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Demand driven growth and two class capital distribution with applications to the United States[☆]

Rishabh Kumar¹, Christian Schoder², Siavash Radpour³

Abstract

We present a structuralist growth and distribution model of capitalists and workers. Our model highlights the role of class-differentiated savings propensities as well as an independent accumulation function in determining the dynamics of wealth distribution in the long run. At the steady state, investment parameters do not influence the distribution of wealth but there exists a long run paradox of thrift effect, which distributes wealth to capitalists whilst simultaneously exerting downward pressure on the long run state of aggregate demand. Applied to annual US data from 1950-2015 using the Metropolis-Hastings algorithm we find that the share of capitalist wealth will stabilize at approximately 68%, fairly close to the Kotlikoff-Summers dynastic capital range. We estimate the ratio of worker-capitalist saving propensities to be as low as 6%. Our paper has implications for the interaction between wealth inequality and long run economic stagnation in low-growth mature economies.

Keywords: Wealth Distribution, Economic Growth, Paradox of Thrift

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1. Introduction

In this paper we try to answer important issues regarding the interaction of aggregate demand and the concentration of wealth. While there has been a resurgence on the literature⁴ which discuss theoretical and empirical aspects of wealth inequality, less attention has been devoted to addressing the impact of wealth concentration on macroeconomic performance. Traditional models either choose to optimize saving or assume that the long run rate of economic growth is exogenous and determined by demographic parameters. In many such models either wealth ends up completely in the hands of one group which has the highest incentive to save or the distribution of wealth is directly proportional to the marginal contribution of different actors - evidence points to neither of these facts being empirically verified.

. Our paper proceeds in a systematic manner, motivated by two important stylized trends captured in Figure B.1 and Figure B.2. The former chart shows a relatively stable distribution of profits in US national income, with a slight increase in the last half decade. The latter chart shows (for the post war US economy) a slight decline in the long run ratio of output to the fixed private capital stock utilized in the production process. Thus while the share of wages and profit is relatively unchanged, the productivity of capital has declined. These trends square up with the recently highlighted secular stagnation argument, promoted in Summers (2014) where the author proceeds to link the decline in aggregate demand to a concentration of income and wealth amongst *high-savers* (amongst other reasons).

[Figure 1 about here.]

[Figure 2 about here.]

We distinguish our analysis in this paper in two ways. First we present a theoretical two-class model (capitalists and workers) where output is demand-determined and based on an independent accumulation function which responds to profitability. Our models set limits for the proportion of wealth held by either class exploiting the fact that while capitalists by definition only earn capital income, workers (who also engage in saving) have access to both wages and returns to capital claims generated by their past saving.

⁴See for example the summary in Piketty (2015)

By avoiding a neoclassical production function structure we are able to highlight the role of savings behavior on both long run distribution and output growth. We use US data to calibrate this model using Bayesian estimation techniques and compute the rate of capitalist saving as well as predict steady state values for the share of wealth owned capitalists and workers. We ensure consistency between our highly simplified one-good model by using appropriate data related to the fixed private capital stock, as opposed to the sum of all private assets.

. We show that (theoretically) in a demand driven growth model, investment parameters do not have any impact on the distribution of inter-class wealth but instead only influence the rate of profit and productivity of capital. Similar to the model of Pasinetti (1962), workers cannot influence growth or rates of return but only influence their steady state share of wealth. Our empirical findings imply that capitalists' share wealth will stabilize at 68%, based on historical trends and driven by saving approximately 73% of their (capitalists) income. Independent of profitability, our computations show that the US economy has a long run rate of growth of roughly 1.5%. These estimates have important implications for policies seeking to achieve both equity and improved macroeconomic performance.

. The rest of this paper is organized as follows. In section 2, we discuss theoretical and empirical literature related to our analysis. In section 3 we develop our theoretical model then proceeding the estimate and calibrate it in section 4. Section 5 concludes by discussing the implications of our propositions and findings.

2. Related Literature

2.1. Literature on capital accumulation

Our paper and model follow in the tradition of *structuralist* growth models where both the long and short run are influenced endogenously by aggregate demand rather than an exogenous natural growth rate. A range of possible models and specifications are available for the interested reader in Taylor (2009). In particular, for mature low growth economies such as the US or Western Europe the issue of chronic long run stagnation of aggregate demand⁵ are more appropriate in our perspective, as opposed to the benchmark

⁵See for example Summers (2014)

neoclassical growth models which follow Solow (1956). The model closest to our specification is perhaps the two class growth and distribution models in Pasinetti (1962) and extended in Taylor (2014). Were we to consider issues of capital accumulation even in the neoclassical tradition, recent evidence points to possible *savings gluts*⁶ generating a drag on the profitability of accumulation. For example Geerolf (2013) overturns the original results on the excess of the profit share over investment in GDP as found by Abel et al. (1989) for rich economies.

. The second range of literature we draw upon highlight the definition of capital. In our model, we exclusively focus on a one-good model of accumulation so that output can be consumed or saved as *wealth*. In general, the evolution of capital stock to national income can occur due to both savings or due to a relative rise/fall in asset prices. We stress that this issue is related to which assets are counted in the definition of wealth. For example, by counting all private wealth as capital Piketty and Zucman (2014) show a greater decline in the output capital ratio. This non-standard definition leads to inconsistent outcomes regarding the share of capital income and rate of return (both increasing), implying a high elasticity of substitution between labor and capital and has been criticized in many quarters⁷.

2.2. Literature on distribution

We draw on the findings of Saez and Zucman (2016) regarding rising US wealth inequality. Empirical estimates of saving also point to an increase in the rate of saving going up the income distribution, particularly for the US in Dynan et al. (2004) and Kumar (2016). However, our focus is less on identifying fractiles of the population amongst whom wealth is being concentrated and instead on distinguishing classes based on sources of income. Data does not permit an easy identification of the so-called capitalist class although many workers fulfill out definition by engaging in savings only for lifecycle purposes. At the same time, this allows us to highlight our assumptions regarding differential saving between capitalists and workers. Perhaps one simple way of justifying this possibility is that models based on simple lifecycle and bequest motives fail to capture the rate of saving observed in US

⁶Themselves linked to income and wealth concentration amongst higher saving classes.

⁷See for example Semieniuk (2017) and Chirinko (2008) for estimates of the elasticity of substitution

data. This is best captured in Carroll (1998) who proposes a *capitalist spirit* model where the explicit motivation to accumulate wealth directly enters the utility function and fits the data much better. We rely on such motivations to model a two class economy where capitalists are similar to a dynastic class whose motivation for accumulation is over and above dynastic transmission or lifecycle saving.

2.3. Literature on demand driven models

Finally we follow in the tradition of models which link demand and distribution in a unified framework. Two strands of this literature are important for our purposes. First, we closely follow the model Pasinetti (1962) which is itself representative of rich debate⁸ on growth and distribution. Second, the issue of macroeconomic stability was related to investment and saving behavior by Bhaduri and Marglin (1990) which we utilize in our analysis. Setterfield (2017) reviews the features of models⁹ that follow in the Bhaduri-Marglin tradition in a comprehensive manner.

3. A growth model with capitalists and workers

We define a simple highly stylized aggregate economy populated by capitalists and workers. Both classes engage in saving with the conventional distinction that capitalists save (s_c) at a higher rate than workers (s_w) so that $s_c > s_w$. Capitalists earn capital income on their stock of capital (K_c) and workers earn wages as well as returns on the capital stock (K_w) accumulated through their past saving. We assume throughout a uniform rate of return r regardless of the class of its owner so that capital income earned is only distinguished by the size of capital (rK_w, rK_c). The functional distribution of income π is defined by the share of profits (or capital income) in total income X . By definition, the share of wages $1 - \pi$ accrues only to workers while both classes extend claims on capital income. Since we only discuss a one good framework, output can be consumed or saved as wealth hence we use the terms wealth and capital interchangeably. We denote the share of

⁸In the 1960s, the debate initiated by Pasinetti was followed by neoclassical responses by Samuelson and Modigliani (1966) and Stiglitz (1969). Nearly all these models assumed the long run rate of growth to be exogenous to macroeconomic factors.

⁹This issue of the Review of Keynesian Economics (Edward Elgar Publishing) has a range of articles devoted to this issue.

capitalist wealth as $Z = \frac{K_c}{K}$ so that the share of wealth owned by workers is simply $1 - Z$. Aggregate savings S can be decomposed into worker savings (S_w) and capitalist savings (S_c):

$$S = S_w + S_c = s_w \underbrace{((1 - \pi)X + rK_w)}_{\text{income earned by workers}} + s_c r K_c$$

In simple accounting terms, the rate of profit can be decomposed into the share of profits and the ratio of output to capital (or the inverse of the wealth-income ratio in a one good economy) so that $r = \pi \frac{X}{K}$. We symbolize the output capital ratio as u so $r = \pi u$. Dividing throughout by X , we get the aggregate saving rate, s :

$$s = \underbrace{s_c Z \pi}_{\text{weighted capitalist saving rate}} + \underbrace{s_w (1 - Z) \pi + s_w (1 - \pi)}_{\text{weighted worker saving rate}} \quad (1)$$

To distinguish our analysis from the benchmark neoclassical models, we endogenize the accumulation rate or the investment-capital ratio as a linear function of the (net of depreciation) rate of profit. This is standard in the structuralist literature, separating the motives to save (or abstain from consumption) from the entrepreneurial impulse to accumulate and follows, for example the model of Taylor (2014). Investment (net-of-depreciation) as a ratio of the capital stock is denoted by g :

$$g = g_0 + \alpha r = g_0 + \alpha \pi u = g(\pi, Z) \quad (2)$$

with $g = \hat{K} = \dot{K}/K$ as the accumulation rate (or the investment-capital ratio), there are two parts to accumulation. g_0 is an independent or exogenous term¹⁰ while $\alpha > 0$ is the sensitivity of investment to profitability.¹¹ Since effective demand must be set to zero, or investment must be identical to savings therefore $gK \equiv sX$ or $u = g/s$. Using expressions 1 and 2 we derive the reduced form expression for the output capital ratio:

$$u = \frac{g_0}{s_w + ((s_c - s_w)Z - \alpha)\pi} = u(\pi, Z) \quad (3)$$

Note that in the short run with π and Z given, the paradox of thrift applies, i.e for any level of saving $s'_i > s_i$ output is lower relative to the capital stock i.e $u' < u$.

¹⁰Analogous to animal spirits in the Keynesian tradition

¹¹The symbols dots and hats indicating time derivatives and growth rates respectively.

3.1. Short run stability and response to the profit share

Expression 3 allows to derive important properties of the macroeconomic systems and parameters. An important feature for the stability of a demand driven model is the so-called Keynesian stability system as for example in Bhaduri and Marglin (1990). These conditions dictate that the difference in the responses of investment and savings must be negative for any increase in output. Dividing investment, savings and output by the capital stock, this implies that:

$$\frac{\partial g}{\partial u} - \frac{\partial su}{\partial u} < 0$$

or $s_c\pi Z + s_w(1 - Z)\pi + s_w(1 - \pi) - \alpha\pi > 0$

For any $0 < \pi < 1$ and $1 > s_c > s_w > 0$, these conditions are fulfilled for $s_c > \alpha$, i.e the capitalist propensity to save must exceed the response of investment to the rate of profit. In the forthcoming section, we show this to be fulfilled quite easily.

The second important property of the macroeconomic system is the response of the output-capital ratio to a change in the functional income distribution. An economy is defined as profit-led (wage-led) if an increase in the share of profit (wages) increases the output capital ratio or put simply in our notations:

$$\text{Profit led: } \frac{\partial u}{\partial \pi} > 0$$

$$\text{Wage led: } \frac{\partial u}{\partial \pi} < 0$$

Taking the partial derivative of 3 against π for any $g_0 > 0$, we can define the profit or wage led character of the economy at any given level of the capitalist wealth share Z :

$$\text{Profit led: } Z < \frac{\alpha}{s_c - s_w} \text{ or Wage led: } Z > \frac{\alpha}{s_c - s_w} \quad (4)$$

The inequality conditions specified in (4) will be important for the long run steady state as well, which we discuss in further sections. For the moment, this completes the analytics of the short run demand driven system.

3.2. Long run steady state

We are interested in wealth inequality between capitalists and workers, for which we allow Z to evolve in the long run. The law of motion for Z is a simple accounting framework which states that the share of capitalist wealth increases (decreases) if they accumulate faster (slower) than the aggregate economy - or $\dot{Z} = \hat{K}_c - \hat{K}$. Since capitalists only earn capital income, their accumulation rate is given by $\hat{K}_c = s_c \frac{rK_c}{K}$ so that:

$$\frac{\dot{Z}}{Z} = s_c \pi u - g$$

As previously defined, $u = u(\pi, Z)$ and $g = g(\pi, z)$. We fix the functional distribution of income $\pi = \bar{\pi}$ to define a single differential equation for the long run:

$$\dot{Z} = Z (s_c \bar{\pi} u(Z) - g_0 - \alpha \bar{\pi} u(Z)) = \phi(Z) \quad (5)$$

Any theory of the distribution of income, whether determined by a neoclassical production function technology or via alternate theories of the tradeoff between profits and wages is a *closure* distinguishing growth models. We impose a strong dynamic restriction by fixing the functional income distribution to keep the analysis parsimonious yet interesting due to its demand driven nature.¹² When aggregate demand goes up and capital is more efficiently utilized to produce higher output, u increases and the rate of profit also increases since $r = \bar{\pi}u$. In Appendix A, we discuss the implications of a fully accommodating functional income distribution which transforms the *paradox of thrift* into a long run *paradox of wealth* and is closely related to our short run inequalities in expression (4) which distinguish wage-led and profit-led regimes. Based on equation (5) we define the steady state:

Theorem 1. For the law of motion of wealth distribution in (5), there exists a $Z^* > 0$ such that $\dot{Z} = 0$ so that $g^* = g(Z^*)$ and $u^* = u(Z^*)$.

¹²In neoclassical growth, for a Cobb Douglas production function $X = K^\pi L^{1-\pi}$ the share of profits is given by $\pi = \bar{\pi}$ but the rate of growth is also given at $g = \bar{g}$. Our long run dynamics are distinguished by the linear accumulation function which separate investment and saving decisions. Stiglitz (1969) presents a neoclassical theory of wealth distribution on similar lines as our analysis but under a given growth rate and linear savings functions.

The steady state expression for Z^* can be derived by setting $\phi(Z) = 0$ which gives:

$$Z^* = \frac{s_w - s_c \bar{\pi}}{(s_w - s_c) \bar{\pi}} \quad (6)$$

From the above, the responses of the distribution of wealth to savings propensities of either class are straightforward. For any given functional income distribution $0 < \bar{\pi} < 1$, the steady state share of capitalists wealth Z^* responds positively to capitalist saving i.e $\frac{\partial Z^*}{\partial s_c} > 0$ and negatively to worker saving i.e $\frac{\partial Z^*}{\partial s_w} < 0$ for $s_c > s_w$. At the steady state, the wealth of capitalists grows at the same rate as the aggregate rate of capital accumulation, i.e $s_c r^* = g^*$, which gives:

$$r^* = \frac{g^*}{s_c} \quad (7)$$

Note that this expression is the famous Cambridge equation due to Pasinetti (1962). In his original model however Pasinetti defined the rate of growth as given, similar to neoclassical growth models. In our model, the rate of return is intrinsically linked to investment parameters through g . By substituting $g^* = g(Z^*)$ we get:

$$r^* = \frac{g_0}{s_c - \alpha}, u^* = \frac{g_0}{(s_c - \alpha) \bar{\pi}} \quad (8)$$

Thus in the case that $\alpha = 0$ so that the accumulation is given ($g = g_0$) we get an analogous expression to Pasinetti. Regardless, the saving behavior of workers does not influence the steady state rate of profit or the output-capital ratio. Secondly, the Keynesian stability condition ($s_c - \alpha$) defined previously ensures feasible (non-negative) values of the steady state rate of profit and the output-capital ratio. We can combine the expressions (6) and (8) to get the following important properties:

Remark 1. *The distribution of wealth between capitalists and workers is independent of the investment parameters for the aggregate economy α and g_0 . On the other hand at the steady state the rate of return is positively related to α and output per unit of capital stock is higher for larger values of g_0 and α .*

In Appendix B we provide parametric values necessary to obtain a range of distributions such as an egalitarian wealth distribution as well as complete ownership of wealth by workers or capitalists. In further sections, our empirical analysis will show that parameters are well within the bounds to generate a distribution rather than single class capital ownership.

3.2.1. Stability of Z^*

Finally, to conclude the theoretical framework we derive the conditions for stability of the dynamic system for Z defined in a single variable:

$$\frac{\partial \dot{Z}}{\partial Z} \Big|_{Z=Z^*} < 0$$

Which from substitution of (6) into (5) yields

$$\frac{g_0 (s_w - \bar{\pi} s_c)}{\bar{\pi} (s_c - \alpha)} < 0$$

The stability conditions are fulfilled for $g \in [0, 1]$, $0 < s_w < s_c < 1$ and $s_c > \alpha$ defined previously with the additional restriction $\bar{\pi} > \frac{s_w}{s_c}$ or the share of profits in total income should be greater than the ratio of savings propensities of workers to capitalists. So for example, if workers save 10% of their income and capitalists save 50% of their income then the long run distribution of wealth is stable if profits are greater than $\frac{0.1}{0.5} = 20\%$ of national income. Under this parametric restriction, we know from Appendix B that there is a wealth distribution as opposed to complete ownership of wealth by either class.

Thus our stylized economy behaves in the following way - for the parameters $[s_c, s_w, g_0, \alpha]$ and a given functional income distribution $\bar{\pi}$, the distribution of wealth stabilizes in the long run at Z^* starting from any Z_0 . Output grows at the rate of accumulation of capital stock g^* to maintain a constant output-capital ratio u^* . Note that we do not explicitly model any differential fertility rates between the two classes and focus on the the aggregate behavior. Our results would be different if the natural growth rates of either class differ from each other or even from the long run accumulation rate g^* but we do not discuss these in the context of this model. Our theoretical framework is highly stylized and only intended to illustrate the dynamics of wealth distribution in a two class demand-driven setting. Without calibrating the parameters of this model, there is little more we can say about its implications for actual economies.

4. Calibration to US economy: 1950-2015

The Post-Keynesian literature offers different approaches to calibration of growth models, none of which we find entirely satisfying for our purposes: early Post-Keynesians typically calibrate the model such that selected

moments of the data, typically steady-state ratios, are matched. Hein and Schoder (2011), among others, estimate the model parameters using a single-equation Ordinary Least Squares approach. However, as argued by Schoder (2017a) this approach is likely to yield biased results and ignores the uncertainty of estimates.

To parameterize the current model we follow Schoder (2017b) and estimate the k model parameters θ using Bayesian inference. That is, we combine our prior beliefs about the probability distribution of the model parameters $p(\theta)$ with the likelihood that these parameters give rise to the data observed $p(y|\theta)$. This allows us to make a probability statement about the parameters given the data, i.e. $p(\theta|y)$. This principle of Bayesian inference is condensed in Bayes' rule which states that

$$p(\theta|y) = \frac{p(\theta)p(y|\theta)}{p(y)}.$$

Note that θ is a vector of parameters. Yet, we would like to make a probability statement about the single parameter, say θ_i . This is achieved by a simple probability rule, i.e.

$$p(\theta_i|y) = \int p(\theta|y)d\theta_1 \cdots \theta_{i-1}\theta_{i+1} \cdots \theta_k.$$

The required integration cannot be done analytically, in general. Hence we apply a simulation method to draw from $p(\theta_i|y)$ for all i . In particular we apply the Metropolis-Hastings algorithm, a Markov chain Monte Carlo simulation method details of which can be found in any introductory text on Bayesian statistics (Greenberg, 2012).

4.1. Estimation

[Table 1 about here.]

We utilize historical US data for estimation and calibration, taking g , r , and u as observed variables and the rest as unobserved. Subsequently we systematically approximate the values of observed variables for the period 1950 to 2015, using data from NIPA tables and depreciation and capital stock data reported by the Bureau of Economic Analysis (BEA). Note that throughout we have emphasized a one-good model for our stylized economy and hence abstract from asset price effects. Therefore we use the stock of private fixed

assets as our measure of capital stock - this also permits us to avoid the volatility caused by fluctuations in the value of financial assets. We use the ratio of the investment in private fixed assets (I) net of depreciation (D) to the stock of private fixed assets (PFA) as a proxy for the accumulation rate g . The ratio of income from profit (PI) as the sum of pre-tax rental income, net interest payments, and corporate profit adjusted for capital consumption to the stock of private fixed assets (PFA) is used as our measure of r . We exclude proprietors income from our calculations to avoid mixed income definitions. Table B.1 presents a detailed list of variables, and NIPA tables and other BEA data used in their calculations. Since s_c and s_w always appear in our equations together, one of them should be manually calibrated. We assume the saving propensity of wage earners, s_w at 5%, a rough average of the average personal saving rate in the NIPA accounts which varies temporally between 2-8%. Thus our estimations of the capitalist saving propensity s_c can be considered conditional on the specified value of s_w or a fixed ratio $\frac{s_w}{s_c}$.

[Table 2 about here.]

Table B.2 presents the estimation results for 50,000 iterations and 2 Markov chains. These specifications are sufficient to guarantee convergence of the simulated distribution to the distribution of interest i.e. $p(\theta|y)$. The Bayesian approach combines prior beliefs with data. The prior beliefs are incorporated in the prior distributions $p(\theta_i)$ for all i . These are reported in the second to fourth column of the parameters estimated. We choose rather large variances for the prior distribution which means that our confidence in our prior beliefs are rather weak. In particular, we follow the conventional literature and assume an inverse gamma distribution for the estimated standard deviations of the disturbances.

. For parameters which are constrained by 0 and 1 we assume a beta distribution and for parameters constrained to be positive we assume a gamma distribution. Columns 5 and 6 report moments of interest of the posterior distributions $p(\theta_i|y)$. The estimated mean for the capitalist propensity to save (s_c) is 0.734 and falls in the range of 0.641 and 0.818 with a 90% probability. The mean of the profit share (π) is estimated to be 0.188 with a narrow probability interval. This is not surprising as this is consistent with the mean implied by $\pi = r/u$ recalling that both r and u are observed. For

the intercept and slope parameters (g_0, α) of the investment function we find as the means 0.015 and 0.214, respectively. The remaining parameters are of less economic interest as they capture the persistence of the shock processes (ρ_π, ρ_{sc} , and ρ_g) as well as the standard deviations of the innovations (σ_π, σ_{sc} , and σ_g).

4.2. Dynamics of US capital distribution

[Figure 3 about here.]

In Figure B.3 we plot the phase diagram for the differential equation in 5 with the parameters calibrated to our estimated values. The parameters fulfill all our assumptions for stability hence the steady state distribution of wealth is limited in the interval $Z^* \in [0, 1]$ at 0.684. That is, calibrated to historical US data our model predicts that in the long run, capitalists will own approximately 68% of the aggregate *productive* capital stock. At $Z > Z^*$, for $\pi = \bar{\pi}$ the output capital ratio is lower and exerts downward pressure on the rate of profit. Since capitalists only earn capital income, this reduces their share of income and for unchanged s_c returns the distribution of wealth to Z^* . For $Z < Z^*$ aggregate demand is higher and increases the rate of profit and capitalists' income hence increasing their share of wealth until the distribution of wealth reaches the steady state.

Our model predicts a higher wealth share than predicted by the estimates of Saez and Zucman (2016) for the Top 1% of US wealth holders. This is understandable - even top wealth holders are not strictly capitalists and comprise very well paid professionals and managers. The pure (abstract) capitalist class itself would be some proportion of this fraction of the population with income derived solely from claims on means of production. Secondly, due to the complexities of wealth ownership through pension investments, hedge funds and other financial instruments it is unclear from administrative data as to exactly who exerts claims on the productive capital stock as opposed to the non standard definition of capital in Piketty and Zucman (2014). It is quite possible that the distribution of wealth maybe less concentrated (say 40%) than the productive capital stock. This can for example occur due to a more egalitarian ownership of residential assets which may not be part of the production process, but nevertheless are considered wealth. However we are reassured that our model and estimates are within the plausible limits of the Modigliani-Kotlikoff-Summers 20-80% range for US dynastic capital and closer to the estimate of 80% in Kotlikoff and Summers (1981).

Figures B.4 to B.6 depict how the economy responds to a disturbance to the investment function, the profit share, and the capitalists propensity to save, respectively. On abscissa shows the years after the impulse and the ordinate shows for each variable the level deviations from the steady state. Since the underlying model parameters are associated with some uncertainty reflected by the variance of the distribution, there is also uncertainty associated with the impulse response functions (IRFs). We report the mean responses as well as the 90% confidence bands. The size of the impulse is the mean of the estimated standard deviation of the respective shock.

Figure B.4 illustrates the response of the accumulation rate, the utilization rate, and the profit rate to a one-time increase of ϵ_g by 0.0044 which is the increase in the accumulation rate on impact. Since the rise in the utilization rate increases the profit rate, the accumulation rate increases further in the second year despite the fact that the impulse is present in only the first year. The persistence in the data comes from the multiplier effect and the persistence in the shock process v_g . As discussed in our theoretical model, the distribution of wealth is independent of the parameters of the investment function so that only shocks to the functional income distribution and relative saving propensities influence the steady state capitalist wealth share.

Figure B.5 plots the response to a one standard deviation impulse in the profit share shock process. The size of the impulse is around 0.02 reflecting an increase in the profit share by 2 percentage points. This contracts utilization through lower consumption demand given the saving differential between workers and capitalists. Yet, investment increases only moderately. Due to the redistribution of income, the overall saving rate goes up. The capitalist wealth share increases persistently because capitalists accumulate more wealth by drawing upon the temporary expansion of their income source.

Finally, Figure B.6 depicts the macroeconomic response to a one standard deviation increase in the capitalist's propensity to save. The impulse is an increase of s_c by around 20 percentage points. This increases the overall saving rate and therefore reduces output per unit of capital stock, the profit rate, and the accumulation rate - thus the downward pressure on aggregate demand reduces aggregate profitability. The higher propensity to save however allows capitalists to increase their steady state share of capital stock and compensate for the loss of income from the lower rate of return.

[Figure 4 about here.]

[Figure 5 about here.]

[Figure 6 about here.]

5. Discussion

We briefly discuss the implications of our results. Our highly abstract model is able to capture important aspects of growth and distribution as evidenced by our computed estimates. Our computations of the capitalist wealth share (or the proportion of US fixed private assets) as a possible steady state draws attention to the ownership of productive resources in an advanced capitalist society. We show in our estimation that such concentration of capital is possible primarily through an extraordinarily high rate of saving from the accumulated surplus. Our impulse analysis suggests that negative (positive) shocks to the labor (profit) share have the effect of redistributing wealth to the capitalist class while only mildly boosting investment. At the same time, an upward shock in the capitalist saving propensity lowers macroeconomic performance whilst permanently worsening wealth inequality.

We highlight two important discussions which are lacking in our paper. Firstly, for simplicity we assume that everyone earns the same rate of return which is not entirely realistic. In capital markets, the size and composition of portfolios play an important role in generating a differential rate of return. In such cases, multiplicative shocks to portfolios would be important determinant of both trends and cycles of capital accumulation. Secondly we abstain from discussing the impact of labor markets on the distribution of income and wealth as well as the role of demographic parameters. We admit this is an important line of reasoning, especially because we have shown the functional income distribution (itself related to growth of employment and bargaining power) to be a critical determinant of capital inequality. We believe our calibrated models explain our initial questions in a new and sufficient manner for the moment and will continue to add new dimensions just suggested in further research.

6. References

Abel, A. B., Mankiw, N. G., Summers, L. H., Zeckhauser, R. J., 1989. Assessing dynamic efficiency: Theory and evidence. *Review of Economic Studies* 56 (1).

- Bhaduri, A., Marglin, S., 1990. Unemployment and the real wage: the economic basis for contesting political ideologies. *Cambridge Journal of Economics* 14 (4), 375–393.
- Carroll, C. D., 1998. Why do the rich save so much?
- Chirinko, R. S., 2008. Sigma: The long and short of it. *Journal of Macroeconomics* 30 (2), 671–686.
- Dynan, K. E., Skinner, J., Zeldes, S. P., 2004. Do the rich save more? *Journal of Political Economy* 112 (2), 397–444.
- Geerolf, F., 2013. Reassessing dynamic efficiency. manuscript, Toulouse School of Economics.
- Greenberg, E., 2012. Introduction to Bayesian Econometrics. Introduction to Bayesian Econometrics. Cambridge University Press.
URL <https://books.google.com.co/books?id=ErUAOdGWFRAC>
- Hein, E., Schoder, C., November 2011. Interest rates, distribution and capital accumulation – a post-kaleckian perspective on the us and germany. *International Review of Applied Economics* 25 (6), 693–723.
URL <http://ideas.repec.org/a/taf/irapec/v25y2011i6p693-723.html>
- Kotlikoff, L. J., Summers, L. H., 1981. The role of intergenerational transfers in aggregate capital accumulation. *Journal of Political Economy* 89 (4), 706–732.
- Kumar, R., 2016. Personal savings from top incomes and household wealth accumulation in the united states: Results from disaggregated national accounts. *International Journal of Political Economy* 45 (3), 224–240.
- Pasinetti, L. L., 1962. Rate of profit and income distribution in relation to the rate of economic growth. *The Review of Economic Studies*, 267–279.
- Piketty, T., 2015. Putting distribution back at the center of economics: Reflections on “capital in the twenty-first century”. *The Journal of Economic Perspectives* 29 (1), 67–88.
- Piketty, T., Zucman, G., 2014. Capital is back: Wealth-income ratios in rich countries, 1700-2010. *The Quarterly Journal of Economics*.

- Saez, E., Zucman, G., 2016. Wealth inequality in the united states since 1913: Evidence from capitalized income tax data. *The Quarterly Journal of Economics* 131 (2), 519–578.
- Samuelson, P. A., Modigliani, F., 1966. The pasinetti paradox in neoclassical and more general models. *The Review of Economic Studies*, 269–301.
- Schoder, C., 2017a. Bayesian methods for quantitative analysis in political economy. *the new school for social research*.
- Schoder, C., 2017b. Estimating keynesian models of business fluctuations using bayesian maximum likelihood. *Review of Post Keynesian Economics* forthcoming.
- Semieniuk, G., 2017. Piketty's elasticity of substitution: A critique. *Review of Political Economy* 29 (1), 64–79.
- Setterfield, M., 2017. Wage-versus profit-led growth after 25 years: an introduction to the third special issue. *Review of Keynesian Economics* 5 (2), 143–145.
- Solow, R. M., 1956. A contribution to the theory of economic growth. *The Quarterly Journal of Economics*, 65–94.
- Stiglitz, J. E., 1969. Distribution of income and wealth among individuals. *Econometrica: Journal of the Econometric Society*, 382–397.
- Summers, L. H., 2014. Us economic prospects: Secular stagnation, hysteresis, and the zero lower bound. *Business Economics* 49 (2), 65–73.
- Taylor, L., 2009. *Reconstructing macroeconomics: Structuralist proposals and critiques of the mainstream*. Harvard University Press.
- Taylor, L., 2014. The triumph of the rentier? thomas piketty vs. luigi pasinetti and john maynard keynes. *International Journal of Political Economy* 43 (3), 4–17.

Appendix A. Specification with full adjustment of the profit share

In our benchmark model, we have assumed a fixed functional income distribution $\pi = \bar{\pi}$. Within the variables we have defined, there exists a possible steady state where we can relax this assumption and allow the profit share to fully accommodate the dynamics of growth and distribution. Instead the forcing variable in the long run is a given steady state growth rate g^* . To see this, suppose in the long run there exists a $\pi = \pi^*$ such that the economy grows at a rate $g = g^* = s_c r^*$ and hence the distribution of wealth is constant at $Z = Z^*$. Solving for π^* such that $g = g_0 + \alpha\pi u = g^*$ and substituting in expression (3) we get:

$$u^* = \frac{Z^*(g_0 - g^*)(s_c - s_w) + \alpha g^*}{\alpha s_w} \quad (\text{A.1})$$

Since the economy is at a steady state, hence the output-capital ratio should necessarily be such that $u = u^* = u(\pi^*, Z^*)$. Thus we can equate expressions (3) and (A.1) to solve for Z^* :

$$\begin{aligned} u^* &= \frac{Z^*(g_0 - g^*)(s_c - s_w) + \alpha g^*}{\alpha s_w} \\ &= \frac{g_0}{s_w + ((s_c - s_w)Z^* - \alpha)\pi^*} \end{aligned}$$

So that:

$$Z^* = \frac{\alpha}{s_c - s_w} \quad (\text{A.2})$$

Thus assuming this set of assumption delivers a *paradox of wealth*. In expression (A.2) an increase in the saving propensity of any class distributes wealth against them. If capitalists increase their saving rate, then Z^* declines while if workers increase their rate of saving then $(1 - Z^*)$ declines. At the same time, we now see the response of investment to profitability α directly influence the distribution of wealth at the steady state, contrary to our original results.

Appendix B. Possible wealth distributions

Lemma 1. *If capitalists comprise some fraction b of the population, then a steady state egalitarian wealth distribution would be represented by $Z^* = b$ or $\frac{s_w - s_c \bar{\pi}}{(s_w - s_c)\bar{\pi}} = b$. For $\bar{\pi} = \frac{s_w}{s_c - b(s_c - s_w)}$, capitalists will own a share of wealth equal to their proportion in the population.*

To see this, suppose the $Z^* = b$. Substituting (6):

$$Z^* = \frac{s_w - s_c \bar{\pi}}{(s_w - s_c) \bar{\pi}} = b$$

Solving the above for $\bar{\pi}$ we get the proposed result

Lemma 2. *For $s_w = 0$ or $\bar{\pi} = 1$, wealth is entirely owned by capitalists or $Z^* = 1$. If $s_w = 0$ then this implies workers do not accumulate capital stock while if $\bar{\pi} = 1$ then all income is comprised of profits or the entire economy is made up of capitalists.*

This is verified by setting $Z^* = \frac{s_w - s_c \bar{\pi}}{(s_w - s_c) \bar{\pi}} = 1$ viz only possible if $s_w = 0$ or $\bar{\pi} = 1$.

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Corporate Profit, Rent, and Interest Share of National Income

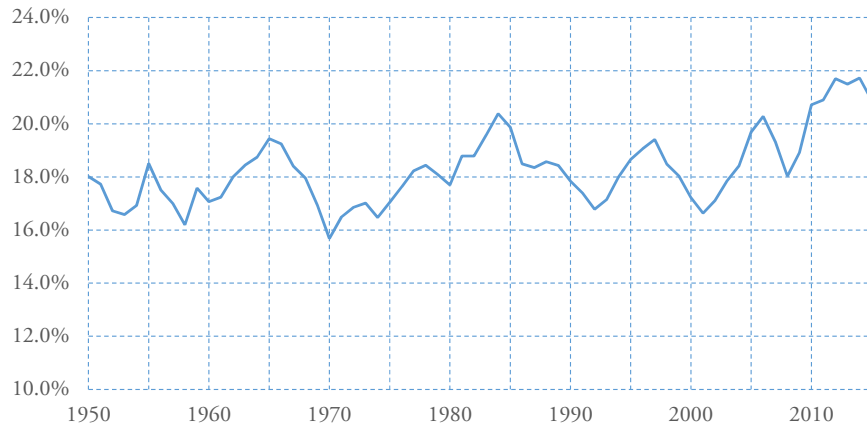


Figure B.1: The share of profits in US national income. Source: NIPA, Bureau of Economic Analysis

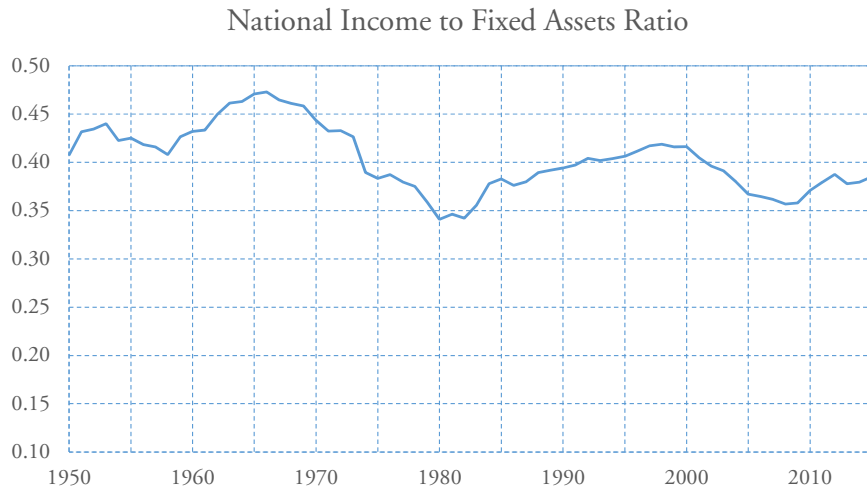


Figure B.2: Ratio of capital stock to US national income. Source: NIPA, Bureau of Economic Analysis

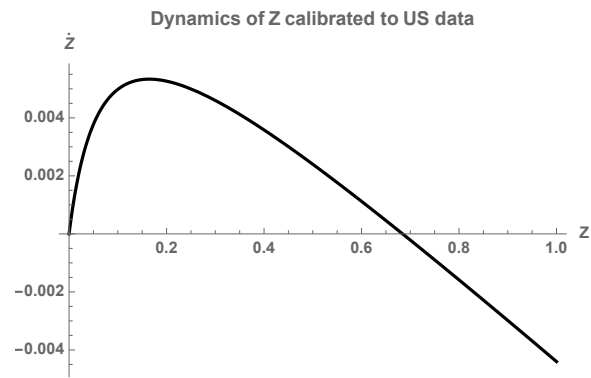


Figure B.3: Phase diagram for Z with parameters calibrated to US data for 1950-1915

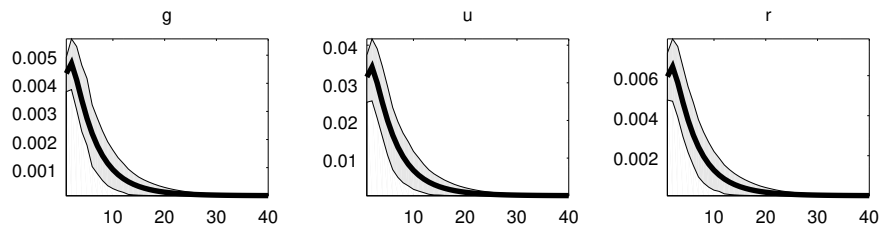


Figure B.4: Macroeconomic responses to a one standard deviation innovation to the investment shock process.

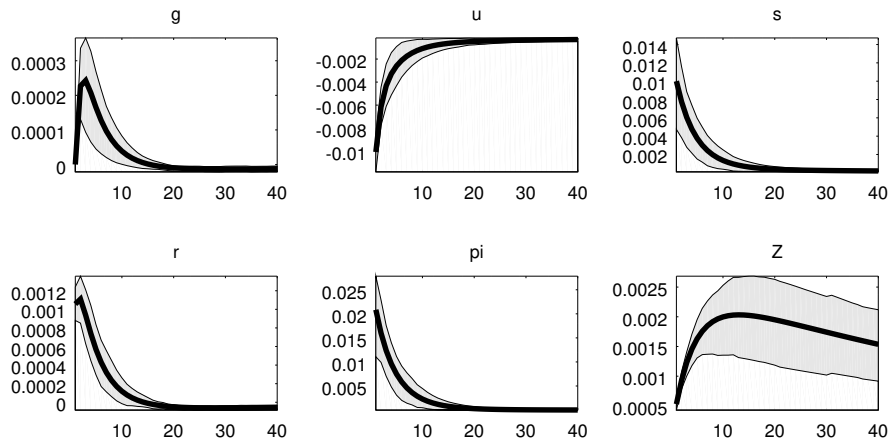


Figure B.5: Macroeconomic responses to a one standard deviation innovation to the profit share shock process.

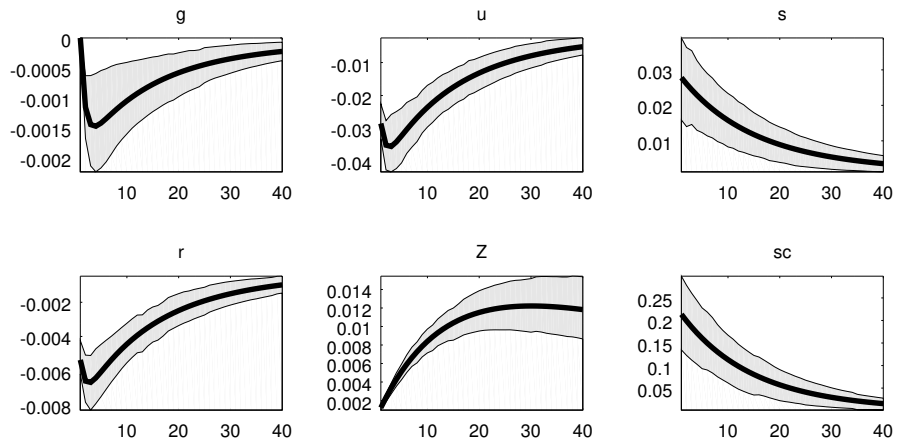


Figure B.6: Macroeconomic responses to a one standard deviation innovation to the capitalist's saving propensity process.

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Table B.1: Variable Definition and Data Sources for Observed Variables

Variable	Definitions and Data Sources
g	(Investment in private fixed assets (I) - Depreciation (D)) / Stock of private fixed assets (PFA)
u	National Income (NI) / Stock of private fixed assets (PFA)
r	Income from profits (PI) / Stock of private fixed assets (PFA)
I	Investment in private fixed assets BEA Fixed Assets Accounts tables, Table 1.5, Line 3
D	Current-cost depreciation of private fixed assets BEA Fixed Assets Accounts tables, Table 1.3, Line 3
PFA	Current-cost net stock of private fixed assets BEA Fixed Assets Accounts tables, Table 1.1, Line 3
PI	Income from profit \approx rent + interest + corporate profit NIPA Table 1.12, lines 12,13, and 18
NI	National Income NIPA Table 1.12, Line 1

Table B.2: Estimation results

Parameters	prior dist.	prior mean	prior stddev.	post. mean	90% HDP IV
s_{c0}	\mathcal{B}	.40	.10	.734	[.641, .818]
π_0	\mathcal{B}	.30	.10	.188	[.181, .196]
g_0	\mathcal{G}	.01	.01	.015	[.008, .021]
α	\mathcal{G}	.30	.10	.214	[.129, .294]
ρ_π	\mathcal{B}	.50	.10	.769	[.683, .865]
ρ_{sc}	\mathcal{B}	.50	.10	.931	[.908, .952]
ρ_g	\mathcal{B}	.50	.10	.789	[.710, .865]
σ_π	\mathcal{IG}	.01	.01	.021	[.710, .030]
σ_{sc}	\mathcal{IG}	.01	.01	.214	[.133, .298]
σ_g	\mathcal{IG}	.01	.01	.004	[.003, .005]

Notes: \mathcal{B} , \mathcal{U} , \mathcal{G} and \mathcal{IG} denote the Beta, Uniform, Gamma and Inverse Gamma distributions, respectively.