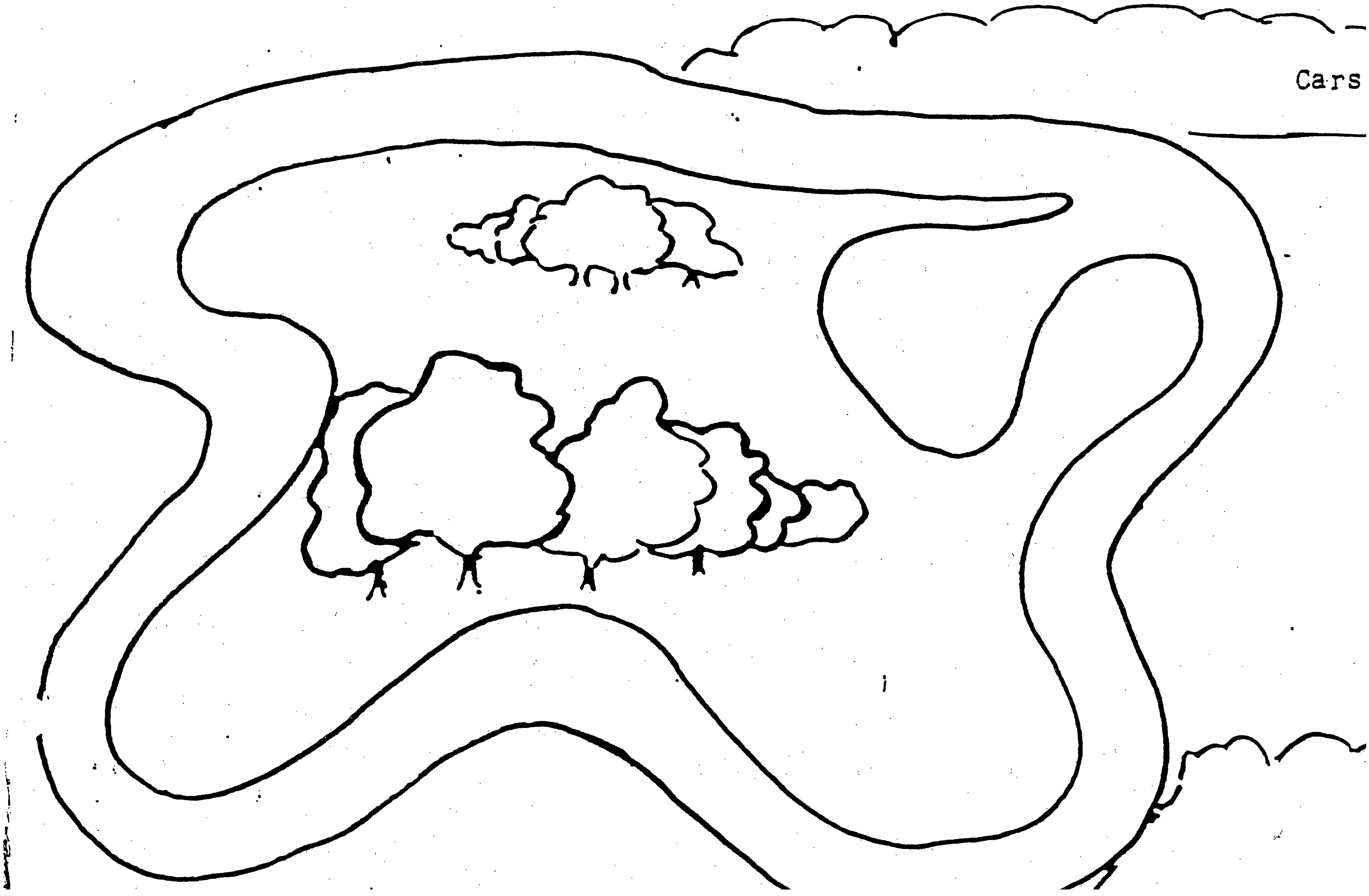
Outer Space




Rainbow



Marbles

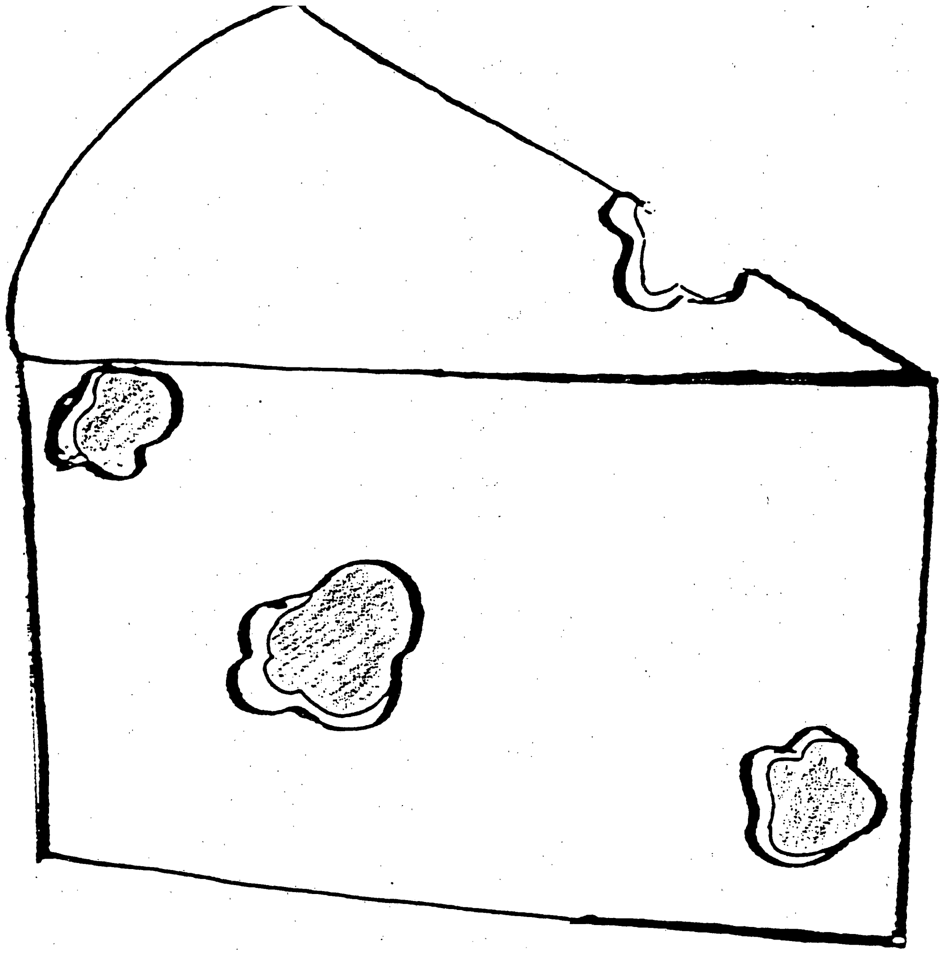


Cars

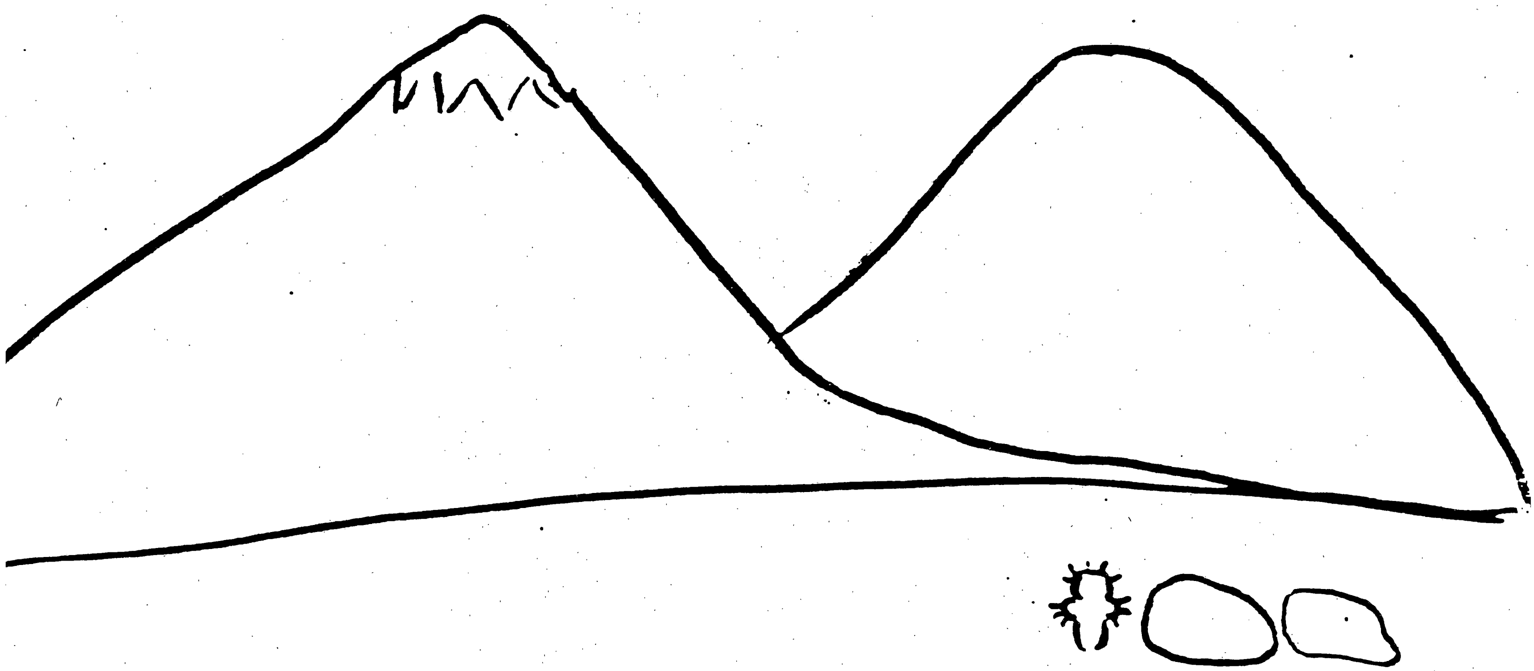
Birds

Caps

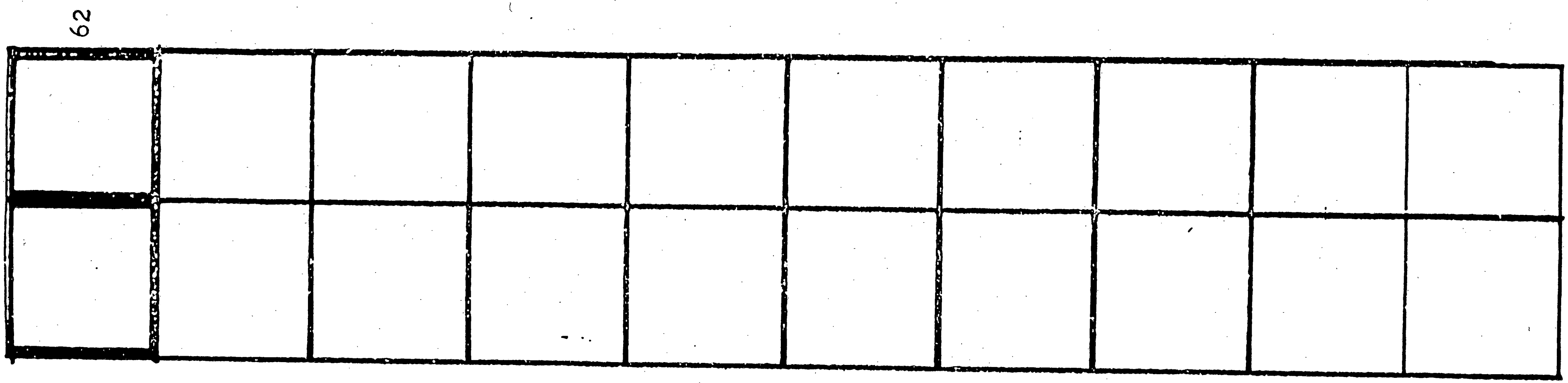
Mice



Rocks

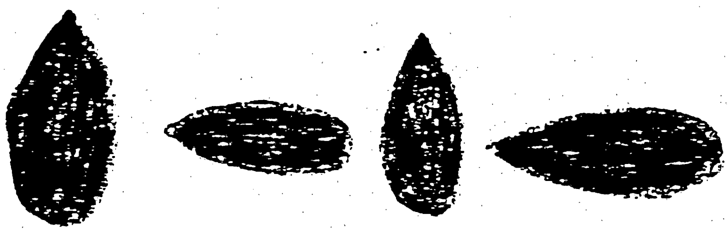


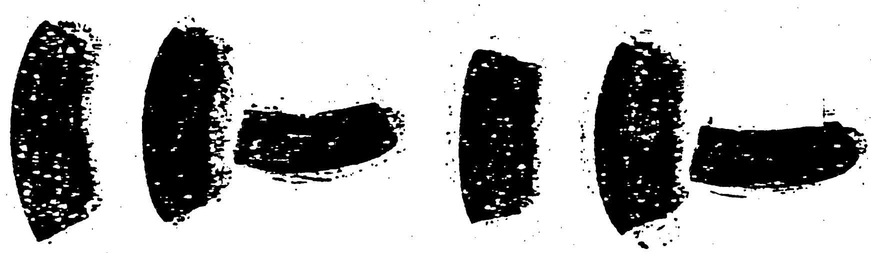
Horizontal graph

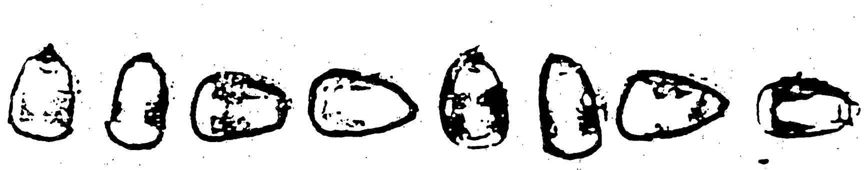


Vertical graph

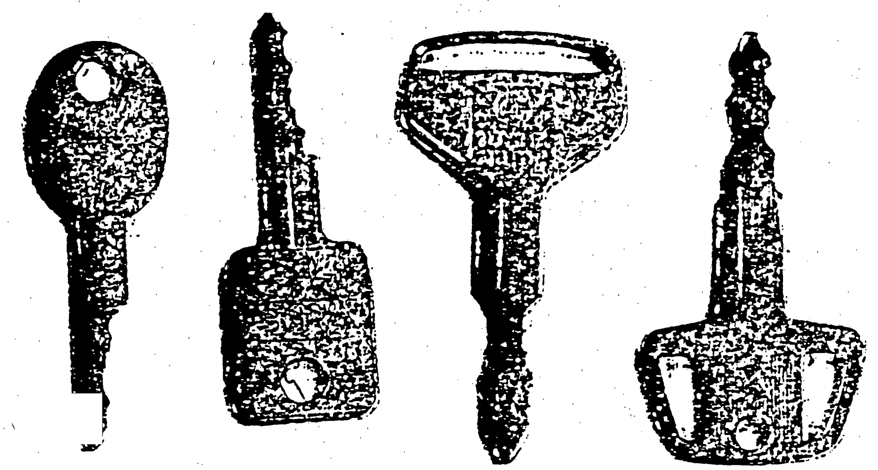
Patterns

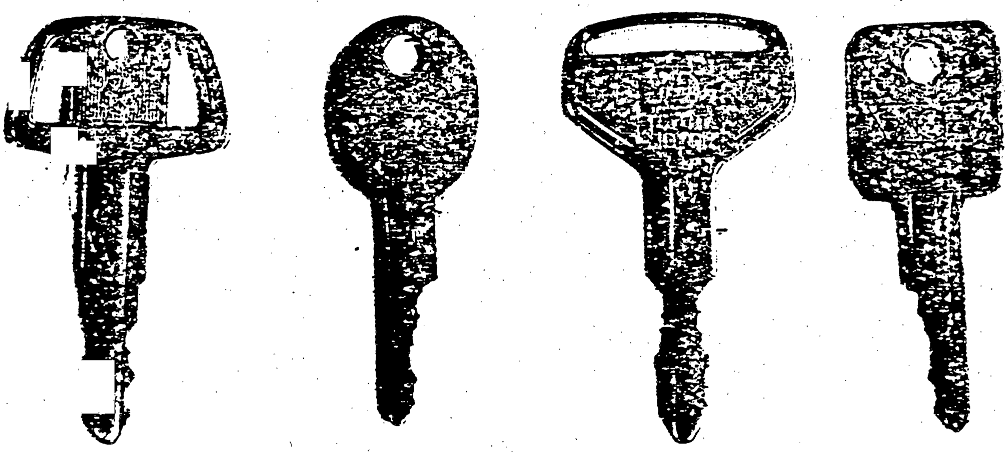


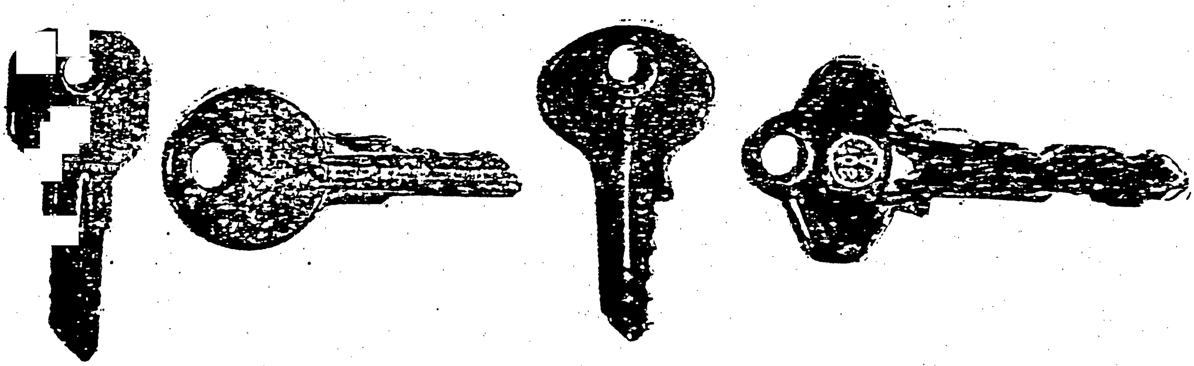




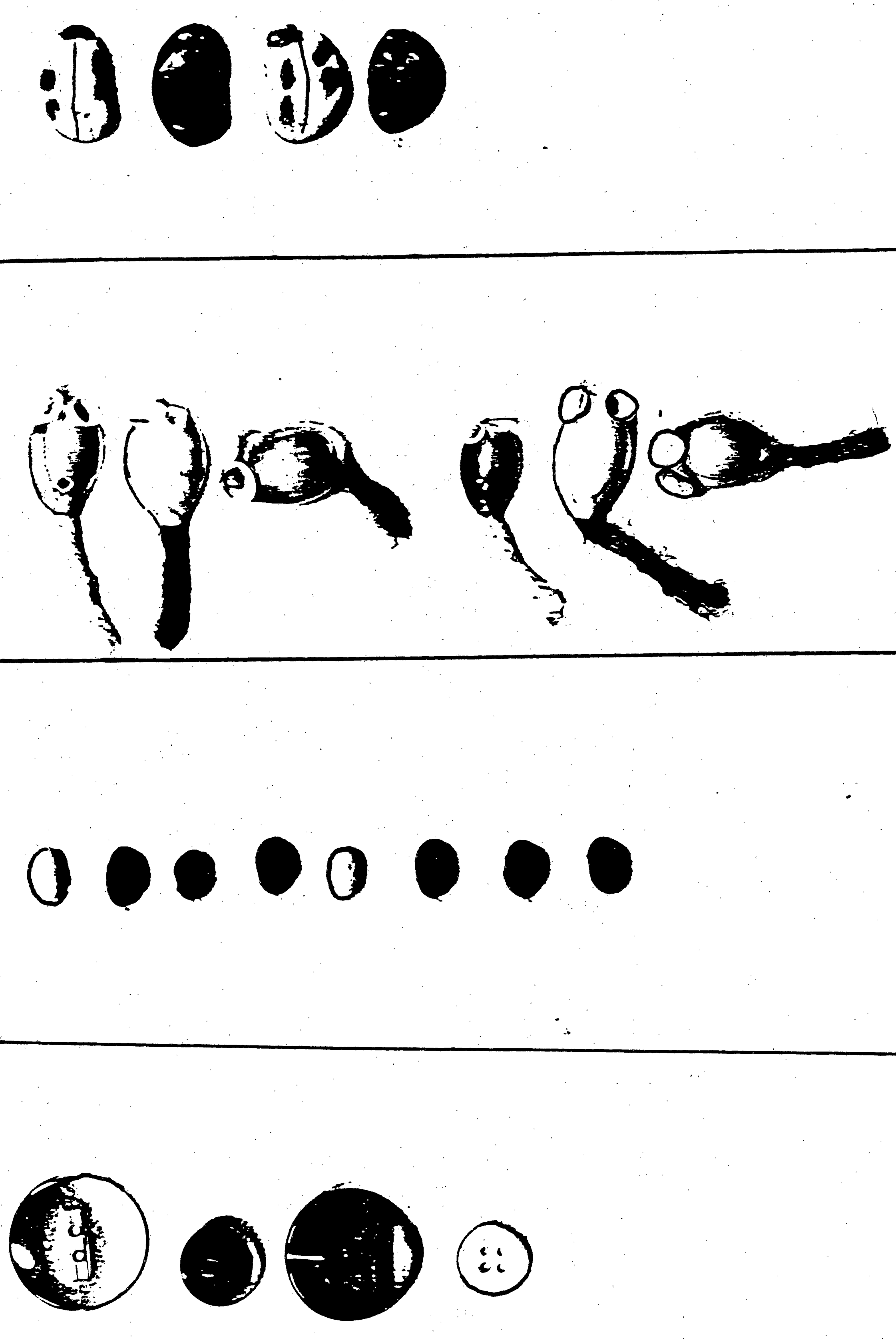
Patterns







Patterns



$\begin{array}{r} 5 \\ -1 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ -3 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ -3 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ -0 \\ \hline \end{array}$
$\begin{array}{r} 3 \\ -2 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ -4 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ -5 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ -3 \\ \hline \end{array}$
$\begin{array}{r} 5 \\ -0 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ -2 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ -2 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ -4 \\ \hline \end{array}$
$\begin{array}{r} 4 \\ -2 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ -0 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ -0 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ -1 \\ \hline \end{array}$

0 +0	5 +3	2 +2	3 +4
5 +4	5 +1	1 +2	5 +3
4 +4	2 +5	5 +3	3 +5
2 +4	3 +4	4 +3	5 +2

Books and Catalogs:

Clifford: The Small Red Puppy, Norman Bridwell

New York: Scholastic Inc.

Corduroy, Don Freeman

New York: Viking-Penguin Inc.

Curious George, H. A. Rey

Boston: Houghton Mifflin Company

It Looked Like Spilt Milk, Charles G. Shaw

New York: Harper Row

Ten Apples Up on Top, Theodore Le Sieg

New York: Beginner

Math Learning Center
Post Office Box 3226
Salem, OR 97302

Free catalog available upon request.

Workjobs II: Number Activities for Early Childhood, Mary Baratta-Lorton

Available from: Addison-Wesley Publishing Company
Jacob Way
Reading, MA 01867

Catalog Number: SA04302 - $11.00 plus tax.

Free catalog available upon request.
Ask for Addison-Wesley Supplementary Math K-8.

V. Bibliography

Bangs, Tina. In Language and Learning Disorders of the Preacademic Child. Englewood Cliffs, NJ: Prentice Hall, 1982.

Baratta-Lorton, Mary. Mathematics Their Way. Menlo Park, CA: Addison-Wesley, 1976.

Workjobs. Menlo Park, CA: Addison-Wesley, 1972.

Workjobs II. Menlo Park, CA: Addison-Wesley, 1979.

Boyer, Carl. A History of Mathematics. New York: Wiley & Son, 1968.

Brainerd, Charles, ed. Children's Logical and Mathematical Cognition: Progress in Cognitive Development. New York: Springer-Verlag, 1982.

Brownell, Herbert. "Psychological Considerations in Learning and Teaching of Arithmetic in Readings." In The History of Mathematics Education. Edited by James Bidwell and Robert Clason. Washington D.C.: National Council of Teachers of Mathematics, 1970.

Callahan, Leroy G. and Donald L. MacMillan. Teaching Mathematics to Slow Learning and Mentally Retarded Children. Reston, VA: The National Council of Teachers of Mathematics, Inc., 1981.

Cawley, John, ed. Developmental Teaching of Mathematics for the Learning Disabled. Rockville, MD: Aspen System Corp., 1984.

Copeland, Richard. How Children Learn Math. New York: McMillan, 1979.

Dienes, Zoltan. Mathematics in the Primary School. New York: McMillan, 1969.

Elkind, David. "Misunderstanding About How Children Learn." In Today's Education 61, March, 1972.

Erikson, Erik H. Childhood and Society. New York: W. W. Norton & Company, Inc., 1963.

Fuson, Karen C. "Response." In Mathematics Education Research: Implications for the 80s. Edited by Elizabeth Fennema. Alexandria: Association for Supervision and Curriculum Development, 1981.

Gappa, Sylvia and Deirdre Glynn. Creating and Managing the Self-contained Special Education Class. Belmont, CA: Pitman Learning, 1981.

Ginsberg, Herbert and Sylvia Opper. Piaget's Theory of Intellectual Development. Englewood Cliffs, NJ: Prentice Hall, 1969.

Glennon, Vincent and Douglas Cruickshank. "Teaching Mathematics to Children and Youth With Perceptual and Cognitive Processing Deficits." In The Mathematical Education of Exceptional Children and Youth. Edited by Vincent Glennon. Reston, VA: National Council of Teachers of Mathematics, 1981.

Hedden, James W. "Bridging the Gap Between the Concrete and the Abstract." In Arithmetic Teacher 33, February, 1986.

Hendrickson, A. Dean. "Prevention or Cure? Another Look at Mathematics Learning Problems." In Interdisciplinary Voices in Learning Disabilities and Remedial Education. Edited by D. Carnine, D. Elkind, A. D. Hendrickson, D. Meichenbaum, R. Seiben, and F. Smith. Austin: Pro-Ed, 1983.

Hiebert, James. "Children's Thinking in Mathematics." In Education Research: Implications for the 80s. Edited by Elizabeth Fennema. Alexandria: Association for Supervision and Curriculum Development, 1981.

Hynes, Michael. "Selection Criteria." In Arithmetic Teacher 33, February, 1986.

Ingebo, George. (1984) Third Grade Math Their Way Pilot Project in Portland Schools: 1983-84 Evaluation Report. Columbus. (ERIC Informational Analysis Center for Science, Mathematics, and Environmental Education, ED 253-592)

Kamii, Constance Kazuko. Young Children Reinvent Arithmetic. New York: Teachers College Press, 1985.

Labinowicz, Ed. The Piaget Primer. Menlo Park, CA: Addison-Wesley, 1980.

Learning From Children: New Beginnings for Teaching Numerical Thinking. Menlo Park, CA: Addison-Wesley, 1985.

Lamon, William E., ed. Learning and the Nature of Mathematics. Palo Alto, CA: Science Research Associates. 1972.

Leinwand, Steven. "Curricular Improvement Versus Standardized Testing." In Arithmetic Teacher 33, April, 1986.

Lesh, Richard A. "Applied Problem Solving in Early Mathematics Learning." Northwestern University, 1979. (Unpublished working paper)

Lieberthal, Edwin M. The Complete Book of Fingermath. New York: McGraw-Hill, 1979.

Lovell, Kenneth. The Growth of Understanding Mathematics: Kindergarten Through Grade Three. San Francisco: Holt, Rinehart & Winston, 1971.

Morris, Robert, ed. Studies in Math Education. Paris: UNESCO, 1984.

Myers, Patricia I. and Donald D. Hammill. Learning Disabilities. Austin: Pro-Ed, 1982.

Methods for Learning Disorders. New York: Wiley & Son, 1976.

National Joint Committee for Learning Disabilities. "Learning Disabilities: Issues on Definition." Towson, MD: National Joint Committee for Learning Disabilities, 1981. (Unpublished manuscript)

Nuffield Foundation. New York: Wiley & Son, 1972.

Piaget, Jean. The Child's Perception of Numbers. London: Lund, Humphries & Co., LTD., 1961.

Poplin, Mary. "Summary, Rationalization, Apologies, and Farewell: What We Don't Know About the Learning Disabled." In Learning Disability Quarterly 7, Spring, 1984.

Post, Thomas R. "The Role of Manipulative Materials in the Learning of Mathematical Concepts." In Selected Issues in Mathematics Education. Edited by Mary Montgomery Lindquist. Berkeley: McCulchan, 1980.

Price, Jack and J.D. Cawronski, eds. Changing School Mathematics: A Responsive Process. Reston, VA: The National Council of Teachers of Math inc., 1981.

Pulaski, Mary Ann Spencer. Understanding Piaget. San Francisco: Harper & Row, 1971.

Resnick, Lauren B. "A Developmental Theory of Number Understanding." In The Development of Mathematical Thinking. Edited by Herbert P. Ginsburg. San Diego: Academic Press, 1983.

Reys, Robert E., Marilyn N. Suydam, and Mary Lindquist. Helping Children Learn Mathematics. Englewood Cliffs, NJ: Prentice Hall, 1984.

Richardson, Kathy. Developing Number Concepts Using Unifix Cubes. Menlo Park, CA: Addison-Wesley, 1984.

Sapir, Selma G. The Clinical Teaching Model. New York: Brunner/Mazel, 1985.

Schuman, Kay. (1986, August) Mathematics Their Way workshop. San Bernardino Unified School District.

Scott, Lloyd F. "Increasing Mathematics Learning Through Improving Instructional Organization." In Learning and the Nature of Mathematics. Edited by William E. Lamon. Palo Alto, CA: Science Research Associates, 1972.

Sears, Carol. "Mathematics for the Learning Disabled Child in the Regular Classroom." In Arithmetic Teacher 33, January, 1986.

Silbert, Jerry; Douglas Carnine and Marcy Stein. Direct Instruction Mathematics. Columbus: Merrill Publishing Co., 1981.

Sternberg, Les and George Fair. "Mathematics Programs and Materials." In The Mildly Handicapped Student. Edited by Ted Miller and Earl Davis. San Diego: Grune & Stratton, 1982.

Suydam, Marilyn N. "Manipulative Materials and Achievement." In Arithmetic Teacher, 33, February, 1986.

Suydam, Marilyn N. and John L. Higgins. Activity-based Learning in Elementary School Mathematics: Recommendations from Research. Columbus: ERIC: Informational Analysis Center for Science, Mathematics, and Environmental Education, ED 120 013, 1976.

Swelt, Sheila. "Math as a Teaching Tool for the Learning Disabled." In Readings in Learning Disabilities. Edited by Robert Piazza. Guilford, CT: Special Learning Corporation, 1980.

Traver, Sara G. "Cognitive Behavior Modification, Direct Instruction and Holistic Approaches to the Education of Students with Learning Disabilities." In Journal of Learning Disabilities. June/July, 1986: 19:6.

Uzgiris, Ina and J. McVickers Hunt. Assessment in Infancy. Chicago: University of Illinois Press, 1975.

van Erp, J. W. and L. Heshusius. "Action Psychology: Learning as the Interiorization of Action in Early Instruction of Mathematically Disabled Learners." In Journal of Learning Disabilities. May, 1986: 19:5.

Weaver, J. F. and William C. Morse. "Teaching Mathematics to Socially and Emotionally Impaired Pupils." In The Mathematical Education of Exceptional Children and Youth. Reston, VA: National Council of Teachers of Math, 1981.

Willis, Lynn. (1984, July) Mathematics Their Way workshop. Center For Innovation in Education Inc., Crestline.

Worth, Jean. "By Way of Introduction." In Arithmetic Teacher 33, February, 1986.