ALGEBRA 1 STUDENTS’ ABILITY TO RELATE THE DEFINITION OF A FUNCTION TO ITS REPRESENTATIONS

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ALGEBRA 1 STUDENTS’ ABILITY TO RELATE THE DEFINITION OF A FUNCTION TO ITS REPRESENTATIONS

A Project
Presented to the
Faculty of
California State University,
San Bernardino

In Partial Fulfillment
of the Requirements for the Degree
Master of Arts
In
Teaching:
Mathematics

by
Sarah Carrillo Thomson
June 2015
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ABSTRACT

One hundred high school Algebra students from a southern California school participated in this study to provide information on students’ ability to relate the definition of function to its representations. The goals of the study were (1) to explore the extent to which students are able to distinguish between representations of functions/non-functions; (2) to compare students’ ability to distinguish between familiar/unfamiliar representations of functions/non-functions; (3) to explore the extent to which students are able to apply the definition of function to verify function representations; and (4) to explore the extent to which students are able to provide an adequate definition of function. Data was collected from written responses on a math survey consisting of items that asked students to decide if given illustrations are representations of functions, to explain how the decision was made, and to supply the domain and range when applicable. The questions included seven types of illustrations: graphs, equations, ordered pairs, tables, statements, arrow diagrams, and arbitrary mappings. Findings indicated that students were more able to correctly identify familiar than unfamiliar function representations. The easiest representation for students to correctly identify was the graph of a linear function and the most difficult was the graph of a piecewise function. A conjecture as to why this occurred is that the formal definition of function is not often emphasized or referenced when function and its representations are introduced so students do not have a deep understanding of how the function definition is related to its
representations. The explanation, domain, and range responses were sketchy. A conjecture as to why this occurred is that in general, students have difficulty expressing themselves orally and in writing or perhaps students had not learned about domain and range. A separate question asked students, “What is a function?” To this question, students provided a variety of responses. It is suggested that conducting further studies that include student interviews and participants from multiple teachers, would provide increased understanding of how students learn the definition of function and the extent to which they are able to relate it to its representations.
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CHAPTER ONE
INTRODUCTION

A Classroom Experience

More than 36 years ago, when I taught Algebra 1 for the first time, I thought that teaching functions would be quite simple. During my first lesson on functions, I introduced the definition as a relationship between two sets such that every element in the first set is mapped to exactly one element in the second set. I compared a function to a true love story where for every $x$ there was one and only one $y$. My students enjoyed the story and seemed to have grasped the notion that each input was related to exactly one output. So, I moved on to the graphs of functions and how to identify them. I explained how to use the vertical line test and this aligned nicely with the true love story. I modeled use of the vertical line test with the graph of $y = x$. So far, teaching about functions was simple and all was well. The next day, students were asked to apply the definition of function to distinguish between graphs of functions and non-functions. I thought this would follow easily from the “definition” of function that was learned the day before. Students were given the graphs of $y = 3x$, $y = x^2$, $y = \frac{1}{x}$, and $x = 4$. The first graph looked similar to the previous day’s example so it brought success throughout the room; then, the struggles started. Students applied the vertical line test to $y = x^2$ and decided that it was not a function graph because
two different $x$ values shared the same $y$ value. Students applied the vertical line test to $y = \frac{1}{x}$, and decided it was not a function because $x$ values near zero appeared to have no assigned $y$ values since the $y$ values could not be seen on the calculator’s viewing screen. Students attempted to apply the vertical line test to $x = 4$ and they were confused because they didn’t know how to run a vertical line through a vertical line. Students were also confused because there were no $y$’s in the equation. I explained that a vertical line has an infinite number of $y$ values assigned to one $x$ value so $x = 4$ is not a function. Furthermore, I explained that the other two graphs were representations of functions even though in one graph, some $x$ values had the same $y$ values, and in the other graph, for one $x$ value, the $y$ value was undefined. My words were met with blank stares and after about 5 seconds of silence, I heard from around the classroom the four little words that I came to dread with each succeeding lesson, “I don’t get it.” My students didn’t learn much about functions that day; but I learned something about teaching functions. Teaching functions was not going to be easy.

I tried to understand why students struggled with the notion of function and its representations and I tried many different approaches to teach about functions and some approaches worked better than others. However, I knew that I was only treating the symptoms, not the cause. I envisioned my students floating on a precarious vessel on a stormy ocean of misunderstandings while I tried to plug
holes as they randomly sprouted. At that time, in the absence of organized curriculum standards, educational research and data, my quest to provide the best chance of success for my students was a challenge, an isolated endeavor and often based on intuition. For many years, I wondered why students struggled with learning about functions and I often felt overwhelmed as I struggled with how to obtain better understanding that would help facilitate increased student success.

National Struggles

The concern for improving student success is not new and in the early years of education, there was not much joint effort to standardize content or understand cognitive development. Teachers were expected to somehow know what to teach, how to teach, and to produce highly successful and proficient students. I recall asking an administrator in the mid-seventies, “What shall I teach?” To my bewilderment, the reply was, “Whatever you want.” Things have come a long way since then, but not without some hit and miss efforts and some harsh reality checks at all educational levels.

In 1983, the struggles that manifested in individual classrooms amalgamated into what the National Commission on Excellence in Education (NCEE) proclaimed as a national rising tide of mediocrity in public schools (NCEE, 1983). This proclamation brought national attention to low performance in schools; the intensified attention motivated educational reform that sought to
increase student achievement (NCEE, 1983). Some important improvements were made in educational planning, infrastructure, and instruction; however, direct positive effects on learning in the classroom were inconsistent. Fifteen years after the beginning of the reform, the tide of minimum student achievement had not receded (Bennett, et al., 1998).

Near the end of the 20th century, students’ under performance and struggles were thought to be mostly attributed to lack of rigorous academic standards. In response, in 1997, California Content Standards (CCS) were adopted and a Standardized Testing and Reporting (STAR) program was authorized. From 1997 to 2012, the expectation was that CCS would instill rigor in content and produce high student achievement by explicitly stating what content and concepts should be taught at every grade level (CDE, 2013). However, annual test results continued to bring disappointing news. In 2002, STAR revealed disappointing results: 65% of California Algebra students scored basic, below basic, or far below basic (CDE, 2012). Between 2003 and 2012, high school Algebra scores improved but the rate of improvement declined yearly and in 2013, Algebra STAR scores declined for the first time in more than a decade: 74% of ninth grade students, 87% of tenth grade (mostly repeating) students, and 90% of eleventh grade (mostly repeating) students scored at basic, below basic or far below basic (CDE, 2013).

On a global scale, low performance was equally disheartening: results of the 2012 Programme for International Assessment (PISA), which is an
international assessment given every three years to 15 years olds in participating countries to assess competency in mathematics, science, and reading, indicated that the US ranked 35th in mathematics proficiency compared to other developed countries (U.S. Department of Education (ED), 2013). In 2012, the test had special focus on mathematics and 65 countries participated in the assessment (ED, 2013). The US was outperformed by participants from East Asian countries; Shanghai ranked highest followed by Singapore, Chinese Taipei, Hong Kong, Korea and Japan (Sedghi, Arnett & Chalabi, 2013).

2012 PISA results further revealed that on a 6 level mathematics proficiency scale, the US had an alarming 25.8 share of low performers that scored below Level 2 contrasted with an 8.8 share of top performers scoring at Level 5 or 6. (ED, 2013). The report also indicated a disappointing trend in mathematics proficiency for US participants: compared to 2003, the number of students in the 90th, 75th, and 50th percentiles dropped slightly in each level, the number in the 25th percentile remained the same, and the number increased slightly in the bottom 10th percentile (ED, 2013).

Secretary of State, Arne Duncan, responded to the results:

The big picture of U.S. performance on the 2012 PISA is straightforward and stark: It is a picture of educational stagnation. That brutal truth, that urgent reality, must serve as a wake-up call against educational complacency…In a knowledge based, global economy, our students are
basically losing ground. We’re running in place, as other high-performing countries start to lap us (ED, 2013).

Through the years, it became increasingly obvious that eliminating mediocrity in education would take more than the creation of a list of standards.

Statement of the Problem

Simply stated, the problem is that in American classrooms, students are struggling with learning and understanding fundamental concepts in mathematics and there is no doubt that for the last three decades the problem of underachievement in U.S. classrooms has become increasingly grave and is now manifested on a global scale. In the Fall of 2012, a Report of the Council on Foreign Relations warned that the US “will not be able to keep pace—much less lead—globally unless it moves to fix the [students’ underachievement] problems it has allowed to fester for too long” (U.S. Education Reform and National Security, 2012).

On state and national levels, huge collaborative efforts and resources were extended to develop Common Core State Standards (CCSS) and Standards for Mathematical Practices (SMP). Extensive research on cognitive development played a big role in the development of CCSS and SMP; progression in content expertise is based on how students’ mathematical knowledge, skill, and understanding develop over time. CCSS and SMP require that content be taught at a deeper level and with more precision. At the same
time, CCSS has given educators more flexibility to use their own expertise in developing lesson plans and curriculum to address the individual needs of the students in their classrooms. As a result of these expectations, teachers face increased responsibility to gain understanding of how students learn.

Teachers presently extend huge efforts to facilitate learning in their classrooms, however, many teachers do not understand how students learn. There exists a disconnect between instruction which is rooted in expert content knowledge, and students’ abecedarian ability to grasp knowledge; and as a result, instructors do not know that what they say is not always perceived or assimilated by students as they intended (Wu, 2006). Instructors may not be aware of a need to reference old knowledge, point out connections, and plan activities for clarification to assure that students acquire expertise in the content. Without understanding how students learn, instructors’ efforts may be ineffective; students may fail to develop precise, clear, and complete understanding of mathematical concepts; and, when the concepts are fundamental to the study of mathematics, the problem can be highly debilitating to students’ success at all levels of mathematics.

Understanding how students learn mathematics is a huge and complex requirement. One way to begin this quest is to gain understanding of how students perceive and learn fundamental concepts and definitions. One of the fundamental concepts in Algebra is functions and it establishes a foundation for future courses. Thus, gaining understanding on how students perceive and learn
the concept of function, its definition, and its relation to representations is an important endeavor.

Purpose of the Study

In order to gain increased insight into how students learn fundamental mathematical concepts, this study was conducted to investigate the concept of function, in particular, its definition and students’ ability to relate the definition to its representations.

This study examined students’ ability to relate the definition of a function to distinguish between function and non-function representations. The specific goals of this study are:

1. To study the extent to which students are able to distinguish between representations of functions and non-functions,
2. To compare students’ ability to distinguish between function and non-function representations to the students’ familiarity with the representations,
3. To explore the extent to which students are able to apply the definition of function to verify representations of functions, and
4. To explore the extent to which students are able to provide a complete, clear, and precise definition of a function.

The assessment instrument used for this study was a function survey that was developed by the researcher for the purpose of this study. The survey
consists of items that ask students to decide if given illustrations are representations of functions, to explain how the decision was made, and to supply the domain and range when applicable. The survey also asked students for a one or two sentence answer to the question, “What is a function?”

Participant Description

The participants in this study were 100 ninth grade high school students enrolled in regular, college-preparatory Algebra 1 ninth-grade classes. The students were enrolled with the same Algebra 1 teacher during various periods of the day. The teacher and students volunteered to participate in the study and the students were randomly selected.

The high school that the students attended is located in an urban area in Southern California. The school has a large population with approximately 80% Hispanic students.

The study was performed near the end of the school year after STAR testing. The students had received instruction in all of the 1997 Algebra Mathematics Content Standards which include the function concept, function definition, and representations of functions.

None of the participating students required special instructional or curricular modifications.
Significance of the Study

The need to increase student performance in Mathematics is not new and various endeavors have been extended to increase student performance in Mathematics. Ultimately, the role of making the biggest difference lies at the ground level, i.e. with the instructor in the classroom. Classroom instructors are diligent in preparing instruction and knowing their students’ needs; however, research reveals that even with what is taken to be good instruction, many students understand less than we think they do (Wu, 2006). Thus it is increasingly important that instructors focus on the most important concepts, concentrate on the quality of information presented, and understand how students learn.

In California, in 2014, CCS and STAR were eliminated and along with 48 states, California adopted Common Core State Standards (CCSS) and a Smarter Balanced Assessment System (CDE, 2014). These adoptions bring consistent rigorous content standards across states and provide more guidance and support to maximize the expertise of the instructor in the classroom. CCSS and the accompanying Standards for Mathematical Practice (SMP) ask teachers to narrow effort and energy in order to instill expertise at a deeper conceptual level. Rather than provide a list of what to teach, CCSS and SMP organize mathematics content and teaching so that students understand mathematics and do not just memorize procedures (CDE, 2013). CCS incorporates the view of its global competitors that it is not enough for students to “simply reproduce what
they have learned, but …to extrapolate from what they know and apply their knowledge creatively in novel situations” (Schleicher, 2014).

Changes initiated by the CDE (2013) have resulted in paramount changes to ensure standards align mathematical content and structure with students’ cognitive development. The changes require that instructors focus on the learner and continually be aware of the effectiveness of teaching taking place in the classroom. Instructors must know if their instruction is producing students with the required mathematical expertise and habits of mind. One way to ensure or increase effectiveness is to have a good understanding of how students learn and to use that understanding in preparing learning experiences in the classroom (Wu, 2006). The findings from this study provided some information that will bring insight and understanding to educators about how students learn a fundamental aspect of a concept in a domain of the CCS: the definition of function and its relation to its representations. “As Confrey (2007) points out, developing ‘sequenced obstacles and challenges for students...absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise’” (Confrey, Maloney, & Corley, 2015).

Limitations of the Study

There are two factors that contributed to the limitations of this study. The first involved not having a close rapport between the researcher and the students. The survey was conducted in a classroom in a school not affiliated with
the researcher so it is unknown as to whether the students’ responses were affected by their knowledge that the survey results were for a study and would not impact their class grades. The second factor involved the researcher’s missing information over the extent to which the concept of function, its definition, and its multiple representations were taught to the participating students.
CHAPTER TWO

LITERATURE REVIEW

What Factors Affect Students’ Ability to Relate the Definition of Function to its Representations?

Effective teachers are aware of the value of understanding how students learn; however, research reveals that students bring a multitude of complex learning characteristics to the classroom. The better educators understand how students learn, the better chance they have of meeting the wide range of students’ learning needs. “An educator must consider the characteristics of the students at their institution, the mindset of the generation, the variety of learning styles, and the cognitive development of students” (Hansen, n.d.). Many educators in mathematics strive to know what factors impact students’ ability to acquire deep understanding of the function concept and how to use that knowledge to close the achievement gap between math students who “get it” and those who “don’t get it.” In response to that need, various studies have been conducted using a variety of approaches to investigate how students learn the concept of function, its definition, and its relation to its representations. Some of the investigations looked for connections between learning and factors such as nature of a definition, first impressions, the roles and features of a mathematical definition, procedure versus process, operative versus inoperative definition, and the importance of function and its representations (Sicker, 2002; Sajka, 2003;

Nature of a Definition

Spoken words do not leave fossils but studying the development of human vocal chords indicates that human language in one form or another has been around for over 100,000 years, and written language can be traced back to about 5000 years (Jackendoff, 2006). Currently, there are over six thousand language schemes in use in the world today although only a minority of these have a written format (Jackendoff, 2006). Studies show that infants begin to recognize and learn language before birth, and after birth infants seem to have a remarkable ability to learn words and appropriate meanings from hearing others use language (Skwarecki, 2013). Thus, everyday human language is naturally passed from one generation to the next.

But what is the connection between this language synopsis and students’ ability to relate the definition of function to its representations? One answer is that acquiring the language of mathematics does not typically occur in the same way as acquiring everyday language which is naturally learned from hearing others use the language. Ironically, the notion of “not being” or “not occurring” the same way, exemplifies a very unique feature of human language—the ability to express the negation or what is not the case (Jackendoff, 2006). Thus, while the language of mathematics fulfills the usual requirements of communication, it is a
sub-system of language whose essence is not the same as the main system because it creates usage rather than reports usage, as explained below (Edwards & Ward, 2008). This difference in the nature of mathematical definitions is one of the reasons why students struggle with learning concept definitions. Many studies have been conducted to understand the nature of mathematical definitions and how it affects learning.

Edwards and Ward (2008) compared the nature of everyday definitions with the nature of mathematical definitions. They suggested that in general, students encounter two types of definitions with contrasting natures: extracted and stipulated (Edwards & Ward, 2008). In everyday life, students generally acquire extracted definitions which are formed by observed experiences or from a collection of evidence. Extracted definitions have a truth value; they either accurately report behavior/experiences or they inaccurately report behavior/experiences (Edwards & Ward, 2008).

Mathematical definitions, on the other hand, fall into the stipulated category and it is assumed that these definitions will root and grow on a clean slate without interference from any previous non-technical or non-contradictory use (Edwards & Ward, 2008). These stipulated definitions theoretically create a usage for the learner. They are precisely defined and accepted by the field; thus they are completely objective in nature and unlike extracted definitions, they have no optional truth value because they create precise and well-defined usage. Formal definitions are not left to contextual interpretation; they are stipulated and
there is no variance in how the definition is to be interpreted (Edwards & Ward, 2008). By their nature, stipulated definitions are difficult to trace within the cognitive structure of students’ minds; therefore, it is difficult to determine the level to which students learn and internalize stipulated definitions. Sometimes students are able to precisely state a definition but do not really understand the meaning of their statements and are not able to utilize them appropriately or extensively (Edwards & Ward, 2008).

Throughout the day, students continually practice, monitor, and assess extracted definitions; their nature promotes ease of competent acquisition. On the other hand, stipulated definitions are typically practiced, monitored, and assessed within a specific period during teacher-structured experiences. The nature of mathematical definitions complicates and may hinder acquisition of competency in utilization of stipulated definitions because, in a sense, they are imposed upon students. Understanding the nature of mathematical definitions will provide insight in planning instruction that will increase student achievement.

First Impression

First impressions are rapidly formed and if flawed, are often difficult to change. New experiences that may contradict the first impression become bound only to the context in which they were made and the first impression continues to prevail in other contexts (Wyer, 2010). Similarly, first exposure to the definition of function creates a conception that if linked only to familiar representations will be
difficult to change. New experiences with unfamiliar representations will likely be bound only to the context of the new situation and the initial perception will continue to prevail in other contexts.

Researchers have suggested that students' initial experience with the definition of function and relating it to its representation has an impact on the level of understanding that a student develops over time. In general, it has been found that minimal initial relation of the definition of function to a variety of its representations results in lower level understanding of the concept definition and inability to verify unfamiliar representations of functions (DeMarois, 1996; Tall & Bakar, 1991; Wyer, 2010; Zaslavsky & Shir, 2005).

Roles and Features of a Mathematical Definition

Learning and understanding mathematical definitions are essential to learning mathematical concepts; the power of definitions is captured in the roles and features of the definitions. Features of definitions include two formats: imperative and optional (Zaslavsky & Shir, 2005).

As previously mentioned, students may be able to memorize a definition and recite it verbatim, but not understand what the definition means. One way to gain insight into how well students understand a definition, is to study students' views and preferences of alternative definitions. Alternative definitions of a concept are equivalent statements that vary along optional features. Zaslavsky and Shir (2005) conducted a study to look at students' conceptions of
mathematical definitions by having students consider possible alternative definitions of some math concepts. The following characterizations can be attributed to a definition:

Four main roles:

1) Introduce the concept and convey its associated properties.
2) Provide fundamental components for concept development.
3) Establish a foundation for problem solving and proofs.
4) Create uniformity in communicating about the concept (Zaslavsky & Shir, 2005).

Three imperative features:

1) All conditions of the definition must be capable of coexisting.
2) The meaning should be uniquely interpreted.
3) When applicable, the definition should be invariant under change of representation and based on previously defined concepts in a non-circular way (Zaslavsky & Shir, 2005).

In general, these characterizations are the commonly accepted roles and imperative features of a definition, however, optional features are sometimes also attributed to a definition. Optional features are those features that may be omitted from a definition without losing the integrity of the definition; on the other hand, if an imperative feature is omitted, the result is a non-example of a concept definition. For example, a student may state that a function is a graph that passes the vertical line test. This statement is an example of a non-definition.
since it does not include all the imperative roles and features commonly attributed to the function concept definition.

Zaslavsky and Shir (2005) found that during leader-led group discussions of alternative definitions, students made little or no reference to the imperative features of a given definition. Students made even less comments about connections between alternative representations inferred by a given formal definition and those inferred by alternative definitions. In general, students acquire limited conceptions and limited understanding of which essentials are required for mathematical definitions; these limitations are the result of learning experiences and discussions focused primarily around limited roles and features of a definition. Students are commonly exposed to only the textbook definition of a concept and rarely to alternative or non-examples of definitions. This affects the level of understanding of a concept and definition that a student develops over time (Zaslavsky & Shir, 2005).

When students create their own definitions from guided learning experiences, students gain higher level internalization and expertise in a concept. Zaslavsky and Shir (2005) observed that when students were left on their own to discuss their views on alternative definitions to a given definition, students gained higher level understanding and clarification of their own perception as they needed to generate increased numbers of examples or counterexamples to support their personal claims. When students are more comfortable with a concept, students tend to focus on optional features of a definition such as
clarification and comprehension for the user. When students are less familiar with the concept, they tend to focus on the correctness or necessary and sufficient conditions specified in the statement, even though they may not understand the meaning of the terms used in the definition. This supports the view that “Generating examples [and counterexamples] is an important cognitive activity, as the ability to generate examples [and counterexamples] as needed is one of the distinctions between novices and experts…” (Zaslavsky & Shir, 2005).

Procedure versus Process

DeMarois (1996) studied students’ level of understanding of the function concept by comparing cognitive endeavors that involved mathematical processes to those that involved mathematical procedures. DeMarois (1996) used Gray and Tall’s (1994) procept theory to separate students across a proceptual divide according to their level of understanding of the function concept. In the procept theory, an amalgam of a process, a concept and a symbol serve as the criteria for making the separation. According to the theory, when a symbol (representation) evokes either a process or a concept the student holds a high level of understanding which allows the student to verify representations of functions in unfamiliar formats. Higher level understanding allows flexibility in applying the definition of function in new contexts. On the other hand, when a symbol (representation) evokes a procedure, the student holds a low level understanding that impedes the ability to verify function representations in
unfamiliar contexts. Lower level understanding relies on a concept image which evolves from a mishmash of disconnected procedures that have been memorized with limited understanding about connections between the facets of the definition. This results in difficulty when thinking about or understanding the relationships between the concept of function, the definition of function, and its representations (DeMarois, 1996).

Operative versus Inoperative Definition

Other studies have investigated the relationship between the acquisition of students’ operative or inoperative definition of function and prototypes. Some findings indicated that students form habits of mind through frequent exposure to familiar patterns and they develop prototypes for the function concept similarly to the way they develop prototypes for everyday concepts (Tall & Bakar, 1991). When the definition of function is introduced but not emphasized or re-referenced, students develop an inoperative definition of function and the mind attempts to solve problems by resonating with mental prototypes (Tall & Bakar, 1991). For example, if a student is asked whether a particular graph represents a function and the graph produces a mental hit, the student experiences that sensation and the student responds accordingly. If the student does not have an operative definition, then the mind attempts to resonate with mental prototypes that have been collected in the past. Without a hit, the student experiences confusion and mentally searches to formulate the reason for failure to obtain a
resonance; the student’s mental search can cause frustration and may result in an incorrect response or no response at all (Tall & Baker, 1991).

When a student looks at an unfamiliar graph of a function, the student may mistake it for a graph of a non-function, because his/her previous experiences with graphs have been that graphs have familiar shapes or are usually described by a formula. Sometimes, the resonance may evoke inappropriate properties of prototypes, then the student may give an incorrect response (Tall & Bakar, 1991). For example, a student may think that a constant function is not a function because previously encountered prototypes depended on variables and variables vary. A constant function such as \( y = 3 \) always yields 3 as an output regardless of the input, so a student may consider that this invariance produces a non-function; its horizontal linear graph may also be falsely considered a non-function representation because of its linear, horizontal, or non-varying appearance. Thus, even positive resonances may result in incorrect responses.

Tall and Bakar (1991) found that students are sometimes provided with limited experiences to formulate formal depths of logical meaning for the definition of function. Sometimes, formalities are introduced, but the formalities are not emphasized consistently or are not related to experiences, activities, or to work intended to develop depth in learning the concept of function; “…the collection of activities inadvertently colours the meaning of the function concept with impressions that are different from the mathematical meaning which, in turn, can store up problems for later stages of development” (Tall & Baker, 1991, p. 22).
Acquiring an operative definition of function allows students to successfully relate the definition of function to its representations.

**Importance of Function and Function Representations**

One might ask why is it important to put effort into understanding students’ ability to relate the definition of function to its representations; after all, theoretically speaking, the concept of function is a simple idea. What could be simpler, than pairing exactly one and only one output to every input between whatever sets one wishes? In reality, research reveals that the concept of function remains one of the most difficult concepts for students to learn; however, when mastered, the notion of function, along with its relation to the vast array of function representations, opens up many opportunities for problem solving and describing just about any event in the world (Bayazit, 2011).

The concept of function is one of the most basic concepts of mathematics and one of the most amazing and powerful because of its diversity of interpretations and representations. With its associated sub-notions, the variety and range of representations of functions is immense and useful in real life to understand almost any phenomenon. However, by its very simple nature, the definition of function is sometimes glided over or perhaps assumed intuitive. Sajka (2003) discussed this view in her work on students’ understanding of functions. She suggested that the power of function is due in part to the very seemingly simple nature of the formal definition of function, particularly with its
arbitrariness property, i.e. a property that allows function relationships to be arbitrary and the domain and co-domain to be any arbitrary sets. This property allows function representation of situations that range from simple to highly sophisticated. Sajka (2003) stated, “Function is one of the basic concepts of mathematics, amazing in the diversity of its interpretations and representations. Much time and attention have been spent on it in the didactic process, yet it remains a difficult concept” (p. 231).

Students who develop a thorough understanding of the function concept and its relation to representations of various and complex formats are better prepared for life’s challenges and opportunities in today’s rapidly changing world. Ainsworth (1999), and Amit and Fried (2005), agreed on the importance of learning about functions and its representations in order to understand and solve problems in real life. The notion of cause-and-effect is highly related to students’ understanding of function and its representations; the notion of dependent and independent factors is critical in a multitude of world situations including economics, politics, science, medicine, peace, and survival.

Amit and Fried (2005) further suggested that there is a correlation between learning the function concept along with its representations and developing higher level problem solving skills. Tall and Vinner (1981) had a similar suggestion from their study on functions in the context of limits and continuity. The researchers explored debilitations that might arise from the total cognitive structure associated with the concept and how the structure is built.
through the years. Tall and Vinner (1981) found that sometimes subconscious conflict factors cause a vague sense of uneasiness when solving problems or doing research and that it may be a considerable time later until the reason for conflict is consciously understood. For example, a significant finding was that a weak understanding of the representation of continuous functions led to struggles with understanding limits in higher level course work (Tall & Vinner, 1981).

DeMarois and Tall (1996) also found that students’ ability to move comfortably between facets of a definition is imperative for success in higher level courses where functions may need to be treated as objects (DeMarois & Tall, 1996). In “Facets and Layers of the Function Concept,” DeMarois and Tall (1996) found the ability to do so was a higher-order function and they expressed an appreciation for the links involved in forming connections between multiple representations of function. Ainsworth (1999) had similar suggestions in a study where she determined that translation across multiple representations supports deeper understanding and higher order cognitive processes.

The findings above suggest that struggling with function representations in Algebra may be detrimental to learning more sophisticated and complex function-based concepts in mathematics (DeMarois & Tall, 1996; Tall & Vinner, 1981; Ainsworth, 2000).
CHAPTER THREE

METHODOLOGY

The goals of this study were stated in the introduction and are restated here:

1. To study the extent to which students are able to distinguish between representations of functions and non-functions,
2. To compare students’ ability to distinguish between function and non-function representations to the students’ familiarity with the representations,
3. To explore the extent to which students are able to apply the definition of function to verify representations of functions, and
4. To explore the extent to which students are able to provide a clear and precise definition of a function.

Sample

The participants in this study were 100 ninth grade Algebra 1 high school students. The students were all enrolled in regular, college-preparatory Algebra 1 classes with the same teacher during various periods of the day. The teacher and students volunteered to participate and the students were randomly selected.

The study was conducted near the end of the school year after STAR testing. The teacher indicated that participating students had received instruction according to the 1997 California Mathematics Content Standards for Algebra.
which include function definition and representations of functions. None of the
students that participated in this study required instructional or curricular
modifications.

The high school that the students attended has a large population with
approximately 80% Hispanic, 10% White, 5% African American, and 5% Asian
students (CDE, 2011). Approximately 75% of the students at the school are
economically disadvantaged (CDE, 2011).

The school’s Academic Performance Index (API) for the year of the study
was over 700 and the CST scores for ninth grade Algebra 1 students were 46%
below and far below basic combined, 27% basic, and 27% proficient and
advanced proficient combined (CDE, 2011). The passing rate for CAHSEE for
tenth graders was 87% (CDE, 2011).

The school is located in an urban area in Southern California.
Approximately 25% of the constituents have some high school education, about
50% have high school diplomas, roughly 18% have associate degrees, and
about 7% have graduate degrees (U.S. Census, 2013).

Terminology
Throughout this study, the following terms will be used:

1. *Familiar representations* will be used to describe representations that
   are likely to be frequently referenced and practiced in learning
   functions in Algebra 1 high school classes. Familiar representations
include linear, quadratic, and continuous graphs. Also included are equations with $x$ and $y$ variables, mapped sets involving numbers, and two column tables.

2. **Unfamiliar representations** are interpreted to mean the type of representations that are most likely minimally referenced and practiced in Algebra 1 high school classes. These include representations with variables different from $x$ and $y$, such as $z$, $t$, and function notation $f(x)$, and sets with arbitrary elements that are not necessarily numeric.

3. Definition of function is used for the mathematical **definition of a function**: an arbitrary relation from a set of possible inputs to a set of possible outputs where each input is related to exactly one output.

**Instrumentation**

A function survey was developed by the researcher for this high school student study. The format of the survey is similar to one designed by Elia and Spyron (2006) for their research on how students perceive functions and how students relate the definition of function to its representations. Elia and Spyron (2006) used their survey to study university students so a main difference between their survey and the survey for this high school study is in the level of mathematical sophistication. The researcher for this high school study aligned the question content with the 2006 Mathematics Framework for California Public Schools.
The survey designed for this study consisted of 29 questions. The first 26 questions (Q1 – Q26) were to collect data involving students’ ability to relate the function definition to its representations, and to provide an adequate definition of function. The last three questions were used to collect student data regarding previous math courses taken, grades, and general views about mathematics.

Q1 – Q25 in the survey included three tasks. The first task was to decide if the given representation was that of a function or non-function. Students responded to this task by circling Yes or No. The second task was to explain how the answer to the first task was decided. Students wrote their answers to this question on the survey. The third task was to give the domain and range in case the representation was that of a function. Students were provided a space on the survey to write their responses to the third task.

Q1 – Q25 included seven types of familiar and unfamiliar representations. The types of representations were grouped as follows: graphs, equations, ordered pairs, tables, statements, arrow diagrams, and arbitrary set mappings. Each group was designed to provide data on students’ ability to relate a different type of representation to the function definition and to explore students’ ability to distinguish between representations of familiar versus unfamiliar representations. Table 1 shows the seven types of representations, the number of questions of each type, the question numbers, the assessed task of each question, and an example of each type.
### Table 1

*Types of Function Representations*

<table>
<thead>
<tr>
<th>Type of Representation (number of questions)</th>
<th>Question Numbers</th>
<th>Assessed task</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph (7)</td>
<td>1 – 7</td>
<td>Distinguish between graphical representation of functions/non-functions</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>Equations (5)</td>
<td>8 – 12</td>
<td>Distinguish between equation of functions/non-functions</td>
<td>$y = 2x - 36$</td>
</tr>
<tr>
<td>Ordered Pairs (2)</td>
<td>13 – 14</td>
<td>Distinguish between ordered pairs of functions/non-functions</td>
<td>${(3,2),(5,4),(7,-8),(8,10),(9,4)}$</td>
</tr>
<tr>
<td>Tables (2)</td>
<td>15 -16</td>
<td>Distinguish between table values of functions/non-functions</td>
<td><img src="image" alt="Table" /></td>
</tr>
<tr>
<td>Statements (3)</td>
<td>17 – 19</td>
<td>Distinguish between statements describing functions/non-functions</td>
<td>“An input is assigned to three different outputs.”</td>
</tr>
<tr>
<td>Arrow Diagrams (3)</td>
<td>20 – 22</td>
<td>Distinguish between arrow diagrams of functions/non-functions</td>
<td><img src="image" alt="Arrow Diagram" /></td>
</tr>
<tr>
<td>Arbitrary set mappings (3)</td>
<td>23 – 25</td>
<td>Distinguish between arbitrary set mappings of functions/non-functions</td>
<td><img src="image" alt="Arbitrary Set Mapping" /></td>
</tr>
<tr>
<td>Definition (1)</td>
<td>26</td>
<td>Provide definition of function</td>
<td></td>
</tr>
<tr>
<td>Background Information (3)</td>
<td>27 – 29</td>
<td>Provide prior courses taken, views about math, perceived math grade</td>
<td></td>
</tr>
</tbody>
</table>
Variations of representations were included within some types of representations. The variety within each type consisted of familiar and unfamiliar representations. A brief description of the variations within each type, and whether the given representation was of a function or non-function are shown in Table 2.
### Table 2

**Variations within Types of Representations**

<table>
<thead>
<tr>
<th>Type of Representation (Question Numbers)</th>
<th>Variation</th>
<th>Function or Non-function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphs (1 – 7)</td>
<td>1. hyperbola</td>
<td>1. non-function</td>
</tr>
<tr>
<td></td>
<td>2. linear</td>
<td>2. function</td>
</tr>
<tr>
<td></td>
<td>3. cubic</td>
<td>3. function</td>
</tr>
<tr>
<td></td>
<td>4. discrete</td>
<td>4. function</td>
</tr>
<tr>
<td></td>
<td>5. piecewise</td>
<td>5. function</td>
</tr>
<tr>
<td></td>
<td>6. vertical line</td>
<td>6. non-function</td>
</tr>
<tr>
<td></td>
<td>7. greatest integer</td>
<td>7. function</td>
</tr>
<tr>
<td>Equations (8 – 12)</td>
<td>8. includes $x, y$ variables</td>
<td>8. function</td>
</tr>
<tr>
<td></td>
<td>9. includes $x$, not $y$ variable</td>
<td>9. non-function</td>
</tr>
<tr>
<td></td>
<td>10. includes $y$, not $x$ variable</td>
<td>10. function</td>
</tr>
<tr>
<td></td>
<td>11. includes notation $f(x)$</td>
<td>11. function</td>
</tr>
<tr>
<td></td>
<td>12. includes $t$ variable, decimal number</td>
<td>12. function</td>
</tr>
<tr>
<td>Ordered Pairs (13 – 14)</td>
<td>13. first or second element: input not related to exactly one output (fails).</td>
<td>13. non-function</td>
</tr>
<tr>
<td></td>
<td>14. first element set is domain: input related to exactly one output (passes)</td>
<td>14. function; non-function</td>
</tr>
<tr>
<td>Tables (15 – 16)</td>
<td>15. passes if $x$ set is domain; fails if $y$ set is domain</td>
<td>15. non-function; function</td>
</tr>
<tr>
<td></td>
<td>16. passes if $x$ set is domain; fails if $y$ set is domain</td>
<td>16. function; non-function</td>
</tr>
<tr>
<td>Statements (17 – 19)</td>
<td>17. written statement</td>
<td>17. function</td>
</tr>
<tr>
<td></td>
<td>18. written statement</td>
<td>18. non-function</td>
</tr>
<tr>
<td></td>
<td>19. different context</td>
<td>19. function</td>
</tr>
<tr>
<td>Arrow Diagrams (20 – 22)</td>
<td>20. related numbers</td>
<td>20. non-function</td>
</tr>
<tr>
<td></td>
<td>21. one extra element in range</td>
<td>21. function</td>
</tr>
<tr>
<td></td>
<td>22. alpha character in range</td>
<td>22. function</td>
</tr>
<tr>
<td>Arbitrary Mappings (23 – 25)</td>
<td>23. arbitrary elements</td>
<td>23. function</td>
</tr>
<tr>
<td></td>
<td>24. arbitrary elements</td>
<td>24. function</td>
</tr>
<tr>
<td></td>
<td>25. arbitrary elements</td>
<td>25. non-function</td>
</tr>
</tbody>
</table>
Q26 asked students to respond, using one or two sentences, to the question, “What is a function?” A list of response descriptors (Appendix B) for analyzing students’ responses to this question was developed by the researcher. The list includes some descriptors used by Arnold (1992) for a similar question in one of his studies on students’ understanding of the function definition.

Q27, Q28, Q29 asked students for information that would allow exploration of the relationships between students’ knowing the definition of function and students’ prior course work, views about mathematics, and perceived grades in Algebra. Data from Q27 – Q28 are left for future exploration.

Data Analysis

Part (a) of Q1 – Q25 asked students if a given representation was that of a function. The responses for Part (a) of Q1 – Q25 provided data to explore the extent to which students were able to distinguish between representations of functions and non-functions and to explore the influence of familiarity on students’ ability to distinguish between function/non-function representations. The Yes and No dichotomous responses to Part (a) of the questions were calculated using 1 for a correct answer, and 0 for an incorrect answer, or no response. Percent for correct/incorrect Part (a) responses were calculated and compared as a whole including all twenty-five questions and within each type of representation (graphs, equations, ordered pairs, tables, statements, arrow diagrams, and arbitrary set relationships). The results were separated by types in
order to compare percent correct/incorrect of item variations within each type of representation.

Within each type of representation, data was compared to determine if there was a relationship between the familiarity of the representation and students’ ability to distinguish between function and non-function representations. The data was used to generate bar graphs showing percent of correct and incorrect responses. Familiar and unfamiliar representations were compared based on whether students are likely to encounter them in regular mathematics textbooks for Algebra 1, lower level math courses, or classroom instruction as per classroom observations, California Content Standards in Mathematics and Common Core State Standards in Mathematics.

Part (b) of Q1 – Q25 in the Math Survey asked students to explain how they determined their response to Part (a). The responses for Part (b) were scored using a rubric (Appendix A) to examine the extent to which students were able to apply adequate explanations for distinguishing between function and non-function representations.

Responses to Parts (a) and (b) were further analyzed to explore the correlation between a students’ ability to correctly decide whether or not a representation was that of a function and the students’ ability to relate the decision to the definition of function.

Part (c) of the Math Survey asked students to provide the domain and range of functions. These were scored as correct or incorrect depending on the
correct or incorrect order of mapping provided by the student response. A score of “1” was assigned for a correct answer and “0” for an incorrect answer or no response. These scores were examined to add to the data about students’ ability to provide the correct order of mapping in a function representation.

Winsteps Software (Linacre, 2011), which is based on the Georg Rasch measurement theory, was used to analyze response data from the first twenty-five questions on the survey to obtain mean, standard deviation, and item difficulty for each item. Determining the item difficulty value involves a joint maximum likelihood process upon a matrix of responses of persons to items; the item difficulty measures are invariant of other items in the data matrix and of the persons who responded to the items. The total scores, for items and persons, are the sufficient statistics for obtaining item difficulty and person ability measures. The Rasch analysis produces a linear interval scale that measures student ability and item difficulty on a common scale measured in logit units (log-odds) (Rasch, 1980). The higher the person measure, the higher is the person ability; the higher the item difficulty measure, the more difficult is the item. The item difficulty and student ability measures are shown on a common scale in Figure 1, Variable Map of Yes/No Items.

The reliability coefficient for students’ ability measures was also calculated. The rate of success demonstrated by students for each question type and a comparison of differences in proportion right was reported. Finally, Rasch’s measurement model was used to analyze the data to examine the
quality of the items. The analysis also provided a visual representation of the item difficulties and person abilities on an interval scale.
CHAPTER FOUR

FINDINGS AND RESULTS

Data were obtained from the student responses on the survey. Analysis of the data provided information about students’ ability to distinguish between functions and non-function representations, to relate the definition of function to its representations, and to provide an adequate definition of function.

Three different data analyses were conducted using data from student responses to the first twenty-five questions. Winsteps Software (Linacre, 2011) was used for performing the analyses. The Winsteps program is based on the Georg Rasch measurement theory. The Rasch analysis produces a linear interval scale that measures student ability and item difficulty on a common scale measured in logit units (log-odds) (Rasch, 160, 1980).

The first analysis selected data from Part (a) of the questions. Part (a) asked students to respond with yes or no as to whether a given illustration was that of a function or not. A correct answer received a score of 1 and an incorrect or blank answer received a score of 0. The second analysis selected data from Part (b) of the questions which asked students to explain how they decided the answer to Part (a). The data selected in Part (b) consisted of student scores based on the Explanation Rubric (Appendix A). The third analysis selected the combined total of Parts (a) and (b).
Descriptive Statistics of Measured Person and Measured Item for Yes/No Items

Summary Table 3 shows descriptive statistics for Part (a) of Items, Q1 to Q25, (Q1a – Q25a). The results shown report for 100 measured person a mean of -.11 logits and a standard deviation of .79 logits. The reliability measure was .63 for the students and the students’ measures range from -3.45 logits to 1.58 logits. The student with the lowest measure of -3.45 logits was 1.2 logits lower than the next two persons whose measures were -2.25 logits. The fourth lowest student measure was -1.9 logits. Without these four lowest outliers, all the remaining person measures were located within 2 logits below and 2 logits above the person mean, with the lowest at 1.9 logits below the mean and the highest at 1.89 logits above the mean.

The mean measure for item difficulty was arbitrarily set at 0 logits and the item difficulties ranged from -1.32 to 1.89 with a standard deviation of .83 logits.
Table 3

Descriptive Statistics of Yes/No Items

<table>
<thead>
<tr>
<th></th>
<th>TOTAL SCORE</th>
<th>RASCH MEASURE</th>
<th>INFIT MNSQ</th>
<th>OUTFIT MNSQ</th>
<th>ZSTD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MEAN</strong></td>
<td>12.1</td>
<td>-.11</td>
<td>1.00</td>
<td>1.01</td>
<td>.0</td>
</tr>
<tr>
<td><strong>S.D.</strong></td>
<td>3.7</td>
<td>.79</td>
<td>.18</td>
<td>.33</td>
<td>1.1</td>
</tr>
<tr>
<td><strong>MAX</strong></td>
<td>20.0</td>
<td>1.58</td>
<td>1.52</td>
<td>3.18</td>
<td>2.9</td>
</tr>
<tr>
<td><strong>MIN</strong></td>
<td>1.0</td>
<td>-3.45</td>
<td>.62</td>
<td>.37</td>
<td>-2.6</td>
</tr>
<tr>
<td><strong>REAL RMSE</strong></td>
<td>.48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TRUE SD</strong></td>
<td>.62</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>PERSON RELIABILTY</strong></td>
<td>.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>S.E. OF PERSON MEAN</strong></td>
<td>.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>TOTAL SCORE</th>
<th>RASCH MEASURE</th>
<th>INFIT MNSQ</th>
<th>OUTFIT MNSQ</th>
<th>ZSTD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MEAN</strong></td>
<td>48.5</td>
<td>0.00</td>
<td>1.00</td>
<td>1.01</td>
<td>.0</td>
</tr>
<tr>
<td><strong>S.D.</strong></td>
<td>16.7</td>
<td>.83</td>
<td>.08</td>
<td>.13</td>
<td>.9</td>
</tr>
<tr>
<td><strong>MAX</strong></td>
<td>75.0</td>
<td>1.89</td>
<td>1.13</td>
<td>1.39</td>
<td>1.6</td>
</tr>
<tr>
<td><strong>MIN</strong></td>
<td>14.0</td>
<td>-1.32</td>
<td>.86</td>
<td>.78</td>
<td>-1.8</td>
</tr>
<tr>
<td><strong>REAL RMSE</strong></td>
<td>.23</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TRUE SD</strong></td>
<td>.80</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SEPARATION</strong></td>
<td>3.46</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>ITEM RELIABILITY</strong></td>
<td>.92</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>MODEL RMSE</strong></td>
<td>.23</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TRUE SD</strong></td>
<td>.80</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SEPARATION</strong></td>
<td>3.52</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>ITEM RELIABILITY</strong></td>
<td>.93</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>S.E. OF ITEM MEAN</strong></td>
<td>.17</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A Variable Map of Q1a – Q25a, was produced and is shown in Figure 1. The map shows the item and person distributions on the common logit scale. Of the first twenty-five questions, the easiest ones were Q2a, which asked students to decide if an illustration was that of a function, and Q8a, which asked student to decide if $y = 2x - 36$ is an equation of a function. Q2a was 1.32 logits below the mean and Q8 was 1.20 logits below the mean. These two questions were within .12 logits of each other.

The most difficult items were Q5a, which asked students to decide if a piecewise graph was that of a function, and Q4a, which asked students to decide if a discrete graph was that of a function. Q5a was 1.89 logits above the mean and Q4a was 1.73 logits above the mean. These two items were .16 logits apart in difficulty.

The variable map shows that Q1a – Q25a, which asked students to decide if the representation was that of a function, was well targeted to the abilities of the students, i.e. the mean of item calibration was approximately equal to the mean of the students’ ability measure. Also, the spread of the items and the spread of the people along the scale were comparable and there were no large gaps between items where students are located.
Figure 1. Variable Map of Yes/No Items
A misfit analysis for Yes/No items, Q1a – Q25a, was conducted and reported in Table 4. The point measure correlations were positive and closely matched the expected values. The expected mean squared value is 1 and the outfit mean squares ranged from .78 to 1.39. The analysis indicated that the Yes/No items fit the Rasch model well.
Table 4

Item Misfit Order

ITEM STATISTICS: MISFIT ORDER

<table>
<thead>
<tr>
<th>ITEM</th>
<th>RASCH MEASURE</th>
<th>OUTFIT MNSQ</th>
<th>ZSTD</th>
<th>PTMEASURE-A CORR.</th>
<th>EXP</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.89</td>
<td>1.39</td>
<td>1.2</td>
<td>.21</td>
<td>.21</td>
</tr>
<tr>
<td>7</td>
<td>.75</td>
<td>1.17</td>
<td>1.1</td>
<td>.14</td>
<td>.29</td>
</tr>
<tr>
<td>11</td>
<td>-.64</td>
<td>1.17</td>
<td>1.6</td>
<td>.18</td>
<td>.33</td>
</tr>
<tr>
<td>2</td>
<td>-1.32</td>
<td>1.13</td>
<td>.8</td>
<td>.18</td>
<td>.32</td>
</tr>
<tr>
<td>17</td>
<td>.04</td>
<td>1.13</td>
<td>1.3</td>
<td>.18</td>
<td>.32</td>
</tr>
<tr>
<td>10</td>
<td>.32</td>
<td>1.12</td>
<td>1.1</td>
<td>.19</td>
<td>.31</td>
</tr>
<tr>
<td>19</td>
<td>.18</td>
<td>1.09</td>
<td>.9</td>
<td>.18</td>
<td>.31</td>
</tr>
<tr>
<td>12</td>
<td>.75</td>
<td>1.04</td>
<td>.3</td>
<td>.20</td>
<td>.29</td>
</tr>
<tr>
<td>6</td>
<td>.09</td>
<td>1.06</td>
<td>.6</td>
<td>.28</td>
<td>.32</td>
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<td>3</td>
<td>1.18</td>
<td>1.05</td>
<td>.3</td>
<td>.23</td>
<td>.26</td>
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<td>1</td>
<td>-.45</td>
<td>1.01</td>
<td>1</td>
<td>.30</td>
<td>.33</td>
</tr>
<tr>
<td>8</td>
<td>-1.20</td>
<td>1.03</td>
<td>.2</td>
<td>.39</td>
<td>.33</td>
</tr>
<tr>
<td>9</td>
<td>-.36</td>
<td>.99</td>
<td>.0</td>
<td>.32</td>
<td>.33</td>
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<tr>
<td>21</td>
<td>.18</td>
<td>1.00</td>
<td>.0</td>
<td>.30</td>
<td>.31</td>
</tr>
<tr>
<td>20</td>
<td>-.83</td>
<td>.99</td>
<td>-.1</td>
<td>.33</td>
<td>.33</td>
</tr>
<tr>
<td>24</td>
<td>.96</td>
<td>.97</td>
<td>-.1</td>
<td>.32</td>
<td>.28</td>
</tr>
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<td>16</td>
<td>-.78</td>
<td>.90</td>
<td>-.9</td>
<td>.40</td>
<td>.33</td>
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<tr>
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<td>-.09</td>
<td>.94</td>
<td>-.7</td>
<td>.38</td>
<td>.32</td>
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<td>4</td>
<td>1.73</td>
<td>.83</td>
<td>-.5</td>
<td>.32</td>
<td>.23</td>
</tr>
<tr>
<td>18</td>
<td>-.73</td>
<td>.92</td>
<td>-.7</td>
<td>.41</td>
<td>.33</td>
</tr>
<tr>
<td>22</td>
<td>.27</td>
<td>.87</td>
<td>-1.2</td>
<td>.42</td>
<td>.31</td>
</tr>
<tr>
<td>13</td>
<td>-.31</td>
<td>.85</td>
<td>-1.6</td>
<td>.45</td>
<td>.32</td>
</tr>
<tr>
<td>15</td>
<td>-.50</td>
<td>.87</td>
<td>-1.3</td>
<td>.45</td>
<td>.33</td>
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<tr>
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<td>-.9</td>
<td>.42</td>
<td>.32</td>
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<tr>
<td>14</td>
<td>-.93</td>
<td>.78</td>
<td>-1.8</td>
<td>.51</td>
<td>.33</td>
</tr>
</tbody>
</table>
Descriptive Statistics of Measured Person and Measured Item for Explanation Items

Summary Table 5 shows descriptive statistics for student explanations, Part (b), of Items 1 to 25 (Q1b – Q25b). The results shown report for 100 measured person a mean of -1.95 logits and a standard deviation of 1.28 logits. The reliability measure was .87 for the students and the students’ measures ranged from -5.01 logits to -.12 logits. The mean measure for item difficulty was arbitrarily set at 0 logits and the item difficulties ranged from -1.46 to 2.79 with a standard deviation of .99 logits.
Table 5

*Descriptive Statistics of Explanations*

### SUMMARY OF 100 MEASURED PERSON

<table>
<thead>
<tr>
<th></th>
<th>TOTAL SCORE</th>
<th>RASCH MEASURE</th>
<th>INFIT MNSQ</th>
<th>OUTFIT MNSQ</th>
<th>ZSTD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MEAN</strong></td>
<td>16.5</td>
<td>-1.95</td>
<td>1.03</td>
<td>1.05</td>
<td>.0</td>
</tr>
<tr>
<td><strong>S.D.</strong></td>
<td>9.7</td>
<td>1.28</td>
<td>.36</td>
<td>.53</td>
<td>1.2</td>
</tr>
<tr>
<td><strong>MAX</strong></td>
<td>36.0</td>
<td>-.12</td>
<td>1.94</td>
<td>4.27</td>
<td>2.8</td>
</tr>
<tr>
<td><strong>MIN</strong></td>
<td>1.0</td>
<td>-5.01</td>
<td>.26</td>
<td>.25</td>
<td>-3.7</td>
</tr>
</tbody>
</table>

REAL RMSE .46  TRUE SD 1.19  SEPARATION 2.59  ITEM RELIABILITY .87
MODEL RMSE .49  TRUE SD 1.20  SEPARATION 2.79  ITEM RELIABILITY .89
S.E. OF ITEM MEAN = .13

### SUMMARY OF 25 MEASURED ITEM

<table>
<thead>
<tr>
<th></th>
<th>TOTAL SCORE</th>
<th>RASCH MEASURE</th>
<th>INFIT MNSQ</th>
<th>OUTFIT MNSQ</th>
<th>ZSTD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MEAN</strong></td>
<td>62.1</td>
<td>0.00</td>
<td>.98</td>
<td>1.05</td>
<td>.1</td>
</tr>
<tr>
<td><strong>S.D.</strong></td>
<td>26.9</td>
<td>.99</td>
<td>.32</td>
<td>.41</td>
<td>1.6</td>
</tr>
<tr>
<td><strong>MAX</strong></td>
<td>119.0</td>
<td>2.79</td>
<td>1.83</td>
<td>2.22</td>
<td>4.0</td>
</tr>
<tr>
<td><strong>MIN</strong></td>
<td>6.0</td>
<td>-1.46</td>
<td>.45</td>
<td>.54</td>
<td>-2.5</td>
</tr>
</tbody>
</table>

REAL RMSE .21  TRUE SD .97  SEPARATION 4.67  ITEM RELIABILITY .96
MODEL RMSE .20  TRUE SD .97  SEPARATION 4.91  ITEM RELIABILITY .96
S.E. OF ITEM MEAN = .20
A Variable Map for Explanations, for Part (b) of Items Q1 – Q25 (Q1b – Q25b), was produced and is shown in Figure 2. The figure displays the item and person distributions on the common logit scale. The most difficult items for Part (b) were Q7b, which asked students to explain how they decided if a step graph was that of a function, and Q3b, which asked students to explain how they decided if a polynomial graph was that of a function. Q7b was approximately 2.75 logits above the item mean, and Q3b was about 2.5 logits above the item mean.

The item with the least difficulty measure is Q15b, which asked students to explain how they decided if a table of $x, y$ values represented a function. This item had the least difficulty measure, and is located 1.5 logits above the students’ ability mean.

The highest person ability measures is located at -.12 logits below the item difficulty mean. The variable map measures support that the written explanations requested in Part (b) of the survey items were not well targeted to the abilities of the students; all of the item difficulty measures are located above the mean ability measure of students.
Figure 2. Variable Map of Explanations
Responses to Q1b – Q25b, to explain how the decision of whether or not the given representation was that of a function or non-function suggested inconsistency and a variety of views. Examples of explanations given are: (1) yes, positive slope; (2) no, there is nothing on the y axis all I see is the x axis on negative two; (3) yes, you put an equation into a graph and got results of a line out; (4) yes, it has an x and y value; (5) yes, you could draw a line through it, & it hits only 1 time; (6) yes, because x doesn’t repeat; (7) yes, it crosses a number on the x-axis; (8) yes, the line crosses both the y and x; and (9) no, straight lines cannot form functions. Explanations were scored using the Explanation Rubric (Appendix A) and the average score was 1.5. According to the criteria in the rubric, this score suggests that the following characteristics may have been included in the responses: some explanation, with some or minimal mathematical basis; none or some connection between sets; and/or one word explanations based on some mathematical basis.

Percent Comparison of Yes/No Items

The items were further analyzed in total and by types using percent correct/incorrect. There were seven types of functions and representations in the survey: graphs, equations, ordered pairs, tables, statements, arrow diagrams, and arbitrary mappings. The percent correct/incorrect provided data to compare students’ ability to relate the definition to familiar representations within types and as a whole.
Figure 3 shows the percent of students who responded correctly and incorrectly to Part (a) of the first type of questions, which involved illustrations of graphs. There were seven graph type questions: (Q1a) hyperbola, (Q2a) linear, (Q3a) polynomial of degree higher than 1, (Q4a) discrete, (Q5a) piecewise, and (Q6a) step functions. The question, within this group type, with the most correct responses was Q2a; in this question, 75% of students were able to identify a linear graph as that of a function. Q1a had the next highest percent of correct responses in this group type. In this question, 58% of students correctly decided that the graph of a hyperbola did not represent a function. Students had the most difficulty with Q5a, deciding if a piecewise graph represents a function, and Q4a, deciding if a discrete graph represents a function. Q5a received 14% correct responses and Q4a received 16% correct responses.
Figure 3. Graphs of Functions/Non-functions

Figure 4 displays the percent of students who responded correctly and incorrectly to Part (a) of the second type of questions, equations. There were five of this type questions and they asked students to decide if given equations described functions. The equations included (Q8a) variables $x$ and $y$, (Q9a) variable $x$ only, (Q10a) variable $y$ only, (Q11a) notation $f(x)$, and (Q12a) variables $z$, $t$, a fraction, and a decimal. For the equations section, the highest correct responses, 73%, were for Q8a, in which the given equation was $y = 2x - 36$. The most difficult question, with 32% correct responses, was Q12a. The equation given in this question was $z = \frac{3}{4}t + 24.72$. 

50
The next type of questions on the survey involved ordered pairs. As shown in Figure 5, there were two questions of this type, Q13a – Q14a. The percent correct and incorrect responses are also shown in Figure 5. Both questions had a set of five ordered pairs. The pairs considered in Q13a, contained a first element that was paired with two different second elements; and 55% of students decided correctly that the ordered pairs did not represent a function. In Q14a, the first elements were uniquely paired with second elements; and in Q14a, 68% of the students were able to determine that the pairs represented a function.

![Distinguish between equation of functions/ non-functions](image)

**Figure 4. Equations of Functions/Non-functions**

<table>
<thead>
<tr>
<th>Question</th>
<th>Correct</th>
<th>Not Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q13</td>
<td>41%</td>
<td>59%</td>
</tr>
<tr>
<td>Q14</td>
<td>38%</td>
<td>62%</td>
</tr>
<tr>
<td>Q15</td>
<td>32%</td>
<td>68%</td>
</tr>
<tr>
<td>Q16</td>
<td>27%</td>
<td>73%</td>
</tr>
<tr>
<td>Q17</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td>Q18</td>
<td>49%</td>
<td>51%</td>
</tr>
</tbody>
</table>

The bar chart in Figure 4 illustrates the distribution of correct and incorrect responses for questions related to distinguishing between equations of functions and non-functions.
The next two questions, Q15a and Q16a, on the survey contained values in table form, and the correct/incorrect responses are reported in Figure 6. Both questions gave values for $x$ and $y$ in a T-table format. The T-table in Q15a had the same $x$ values assigned to different $y$ values. The T-table in Q16a had unique assignments of $x$ values to $y$ values. The T-table in Q16a had $x$ values from $x = -1$ to $x = 4$; these values are typically included in textbook T-tables; the $x$ values in Q15a were not sequential, and not commonly seen this way.

In Q16a, more students, 65%, were able to identify the table as representing a function, and in Q15a, less students, 59%, were able to identify the table as not representing a function.
In the next three questions of the survey, Q16a – Q18a, students were asked to decide if given statements represented a function or not. The percent of correct and incorrect responses are shown in Figure 7.

The statements in Q17a and Q18a both contained the terms input and output, which are familiar terms, typically encountered in Algebra classes, or in lower math courses. In Q18a, the statement was that an input is assigned to three different outputs, and 64% of students correctly decided that this statement did not describe a function. The statement in Q17a was that an output was four more than an input, and 47% of students correctly responded that this statement described a function. The statement in Q19a involved giving a capital city when
given a state, and 44% of students correctly decided that this statement described a function.

\[ \text{Figure 7. Statements Describing Functions/Non-functions} \]

In the next three questions of the survey, Q20a – Q22a, students were asked to decide if illustrations which used arrows to map one set to another set represented a function. Arrow diagrams are considered familiar type mappings which students typically begin to encounter in early grades. The domain and co-domain sets included integers and an alpha character. The percent of correct and incorrect responses for these items are shown in Figure 8.

All three questions in this type of illustration had four elements in the first set and four elements in the second set. The second sets in Q20a and Q21a,
were composed of numbers only. In Q20a, an element in the first set was mapped to two elements in the second set, and 66% of students correctly decided that Q20a did not represent a function. Q21a was a function representation, but not one-to-one or onto, and 44% of students decided correctly that this was a function representation. Q22a had three numbers and one alpha character in the second set, and this function representation was both one-to-one and onto. 42% of students correctly decided this was a function representation.

![Distinguish between arrow diagrams of functions/non-functions](image)

**Figure 8. Arrow Diagrams Representing Functions/Non-functions**

The last type of representations, in Q23a – Q25a, were composed of arbitrary sets with arbitrary mappings. The elements in these sets were animals,
a tree, and chairs. While these elements are familiar, the mappings from the first to the second elements were arbitrary, and did not necessarily reflect typical everyday relationships between the sets’ elements. The percent correct and incorrect are shown in Figure 9. The mapping in Q23a was one-to-one and onto, and 52% of students correctly decided that this was a function representation. The mapping in Q24a was onto, but not one-to-one, and 28% of students correctly decided that this was a function representation. In Q25a, elements in the first set were mapped to similar elements in the second set, however, one element in the first set was mapped to two different elements in the second set, and 50% of students correctly responded that this was not a function representation.

**Figure 9. Arbitrary Mappings Representing Functions/Non-functions**

![Distinguish between arbitrary set mapping of functions/non-functions](chart)
Figure 10 displays the responses to Q1a – Q25a in one graph. Overall, Q2a received the most, 75%, correct responses. In Q2a students were given the graph of a linear function. The graph of a linear function is typically used to relate the definition of function to a representation when the function definition is introduced in Algebra classes. The next highest, 73%, correct responses were for Q8a and in this question, students were asked if a linear equation, $y = 2x - 36$, was that of a function. Students often encounter this type of equation in Algebra courses.

The questions with the least correct responses overall were Q5a, with 14% correct responses, and Q4a, with 16% correct responses. Both of these questions involved graphical representations of functions that are typically unfamiliar to Algebra students; in Q5a, a piecewise function graph is given, and in Q4a, a discrete graph is illustrated. In Q5a, a piecewise function graph is given, and in Q4a, a discrete graph is illustrated. The mean for correct response for the twenty five Yes/No items was 48.5%.
### Figure 10. Function/Non-function Representations – All 25 Questions

![Bar chart showing the distribution of correct and not correct responses for each of the 25 questions.](image_url)
Definition of Function

Q26 asked students, “What is a function?” Responses were inconsistent and reflected a variety of views. The data did not fit the function descriptors (Appendix B) that had been created for exploring students’ thoughts about the definition of function. These are some examples of responses given to Q26: (1) a rule for a domain and range, (2) one input has exactly one output, (3) a group of numbers that you can graph, (4) when a number repeats in the x-int, (5) when you factor a number into an answer, (6) a graph line that crosses an x and y axis, (7) a formula used to solve your question, (8) an equation that does not go over twice on its range; and (9) something that works.

A modified list of function descriptors was created and used to categorize responses to Q26. The modified list includes a key idea or word from some of the original function descriptors (Appendix B). Using the modified list of descriptors, 75% of the responses were categorized and the results are shown in Table 6.

25% of the responses were not included in Table 6 because the responses were either blank or stated, “I don’t know.”
Table 6

*Number of Responses – Key Words Function Descriptors*

<table>
<thead>
<tr>
<th>Key Words/Ideas Function Descriptors</th>
<th>Number of responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formula</td>
<td>4</td>
</tr>
<tr>
<td>Equation</td>
<td>18</td>
</tr>
<tr>
<td>Process</td>
<td>8</td>
</tr>
<tr>
<td>Graph</td>
<td>14</td>
</tr>
<tr>
<td>Rule</td>
<td>23</td>
</tr>
<tr>
<td>Problem</td>
<td>6</td>
</tr>
<tr>
<td>Total Responses</td>
<td>73</td>
</tr>
</tbody>
</table>

Figure 11 shows a bar graph generated from the information in Table 6. The descriptor counts were highest for rule, equation, and graph. The lowest counts were for process, problem, and formula.

*Figure 11. Number of Responses – Key Words Function Descriptors*
Most/Least Correct Responses to Yes/No Items

Table 7 includes the four items receiving the most correct responses, and the four items receiving the least correct responses, to Part (a) of Q1a – Q25a. The representations and percent of correct responses for each item are shown in Table 7. The top four items Q2a, Q8a, Q14a, and Q25a, are those that received the most correct responses and the bottom four items, Q24a, Q3a, Q4a, and Q5a, are those that received the least correct responses. The top four items include representation types that are most frequently encountered in Algebra classrooms and textbooks, i.e. familiar representations. These familiar representations include: Q2a, a linear function; Q8a, a simple equation in terms of x and y; Q14a, a set of ordered pairs, and Q20a, an arrow mapping with numeric elements.

The bottom four items, Q24a, Q3a, Q4a, and Q20a, are the items that received the least correct responses. Two of these four representations, Q4a and Q5a, are graphs that are unlikely to be encountered in Algebra 1 classrooms, i.e. unfamiliar representations of functions. The other two items in the bottom list, Q24a and Q3a, are also categorized as unfamiliar. In Q24a, the mapping is arbitrary without regard to everyday relationships between elements; and in Q3a, the graph is not linear or quadratic.

Seven of the eight questions in the table illustrate functions or representations of functions; the correct answer to each of these questions, was
Yes. The illustration in one item, Q20, does not illustrate a function; and the correct answer to this question, was No.
Table 7.

Most/Least Correct Responses to Yes/No Items

<table>
<thead>
<tr>
<th>Item Part (a)</th>
<th>Question</th>
<th>Representation</th>
<th>% Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>Does this graph represent ( y ) as a function of ( x )?</td>
<td><img src="image" alt="Graph" /></td>
<td>75%</td>
</tr>
<tr>
<td>8.</td>
<td>Is ( y = 2x - 36 ) the equation of a function?</td>
<td></td>
<td>73%</td>
</tr>
<tr>
<td>14.</td>
<td>Does this set represent a function?</td>
<td>( {(3, 2), (5, 4), (7, -8), (8, 10), (9, 4)} )</td>
<td>68%</td>
</tr>
<tr>
<td>20.</td>
<td>Does the following diagram illustrate a function?</td>
<td><img src="image" alt="Diagram" /></td>
<td>66%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Item Part (a)</th>
<th>Question</th>
<th>Representation</th>
<th>% Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.</td>
<td>Does this graph represent ( y ) as a function of ( x )?</td>
<td><img src="image" alt="Graph" /></td>
<td>24%</td>
</tr>
<tr>
<td>4.</td>
<td>Does this graph represent ( y ) as a function of ( x )?</td>
<td><img src="image" alt="Graph" /></td>
<td>16%</td>
</tr>
<tr>
<td>5.</td>
<td>Does this graph represent ( y ) as a function of ( x )?</td>
<td><img src="image" alt="Graph" /></td>
<td>14%</td>
</tr>
</tbody>
</table>
CHAPTER FIVE

DISCUSSIONS AND CONCLUSIONS

This study provided a snap-shot of the abilities of a group of students to relate the definition of function to its representations by answering questions on a math survey. The students were enrolled in one teacher’s classes and the survey was taken near the end of the school year. The results may have been impacted by a variety of factors, such as students’ knowing that the survey bore no weight on their class grade or that the students were not familiar with the researcher. In general, students tend to perform better when they know that an activity will impact their grade or when they have a rapport with the teacher. And, it is a challenge even in one’s own classroom, to maintain students’ enthusiasm and serious engagement in activities during the last month of school. Nevertheless, the survey results provided important information about how students in a typical classroom with similar geographic settings, may relate a function definition to it to representations.

Familiar Representations

It may seem logical that if students do not know the definition of function then they will not be able to distinguish between representations of functions and non-functions. In this study, findings support the belief that this may not necessarily occur. Findings revealed that students were not able to provide an
adequate definition of function, yet some items in Table 7 received a high percent of correct responses. What might account for the ability of 75% of students to answer Q2 correctly, and 73% to answer Q8 correctly? And on the opposite end, that only 14% answered Q5 correctly and only 16% answered Q4 correctly? A conjecture is that students relied on memory of previously seen examples to decide. Students typically encounter graphical representations of functions when they study the definition of function; the most familiar are graphs of linear or quadratic functions. The item receiving the most correct responses and the three receiving the least correct responses were graphs; and students typically are most familiar with the graph of a linear function. A conjecture is that students may have a strong memory of a linear graph so that perhaps any graph that is not similar to it may be deemed a non-function representation. Findings shown in Table 7 support that belief.

Function Definition and Its Relation to its Representations

Findings suggest that students may have a concept image of function based on what DeMarois (1996) described as a mixture of disconnected procedures that have been memorized with limited understanding about connections between the facets of the definition. Research suggests that this may occur when the formal definition of function is introduced but not referenced frequently, when there are limited opportunities to compare examples with non-examples, and/or when there are limited discussions about alternative definitions.
In these cases, the stipulated mathematical definition may take on properties of an extracted definition and a personal definition evolves and reports usage based on individual observations, interpretations, and experiences. Some aspects of the definition that are developed in this manner may be mathematically correct; however, typically, the definition held is not complete, clear, and precise. For example, students may have had earlier experiences with the following notions and hold any of these, or a combination of these, to be the definition of function: (1) a function machine where an action takes a value and churns it into a different value; (2) plotted points that connect to make a familiar continuous graph; (3) an equation; (4) a rule; (5) a formula; or (6) a table of values with a sequential domain such as \([-5, 5]\).

Another possible cause that may have contributed to students’ inability to provide a complete, clear, and precise definition of function is a general inability to express oneself clearly orally or in writing.

It is interesting to note that approximately 73% of the students’ responses to Q26, included at least one of the key words or ideas in the modified descriptor list in Table 6. The bar chart in Figure 10 shows that the most common term that students associated with the definition of function is “rule” and the next most common word is “equation.” Rule is a common word frequently used in everyday language; equation and graph are words that are frequently used in math classes. These findings suggest that participants in the study, may have acquired some anchor words for the concept of function. Gaining understanding of how
students learn may provide guidance on how to develop from the anchor words, a deeper understanding of the roles and features of the definition of function and the unique relationship between domains and co-domains.

One goal of this study was to compare students’ ability to distinguish between function and non-function representations. As discussed above, findings suggest that the most familiar representations received the most correct responses and the least familiar representations received the least correct responses.

Another goal of this study was to explore the extent to which students are able to distinguish between representations of functions and non-functions. The findings for this goal revealed a mean person ability approximately equal to the item difficulty mean. Findings suggest that the students’ ability to distinguish between representations of functions and non-functions was well matched to the difficulty of the items.

Another goal of this study was to explore the extent to which students apply the definition of function to verify representations of function. Findings in this study suggest that students rely more on memorized previously seen examples of representations, rather than utilizing knowledge about the definition of function, to verify representations of functions.

The last goal of this study was to explore the extent to which students are able to provide a complete, clear, and precise definition of function. Findings in this study support the belief that at this point in the study of functions and their
representations, students struggled to provide a complete, clear, and precise definition of function. These findings support what Edwards & Ward (2008) found in their study that students struggle with stipulated definitions.

Conclusion

This paper examined students’ ability to relate the definition of function to its representations and it explored how students distinguish between representations of functions and non-functions. This paper also discussed research related to how students learn the concept of function and its definition.

Findings from this study suggest that students struggle with understanding and grasping the richness and power of the function definition, and that students have limited ability, especially in unfamiliar contexts, to relate the definition of function to its representations. It is suggested that further explorations that include student interviews and groups of participants from multiple teachers’ classes, may provide increased knowledge and understanding about how students learn the definition of function and how they relate it to its representations. Such pursuits may also effectuate invaluable guidance to help close the gap between those students who “get it” and those who “don’t get it.”
APPENDIX A

EXPLANATION RUBRIC
## EXPLANATION RUBRIC

**Connect Explanation to Function Definition**

<table>
<thead>
<tr>
<th>Score</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definition</td>
<td>No explanation is given or explanation given has no relevant mathematical basis.</td>
<td>Some explanation is given with minimal relevant mathematical basis. No connection between sets given.</td>
<td>Some explanation is given with some relevant mathematical basis. Some connection between sets included. One word explanation based on some mathematical basis involving set connection (rule, graph, etc.)</td>
<td>Adequate explanation is given with adequate relevant mathematical basis. Adequate connection between sets included</td>
</tr>
</tbody>
</table>
APPENDIX B

DESCRIPTORS
DESCRIPTORS

Function Descriptors

1. A rule or relationship between variables
2. A rule which can be expressed algebraically
3. An algebraic formula
4. An equation
5. A process which can change one number to another number
6. A graph
7. A rule which can be graphed
8. A vertical line test
9. A rule that is arbitrary
REFERENCES


http://web.ntnu.edu.tw/~699450216/shares/Zhong_Xiaolan/2011_11_23/%A6h%AD%AB%AA%ED%BCx%A4%E5%C4m/ainsworth.pdf.pdf


http://www.hoover.org/publications/policy-review


http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.46.5260&rep=rep1&type=pdf


JSTOR.
http://www.duq.edu/about/centers-and-institutes/center-for-teaching-excellence/teaching-and-learning/understanding-your-students

http://ase.tufts.edu/cogstud/jackendoff/papers/howdidlanguagebegin.pdf

Dr. Jesunathadas, College of Education, California State University, San Bernardino, CA.

Motivating Struggling Math Students. In *Educational Partnerships, Inc.*, Retrieved from
http://gearup.ous.edu/sites/default/files/Research-Briefs/
ResearchBriefMotivatingMathStudents.pdf


http://www2.ed.gov/pubs/NatAtRisk/risk.html


http://lib-srvc9.lib.plymouth.ac.uk:8080/pearl_xmlui/handle/10026.1/2947