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# **Optimal Equipment Replacement and Scrapping Under Improving Technology**

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## **ABSTRACT**

*The rational replacement management of hi-tech equipment is an important problem of technology management. This paper analyzes and compares two alternative policies for determining the service life and replacement demand of industrial equipment under improving technology. These policies lead to different estimates of the impact of new technology on the replacement policies and equipment service life.*

## **INTRODUCTION**

In order to secure the services of durables at minimum cost, producers and consumers confront invariably the question: How frequently should a stock of old durables be replaced by a stock of new ones? Clearly, the old durables should not be replaced too soon because the cost of acquiring them will occur too frequently and this will raise the unit cost of their services. However, the durables should not be replaced too late either, because their rising operating costs and the higher productivity of durables of newer vintages render them economically inferior. So, to tackle the issues involved in determining the optimal service life of durables, researchers in the fields of management and economics have adopted over the years various approaches.

The terms "capital", "equipment", and "machine" have been employed frequently in the relevant literature to indicate that the good under consideration has the properties of a producer's durable. In this paper, we use these terms interchangeably. The same comment holds also for "economic life", "service life", "life", and "lifetime".

Preinreich (1940) was the first to show how the optimal life of durables can be determined. More specifically, according to his theorem, the optimal economic life of a single machine should be computed together with the economic life of each machine in the chain of future replacements extending as far into the future as the owner's profit horizon. However, the theorem was formulated under two crucial assumptions. The first of them abstracted from a technological progress and postulated that newer machines of identical type replaced older machines (like-for-like). This assumption contradicted casual observations and was ultimately relaxed by Smith

(1961) who generalized the above result of Preinreich (1940) to the case where the older machines were replaced by more productive machines embodying the most recent advances in science and technology.

The second assumption concerned the horizon of the reinvestment process and required the owner of the machine to choose its duration on the basis of their perception on how long the investment opportunity might remain profitable. Later, depending on the specification of the owner's profit horizon, different models emerged for the determination of the optimal lifetime of assets. In particular, by limiting the owner's profit horizon to a single investment cycle, researchers in the field of capital budgeting obtained the so-called "abandonment" class of models and used it to derive strict rules regarding the optimal asset life. Initially, Robichek and Van Horne (1967) suggested that an asset should be abandoned during any period, in which the present value of future cash flows did not exceed its abandonment value. Then, based on the possibility that the function of cash flows might not have a single peak, Dyl and Long (1969) argued that abandonment should not occur at the earliest possible date that the above abandonment condition was satisfied, but rather at the date that yielded the highest net present value over all future abandonment opportunities. Later, Howe and McCabe (1983) highlighted the patterns of cash flows and scrap values under which the "abandonment" model led to a unique global optimum of the abandonment time. They also characterized the complete range of models that could be obtained by varying the owner's profit horizon and clarified the practical guidelines for the choice between "abandonment" and "replacement" models.

Theoretical economists, on the other hand, continued to work in the tradition of Terborgh (1949) and Smith (1961) by assuming invariably that the owner's *profit horizon is infinite*. This in turn led them to concentrating on a single class of replacement models, all of which presumed that the infinite reinvestments *took place at equal time intervals*. This pervasive conceptualization was adopted in all significant contributions in the area from Brems (1968) to Nickel (1975), Rust (1987) and to Mauer and Ott (1995). The persistence of this approach was probably encouraged by the proof that Elton and Gruber (1976) provided regarding the optimality of an equal life policy for a machine subject to technological improvement. However, subsequent research has established that the equal life policy is only a special case of a much more general set of variable life replacement policies. More recently, Van Hilten (1991), Hritonenko and Yatsenko (1996, 2005, 2007, 2008), Regnier, Sharp, and Tovey (2004) relaxed these assumptions and considered both the variable replacement period and the finite profit horizon.

The practical importance of the finite-horizon replacement problem was highlighted by Hartman and Murphy (2006), who explored the replacement policy which occurs when companies only require an asset for a specified length of time, usually to fulfill a specific contract and identify when this policy deviates significantly from optimal. Bitros and Flytzanis (2005) demonstrated that the *infinite-horizon* replacement policies and the abandonment policies of *transitory replacements* ending with scrapping "abandonment" lead to different results regarding the profit horizon, the duration of replacements, the timing of scrapping, and the impact of output and market structure on service lives. In doing so, they assumed that the technological progress had the form of random breakthroughs, which at the time of their occurrence rendered all existing equipment inoperable.

In the real world, there are two fundamentally different modes of technical progress (e.g., Simpson,

Toman, and Ayres 2005, p.144): a “*normal mode*”, in which technological improvements occur incrementally and more or less automatically as a result of accumulated experience and learning, and the *radical innovations (technological breakthroughs)*. The normal mode is characterized by a simple positive feedback between increasing consumption, increasing investment, increasing scale, and learning-by-doing (or experience). The production technology itself may also gradually become more efficient. It results in gradually declining costs and prices, stimulating further increases in consumption and, hence, economic growth. The second mode involves multiple competing and evolving production technologies (plastics and synthetic fibers substituting iron and steel, automobiles replacing horses and carriages, air transport displacing railways, and so on). The substitution of major general-purpose technologies can be both very productive and very traumatic.

The goal of this paper is to explore and analyze differences between the abandonment and replacement management policies of Bitros and Flytzanis (2005) in the case when the technological change is in the normal mode and of the embodied type. As analytic tool, we adopt a *vintage capital model* in which the units of equipment brought into operation are more productive than the ones already in place because they embody the most recent advances in science and technology. We expect that this setting is promising for our purpose because the optimal lifetime of equipment in each vintage depends on the horizon of the reinvestment opportunity as well as on the date of its introduction into operations. On the other hand, this setting poses essential challenges because it requires solving a non-linear dynamic optimal control problem. Mathematical fundamentals of such non-linear control problems have been established in Hritonenko and Yatsenko (2007, 2008, 2009).

The rest of the paper is organized as follows. In the next section, we set up the model. Its vintage specification allows for improvements in the productivity of consecutive vintages of equipment. This, in turn, leads to a new non-linear optimization problem that involves the optimal control of the lifetime of specific vintages. Then, in the following two sections, we investigate the implications of two different approaches to the administration of equipment. Section 3 focuses on the strategy of *infinite-horizon replacements*, implying that the equipment is being replaced indefinitely, whereas Section 4 concentrates on the strategy of *transitory replacements*, where the equipment is replaced a finite number of times, ending with abandonment or scrapping. Lastly, in Section 5, we conclude with a synopsis of main findings.

## OPTIMIZATION MODEL

At the end of the service life of equipment, there are always two options: to replace it and continue doing so up to some profit horizon, or to abandon (scrap) it and terminate operations. To examine their implications, we assume that during year  $\tau$  the representative firm acquires  $K(\tau)$  units of new capital, which possess the same efficiency  $b(\tau)$  because they belong to the same vintage and embody the same technology. The output of the new capital  $K(\tau)$  is denoted as  $X(\tau) = b(\tau)K(\tau)$ . The units of capital built later in the year  $t > \tau$  are more productive because they embody the latest advances of science and technology. To describe this process, we assume that the capital efficiency (output-capital ratio) is

$$b(t) = b(\tau)e^{\mu(t-\tau)}, \quad (1)$$

where  $\lambda > 0$  is the constant exogenous rate of technological change. Thus, the efficiency of capital in each vintage depends on the date of its construction.

To emphasize the role of the new technology in optimal equipment replacement, we will assume that the representative firm acquires only the newest vintage of equipment and removes from service the oldest equipment that has become obsolete. Then, the total output produced in year  $t$  is described as:

$$X(t) = b(0) \int_{a(t)}^t e^{\mu\tau} K(\tau) d\tau, \quad a(t) < t, \quad (2)$$

where the purchasing time  $a(t)$  of the equipment scrapped at time  $t$  is known as the *capital scrapping time*, following Malcomson (1975), van Hilten (1991), Boucekine et al. (1997, 1998), Hritonenko and Yatsenko (1996, 2005, 2007), Greenwood et al. (2000). The integral over  $[a(t), t]$  in (2) implies that at time  $t$  the firm uses only the equipment units placed into service between  $a(t)$  and  $t$ . Specifically, expression (2) states that the capital bought at time  $a(t)$  is scrapped at the current time  $t$ . The time  $t - a(t)$  is the service life (lifetime) of equipment bought at time  $a(t)$ . Introducing the market price  $p(t)$  of output  $X(t)$ , we can represent the net operating revenue of the firm as:

$$Q(t) = p(t)b(0) \int_{a(t)}^t e^{\mu\tau} K(\tau) d\tau - w(t)L(t), \quad (3)$$

where  $L(t)$  is the total labor employed in year  $t$  and  $w(t)$  is the wage rate. In this paper, we restrict ourselves to the labor expenses only, although other operating costs can be also considered. Assuming that  $m(\tau)$  units of labor operate each equipment unit introduced at time  $\tau$ , the total labor demand of the firm is described as:

$$L(t) = \int_{a(t)}^t K(\tau)m(\tau) d\tau. \quad (4)$$

We shall notice that a resource constraint similar to (4) can be imposed on any other critical resource of a firm such as energy, finances, operating space, or even repair facilities. For example, in energy production, a crucial restriction is set by the environment contamination limits.

Compared to other models reviewed in the previous section, the vintage capital model (1)-(4) provides a convenient tool to consider the optimal lifetime of equipment as an unknown (endogenous) variable. To determine this endogenous variable, we formulate an optimization problem by assuming that the present value of total profits over the planning horizon  $[t_0, T_{\max}]$

$$\Pi = \int_{t_0}^{T_{\max}} e^{-rt} [Q(t) - q(t)K(t)] dt \quad (5)$$

is maximized under the given labor resource (4). Here  $q(t)$  is the acquisition price of the new equipment unit and  $r$  is the discount rate. We assume that the residual (salvage) value of the

scrapped equipment is negligible compared with its acquisition price. The dynamics of  $q(t)$  is also determined by technological change and, together with the output-capital coefficient described in (1), appears to be critical for determining the optimal service life of equipment. In this paper, we assume the dynamics of  $q(t)$  and  $p(t)$  to be different:

$$q(t) = q(0) \exp(\eta t), \quad p(t) = p(0) \exp(\zeta t), \quad (6)$$

where the constants  $\eta$  and  $\zeta$  may be positive or negative.

In the formulated optimization problem, the unknown controls are the investment  $K(t)$  and the scrapping time  $a(t)$ . In contrast to the simple model of infinite equal time replacements employed by Bitros (2005) to compare the two policies, the *vintage capital model* (1)-(6) considers the variable equipment lifetime (service life)  $T(t) = t - a(t)$ . The output-capital ratio  $b(\tau)$ , the labour-capital ratio  $m(\tau)$ , the total labor  $L(t)$ , the acquisition price of capital  $q(t)$ , and the product price  $p(t)$  are given on  $t \in [t_0, T_{\max})$ . It is convenient to assume that one man operates one unit of equipment. Then,  $m(\tau) = 1$  in (4),  $b(\tau)$  becomes the output-labor coefficient, and  $q(t)$  in (5) is the *relative price of a labor unit of equipment* as in Greenwood et al. (2000). To simplify the optimization analysis, we also assume that  $L(t) = \text{const}$ .

**Remark 1.** Bitros and Flytzanis (2005) considered a similar problem in a simpler time-invariant framework with an impatience assumption and in the absence of embodied technological change.

Let us impose some necessary restrictions on the unknown variables. First of all, we set  $0 \leq K(t) \leq K_{\max}(t)$ . This implies that the *maximal possible investment*  $K_{\max}(t)$  is determined by external financial constraints faced by the representative firm. It is also natural to assume that the scrapped equipment cannot be used again, i.e.  $a'(t) \geq 0$ . Finally, as model (3)-(5) is defined on the future interval  $[t_0, T_{\max})$ , a specific vintage structure of the equipment should be known at the initial time  $t_0$ . This structure is defined by the given investment  $K(\tau) = K_0(\tau)$  undertaken throughout the pre-history interval  $[a(t_0), t_0]$ .

Thus, the problem is to find the unknown functions  $K(t)$  and  $a(t)$ ,  $t \in [t_0, T_{\max})$ ,  $T_{\max} \leq \infty$ , which maximize the objective functional (5) under the constraint-equalities (3) and (4), the constraints-inequalities:

$$0 \leq K(t) \leq K_{\max}(t), \quad (7)$$

$$a'(t) \geq 0, \quad a(t) < t, \quad (8)$$

and the initial conditions:

$$a(t_0) = a_0 < t_0, \quad K(\tau) = K_0(\tau), \quad \tau \in [a_0, t_0]. \quad (9)$$

Malcomson (1975) was first to introduce the vintage capital model (2)-(3) to find the optimal capital replacement policy of an individual firm with vintage technology under the embodied

technological progress. In the same framework, van Hilten (1991) investigated an infinite-horizon optimization problem where the constant lifetime of capital was optimal. Hritonenko and Yatsenko (1996, 2005, 2008) provided a qualitative analysis of the problem (2)-(9).

**Remark 2.** It would be interesting to assume that the representative firm faces a demand curve of the constant elasticity type as in (Samaniego 2008). However, this assumption introduces a scale effect and makes the solution of the problem (1)-(9) considerably more difficult. In particular, the optimal lifetime of equipment would depend on the amount of output produced. So, we leave this specification for future research.

Let us turn now to the investigation of the possible differences between the two approaches to the management of capital. In the optimization problem (1)-(9), the policy of *infinite-horizon replacements* corresponds to the case  $T_{\max}=\infty$ , whereas the policy of *transitory replacements* ending with scrapping corresponds to the case  $T_{\max}<\infty$ . The structure of the solutions  $(K^*(t), a^*(t))$  appears to be quite different under  $T_{\max}=\infty$  and  $T_{\max}<\infty$ . Section below is devoted to the analysis of the *infinite horizon replacement* policy, whereas the investigation of the policy of *transitory replacements* ending with scrapping is relegated to Section 4.

### INFINITE-HORIZON REPLACEMENTS

The formulated optimization problem is meaningful at  $T_{\max}=\infty$  if the value of the improper integral in (5) is finite (otherwise, there is no sense to maximize it). One can see that the integral is finite when

$$r > \mu, \quad r > \mu + \zeta, \quad r > \eta, \quad (10)$$

Formulae (10) reflect the natural condition that the discounting factor  $r$  needs to be greater than the TC rates in order to have a finite value of the profit on the infinite horizon.

The structure of problem (1)-(9) solutions appears to be quite simple in the case  $T_{\max}=\infty$  of infinite horizon. As shown by Hritonenko and Yatsenko (1996, 2008), the problem (1)-(9) at  $T_{\max}=\infty$  has a unique solution  $(K^*(t), a^*(t))$ ,  $t \in [t_0, \infty)$ , such that the optimal scrapping time  $a^*(t)$  coincides with a special trajectory (*turnpike*)  $\tilde{a}(t)$ ,  $t \in [t_0, \infty)$ , on almost all planning horizon  $[t_0, \infty)$  except for an initial (transition) period  $[t_0, \mu)$ . The *turnpike*  $\tilde{a}$  determined from the following nonlinear integral equation

$$\int_t^{a^{-1}(t)} e^{(-r+\zeta)\tau} [1 - e^{-\mu(t-a(\tau))}] d\tau = C e^{(-r-\mu+\eta)t}, \quad (11)$$

over the interval  $[t_0, \infty)$ , where  $a^{-1}(t)$  is the inverse function of  $a(t)$  and

$$C = q(0)/b(0)/p(0). \quad (12)$$

The inverse function  $a^{-1}(t)$  in (11) exists because  $a'(t) \geq 0$  by (8). The *economic interpretation* of equality (11) is that the profit of introducing new equipment unit and using it during its future lifetime with removing an older unit must be equal to the price of the new equipment unit.

**Proposition 1.** (Hritonenko & Yatsenko, 2009). If (10) holds and  $\mu > 0$ , then equation (11) has the unique solution  $\tilde{a}(t)$ , at least, in the following cases:

- If  $\mu + \zeta = \eta$  and  $C(r - \zeta) < 1$ , then  $\tilde{a}(t) \equiv t - T$ ,  $t \in [t_0, \infty)$ , where the positive constant  $T$  is determined by the following non-linear equation:

$$(r - \zeta)\exp(-c_1 T) - \mu \exp(-(r - \zeta)T) = (r - \zeta - \mu)(1 - C(r - \zeta)) \quad (13)$$

The constant lifetime is approximately  $T \approx (2C/\mu)^{1/2}$  at small  $\mu \ll 1$ .

- If  $\mu + \zeta > \eta$  and  $C(r - \zeta)(r - \eta) \exp[(\eta - \mu - \zeta)t] < (r - \zeta - \mu)$ , then equation (11) has the unique monotonically increasing solution  $\tilde{a}(t)$  on an interval  $[t_0, \infty)$ , such that  $T(t) = t - \tilde{a}(t) > 0$  and  $T(t) \rightarrow 0$  at  $t \rightarrow \infty$ .
- If  $\mu + \zeta < \eta$ , then equation (13) has the unique solution  $\tilde{a}(t)$  only over a certain finite interval  $[t_0, t_{cr})$ , where  $t_{cr} \approx \ln[(r - \zeta - \mu)/(r - \zeta)/(r - \eta)/C]/(\eta - \mu - \zeta)$ . On the interval  $[t_0, t_{cr})$ , the solution  $\tilde{a}(t)$  increases on  $[t_0, t_c)$ , becomes decreasing on  $(t_c, t_{cr})$  at some  $t_c < t_{cr}$ , and  $\tilde{a}(t) \rightarrow -\infty$ ,  $T(t) \rightarrow \infty$  at  $t \rightarrow t_{cr}$ .

**Definition.** We refer the *proportional TC* to the case

$$\mu + \zeta = \eta, \quad (14)$$

when the sum of the TC and product price rates is equal to the growth rate of equipment price. Then, the revenue/cost ratio  $b(t)p(t)/q(t)$  is constant (12).

This Proposition 1 can be used for estimating the rational replacement strategies for industrial equipment based on economic obsolescence at a separate firm (plant, enterprise) level. In particular, the below properties of a firm's optimal capital replacement strategy follow immediately:

1. Except for a possible initial (transitory) period, the rational lifetime of capital equipment  $T(t) = t - \tilde{a}(t)$  does not depend on the production scale and the initial equipment structure and is defined only by the rates of TC, prices, and discount.
2. The rational equipment lifetime may be finite only if the equipment productivity  $b(t)$  increases, i.e., if  $\mu > 0$ . Otherwise, no replacement is necessary (in the absence of deterioration).
3. The ratio (proportion) between the productivity  $b(t)p(t)$  and equipment price  $q(t)$  determines the dynamics of the rational capital lifetime. If  $\mu + \zeta > \eta$  (i.e., the revenue  $b(t)p(t)/q(t)$  per new equipment unit cost increases), then the rational capital lifetime decreases. If  $\mu + \zeta < \eta$  (i.e.,  $b(t)p(t)/q(t)$  decreases), then the rational capital lifetime increases and becomes infinite at some finite instant  $t_{cr}$ . The replacements become less frequent and finally stop.

4. In the case of proportional TC (and only this case), the rational capital lifetime is constant. This constant lifetime depends only on the discount rate and the ratio  $b(t)p(t)/q(t)$  between the productivity and equipment price.

Following (Bitros and Flytzanis 2005), we can say that the equipment is:

- *finitely replaceable* if it has a finite number  $N > 0$  profitable replacements;
- *infinitely replaceable* if  $N = \infty$ ;
- *non-replaceable* if no replacements is profitable.

Then, in accordance with the above Properties 1-4, the following result holds.

**Proposition 2.** The vintage equipment is:

- infinitely replaceable if  $\mu + \zeta \geq \eta$ ,
- non-replaceable if  $\mu + \zeta < \eta$  and  $t_{cr} < t_0$ ,
- finitely replaceable if  $\mu + \zeta < \eta$  and  $t_{cr} \gg t_0$ . Then, the exact number  $N$  of profitable replacements depends on the proportion between the horizon length  $T_0 - t_0$  and the optimal  $T^*(t)$  (where  $T^*(t)$  is determined from equation (11)).

Now, let us describe the optimal dynamics of the corresponding replacement investment  $K^*(t)$ . During an initial transitory period  $[t_0, t_1]$ ,  $t_1 \geq t_0$ , the replacement is maximum possible  $K^*(t) = K_{\max}(t)$  if  $a_0 < \tilde{a}(t_0)$  or minimum possible  $K^*(t) = 0$  if  $a_0 > \tilde{a}(t_0)$ . Differentiating (4), we get the equation  $K(a(t))a'(t) = K(t) - L'(t)$  for optimal investment  $K$  under known optimal  $a$ . Recalling our assumption  $L \equiv \text{const}$ , this equation becomes

$$K(a(t))a'(t) = K(t). \quad (15)$$

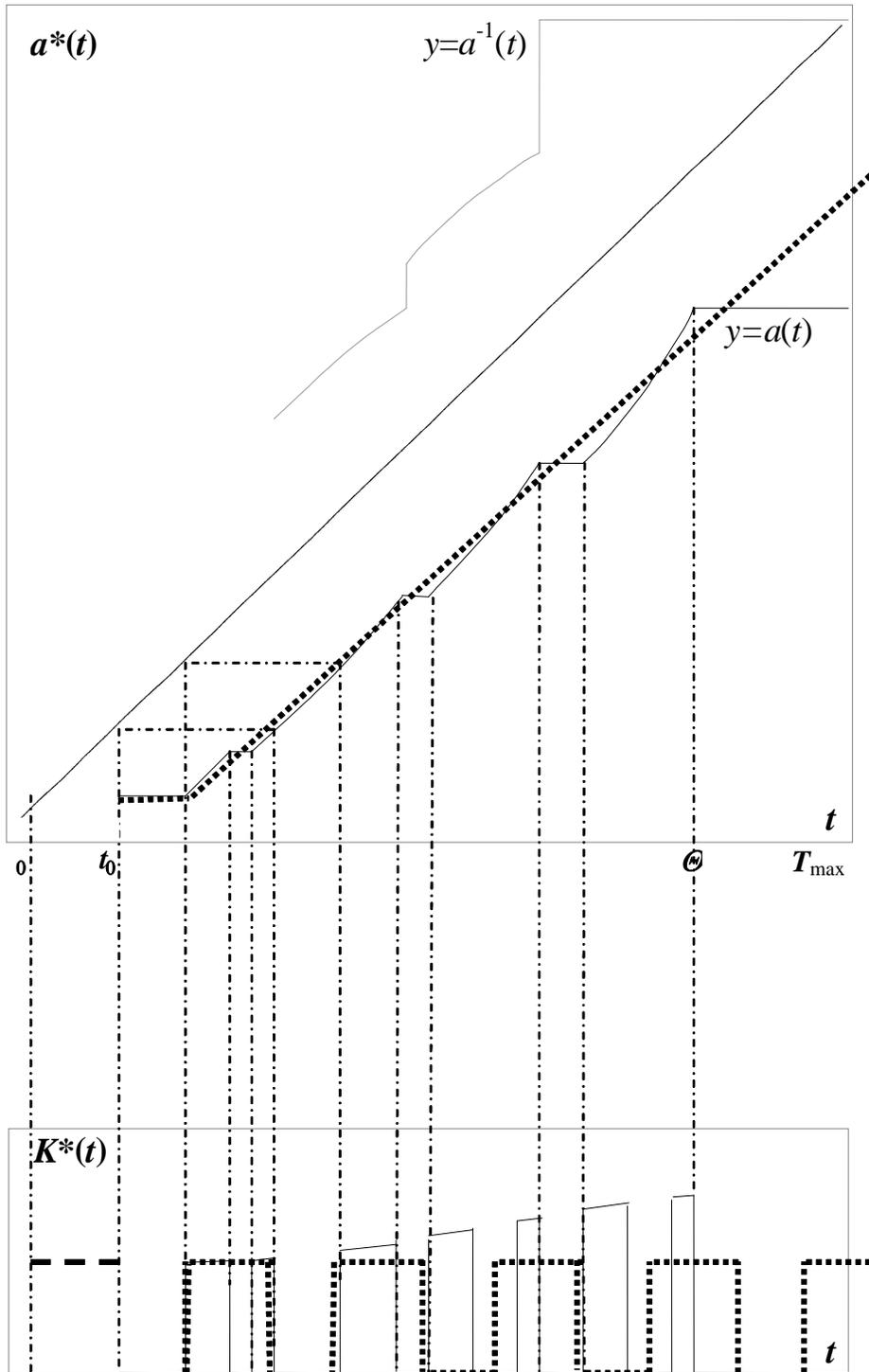
Hence, under the constant labour  $L \equiv \text{const}$ , the minimum replacement regime  $K^*(t) = 0$  is  $a^*(t) = 0$ ,  $a^*(t) = \text{const}$ . It means that no working equipment is being scrapped.

In the general case, the *optimal investment* trajectory  $K^*(t)$  is boundary (minimum or maximum) at a beginning (transitory) part  $[t_0, t_1]$  of the planning horizon. After that,  $K^*(t)$  is found from (4) as  $K^*(t) = K(\tilde{a}(t))d\tilde{a}/dt$  under the known optimal  $\tilde{a}$ . The last formula shows that the initial boundary-valued section of  $K^*$  is reproduced throughout the whole horizon  $[t_0, T)$ . In particular, in the case  $\mu + \zeta = \eta$ , we obtain  $\tilde{a}(t) = t - T$  and a strictly periodic  $K^*(t) = K(t - T)$ . The repetition pattern with bursts and slumps in  $K^*$  could be observed in Figure 1. These “spikes” (*replacement echoes* in Hritonenko and Yatsenko (1996) and Boucekkin et al. (1997)) were confirmed in recent years by economists studying investment at the plant level.

In the case (14) of the proportional TC, the optimal equipment lifetime  $\tilde{a}(t)$  is constant and the replacement echoes repeat indefinitely in the same shape (as in Figure 1). In the general case, if the TC rates  $\mu + \zeta$  and  $\eta$  are different, then the optimal equipment lifetime is not constant and decreases or increases depending on the sign of  $\mu + \zeta - \eta$  and the replacement echoes amplitude

and shape vary. If  $\tilde{a}(t)$  increases, then replacement intervals increase and, by (14), the amplitude of investment echoes decreases from one interval to other, and converse.

**Figure 1: Optimal dynamics of the scrapping time  $a^*$  and investment  $K^*$  for the *infinite-horizon* replacement (dashed lines) and the *transitory replacement* ending with scrapping.**



The replacement echoes are absent in the ideal case  $a_0 = \tilde{a}(t_0)$  of a “perfect” replacement prehistory. However, in practice, the perfect prehistory is not possible because every firm starts with buying some significant amount of equipment during the date of its establishment. This process is known as *entry echoes* – see (Samaniego 2008, Jovanovic and Tse 2010). The entry echoes cause the *exit echoes* near the end of a finite planning horizon (abandonment time), which is explored in the next section.

### TRANSITORY REPLACEMENTS ENDING WITH SCRAPPING

No equipment is scrapped. The optimal lifetime  $T^*(T_{\max}) = T_{\max} - a^*(T_{\max}) = T^*(\Theta)$  of the oldest equipment at the horizon end  $t = T_{\max}$  is always larger than the optimal lifetime  $T(T_{\max})$  in the indefinite-replacement case from Section 3.

The presence of the *zero-investment period* causes essential changes (end-of-horizon effects) in the behavior of the optimal trajectories  $K^*(t)$  and  $a^*(t)$  over the whole  $[t_0, T_{\max}]$  (see Hritonenko and Yatsenko 2008). However, the endHere, we assume that the equipment is managed optimally for a finite number of operating periods with terminal scrapping, i.e.,  $T_{\max} < \infty$ . Then the structure of the optimization problem solutions is more complicated as compared with the  $T_{\max} = \infty$  case.

A key new feature is the existence of the *zero-investment period*  $[\Theta, T_{\max}]$ ,  $t_0 \leq \Theta < T_{\max}$ , at the end of the planning horizon  $[t_0, T_{\max}]$ , first discovered by Hritonenko and Yatsenko (1996). During this period, the investment is minimum possible because there is no sense to invest into new capital given the firm quits production at  $T_{\max}$ . This effect is well known in different optimization economic models. Under the condition of constant labour,  $L = \text{const}$ , the minimum investment regime is  $K^*(t) = 0$  and  $da^*/dt = 0$ ,  $a^*(t) = a^*(\Theta) = \text{const}$  at  $t \in [\Theta, T_{\max}]$ , i.e., no new investment is made and -of-horizon effects weaken when the duration  $T_{\max} - t_0$  becomes larger. In particular, the optimal lifetime  $T^*(t) = t - a^*(t)$  strives to the “indefinite-replacement” optimal lifetime  $T(t) = t - \tilde{a}(t)$  as  $T - t \rightarrow \infty$ . Mathematically (see Hritonenko and Yatsenko 2008), at  $\mu + \zeta \geq \eta$ , the finite-horizon optimization problem (1)-(9) has the unique solution  $(K^*, a^*)$  such that the optimal  $K^*$  has zero-investment parts  $K^*(t) = 0$  on a finite set of backward repetitive intervals  $(\alpha_i, \beta_i)$ ,  $\beta_{i+1} < \alpha_i < \beta_i$ ,  $i = 1, 2, 3, \dots$ ,  $\alpha_1 = \Theta$ ,  $\beta_1 = T_{\max}$ .

The solution  $a^*$  and  $K^*$  of the problem (1)-(9) is shown in Figure 1 (solid lines). The infinite-horizon replacement solution is indicated with the dotted line (where  $t - \tilde{a}(t) = \text{const}$  as in the proportional TC case  $\mu + \zeta = \eta$ ). The finite-horizon equipment lifetime  $a^*(t)$  tends to the infinite-horizon turnpike when the horizon endpoint  $T_{\max}$  and the time  $T_{\max} - t$  increase. The finite-horizon optimal policy possesses sharper changes at certain “critical” instants, that depend on the length of the planning horizon  $[t_0, T_{\max}]$ . These changes are referred to as the *anticipation echoes* in Yatsenko and Hritonenko (2008). They are caused by the zero-investment period  $[\Theta, T_{\max}]$  and propagate backward through the entire planning horizon  $[t_0, T_{\max}]$ . As shown in Figure 1, these “*anticipation echoes*” propagate backward throughout the whole horizon  $[t_0, T]$  starting from the zero-investment period  $(\Theta, T]$ . The intensity of the anticipation echoes in the optimal trajectory  $a^*(t)$  decreases as  $T - t$  increases. Namely, the optimal capital lifetime  $a^*$  becomes smoother at the

beginning of  $[t_0, T]$  for larger horizons. However, the above described optimal replacement policy remains the same near the abandonment point  $T_{\max}$  regardless of the planning horizon length.

The beginning  $\Theta$  of the "zero-investment period"  $[\Theta, T_{\max}]$  is found from the condition

$$\int_{\Theta}^{T_{\max}} e^{-c_3\tau} [e^{c_1\theta} - e^{c_1a_1(\theta)}] d\tau - Ce^{(-c_3+c_2)\theta} = 0$$

and  $\Theta$  is obviously smaller than  $T_{\max}$ .

The relationship between the values  $t_0$ ,  $T_{\max}$ , and  $\Theta$  determines the practical properties of the optimal replacement policy.

Namely, if the calculated above  $\Theta$  appears to be less than  $t_0$ , then the equipment is *non-replaceable* over the planning horizon  $[t_0, T_{\max}]$  because  $[t_0, T_{\max}]$  is too short. If  $\Theta > t_0$ , then the corresponding equipment is *finitely replaceable* on the *finite* interval  $[t_0, T_{\max}]$ . If the equipment is *finitely replaceable* on a *finite* interval, then it may be *infinitely replaceable* on the *infinite* interval or may be not (see Section 3). However, if the equipment is *infinitely replaceable*, then it is *finitely replaceable* over a large enough finite horizon  $[t_0, T_{\max}]$ . This result clarifies the profitability conditions of (Bitros 2005) for infinite-horizon replacement and transitory replacement policies on large planning horizons  $[t_0, T_{\max}]$ .

**Remark 3.** The profit horizon in the transitory case can be endogenously determined by the equipment parameters and the external market environment (Bitros 2005). Namely, taking the given initial equipment distribution on the pre-history interval  $[a(t_0), t_0]$  into account, it may appear that it is more profitable to extend (or decrease) slightly the interval  $[t_0, T_{\max}]$ . So, the transitory approach to replacement is more flexible. For example, in capital budgeting it will allow us to consider the endogenous influence of profit horizon on the selection of projects (and inversely). Mathematically, it requires adding the value  $T_{\max}$  as the additional control variable of the optimization problem.

## CONCLUSION

Our objective in this paper is to compare the differences between the policy of *infinite-horizon* replacements and that of *transitory replacements* ending with scrapping in the presence of embodied technological change. To accomplish it, we adopt the *vintage capital model*, in which the new units of equipment brought into operation are more productive than those already in place due to advances in science and technology. Our main findings show that, if the vintage equipment is *infinite-horizon replaceable*, it is always *finite-horizon replaceable* over a *large finite* horizon. The *infinite-horizon* policy predicts shorter replacement durations than the *transitory* replacement policy does. This difference is significant near the end of the planning horizon in the case of the *transitory* replacement policy and becomes smaller when the planning horizon ends in the more distant future.

As in Bitros and Flytzanis (2005), the replacement period in the *infinite-horizon* case adjusts gradually to possible changes in economic parameters. In the case of the *transitory* replacement

policy, in addition to this smooth change, we have also sharp changes in optimal policies when the parameters (such as the planning horizon length) cross certain critical values. These changes are caused by the existence of the *zero-investment period* at the end of the planning horizon, when no new investment is made and no capital is scrapped. Because of the zero-investment period, the optimal replacement possesses the *zero-investment echoes* that propagate backward through the entire planning horizon. This fact demonstrates that the *multi-step transitory* replacement is much more complex and flexible management policy, which is often overlooked in economic theory and management practice.

In summary, in cases when the owner's profit horizon is given (for example, by a contract length), the rational investment policy is heavily affected near the horizon end when no capital modernization is profitable, no new investment should be made, and no workable equipment is scrapped. Other management implications for long enough planning horizons include the possibility of careful planning of the investments during the period far before policy the horizon - during the times when a capital is being installed that will be used until the horizon end. In addition, when possible, the owner's profit horizon length shall be considered as an endogenous variable along with other variables in the optimization process (instead of treating the horizon as given).

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