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Method of Joints: Theory and Practice of Designing, Building, and Testing Trusses

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theory and practice of designing, building, and testing trusses

by Andrew J. Hughes and Chris Merrill, DTE

Students should be able to readily see the connection between using their hands and minds to solve a fun, authentic problem.

Figure 10. Classroom testing setup.

The authors of this article, like many of us, are proponents of engineering education but are also proponents of shop skills, craftsmanship, technological literacy, and the tacit knowledge and skills developed through applying sound theories during practical hands-on learning. The authors believe that engineering is an important aspect of our discipline, but so are the application of thinking, tool skills, measurement, geometric construction, manufacturing, instrumentation, testing and analysis, mathematical and scientific theories, and many other hands-on, minds-on skillsets that all need to maintain association with our discipline. As the authors are proponents for engineering education that is done well, they have provided an explanation of truss design using the Method of Joints that combines the application of practical hands-on learning with sound mathematical and scientific theory. The Method of Joints is a static principle stating that all joints in a truss must be in equilibrium. This means that forces on truss members of each joint must combine at the joint to equal zero for all joints. The Method of Joints will allow students to design trusses to meet specified criteria using mathematical models. The process of designing a truss to meet specific requirements involves students applying the Method of Joints to create and apply computational models. In the activity students will be able to design and predict the amount of weight causing truss failure, then test their truss to validate their predictions' accuracy. If students minimize possible errors by building their trusses exactly as designed and calculated, most trusses will fail within 5% of the calculated amount. In two previous articles, component force systems were covered to help with the understanding of forces systems involved in truss design and the Method of Sections was also presented as another method for solving for forces in a truss (Huges & Merrill, ITEEA 2020, pp. 16-22). For a more thorough understanding of truss design, it is recommended that the reader review these two previous articles.

Listed on page 29 are the standards and benchmarks that this lesson and activity were developed from/based on. The number of standards and benchmarks may seem to unrealistic, but as you progress through this lesson and activity, you will see that each is appropriate. Based on the authors' experience, it is appropriate to cover this content with middle school students but may require more time and the simplification of some aspects of the truss design.

Standards for Technological and Engineering Literacy:

- ***STEL*** **7. Design in Technology and Engineering Education**
  - o **7R:** Refine design solutions to address criteria and constraints.
  - o **7Y:** Optimize a design by addressing desired qualities within criteria and constraints.
  - o **7Q:** Apply the technology and engineering design process.
  - o **7U:** Evaluate the strengths and weaknesses of different design solutions.
  - o **7AA:** Illustrate principles, elements, and factors of design.

Next Generation Science Standards:

- **Middle (MS) and High School (HS)**
  - o **MS-PS2-2:** Plan an investigation to provide evidence that the change in an object's motion depends on the sum of the forces on the object and the mass of the object.
  - o **MS-ETS1-1:** Define the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles and potential impacts on people and the natural environment that may limit possible solutions.
  - o **MS-ETS1-4:** Develop a model to generate data for iterative testing and modification of a proposed object, tool, or process such that an optimal design can be achieved.
  - o **HS-PS2-1:** Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.
  - o **HS-PS3-1:** Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other components and energy flows in and out of the system are known.
  - o **HS-ETS1-2:** Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
  - o **HS-ETS1-3:** Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics as well as possible social, cultural, and environmental impacts.

The 20-pound force in the middle of the truss is an arbitrary load (Figure 1).

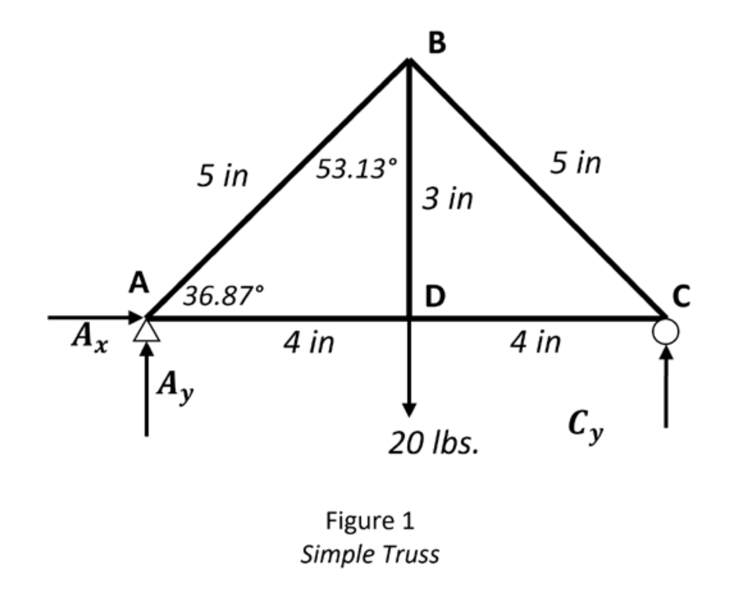


Figure 1. Simple Truss.

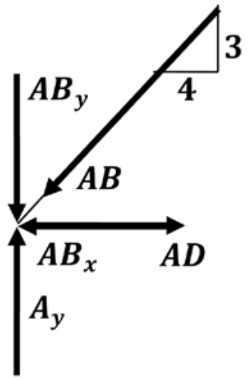
$$+\Sigma M_A = -F(d) + F(d) = 0$$
$$+\Sigma M_A = -20\ lb\ (4\ in) + C_y\ (8\ in) = 0$$
$$C_y\ (8\ in) = 80\ lb\ in$$
$$C_y = 10\ lbs.$$
$$+\Sigma M_C = F(d) - F(d) = 0$$
$$+\Sigma M_C = 80\ lb\ in - A_y\ (8in) = 0$$
$$-A_y\ (8in) = -80lb\ in$$
$$A_y = 10\ lbs.$$

Figure 2. External Reactions Joints A and C.

Note: The rotational arrow and plus sign symbols are an indication of positive rotational direction and not magnitude. Forces causing rotation counterclockwise will be positive, and forces causing rotation clockwise, negative. If the final result of the external forces (in this case A_y and C_y) were negative, that indicates that the assumed direction of those forces is incorrect. If A_y equaled a negative 10 pounds, that means that the A_y force would be going down on the y-axis.

What is a Truss?

"Trusses are frameworks composed of slender members joined together at the ends of the members" (Cheng, 1997, p. 144). Trusses are often made from wood or metal, and the members in metal trusses use a variety of cross-sectional shapes. The members in wooden trusses are joined at their ends by a nailing brace or gusset. Metal truss members are joined at the ends by bolting, riveting, or welding to a gusset plate. Trusses are easier to design and solve



$$\text{Step 1: } +\uparrow \Sigma F_y = A_y - AB_y = 0$$

$$AB_y = 10\ lbs.$$

$$\text{Step 2: } \frac{3}{4} = \frac{AB_y}{AB_x}$$

$$AB_x = 13.33\ lbs$$

Step 3: Pythagorean Theorem

$$AB^2 = (AB_x{}^2 + AB_y{}^2)$$

$$AB = 16.67\ lbs.\ \text{(compression)}$$

$$\text{Step 4: } \overset{+}{\longrightarrow} \Sigma F_x = -AB_x + AD = 0$$

$$AD = 13.33\ lbs\ \text{(tension)}$$

Figure 3. Equilibrium of Joint A.

Note: There are two reasons that it is known that member AB is in compression: (1) the assumed direction of the force AB is going into the joint and (2) the result of Step 3 is positive. If the result at Step 3 was negative, the assumed direction of the force in member AB would be incorrect. Similarly, it is known that AD is in tension because (1) the assumed direction of the force is going away from the joint and (2) the result of Step 4 was positive.

for statically because concurrent and nonconcurrent coplanar force systems are easier to solve than spatial force systems. The three common types of force systems include: (1) concurrent coplanar, (2) nonconcurrent coplanar, and (3) spatial (i.e., noncoplanar). In concurrent coplanar force systems, all forces act through the same point and are on the same plane, but the forces are not collinear or parallel. In noncurrent coplanar force systems, all forces are on the same plane but act through different points and may be collinear or parallel. In spatial force systems, all forces are not on the same plane or act through the same point. Bridges (a combination of multiple trusses) can be solved as spatial force systems, but that is beyond the scope of this article. However, a bridge can be solved as two side-by-side planar trusses connected by lateral bracing and floor beams that support a decking material. This means that it is possible to solve for the force causing bridge failure simply by solving individual trusses on each side of the bridge.

There are three assumptions held when solving a truss: (a) the members are connected at their ends to form joints that behave like frictionless pins; (b) all forces acting on the truss are applied at the joints; and (c) the members, joints, and loads all lie in the same plane (Morrow & Kokernak, 2004, p. 155). What do these assumptions mean for student-made trusses? First, students need to adequately glue and possibly use small nails to pin each joint. Second, students will apply external loading only on joints. Third, the compressive members of a truss will bend, and members need to be allowed to bend; truss members will most likely fail as a result of bending and not pure compression. Fourth, all joints need to remain in the same plane to fail at the predicted amount.

Using Method of Joints to Solve a Truss

While learning about using methods of joints to solve trusses, it is generally considered important to learn about forces and solving force systems. Aspects of solving both concurrent and noncurrent coplanar force systems are used when solving trusses using the method of joints. However, concurrent coplanar force systems are primarily used during solving trusses using the method of joints. Trusses can be statically determinate or indeterminate, based on the relationship between the number of joints, members, and reaction forces. For a truss to be statically determinate, the number of joints (j) multiplied by 2 must equal the number of members (m) added to the number of reaction forces (r) (2j = m + r) (Cheng, 1997). For example, the simple truss in Figure 1 has 4 joints (A, B, C, and D), 5 members (AB, AD, BC, BD, and CD), and 3 reaction forces (A_x, A_y, and C_y), making the truss statically determinate. There are two characteristics of a simple truss: (1) the truss must be statically determinate and (2) the truss must be supported by a pinned (fixed) support (Joint A) and supported by a roller support (Joint C) (as illustrated in Figure 1). The pinned joint can support an external force on both the x and y-axis (A_x and A_y). The roller joint can support only an external force on the y-axis (C_y). The reaction forces must be determined prior to using the method of joints to solve the truss.

Using the 20-pound external load, moment equations are used to determine forces in A_y and C_y (Figure 2). A moment is a force multiplied by a perpendicular distance (m = f x d). The force of A_x equals 0 because the 20-pound external load is acting directly on the y-axis and is not inducing any force on the x-axis. The ΣM_A value reads as *the sum of moments about joint A*. This is visualized as putting a pin at joint A to allow for rotation and then determining the forces multiplied by their perpendicular distances from A that cause rotation about A. Since the 20 pounds is positioned directly in the middle of the truss, it makes sense that A_y and C_y would each equal 10 pounds to have the force system in equilibrium. Now that the external forces are known, it is time to determine the force in each member using the method of joints (Figures 3, 5, 6, and 7).

$$\uparrow^{+}\ \Sigma F_y = A_y - AB\sin(36.87) = 0$$

$$AB = \frac{10\ lbs.}{\sin(36.87)}$$

$$AB = 16.67\ lbs.\ \text{(compression)}$$

OR another option:

$$\uparrow^{+}\ \Sigma F_y = A_y - AB\left(\frac{3}{5}\right) = 0$$

$$AB = \frac{10\ lbs.}{\left(\frac{3}{5}\right)}$$

$$AB = 16.67\ lbs.\ \text{(compression)}$$

$$\xrightarrow{+}\ \Sigma F_x = AD - AB\cos(36.87) = 0$$

$$AD = 16.67\ lbs.\cos(36.87)$$

$$AD = 13.33\ lbs.\ \text{(tension)}$$

OR another option:

$$\xrightarrow{+}\ \Sigma F_x = AD - AB\left(\frac{4}{5}\right) = 0$$

$$AD = 16.67\ lbs.\left(\frac{4}{5}\right)$$

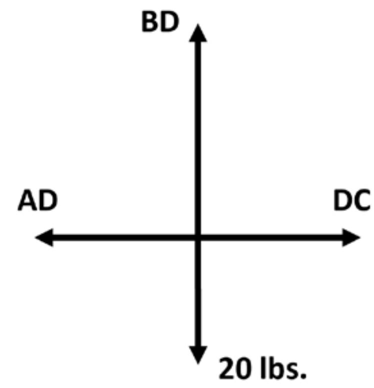
$$AB = 13.33\ lbs.\ \text{(tension)}$$

Figure 4. Other ways to write the ΣF_y and ΣF_x equations for Joint A.

In Figure 3, 5, 6, and 7 equilibrium equations (i.e., ΣF_x and ΣF_y) are written for all forces on the x-axis and y-axis. The ΣF_x value reads as *the sum of forces on the x-axis*. The ΣF_x and ΣF_y equations must equal zero for forces in the system to be in balance. Truss member forces not directly on the x- or y-axis are commonly separated into their x and y component forces (for example AB_x and AB_y in Figure 3). Separating truss members' forces into their x and y component forces helps students visualize and solve the force system. However, once students become more familiar with solving force systems (e.g., trusses), writing equilibrium equations using trigonometric functions for right triangles is faster and more beneficial when simultaneously solving two linear equations with two unknowns.

When solving trusses or any two-dimensional equilibrium problems (i.e., concurrent coplanar and nonconcurrent coplanar force systems), occasionally there is a need to solve two simultaneous linear equations with two unknowns. Based on the authors' experience, this tends to happen more often when using the method of joints to solve trusses. There are three methods to simultaneously solve two linear equations with two unknowns: (a) elimination by substitution, (b) elimination by addition and subtraction, and (c) Cramer's rule (i.e., determinant). Both matrices and determinants (basically the result of square matrix) are important aspects of linear mathematics. In the example provided, solving simultaneous linear equations with two unknowns was not needed. Using trigonometric functions for right triangles, there are additional ways to write the equilibrium equations used for determining the force in truss members (Figure 4). In Figure 4, the AB member is not separated into the x and y component forces. Instead the ΣF_x equation is written one way using cosine and another way using the tangent ratio $\left(\frac{\text{adj.}}{\text{hyp.}}\right)$ (Figure 4). While the ΣF_y equation is written one way using sine and another way using the tangent ratio $\left(\frac{\text{opp.}}{\text{hyp.}}\right)$ (Figure 4), the other ways to write the equilibrium equations make using any of the three methods for simultaneous equation solving much easier.

There is a ratio between the external load and the force produced by this external load in each member. Once the force in each member is calculated based on the external load, the ratio can be determined (Figure 8), making it easier to determine the force in each member with a known external load. For example, if the



Step 1: $\xrightarrow{+}\ \Sigma F_x = CD - AD = 0$

$$CD = 13.33\ lbs.\ \text{(tension)}$$

Step 2: $\uparrow^{+}\ \Sigma F_y = BD - 20\ lbs = 0$

$$BD = 20\ lbs.\ \text{(tension)}$$

Figure 5. Equilibrium of Joint D.

external load was 30 pounds and was applied at joint D, the force in member AB would be 24.99 pounds (e.g., 30 lbs. x .833); this is especially useful in determining the amount of weight the truss can hold and which member is going to fail first.

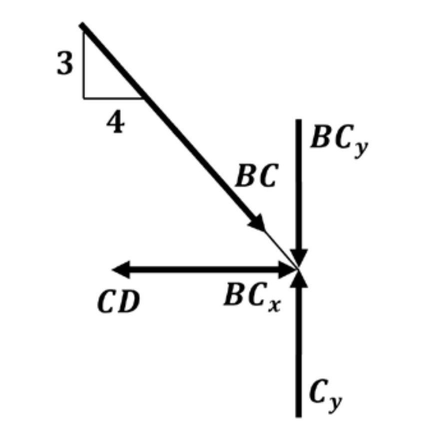
Truss Materials for Activity

To select an appropriate material, various craft sticks and popsicle sticks were thoroughly tested by students; the strength of each member (truss material) must be known to determine the amount of weight a truss can hold before a known member will fail. Specifically sized pine craft sticks were selected due to quality and repeatability during compressive and tensile strength testing. The craft sticks are sold in bags of 75, sized 3/32" x .25" x 5.75" (Thickness x Width x Length), with squared ends (they are not popsicle sticks). The craft sticks will fail at 30 pounds in tension and at varying compressive forces based on final cut length (Figure 9). Note: Sometimes the cut length is longer to account for member overlap for gluing purposes at joints. In these cases, the members' length should be measured from the center of each corresponding joint. Although unlikely, based on authors' experience, it is possible for students to design a truss that will have a member fail in tension prior to a member failing in compression. When using only wood glue and the specified craft sticks, trusses designed to fail in tension will result in joint failure if joints are not pinned.

In the case of the simple truss from Figure 1, member AB will be the member that causes truss failure as the longest compressive member with the most force. Based on Figure 9, a member made of the specified craft sticks and is 5 inches long, like member AB, will fail at 9.28 pounds of force. Using the ratio from Figure 8, the truss will be able to hold 11.14 pounds before member AB fails $\left(\frac{9.28\ lbs.}{.833}\right)$. If students minimize possible errors by building their trusses exactly as designed and calculated, most trusses will fail within 5% of the calculated amount. Figure 10 (page 28) shows the basic classroom setup used for testing trusses. The setup consists of 2 C-clamps, locking pliers, a 5-gallon bucket where sand or gravel weight is added, and two classroom tables set up parallel to each other—in this case 8 inches apart. After the truss fails, a luggage scale is used to measure the combined weight of the locking pliers, bucket, and sand or gravel.

Conclusion

Constructing trusses is a useful activity in technology and engineering education because it is hands-on (whether made from craft sticks or 2 X materials) and reinforces the use of mathematical formulas and scientific principles (Figure 11, page 34). Students should be able to readily see the connection between using their hands and minds to solve a fun, authentic problem.



Step 1: $\xrightarrow{+}$ $\Sigma F_x = -CD + BC_x = 0$

$$BC_x = 13.33\ lbs.$$

Step 2: $+\uparrow$ $\Sigma F_y = -BC_y + C_y = 0$

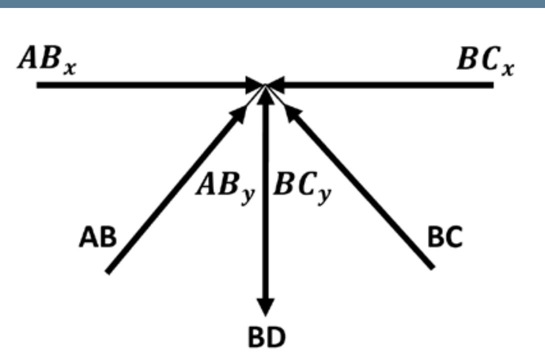
$$BC_y = 10\ lbs$$

Step 3: Pythagorean Theorem

$$BC^2 = BC_x^{\ 2} + BC_y^{\ 2}$$

$$BC = 16.67\ lbs.\ \text{(compression)}$$

Figure 6. Equilibrium of Joint C.



Step 1: $\xrightarrow{+}$ $\Sigma F_x = AB_x - BC_x = 0$

$$\Sigma F_x = 0$$

The truss is in equilibrium in the x-axis

Step 2: $+\uparrow$ $\Sigma F_y = BC_y + AB_y - BD = 0$

$$\Sigma F_y = 0$$

The truss is in equilibrium in the y-axis

Figure 7. Checking Answers Joint B.

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If a 20-pound external load is applied to the truss in Figure 1, the forces in each member and corresponding ratios equal:

AB = 16.67 lbs. making the ratio $\frac{16.66\ lbs.}{20\ lbs.} = .833$

BC = 16.67 lbs. making the ratio $\frac{16.66\ lbs.}{20\ lbs.} = .833$

CD = 13.33 lbs. making the ratio $\frac{13.33\ lbs.}{20\ lbs.} = .67$

BD = 20 lbs. making the ratio $\frac{20\ lbs.}{20\ lbs.} = 1$

AD = 13.33 lbs. making the ratio $\frac{13.33\ lbs.}{20\ lbs.} = .67$

Figure 8. Ratio of Member Forces to External Load.

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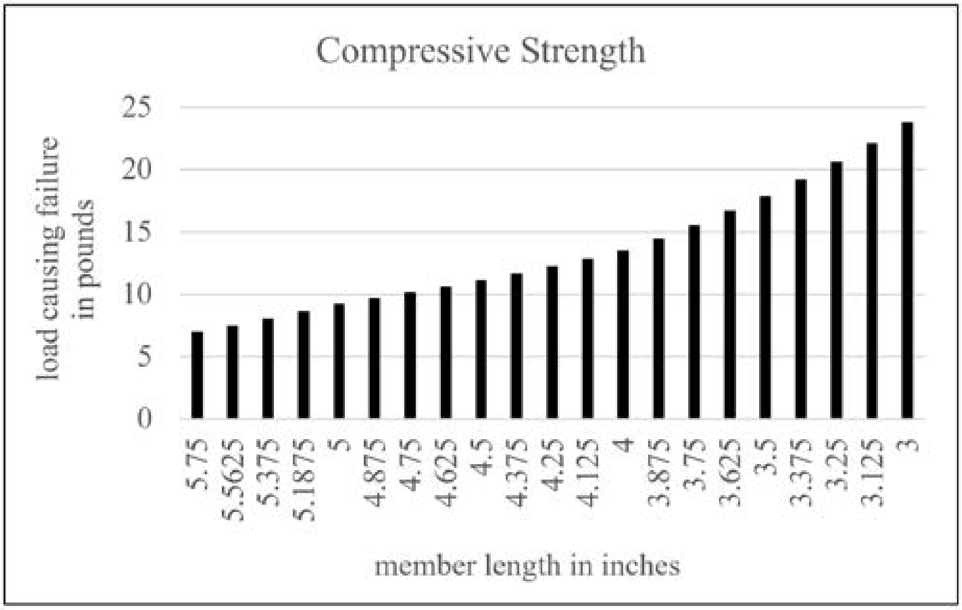


Member Length (inches)	Load Causing Failure (pounds)
5.75	7
5.5625	7.51
5.375	8.06
5.1875	8.65
5	9.28
4.875	9.715
4.75	10.17
4.625	10.65
4.5	11.16
4.375	11.69
4.25	12.26
4.125	12.86
4	13.5
3.875	14.497
3.75	15.56
3.625	16.69
3.5	17.9
3.375	19.21
3.25	20.62
3.125	22.147
3	23.8

Figure 9. Compressive Strength of Craft Sticks.

OVERVIEW: Participants design and construct a model structure that is destructively tested to determine design efficiency. This event is completed at the home school and brought to the state conference for judging/testing.

I. CONTEST PURPOSE

The purpose of the Structural Engineering Contest is to provide a means for TSA members to demonstrate their ability to design and fabricate a structure after having been assigned span and width specifications.

II. ELIGIBILITY FOR ENTRY

- All IL-TSA members in good standing are eligible to enter the Structural Engineering Contest.
- One entry per student (unlimited entries from chapter) is allowed for this competitive event.

III. LEVELS OF COMPETITION

There are two (2) levels of competition in the Structural Engineering Contest, Level I (Grades 6-9) and Level II (Grades 10-12), as described in the General Rules.

IV. TIME LIMITATIONS

Since the structural artifacts are constructed prior to the state conference, there is no time limitation other than the artifacts must have been constructed during the current school year.

V. SPECIFIC REGULATIONS

- Trusses must be constructed according to specifications.
- Trusses must be constructed according to the following definitions and graphic illustrations, which are an integral part of the contest regulations.
- **Span:** The overall span of the truss must be 12".
- All stock used to construct the truss may only have adhesive at joints (no adhesive spread over or coating of stock is allowed).
- **Materials:** The amount of basswood or balsawood 1/8" x 1/8" or 3/32" x 3/32" stock is unlimited, and one 3" x 5" note card may be used to construct the truss. Any type of liquid adhesive can be used in the construction of the truss.
- The structure must include 2 or 4 individual trusses, attached together to create the truss structure, but must not be wider than 3".
- **Lamination:** Two pieces of 1/8" x 1/8" or 3/32" x 3/32" stock glued together surface-to-surface with the wood grain running parallel. Lamination of more than two pieces is not permitted.

Figure 11.
2 x 4 Trusses in School Play Set.

Failure Weight: The greatest weight recorded during testing before failure of the structure. Failure to comply: If a structure fails to comply with any regulation, a penalty reduction of twenty percent (20%) of the greatest weight held in the contest is subtracted from the individual's failure weight.

Gusset: A panel or bracket attached to corners or intersections of truss components to add strength or stiffness. Note cards may be cut and used as gussets to strengthen the joints of truss structures. Note card gussets on trusses are to be no larger than the diameter of a current issue American quarter-dollar coin. The gussets may not touch another note card gusset or overlap other trusses. They may not be sandwiched between two (2) laminated members. The tester will be set at 10" for the 12" truss.

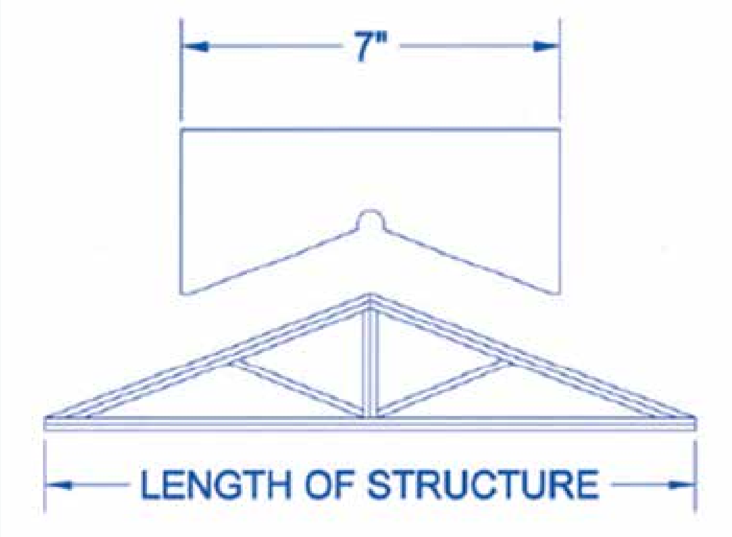
The roof truss will need to be fabricated on a 5/12 slope. This simply means that the truss would rise vertically 5 units for every 12 units of horizontal run.

No part of the structure may extend below the bottom chord of the truss.

Truss members to simulate decking may be used above the truss to add stability. Internal members may be used to simulate cross bracing, but be certain to avoid blocking the center (the location of the testing rod). An opening of ½" x ½" must be present in the

design in order for the truss to be tested correctly.
The roof truss must be a triangle.

The peak of the truss is to be centered in the length of the truss.



VII. REQUIRED CONTEST PERSONNEL AND EQUIPMENT

- Contest Coordinator
- Assistant to help with testing
- Testing equipment
    - o Appropriate testing equipment will be supplied
    - o Scales
    - o Calculator
- Evaluation forms

VIII. CRITERIA FOR JUDGING

- The structure is weighed before testing, and the weight is recorded on the evaluation form.
- An increasing load is applied to the structure via the test block until the structure fails.
- The failure weight is recorded on the evaluation form.
- The efficiency is determined by the failure weight x 4.54 divided by the weight of the structure in grams.
- The efficiency is rounded off to three (3) decimal places and recorded on the evaluation form.
- The highest numeric efficiency is the winner. In case of an efficiency tie, the greatest weight held by the tied entries will be declared the winner.
- Structures that violate guidelines will receive a deduction of 20% of the greatest weight held for the first violation.
- Structures are not to be tested if
    - o There are two (2) or more rule violations.
    - o The structure cannot be placed on the tester.
    - o The testing hook cannot be placed in the center of the structure.
    - o Straight pins are left in the structure.
    - o There is a failure to wear safety eyewear.
    - o There is evidence of conduct unbecoming a TSA conference participant during check-in, fabrication, or testing.

The structure is destructively tested using the breaker block shown below.