Nonlinear dynamics of piezoelectric high displacement actuators in cantilever mode

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Nonlinear dynamics of piezoelectric high displacement actuators in cantilever mode

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Experimental results of the nonlinear dynamic response of a piezoelectric high displacement actuator known as thin-layer composite unimorph ferroelectric driver and sensor were compared to a theoretical model, which utilizes the multiple scales method to connect the effective spring constant to higher-order stiffness constants $c_4$ of the piezoelectric layer. This type of actuator has prestress gradients resulting from the manufacturing process that have been reported to play an important role in enhanced actuation. A value of $c_4=−4.7 \times 10^{20} \text{N/m}^2$ was obtained for the higher-order lead zirconate titanate (PZT) stiffness coefficient, which is higher than other published results for PZT without prestress gradients. Peak resonance displacements over 1 mm were obtained for even small (100 $V_{pp}$) applied fields. The analysis showed a slight voltage dependence that was not specifically accounted for in the theory. This was confirmed by recasting data from other published results and further confirmed by dc offset studies reported here. © 2005 American Institute of Physics. [DOI: 10.1063/1.2041844]

I. INTRODUCTION

Piezoelectric actuators play an increasingly important role in a variety of technological applications. For example, piezoelectric stack actuators are used in a variety of ways in scanning microscopes. As the limits of scanning microscopes are pushed, the higher-order nonlinear material properties of the piezoelectric material play an increasingly important role. More recently, cantilevers with piezoelectric patches have been used as probes in some atomic force microscopes where a clear understanding of the frequency response is essential. In many other applications, it is desirable to operate the actuators at or near resonance where maximum displacements occur. However, these are precisely the conditions under which nonlinear effects are most pronounced. Historically, one disadvantage of traditional piezoelectric actuators is the fact that they produce small displacements compared to other actuator technologies. Efforts have been made to trade off the high pressure forces developed by piezoelectrics for greater displacements. Unfortunately, mechanical leveraging systems suffer numerous shortcomings. Early efforts to find alternatives to mechanical leveraging systems led to the development of Moonie actuators. Later, Haertling advanced the concept of a composite design, which has economic advantages in manufacturing. The actuator is known as reduced and inter-

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FIG. 1. Typical high displacement actuator construction. After bonding of adhesive at elevated temperature, the actuator obtains a characteristic curvature.
TABLE I. Properties of 11R THUNDER actuator.

<table>
<thead>
<tr>
<th>Material</th>
<th>t (mm)</th>
<th>b (mm)</th>
<th>L (mm)</th>
<th>ρ (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PZT</td>
<td>0.200</td>
<td>15.0</td>
<td>77.0</td>
<td>7.80×10²</td>
</tr>
<tr>
<td>Stainless</td>
<td>0.152</td>
<td>15.0</td>
<td>77.0</td>
<td>7.87×10²</td>
</tr>
</tbody>
</table>

properties. To address this question, we report experimental results of the frequency response of THUNDER actuators in cantilever mode for various excitation voltages. The results are compared to a theoretical development that connects the effective spring constant to the higher-order stiffness constants of the piezoelectric material.

II. THEORY

A theoretical treatment of the nonlinear response of a cantilevered beam can be found in the book by Nayfeh and Mook\textsuperscript{13} as well as in other places. Wolf and Gottlieb adapted the theory to the case of a noncontacting atomic force microscope cantilever\textsuperscript{11} and to a cantilever beam actuated by piezoelectric layers.\textsuperscript{12} The following is, in large part, a summary of their theoretical development. Readers wishing to fully explore the details of the theoretical development are strongly encouraged to study Ref. 12. Slightly different notation is used. For a piezoelectric cantilever actuator, such as the one considered here, the nonlinear constitutive equations, modified for the particular geometry can be expressed as follows:

\[ T_1 = -e_{31}^E E_3 + c_2 S_1 + \frac{1}{2} e_{33}^S S_1^2 + \frac{1}{6} e_{31}^S S_1^2, \]
\[ D_3 = e_{33}^L E_3 + e_{31}^L S_1, \]

where \( T_1 \) and \( S_1 \) are the longitudinal (along the direction) stress and strain, respectively. In addition, \( D_3 \) and \( E_3 \) are the electric displacement and electric field in the transverse direction (along the thin direction). The coefficients \( c_2 \) and \( c_3 \) are the nonlinear stiffness coefficients of particular interest in this investigation.

The linear material constants for the piezoelectric are as follows:

\[ c_2 = 1/S_{11}^E, \]
\[ e_{31}^L = d_{31}/S_{11}^E, \]
\[ e_{33}^L = e^L_{33}(1 - k_{33}^2). \]

Here \( c_2 \) is the linear elastic stiffness coefficient and \( S_{11}^E \) was defined earlier. The constant \( k_{33}^2 \) is the electromechanical coupling factor, \( d_{31} \) is the piezoelectric constant perpendicular to the direction of the electric field, \( e_{32}^L \) is the piezoelectric stress constant, and \( e_{33}^L \) is the permittivity. The superscripts indicate constant (zero) stress (\( T \)), constant (short circuit) electric field (\( E \)), and constant strain (\( L \)).

According to the Bernoulli-Euler hypothesis, it is assumed that the layer of thickness \( dz \) remains plane and perpendicular to the neutral axis of the cantilever. The neutral axis \( \eta \) is given by the following expression:

\[ \eta = \frac{1}{2} \left( \frac{r_p Y_p - r_p c_2}{t_p Y_p + t_p c_2} \right), \]

where \( t_p \) is the thickness of the piezoelectric, \( t_p \) is the thickness of the substrate, and \( Y_p \) is the Young’s modulus of the substrate. The equations of motion for the beam are derived from Hamilton’s principle. The material properties of the piezoelectric layers are represented by the electric enthalpy density. The enthalpy can be expressed in terms of a fourth-order polynomial that provides a connection between the piezoelectric material and its response to the applied electric field. The constants \( c_3 \) and \( c_4 \) are nonlinear material constants. The electric enthalpy per unit length can be expressed in terms of a polynomial with the following coefficients:

\[ H_{20} = -\frac{b}{t_p} e_{33}^L, \]
\[ H_{11} = -\frac{1}{2} b(t_2 + t_p) c_2, \]
\[ H_{02} = \frac{1}{12} b \left( t_p^2 e_{31}^L + 4(t_3^2 + 3t_p^2 \eta + 3t_p \eta^2)c_2 + 4(t_3^2 - 3t_p^2 \eta + 3t_p \eta^2)Y_p \right), \]
\[ H_{03} = \frac{1}{8} b(t_2^3 + 4t_p^3 \eta + 6t_p^3 \eta^2 + 4t_p \eta^3)c_3, \]
\[ H_{04} = \frac{1}{30} b(t_p^4 + 5t_p^4 \eta + 10t_p^4 \eta^3 + 5t_p \eta^4)c_4, \]

where \( b \) is the width of the cantilever. The Lagrangian can be written in terms of the electric enthalpy per unit length. Taking the variation of the Lagrange density and expanding the angular displacement of the beam to third order in the linear displacement lead to the equations of motion for the system. (See Ref. 12 for details.)

The following dimensionless spatial \( \xi \) and temporal \( \tau \) quantities are useful for the remainder of the theoretical development:

\[ \xi = \frac{x}{L}, \]

and

\[ \tau = t \sqrt{\frac{H_{02}}{\rho_L L^4}}, \]

where the dimensional spatial \( x \) and temporal \( t \) quantities have appropriate units. The length of the actuator is given by \( L \) and \( \rho_L \) is the average linear density (mass/length) of the actuator. \( H_{02} \) is given by Eq. (4). Incidentally, Eq. (6) corrects a typographical error in Wolf and Gottlieb’s paper.\textsuperscript{12}

In accordance with the multiple scale method, the response of the cantilever can be represented via three different time scales, which are distinguished in their order of magni-
tude by a small dimensionless parameter $\delta$. The linear displacement can also be expanded in terms of the parameter $\delta$. This method eventually leads to the eigenmode solutions for the clamped beam,

$$\Phi_n(\xi) = \cosh(z_n \xi) - \cos(z_n \xi) - A_n [\sinh(z_n \xi) - \sin(z_n \xi)]$$

(7)

with $z_n = \omega_n^{1/2}$, where $\omega_n$ is the $n$th (angular) eigenmode frequency and

$$A_n = \frac{\cosh(z_n) + \cos(z_n)}{\sinh(z_n) + \sin(z_n)}$$

(8)

is determined numerically from the $n$th root of the characteristic equation

$$1 + \cosh(z_n) \cos(z_n) = 0.$$  

(9)

We can assume a driving voltage at a frequency $\Omega$ and amplitude $U_{\text{max}}$. Furthermore, we can introduce a dimensionless damping coefficient $\mu$. It is also customary to introduce a detuning parameter $\sigma$ where

$$\Omega = \omega_1 + \delta \sigma$$

and the corresponding normalized damping coefficient $\bar{\mu}$ = $\delta \mu$. The bar indicates that it is normalized.

With these definitions and further analysis, we ultimately arrive at the evolution equation, which can be solved to yield the following important result for the response amplitude $a_1$ for the primary (as denoted by the subscripts 1) resonance peak:

$$\sigma = \frac{\gamma_c \omega_1}{a_1} \pm \frac{1}{2} \sqrt{\left( \frac{\gamma_{12} \alpha_{12} U_{\text{max}}}{a_1 \omega_1} \right)^2 - \mu^2}.$$  

(11)

The $+$ in Eq. (11) corresponds to frequencies below the resonance peak. The $-$ in Eq. (11) corresponds to frequencies above the resonance peak. The effective spring constant $\gamma_c$ is given by the following expression:

$$\gamma_c = \left( \frac{3}{8} \alpha_1 + \frac{9}{8} \gamma_{42} \alpha_{42} - \frac{1}{2} \gamma_{32}^2 \alpha_{32} - \frac{1}{2} \alpha_2 \alpha_{12} \right)$$

(12)

with

$$\alpha_1 = \int_0^1 \Phi_1(\xi) \Phi_1'(\xi) \Phi_1''(\xi) d\xi,$$

(13)

$$\alpha_{42} = \int_0^1 \Phi_1(\xi) \left( \Phi_1''(\xi) \right)^2 d\xi$$

(14)

$$\alpha_2 = \int_0^1 \Phi_1(\xi) \left[ \frac{1}{2} \int_0^\xi \Phi_1'(\xi) \Phi_1''(\xi) d\xi \right]' d\xi,$$

(15)

$$\alpha_{12} = \Phi_1'(1).$$

(16)

The primes indicate spatial derivatives. The gammas are given by the following expressions.
A. Mechanical

Actuators were mounted mechanically in a cantilever configuration with displacement in the horizontal plane to avoid acceleration errors at high displacement and/or frequency values. Whenever possible, standard, easily accessible components were used rather than specially fabricated components in order to promote repeatability by other researchers. Two steel machine flats were clamped inside the jaws of a Craftsman® 24073 drill press vise, which was mounted on a Newport® EG 32-2 optical bench. It was not necessary to utilize the vibration isolation capabilities of the optical bench. The actuator was clamped between the machine flats in order to provide a hard cantilever clamp. A variety of measured torques for the vise were employed to verify that the torque did not affect the results. It was critical to avoid mechanical parasitic effects due to the electrical leads. This was accomplished by feeding the electrical contacts through the mechanical clamps. The details of the electrical leads are described below.

B. Electrical

The original actuator electrical leads supplied by the factory were removed and thin flexible copper traces were used as leads. The traces were approximately 2 mm wide and 0.2 mm thick. They were bonded to the top aluminum electrode using Master Bond® EP211TDCHT metallic adhesive. The thin outline and increased width of the copper leads could be improved as it is both difficult to implement and avoid initial transient effects. Both the wait time and the rate to avoid over-sampling. Typical frequency spans are necessary to utilize the vibration isolation capabilities of the optical bench. The actuator was clamped between the machine flats in order to provide a hard cantilever clamp. A variety of measured torques for the vise were employed to verify that the torque did not affect the results. It was critical to avoid mechanical parasitic effects due to the electrical leads. This was accomplished by feeding the electrical contacts through the mechanical clamps. The details of the electrical leads are described below.

The applied voltage was monitored by one of the channels of a National Instruments® data acquisition card (NI DAQ 6024E) after a passive voltage divider. A laser charge-coupled device (CCD) micrometer (Micro-Epsilon® IDL-1400) was used to measure the displacement. The current output of the micrometer was passed through a resistor providing a conversion from current to voltage. The voltage across the resistor was then monitored by another channel of the data acquisition card. We calibrated the laser micrometer against a Michelson-Morley interferometer over the entire range of the micrometer (5 mm). This was accomplished by automating the fringe counting process, thereby providing an extremely accurate distance measurement. The calibration curve was very linear, so a simple multiplicative factor was used to convert micrometer voltage to a displacement. An oscilloscope (HP5403B) was used to independently monitor the applied high voltage and micrometer output voltage. In addition to serving as an independent verification of the acquired data, this configuration was used to monitor the phase relation between the applied high voltage and the micrometer displacement voltage. In this way any signal coupling, degradation, or anomalous behavior between the supply and the actuator could be observed in situ.

Data collection and reduction was integrated into the LabVIEW® code and allowed for frequency and voltage stepping. The reduced data profiles then consist of a spectrum of voltages and frequencies/tones centered about the expected resonant frequency (see Fig. 4). A single tone data set (raw data) consists of 512 points sampled at 990 S/s (samples per second), where the update rate of the IDL-1400 is 1024 S/s. Since the sampled frequencies were much less than the DAQ or IDL-1400 sampling rate, we reduced the DAQ rate to avoid over-sampling. Typical frequency spans are +/−20 Hz, about the center tone, to allow for asymptotic behavior to appear. The first resonant modes for actuators considered here were below 120 Hz. In addition, a wait time was associated with each frequency, prior to collection, to avoid initial transient effects. Both the wait time and the center frequency were determined empirically prior to operation. These parameters and sampling rates provided ample data density for the reduction process. The micrometer response voltage and dc offset for a given tone were first cal-
culated, and then the single tone frequency was obtained using standard fast Fourier transform (FFT) techniques. This provided a single data point (V, Hz) in the dynamic response curve. Then, for a single applied voltage, we obtain a range of frequency and micrometer response voltage points that compose a single response curve in the voltage domain (see Fig. 4). The data were then converted from volts to meters using the previously mentioned conversion factors. In addition, for each response curve the peak and center frequencies were calculated using a standard singular value decomposition peak fitting routine. From this and the original data, frequency versus voltage and peak displacement versus voltage plots were obtained. The error in the data sets follows largely from the calibrations as the fitting algorithms yield very small error contributions. The details of the calibrations and complete error analysis are omitted. However, we considered a spectrum of error sources including temperature, power, and drift of the electronics in addition to parasitic energy losses, inductive/capacitive coupling, and the full spectrum response of all instruments beyond the required ranges. Furthermore, the amplifier was carefully monitored to ensure that it did not deviate from the expected voltage as a result of the highly reactive load near resonance. The final worst-case errors for frequency, applied voltage, and displacement, respectively, are as follows: +/-0.1 Hz, +/-0.1 V, and +/-0.1 mm.

IV. RESULTS

We focused on the Face International 11R THUNDER® actuator. The other models showed similar results. However, they were not as extensively investigated. Six 11R actuators were investigated with similar results for each actuator. A typical set of experimental (data points) nonlinear response curves are shown in Fig. 4 along with the fit to theory (smooth curve).

The resonance curves shown in Fig. 4 clearly show a nonlinear response. The response amplitude was kept small to avoid the jump phenomena typical of nonlinear systems. The peaks shifted towards lower frequencies with increasing drive voltage amplitudes, indicating a softening of the effective spring constant. The results compare favorably with the theory by Wolf and Gottlieb.\(^\text{12}\)

By inspecting Eq. (11), it can be seen that for a given drive voltage, the peak amplitude \(a_{1p}\) occurs when the two terms inside the square root are equal,

\[
\mu_1^2 = \left( \frac{\gamma_1 \alpha_{12} U_{\text{max}}}{a_{1p} \omega_1} \right)^2.
\]

This expression can be rearranged to give a linear relation between the peak response amplitude \(a_{1p}\) and the maximum applied voltage \(U_{\text{max}}\),

\[
a_{1p} = \frac{\gamma_1 \alpha_{12} U_{\text{max}}}{\mu_1 \omega_1}.
\]

Figure 5 represents typical experimental results showing a linear relation in good agreement with theory.

In order to further compare quantitatively the experimental results with theory, it is necessary to obtain an independent value for the dimensionless damping coefficient \(\mu\). This was done experimentally by manually displacing the cantilever tip and suddenly releasing it, resulting in a free decay vibration, which was monitored electronically via the direct piezoelectric effect and monitored mechanically via the laser micrometer. A typical decay response is shown in Fig. 6. Inspection of the shape of the curve confirms the assumption of linear damping as opposed to quadratic, Coulomb, or hysteric damping. (See Sec. 3.3.3 in Ref. 10.) The damping curves yield an independent value for the damping coefficient of 6.90 1/s and using Eq. (6), a dimensionless value of \(\mu = 0.119\). This, in turn, yields a value for the small order parameter \(\delta = 0.0279\) that is used in further calculations discussed later.

Now turning our attention to the first term in Eq. (11) which is left after setting the last term equal to zero, we have the following linear relation between the detuning parameter and the square of the peak amplitude,

\[
\sigma = \frac{\gamma_1 a_{1p}^2}{\omega_1}.
\]

The curve prescribed by this equation is commonly referred to as the backbone curve. Figure 7 shows some typical ex-

![Figure 5](image-url)  
**FIG. 5.** Typical maximum amplitude response as a function of applied voltage (peak to peak) for an 11R actuator. Data is for an 11R actuator other than actuators presented in other figures.

![Figure 6](image-url)  
**FIG. 6.** Typical free decay of 11R actuator. The decay is monitored by the laser micrometer and by the actuator in sensor mode. The micrometer decay includes an unimportant offset.
perimental results. A straight line could be drawn through the data points and still remain within experimental error. However, there is a clear indication of curvature, which will be discussed later. Assuming a linear fit, the experimental data gives a value of $-2.6 \times 10^5$ Hz/m$^2$ for the slope. This slope can then be used to calculate a value for the effective spring constant $\gamma_c$. The effective spring constant is negative (softening). It is important to keep in mind that the relations given in Eq. (5) and (6) must be used to compare experimental and theoretical values.

The effective spring constant is given by Eq. (12). Other experiments have shown that that the relative stiffness coefficient $K_{33}$ is small so the third term will be dropped resulting in the following expression:

$$\gamma_c = \left( \frac{1}{5} \alpha_1^2 + \frac{9}{8} \gamma_{42} \alpha_2^2 - \frac{1}{2} \alpha_2 \omega_2 \right).$$

(22)

Assuming a straight line for Fig. 7, the slope leads to a value of $c_4 = -4.8 \times 10^{20}$ N/m$^2$ for the higher-order stiffness coefficient. Other experimental values for $c_4$ are scarce in the literature. Furthermore, they vary widely from $-2.4 \times 10^{16}$ to $-1.4 \times 10^{18}$ N/m$^2$ depending on the particular PZT formulation used. One publication by von Wagner and Hagedorn reports $c_4 = -1.4 \times 10^{18}$ N/m$^2$. However, different PZT formulations were used. Morgan Electroceramics PZT-5A ceramics were used in our work and von Wagner and Hagedorn used PIC 151 manufactured by PI-Ceramic. The higher $c_4$ value in the case of high displacement THUNDER actuators may be associated with the prestress. However, further studies need to be conducted to be certain. Our value for $c_4$ is larger than those reported by others. Referring to Eq. (1), it is interesting to examine the relative contributions of the term involving the higher-order stiffness coefficient $c_4$ with the linear term involving the linear coefficient $c_2$. Even for strains as small as 250 μm/m (typical strains for PZT can be as high as 1200 μm/m), the contribution of the nonlinear term is of the same order of magnitude as the linear term. Unfortunately, the need to avoid experimental complications associated with the jump phenomena at high amplitudes results in decreased dynamic range and consequently decreased sensitivity to the value of the effective spring constant. In future experiments it may be possible to improve sensitivity by exploring higher-order resonance modes. However, these results indicate that the effective spring constant depends on voltage. This dependence should be more pronounced at higher amplitudes (higher drive voltages). The curvature in Fig. 7 could be explained by a voltage-dependent spring constant $\gamma_c$, which is not specifically accounted for in the theory.

Comparable results were reported by Wang et al. from Cross’s laboratory for RAINBOW actuators and by Ounaies et al. for THUNDER actuators. Wang et al. investigated nonlinear behavior in RAINBOW actuators, which were the predecessors of THUNDER actuators. Although their theoretical treatment is quite different from the one presented here, the experimental results are similar. Figure 8 shows an adaptation of their experimental data to the analysis used here. Their data shows the same voltage-dependent behavior as seen in our data Fig. 7.

In an attempt to separate, to the extent possible, mechanical response characteristics from electrical response characteristics we applied a small ac voltage with a large dc voltage. The results are shown in Fig. 9. These results show a clear dc voltage dependence. The very strong dependence at large negative dc voltages may be due to the onset of depoling.

V. CONCLUSIONS

The coupling of electric and mechanical fields is the defining characteristic of piezoelectric materials. First-order coefficients have been thoroughly investigated even though relatively recent surprises have been discovered in BaTiO$_3$, which is one of the most widely studied piezoelectric materials. Higher-order coefficients are not as widely known.

![FIG. 7](image7.png)

**FIG. 7.** Peak resonance frequency plotted against maximum amplitude squared for a typical 11R actuator response. The horizontal error bars are smaller than the data point symbol.

![FIG. 8](image8.png)

**FIG. 8.** Peak resonance frequency plotted against maximum amplitude based on results from Wang et al. (see Ref. 15). The smooth curve is a guide for the eye.

![FIG. 9](image9.png)

**FIG. 9.** Typical resonance response as a function of dc offset voltage for an 11R actuator. The smooth curve is a guide for the eye.
However, they are important in applied studies as well as those that are more fundamental. This report connects a non-linear theory with experimental results on prestress-gradient piezoelectric actuators. Our results compare favorably with the theory, as well as other experimental results, providing insight into piezoelectric material properties. The results indicate that the prestress gradients have minimal impact on the higher-order stiffness coefficients. However, the effective spring constant exhibits a voltage dependence that is not accounted for. This conclusion is confirmed by reconstituted data from other published results as well as our dc offset results. This may warrant further study.

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