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# Solving Concurrent and Nonconcurrent Coplanar Force Systems: Balancing Theory and Practice in the Technology and Engineering Education Classroom

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# balancing theory and practice in the T&EE classroom

by Andrew J. Hughes and Chris Merrill, DTE

*T&EE's implementation of statics with an engineering approach could promote students' ability to transfer learning from scientific theory into conceptualized practical application within an engineering design problem.*

The basic concepts inherent to statics, including unbalanced and balanced forces and instability and stability of physical systems, have traditionally been covered in middle and high school physical science courses (Physical Science as indicated in *Next Generation Science Standards*). Yet, these concepts are covered using a physical science approach that has minor but significant differences in terminology, structure, and focus when compared with an engineering approach. Since a robust understanding of statics is considered an essential component for most engineering disciplines, Technology and Engineering Education's (T&EE) implementation of statics with an engineering approach could promote students' ability to transfer learning from scientific theory into conceptualized practical application within an engineering design problem. During the utilitarian period of our

**Crushed cans.** *Photo credit: Ruth Hartnup, Vancouver, Canada, creativecommons.org.*

discipline (i.e., Manual Training [Arts] and Industrial Arts), scientific theories were applied to practical static problems like tree stands, dirt-bike stands, can crushers, wall brackets for hanging objects, scissor lifts, log splitters, dumb trailers, furniture, and other similar projects and mechanisms. Moving away from a more utilitarian rationale and towards an academic one, Technology Education and now T&EE needs to find the balance between theoretical and practical learning. The intention of this article is to provide the reader with a better understanding of an engineering approach to statics involving the terminology, structure, and focus aligned with applying theory to practical hands-on learning activities.

## Statics

Statics is a branch of mechanics that involves studying forces applied to physical systems that are in equilibrium (Morrow & Kokernak, 2004). Dynamics and strength of materials are the other two branches of mechanics. In statics, all physical systems are considered to be rigid bodies that experience no change in size or shape from applied forces. Forces, considered a push or pull, act on physical systems resulting in no acceleration due to the system being in static equilibrium. Systems in static equilibrium are either moving at a constant velocity or not moving based on applied forces (i.e., no acceleration). Archimedes (287-212 BCE) is often credited for the first written theories of statics from two experiments, (1) equilibrium of a lever and the (2) law of buoyancy. Yet, the basic understanding of forces and static equilibrium date back to at least the earliest construction of human-made structures (e.g., Gobekli Tepe 12000 BCE, Ggantija 3700 BCE, Egyptian Pyramids 2630-2611 BCE, and many others). Modern statics is based on Simon Stevin's (i.e., Stevinus) theorem of the triangle of forces (about 1600 CE), which was equivalent to the parallelogram of forces (i.e., parallelogram law) presented by Bernard Lamy (1679 CE) and later proofed by Newton (1729 CE) and others. Pierre Varignon, Leonhard Euler, the Bernoulli family, Jean le Rond d'Alembert, Immanuel Kant, and others made many other important contributions in mathematics that helped conceptualize statics throughout the 1600s and 1700s. Theories and principles in engineering are commonly identified by the last name of these and other influential people (e.g., Newton's Law).

## Forces and Force Systems

Understanding forces includes various aspects, some that will and some that will not be discussed in this article. The nature and types of forces, Newton's Laws, and the principle of transmissibility will not be specifically covered here but should be considered important knowledge for students learning about statics. This article will cover force quantities and types of force systems. There are two force quantities: (1) scalar and (2) vector. Scalar quantities include length, area, volume, mass, and others. Algebra is primarily used when working with scalar quantities. Vector quantities combine magnitude, direction, and point of application. Vector force systems are solved using the parallelogram law, which basically states vectors are added geometrically. For this article, only vector quantities will be used.

## Standards and Benchmarks

The standards and benchmarks utilized with the included can-crusher activity are:

***Standards for Technological Literacy:***

- **Standard 5:** Students will develop an understanding of the effects of technology on the environment.
    - **Benchmark D:** The management of waste produced by technological systems is an important societal issue.
    - **Benchmark K:** Humans devise technologies to reduce the negative consequences of other technologies.
- **Standard 8:** Students will develop an understanding of the attributes of design.
    - **Benchmark G:** Requirements for a design are made up of criteria and constraints.
    - **Benchmark J:** The design needs to be continually checked and critiqued, and the ideas of the design must be redefined and improved.
- **Standard 9:** Students will develop an understanding of engineering design.
    - **Benchmark H:** Modeling, testing, evaluating, and modifying are used to transform ideas into practical solutions.
    - **Benchmark I:** Established design principles are used to evaluate existing designs, to collect data, and to guide the design process.
    - **Benchmark L:** The process of engineering design takes into account a number of factors.

***Next Generation Science Standards:***

- Middle (MS) and High School (HS)
    - **MS-PS2-2:** Plan an investigation to provide evidence that the change in an object's motion depends on the sum of the forces on the object and the mass of the object.
    - **MS-ETS1-1:** Define the criteria and constraints of a design problem with sufficient precision to ensure a successful solution, taking into account relevant scientific principles and potential impacts on people and the natural environment that may limit possible solutions.
    - **MS-ETS1-4:** Develop a model to generate data for iterative testing and modification of a proposed object, tool, or process such that an optimal design can be achieved.
    - **HS-PS2-1:** Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.
    - **HS-PS3-1:** Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other components and energy flows in and out of the system are known.
    - **HS-ETS1-2:** Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.
    - **HS-ETS1-3:** Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics as well as possible social, cultural, and environmental impacts.

The three common types of force systems include: (1) concurrent coplanar, (2) noncurrent coplanar, and (3) spatial (i.e., noncoplanar). Other common force system descriptors used in this article include collinear and parallel. In concurrent coplanar force systems (CCFS), all forces act through the same point and are on the same plane, but the forces are not collinear or parallel. In noncurrent coplanar force systems (NCFS), all forces are on the same plane but act through different points and maybe collinear or parallel. In spatial force systems, all forces are not on the same plane or act through the same point.

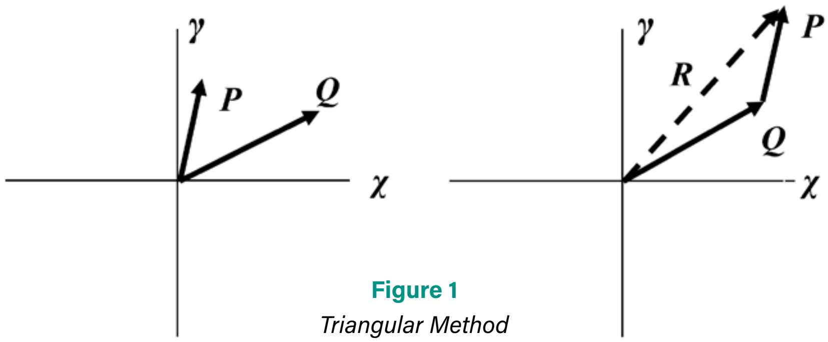



**Figure 1**
*Triangular Method*

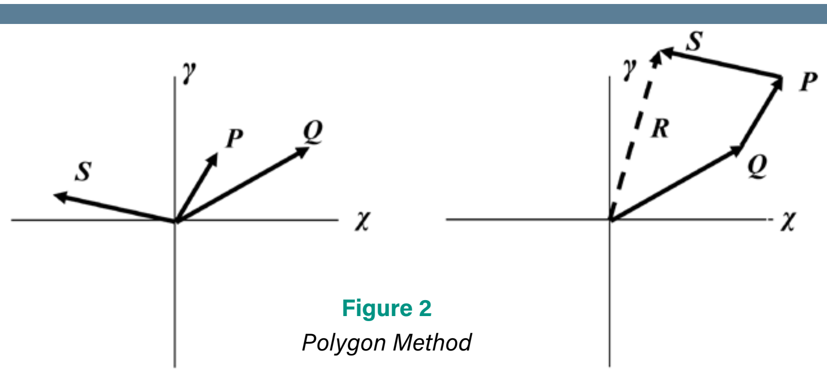
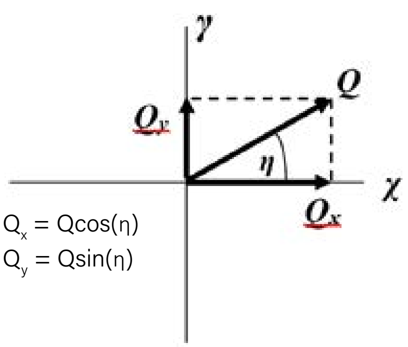


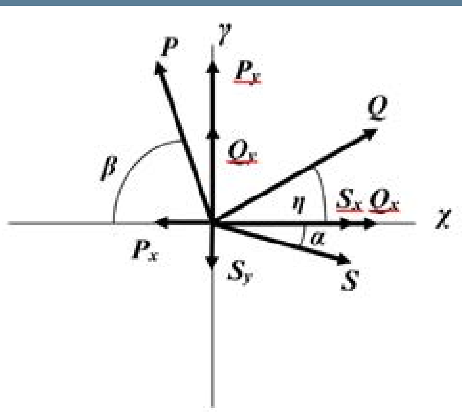

**Figure 2**
*Polygon Method*




$Q_x = Q\cos(\eta)$
$Q_y = Q\sin(\eta)$

**Figure 3**
*Rectangular Components*

Note: The $Q_x$ and $Q_y$ forces can be added tip to tail as seen in the triangular method (Figure 1).




$Q_x = Q\cos(\eta) = 30\text{lbs.} \cos(30) = 25.98$ lbs
$Q_y = Q\sin(\eta) = 30\text{lbs.} \sin(30) = 15$ lbs
$P_x = P\cos(\beta) = 45\text{lbs.} \cos(75) = 11.65$ lbs
$P_y = P\sin(\beta) = 45\text{lbs.} \cos(75) = 43.47$ lbs
$S_x = S\cos(\alpha) = 20\text{lbs.} \cos(15) = 19.32$ lbs
$S_y = S\sin(\alpha) = 20\text{lbs.} \cos(15) = 5.18$ lbs

**Figure 4**
*Multiple Rectangular Components*

## Solving Concurrent Coplanar Force Systems

There are three methods that can be used to help visualize and solve for resultant forces in CCFS: (1) triangular, (2) polygon, and (3) rectangular component methods. A resultant (R) is the combination of all forces in the force system into one representative force that will produce the same effect as all the other forces within the system. The triangular method can only be used for CCFS with three forces including a resultant force (Figure 1), while the polygon method is only used when more than three forces exist, including a resultant force (Figure 2). When using the triangular or polygon method, notice that forces are attached tip to tail. The resultant starts at the tail of the first force and ends at the tip of the last force. Trigonometric methods (e.g., law of sines and cosines) and special angle equalities (e.g. vertical, alternate interior, and corresponding angles) are used to solve for the resultant force quantities represented in Figures 1 and 2.

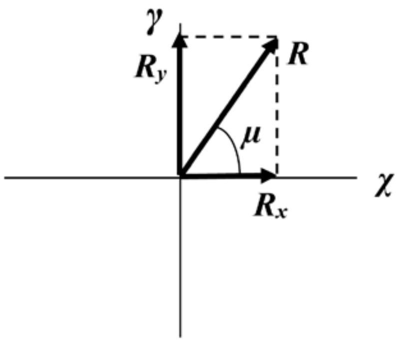
The rectangular component method is often seen as less challenging than the triangular and polygon methods because only trigonometric functions for right triangles are used, and the resultant's component forces are solved algebraically. The rectangular component method separates a force into two component forces, one force on the x-axis and one force on the y-axis (Figure 3). The two rectangular components are the same as the single force. The single force Q is the resultant of the two component forces $Q_x$ and $Q_y$ (Figure 3). The ability to determine component forces of a single force allows for easy algebraic combination of multiple forces in a force system (Figure 4). For example, if forces Q = 30 lbs., P = 45 lbs., S = 20 lbs., angle $\eta = 30°$, angle $\beta = 75°$, and angle $\alpha = 15°$ then rectangular component forces $Q_x = 25.98$ lbs., $Q_y = 15$ lbs., $P_x = 11.65$ lbs., $P_y = 43.47$ lbs., $S_x = 19.32$ lbs., and $S_y = 5.18$ lbs. Next, all forces on the x-axis can be added together, and all forces on the y-axes can be added together (Figure 5). The arrow and plus sign symbols are used

$$\xrightarrow{+}\ \Sigma R_x = Qx + Sx - Px = 25.98 \text{ lbs.} + 19.32 \text{ lbs.} - 11.65 \text{ lbs.} = 33.65 \text{ lbs.}$$

$$+\!\uparrow\ \Sigma R_y = Qy + Py - Sy = 15 \text{ lbs.} + 43.47 \text{ lbs.} - 5.18 \text{ lbs.} = 53.29 \text{ lbs.}$$

**Figure 5**
*Summarizing Multiple Rectangular Components*

Note: The arrow and plus sign symbols are an indication of positive direction and not magnitude. Summed forces going to the right and up will be considered positive, and forces going left and down will be considered negative. **The summed forces being positive or negative indicates direction only and not a positive or negative magnitude**. All forces have positive magnitude when solving force systems.




$$(R)^2 = (R_x)^2 + (R_y)^2 = (33.65 \text{ lbs.})^2 + (53.29 \text{ lbs.})^2 = \sqrt{3972.14\ lbs^2}$$
$$R = 63.02 \text{ lbs.}$$

$$\tan\mu = \frac{R_y}{R_x}$$

$$\mu = \tan^{-1}\frac{53.29\ lbs.}{33.65\ lbs.} = 57.7°$$

**Figure 6**
*Resultant of Multiple Rectangular Components*

Note: If R is in a quadrant other than 1 (based on the summed values of $R_x$ and $R_y$ above having positive or negative direction indications), the reference angle (θ) may need further calculations.

First quadrant ($R_x$ and $R_y$ have both positive direction): θ = μ
Second quadrant ($R_x$ has negative direction, and $R_y$ has positive direction): θ = 180 - μ
Third quadrant ($R_x$ and $R_y$ have both negative direction): θ = 180 + μ
Fourth quadrant ($R_x$ has positive direction, and $R_y$ has negative direction): θ = 360 - μ

to help a person remember that all forces considered positive are going right and up and all forces considered negative are going left and down. Since $R_x$ and $R_y$ are positive, that means that $R_x$ is acting to the right and $R_y$ is acting up on the corresponding axes. Now $R_x$ and $R_y$ should be combined into R using Pythagorean theorem as the single resultant force of forces Q, P, and S (Figure 6). Now that the basics of CCFS have been covered, the article transitions into NCFS problems.

## Solving Nonconcurrent Coplanar Force Systems

The first step in solving most CCFS and NCFS problems is drawing a free body diagram (Figure 7). In NCFS, the forces applied to a physical body produce a moment (i.e., force multiplied by perpendicular distance). A moment is produced relative to a central point (i.e., point of rotation, reference point, or reference axis) in NCFS by the application of a force at a different point with a perpendicular distance to the central point (Figure 7). In Figure 7, force F can be applied anywhere along a line perpendicular to point A at angle η with the same effect being produced on the nut (principle of transmissibility). The moment effect caused by force F can be solved in two different but equivalent ways: $M_A=Fd_1$ or $M_A=F_yd_2$ (i.e., the theorem of moments). When force F is broken down into rectangular component forces $F_x$ and $F_y$, the force of $F_x$ acts through point A, resulting in no moment effect (i.e., point A at the center of the nut experiences no change from force $F_x$). Basically, the $F_x$ component is not adding to the moment (torque) effect on the nut at point A. Additionally, in a practical sense, using the $F_y$ component force to calculate the moment would be uncommon due to the principle of transmissibility and the related difficulty with determining the true perpendicular distance $d_2$.

The calculation of multiple forces in an NCFS is the addition and subtraction of multiple moments. Applying a 20-pound external load to a physical body, moment equations are used to determine supporting forces $A_y$ and $C_y$ (Figure 8). Force $A_x$ = 0 because the 20-pound external load is acting directly on the y-axis, resulting in

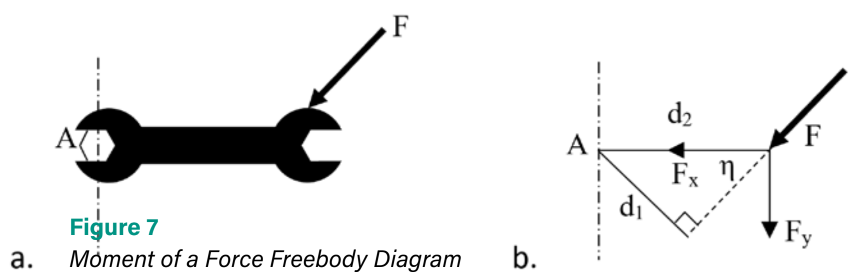
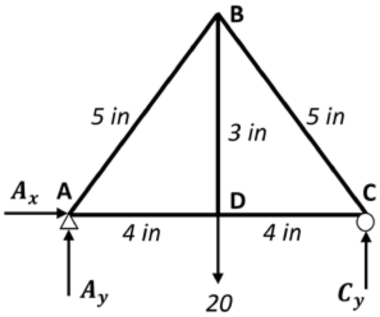



**Figure 7**
*Moment of a Force Freebody Diagram*




$$\circlearrowleft + \Sigma M_A = -F(d) + C_y(d) = 0$$

$$\circlearrowleft + \Sigma M_A = -20\ lb\ (4\ in) + C_y\ (8\ in) = 0$$

$$C_y\ (8\ in) = 80\ lb\ in$$
$$C_y = 10\ lbs.$$

$$\circlearrowleft + \Sigma M_C = F(d) - A_y(d) = 0$$

$$\circlearrowleft + \Sigma M_C = 80\ lb\ in - A_y\ (8in) = 0$$

$$-A_y\ (8in) = -80 lb\ in$$
$$A_y = 10\ lbs.$$

Note: The rotational arrow and plus sign symbols are an indication of positive rotational direction and not magnitude. Forces causing rotation counterclockwise will be positive, and forces causing rotation clockwise negative. If the final result of the external forces (in this case $A_y$ and $C_y$) were negative, that indicates that the assumed direction of those forces is incorrect. If $A_y$ equaled a negative 10 pounds, that means that the $A_y$ force would be going down on the y-axis instead of up.

**Figure 8.** *Determining External Forces*

no external force on the x-axis. The ΣM_A value reads as *the sum of moments about joint A*. This is visualized as putting a nail at the center of joint A allowing for rotation about joint A. Determining ΣM_A involves the forces causing rotation about A multiplied by their perpendicular distances from A. Since the 20 pounds is positioned directly in the middle of the truss, it makes sense that $A_y$ and $C_y$ would each equal 10 pounds to have the force system in equilibrium.

## Hands-On Practical Application

The knowledge of force systems can be applied to understanding many situations in a practical T&EE classroom including numerous aspects of robotics (i.e., chassis, weapon, and drive system/maneuverability design), structural members (i.e., beams, columns, and trusses) and the design and creation of many physical objects and mechanisms (i.e., can crusher, Geneva mechanism, drive train, gear train, and many more) (Figures 9, 10, and 11). In Figure 9, a student is designing a jack stand with three legs. The weight of 2500 pounds is divided by the 3 legs of the stand. The 60° angle results in $F_x$ = 721.68 lbs. and $F_y$ = 416.665 lbs. There were two main considerations the student thought about during this design; one was the double shear stress on the pin used to adjust height, and the other was the $F_x$ force. The pin used to adjust the jack-stand height should have an increased safety factor due to the potential of someone using the jack stand to support a vehicle they are working underneath. Without any safety factor, the pin would need to support 1250 pounds in single shear (2500 lbs. for double shear). Considering that a 1/4″-20 grade 2 bolt will fail at a single sheer stress of about 2350 pounds, this choice would provide a safety factor of 1.88 (2350÷1250), although the authors would not consider this an acceptable safety factor considering the risk and potential to load the jack stand with more than 2500 lbs. Additionally, the student considered that the $F_x$ force was trying to pull the legs off the jack-stand body. This meant that the welds attaching the jack-stand leg to the body needed to be substantial enough to withstand the loading, again with an appropriate safety factor. The jack-stand body also needed to be made of a material that would not deform under loading, as this could impede the height adjustment functionality. While not included in this article, allowable stresses on the welds, shear, and bearing stresses for the height-adjusting pin, and allowable stresses on the jack-stand body were also calculated so that appropriately strong materials could be selected. After building a prototype, the student was able to see that the 60° angle may be too much due to the increased floor space the jack stand required and carefully considered reducing the angle between the jack-stand body and each leg.

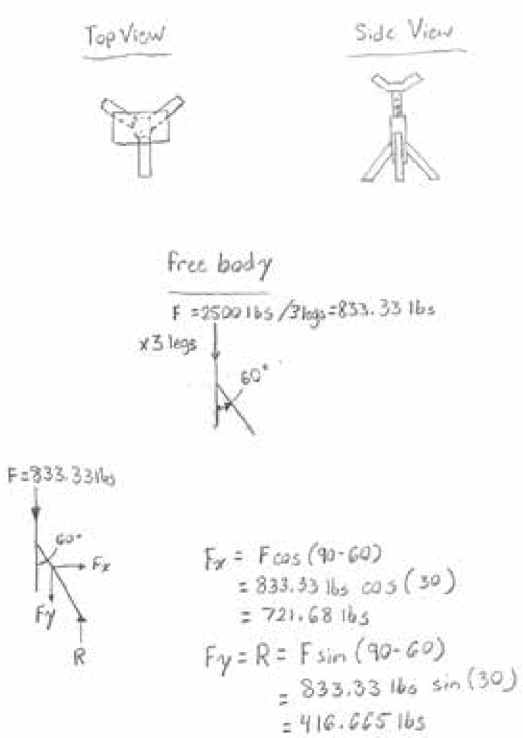



**Figure 9**
*High School Student's Jack-Stand Design.*

In Figure 10, there are cables attached to motors and lever arms that are being used to lift objects. Based on the current design math, the motors would need to produce about 28 in.-lbs. of torque to lift the target weight of 25 pounds. The targeted torque value was 3 in.-lbs. The student was able to use the calculated information to think about and redesign based on characteristics like changing the ratios of the lever arms, position of the motors, and how forces changed based on angles. The student approached the problem scientifically, making one change at a time and then recalculating to determine the impact of that change.

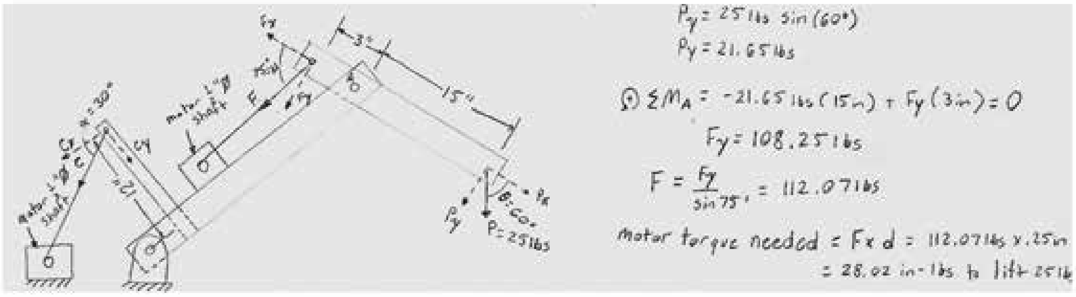



**Figure 10.** *High School Student's Robotic Arm Design*

Another example is the design and creation of an aluminum can crusher (included design challenge). The can crusher that students design may be quite different than the one presented (Figure 11). However, this can-crusher example is being presented to show how forces act in a force system. Figure 11a is a diagram of a common can-crusher design. Force F would be applied at point A causing the counterclockwise rotation of lever arm AD. After lever arm AD passes through a linear alignment with lever arm BC (Figure 11b), force F would start increasing to overcome force R (the resistance of the can) (Figure 11c). When lever arm BC is perfectly vertical, as it is throughout the entire process of crushing the can, force R basically acts directly through BC and point C (Figure 12). (Note: Viewing the problem this way basically ignores the forces in BE and BC as well as the corresponding moments produced. The forces in BE and BC will need to be considered.) Since distances d1 and d2 are known, the component forces for R are determined. Notice that $R_x$ is equal to 241.48 pounds but acts in line with the point of rotation D. This means that $R_x$ does not add to any moment. As mentioned in the note above, the forces (and corresponding effects) on lever arms BC (tension), BE (bending), and the fasteners at points B and C (shear) are substantial until deformation of the can occurs. Determining how the forces "flow" throughout the structure of the can crusher will help students design by adjusting lever arm ratios and identify components that need strength adjustments. Also notice that $R_y$ is only 64.7 pounds and acts at a perdenciular distance of 2 inches ($d_1$) from point D. Performing a sum of moments about point D, force F is determined to be 10.78 pounds. This means that only a force of 10.78 pounds at point A is required to crush an aluminum can requiring 250 pounds of force (R) in the presented can crusher.

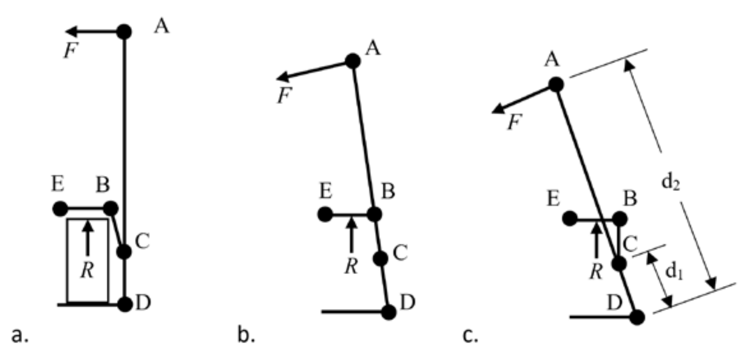
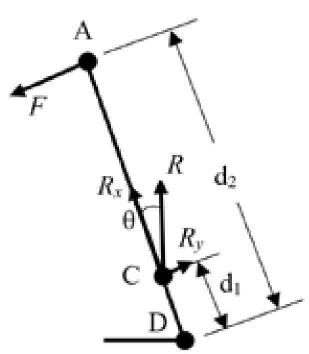



**Figure 11.** *Aluminum Can Crusher Diagram*




$$R_x = R\cos(\theta) = 250\text{lbs.} \cos(15) = 241.48 \text{ lbs}$$
$$R_y = R\sin(\theta) = 250\text{lbs.} \sin(15) = 64.7 \text{ lbs}$$
$$\circlearrowleft + \Sigma M_D = F(d_2) - R_y(d_1) = 0$$
$$\circlearrowleft + \Sigma M_D = F\,(12\ in) - 64.7\ lbs\,(2\ in) = 0$$
$$F\,(12\ in) = 129.4\ lb\ in$$
$$F = 10.78\ lbs.$$

**Figure 12.** *Free Body Diagram*

## Conclusion

Why is crushing aluminum cans so important? There are about 6,375 aluminum cans sold every second in the world. That is about 550 million every day. If these aluminum cans are not crushed, each day that equates to a volume of more than 1,148,000 cubic feet of waste (about the same volume as 285 school buses) that waste management facilities must accommodate. If the cans are crushed, the volume of 1,148,000 cubic feet can be reduced by at least 75% to 287,000 cubic feet each day in the world. During the can-crusher activity, the discussion centers on recycled materials and what happens to them after they are recycled. In the case of aluminum cans, they are 100% recyclable. However, other recycled materials like plastics are no longer being imported by China. As a result, these plastics are entering landfills across the United States. Another design challenge using force system calculations is a device or system that sorts, washes, and shreds recycled plastic at the school.

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## Aluminum-Can Crusher Design Challenge

The challenge is to design an aluminum-can crusher based on the constraints. Your aluminum-can crusher must be built with the following constraints:

**Constraints:**

1. Ability to crush an aluminum can with a range of 40 to 250 pounds and average 170 pounds of force.
2. Ability to crush a maximum number of aluminum cans in a given time, fast cycle time.
3. Cost-effective in terms of cost to manufacture and selling price.
4. Ability to be efficiently mass-produced.
5. Ability to be mounted, stored, and used effectively and efficiently by a person with average capability.

**Points of Interest:**

1. The can crusher will be graded on these areas:
   a. Effectiveness and efficiency of crushing an aluminum can.
   b. Manufacturability and salability.
   c. Usability and design.
2. Constraints are requirements.

**Items to Consider During Design:**

1. What mechanism design will allow a person with average capability to apply 250 pounds of force to crush a can?
2. How will you balance the cost to manufacture with the selling price?
3. Looking at the calculations, which variables do you control?
4. How will you design the crusher while considering mounting, storage, and usability?
5. How can you design the crusher to reduce cycle time?

**Information About Aluminum Can Recycling:**

Why is crushing aluminum cans so important? There are about 6,375 aluminum cans sold every second in the world. That is about 550 million every day. If these aluminum cans are not crushed, each day that equates to a volume more than 1,148,000 cubic feet of waste (about the same volume as 285 school buses) that waste management facilities must accommodate. If the cans are crushed, the volume of 1,148,000 cubic feet can be reduced by at least 75% to 287,000 cubic feet each day in the world.