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Symmetric Presentations and Generation

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SYMMETRIC PRESENTATIONS AND GENERATION

A Thesis

Presented to the

Faculty of

California State University,

San Bernardino

In Partial Fulfillment

of the Requirements for the Degree

Master of Arts

in

Mathematics

by

Dustin Grindstaff

June 2015

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ABSTRACT

The aim of this thesis is to generate original symmetric presentations for finite non-abelian simple groups. We will discuss many permutation progenitors, including $2^{*14} : D_{28}$, $2^{*9} : (3 \bullet (3^2))$, $3^{*9} : (3 \bullet (3^2))$, $2^{*21} : ((7 \times 3) : 2)$ as well as monomial progenitors, including $7^{*5} :_m A_5$, $3^{*5} :_m S_5$. Their homomorphic images include the sporadic Mathieu groups M_{11} and M_{12} , the sporadic Janko group $3 \bullet J_2$, the Symplectic group $2 \times S(4, 5)$, as well as, many linear and alternating groups. We will give proofs of the isomorphism types of each progenitor, either by hand using double coset enumeration or using MAGMA. We have also constructed Cayley diagrams of the following groups, $2^5 : S_5$ over S_5 , $PGL(3, 4)$ over M_{10} , $PSL(2, 8)$ over D_{14} and M_{12} over the maximal subgroup $2 \times S_5$. We have developed a lemma using relations to factor permutation progenitors of the form $m^{*n} : N$ to give the group $m^n : N$. We will present a program written in MAGMA that, when given a target finite non-abelian simple group, generates possible progenitors that will give the target simple group. Iwasawa's lemma is also discussed and used to show $PSL(2, 8)$ and M_{12} are simple groups.

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Chapter 1

Introduction

The goal of this thesis is to find homomorphic images of finite non-abelian simple groups. We do this through the use of a progenitor, $m^{*n} : N$, where N is transitive on n letters. To find finite homomorphic images, we factor the progenitor by appropriate relations of the form $\pi w(t_1, \dots, t_n)$, where $\pi \in N$ and w is a word in the symmetric generators. This will give us finite homomorphic images of the infinite group $m^{*n} : N$. It takes immense effort to find suitable relations to factor the progenitor by. Through the course of our research, we have developed a lemma for writing relations that can be used to factor $m^{*n} : N$ to give the group $m^n : N$. Of our own means and along with two previously developed methods, we will factor these progenitors by suitable relations to generate homomorphic images.

We can then determine the isomorphism type by solving the extension problem using the groups composition factors. Through the process of double coset enumeration, we can construct a graphical representation of the group, called a Cayley diagram. We can then prove the order of the group using the Cayley diagram and the First Isomorphism Theorem.

During our research, the question was posed that if we are given a target finite non-abelian simple group, can we come up with control groups N , when written in the progenitor $2^{*n} : N$, can give the target simple group. Using a theorem proved by Robert T. Curtis and expanding on it, we were able to develop an algorithm and prove its validity. We implemented this algorithm in MAGMA. This program generates possible progenitors that will give a target non-abelian simple group when factored by relations.

Chapter 2

Writing Progenitors

2.1 Related Theorems and Definitions

Definition 2.1. Operation

Let G be a set. A (binary) **operation** on G is a function that assigns each ordered pair of elements of G an element on G . [Rot95]

Definition 2.2. Semigroup

A **semigroup** $(G, *)$ is a nonempty set G equipped with an associative operation $*$. [Rot95]

Definition 2.3. Group

A **group** is a semigroup G containing an element e such that

$$(i) \ e * a = a = a * e \text{ for all } a \in G$$

$$(ii) \text{ for every } a \in G, \text{ there is an element } b \in G \text{ with } a * b = e = b * a. [\text{Rot95}]$$

Definition 2.4. Free Group

If X is a nonempty subset of a group F , then F is a **free group** with basis X if, for every group G and every function $f : X \rightarrow G$, there exists a unique homomorphism $\phi : F \rightarrow G$ extending f . Moreover, X generates F . [Rot95]

Definition 2.5. Presentation

Let X be a set and let Δ be a family of words on X . A group G has **generators** X and **relations** Δ if $G \cong F/R$, where F is the free group with basis X and R is the normal subgroup of F generated by Δ . The ordered pair $(X|\Delta)$ is called a **presentation** of G . [Rot95]

Definition 2.6. Progenitor

Let G be a group and $T = \{t_1, t_2, \dots, t_n\}$ be a symmetric generating set for G with $|t_i| = m$. Then if $N = N_G(\bar{T})$, then we define the **progenitor** to be the semi direct product $m^{*n} : N$, where m^{*n} is the free product of n copies of the cyclic group C_m . [Cur07]

Definition 2.7. Character

Let $A(x) = (a_{ij}(x))$ be a matrix representation of G of degree m . We consider the characteristic polynomial of $A(x)$, namely

$$\det(\lambda I - A(x)) = \begin{pmatrix} \lambda - a_{11}(x) & -a_{12}(x) & \cdots & -a_{1m}(x) \\ \lambda - a_{11}(x) & -a_{12}(x) & \cdots & -a_{1m}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \lambda - a_{m1}(x) & -a_{m2}(x) & \cdots & \lambda - a_{mm}(x) \end{pmatrix}$$

This is a polynomial of degree m in λ , and inspection shows that the coefficient of $-\lambda^{m-1}$ is equal to

$$\phi(x) = a_{11}(x) + a_{22}(x) + \dots + a_{mm}(x)$$

It is customary to call the right-hand side of this equation the trace of $A(x)$, abbreviated to $\text{tr}A(x)$, so that

$$\phi(x) = \text{tr}A(x)$$

We regard $\phi(x)$ as a function on G with values in K , and we call it the **character** of $A(x)$. [Led87]

Theorem 2.8. The number of irreducible character of G is equal to the number of conjugacy classes of G . [Led87]

Definition 2.9. Degree of a Character

The sum of squares of the degrees of the distinct irreducible characters of G is equal to $|G|$. The **degree of a character** χ is $\chi(1)$. Note that a character whose degree is 1 is called a linear character. [Led87]

Definition 2.10. Lifting Process

Let N be a normal subgroup of G and suppose that $A_0(Nx)$ is a representation of degree m of the group G/N . Then $A(x) = A_0(Nx)$ defines a representation of G/N lifted from G/N . If $\phi_0(Nx)$ is a character of $A_0(Nx)$, then $\phi(x) = \phi_0(Nx)$ is the lifted character of $A(x)$. Also, if $u \in N$, then $A(u) = I_m$, $\phi(u) = m = \phi(1)$. The lifting process preserves irreducibility. [Led87]

Definition 2.11. Induced Character

Let $H \leq G$ and $\phi(u)$ be a character of H and define $\phi(x) = 0$ if $x \in H$, then

$$\phi^G(x) = \begin{cases} \phi(x), & x \in H \\ 0, & x \notin H \end{cases}$$

is an induced character of G . [Led87]

Definition 2.12. Formula for Induced Character

Let G be a finite group and H be a subgroup such that $[G : H] = n$. Let C_α , $\alpha = 1, 2, \dots, m$ be the conjugacy classes of G with $|C_\alpha| = h_\alpha$, $\alpha = 1, 2, \dots, m$. Let ϕ be a character of H and ϕ^G be the character of G induced from the character ϕ of H up to G . The values of ϕ^G on the m classes of G are given by:

$$\phi_\alpha^G = \frac{n}{h_\alpha} \sum_{w \in C_\alpha \cap H} \phi(w), \alpha = 1, 2, 3, \dots, m.$$

[Led87]

2.2 Permutation Progenitor Examples

2.2.1 $2^{*5} : S_5$

We want write the progenitor $2^{*5} : S_5$. Here, our control group N is S_5 .

Therefore we write a presentation for S_5 given by,

$$S_5 = \langle x, y \mid x^5, y^2, (x * y)^4, (x, y)^3 \rangle.$$

We can quickly check this presentation in MAGMA with the following code.

```
> G<x,y>:=Group<x,y|x^5,y^2,(x*y)^4,(x,y)^3>;
> f,G1,k:=CosetAction(G,sub<G|Id(G)>);
> s,t:=IsIsomorphic(G1,Sym(5));
> s;
true
```

We need to add the free product 2^{*5} to this group to form our progenitor. Since our progenitor has 2^{*5} , we also know that we have 5 t 's of order 2. We will add a t of order 2 and let t commute with the point stabilizer of 1 in N . This step will label our t as t_1 and ensure that t_1 has exactly 5 conjugates under conjugation by N . To determine the stabilizer of 1 in N , we need to represent N with permutations. In this case, we will use $N = \langle x, y \rangle$, where $x = (1, 2, 3, 4, 5)$ and $y = (1, 2)$.

With the help of MAGMA, we can see that the stabilizer of 1 in N is equal to $\langle (2, 3), (3, 4), (4, 5) \rangle$. Representing these permutations as words, we have $y^x = (2, 3)$, $y^{x^2} = (3, 4)$, $y^{x^3} = (4, 5)$,

therefore our finished progenitor follows.

```
G<x,y,t>:=Group<x,y,t|x^5,y^2,(x*y)^4,(x,y)^3,t^2,(t,y^x),(t,y^x^2),
(t,y^x^3)>
```

2.2.2 $3^{*9} : 3^{\bullet}(3^2)$

Our control group $N = 3^{\bullet}(3^2)$ and $t'_i s$ are of order 3. We write a presentation for N and check in MAGMA.

```
> G<a,b,c>:=Group<a,b,c|a^3,b^3,(a,b)=c,c^3,(a,c),(b,c)>;
> #G;
27
```

Note: This presentation has been checked by solving the extension problem discussed in chapter 4. This example is given in section 4.2.3.

Now that we have a presentation for our control group, we just need to add a t of order 3 and let t commute with the one point stabilizer in N . The generators of N in this case with the stabilizer of 1 in N are given in the following MAGMA code. The SchreierSystem code is used to change the permutations in the stabilizer into words in the generators of N .

```
> S:=Sym(9);
> aa:=S!(1,3,8)(2,4,5)(6,7,9);
> bb:=S!(1,2,5)(3,7,8)(4,6,9);
> cc:=S!(1,4,7)(2,6,8)(3,5,9);
> N:=sub<S|aa,bb,cc>;
> #N;
27
> Sch:=SchreierSystem(G,sub<G|Id(G)>);
> ArrayP:=[Id(N): i in [1..#N]];
> for i in [2..#N] do
for> P:=[Id(N): l in [1..#Sch[i]]];
for> for j in [1..#Sch[i]] do
for|for> if Eltseq(Sch[i])[j] eq 1 then P[j]:=aa; end if;
for|for> if Eltseq(Sch[i])[j] eq 2 then P[j]:=bb; end if;
for|for> if Eltseq(Sch[i])[j] eq 3 then P[j]:=cc; end if;
for|for> if Eltseq(Sch[i])[j] eq -1 then P[j]:=aa^-1; end if;
for|for> if Eltseq(Sch[i])[j] eq -2 then P[j]:=bb^-1; end if;
for|for> if Eltseq(Sch[i])[j] eq -3 then P[j]:=cc^-1; end if;
for|for> end for;
for> PP:=Id(N);
for> for k in [1..#P] do
for|for> PP:=PP*P[k]; end for;
for> ArrayP[i]:=PP;
for> end for;
>
> Stabiliser(N,1);
```

```

Permutation group acting on a set of cardinality 9
Order = 3
      (2, 8, 6)(3, 5, 9)
> for i in [1..#N] do
for> if ArrayP[i] eq S!(2,8,6)(3,5,9) then Sch[i]; end if; end for;
a * b * c

```

Therefore, we let t commute with $a * b * c$ and the progenitor is complete. The finished progenitor for $3^{*9} : 3^{\bullet}(3^2)$ follows:

```

G<a,b,c,t>:=Group<a,b,c,t|a^3,b^3,(a,b)=c,c^3,(a,c),(b,c),t^3,(t,a*b*c)>;

```


2.3 Monomial Progenitors

2.3.1 $7^{*5} :_m A_5$

We will write a presentation for the monomial progenitor $7^{*5} :_m A_5$. Note: 7^{*5} represents 5 t 's of order 7. Let $G = A_5 = \langle (1, 2)(3, 4), (1, 3, 5) \rangle$. First we need to induce a non-trivial linear character from a subgroup H of G such that $\frac{|G|}{|H|} = 5$ (the number of t 's in our presentation). $|G| = 60$, therefore we must have $|H| = 12$.

Let $H = A_4 = \langle (1, 2)(3, 4), (1, 2, 3) \rangle$.

From MAGMA, we have the following character tables, with J representing the cube root of unity.

Table 2.1: Character Table of A_5

Conjugacy Classes	C_1	C_2	C_3	C_4	C_5
Order	1	2	3	5	5
χ_1	1	1	1	1	1
χ_2	3	-1	0	$Z1$	$Z1_2$
χ_3	3	-1	0	$Z1_2$	$Z1$
χ_4	4	0	1	-1	-1
χ_5	5	1	-1	0	0

Table 2.2: Character Table of A_4

Conjugacy Classes	C_1	C_2	C_3	C_4
Order	1	2	2	2
ϕ_1	1	1	1	1
ϕ_2	1	1	J	$-1 - J$
ϕ_3	1	1	$-1 - J$	J
ϕ_4	3	-1	0	0

Now, we must induce a non-trivial linear character of H , either χ_2 or χ_3 in this case. We will induce χ_2 up to G . Next, we find the right transversals of H in G . Using MAGMA, these are $\{e, (1, 3, 5), (1, 5, 3), (1, 4, 3, 5, 2), (1, 5, 4, 3, 2)\}$. Label these transversals as

$$T_1 = e$$

$$T_2 = (1, 3, 5)$$

$$T_3 = (1, 5, 3)$$

$$T_4 = (1, 4, 3, 5, 2)$$

$$T_5 = (1, 5, 4, 3, 2).$$

Now, the matrices $A(x)$ and $A(y)$ will be a representation of G induced from the character in H , χ_2 . The general forms for $A(x)$ and $A(y)$ are

$$A(x) = \begin{bmatrix} B(T_1 x T_1^{-1}) & B(T_1 x T_2^{-1}) & B(T_1 x T_3^{-1}) & B(T_1 x T_4^{-1}) & B(T_1 x T_5^{-1}) \\ B(T_2 x T_1^{-1}) & B(T_2 x T_2^{-1}) & B(T_2 x T_3^{-1}) & B(T_2 x T_4^{-1}) & B(T_2 x T_5^{-1}) \\ B(T_3 x T_1^{-1}) & B(T_3 x T_2^{-1}) & B(T_3 x T_3^{-1}) & B(T_3 x T_4^{-1}) & B(T_3 x T_5^{-1}) \\ B(T_4 x T_1^{-1}) & B(T_4 x T_2^{-1}) & B(T_4 x T_3^{-1}) & B(T_4 x T_4^{-1}) & B(T_4 x T_5^{-1}) \\ B(T_5 x T_1^{-1}) & B(T_5 x T_2^{-1}) & B(T_5 x T_3^{-1}) & B(T_5 x T_4^{-1}) & B(T_5 x T_5^{-1}) \end{bmatrix}$$

$$A(y) = \begin{bmatrix} B(T_1 y T_1^{-1}) & B(T_1 y T_2^{-1}) & B(T_1 y T_3^{-1}) & B(T_1 y T_4^{-1}) & B(T_1 y T_5^{-1}) \\ B(T_2 y T_1^{-1}) & B(T_2 y T_2^{-1}) & B(T_2 y T_3^{-1}) & B(T_2 y T_4^{-1}) & B(T_2 y T_5^{-1}) \\ B(T_3 y T_1^{-1}) & B(T_3 y T_2^{-1}) & B(T_3 y T_3^{-1}) & B(T_3 y T_4^{-1}) & B(T_3 y T_5^{-1}) \\ B(T_4 y T_1^{-1}) & B(T_4 y T_2^{-1}) & B(T_4 y T_3^{-1}) & B(T_4 y T_4^{-1}) & B(T_4 y T_5^{-1}) \\ B(T_5 y T_1^{-1}) & B(T_5 y T_2^{-1}) & B(T_5 y T_3^{-1}) & B(T_5 y T_4^{-1}) & B(T_5 y T_5^{-1}) \end{bmatrix}$$

where $B(x)$ is the value of the induced character for the class of H containing x , recall if $x \notin H$, then $B(x) = 0$.

Note: The cube roots of unity are represented as 1,2 and 4 from a field of order 7.

Now, $x = (1, 2)(3, 4)$ and $y = (1, 3, 5)$ and the classes of H are

Class 1 = e

Class 2 = (1,2)(3,4),(1,3)(2,4),(1,4)(2,3)

Class 3 = (1,3,4),(2,4,3),(1,4,2),(1,2,3)

Class 4 = (1,4,3),(2,3,4),(1,2,4),(1,3,2).

Therefore, we have

$$B(T_1 x T_1^{-1}) = B(e(1, 2)(3, 4)e) = B((1, 2)(3, 4)) = 1$$

$$B(T_1 x T_2^{-1}) = B(e(1, 2)(3, 4)(1, 5, 3)) = B((1, 2, 5, 3, 4)) = 0$$

$$B(T_1 x T_3^{-1}) = B(e(1, 2)(3, 4)(1, 3, 5)) = B((1, 2, 3, 4, 5)) = 0$$

$$B(T_1 x T_4^{-1}) = B(e(1, 2)(3, 4)(1, 2, 5, 3, 4)) = B((1, 5, 3)) = 0$$

$$B(T_1 x T_5^{-1}) = B(e(1, 2)(3, 4)(1, 2, 3, 4, 5)) = B((1, 3, 5)) = 0$$

$$\begin{aligned}
B(T_2 x T_1^{-1}) &= B((1, 3, 5)(1, 2)(3, 4)e) = B((1, 4, 3, 5, 2)) = 0 \\
B(T_2 x T_2^{-1}) &= B((1, 3, 5)(1, 2)(3, 4)(1, 5, 3)) = B((1, 4)(2, 5)) = 0 \\
B(T_2 x T_3^{-1}) &= B((1, 3, 5)(1, 2)(3, 4)(1, 3, 5)) = B((1, 4, 5, 2, 3)) = 0 \\
B(T_2 x T_4^{-1}) &= B((1, 3, 5)(1, 2)(3, 4)(1, 2, 5, 3, 4)) = B(e) = 1 \\
B(T_2 x T_5^{-1}) &= B((1, 3, 5)(1, 2)(3, 4)(1, 2, 3, 4, 5)) = B((1, 5, 3)) = 0 \\
B(T_3 x T_1^{-1}) &= B((1, 5, 3)(1, 2)(3, 4)e) = B((1, 5, 4, 3, 2)) = 0 \\
B(T_3 x T_2^{-1}) &= B((1, 5, 3)(1, 2)(3, 4)(1, 5, 3)) = B((1, 3, 2, 5, 4)) = 0 \\
B(T_3 x T_3^{-1}) &= B((1, 5, 3)(1, 2)(3, 4)(1, 3, 5)) = B((2, 3)(4, 5)) = 0 \\
B(T_3 x T_4^{-1}) &= B((1, 5, 3)(1, 2)(3, 4)(1, 2, 5, 3, 4)) = B((1, 3, 5)) = 0 \\
B(T_3 x T_5^{-1}) &= B((1, 5, 3)(1, 2)(3, 4)(1, 2, 3, 4, 5)) = B((e)) = 1 \\
B(T_4 x T_1^{-1}) &= B((1, 4, 3, 5, 2)(1, 2)(3, 4)e) = B((1, 3, 5)) = 0 \\
B(T_4 x T_2^{-1}) &= B((1, 4, 3, 5, 2)(1, 2)(3, 4)(1, 5, 3)) = B((e)) = 1 \\
B(T_4 x T_3^{-1}) &= B((1, 4, 3, 5, 2)(1, 2)(3, 4)(1, 3, 5)) = B((1, 5, 3)) = 0 \\
B(T_4 x T_4^{-1}) &= B((1, 4, 3, 5, 2)(1, 2)(3, 4)(1, 2, 5, 3, 4)) = B((1, 4)(2, 5)) = 0 \\
B(T_4 x T_5^{-1}) &= B((1, 4, 3, 5, 2)(1, 2)(3, 4)(1, 2, 3, 4, 5)) = B((1, 4, 5, 2, 3)) = 0 \\
B(T_5 x T_1^{-1}) &= B((1, 5, 4, 3, 2)(1, 2)(3, 4)e) = B((1, 5, 3)) = 0 \\
B(T_5 x T_2^{-1}) &= B((1, 5, 4, 3, 2)(1, 2)(3, 4)(1, 5, 3)) = B((1, 3, 5)) = 0 \\
B(T_5 x T_3^{-1}) &= B((1, 5, 4, 3, 2)(1, 2)(3, 4)(1, 3, 5)) = B((e)) = 1 \\
B(T_5 x T_4^{-1}) &= B((1, 5, 4, 3, 2)(1, 2)(3, 4)(1, 2, 5, 3, 4)) = B((1, 3, 2, 5, 4)) = 0 \\
B(T_5 x T_5^{-1}) &= B((1, 5, 4, 3, 2)(1, 2)(3, 4)(1, 2, 3, 4, 5)) = B((2, 3)(4, 5)) = 0.
\end{aligned}$$

Substituting the above values into $A(x)$, we have the following matrix.

$$A(x) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Using the same method for $A(y)$ we have the following matrix.

$$A(y) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

Now, $\langle A(x), A(y) \rangle$ gives a faithful representation of A_5 because, $|A(x)| = 2$, $|A(y)| = 3$ and $|A(x)A(y)| = 5$.

Using the following rule, we can write permutation representations of the above matrices. For each matrix entry, $a_{ij} = n$ implies t_i goes to t_j^n

The labeling of the t'_i s is given in the following table.

Table 2.3: Labeling t'_i s

1. t_1	6. t_1^2	11. t_1^3	16. t_1^4	21. t_1^5	26. t_1^6
2. t_2	7. t_2^2	12. t_2^3	17. t_2^4	22. t_2^5	27. t_2^6
3. t_3	8. t_3^2	13. t_3^3	18. t_3^4	23. t_3^5	28. t_3^6
4. t_4	9. t_4^2	14. t_4^3	19. t_4^4	24. t_4^5	29. t_4^6
5. t_5	10. t_5^2	15. t_5^3	20. t_5^4	25. t_5^5	30. t_5^6

We will start by looking at the non-zero entries of $A(x)$. $a_{11} = 1$, therefore t_1 goes to t_1 and similarly for all powers of t_1 . Next in row 2 of $A(x)$, we have the entry $a_{24} = 1$, therefore t_2 goes to t_4 and similarly all powers of t_2 go to corresponding powers of t_4 .

Following this pattern, we develop the following charts.

Table 2.4: Permutations of the t'_i s using $A(x)$

t_1	t_2	t_3	t_4	t_5	t_1^2	t_2^2	t_3^2	t_4^2	t_5^2	t_1^3	t_2^3	t_3^3	t_4^3	t_5^3
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
1	4	5	2	3	6	9	10	7	8	11	14	15	12	13
t_1^4	t_2^4	t_3^4	t_4^4	t_5^4	t_1^5	t_2^5	t_3^5	t_4^5	t_5^5	t_1^6	t_2^6	t_3^6	t_4^6	t_5^6
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
16	19	20	17	18	21	24	25	22	23	26	29	30	27	28

Using the chart above, we develop the following permutation representation for $A(x)$,
 $xx = (2, 4)(3, 5)(7, 9)(8, 10)(12, 14)(13, 15)(17, 19)(18, 20)(22, 24)(23, 25)(27, 29)(28, 30)$.

Table 2.5: Permutations of the t'_i s using $A(y)$

t_1	t_2	t_3	t_4	t_5	t_1^2	t_2^2	t_3^2	t_4^2	t_5^2	t_1^3	t_2^3	t_3^3	t_4^3	t_5^3
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
2	3	1	9	20	7	8	6	19	5	12	13	11	29	25
t_1^4	t_2^4	t_3^4	t_4^4	t_5^4	t_1^5	t_2^5	t_3^5	t_4^5	t_5^5	t_1^6	t_2^6	t_3^6	t_4^6	t_5^6
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
17	18	16	4	10	22	23	21	14	30	27	28	26	24	15

Using the chart above, we develop the following permutation representation for $A(y)$,
 $yy = (1, 2, 3)(6, 7, 8)(11, 12, 13)(16, 17, 18)(21, 22, 23)(26, 27, 28)(4, 9, 19)(5, 20, 10)$
 $(14, 29, 24)(15, 25, 30)$.

Now the presentation for G follows.

$$\langle x, y, t \mid x^2, y^3, (x * y)^5, t^7, (t, \text{Normalizer}(N, \langle t_1 \rangle)) \rangle$$

The $\text{Normalizer}(N, \langle t_1 \rangle)$ is the stabilizer of all powers of t_1 in N . Thus, we want the permutations in N that stabilize $\{1, 6, 11, 16, 21, 26\}$ set wise.

Using MAGMA, we can find the stabilizer of $\{1, 6, 11, 16, 21, 26\}$ in N , which is the group generated by the following permutations.

$$(2,5)(3,4)(7,10)(8,9)(12,15)(13,14)(17,20)(18,19)(22,25)(23,24)(27,30)(28,29)$$

$$(2,3)(4,5)(7,8)(9,10)(12,13)(14,15)(17,18)(19,20)(22,23)(24,25)(27,28)(29,30)$$

$$(1,16,6)(2,8,19)(3,9,17)(4,7,18)(5,10,20)(11,21,26)(12,28,24)(13,29,22)(14,27,23)(15,30,25)$$

Using MAGMA to change the permutations into words in terms of x and y , and letting t commute with the generators of the stabilizer, we have the presentation of the progenitor $7^{*5} :_m A_5$.

$$\text{G}\langle x, y, t \rangle := \text{Group}\langle x, y, t \mid x^2, y^3, (x*y)^5, t^7, (t, y*x*y^{-1}*x*y*x*y^{-1}*x*y), \\ (t, x*y*x*y^{-1}*x*y*x*y^{-1}*x*y), (t, x*y*x*y^{-1}*x*y^{-1}) \rangle;$$

The next step would be to factor these presentations by appropriate relations.

2.4 Writing Relations

Now that we have a firm grasp of how to write a progenitor, we will look at a few methods for finding appropriate relations to factor our progenitors by. The first lemma was developed during our research and is used as more of a check to determine if the progenitor was written correctly. The second and third methods were developed by Robert T. Curtis throughout his research and they are simply explained here.

2.4.1 Factoring Lemma

Lemma 2.13. *Factoring Lemma*

*Factoring the progenitor $m^{*n} : N$ by (t_i, t_j) for $1 \leq i < j \leq n$ gives the group $m^n : N$.*

Proof. We need to show that the two presentations are equivalent. Let $N = \langle x_1, \dots, x_r \rangle$ where x_k , $1 \leq k \leq r$, are the generators of N .

We first write the presentation for $m^n : N$. m^n is the direct product of n elements of order m . We will list these as $t'_i s$, $1 \leq i \leq n$ and all $t'_i s$ will commute with each other (definition of a direct product).

Next, we have to list the generators of N and the action of the generators of N on the $t'_i s$. The presentation follows: $\langle t_1, \dots, t_n, x_1, \dots, x_r | t_i^m, (t_i, t_j), N, t_i^{x_k} \rangle$ where $1 \leq i < j \leq n$ and $1 \leq k \leq r$.

The presentation for $m^{*n} : N$ along with the relation (t_1, t_2) for $1 \leq i < j \leq n$ is written: $\langle x_1, \dots, x_r, t | N, t^m, (t, N^1), (t_1, t_2) \rangle$ where N^1 denotes the stabilizer of 1 in N .

Now, to show the presentations are equivalent, we need to show the following three things.

1) t^m is equivalent to t_i^m , since we can conjugate our $t'_i s$ by all of N . Given that N is transitive on n letters, there is a permutation that will send t_1 to all the other $t'_i s$.

2) (t_1, t_2) is equivalent to (t_i, t_j) for $1 \leq i < j \leq n$, since we can conjugate (t_1, t_2) by all of N . Similar to step (1), there will be a permutation that sends t_1 and t_2 to all the other $t'_i s$.

3) (t, N^1) is equivalent to $t_i^{x_k}$ for $1 \leq i \leq n$ and $1 \leq k \leq r$, since (t, N^1) implies $t^{N^1} = t$. Now conjugating this relationship by all of N determines the action of the generators of N on all the $t'_i s$.

Therefore, the presentations are equivalent. \square

This lemma is particularly useful because we can factor our progenitors using this lemma as a way to verify that our progenitors are written correctly.

Example: $\frac{2^{*3}:S_3}{t_0 t_1 = t_1 t_0} = \langle x, y, t | x^3, y^2, (x * y)^2, t^2, (t, t^x), (t, y) \rangle$.

$x \sim (1, 2, 3)$ and $y \sim (2, 3)$ and $t \sim t_1$.

$t_1^2 = 1$ implies $t_1 = t_1$ therefore $t_1^x = t_1^x \implies t_2 = t_2 \implies t_2^2 = 1$

$t_1^2 = 1$ implies $t_1 = t_1$ therefore $t_1^{x^2} = t_1^{x^2} \implies t_3 = t_3 \implies t_3^2 = 1$

This shows 3 elements of order 2.

$(t, t^x) = (t_1, t_2) \implies t_1 t_2 = t_2 t_1$

$(t_1 t_2 = t_2 t_1)^y \implies t_1 t_3 = t_3 t_1$

$(t_1 t_2 = t_2 t_1)^x \implies t_2 t_3 = t_3 t_2$

Thus, all 3 elements of order 2 commute.

Now we have 2^3 and we need to show the extension problem of $2^3 : S_3$.

Note $N^1 = y = (2, 3)$.

$(t, y) \implies (2, 3)t_1(2, 3) = t_1 \implies t_1^{(2,3)} = t_1$.

Now conjugate by $x = (1, 2, 3)$.

$t_1^{(2,3)(1,2,3)} = t_1^{(1,2,3)} \implies t_1^x = t_2$.

Continuing this process we see

$t_1^{xx} = t_2^x \implies t_2^x = t_3$ and $t_1^{xy} = t_2^y \implies t_2^y = t_3$

$t_2^{yx} = t_3^x \implies t_3^x = t_1$ and $t_2^{yy} = t_3^y \implies t_3^y = t_2$

Therefore, we have the action of the generators of S_3 on 2^3 .

Thus, the progenitor shown above is isomorphic to $2^3 : S_3$.

We can show another example of this lemma using the first symmetric progenitor written in this chapter. The progenitor we wrote was $2^{*5} : S_5$. Now, if we factor this progenitor by letting all the 5 t 's commute, the progenitor should generate the group $2^5 : S_5$. The order of $2^5 : S_5$ is 3840. We will show this using MAGMA.

> $S := \text{Sym}(5)$;

```

> xx:=S!(1,2,3,4,5);
> yy:=S!(1,2);
> N:=sub<S|xx,yy>;
> N1:=Stabiliser(N,1);
> N1;
Permutation group N1 acting on a set of cardinality 5
Order = 24 = 2^3 * 3
      (2, 3)
      (3, 4)
      (4, 5)

> G<x,y,t>:=Group<x,y,t|x^5,y^2,(x*y)^4,(x,y)^3,t^2,(t,y^x),(t,y^x^2),(t,\
y^x^3),(t,t^x)>;
> #G;
3840

```

2.4.2 First Order Relations

Given a progenitor of the form $m^{*n} : N$, a method for generating all first order relations of the form $(xt_i)^k = 1$ where $x \in N$ is outlined below.

First, we must find the conjugacy classes of N and compute the centralizer of a representative from each class in N . We then calculate the orbits for each one of the centralizers. Next, we take the representative from each class and right multiply by a t_i from each orbit. We have developed a small program in MAGMA to generate such representatives and orbits.

Note: The ScheierSystem code for N must be ran for this program to work.

```

CL:=Classes(N);
for ii in [2..NumberOfClasses(N)] do
for i in [1..#N] do
if ArrayP[i] eq CL[ii][3] then Sch[i]; end if; end for;
C12:=Centraliser(N,CL[ii][3]);
Orbits(C12);
end for;

```

We will demonstrate the method with the example $2^{*4} : S_4$. First we calculate the conjugacy classes of S_4 , which follow:

```

> N:=Sym(4);
> CL:=Classes(N);
> CL;
Conjugacy Classes of group N

```



```

-----
[1]      Order 1      Length 1
      Rep Id(N)

[2]      Order 2      Length 3
      Rep (1, 2)(3, 4)

[3]      Order 2      Length 6
      Rep (1, 2)

[4]      Order 3      Length 8
      Rep (1, 2, 3)

[5]      Order 4      Length 6
      Rep (1, 2, 3, 4)

```

Now, we can calculate the centralizer of each class representative and the orbit of the centralizer.

```

> for i in [2..#CL] do
for> C12:=Centraliser(N,CL[i][3]);
for> CL[i][3]; Orbits(C12);
for> end for;
(1, 2)(3, 4)
[
  GSet{@ 1, 2, 4, 3 @}
]
(1, 2)
[
  GSet{@ 1, 2 @},
  GSet{@ 3, 4 @}
]
(1, 2, 3)
[
  GSet{@ 4 @},
  GSet{@ 1, 2, 3 @}
]
(1, 2, 3, 4)
[
  GSet{@ 1, 2, 3, 4 @}
]

```

Recall our relations will take the form $(xt_i)^k = 1$, such that x is the representative from each class and t_i is a t from each orbit of the centralizer of that representative.

Following this rule, the list of first order relations follow:

$$((1, 2)(3, 4)t_1)^a = 1$$

$$((1, 2)t_1)^b = 1$$

$$((1, 2)t_3)^c = 1$$

$$((1, 2, 3)t_1)^d = 1$$

$$((1, 2, 3)t_4)^e = 1$$

$$((1, 2, 3, 4)t_1)^f = 1$$

Of course, when writing the progenitor we will need to convert these permutations into words in terms of the generators of N .

2.4.3 Curtis' Lemma

An outline of the lemma developed by Robert T. Curtis follows:

If we have a progenitor of the form $2^{*n} : N$, where N is transitive on n letters. First we calculate the stabilizer of two points in our control group N . Without loss of generality, let the two points be 1 and 2, and call this stabilizer N_{12} . Now we calculate the centralizer of N_{12} in N , call this group C_{12} . Note: $Centralizer(N, H) = \{n \in N | h^n = h \forall h \in H\}$. We can use all elements of C_{12} for the lemma, but it is sufficient to use just the generators of C_{12} . Therefore, we look at the generators of C_{12} and produce relations based on the following rule.

Let x be a generator of C_{12} , then

If x fixes 1 and 2 then we write $(t_1 t_2)^k = x$ where k is even.

If x sends 1 and 2 to each other then we write $(x t_1)^k = 1$ where k is odd.

Example:

We want to write relations for the progenitor $2^{*5} : A_5$, therefore we want to look at the point stabilizer of 1 and 2 in A_5 , then calculate the centralizer of N_{12} in A_5 . This group is easily shown using MAGMA.

```
> N:=Alt(5);
> N12:=Stabiliser(N, [1,2]);
> N12;
Permutation group N12 acting on a set of cardinality 5
Order = 3
```

```
(3, 4, 5)
> C12:=Centraliser(N,N12);
> C12;
Permutation group C12 acting on a set of cardinality 5
Order = 3
(3, 4, 5)
```

Now, $C_{12} = \langle (3, 4, 5) \rangle$, so let $x = (3, 4, 5)$. x fixes 1 and 2, so the relation will be $(t_1 t_2)^k = x$ and we would let k be even.

Chapter 3

Double Coset Enumeration

3.1 Related Theorems and Definitions

Definition 3.1. Double Coset

Let H and K be subgroups of the group G and define a relation on G as follows:

$$x \sim y \iff \exists h \in H \text{ and } k \in K \text{ such that } y = h x k$$

where \sim is an equivalence relation and the equivalence classes are sets of the following form

$$HxK = \{h x k \mid h \in H, k \in K\} = \cup_{k \in K} H x k = \cup_{h \in H} h x K$$

Such a subset of G is called a **double coset**. [Cur07]

Definition 3.2. Point Stabilizer

Let G be a group of permutations of a set S . For each $g, s \in S$, let $g^s = g$, then we call the set of $s \in S$ the **point stabilizer** of $g \in G$. [Cur07]

Definition 3.3. Coset Stabilizing Group

The **coset stabilizing group** of a coset Nw is defined as

$$N^{(w)} = \{\pi \in N \mid Nw\pi = Nw\}$$

where $n \in N$ and w is a reduced word in the t'_i 's. [Cur07]

Theorem 3.4. Number of single cosets in NwN

From above we see that,

$$\begin{aligned} N^{(w)} &= \{\pi \in N \mid Nw\pi = Nw\} = \{\pi \in N \mid Nw\pi w^{-1} = N\} \\ &= \{\pi \in N \mid (Nw)^\pi = Nw\} \end{aligned}$$

and the number of single cosets in NwN is given by $[N : N^{(w)}]$. [Cur07]

Definition 3.5. Orbits

Let G be a group of permutations of a set S . For each $s \in S$, let $\text{orb}_G(s) = \{\phi(s) \mid \phi \in G\}$. The set $\text{orb}_G(s)$ is a subset of S called the **orbits** of s under G . We use $|\text{orb}_G(s)|$ to denote the number of elements in $\text{orb}_G(s)$. [Rot95]

3.2 Construction of $2^5 : S_5$ over S_5

$$2^5 : S_5 \cong \frac{2^{*5}:S_5}{t_3 t_5 t_3 t_5 = 1}$$

$$\text{Progenitor} = 2^{*5} : S_5 \implies \langle t_1 \rangle * \langle t_2 \rangle * \langle t_3 \rangle * \langle t_4 \rangle * \langle t_5 \rangle : S_5,$$

where the t'_i s have order 2.

We have $t_3 t_5 t_3 t_5 = 1$. By multiplying both sides of the equation on the right by $t_5 t_3$ we can obtain the equivalent relation $t_3 t_5 = t_5 t_3$.

Control Group $N = S_5$.

Definition of double coset: $NtN = \{Ntn | n \in N\}$. A Cayley diagram will be constructed to track the manual double coset enumeration of G over S_5 .

We first need to calculate the total number of unique cosets of N in G . This is the index of G in N . The index will be the order of G divided by the order of N .

$$\frac{|G|}{|N|} = \frac{3840}{120} = 32.$$

Now we know that we will have 32 unique single cosets.

Constructing the Cayley Diagram.

Circle One: First Double Coset.

We start constructing the Cayley diagram with one circle, this is the double coset NeN , it contains one single coset and it is labeled $[*]$. N is transitive on $\{1, 2, 3, 4, 5\}$ so it has a single orbit $\{t_1, t_2, t_3, t_4, t_5\}$. Next, take a representative from the orbit and see which double coset it belongs to. We pick 3 because of the given relation. $Nt_3N = \{Nt_3n | n \in N\} = \{Nt_3, Nt_4, Nt_5\}$. Since all five t'_i s are in the same orbit, we know all five t'_i s will extend to the next double coset.

Circle Two: Second Double Coset.

Now, there are five t'_i s extending to the second double coset, we label the double coset $[3]$, which represents Nt_3N . We next find the coset stabilizer, $N^{(3)}$ in S_5 . The coset stabilizer, $N^{(3)}$ will be elements of S_5 that fix 3. Therefore,

$$\begin{aligned} N^{(3)} = \{ & e, (1, 2), (1, 5, 2), (1, 4, 5, 2), (2, 5), (1, 2, 5), (1, 5), (1, 4, 5), (2, 4, 5), \\ & (1, 2, 4, 5), (1, 5)(2, 4), (1, 4, 2, 5), (4, 5), (1, 2)(4, 5), (1, 5, 4, 2), (1, 4, 2), (2, 5, 4), \\ & (1, 2, 5, 4), (1, 5, 4), (1, 4), (2, 4), (1, 2, 4), (1, 5, 2, 4), (1, 4)(2, 5) \} \end{aligned}$$

Now, the number of single cosets in Nt_3N will be at most $\frac{|N|}{|N^{(3)}|} = \frac{120}{24} = 5$.

We can see which t'_i s share orbits, by seeing which t'_i s share permutations in $N^{(3)}$. The orbits for $N^{(3)}$ will be $\{t_3\}$ and $\{t_1, t_2, t_4, t_5\}$. Choose a representative from each orbit to find the behavior for all elements of that orbit.

$Nt_3t_3 = NeN$ (t_i 's have order 2) : this means t_3 goes back to $[*]$.

$Nt_3t_5 \in Nt_3t_5N$: There is no relation that sends two t_i 's to one t_i . This means t_5 extends to the next double coset. All of the t_i 's in the same orbit will extend also. Therefore, $\{t_1, t_2, t_4, t_5\}$ all extend to the next double coset.

Circle Three: Third Double Coset.

We choose t_5 as a representative from this orbit. We label the third double coset $[35]$ and compute the coset stabilizer, $N^{(35)}$. We need $n \in N \ni N(t_3t_5)^n = Nt_3t_5$. Using the given relation ($t_3t_5 = t_5t_3$), we can see that the coset stabilizer will be all elements of S_5 that fix 3 and 5, as well as, the elements of S_5 that send 3 to 5 and 5 to 3. Therefore, $N^{(35)} = \{e, (1, 2), (1, 4), (2, 4), (1, 2, 4), (1, 4, 2), (3, 5), (1, 2)(3, 5), (1, 4)(3, 5), (2, 4)(3, 5), (1, 2, 4)(3, 5), (1, 4, 2)(3, 5)\}$.

Now, the number of single cosets in Nt_3t_5N will be at most, $\frac{|N|}{|N^{(35)}|} = \frac{120}{12} = 10$

Next, we calculate the transversals for $[35]$ by finding the right cosets of $N^{(35)}$ in N .

$$N^{(35)}(e) = N^{(35)}$$

$$N^{(35)}(13) = \{(13), (123), (143), (13)(24), (1243), (1423), (135), (1235), (1435), (135)(24), (12435), (14235)\}$$

$$N^{(35)}(15) = \{(12), (125), (145), (15)(24), (1245), (1425), (153), (1253), (1453), (153)(24), (12453), (14253)\}$$

$$N^{(35)}(23) = \{(23), (132), (14)(23), (243), (1324), (1432), (235), (1352), (14)(235), (2435), (13524), (14352)\}$$

$$N^{(35)}(25) = \{(25), (152), (14)(25), (245), (1524), (1452), (253), (1532), (14)(253), (2453), (15324), (14532)\}$$

$$N^{(35)}(34) = \{(34), (12)(34), (134), (234), (1234), (1342), (354), (12)(354), (1354), (2354), (12354), (13542)\}$$

$$N^{(35)}(45) = \{(45), (12)(45), (154), (254), (1254), (1542), (345), (12)(345), (1534), (2534), (12534), (15342)\}$$

$$N^{(35)}(1543) = \{(1543), (12543), (13)(45), (15423), (123)(45), (13)(254), (15)(34), (125)(34), (1345), (15)(234), (12345), (13425)\}$$

$$N^{(35)}(1325) = \{(1325), (15)(23), (14325), (13245), (15)(243), (23)(145), (13)(25), (1523), (143)(25), (13)(245), (15243), (14523)\}$$

$$N^{(35)}(2345) = \{(2345), (13452), (15234), (25)(34), (134)(25), (152)(34), (23)(45),$$

$$(132)(45), (154)(23), (2543), (13254), (15432)\}$$

Now, we find the relations based on two t'_i s by conjugating our original relation ($t_3t_5 = t_5t_3$), with a representative of each transversal.

Notation: $35 \sim 53$ means $Nt_3t_5 = Nt_5t_3$.

$$(35 \sim 53)^{(e)} \Leftrightarrow (35 \sim 53)$$

$$(35 \sim 53)^{(13)} \Leftrightarrow (15 \sim 51)$$

$$(35 \sim 53)^{(15)} \Leftrightarrow (31 \sim 13)$$

$$(35 \sim 53)^{(23)} \Leftrightarrow (25 \sim 52)$$

$$(35 \sim 53)^{(25)} \Leftrightarrow (32 \sim 23)$$

$$(35 \sim 53)^{(34)} \Leftrightarrow (45 \sim 54)$$

$$(35 \sim 53)^{(45)} \Leftrightarrow (34 \sim 43)$$

$$(35 \sim 53)^{(1543)} \Leftrightarrow (14 \sim 41)$$

$$(35 \sim 53)^{(1325)} \Leftrightarrow (21 \sim 12)$$

$$(35 \sim 53)^{(2345)} \Leftrightarrow (42 \sim 24)$$

This gives the 10 distinct right cosets in the double coset Nt_3t_5N . The orbits for $N^{(35)}$ on $\{1, 2, 3, 4, 5\}$ are $\{t_3, t_5\}$ and $\{t_1, t_2, t_4\}$. Choose a representative from each orbit to find the behavior for all elements of that orbit.

$Nt_3t_5t_5 = Nt_3N$: this means t_3, t_5 go back to [3]. $Nt_3t_5t_1 = Nt_3t_5t_1N$: there is no relation that takes three t'_i s back to two t'_i s, so t_1, t_2, t_4 all extend to the next double coset.

Circle Four: Fourth Double Coset.

We choose t_1 as a representative from this orbit. Now, label the fourth double coset [351] and look for the coset stabilizer, $N^{(351)}$. We need $n \in N \ni N(t_3t_5t_1)^n = Nt_3t_5t_1$. Using the relations that we developed for two t'_i s, we can form relations for $t_3t_5t_1$.

Notation: $351 \sim 531$ means $Nt_3t_5t_1 = Nt_5t_3t_1$.

$$351 \sim 531 \sim 513 \sim 153 \sim 135 \sim 315$$

Now, the coset stabilizer will be the point stabilizer N^{351} and any permutation that maintains the above relation.

$$N^{(351)} = \{e, (13), (15), (35), (135), (153), (24), (24)(13), (24)(15), (24)(35), (24)(135), (24)(153)\}.$$

Now, the number of single cosets in $Nt_3t_5t_1N$ will be at most $\frac{|N|}{|N^{(351)}|} = \frac{120}{12} = 10$.

Now, we find the transversals for [351] by calculating the right cosets of $N^{(351)}$.

$$\begin{aligned}
N^{(351)}(e) &= \{e, (13), (15), (35), (135), (153), (24), (24)(13), (24)(15), (24)(35), (24)(135), \\
&\quad (24)(153)\} \\
N^{(351)}(12) &= \{(12), (132), (152), (12)(35), (1352), (1532), (124), (1324), (1524), (124)(35), \\
&\quad (13524), (15324)\} \\
N^{(351)}(14) &= \{(14), (134), (154), (14)(35), (1354), (1534), (142), (1342), (1542), (142)(35), \\
&\quad (13542), (15342)\} \\
N^{(351)}(23) &= \{(23), (123), (15)(23), (235), (1235), (1523), (243), (1243), (15)(243), (2435), \\
&\quad (12435), (15243)\} \\
N^{(351)}(25) &= \{(25), (13)(25), (125), (253), (1325), (1253), (245), (13)(245), (1245), (2453), \\
&\quad (13245), (12453)\} \\
N^{(351)}(34) &= \{(34), (143), (15)(34), (354), (1435), (1543), (234), (1423), (15)(234), (2354), \\
&\quad (14235), (15423)\} \\
N^{(351)}(45) &= \{(45), (13)(45), (145), (345), (1345), (1453), (254), (13)(254), (1425), (2534), \\
&\quad (13425), (14253)\} \\
N^{(351)}(2543) &= \{(2543), (12543), (14325), (25)(34), (125)(34), (143)(25), (23)(45), \\
&\quad (123)(45), (145)(23), (2345), (12345), (14523)\} \\
N^{(351)}(1452) &= \{(1452), (13452), (12)(45), (14532), (132)(45), (12)(345), (14)(25), \\
&\quad (134)(25), (1254), (14)(253), (13254), (12534)\} \\
N^{(351)}(1234) &= \{(1234), (14)(23), (15234), (12354), (14)(235), (154)(23), (12)(34), (1432), \\
&\quad (152)(34), (12)(354), (14352), (15432)\}
\end{aligned}$$

Now, we find the relations based on three t'_i s by conjugating our above relationship ($t_3t_5t_1 = t_5t_3t_1 = t_5t_1t_3 = t_1t_5t_3 = t_1t_3t_5 = t_3t_1t_5$), with a representative of each transversal.

$$\begin{aligned}
(351 \sim 531 \sim 513 \sim 153 \sim 135 \sim 315)^{(e)} &\Leftrightarrow (351 \sim 531 \sim 513 \sim 153 \sim 135 \sim 315) \\
(351 \sim 531 \sim 513 \sim 153 \sim 135 \sim 315)^{(12)} &\Leftrightarrow (352 \sim 532 \sim 523 \sim 253 \sim 235 \sim 325) \\
(351 \sim 531 \sim 513 \sim 153 \sim 135 \sim 315)^{(14)} &\Leftrightarrow (354 \sim 534 \sim 543 \sim 453 \sim 435 \sim 345) \\
(351 \sim 531 \sim 513 \sim 153 \sim 135 \sim 315)^{(23)} &\Leftrightarrow (251 \sim 521 \sim 512 \sim 152 \sim 125 \sim 215) \\
(351 \sim 531 \sim 513 \sim 153 \sim 135 \sim 315)^{(25)} &\Leftrightarrow (321 \sim 231 \sim 213 \sim 123 \sim 132 \sim 312) \\
(351 \sim 531 \sim 513 \sim 153 \sim 135 \sim 315)^{(34)} &\Leftrightarrow (451 \sim 541 \sim 514 \sim 154 \sim 145 \sim 415) \\
(351 \sim 531 \sim 513 \sim 153 \sim 135 \sim 315)^{(45)} &\Leftrightarrow (341 \sim 431 \sim 413 \sim 143 \sim 134 \sim 314) \\
(351 \sim 531 \sim 513 \sim 153 \sim 135 \sim 315)^{(2543)} &\Leftrightarrow (241 \sim 421 \sim 412 \sim 142 \sim 124 \sim 214)
\end{aligned}$$

$$(351 \sim 531 \sim 513 \sim 153 \sim 135 \sim 315)^{(1452)} \Leftrightarrow (324 \sim 234 \sim 243 \sim 423 \sim 432 \sim 342)$$

$$(351 \sim 531 \sim 513 \sim 153 \sim 135 \sim 315)^{(1234)} \Leftrightarrow (452 \sim 542 \sim 524 \sim 254 \sim 245 \sim 425)$$

The orbits for $N^{(351)}$ on $\{1, 2, 3, 4, 5\}$ are $\{t_3, t_5, t_1\}$ and $\{t_2, t_4\}$. Choose a representative from each orbit to find the behavior for all elements of that orbit.

$$Nt_3t_5t_1t_1 = Nt_3t_5N : \text{so } \{t_3, t_5, t_1\} \text{ all go back to } [35].$$

$Nt_3t_5t_1t_2 = Nt_3t_5t_1t_2N$: there is no relation that takes four t'_i s back to three t'_i s, so $\{t_2, t_4\}$ all extend to the next double coset.

Circle Five: Fifth Double Coset.

We choose t_2 as a representative from the second orbit. Now, label the fifth double coset $[3512]$ and look for the coset stabilizer, $N^{(3512)}$. We need $n \in N \ni N(t_3t_5t_1t_2)^n = Nt_3t_5t_1t_2$. Using the relations that we developed for 3 t'_i s, we can form relations for $t_3t_5t_1t_2$.

$$\text{Notation: } 3512 \sim 5312 \text{ means } Nt_3t_5t_1t_2 = Nt_5t_3t_1t_2.$$

The following are equivalent: $1235 \sim 1253 \sim 1325 \sim 1352 \sim 1523 \sim 1532 \sim 2135 \sim 2153 \sim 2315 \sim 2351 \sim 2513 \sim 2531 \sim 3125 \sim 3152 \sim 3215 \sim 3251 \sim 3512 \sim 3521 \sim 5123 \sim 5132 \sim 5213 \sim 5231 \sim 5312 \sim 5321$

Now, the coset stabilizer will be the point stabilizer N^{3512} and any permutation that maintains the above relation. Which is essentially the point stabilizer, N^4 .

$$N^{(3512)} = \{e, (12), (13), (15), (23), (25), (35), (12)(35), (13)(25), (15)(23), (123), (125), (132), (135), (152), (153), (235), (253), (1235), (1352), (1325), (1532), (1523), (1253)\}$$

Now, the number of single cosets in $Nt_3t_5t_1t_2N$ will be at most

$$\frac{|N|}{|N^{(3512)}|} = \frac{120}{24} = 5.$$

Now, we find the transversals for $[3512]$ by calculating the right cosets of $N^{(3512)}$.

$$N^{(3512)}(e) = \{e, (12), (13), (15), (23), (25), (35), (12)(35), (13)(25), (15)(23), (123), (125), (132), (135), (152), (153), (235), (253), (1235), (1352), (1325), (1532), (1523), (1253)\}$$

$$N^{(3512)}(14) = \{(14), (124), (134), (154), (14)(23), (14)(25), (14)(35), (124)(35), (134)(25), (154)(23), (1234), (1254), (1324), (1354), (1524), (1534), (235)(14), (253)(14), (12354), (13524), (13254), (15324), (15234), (12534)\}$$

$$N^{(3512)}(24) = \{(24), (142), (13)(24), (15)(24), (234), (254), (35)(24), (142)(35), (254)(13), (234)(15), (1423), (1425), (1342), (135)(24), (1542), (153)(24), (2354), (2534), (14235), (13542), (13425), (15342), (15423), (14253)\}$$

$$\begin{aligned}
N^{(3512)}(34) &= \{(34), (12)(34), (143), (15)(34), (243), (25)(34), (354), (12)(354), (25)(143), \\
&\quad (15)(243), (1243), (34)(125), (1432), (1435), (152)(34), (1543), (2435), (2543), \\
&\quad (12435), (14352), (14325), (15432), (15243), (12543)\} \\
N^{(3512)}(45) &= \{(45), (12)(45), (13)(45), (145), (23)(45), (245), (345), (12)(345), (13)(245), \\
&\quad (145)(23), (123)(45), (1245), (132)(45), (1345), (1452), (1453), (2345), (2453), \\
&\quad (12345), (13452), (13245), (14532), (14523), (12453)\}
\end{aligned}$$

Now, we find the relations based on four $t'_i s$ by conjugating our above relations with a representative of each transversal.

$$\begin{aligned}
&\begin{pmatrix} 1235 \sim & 1253 \sim & 1325 \sim & 1352 \sim & 1523 \sim & 1532 \sim \\ 2135 \sim & 2153 \sim & 2315 \sim & 2351 \sim & 2513 \sim & 2531 \sim \\ 3125 \sim & 3152 \sim & 3215 \sim & 3251 \sim & 3512 \sim & 3521 \sim \\ 5123 \sim & 5132 \sim & 5213 \sim & 5231 \sim & 5312 \sim & 5321 \end{pmatrix}^{(e)} = \\
&\begin{pmatrix} 1235 \sim & 1253 \sim & 1325 \sim & 1352 \sim & 1523 \sim & 1532 \sim \\ 2135 \sim & 2153 \sim & 2315 \sim & 2351 \sim & 2513 \sim & 2531 \sim \\ 3125 \sim & 3152 \sim & 3215 \sim & 3251 \sim & 3512 \sim & 3521 \sim \\ 5123 \sim & 5132 \sim & 5213 \sim & 5231 \sim & 5312 \sim & 5321 \end{pmatrix} \\
&\begin{pmatrix} 1235 \sim & 1253 \sim & 1325 \sim & 1352 \sim & 1523 \sim & 1532 \sim \\ 2135 \sim & 2153 \sim & 2315 \sim & 2351 \sim & 2513 \sim & 2531 \sim \\ 3125 \sim & 3152 \sim & 3215 \sim & 3251 \sim & 3512 \sim & 3521 \sim \\ 5123 \sim & 5132 \sim & 5213 \sim & 5231 \sim & 5312 \sim & 5321 \end{pmatrix}^{(14)} = \\
&\begin{pmatrix} 4235 \sim & 4253 \sim & 4325 \sim & 4352 \sim & 4523 \sim & 4532 \sim \\ 2435 \sim & 2453 \sim & 2345 \sim & 2354 \sim & 2543 \sim & 2534 \sim \\ 3425 \sim & 3452 \sim & 3245 \sim & 3254 \sim & 3542 \sim & 3524 \sim \\ 5423 \sim & 5432 \sim & 5243 \sim & 5234 \sim & 5342 \sim & 5342 \end{pmatrix} \\
&\begin{pmatrix} 1235 \sim & 1253 \sim & 1325 \sim & 1352 \sim & 1523 \sim & 1532 \sim \\ 2135 \sim & 2153 \sim & 2315 \sim & 2351 \sim & 2513 \sim & 2531 \sim \\ 3125 \sim & 3152 \sim & 3215 \sim & 3251 \sim & 3512 \sim & 3521 \sim \\ 5123 \sim & 5132 \sim & 5213 \sim & 5231 \sim & 5312 \sim & 5321 \end{pmatrix}^{(24)} = \\
&\begin{pmatrix} 1435 \sim & 1453 \sim & 1345 \sim & 1354 \sim & 1543 \sim & 1534 \sim \\ 4135 \sim & 4153 \sim & 4315 \sim & 4351 \sim & 4513 \sim & 4531 \sim \\ 3145 \sim & 3154 \sim & 3415 \sim & 3451 \sim & 3514 \sim & 3541 \sim \\ 5143 \sim & 5134 \sim & 5413 \sim & 5431 \sim & 5314 \sim & 5341 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
& \left(\begin{array}{cccccc} 1235 \sim & 1253 \sim & 1325 \sim & 1352 \sim & 1523 \sim & 1532 \sim \\ 2135 \sim & 2153 \sim & 2315 \sim & 2351 \sim & 2513 \sim & 2531 \sim \\ 3125 \sim & 3152 \sim & 3215 \sim & 3251 \sim & 3512 \sim & 3521 \sim \\ 5123 \sim & 5132 \sim & 5213 \sim & 5231 \sim & 5312 \sim & 5321 \sim \end{array} \right)^{(34)} = \\
& \left(\begin{array}{cccccc} 1245 \sim & 1254 \sim & 1425 \sim & 1452 \sim & 1524 \sim & 1542 \sim \\ 2145 \sim & 2154 \sim & 2415 \sim & 2451 \sim & 2514 \sim & 2541 \sim \\ 4125 \sim & 4152 \sim & 4215 \sim & 4251 \sim & 4512 \sim & 4521 \sim \\ 5124 \sim & 5142 \sim & 5214 \sim & 5241 \sim & 5412 \sim & 5421 \sim \end{array} \right) \\
& \left(\begin{array}{cccccc} 1235 \sim & 1253 \sim & 1325 \sim & 1352 \sim & 1523 \sim & 1532 \sim \\ 2135 \sim & 2153 \sim & 2315 \sim & 2351 \sim & 2513 \sim & 2531 \sim \\ 3125 \sim & 3152 \sim & 3215 \sim & 3251 \sim & 3512 \sim & 3521 \sim \\ 5123 \sim & 5132 \sim & 5213 \sim & 5231 \sim & 5312 \sim & 5321 \sim \end{array} \right)^{(45)} = \\
& \left(\begin{array}{cccccc} 1234 \sim & 1243 \sim & 1324 \sim & 1342 \sim & 1423 \sim & 1432 \sim \\ 2134 \sim & 2143 \sim & 2314 \sim & 2341 \sim & 2413 \sim & 2431 \sim \\ 3124 \sim & 3142 \sim & 3214 \sim & 3241 \sim & 3412 \sim & 3421 \sim \\ 4123 \sim & 4132 \sim & 4213 \sim & 4231 \sim & 4312 \sim & 4321 \sim \end{array} \right)
\end{aligned}$$

The orbits for $N^{(351)}$ on $\{1, 2, 3, 4, 5\}$ are $\{t_3, t_5, t_1, t_2\}$ and $\{t_4\}$. Choose a representative from each orbit to find the behavior for all elements of that orbit.

$Nt_3t_5t_1t_2t_2 = Nt_3t_5t_1N$: so $\{t_3, t_5, t_1, t_2\}$ all go back to the fourth double coset. $Nt_3t_5t_1t_2t_4 = Nt_3t_5t_1t_2t_4N$: there is no relation that takes five t'_i s back to four t'_i s, so $\{t_4\}$ extends to the next double coset.

Circle Six: Sixth Double Coset.

We take the lone t_4 from this orbit. Now, label the sixth double coset $[35124]$ and look for the coset stabilizer, $N^{(35124)}$. $N^{(35124)} = \{e\}$

Now, the number of single cosets in $Nt_3t_5t_1t_2t_4N$ will be at most

$$\frac{|N|}{|N^{(35124)}|} = \frac{120}{120} = 1.$$

Therefore, the only one orbit of $N^{(35124)}$ on $\{1, 2, 3, 4, 5\}$ is $\{t_3, t_5, t_1, t_2, t_4\}$. Choose a representative from this orbit to find the behavior for all elements of this orbit.

$Nt_3t_5t_1t_2t_4t_4 = Nt_3t_5t_1t_2N$: so $\{t_3, t_5, t_1, t_2, t_4\}$ all go back to the fifth double coset.

A Cayley diagram for this group follows.

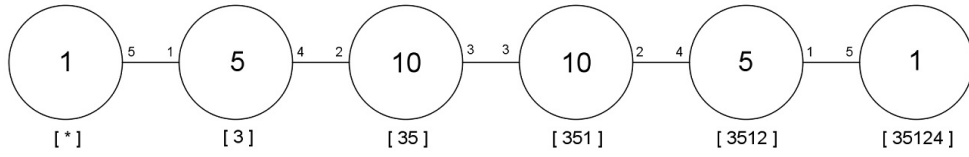


Figure 3.1: Cayley Diagram of $2^5 : S_5$ over S_5

3.3 Construction of $PGL(3, 4)$ over M_{10}

$$PGL(3, 4) \cong \frac{2^{*10}:M_{10}}{(yt)^5=1}$$

Progenitor : $2^{*10} : M_{10} \implies \langle t_1 \rangle * \dots * \langle t_{10} \rangle : M_{10}$ (t'_i s have order 2).

Relation : $(yt)^5 = 1$, with $y = (1, 2)(3, 4, 7, 9, 10, 8, 6, 5)$

$$\implies (yt)^5 = y^5 t_{10}^4 t_{10}^3 t_{10}^2 t_{10}^y t_{10} = y^5 t_3 t_5 t_6 t_8 t_{10} = 1$$

A Cayley diagram will be constructed to track the manual double coset enumeration of G over M_{10} .

We first need to calculate the total number of unique cosets of N in the group G . This is called the index. Let $N = M_{10}$. The index will be the order of G divided by the order of N .

$$\frac{|G|}{|N|} = \frac{40320}{720} = 56$$

Now we know that we will have 56 unique single cosets.

Constructing the Cayley Diagram.

Circle One: First Double Coset.

We start constructing the Cayley diagram with the first double coset, NeN . This coset contains one single coset and it is labeled $[*]$. N is transitive on $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ so it has a single orbit $\{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{10}\}$. Next, take a representative from the orbit and see which double coset it belongs to. We pick 10 because of the given relation. $Nt_{10}N = \{Nt_{10}n | n \in N\} = \{Nt_1, Nt_2, Nt_3, Nt_4, Nt_5, Nt_6, Nt_7, Nt_8, Nt_9, Nt_{10}\}$. Since all ten t'_i s are in the same orbit, we know all ten t'_i s will extend to the next double coset.

Circle Two: Second Double Coset.

Now, there are ten t'_i s extending to the second double coset. We label the double coset $[0]$, which represents $Nt_{10}N$. We next find the coset stabilizer, $N^{(0)}$ in M_{10} . The coset stabilizer, $N^{(0)}$ will be elements of M_{10} that fix (10). Therefore,

$$N^{(0)} = \langle (2, 3)(4, 6)(5, 7)(8, 9), (1, 8, 4, 2)(3, 9, 7, 5), (2, 4, 3, 6)(5, 9, 7, 8),$$

$(2, 9, 3, 8)(4, 5, 6, 7) \rangle$. Now, the number of single cosets in $Nt_{10}N$ will be at most

$$\frac{|N|}{|N^{(0)}|} = \frac{720}{72} = 10.$$

We can see which t'_i s share orbits, by seeing which t'_i s share permutations in $N^{(0)}$. The orbits for $N^{(0)}$ on $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ are $\{t_{10}\}$ and $\{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9\}$. Choose a representative from each orbit to find the behavior for all elements of that orbit.

$Nt_{10}t_{10} = Ne \in NeN$ (t'_i s have order 2) : this means t_{10} goes back to $[*]$.

$Nt_{10}t_1 \in Nt_{10}t_1N$: There is no relation that sends two t'_i s to one t_i . This means t_1 extends to the next double coset. All of the t'_i s in the same orbit will extend also. Therefore, $\{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9\}$ all extend to the next double coset.

Circle Three: Third Double Coset.

We choose t_1 as a representative from this orbit. We label the third double coset [01] and look for the coset stabilizer, $N^{(01)}$ in M_{10} . We have

$t_{10}t_1 = (8, 9)(6, 4)(7, 5)(3, 2)t_1t_{10}$. Using this relation, we can see that the coset stabilizer will be all elements of M_{10} that fix 10 and 1 and the elements of M_{10} that send 10 to 1 and 1 to 10. Therefore,

$$N^{(01)} = \langle (2, 3)(4, 6)(5, 7)(8, 9), (2, 9, 3, 8)(4, 5, 6, 7), (2, 5, 3, 7)(4, 8, 6, 9), (1, 10)(4, 7)(5, 6)(8, 9) \rangle.$$

Now, the number of single cosets in $Nt_{10}t_1N$ will be at most

$$\frac{|N|}{|N^{(01)}|} = \frac{720}{16} = 45.$$

Next, the orbits of $N^{(01)}$ in M_{10} are $\{t_1, t_{10}\}$ and $\{t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9\}$.

Choose a representative from each orbit to find the behavior for all elements of that orbit.

$Nt_{10}t_1t_1 = Nt_{10} \in Nt_{10}N$: This means $\{t_1, t_{10}\}$ go back to [0].

We have the relation $t_{10}t_1 = (10, 5, 2, 6, 9, 1, 7, 8)(3, 4)t_9t_8$, therefore

$Nt_{10}t_1t_2 \in Nt_{10}t_1N$: this means that $\{t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9\}$ stay in the double coset [01].

The resulting Cayley diagram follows:

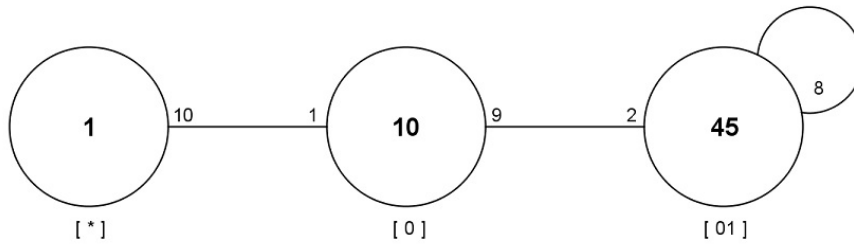


Figure 3.2: Cayley Diagram of $PGL(3,4)$ over M_{10}

3.4 Constructing $PSL(2, 8)$ over D_{14}

$$PSL(2, 8) \cong \frac{2^7:D_{14}}{(x^2ytt^x)^7, (xt^x)^3}$$

Constructing the Cayley diagram for larger groups can be quite difficult, but we can use MAGMA to do most of the calculations for us. We will do this example using MAGMA and discuss the steps.

First we write the progenitor for G and calculate the coset action. Then we can write the permutations that generate D_{14} . Followed by calculating the image of N in G and the number of double cosets of G over N .

```
> a:=0;b:=0;c:=7;e:=3;f:=0;
> G<x,y,t>:=Group<x,y,t|x^7,y^2,(x*y)^2,t^2,(t,y),(x^2*y*t*t^x)^7,
(x*t^x)^3>;
> f,G1,k:=CosetAction(G,sub<G|x,y>);
> #G;
504
> S:=Sym(7);
> xx:=S!(1,2,3,4,5,6,7);
> yy:=S!(2,7)(3,6)(4,5);
> N:=sub<S|xx,yy>;
> #N;
14
> IN:=sub<G1|f(x),f(y)>;
> DN:=DerivedGroup(N);
> #DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);
5
> #N; #G/#N;
14
36
```

Now, we must declare the seven t 's in MAGMA using the following code:

```
> ts := [Id(G1): i in [1 .. 7] ];
> ts[1]:=f(t); ts[2]:=f(t^x); ts[3]:=f(t^(x^2)); ts[4]:=f(t^(x^3));
> ts[5]:=f(t^(x^4)); ts[6]:=f(t^(x^5)); ts[7]:=f(t^(x^6));
```

Before starting the double coset enumeration, we must include a small program that counts the number of new single cosets and the SchreierSystem program which converts permutations into words composed of the generators of our group. These programs have been included in the appendix. We will apply the same techniques that were used in the first two double coset enumerations done in this chapter.

The first double coset is NeN , so we use MAGMA to determine the orbits of this double coset. This double coset should only have one orbit, since our t 's are order two and N is transitive on our seven t 's. So all seven t 's extend to the next double coset. We will label this double coset Nt_1N . We can then determine the orbits of the second double coset by using the *Orbits(N1)* command. The following code is used to accomplish this.

```
> Orbits(N);
[
  GSet{@ 1, 2, 3, 7, 4, 6, 5 @}
]
>
> N1:=Stabiliser(N,1);
> #N/#N1;
7
> Orbits(N1);
[
  GSet{@ 1 @},
  GSet{@ 2, 7 @},
  GSet{@ 3, 6 @},
  GSet{@ 4, 5 @}
]
```

Now, we must find out where a representative from each orbit will go in our Cayley diagram. Since our t 's are order 2, t_1 will go back to the double coset NeN . Now, the other three orbits can either go to new double cosets or loop around and come back to the same double coset. To check the orbit containing 2, we calculate the stabilizer of 2 in $N1$, then we check to see if any other permutations will be in the coset stabilizer of $N12$. We run the following code in MAGMA.

```
> N12:=Stabiliser(N,[1,2]);
> SSS:={[1,2]}; SSS:=SSS^N;
> #(SSS);
14
> Seqq:=Setseq(SSS);
> for i in [1..#SSS] do
for> for n in IN do
for|for> if ts[1]*ts[2] eq
for|for|if> n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
for|for|if> then print Rep(Seqq[i]);
for|for|if> end if; end for; end for;
[ 1, 2 ]
```

[5, 4]

This tells us that the single cosets Nt_1t_2 and Nt_5t_4 are equivalent. Therefore any permutations that send 1 to 5 and 2 to 4 are going to be in the coset stabilizer. The following loop will do this check for us and store the permutation into $N12s$, which is our coset stabilizing set.

```
> N12s:=N12;
> for g in N do if 1^g eq 5 and 2^g eq 4 then N12s:=sub<N|N12s,g>; end if;
end for;
> #N12s;
2
```

We can then use the following code to calculate the transversals of this double coset to see if we have a new double coset.

```
> T12:=Transversal(N,N12s);
> for i in [1..#T12] do
for> ss:=[1,2]^T12[i];
for> cst[prodim(1, ts, ss)] := ss;
for> end for;
> m:=0; for i in [1..36] do if cst[i] ne []
for|if> then m:=m+1; end if; end for; m;
7
```

This tells us that we have 7 unique single cosets, we know that this orbit has to go to an existing double coset. We only have one double coset at this point, so we use a loop to see if the double coset Nt_1N is equal to Nt_1t_2N . We also can store the permutations g and h that generate this equality and use the SchreierSystem to convert them into words. These relations are used in the proving this group is simple using Iwasawa's Lemma in chapter 7.

```
> for g,h in IN do if ts[1] eq g*(ts[1]*ts[2])^h then "true"; break;
gg:=g;hh:=h; end if; end for;
true
true
> for i in [1..#N] do
for> if ArrayP[i] eq gg then Sch[i]; end if; end for;
x^-3
> for i in [1..#N] do
for> if ArrayP[i] eq hh then Sch[i]; end if; end for;
x^-2
```

This technique is used for each orbit of each new double coset and continuing in this process we have generated the following Cayley diagram. The remaining code for this double coset enumeration can be seen in the appendix.

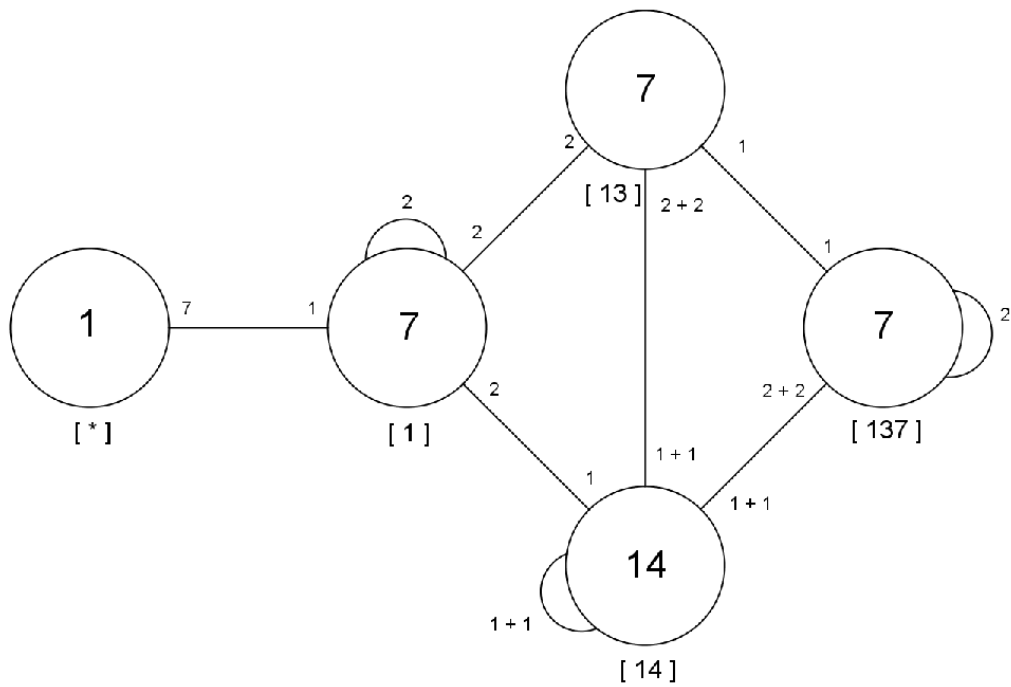


Figure 3.3: Cayley Diagram of $PSL(2, 8)$ over D_{14}

3.5 Constructing M_{12} over the Maximal Subgroup $2 \times S_5$

We have found that we can perform the double coset enumeration of a group over a maximal subgroup and still retain the same information. We let H be a subgroup of G such that $N \leq H \leq G$. We can write G as set of single cosets in H . Now $H = NwN$ and $G = HwN$, where w is a word of the symmetric generators.

Our group is a finite homomorphic image of M_{12} . Our ultimate goal is to prove M_{12} is simple using Iwasawa's lemma, which is discussed in chapter 7. Therefore, we want to pick a maximal subgroup of G , such that G is primitive over M and $\exists k \in M$ such that $\langle k^G \rangle = G$. We use the following code to find M and determine if it contains the required k .

First we look in the maximal subgroups of G and check that either M or a conjugacy class of M contain the generators of N .

```
> G<x,y,t>:=Group<x,y,t|x^3,y^2,(x*y)^2,t^5,(t,y),(x*t)^6,(y^(x^2)*t)^5>;
> f,G1,k:=CosetAction(G,sub<G|x,y>);
> CompositionFactors(G1);
  G
  | M12
  1
>
> M:=MaximalSubgroups(G1);
> M;
Conjugacy classes of subgroups
-----
```

```
[ 1]   Order 72           Length 1320
      Permutation group acting on a set of cardinality 15840
      Order = 72 = 2^3 * 3^2
[ 2]   Order 660          Length 144
      Permutation group acting on a set of cardinality 15840
      Order = 660 = 2^2 * 3 * 5 * 11
[ 3]   Order 240          Length 396
      Permutation group acting on a set of cardinality 15840
      Order = 240 = 2^4 * 3 * 5
[ 4]   Order 192          Length 495
      Permutation group acting on a set of cardinality 15840
      Order = 192 = 2^6 * 3
[ 5]   Order 192          Length 495
      Permutation group acting on a set of cardinality 15840
      Order = 192 = 2^6 * 3
```

```

[ 6]   Order 432           Length 220
      Permutation group acting on a set of cardinality 15840
      Order = 432 = 2^4 * 3^3
[ 7]   Order 432           Length 220
      Permutation group acting on a set of cardinality 15840
      Order = 432 = 2^4 * 3^3
[ 8]   Order 1440          Length 66
      Permutation group acting on a set of cardinality 15840
      Order = 1440 = 2^5 * 3^2 * 5
[ 9]   Order 1440          Length 66
      Permutation group acting on a set of cardinality 15840
      Order = 1440 = 2^5 * 3^2 * 5
[10]   Order 7920          Length 12
      Permutation group acting on a set of cardinality 15840
      Order = 7920 = 2^4 * 3^2 * 5 * 11
[11]   Order 7920          Length 12
      Permutation group acting on a set of cardinality 15840
      Order = 7920 = 2^4 * 3^2 * 5 * 11
>
> for i in [1..11] do if f(x) in M[i]'subgroup and f(y) in M[i]'subgroup
then i; end if; end for;
> for i in [1..11] do C:=Conjugates(G1,M[i]'subgroup);
C:=SetToSequence(C);
for j in [1..#C] do if f(x) in C[j] and f(y) in C[j] then i,j;
for|for|if> end if; end for; end for;
1 1318
2 27
2 65
2 81
2 136
3 48
3 258
3 377
4 66
4 87
4 132
4 488

```

Now, we need to see which M contains the required k . We see from the above code that the $M[3]$ contains a conjugacy class, namely the 48th conjugate group, which contains the generators of N . We make M a subgroup of G generated by its conjugates and label it H . Next we look in the normal lattice to check which subgroups of H are abelian. Then check to see if G is generated by that subgroups conjugates in G .

```

> C:=Conjugates(G1,M[3]‘subgroup);
> C:=SetToSequence(C);
> C48:=C[48];
> #C48;
240
> NumberOfGenerators(C48);
3
>
> H:=sub<G1|C48>;
> NL:=NormalLattice(H);
> NL;

```

Normal subgroup lattice

```

[7] Order 240 Length 1 Maximal Subgroups: 4 5 6
---
[6] Order 120 Length 1 Maximal Subgroups: 3
[5] Order 120 Length 1 Maximal Subgroups: 2 3
[4] Order 120 Length 1 Maximal Subgroups: 3
---
[3] Order 60 Length 1 Maximal Subgroups: 1
---
[2] Order 2 Length 1 Maximal Subgroups: 1
---
[1] Order 1 Length 1 Maximal Subgroups:

```

```

> for i in [1..#NL] do if IsAbelian(NL[i]) then i; end if; end for;
1
2
> G1 eq sub<G1|NL[2]^G1>;
true

```

We can check to see if G is primitive over H by computing the cosets of G over H and using the `IsPrimitive` command in MAGMA.

```

> f2,G2,k2:=CosetAction(G1,H);
> IsPrimitive(G2);
true

```

Now, that we know the M that we chose meets all the requirements to use Iwasawa's lemma, we have to determine which element in G with x generates M . We use the following code to determine that element and then use the `SchreierSystem` on G to find

that element in terms of x , y and t . Then we add that element to our presentation for G in order to have a presentation for M .

```

> for g in C48 do if sub<G1|f(x),f(y),g> eq C48 then gg:=g; end if;
end for;
> Order(gg);
2
> sub<G1|f(x),f(y),gg> eq C48;
true
> s:=IsIsomorphic(C48,M[3] 'subgroup);
> s;
true
> A:=f(x);
> B:=f(y);
> C:=f(t);
>
> N:=sub<G1|f(x),f(y),f(t)>;
> NN<x,y,t>:=Group<x,y,t|x^3,y^2,(x*y)^2,t^5,(t,y),(x*t)^6,(y^(x^2)*t)^5>;
> Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
> ArrayP:=[Id(N): i in [1..#N]];
> for i in [2..#N] do
for> P:=[Id(N): l in [1..#Sch[i]]];
for> for j in [1..#Sch[i]] do
for|for> if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
for|for> if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;
for|for> if Eltseq(Sch[i])[j] eq 3 then P[j]:=C; end if;
for|for> if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
for|for> if Eltseq(Sch[i])[j] eq -3 then P[j]:=C^-1; end if;
for|for> end for;
for> PP:=Id(N);
for> for k in [1..#P] do
for|for> PP:=PP*P[k]; end for;
for> ArrayP[i]:=PP;
for> end for;
>
> for i in [1..#N] do
for> if ArrayP[i] eq gg then Sch[i]; end if; end for;
y * x * t^-2 * x * t^-1 * x^-1 * t * x * t^2 * x
> G<x,y,t>:=Group<x,y,t|x^3,y^2,(x*y)^2,t^5,(t,y),(x*t)^6,(y^(x^2)*t)^5>;
> M:=sub<G|x,y,y*x*t^-2*x*t^-1*x^-1*t*x*t^2*x>;
> #M;
240
> N:=sub<G|x,y>;
> #N;

```

```
6
> #DoubleCosets(G,M,N);
85
```

The double coset enumeration will be done in MAGMA using the same techniques described in the previous section. The MAGMA code for the double coset enumeration is in the appendix. The Cayley diagram of M_{12} over $2 \times S_5$ follows. The joints for the double cosets of length greater than 2 have been excluded for clarity purposes.



Figure 3.4: Cayley Diagram of M_{12} over $2 \times S_5$

Chapter 4

The Extension Problem (Isomorphism Types)

4.1 Related Theorems and Definitions

To determine the isomorphism types of the progenitors we need to solve the extension problem. The following are the definitions and theorems we will be referring to in this section.

Definition 4.1. *Extension*

If K and Q are groups, then an **extension** of K by Q is a group G having a normal subgroup $K_1 \cong K$ with $G/K_1 \cong Q$. [Rot95]

Definition 4.2. *Direct Product*

If H and K are groups, then their **direct product**, denoted by $H \times K$, is the group with elements all ordered pairs (h, k) , where $h \in H$ and $k \in K$ and with operation

$$(h, k)(h', k') = (hh', kk'). [Rot95]$$

Definition 4.3. *Complement*

Let K be a (not necessarily normal) subgroup of a group G . Then a subgroup $Q \leq G$ is a **complement** of K in G if $K \cap Q = 1$ and $KQ = G$. [Rot95]

Definition 4.4. *Semi-Direct Product*

A group G is a **semi-direct product** of the subgroups K by the subgroups Q , denoted by $G = K : Q$, if K is normal in G and K has a complement $Q_1 \cong Q$. [Rot95]

Definition 4.5. Central Extension

A *central extension* of K by Q is an extension G of K by Q with $K \leq Z(G)$. [Rot95]

Definition 4.6. Mixed Extension

A *mixed extension* is a combination of a central extension with a semi-direct product, where the center of the group is not the largest abelian subgroup. We can then factor the group by the largest abelian subgroup and solve the extension problem with a mixture of semi-direct product and central extension properties.

Definition 4.7. Normal Series

A *normal series* of a group G is a sequence of subgroups

$$G = G_0 \geq G_1 \geq \cdots \geq G_n = 1$$

in which $G_{i+1} \triangleleft G_i$ for all i . The **factor groups** of this normal series are the groups G_i/G_{i+1} for $i = 0, 1, \dots, n-1$; the **length** of the normal series is the number of inclusions; that is, the length is the number of nontrivial factor groups. [Rot95]

Definition 4.8. Composition Series

A *composition series* is a normal series

$$G = G_0 \geq G_1 \geq \cdots \geq G_n = 1$$

in which, for all i either G_{i+1} is a maximal normal subgroup of G_i or $G_{i+1} = G_i$. [Rot95]

Theorem 4.9. Jordan Hölder

Every two composition series of a group G are equivalent. [Rot95]

Definition 4.10. Composition Factors

If G has a composition series, then the factor groups of this series are called the **composition factors** of G . [Rot95]

4.2 Extension Examples

4.2.1 Direct Product

Lets look at the group $G = \frac{2^7:D_{14}}{(at_1t_2)^2,(b^2t)^9}$.

This group has the following composition series.

```
> CompositionFactors(G1);
  G
  | A(1, 8)                = L(2, 8)
  *
  | Cyclic(2)
  1
```

Now, in order for this group to be a direct product of C_2 and $PSL(2, 8)$, both subgroups have to be normal in G . Therefore, we determine the normal lattice of G using MAGMA.

```
> NL:=NormalLattice(G1);
> NL;
```

Normal subgroup lattice

```
-----
[4] Order 1008 Length 1 Maximal Subgroups: 2 3
----
[3] Order 504 Length 1 Maximal Subgroups: 1
----
[2] Order 2 Length 1 Maximal Subgroups: 1
----
[1] Order 1 Length 1 Maximal Subgroups:
```

We can see that we have normal subgroups of order 2 and order 504 that are not contained in each other, therefore we could have a direct product of C_2 and $PSL(2, 8)$. We run the following code in MAGMA to determine if we are correct.

```
> D:=DirectProduct(NL[2],NL[3]);
> s,t:=IsIsomorphic(G1,D);
> s;
true
```

Now, we need to write a presentation for $PSL(2, 8)$, which follows with a check in MAGMA to make sure our presentation is correct.

```

> H<a,b>:=Group<a,b|a^7,b^3,(a^-1*b*a^-1*b^-1)^2,
(a*b^-1*a^-1*b^-1*a)^2>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,PSL(2,8));
> s;
true

```

Now, we add a generator of order 2 and make that generator commute with the generators of $PSL(2,8)$. We then can see if the presentation we wrote is isomorphic to our original group G .

```

> H<a,b,c>:=Group<a,b,c|a^7,b^3,(a^-1*b*a^-1*b^-1)^2,
(a*b^-1*a^-1*b^-1*a)^2,c^2,(a,c),(b,c)>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,G1);
> s;
true

```

Therefore, $G \cong (2 \times PSL(2,8))$.

4.2.2 Semi-Direct Product

Lets look at the group $G = \frac{2^9:3^*(3 \times 3)}{(bct_3t_4)^3, (ct_1t_2)^4, (abt_1)^3}$.

This group has the following composition series.

```
> CompositionFactors(G1);
  G
  |  Cyclic(3)
  *
  |  A(2, 4)           = L(3, 4)
  1
```

Now, we use MAGMA to see that the center of G is order 1 and print out the normal lattice of G .

```
> Center(G1);
Permutation group acting on a set of cardinality 2240
Order = 1

> NL:=NormalLattice(G1);
> NL;
```

Normal subgroup lattice

```
-----
[3]  Order 60480  Length 1  Maximal Subgroups: 2
---
[2]  Order 20160  Length 1  Maximal Subgroups: 1
---
[1]  Order 1      Length 1  Maximal Subgroups:
```

We can see that $NL[2]$ is the normal subgroup $PSL(3, 4)$, but there is no normal subgroup of order 3. Therefore, we have a semi-direct product. We must find the element of order 3 that extends $PSL(3, 4)$ and determine its action on the generators of $PSL(3, 4)$. First we write a presentation for $PSL(3, 4)$.

```
> H<a,b>:=Group<a,b|a^3,b^7,(a^-1,b^-1)^2,(a^-1*b^-2)^4,
(a*b^-2)^4,(b^-1*a^-1)^7,(b^-1*a)^7>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,NL[2]);
> s;
true
```

We can use MAGMA to find the element of order 3 that extends $NL[2]$ to G using the following loop.

```
> for i in NL[3] do if i notin NL[2] and Order(i) eq 3 and sub<G1|i,NL[2]>
eq G1 then C:=i; break; end if; end for;
```

This labels the permutation as C and we use the SchreierSystem code in MAGMA to determine that

$$a^c = b * a^{-1} * b^{-3} * a^{-1} * b^{-2}$$

$$b^c = b^2.$$

Now we add a third generator to our presentation for $NL[2]$ and add the action of the third generator on the first two. We develop the following presentation for the semi-direct product and determine if it is isomorphic to G .

```
> H<a,b,c>:=Group<a,b,c|a^3,b^7,(a^-1,b^-1)^2,(a^-1*b^-2)^4,
(a*b^-2)^4,(b^-1*a^-1)^7,(b^-1*a)^7,
> c^3,a^c=b*a^-1*b^-3*a^-1*b^-2,b^c=b^2>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,G1);
> s;
true
```

Therefore, $G \cong (PSL(3, 4) : 3)$.

4.2.3 Central Extension

In order to determine if an extension is a central extension, we need to compute the center of our group. We then factor the group by the center and determine which elements of the factor group can be written in terms of the center.

Lets look at the group N with the following composition series and normal lattice.

```
> CompositionFactors(N);
  G
  | Cyclic(3)
  *
  | Cyclic(3)
  *
  | Cyclic(3)
  1
> NL:=NormalLattice(N);
> NL;
```

Normal subgroup lattice

```
-----
[7] Order 27 Length 1 Maximal Subgroups: 3 4 5 6
---
[6] Order 9 Length 1 Maximal Subgroups: 2
[5] Order 9 Length 1 Maximal Subgroups: 2
[4] Order 9 Length 1 Maximal Subgroups: 2
[3] Order 9 Length 1 Maximal Subgroups: 2
---
[2] Order 3 Length 1 Maximal Subgroups: 1
---
[1] Order 1 Length 1 Maximal Subgroups:
```

Now, we use MAGMA to determine if N has a center and determine which element in the normal lattice is the center.

```
> Center(N);
Permutation group acting on a set of cardinality 9
Order = 3
(1, 7, 4)(2, 8, 6)(3, 9, 5)
> Center(N) eq NL[2];
true
```

We can factor N by the center and determine the isomorphism type of the resulting factor group, Q . The composition factors and normal lattice of Q follow.

```
> Q,ff:=quo<N|NL[2]>;
> CompositionFactors(Q);
  G
  |  Cyclic(3)
  *
  |  Cyclic(3)
  1
> nl:=NormalLattice(Q);
> nl;
```

Normal subgroup lattice

```
[6]  Order 9  Length 1  Maximal Subgroups: 2 3 4 5
----
[5]  Order 3  Length 1  Maximal Subgroups: 1
[4]  Order 3  Length 1  Maximal Subgroups: 1
[3]  Order 3  Length 1  Maximal Subgroups: 1
[2]  Order 3  Length 1  Maximal Subgroups: 1
----
[1]  Order 1  Length 1  Maximal Subgroups:
```

From the normal lattice, we believe Q is the direct product of C_3 and C_3 . We then write a presentation for Q and check that it is correct using MAGMA.

```
> H<a,b>:=Group<a,b|a^3,b^3,(a,b)>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,Q);
> s;
true
```

Now, we need to convert our generators a,b to elements in N using the transversals of $NL[2]$ in N .

```
> A:=t(f1(a));
> B:=t(f1(b));
> T:=Transversal(N,NL[2]);
> ff(T[2]) eq A;
true
> ff(T[3]) eq B;
true
```



```
//Assigning a,b to Elements in N
> A:=T[2];
> B:=T[3];
> C:=NL[2].1;
```

Now we determine if any of the generators and relations from our presentation for Q can be written in terms of the center of G using the following MAGMA code.

```
> for i in [1..2] do if A^3 eq C^i then i; end if; end for;
> for i in [1..2] do if B^3 eq C^i then i; end if; end for;
> for i in [1..2] do if (A,B) eq C^i then i; end if; end for;
1
```

We can now add a generator of order three to our presentation for Q . From the above code, $(A, B) = C$, so we also add that to the presentation, as well as, C commutes with A and B . Then we determine if our presentation is isomorphic to N .

```
> H<a,b,c>:=Group<a,b,c|a^3,b^3,(a,b)=c,c^3,(a,c),(b,c)>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,N);
> s;
true
```

Therefore, $N \cong (3 \bullet (3 \times 3))$.

4.2.4 Mixed Extension

A mixed extension is written on a group that has a center, but the center is not the largest abelian subgroup. Then we factor out the largest abelian subgroup containing the center and write the isomorphism type as a mixture of a central extension and a semi-direct product.

Let's look at the group $G \cong \frac{2^{*16}:2^{\bullet}(((2 \times 2):3):2)}{(b^*t^a)^4, (a^*t^c)^4}$

This group has the following composition series and normal lattice.

```
> CompositionFactors(G1);
  G
  | Cyclic(2)
  *
  | Alternating(6)
  *
  | Cyclic(2)
  *
  | Cyclic(2)
  1
> NL:=NormalLattice(G1);
> NL;
```

Normal subgroup lattice

```
-----
[9] Order 2880 Length 1 Maximal Subgroups: 6 7 8
---
[8] Order 1440 Length 1 Maximal Subgroups: 5
[7] Order 1440 Length 1 Maximal Subgroups: 5
[6] Order 1440 Length 1 Maximal Subgroups: 3 5
---
[5] Order 720 Length 1 Maximal Subgroups: 2 4
---
[4] Order 360 Length 1 Maximal Subgroups: 1
---
[3] Order 4 Length 1 Maximal Subgroups: 2
---
[2] Order 2 Length 1 Maximal Subgroups: 1
---
[1] Order 1 Length 1 Maximal Subgroups:
```

Now we can use MAGMA to determine if this group has a center and if it has

a larger abelian subgroup.

```
> Center(G1);
Permutation group acting on a set of cardinality 120
Order = 2
> Center(G1) eq NL[2];
true
> IsAbelian(NL[3]);
true
```

Now we factor G by the largest abelian subgroup, which in our case is $NL[3]$. We anticipate that our quotient could be S_6 and so we check to see if they are isomorphic.

```
> Q,ff:=quo<G1|NL[3]>;
> s,t:=IsIsomorphic(Q,Sym(6));
> s;
true
```

Next, we need a presentation for S_6 , we can get a presentation using the FP-Group command in MAGMA. We write that presentation and check that is isomorphic to our Q .

```
> H<a,b>:=Group<a,b|a^6,b^2,(b*a^-1)^5,(a*b*a^-2*b*a)^2,(a^-1*b*a*b)^3>;
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,Q);
> s;
true
```

Now, we need to convert our generators of H into elements in Q . The mapping in the coset action on H was declared as $f1$ in the above code. So we take $f1(a)$ and $f1(b)$ and now these permutations are in $H1$. Next we have to take t of these elements since t is our mapping for the isomorphism between $H1$ and Q . Now, we will have a and b represented in Q . We call these elements A and B . This is done using the following code.

```
> A:=t(f1(a));
> B:=t(f1(b));
```

We need to know which elements in $G1$ are A and B , so we can find the action of these on the generators of $NL[3]$. This will be the semi-direct product part of our extension. To do this, we calculate the transversals of $G1$ and $NL[3]$ and run a loop in

MAGMA to determine which transversals represent A and B in $G1$. The following two loops do this for us.

```
> T:=Transversal(G1,NL[3]);
> for i in [1..#T] do if ff(T[i]) eq A then i; end if; end for;
536
> for i in [1..#T] do if ff(T[i]) eq B then i; end if; end for;
561
```

Now we declare these transversals as A and B , then let C and D be the generators of $NL[3]$. To complete the semidirect product, we must calculate the action of A and B (the generators of the group we are extending by) on C and D (the generators of the group we are extending from).

```
> A:=T[536];
> B:=T[561];
> C:=NL[3].2;
> D:=NL[3].3;

> for k in [0..5] do for i,j,l in [0..1] do if C^A eq C^i*D^j*A^k*B^l
then k,l,i,j; end if; end for; end for;
0 0 1 1
> for k in [0..5] do for i,j,l in [0..1] do if C^B eq C^i*D^j*A^k*B^l
then k,l,i,j; end if; end for; end for;
0 0 1 1
> for k in [0..5] do for i,j,l in [0..1] do if D^A eq C^i*D^j*A^k*B^l
then k,l,i,j; end if; end for; end for;
0 0 0 1
> for k in [0..5] do for i,j,l in [0..1] do if D^B eq C^i*D^j*A^k*B^l
then k,l,i,j; end if; end for; end for;
0 0 0 1
>
```

From this code, we know the following:

$$c^a = c * d$$

$$c^b = c * d$$

$$d^a = d$$

$$d^b = d$$

Now, for the central part of the extension, we must see what generators or relations from our presentation for Q can be written as the generators of $NL[3]$ (C and

D). Therefore, we run loops in MAGMA to see if this happens.

```
> for i,j in [0..1] do if A^6 eq C^i*D^j then i,j; end if; end for;
0 1
> for i,j in [0..1] do if B^2 eq C^i*D^j then i,j; end if; end for;
0 0
> for i,j in [0..1] do if (B*A^-1)^5 eq C^i*D^j then i,j; end if;
end for;
1 1
> for i,j in [0..1] do if (A*B*A^-2*B*A)^2 eq C^i*D^j then i,j; end if;
end for;
0 0
> for i,j in [0..1] do if (A^-1*B*A*B)^3 eq C^i*D^j then i,j; end if;
end for;
0 1
>
```

From this code, we know the following:

$$a^6 = d$$

$$(b * a^{-1})^5 = c * d$$

$$(a^{-1} * b * a * b)^3 = d$$

We add the new relations and generators to our presentation for S_6 and check with MAGMA to make sure we are correct.

```
> H<a,b,c,d>:=Group<a,b,c,d|a^6=d,b^2,c^2,d^2,(c,d),
(b*a^-1)^5=c*d,(a*b*a^-2*b*a)^2,(a^-1*b*a*b)^3=d,
c^a=c*d,c^b=c*d,(a,d),(b,d)>;
> #H;
2880
> f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
> s,t:=IsIsomorphic(H1,G1);
> s;
true
>
```

Therefore, G is a mixed extension of 2^2 by S_6 .

$$G \cong 2^{2\bullet} : S_6.$$

These techniques are used to develop the charts of isomorphism types in chapter 6. The computer based proofs are included in the appendix for each isomorphism type.

Chapter 5

Program to Generate Progenitors

During our research, the question was asked, given a finite non-abelian simple group, can we find a method to generate control groups for progenitors when factored by the appropriate relations will give the finite non-abelian simple group as its image. This question was motivated by Robert T. Curtis' research in which he proved the following theorems.

Theorem 5.1. *If $G = \langle t_1, t_2, \dots, t_n \rangle$ where $|t_i| = 2$, for $1 \leq i \leq n$, and $N = \text{Normalizer}(G, \langle t_1 \rangle, \langle t_2 \rangle, \dots, \langle t_n \rangle)$ where N acts transitively on $\{\langle t_1 \rangle, \langle t_2 \rangle, \dots, \langle t_n \rangle\}$, then G is a homomorphic image of the progenitor $2^{*n} : N$. [Cur07]*

Theorem 5.2. *Any finite non-abelian simple group is an image of a progenitor of form $P = 2^{*n} : N$, where N is a transitive subgroup of the symmetric group of S_n . [Cur07]*

Using the above theorems, we will prove a corollary that doesn't require the subgroup of the finite non-abelian simple group to be a maximal subgroup. We reasoned that this subgroup didn't have to be maximal as long as we could find an involution that together with our chosen subgroup would generate all of G . First, we need to prove two theorems and then the corollary follows.

Theorem 5.3. *If $S \leq R$ and $[R : S] = n$, then there is a homomorphism $\rho : R \rightarrow S_n$ with $\ker \rho \leq S$. [Rot95]*

Proof. If $a \in R$ and X is the set of all the right cosets of R in S , we define a function $\rho_a : X \rightarrow X$ by $Sr \mapsto Sra$ for all $r \in R$. We know that $\rho : X \rightarrow X$ is 1-1 if and only

if there exists a function $g : X \rightarrow X$ such that $\rho(g) = 1_X$. Therefore we need to show $(\rho_a)^{-1} = \rho_{a^{-1}}$. So, given that $\rho_a : Sr \mapsto Sra$, then $(\rho_a)^{-1} : Sra \mapsto Sr$. Now,

$$\rho_{a^{-1}} : Sr \rightarrow Sra^{-1}$$

Taking the composition $\rho_a \rho_{a^{-1}}$ we have,

$$\rho_a \rho_{a^{-1}} : Sr \rightarrow Sra^{-1} \rightarrow Sr$$

Therefore $(\rho_a)^{-1} = \rho_{a^{-1}}$ and $\rho_a \in S_X$ for all $a \in R$. We now note that, $a \mapsto \rho_a : X \rightarrow X$ is a homomorphism given by the mapping $\rho : R \rightarrow S_X \cong S_n$. To show this mapping is a homomorphism we let $Sr \in X$ then if $a, b \in R$ we have $(ab)\rho = (Sr)(ab)\rho = (Sra)(b\rho) = Srab = a\rho b\rho$. If $a \in \ker \rho$ this implies $Sra = Sr$ for all $r \in R$. Letting $r = Id$, then we have $Sa = S$ and by properties of cosets we know $a \in S$ and $\ker \rho \leq S$. \square

Theorem 5.4. *If $S \leq R$, then R acts transitively on the set of all right cosets of R in S . [Rot95]*

Proof. Let X be the set of all right cosets of R in S and assume $Sr \in X$. To show that R acts transitively on the set of right cosets we must find $r \in R$ such that $S \rightarrow Sr$, but $r \in R$ so there exists $r \in R$ such that $S \rightarrow Sr$, thus R acts transitively on the set of all right cosets of R in S . \square

Corollary 5.5. *Let G be a non-abelian, simple group with $R \not\cong G$ and assume $\exists c \in G$ such that $|c| = 2$ and $G = \langle R, c \rangle$. Then G is a homomorphic image of $2^{*n} : R$, where R is a transitive subgroup of S_n . Moreover, R has a faithful permutation representation of the cosets of R in S , where S is the centralizer of $c \in R$.*

Proof. Let G be non-abelian and simple. Let $R \not\cong G$ with $c \in G$ such that $|c| = 2$ and $G = \langle R, c \rangle$. We will now show that $G = \langle c^R \rangle$.

$\langle c^R \rangle$ is normalized by R and c . Therefore, $G = \langle c^R \rangle$, otherwise

$\langle c^R \rangle \neq 1 \trianglelefteq G$, but G is simple.

Thus $G = \{c_1, c_2, \dots, c_n\}$, $|c_i| = 2$ for $1 \leq i \leq n$. By Theorem 5.3, we can define a homomorphism $\rho : 2^{*n} : R \rightarrow G$ given by $\rho(c_i) = c_i$ and $\rho(R) = R$. We note that

$\rho(R) = R$, c_i has n conjugates under $\rho(R)$, and $\rho(R)$ acts as R on the n conjugates of c_i by conjugation implying that G is a homomorphic image of $2^{*n} : R$. To show that R is a transitive subgroup of S_n we must show R acts faithfully on the set $\{c_1, c_2, \dots, c_n\}$ by conjugation. Clearly, R is transitive on n letters, since $\{c_1, c_2, \dots, c_n\}$ was generated by c_i^R . Lastly, to show that R acts faithfully on $\{c_1, c_2, \dots, c_n\}$, we need to show the only element that commutes with each c_i must be the identity element. Assume by contradiction, that $\exists r \in R \neq Id$ such that $c_i^R = t_i$ for $1 \leq i \leq n$. Therefore, $c_i r = r c_i$ for $1 \leq i \leq n$, but $G = \langle c_1, c_2, \dots, c_n \rangle$. Thus r commutes with $g, \forall g \in G$. Therefore, $r \in Z(G)$ but G is simple and $Z(G) \trianglelefteq G$ implies $Z(G) = G$, but G is non-abelian, contradiction. Therefore, R is a transitive subgroup of S_n that acts faithfully.

Now R is written on the same number of letters as G . However, we want to find a transitive and faithful permutation representation of R of a degree that's equal to the number of conjugates of c . Allowing $S \leq R$, with S being the centralizer of c in R , we find that the right cosets of R in S will always generate a transitive and faithful permutation representation. To show this we must show that S is a subgroup of R . Note, S is not empty since $e \in S, (c_i^e = c_i)$. Now let $r_1 \in S, r_2 \in S$ then show $r_1 * r_2^{-1} \in S$. Now if $r \in S$ then $r^{-1} \in S$ since

$$\begin{aligned} c_i^r &= c_i \\ r^{-1} c_i r &= c_i \\ (r^{-1} c_i r)^{r^{-1}} &= c_i^{r^{-1}} \\ r r^{-1} c_i r r^{-1} &= c_i^{r^{-1}} \\ c_i &= c_i^{r^{-1}} \end{aligned}$$

Now $r_1 \in S \implies c_i^{r_1} = c_i$ and $r_2 \in S \implies r_2^{-1} \in S$, from above. Thus $r_2^{-1} \in S \implies c_i^{r_2^{-1}} = c_i$. So,

$$c_i^{r_1} * c_i^{r_2^{-1}} = c_i * c_i = e \in K$$

Thus by the one step subgroup test S is a subgroup of R . By Theorem 5.4 we know that R in S is transitive on the n letters. It is left to show that the action of R on the cosets of R in S is faithful. We note that $Sr_i = Sr_j \iff c_i^{r_i} = c_i^{r_j}$, since if

$$\begin{aligned}
Sr_i &= Sr_j \\
\iff Sr_i r_j^{-1} &= S \\
\iff r_i r_j^{-1} &\in S \\
\iff c_i^{r_i r_j^{-1}} &= c_i \\
\iff c_i^{r_i} &= c_i^{r_j}
\end{aligned}$$

Thus, if $\exists r \in R$ such that $Sr_i r = Sr_i$ then $c_i^{r_i r} = c_i^{r_i} \implies [c_i^{r_i}]^r = c_i^{r_i}$. So $c_i^h = c_i$ for all $1 \leq i \leq n$ implies $r \in Z(G)$ since $G = \langle c_1, c_2, \dots, c_n \rangle$. Now G is simple gives $r = 1$. Therefore R acts faithfully on the cosets of R in S . \square

Leonard Lamp and I used the above corollary to write the following program using MAGMA to generate progenitors of the form $2^{*n} : N$, that when factored by relations will give homomorphic images of a target finite non-abelian simple group.

```

load "Simple Group";
count:=0;
SG:=Subgroups(G);
for i in [1..#SG] do for c in G do
if Order(c) eq 2 and c notin SG[i]'subgroup and
sub<G|SG[i]'subgroup,c> eq G then
R:=SG[i]'subgroup;
S:=Centraliser(R,c);
f,N,k:=CosetAction(R,S);
"=====";
"2 *", Index(R,S), ": N";
"c =", c;
"N = \n", N;
"\n", CompositionFactors(N);
"\n", FPGroup(N);
"\nStabiliser of 1 in N\n", Stabiliser(N,1);
"\n\n\n";
count:=count+1;
break;
end if; end for; end for;
count;

```

Chapter 6

Progenitor Charts

6.1 $2^{*14} : D_{28}$

```

G<x,y,t>:=Group<x,y,t|x^14,y^2,(x*y)^2,t^2,(t,y),
(x*t)^a,
(x*y*t^x)^b,
(x^2*y*t*t^x)^c,
(t*t^x*t^(x^3))^d,
(x*t^x)^e,
(y*t)^f
>;

```

Table 6.1: $2^{*14} : D_{28}$

a	b	c	d	e	f	G
0	4	4	0	3	2	$2^{\bullet}PGL(2, 13)$
0	6	6	3	3	2	$2^{\bullet}PSL(2, 13)$
3	7	7	3	3	2	$PSL(2, 8)$
0	0	8	3	3	2	$PGL(2, 7)$
0	9	9	3	3	2	$PSL(2, 71)$
0	10	10	3	3	2	$PGL(2, 29)$
0	5	3	7	5	2	$PSL(2, 29)$

6.2 $2^{*16} : 2^{\bullet}(((2 \times 2) : 3) : 2)$

```
G<a,b,c,d,e,t>:=Group<a,b,c,d,e,t|a^4,c^4,d^4,e^2,a^-2*e,c^-1*a^2*c^-1,
  d^-1*a^2*d^-1,b^3*e,b*c^-1*b^-1*d,a^-1*d^-1*a*d^-1,
  c^-1*d^-1*c*d^-1,a^-1*b^-1*a^-1*b^-1*c,t^2,(t,c*b^-1),
  (c*t*t^d)^r1,
  (b*t*t^a)^r2,
  (b*t*t^(c*a))^r3,
  (b*t^a)^r4,
  (a*t^c)^r5>;
```

Table 6.2: $2^{*16} : 2^{\bullet}(((2 \times 2) : 3) : 2)$

r1	r2	r3	r4	r5	G
0	3	15	4	3	$(3 \times A_5) : 2$
0	0	4	4	4	$2^2 \bullet : S_6$
0	0	0	4	5	$(5 : 2) \times A_7$
0	4	7	4	5	$2 \times A_7$
0	5	0	4	4	$2 \times S_6$
0	6	5	0	4	$((A_5 \times A_5) : 2) : 2$

6.3 $2^{*21} : ((7 \times 3) : 2)$

```
G<a,b,c,t>:=Group<a,b,c,t|a^2,c^3,b^-1*a*b*a,(b,c),(a*c^-1)^2,b^-7,
  t^2,(t,a*c^-1),
  (a*t)^r1,
  (b^2*t)^r2,
  (b*c*t)^r3,
  (b*t*t^c*t^(c^2))^r4,
  (a*b*c*t^b*t*t^b)^r5,
  (c*t^(b^2)*t^a)^r6,
  (b*t)^r7>;
```

Table 6.3: $2^{*21} : ((7 \times 3) : 2)$

r1	r2	r3	r4	r5	r6	r7	G
0	0	0	0	0	2	3	$PSL(2, 64) : 2$
0	0	0	0	7	4	2	$PGL(2, 7)$
0	0	0	0	7	6	2	$PGL(2, 13)$
0	0	0	0	8	4	2	$2^{\bullet}PGL(2, 7)$
0	0	0	0	11	4	2	$PGL(2, 27)$
0	0	0	3	9	0	2	$PSL(2, 19)$
0	0	0	5	7	15	2	$PSL(2, 29)$
0	0	0	7	13	7	2	$PSL(2, 13)$

6.4 $2^{*9} : (3^{\bullet}3^2)$

$G\langle a, b, c, t \rangle := \text{Group}\langle a, b, c, t \mid a^3, b^3, c^3, (a, c), (b, c), a*b^{-1}*a^{-1}*b*c, t^2, (t, a*b*c), (a*t*t^b*t^c)^{r1}, (a*b*c*t*t^a*t^b)^{r2}, (a*b*t)^{r3}, (b*c*t^a*t^c)^{r4}, (c*t*t^b)^{r5} \rangle;$

Table 6.4: $2^{*9} : (3^{\bullet}3^2)$

r1	r2	r3	r4	r5	G
0	0	3	4	3	$PGL(3, 4) : 2$
0	3	2	0	5	$3^{\bullet}(PSL(2, 19))$
0	4	3	5	0	M_{12}
0	7	2	7	6	$PSL(2, 13)$
0	8	2	8	4	$PGL(2, 7)$
0	11	2	11	4	$PSL(2, 23)$

6.5 $3^{*9} : (3^{\bullet}3^2)$

$G\langle a, b, c, t \rangle := \text{Group}\langle a, b, c, t \mid a^3, b^3, c^3, (a, c), (b, c), a*b^{-1}*a^{-1}*b*c, t^3, (t, a*b*c), (a*t*t^b*t^c)^{r1}, (a*b*c*t*t^a*t^b)^{r2}, (a*b*t)^{r3} \rangle;$

(b*c*t^a*t^c)^r4,
(c*t*t^b)^r5>;

Table 6.5: $3^{*9} : (3^{\bullet}3^2)$

r1	r2	r3	r4	r5	G
0	0	0	2	4	$3^{\bullet}PSL(2,7)$
0	0	0	2	5	$3^{2^{\bullet}}A_6$
0	0	2	4	0	$2^2 : PGL(3,4)$
0	0	2	7	3	$3^{\bullet}J_2$
0	5	3	2	5	$3^{\bullet}A_6$
0	8	12	2	4	$PSL(2,7)$

6.6 $2^{*7} : D_{14}$

G<a,b,t>:=Group<a,b,t|a^2,b^-7,(b^-1*a)^2,t^2,(t,a),
(a*t^(b^2)*t*t^(b^3))^r1,
(a*t^(b^3)*t*t^b)^r2,
(a*b*t^b*t^(b^2)*t^(b^3))^r3,
(b*t^b*t)^r4,
(a*t^b*t^(b^2))^r5,
(a*t^b*t)^r6>;

Table 6.6: $2^{*7} : D_{14}$

r1	r2	r3	r4	r5	r6	G
0	0	2	0	0	8	$2^2 : PGL(2,7)$
0	0	2	0	0	9	$2^{\bullet} : PSL(2,8)$
0	0	2	0	9	0	$2^{\bullet} : PSL(2,71)$
0	0	3	0	4	0	$PGL(2,139)$
0	0	4	0	0	3	$2 \times PSL(2,169)$
0	0	4	0	3	0	$PGL(2,13)$
0	0	5	0	3	0	$PSL(2,49)$

6.7 $2^{*5} : D_{10}$

G<a,b,t>:=Group<a,b,t|a^(-5),b^2,(a^-1*b)^2,t^2,(t,b),

```
(b*t^(a^4)*t)^r1,
(b*t^(a^3)*t*t^(a^3))^r2,
(b*t^(a^2)*t^a*t^(a^3))^r3,
((a*b)*t^(a^3)*t*t^(a^2))^r4,
(b*t^(a^2)*t*t^(a^2))^r5,
(b*t^(a^4)*t*t^(a^4))^r6>;
```

Table 6.7: $2^{*5} : D_{10}$

r1	r2	r3	r4	r5	r6	G
0	0	0	0	2	3	$PGL(2,41)$
0	0	0	3	0	3	$PGL(2,11)$
0	0	0	3	3	0	$PGL(2,61)$
0	0	0	5	2	6	$2 \times S(4,5)$
0	0	6	3	0	0	$A_5 \times PGL(2,11)$
0	0	7	3	0	0	$2^{\bullet}PSL(2,29)$

6.8 $2^{*8} : (2^3 \bullet : 2)$

```
G<a,b,c,d,t>:=Group<a,b,c,d,t|a^4,b^2,c^4,d^2,a^-2*b,c^-1*a^2*c^-1,
a^-1*c^-1*a*c^-1,a^-1*d*c^-1*d,t^2,(t,d),
(c*d*t)^r1,
(d*t^(a^3)*t*t^(a^3))^r2,
(b*t^b*t^a*t^c)^r3,
((a*b)*t^(a^2*c)*t*t^(a^2))^r4,
(b*t^c*t*t^(a^2))^r5,
(b*t^(a^4)*t*t^b)^r6>;
```

Table 6.8: $2^{*8} : 2^3 \bullet : 2$

r1	r2	r3	r4	r5	r6	G
0	0	0	3	0	3	$PSL(3,3) : 2$
0	0	2	6	5	3	$(2^2 : 2) \times M_{11}$
0	0	3	6	2	2	$2^{2^{\bullet}} : PGL(2,19)$
0	0	3	7	2	2	$PGL(2,27)$

Chapter 7

Iwasawa's Lemma

7.1 Related Theorems and Definitions

Definition 7.1. *G*-set

If X is a set and G is a group, then X is a ***G*-set** if there is a function $\alpha : G \times X \rightarrow X$ (called an action), denoted by $\alpha : (g, x) \mapsto gx$, such that:

(i) $1 * x = x$ for all $x \in X$

(ii) $g(hx) = (gh)x$ for all $g, h \in G$ and $x \in X$. We say that G acts on X . If $|X| = n$, then n is called the **degree** of the G -set X . [Rot95]

Definition 7.2. Block

If X is a G -set, then a **block** is a subset B of X such that, for each $g \in G$, either $gB = B$ or $gB \cap B = \emptyset$. Note $gB = \{gx : x \in B\}$. **Nontrivial** blocks are \emptyset, X , and one-point subsets. [Rot95]

Definition 7.3. Transitive *G*-set

A G -set X is transitive if it has only one orbit; that is, for every $x, y \in X$, there exists $\sigma \in G$ with $y = \sigma x$. [Rot95]

Definition 7.4. If X is a transitive G -set of degree n , and if $x \in X$, then

$$|G| = n|G_x|$$

[Rot95]

Definition 7.5. A G -set X is transitive if it has only one orbit; that is, for every $x, y \in X$, there exists $\sigma \in G$ with $y = \sigma x$. [Rot95]

Definition 7.6. Primitive

A transitive G – set X is **primitive** if it contains no nontrivial block; otherwise, it is **imprimitive**. [Rot95]

Definition 7.7. Let X be a finite G – set, and let $H \leq G$ act transitively on X . Then $G = HG_x$ for each $x \in X$. [Rot95]

Definition 7.8. Let X be a G – set and $x, y \in X$.

(i) If $H \leq G$, then $H_x \cap H_y \neq \emptyset \implies H_x = H_y$

(ii) If H is normal in G , then the subsets Hx are blocks of X . [Rot95]

Definition 7.9. (i) If X is a faithful primitive G – set of degree $n \geq 2$. If H is normal in G and if $H \neq 1$, then X is a transitive H – set. [Rot95]

Theorem 7.10. Iwasawa's Lemma

Let $G' = G$ (such a group is called **perfect**) and let X be a faithful primitive G – set. If there is $x \in X$ and an abelian normal subgroup K of G_x whose conjugates $\{ghg^{-1}\}$ generate G , then G is simple. [Rot95]

7.2 Example: Prove $PSL(2, 8)$ is Simple using Iwasawa's Lemma

In the charts shown in chapter 7, we saw that we can write $PSL(2, 8)$ as the progenitor $2^{*7} : D_{14}$ factored by relations. We start the proof by doing the double coset enumeration of $PSL(2, 8)$ over D_{14} . Using the technique discussed in chapter 5, we construct the following Cayley Diagram. (The MAGMA code used in the double coset enumeration can be found in the appendix).

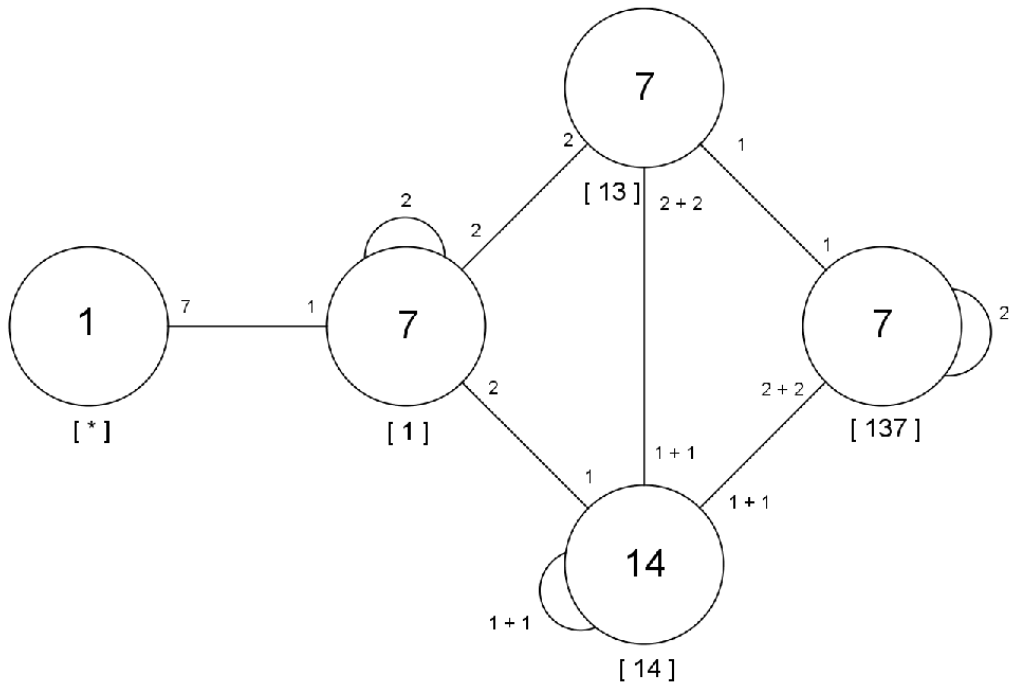


Figure 7.1: Cayley Diagram of $PSL(2, 8)$ over D_{14}

For this proof, we will use the following labeling.

$$N = D_{14}$$

$$G = PSL(2, 8)$$

Step one: Show that G acts faithfully and primitively on X , the set of double cosets of G over N .

Therefore, $X = \{N, Nt_1N, Nt_1t_3N, Nt_1t_4N, Nt_1t_3t_7N\}$ and $|X| = 36$. G acts on X implies there exists a homomorphism f , such that $f : G \rightarrow S_X$, where X is 36.

By the First Isomorphism Theorem

$\frac{G}{\ker f} \cong f(G)$. If $\ker f = 1$, then $G \cong f(G)$. We note, the only elements of X that fix N are elements of N . This implies that $G_1 = N$.

$$\begin{aligned} |G| &= 36 \times |G_1| \\ |G| &= 36 \times |N| \\ |G| &= 36 \times 14 \\ |G| &= 504 \end{aligned}$$

Now, from the Cayley diagram, we know $|G| \leq 504$. Therefore, using the First Isomorphism Theorem we can say that $\ker f$ has to be 1. Hence, G acts faithfully on X .

To show that G acts primitively on X , we can show that X has no blocks of imprimitivity. Therefore, show that X only has trivial blocks.

We know that if B is a non-trivial block, then $|B| \mid |X|$.

Therefore, $|B|$ will be 2,3,4,6,9,12 or 18.

We also know, if one of the single cosets of G over N is in B , then the entire double coset containing that single coset is in B .

Therefore, by inspection of the Cayley diagram, we can determine that the only blocks possible will have an order that is some combinations of 1,7 and 14. Hence, the only possibilities will be the trivial blocks of order 1 or the trivial block X . So we can say that G acts primitively on X .

For the next two steps, we will be using relations developed during the double coset enumeration of $PSL(2, 8)$ over D_{14} .

Step two: Show that G is perfect.

We want to show that $G = G'$. First, we will show that G is generated by the t 's in our presentation. $G = \langle x, y, t \rangle$

Expand the relations that are in our progenitor,

$$\text{relation 1: } (x^2yt_1t_2)^7 = e$$

$$x^2yt_1t_2x^2yt_1t_2x^2yt_1t_2x^2yt_1t_2x^2yt_1t_2x^2yt_1t_2x^2yt_1t_2 = e$$

$$x^2 y t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 = e$$

$$x^2 y = t_2 t_1 t_5 t_6 t_2 t_1 t_5 t_6 t_2 t_1 t_5 t_6 t_2 t_1$$

$$\text{relation 2: } (x t_2)^3 = e$$

$$x t_2 x t_2 x t_2 = e$$

$$x^3 t_4 t_3 t_2 = e$$

$$x^3 = t_2 t_3 t_4$$

$$\text{relation 3: } (t_1 t_2 t_4)^3 = e$$

$$t_1 t_2 t_4 t_1 t_2 t_4 t_1 t_2 t_4 = e$$

Now, we have the following relation from our double coset enumeration

$$t_1 t_5 = x^3 y t_2 t_4$$

We can use this relation together with relation 1 to solve for x . $t_1 t_5 t_4 t_2 = x \cdot x^2 y$

$$t_1 t_5 t_4 t_2 = x \cdot t_2 t_1 t_5 t_6 t_2 t_1 t_5 t_6 t_2 t_1 t_5 t_6 t_2 t_1$$

$$x = t_1 t_5 t_4 t_2 t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2$$

Now, we solve for y in relation 1

$$y = x^5 t_2 t_1 t_5 t_6 t_2 t_1 t_5 t_6 t_2 t_1 t_5 t_6 t_2 t_1$$

Since x can be written as a series of t 's, we now know y can also be written as a series of t 's by substituting the above value we found for x .

Therefore, we have shown that G is generated by t 's and now

$G = \langle t_1, t_2, t_3, t_4, t_5 \rangle$. Now to show $G' = G$, we need to show that a single $t \in G'$, then by conjugation, we can show all t 's are in G' .

$N' = \langle x \rangle$ and $N \subseteq G \rightarrow N' \subseteq G'$, so

$$x^3 = t_2 t_3 t_4 \in G' \rightarrow t_4 t_2 t_3 \in G' \rightarrow t_3 t_4 t_2 \in G' \text{ and } e = t_1 t_2 t_4 t_1 t_2 t_4 t_1 t_2 t_4 \in G'$$

$$\begin{aligned}
t_1 t_2 t_4 t_1 t_2 t_4 t_1 t_2 t_4 \cdot t_4 t_2 t_3 &\in G' \\
t_1 t_2 t_4 t_1 t_2 t_4 t_1 t_3 &\in G' \\
(t_1 t_2 t_4 t_1 t_2 t_4 t_1 t_3)^{t_3 t_1} &\in G' \\
t_1 t_3 t_1 t_2 t_4 t_1 t_2 t_4 &\in G' \\
t_1 t_3 t_1 \cdot t_3 t_3 \cdot t_2 t_4 t_1 t_2 t_4 &\in G' \\
[t_1, t_3] t_3 t_2 t_4 t_1 t_2 t_4 &\in G' \\
t_3 t_2 t_4 t_1 t_2 t_4 &\in G' \\
t_4 t_2 t_3 \cdot t_3 t_2 t_4 t_1 t_2 t_4 &\in G' \\
t_1 t_2 t_4 &\in G' \\
t_1 t_2 t_4 \cdot t_4 t_2 t_3 &\in G' \\
t_1 t_3 &\in G'
\end{aligned}$$

$$\begin{aligned}
x = t_1 t_5 t_4 t_2 t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 &\in G' \\
t_1 t_5 t_4 t_2 t_1 t_2 \cdot t_1 t_1 \cdot t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 &\in G' \\
t_1 t_5 t_4 [t_2, t_1] t_1 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 &\in G' \\
t_1 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_1 t_4 t_5 &\in G' \\
t_1 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_1 \cdot t_2 t_2 \cdot t_5 t_4 &\in G' \\
t_1 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_5 [t_1, t_2] t_2 t_5 t_4 &\in G' \\
t_1 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_5 t_2 t_5 t_4 &\in G' \\
t_1 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_5 t_2 t_5 \cdot t_2 t_2 \cdot t_4 &\in G' \\
t_1 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_6 [t_5, t_2] t_2 t_4 &\in G' \\
t_1 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_2 t_6 t_2 t_4 &\in G' \\
t_1 t_6 t_5 t_1 t_2 t_6 t_5 t_1 \cdot t_6 t_6 \cdot t_2 t_6 t_2 t_4 &\in G' \\
t_1 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_6 [t_6, t_2] t_4 &\in G' \\
t_1 t_6 t_5 t_1 t_2 t_6 t_5 t_1 t_6 t_4 &\in G' \\
t_1 t_6 t_5 t_1 \cdot t_5 t_6 t_6 t_5 \cdot t_2 t_6 t_5 t_1 t_6 t_4 &\in G' \\
[t_1, t_5 t_6] t_6 t_5 \cdot t_2 t_6 t_5 t_1 t_6 t_4 &\in G' \\
t_6 t_5 t_2 t_6 t_5 t_1 t_6 t_4 &\in G' \\
t_6 t_5 t_2 t_6 \cdot t_2 t_5 t_5 t_2 \cdot t_5 t_1 t_6 t_4 &\in G' \\
[t_6, t_2 t_5] t_5 t_2 \cdot t_5 t_1 t_6 t_4 &\in G' \\
t_5 t_2 t_5 t_1 t_6 t_4 &\in G' \\
t_2 t_2 \cdot t_5 t_2 t_5 t_1 t_6 t_4 &\in G' \\
t_2 [t_2, t_5] t_1 t_6 t_4 &\in G' \\
t_2 t_1 t_6 t_4 &\in G' \\
t_6 t_4 t_2 t_1 &\in G'
\end{aligned}$$

Now,

$$\begin{aligned}
t_1 t_3 \cdot t_3 t_4 t_2 &\in G' \\
t_1 t_4 t_2 &\in G' \\
t_6 t_4 t_2 t_1 \cdot t_1 t_4 t_2 &\in G' \\
t_6 t_4 t_2 t_4 t_2 &\in G' \\
t_6 [t_4, t_2] &\in G' \\
t_6 &\in G'
\end{aligned}$$

Therefore, by conjugation $\{t_1, t_2, t_3, t_4, t_5, t_6, t_7\} \in G'$ and

$\langle t_1, t_2, t_3, t_4, t_5, t_6, t_7 \rangle \subseteq G'$. But, we know that $G' \subseteq G$, hence $G' = G$.

Step 3: Show $\exists k \in G \ni k \trianglelefteq G_x$, k is abelian and the conjugates of k generate G .

Let $k = \langle x \rangle$

$$x^6 = t_1 t_2 t_4 t_5 \in k \rightarrow t_2 t_4 t_5 t_1 \in k$$

$$x^3 = t_2 t_3 t_4 \in k \rightarrow t_3 t_4 t_2 \in k$$

$$t_3 t_4 t_2 \cdot t_2 t_4 t_5 t_1 \in k$$

$$t_3 t_5 t_1 \in k$$

$$x^4 = t_1 t_4 t_3 t_2 t_5 \in k \rightarrow t_2 t_5 t_1 t_4 t_3 \in k$$

$$t_2 t_5 t_1 t_4 t_3 \cdot t_3 t_4 t_2 \in k$$

$$t_2 t_5 t_1 t_2 \in k$$

$$t_5 t_1 \in k \rightarrow t_1 t_5 \in k$$

$$t_3 t_5 t_1 \cdot t_1 t_5 \in k$$

$$t_3 \in k$$

Therefore, by conjugation $\{t_1, t_2, t_3, t_4, t_5, t_6, t_7\} \in k$ and we have already shown that $G = \langle t_1, t_2, t_3, t_4, t_5, t_6, t_7 \rangle$, thus the conjugates of k generate G as required.

We have shown all three requirements for Iwasawa's Lemma, thus $PSL(2, 8)$ is simple.

7.3 Prove M_{12} is Simple using Iwasawa's Lemma

Prove M_{12} is simple using Iwasawa's lemma. We have completed the double coset enumeration of M_{12} over $2 \times S_5$ in chapter 3 and the Cayley diagram from this group can be found in section 3.5.

Part (1) Show M_{12} acts transitively and primitively.

For this proof, we will use the following labeling.

$$N = 2 \times S_5$$

$$G = M_{12}$$

Step one: Show that G acts faithfully and primitively on X , the set of double cosets of G over M .

$|X| = 396$ and G acts on X implies there exists a homomorphism f , such that $f : G \rightarrow S_X$, where X is 396.

By the First Isomorphism Theorem,

$\frac{G}{\ker f} \cong f(G)$. If $\ker f = 1$, then $G \cong f(G)$. We note, the only elements of X that fix M are elements of M . This implies that $G_1 = M$.

$$\begin{aligned} |G| &= 396 \times |G_1| \\ |G| &= 396 \times |M| \\ |G| &= 396 \times 240 \\ |G| &= 95040 \end{aligned}$$

Now, from the double coset enumeration, we know $|G| \leq 95040$. Therefore, using the First Isomorphism Theorem we can say that $\ker f$ has to be 1. Hence, G acts faithfully on X .

To show that G acts primitively on X , we can show that X has no blocks of imprimitivity, going through the entire Cayley diagram. However, it would not be practical to go through all the possibilities for blocks of imprimitivity, so MAGMA is used for this step, which is included in the code for the double coset enumeration.

```
> f2,G2,k2:=CosetAction(G1,H);
> IsPrimitive(G2);
true
```

Part (2), Show $G' = G$

Relations (From DCE),

$$\begin{aligned} x &= t_3 t_1 t_2^2 t_1^3 t_2^4 t_3 t_1^2 t_3^3 t_1^3 && \in G' \\ x^2 &= t_3 t_1^3 t_2^3 t_3^2 t_1^2 t_2^4 t_3^3 t_1^4 t_3 t_1^2 && \in G' \\ e &= t_3^2 t_1^2 t_3^3 t_1^3 t_3^2 t_1^4 t_3 && \in G' \end{aligned}$$

$$\begin{aligned} x &= t_3 t_1 t_2^2 t_1^3 t_2^4 t_3 t_1^2 t_3^3 t_1^3 && \in G' \\ t_3 \cdot t_1^4 t_1 \cdot t_1 t_2^2 t_1^3 t_2^3 t_2 t_3 t_1^2 t_3^3 t_1^3 &&& \in G' \\ t_3 t_1^4 t_1^2 t_2^2 t_1^3 t_2^3 t_2 t_3 t_1^2 t_3^3 t_1^3 &&& \in G' \\ t_3 t_1^4 [t_1^3, t_2^3] t_2 t_3 t_1^2 t_3^3 t_1^3 &&& \in G' \\ t_2 t_3 t_1^2 t_3^3 t_1^3 t_3 t_1^4 &&& \in G' \\ t_2 t_3 t_1^2 t_3^3 t_3 \cdot t_3 t_3^4 \cdot t_1^4 &&& \in G' \\ t_2 t_3 t_1^2 t_3^3 t_3^3 t_3^4 t_3^4 &&& \in G' \\ t_2 t_3 [t_1^3, t_3^2] t_3^4 t_3^4 &&& \in G' \\ t_3^4 t_1^4 t_2 t_3 &&& \in G' \\ t_3^4 t_1^4 t_2 t_3 \cdot t_2 t_1 t_1^4 t_2 &&& \in G' \\ [t_3, t_2 t_1] t_1^4 t_2 &&& \in G' \\ t_1^4 t_2 &&& \in G' \\ t_2 t_1^4 &&& \in G' \end{aligned}$$

Now,

$$\begin{aligned} t_3 t_1^3 t_2^3 t_3^2 t_1^4 t_2^3 t_1^4 t_3 t_1^2 &&& \in G' \\ t_3 t_1^3 t_2^3 t_3^2 t_1^2 \cdot t_3^2 t_2^3 t_3^2 \cdot t_2^4 t_3^4 t_3 t_1^2 &&& \in G' \\ t_3 [t_1^2, t_3^2] t_2^3 t_2^4 t_3^3 t_1^4 t_3 t_1^2 &&& \in G' \\ t_2^3 t_2^4 t_3^3 t_1^4 t_3 t_1^2 t_3 &&& \in G' \\ t_2^3 t_2^4 t_3^3 t_1^3 t_3 t_1^2 t_3 \cdot t_3^2 t_3^2 &&& \in G' \\ t_2^3 t_2^4 t_3^3 t_1 [t_1^2, t_3^4] t_3^2 &&& \in G' \\ t_3^2 t_2^3 t_2^4 t_3^3 t_1 &&& \in G' \\ t_3^2 t_2^2 t_3^2 t_2^4 t_3^3 t_1 &&& \in G' \\ t_3^2 t_2^2 [t_2^4, t_3^3] t_1 &&& \in G' \\ t_1 t_2^2 t_2^2 &&& \in G' \end{aligned}$$

Multiplying elements (1) and (2), we see

$$\begin{aligned} t_2 t_1^4 \cdot t_1 t_2^2 t_2^2 &&& \in G' \\ t_2 t_2^2 t_2^2 &&& \in G' \\ t_2^3 t_2^2 &&& \in G' \end{aligned}$$

Now reduce the third relation

$$\begin{aligned}
t_3^2 t_1^2 t_3^3 t_1^3 t_3^2 t_1^4 t_3^4 &\in G' \\
[t_3^3, t_1^3] t_3^3 t_1^2 t_3^4 t_1^4 &\in G' \\
t_3^3 t_1^2 t_3^4 t_1^4 &\in G' \\
t_3^3 t_1 t_1 t_3^4 t_1^4 &\in G' \\
t_3^3 t_1 [t_1^4, t_3^4] &\in G' \\
t_3^3 t_1 &\in G'
\end{aligned}$$

Multiply elements (3) and (4),

$$\begin{aligned}
t_2^3 t_3^2 \cdot t_3^3 t_1 &\in G' \\
t_2^3 t_1 &\in G' \\
t_1 t_2^3 &\in G'
\end{aligned}$$

Multiply elements (1) and (5),

$$\begin{aligned}
t_2 t_1^4 t_1 t_2^3 &\in G' \\
t_2^4 &\in G'
\end{aligned}$$

Now, by multiplying by t_2^4 , we can show all powers of t_2 are in G' .

$$\begin{aligned}
t_2^4 &\in G' \\
t_2^4 \cdot t_2^4 &\in G' \\
t_2^8 &\in G' \\
t_2^3 &\in G' \\
t_2^3 \cdot t_2^4 &\in G' \\
t_2^7 &\in G' \\
t_2^2 &\in G' \\
t_2^4 t_2^2 &\in G' \\
t_2^6 &\in G' \\
t_2 &\in G'
\end{aligned}$$

Therefore, $\{t_2, t_2^2, t_2^3, t_2^4\} \in G'$

By conjugation, $\{t_1, t_1^2, t_1^3, t_1^4, t_2, t_2^2, t_2^3, t_2^4, t_3, t_3^2, t_3^3, t_3^4\} \in G'$

$\langle t_1, t_1^2, t_1^3, t_1^4, t_2, t_2^2, t_2^3, t_2^4, t_3, t_3^2, t_3^3, t_3^4 \rangle = G \subseteq G'$ and $G' \subseteq G \implies G' = G$

Step 3: Show $\exists k \in G \ni k \trianglelefteq G_x$, k is abelian and the conjugates of k generate G .

It is left to show step 3, at which point the proof is complete.

Appendix A

MAGMA Code: DCE of $PSL(2, 8)$ over D_{14}

```

a:=0;b:=0;c:=7;e:=3;f:=0;
G<x,y,t>:=Group<x,y,t|x^7,y^2,(x*y)^2,t^2,(t,y),(x^2*y*t*t^x)^7,
(x*t^x)^3>;
f,G1,k:=CosetAction(G,sub<G|x,y>);
#G;

S:=Sym(7);
xx:=S!(1,2,3,4,5,6,7);
yy:=S!(2,7)(3,6)(4,5);
N:=sub<S|xx,yy>;
#N;

IN:=sub<G1|f(x),f(y)>;
DN:=DerivedGroup(N);

#DoubleCosets(G,sub<G|x,y>,sub<G|x,y>);

#N; #G/#N;

prodim := function(pt, Q, I)
v := pt;
for i in I do
v := v^(Q[i]);
end for;
return v;
end function;

```

```

ts := [Id(G1): i in [1 .. 7] ];
ts[1]:=f(t); ts[2]:=f(t^x); ts[3]:=f(t^(x^2));ts[4]:=f(t^(x^3));
ts[5]:=f(t^(x^4)); ts[6]:=f(t^(x^5)); ts[7]:=f(t^(x^6));

cst := [null : i in [1 .. Index(G,sub<G|x,y>)]] where null
is [Integers() | ];
for i := 1 to 7 do
cst[prodim(1, ts, [i])] := [i];
end for;
m:=0;
for i in [1..36] do if cst[i] ne [] then m:=m+1; end if; end for; m;

xxx:=f(x);
yyy:=f(y);
N:=sub<G1|xxx,yyy>;
NN<x,y>:=Group<x,y|x^7,y^2,(x*y)^2>;
Sch:=SchreierSystem(NN,sub<NN|Id(NN)>);
ArrayP:=[Id(N): i in [1..#N]];
for i in [2..#N] do
P:=[Id(N): 1 in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=xxx; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=yyy; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=xxx^-1; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;

Orbits(N);

N1:=Stabiliser(N,1);
#N/#N1;
Orbits(N1);

/*                               */

```

```
/* WORDS OF LENGTH TWO */
/* */
```

```
N12:=Stabiliser(N,[1,2]);
SSS:={1,2}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
```

```
N12s:=N12;
for g in N do if 1^g eq 5 and 2^g eq 4 then N12s:=sub<N|N12s,g>;
end if; end for;
#N12s;
```

```
T12:=Transversal(N,N12s);
for i in [1..#T12] do
ss:={1,2}^T12[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..36] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N12s);
```

```
for g in IN do if ts[1]*ts[2] eq g*ts[5]*ts[4] then "true";
gg:=g; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
```

```
N13:=Stabiliser(N,[1,3]);
SSS:={1,3}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IN do
```

```

if ts[1]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;

N13s:=N13;
for g in N do if 1^g eq 6 and 3^g eq 4 then N13s:=sub<N|N13s,g>;
end if; end for;
#N13s;

T13:=Transversal(N,N13s);
for i in [1..#T13] do
ss:=[1,3]^T13[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..36] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N13s);

for g in IN do if ts[1]*ts[3] eq g*ts[6]*ts[4] then "true";
gg:=g; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

N14:=Stabiliser(N,[1,4]);
SSS:={1,4}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;

N14s:=N14;

T14:=Transversal(N,N14s);
for i in [1..#T14] do
ss:=[1,4]^T14[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..36] do if cst[i] ne []

```

```

then m:=m+1; end if; end for; m;
Orbits(N14s);

/*                                     */
/*  WORDS OF LENGTH THREE           */
/*                                     */

N137:=Stabiliser(N,[1,3,7]);
SSS:={[1,3,7]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;

N137s:=N137;
for g in N do if 1^g eq 6 and 3^g eq 4 and 7^g eq 7 then
N137s:=sub<N|N137s,g>; end if; end for;
#N137s;

T137:=Transversal(N,N137s);
for i in [1..#T137] do
ss:=[1,3,7]^T137[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..36] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N137s);

for g in IN do if ts[1]*ts[3]*ts[7] eq g*ts[6]*ts[4]*ts[7]
then "true"; gg:=g; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

N131:=Stabiliser(N,[1,3,1]);
SSS:={[1,3,1]}; SSS:=SSS^N;
#(SSS);

```

```

Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;

N131s:=N131;

T131:=Transversal(N,N131s);
for i in [1..#T131] do
ss:=[1,3,1]^T131[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..36] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N131s);

for g,h in IN do if ts[1]*ts[3]*ts[1] eq g*(ts[1]*ts[4])^h then
"true";gg:=g;hh:=h; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N132:=Stabiliser(N,[1,3,2]);
SSS:=[1,3,2]; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[3]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;

N132s:=N132;

T132:=Transversal(N,N132s);
for i in [1..#T132] do
ss:=[1,3,2]^T131[i];
cst[prodim(1, ts, ss)] := ss;

```

```

end for;
m:=0; for i in [1..36] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N132s);

N141:=Stabiliser(N,[1,4,1]);
SSS:={1,4,1}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IN do
if ts[1]*ts[4]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;

N141s:=N141;
for g in N do if 1^g eq 3 and 4^g eq 7 and 1^g eq 3 then
N141s:=sub<N|N141s,g>; end if; end for;
#N141s;

T141:=Transversal(N,N141s);
for i in [1..#T141] do
ss:=[1,4,1]^T141[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..36] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N141s);

for i in [1..7] do for g,h in IN do if ts[1]*ts[3]*ts[i] eq
g*(ts[1]*ts[3])^h then i; end if; end for; end for;
for i in [1..#N] do
if ArrayP[i] eq S!(1,6,4,2,7,5,3) then Sch[i]; end if; end for;

for g in IN do if ts[1]*ts[4]*ts[3] eq g*ts[5]*ts[2] then "true";
gg:=g; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

```

```
for g,h in IN do if ts[1]*ts[4]*ts[3] eq g*(ts[1]*ts[4])^h then
"true";gg:=g;hh:=h; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;
```


Appendix B

MAGMA Code: DCE of M_{12} over $2 \times S_5$

```
G<x,y,t>:=Group<x,y,t|x^3,y^2,(x*y)^2,t^5,(t,y),(x*t)^6,(y^(x^2)*t)^5>;
f,G1,k:=CosetAction(G,sub<G|x,y,y*x*t^-2*x*t^-1*x^-1*t*x*t^2*x>);
```

```
S:=Sym(12);
xx:=S!(1,2,3)(4,5,6)(7,8,9)(10,11,12);
yy:=S!(1,2)(4,5)(7,8)(10,11);
N:=sub<S|xx,yy>;
```

```
a:=f(x);
b:=f(y);
c:=f(t);
```

```
M:=sub<G|x,y,y*x*t^-2*x*t^-1*x^-1*t*x*t^2*x>;
IN:=sub<G1|f(x),f(y)>;
IM:=sub<G1|f(x),f(y),f(y*x*t^-2*x*t^-1*x^-1*t*x*t^2*x)>;
DM:=DerivedGroup(IM);
```

```
#DoubleCosets(G,M,sub<G|x,y>);
```

```
#M; #G/#M;
```

```
ts := [Id(G1): i in [1 .. 12] ];
ts[3]:=f(t); ts[1]:=f(t^x); ts[2]:=f(t^(x^2));
```

```

ts[4]:=(ts[1])^2; ts[5]:=(ts[2])^2; ts[6]:=(ts[3])^2;
ts[7]:=(ts[1])^3; ts[8]:=(ts[2])^3; ts[9]:=(ts[3])^3;
ts[10]:=(ts[1])^4; ts[11]:=(ts[2])^4; ts[12]:=(ts[3])^4;

```

```

prodim := function(pt, Q, I)
v := pt;
for i in I do
v := v^(Q[i]);
end for;
return v;
end function;

```

```

cst := [null : i in [1 .. Index(G,M)]] where null is [Integers() | ];
for i := 1 to 12 do
cst[prodim(1, ts, [i])] := [i];
end for;
m:=0;
for i in [1..396] do if cst[i] ne [] then m:=m+1; end if; end for; m;

```

```

NNN:=sub<G1|a,b,c>;
Sch:=SchreierSystem(G,sub<G|Id(G)>);
ArrayP:=[Id(NNN): i in [1..#NNN]];
for i in [2..#NNN] do
P:=[Id(NNN): 1 in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=a; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=b; end if;
if Eltseq(Sch[i])[j] eq 3 then P[j]:=c; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=a^-1; end if;
if Eltseq(Sch[i])[j] eq -3 then P[j]:=c^-1; end if;
end for;
PP:=Id(NNN);
for k in [1..#P] do
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;

```

```

Orbits(N);

```

```

N3:=Stabiliser(N,3);
#N/#N3;
Orbits(N3);

```

```

N6:=Stabiliser(N,6);
#N/#N6;
Orbits(N6);

N9:=Stabiliser(N,9);
#N/#N9;
Orbits(N9);

N12:=Stabiliser(N,12);
#N/#N12;
Orbits(N12);

/*          */
/*  WORDS OF LENGTH TWO  */
/*          */

N31:=Stabiliser(N,[3,1]);
SSS:={[3,1]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N31s:=N31;
T31:=Transversal(N,N31s);
for i in [1..#T31] do
ss:=[3,1]^T31[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N31s);

N34:=Stabiliser(N,[3,4]);
SSS:={[3,4]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[4] eq

```

```

n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N34s:=N34;
T34:=Transversal(N,N34s);
for i in [1..#T34] do
ss:=[3,4]^T34[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N34s);

N37:=Stabiliser(N,[3,7]);
SSS:={[3,7]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N37s:=N37;
T37:=Transversal(N,N37s);
for i in [1..#T37] do
ss:=[3,7]^T37[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N37s);

N310:=Stabiliser(N,[3,10]);
SSS:={[3,10]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N310s:=N310;

```

```

T310:=Transversal(N,N310s);
for i in [1..#T310] do
ss:=[3,10]^T310[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N310s);

N61:=Stabiliser(N,[6,1]);
SSS:={[6,1]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N61s:=N61;
T61:=Transversal(N,N61s);
for i in [1..#T61] do
ss:=[6,1]^T61[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N61s);

N64:=Stabiliser(N,[6,4]);
SSS:={[6,4]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N64s:=N64;
for g in N do if 6^g eq 4 and 4^g eq 6 then N64s:=sub<N|N64s,g>;
end if; end for;
#N64s;
T64:=Transversal(N,N64s);

```

```

for i in [1..#T64] do
ss:=[6,4]^T64[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N64s);

for g in IM do if ts[6]*ts[4] eq g*(ts[4]*ts[6]) then "true";
gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

N67:=Stabiliser(N,[6,7]);
SSS:={[6,7]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N67s:=N67;
T67:=Transversal(N,N67s);
for i in [1..#T67] do
ss:=[6,7]^T67[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N67s);

N610:=Stabiliser(N,[6,10]);
SSS:={[6,10]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[10] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N610s:=N610;

```

```

T610:=Transversal(N,N610s);
for i in [1..#T610] do
ss:=[6,10]^T610[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N610s);

```

```

N91:=Stabiliser(N,[9,1]);
SSS:={[9,1]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N91s:=N91;
T91:=Transversal(N,N91s);
for i in [1..#T91] do
ss:=[9,1]^T91[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N91s);

```

```

N94:=Stabiliser(N,[9,4]);
SSS:={[9,4]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N94s:=N94;
T94:=Transversal(N,N94s);
for i in [1..#T94] do
ss:=[9,4]^T94[i];
cst[prodim(1, ts, ss)] := ss;

```

```

end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N94s);

N97:=Stabiliser(N,[9,7]);
SSS:={[9,7]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N97s:=N97;
T97:=Transversal(N,N97s);
for i in [1..#T97] do
ss:=[9,7]^T97[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N97s);

N910:=Stabiliser(N,[9,10]);
SSS:={[9,10]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[10] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N910s:=N910;
for g in N do if 9^g eq 7 and 10^g eq 12 then N910s:=sub<N|N910s,g>;
end if; end for;
#N910s;
T910:=Transversal(N,N910s);
for i in [1..#T910] do
ss:=[9,10]^T910[i];
cst[prodim(1, ts, ss)] := ss;
end for;

```



```

m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N910s);

for g in IM do if ts[9]*ts[10] eq g*(ts[7]*ts[12]) then "true";
gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

N121:=Stabiliser(N,[12,1]);
SSS:={[12,1]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N121s:=N121;
T121:=Transversal(N,N121s);
for i in [1..#T121] do
ss:=[12,1]^T121[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N121s);

N124:=Stabiliser(N,[12,4]);
SSS:={[12,4]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N124s:=N124;
T124:=Transversal(N,N124s);
for i in [1..#T124] do
ss:=[12,4]^T124[i];
cst[prodim(1, ts, ss)] := ss;

```

```

end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N124s);

N127:=Stabiliser(N,[12,7]);
SSS:={[12,7]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N127s:=N127;
T127:=Transversal(N,N127s);
for i in [1..#T127] do
ss:=[12,7]^T127[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N127s);

N1210:=Stabiliser(N,[12,10]);
SSS:={[12,10]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[10] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]
then print Rep(Seqq[i]);
end if; end for; end for;
N1210s:=N1210;
T1210:=Transversal(N,N1210s);
for i in [1..#T1210] do
ss:=[12,10]^T1210[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1210s);

```

```

/*                                     */
/*  WORDS OF LENGTH THREE             */
/*                                     */

N312:=Stabiliser(N,[3,1,2]);
SSS:={[3,1,2]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[1]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N312s:=N312;
T312:=Transversal(N,N312s);
for i in [1..#T312] do
ss:=[3,1,2]^T312[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N312s);

N313:=Stabiliser(N,[3,1,3]);
SSS:={[3,1,3]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[1]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N313s:=N313;
T313:=Transversal(N,N313s);
for i in [1..#T313] do
ss:=[3,1,3]^T313[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N313s);

```

```

for a in [3,6,9,12] do for b in [1,4,7,10]
do for g in IM do for h in IN
do if ts[3]*ts[1]*ts[3] eq g*(ts[a]*ts[b])^h then a,b; break;
end if; end for; end for; end for; end for;
for g in IM do for h in IN do if ts[3]*ts[1]*ts[3] eq
g*(ts[12]*ts[10])^h
then "true"; gg:=g; hh:=h; break; end if; end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N315:=Stabiliser(N,[3,1,5]);
SSS:={[3,1,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[1]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N315s:=N315;
for g in N do if 3^g eq 3 and 1^g eq 2 and 5^g eq 4 then
N315s:=sub<N|N315s,g>; end if; end for;
#N315s;
T315:=Transversal(N,N315s);
for i in [1..#T315] do
ss:=[3,1,5]^T315[i];
cst[prod(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N315s);

for g in IM do if ts[3]*ts[1]*ts[5] eq g*(ts[3]*ts[2]*ts[4])
then "true"; gg:=g; break; end if; end for;

for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

N316:=Stabiliser(N,[3,1,6]);
SSS:={[3,1,6]}; SSS:=SSS^N;

```

```

#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[1]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N316s:=N316;
T316:=Transversal(N,N316s);
for i in [1..#T316] do
ss:=[3,1,6]^T316[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N316s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do
if ts[3]*ts[1]*ts[6] eq g*(ts[a]*ts[b])^h then a,b; break;
end if; end for; end for; end for; end for;
for g in IM do for h in IN do if ts[3]*ts[1]*ts[6] eq
g*(ts[3]*ts[4])^h then "true";
gg:=g; hh:=h; break; end if; end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N318:=Stabiliser(N,[3,1,8]);
SSS:=[3,1,8]; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[1]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N318s:=N318;
for g in N do if 3^g eq 2 and 1^g eq 1 and 8^g eq 9 then
N318s:=sub<N|N318s,g>;
end if; end for;

```

```

#N318s;
T318:=Transversal(N,N318s);
for i in [1..#T318] do
ss:=[3,1,8]^T318[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N318s);

for g in IM do if ts[3]*ts[1]*ts[8] eq g*(ts[2]*ts[1]*ts[9])
then "true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

N319:=Stabiliser(N,[3,1,9]);
SSS:={[3,1,9]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[1]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N319s:=N319;
T319:=Transversal(N,N319s);
for i in [1..#T319] do
ss:=[3,1,9]^T319[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N319s);

N3111:=Stabiliser(N,[3,1,11]);
SSS:={[3,1,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[1]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);

```

```

end if; end for; end for;
N3111s:=N3111;
T3111:=Transversal(N,N3111s);
for i in [1..#T3111] do
ss:=[3,1,11]^T3111[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3111s);

N3112:=Stabiliser(N,[3,1,12]);
SSS:={[3,1,12]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[1]*ts[12] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3112s:=N3112;
T3112:=Transversal(N,N3112s);
for i in [1..#T3112] do
ss:=[3,1,12]^T3112[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3112s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do
if ts[3]*ts[1]*ts[12] eq g*(ts[a]*ts[b])^h then a,b; break;
end if; end for; end for; end for; end for;
for g in IM do for h in IN do if ts[3]*ts[1]*ts[12] eq
g*(ts[9]*ts[4])^h then "true";
gg:=g; hh:=h; break; end if; end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N342:=Stabiliser(N,[3,4,2]);

```

```

SSS:={[3,4,2]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[4]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N342s:=N342;
T342:=Transversal(N,N342s);
for i in [1..#T342] do
ss:=[3,4,2]^T342[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N342s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do
if ts[3]*ts[4]*ts[2] eq g*(ts[a]*ts[b])^h then a,b; break;
end if; end for; end for; end for; end for;
for g in IM do for h in IN do if ts[3]*ts[4]*ts[2] eq
g*(ts[9]*ts[4])^h then "true";
gg:=g; hh:=h; break; end if; end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N343:=Stabiliser(N,[3,4,3]);
SSS:={[3,4,3]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[4]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N343s:=N343;
T343:=Transversal(N,N343s);
for i in [1..#T343] do

```



```

ss:=[3,4,3]^T343[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N343s);

for a in [2,5,8,9,11] do for g in IM do for h in IN do if
ts[3]*ts[4]*ts[3] eq g*(ts[3]*ts[1]*ts[a])^h then a; break;
end if; end for; end for; end for;
for g in IM do for h in IN do if ts[3]*ts[4]*ts[3] eq g*
(ts[3]*ts[1]*ts[9])^h then "true"; gg:=g; hh:=h; break;
end if; end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N345:=Stabiliser(N,[3,4,5]);
SSS:={[3,4,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[4]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N345s:=N345;
for g in N do if 3^g eq 3 and 4^g eq 5 and 5^g eq 4 then
N345s:=sub<N|N345s,g>; end if; end for;
#N345s;
T345:=Transversal(N,N345s);
for i in [1..#T345] do
ss:=[3,4,5]^T345[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N345s);

for g in IM do if ts[3]*ts[4]*ts[5] eq g*(ts[3]*ts[5]*ts[4])
then "true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do

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if ArrayP[i] eq gg then Sch[i]; end if; end for;

N346:=Stabiliser(N,[3,4,6]);
SSS:={[3,4,6]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[4]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N346s:=N346;
T346:=Transversal(N,N346s);
for i in [1..#T346] do
ss:=[3,4,6]^T346[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N346s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do for
h in IN do if ts[3]*ts[4]*ts[6] eq g*(ts[a]*ts[b])^h then a,b;
break; end if; end for; end for; end for; end for;
for g in IM do for h in IN do if ts[3]*ts[4]*ts[6] eq
g*(ts[9]*ts[4])^h
then "true"; gg:=g; hh:=h; break; end if; end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N348:=Stabiliser(N,[3,4,8]);
SSS:={[3,4,8]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[4]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N348s:=N348;

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```

T348:=Transversal(N,N348s);
for i in [1..#T348] do
ss:=[3,4,8]^T348[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N348s);

N349:=Stabiliser(N,[3,4,9]);
SSS:={[3,4,9]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[4]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N349s:=N349;
T349:=Transversal(N,N349s);
for i in [1..#T349] do
ss:=[3,4,9]^T349[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N349s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[3]*ts[4]*ts[9] eq g*(ts[a]*ts[b])^h then
a,b; break; end if; end for; end for; end for;
for g in IM do for h in IN do if ts[3]*ts[4]*ts[9] eq
g*(ts[3]*ts[1])^h then "true"; gg:=g; hh:=h; break; end if;
end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N3411:=Stabiliser(N,[3,4,11]);
SSS:={[3,4,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);

```

```

for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[4]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3411s:=N3411;
T3411:=Transversal(N,N3411s);
for i in [1..#T3411] do
ss:=[3,4,11]^T3411[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3411s);

N3412:=Stabiliser(N,[3,4,12]);
SSS:={[3,4,12]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[4]*ts[12] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3412s:=N3412;
T3412:=Transversal(N,N3412s);
for i in [1..#T3412] do
ss:=[3,4,12]^T3412[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3412s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[3]*ts[4]*ts[12] eq g*(ts[a]*ts[b])^h then
a,b; break; end if; end for; end for; end for; end for;
for g in IM do for h in IN do if ts[3]*ts[4]*ts[12] eq
g*(ts[12]*ts[10])^h then "true"; gg:=g; hh:=h; break; end if;
end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

```

```

for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N372:=Stabiliser(N,[3,7,2]);
SSS:={[3,7,2]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N372s:=N372;
for g in N do if 3^g eq 2 and 7^g eq 7 and 2^g eq 3 then
N372s:=sub<N|N372s,g>; end if; end for;
#N372s;
T372:=Transversal(N,N372s);
for i in [1..#T372] do
ss:=[3,7,2]^T372[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N372s);

for g in IM do if ts[3]*ts[7]*ts[2] eq g*(ts[2]*ts[7]*ts[3])
then "true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

N373:=Stabiliser(N, [3,7,3]);
SSS:={[3,7,3]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;

N373s:=N373;
for g in N do if 3^g eq 1 and 7^g eq 9 and 3^g eq 1 then

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```

N373s:=sub<N|N373s,g>; end if; end for;
#N373s;
T373:=Transversal(N,N373s);
for i in [1..#T373] do
ss:=[3,7,3]^T373[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N373s);

for g in IM do if ts[3]*ts[7]*ts[3] eq g*(ts[1]*ts[9]*ts[1])
then "true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

N375:=Stabiliser(N,[3,7,5]);
SSS:={[3,7,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N375s:=N375;
T375:=Transversal(N,N375s);
for i in [1..#T375] do
ss:=[3,7,5]^T375[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N375s);

N376:=Stabiliser(N,[3,7,6]);
SSS:={[3,7,6]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]

```

```

then print Rep(Seqq[i]);
end if; end for; end for;
N376s:=N376;
T376:=Transversal(N,N376s);
for i in [1..#T376] do
ss:=[3,7,6]^T376[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N376s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[3]*ts[7]*ts[6] eq g*(ts[a]*ts[b])^h then
a,b; break;
end if; end for; end for; end for; end for;
for g in IM do for h in IN do if ts[3]*ts[7]*ts[6] eq
g*(ts[12]*ts[7])^h then "true"; gg:=g; hh:=h; break; end if;
end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N378:=Stabiliser(N,[3,7,8]);
SSS:={[3,7,8]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N378s:=N378;
for g in N do if 3^g eq 2 and 7^g eq 7 and 8^g eq 9 then
N378s:=sub<N|N378s,g>; end if; end for;
#N378s;
T378:=Transversal(N,N378s);
for i in [1..#T378] do
ss:=[3,7,8]^T378[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []

```

```

then m:=m+1; end if; end for; m;
Orbits(N378s);

for g in IM do if ts[3]*ts[7]*ts[8] eq g*(ts[2]*ts[7]*ts[9])
then "true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

N379:=Stabiliser(N,[3,7,9]);
SSS:={[3,7,9]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N379s:=N379;
T379:=Transversal(N,N379s);
for i in [1..#T379] do
ss:=[3,7,9]^T379[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N379s);

N3711:=Stabiliser(N,[3,7,11]);
SSS:={[3,7,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3711s:=N3711;
T3711:=Transversal(N,N3711s);
for i in [1..#T3711] do
ss:=[3,7,11]^T3711[i];
cst[prodim(1, ts, ss)] := ss;
end for;

```



```

m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3711s);

for a in [1,4,7] do for b in [2,3,5,8,9,11] do for g in IM do
for h in IN do if ts[3]*ts[7]*ts[11] eq g*(ts[3]*ts[a]*ts[b])^h
then a,b; break;
end if; end for; end for; end for; end for;
for g in IM do for h in IN do if ts[3]*ts[7]*ts[11] eq
g*(ts[3]*ts[4]*ts[11])^h then "true"; gg:=g; hh:=h; break;
end if; end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N3712:=Stabiliser(N,[3,7,12]);
SSS:={[3,7,12]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7]*ts[12] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3712s:=N3712;
T3712:=Transversal(N,N3712s);
for i in [1..#T3712] do
ss:=[3,7,12]^T3712[i];
cst[prod(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3712s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[3]*ts[7]*ts[12] eq g*(ts[a]*ts[b])^h then
a,b; break;
end if; end for; end for; end for; end for;
for g in IM do for h in IN do if ts[3]*ts[7]*ts[12] eq
g*(ts[6]*ts[10])^h then "true"; gg:=g; hh:=h; break; end if;
end for; end for;
for i in [1..#NNN] do

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if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N3102:=Stabiliser(N,[3,10,2]);
SSS:={[3,10,2]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3102s:=N3102;
T3102:=Transversal(N,N3102s);
for i in [1..#T3102] do
ss:=[3,10,2]^T3102[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3102s);

N3103:=Stabiliser(N,[3,10,3]);
SSS:={[3,10,3]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3103s:=N3103;
T3103:=Transversal(N,N3103s);
for i in [1..#T3103] do
ss:=[3,10,3]^T3103[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3103s);

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N3105:=Stabiliser(N,[3,10,5]);
SSS:={[3,10,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3105s:=N3105;
T3105:=Transversal(N,N3105s);
for i in [1..#T3105] do
ss:=[3,10,5]^T3105[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3105s);

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N3106:=Stabiliser(N,[3,10,6]);
SSS:={[3,10,6]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3106s:=N3106;
for g in N do if 3^g eq 1 and 10^g eq 12 and 6^g eq 4 then
N3106s:=sub<N|N3106s,g>; end if; end for;
#N3106s;
T3106:=Transversal(N,N3106s);
for i in [1..#T3106] do
ss:=[3,10,6]^T3106[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;

```

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Orbits(N3106s);

for g in IM do if ts[3]*ts[10]*ts[6] eq g*(ts[1]*ts[12]*ts[4])
then "true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

N3108:=Stabiliser(N,[3,10,8]);
SSS:={[3,10,8]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3108s:=N3108;
T3108:=Transversal(N,N3108s);
for i in [1..#T3108] do
ss:=[3,10,8]^T3108[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3108s);

N3109:=Stabiliser(N,[3,10,9]);
SSS:={[3,10,9]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3109s:=N3109;
for g in N do if 3^g eq 1 and 10^g eq 12 and 9^g eq 7 then
N3109s:=sub<N|N3109s,g>; end if; end for;
#N3109s;
T3109:=Transversal(N,N3109s);
for i in [1..#T3109] do
ss:=[3,10,9]^T3109[i];

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```

cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3109s);

for g in IM do if ts[3]*ts[10]*ts[9] eq g*(ts[1]*ts[12]*ts[7])
then "true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

N31011:=Stabiliser(N,[3,10,11]);
SSS:={[3,10,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N31011s:=N31011;
T31011:=Transversal(N,N31011s);
for i in [1..#T31011] do
ss:=[3,10,11]^T31011[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N31011s);

N31012:=Stabiliser(N,[3,10,12]);
SSS:={[3,10,12]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[12] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N31012s:=N31012;
T31012:=Transversal(N,N31012s);
for i in [1..#T31012] do

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ss:=[3,10,12]^T31012[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N31012s);

N612:=Stabiliser(N,[6,1,2]);
SSS:=[6,1,2]; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[1]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N612s:=N612;
T612:=Transversal(N,N612s);
for i in [1..#T612] do
ss:=[6,1,2]^T612[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N612s);

N613:=Stabiliser(N,[6,1,3]);
SSS:=[6,1,3]; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[1]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N613s:=N613;
T613:=Transversal(N,N613s);
for i in [1..#T613] do
ss:=[6,1,3]^T613[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []

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```

then m:=m+1; end if; end for; m;
Orbits(N613s);

for a in [3,6] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12]
do for g in IM do for h in IN do if ts[6]*ts[1]*ts[3] eq
g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for;
for g in IM do for h in IN do if ts[6]*ts[1]*ts[3] eq
g*(ts[3]*ts[10]*ts[12])^h then "true"; gg:=g; hh:=h; break; end if;
end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N615:=Stabiliser(N,[6,1,5]);
SSS:={[6,1,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[1]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N615s:=N615;
T615:=Transversal(N,N615s);
for i in [1..#T615] do
ss:=[6,1,5]^T615[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N615s);

N616:=Stabiliser(N,[6,1,6]);
SSS:={[6,1,6]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[1]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]

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```

then print Rep(Seqq[i]);
end if; end for; end for;
N616s:=N616;
T616:=Transversal(N,N616s);
for i in [1..#T616] do
ss:=[6,1,6]^T616[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N616s);

N618:=Stabiliser(N,[6,1,8]);
SSS:={[6,1,8]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[1]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N618s:=N618;
T618:=Transversal(N,N618s);
for i in [1..#T618] do
ss:=[6,1,8]^T618[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N618s);

N619:=Stabiliser(N,[6,1,9]);
SSS:={[6,1,9]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[1]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N619s:=N619;
T619:=Transversal(N,N619s);

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for i in [1..#T619] do
ss:=[6,1,9]^T619[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N619s);

N6111:=Stabiliser(N,[6,1,11]);
SSS:={[6,1,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[1]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N6111s:=N6111;
T6111:=Transversal(N,N6111s);
for i in [1..#T6111] do
ss:=[6,1,11]^T6111[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N6111s);

N6112:=Stabiliser(N,[6,1,12]);
SSS:={[6,1,12]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[1]*ts[12] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N6112s:=N6112;
T6112:=Transversal(N,N6112s);
for i in [1..#T6112] do
ss:=[6,1,12]^T6112[i];
cst[prodim(1, ts, ss)] := ss;
end for;

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```

m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N6112s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[6]*ts[1]*ts[12] eq g*(ts[a]*ts[b])^h
then a,b; break;
end if; end for; end for; end for; end for;
for g in IM do for h in IN do if ts[6]*ts[1]*ts[12] eq
g*(ts[12]*ts[4])^h then "true"; gg:=g; hh:=h; break; end if;
end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N642:=Stabiliser(N,[6,4,2]);
SSS:={[6,4,2]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[4]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N642s:=N642;
for g in N do if 6^g eq 4 and 4^g eq 6 and 2^g eq 2 then
N642s:=sub<N|N642s,g>; end if; end for;
#N642s;
T642:=Transversal(N,N642s);
for i in [1..#T642] do
ss:=[6,4,2]^T642[i];
cst[prod(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N642s);

for a in [3,6] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12]
do for g in IM do for h in IN do if ts[6]*ts[4]*ts[2] eq
g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;

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```

for g in IM do for h in IN do if ts[6]*ts[4]*ts[2] eq
g*(ts[3]*ts[4]*ts[5])^h then "true"; gg:=g; hh:=h; break; end if;
end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N645:=Stabiliser(N,[6,4,5]);
SSS:={[6,4,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[4]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N645s:=N645;
for g in N do if 6^g eq 4 and 4^g eq 5 and 5^g eq 6 then
N645s:=sub<N|N645s,g>; end if; end for;
for g in N do if 6^g eq 6 and 4^g eq 5 and 5^g eq 4 then
N645s:=sub<N|N645s,g>; end if; end for;
for g in N do if 6^g eq 5 and 4^g eq 6 and 5^g eq 4 then
N645s:=sub<N|N645s,g>; end if; end for;
for g in N do if 6^g eq 5 and 4^g eq 4 and 5^g eq 6 then
N645s:=sub<N|N645s,g>; end if; end for;
for g in N do if 6^g eq 4 and 4^g eq 6 and 5^g eq 5 then
N645s:=sub<N|N645s,g>; end if; end for;
#N645s;
T645:=Transversal(N,N645s);
for i in [1..#T645] do
ss:=[6,4,5]^T645[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N645s);

for g in IM do if ts[6]*ts[4]*ts[5] eq g*(ts[4]*ts[5]*ts[6]) then
"true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for g in IM do if ts[6]*ts[4]*ts[5] eq g*(ts[6]*ts[5]*ts[4]) then

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"true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for g in IM do if ts[6]*ts[4]*ts[5] eq g*(ts[5]*ts[6]*ts[4]) then
"true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for g in IM do if ts[6]*ts[4]*ts[5] eq g*(ts[5]*ts[4]*ts[6]) then
"true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for g in IM do if ts[6]*ts[4]*ts[5] eq g*(ts[4]*ts[6]*ts[5]) then
"true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

N648:=Stabiliser(N,[6,4,8]);
SSS:={[6,4,8]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[4]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N648s:=N648;
for g in N do if 6^g eq 4 and 4^g eq 6 and 8^g eq 8 then
N648s:=sub<N|N648s,g>; end if; end for;
#N648s;
T648:=Transversal(N,N648s);
for i in [1..#T648] do
ss:=[6,4,8]^T648[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N648s);

for g in IM do if ts[6]*ts[4]*ts[8] eq g*(ts[4]*ts[6]*ts[8]) then
"true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;

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```

N6411:=Stabiliser(N,[6,4,11]);
SSS:={[6,4,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[4]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N6411s:=N6411;
for g in N do if 6^g eq 4 and 4^g eq 6 and 11^g eq 11 then
N6411s:=sub<N|N6411s,g>; end if; end for;
#N6411s;
T6411:=Transversal(N,N6411s);
for i in [1..#T6411] do
ss:=[6,4,11]^T6411[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N6411s);

for g in IM do if ts[6]*ts[4]*ts[11] eq g*(ts[4]*ts[6]*ts[11])
then "true"; gg:=g; break; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12]
do for g in IM do for h in IN do if ts[6]*ts[4]*ts[11] eq
g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for g in IM do for h in IN do if ts[6]*ts[4]*ts[11] eq
g*(ts[3]*ts[1]*ts[5])^h then "true"; gg:=g; hh:=h; break; end if;
end for; end for;
for i in [1..#NNN] do
if ArrayP[i] eq gg then Sch[i]; end if; end for;
for i in [1..#NNN] do
if ArrayP[i] eq hh then Sch[i]; end if; end for;

N672:=Stabiliser(N,[6,7,2]);
SSS:={[6,7,2]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);

```

```

for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[7]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N672s:=N672;
T672:=Transversal(N,N672s);
for i in [1..#T672] do
ss:=[6,7,2]^T672[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N672s);

```

```

N673:=Stabiliser(N,[6,7,3]);
SSS:={[6,7,3]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[7]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N673s:=N673;
T673:=Transversal(N,N673s);
for i in [1..#T673] do
ss:=[6,7,3]^T673[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N673s);

```

```

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12]
do for g in IM do for h in IN do if ts[6]*ts[7]*ts[3] eq
g*(ts[a]*ts[b]*ts[c])^h then a,b; break;
end if; end for; end for; end for; end for;

```

```

N675:=Stabiliser(N,[6,7,5]);
SSS:={[6,7,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[7]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N675s:=N675;
T675:=Transversal(N,N675s);
for i in [1..#T675] do
ss:=[6,7,5]^T675[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N675s);

N676:=Stabiliser(N,[6,7,6]);
SSS:={[6,7,6]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[7]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N676s:=N676;
T676:=Transversal(N,N676s);
for i in [1..#T676] do
ss:=[6,7,6]^T676[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N676s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for
h in IN do if ts[6]*ts[7]*ts[6] eq g*(ts[a]*ts[b]*ts[c])^h
then a,b,c; break;

```

```

end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[6]*ts[7]*ts[6] eq g*(ts[a]*ts[b])^h
then a,b; break;
end if; end for; end for; end for; end for;

N678:=Stabiliser(N,[6,7,8]);
SSS:={[6,7,8]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[7]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N678s:=N678;
for g in N do if 6^g eq 6 and 7^g eq 8 and 8^g eq 7 then
N678s:=sub<N|N678s,g>; end if; end for;
#N678s;
T678:=Transversal(N,N678s);
for i in [1..#T678] do
ss:=[6,7,8]^T678[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N678s);

N679:=Stabiliser(N,[6,7,9]);
SSS:={[6,7,9]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[7]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N679s:=N679;
T679:=Transversal(N,N679s);
for i in [1..#T679] do
ss:=[6,7,9]^T679[i];
cst[prodim(1, ts, ss)] := ss;

```



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end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N679s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[6]*ts[7]*ts[9] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do for
h in IN do if ts[6]*ts[7]*ts[9] eq g*(ts[a]*ts[b])^h
then a,b; break;
end if; end for; end for; end for; end for; end for;

N6711:=Stabiliser(N,[6,7,11]);
SSS:={[6,7,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[7]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N6711s:=N6711;
T6711:=Transversal(N,N6711s);
for i in [1..#T6711] do
ss:=[6,7,11]^T6711[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N6711s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[6]*ts[7]*ts[11] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c;
break; end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do for
h in IN do if ts[6]*ts[7]*ts[11] eq g*(ts[a]*ts[b])^h
then a,b; break;
end if; end for; end for; end for; end for; end for;

N6712:=Stabiliser(N,[6,7,12]);

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SSS:={[6,7,12]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[7]*ts[12] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N6712s:=N6712;
T6712:=Transversal(N,N6712s);
for i in [1..#T6712] do
ss:=[6,7,12]^T6712[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N6712s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[6]*ts[7]*ts[12] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM
do for h in IN do if ts[6]*ts[7]*ts[12] eq g*(ts[a]*ts[b])^h
then a,b; break;
end if; end for; end for; end for; end for;

N6102:=Stabiliser(N,[6,10,2]);
SSS:={[6,10,2]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[10]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N6102s:=N6102;
T6102:=Transversal(N,N6102s);
for i in [1..#T6102] do
ss:=[6,10,2]^T6102[i];
cst[prodim(1, ts, ss)] := ss;
end for;

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```

m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N6102s);

N6103:=Stabiliser(N,[6,10,3]);
SSS:={[6,10,3]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[10]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N6103s:=N6103;
T6103:=Transversal(N,N6103s);
for i in [1..#T6103] do
ss:=[6,10,3]^T6103[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N6103s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[6]*ts[10]*ts[3] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[6]*ts[10]*ts[3] eq g*(ts[a]*ts[b])^h
then a,b; break;
end if; end for; end for; end for; end for;

N6105:=Stabiliser(N,[6,10,5]);
SSS:={[6,10,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[10]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N6105s:=N6105;

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T6105:=Transversal(N,N6105s);
for i in [1..#T6105] do
ss:=[6,10,5]^T6105[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N6105s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[6]*ts[10]*ts[5] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[6]*ts[10]*ts[5] eq g*(ts[a]*ts[b])^h
then a,b; break;
end if; end for; end for; end for; end for;

N6106:=Stabiliser(N,[6,10,6]);
SSS:={[6,10,6]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[10]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N6106s:=N6106;
for g in N do if 6^g eq 4 and 10^g eq 12 and 6^g eq 4 then
N6106s:=sub<N|N6106s,g>; end if; end for;
#N6106s;
T6106:=Transversal(N,N6106s);
for i in [1..#T6106] do
ss:=[6,10,6]^T6106[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N6106s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[6]*ts[10]*ts[6] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;

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end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[6]*ts[10]*ts[6] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N6108:=Stabiliser(N,[6,10,8]);
SSS:={[6,10,8]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[10]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N6108s:=N6108;
T6108:=Transversal(N,N6108s);
for i in [1..#T6108] do
ss:=[6,10,8]^T6108[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N6108s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[6]*ts[10]*ts[8] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[6]*ts[10]*ts[8] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N6109:=Stabiliser(N,[6,10,9]);
SSS:={[6,10,9]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[10]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N6109s:=N6109;

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T6109:=Transversal(N,N6109s);
for i in [1..#T6109] do
ss:=[6,10,9]^T6109[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N6109s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[6]*ts[10]*ts[9] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[6]*ts[10]*ts[9] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N61011:=Stabiliser(N,[6,10,11]);
SSS:={[6,10,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[10]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N61011s:=N61011;
T61011:=Transversal(N,N61011s);
for i in [1..#T61011] do
ss:=[6,10,11]^T61011[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N61011s);

N61012:=Stabiliser(N,[6,10,12]);
SSS:={[6,10,12]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[6]*ts[10]*ts[12] eq

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n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N61012s:=N61012;
T61012:=Transversal(N,N61012s);
for i in [1..#T61012] do
ss:=[6,10,12]^T61012[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N61012s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[6]*ts[10]*ts[12] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[6]*ts[10]*ts[12] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N912:=Stabiliser(N,[9,1,2]);
SSS:={[9,1,2]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[1]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N912s:=N912;
T912:=Transversal(N,N912s);
for i in [1..#T912] do
ss:=[9,1,2]^T912[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N912s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[9]*ts[1]*ts[2] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;

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end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[1]*ts[2] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N915:=Stabiliser(N,[9,1,5]);
SSS:={[9,1,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[1]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N915s:=N915;
T915:=Transversal(N,N915s);
for i in [1..#T915] do
ss:=[9,1,5]^T915[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N915s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[9]*ts[1]*ts[5] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[1]*ts[5] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N916:=Stabiliser(N,[9,1,6]);
SSS:={[9,1,6]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[1]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;

```



```

N916s:=N916;
for g in N do if 9^g eq 7 and 1^g eq 3 and 6^g eq 4 then
N916s:=sub<N|N916s,g>; end if; end for;
#N916s;
T916:=Transversal(N,N916s);
for i in [1..#T916] do
ss:=[9,1,6]^T916[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N916s);

N918:=Stabiliser(N,[9,1,8]);
SSS:={[9,1,8]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[1]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N918s:=N918;
T918:=Transversal(N,N918s);
for i in [1..#T918] do
ss:=[9,1,8]^T918[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N918s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[9]*ts[1]*ts[8] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[1]*ts[8] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N919:=Stabiliser(N,[9,1,9]);
SSS:={[9,1,9]}; SSS:=SSS^N;
#(SSS);

```

```

Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[1]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N919s:=N919;
T919:=Transversal(N,N919s);
for i in [1..#T919] do
ss:=[9,1,9]^T919[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N919s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[9]*ts[1]*ts[9] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[1]*ts[9] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N9111:=Stabiliser(N,[9,1,11]);
SSS:={[9,1,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[1]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N9111s:=N9111;
T9111:=Transversal(N,N9111s);
for i in [1..#T9111] do
ss:=[9,1,11]^T9111[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N9111s);

```

```

N9112:=Stabiliser(N,[9,1,12]);
SSS:={[9,1,12]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[1]*ts[12] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N9112s:=N9112;
T9112:=Transversal(N,N9112s);
for i in [1..#T9112] do
ss:=[9,1,12]^T9112[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N9112s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[9]*ts[1]*ts[12] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[1]*ts[12] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N942:=Stabiliser(N,[9,4,2]);
SSS:={[9,4,2]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[4]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N942s:=N942;
for g in N do if 9^g eq 9 and 4^g eq 5 and 2^g eq 1 then
N942s:=sub<N|N942s,g>; end if; end for;
#N942s;
T942:=Transversal(N,N942s);

```

```

for i in [1..#T942] do
ss:=[9,4,2]^T942[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N942s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[9]*ts[4]*ts[2] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[4]*ts[2] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N943:=Stabiliser(N,[9,4,3]);
SSS:={[9,4,3]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[4]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N943s:=N943;
T943:=Transversal(N,N943s);
for i in [1..#T943] do
ss:=[9,4,3]^T943[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N943s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[9]*ts[4]*ts[3] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[4]*ts[3] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

```

```

N945:=Stabiliser(N,[9,4,5]);
SSS:={[9,4,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[4]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N945s:=N945;
T945:=Transversal(N,N945s);
for i in [1..#T945] do
ss:=[9,4,5]^T945[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N945s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[9]*ts[4]*ts[5] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[4]*ts[5] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N946:=Stabiliser(N,[9,4,6]);
SSS:={[9,4,6]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[4]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N946s:=N946;
T946:=Transversal(N,N946s);
for i in [1..#T946] do
ss:=[9,4,6]^T946[i];
cst[prodim(1, ts, ss)] := ss;
end for;

```

```

m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N946s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[9]*ts[4]*ts[6] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[4]*ts[6] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N949:=Stabiliser(N,[9,4,9]);
SSS:={[9,4,9]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[4]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N949s:=N949;
T949:=Transversal(N,N949s);
for i in [1..#T949] do
ss:=[9,4,9]^T949[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N949s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[9]*ts[4]*ts[9] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[4]*ts[9] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N975:=Stabiliser(N,[9,7,5]);
SSS:={[9,7,5]}; SSS:=SSS^N;
#(SSS);

```

```

Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[7]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N975s:=N975;
T975:=Transversal(N,N975s);
for i in [1..#T975] do
ss:=[9,7,5]^T975[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N975s);

N978:=Stabiliser(N,[9,7,8]);
SSS:={[9,7,8]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[7]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N978s:=N978;
for g in N do if 9^g eq 9 and 7^g eq 8 and 8^g eq 7 then
N978s:=sub<N|N978s,g>; end if; end for;
#N978s;
T978:=Transversal(N,N978s);
for i in [1..#T978] do
ss:=[9,7,8]^T978[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N978s);

N979:=Stabiliser(N,[9,7,9]);
SSS:={[9,7,9]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);

```

```

for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[7]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N979s:=N979;
T979:=Transversal(N,N979s);
for i in [1..#T979] do
ss:=[9,7,9]^T979[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N979s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[9]*ts[7]*ts[9] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[7]*ts[9] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N9711:=Stabiliser(N,[9,7,11]);
SSS:={[9,7,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[7]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N9711s:=N9711;
T9711:=Transversal(N,N9711s);
for i in [1..#T9711] do
ss:=[9,7,11]^T9711[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N9711s);

```



```

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[9]*ts[7]*ts[11] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[7]*ts[11] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

```

```

N9102:=Stabiliser(N,[9,10,2]);
SSS:={[9,10,2]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[10]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N9102s:=N9102;
for g in N do if 9^g eq 7 and 10^g eq 12 and 2^g eq 2 then
N9102s:=sub<N|N9102s,g>; end if; end for;
#N9102s;
T9102:=Transversal(N,N9102s);
for i in [1..#T9102] do
ss:=[9,10,2]^T9102[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N9102s);

```

```

N9105:=Stabiliser(N,[9,10,5]);
SSS:={[9,10,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[10]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N9105s:=N9105;
for g in N do if 9^g eq 7 and 10^g eq 12 and 5^g eq 5 then
N9105s:=sub<N|N9105s,g>; end if; end for;

```

```

#N9105s;
T9105:=Transversal(N,N9105s);
for i in [1..#T9105] do
ss:=[9,10,5]^T9105[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N9105s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[9]*ts[10]*ts[5] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[10]*ts[5] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N9108:=Stabiliser(N,[9,10,8]);
SSS:={[9,10,8]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[10]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N9108s:=N9108;
for g in N do if 9^g eq 7 and 10^g eq 12 and 8^g eq 8 then
N9108s:=sub<N|N9108s,g>; end if; end for;
#N9108s;
T9108:=Transversal(N,N9108s);
for i in [1..#T9108] do
ss:=[9,10,8]^T9108[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N9108s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[9]*ts[10]*ts[8] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;

```

```

end if; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[9]*ts[10]*ts[8] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[12]*ts[1]*ts[8] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do for
h in IN do if ts[12]*ts[1]*ts[8] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N1218:=Stabiliser(N,[12,1,8]);
SSS:={[12,1,8]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[1]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N1218s:=N1218;
T1218:=Transversal(N,N1218s);
for i in [1..#T1218] do
ss:=[12,1,8]^T1218[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1218s);

N12111:=Stabiliser(N,[12,1,11]);
SSS:={[12,1,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[1]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N12111s:=N12111;

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```

T12111:=Transversal(N,N12111s);
for i in [1..#T12111] do
ss:=[12,1,11]^T12111[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N12111s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[12]*ts[1]*ts[11] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in
IM do for h in IN do if ts[12]*ts[1]*ts[11] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N1243:=Stabiliser(N,[12,4,3]);
SSS:={[12,4,3]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[4]*ts[3] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N1243s:=N1243;
T1243:=Transversal(N,N1243s);
for i in [1..#T1243] do
ss:=[12,4,3]^T1243[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1243s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[12]*ts[4]*ts[3] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM
do for h in IN do if ts[12]*ts[4]*ts[3] eq g*(ts[a]*ts[b])^h

```

```

then a,b; break; end if; end for; end for; end for; end for;

N1245:=Stabiliser(N,[12,4,5]);
SSS:={[12,4,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[4]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N1245s:=N1245;
T1245:=Transversal(N,N1245s);
for i in [1..#T1245] do
ss:=[12,4,5]^T1245[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1245s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[12]*ts[4]*ts[5] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[12]*ts[4]*ts[5] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N1246:=Stabiliser(N,[12,4,6]);
SSS:={[12,4,6]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[4]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N1246s:=N1246;
T1246:=Transversal(N,N1246s);
for i in [1..#T1246] do

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```

ss:=[12,4,6]^T1246[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1246s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[12]*ts[4]*ts[6] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[12]*ts[4]*ts[6] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N12411:=Stabiliser(N,[12,4,11]);
SSS:={[12,4,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[4]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N12411s:=N12411;
T12411:=Transversal(N,N12411s);
for i in [1..#T12411] do
ss:=[12,4,11]^T12411[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N12411s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do
if ts[12]*ts[7]*ts[5] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[12]*ts[7]*ts[5] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N1275:=Stabiliser(N,[12,7,5]);

```

```

SSS:={[12,7,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[7]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N1275s:=N1275;
T1275:=Transversal(N,N1275s);
for i in [1..#T1275] do
ss:=[12,7,5]^T1275[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1275s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN
do if ts[12]*ts[7]*ts[8] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM
do for h in IN do if ts[12]*ts[7]*ts[8] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N1278:=Stabiliser(N,[12,7,8]);
SSS:={[12,7,8]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[7]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N1278s:=N1278;
for g in N do if 12^g eq 12 and 7^g eq 8 and 8^g eq 7 then
N1278s:=sub<N|N1278s,g>; end if; end for;
#N1278s;
T1278:=Transversal(N,N1278s);
for i in [1..#T1278] do
ss:=[12,7,8]^T1278[i];

```

```

cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N1278s);

for a in [3,6,9,12] do for b in [1,4,7,10] do for
c in [2,3,5,6,8,9,11,12] do for g in IM do for h in IN do if
ts[12]*ts[10]*ts[5] eq g*(ts[a]*ts[b]*ts[c])^h then a,b,c; break;
end if; end for; end for; end for; end for; end for;
for a in [3,6,9,12] do for b in [1,4,7,10] do for g in IM do
for h in IN do if ts[12]*ts[10]*ts[5] eq g*(ts[a]*ts[b])^h
then a,b; break; end if; end for; end for; end for; end for;

N12105:=Stabiliser(N,[12,10,5]);
SSS:={[12,10,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[10]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]
then print Rep(Seqq[i]);
end if; end for; end for;
N12105s:=N12105;
for g in N do if 12^g eq 11 and 10^g eq 10 and 5^g eq 6 then
N12105s:=sub<N|N12105s,g>; end if; end for;
#N12105s;
T12105:=Transversal(N,N12105s);
for i in [1..#T12105] do
ss:=[12,10,5]^T12105[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N12105s);

/*                               */
/*  WORDS OF LENGTH FOUR  */
/*                               */

N3127:=Stabiliser(N,[3,1,2,7]);
SSS:={[3,1,2,7]}; SSS:=SSS^N;

```



```

#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[1]*ts[2]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3127s:=N3127;
T3127:=Transversal(N,N3127s);
for i in [1..#T3127] do
ss:=[3,1,2,7]^T3127[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3127s);

N3486:=Stabiliser(N,[3,4,8,6]);
SSS:=[3,4,8,6]; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[4]*ts[8]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3486s:=N3486;
T3486:=Transversal(N,N3486s);
for i in [1..#T3486] do
ss:=[3,4,8,6]^T3486[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3486s);

N3487:=Stabiliser(N,[3,4,8,7]);
SSS:=[3,4,8,7]; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);

```

```

for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[4]*ts[8]*ts[7] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3487s:=N3487;
T3487:=Transversal(N,N3487s);
for i in [1..#T3487] do
ss:=[3,4,8,7]^T3487[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3487s);

N3489:=Stabiliser(N,[3,4,8,9]);
SSS:=[3,4,8,9]; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[4]*ts[8]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3489s:=N3489;
T3489:=Transversal(N,N3489s);
for i in [1..#T3489] do
ss:=[3,4,8,9]^T3489[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3489s);

N3721:=Stabiliser(N,[3,7,2,1]);
SSS:=[3,7,2,1]; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do

```

```

if ts[3]*ts[7]*ts[2]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3721s:=N3721;
T3721:=Transversal(N,N3721s);
for i in [1..#T3721] do
ss:=[3,7,2,1]^T3721[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3721s);

N3732:=Stabiliser(N,[3,7,3,2]);
SSS:={[3,7,3,2]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7]*ts[3]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3732s:=N3732;
T3732:=Transversal(N,N3732s);
for i in [1..#T3732] do
ss:=[3,7,3,2]^T3732[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3732s);

N37311:=Stabiliser(N,[3,7,3,11]);
SSS:={[3,7,3,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7]*ts[3]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*

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```

ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N37311s:=N37311;
T37311:=Transversal(N,N37311s);
for i in [1..#T37311] do
ss:=[3,7,3,11]^T37311[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N37311s);

N3795:=Stabiliser(N,[3,7,9,5]);
SSS:=[3,7,9,5]; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7]*ts[9]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3795s:=N3795;
T395:=Transversal(N,N3795s);
for i in [1..#T3795] do
ss:=[3,7,9,5]^T3795[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3795s);

N3798:=Stabiliser(N,[3,7,9,8]);
SSS:=[3,7,9,8]; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7]*ts[9]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);

```

```

end if; end for; end for;
N3798s:=N3798;
T3798:=Transversal(N,N3798s);
for i in [1..#T3798] do
ss:=[3,7,9,8]^T3798[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3798s);

N37911:=Stabiliser(N,[3,7,9,11]);
SSS:={[3,7,9,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[7]*ts[9]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N37911s:=N37911;
T37911:=Transversal(N,N37911s);
for i in [1..#T37911] do
ss:=[3,7,9,11]^T37911[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N37911s);

N310210:=Stabiliser(N,[3,10,2,10]);
SSS:={[3,10,2,10]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[2]*ts[10] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N310210s:=N310210;

```

```

T310210:=Transversal(N,N310210s);
for i in [1..#T310210] do
ss:=[3,10,2,10]^T310210[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N310210s);

N31038:=Stabiliser(N,[3,10,3,8]);
SSS:={[3,10,3,8]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[3]*ts[8] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N31038s:=N31038;
T31038:=Transversal(N,N31038s);
for i in [1..#T31038] do
ss:=[3,10,3,8]^T31038[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N31038s);

N310311:=Stabiliser(N,[3,10,3,11]);
SSS:={[3,10,3,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[3]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N310311s:=N310311;
T310311:=Transversal(N,N310311s);
for i in [1..#T310311] do

```

```

ss:=[3,10,3,11]^T310311[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N310311s);

N31051:=Stabiliser(N,[3,10,5,1]);
SSS:={[3,10,5,1]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[5]*ts[1] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N31051s:=N31051;
T31051:=Transversal(N,N31051s);
for i in [1..#T31051] do
ss:=[3,10,5,1]^T31051[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N31051s);

N31054:=Stabiliser(N,[3,10,5,4]);
SSS:={[3,10,5,4]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[5]*ts[4] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N31054s:=N31054;
T31054:=Transversal(N,N31054s);
for i in [1..#T31054] do
ss:=[3,10,5,4]^T31054[i];
cst[prodim(1, ts, ss)] := ss;

```

```

end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N31054s);

N31056:=Stabiliser(N,[3,10,5,6]);
SSS:={[3,10,5,6]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[5]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N31056s:=N31056;
T31056:=Transversal(N,N31056s);
for i in [1..#T31056] do
ss:=[3,10,5,6]^T31056[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N31056s);

N31059:=Stabiliser(N,[3,10,5,9]);
SSS:={[3,10,5,9]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[5]*ts[9] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N31059s:=N31059;
T31059:=Transversal(N,N31059s);
for i in [1..#T31059] do
ss:=[3,10,5,9]^T31059[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []

```



```

then m:=m+1; end if; end for; m;
Orbits(N31059s);

```

```

N310512:=Stabiliser(N,[3,10,5,12]);
SSS:={[3,10,5,12]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[5]*ts[12] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N310512s:=N310512;
T310512:=Transversal(N,N310512s);
for i in [1..#T310512] do
ss:=[3,10,5,12]^T310512[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N310512s);

```

```

N31062:=Stabiliser(N,[3,10,6,2]);
SSS:={[3,10,6,2]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[6]*ts[2] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N31062s:=N31062;
T31062:=Transversal(N,N31062s);
for i in [1..#T31062] do
ss:=[3,10,6,2]^T31062[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;

```

```

Orbits(N31062s);

N310611:=Stabiliser(N,[3,10,6,11]);
SSS:={[3,10,6,11]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[6]*ts[11] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N310611s:=N310611;
T310611:=Transversal(N,N310611s);
for i in [1..#T310611] do
ss:=[3,10,6,11]^T310611[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N310611s);

N310125:=Stabiliser(N,[3,10,12,5]);
SSS:={[3,10,12,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[12]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N310125s:=N310125;
T310125:=Transversal(N,N310125s);
for i in [1..#T310125] do
ss:=[3,10,12,5]^T310125[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N310125s);

```

```

N3101210:=Stabiliser(N,[3,10,12,10]);
SSS:={[3,10,12,10]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[3]*ts[10]*ts[12]*ts[10] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N3101210s:=N3101210;
T3101210:=Transversal(N,N3101210s);
for i in [1..#T3101210] do
ss:=[3,10,12,10]^T3101210[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N3101210s);

N9165:=Stabiliser(N,[9,1,6,5]);
SSS:={[9,1,6,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[1]*ts[6]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N9165s:=N9165;
T9165:=Transversal(N,N9165s);
for i in [1..#T9165] do
ss:=[9,1,6,5]^T9165[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N9165s);

N9756:=Stabiliser(N,[9,7,5,6]);
SSS:={[9,7,5,6]}; SSS:=SSS^N;

```

```

#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[9]*ts[7]*ts[5]*ts[6] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N9756s:=N9756;
T9756:=Transversal(N,N9756s);
for i in [1..#T9756] do
ss:=[9,7,5,6]^T9756[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N9756s);

N12185:=Stabiliser(N,[12,1,8,5]);
SSS:={[12,1,8,5]}; SSS:=SSS^N;
#(SSS);
Seqq:=Setseq(SSS);
for i in [1..#SSS] do
for n in IM do
if ts[12]*ts[1]*ts[8]*ts[5] eq
n*ts[Rep(Seqq[i])[1]]*ts[Rep(Seqq[i])[2]]*ts[Rep(Seqq[i])[3]]*
ts[Rep(Seqq[i])[4]]
then print Rep(Seqq[i]);
end if; end for; end for;
N12185s:=N12185;
T12185:=Transversal(N,N12185s);
for i in [1..#T12185] do
ss:=[12,1,8,5]^T12185[i];
cst[prodim(1, ts, ss)] := ss;
end for;
m:=0; for i in [1..396] do if cst[i] ne []
then m:=m+1; end if; end for; m;
Orbits(N12185s);

```

Appendix C

MAGMA Code: Progenitor

$$7^*5 :_m A_5$$

```

S:=Alt(5);
xx:=S!(1,2)(3,4);
yy:=S!(1,3,5);
G:=sub<S|xx,yy>;
H:=sub<G|(1,2)(3,4),(1,2,3)>;
C:=Classes(G);
Cprime:=Classes(H);
CT:=CharacterTable(G);
ch:=CharacterTable(H);
CT, ch;
I:=Induction(ch[2],G);
I;
I eq CT[5];
T:=Transversal(G,H);

//Matrix Code for third roots of unity
//INPUTS

char:=2; //Character of H

w:=RootOfUnity(3);
M:=GL(#T,7);
for m in [1..NumberOfGenerators(G)] do
AA:= [0:i in [1..(#T)^2]];
for j in [1..#T] do

```

```

for i in [1..#T] do if T[j]*(G.m)*T[i]^-1 in H then
if ch[char](T[j]*(G.m)*T[i]^-1) eq w then AA[#T*(j-1)+i]:=2; end if;
if ch[char](T[j]*(G.m)*T[i]^-1) eq w^2 then AA[#T*(j-1)+i]:=4; end if;
if ch[char](T[j]*(G.m)*T[i]^-1) eq 1 then AA[#T*(j-1)+i]:=1; end if;
end if; end for;end for;
"MATRIX G.",m; M!AA; end for;

S:=Sym(30);
kx:=S!(2,4)(3,5)(7,9)(8,10)(12,14)(13,15)(17,19)(18,20)(22,24)(23,25)
(27,29)(28,30);
ky:=S!(1,2,3)(6,7,8)(11,12,13)(16,17,18)(21,22,23)(26,27,28)(4,9,19)
(5,20,10)(14,29,24)(15,25,30);
K:=sub<S|kx,ky>;
s:=IsIsomorphic(G,K);
s;

```

Appendix D

MAGMA Code: Composition

Factors for $2^{*14} : D_{28}$

```

a:=0; b:=4; c:=4; d:=0; e:=3; f:=2;
G<x,y,t>:=Group<x,y,t|x^14,y^2,(x*y)^2,t^2,(t,y),(x*t)^a,(x*y*t^x)^b,
(x^2*y*t*t^x)^c,(t*t^x*t^(x^3))^d,(x*t^x)^e,(y*t)^f>;
#G;

f,G1,k:=CosetAction(G,sub<G|Id(G)>);
#k;
CompositionFactors(G1);
NL:=NormalLattice(G1);
NL;
Center(G1);
Center(G1) eq NL[2];
Q,ff:=quo<G1|NL[2]>;
CompositionFactors(Q);
s,t:=IsIsomorphic(Q,PGL(2,13));
s;

//-----

a:=0; b:=6; c:=6; d:=3; e:=3; f:=2;
G<x,y,t>:=Group<x,y,t|x^14,y^2,(x*y)^2,t^2,(t,y),(x*t)^a,(x*y*t^x)^b,
(x^2*y*t*t^x)^c,(t*t^x*t^(x^3))^d,(x*t^x)^e,(y*t)^f>;
#G;

f,G1,k:=CosetAction(G,sub<G|Id(G)>);

```

```

#k;
CompositionFactors(G1);
NL:=NormalLattice(G1);
NL;
Center(G1);
Center(G1) eq NL[2];
Q,ff:=quo<G1|NL[2]>;
CompositionFactors(Q);

//-----

a:=3; b:=7; c:=7; d:=3; e:=3; f:=2;
G<x,y,t>:=Group<x,y,t|x^14,y^2,(x*y)^2,t^2,(t,y),(x*t)^a,(x*y*t^x)^b,
(x^2*y*t*t^x)^c,(t*t^x*t^(x^3))^d,(x*t^x)^e,(y*t)^f>;
#G;

f,G1,k:=CosetAction(G,sub<G|Id(G)>);
#k;
CompositionFactors(G1);

//-----

a:=0; b:=0; c:=8; d:=3; e:=3; f:=2;
G<x,y,t>:=Group<x,y,t|x^14,y^2,(x*y)^2,t^2,(t,y),(x*t)^a,(x*y*t^x)^b,
(x^2*y*t*t^x)^c,(t*t^x*t^(x^3))^d,(x*t^x)^e,(y*t)^f>;
#G;

f,G1,k:=CosetAction(G,sub<G|Id(G)>);
#k;
CompositionFactors(G1);
s,t:=IsIsomorphic(G1,PGL(2,7));
s;

//-----

a:=0; b:=9; c:=9; d:=3; e:=3; f:=2;
G<x,y,t>:=Group<x,y,t|x^14,y^2,(x*y)^2,t^2,(t,y),(x*t)^a,(x*y*t^x)^b,
(x^2*y*t*t^x)^c,(t*t^x*t^(x^3))^d,(x*t^x)^e,(y*t)^f>;
#G;

f,G1,k:=CosetAction(G,sub<G|Id(G)>);
#k;
CompositionFactors(G1);

```



```
//-----

a:=0; b:=10; c:=10; d:=3; e:=3; f:=2;
G<x,y,t>:=Group<x,y,t|x^14,y^2,(x*y)^2,t^2,(t,y),(x*t)^a,(x*y*t^x)^b,
(x^2*y*t*t^x)^c,(t*t^x*t^(x^3))^d,(x*t^x)^e,(y*t)^f>;
#G;

f,G1,k:=CosetAction(G,sub<G|Id(G)>);
#k;
CompositionFactors(G1);
s,t:=IsIsomorphic(G1,PGL(2,29));
s;

//-----

a:=0; b:=5; c:=3; d:=7; e:=5; f:=2;
G<x,y,t>:=Group<x,y,t|x^14,y^2,(x*y)^2,t^2,(t,y),(x*t)^a,(x*y*t^x)^b,
(x^2*y*t*t^x)^c,(t*t^x*t^(x^3))^d,(x*t^x)^e,(y*t)^f>;
#G;

f,G1,k:=CosetAction(G,sub<G|Id(G)>);
#k;
CompositionFactors(G1);
```

Appendix E

MAGMA Code: Composition

Factors for

$$2^{*16} : 2^{\bullet}(((2 \times 2) : 3) : 2)$$

```

r1:=0;r2:=3;r3:=15;r4:=4;r5:=3;
G<a,b,c,d,e,t>:=Group<a,b,c,d,e,t|a^4,c^4,d^4,e^2,a^-2*e,c^-1*a^2*c^-1,
    d^-1*a^2*d^-1,b^3*e,b*c^-1*b^-1*d,a^-1*d^-1*a*d^-1,
    c^-1*d^-1*c*d^-1,a^-1*b^-1*a^-1*b^-1*c,t^2,(t,c*b^-1),
    (c*t*t^d)^r1,
    (b*t*t^a)^r2,
    (b*t*t^(c*a))^r3,
    (b*t^a)^r4,
    (a*t^c)^r5>;

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c,d,e>);
#k;
CompositionFactors(G1);

NL:=NormalLattice(G1);
NL;

MinimalNormalSubgroups(G1);

D:=DirectProduct(NL[2],NL[3]);
s,t:=IsIsomorphic(D,NL[4]);
s;

```

```

H<a,b,c>:=Group<a,b,c|a^3,b^2,c^3,(b*c)^5,(a,b),(a,c)>;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,NL[4]);
s;

for i in G1 do if i notin NL[4] and Order(i) eq 2 and sub<G1|NL[4],i>
eq G1 then D:=i; end if; end for;

A:=t(f1(a));
B:=t(f1(b));
C:=t(f1(c));
N:=sub<G1|A,B,C>;

Sch:=SchreierSystem(H,sub<H|Id(H)>);
ArrayP:=[Id(N): i in [1..#N]];
for i in [2..#N] do
P:=[Id(N): 1 in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;
if Eltseq(Sch[i])[j] eq 3 then P[j]:=C; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
if Eltseq(Sch[i])[j] eq -3 then P[j]:=C^-1; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;

for i in [1..#N] do
if ArrayP[i] eq A^D then Sch[i]; end if; end for;

for i in [1..#N] do
if ArrayP[i] eq B^D then Sch[i]; end if; end for;

for i in [1..#N] do
if ArrayP[i] eq C^D then Sch[i]; end if; end for;

H<a,b,c,d>:=Group<a,b,c,d|a^3,b^2,c^3,(b*c)^5,(a,b),(a,c),d^2,a^d=a^-1,
(b,d),c^d=c^-1*b*c*b*c^-1*b>;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,G1);

```

```

s;

//-----

r1:=0;r2:=0;r3:=4;r4:=4;r5:=4;
G<a,b,c,d,e,t>:=Group<a,b,c,d,e,t|a^4,c^4,d^4,e^2,a^-2*e,c^-1*a^2*c^-1,
    d^-1*a^2*d^-1,b^3*e,b*c^-1*b^-1*d,a^-1*d^-1*a*d^-1,
    c^-1*d^-1*c*d^-1,a^-1*b^-1*a^-1*b^-1*c,t^2,(t,c*b^-1),
    (c*t*t^d)^r1,
    (b*t*t^a)^r2,
    (b*t*t^(c*a))^r3,
    (b*t^a)^r4,
    (a*t^c)^r5>;

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c,d,e>);
#k;
CompositionFactors(G1);

NL:=NormalLattice(G1);
NL;

Center(G1);

MinimalNormalSubgroups(G1);

D:=DirectProduct(NL[3],NL[4]);
s,t:=IsIsomorphic(D,NL[6]);
s;

H<a,b,c,d>:=Group<a,b,c,d|a^2,b^2,(a,b),c^2,d^4,(c*d)^5,(c*d^2)^5,
    (a,c),(a,d),(b,c),(b,d)>;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,NL[6]);
s;

Q,ff:=quo<G1|NL[3]>;
s,t:=IsIsomorphic(Q,Sym(6));
s;

```

```

FPGroup(Sym(6));

H<a,b>:=Group<a,b|a^6,b^2,(b*a^-1)^5,(a*b*a^-2*b*a)^2,(a^-1*b*a*b)^3>;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,Q);
s;

T:=Transversal(G1,NL[3]);
A:=t(f1(a));
B:=t(f1(b));

for i in [1..#T] do if ff(T[i]) eq A then i; end if; end for;
for i in [1..#T] do if ff(T[i]) eq B then i; end if; end for;

A:=T[536];
B:=T[561];
C:=NL[3].2;
D:=NL[3].3;

for i,j in [0..1] do if A^6 eq C^i*D^j then i,j; end if; end for;
for i,j in [0..1] do if B^2 eq C^i*D^j then i,j; end if; end for;
for i,j in [0..1] do if (B*A^-1)^5 eq C^i*D^j then i,j; end if;
end for;
for i,j in [0..1] do if (A*B*A^-2*B*A)^2 eq C^i*D^j then i,j;
end if; end for;
for i,j in [0..1] do if (A^-1*B*A*B)^3 eq C^i*D^j then i,j;
end if; end for;

for k in [0..5] do for i,j,l in [0..1] do if C^A eq C^i*D^j*A^k*B^l
then k,l,i,j; end if; end for; end for;
for k in [0..5] do for i,j,l in [0..1] do if C^B eq C^i*D^j*A^k*B^l
then k,l,i,j; end if; end for; end for;
for k in [0..5] do for i,j,l in [0..1] do if D^A eq C^i*D^j*A^k*B^l
then k,l,i,j; end if; end for; end for;
for k in [0..5] do for i,j,l in [0..1] do if D^B eq C^i*D^j*A^k*B^l
then k,l,i,j; end if; end for; end for;

H<a,b,c,d>:=Group<a,b,c,d|a^6=d,b^2,c^2,d^2,(b*a^-1)^5=c*d,
(a*b*a^-2*b*a)^2,(a^-1*b*a*b)^3=d,c^a=c*d,c^b=c*d,(a,d),(b,d)>;
#H;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,G1);

```

```

s;

//-----

r1:=0;r2:=0;r3:=0;r4:=4;r5:=5;
G<a,b,c,d,e,t>:=Group<a,b,c,d,e,t|a^4,c^4,d^4,e^2,a^-2*e,c^-1*a^2*c^-1,
    d^-1*a^2*d^-1,b^3*e,b*c^-1*b^-1*d,a^-1*d^-1*a*d^-1,
    c^-1*d^-1*c*d^-1,a^-1*b^-1*a^-1*b^-1*c,t^2,(t,c*b^-1),
    (c*t*t^d)^r1,
    (b*t*t^a)^r2,
    (b*t*t^(c*a))^r3,
    (b*t^a)^r4,
    (a*t^c)^r5>;

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c,d,e>);
#k;
CompositionFactors(G1);

NL:=NormalLattice(G1);
NL;

Center(G1);

A:=NL[2].2;

for i in NL[3] do if Order(i) eq 2 and i notin NL[2] and sub<G1|NL[2],i>
eq NL[3] then B:=i; end if; end for;

for i,j in [0..4] do if A^B eq A^i*B^j then i,j; end if; end for;

H<a,b>:=Group<a,b|a^5,b^2,a^b=a^4>;
#H;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,NL[3]);
s;

FPGroup(NL[4]);

H<a,b,c,d>:=Group<a,b,c,d|a^5,b^2,a^b=a^4,c^3,d^6,(a,c),(a,d),(b,c),
(b,d),(c^-1*d*c^-1*d^-1*c*d*c*d^-1),(c*d^-2*c^-1*d^-2)^2,
(c^-1*d^2*c^-1*d^-2)^2,(d^-1*c^-1)^7>;
#H;

```

```

f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,G1);
s;

//-----

r1:=0;r2:=4;r3:=7;r4:=4;r5:=5;
G<a,b,c,d,e,t>:=Group<a,b,c,d,e,t|a^4,c^4,d^4,e^2,a^-2*e,c^-1*a^2*c^-1,
    d^-1*a^2*d^-1,b^3*e,b*c^-1*b^-1*d,a^-1*d^-1*a*d^-1,
    c^-1*d^-1*c*d^-1,a^-1*b^-1*a^-1*b^-1*c,t^2,(t,c*b^-1),
    (c*t*t^d)^r1,
    (b*t*t^a)^r2,
    (b*t*t^(c*a))^r3,
    (b*t^a)^r4,
    (a*t^c)^r5>;

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c,d,e>);
#k;
CompositionFactors(G1);

NL:=NormalLattice(G1);
NL;

D:=DirectProduct(NL[2],NL[3]);
s,t:=IsIsomorphic(D,G1);
s;

//-----

r1:=0;r2:=5;r3:=0;r4:=4;r5:=4;
G<a,b,c,d,e,t>:=Group<a,b,c,d,e,t|a^4,c^4,d^4,e^2,a^-2*e,c^-1*a^2*c^-1,
    d^-1*a^2*d^-1,b^3*e,b*c^-1*b^-1*d,a^-1*d^-1*a*d^-1,
    c^-1*d^-1*c*d^-1,a^-1*b^-1*a^-1*b^-1*c,t^2,(t,c*b^-1),
    (c*t*t^d)^r1,
    (b*t*t^a)^r2,
    (b*t*t^(c*a))^r3,
    (b*t^a)^r4,
    (a*t^c)^r5>;

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c,d,e>);

```

```

#k;
CompositionFactors(G1);

NL:=NormalLattice(G1);
NL;

s,t:=IsIsomorphic(NL[4],Sym(6));
s;

D:=DirectProduct(NL[2],NL[4]);
s,t:=IsIsomorphic(D,G1);
s;

//-----

r1:=0;r2:=6;r3:=5;r4:=0;r5:=4;
G<a,b,c,d,e,t>:=Group<a,b,c,d,e,t|a^4,c^4,d^4,e^2,a^-2*e,c^-1*a^2*c^-1,
    d^-1*a^2*d^-1,b^3*e,b*c^-1*b^-1*d,a^-1*d^-1*a*d^-1,
    c^-1*d^-1*c*d^-1,a^-1*b^-1*a^-1*b^-1*c,t^2,(t,c*b^-1),
    (c*t*t^d)^r1,
    (b*t*t^a)^r2,
    (b*t*t^(c*a))^r3,
    (b*t^a)^r4,
    (a*t^c)^r5>;

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c,d,e>);
#k;
CompositionFactors(G1);

NL:=NormalLattice(G1);
NL;

D:=DirectProduct(Alt(5),Alt(5));
s,t:=IsIsomorphic(D,NL[2]);
s;

H<a,b,c,d>:=Group<a,b,c,d|a^3,b^3,(a*b^-1*a^-1*b^-1)^2,
    (a^-1*b*a^-1*b^-1)^2,c^3,d^3,(c*d^-1*c^-1*d^-1)^2,
    (c^-1*d*c^-1*d^-1)^2,(a,c),(a,d),(b,c),(b,d)>;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,NL[2]);
s;

```



```

A:=t(f1(a));
B:=t(f1(b));
C:=t(f1(c));
D:=t(f1(d));
N:=sub<G1|A,B,C,D>;
#N;

for i in NL[3] do if i notin NL[2] and Order(i) eq 2 and sub<G1|NL[2],i>
eq NL[3] then E:=i; end if; end for;
Order(E);

Sch:=SchreierSystem(H,sub<H|Id(H)>);
ArrayP:=[Id(N): i in [1..#N]];
for i in [2..#N] do
P:=[Id(N): 1 in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;
if Eltseq(Sch[i])[j] eq 3 then P[j]:=C; end if;
if Eltseq(Sch[i])[j] eq 4 then P[j]:=D; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=B^-1; end if;
if Eltseq(Sch[i])[j] eq -3 then P[j]:=C^-1; end if;
if Eltseq(Sch[i])[j] eq -4 then P[j]:=D^-1; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;

for i in [1..#N] do
if ArrayP[i] eq A^E then Sch[i]; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq B^E then Sch[i]; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq C^E then Sch[i]; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq D^E then Sch[i]; end if; end for;

H<a,b,c,d,e>:=Group<a,b,c,d,e|a^3,b^3,(a*b^-1*a^-1*b^-1)^2,
(a^-1*b*a^-1*b^-1)^2,c^3,d^3,(c*d^-1*c^-1*d^-1)^2,
(c^-1*d*c^-1*d^-1)^2,(a,c),(a,d),(b,c),(b,d),e^2,
a^e=c^-1*d^-1*c,b^e=(c^-1,d),c^e=b^a,d^e=(b^-1,a^-1)>;

```

```

f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,NL[3]);
s;

A:=t(f1(a));
B:=t(f1(b));
C:=t(f1(c));
D:=t(f1(d));
E:=t(f1(e));
N:=sub<G1|A,B,C,D,E>;
#N;

for i in G1 do if i notin NL[3] and Order(i) eq 2 and sub<G1|NL[3],i>
eq G1 then F:=i; break; end if; end for;
Order(F);

Sch:=SchreierSystem(H,sub<H|Id(H)>);
ArrayP:=[Id(N): i in [1..#N]];
for i in [2..#N] do
P:=[Id(N): 1 in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;
if Eltseq(Sch[i])[j] eq 3 then P[j]:=C; end if;
if Eltseq(Sch[i])[j] eq 4 then P[j]:=D; end if;
if Eltseq(Sch[i])[j] eq 5 then P[j]:=E; end if;
if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=B^-1; end if;
if Eltseq(Sch[i])[j] eq -3 then P[j]:=C^-1; end if;
if Eltseq(Sch[i])[j] eq -4 then P[j]:=D^-1; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;

for i in [1..#N] do
if ArrayP[i] eq A^F then Sch[i]; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq B^F then Sch[i]; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq C^F then Sch[i]; end if; end for;
for i in [1..#N] do

```

```

if ArrayP[i] eq D^F then Sch[i]; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq E^F then Sch[i]; end if; end for;

H<a,b,c,d,e,f>:=Group<a,b,c,d,e,f|a^3,b^3,(a*b^-1*a^-1*b^-1)^2,
(a^-1*b*a^-1*b^-1)^2,c^3,d^3,(c*d^-1*c^-1*d^-1)^2,
(c^-1*d*c^-1*d^-1)^2,(a,c),(a,d),(b,c),(b,d),e^2,a^e=c^-1*d^-1*c,
b^e=(c^-1,d),c^e=b^a,d^e=(b^-1,a^-1),f^2,a^f=b^-1*e*c*e,
b^f=e*d*e,c^f=d*c*d^-1,d^f=d^-1,e^f=a*b*a^-1*e*a*b^-1*a^-1>;
f1,H1,k1:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,G1);
s;

```

Appendix F

MAGMA Code: Composition

Factors for $2^{*21} : ((7 \times 3) : 2)$

```

r1:=0;r2:=0;r3:=0;r4:=0;r5:=0;r6:=2;r7:=3;
G<a,b,c,t>:=Group<a,b,c,t|a^2,c^3,b^-1*a*b*a,(b,c),(a*c^-1)^2,b^-7,
t^2,(t,a*c^-1),
(a*t)^r1,
(b^2*t)^r2,
(b*c*t)^r3,
(b*t*t^c*t^(c^2))^r4,
(a*b*c*t^b*t*t^b)^r5,
(c*t^(b^2)*t^a)^r6,
(b*t)^r7>;

```

```

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);
#k;
CompositionFactors(G1);

```

```
MinimalNormalSubgroups(G1);
```

```
//-----
```

```

r1:=0;r2:=0;r3:=0;r4:=0;r5:=7;r6:=4;r7:=2;
G<a,b,c,t>:=Group<a,b,c,t|a^2,c^3,b^-1*a*b*a,(b,c),(a*c^-1)^2,b^-7,
t^2,(t,a*c^-1),
(a*t)^r1,

```

```

(b^2*t)^r2,
(b*c*t)^r3,
(b*t*t^c*t^(c^2))^r4,
(a*b*c*t^b*t*t^b)^r5,
(c*t^(b^2)*t^a)^r6,
(b*t)^r7>;

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);
#k;
CompositionFactors(G1);

MinimalNormalSubgroups(G1);
s,t:=IsIsomorphic(G1,PGL(2,7));
s;

//-----

r1:=0;r2:=0;r3:=0;r4:=0;r5:=7;r6:=6;r7:=2;
G<a,b,c,t>:=Group<a,b,c,t|a^2,c^3,b^-1*a*b*a,(b,c),(a*c^-1)^2,b^-7,
t^2,(t,a*c^-1),
(a*t)^r1,
(b^2*t)^r2,
(b*c*t)^r3,
(b*t*t^c*t^(c^2))^r4,
(a*b*c*t^b*t*t^b)^r5,
(c*t^(b^2)*t^a)^r6,
(b*t)^r7>;

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);
#k;
CompositionFactors(G1);

s,t:=IsIsomorphic(G1,PGL(2,13));
s;

//-----

r1:=0;r2:=0;r3:=0;r4:=0;r5:=8;r6:=4;r7:=2;
G<a,b,c,t>:=Group<a,b,c,t|a^2,c^3,b^-1*a*b*a,(b,c),(a*c^-1)^2,b^-7,
t^2,(t,a*c^-1),
(a*t)^r1,

```

```

(b^2*t)^r2,
(b*c*t)^r3,
(b*t*t^c*t^(c^2))^r4,
(a*b*c*t^b*t*t^b)^r5,
(c*t^(b^2)*t^a)^r6,
(b*t)^r7>;

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);
#k;
CompositionFactors(G1);

Center(G1);
NL:=NormalLattice(G1);
NL;

Q,ff:=quo<G1|NL[2]>;

s,t:=IsIsomorphic(Q,PGL(2,7));
s;

nl:=NormalLattice(Q);
nl;

H<a,b>:=Group<a,b|a^3,b^3,(a^-1*b^-1)^4,
a^-1*b^-1*a*b^-1*a^-1*b^-1*a*b^-1*a^-1*b*a*b>;
f2,H1,k2:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,Q1);
s;

for i in nl[3] do if i notin nl[2] and Order(i) eq 2 and sub<Q1|nl[2],i>
eq Q1 then Z:=i; break; end if; end for;

A:=t(f2(a));
B:=t(f2(b));
N:=sub<Q1|A,B>;

Sch:=SchreierSystem(H,sub<H|Id(H)>);
ArrayP:=[Id(N): i in [1..#N]];
for i in [2..#N] do
P:=[Id(N): 1 in [1..#Sch[i]]];
for j in [1..#Sch[i]] do
if Eltseq(Sch[i])[j] eq 1 then P[j]:=A; end if;
if Eltseq(Sch[i])[j] eq 2 then P[j]:=B; end if;

```

```

if Eltseq(Sch[i])[j] eq -1 then P[j]:=A^-1; end if;
if Eltseq(Sch[i])[j] eq -2 then P[j]:=B^-1; end if;
end for;
PP:=Id(N);
for k in [1..#P] do
PP:=PP*P[k]; end for;
ArrayP[i]:=PP;
end for;

for i in [1..#N] do
if ArrayP[i] eq A^Z then Sch[i]; end if; end for;
for i in [1..#N] do
if ArrayP[i] eq B^Z then Sch[i]; end if; end for;

H<a,b,c>:=Group<a,b,c|a^3,b^3,(a^-1*b^-1)^4,
a^-1*b^-1*a*b^-1*a^-1*b^-1*a*b^-1*a^-1*b*a*b,
c^2,a^c=a*b^-1*a*b^-1*a^-1*b*a^-1,b^c=b^-1*a*b^-1*a^-1*b>;
f2,H1,k2:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,Q);
s;

A:=t(f2(a));
B:=t(f2(b));
C:=t(f2(c));
T:=Transversal(G1,NL[2]);

for i in [1..#T] do if ff(T[i]) eq A then i; end if; end for;
for i in [1..#T] do if ff(T[i]) eq B then i; end if; end for;
for i in [1..#T] do if ff(T[i]) eq C then i; end if; end for;

A:=T[51];
B:=T[3];
C:=T[315];
D:=NL[2].2;

for i in [1..2] do if A^3 eq D^i then i; end if; end for;
for i in [1..2] do if B^3 eq D^i then i; end if; end for;
for i in [1..2] do if (A^-1*B^-1)^4 eq D^i then i; end if; end for;
for i in [1..2] do if A^-1*B^-1*A*B^-1*A^-1*B^-1*A*B^-1*A^-1*B*A*B
eq D^i then i; end if; end for;
for i in [1..2] do if C^2 eq D^i then i; end if; end for;

H<a,b,c,d>:=Group<a,b,c,d|a^3,b^3,(a^-1*b^-1)^4,
a^-1*b^-1*a*b^-1*a^-1*b^-1*a*b^-1*a^-1*b*a*b,

```

```

c^2,a^c=a*b^-1*a*b^-1*a^-1*b*a^-1,b^c=b^-1*a*b^-1*a^-1*b,
d^2,(a,d),(b,d),(c,d)>;
f2,H1,k2:=CosetAction(H,sub<H|Id(H)>);
s,t:=IsIsomorphic(H1,G1);
s;

//-----

r1:=0;r2:=0;r3:=0;r4:=0;r5:=9;r6:=5;r7:=2;
G<a,b,c,t>:=Group<a,b,c,t|a^2,c^3,b^-1*a*b*a,(b,c),(a*c^-1)^2,b^-7,
t^2,(t,a*c^-1),
(a*t)^r1,
(b^2*t)^r2,
(b*c*t)^r3,
(b*t*t^c*t^(c^2))^r4,
(a*b*c*t^b*t*t^b)^r5,
(c*t^(b^2)*t^a)^r6,
(b*t)^r7>;

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);
#k;
CompositionFactors(G1);

NL:=NormalLattice(G1);
NL;

Center(G1);
Center(G1) eq NL[2];
Q,ff:=quo<G1|NL[2]>;

CompositionFactors(Q);
nl:=NormalLattice(Q);
nl;

s,t:=IsIsomorphic(Q,PGL(2,19));
s;
Center(Q);

//-----

r1:=0;r2:=0;r3:=0;r4:=0;r5:=11;r6:=4;r7:=2;
G<a,b,c,t>:=Group<a,b,c,t|a^2,c^3,b^-1*a*b*a,(b,c),(a*c^-1)^2,b^-7,
t^2,(t,a*c^-1),

```



```

(a*t)^r1,
(b^2*t)^r2,
(b*c*t)^r3,
(b*t*t^c*t^(c^2))^r4,
(a*b*c*t^b*t*t^b)^r5,
(c*t^(b^2)*t^a)^r6,
(b*t)^r7>;

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);
#k;
CompositionFactors(G1);

s,t:=IsIsomorphic(G1,PGL(2,23));
s;

NL:=NormalLattice(G1);
NL;

Center(G1);
Center(G1) eq NL[2];
Q,ff:=quo<G1|NL[2]>;

CompositionFactors(Q);
nl:=NormalLattice(Q);
nl;

s,t:=IsIsomorphic(Q,PGL(2,19));
s;
Center(Q);

//-----

r1:=0;r2:=0;r3:=0;r4:=3;r5:=9;r6:=0;r7:=2;
G<a,b,c,t>:=Group<a,b,c,t|a^2,c^3,b^-1*a*b*a,(b,c),(a*c^-1)^2,b^-7,
t^2,(t,a*c^-1),
(a*t)^r1,
(b^2*t)^r2,
(b*c*t)^r3,
(b*t*t^c*t^(c^2))^r4,
(a*b*c*t^b*t*t^b)^r5,
(c*t^(b^2)*t^a)^r6,
(b*t)^r7>;

```

```

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);
#k;
CompositionFactors(G1);

//-----

r1:=0;r2:=0;r3:=0;r4:=5;r5:=7;r6:=15;r7:=2;
G<a,b,c,t>:=Group<a,b,c,t|a^2,c^3,b^-1*a*b*a,(b,c),(a*c^-1)^2,b^-7,
t^2,(t,a*c^-1),
(a*t)^r1,
(b^2*t)^r2,
(b*c*t)^r3,
(b*t*t^c*t^(c^2))^r4,
(a*b*c*t^b*t*t^b)^r5,
(c*t^(b^2)*t^a)^r6,
(b*t)^r7>;

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);
#k;
CompositionFactors(G1);

//-----

r1:=0;r2:=0;r3:=0;r4:=7;r5:=13;r6:=7;r7:=2;
G<a,b,c,t>:=Group<a,b,c,t|a^2,c^3,b^-1*a*b*a,(b,c),(a*c^-1)^2,b^-7,
t^2,(t,a*c^-1),
(a*t)^r1,
(b^2*t)^r2,
(b*c*t)^r3,
(b*t*t^c*t^(c^2))^r4,
(a*b*c*t^b*t*t^b)^r5,
(c*t^(b^2)*t^a)^r6,
(b*t)^r7>;

#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);
#k;
CompositionFactors(G1);

```

Appendix G

MAGMA Code: Composition

Factors for $2^9 : 3^\bullet(3 \times 3)$

```
//-----

r1:=0; r2:=0; r3:=3; r4:=4; r5:=3;
G<a,b,c,t>:=Group<a,b,c,t|a^3,b^3,c^3,(a,c),(b,c),a*b^-1*a^-1*b*c,
t^2,(t,a*b*c),
(a*t*t^b*t^(c*b))^r1,
(a*b*c*t*t^a*t^b)^r2,
(a*b*t)^r3,
(b*c*t^a*t^c)^r4,
(c*t*t^b)^r5>;
#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);

CompositionFactors(G1);
s,t:=IsIsomorphic(G1,PGL(3,4));
s;

//-----

r1:=0; r2:=3; r3:=2; r4:=0; r5:=5;
G<a,b,c,t>:=Group<a,b,c,t|a^3,b^3,c^3,(a,c),(b,c),a*b^-1*a^-1*b*c,
t^2,(t,a*b*c),
(a*t*t^b*t^(c*b))^r1,
(a*b*c*t*t^a*t^b)^r2,
(a*b*t)^r3,
```

```

(b*c*t^a*t^c)^r4,
(c*t*t^b)^r5>;
#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);

CompositionFactors(G1);
Center(G1);

//-----

r1:=0; r2:=4; r3:=3; r4:=5; r5:=0;
G<a,b,c,t>:=Group<a,b,c,t|a^3,b^3,c^3,(a,c),(b,c),a*b^-1*a^-1*b*c,
t^2,(t,a*b*c),
(a*t*t^b*t^(c*b))^r1,
(a*b*c*t*t^a*t^b)^r2,
(a*b*t)^r3,
(b*c*t^a*t^c)^r4,
(c*t*t^b)^r5>;
#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);

CompositionFactors(G1);

//-----

r1:=0; r2:=7; r3:=2; r4:=7; r5:=6;
G<a,b,c,t>:=Group<a,b,c,t|a^3,b^3,c^3,(a,c),(b,c),a*b^-1*a^-1*b*c,
t^2,(t,a*b*c),
(a*t*t^b*t^(c*b))^r1,
(a*b*c*t*t^a*t^b)^r2,
(a*b*t)^r3,
(b*c*t^a*t^c)^r4,
(c*t*t^b)^r5>;
#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);

CompositionFactors(G1);

//-----

r1:=0; r2:=8; r3:=2; r4:=8; r5:=4;
G<a,b,c,t>:=Group<a,b,c,t|a^3,b^3,c^3,(a,c),(b,c),a*b^-1*a^-1*b*c,
t^2,(t,a*b*c),

```

```

(a*t*t^b*t^(c*b))^r1,
(a*b*c*t*t^a*t^b)^r2,
(a*b*t)^r3,
(b*c*t^a*t^c)^r4,
(c*t*t^b)^r5>;
#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);

CompositionFactors(G1);
s,t:=IsIsomorphic(G1,PGL(2,7));
s;

//-----

r1:=0; r2:=11; r3:=2; r4:=11; r5:=4;
G<a,b,c,t>:=Group<a,b,c,t|a^3,b^3,c^3,(a,c),(b,c),a*b^-1*a^-1*b*c,
t^2,(t,a*b*c),
(a*t*t^b*t^(c*b))^r1,
(a*b*c*t*t^a*t^b)^r2,
(a*b*t)^r3,
(b*c*t^a*t^c)^r4,
(c*t*t^b)^r5>;
#G;
f,G1,k:=CosetAction(G,sub<G|a,b,c>);

CompositionFactors(G1);

```

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