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Mechanism Design and Analysis: Developing an Understanding of Mechanism Motion Through Graphical Modeling

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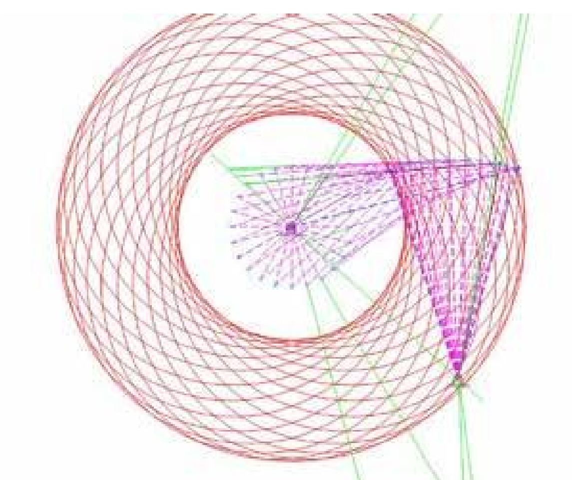

The design of mechanisms is important for many common activities in technology and engineering education.

The intention of this article is to provide Technology and Engineering Educators (T&EEs) with foundational knowledge of mechanism design and analysis and the ability to develop middle and high school students' mechanism knowledge during practical hands-on learning activities in the STEM classroom. T&EE's implementation of mechanism design and analysis could promote students' increased depth of mechanical knowledge and ability to apply this knowledge during engineering design challenges. In this article, the authors present an introduction to four-bar mechanism design and analysis using CAD software to produce graphical representations. After designing mechanisms graphically, students should be allowed to produce their mechanisms using tools like 3D printers.

Mechanical Engineering as a discipline started during the industrial revolution in Europe around the late 1700s and early 1800s; yet the application of mechanical devices like wedges dates to the Prehistoric era. Throughout the Prehistoric and Ancient eras, simple mechanical devices like the lever, wheel and axle, pulley, inclined plane, wedge, and screw, now known as simple machines, were increasingly used. In practice, the words machine and mechanism are frequently used interchangeably, yet there are clear but subtle differences. Complex machines are combinations of two or more simple machines. **Machines** are associated with the ability to do work involving the transmission of energy and transformation of forces, *but not motion*. An internal combustion engine is an example of a machine that includes several different mechanisms. **Mechanisms** are systems made up of rigid bodies that are connected and arranged in a specific way *to produce a desired motion*. A mechanical watch is an example of a mechanism. Machines and mechanisms inhabit the same body, and, when combined during design, include consideration of motion and force to accomplish a specific objective. Another example of a machine that has multiple mechanisms is a person riding a bicycle. The person's leg and the bike's crank, seat tube, and post make a four-bar mechanism. Additionally, bicycles have gear train mechanisms.

Throughout the Ancient, Medieval, and Modern Eras developments in technology, mathematics, physics, and material science helped shape our understanding of the human-made world. For example, the basic concept of mechanical advantage was previously utilized but the concept was not formally expressed until Archimedes (287 BCE–212 BCE). Other concepts related to simple machines like mechanical trade-offs, statics, and dynamics of mechanical devices were not expressed until around the late 1500s and later. Mechanics is usually divided into two branches, fluids and solids. Solids is then divided into rigid and deformable bodies; rigid bodies into

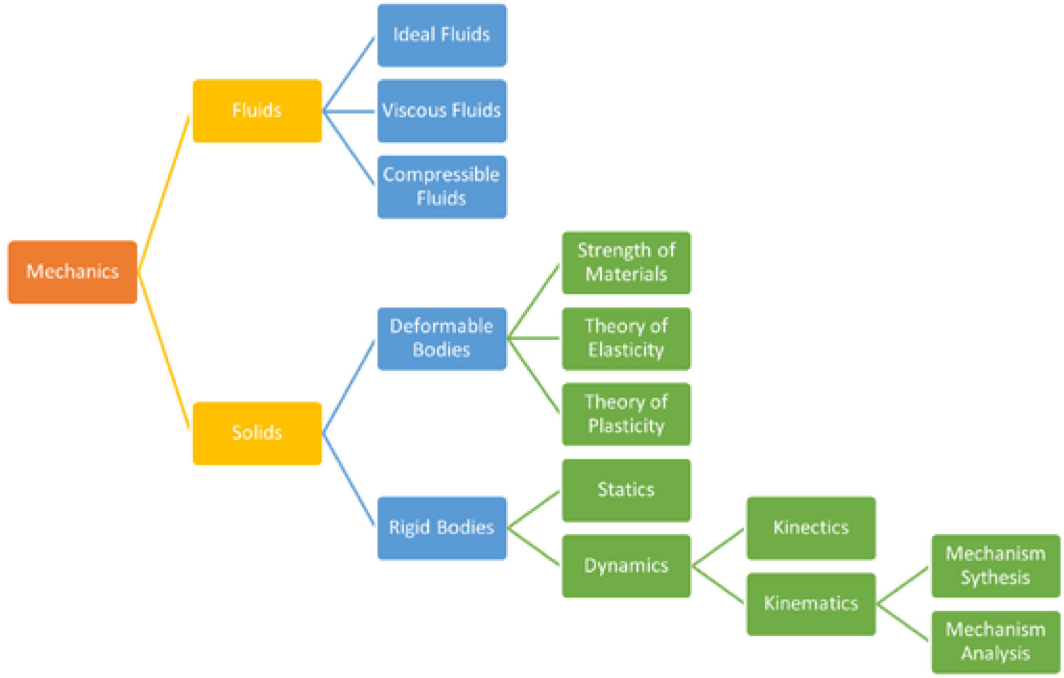

Figure 1.
Branches of Mechanics

statics and dynamics, and dynamics into kinetics and kinematics (Barton, 1982) (Figure 1).

Kinematics is the study of motion and basic geometry of mechanisms, often including the velocity and acceleration of mechanism components (i.e., members or links) but *does not include the forces that cause or affect motion*. **Kinetics**, on the other hand, *includes the analysis of forces* on a mechanism's components to determine both the internal and external mechanism forces. Due to the inclusion of force analysis in kinetics, students learning about mechanisms more commonly begin by studying kinematics, first focusing on mechanism motion. Kinematics consists of both mechanism analysis and synthesis. As the names imply, mechanism analysis is the study of a mechanism's motion, and mechanism synthesis is the design of a mechanism to yield desired motion characteristics. The authors' favorite aspect of kinematics is that it allows students to *conceptually visualize* mechanical motion using *graphical models*.

Kinematics

In the study of kinematics, it is important to begin by developing an understanding of the motion characteristics for given mechanisms (i.e., mechanism analysis). To help students develop an understanding of motion characteristics, teachers can have students use CAD software to draw graphical models of various common mechanisms. Then teachers can have students design and produce their own mechanisms. Finally, teachers can add analysis of velocities and accelerations to the understanding of motion using methods like effective component, instant center, relative, difference, calculus, graphical, and/or any combination of these methods.

Figure 2 is a four-bar mechanism (i.e., four-bar linkage). In Figure 2, *Label 1* represents *Member 1*. Member 1 represents both fixed points of rotation (i.e., the mechanism's frame). *Label 2* represents *Member 2* (i.e., the crank). *Label 3* represents *Member 3* (i.e., the coupler). Finally, *Label 4* represents *Member 4* (i.e., the follower). The connections between members 1 and 2, 2 and 3, 3 and 4, and 1 and 4 are considered pivots as well as *kinematic pairs*, and more specifically *lower pairs*. Lower pairs have surfaces in contact; for example, the *pivot* surface of member 3 is in contact with the *pivot* surface of member 4. There are also *higher pairs*. Higher pairs only have one point in contact, for example a cam and follower.

To determine the motion characteristics of this four-bar mechanism, students will create a graphical model similar to Figure 2 using CAD software. The students will be able to conceptually picture the four-bar mechanism's motion by creating graphical drawings representing every 15 degrees of Member 2's rotation (Figure 3).

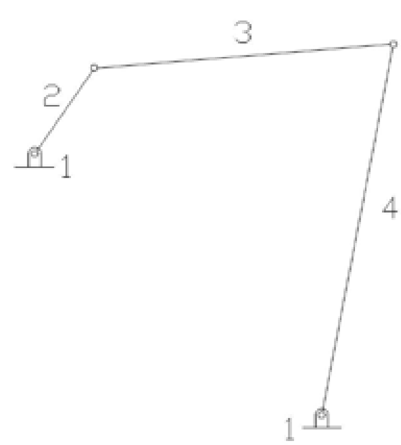

Figure 2.
Labeled Four-Bar Mechanism

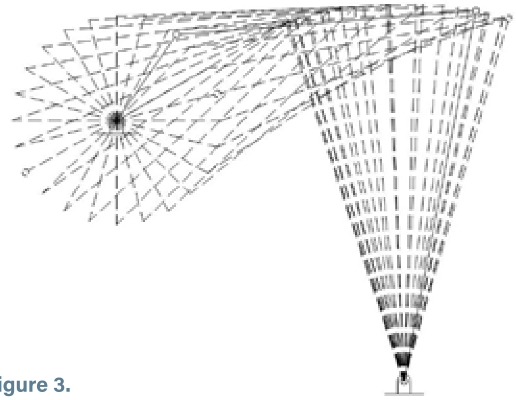

Figure 3.
Motion Characteristics

The students can use different layers, line color, and line type to have a colorful, some might say artistic, graphical representation of the four-bar mechanism's motion (see image on page 21). At this point students start to make connections between the graphical representation and the actual motion of a four-bar mechanism. This is a good time to show students a four-bar mechanism that the teacher has made based on Figure 2. The four-bar mechanism the authors used was made from wood, but the mechanism could also be 3D printed (Figure 4). Additionally, mechanisms could be made using construction paper or cardboard. Figure 5 represents the extent of this four-bar mechanism operation. In Figure 5, Member 2 is graphically rotated until Member 4 is as far left and right as possible based on the current mechanism design (i.e., the relationship between Members 1, 2, 3, and 4).

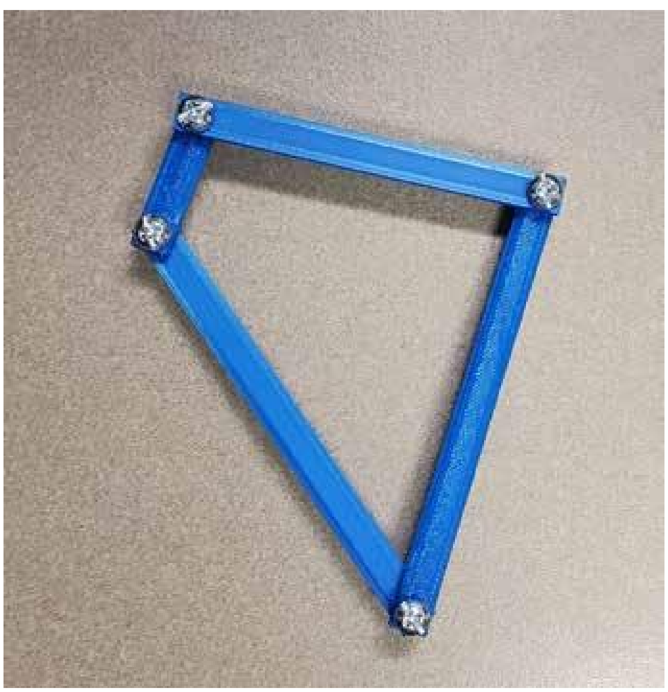

Figure 4.
3D-Printed Four-Bar Mechanism

After students spend time using CAD to graphically understand a mechanism's motion and are in the process of developing conceptual understanding, the mechanism can be modified by the students using CAD to see the impact of specific changes to the mechanism's motion. There are two common changes that help students further develop conceptual understanding of a mechanism. The most common is inversion. For example, in Figure 2, Member 1 is the frame or fixed member. The students could allow Member 1 to move and could fix any one of the other members one at a time. This is called inversion and completely changes the mechanism's motion. The other common change is the length of any one member at a time.

The four-bar mechanism in Figure 2 can also be considered a *constrained kinematic chain*. Gruebler and Kutzbach's criterion (i.e., mobility formula) are similar and are used to describe the mobility of a mechanism. Gruebler's equation is used to calculate the degrees of freedom, $F=3(n-1)-2l-h$. Where *F* is the total degrees of freedom, *n* is the number of links, *l* is the number of lower pairs, and *h* is the number of higher pairs. In Figure 2, the four-bar mechanism, there are four (4) links and four (4) lower pairs for a total of one (1) degree of freedom. This basically means that only one member needs to be controlled in order to control the motion of the entire mechanism. Knowing the degrees of freedom will help students determine if the kinematic chain is locked, constrained, or unconstrained. A locked mechanism has zero degrees of freedom. A constrained mechanism has one (1) degree of freedom, like the four-bar mechanism, meaning that one input produces defined relative motion between all links (Figure 3). An unconstrained mechanism is one with two or more degrees of freedom, meaning that one input will result in the mechanism's links taking undefined paths.

Instantaneous Centers

The instantaneous center method (i.e., instant center method) or velocities by centro method is the technique to calculate the veloc-

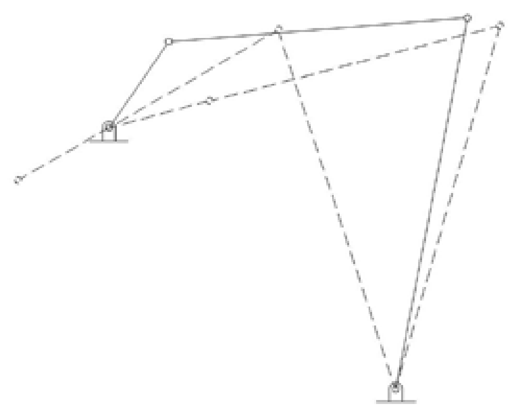

Figure 5.
Four-bar Mechanism Extents

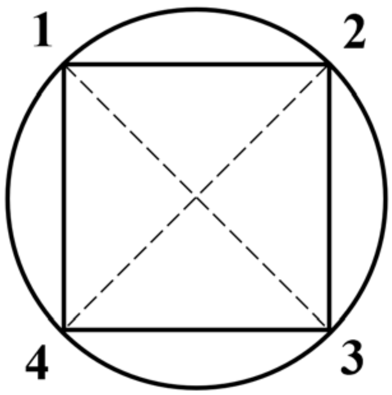

Figure 6.
Four-bar Mechanism Instant Centers

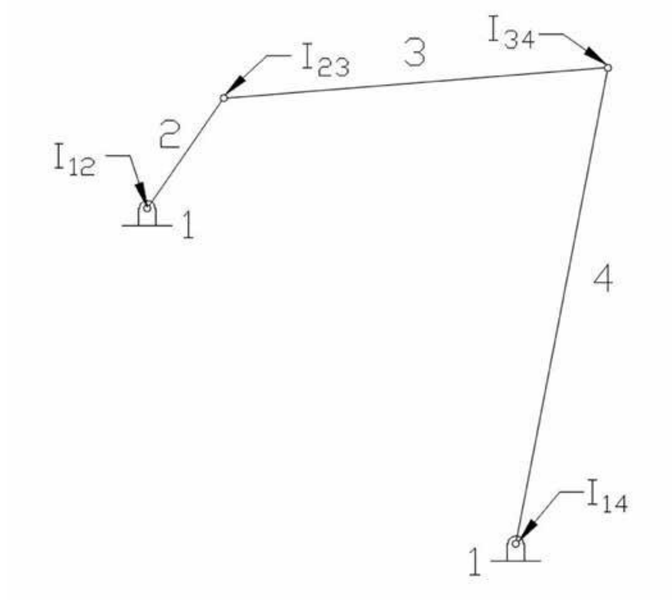

Figure 7.
Known Instant Centers

ities of a mechanism's members based on the dimensions of the mechanism. The instant center method helps to describe a mechanism's motion at any given instance as rotational motion around an instant center (i.e., point). An easy way to think about an instant center (i.e., centro) is as a point around which the members of the mechanism rotate. For simple mechanisms, the ability to analyze the mechanism is greatly simplified by the ability to visualize members' rotation around points. The key to the instant center method is finding and visualizing these instant center points that members are rotating around. The process of finding these instant centers is quite simple. The students must determine how many of these instant centers exist using this equation: number of instant centers = $\frac{(n\,(n-1)}{2}$, where n is the number of members in the mechanism. For the four-bar mechanism in this article, there are four members which, based on the equation, means there are six instant centers.

The students will use Kennedy's theorem and the derived circle method to determine the labeling and location of the 6 instant centers (Figure 6). Figure 6 shows a circle diagram, where the numbers 1, 2, 3, and 4 represent members in the four-bar mechanism.

The solid lines between 1 and 2, 2 and 3, 3 and 4, and 1 and 4 represent known instant centers, or instant centers that are easily visible (i.e., primary centros) (Figure 7).

The hidden line between 1 and 3 and 2 and 4 represent instant centers that need to be located (i.e., secondary centros) (Figure 8). In Figure 8, all instant centers are visible. Kennedy's theorem basically states that any three bodies (i.e., members) having plane motion relative to one another have three instant centers, and the instant centers all lie on a straight line. For example, in Figure 8, you can see that members 1, 2, and 3 have instant centers 12 (said as *instant center one two*), 23, and 13. In Figure 8, you can see that instant centers 12, 23, and 13 all lie on a straight line; this is basically Kennedy's theorem.

There are 3 different types of instant centers. Instant center 12 and 14 are *fixed* instant centers because they exist on fixed points around which members 2 and 4 rotate. Instant centers 23 and 34 are considered *permanent* instant centers. Member 2 is always connected to member 3 and instant center 23 permanently exists in a circular path (i.e., centrode) defined by the rotation of member 2 and curvilinear motion of member 3. Additionally, member 3 is always connected to member 4, instant center 34 permanently exists along an arched line (i.e., centrode) which is defined by rotational

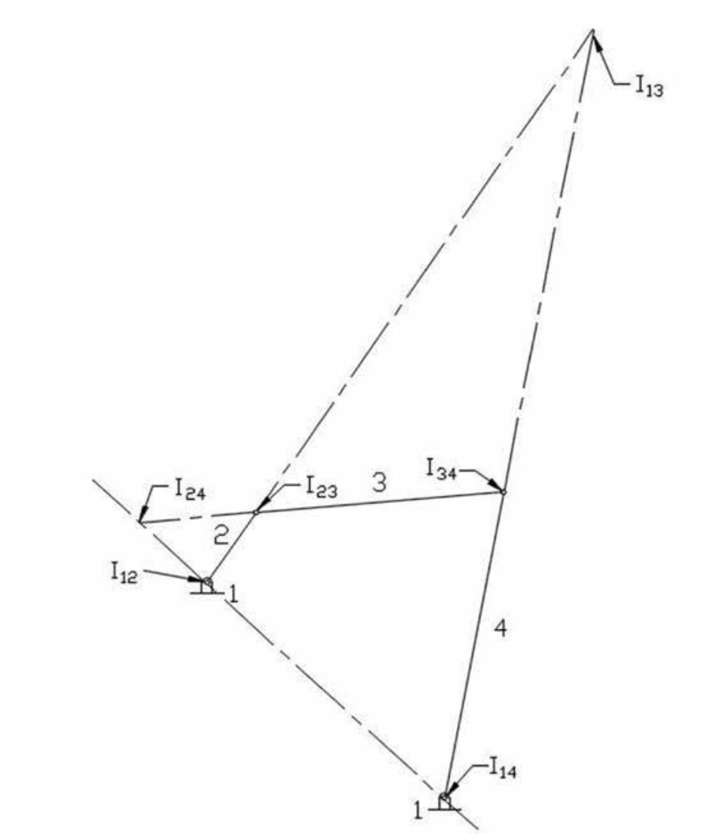

Figure 8
Instant Centers Located

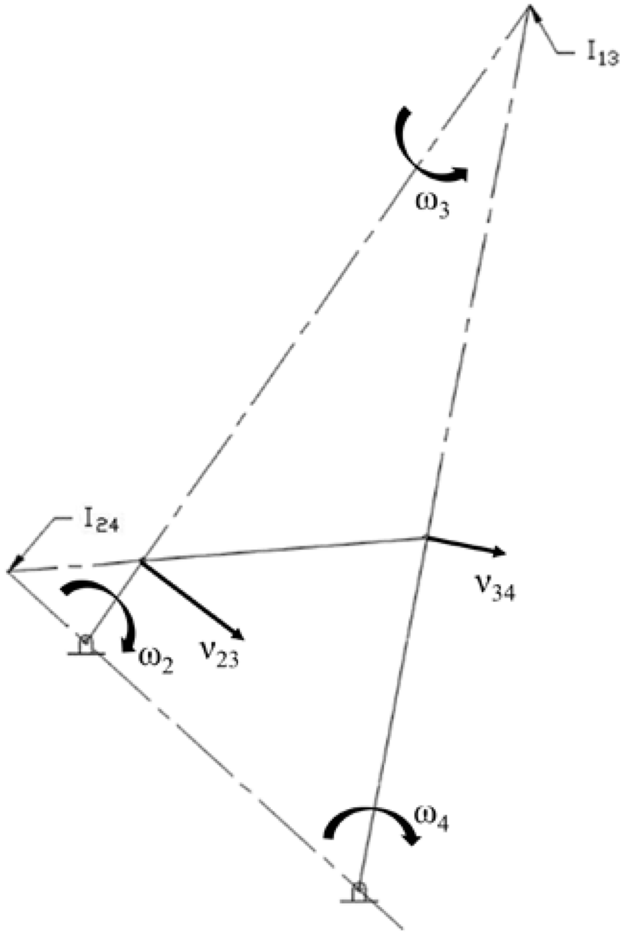

Figure 9
Link to Link Method

motion (i.e., oscillation) of member 4. Instant centers 13 and 24 are considered *imaginary* instant centers, which exist outside the mechanism at instances of the mechanism's operation and have similar characteristics as fixed or permanent instant centers. Instant centers 13 and 24 move along the corresponding center lines as the mechanism moves (Figure 8).

Determining Velocities Based on Instantaneous Centers

When first learning to analyze the velocities of a mechanism's member, students must have developed the ability to graphically and conceptually visualize the mechanism in terms of pure rotation as well as the application of the appropriate mathematical techniques. When educators are applying the instant center method in their classrooms, it is important to start with the graphical visualization and then further progress with the conceptual visualization of a mechanism's motion. The use of CAD software really helps encourage this progression. The next step is applying simple mathematical relationships to determine the velocities of the mechanism's members during specific instances of the mechanism's motion.

Link to Link Method

Using the instant center method allows the student to determine velocity by converting rotatory motion into rectilinear motion. For analysis, converting rotatory motion into rectilinear motion requires the student to determine the radius graphically and visualize that linear velocity acts tangential to the path of rotation (Figure 9).

For example, the linear velocity ν_{23} is equal to the length of member 2 multiplied by angular velocity of member 2 ($\nu_{23}=\text{length of member 2 x } \omega_2$) (Figure 9). If Member 2 is 1 inch long (distance determined in CAD) and rotating clockwise at a constant angular velocity of 200 revolutions per minute (RPM), ν_{23} is equal to 200 inches per minute (in/min). Instant center 23 can also be treated as point on Member 3. Member 3 has the instant center 13, therefore the angular velocity of member 3 (ω_3) is equal to ν_{23} divided by the distance between instant centers 23 and 13 $\left(\omega_3=\frac{\nu_{23}}{\text{distance between 23 and 13}}\right)$. The angular velocity ω_3 is equal to 200 in/min divided by 6.83 inches or 29.28 RPM. The linear velocity ν_{34} is equal to the distance between instant centers 13 and 34 multiplied by angular velocity of member 3 ($\nu_{34}=\text{distance between 13 and 34 x } \omega_3$). Therefore, ν_{34} is equal to 5.47 inches multiplied by

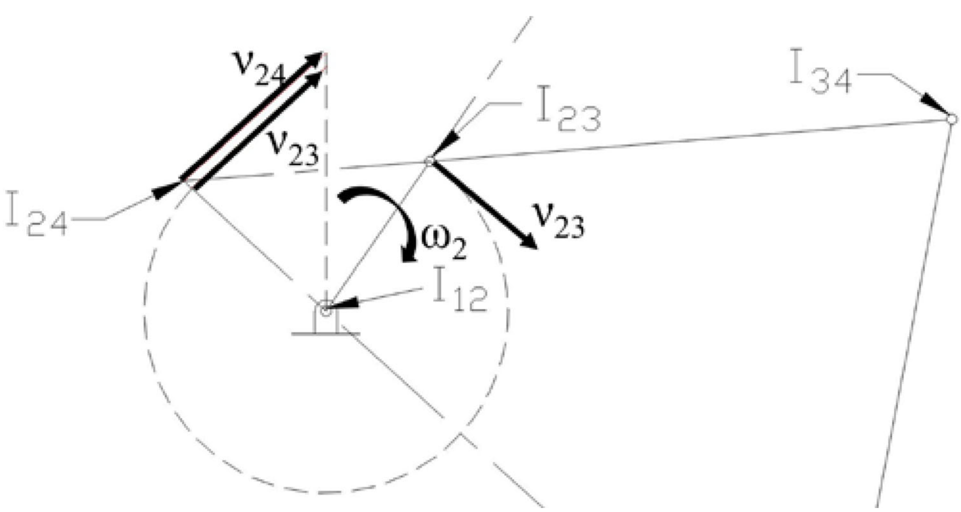

Figure 10
12, 23, and 24 Radius of Rotation Triangle

29.28 RPM or 160.16 in/min. Finally, angular velocity of member 4 (ω_4) is equal to linear velocity $\nu 34$ divided by the length of member 4 $\left(\omega_4 = \frac{\nu_{34}}{\text{length of member 4}}\right)$.

Angular velocity ω_4 is equal to 160.16 in/min divided by 3.62 inches or 44.24 RPM. In certain instances of member 2's rotation, it will be impossible to determine the distance from imaginary instant centers to the fixed or permanent instant centers. This distance could be infinite or too large to practically dimension using CAD software. In these cases, the link to link method may not be appropriate and a different method is needed.

Radius of Rotation Method

Another common method associated with determining velocities using instant centers is the *radius of rotation method*. The radius of rotation method is being applied here with the assumption that the distance to instant center 13 cannot be easily determined. Using the radius of rotation method, velocities are proportional based on each velocity's radius of rotation. For example, if one velocity is known, another velocity can be determined using mathematical or graphical proportions. In the four-bar mechanism, there are 3 links that move, links 2, 3, and 4. Each link has 3 instant centers. For example, link 2 has instant centers 12, 23, and 24 (Figure 10); link 3 has instant centers 13, 23, and 34; and link 4 has instant centers 14, 24, and 34 (Figure 11).

Notice that these instant center groupings form triangles. Given the angular velocity of member 2 ($\omega 2$), the linear velocity $\nu 23$ can be determined as in the previous section ($\nu_23=$length of member 2 x ω_2) or $\nu 23$ is equal to 200 inches per minute (in./min.). The aim is to calculate linear velocity $\nu 34$ without knowing any distance to instant center 13. The linear velocity $\nu 24$ is equal to the linear velocity $\nu 23$ multiplied by the radius of rotation for instant center 24 divided by the radius of rotation for instant center 23 ($\nu\_24=\nu\_23 ((12\text{-}24)/(12\text{-}23))$) (Figure 9). The radii of rotation in this example, denoted as 12-24 and 12-23, are actually indicating the distances between the center of rotation (i.e., instant center 12) and the point at which the linear velocities ($\nu 24$ and $\nu 23$) act (i.e., instant center 24 and 23). Basically, the distance between instant center 12 and 24 as well as the distance between 12 and 23. The linear velocity $\nu 24$ is equal to 200 in./min. multiplied by 1.0529 in. divided by 1 in. (distances determined in CAD) or 210.58 in/min. The linear velocity $\nu 34$ is equal to the linear velocity $\nu 24$ multiplied by the radius of rotation for instant center 34 divided by the radius of rotation for instant center 24 ($\nu\_34=\nu\_24 ((14\text{-}34)/(14\text{-}24))$). The linear velocity $\nu 34$ is equal to 210.58 in./min. multiplied by 3.62 in. divided by 4.77 in. (distances determined in CAD) or 159.8 in/min. Notice the slight difference between $\nu 34$ using the link to link and radius of rotation methods for determining velocity. This is likely due to rounding certain values during calculation. It is interesting to note that the graphical method provides a more precise answer.

Conclusion

Students' ability to understand kinematics, especially the motion of a mechanism is enhanced by their ability to graphically and then conceptually visualize mechanism motion. In this article, the authors presented the most common mechanism to start with in the understanding of kinematics, the four-bar mechanism. The goal of this content is to have students design mechanisms graphically based on desired motion characteristics and then have the students produce their mechanisms. The design of mechanisms is important for many common activities in technology and engineering education. In future articles, other mechanisms will be covered to help develop a more thorough understanding of mechanism design and analysis.

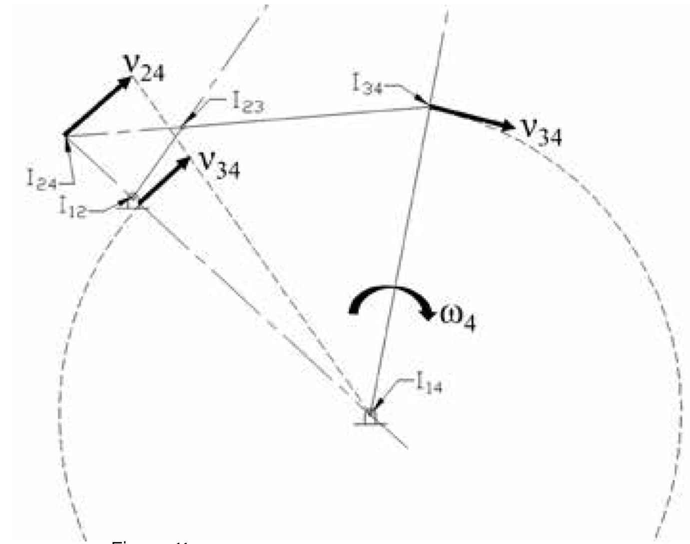

Figure 11
14, 24, and 34 Radius of Rotation Triangle

References

Barton, L. O. (1984). *Mechanical analysis: Simplified graphical and analytical techniques.* New York, NY: Marcel Dekker, INC.

Constans, E. & Dyer, K. B. (2018). *Introduction to mechanism design: With computer applications.* CRC Press.

Rao, J. S. & Dukkipati, R. V. (2015). *Mechanism and machine theory.* New York: Wiley.

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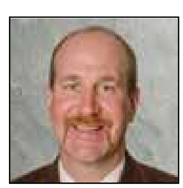

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