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Mathematics curriculum implementation for the sixth grade

Steven Anthony Knap

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MATHEMATICS CURRICULUM IMPLEMENTATION
FOR THE SIXTH GRADE

A Project
Presented to the
Faculty of
California State University,
San Bernardino

In Partial Fulfillment
of the Requirements for the Degree
Master of Arts
in
Education: Elementary

by
Steven Anthony Knap
June 1995
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FOR THE SIXTH GRADE

A Project
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Approved by:

Dr. Richard Griffiths,  
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Abstract

The model mathematics curriculum as described in *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics, 1989) and *Mathematics Framework for California Public Schools* (California Department of Education, 1992) is summarized. A rationale for the model mathematics curriculum is provided through a review of the literature as it relates to constructivist learning theory. A handbook provides lesson plans designed to implement the model mathematics curriculum at the sixth grade level.
Acknowledgements

This project is dedicated to my mother and father, Delphine and Florian Knap. They instilled a deep respect for learning through their steadfast help in my education.

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# Table of Contents

ABSTRACT................................................................. iii
ACKNOWLEDGEMENTS..................................................... iv
REVIEW OF THE LITERATURE........................................ 1
  Introduction.......................................................... 1
  New Challenges....................................................... 6
  Dimensions of Mathematical Power............................... 9
  Goals that Support Mathematical Power.......................... 14
  Implementing Mathematical Power.................................. 17
  Constructivist Learning............................................. 20
  Whole Concept Learning............................................. 23
  Challenges Regarding Implementation............................ 25
REFERENCES............................................................. 29
MATHMATICS HANDBOOK.................................................. 33
  Curriculum Implementation......................................... 33
  Make a Million....................................................... 34
  Check Writing......................................................... 36
  Handshakes............................................................ 40
  Visualizing Multiplication......................................... 42
  Digit Combinations................................................... 44
  Stating Equality Relationships................................... 45
  Border Problems..................................................... 47
  Playing Card Activity............................................... 50
  Squares and Cubes................................................... 52
  Visualizing Division................................................ 54
<table>
<thead>
<tr>
<th>Activity</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Going Shopping</td>
<td>55</td>
</tr>
<tr>
<td>Decimal Building Game</td>
<td>57</td>
</tr>
<tr>
<td>Match Up</td>
<td>59</td>
</tr>
<tr>
<td>Visualizing Decimal Addition</td>
<td>61</td>
</tr>
<tr>
<td>Visualizing Decimal Multiplication</td>
<td>63</td>
</tr>
<tr>
<td>Multiplying with Decimals</td>
<td>65</td>
</tr>
<tr>
<td>Factors</td>
<td>67</td>
</tr>
<tr>
<td>Properties of Nine</td>
<td>69</td>
</tr>
<tr>
<td>Restaurant Menus</td>
<td>70</td>
</tr>
<tr>
<td>Metric Measurement</td>
<td>72</td>
</tr>
<tr>
<td>Fraction Mobiles</td>
<td>74</td>
</tr>
<tr>
<td>Rapidly Multiplying by 5</td>
<td>75</td>
</tr>
<tr>
<td>Classroom Measurements</td>
<td>78</td>
</tr>
<tr>
<td>Fraction Concept Strategies</td>
<td>80</td>
</tr>
<tr>
<td>Domino Fractions</td>
<td>82</td>
</tr>
<tr>
<td>Fraction Products</td>
<td>84</td>
</tr>
<tr>
<td>Circle Graphs</td>
<td>86</td>
</tr>
<tr>
<td>Hands-on Averages</td>
<td>88</td>
</tr>
<tr>
<td>Finding Data</td>
<td>90</td>
</tr>
<tr>
<td>Bean City</td>
<td>92</td>
</tr>
<tr>
<td>Find It Fast</td>
<td>94</td>
</tr>
<tr>
<td>Round Houses</td>
<td>96</td>
</tr>
<tr>
<td>How to Bag It?</td>
<td>98</td>
</tr>
<tr>
<td>Angle Check</td>
<td>101</td>
</tr>
<tr>
<td>Sorting Quadrilaterals</td>
<td>102</td>
</tr>
</tbody>
</table>
Review of the Literature

Introduction

The National Council of Teachers of Mathematics, in *Curriculum and Evaluation Standards for School Mathematics* (1989), states that modern society is undergoing a transformation from an industrial society to an information one. This transformation, coupled with the rapid proliferation of technological advances such as computers, calculators, and communication devices, has necessitated reform of the requirements for preparation of mathematically literate individuals. If today's students are to function as productive citizens in the twenty-first century, reform must occur in the goals, instruction, and evaluation of mathematics education (National Council of Teachers of Mathematics [NCTM], 1989). The social goals and economic requirements for the new information society include mathematically literate employees, individuals open to learning throughout life, opportunities for all members of society, and an electorate that is informed and capable of critical thought (NCTM, 1989).

A publication of the California Department of Education, *Mathematics Framework for California Public Schools* (1992), states that students must become adept at reasoning and develop their individual talents in order to appreciate and enjoy mathematics. Moreover, employers
today demand workers and managers with deeper understandings of mathematical concepts than have traditionally been developed in students. In contrast to the limited proficiency required of the industrial age, when products were produced the same way for many consecutive years, today's economy requires constant change and innovation in product development and production. Workers today, and in the future, will be required to understand the complex function of communication technologies, to pose pertinent questions, to successfully assimilate new information, and to work cooperatively with others (NCTM, 1989).

Modern business is becoming increasingly vocal about the necessity of schools producing graduates with qualitatively different mathematics training than that of their predecessors. Lappan and Ferrini-Mundy (1993) argue that in response to new national education goals, current programs must not be merely revised. Overall mathematical goals for students, roles of teachers, and assessment practices must be completely rethought. The expectations employers will have for future employees in industry were summarized by the industrial mathematician, Henry Pollack, in 1987. They appear in Curriculum and Evaluation Standards for School Mathematics (1989) and include:

1. The ability to set up problems with the
appropriate operations
2. Knowledge of a variety of techniques to approach and work on problems
3. Understanding of the underlying mathematical features of a problem
4. The ability to work with others on problems
5. The ability to see the applicability of mathematical ideas to common and complex problems
6. Preparation for open situations, since most real problems are not well formulated
7. Belief in the utility and value of mathematics (p. 4)

It would seem that the above abilities are different than those acquired through traditional mathematics instruction. Students at the middle grade level are not being served very well in respect to the expectations of the business community (Lappan & Ferrini-Mundy, 1993). This suggests that if students are to arrive at deeper understandings of mathematical concepts, they must receive instruction based on revised content and teaching processes.

Due to technological changes, employment opportunities are likely to change rapidly. It is likely that workers will have to change jobs as many as five times in a span of 3.
twenty-five years (NCTM, 1989). This suggests that a workforce capable of retraining in communication skills and receptive to lifelong learning becomes a necessity. Mathematics instruction can play a role in helping individuals acquire a dynamic form of literacy (NCTM, 1989). One aspect of mathematics training that can be employed in this area is that of problem solving. As a framework for representing a problem, expressing problems in mathematical terms, and conjecture and exploration of possible solutions, instruction in problem solving may help individuals adapt to changes in a technological economy.

All people have a right to develop their talents regardless of their race or sex. Mathematics has traditionally been the province of white males (NCTM, 1989). Mathematics education will become a necessity for gainful employment and individuals of all ethnic groups and of both sexes must have equal access to it. This is not only an issue of fairness, but, in an increasingly competitive world, one of economic necessity (NCTM, 1989).

The social goal of an informed electorate is dependent on reform in mathematics instruction. A democratic form of government is dependent on a society informed about challenges of the future. Protecting the environment, spending on defense, use of nuclear power, exploration of space, and taxing ourselves to pay for government programs...
often involve complex mathematical concepts (NCTM, 1989). This suggests that an understanding of mathematics and problem solving techniques will be essential in arriving at decisions regarding these and other issues.

If the demands of an information society are to be met, the goals, standards, and techniques of mathematics instruction must no longer be grounded in traditions established in the past industrial era. New ways must be developed and made a part of existing educational institutions. Lappan and Ferrini-Mundy (1993) argue that changes in mathematics instruction will be of great magnitude and will change the structure of schools, of how teachers perform in their profession and are held accountable, and how students are assessed. Although change will most likely take place slowly, it may receive impetus from a careful evaluation of innovations and further research concerning the nature of learning (Lappan & Ferrini-Mundy, 1993). If reform is to take place on a large scale, it would seem imperative that more teachers be made aware of new knowledge concerning the way in which children acquire mathematical concepts.

How to reform a mathematics curriculum is a question that will be facing many teachers and schools in the coming years. This project will provide a rationale for reform based on educational research regarding how children learn.
It will also address the effective use of manipulatives and projects in the implementation of a model mathematics curriculum. This project will attempt to provide teachers a description of, rationale for, and effective lessons in implementing a model sixth grade mathematics curriculum as described in *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), and *Mathematics Framework for California Public Schools* (California Department of Education, 1992).

New Challenges

As educators attempt to prepare students for productive lives as members of the work force, many challenges arise. One such challenge is the necessity of preparing children to work with numbers and mathematical concepts. It is important that children develop abilities in areas of mathematics that will enable them to become adept problem solvers and creative innovators in an increasingly changing, technological, and complex society. It is essential that educators focus on areas that encompass much more than the attainment of proficiency in computational skills. Although it is worthwhile to help students become competent in these traditional areas of emphasis, it is necessary to go beyond these basic skills and encourage and aid students in the development of deeper understandings of underlying concepts, their applications.
to real world problems, and the ability to use mathematics creatively. To view the major role of mathematics education as one of imparting rote skills of computation is to discount the efficiency with which the same results can be obtained through the use of calculators and computers.

The ideal curriculum for the teaching of mathematics in the public schools of California is described in the Mathematics Framework for California Public Schools (1992) (hereafter referred to as the Mathematics Framework). The major thrust of this document published by the California Department of Education is one of reform. It describes a program with which teachers, as professionals, should become familiar. The recommendations of the Mathematics Framework (1992) and a description of the major components of an ideal mathematics program will be summarized. Related documents from which parts of the Mathematics Framework (1992) were derived will be cited. It is hoped that all mathematics educators will become knowledgeable of the standards described in the Mathematics Framework (1992), thoughtfully consider their merit in relation to their particular circumstances, and implement them in a manner conducive to the future success of their students.

The Mathematics Framework (1992) supports a move away from a detailed study of the mechanics of computation. Rather, the emphasis is placed on deeper understandings of
concepts central to mathematics and a broader study of the various strands of mathematics. The shift is away from the fragmentation of the mathematics curriculum and toward a more holistic approach. A holistic approach includes a well defined educational purpose, a range of learning experiences that are effective and available, and a wholeness among curricular activities (Wiles & Bondi, 1993). Mathematics is no longer to be viewed as a set of procedures designed to arrive at a correct answer. Although computation is a necessary part of the elementary mathematics curriculum, it has traditionally been taught as a set of procedures, rather than as a vehicle for more sophisticated thinking skills (Campbell & Fey, 1988).

The Mathematics Framework (1992) emphasizes reform in the way mathematics is taught as the only way of preparing students for the future. Once they leave the public school system, they will be faced with the increasingly complex challenges in mathematics found in most occupations and college courses. A new direction for school mathematics has been necessitated by the changing work place, the impact of technological advances, and the need of an informed citizenry in a democratic society (Frye, 1989). Also, students will find their lives enriched through an appreciation of the beauty of mathematics which is more likely to be realized through a reformed curriculum.
Dimensions of Mathematical Power

The major educational goal of the ideal mathematics curriculum as described in the Mathematics Framework (1992) is to develop mathematical power. Mathematical power is defined as the ability to discern mathematical relationships, reason logically, and use mathematical techniques effectively. Mathematically powerful students think and communicate, drawing on mathematical ideas and using mathematical tools and techniques (Mathematics Framework, 1992). The development of mathematical power will enable the student to function effectively in situations outside the classroom. Complex, ambiguous problems are encountered in real life situations. The individual facing this type of problem cannot rely on a structured set of rules or procedures to follow as a means for solution. The individual may have to define the problem, find several solutions, and revise those solutions over a prolonged period of time. Mathematical power, rather than competence in rote calculations, will enable the individual to eventually arrive at a successful solution that has meaning for the individual.

Mathematical power is a concept that has four dimensions in relation to its role as the primary goal of an ideal mathematics curriculum. Expectations for students must change dramatically if mathematical power is to be
attained. Students will be expected to work on investigations rather than merely solve problems provided by the teacher. As stated in the Quality Criteria for Middle Grades (1990), students are to work on individual and group projects, some of which the students develop themselves.

As students work on projects, they utilize the first dimension of mathematical power, mathematical thinking. Mathematical modes of thought include modeling, optimization, symbolism, inference, logical analysis, and abstraction (National Research Council, 1989). Mathematical thinking has also been described as reasoning, problem solving, and making connections (NCTM, 1989). Although different terms can be used to describe mathematical thinking, it is evident that such thinking is higher-order in nature. Higher-order thinking has been described by a psychologist, Lauren Resnick, in Education and Learning to Think (1987) as having some of the following qualities. Higher-order thinking is rigorous, complex, uncertain, and nonalgorithmic (the steps are not known in advance). It results in multiple solutions involving nuanced judgements, and multiple, conflicting criteria. Higher-order thinking also involves imposing meaning on apparent disorder and self-regulation, where the learner, rather than someone else, controls the process.
The dimension of mathematical thinking will greatly challenge students and will come about only through a long process of continual gradual change.

Mathematical communication is the second dimension of mathematical power that will alter the traditional expectations for both students and teachers. According to Caught in the Middle: Educational Reform for Young Adolescents in California Public Schools, a 1987 publication of the California Department of Education, the capacities for critical thought and effective communication should be developed in every middle grade student. Students often clarify their thinking through communication. Although not all mathematical explanations by students are equal in quality, a teacher who has a genuine commitment to communicate with students guides them into a highly complex thinking activity (Cobb et al., 1991). Students not only communicate with teachers, but also share their thinking with peers, parents, and other adults.

The Mathematics Framework (1992) notes that some of the many modes of communication include informal conversations, written text, diagrams, symbols, numbers, graphs, tables, models, and algebraic expressions. An important aspect of communication is that feedback allows students the opportunity to obtain useful suggestions and
information that will enable them to revise and improve their work. Communication also helps teachers assess their students' thinking and grasp of important concepts. Results of research in this area suggest that when teachers understand their students thinking they alter their instruction and classroom practices to better accommodate strategies that enable students to solve problems more accurately (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). In addition, a teacher's interactions in the exploration of ideas establishes concepts as acceptable and plausible, leading students to the construction of the knowledge intended (Driver, Asoko, Leach, Mortimer, & Scott, 1994). Communication, therefore, is important for a variety of reasons, and teachers will need to become adept at asking leading questions and listening critically.

The third dimension of mathematical power is that of mathematical ideas. The Mathematics Framework (1992) describes mathematical ideas as the content, or specific subject matter, of the mathematics curriculum. The other dimensions of mathematical power seem to emphasize how concepts are taught, with mathematical ideas emphasizing what is taught. The concept of mathematical ideas is described in two ways. One way is in reference to the different strands of mathematics. The strands of mathematics include functions, algebra, geometry,
statistics and probability, discrete mathematics, measurement, number, and logic and language. By drawing on the eight strands for a year's course of study, a broad base of knowledge and experience is assured. Every student at every grade is to receive instruction in each of the strands. This equity of instruction is to be provided in heterogeneous groups. Ability grouping tends to relegate lower ability students to computational strands only. In addition, the research of Mergendoller and Mitman (1992) indicates that the effects of ability grouping on achievement is so small that it is meaningless in practical terms.

Another aspect of the dimension of mathematical ideas is that of unifying ideas. These deep, pervasive concepts are found throughout the various strands. They include ideas such as patterns and proportionality. They are abstract and underly and unify the different strands. These ideas are to be developed throughout all the years of instruction.

The fourth dimension of mathematical power as described in the Mathematics Framework (1992) is that of mathematical tools and techniques. In order to achieve wholeness and complete work, students must put their ideas and thinking into practice. In order to accomplish this objective, students may depend on methods such as
recognition of patterns and relationships through the use of tables, illustrations, graphs, formulas, or spreadsheet software. Students may decide to use techniques such as models, measurement or counting tools, or memory aids such as graphs, tables, and calculators. An important aspect of this dimension of mathematical power, whether it be addressed through the use of paper and pencil or the computer, is that the proper tool or technique be used to accomplish the particular task at hand. Teachers can engage students in active learning by employing tools such as manipulatives and computer based activities (Ventura, 1990).

Goals that Support Mathematical Power

In addition to the four dimensions of mathematical power, three goals for students that support mathematical power are described in the Mathematics Framework (1992). The goals that support mathematical power are the ability to work both collaboratively and independently, a positive disposition toward mathematics, and a knowledge of mathematics' connection to history and society.

The traditional way students have been assigned work in mathematics has been to complete a set of problems in isolation without interacting with other students. This goes contrary to the manner in which students will be required to work in most future occupations. People will
need to develop skills that will allow them to work collaboratively. In addition to the development of social skills, greater learning of mathematics can take place as students share points of view and strategies. As stated in Caught in the Middle (1987), a student has many more opportunities to respond to questions as a member of a small group than while working independently. In addition, students who are made aware of what to look for in other students' comments and learn how to give constructive suggestions, experience more frequent, higher quality feedback which is an effective technique of instruction.

The results of one study confirm the usefulness of working in cooperative groups. Nattiv (1994) found a strong positive correlation between achievement and students engaging in both giving and receiving help from peers in cooperative groups. Students of all ability levels benefited from helping behaviors. Student collaboration may also have benefits for the teacher. Energy that would once have been manifested in disruptive behavior can be directed toward aspects of learning such as communication and problem solving. The Mathematics Framework (1992) also points out that students become more responsible for their own learning which decreases the role of the teacher as the only authority or resource. The teacher remains responsible as a facilitator for guiding
and advising students (Mathematics Framework, 1992).

The second goal for students that supports mathematical power is that of a positive disposition toward mathematics. The Mathematics Framework (1992) characterizes students with a positive disposition toward mathematics as persistent, adventurous, curious, and confident. These feelings are fostered by teachers who carefully guide their students through purposeful, challenging assignments without giving them the answers. Students are encouraged to experiment and make several attempts at a solution. This helps students to eventually come to view mathematics as a way of making sense of real life situations and achieving their own purposes through mathematics.

The third goal for students that supports mathematical power is that of establishing a connection between mathematics and history and society. The Mathematics Framework (1992) states that all students, through the introduction of new materials, should be able to connect their own culture and individual perspectives with the continuing development of mathematics. This can be accomplished by providing examples from different cultures, and different historical periods, of ways people have used mathematics as part of their daily lives. Students should be able to recognize the role of diverse groups and
individuals in the development of mathematics. By relating these same perspectives on mathematics to the present day and to our culture and society, students gain an increased awareness of the role of mathematics in modern life and today's careers.

Implementing Mathematical Power

The Mathematics Framework (1992) has set forth four dimensions of mathematical power and three goals for students that support mathematical power. They have not been implemented in most classrooms as it is very difficult to reach all elementary school teachers (Hatfield & Price, 1992). Even the most thoroughly planned efforts at national reform have failed (Hatfield & Price, 1992). This suggests that it will be a very difficult task for teachers to change their own and their students' attitudes and expectations for the study of mathematics as outlined by the Mathematics Framework (1992).

Research has found that teachers who attempt to have children accept obligations that differ from those of traditional mathematics instruction have been successful (Nicholls, Cobb, Wood, Yackel, & Patashnik, 1990). It has been necessary, however, for the teacher to develop nontraditional interactions with students in order to accomplish this (Nicholls et al., 1990). The nontraditional interactions successful teachers adopted
include taking students' mathematical interpretations seriously and resisting the impulse to guide students to a predetermined solution (Nicholls et al., 1990). These findings seem to support the recommendations of the Mathematics Framework (1992).

The expectations for student work include the completion of projects of longer duration and greater involvement and personal responsibility than has traditionally been required. These projects are to focus students on thinking in ways that are different from the traditional drill and practice type of assignment. The mathematics model curriculum, as described in the Mathematics Framework (1992) will:

1. Foster a common expectation of quality work.
2. Prepare students for complete work.
3. Give the student the time and tools they need to do that work.
4. Require that the mathematical work be done according to a comprehensible standard.
5. Help students develop a habit of draft, feedback, and revision.
6. Provide tasks worthy of quality large-scale work. (p. 26)

Reform of the mathematics curriculum raises some important issues regarding assessment. One issue involves
developing new ways of evaluating student work and understanding as they relate to normal instruction. As the nature of student work changes, so must the criteria for evaluating that work change. The *Mathematics Framework* (1992) recommends that authentic assessment, or performance assessment be used. This type of assessment takes place as students participate in learning activities, rather than by stopping and taking a test. In this way, assessment is integrated with normal instruction (*Mathematics Framework*, 1992). Three alternative forms of assessment include open-ended tasks, observation of student work, and student portfolios (*Mathematics Framework*, 1992). Other possible assessment techniques include journals, class presentations, and written reports (Hatfield & Price, 1992). Teachers will be involved in a greater variety of assessment activities than they have been traditionally. Correcting computational work for accuracy will be replaced by seeking evidence of the proper use of mathematical thinking, ideas, communication, and tools and techniques.

A second major issue regarding assessment has to do with the role of standardized tests in assessing school-wide achievement. Traditional achievement tests and reformed methods of mathematics instruction are not in close alignment (Nicholls et al., 1990). It would seem that many mathematics teachers would be concerned that
their students achieve a high level of proficiency in mathematics as measured by standardized achievement tests. This suggests that if instructional practices are to change, the standardized tests must also change to measure the new desired outcomes of mathematical power.

Initially, there is likely to be a great deal of difficulty in implementing the goals of the model mathematics curriculum as defined by the Curriculum and Evaluation Standards for School Mathematics (1989) and the Mathematics Framework (1992). However, when one considers the changes in society, most educators would agree that such reform is a worthwhile goal. It is important to note that new theories regarding how children learn support the reform movement in mathematics education. Evidence of the effectiveness of a reformed mathematics curriculum can be found in several sources. These studies will be summarized and related to the model mathematics curriculum previously described.

Constructivist Learning

Before teachers embark on the road to implementation of the model mathematics curriculum, some evidence should be brought to light that justifies the expenditure of the considerable resources necessary to complete such a journey. Teachers will be willing to make the effort to bring about reform if it is shown that their hard work will
pay off for their students in greater understanding of concepts and their applications. The way that children learn and how it relates to the implementation of the model mathematics curriculum is an important topic as it lends support and provides a rationale for carrying out those reforms.

Research by Jean Piaget, and others, indicates that children are active learners and create their own understanding of the world (Kamii, Lewis, & Jones, 1991). This constructivist view of learning holds that children acquire number concepts through a process of formation that occurs from within rather than from internalizing the external environment as has been traditionally believed (Kamii et al., 1991). Constructivism has become a widely accepted view in recent years (Cobb, 1994).

According to Piaget, logico-mathematical knowledge involves recognition of a relationship between observable characteristics of two objects (Kamii et al., 1991). For example, a blue bead and a red bead are different. The fact that a difference exists is not a part of either bead in itself. It is only in relating the two, within an individual's thought process, that a difference is recognized. This logico-mathematical knowledge, as it relates to number sense, usually begins to develop by the age of five or six. This construction comes from that
person's natural ability to think. The natural development of logico-mathematical thinking and number sense is an ongoing process and takes many years before complex relationships are constructed (Kamii et al., 1991).

A constructivist method of teaching arithmetic has been developed that coincides with the model mathematics curriculum in several ways. The teaching of algorithms, in a rote manner, is rejected. Rote learning is in conflict with the way children naturally think (Kamii et al., 1991). Performing rote computation devoid of meaning deprives children of the opportunity of building their own number sense. An understanding of place value can best be arrived at by allowing children to develop their own processes for doing arithmetic. Only after number sense and place value have been constructed within each child's thought process can algorithms be used efficiently and in a way that does not negate or obscure meaning (Kamii et al., 1991).

The encouragement of children to construct their own thinking leads to the development of number sense, mental arithmetic, and estimation. Although algorithms can still be useful to someone possessing a thorough knowledge of place value, in today's world calculators will most often be used to perform computation. If number sense is well developed, errors from striking the wrong key can be
recognized immediately. The ability to estimate and reason with numbers will be more important than rapid and accurate calculation through the use of paper and pencil. Research has also found that it is not necessary to delay the introduction of word problems until computation has been mastered (Carpenter & Moser, 1984).

Although constructivism has become widely accepted, there is an ongoing debate among researchers as to the role the process of enculturation plays in learning. Cobb (1994) argues that individual construction and the process of enculturation can both be at work in mathematical learning. A point upon which both sides appear to agree is the vital role of activity in mathematical learning and progress (Cobb, 1994). Thus, proponents of constructivism stress students' sensory-motor and conceptual activity (Cobb, 1994).

Whole Concept Learning

Another way to implement the model curriculum and make mathematics interesting and meaningful is the whole concept approach. It is derived from, and shares many principles with, the whole language curriculum. Whole concept mathematics stresses the importance of real-life situations, integration with other disciplines, holistic concepts, mixed ability grouping, and cooperative learning (Brown, 1991). Cobb (1994) notes that interactions with
others is frequently a part of the learning process which supports the efficacy of cooperative learning.

Whole concept learning rejects the traditional emphasis on rote learning of algorithms, sequential and isolated learning, and skill attainment. Rather, the emphasis is on understanding content, applications, organization of knowledge, interrelationships, and problem solving (Brown, 1991). A blind reliance on developmental stages and chronological age of students as determinants of content of the curriculum is also criticized. It is believed that a reliance on stages can have a negative effect on expectations regarding a student's ability to learn (Brown, 1991). It is deemed more effective to determine how students organize existing knowledge and match instruction to that mental framework (Brown, 1991).

Consistent with the constructivist view, whole concept mathematics recognizes the ability of children to create knowledge from within. The teacher's role is to provide experiences and situations that lead to deep active thought regarding mathematical concepts. Children are to address the situations presented through the use of diverse modes of expression. A major goal of this type of instruction is to encourage students to explore concepts and offer opinions in ways acceptable to the group as a whole (Brown, 1991). It is important for students to explore ways to
test their ideas (The Cognitive and Technology Group at Vanderbilt, 1991).

In summary, some major principles emerge from research literature that provide a rationale for the recommendations of the Mathematics Framework (1992). Paramount among them is the idea that children naturally develop concepts about the world around them, including mathematical concepts, from within rather than through an external source. In order to develop mathematical concepts fully, students must be challenged with activities that have meaning for them. Hopefully, implementation of a rich mathematics curriculum that includes all four dimensions of mathematical power will enable students to become fully prepared for the challenges of the information society of the twenty-first century.

Challenges Regarding Implementation

There are many changes that need to be instituted if the curriculum described in the Mathematics Framework (1992) is to be implemented. A major challenge concerns the perception teachers have of how mathematics should be taught. Most teachers at the elementary school level have taught mathematics as a series of steps to follow in order to arrive at the correct answer (Etchberger & Shaw, 1992). One of the greatest challenges is to motivate teachers to reflect on whether they are preparing children to be
problem solvers or merely programming them to get the right answer (Etchberger & Shaw, 1992). It has, however, been extremely difficult to reach teachers. For the past thirty years, a wide variety of curricular and instructional reforms have failed (Hatfield & Price, 1992). Teachers are the key to providing a reformed instructional setting (Frye, 1989). One way to encourage reform may include an increased focus on teacher inservice training emphasizing mathematics curriculum implementation. Once trained and empowered to carry out reform, teachers may be more likely to become essential developers and builders of curriculum (Frye, 1989).

Another challenge that needs to be overcome lies in the area of textbooks. Teachers attempting to find mathematics materials or textbooks oriented toward constructivist learning through the use of meaningful activities are often frustrated (Etchberger & Shaw, 1992). Most textbooks follow a constraining outline that adheres to a strict scope and sequence (Etchberger & Shaw, 1992). Hopefully, textbook publishers will be responsive to educators requesting materials designed to implement the reforms recommended by the Mathematics Framework (1992). Teachers must also be encouraged and empowered to go beyond the constraints of the textbook curriculum. If appropriate materials are provided on a grade level basis, perhaps
teachers will be more likely to introduce innovative instructional strategies.

Teachers must also be assured that increased emphasis on standards for achievement not result in standardized tests designed to measure only traditional skills. Standards must be agreed upon and assessment geared toward those standards. The goals and content of the reformed mathematics curriculum should be reflected in all tests designed to measure achievement in mathematics (Gawronski, 1991).

Another area that is a challenge for implementation of the model mathematics curriculum is that of leadership. Reforming a school's mathematics program will not be possible unless teachers become involved in leadership roles (Bruni, 1991). Teachers at each school must emerge who are willing to take professional responsibility for guiding their grade level programs toward the recommended reforms (Bruni, 1991). An enthusiastic, empowered teacher can be a significant force in leading a process of gradual change. Three phases can be envisioned. They include "unfreezing", in which people are encouraged to become open to change. The second phase involves implementing change. The third phase involves "refreezing" in which the change becomes institutionalized as support is lent to those involved to make certain the changes are permanent.
(Hatfield & Price, 1992).

There are many opportunities to better serve the students of today through reform of the mathematics curriculum. However, support must be provided by the community and school administration if reform is to occur (Lappan & Ferrini-Mundy, 1993). The establishment of rich, engaging mathematical contexts that provide opportunities for developing mathematical power is also essential. It is hoped that the development of a teacher handbook that provides such a curriculum resource will act as a catalyst for implementation of the recommendations of the Mathematics Framework (1992) at the sixth grade level.
References


Bruni, J. V. (1991). One point of view: We must have "designated math leaders" in the elementary school! Arithmetic Teacher, 39(1), 7-9.


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Routledge & Kegan Paul.


Mathematics Handbook
Curriculum Implementation

This handbook consists of a variety of lesson plans designed to help teachers implement the model mathematics curriculum as described in the Mathematics Framework (1992). The lesson plans provided are intended for use at the sixth grade level. Hopefully, they will prove to be a practical and effective supplement to textbooks currently in use and will provide activities corresponding to concepts taught throughout the school year.

The purpose of each lesson is to teach at least one mathematical idea in a way that implements the goal of mathematical power for all students. The activities will include the use of manipulatives aimed at helping students arrive at deep understandings of mathematical concepts. Projects will attempt to provide students with the opportunity to work both independently and cooperatively on meaningful applications.

Teachers are encouraged to try new techniques and thus enable their students to increase their proficiency in mathematics. Hopefully, use of the activities and projects provided will lead to intellectually stimulating and enjoyable experiences for both teachers and students and provide a first step toward implementation of the model mathematics curriculum.
Make a Million

**Topic:** Place value

**Objective:** Students will understand place value and construct an image of a million.

**Materials:** Each group of four students will need a set of Base Ten Blocks consisting of 1 thousands-block, 1 hundreds-block, 18 tens-blocks, and 37 ones-blocks. Each group will also need a large sheet of paper, a meter stick and a thin felt tip pen.

**Procedure:**

Step 1: Introduce the problem by telling students that although they do not have a million blocks in their group's set, they can use the blocks to help them make a drawing of a model of a million blocks.

Step 2: Ask students what they would need to know to make a model of a million to show someone. They may respond with ideas such as needing to know the size of each block and their arrangement.

Step 3: Explore the Base Ten Blocks with the students by having each group put them into like groups and in proper order. Review the names of the groups and the relationship between them. Students should note that each block is ten times greater than the next smaller block.

Step 4: Challenge each group to use the blocks as a basis to figure out what a million blocks would look like. Ask
students to describe the model's shape and dimensions. Ask them how they would know these characteristics.

Step 5: Have each group record their solution through a drawing and an accompanying written description. Students must show their reasoning through a series of steps that develops the meaning of each place value from the ones place through the millions place.

Step 6: The lesson may be extended by having students construct their model using cardboard and tape or by asking them to describe a billions block.

Check Writing

Topic: Numbers as words

Objective: Students will write numbers using words and subtract using money.

Materials: Each student will need newspapers with advertisements, check and register worksheets, and a pen.

Procedure:

Step 1: Introduce the lesson by telling students that the proper use of a personal checking account is a safe and convenient way to pay bills. Demonstrate how to write a check and keep a running balance by making a transparency of the check and register worksheet and filling out the appropriate information using the overhead projector.

Step 2: Tell students to imagine they are buying five gift items for people they know. Each gift will be paid for with a separate check. The students are to "shop" for their gift items by finding them in newspaper advertisements. They are to write a check for the store and price found in the advertisement. Students are to start with a balance of $350.00 and record each check on the register worksheet. They are to subtract the amount of each check so as to keep an accurate balance.

Step 3: After students have completed their checks and register, they are to exchange materials with a partner and check each other's work for completeness and accuracy.
(Your Name & Address)  

123

Pay to the

__________________________

Order of ________________________ $ ____________

__________________________ Dollars

For ________________________

__________________________

(Your Name & Address)  

124

Pay to the

__________________________

Order of ________________________ $ ____________

__________________________ Dollars

For ________________________

__________________________

<table>
<thead>
<tr>
<th>Number</th>
<th>Date</th>
<th>Description</th>
<th>Amount</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Beginning</td>
<td>$350.00</td>
<td></td>
</tr>
</tbody>
</table>

38.
Handshakes

**Topic:** Patterns and counting

**Objective:** Students will recognize the use of tables, simulations, and drawings as effective counting techniques.

**Materials:** Paper and pencils are needed for this activity.

**Procedure:**

Step 1: Group the students into small groups so that they will be able to discuss strategies and solutions. Students must verify their results, rather than the teacher.

Step 2: Remind students that it may be useful to use a table, simulation, or drawing to solve some problems. Have each group appoint a recorder and a reporter as they will be asked to share their solutions with the class.

Step 3: Write the following question on the chalkboard.

How many handshakes will occur if every one of 15 people shakes hands with each of the others once?

Step 4: Allow students to explore various strategies for solving the problem.

Step 5: Have a reporter from each group share their group's thinking and record of their activities.

Step 6: The lesson may be extended by asking the students to generalize the problem for any number of people. This may result in the students arriving at the following formula: \( h = \frac{n(n-1)}{2} \) where \( h \) is the number of handshakes and \( n \) is the number of people.

40.
Source: National Council of Teachers of Mathematics.
Visualizing Multiplication

**Topic:** Two-digit by two-digit multiplication

**Objective:** Students will arrive at a deeper understanding of multiplication and place value.

**Materials:** Each group of four students will need a set of base ten blocks to include 2 hundreds-blocks, 25 tens-blocks, and 120 ones-blocks. They will also need a large sheet of paper, a pencil, and a ruler.

**Procedure:**

Step 1: Introduce the lesson by telling the students that they are going to use blocks to represent a multiplication problem and record their results by making a drawing and an accompanying written explanation.

Step 2: Show this problem to the class: \(18 \times 14 = n\).

Step 3: Direct the students to work in their groups to arrange the blocks in a way that clearly represents the multiplication problem. Remind them to record their results by making a drawing with a written explanation.

Step 4: Extend the lesson by having the students show the product using the fewest number of pieces possible and record their results as before. They should have 2 hundreds-blocks, 5 tens-blocks, and 2 ones-blocks for a total of 9 pieces.

Step 5: On their recording paper, have the students solve the problem using numbers and write the solution.
Digit Combinations

**Topic:** Exploring numbers and multiplication

**Objective:** The students will deepen their understanding of place value, multiplication, and number sense through this open-ended activity.

**Materials:** One calculator for each student, paper, and a pencil will be needed for this activity.

**Procedure:**

Step 1: Challenge students to explore the following situation using a calculator to perform their computation. Remind them to record each attempt at a solution with their pencil and paper.

Step 2: Tell the students to select five different digits. They are to use the five digits to form a two-digit and a three-digit number so that the product of the two numbers is the largest possible. Then they are to find the arrangement that gives the smallest possible product.

Step 3: Students can be further challenged to generalize their solution to any five digits and any number of digits.

Step 4: Give the students the opportunity of sharing their results with the class.

Stating Equality Relationships

**Topic:** The distributive property

**Objective:** The students will recognize equality relationships and develop an understanding of the distributive property.

**Materials:** Each group of two students will need a sheet of one centimeter square grid paper, scissors, paste, paper, and a pencil.

**Procedure:**

Step 1: Direct each group to cut out two identical rectangles from the grid paper. One is to be cut into two smaller rectangles, the other is to be left alone. Paste the rectangles on the paper.

Step 2: Tell the students to count the number of squares in each rectangle.

Step 3: Challenge the students to recognize and record as many different ways of writing equality statements using the relationships between the rectangles as possible.

Step 4: After the students have had an opportunity to explore the various relationships, write the distributive property on the chalkboard. Give an example using numbers.

Step 5: Allow one student from each group to go to the chalkboard and write one relationship they discovered that they think illustrates the distributive property. Discuss the different ways of expressing the relationship involved.
Border Problems

Topic: Mathematical expressions

Objective: Students will recognize patterns and derive mathematical expressions to describe them.

Materials: Each pair of students will need square grid paper, markers, scissors, paper, and a pencil.

Procedure:
Step 1: Cut out an 8 by 8 square from grid paper large enough for the entire class to see. Color the border with one of the markers.

Step 2: Show the students the square. Ask them how many unit squares it has (8 X 8, or 64). Ask them how many unit squares are on the border (28). Ask for different mathematical expressions for describing the number of unit squares on the border.

Step 3: Write some of the student responses where everyone can see them. Some possible expressions include:

\[
\begin{align*}
8 + 8 + 6 + 6 \\
(2 \times 8) + (2 \times 6) \\
7 + 7 + 7 + 7 \\
4 \times 7 \\
8 + 7 + 7 + 6
\end{align*}
\]

Step 4: Have the students cut out a 9 by 9 square and color the square to illustrate one way of thinking about the border. They may use more than one color to accomplish
this. One possibility, which would result from coloring
the four corners one color and the four segments between
them another color is \((4 \times 1) + (4 \times 7) = 32\).
Step 5: Assign groups to explore different size squares to
cut out, color, and write mathematical expressions to
describe the border. Ask the students to write
mathematical expressions to describe the interiors of the squares also.
Step 6: Ask the students how many unit squares are in a square grid if the interior of the grid is 81 unit squares. Ask how many unit squares the square has on its border.
Step 7: Ask the students if the number of unit squares of a square grid can be equal to 40. Ask them to explain their reasoning.
Step 8: Guide students in discovering the relationships between the number of unit squares on the border, the interior, and in the entire square. One example is the total number of unit squares is equal to the sum of the border unit squares and the interior unit squares.
Step 9: The lesson may be extended by describing relationships for an \( n \) by \( n \) square. They may include:
\[ 4 + [4 \times (n - 2)], n + n + (n - 2) + (n - 2), (n - 1) \times 4, \] or \( 4 \times n - 4 \).
Step 10: The lesson may be further extended by having the
class find expressions to describe the number of unit squares in the interior of an n by n square once the boundary is known. Guide the students to discover this generalization: In an n by n square, there are \( n \times n \) unit squares with \( 4 \times (n - 1) \) of these squares on the border and \( n \times n - 4 \times (n - 1) \) in the interior. The expression \((n - 2) \times (n - 2)\) also describes the number of unit squares in the interior.

Playing Card Activity

**Topic**: Order of operations

**Objective**: Students will write numerical expressions for a given number using the order of operations.

**Materials**: Each pair of students will need playing cards, paper, and a pencil.

**Procedure**:

*Step 1*: Each pair of students will begin by writing down all they know about the order of operations.

*Step 2*: The teacher will explain the numerical value of the cards by posting the following.

<table>
<thead>
<tr>
<th>Card</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ace</td>
<td>1</td>
</tr>
<tr>
<td>2 - 10</td>
<td>face value</td>
</tr>
<tr>
<td>jack</td>
<td>11</td>
</tr>
<tr>
<td>queen</td>
<td>12</td>
</tr>
<tr>
<td>king</td>
<td>13</td>
</tr>
</tbody>
</table>

*Step 3*: The students turn over six cards. The numerical value of the sixth card is the target number. The other five cards are to be used to generate the target number.

*Step 4*: The students arrange the cards so as to derive a numerical expression equal to the target number. Students record their expression. This is one example.

- First five cards: 5, 4, jack, 2, 2
- Sixth (target) card: 5

50.
Expression:  \( 5 \times 2 + 4 - 11 + 2 = 5 \)
\( 10 + 4 - 11 + 2 = 5 \)
\( 14 - 11 + 2 = 5 \)
\( 3 + 2 = 5 \)
\( 5 = 5 \)

Step 5: Students shuffle the cards and repeat the activity. They may want to determine their success ratio after a given amount of time.

Squares and Cubes

**Topic:** Numbers that are squares and cubes up to 100

**Objective:** Students will develop a mental image of numbers that are squares and cubes up to 100.

**Materials:** Each group of four students will need 100 small cubes, a large piece of paper, a ruler, and a pencil.

**Procedure:**

Step 1: Introduce the lesson by telling the students that when a number is multiplied by itself the product is called a square. Also state that if a number is multiplied by itself and then by itself again the product is called a cube.

Step 2: Show the students that numbers that are squares can be found by arranging the small cubes to form a square. For example, 9 is a square because 9 cubes can be arranged to form a square. The multiplication fact $3 \times 3 = 9$ describes that square.

Step 3: Show the students that numbers that are cubes can be found by arranging the small cubes to form a cube. For example, 8 is a cube because 8 cubes can be arranged to form a cube. The multiplication fact $2 \times 2 \times 2 = 8$ describes that cube.

Step 4: Tell each group that they are to work together to find all the numbers from 1 to 100 that are squares or cubes and write the multiplication facts that describe
them. They are to record their results by drawing a picture to go along with each square or cube. Suggest that students first write down all the numbers from 1 to 100 in a chart that is a 10 by 10 grid.

Step 5: When the students have found all the squares and cubes up to 100, have them write a summary of their work and report to the class.

Visualizing Division

**Topic:** Two-digit by two-digit division

**Objective:** Students will arrive at a deeper understanding of the concept of division.

**Materials:** Each group of four students will need a set of base ten blocks to include 1 hundreds-block, 18 tens-blocks, and 120 ones-blocks. Each group will also need a large piece of paper, a ruler, and a pencil.

**Procedure:**

Step 1: Introduce the lesson by telling the students that they are going to use blocks to represent a division problem and record their results by making a drawing with an accompanying written explanation.

Step 2: Show this problem to the class: \( 158 + 14 = n \).

Step 3: Direct the students to work in their groups to arrange the blocks in a way that clearly represents the division problem. Remind them to record their results by making a drawing with a written explanation.

Step 4: On their recording paper, have the students solve the problem using numbers and write the solution.


54.
Going Shopping

**Topic:** Problem solving

**Objective:** Students will use basic operations and the data provided to create and solve story problems.

**Materials:** Each student will need a calculator, an index card, paper, a pencil, and a copy of the following chart.

### Summer Sporting Equipment

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skateboard</td>
<td>$96.59</td>
</tr>
<tr>
<td>Skateboard wheels</td>
<td>23.50 (set)</td>
</tr>
<tr>
<td>Skateboard knee pads</td>
<td>6.25 (pair)</td>
</tr>
<tr>
<td>Skateboard arm guards</td>
<td>8.95 (pair)</td>
</tr>
<tr>
<td>Surfboard</td>
<td>181.50</td>
</tr>
<tr>
<td>Boogie board</td>
<td>79.95</td>
</tr>
<tr>
<td>Goggles</td>
<td>17.50</td>
</tr>
<tr>
<td>Wet suit</td>
<td>124.99</td>
</tr>
</tbody>
</table>

**Procedure:**

Step 1: Ask each student to write a story problem using the data provided. Encourage the students to be creative in setting up a situation and in the procedures required to solve the problem.

Step 2: When the students have finished writing their story problem, have them exchange papers and attempt to solve the problem they receive. Provide time for partners to clarify and revise the problem if necessary.
Step 3: Call on students to read the problems and discuss the process they used to solve the problem. Have other students discuss alternative strategies.

Step 4: Allow the students to establish a story problem file by having them write their problem on an index card.

Step 5: Have the students sort the cards by level of difficulty.

Step 6: The lesson may be extended by having students write in their math journals answers to questions such as these. What kinds of problems are difficult to solve? What makes a problem difficult? How can you tell when a problem is easy?

Step 7: This lesson can be repeated with the variation of having students bring in their own price data. They may wish to bring in items such as catalogs, menus, flyers, or newspaper ads.

Decimal Building Game

**Topic:** Decimal place value

**Objective:** The students will recognize the value of digits in the tenths and hundredths place. They will say and write decimal numbers in words and expanded form.

**Materials:** Each pair of students will need a die, paper, a pencil, and a set of base ten blocks consisting of 1 flat (whole unit), 10 rods (tenths), and 70 small cubes (hundredths).

**Procedure:**

Step 1: Introduce the lesson by telling students that they will take turns building and recording the names of decimal numbers.

Step 2: Explain that each team member will take a turn to roll the die twice. The first digit rolled will be the tens digit and the second digit rolled will be the ones digit of the number of small cubes (hundredths) to be counted out. For example, if a 5 is rolled and then a four is rolled, 54 small cubes will be counted out.

Step 3: Explain that after the small cubes are counted out, the player will place them on the flat block, which represents the whole unit.

Step 4: Direct the player to exchange every row of 10 small cubes on top of the flat for one rod, which represents one tenth.
Step 5: Tell the students that after the tenths have been built they are to combine them with the remaining small cubes and state the number name and value of each digit in words. For example, the player will say, "54 hundredths is 5 tenths and 4 hundredths."

Step 6: Finally, the player will write the decimal number in words and expanded form. For example, fifty-four hundredths; $0.54 = 0.5 + 0.04$. As students take turns, check for correct usage of word names and expanded form.

Match Up

**Topic:** Comparing decimal numbers

**Objective:** Students will be able to write comparison relationships using "less than" and "greater than" with decimal numbers to the hundredths place.

**Materials:** Each group of four students will need a set of base ten blocks consisting of four flats, 18 rods, and 18 small cubes. They will also need paper and pencil to record their answers.

**Procedure:**

Step 1: Write the numbers 2.34 and 2.61 on the chalkboard.

Step 2: Ask each group to represent the two numbers using the base ten blocks. The flats are ones, the rods are tenths, and the small cubes are hundredths. Students should show 2 flats, 3 rods, and 4 cubes for the first number and 2 flats, 6 rods, and 1 cube for the second number. A rod should be used in place of 10 cubes whenever possible.

Step 3: Guide the students to compare the two amounts of blocks by helping them to recognize that both sets of blocks have 2 flats, so the rods should be checked. The first number has 3 rods (tenths) and the second number has 6 rods (tenths), which is more. Students should observe that in the second number, the six rods (tenths) are worth more than the 3 rods (tenths) in the first number. They...
should also recognize that 9 is the maximum number of cubes that can be present in any number without converting them into a rod. Therefore, no comparison of the cubes needs to be made and the second number is greater than the first number.

Step 4: Write on the chalkboard: $2.34 < 2.61$. State in words, "Two and thirty-four hundredths of one is less than two and sixty-one hundredths of one. The phrase "of one" helps the students realize that both numbers are based on the same unit, or whole. Have students record the statement on their papers. Have the students record the statement $2.61 > 2.34$. Point out that the statement must be read from left to right and in words it says "two and sixty-one hundredths is greater than two and thirty-four hundredths."

Step 5: Write several pairs of decimal numbers on the board. Be sure to include pairs such as 1.8 and 1.75, 0.95 and 1.3, and 2.04 and 2. However, be careful not to assign numbers with more than four ones in each pair as there are only four flats in each set of base ten blocks.


60.
Visualizing Decimal Addition

**Topic:** Decimal place value and addition

**Objective:** Students will explore ways of representing decimal addition and add to their understanding of decimal place value.

**Materials:** Each group of four students will need a set of base ten blocks to include 1 thousands-block, 9 hundreds-blocks, and 9 tens-blocks. Each group will also need a large piece of paper, a ruler, and a pencil.

**Procedure:**

**Step 1:** Introduce the lesson by telling the students that they are going to use blocks to represent an addition problem and record their results by making a drawing with an accompanying written explanation. The problem will be one that involves finding the sum of decimal addends.

**Step 2:** Show this problem to the class: $3.42 + 1.31 = n$.

**Step 3:** Tell the students to work in their groups to represent the problem with the base ten blocks. They should use the blocks to find the solution to the problem.

**Step 4:** Discuss the different ways the groups used the blocks to make a model of the addition problem and find its solution. Some groups may use the flat block to represent one, other groups may use the large cube for one. Discuss how either way will model the problem correctly if the relationship between the blocks is consistent.
Step 5: List these problems on the chalkboard: 3.2 + 2.1, 11.3 + 3.2, 11.26 + 14.73, and 12.6 + 2.12. Direct the students to make a model of each problem using the base ten blocks and find the solution using the blocks. Remind students to record their work by making a picture with a written explanation for each problem.

Step 6: Share the students' work with the class after the groups have had adequate time to work out and record their solutions.

Visualizing Decimal Multiplication

**Topic:** Multiplication of decimal factors

**Objective:** Students will use words to state a decimal multiplication problem, model the problem, and gain a greater understanding of the concept of multiplication.

**Materials:** Each group of four students will need a set of base ten blocks to include 1 hundreds-block, 18 tens-blocks, and 37 ones-blocks. They will also need a large piece of paper, a ruler, and a pencil.

**Procedure:**

**Step 1:** Introduce the lesson by telling the students that they are going to use blocks to represent a multiplication problem and record their results by making a drawing with an accompanying written explanation. The problem will be one that involves finding the product of decimal factors.

**Step 2:** Show this problem to the class: $1.2 \times 0.3 = n$. Tell the students that one way of thinking about this problem is to start with one and two tenths and take it only three tenths of one time, which will result in a product less than 1.2. Another way of thinking about it would be to think of the problem as asking for the number that is three tenths of one and two tenths.

**Step 3:** Challenge the students to model the problem with base ten blocks and find the solution.

**Step 4:** After an adequate time, discuss with the class the
ways they modeled the problem. Guide them to the solution based on making 1.2 into tenths by exchanging the flat block for ten rods and placing one rod in each of ten groups and exchanging the two rods for twenty small cubes and placing two of the small cubes with each of the rods. Students should then take three of the groups so they have three rods and six small cubes which represents the solution of three tenths and six hundredths or 0.36.

Step 5: Write these problems on the chalkboard: 14 X 3, 1.4 X 3, 14 X 0.3, and 1.4 X 0.3. Direct the students to say these problems in words, model them with base ten blocks, and find the solution. Remind the students that they must record their work by making a drawing and written explanation for each problem.

Step 6: Share the students' work with the class after the groups have had adequate time to work out and record their solutions.


Multiplying with Decimals

**Topic:** Decimal placement in products

**Objective:** Students will use number sense to properly locate the decimal point in a product.

**Materials:** Each student will need a pencil and a copy of a worksheet having several multiplication problems involving decimal numbers.

**Procedure:**

Step 1: Tell students that an effective way of approaching a multiplication problem in which a decimal is present in one or more of the factors is to initially disregard the decimal points and perform the computation with the digits only. After the intermediary product is determined, test it for reasonableness by using number sense. If the product does not make sense according to their understanding of multiplication, locate the only place the decimal could reasonably belong.

Step 2: Do the following example for the students on the chalkboard or overhead projector: \(2.4 \times 1.50 = n\). First, view the problem as \(24 \times 15\). Determine the product mentally by thinking \(24 \times 15 = 240 + 120 = 360\). Then, test the answer for reasonableness by thinking back to the original problem which can be thought of as \(2.4 \ "taken" 1.5\) times. The decimal must be placed between the 3 and the 6 as 3.6 is the only logical choice which satisfies the
meaning of the original problem.

Step 3: Provide the students with problems such as these with which to practice.

\[
\begin{align*}
80 \times 0.3 &= \\
4.6 \times 200 &= \\
700 \times 0.5 &= \\
2.5 \times 300 &= \\
3.9 \times 20 &= \\
1.2 \times 120 &= \\
1,800 \times 0.03 &= \\
\end{align*}
\]

Factors

**Topic:** Finding factors of a number

**Objective:** Students will find all the factors of a number through the use of small cubes.

**Materials:** Each group of four students will need 112 centimeter cubes, a piece of paper, and a pencil.

**Procedure:**

Step 1: Explain to students that it is often useful when working with fractions to be able to determine the factors of numbers. Remind them that numbers that can be multiplied together to equal a number are factors of that number or product.

Step 2: Direct each group to find all the factors of 6 and record them on their paper. Arrange all 6 small cubes into the shape of a rectangle. Students may arrange the cubes to form a rectangle 1 by 6. Since $1 \times 6 = 6$, 1 and 6 are factors of 6.

Step 3: Ask the students if there is another way to arrange 6 cubes so as to form a rectangle. They may respond by stating that a rectangle 2 by 3 can be formed. Since $2 \times 3 = 6$, 2 and 3 are also factors of 6. Students should record all the factors of 6 as 1, 2, 3, and 6 and include a drawing with a written explanation as part of their record.

Step 4: Direct the students to find all the factors of 9.
They should form rectangles that are 1 by 9 and 3 by 3. They should record all the factors of 9 as 1, 3, and 9.

Step 5: Provide several more examples for students. The factors of 12, 14, 15, 18, 35, 39, and 41 are some they may wish to explore.

Properties of Nine

Topic: Using fingers as a calculator

Objective: Students will appreciate an amusing and useful way of approaching multiplication by nine.

Materials: Students will need their fingers and a list of the "nine times facts" for this activity.

Procedure:
Step 1: Introduce the lesson by telling students that they can use their fingers as a calculator and a convenient way of checking their multiplication facts for the products of 9 X 1 through 9 X 10.

Step 2: Have students spread the fingers of both hands out in front of them with the palms down.

Step 3: Direct the students to mentally number their fingers from 1 to 10 from left to right.

Step 4: Explain that to find 9 X 7 they can lift finger number 7. There are 6 fingers to the left of the lifted finger and 3 to the right. These two digits form the number 63 which is the product of 9 X 7.

Step 5: Have the students experiment with all the "9 times facts" through 9 X 10. Note that there are 0 fingers to the right of finger number 10.

Restaurant Menus

**Topic:** Computation with money

**Objective:** Students will learn an important application of addition and subtraction of decimal numbers and finding the percent of a number.

**Materials:** Each student will need a restaurant menu, paper, and a pencil.

**Procedure:**

Step 1: Plan in advance for this lesson and ask students to save and bring to class restaurant menus. They may find them in flyers restaurants mail to their homes or take them, with permission, from local restaurants.

Step 2: Give students a designated amount of play money, perhaps $7.00 per person, for each family member. The amount can be adjusted to match the price range on the menu they are to use.

Step 3: Challenge the students to use their budget and imaginations to take their family out to eat at the restaurant for which they have a menu.

Step 4: Remind the students that they must total their menu selections and then multiply by the decimal form of the sales tax in their area and add that amount to the bill. They must also figure the gratuity at 15% of the bill before the tax was added and include that as part of the expense of eating at the restaurant.
Step 5: The students are to record the items they select and the computations they perform to arrive at a final bill. They are to share their results with the class.

Step 6: Discuss related issues such as how restaurant meals may affect a family's budget and the need to leave an appropriate gratuity for the server.

Metric Measurement

**Topic:** Measuring with metric units

**Objective:** Students will gain a mental image of the units used in the metric system of measurement.

**Materials:** The students will work at various stations which will need these items: a metric measuring tape, a class set of large centimeter grid paper, polystyrene peanuts, spoons, a graduated cylinder, a triple beam balance, recording sheets, pencils, and a variety of common items to weigh.

**Procedure:**

Step 1: Explain to the students that they will engage in a series of competitions involving activities using metric units of length, volume, and weight. They will be assigned to equal groups and rotate through a variety of events.

Step 2: Remind the students to accurately record their scores so a winner can be determined at the end of the competition.

Step 3: Students work their way through events such as "leaping lizards" where they measure their standing broad jump, "big foot" where they determine the area of their foot using grid paper, "happy face" where they measure the size of their smiles, "heads up" where they measure the circumference of their head, "peanut toss" where they throw polystyrene peanuts, "splashdown" where they carry
spoonfuls of water in a race to fill graduated cylinders, and "tip the scales" where they estimate and measure weights of given items.

Step 4: When the students have completed the activities, assign a group of students to evaluate the scores of each competitor and determine a winner.

Fraction Mobiles

**Topic:** Equivalent fractions

**Objective:** Students will recognize equivalent fractions.

**Materials:** Each group will need one metal hanger, eight bamboo skewers, string, and a poster board fraction set consisting of a 6 by 8 piece (one whole), four one-half pieces, six one-fourth pieces, eight one-eighth pieces, six one-sixth pieces, and six one-third pieces. Include both rectangles and triangles to represent the fractions.

**Procedure:**

Step 1: Tell the students that they will be creating a mobile with the theme "equivalent fractions".

Step 2: Direct the students to find the largest piece in the fraction packet and label it "1".

Step 3: The students are then to label all the other pieces using the appropriate fraction name.

Step 4: Each group is to create a mobile using the hanger as a frame, bamboo skewers as horizontal pieces, string, and fraction pieces. The mobile should be visually pleasing and balanced. Balance can be achieved by hanging equivalent fraction pieces from opposite ends of the bamboo skewers.

Step 5: Display the student work in the classroom.

Rapidly Multiplying by 5

**Topic:** An amusing trick for rapidly multiplying by 5

**Objective:** The students will develop number sense.

**Materials:** Students will need a worksheet of multiplication problems using 5 as a factor and a pencil.

**Procedure:**

Step 1: Introduce the lesson by telling the students that there is a way of multiplying numbers by 5 that is fun and faster than they can do normally.

Step 2: Explain that to multiply a number by 5, all one need do is divide it by 2 and add zeros necessary for the answer to make sense.

Step 3: Illustrate the trick with the example 24 X 5 by following these steps.

```
24 X 5
24 ÷ 2 = 12
12 is too small
add a 0 to 12
120 is the answer
```

The method requires students to estimate the product and students are thus motivated to develop number sense.

Step 4: Provide another example.

```
46 X 5
46 ÷ 2 = 23
23 is too small
```

75.
add a 0 to 23
230 is the answer

Step 5: Guide students through this extension of the lesson. It requires more methods of working with numbers.

35 X 50
35 X 5 (disregard 0)
35 ÷ 2 = 17.5
17.5 is too small
1750 is the answer

Step 6: This example provides a decimal in a factor.

36.2 X 5
362 X 5 (disregard the decimal)
362 ÷ 2 = 181
181 seems reasonable
181 is the answer.

Step 7: Provide the students examples such as these with which to practice.

16 X 5 = 62 X 5 =
38 X 5 = 28 X 5 =
88 X 5 = 66 X 5 =
42 X 5 = 94 X 5 =
5 X 74 = 5 X 54 =
5 x 58 = 5 X 82 =
5 X 22 = 5 X 96 =
5 X 76 = 5 X 44 =

76.
Step 8: Here are some challenging examples with odd factors and decimals.

85 x 5 = 0.5 x 12.2 =
49 x 5 = 500 x 0.79 =
33 x 5 = 5 x 510 =
97 x 5 = 2.1 x 50 =
50 x 55 = 6.8 x 500 =

Classroom Measurements

**Topic:** Metric measurement

**Objective:** Students will construct a strong mental image of the size of metric units of length.

**Materials:** Each group of two or three students will need a meter stick, a centimeter ruler, a chart to help them convert metric units of length, and a math lab worksheet.

**Procedure:**

Step 1: Introduce the lesson by telling students that they will be measuring various items in the classroom in meters, decimeters, centimeters, and millimeters using the materials provided.

Step 2: Tell the students that when their team is finished measuring they are to find another team with which to compare answers. If they are different, they are to find out why and correct any errors they may find. Then they are to show the teacher their work.

Step 3: Direct the students to measure items in the classroom and convert measurements from one unit to another.

Step 4: Remind students that they will find it easier if they start in different areas of the room as this will help avoid crowding. Also remind students to try to recognize the relationships between the units as they carry out the measuring activity.
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Fraction Concept Strategies

**Topic:** Problems involving fraction concepts

**Objective:** Students will develop an understanding of the meaning of fractions.

**Materials:** Each pair of students will need a ruler, paper in four different colors, large white paper, scissors, paste, and a pencil.

**Procedure:**

Step 1: Introduce the lesson by challenging students to create a visually attractive display that illustrates their reasoning in finding the solutions to two problems involving fractions.

Step 2: Direct the students to cut out a strip of paper one inch by eight inches. This is to represent one whole.

Step 3: Direct the students to label and display one whole by pasting the strip to the large white paper and labeling it "1".

Step 4: Challenge the students to complete their display by using other colors to show three-fourths and one and one-fourth. They are to show all steps in their reasoning using a different color for each fraction. They are to label each piece with the correct fraction name. Note that the students should include the step of describing and labeling one whole as four-fourths.

Step 5: When students have completed the first problem,
direct them to draw a line segment three inches long and label it "3/5".

Step 6: Challenge students to devise a way of finding one whole using the three inch line as a basis for their display. Remind them to show and label all the steps in their reasoning.

Domino Fractions

**Topic:** Addition of fractions

**Objective:** Students will add fractions represented by dots on dominoes to arrive at the sum of a given number.

**Materials:** Each group of three students will need a set of dominoes, paper, and a pencil.

**Procedure:**

**Step 1:** Have the students remove the domino tiles that are doubles, that is, those with the same number of dots at both ends. Also have the students remove the tiles that have a blank, that is, those with no dots on one end. There should be fifteen tiles remaining.

**Step 2:** Tell the students that they are to use the remaining dominoes to form proper fractions by orienting them vertically with the lesser number of dots for each domino at the top.

**Step 3:** Challenge the students to arrange the dominoes in three separate rows with each row of five dominoes adding up to 2 1/2. One solution is provided below.

\[
\begin{align*}
3/4 + 1/4 + 3/6 + 1/2 + 2/4 &= 2 1/2 \\
5/6 + 2/6 + 1/3 + 4/5 + 1/5 &= 2 1/2 \\
4/6 + 1/6 + 2/3 + 3/5 + 2/5 &= 2 1/2 
\end{align*}
\]

**Step 4:** Direct the students to make a drawing of their solution and write the equations that show sums of 2 1/2.

**Step 5:** The lesson may be extended by directing the
students to orient the dominoes vertically so as to form a combination of proper and improper fractions. Challenge them to arrange the dominoes into three separate rows with each row of five having a sum of 10. One solution is shown below.

\[
\begin{align*}
1/3 + 6/1 + 3/4 + 5/3 + 5/4 &= 10 \\
2/1 + 5/1 + 2/6 + 6/3 + 4/6 &= 10 \\
4/1 + 2/3 + 4/2 + 5/2 + 5/6 &= 10
\end{align*}
\]

Step 6: Share some of the solutions that students generate with the class.

Fraction Products

Topic: Multiplying with fractions and mixed numbers

Objective: Students will gain an understanding of the underlying concept of multiplication using fractions and mixed numbers.

Materials: Each pair of students will need drawing paper, a ruler, a pencil, and colored markers.

Procedure:

Step 1: Introduce the lesson by telling students that they are to use the materials provided to determine the solution to a problem involving a fraction and a mixed number.

Step 2: Ask the students to explain how to find 3/4 of 1 2/3 by making a drawing and a written explanation.

Step 3: Suggest a square as a model for one whole.

Step 4: Emphasize that it is important to make a clear drawing accompanied by a good written explanation.

Step 5: After the activity, share some answers with the class. Note that although the problem may have been thought of in different ways, the solution is the same.

Step 6: Explain to students that 3/4 of 1 2/3 is one way of expressing the multiplication problem 3/4 X 1 2/3 which can be solved as they did, with a drawing, or through the use of an algorithm.

Step 7: Introduce the algorithm, it may make more sense to the students after having done the activity.
Circle Graphs

**Topic:** Constructing a circle graph

**Objective:** Students will gather data and represent relative numbers using fractions and a circle graph.

**Materials:** Each team of four students will need centimeter rulers, long paper strips such as adding machine tapes, a large piece of paper, a pencil, markers, and tape.

**Procedure:**

Step 1: Have the students cut the paper strips into lengths 2 centimeters wide and 100 centimeters long. They are to mark the strip with centimeter marks and number them from 1 to 100.

Step 2: Brainstorm about questions the class would like to ask their classmates. Choose one of the questions with which to survey the class.

Step 3: Conduct the survey and record the data in a chart. It will be useful to have approximately four different responses expressed.

Step 4: Direct the students to let their paper strips represent the relative number of responses each response received. For example, if 8 of 33 students questioned shared the same response, the students would let each response represent 3 centimeters of the paper strip. Those 8 responses would then be recorded by coloring 24 centimeters of the tape one color. After all the responses
were recorded, only 1 centimeter would be left unused.

Step 5: Have the students tape the ends of their paper strip together so as to form a circle.

Step 6: Direct the students to trace around the circle and mark where each different response begins and ends.

Step 7: Direct the students to locate the center of the circle and draw segments from the center to the edge of the circle where the boundaries of each different response have been marked. This will make the "slices" of the pie chart.

Step 8: Tell the students to label each section with the appropriate fraction and response.

Step 9: Discuss the results with the class, using the work of each group as a focal point.

Step 10: The lesson may be extended by allowing each group to choose a question, devise and conduct a survey, and construct a circle graph. They may then be displayed throughout the classroom.

Source: National Council of Teachers of Mathematics.

Hands-on Averages

**Topic:** Collecting data and finding the average

**Objective:** Students will gain an understanding of the concept of average and find averages.

**Materials:** Each group of four students will need several meters of string, scissors, masking tape, a meter stick, paper, and a pencil.

**Procedure:**

Step 1: Divide the class into groups of four. Direct the groups to take measurements of the members of their group and the members of one other group. There will be a total of eight measurements for each set of data.

Step 2: Tell the students to use the string to find the total length for each type of measurement. Stress that they are to record the sum of all eight individual measurements for each type of measurement on one string. After all eight measurements are included on the length of string they are to cut the string and label its type with the masking tape.

Step 3: Direct the students to measure the following:

1. Pencil lengths
2. Head circumferences
3. Foot lengths
4. Hand spans
5. Heights
Step 4: After the students have determined the total length for each type of measurement direct them to find the average for each by folding each string into eighths.

Step 5: Determine the averages to the nearest centimeter by measuring the five folded strings and record the results.

Step 6: Discuss the resulting averages. Note any differences between the group averages and possible reasons for them. Also note the similarities and differences between the averages and the individual measurements.

Finding Data

**Topic:** Mean, median, and mode

**Objective:** Students will gain a deeper understanding of the statistical concepts of mean, median, and mode.

**Materials:** Each group of three or four students will need approximately forty small squares of paper, a pencil, and a piece of paper on which to record their answer.

**Procedure:**

Step 1: Introduce the lesson by stating that students are often asked to find the mean, median, and mode for a given set of data. In this activity they will be given the mean, median, mode, highest number, and lowest number for a set of data. Their task will be to use the information provided to find at least one set of data from which the given numbers could possibly have come.

Step 2: Provide the students the following information:

1. Nine people have some candy.
2. The most anyone has is 9 pieces.
3. At least one person has no candy.
4. The average, or mean, number of pieces is 4.
5. The median number of pieces is also 4.
6. The mode of the numbers is 2.

Step 3: If students need help understanding the problem, state that if everyone shared the candy equally, they would each have 4 pieces; if the people were arranged in order
from the person having the least number of pieces to the person having the greatest number of pieces, the person in the middle would have 4 pieces; and more people have 2 pieces of candy than any other number.

Step 4: Allow the students to use the small paper squares to simulate pieces of candy. They may wish to arrange the squares in columns to show how many pieces of candy each person would have. Remind the students to record their solutions.

Step 5: Discuss the students' work, reasoning, and solutions. One possible solution is that one person has 0 pieces, three people have 2 pieces, one person has 4 pieces, two people have 5 pieces, one person has 7 pieces, and one person has 9 pieces. Another solution is that one person has 0 pieces, three people have 2 pieces, two people have 4 pieces, one person has 6 pieces, one person has 7 pieces, and one person has 9 pieces.

Bean City

**Topic:** Ratios

**Objective:** The students will use estimation skills and write ratios in this problem solving activity.

**Materials:** Each group of four students will need a variety of measuring and weighing devices, an opaque bag filled with 1000 beans comprised of a combination of navy beans, pinto beans, and lima beans. Each group will also need a piece of paper on which to record their work and results.

**Procedure:**

Step 1: Introduce the lesson by telling the students that they will act as census takers in a city called Bean City. All the residents are either navy beans, pinto beans, or lima beans. The city has a total of 1000 residents. They must find out how many of each type of resident are present in Bean City. Tell the students that it would be too costly to go to every resident of Bean City to find out their type. They must not count every Bean City resident. They must devise a way of obtaining a representative sample of the population and estimate the actual population from the sample. They may count the sample and write ratios to estimate the actual population.

Step 2: Brainstorm a variety of strategies and guide students to include weighing as part of their procedure.
Step 3: Remind the students to appoint a scribe for their group. This person is to record discussions, procedures, data, and final estimates.

Step 4: Allow the groups to work on obtaining a sample and gathering data. Briefly discuss the estimates as the groups carry out and complete their work.

Step 5: After each group has arrived at an estimate of the numbers of three types of beans in their bag allow them to count the beans and determine the exact number of each type present.

Step 6: Discuss which sampling strategies resulted in the most accurate results and possible reasons.

Find It Fast

**Topic:** Percentages

**Objective:** The students will find percent of a number mentally.

**Materials:** The students will not need any materials.

**Procedure:**

Step 1: Introduce the lesson by reminding students that many situations call for quick mental calculations. One example is computing a gratuity after receiving a check at a restaurant.

Step 2: Write on the chalkboard or on an overhead transparency the following percentage expressions, one at a time:

- 50 percent of 30; 10 percent of 12
- 20 percent of 50; 150 percent of 40
- 17 percent of 10; 13 percent of 10
- 40 percent of 75; 35 percent of 20
- 16 percent of 25; 62 percent of 50
- 125 percent of 40; 30 percent of 50

Step 3: Ask the students for a quick way to arrive at an answer. Discuss the reasoning behind their strategies. Students may respond in ways that include treating 50 percent as one-half of a number, 25 percent as one-fourth, 10 percent as dividing by ten, or finding 17 percent of 10 as finding 10 percent of 17.
Step 4: Extend the lesson by encouraging the students to develop their own percentage problems. They may wish to share them with their classmates at the beginning or end of the next class.

Round Houses

**Topic:** Perimeter and area

**Objective:** The students will investigate the relationship between perimeter and area for common shapes.

**Materials:** Each group of two or three students will need string, one-quarter-inch or centimeter grid paper, paper, and a pencil.

**Procedure:**

Step 1: Introduce the lesson by telling students that they are to imagine that they live in a society where building materials are scarce. They are to build a house but have a limited amount of material for the walls. Therefore, they are to design a floor shape that will provide the largest floor space for the wall material available.

Step 2: Tell the students that in mathematical terms, they are to find the greatest area for the given perimeter.

Step 3: Tell the students that they will draw shapes on the grid paper with a perimeter of thirty-two units. They are to count the squares inside the shape to determine the area of the shape.

Step 4: Direct the students to cut a piece of string thirty-two units long. Direct the students to use the string to form a closed shape on the grid paper and trace around it to generate a drawing.

Step 5: The students will draw a circle, a square, two
rectangles that are not squares, and a triangle.

Step 6: Allow the students to draw the shapes and count the small squares to arrive at an estimate of the area of each figure.

Step 7: Direct the students to record their results in a chart.

Step 8: Discuss the results of the investigation with the students. Points to consider include the shape with the greatest area, the rectangle with the greatest area, and other factors that influence a decision on what shape to build a house.

How to Bag It?

**Topic:** Graphing information

**Objective:** The student will discuss different types of bags, gather data, make and interpret a graph, and develop a recycling plan.

**Materials:** Each group of three to five students will need an activity sheet, large paper, markers, and a pencil.

**Procedure:**

Step 1: Introduce the lesson by telling students that almost every shopper leaves the store with some type of bag. Producing bags uses resources such as wood and oil. Many bags are discarded which adds to the volume of garbage dumped in landfills.

Step 2: Direct the students to use the activity sheet to discuss the advantages and disadvantages of different kinds of bags. Each group is to appoint a recorder to write down their ideas.

Step 3: Have each group share their ideas with the class.

Step 4: Review different kinds of graphs with the class. Some types they will be familiar with include bar graphs, picture graphs, and circle graphs.

Step 5: Ask each group to poll ten classmates concerning the type of bag each considers the best on the basis of the previous discussion.

Step 6: After the groups have gathered their poll data,
have the students use the activity sheet to choose a type of graph with which to represent their data.

Step 7: Allow the groups to construct their graphs.

Step 8: Have the students present their graphs to the class and share their interpretations.

Step 9: Brainstorm ways to use bags with conservation as a major consideration. Ask each student to devise a plan for improving their personal use of bags.

Step 10: Appoint a committee of students to display the graphs and activity sheets in the classroom. The activity sheet, which provides a statement of the problem and a chart for students to use, is as follows.

Most purchases are bagged in plastic or paper. People environmentally aware question whether so many bags are needed. They remind us of such facts as the following: A fifteen-year-old tree makes only 700 grocery bags. Plastic bags, typically discarded, take up little landfill space but don't biodegrade.

Some people, like those in other countries, take their own plastic or cloth bags to carry their purchases. Others "just say no" to bags for small purchases, telling the clerk they don't need a bag.

1. What are some pros and cons of using different kinds of bags? Discuss this question with a group. Make notes in the chart.
2. After your discussion, poll ten classmates to see which kind of bag they think is best to use. Choose a type of graph. Show your data in a graph on a separate piece of paper.

3. What made you choose the type of graph you made?

4. Interpret your graph. Write several things the graph shows.

5. On the basis of your discussion and on what you found in your poll of classmates, think of a plan to conserve energy and material associated with manufacturing bags. Write a summary of your plan.

Angle Check

**Topic:** Acute, right, and obtuse angles

**Objective:** The students will gain a concrete understanding of acute, right, and obtuse angles.

**Materials:** Each student will need a worksheet with several examples of each type of angle and an index card.

**Procedure:**

Step 1: Tell the students that they will be using the index card to see if the angles on the worksheet are acute, right, or obtuse angles.

Step 2: Direct the students to align the corner of the index card with the vertex of the angle they are checking and one edge of the index card with one of the rays of the angle.

Step 3: Tell students that if the remaining ray lies inside the angle formed by the two edges of the card, the angle is acute. If the remaining ray lies outside the angle of the card, the angle is obtuse, and if the angle matches the angle of the card, the angle is a right angle.

Step 4: Allow the students to use this method to classify the drawn angles and write the correct name.

Sorting Quadrilaterals

**Topic:** Properties of quadrilaterals

**Objective:** The students will draw and classify quadrilaterals. They will also test quadrilaterals for congruency.

**Materials:** Each pair of students will need geopaper divided into three by three sections with each section having an array of nine dots. The students will also need tracing paper, a ruler and a pencil.

**Procedure:**

Step 1: Tell the students to use the materials provided to draw as many different quadrilaterals as possible. They must not draw congruent quadrilaterals.

Step 2: Direct the students to test their quadrilaterals for congruency by tracing them on tracing paper. If they can superimpose any of the quadrilaterals, even through turning or flipping the tracing paper, then they are congruent and not different quadrilaterals. The students should use a different set of four dots for each quadrilateral.

Step 3: Have the students classify the sixteen possible quadrilaterals as squares, rectangles, parallelograms, trapezoids, convex, or concave.

Step 4: Share examples of student work with the class. The sixteen possible quadrilaterals are shown here.
Source: National Council of Teachers of Mathematics.

What's in the Bag?

**Topic:** Probability

**Objective:** Students will arrive at a concrete understanding of probability and write probability statements in this problem solving activity.

**Materials:** Each group of four students will need ten colored tiles to include a combination of four different colors, a paper lunch bag, paper, and a pencil.

**Procedure:**

Step 1: Prepare a lunch bag for each group by placing ten tiles in each, include a combination of four different colors.

Step 2: Give each group a bag, remind them not to look inside to see what they have.

Step 3: Tell the students that there are ten tiles and four different colors of tiles in each bag. They are to determine the four colors and the number of tiles of each color present without looking in the bag.

Step 4: Tell the students that they may remove a tile from the bag, record the color, and immediately return it to the bag. They may repeat the process as many times as they like but must never have more than one tile outside the bag at a time and must not look in the bag.

Step 5: After the students have written down their predictions, ask them to explain their reasoning.
Step 6: Have the groups empty their bags and count the number of tiles of each color present.

Step 7: Have the students write a probability statement for each color of tile. For example, if there are four blue tiles, they may write that the probability of choosing a blue tile is 4/10.


105.
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